Computer Algebra
Independent Integration Tests

Summer 2023 edition

12-table-of-integrals

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [163]. This is test number [212].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (163)	0.00 (0)
Mathematica	100.00 (163)	0.00 (0)
Fricas	100.00 (163)	0.00 (0)
Giac	100.00 (163)	0.00 (0)
Maple	97.55 (159)	2.45 (4)
Mupad	92.64 (151)	7.36 (12)
Maxima	92.64 (151)	7.36 (12)
Sympy	90.80 (148)	9.20 (15)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

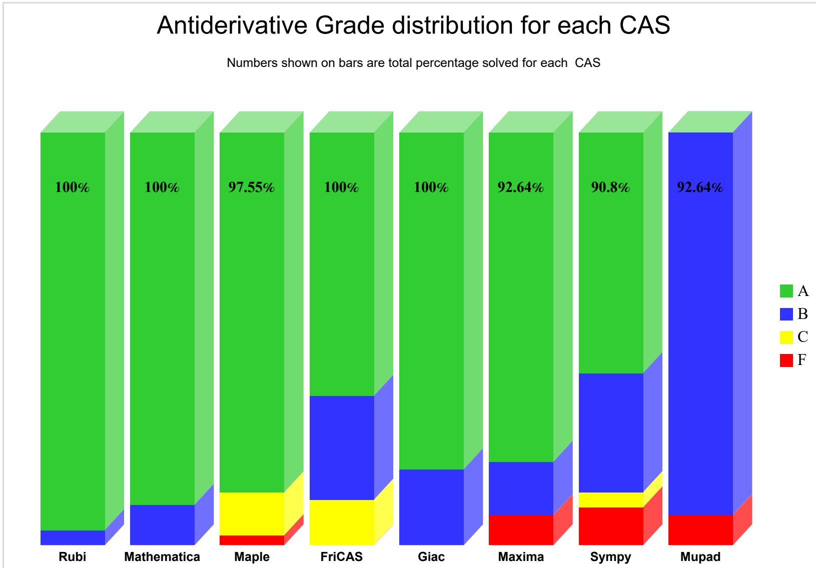
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

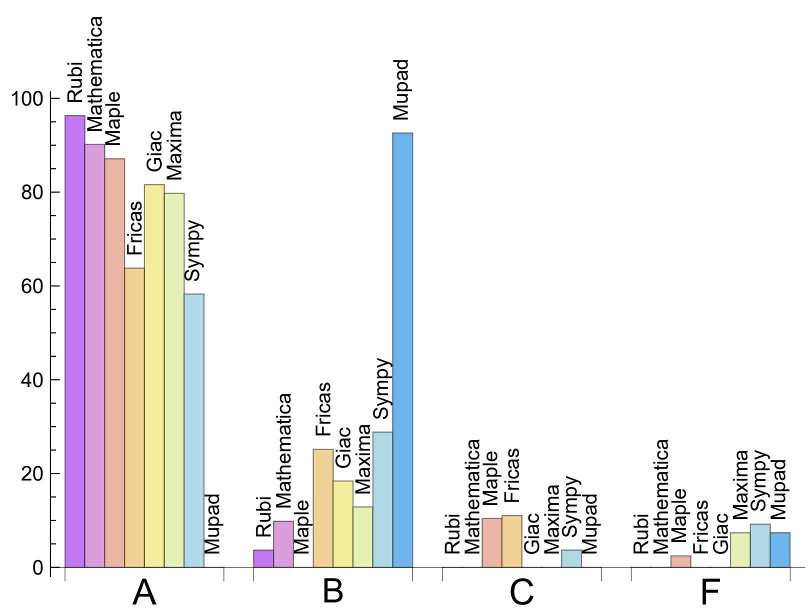
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.319	3.681	0.000	0.000
Mathematica	90.184	9.816	0.000	0.000
Maple	87.117	0.000	10.429	2.454
Giac	81.595	18.405	0.000	0.000
Maxima	79.755	12.883	0.000	7.362
Fricas	63.804	25.153	11.043	0.000
Sympy	58.282	28.834	3.681	9.202
Mupad	0.000	92.638	0.000	7.362

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Giac	0	0.00	0.00	0.00
Maple	4	100.00	0.00	0.00
Mupad	12	0.00	100.00	0.00
Maxima	12	91.67	0.00	8.33
Sympy	15	93.33	6.67	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.04
Mathematica	0.07
Maple	0.12
Maxima	0.24
Fricas	0.24
Giac	0.36
Sympy	4.25
Mupad	8.17

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	39.21	0.94	30.00	0.95
Mupad	42.09	0.98	33.00	1.00
Mathematica	53.37	1.32	38.00	1.00
Maxima	58.46	1.29	36.00	1.01
Giac	63.47	1.52	37.00	1.10
Rubi	63.88	1.16	41.00	1.04
Fricas	100.71	2.09	62.00	1.62
Sympy	133.52	2.32	38.00	1.09

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

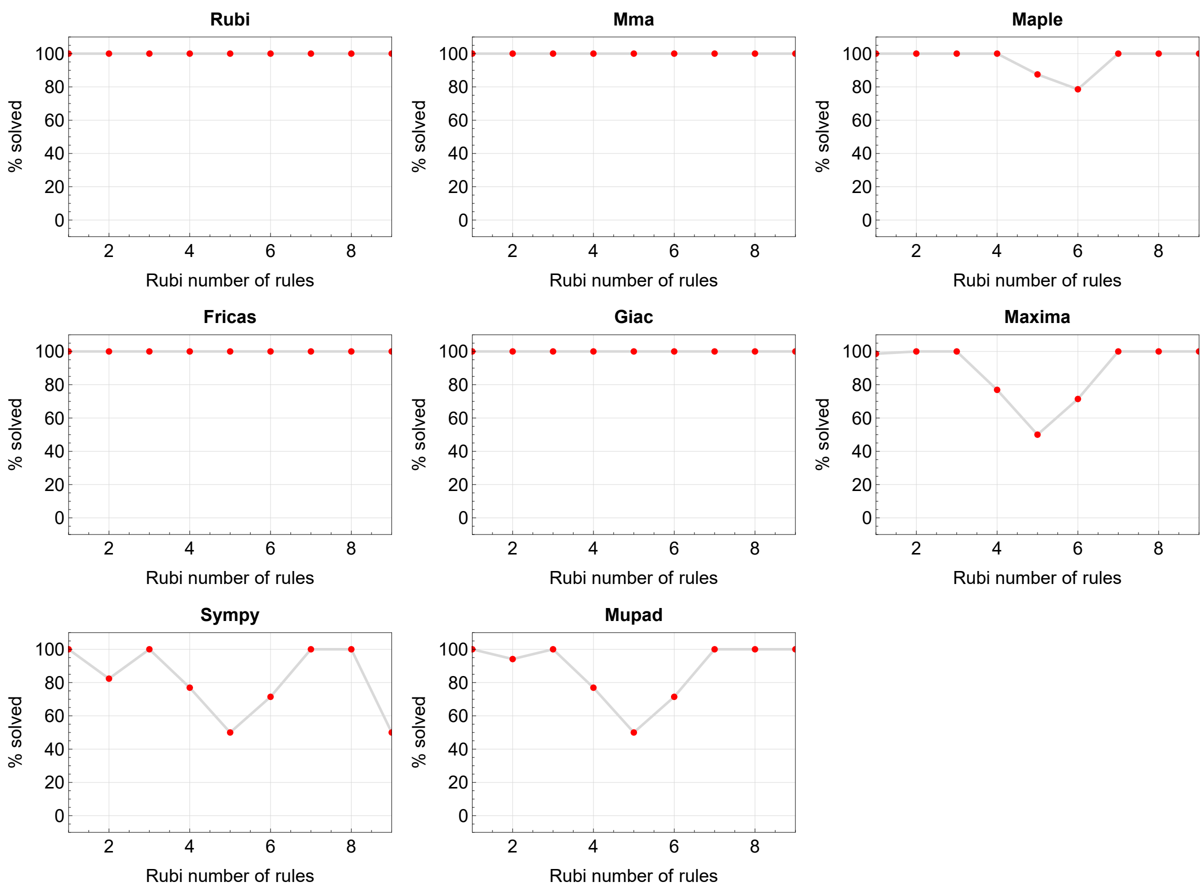


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

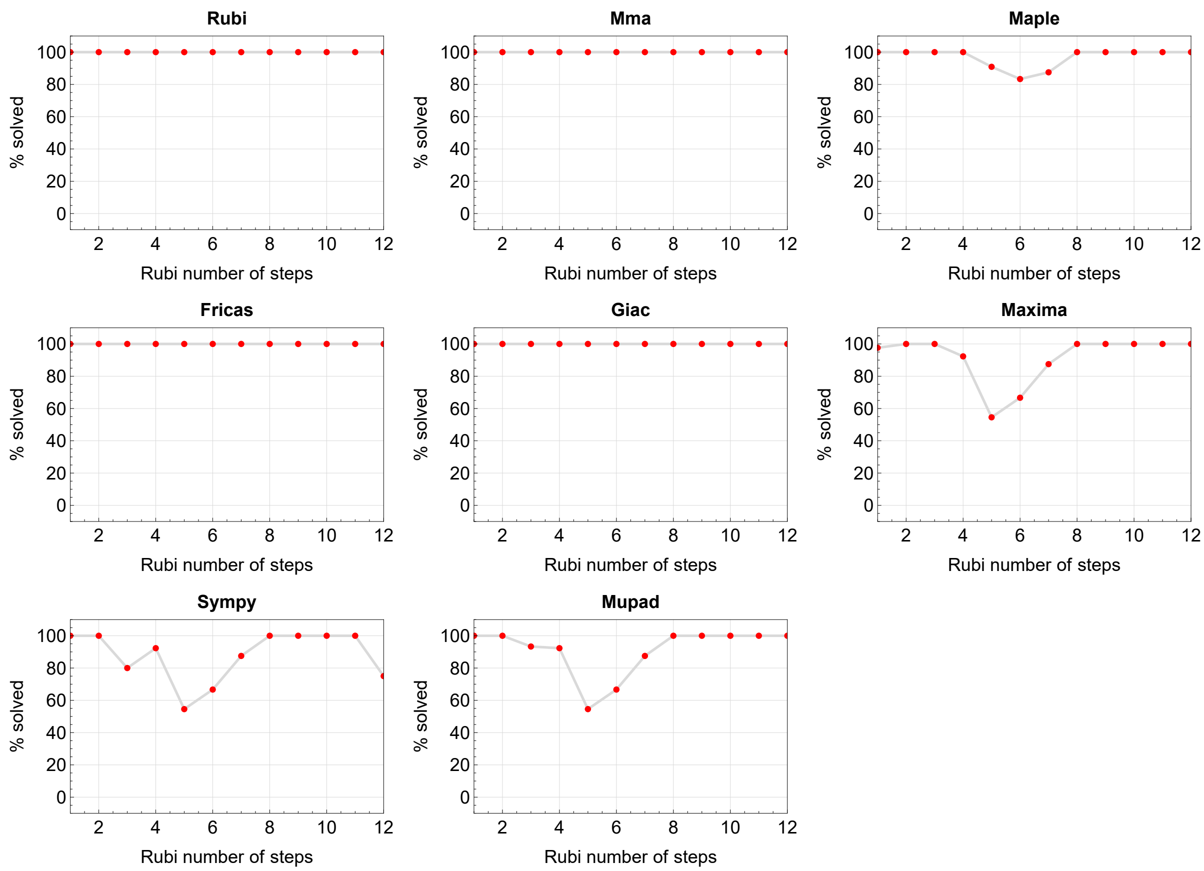


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

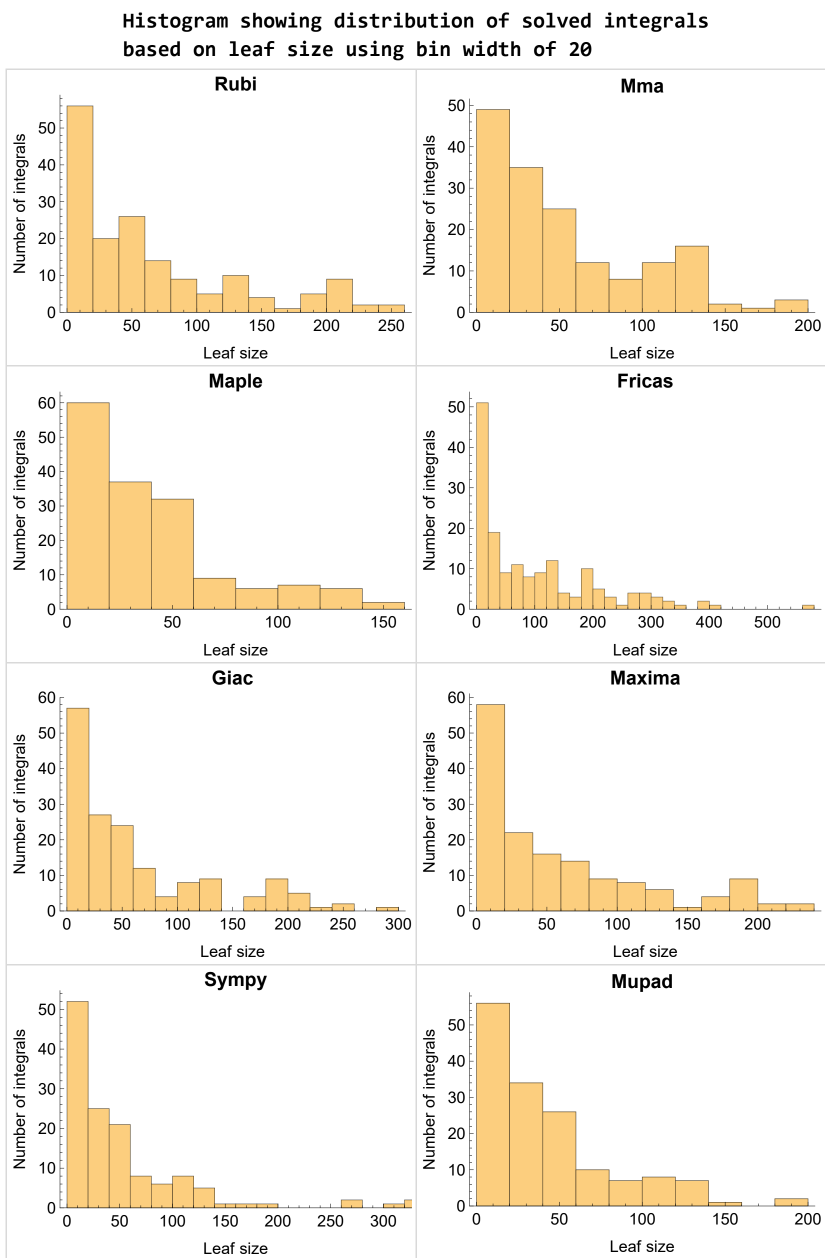


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

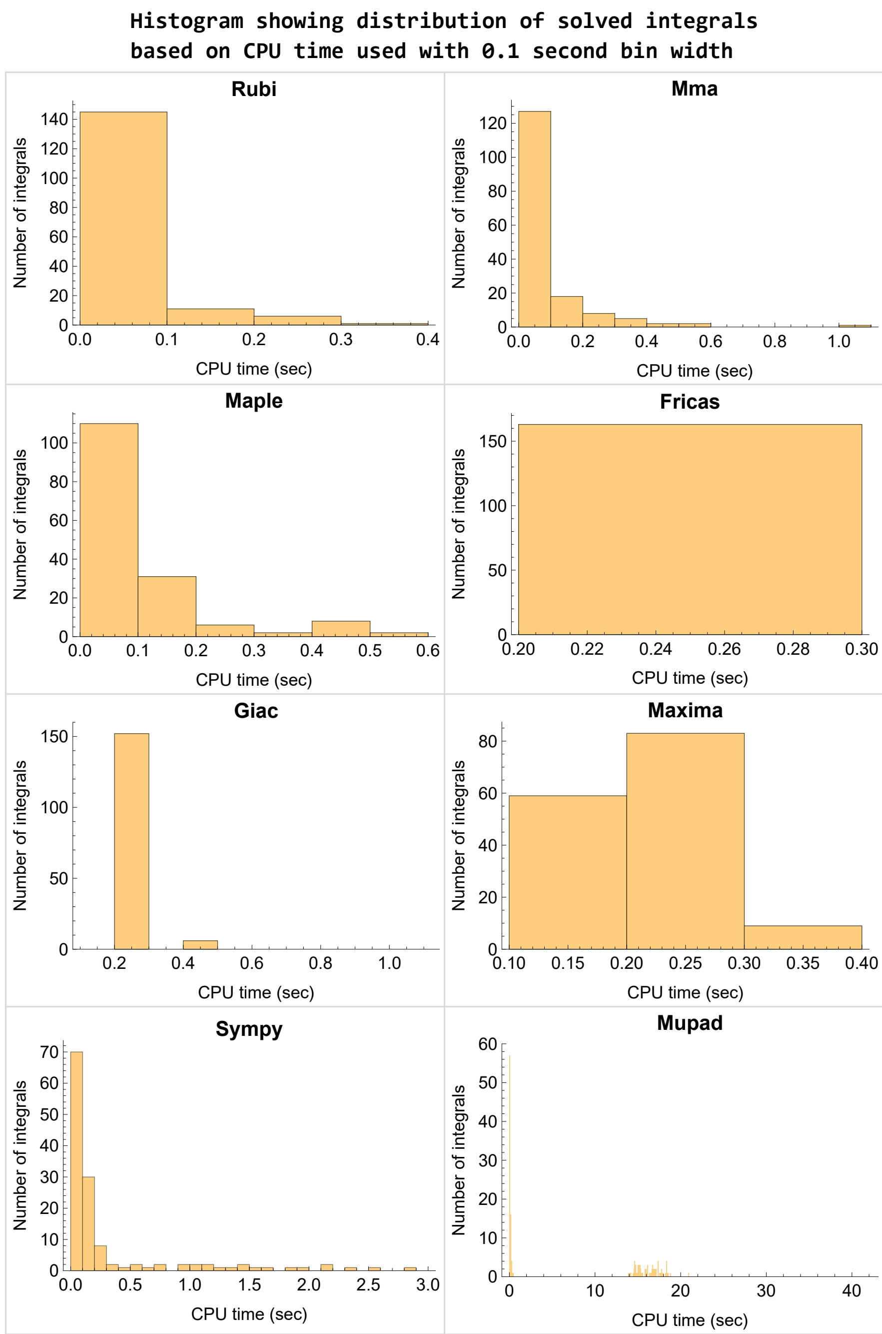


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

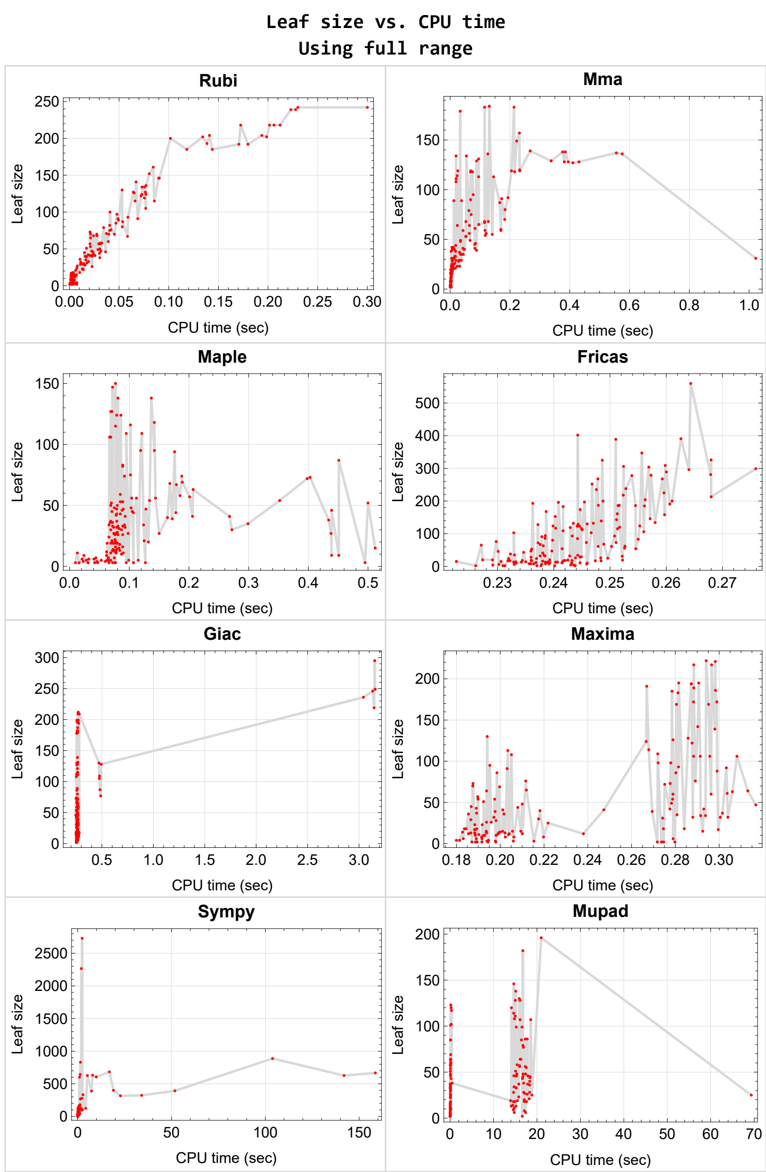


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

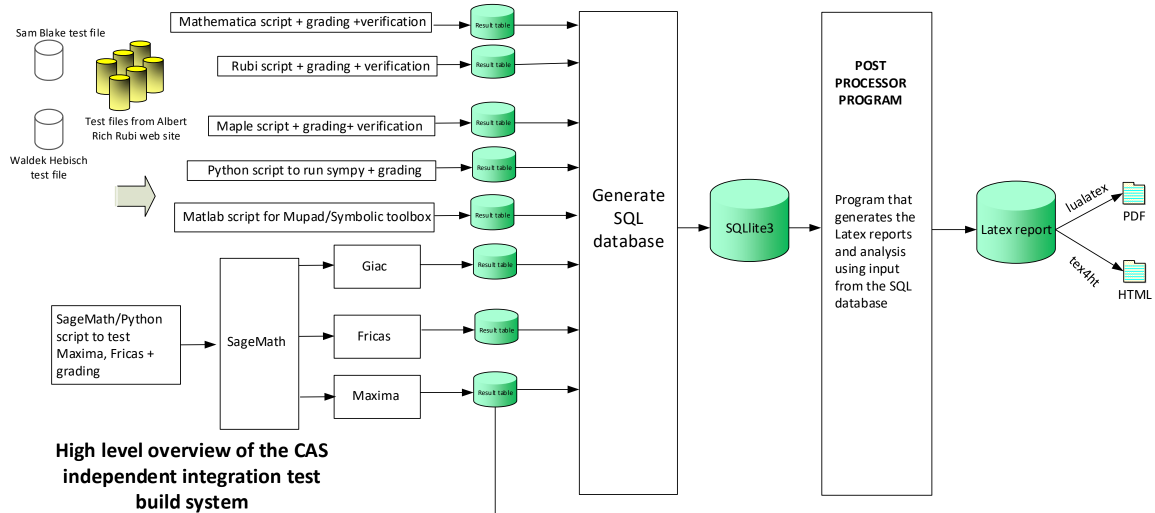
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 80, 82, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163 }

B grade { 77, 79, 81, 83, 86, 137 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 15, 20, 21, 22, 23, 24, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 80, 82, 84, 85, 87, 88, 89, 90, 91, 94, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163 }

B grade { 12, 13, 16, 17, 18, 19, 25, 26, 77, 79, 81, 83, 86, 92, 93, 95 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 64, 69, 71, 74, 78, 80, 82, 84, 85, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 153, 154, 157, 159, 160, 161, 162, 163 }

B grade { }

C grade { 62, 63, 65, 66, 67, 68, 70, 72, 73, 75, 76, 77, 79, 81, 83, 86, 90 }

F normal fail { 151, 155, 156, 158 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 15, 19, 20, 21, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 40, 41, 45, 46, 47, 48, 49, 50, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 80, 82, 84, 85, 87, 88, 89, 91, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 133, 134, 135, 139, 140, 141, 142, 144, 145, 146, 147, 152, 153, 154, 157, 160, 161, 162, 163 }

B grade { 8, 11, 12, 13, 14, 16, 17, 18, 22, 23, 24, 25, 26, 38, 39, 42, 43, 44, 51, 52, 53, 54, 55, 56, 57, 58, 59, 92, 93, 136, 137, 138, 143, 148, 149, 150, 151, 155, 156, 158, 159 }

C grade { 77, 79, 81, 83, 86, 90, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 17, 18, 19, 20, 21, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 80, 82, 84, 85, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 152, 153, 154, 157, 160, 161, 162, 163 }

B grade { 13, 14, 16, 22, 23, 26, 39, 43, 44, 77, 79, 81, 83, 86, 92, 93, 121, 122, 123, 124, 127 }

C grade { }

F normal fail { 143, 144, 145, 148, 149, 150, 151, 155, 156, 158, 159 }

F(-1) timedout fail { }

F(-2) exception fail { 61 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 80, 82, 84, 85, 87, 88, 89, 90, 91, 94, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 123, 124, 125, 126, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 153, 154, 157, 160, 161, 162, 163 }

B grade { 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 77, 79, 81, 83, 86, 92, 93, 95, 121, 122, 127, 128, 151, 155, 156, 158, 159 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 152, 153, 154, 157, 161, 162, 163 }

C grade { }

F normal fail { }

F(-1) timedout fail { 143, 144, 145, 148, 149, 150, 151, 155, 156, 158, 159, 160 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 15, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 41, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 94, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 121, 122, 123, 124, 133, 139, 140, 141, 142, 146, 147, 163 }

B grade { 7, 8, 13, 14, 16, 22, 23, 24, 25, 34, 38, 39, 40, 42, 43, 44, 60, 61, 78, 82, 92, 93, 95, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136 }

C grade { 152, 153, 154, 157, 161, 162 }

F normal fail { 137, 138, 143, 144, 145, 148, 149, 150, 151, 155, 156, 158, 159, 160 }

F(-1) timedout fail { 132 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	11	10	12	11	20
N.S.	1	1.00	1.00	1.00	1.00	0.91	1.09	1.00	1.82
time (sec)	N/A	0.003	0.002	0.013	0.190	0.233	0.019	0.261	17.099

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	3	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.50	1.00
time (sec)	N/A	0.000	0.000	0.010	0.187	0.237	0.036	0.255	0.012

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	3	2	2	2	2	2
N.S.	1	1.00	1.00	1.00	0.67	0.67	0.67	0.67	0.67
time (sec)	N/A	0.001	0.000	0.016	0.189	0.238	0.029	0.257	0.011

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	8	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.003	0.001	0.024	0.220	0.239	0.044	0.252	17.111

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.002	0.001	0.028	0.195	0.243	0.035	0.251	0.023

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.003	0.001	0.031	0.198	0.240	0.042	0.255	0.032

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	6	8	7	6	4
N.S.	1	1.00	1.00	1.25	1.50	2.00	1.75	1.50	1.00
time (sec)	N/A	0.006	0.003	0.099	0.189	0.236	0.040	0.266	0.019

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	7	5	2	2
N.S.	1	1.00	1.00	1.50	1.00	3.50	2.50	1.00	1.00
time (sec)	N/A	0.006	0.001	0.107	0.192	0.246	0.048	0.255	0.031

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	5	4	4	3	4	4
N.S.	1	1.00	1.00	2.50	2.00	2.00	1.50	2.00	2.00
time (sec)	N/A	0.007	0.003	0.069	0.180	0.244	0.040	0.254	0.044

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	6	6	5	6	6
N.S.	1	1.00	1.00	1.25	1.50	1.50	1.25	1.50	1.50
time (sec)	N/A	0.008	0.002	0.053	0.183	0.238	0.044	0.258	17.397

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	3	11	5	6	5
N.S.	1	1.00	1.00	1.20	0.60	2.20	1.00	1.20	1.00
time (sec)	N/A	0.003	0.002	0.021	0.206	0.241	0.037	0.252	0.027

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	7	4	3	11	3	4	3
N.S.	1	1.00	2.33	1.33	1.00	3.67	1.00	1.33	1.00
time (sec)	N/A	0.003	0.002	0.046	0.215	0.241	0.049	0.258	0.029

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	5	17	6	15	19	15	17	5
N.S.	1	0.71	2.43	0.86	2.14	2.71	2.14	2.43	0.71
time (sec)	N/A	0.002	0.004	0.041	0.201	0.252	0.068	0.252	0.040

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	3	3	7	15	17	15	17	11
N.S.	1	0.50	0.50	1.17	2.50	2.83	2.50	2.83	1.83
time (sec)	N/A	0.003	0.000	0.044	0.194	0.248	0.043	0.278	0.045

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.001	0.004	0.076	0.275	0.244	0.042	0.253	16.620

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	19	3	13	13	12	15	2
N.S.	1	1.00	9.50	1.50	6.50	6.50	6.00	7.50	1.00
time (sec)	N/A	0.001	0.003	0.072	0.202	0.239	0.045	0.248	0.067

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	20	3	2	18	2	17	2
N.S.	1	1.00	10.00	1.50	1.00	9.00	1.00	8.50	1.00
time (sec)	N/A	0.001	0.002	0.127	0.272	0.234	0.057	0.271	0.010

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	16	3	2	14	2	25	2
N.S.	1	1.00	8.00	1.50	1.00	7.00	1.00	12.50	1.00
time (sec)	N/A	0.001	0.001	0.077	0.274	0.235	0.066	0.263	0.002

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	38	11	14	14	10	26	10
N.S.	1	1.00	3.17	0.92	1.17	1.17	0.83	2.17	0.83
time (sec)	N/A	0.002	0.003	0.090	0.200	0.233	0.065	0.260	0.211

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	11	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	5.50	1.00
time (sec)	N/A	0.004	0.001	0.047	0.194	0.226	0.069	0.253	0.025

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	11	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	5.50	1.00
time (sec)	N/A	0.003	0.002	0.042	0.189	0.231	0.069	0.262	0.021

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	10	20	15	10	4
N.S.	1	1.00	1.00	1.25	2.50	5.00	3.75	2.50	1.00
time (sec)	N/A	0.007	0.002	0.115	0.193	0.227	0.248	0.269	0.014

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	10	20	14	10	2
N.S.	1	1.00	1.00	1.50	5.00	10.00	7.00	5.00	1.00
time (sec)	N/A	0.007	0.001	0.095	0.192	0.229	0.290	0.266	0.002

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	18	7	11	3
N.S.	1	1.00	1.00	1.33	1.00	6.00	2.33	3.67	1.00
time (sec)	N/A	0.004	0.001	0.023	0.194	0.236	0.056	0.273	0.037

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	7	4	3	18	12	12	3
N.S.	1	1.00	2.33	1.33	1.00	6.00	4.00	4.00	1.00
time (sec)	N/A	0.004	0.001	0.060	0.198	0.242	0.145	0.273	0.037

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	5	17	6	17	17	5	14	5
N.S.	1	0.71	2.43	0.86	2.43	2.43	0.71	2.00	0.71
time (sec)	N/A	0.004	0.004	0.032	0.189	0.232	0.091	0.274	0.020

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	20	20	18	18
N.S.	1	1.00	1.00	1.06	1.00	1.11	1.11	1.00	1.00
time (sec)	N/A	0.004	0.007	0.079	0.184	0.239	0.018	0.278	17.062

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	11	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00
time (sec)	N/A	0.001	0.002	0.078	0.189	0.246	0.019	0.258	0.039

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	17	14	19	18
N.S.	1	1.00	1.00	1.06	1.00	0.94	0.78	1.06	1.00
time (sec)	N/A	0.010	0.004	0.069	0.184	0.247	0.045	0.269	17.613

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	29	29	26	30	29
N.S.	1	1.00	1.00	0.97	0.94	0.94	0.84	0.97	0.94
time (sec)	N/A	0.018	0.005	0.069	0.187	0.245	0.051	0.271	0.063

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	13	10	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.08	0.83	1.00	1.00
time (sec)	N/A	0.002	0.003	0.072	0.199	0.238	0.057	0.267	0.035

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	23	20	24	26	28	20	42	23
N.S.	1	0.96	0.83	1.00	1.08	1.17	0.83	1.75	0.96
time (sec)	N/A	0.013	0.009	0.066	0.196	0.236	0.079	0.253	0.061

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	34	36	47	31	50	36
N.S.	1	1.00	0.88	1.03	1.09	1.42	0.94	1.52	1.09
time (sec)	N/A	0.019	0.019	0.066	0.185	0.240	0.075	0.266	18.302

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	24	26	12	26
N.S.	1	1.00	1.00	0.93	0.86	1.71	1.86	0.86	1.86
time (sec)	N/A	0.002	0.003	0.084	0.210	0.237	0.082	0.255	17.761

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	17	20	19	32	32	32	18	32
N.S.	1	0.74	0.87	0.83	1.39	1.39	1.39	0.78	1.39
time (sec)	N/A	0.002	0.008	0.063	0.197	0.238	0.081	0.258	0.050

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	41	33	36	48	61	46	37	46
N.S.	1	1.11	0.89	0.97	1.30	1.65	1.24	1.00	1.24
time (sec)	N/A	0.022	0.016	0.072	0.210	0.250	0.086	0.262	17.906

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	50	40	45	57	83	58	44	43
N.S.	1	0.88	0.70	0.79	1.00	1.46	1.02	0.77	0.75
time (sec)	N/A	0.031	0.048	0.071	0.190	0.255	0.117	0.253	0.224

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	35	37	12	37
N.S.	1	1.00	1.00	0.93	0.86	2.50	2.64	0.86	2.64
time (sec)	N/A	0.002	0.004	0.078	0.185	0.233	0.112	0.261	0.052

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	30	20	19	43	43	44	18	44
N.S.	1	1.20	0.80	0.76	1.72	1.72	1.76	0.72	1.76
time (sec)	N/A	0.014	0.007	0.075	0.189	0.235	0.101	0.254	18.366

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	17	31	30	54	54	56	29	56
N.S.	1	0.50	0.91	0.88	1.59	1.59	1.65	0.85	1.65
time (sec)	N/A	0.002	0.013	0.272	0.190	0.255	0.114	0.259	17.317

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	58	44	47	70	94	70	46	45
N.S.	1	1.16	0.88	0.94	1.40	1.88	1.40	0.92	0.90
time (sec)	N/A	0.031	0.019	0.079	0.187	0.238	0.140	0.255	16.739

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	46	49	12	48
N.S.	1	1.00	1.00	0.93	0.86	3.29	3.50	0.86	3.43
time (sec)	N/A	0.002	0.004	0.075	0.189	0.230	0.127	0.257	16.809

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	30	20	19	54	54	56	31	18
N.S.	1	1.20	0.80	0.76	2.16	2.16	2.24	1.24	0.72
time (sec)	N/A	0.015	0.007	0.078	0.197	0.241	0.130	0.251	0.109

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	47	31	30	65	65	68	46	22
N.S.	1	1.24	0.82	0.79	1.71	1.71	1.79	1.21	0.58
time (sec)	N/A	0.022	0.010	0.087	0.212	0.227	0.132	0.250	0.099

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	17	42	41	76	76	80	61	48
N.S.	1	0.36	0.89	0.87	1.62	1.62	1.70	1.30	1.02
time (sec)	N/A	0.002	0.011	0.206	0.212	0.230	0.155	0.267	17.388

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	18	18	16	18	16	10	20	15
N.S.	1	1.20	1.20	1.07	1.20	1.07	0.67	1.33	1.00
time (sec)	N/A	0.004	0.007	0.077	0.189	0.238	0.061	0.259	0.069

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	28	28	26	28	26	19	30	25
N.S.	1	1.17	1.17	1.08	1.17	1.08	0.79	1.25	1.04
time (sec)	N/A	0.016	0.007	0.081	0.198	0.248	0.078	0.259	69.344

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	42	42	41	40	41	31	45	38
N.S.	1	1.14	1.14	1.11	1.08	1.11	0.84	1.22	1.03
time (sec)	N/A	0.019	0.006	0.268	0.218	0.238	0.091	0.254	0.459

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	42	35	43	45	63	37	52	45
N.S.	1	1.05	0.88	1.08	1.12	1.58	0.92	1.30	1.12
time (sec)	N/A	0.025	0.044	0.085	0.187	0.238	0.112	0.255	18.449

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	58	53	57	64	86	54	74	57
N.S.	1	1.02	0.93	1.00	1.12	1.51	0.95	1.30	1.00
time (sec)	N/A	0.033	0.052	0.201	0.194	0.238	0.133	0.257	0.079

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	43	37	41	51	80	46	43	43
N.S.	1	1.13	0.97	1.08	1.34	2.11	1.21	1.13	1.13
time (sec)	N/A	0.024	0.032	0.093	0.192	0.240	0.132	0.252	16.672

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	57	53	56	69	109	66	60	63
N.S.	1	1.12	1.04	1.10	1.35	2.14	1.29	1.18	1.24
time (sec)	N/A	0.031	0.054	0.144	0.200	0.242	0.160	0.259	0.107

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	76	68	72	86	130	78	73	79
N.S.	1	1.19	1.06	1.12	1.34	2.03	1.22	1.14	1.23
time (sec)	N/A	0.042	0.053	0.398	0.198	0.246	0.177	0.248	16.776

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	57	48	52	73	124	70	54	60
N.S.	1	1.12	0.94	1.02	1.43	2.43	1.37	1.06	1.18
time (sec)	N/A	0.029	0.035	0.500	0.188	0.240	0.166	0.251	0.196

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	70	64	69	91	153	90	71	85
N.S.	1	1.13	1.03	1.11	1.47	2.47	1.45	1.15	1.37
time (sec)	N/A	0.045	0.064	0.189	0.203	0.240	0.210	0.259	16.429

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	93	79	83	108	174	104	86	101
N.S.	1	1.18	1.00	1.05	1.37	2.20	1.32	1.09	1.28
time (sec)	N/A	0.059	0.063	0.089	0.205	0.245	0.222	0.257	0.100

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	71	59	63	95	168	94	89	77
N.S.	1	1.11	0.92	0.98	1.48	2.62	1.47	1.39	1.20
time (sec)	N/A	0.037	0.043	0.207	0.195	0.239	0.214	0.261	16.873

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	87	75	82	113	197	114	108	107
N.S.	1	1.13	0.97	1.06	1.47	2.56	1.48	1.40	1.39
time (sec)	N/A	0.054	0.070	0.089	0.203	0.245	0.238	0.251	16.120

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	105	90	94	130	218	128	128	123
N.S.	1	1.18	1.01	1.06	1.46	2.45	1.44	1.44	1.38
time (sec)	N/A	0.077	0.069	0.176	0.194	0.252	0.271	0.263	0.176

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	24	24	16	15	67	53	15	16
N.S.	1	1.20	1.20	0.80	0.75	3.35	2.65	0.75	0.80
time (sec)	N/A	0.007	0.009	0.089	0.292	0.238	0.059	0.259	0.067

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	27	24	26	0	29	102	23	23
N.S.	1	1.08	0.96	1.04	0.00	1.16	4.08	0.92	0.92
time (sec)	N/A	0.008	0.009	0.089	0.000	0.245	1.152	0.265	15.819

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	115	89	27	98	299	20	112	99
N.S.	1	1.22	0.95	0.29	1.04	3.18	0.21	1.19	1.05
time (sec)	N/A	0.085	0.038	0.098	0.272	0.276	0.065	0.263	15.236

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	115	89	27	98	304	24	112	111
N.S.	1	1.14	0.88	0.27	0.97	3.01	0.24	1.11	1.10
time (sec)	N/A	0.066	0.012	0.079	0.278	0.257	0.058	0.254	15.008

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.004	0.004	0.073	0.188	0.241	0.057	0.257	15.157

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	119	108	34	106	106	22	111	114
N.S.	1	1.19	1.08	0.34	1.06	1.06	0.22	1.11	1.14
time (sec)	N/A	0.075	0.020	0.124	0.308	0.256	0.077	0.265	14.649

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	124	111	37	109	123	32	114	120
N.S.	1	1.10	0.98	0.33	0.96	1.09	0.28	1.01	1.06
time (sec)	N/A	0.077	0.020	0.079	0.272	0.245	0.075	0.262	0.244

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	134	118	46	122	389	39	127	128
N.S.	1	1.20	1.05	0.41	1.09	3.47	0.35	1.13	1.14
time (sec)	N/A	0.076	0.075	0.088	0.288	0.251	0.120	0.258	16.108

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	136	119	48	124	402	44	129	138
N.S.	1	1.10	0.96	0.39	1.00	3.24	0.35	1.04	1.11
time (sec)	N/A	0.077	0.069	0.103	0.267	0.244	0.107	0.260	15.118

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	15	14	14
N.S.	1	1.00	1.00	0.94	0.88	0.94	0.94	0.88	0.88
time (sec)	N/A	0.004	0.006	0.083	0.194	0.223	0.098	0.263	14.417

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	134	118	43	114	391	39	130	108
N.S.	1	1.17	1.03	0.37	0.99	3.40	0.34	1.13	0.94
time (sec)	N/A	0.073	0.068	0.079	0.268	0.263	0.112	0.256	14.816

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	22	22	21	23	18	15	22	18
N.S.	1	1.05	1.05	1.00	1.10	0.86	0.71	1.05	0.86
time (sec)	N/A	0.013	0.009	0.125	0.188	0.243	0.115	0.266	14.924

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	122	114	53	106	103	29	121	102
N.S.	1	1.11	1.04	0.48	0.96	0.94	0.26	1.10	0.93
time (sec)	N/A	0.072	0.023	0.089	0.291	0.233	0.080	0.262	0.286

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	124	119	54	106	143	32	115	128
N.S.	1	1.17	1.12	0.51	1.00	1.35	0.30	1.08	1.21
time (sec)	N/A	0.073	0.026	0.352	0.288	0.251	0.099	0.265	15.019

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	37	47	34	45	34
N.S.	1	1.00	0.87	0.92	0.97	1.24	0.89	1.18	0.89
time (sec)	N/A	0.031	0.018	0.299	0.189	0.236	0.165	0.265	14.732

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	146	131	73	126	146	56	139	120
N.S.	1	1.11	0.99	0.55	0.95	1.11	0.42	1.05	0.91
time (sec)	N/A	0.090	0.095	0.403	0.279	0.257	0.151	0.263	14.124

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	146	129	74	128	187	58	131	146
N.S.	1	1.16	1.02	0.59	1.02	1.48	0.46	1.04	1.16
time (sec)	N/A	0.091	0.088	0.092	0.286	0.252	0.173	0.267	14.672

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	C	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	185	134	27	169	112	20	179	33
N.S.	1	2.94	2.13	0.43	2.68	1.78	0.32	2.84	0.52
time (sec)	N/A	0.144	0.054	0.074	0.280	0.245	0.069	0.265	0.108

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	29	29	19	18	72	56	18	19
N.S.	1	1.16	1.16	0.76	0.72	2.88	2.24	0.72	0.76
time (sec)	N/A	0.014	0.008	0.080	0.275	0.237	0.075	0.266	13.949

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	C	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	185	134	27	169	124	26	179	35
N.S.	1	2.94	2.13	0.43	2.68	1.97	0.41	2.84	0.56
time (sec)	N/A	0.118	0.020	0.079	0.295	0.255	0.076	0.261	0.102

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.004	0.005	0.079	0.187	0.234	0.068	0.258	14.079

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	C	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	202	183	46	189	183	39	194	58
N.S.	1	2.49	2.26	0.57	2.33	2.26	0.48	2.40	0.72
time (sec)	N/A	0.134	0.115	0.439	0.288	0.242	0.140	0.275	0.113

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	49	49	40	39	129	83	39	37
N.S.	1	1.02	1.02	0.83	0.81	2.69	1.73	0.81	0.77
time (sec)	N/A	0.029	0.035	0.164	0.269	0.244	0.142	0.268	0.049

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	C	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	204	184	48	191	196	46	196	60
N.S.	1	2.37	2.14	0.56	2.22	2.28	0.53	2.28	0.70
time (sec)	N/A	0.141	0.131	0.075	0.267	0.241	0.139	0.260	0.119

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	15	14	14
N.S.	1	1.00	1.00	0.94	0.88	0.94	0.94	0.88	0.88
time (sec)	N/A	0.004	0.006	0.512	0.183	0.231	0.102	0.252	0.031

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	22	22	21	23	18	15	24	18
N.S.	1	1.05	1.05	1.00	1.10	0.86	0.71	1.14	0.86
time (sec)	N/A	0.014	0.009	0.077	0.192	0.236	0.129	0.260	14.579

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	C	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	193	179	50	183	128	29	187	51
N.S.	1	2.68	2.49	0.69	2.54	1.78	0.40	2.60	0.71
time (sec)	N/A	0.139	0.034	0.080	0.281	0.237	0.110	0.265	15.075

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	5	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.25	1.00
time (sec)	N/A	0.001	0.001	0.063	0.181	0.229	0.017	0.256	0.024

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.001	0.003	0.066	0.280	0.231	0.039	0.251	0.003

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	41	40	33	34	34	41	35	31
N.S.	1	0.95	0.93	0.77	0.79	0.79	0.95	0.81	0.72
time (sec)	N/A	0.026	0.014	0.072	0.294	0.244	0.058	0.263	0.117

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	85	64	22	72	61	73	72	33
N.S.	1	1.31	0.98	0.34	1.11	0.94	1.12	1.11	0.51
time (sec)	N/A	0.047	0.024	0.076	0.275	0.248	0.064	0.265	15.201

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	7	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75
time (sec)	N/A	0.001	0.003	0.065	0.192	0.233	0.021	0.266	0.027

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	19	3	13	13	12	15	2
N.S.	1	1.00	9.50	1.50	6.50	6.50	6.00	7.50	1.00
time (sec)	N/A	0.001	0.003	0.495	0.194	0.240	0.046	0.256	0.003

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	19	5	13	13	12	15	4
N.S.	1	1.00	4.75	1.25	3.25	3.25	3.00	3.75	1.00
time (sec)	N/A	0.001	0.002	0.065	0.199	0.233	0.036	0.252	0.084

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	40	40	33	32	32	41	33	46
N.S.	1	0.93	0.93	0.77	0.74	0.74	0.95	0.77	1.07
time (sec)	N/A	0.023	0.010	0.089	0.304	0.238	0.059	0.270	0.107

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	13	25	10	17	17	17	19	9
N.S.	1	1.44	2.78	1.11	1.89	1.89	1.89	2.11	1.00
time (sec)	N/A	0.004	0.005	0.087	0.300	0.242	0.058	0.253	14.570

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	5	9	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.62	1.12	1.00
time (sec)	N/A	0.005	0.002	0.439	0.192	0.236	0.026	0.262	0.026

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.002	0.003	0.062	0.202	0.236	0.026	0.259	0.035

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	41	40	33	34	34	41	35	46
N.S.	1	1.02	1.00	0.82	0.85	0.85	1.02	0.88	1.15
time (sec)	N/A	0.023	0.007	0.081	0.292	0.238	0.055	0.259	14.693

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.003	0.005	0.075	0.279	0.230	0.036	0.259	14.774

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	7	11	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.58	0.92	0.83
time (sec)	N/A	0.005	0.003	0.069	0.198	0.243	0.025	0.262	0.031

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	8	9	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.67	0.75	0.67
time (sec)	N/A	0.002	0.003	0.451	0.206	0.239	0.029	0.256	0.061

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	31	32	32	41	33	46
N.S.	1	1.00	1.00	0.76	0.78	0.78	1.00	0.80	1.12
time (sec)	N/A	0.023	0.008	0.083	0.301	0.243	0.055	0.257	15.188

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	8	23	7	17	17	15	18	6
N.S.	1	0.40	1.15	0.35	0.85	0.85	0.75	0.90	0.30
time (sec)	N/A	0.004	0.004	0.065	0.188	0.232	0.038	0.270	0.003

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	13	13	12	15	11	10	15	11
N.S.	1	1.08	1.08	1.00	1.25	0.92	0.83	1.25	0.92
time (sec)	N/A	0.006	0.003	0.062	0.206	0.239	0.035	0.251	14.775

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	15	15	12	15	11	10	16	11
N.S.	1	1.07	1.07	0.86	1.07	0.79	0.71	1.14	0.79
time (sec)	N/A	0.007	0.004	0.076	0.209	0.245	0.041	0.259	0.079

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	26	25	26	25	24	20	26	26
N.S.	1	1.04	1.00	1.04	1.00	0.96	0.80	1.04	1.04
time (sec)	N/A	0.023	0.012	0.076	0.222	0.232	0.069	0.254	0.065

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	36	26	27	36	26	128	46	25
N.S.	1	1.44	1.04	1.08	1.44	1.04	5.12	1.84	1.00
time (sec)	N/A	0.011	0.016	0.438	0.202	0.244	0.160	0.258	0.127

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	44	38	38	44	38	138	46	37
N.S.	1	1.26	1.09	1.09	1.26	1.09	3.94	1.31	1.06
time (sec)	N/A	0.030	0.018	0.434	0.208	0.232	0.361	0.246	15.832

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	29	29	19	18	68	73	18	19
N.S.	1	1.81	1.81	1.19	1.12	4.25	4.56	1.12	1.19
time (sec)	N/A	0.011	0.030	0.073	0.284	0.249	0.377	0.250	15.501

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	40	40	32	31	85	88	31	28
N.S.	1	1.29	1.29	1.03	1.00	2.74	2.84	1.00	0.90
time (sec)	N/A	0.016	0.046	0.079	0.274	0.244	0.269	0.262	14.628

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	53	49	42	42	103	107	45	37
N.S.	1	1.18	1.09	0.93	0.93	2.29	2.38	1.00	0.82
time (sec)	N/A	0.021	0.067	0.078	0.278	0.247	0.530	0.257	0.061

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	68	61	53	54	132	122	59	48
N.S.	1	1.19	1.07	0.93	0.95	2.32	2.14	1.04	0.84
time (sec)	N/A	0.028	0.083	0.085	0.279	0.247	1.803	0.271	15.384

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	45	45	36	35	116	277	35	33
N.S.	1	1.50	1.50	1.20	1.17	3.87	9.23	1.17	1.10
time (sec)	N/A	0.015	0.082	0.068	0.281	0.245	2.192	0.264	15.210

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	46	46	37	37	115	269	36	34
N.S.	1	1.48	1.48	1.19	1.19	3.71	8.68	1.16	1.10
time (sec)	N/A	0.024	0.085	0.069	0.302	0.251	1.438	0.257	0.047

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	57	54	47	49	134	332	46	46
N.S.	1	1.14	1.08	0.94	0.98	2.68	6.64	0.92	0.92
time (sec)	N/A	0.021	0.117	0.090	0.278	0.258	2.823	0.266	15.302

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	70	68	56	63	161	389	65	58
N.S.	1	1.01	0.99	0.81	0.91	2.33	5.64	0.94	0.84
time (sec)	N/A	0.027	0.128	0.112	0.306	0.251	7.325	0.267	0.080

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	70	59	59	60	186	632	47	57
N.S.	1	1.23	1.04	1.04	1.05	3.26	11.09	0.82	1.00
time (sec)	N/A	0.040	0.117	0.085	0.279	0.251	7.923	0.254	15.986

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	73	60	52	64	186	627	52	56
N.S.	1	1.16	0.95	0.83	1.02	2.95	9.95	0.83	0.89
time (sec)	N/A	0.021	0.170	0.070	0.313	0.255	5.172	0.256	16.017

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	70	59	50	61	185	605	47	58
N.S.	1	0.80	0.68	0.57	0.70	2.13	6.95	0.54	0.67
time (sec)	N/A	0.021	0.170	0.069	0.304	0.256	9.858	0.258	15.438

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	82	70	56	73	200	683	59	69
N.S.	1	0.81	0.69	0.55	0.72	1.98	6.76	0.58	0.68
time (sec)	N/A	0.040	0.182	0.104	0.278	0.249	16.889	0.255	0.137

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	192	92	106	172	120	104	182	37
N.S.	1	1.94	0.93	1.07	1.74	1.21	1.05	1.84	0.37
time (sec)	N/A	0.180	0.193	0.067	0.288	0.252	1.544	0.263	17.877

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	192	91	106	172	132	104	182	38
N.S.	1	1.90	0.90	1.05	1.70	1.31	1.03	1.80	0.38
time (sec)	N/A	0.171	0.172	0.069	0.299	0.243	0.910	0.261	16.937

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	202	118	115	185	117	110	178	55
N.S.	1	1.80	1.05	1.03	1.65	1.04	0.98	1.59	0.49
time (sec)	N/A	0.199	0.215	0.077	0.278	0.240	1.655	0.255	17.526

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	204	119	116	186	158	124	178	54
N.S.	1	1.81	1.05	1.03	1.65	1.40	1.10	1.58	0.48
time (sec)	N/A	0.194	0.205	0.102	0.298	0.260	4.244	0.255	0.098

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	218	128	124	194	193	316	199	64
N.S.	1	1.72	1.01	0.98	1.53	1.52	2.49	1.57	0.50
time (sec)	N/A	0.206	0.382	0.079	0.287	0.236	22.759	0.264	0.132

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	218	128	127	194	204	400	199	64
N.S.	1	1.69	0.99	0.98	1.50	1.58	3.10	1.54	0.50
time (sec)	N/A	0.201	0.395	0.069	0.287	0.256	19.034	0.260	16.305

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	218	127	127	195	191	323	199	64
N.S.	1	1.76	1.02	1.02	1.57	1.54	2.60	1.60	0.52
time (sec)	N/A	0.212	0.411	0.071	0.291	0.261	34.062	0.271	16.135

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	218	128	124	195	200	393	199	64
N.S.	1	1.73	1.02	0.98	1.55	1.59	3.12	1.58	0.51
time (sec)	N/A	0.172	0.431	0.086	0.282	0.261	51.807	0.257	0.087

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	239	138	147	217	268	627	209	86
N.S.	1	1.65	0.95	1.01	1.50	1.85	4.32	1.44	0.59
time (sec)	N/A	0.223	0.376	0.072	0.288	0.248	141.962	0.269	17.311

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	239	138	150	217	281	887	209	86
N.S.	1	1.63	0.94	1.02	1.48	1.91	6.03	1.42	0.59
time (sec)	N/A	0.228	0.385	0.077	0.297	0.268	103.847	0.280	17.778

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	242	137	138	221	279	666	211	85
N.S.	1	1.74	0.99	0.99	1.59	2.01	4.79	1.52	0.61
time (sec)	N/A	0.300	0.556	0.081	0.298	0.257	158.653	0.269	0.129

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	242	136	138	222	289	0	212	85
N.S.	1	1.37	0.77	0.78	1.25	1.63	0.00	1.20	0.48
time (sec)	N/A	0.230	0.576	0.137	0.294	0.260	0.000	0.270	0.099

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.002	0.004	0.072	0.204	0.237	0.031	0.255	0.031

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	32	23	21	26	19	162	23	25
N.S.	1	1.19	0.85	0.78	0.96	0.70	6.00	0.85	0.93
time (sec)	N/A	0.011	0.024	0.071	0.194	0.243	0.633	0.265	18.835

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	51	35	32	41	31	600	37	37
N.S.	1	1.31	0.90	0.82	1.05	0.79	15.38	0.95	0.95
time (sec)	N/A	0.017	0.035	0.074	0.201	0.252	0.957	0.262	0.069

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	21	21	20	12	53	27	12	21
N.S.	1	1.50	1.50	1.43	0.86	3.79	1.93	0.86	1.50
time (sec)	N/A	0.006	0.017	0.132	0.238	0.236	0.756	0.256	18.221

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	46	28	27	30	62	0	29	28
N.S.	1	2.19	1.33	1.29	1.43	2.95	0.00	1.38	1.33
time (sec)	N/A	0.036	0.028	0.150	0.218	0.253	0.000	0.257	18.310

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	67	41	39	41	73	0	46	40
N.S.	1	1.72	1.05	1.00	1.05	1.87	0.00	1.18	1.03
time (sec)	N/A	0.059	0.043	0.172	0.247	0.251	0.000	0.259	18.333

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	23	23	18	32	56	24	21	17
N.S.	1	0.55	0.55	0.43	0.76	1.33	0.57	0.50	0.40
time (sec)	N/A	0.013	0.032	0.084	0.288	0.253	0.559	0.259	0.061

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	35	35	28	42	73	68	32	27
N.S.	1	0.65	0.65	0.52	0.78	1.35	1.26	0.59	0.50
time (sec)	N/A	0.017	0.039	0.080	0.293	0.246	0.796	0.264	17.177

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	39	39	32	47	93	44	41	31
N.S.	1	0.64	0.64	0.52	0.77	1.52	0.72	0.67	0.51
time (sec)	N/A	0.017	0.088	0.093	0.317	0.251	1.073	0.263	0.130

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	65	55	44	88	119	97	66	48
N.S.	1	0.72	0.61	0.49	0.98	1.32	1.08	0.73	0.53
time (sec)	N/A	0.021	0.141	0.107	0.299	0.243	2.131	0.255	0.083

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	75	65	54	0	235	0	44	0
N.S.	1	1.15	1.00	0.83	0.00	3.62	0.00	0.68	0.00
time (sec)	N/A	0.041	0.096	0.134	0.000	0.248	0.000	0.257	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	80	67	68	0	252	0	56	0
N.S.	1	0.86	0.72	0.73	0.00	2.71	0.00	0.60	0.00
time (sec)	N/A	0.053	0.115	0.168	0.000	0.247	0.000	0.269	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	91	80	67	0	278	0	64	0
N.S.	1	0.67	0.59	0.50	0.00	2.06	0.00	0.47	0.00
time (sec)	N/A	0.049	0.184	0.179	0.000	0.254	0.000	0.262	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	41	41	34	60	93	44	47	33
N.S.	1	0.61	0.61	0.51	0.90	1.39	0.66	0.70	0.49
time (sec)	N/A	0.021	0.083	0.092	0.296	0.243	1.144	0.267	0.059

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	68	56	45	92	123	102	69	51
N.S.	1	0.84	0.69	0.56	1.14	1.52	1.26	0.85	0.63
time (sec)	N/A	0.024	0.121	0.105	0.303	0.244	2.516	0.259	0.070

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	60	56	47	0	268	0	37	0
N.S.	1	1.05	0.98	0.82	0.00	4.70	0.00	0.65	0.00
time (sec)	N/A	0.039	0.064	0.128	0.000	0.259	0.000	0.265	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	89	68	58	0	309	0	64	0
N.S.	1	1.24	0.94	0.81	0.00	4.29	0.00	0.89	0.00
time (sec)	N/A	0.049	0.115	0.185	0.000	0.260	0.000	0.264	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	126	87	74	0	347	0	80	0
N.S.	1	1.38	0.96	0.81	0.00	3.81	0.00	0.88	0.00
time (sec)	N/A	0.065	0.166	0.188	0.000	0.256	0.000	0.262	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	127	113	0	0	225	0	219	0
N.S.	1	1.61	1.43	0.00	0.00	2.85	0.00	2.77	0.00
time (sec)	N/A	0.076	0.145	0.000	0.000	0.260	0.000	3.147	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	91	113	87	86	91	180	87	107
N.S.	1	0.99	1.23	0.95	0.93	0.99	1.96	0.95	1.16
time (sec)	N/A	0.069	0.095	0.451	0.280	0.238	1.214	0.481	18.574

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	97	119	95	93	139	643	105	117
N.S.	1	0.80	0.98	0.78	0.76	1.14	5.27	0.86	0.96
time (sec)	N/A	0.048	0.233	0.142	0.281	0.248	1.304	0.475	0.341

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	127	129	118	139	187	2266	128	196
N.S.	1	0.91	0.92	0.84	0.99	1.34	16.19	0.91	1.40
time (sec)	N/A	0.064	0.338	0.142	0.298	0.248	1.914	0.493	20.986

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	161	157	0	0	325	0	246	0
N.S.	1	1.55	1.51	0.00	0.00	3.12	0.00	2.37	0.00
time (sec)	N/A	0.084	0.232	0.000	0.000	0.249	0.000	3.133	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	200	183	0	0	326	0	295	0
N.S.	1	1.69	1.55	0.00	0.00	2.76	0.00	2.50	0.00
time (sec)	N/A	0.102	0.214	0.000	0.000	0.268	0.000	3.152	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	79	95	75	76	213	155	77	99
N.S.	1	1.01	1.22	0.96	0.97	2.73	1.99	0.99	1.27
time (sec)	N/A	0.034	0.077	0.102	0.289	0.268	1.012	0.489	16.561

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	141	136	0	0	238	0	249	0
N.S.	1	1.48	1.43	0.00	0.00	2.51	0.00	2.62	0.00
time (sec)	N/A	0.067	0.126	0.000	0.000	0.253	0.000	3.157	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	152	139	109	0	560	0	236	0
N.S.	1	1.32	1.21	0.95	0.00	4.87	0.00	2.05	0.00
time (sec)	N/A	0.080	0.268	0.095	0.000	0.264	0.000	3.043	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	47	31	44	11	13	0	12	0
N.S.	1	0.34	0.22	0.32	0.08	0.09	0.00	0.09	0.00
time (sec)	N/A	0.032	1.022	0.178	0.204	0.239	0.000	0.258	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	100	120	95	106	306	831	109	130
N.S.	1	0.95	1.14	0.90	1.01	2.91	7.91	1.04	1.24
time (sec)	N/A	0.041	0.232	0.119	0.296	0.252	1.456	0.477	15.941

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	130	149	109	142	296	2730	130	182
N.S.	1	1.11	1.27	0.93	1.21	2.53	23.33	1.11	1.56
time (sec)	N/A	0.053	0.223	0.121	0.290	0.264	2.366	0.472	16.753

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	40	29	27	39	25	53	39	28
N.S.	1	0.91	0.66	0.61	0.89	0.57	1.20	0.89	0.64
time (sec)	N/A	0.017	0.037	0.076	0.198	0.250	0.411	0.256	16.963

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [89] had the largest ratio of [.8570999999999999973]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	3	0.333
2	A	1	1	1.00	3	0.333
3	A	1	1	1.00	3	0.333
4	A	1	1	1.00	3	0.333
5	A	1	1	1.00	2	0.500
6	A	1	1	1.00	2	0.500
7	A	2	2	1.00	4	0.500
8	A	2	2	1.00	4	0.500
9	A	2	2	1.00	5	0.400
10	A	2	2	1.00	5	0.400
11	A	1	1	1.00	2	0.500
12	A	1	1	1.00	2	0.500
13	A	1	1	0.71	2	0.500
14	A	1	1	0.50	2	0.500
15	A	1	1	1.00	7	0.143
16	A	1	1	1.00	9	0.111
17	A	1	1	1.00	11	0.091
18	A	1	1	1.00	9	0.111
19	A	2	2	1.00	9	0.222
20	A	1	1	1.00	2	0.500
21	A	1	1	1.00	2	0.500
22	A	2	2	1.00	4	0.500
23	A	2	2	1.00	4	0.500
24	A	1	1	1.00	2	0.500

Continued on next page

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	1	1	1.00	2	0.500
26	A	1	1	0.71	2	0.500
27	A	1	1	1.00	7	0.143
28	A	1	1	1.00	7	0.143
29	A	2	1	1.00	9	0.111
30	A	2	1	1.00	11	0.091
31	A	1	1	1.00	7	0.143
32	A	2	1	0.96	9	0.111
33	A	2	1	1.00	11	0.091
34	A	1	1	1.00	7	0.143
35	A	1	1	0.74	9	0.111
36	A	2	1	1.11	11	0.091
37	A	2	1	0.88	11	0.091
38	A	1	1	1.00	7	0.143
39	A	2	1	1.20	9	0.111
40	A	1	1	0.50	11	0.091
41	A	2	1	1.16	11	0.091
42	A	1	1	1.00	7	0.143
43	A	2	1	1.20	9	0.111
44	A	2	1	1.24	11	0.091
45	A	1	1	0.36	11	0.091
46	A	3	3	1.20	11	0.273
47	A	2	1	1.17	11	0.091
48	A	2	1	1.14	11	0.091
49	A	2	1	1.05	11	0.091
50	A	2	1	1.02	11	0.091
51	A	2	1	1.13	11	0.091
52	A	2	1	1.12	11	0.091
53	A	2	1	1.19	11	0.091
54	A	2	1	1.12	11	0.091
55	A	2	1	1.13	11	0.091
56	A	2	1	1.18	11	0.091
57	A	2	1	1.11	11	0.091
58	A	2	1	1.13	11	0.091
59	A	2	1	1.18	11	0.091

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	1	1	1.20	9	0.111
61	A	1	1	1.08	13	0.077
62	A	6	6	1.22	9	0.667
63	A	6	6	1.14	11	0.546
64	A	1	1	1.00	13	0.077
65	A	7	7	1.19	13	0.538
66	A	7	7	1.10	13	0.538
67	A	7	7	1.20	9	0.778
68	A	7	7	1.10	11	0.636
69	A	1	1	1.00	13	0.077
70	A	7	7	1.17	13	0.538
71	A	4	4	1.05	13	0.308
72	A	7	7	1.11	13	0.538
73	A	7	7	1.17	13	0.538
74	A	3	2	1.00	13	0.154
75	A	8	8	1.11	13	0.615
76	A	8	8	1.16	13	0.615
77	B	9	6	2.94	9	0.667
78	A	2	2	1.16	11	0.182
79	B	9	6	2.94	13	0.462
80	A	1	1	1.00	13	0.077
81	B	10	7	2.49	9	0.778
82	A	3	3	1.02	11	0.273
83	B	10	7	2.37	13	0.538
84	A	1	1	1.00	13	0.077
85	A	4	4	1.05	13	0.308
86	B	10	7	2.68	13	0.538
87	A	1	1	1.00	5	0.200
88	A	1	1	1.00	7	0.143
89	A	6	6	0.95	7	0.857
90	A	9	6	1.31	7	0.857
91	A	1	1	1.00	7	0.143
92	A	1	1	1.00	9	0.111
93	A	1	1	1.00	7	0.143
94	A	6	6	0.93	9	0.667

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	3	3	1.44	9	0.333
96	A	2	1	1.00	7	0.143
97	A	1	1	1.00	9	0.111
98	A	6	6	1.02	9	0.667
99	A	2	2	1.00	9	0.222
100	A	2	1	1.00	9	0.111
101	A	1	1	1.00	11	0.091
102	A	6	6	1.00	11	0.546
103	A	2	2	0.40	11	0.182
104	A	4	4	1.08	11	0.364
105	A	4	4	1.07	13	0.308
106	A	2	1	1.04	13	0.077
107	A	3	2	1.44	15	0.133
108	A	2	1	1.26	16	0.062
109	A	2	2	1.81	13	0.154
110	A	3	3	1.29	13	0.231
111	A	4	3	1.18	13	0.231
112	A	5	3	1.19	13	0.231
113	A	3	3	1.50	13	0.231
114	A	3	3	1.48	13	0.231
115	A	4	4	1.14	13	0.308
116	A	5	4	1.01	13	0.308
117	A	4	3	1.23	13	0.231
118	A	4	4	1.16	13	0.308
119	A	4	3	0.80	13	0.231
120	A	5	4	0.81	13	0.308
121	A	10	7	1.94	15	0.467
122	A	10	7	1.90	15	0.467
123	A	11	8	1.80	15	0.533
124	A	11	8	1.81	15	0.533
125	A	11	8	1.72	15	0.533
126	A	11	8	1.69	15	0.533
127	A	11	8	1.76	15	0.533
128	A	11	8	1.73	15	0.533
129	A	12	8	1.65	15	0.533

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	12	8	1.63	15	0.533
131	A	12	9	1.74	15	0.600
132	A	12	9	1.37	15	0.600
133	A	1	1	1.00	9	0.111
134	A	2	1	1.19	11	0.091
135	A	2	1	1.31	13	0.077
136	A	3	3	1.50	11	0.273
137	B	3	2	2.19	13	0.154
138	A	3	2	1.72	15	0.133
139	A	2	2	0.55	13	0.154
140	A	3	3	0.65	13	0.231
141	A	3	3	0.64	13	0.231
142	A	4	4	0.72	13	0.308
143	A	5	4	1.15	15	0.267
144	A	5	5	0.86	15	0.333
145	A	5	4	0.67	15	0.267
146	A	3	3	0.61	13	0.231
147	A	4	3	0.84	13	0.231
148	A	4	4	1.05	15	0.267
149	A	5	5	1.24	15	0.333
150	A	6	5	1.38	15	0.333
151	A	5	5	1.61	15	0.333
152	A	5	5	0.99	13	0.385
153	A	5	5	0.80	13	0.385
154	A	6	6	0.91	13	0.462
155	A	6	6	1.55	15	0.400
156	A	7	6	1.69	15	0.400
157	A	4	4	1.01	13	0.308
158	A	6	6	1.48	15	0.400
159	A	6	6	1.32	15	0.400
160	A	3	2	0.34	15	0.133
161	A	5	5	0.95	13	0.385
162	A	6	5	1.11	13	0.385
163	A	2	1	0.91	15	0.067

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^n \, dx$

Optimal result

Integrand size = 3, antiderivative size = 11

$$\int x^n \, dx = \frac{x^{1+n}}{1+n}$$

```
[Out] x^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

```
[In] Int[x^n,x]
```

```
[Out] x^(1 + n)/(1 + n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{x^{1+n}}{1+n}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int x^n\,dx = \frac{x^{1+n}}{1+n}$$

```
[In] Integrate[x^n,x]
[Out] x^(1 + n)/(1 + n)
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{x\,x^{n}}{n+1}$	11
parallelrisch	$\frac{x\,x^{n}}{n+1}$	11
gosper	$\frac{x^{n+1}}{n+1}$	12
default	$\frac{x^{n+1}}{n+1}$	12
norman	$\frac{x\,{\mathrm e}^{n\ln(x)}}{n+1}$	13

```
[In] int(x^n,x,method=_RETURNVERBOSE)
[Out] x/(n+1)*x^n
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int x^n\,dx = \frac{xx^n}{n+1}$$

```
[In] integrate(x^n,x, algorithm="fricas")
[Out] x*x^n/(n + 1)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int x^n\,dx = \begin{cases} \frac{x^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(x) & \text{otherwise} \end{cases}$$

```
[In] integrate(x**n,x)
[Out] Piecewise((x**(n + 1)/(n + 1), Ne(n, -1)), (log(x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

```
[In] integrate(x^n,x, algorithm="maxima")
[Out] x^(n + 1)/(n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

```
[In] integrate(x^n,x, algorithm="giac")
[Out] x^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 17.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int x^n\,dx = \begin{cases} \ln(x) & \text{if } n = -1 \\ \frac{x^{n+1}}{n+1} & \text{if } n \neq -1 \end{cases}$$

```
[In] int(x^n,x)
[Out] piecewise(n == -1, log(x), n ~= -1, x^(n + 1)/(n + 1))
```

3.2 $\int \frac{1}{x} \, dx$

Optimal result

Integrand size = 3, antiderivative size = 2

$$\int \frac{1}{x} \, dx = \log(x)$$

```
[Out] ln(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{29\}$

$$\int \frac{1}{x} \, dx = \log(x)$$

```
[In] Int[x^(-1),x]
```

```
[Out] Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\text{integral} = \log(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} \, dx = \log(x)$$

```
[In] Integrate[x^(-1),x]
[Out] Log[x]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\ln(x)$	3
norman	$\ln(x)$	3
risch	$\ln(x)$	3
parallelrisch	$\ln(x)$	3

```
[In] int(1/x,x,method=_RETURNVERBOSE)
[Out] ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} \, dx = \log(x)$$

```
[In] integrate(1/x,x, algorithm="fricas")
[Out] log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} \, dx = \log(x)$$

```
[In] integrate(1/x,x)
[Out] log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} \, dx = \log(x)$$

```
[In] integrate(1/x,x, algorithm="maxima")
[Out] log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{x} \, dx = \log(|x|)$$

```
[In] integrate(1/x,x, algorithm="giac")
[Out] log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} \, dx = \ln(x)$$

```
[In] int(1/x,x)
[Out] log(x)
```

3.3 $\int e^x \, dx$

Optimal result

Integrand size = 3, antiderivative size = 3

$$\int e^x \, dx = e^x$$

```
[Out] exp(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2225}

$$\int e^x \, dx = e^x$$

```
[In] Int[E^x,x]
```

```
[Out] E^x
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\text{integral} = e^x$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int e^x\,dx = e^x$$

```
[In] Integrate[E^x,x]
[Out] E^x
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

method	result	size
gosper	e^x	3
lookup	e^x	3
derivativedivides	e^x	3
default	e^x	3
norman	e^x	3
risch	e^x	3
parallelrisch	e^x	3
meijerg	$e^x - 1$	5

```
[In] int(exp(x),x,method=_RETURNVERBOSE)
[Out] exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx = e^x$$

```
[In] integrate(exp(x),x, algorithm="fricas")
[Out] e^x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx = e^x$$

```
[In] integrate(exp(x),x)
[Out] exp(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx = e^x$$

```
[In] integrate(exp(x),x, algorithm="maxima")
[Out] e^x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx = e^x$$

```
[In] integrate(exp(x),x, algorithm="giac")
[Out] e^x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx = \mathrm{e}^x$$

```
[In] int(exp(x),x)
[Out] exp(x)
```

3.4 $\int a^x \, dx$

Optimal result

Integrand size = 3, antiderivative size = 8

$$\int a^x \, dx = \frac{a^x}{\log(x)}$$

```
[Out] a^x/ln(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2225}

$$\int a^x \, dx = \frac{a^x}{\log(a)}$$

```
[In] Int[a^x,x]
```

```
[Out] a^x/Log[a]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\text{integral} = \frac{a^x}{\log(a)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x\,dx = \frac{a^x}{\log(a)}$$

```
[In] Integrate[a^x,x]
[Out] a^x/Log[a]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
gosper	$\frac{a^x}{\ln(a)}$	9
derivativedivides	$\frac{a^x}{\ln(a)}$	9
default	$\frac{a^x}{\ln(a)}$	9
risch	$\frac{a^x}{\ln(a)}$	9
parallelrisch	$\frac{a^x}{\ln(a)}$	9
norman	$\frac{e^{x\ln(a)}}{\ln(a)}$	11
meijerg	$-\frac{1-e^{x\ln(a)}}{\ln(a)}$	16

```
[In] int(a^x,x,method=_RETURNVERBOSE)
[Out] 1/ln(a)*a^x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x\,dx = \frac{a^x}{\log(a)}$$

```
[In] integrate(a^x,x, algorithm="fricas")
[Out] a^x/log(a)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x \, dx = \begin{cases} \frac{a^x}{\log(a)} & \text{for } \log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

```
[In] integrate(a**x,x)
[Out] Piecewise((a**x/log(a), Ne(log(a), 0)), (x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x \, dx = \frac{a^x}{\log(a)}$$

```
[In] integrate(a^x,x, algorithm="maxima")
[Out] a^x/log(a)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x \, dx = \frac{a^x}{\log(a)}$$

```
[In] integrate(a^x,x, algorithm="giac")
[Out] a^x/log(a)
```

Mupad [B] (verification not implemented)

Time = 17.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x \, dx = \frac{a^x}{\ln(a)}$$

```
[In] int(a^x,x)
[Out] a^x/log(a)
```

3.5 $\int \sin(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 4

$$\int \sin(x)\,dx = -\cos(x)$$

```
[Out] -cos(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2718}

$$\int \sin(x)\,dx = -\cos(x)$$

```
[In] Int[Sin[x],x]
```

```
[Out] -Cos[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\cos(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x)\,dx = -\cos(x)$$

```
[In] Integrate[Sin[x],x]
[Out] -Cos[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
lookup	$-\cos\left(x\right)$	5
default	$-\cos\left(x\right)$	5
risch	$-\cos\left(x\right)$	5
parallelrisch	$-1-\cos\left(x\right)$	7
norman	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)^{2}}$	13
meijerg	$\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(x)}{\sqrt{\pi}}\right)$	16

```
[In] int(sin(x),x,method=_RETURNVERBOSE)
[Out] -cos(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x)\,dx = -\cos\left(x\right)$$

```
[In] integrate(sin(x),x, algorithm="fricas")
[Out] -cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \sin(x)\,dx = -\cos(x)$$

```
[In] integrate(sin(x),x)
[Out] -cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x)\,dx = -\cos(x)$$

```
[In] integrate(sin(x),x, algorithm="maxima")
[Out] -cos(x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x)\,dx = -\cos(x)$$

```
[In] integrate(sin(x),x, algorithm="giac")
[Out] -cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x)\,dx = -\cos(x)$$

```
[In] int(sin(x),x)
[Out] -cos(x)
```

3.6 $\int \cos(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 2

$$\int \cos(x)\,dx = \sin(x)$$

```
[Out] sin(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{2717\}$

$$\int \cos(x)\,dx = \sin(x)$$

```
[In] Int[Cos[x],x]
```

```
[Out] Sin[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \sin(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\, dx = \sin(x)$$

```
[In] Integrate[Cos[x],x]
[Out] Sin[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
lookup	$\sin(x)$	3
default	$\sin(x)$	3
meijerg	$\sin(x)$	3
risch	$\sin(x)$	3
parallelrisch	$\sin(x)$	3
norman	$\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^2}$	17

```
[In] int(cos(x),x,method=_RETURNVERBOSE)
[Out] sin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\, dx = \sin(x)$$

```
[In] integrate(cos(x),x, algorithm="fricas")
[Out] sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin(x)$$

```
[In] integrate(cos(x),x)
[Out] sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin(x)$$

```
[In] integrate(cos(x),x, algorithm="maxima")
[Out] sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin(x)$$

```
[In] integrate(cos(x),x, algorithm="giac")
[Out] sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin(x)$$

```
[In] int(cos(x),x)
[Out] sin(x)
```

3.7 $\int \csc^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 4

$$\int \csc^2(x)\,dx = -\cot(x)$$

```
[Out] -cot(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{3852, 8\}$

$$\int \csc^2(x)\,dx = -\cot(x)$$

```
[In] Int[Csc[x]^2,x]
```

```
[Out] -Cot[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int 1\,dx, x, \cot(x)\right)$$
$$= -\cot(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \csc^2(x)\,dx = -\cot(x)$$

```
[In] Integrate[Csc[x]^2,x]
[Out] -Cot[x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$-\cot\left(x\right)$	5
parallelrisch	$-\cot\left(x\right)$	5
risch	$-\frac{2i}{\mathrm{e}^{2ix}-1}$	13
norman	$\frac{-\frac{1}{2}+\frac{\tan\left(\frac{x}{2}\right)^{2}}{2}}{\tan\left(\frac{x}{2}\right)}$	18

```
[In] int(1/sin(x)^2,x,method=_RETURNVERBOSE)
[Out] -cot(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.00

$$\int \csc^2(x)\,dx = -\frac{\cos\left(x\right)}{\sin\left(x\right)}$$

```
[In] integrate(1/sin(x)^2,x, algorithm="fricas")
[Out] -cos(x)/sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$.

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.75

$$\int \csc^2(x)\,dx = -\frac{\cos(x)}{\sin(x)}$$

```
[In] integrate(1/sin(x)**2,x)
[Out] -cos(x)/sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \csc^2(x)\,dx = -\frac{1}{\tan(x)}$$

```
[In] integrate(1/sin(x)^2,x, algorithm="maxima")
[Out] -1/tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \csc^2(x)\,dx = -\frac{1}{\tan(x)}$$

```
[In] integrate(1/sin(x)^2,x, algorithm="giac")
[Out] -1/tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \csc^2(x)\, dx = -\cot(x)$$

```
[In] int(1/sin(x)^2,x)
[Out] -cot(x)
```

3.8 $\int \sec^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 2

$$\int \sec^2(x)\,dx = \tan(x)$$

```
[Out] tan(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3852, 8}

$$\int \sec^2(x)\,dx = \tan(x)$$

```
[In] Int[Sec[x]^2,x]
```

```
[Out] Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int 1\,dx, x, -\tan(x)\right)\\ &= \tan(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\,dx = \tan(x)$$

```
[In] Integrate[Sec[x]^2,x]
[Out] Tan[x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\tan(x)$	3
parallelrisch	$\tan(x)$	3
risch	$\frac{2i}{e^{2ix}+1}$	13
norman	$-\frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2-1}$	17

```
[In] int(1/cos(x)^2,x,method=_RETURNVERBOSE)
[Out] tan(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(2) = 4.

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 3.50

$$\int \sec^2(x)\,dx = \frac{\sin(x)}{\cos(x)}$$

```
[In] integrate(1/cos(x)^2,x, algorithm="fricas")
[Out] sin(x)/cos(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5 vs. $2(2) = 4$.

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 2.50

$$\int \sec^2(x)\,dx = \frac{\sin(x)}{\cos(x)}$$

```
[In] integrate(1/cos(x)**2,x)
[Out] sin(x)/cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\,dx = \tan(x)$$

```
[In] integrate(1/cos(x)^2,x, algorithm="maxima")
[Out] tan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\,dx = \tan(x)$$

```
[In] integrate(1/cos(x)^2,x, algorithm="giac")
[Out] tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\,dx = \tan(x)$$

```
[In] int(1/cos(x)^2,x)
[Out] tan(x)
```

3.9 $\int \sec(x)\tan(x)\,dx$

Optimal result

Integrand size = 5, antiderivative size = 2

$$\int \sec(x)\tan(x)\,dx = \sec(x)$$

```
[Out] sec(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2686, 8}

$$\int \sec(x)\tan(x)\,dx = \sec(x)$$

```
[In] Int[Sec[x]*Tan[x],x]
```

```
[Out] Sec[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int 1\,dx, x, \sec(x)\right) \\ &= \sec(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec(x)\tan(x)\,dx = \sec(x)$$

```
[In] Integrate[Sec[x]*Tan[x],x]
[Out] Sec[x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 5, normalized size of antiderivative = 2.50

method	result	size
derivativedivides	$\frac{1}{\cos(x)}$	5
default	$\frac{1}{\cos(x)}$	5
parallelrisch	$1+\sec(x)$	5
norman	$-\frac{2}{\tan\left(\frac{x}{2}\right)^2-1}$	13
risch	$\frac{2\,\mathrm{e}^{ix}}{\mathrm{e}^{2ix}+1}$	17

```
[In] int(sin(x)/cos(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/cos(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 2.00

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos(x)}$$

```
[In] integrate(sin(x)/cos(x)^2,x, algorithm="fricas")
[Out] 1/cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos(x)}$$

```
[In] integrate(sin(x)/cos(x)**2,x)
[Out] 1/cos(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 4, normalized size of antiderivative = 2.00

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos(x)}$$

```
[In] integrate(sin(x)/cos(x)^2,x, algorithm="maxima")
[Out] 1/cos(x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 2.00

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos(x)}$$

```
[In] integrate(sin(x)/cos(x)^2,x, algorithm="giac")
[Out] 1/cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 4, normalized size of antiderivative = 2.00

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos(x)}$$

```
[In] int(sin(x)/cos(x)^2,x)
[Out] 1/cos(x)
```

3.10 $\int \cot(x)\csc(x)\,dx$

Optimal result

Integrand size = 5, antiderivative size = 4

$$\int \cot(x)\csc(x)\,dx = -\csc(x)$$

```
[Out] -csc(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2686, 8}

$$\int \cot(x)\csc(x)\,dx = -\csc(x)$$

```
[In] Int[Cot[x]*Csc[x],x]
```

```
[Out] -Csc[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int 1\,dx, x, \csc(x)\right)$$
$$= -\csc(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cot(x)\csc(x)\,dx = -\csc(x)$$

```
[In] Integrate[Cot[x]*Csc[x],x]
[Out] -Csc[x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
parallelrisch	$-\csc\left(x\right)$	5
derivativedivides	$-\frac{1}{\sin(x)}$	7
default	$-\frac{1}{\sin(x)}$	7
norman	$\frac{-\frac{1}{2}-\frac{\tan\left(\frac{x}{2}\right)^{2}}{2}}{\tan\left(\frac{x}{2}\right)}$	18
risch	$-\frac{2i e^{ix}}{e^{2ix}-1}$	18

```
[In] int(1/sin(x)^2*cos(x),x,method=_RETURNVERBOSE)
[Out] -csc(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin\left(x\right)}$$

```
[In] integrate(cos(x)/sin(x)^2,x, algorithm="fricas")
[Out] -1/sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin\left(x\right)}$$

```
[In] integrate(cos(x)/sin(x)**2,x)
[Out] -1/sin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin\left(x\right)}$$

```
[In] integrate(cos(x)/sin(x)^2,x, algorithm="maxima")
[Out] -1/sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin\left(x\right)}$$

```
[In] integrate(cos(x)/sin(x)^2,x, algorithm="giac")
[Out] -1/sin(x)
```

Mupad [B] (verification not implemented)

Time = 17.40 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin(x)}$$

```
[In] int(cos(x)/sin(x)^2,x)
[Out] -1/sin(x)
```

3.11 $\int \tan(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 5

$$\int \tan(x)\,dx = -\log(\cos(x))$$

```
[Out] -ln(cos(x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556}

$$\int \tan(x)\,dx = -\log(\cos(x))$$

```
[In] Int[Tan[x],x]
```

```
[Out] -Log[Cos[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\log(\cos(x))$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \tan(x)\,dx = -\log(\cos(x))$$

```
[In] Integrate[Tan[x],x]
[Out] -Log[Cos[x]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
lookup	$-\ln\left(\cos\left(x\right)\right)$	6
default	$-\ln\left(\cos\left(x\right)\right)$	6
derivativedivides	$\frac{\ln\left(1+\tan(x)^{2}\right)}{2}$	10
norman	$\frac{\ln\left(1+\tan(x)^{2}\right)}{2}$	10
parallelrisch	$\frac{\ln\left(1+\tan(x)^{2}\right)}{2}$	10
risch	$ix-\ln\left(\mathrm{e}^{2ix}+1\right)$	16

```
[In] int(tan(x),x,method=_RETURNVERBOSE)
[Out] -ln(cos(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20

$$\int \tan(x)\,dx = -\frac{1}{2}\log\left(\frac{1}{\tan\left(x\right)^{2}+1}\right)$$

```
[In] integrate(tan(x),x, algorithm="fricas")
[Out] -1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \tan(x)\, dx = -\log\left(\cos\left(x\right)\right)$$

```
[In] integrate(tan(x),x)
[Out] -log(cos(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int \tan(x)\, dx = \log\left(\sec\left(x\right)\right)$$

```
[In] integrate(tan(x),x, algorithm="maxima")
[Out] log(sec(x))
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

$$\int \tan(x)\, dx = -\log\left(|\cos\left(x\right)|\right)$$

```
[In] integrate(tan(x),x, algorithm="giac")
[Out] -log(abs(cos(x)))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \tan(x)\, dx = -\ln\left(\cos\left(x\right)\right)$$

```
[In] int(tan(x),x)
[Out] -log(cos(x))
```

3.12 $\int \cot(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \cot(x)\,dx = \log(\sin(x))$$

```
[Out] ln(sin(x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556}

$$\int \cot(x)\,dx = \log(\sin(x))$$

```
[In] Int[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \log(\sin(x))$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$.

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \cot(x)\,dx = \log(\cos(x)) + \log(\tan(x))$$

```
[In] Integrate[Cot[x],x]
[Out] Log[Cos[x]] + Log[Tan[x]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\ln(\sin(x))$	4
default	$\ln(\sin(x))$	4
derivativedivides	$-\frac{\ln(1+\cot(x)^2)}{2}$	10
parallelrisch	$\ln\left(\frac{1}{\sqrt{\sec(x)^2}}\right) + \ln(\tan(x))$	12
norman	$-\frac{\ln(1+\tan(x)^2)}{2} + \ln(\tan(x))$	14
risch	$-ix + \ln(e^{2ix} - 1)$	14

```
[In] int(cot(x),x,method=_RETURNVERBOSE)
[Out] ln(sin(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \cot(x)\,dx = \frac{1}{2}\log\left(-\frac{1}{2}\cos(2x) + \frac{1}{2}\right)$$

```
[In] integrate(cot(x),x, algorithm="fricas")
[Out] 1/2*log(-1/2*cos(2*x) + 1/2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x)\,dx = \log(\sin(x))$$

```
[In] integrate(cot(x),x)
[Out] log(sin(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x)\,dx = \log(\sin(x))$$

```
[In] integrate(cot(x),x, algorithm="maxima")
[Out] log(sin(x))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int \cot(x)\,dx = \log(|\sin(x)|)$$

```
[In] integrate(cot(x),x, algorithm="giac")
[Out] log(abs(sin(x)))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x)\,dx = \ln(\sin(x))$$

```
[In] int(cot(x),x)
[Out] log(sin(x))
```

3.13 $\int \csc(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 7

$$\int \csc(x)\,dx = \log\left(\tan\left(\frac{x}{2}\right)\right)$$

```
[Out] ln(tan(1/2*x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{3855\}$

$$\int \csc(x)\,dx = -\operatorname{arctanh}(\cos(x))$$

```
[In] Int[Csc[x],x]
```

```
[Out] -ArcTanh[Cos[x]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\operatorname{arctanh}(\cos(x))$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$.

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.43

$$\int \csc(x)\,dx = -\log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Csc[x],x]
[Out] -Log[Cos[x/2]] + Log[Sin[x/2]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

method	result	size
norman	$\ln\left(\tan\left(\frac{x}{2}\right)\right)$	6
parallelrisch	$\ln\left(\tan\left(\frac{x}{2}\right)\right)$	6
default	$\ln\left(-\cot\left(x\right)+\csc\left(x\right)\right)$	9
risch	$\ln\left(-1+e^{ix}\right)-\ln\left(e^{ix}+1\right)$	20

```
[In] int(1/sin(x),x,method=_RETURNVERBOSE)
[Out] ln(tan(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(5) = 10$.

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.71

$$\int \csc(x)\,dx = -\frac{1}{2}\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{2}\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)$$

```
[In] integrate(1/sin(x),x, algorithm="fricas")
[Out] -1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.14

$$\int \csc(x)\,dx = \frac{\log\left(\cos\left(x\right)-1\right)}{2} - \frac{\log\left(\cos\left(x\right)+1\right)}{2}$$

```
[In] integrate(1/sin(x),x)
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.14

$$\int \csc(x)\,dx = -\frac{1}{2}\log\left(\cos\left(x\right)+1\right) + \frac{1}{2}\log\left(\cos\left(x\right)-1\right)$$

```
[In] integrate(1/sin(x),x, algorithm="maxima")
[Out] -1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(5) = 10$.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.43

$$\int \csc(x)\,dx = -\frac{1}{2}\log\left(\cos\left(x\right)+1\right) + \frac{1}{2}\log\left(-\cos\left(x\right)+1\right)$$

```
[In] integrate(1/sin(x),x, algorithm="giac")
[Out] -1/2*log(cos(x) + 1) + 1/2*log(-cos(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \csc(x)\,dx = \ln\left(\tan\left(\frac{x}{2}\right)\right)$$

```
[In] int(1/sin(x),x)
[Out] log(tan(x/2))
```

3.14 $\int \sec(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 6

$$\int \sec(x)\,dx = \log(\sec(x) + \tan(x))$$

```
[Out] ln(sec(x)+tan(x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{3855\}$

$$\int \sec(x)\,dx = \operatorname{arctanh}(\sin(x))$$

```
[In] Int[Sec[x],x]
```

```
[Out] ArcTanh[Sin[x]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \operatorname{arctanh}(\sin(x))$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \sec(x)\,dx = \operatorname{arctanh}(\sin(x))$$

```
[In] Integrate[Sec[x],x]
[Out] ArcTanh[Sin[x]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)$	7
norman	$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	18
parallelrisch	$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	18
risch	$\ln\left(i+{\mathrm e}^{i x}\right)-\ln\left({\mathrm e}^{i x}-i\right)$	22

```
[In] int(1/cos(x),x,method=_RETURNVERBOSE)
[Out] ln(sec(x)+tan(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \sec(x)\,dx = \frac{1}{2}\,\log\left(\sin\left(x\right) + 1\right) - \frac{1}{2}\,\log\left(-\sin\left(x\right) + 1\right)$$

```
[In] integrate(1/cos(x),x, algorithm="fricas")
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \sec(x)\,dx = -\frac{\log(\sin(x)-1)}{2} + \frac{\log(\sin(x)+1)}{2}$$

```
[In] integrate(1/cos(x),x)
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(6) = 12$.

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \sec(x)\,dx = \frac{1}{2}\log(\sin(x)+1) - \frac{1}{2}\log(\sin(x)-1)$$

```
[In] integrate(1/cos(x),x, algorithm="maxima")
[Out] 1/2*log(sin(x) + 1) - 1/2*log(sin(x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \sec(x)\,dx = \frac{1}{2}\log(\sin(x)+1) - \frac{1}{2}\log(-\sin(x)+1)$$

```
[In] integrate(1/cos(x),x, algorithm="giac")
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.83

$$\int \sec(x)\,dx = \ln\left(\frac{1}{\cos(x)}\right) + \ln(\sin(x) + 1)$$

```
[In] int(1/cos(x),x)
[Out] log(1/cos(x)) + log(sin(x) + 1)
```

3.15 $\int \frac{1}{1+x^2} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 2

$$\int \frac{1}{1+x^2} \, dx = \arctan(x)$$

```
[Out] arctan(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{209\}$

$$\int \frac{1}{1+x^2} \, dx = \arctan(x)$$

```
[In] Int[(1 + x^2)^(-1),x]
```

```
[Out] ArcTan[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\text{integral} = \arctan(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x^2}\,dx = \arctan(x)$$

```
[In] Integrate[(1 + x^2)^(-1),x]
[Out] ArcTan[x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\arctan\left(x\right)$	3
meijerg	$\arctan\left(x\right)$	3
risch	$\arctan\left(x\right)$	3
parallelrisch	$\frac{i\ln(i+x)}{2}-\frac{i\ln(x-i)}{2}$	18

```
[In] int(1/(x^2+1),x,method=_RETURNVERBOSE)
[Out] arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x^2}\,dx = \arctan\left(x\right)$$

```
[In] integrate(1/(x^2+1),x, algorithm="fricas")
[Out] arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x^2} \, dx = \operatorname{atan}(x)$$

```
[In] integrate(1/(x**2+1),x)
[Out] atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x^2} \, dx = \arctan(x)$$

```
[In] integrate(1/(x^2+1),x, algorithm="maxima")
[Out] arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x^2} \, dx = \arctan(x)$$

```
[In] integrate(1/(x^2+1),x, algorithm="giac")
[Out] arctan(x)
```

Mupad [B] (verification not implemented)

Time = 16.62 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x^2} \, dx = \operatorname{atan}(x)$$

```
[In] int(1/(x^2 + 1),x)
[Out] atan(x)
```

3.16 $\int \frac{1}{1-x^2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 2

$$\int \frac{1}{1-x^2} \, dx = \text{arctanh}(x)$$

```
[Out] arctanh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{212\}$

$$\int \frac{1}{1-x^2} \, dx = \text{arctanh}(x)$$

```
[In] Int[(1 - x^2)^(-1),x]
```

```
[Out] ArcTanh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\text{integral} = \text{arctanh}(x)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 19 vs. $2(2) = 4$.

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 9.50

$$\int \frac{1}{1-x^2}\,dx = -\frac{1}{2}\log(1-x) + \frac{1}{2}\log(1+x)$$

```
[In] Integrate[(1 - x^2)^(-1),x]
[Out] -1/2*Log[1 - x] + Log[1 + x]/2
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\operatorname{arctanh}(x)$	3
meijerg	$\operatorname{arctanh}(x)$	3
norman	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14
risch	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14
parallelrisch	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14

```
[In] int(1/(-x^2+1),x,method=_RETURNVERBOSE)
[Out] arctanh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(2) = 4$.

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{1}{1-x^2}\,dx = \frac{1}{2}\log\left(x+1\right) - \frac{1}{2}\log\left(x-1\right)$$

```
[In] integrate(1/(-x^2+1),x, algorithm="fricas")
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(2) = 4$.

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 6.00

$$\int \frac{1}{1-x^2}\,dx = -\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

```
[In] integrate(1/(-x**2+1),x)
[Out] -log(x - 1)/2 + log(x + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(2) = 4$.

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{1}{1-x^2}\,dx = \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

```
[In] integrate(1/(-x^2+1),x, algorithm="maxima")
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(2) = 4$.

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 7.50

$$\int \frac{1}{1-x^2}\,dx = \frac{1}{2}\log(|x+1|) - \frac{1}{2}\log(|x-1|)$$

```
[In] integrate(1/(-x^2+1),x, algorithm="giac")
[Out] 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-x^2}\,dx = \operatorname{atanh}(x)$$

```
[In] int(-1/(x^2 - 1),x)
[Out] atanh(x)
```

3.17 $\int \frac{1}{\sqrt{1-x^2}} dx$

Optimal result

Integrand size = 11, antiderivative size = 2

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin(x)$$

```
[Out] arcsin(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{222\}$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin(x)$$

```
[In] Int[1/Sqrt[1 - x^2],x]
```

```
[Out] ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\text{integral} = \arcsin(x)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 20 vs. $2(2) = 4$.

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 10.00

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

```
[In] Integrate[1/Sqrt[1 - x^2],x]
[Out] -2*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\arcsin(x)$	3
meijerg	$\arcsin(x)$	3
pseudoelliptic	$-\arctan\left(\frac{\sqrt{-x^2+1}}{x}\right)$	17
trager	$\operatorname{RootOf}(\_Z^2+1)\ln\left(\operatorname{RootOf}(\_Z^2+1)\sqrt{-x^2+1}+x\right)$	27

```
[In] int(1/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(2) = 4$.

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 9.00

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \operatorname{asin}(x)$$

```
[In] integrate(1/(-x**2+1)**(1/2),x)
[Out] asin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \arcsin(x)$$

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(2) = 4.

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 8.50

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \frac{1}{2}\sqrt{-x^2+1}x + \frac{1}{2}\arcsin(x)$$

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \operatorname{asin}(x)$$

```
[In] int(1/(1 - x^2)^(1/2),x)
[Out] asin(x)
```

3.18 $\int \frac{1}{\sqrt{1+x^2}} dx$

Optimal result

Integrand size = 9, antiderivative size = 2

$$\int \frac{1}{\sqrt{1+x^2}} dx = \operatorname{arcsinh}(x)$$

```
[Out] arcsinh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{221\}$

$$\int \frac{1}{\sqrt{1+x^2}} dx = \operatorname{arcsinh}(x)$$

```
[In] Int[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcSinh[x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\text{integral} = \operatorname{arcsinh}(x)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16 vs. $2(2) = 4$.

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 8.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = -\log\left(-x+\sqrt{1+x^2}\right)$$

```
[In] Integrate[1/Sqrt[1 + x^2],x]
[Out] -Log[-x + Sqrt[1 + x^2]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\operatorname{arcsinh}(x)$	3
meijerg	$\operatorname{arcsinh}(x)$	3
trager	$\ln\left(x+\sqrt{x^2+1}\right)$	11
pseudoelliptic	$\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}}{x}\right)$	13

```
[In] int(1/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsinh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(2) = 4$.

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 7.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = -\log\left(-x+\sqrt{x^2+1}\right)$$

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{asinh}(x)$$

```
[In] integrate(1/(x**2+1)**(1/2),x)
[Out] asinh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{arsinh}(x)$$

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] arcsinh(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(2) = 4.

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 12.50

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \frac{1}{2}\sqrt{x^2+1}x - \frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{asinh}(x)$$

```
[In] int(1/(x^2 + 1)^(1/2),x)
[Out] asinh(x)
```

3.19 $\int \frac{1}{\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{1}{\sqrt{-1+x^2}}\,dx = \log\left(x+\sqrt{-1+x^2}\right)$$

```
[Out] ln(x+(x^2-1)^(1/2))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {223, 212}

$$\int \frac{1}{\sqrt{-1+x^2}}\,dx = \operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

```
[In] Int[1/Sqrt[-1 + x^2],x]
```

```
[Out] ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)\\ &= \operatorname{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 38 vs. $2(12) = 24$.

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.17

$$\int \frac{1}{\sqrt{-1+x^2}}\,dx = -\frac{1}{2}\log\left(1-\frac{x}{\sqrt{-1+x^2}}\right)+\frac{1}{2}\log\left(1+\frac{x}{\sqrt{-1+x^2}}\right)$$

```
[In] Integrate[1/Sqrt[-1 + x^2],x]
[Out] -1/2*Log[1 - x/Sqrt[-1 + x^2]] + Log[1 + x/Sqrt[-1 + x^2]]/2
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$\ln\left(x+\sqrt{x^2-1}\right)$	11
trager	$\ln\left(x+\sqrt{x^2-1}\right)$	11
pseudoelliptic	$\operatorname{arctanh}\left(\frac{\sqrt{x^2-1}}{x}\right)$	13
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^2-1)}\,\arcsin(x)}{\sqrt{\operatorname{signum}(x^2-1)}}$	22

```
[In] int(1/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(x+(x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{-1+x^2}}\,dx = -\log\left(-x+\sqrt{x^2-1}\right)$$

```
[In] integrate(1/(x^2-1)^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 - 1))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-1+x^2}}\,dx = \log\left(x+\sqrt{x^2-1}\right)$$

```
[In] integrate(1/(x**2-1)**(1/2),x)
[Out] log(x + sqrt(x**2 - 1))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{-1+x^2}}\,dx = \log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

```
[In] integrate(1/(x^2-1)^(1/2),x, algorithm="maxima")
[Out] log(2*x + 2*sqrt(x^2 - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{-1+x^2}}\,dx = \frac{1}{2}\,\sqrt{x^2-1}x+\frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

```
[In] integrate(1/(x^2-1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^2 - 1)*x + 1/2*log(abs(-x + sqrt(x^2 - 1)))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-1+x^2}}\,dx = \ln\left(x+\sqrt{x^2-1}\right)$$

```
[In] int(1/(x^2 - 1)^(1/2),x)
[Out] log(x + (x^2 - 1)^(1/2))
```

3.20 $\int \sinh(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 2

$$\int \sinh(x)\,dx = \cosh(x)$$

```
[Out] cosh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{2718\}$

$$\int \sinh(x)\,dx = \cosh(x)$$

```
[In] Int[Sinh[x],x]
```

```
[Out] Cosh[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\text{integral} = \cosh(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh(x)$$

```
[In] Integrate[Sinh[x],x]
[Out] Cosh[x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
lookup	$\cosh\left(x\right)$	3
default	$\cosh\left(x\right)$	3
risch	$\frac{{e}^{x}}{2}+\frac{{e}^{-x}}{2}$	12
parallelrisch	$-\frac{2}{\tanh\left(\frac{x}{2}\right)^{2}-1}$	13
meijerg	$-\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh\left(x\right)}{\sqrt{\pi}}\right)$	17

```
[In] int(sinh(x),x,method=_RETURNVERBOSE)
[Out] cosh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh\left(x\right)$$

```
[In] integrate(sinh(x),x, algorithm="fricas")
[Out] cosh(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh(x)$$

```
[In] integrate(sinh(x),x)
[Out] cosh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh(x)$$

```
[In] integrate(sinh(x),x, algorithm="maxima")
[Out] cosh(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(2) = 4.

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 5.50

$$\int \sinh(x)\,dx = \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(sinh(x),x, algorithm="giac")
[Out] 1/2*e^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh(x)$$

```
[In] int(sinh(x),x)
[Out] cosh(x)
```

3.21 $\int \cosh(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 2

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[Out] sinh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{2717\}$

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[In] Int[Cosh[x],x]
```

```
[Out] Sinh[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \sinh(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\, dx = \sinh(x)$$

```
[In] Integrate[Cosh[x],x]
[Out] Sinh[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
lookup	$\sinh\left(x\right)$	3
default	$\sinh\left(x\right)$	3
meijerg	$\sinh\left(x\right)$	3
parallelrisch	$\sinh\left(x\right)$	3
risch	$\frac{e^{x}}{2}-\frac{e^{-x}}{2}$	12

```
[In] int(cosh(x),x,method=_RETURNVERBOSE)
[Out] sinh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\, dx = \sinh\left(x\right)$$

```
[In] integrate(cosh(x),x, algorithm="fricas")
[Out] sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[In] integrate(cosh(x),x)
[Out] sinh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[In] integrate(cosh(x),x, algorithm="maxima")
[Out] sinh(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(2) = 4.

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 5.50

$$\int \cosh(x)\,dx = -\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(cosh(x),x, algorithm="giac")
[Out] -1/2*e^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

```
[In] int(cosh(x),x)
[Out] sinh(x)
```

3.22 $\int \operatorname{csch}^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 4

$$\int \operatorname{csch}^2(x)\,dx = -\coth(x)$$

```
[Out] -coth(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3852, 8}

$$\int \operatorname{csch}^2(x)\,dx = -\coth(x)$$

```
[In] Int[Csch[x]^2,x]
```

```
[Out] -Coth[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\left(i\text{Subst}\left(\int 1\,dx, x, -i\coth(x)\right)\right) \\ &= -\coth(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^2(x)\,dx = -\coth(x)$$

```
[In] Integrate[Csch[x]^2,x]
[Out] -Coth[x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$-\coth\left(x\right)$	5
parallelrisch	$-\coth\left(x\right)$	5
risch	$-\frac{2}{\mathrm{e}^{2x}-1}$	11

```
[In] int(1/sinh(x)^2,x,method=_RETURNVERBOSE)
[Out] -coth(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(4) = 8$.

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 5.00

$$\int \operatorname{csch}^2(x)\,dx = -\frac{2}{\cosh\left(x\right)^2 + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^2 - 1}$$

```
[In] integrate(1/sinh(x)^2,x, algorithm="fricas")
[Out] -2/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \operatorname{csch}^2(x)\,dx = -\frac{\tanh\left(\frac{x}{2}\right)}{2} - \frac{1}{2\tanh\left(\frac{x}{2}\right)}$$

```
[In] integrate(1/sinh(x)**2,x)
[Out] -tanh(x/2)/2 - 1/(2*tanh(x/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(4) = 8$.

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.50

$$\int \operatorname{csch}^2(x)\,dx = \frac{2}{e^{(-2\,x)} - 1}$$

```
[In] integrate(1/sinh(x)^2,x, algorithm="maxima")
[Out] 2/(e^(-2*x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(4) = 8$.

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.50

$$\int \operatorname{csch}^2(x)\,dx = -\frac{2}{e^{(2\,x)} - 1}$$

```
[In] integrate(1/sinh(x)^2,x, algorithm="giac")
[Out] -2/(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^2(x)\,dx = -\coth(x)$$

```
[In] int(1/sinh(x)^2,x)
[Out] -coth(x)
```

3.23 $\int \text{sech}^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 2

$$\int \text{sech}^2(x)\,dx = \tanh(x)$$

```
[Out] tanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3852, 8}

$$\int \text{sech}^2(x)\,dx = \tanh(x)$$

```
[In] Int[Sech[x]^2,x]
```

```
[Out] Tanh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\text{integral} = i\text{Subst}\left(\int 1\,dx, x, -i\tanh(x)\right)$$
$$= \tanh(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(x)\,dx = \tanh(x)$$

```
[In] Integrate[Sech[x]^2,x]
[Out] Tanh[x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\tanh(x)$	3
parallelrisch	$\tanh(x)$	3
risch	$-\frac{2}{e^{2x}+1}$	11

```
[In] int(1/cosh(x)^2,x,method=_RETURNVERBOSE)
[Out] tanh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(2) = 4$.

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 10.00

$$\int \operatorname{sech}^2(x)\,dx = -\frac{2}{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1}$$

```
[In] integrate(1/cosh(x)^2,x, algorithm="fricas")
[Out] -2/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(2) = 4$.

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 7.00

$$\int \operatorname{sech}^2(x)\,dx = \frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}$$

```
[In] integrate(1/cosh(x)**2,x)
[Out] 2*tanh(x/2)/(tanh(x/2)**2 + 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(2) = 4$.

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 5.00

$$\int \operatorname{sech}^2(x)\,dx = \frac{2}{e^{(-2\,x)}+1}$$

```
[In] integrate(1/cosh(x)^2,x, algorithm="maxima")
[Out] 2/(e^(-2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(2) = 4$.

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 5.00

$$\int \operatorname{sech}^2(x)\,dx = -\frac{2}{e^{(2\,x)}+1}$$

```
[In] integrate(1/cosh(x)^2,x, algorithm="giac")
[Out] -2/(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(x)\,dx = \tanh(x)$$

```
[In] int(1/cosh(x)^2,x)
[Out] tanh(x)
```

3.24 $\int \tanh(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \tanh(x)\,dx = \log(\cosh(x))$$

```
[Out] ln(cosh(x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556}

$$\int \tanh(x)\,dx = \log(\cosh(x))$$

```
[In] Int[Tanh[x],x]
```

```
[Out] Log[Cosh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \log(\cosh(x))$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x)\,dx = \log(\cosh(x))$$

```
[In] Integrate[Tanh[x],x]
[Out] Log[Cosh[x]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\ln(\cosh(x))$	4
derivativedivides	$\ln(\cosh(x))$	4
default	$\ln(\cosh(x))$	4
risch	$-x+\ln(e^{2x}+1)$	12
parallelrisch	$-\ln(1-\tanh(x))-x$	14

```
[In] int(tanh(x),x,method=_RETURNVERBOSE)
[Out] ln(cosh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(3) = 6.

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 6.00

$$\int \tanh(x)\,dx = -x + \log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

```
[In] integrate(tanh(x),x, algorithm="fricas")
[Out] -x + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$.

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \tanh(x)\,dx = x - \log\left(\tanh\left(x\right) + 1\right)$$

```
[In] integrate(tanh(x),x)
[Out] x - log(tanh(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x)\,dx = \log\left(\cosh\left(x\right)\right)$$

```
[In] integrate(tanh(x),x, algorithm="maxima")
[Out] log(cosh(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \tanh(x)\,dx = -x + \log\left(e^{(2\,x)} + 1\right)$$

```
[In] integrate(tanh(x),x, algorithm="giac")
[Out] -x + log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x)\,dx = \ln\left(\cosh(x)\right)$$

```
[In] int(tanh(x),x)
[Out] log(cosh(x))
```

3.25 $\int \coth(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \coth(x)\,dx = \log(\sinh(x))$$

```
[Out] ln(sinh(x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{3556\}$

$$\int \coth(x)\,dx = \log(\sinh(x))$$

```
[In] Int[Coth[x],x]
```

```
[Out] Log[Sinh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \log(\sinh(x))$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$.

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \coth(x)\,dx = \log(\cosh(x)) + \log(\tanh(x))$$

```
[In] Integrate[Coth[x],x]
[Out] Log[Cosh[x]] + Log[Tanh[x]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\ln(\sinh(x))$	4
derivativedivides	$\ln(\sinh(x))$	4
default	$\ln(\sinh(x))$	4
risch	$-x+\ln(e^{2x}-1)$	12
parallelrisch	$\ln(\tanh(x))-\ln(1-\tanh(x))-x$	17

```
[In] int(coth(x),x,method=_RETURNVERBOSE)
[Out] ln(sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(3) = 6$.

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 6.00

$$\int \coth(x)\,dx = -x + \log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

```
[In] integrate(coth(x),x, algorithm="fricas")
[Out] -x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(3) = 6$.

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

$$\int \coth(x)\,dx = x - \log\left(\tanh\left(x\right) + 1\right) + \log\left(\tanh\left(x\right)\right)$$

```
[In] integrate(coth(x),x)
[Out] x - log(tanh(x) + 1) + log(tanh(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \coth(x)\,dx = \log\left(\sinh\left(x\right)\right)$$

```
[In] integrate(coth(x),x, algorithm="maxima")
[Out] log(sinh(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(3) = 6$.

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

$$\int \coth(x)\,dx = -x + \log\left(\left|e^{(2\,x)} - 1\right|\right)$$

```
[In] integrate(coth(x),x, algorithm="giac")
[Out] -x + log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \coth(x)\,dx = \ln\left(\sinh(x)\right)$$

```
[In] int(coth(x),x)
[Out] log(sinh(x))
```

3.26 $\int \operatorname{csch}(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 7

$$\int \operatorname{csch}(x)\,dx = \log\left(\tanh\left(\frac{x}{2}\right)\right)$$

```
[Out] ln(tanh(1/2*x))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3855}

$$\int \operatorname{csch}(x)\,dx = -\operatorname{arctanh}(\cosh(x))$$

```
[In] Int[Csch[x],x]
```

```
[Out] -ArcTanh[Cosh[x]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\operatorname{arctanh}(\cosh(x))$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$.

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.43

$$\int \operatorname{csch}(x)\,dx = -\log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Csch[x],x]
[Out] -Log[Cosh[x/2]] + Log[Sinh[x/2]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

method	result	size
default	$-2 \operatorname{arctanh}(e^{x})$	6
parallelrisch	$\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	6
risch	$\ln(e^{x}-1)-\ln(e^{x}+1)$	14

```
[In] int(1/sinh(x),x,method=_RETURNVERBOSE)
[Out] -2*arctanh(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(5) = 10$.

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.43

$$\int \operatorname{csch}(x)\,dx = -\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) + \log\left(\cosh\left(x\right) + \sinh\left(x\right) - 1\right)$$

```
[In] integrate(1/sinh(x),x, algorithm="fricas")
[Out] -log(cosh(x) + sinh(x) + 1) + log(cosh(x) + sinh(x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \operatorname{csch}(x)\,dx = \log\left(\tanh\left(\frac{x}{2}\right)\right)$$

```
[In] integrate(1/sinh(x),x)
[Out] log(tanh(x/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(5) = 10$.

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.43

$$\int \operatorname{csch}(x)\,dx = -\log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(1/sinh(x),x, algorithm="maxima")
[Out] -log(e^(-x) + 1) + log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(5) = 10$.

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.00

$$\int \operatorname{csch}(x)\,dx = -\log\left(e^{x} + 1\right) + \log\left(|e^{x} - 1|\right)$$

```
[In] integrate(1/sinh(x),x, algorithm="giac")
[Out] -log(e^x + 1) + log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \operatorname{csch}(x)\,dx = \ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

```
[In] int(1/sinh(x),x)
[Out] log(tanh(x/2))
```

3.27 $\int (a+bx)^m \, dx$

Optimal result

Integrand size = 7, antiderivative size = 18

$$\int (a+bx)^m \, dx = \frac{(a+bx)^{1+m}}{b(1+m)}$$

```
[Out] (b*x+a)^(1+m)/b/(1+m)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\int (a+bx)^m \, dx = \frac{(a+bx)^{m+1}}{b(m+1)}$$

```
[In] Int[(a + b*x)^m,x]
```

```
[Out] (a + b*x)^(1 + m)/(b*(1 + m))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(a+bx)^{1+m}}{b(1+m)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^m\,dx=\frac{(a+bx)^{1+m}}{b(1+m)}$$

```
[In] Integrate[(a + b*x)^m,x]
[Out] (a + b*x)^(1 + m)/(b*(1 + m))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{(bx+a)^{1+m}}{b(1+m)}$	19
default	$\frac{(bx+a)^{1+m}}{b(1+m)}$	19
risch	$\frac{(bx+a)(bx+a)^{m}}{b(1+m)}$	22
parallelrisch	$\frac{x(bx+a)^{m}ab+(bx+a)^{m}a^{2}}{(1+m)ab}$	36
norman	$\frac{x\,\mathrm{e}^{m\ln(bx+a)}}{1+m}+\frac{a\,\mathrm{e}^{m\ln(bx+a)}}{b(1+m)}$	37

```
[In] int((b*x+a)^m,x,method=_RETURNVERBOSE)
[Out] (b*x+a)^(1+m)/b/(1+m)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (a+bx)^m\,dx=\frac{(bx+a)(bx+a)^m}{bm+b}$$

```
[In] integrate((b*x+a)^m,x, algorithm="fricas")
[Out] (b*x + a)*(b*x + a)^m/(b*m + b)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (a+bx)^m \, dx = \frac{\begin{cases} \frac{(a+bx)^{m+1}}{m+1} & \text{for } m \neq -1 \\ \log{(a+bx)} & \text{otherwise} \end{cases}}{b}$$

```
[In] integrate((b*x+a)**m,x)
[Out] Piecewise(((a + b*x)**(m + 1)/(m + 1), Ne(m, -1)), (log(a + b*x), True))/b
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^m \, dx = \frac{(bx+a)^{m+1}}{b(m+1)}$$

```
[In] integrate((b*x+a)^m,x, algorithm="maxima")
[Out] (b*x + a)^(m + 1)/(b*(m + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^m \, dx = \frac{(bx+a)^{m+1}}{b(m+1)}$$

```
[In] integrate((b*x+a)^m,x, algorithm="giac")
[Out] (b*x + a)^(m + 1)/(b*(m + 1))
```

Mupad [B] (verification not implemented)

Time = 17.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^m\,dx=\frac{(a+b\,x)^{m+1}}{b\,(m+1)}$$

```
[In] int((a + b*x)^m,x)
[Out] (a + b*x)^(m + 1)/(b*(m + 1))
```

3.28 $\int \frac{1}{a+bx} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{a+bx} \, dx = \frac{\log(a+bx)}{b}$$

```
[Out] 1/b*ln(b*x+a)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{31\}$

$$\int \frac{1}{a+bx} \, dx = \frac{\log(a+bx)}{b}$$

```
[In] Int[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\text{integral} = \frac{\log(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx}\,dx=\frac{\log(a+bx)}{b}$$

```
[In] Integrate[(a + b*x)^(-1),x]
[Out] Log[a + b*x]/b
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\ln(bx+a)}{b}$	11
norman	$\frac{\ln(bx+a)}{b}$	11
risch	$\frac{\ln(bx+a)}{b}$	11
parallelrisch	$\frac{\ln(bx+a)}{b}$	11

```
[In] int(1/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*ln(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx}\,dx=\frac{\log\left(bx+a\right)}{b}$$

```
[In] integrate(1/(b*x+a),x, algorithm="fricas")
[Out] log(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{a+bx}\,dx = \frac{\log{(a+bx)}}{b}$$

```
[In] integrate(1/(b*x+a),x)
[Out] log(a + b*x)/b
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx}\,dx = \frac{\log{(bx+a)}}{b}$$

```
[In] integrate(1/(b*x+a),x, algorithm="maxima")
[Out] log(b*x + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+bx}\,dx = \frac{\log{(|bx+a|)}}{b}$$

```
[In] integrate(1/(b*x+a),x, algorithm="giac")
[Out] log(abs(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx}\,dx = \frac{\ln{(a+b\,x)}}{b}$$

```
[In] int(1/(a + b*x),x)
[Out] log(a + b*x)/b
```

3.29 $\int \frac{x}{a+bx} dx$

Optimal result

Integrand size = 9, antiderivative size = 18

$$\int \frac{x}{a+bx} dx = \frac{x}{b} - \frac{a\log(a+bx)}{b^2}$$

```
[Out] x/b-a/b^2*ln(b*x+a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\int \frac{x}{a+bx} dx = \frac{x}{b} - \frac{a\log(a+bx)}{b^2}$$

```
[In] Int[x/(a + b*x),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right) dx \\ &= \frac{x}{b} - \frac{a\log(a+bx)}{b^2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+bx}\,dx = \frac{x}{b} - \frac{a\log(a+bx)}{b^2}$$

```
[In] Integrate[x/(a + b*x),x]
[Out] x/b - (a*Log[a + b*x])/b^2
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
norman	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
risch	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
parallelrisch	$-\frac{a\ln(bx+a)-bx}{b^2}$	19

```
[In] int(x/(b*x+a),x,method=_RETURNVERBOSE)
[Out] x/b-a/b^2*ln(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x}{a+bx}\,dx = \frac{bx - a\log(bx+a)}{b^2}$$

```
[In] integrate(x/(b*x+a),x, algorithm="fricas")
[Out] (b*x - a*log(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{a+bx} \, dx = -\frac{a \log (a+bx)}{b^2} + \frac{x}{b}$$

```
[In] integrate(x/(b*x+a),x)
[Out] -a*log(a + b*x)/b**2 + x/b
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+bx} \, dx = \frac{x}{b} - \frac{a \log (bx+a)}{b^2}$$

```
[In] integrate(x/(b*x+a),x, algorithm="maxima")
[Out] x/b - a*log(b*x + a)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{x}{a+bx} \, dx = \frac{x}{b} - \frac{a \log (|bx+a|)}{b^2}$$

```
[In] integrate(x/(b*x+a),x, algorithm="giac")
[Out] x/b - a*log(abs(b*x + a))/b^2
```

Mupad [B] (verification not implemented)

Time = 17.61 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+bx} \, dx = -\frac{a \ln (a+b\,x) - b\,x}{b^2}$$

```
[In] int(x/(a + b*x),x)
[Out] -(a*log(a + b*x) - b*x)/b^2
```

3.30 $\int \frac{x^2}{a+bx} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{x^2}{a+bx} \, dx = -\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \log(a+bx)}{b^2}$$

```
[Out] 1/2*x^2/b-a*x/b^2+a^2/b^2*ln(b*x+a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{45\}$

$$\int \frac{x^2}{a+bx} \, dx = \frac{a^2 \log(a+bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

```
[In] Int[x^2/(a + b*x),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-\frac{a}{b^2} + \frac{x}{b} + \frac{a^2}{b^2(a+bx)}\right) \, dx \\ &= -\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \log(a+bx)}{b^3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+bx}\,dx = -\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2\log(a+bx)}{b^3}$$

```
[In] Integrate[x^2/(a + b*x),x]
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{-\frac{1}{2}x^2b+ax}{b^2}+\frac{a^2\ln(bx+a)}{b^3}$	30
norman	$\frac{x^2}{2b}-\frac{ax}{b^2}+\frac{a^2\ln(bx+a)}{b^3}$	30
risch	$\frac{x^2}{2b}-\frac{ax}{b^2}+\frac{a^2\ln(bx+a)}{b^3}$	30
parallelrisch	$\frac{b^2x^2+2a^2\ln(bx+a)-2bax}{2b^3}$	30

```
[In] int(x^2/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/b^2*(-1/2*x^2*b+a*x)+a^2/b^3*ln(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a+bx}\,dx = \frac{b^2x^2 - 2\,abx + 2\,a^2\log(bx+a)}{2\,b^3}$$

```
[In] integrate(x^2/(b*x+a),x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{a+bx}\,dx = \frac{a^2\log{(a+bx)}}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

```
[In] integrate(x**2/(b*x+a),x)
[Out] a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a+bx}\,dx = \frac{a^2\log{(bx+a)}}{b^3} + \frac{bx^2-2\,ax}{2\,b^2}$$

```
[In] integrate(x^2/(b*x+a),x, algorithm="maxima")
[Out] a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{a+bx}\,dx = \frac{a^2\log{(|bx+a|)}}{b^3} + \frac{bx^2-2\,ax}{2\,b^2}$$

```
[In] integrate(x^2/(b*x+a),x, algorithm="giac")
[Out] a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a+bx}\,dx = \frac{2\,a^2\,\ln{(a+b\,x)}+b^2\,x^2-2\,a\,b\,x}{2\,b^3}$$

```
[In] int(x^2/(a + b*x),x)
[Out] (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3)
```

3.31 $\int \frac{1}{(a+bx)^2} dx$

Optimal result

Integrand size = 7, antiderivative size = 12

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{b(a+bx)}$$

```
[Out] -1/b/(b*x+a)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{b(a+bx)}$$

```
[In] Int[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{b(a+bx)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2}\,dx = -\frac{1}{b(a+bx)}$$

```
[In] Integrate[(a + b*x)^(-2),x]
[Out] -(1/(b*(a + b*x)))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\frac{1}{b(bx+a)}$	13
default	$-\frac{1}{b(bx+a)}$	13
norman	$\frac{x}{a(bx+a)}$	13
risch	$-\frac{1}{b(bx+a)}$	13
parallelrisch	$-\frac{1}{b(bx+a)}$	13

```
[In] int(1/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/b/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a+bx)^2}\,dx = -\frac{1}{b^2x+ab}$$

```
[In] integrate(1/(b*x+a)^2,x, algorithm="fricas")
[Out] -1/(b^2*x + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{ab+b^2x}$$

```
[In] integrate(1/(b*x+a)**2,x)
[Out] -1/(a*b + b**2*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{(bx+a)b}$$

```
[In] integrate(1/(b*x+a)^2,x, algorithm="maxima")
[Out] -1/((b*x + a)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{(bx+a)b}$$

```
[In] integrate(1/(b*x+a)^2,x, algorithm="giac")
[Out] -1/((b*x + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{b\ (a+b\,x)}$$

```
[In] int(1/(a + b*x)^2,x)
[Out] -1/(b*(a + b*x))
```

3.32 $\int \frac{x}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \frac{x}{(a+bx)^2} \, dx = -\frac{x}{b(a+bx)} + \frac{\log(a+bx)}{b^2}$$

```
[Out] -x/b/(b*x+a)+1/b^2*ln(b*x+a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{45\}$

$$\int \frac{x}{(a+bx)^2} \, dx = \frac{a}{b^2(a+bx)} + \frac{\log(a+bx)}{b^2}$$

```
[In] Int[x/(a + b*x)^2,x]
[Out] a/(b^2*(a + b*x)) + Log[a + b*x]/b^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-\frac{a}{b(a+bx)^2} + \frac{1}{b(a+bx)}\right) \, dx \\ &= \frac{a}{b^2(a+bx)} + \frac{\log(a+bx)}{b^2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x}{(a+bx)^2}\,dx = \frac{\frac{a}{a+bx}+\log(a+bx)}{b^2}$$

```
[In] Integrate[x/(a + b*x)^2,x]
[Out] (a/(a + b*x) + Log[a + b*x])/b^2
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{a}{b^2(bx+a)}+\frac{\ln(bx+a)}{b^2}$	24
norman	$\frac{a}{b^2(bx+a)}+\frac{\ln(bx+a)}{b^2}$	24
risch	$\frac{a}{b^2(bx+a)}+\frac{\ln(bx+a)}{b^2}$	24
parallelrisch	$\frac{\ln(bx+a)xb+a\ln(bx+a)+a}{b^2(bx+a)}$	31

```
[In] int(x/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] a/b^2/(b*x+a)+1/b^2*ln(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{x}{(a+bx)^2}\,dx = \frac{(bx+a)\log(bx+a)+a}{b^3x+ab^2}$$

```
[In] integrate(x/(b*x+a)^2,x, algorithm="fricas")
[Out] ((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x}{(a+bx)^2}\,dx = \frac{a}{ab^2+b^3x} + \frac{\log(a+bx)}{b^2}$$

```
[In] integrate(x/(b*x+a)**2,x)
[Out] a/(a*b**2 + b**3*x) + log(a + b*x)/b**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x}{(a+bx)^2}\,dx = \frac{a}{b^3x+ab^2} + \frac{\log(bx+a)}{b^2}$$

```
[In] integrate(x/(b*x+a)^2,x, algorithm="maxima")
[Out] a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{x}{(a+bx)^2}\,dx = -\frac{\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b} - \frac{a}{(bx+a)b}}{b}$$

```
[In] integrate(x/(b*x+a)^2,x, algorithm="giac")
[Out] -(log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - a/((b*x + a)*b))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{x}{(a+bx)^2}\,dx = \frac{\ln(a+b\,x)}{b^2} + \frac{a}{b^2\,(a+b\,x)}$$

```
[In] int(x/(a + b*x)^2,x)
[Out] log(a + b*x)/b^2 + a/(b^2*(a + b*x))
```

3.33 $\int \frac{x^2}{(a+bx)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{x^2}{(a+bx)^2} dx = \frac{x}{b^2} - \frac{a^2}{b^3(a+bx)} - \frac{2a\log(a+bx)}{b^3}$$

```
[Out] x/b^2-a^2/b^3/(b*x+a)-2*a/b^3*ln(b*x+a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\int \frac{x^2}{(a+bx)^2} dx = -\frac{a^2}{b^3(a+bx)} - \frac{2a\log(a+bx)}{b^3} + \frac{x}{b^2}$$

```
[In] Int[x^2/(a + b*x)^2,x]
[Out] x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{b^2} + \frac{a^2}{b^2(a+bx)^2} - \frac{2a}{b^2(a+bx)}\right) dx \\ &= \frac{x}{b^2} - \frac{a^2}{b^3(a+bx)} - \frac{2a\log(a+bx)}{b^3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(a+bx)^2}\,dx = \frac{bx - \frac{a^2}{a+bx} - 2a\log(a+bx)}{b^3}$$

```
[In] Integrate[x^2/(a + b*x)^2,x]
[Out] (b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])/b^3
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{x}{b^2} - \frac{a^2}{b^3(bx+a)} - \frac{2a\ln(bx+a)}{b^3}$	34
risch	$\frac{x}{b^2} - \frac{a^2}{b^3(bx+a)} - \frac{2a\ln(bx+a)}{b^3}$	34
norman	$\frac{\frac{x^2}{b} - \frac{2a^2}{b^3}}{bx+a} - \frac{2a\ln(bx+a)}{b^3}$	38
parallelrisch	$-\frac{2\ln(bx+a)xab - b^2x^2 + 2a^2\ln(bx+a) + 2a^2}{b^3(bx+a)}$	49

```
[In] int(x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] x/b^2-a^2/b^3/(b*x+a)-2*a/b^3*ln(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{x^2}{(a+bx)^2}\,dx = \frac{b^2x^2 + abx - a^2 - 2\,(abx + a^2)\log\left(bx+a\right)}{b^4x + ab^3}$$

```
[In] integrate(x^2/(b*x+a)^2,x, algorithm="fricas")
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(a+bx)^2}\,dx = -\frac{a^2}{ab^3+b^4x} - \frac{2a\log{(a+bx)}}{b^3} + \frac{x}{b^2}$$

```
[In] integrate(x**2/(b*x+a)**2,x)
[Out] -a**2/(a*b**3 + b**4*x) - 2*a*log(a + b*x)/b**3 + x/b**2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(a+bx)^2}\,dx = -\frac{a^2}{b^4x+ab^3} + \frac{x}{b^2} - \frac{2\,a\log{(bx+a)}}{b^3}$$

```
[In] integrate(x^2/(b*x+a)^2,x, algorithm="maxima")
[Out] -a^2/(b^4*x + a*b^3) + x/b^2 - 2*a*log(b*x + a)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.52

$$\int \frac{x^2}{(a+bx)^2}\,dx = \frac{2\,a\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^3} + \frac{bx+a}{b^3} - \frac{a^2}{(bx+a)b^3}$$

```
[In] integrate(x^2/(b*x+a)^2,x, algorithm="giac")
[Out] 2*a*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^3 + (b*x + a)/b^3 - a^2/((b*x
+ a)*b^3)
```

Mupad [B] (verification not implemented)

Time = 18.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(a+bx)^2}\,dx = \frac{x}{b^2} - \frac{a^2}{x\,b^4 + a\,b^3} - \frac{2\,a\,\ln\left(a+b\,x\right)}{b^3}$$

```
[In] int(x^2/(a + b*x)^2,x)
[Out] x/b^2 - a^2/(a*b^3 + b^4*x) - (2*a*log(a + b*x))/b^3
```

3.34 $\int \frac{1}{(a+bx)^3} dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \frac{1}{(a+bx)^3} dx = -\frac{1}{2b(a+bx)^2}$$

```
[Out] -1/2/b/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\int \frac{1}{(a+bx)^3} dx = -\frac{1}{2b(a+bx)^2}$$

```
[In] Int[(a + b*x)^(-3),x]
```

```
[Out] -1/2*1/(b*(a + b*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{2b(a+bx)^2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^3}\,dx = -\frac{1}{2b(a+bx)^2}$$

```
[In] Integrate[(a + b*x)^(-3),x]
[Out] -1/2*1/(b*(a + b*x)^2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{1}{2b(bx+a)^2}$	13
default	$-\frac{1}{2b(bx+a)^2}$	13
norman	$-\frac{1}{2b(bx+a)^2}$	13
risch	$-\frac{1}{2b(bx+a)^2}$	13
parallelrisch	$-\frac{1}{2b(bx+a)^2}$	13

```
[In] int(1/(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] -1/2/b/(b*x+a)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{1}{(a+bx)^3}\,dx = -\frac{1}{2\,(b^3x^2 + 2\,ab^2x + a^2b)}$$

```
[In] integrate(1/(b*x+a)^3,x, algorithm="fricas")
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{1}{(a+bx)^3}\,dx = -\frac{1}{2a^2b + 4ab^2x + 2b^3x^2}$$

```
[In] integrate(1/(b*x+a)**3,x)
[Out] -1/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^3}\,dx = -\frac{1}{2\,(bx+a)^2b}$$

```
[In] integrate(1/(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2/((b*x + a)^2*b)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^3}\,dx = -\frac{1}{2\,(bx+a)^2b}$$

```
[In] integrate(1/(b*x+a)^3,x, algorithm="giac")
[Out] -1/2/((b*x + a)^2*b)
```

Mupad [B] (verification not implemented)

Time = 17.76 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{1}{(a+bx)^3} \, dx = -\frac{1}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2}$$

```
[In] int(1/(a + b*x)^3,x)
[Out] -1/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x)
```

3.35 $\int \frac{x}{(a+bx)^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int \frac{x}{(a+bx)^3} dx = \frac{-\frac{a}{2b^2} - \frac{x}{b}}{(a+bx)^2}$$

```
[Out] -(x/b+1/2*a/b^2)/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {37}

$$\int \frac{x}{(a+bx)^3} dx = \frac{x^2}{2a(a+bx)^2}$$

```
[In] Int[x/(a + b*x)^3,x]
```

```
[Out] x^2/(2*a*(a + b*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\text{integral} = \frac{x^2}{2a(a+bx)^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x}{(a+bx)^3}\,dx = -\frac{a+2bx}{2b^2(a+bx)^2}$$

```
[In] Integrate[x/(a + b*x)^3,x]
[Out] -1/2*(a + 2*b*x)/(b^2*(a + b*x)^2)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
gosper	$-\frac{2bx+a}{2b^{2}\left(bx+a\right)^{2}}$	19
parallelrisch	$\frac{-2bx-a}{2b^{2}\left(bx+a\right)^{2}}$	21
norman	$\frac{-\frac{x}{b}-\frac{a}{2b^{2}}}{\left(bx+a\right)^{2}}$	22
risch	$\frac{-\frac{x}{b}-\frac{a}{2b^{2}}}{\left(bx+a\right)^{2}}$	22
default	$-\frac{1}{b^{2}\left(bx+a\right)}+\frac{a}{2b^{2}\left(bx+a\right)^{2}}$	27

```
[In] int(x/(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] -1/2*(2*b*x+a)/b^2/(b*x+a)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{x}{(a+bx)^3}\,dx = -\frac{2\,bx+a}{2\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

```
[In] integrate(x/(b*x+a)^3,x, algorithm="fricas")
[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{x}{(a+bx)^3}\,dx = \frac{-a-2bx}{2a^2b^2+4ab^3x+2b^4x^2}$$

```
[In] integrate(x/(b*x+a)**3,x)
[Out] (-a - 2*b*x)/(2*a**2*b**2 + 4*a*b**3*x + 2*b**4*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{x}{(a+bx)^3}\,dx = -\frac{2\,bx+a}{2\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

```
[In] integrate(x/(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx)^3}\,dx = -\frac{2\,bx+a}{2\,(bx+a)^2b^2}$$

```
[In] integrate(x/(b*x+a)^3,x, algorithm="giac")
[Out] -1/2*(2*b*x + a)/((b*x + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{x}{(a+bx)^3}\,dx = -\frac{\frac{a}{2\,b^2}+\frac{x}{b}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

```
[In] int(x/(a + b*x)^3,x)
[Out] -(a/(2*b^2) + x/b)/(a^2 + b^2*x^2 + 2*a*b*x)
```

3.36 $\int \frac{x^2}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{x^2}{(a+bx)^3}\,dx = \frac{\frac{3a^2}{2b^3}+\frac{2ax}{b^2}}{(a+bx)^2}+\frac{\log(a+bx)}{b^3}$$

```
[Out] (2*a*x/b^2+3/2*a^2/b^3)/(b*x+a)^2+1/b^3*ln(b*x+a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\int \frac{x^2}{(a+bx)^3}\,dx = -\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}$$

```
[In] Int[x^2/(a + b*x)^3,x]
[Out] -1/2*a^2/(b^3*(a + b*x)^2) + (2*a)/(b^3*(a + b*x)) + Log[a + b*x]/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{a^2}{b^2(a+bx)^3}-\frac{2a}{b^2(a+bx)^2}+\frac{1}{b^2(a+bx)}\right)dx \\ &= -\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(a+bx)^3}\,dx = \frac{\frac{a(3a+4bx)}{(a+bx)^2} + 2\log(a+bx)}{2b^3}$$

```
[In] Integrate[x^2/(a + b*x)^3,x]
[Out] ((a*(3*a + 4*b*x))/(a + b*x)^2 + 2*Log[a + b*x])/(2*b^3)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

method	result	size
norman	$\frac{\frac{2ax}{b^2}+\frac{3a^2}{2b^3}}{(bx+a)^2}+\frac{\ln(bx+a)}{b^3}$	36
risch	$\frac{\frac{2ax}{b^2}+\frac{3a^2}{2b^3}}{(bx+a)^2}+\frac{\ln(bx+a)}{b^3}$	36
default	$\frac{2a}{b^3(bx+a)}-\frac{a^2}{2b^3(bx+a)^2}+\frac{\ln(bx+a)}{b^3}$	40
parallelrisch	$\frac{2\ln(bx+a)x^2b^2+4\ln(bx+a)xab+2a^2\ln(bx+a)+4bax+3a^2}{2b^3(bx+a)^2}$	60

```
[In] int(x^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] (2*a*x/b^2+3/2*a^2/b^3)/(b*x+a)^2+1/b^3*ln(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int \frac{x^2}{(a+bx)^3}\,dx = \frac{4\,abx + 3\,a^2 + 2\,(b^2x^2 + 2\,abx + a^2)\log(bx+a)}{2\,(b^5x^2 + 2\,ab^4x + a^2b^3)}$$

```
[In] integrate(x^2/(b*x+a)^3,x, algorithm="fricas")
[Out] 1/2*(4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a))/(b^5*x^2 +
 2*a*b^4*x + a^2*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{x^2}{(a+bx)^3}\,dx = \frac{3a^2+4abx}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{\log(a+bx)}{b^3}$$

```
[In] integrate(x**2/(b*x+a)**3,x)
[Out] (3*a**2 + 4*a*b*x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + log(a + b*x)/
b**3
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \frac{x^2}{(a+bx)^3}\,dx = \frac{4\,abx+3\,a^2}{2\,(b^5x^2+2\,ab^4x+a^2b^3)} + \frac{\log(bx+a)}{b^3}$$

```
[In] integrate(x^2/(b*x+a)^3,x, algorithm="maxima")
[Out] 1/2*(4*a*b*x + 3*a^2)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3) + log(b*x + a)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx)^3}\,dx = \frac{\log(|bx+a|)}{b^3} + \frac{4\,ax+\frac{3\,a^2}{b}}{2\,(bx+a)^2b^2}$$

```
[In] integrate(x^2/(b*x+a)^3,x, algorithm="giac")
[Out] log(abs(b*x + a))/b^3 + 1/2*(4*a*x + 3*a^2/b)/((b*x + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 17.91 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{x^2}{(a+bx)^3} \, dx = \frac{\ln(a+b\,x)}{b^3} + \frac{\frac{3\,a^2}{2\,b^3} + \frac{2\,a\,x}{b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

```
[In] int(x^2/(a + b*x)^3,x)
[Out] log(a + b*x)/b^3 + ((3*a^2)/(2*b^3) + (2*a*x)/b^2)/(a^2 + b^2*x^2 + 2*a*b*x
)
```

3.37 $\int \frac{x^3}{(a+bx)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 57

$$\int \frac{x^3}{(a+bx)^3} dx = \frac{-\frac{5a^3}{2b^4} - \frac{2a^2x}{b^3} + \frac{2ax^2}{b^2} + \frac{x^3}{b}}{(a+bx)^2} - \frac{3a\log(a+bx)}{b^4}$$

```
[Out] (x^3/b+2*a/b^2*x^2-2*a^2/b^3*x-5/2*a^3/b^4)/(b*x+a)^2-3*a/b^4*ln(b*x+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\int \frac{x^3}{(a+bx)^3} dx = \frac{a^3}{2b^4(a+bx)^2} - \frac{3a^2}{b^4(a+bx)} - \frac{3a\log(a+bx)}{b^4} + \frac{x}{b^3}$$

```
[In] Int[x^3/(a + b*x)^3,x]
```

```
[Out] x/b^3 + a^3/(2*b^4*(a + b*x)^2) - (3*a^2)/(b^4*(a + b*x)) - (3*a*Log[a + b*
x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{b^3} - \frac{a^3}{b^3(a+bx)^3} + \frac{3a^2}{b^3(a+bx)^2} - \frac{3a}{b^3(a+bx)}\right) dx \\ &= \frac{x}{b^3} + \frac{a^3}{2b^4(a+bx)^2} - \frac{3a^2}{b^4(a+bx)} - \frac{3a\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.70

$$\int \frac{x^3}{(a+bx)^3}\,dx = -\frac{-2bx + \frac{a^2(5a+6bx)}{(a+bx)^2} + 6a\log(a+bx)}{2b^4}$$

```
[In] Integrate[x^3/(a + b*x)^3,x]
[Out] -1/2*(-2*b*x + (a^2*(5*a + 6*b*x))/(a + b*x)^2 + 6*a*Log[a + b*x])/b^4
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{x}{b^3}+\frac{-3a^2x-\frac{5a^3}{2b}}{b^3(bx+a)^2}-\frac{3a\ln(bx+a)}{b^4}$	45
norman	$\frac{\frac{x^3}{b}-\frac{9a^3}{2b^4}-\frac{6a^2x}{b^3}}{(bx+a)^2}-\frac{3a\ln(bx+a)}{b^4}$	47
default	$\frac{x}{b^3}-\frac{3a^2}{b^4(bx+a)}+\frac{a^3}{2b^4(bx+a)^2}-\frac{3a\ln(bx+a)}{b^4}$	49
parallelrisch	$-\frac{6\ln(bx+a)x^2a\,b^2-2b^3x^3+12\ln(bx+a)x\,a^2b+6\ln(bx+a)a^3+12a^2bx+9a^3}{2b^4(bx+a)^2}$	73

```
[In] int(x^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b^3*x+(-3*a^2*x-5/2*a^3/b)/b^3/(b*x+a)^2-3*a/b^4*ln(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.46

$$\int \frac{x^3}{(a+bx)^3}\,dx = \frac{2\,b^3x^3 + 4\,ab^2x^2 - 4\,a^2bx - 5\,a^3 - 6\,(ab^2x^2 + 2\,a^2bx + a^3)\log\left(bx + a\right)}{2\,(b^6x^2 + 2\,ab^5x + a^2b^4)}$$

```
[In] integrate(x^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x^3 + 4*a*b^2*x^2 - 4*a^2*b*x - 5*a^3 - 6*(a*b^2*x^2 + 2*a^2*b*x
 + a^3)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{(a+bx)^3}\,dx = -\frac{3a\log\left(a+bx\right)}{b^4} + \frac{-5a^3 - 6a^2bx}{2a^2b^4 + 4ab^5x + 2b^6x^2} + \frac{x}{b^3}$$

```
[In] integrate(x**3/(b*x+a)**3,x)
```

```
[Out] -3*a*log(a + b*x)/b**4 + (-5*a**3 - 6*a**2*b*x)/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2) + x/b**3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a+bx)^3}\,dx = -\frac{6\,a^2bx + 5\,a^3}{2\,(b^6x^2 + 2\,ab^5x + a^2b^4)} + \frac{x}{b^3} - \frac{3\,a\log\left(bx + a\right)}{b^4}$$

```
[In] integrate(x^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*a^2*b*x + 5*a^3)/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) + x/b^3 - 3*a*log(
b*x + a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{(a+bx)^3}\,dx = \frac{x}{b^3} - \frac{3\,a\log\left(|bx+a|\right)}{b^4} - \frac{6\,a^2bx+5\,a^3}{2\,(bx+a)^2b^4}$$

```
[In] integrate(x^3/(b*x+a)^3,x, algorithm="giac")
[Out] x/b^3 - 3*a*log(abs(b*x + a))/b^4 - 1/2*(6*a^2*b*x + 5*a^3)/((b*x + a)^2*b^
4)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{(a+bx)^3}\,dx = -\frac{3\,a\,\ln\left(a+b\,x\right)-b\,x+\frac{3\,a^2}{a+b\,x}-\frac{a^3}{2\,{\left(a+b\,x\right)}^2}}{b^4}$$

```
[In] int(x^3/(a + b*x)^3,x)
[Out] -(3*a*log(a + b*x) - b*x + (3*a^2)/(a + b*x) - a^3/(2*(a + b*x)^2))/b^4
```

3.38 $\int \frac{1}{(a+bx)^4} dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \frac{1}{(a+bx)^4} dx = -\frac{1}{3b(a+bx)^3}$$

```
[Out] -1/3/b/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\int \frac{1}{(a+bx)^4} dx = -\frac{1}{3b(a+bx)^3}$$

```
[In] Int[(a + b*x)^(-4),x]
```

```
[Out] -1/3*1/(b*(a + b*x)^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{3b(a+bx)^3}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^4}\,dx = -\frac{1}{3b(a+bx)^3}$$

```
[In] Integrate[(a + b*x)^(-4),x]
[Out] -1/3*1/(b*(a + b*x)^3)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{1}{3b(bx+a)^3}$	13
default	$-\frac{1}{3b(bx+a)^3}$	13
norman	$-\frac{1}{3b(bx+a)^3}$	13
risch	$-\frac{1}{3b(bx+a)^3}$	13
parallelrisch	$-\frac{1}{3b(bx+a)^3}$	13

```
[In] int(1/(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] -1/3/b/(b*x+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(12) = 24.

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.50

$$\int \frac{1}{(a+bx)^4}\,dx = -\frac{1}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)}$$

```
[In] integrate(1/(b*x+a)^4,x, algorithm="fricas")
[Out] -1/3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(12) = 24.

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.64

$$\int \frac{1}{(a+bx)^4}\,dx = -\frac{1}{3a^3b+9a^2b^2x+9ab^3x^2+3b^4x^3}$$

```
[In] integrate(1/(b*x+a)**4,x)
[Out] -1/(3*a**3*b + 9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**4*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^4}\,dx = -\frac{1}{3\,(bx+a)^3b}$$

```
[In] integrate(1/(b*x+a)^4,x, algorithm="maxima")
[Out] -1/3/((b*x + a)^3*b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^4}\,dx = -\frac{1}{3\,(bx+a)^3b}$$

```
[In] integrate(1/(b*x+a)^4,x, algorithm="giac")
[Out] -1/3/((b*x + a)^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.64

$$\int \frac{1}{(a+bx)^4}\,dx = -\frac{1}{3\,a^3\,b+9\,a^2\,b^2\,x+9\,a\,b^3\,x^2+3\,b^4\,x^3}$$

```
[In] int(1/(a + b*x)^4,x)
[Out] -1/(3*a^3*b + 3*b^4*x^3 + 9*a^2*b^2*x + 9*a*b^3*x^2)
```

3.39 $\int \frac{x}{(a+bx)^4}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{x}{(a+bx)^4}\,dx = \frac{-\frac{a}{6b^2}-\frac{x}{2b}}{(a+bx)^3}$$

```
[Out] -(1/2*x/b+1/6*a/b^2)/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{45\}$

$$\int \frac{x}{(a+bx)^4}\,dx = \frac{a}{3b^2(a+bx)^3}-\frac{1}{2b^2(a+bx)^2}$$

```
[In] Int[x/(a + b*x)^4,x]
```

```
[Out] a/(3*b^2*(a + b*x)^3) - 1/(2*b^2*(a + b*x)^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{a}{b(a+bx)^4}+\frac{1}{b(a+bx)^3}\right)dx\\ &= \frac{a}{3b^2(a+bx)^3}-\frac{1}{2b^2(a+bx)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{x}{(a+bx)^4}\,dx = -\frac{a+3bx}{6b^2(a+bx)^3}$$

```
[In] Integrate[x/(a + b*x)^4,x]
[Out] -1/6*(a + 3*b*x)/(b^2*(a + b*x)^3)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{3bx+a}{6(bx+a)^3 b^2}$	19
norman	$\frac{-\frac{x}{2b}-\frac{a}{6b^2}}{(bx+a)^3}$	22
risch	$\frac{-\frac{x}{2b}-\frac{a}{6b^2}}{(bx+a)^3}$	22
parallelrisch	$\frac{-3b^2x-ab}{6b^3(bx+a)^3}$	24
default	$-\frac{1}{2b^2(bx+a)^2}+\frac{a}{3b^2(bx+a)^3}$	27

```
[In] int(x/(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] -1/6*(3*b*x+a)/(b*x+a)^3/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(21) = 42.

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \frac{x}{(a+bx)^4}\,dx = -\frac{3\,bx+a}{6\,(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2)}$$

```
[In] integrate(x/(b*x+a)^4,x, algorithm="fricas")
[Out] -1/6*(3*b*x + a)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(19) = 38$.

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{x}{(a+bx)^4}\,dx = \frac{-a-3bx}{6a^3b^2+18a^2b^3x+18ab^4x^2+6b^5x^3}$$

```
[In] integrate(x/(b*x+a)**4,x)
[Out] (-a - 3*b*x)/(6*a**3*b**2 + 18*a**2*b**3*x + 18*a*b**4*x**2 + 6*b**5*x**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(21) = 42$.

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \frac{x}{(a+bx)^4}\,dx = -\frac{3\,bx+a}{6\,(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2)}$$

```
[In] integrate(x/(b*x+a)^4,x, algorithm="maxima")
[Out] -1/6*(3*b*x + a)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{x}{(a+bx)^4}\,dx = -\frac{3\,bx+a}{6\,(bx+a)^3b^2}$$

```
[In] integrate(x/(b*x+a)^4,x, algorithm="giac")
[Out] -1/6*(3*b*x + a)/((b*x + a)^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 18.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{x}{(a+bx)^4}\,dx=-\frac{\frac{a}{6\,b^2}+\frac{x}{2\,b}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

```
[In] int(x/(a + b*x)^4,x)
[Out] -(a/(6*b^2) + x/(2*b))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

3.40 $\int \frac{x^2}{(a+bx)^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int \frac{x^2}{(a+bx)^4} dx = \frac{-\frac{a^2}{3b^3} - \frac{ax}{b^2} - \frac{x^2}{b}}{(a+bx)^3}$$

```
[Out] -(x^2/b+a*x/b^2+1/3*a^2/b^3)/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.50, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {37}

$$\int \frac{x^2}{(a+bx)^4} dx = \frac{x^3}{3a(a+bx)^3}$$

```
[In] Int[x^2/(a + b*x)^4,x]
```

```
[Out] x^3/(3*a*(a + b*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\text{integral} = \frac{x^3}{3a(a+bx)^3}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{(a+bx)^4}\,dx = -\frac{a^2+3abx+3b^2x^2}{3b^3(a+bx)^3}$$

```
[In] Integrate[x^2/(a + b*x)^4,x]
[Out] -1/3*(a^2 + 3*a*b*x + 3*b^2*x^2)/(b^3*(a + b*x)^3)
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
gosper	$-\frac{3b^2x^2+3bax+a^2}{3b^3(bx+a)^3}$	30
parallelrisch	$\frac{-3b^2x^2-3bax-a^2}{3b^3(bx+a)^3}$	32
norman	$\frac{-\frac{x^2}{b}-\frac{ax}{b^2}-\frac{a^2}{3b^3}}{(bx+a)^3}$	33
risch	$\frac{-\frac{x^2}{b}-\frac{ax}{b^2}-\frac{a^2}{3b^3}}{(bx+a)^3}$	33
default	$-\frac{1}{b^3(bx+a)}+\frac{a}{b^3(bx+a)^2}-\frac{a^2}{3b^3(bx+a)^3}$	41

```
[In] int(x^2/(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] -1/3*(3*b^2*x^2+3*a*b*x+a^2)/b^3/(b*x+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int \frac{x^2}{(a+bx)^4}\,dx = -\frac{3\,b^2x^2+3\,abx+a^2}{3\,(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3)}$$

```
[In] integrate(x^2/(b*x+a)^4,x, algorithm="fricas")
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3
*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(27) = 54$.

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

$$\int \frac{x^2}{(a+bx)^4}\,dx = \frac{-a^2-3abx-3b^2x^2}{3a^3b^3+9a^2b^4x+9ab^5x^2+3b^6x^3}$$

```
[In] integrate(x**2/(b*x+a)**4,x)
```

```
[Out] (-a**2 - 3*a*b*x - 3*b**2*x**2)/(3*a**3*b**3 + 9*a**2*b**4*x + 9*a*b**5*x**
2 + 3*b**6*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int \frac{x^2}{(a+bx)^4}\,dx = -\frac{3\,b^2x^2+3\,abx+a^2}{3\,(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3)}$$

```
[In] integrate(x^2/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3
*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{(a+bx)^4}\,dx = -\frac{3\,b^2x^2+3\,abx+a^2}{3\,(bx+a)^3b^3}$$

```
[In] integrate(x^2/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/((b*x + a)^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 17.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

$$\int \frac{x^2}{(a+bx)^4}\,dx = -\frac{a^2+3\,a\,b\,x+3\,b^2\,x^2}{3\,a^3\,b^3+9\,a^2\,b^4\,x+9\,a\,b^5\,x^2+3\,b^6\,x^3}$$

```
[In] int(x^2/(a + b*x)^4,x)
[Out] -(a^2 + 3*b^2*x^2 + 3*a*b*x)/(3*a^3*b^3 + 3*b^6*x^3 + 9*a^2*b^4*x + 9*a*b^5
*x^2)
```

3.41 $\int \frac{x^3}{(a+bx)^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 50

$$\int \frac{x^3}{(a+bx)^4} dx = \frac{\frac{11a^3}{6b^4} + \frac{9a^2x}{2b^2} + \frac{3ax^2}{b^2}}{(a+bx)^3} + \frac{\log(a+bx)}{b^4}$$

```
[Out] (3*a/b^2*x^2+9/2*a^2*x/b^2+11/6*a^3/b^4)/(b*x+a)^3+1/b^4*ln(b*x+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\int \frac{x^3}{(a+bx)^4} dx = \frac{a^3}{3b^4(a+bx)^3} - \frac{3a^2}{2b^4(a+bx)^2} + \frac{3a}{b^4(a+bx)} + \frac{\log(a+bx)}{b^4}$$

```
[In] Int[x^3/(a + b*x)^4,x]
```

```
[Out] a^3/(3*b^4*(a + b*x)^3) - (3*a^2)/(2*b^4*(a + b*x)^2) + (3*a)/(b^4*(a + b*x
)) + Log[a + b*x]/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\text{integral} = \int \left(-\frac{a^3}{b^3(a+bx)^4} + \frac{3a^2}{b^3(a+bx)^3} - \frac{3a}{b^3(a+bx)^2} + \frac{1}{b^3(a+bx)}\right) dx$$
$$= \frac{a^3}{3b^4(a+bx)^3} - \frac{3a^2}{2b^4(a+bx)^2} + \frac{3a}{b^4(a+bx)} + \frac{\log(a+bx)}{b^4}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a+bx)^4} dx = \frac{\frac{a(11a^2+27abx+18b^2x^2)}{(a+bx)^3} + 6\log(a+bx)}{6b^4}$$

```
[In] Integrate[x^3/(a + b*x)^4,x]
[Out] ((a*(11*a^2 + 27*a*b*x + 18*b^2*x^2))/(a + b*x)^3 + 6*Log[a + b*x])/(6*b^4)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

method	result	size
norman	$\frac{\frac{11a^3}{6b^4}+\frac{3a\,x^2}{b^2}+\frac{9a^2x}{2b^3}}{(bx+a)^3}+\frac{\ln(bx+a)}{b^4}$	47
risch	$\frac{\frac{11a^3}{6b^4}+\frac{3a\,x^2}{b^2}+\frac{9a^2x}{2b^3}}{(bx+a)^3}+\frac{\ln(bx+a)}{b^4}$	47
default	$\frac{3a}{b^4(bx+a)}-\frac{3a^2}{2b^4(bx+a)^2}+\frac{\ln(bx+a)}{b^4}+\frac{a^3}{3b^4(bx+a)^3}$	55
parallelrisch	$\frac{6\ln(bx+a)x^3b^3+18\ln(bx+a)x^2a\,b^2+18\ln(bx+a)x\,a^2b+18a\,b^2x^2+6\ln(bx+a)a^3+27a^2bx+11a^3}{6b^4(bx+a)^3}$	88

```
[In] int(x^3/(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] (11/6*a^3/b^4+3*a/b^2*x^2+9/2*a^2/b^3*x)/(b*x+a)^3+1/b^4*ln(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.88

$$\int \frac{x^3}{(a+bx)^4}\,dx = \frac{18\,ab^2x^2 + 27\,a^2bx + 11\,a^3 + 6\,(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3)\log(bx+a)}{6\,(b^7x^3 + 3\,ab^6x^2 + 3\,a^2b^5x + a^3b^4)}$$

```
[In] integrate(x^3/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3 + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*
b*x + a^3)*log(b*x + a))/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

$$\int \frac{x^3}{(a+bx)^4}\,dx = \frac{11a^3 + 27a^2bx + 18ab^2x^2}{6a^3b^4 + 18a^2b^5x + 18ab^6x^2 + 6b^7x^3} + \frac{\log(a+bx)}{b^4}$$

```
[In] integrate(x**3/(b*x+a)**4,x)
```

```
[Out] (11*a**3 + 27*a**2*b*x + 18*a*b**2*x**2)/(6*a**3*b**4 + 18*a**2*b**5*x + 18
*a*b**6*x**2 + 6*b**7*x**3) + log(a + b*x)/b**4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

$$\int \frac{x^3}{(a+bx)^4}\,dx = \frac{18\,ab^2x^2 + 27\,a^2bx + 11\,a^3}{6\,(b^7x^3 + 3\,ab^6x^2 + 3\,a^2b^5x + a^3b^4)} + \frac{\log(bx+a)}{b^4}$$

```
[In] integrate(x^3/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3)/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5
*x + a^3*b^4) + log(b*x + a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(a+bx)^4}\,dx = \frac{\log(|bx+a|)}{b^4} + \frac{18\,abx^2 + 27\,a^2x + \frac{11\,a^3}{b}}{6\,(bx+a)^3b^3}$$

```
[In] integrate(x^3/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^4 + 1/6*(18*a*b*x^2 + 27*a^2*x + 11*a^3/b)/((b*x + a)^3
*b^3)
```

Mupad [B] (verification not implemented)

Time = 16.74 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{(a+bx)^4}\,dx = \frac{\ln(a+b\,x) + \frac{3\,a}{a+b\,x} - \frac{3\,a^2}{2\,(a+b\,x)^2} + \frac{a^3}{3\,(a+b\,x)^3}}{b^4}$$

```
[In] int(x^3/(a + b*x)^4,x)
```

```
[Out] (log(a + b*x) + (3*a)/(a + b*x) - (3*a^2)/(2*(a + b*x)^2) + a^3/(3*(a + b*x
)^3))/b^4
```

3.42 $\int \frac{1}{(a+bx)^5} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \frac{1}{(a+bx)^5} \, dx = -\frac{1}{256b^4(a+bx)^4}$$

```
[Out] -1/256/b^4/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\int \frac{1}{(a+bx)^5} \, dx = -\frac{1}{4b(a+bx)^4}$$

```
[In] Int[(a + b*x)^(-5),x]
```

```
[Out] -1/4*1/(b*(a + b*x)^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{4b(a+bx)^4}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^5} \, dx = -\frac{1}{4b(a+bx)^4}$$

```
[In] Integrate[(a + b*x)^(-5),x]
[Out] -1/4*1/(b*(a + b*x)^4)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{1}{4(bx+a)^4 b}$	13
default	$-\frac{1}{4(bx+a)^4 b}$	13
norman	$-\frac{1}{4(bx+a)^4 b}$	13
risch	$-\frac{1}{4(bx+a)^4 b}$	13
parallelrisch	$-\frac{1}{4(bx+a)^4 b}$	13

```
[In] int(1/(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] -1/4/(b*x+a)^4/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(12) = 24.

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.29

$$\int \frac{1}{(a+bx)^5} \, dx = -\frac{1}{4\,(b^5x^4 + 4\,ab^4x^3 + 6\,a^2b^3x^2 + 4\,a^3b^2x + a^4b)}$$

```
[In] integrate(1/(b*x+a)^5,x, algorithm="fricas")
[Out] -1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(14) = 28$.

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.50

$$\int \frac{1}{(a+bx)^5}\,dx = -\frac{1}{4a^4b + 16a^3b^2x + 24a^2b^3x^2 + 16ab^4x^3 + 4b^5x^4}$$

```
[In] integrate(1/(b*x+a)**5,x)
[Out] -1/(4*a**4*b + 16*a**3*b**2*x + 24*a**2*b**3*x**2 + 16*a*b**4*x**3 + 4*b**5
*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^5}\,dx = -\frac{1}{4\,(bx+a)^4b}$$

```
[In] integrate(1/(b*x+a)^5,x, algorithm="maxima")
[Out] -1/4/((b*x + a)^4*b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^5}\,dx = -\frac{1}{4\,(bx+a)^4b}$$

```
[In] integrate(1/(b*x+a)^5,x, algorithm="giac")
[Out] -1/4/((b*x + a)^4*b)
```

Mupad [B] (verification not implemented)

Time = 16.81 (sec) , antiderivative size = 48, normalized size of antiderivative = 3.43

$$\int \frac{1}{(a+bx)^5} \, dx = -\frac{1}{4\,a^4\,b+16\,a^3\,b^2\,x+24\,a^2\,b^3\,x^2+16\,a\,b^4\,x^3+4\,b^5\,x^4}$$

```
[In] int(1/(a + b*x)^5,x)
[Out] -1/(4*a^4*b + 4*b^5*x^4 + 16*a^3*b^2*x + 16*a*b^4*x^3 + 24*a^2*b^3*x^2)
```

3.43 $\int \frac{x}{(a+bx)^5} dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{x}{(a+bx)^5} \, dx = \frac{-\frac{a}{12b^2} - \frac{x}{3b}}{(a+bx)^4}$$

```
[Out] -(1/3*x/b+1/12*a/b^2)/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\int \frac{x}{(a+bx)^5} \, dx = \frac{a}{4b^2(a+bx)^4} - \frac{1}{3b^2(a+bx)^3}$$

```
[In] Int[x/(a + b*x)^5,x]
```

```
[Out] a/(4*b^2*(a + b*x)^4) - 1/(3*b^2*(a + b*x)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-\frac{a}{b(a+bx)^5} + \frac{1}{b(a+bx)^4}\right) dx \\ &= \frac{a}{4b^2(a+bx)^4} - \frac{1}{3b^2(a+bx)^3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{x}{(a+bx)^5} \, dx = -\frac{a+4bx}{12b^2(a+bx)^4}$$

```
[In] Integrate[x/(a + b*x)^5,x]
```

```
[Out] -1/12*(a + 4*b*x)/(b^2*(a + b*x)^4)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{4bx+a}{12(bx+a)^4b^2}$	19
norman	$\frac{-\frac{x}{3b}-\frac{a}{12b^2}}{(bx+a)^4}$	22
risch	$\frac{-\frac{x}{3b}-\frac{a}{12b^2}}{(bx+a)^4}$	22
parallelrisch	$\frac{-4b^3x-a\,b^2}{12b^4(bx+a)^4}$	26
default	$\frac{a}{4b^2(bx+a)^4}-\frac{1}{3b^2(bx+a)^3}$	27

```
[In] int(x/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(4*b*x+a)/(b*x+a)^4/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(21) = 42.

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int \frac{x}{(a+bx)^5} \, dx = -\frac{4\,bx+a}{12\,(b^6x^4+4\,ab^5x^3+6\,a^2b^4x^2+4\,a^3b^3x+a^4b^2)}$$

```
[In] integrate(x/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*x + a)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^3*b^3*x + a^
4*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(19) = 38$.

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.24

$$\int \frac{x}{(a+bx)^5}\,dx = \frac{-a-4bx}{12a^4b^2+48a^3b^3x+72a^2b^4x^2+48ab^5x^3+12b^6x^4}$$

```
[In] integrate(x/(b*x+a)**5,x)
```

```
[Out] (-a - 4*b*x)/(12*a**4*b**2 + 48*a**3*b**3*x + 72*a**2*b**4*x**2 + 48*a*b**5
*x**3 + 12*b**6*x**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(21) = 42$.

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int \frac{x}{(a+bx)^5}\,dx = -\frac{4\,bx+a}{12\,(b^6x^4+4\,ab^5x^3+6\,a^2b^4x^2+4\,a^3b^3x+a^4b^2)}$$

```
[In] integrate(x/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(4*b*x + a)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^3*b^3*x + a^
4*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \frac{x}{(a+bx)^5}\,dx = -\frac{\frac{4}{(bx+a)^3b}-\frac{3\,a}{(bx+a)^4b}}{12\,b}$$

```
[In] integrate(x/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/12*(4/((b*x + a)^3*b) - 3*a/((b*x + a)^4*b))/b
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{x}{(a+bx)^5} \, dx = -\frac{a+4\,b\,x}{12\,b^2\,{\left(a+b\,x\right)}^4}$$

```
[In] int(x/(a + b*x)^5,x)
```

```
[Out] -(a + 4*b*x)/(12*b^2*(a + b*x)^4)
```

3.44 $\int \frac{x^2}{(a+bx)^5} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{x^2}{(a+bx)^5} \, dx = \frac{-\frac{a^2}{12b^3} - \frac{ax}{3b^2} - \frac{x^2}{2b}}{(a+bx)^4}$$

```
[Out] -(1/2*x^2/b+1/3*a*x/b^2+1/12*a^2/b^3)/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\int \frac{x^2}{(a+bx)^5} \, dx = -\frac{a^2}{4b^3(a+bx)^4} + \frac{2a}{3b^3(a+bx)^3} - \frac{1}{2b^3(a+bx)^2}$$

```
[In] Int[x^2/(a + b*x)^5,x]
```

```
[Out] -1/4*a^2/(b^3*(a + b*x)^4) + (2*a)/(3*b^3*(a + b*x)^3) - 1/(2*b^3*(a + b*x)
^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a^2}{b^2(a+bx)^5} - \frac{2a}{b^2(a+bx)^4} + \frac{1}{b^2(a+bx)^3}\right) dx \\ &= -\frac{a^2}{4b^3(a+bx)^4} + \frac{2a}{3b^3(a+bx)^3} - \frac{1}{2b^3(a+bx)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{(a+bx)^5}\, dx = -\frac{a^2+4abx+6b^2x^2}{12b^3(a+bx)^4}$$

```
[In] Integrate[x^2/(a + b*x)^5,x]
[Out] -1/12*(a^2 + 4*a*b*x + 6*b^2*x^2)/(b^3*(a + b*x)^4)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{6b^2x^2+4bax+a^2}{12(bx+a)^4 b^3}$	30
norman	$\frac{-\frac{x^2}{2b}-\frac{ax}{3b^2}-\frac{a^2}{12b^3}}{(bx+a)^4}$	33
risch	$\frac{-\frac{x^2}{2b}-\frac{ax}{3b^2}-\frac{a^2}{12b^3}}{(bx+a)^4}$	33
parallelrisch	$\frac{-6b^3x^2-4a\,b^2x-a^2b}{12b^4(bx+a)^4}$	35
default	$-\frac{1}{2b^3(bx+a)^2}-\frac{a^2}{4b^3(bx+a)^4}+\frac{2a}{3b^3(bx+a)^3}$	42

```
[In] int(x^2/(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] -1/12*(6*b^2*x^2+4*a*b*x+a^2)/(b*x+a)^4/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(32) = 64$.

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{x^2}{(a+bx)^5}\,dx = -\frac{6\,b^2x^2 + 4\,abx + a^2}{12\,(b^7x^4 + 4\,ab^6x^3 + 6\,a^2b^5x^2 + 4\,a^3b^4x + a^4b^3)}$$

```
[In] integrate(x^2/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*x^2 + 4*a*b*x + a^2)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 +
4*a^3*b^4*x + a^4*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(31) = 62$.

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.79

$$\int \frac{x^2}{(a+bx)^5}\,dx = \frac{-a^2 - 4abx - 6b^2x^2}{12a^4b^3 + 48a^3b^4x + 72a^2b^5x^2 + 48ab^6x^3 + 12b^7x^4}$$

```
[In] integrate(x**2/(b*x+a)**5,x)
```

```
[Out] (-a**2 - 4*a*b*x - 6*b**2*x**2)/(12*a**4*b**3 + 48*a**3*b**4*x + 72*a**2*b*
*5*x**2 + 48*a*b**6*x**3 + 12*b**7*x**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(32) = 64$.

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{x^2}{(a+bx)^5}\,dx = -\frac{6\,b^2x^2 + 4\,abx + a^2}{12\,(b^7x^4 + 4\,ab^6x^3 + 6\,a^2b^5x^2 + 4\,a^3b^4x + a^4b^3)}$$

```
[In] integrate(x^2/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(6*b^2*x^2 + 4*a*b*x + a^2)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 +
4*a^3*b^4*x + a^4*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{(a+bx)^5}\,dx = -\frac{\frac{6}{(bx+a)^2b^2} - \frac{8\,a}{(bx+a)^3b^2} + \frac{3\,a^2}{(bx+a)^4b^2}}{12\,b}$$

```
[In] integrate(x^2/(b*x+a)^5,x, algorithm="giac")
[Out] -1/12*(6/((b*x + a)^2*b^2) - 8*a/((b*x + a)^3*b^2) + 3*a^2/((b*x + a)^4*b^2
))/b
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{x^2}{(a+bx)^5}\,dx = \frac{x^3\,(4\,a+b\,x)}{12\,a^2\,{(a+b\,x)}^4}$$

```
[In] int(x^2/(a + b*x)^5,x)
[Out] (x^3*(4*a + b*x))/(12*a^2*(a + b*x)^4)
```

3.45 $\int \frac{x^3}{(a+bx)^5} dx$

Optimal result

Integrand size = 11, antiderivative size = 47

$$\int \frac{x^3}{(a+bx)^5} dx = \frac{-\frac{a^3}{4b^4} - \frac{a^2x}{b^3} - \frac{3ax^3}{2b^2} - \frac{x^3}{b}}{(a+bx)^4}$$

```
[Out] -(x^3/b+3/2*a*x^3/b^2+a^2/b^3*x+1/4*a^3/b^4)/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.36, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {37}

$$\int \frac{x^3}{(a+bx)^5} dx = \frac{x^4}{4a(a+bx)^4}$$

```
[In] Int[x^3/(a + b*x)^5,x]
```

```
[Out] x^4/(4*a*(a + b*x)^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\text{integral} = \frac{x^4}{4a(a+bx)^4}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{(a+bx)^5}\,dx = -\frac{a^3+4a^2bx+6ab^2x^2+4b^3x^3}{4b^4(a+bx)^4}$$

```
[In] Integrate[x^3/(a + b*x)^5,x]
[Out] -1/4*(a^3 + 4*a^2*b*x + 6*a*b^2*x^2 + 4*b^3*x^3)/(b^4*(a + b*x)^4)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

method	result	size
gosper	$-\frac{4b^3x^3+6a\,b^2x^2+4a^2bx+a^3}{4(bx+a)^4b^4}$	41
parallelrisch	$\frac{-4b^3x^3-6a\,b^2x^2-4a^2bx-a^3}{4b^4(bx+a)^4}$	43
norman	$\frac{-\frac{x^3}{b}-\frac{3a\,x^2}{2b^2}-\frac{a^2x}{b^3}-\frac{a^3}{4b^4}}{(bx+a)^4}$	44
risch	$\frac{-\frac{x^3}{b}-\frac{3a\,x^2}{2b^2}-\frac{a^2x}{b^3}-\frac{a^3}{4b^4}}{(bx+a)^4}$	44
default	$-\frac{1}{b^4(bx+a)}+\frac{3a}{2b^4(bx+a)^2}+\frac{a^3}{4b^4(bx+a)^4}-\frac{a^2}{b^4(bx+a)^3}$	57

```
[In] int(x^3/(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] -1/4*(4*b^3*x^3+6*a*b^2*x^2+4*a^2*b*x+a^3)/(b*x+a)^4/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{x^3}{(a+bx)^5}\,dx = -\frac{4\,b^3x^3+6\,ab^2x^2+4\,a^2bx+a^3}{4\,(b^8x^4+4\,ab^7x^3+6\,a^2b^6x^2+4\,a^3b^5x+a^4b^4)}$$

```
[In] integrate(x^3/(b*x+a)^5,x, algorithm="fricas")
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/(b^8*x^4 + 4*a*b^7*x^3 + 6
*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int \frac{x^3}{(a+bx)^5}\,dx = \frac{-a^3-4a^2bx-6ab^2x^2-4b^3x^3}{4a^4b^4+16a^3b^5x+24a^2b^6x^2+16ab^7x^3+4b^8x^4}$$

```
[In] integrate(x**3/(b*x+a)**5,x)
[Out] (-a**3 - 4*a**2*b*x - 6*a*b**2*x**2 - 4*b**3*x**3)/(4*a**4*b**4 + 16*a**3*b
**5*x + 24*a**2*b**6*x**2 + 16*a*b**7*x**3 + 4*b**8*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{x^3}{(a+bx)^5}\,dx = -\frac{4\,b^3x^3+6\,ab^2x^2+4\,a^2bx+a^3}{4\,(b^8x^4+4\,ab^7x^3+6\,a^2b^6x^2+4\,a^3b^5x+a^4b^4)}$$

```
[In] integrate(x^3/(b*x+a)^5,x, algorithm="maxima")
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/(b^8*x^4 + 4*a*b^7*x^3 + 6
*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.30

$$\int \frac{x^3}{(a+bx)^5}\,dx = -\frac{\frac{4}{(bx+a)b}-\frac{6\,a}{(bx+a)^2b}+\frac{4\,a^2}{(bx+a)^3b}-\frac{a^3}{(bx+a)^4b}}{4\,b^3}$$

```
[In] integrate(x^3/(b*x+a)^5,x, algorithm="giac")
[Out] -1/4*(4/((b*x + a)*b) - 6*a/((b*x + a)^2*b) + 4*a^2/((b*x + a)^3*b) - a^3/(
(b*x + a)^4*b))/b^3
```

Mupad [B] (verification not implemented)

Time = 17.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{(a+bx)^5}\,dx=\frac{\frac{3\,a}{2\,{\left(a+b\,x\right)}^2}-\frac{1}{a+b\,x}-\frac{a^2}{{\left(a+b\,x\right)}^3}+\frac{a^3}{4\,{\left(a+b\,x\right)}^4}}{b^4}$$

```
[In] int(x^3/(a + b*x)^5,x)
[Out] ((3*a)/(2*(a + b*x)^2) - 1/(a + b*x) - a^2/(a + b*x)^3 + a^3/(4*(a + b*x)^4
))/b^4
```

3.46 $\int \frac{1}{x(a+bx)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{1}{x(a+bx)}\,dx = -\frac{\log\left(\frac{a+bx}{x}\right)}{a}$$

```
[Out] -1/a*ln((b*x+a)/x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {36, 29, 31}

$$\int \frac{1}{x(a+bx)}\,dx = \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

```
[In] Int[1/(x*(a + b*x)),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{1}{x}\,dx}{a} - \frac{b\int \frac{1}{a+bx}\,dx}{a}\\ &= \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(a+bx)}\,dx = \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

```
[In] Integrate[1/(x*(a + b*x)),x]
[Out] Log[x]/a - Log[a + b*x]/a
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
parallelrisch	$\frac{\ln(x)-\ln(bx+a)}{a}$	16
default	$-\frac{\ln(bx+a)}{a}+\frac{\ln(x)}{a}$	19
norman	$-\frac{\ln(bx+a)}{a}+\frac{\ln(x)}{a}$	19
risch	$-\frac{\ln(bx+a)}{a}+\frac{\ln(-x)}{a}$	21

```
[In] int(1/x/(b*x+a),x,method=_RETURNVERBOSE)
[Out] (ln(x)-ln(b*x+a))/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{1}{x(a+bx)}\,dx = -\frac{\log(bx+a) - \log(x)}{a}$$

```
[In] integrate(1/x/(b*x+a),x, algorithm="fricas")
[Out] -(log(b*x + a) - log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{x(a+bx)}\,dx = \frac{\log(x) - \log\left(\frac{a}{b}+x\right)}{a}$$

```
[In] integrate(1/x/(b*x+a),x)
[Out] (log(x) - log(a/b + x))/a
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(a+bx)}\,dx = -\frac{\log(bx+a)}{a} + \frac{\log(x)}{a}$$

```
[In] integrate(1/x/(b*x+a),x, algorithm="maxima")
[Out] -log(b*x + a)/a + log(x)/a
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{1}{x(a+bx)}\,dx = -\frac{\log(|bx+a|)}{a} + \frac{\log(|x|)}{a}$$

```
[In] integrate(1/x/(b*x+a),x, algorithm="giac")
[Out] -log(abs(b*x + a))/a + log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+bx)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a}$$

```
[In] int(1/(x*(a + b*x)),x)
[Out] -(2*atanh((2*b*x)/a + 1))/a
```

3.47 $\int \frac{1}{x^2(a+bx)} dx$

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{1}{x^2(a+bx)} dx = -\frac{1}{ax} + \frac{b \log \left(\frac{a+bx}{x}\right)}{a^2}$$

```
[Out] -1/a/x+b/a^2*ln((b*x+a)/x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$\int \frac{1}{x^2(a+bx)} dx = -\frac{b \log(x)}{a^2} + \frac{b \log(a+bx)}{a^2} - \frac{1}{ax}$$

```
[In] Int[1/(x^2*(a + b*x)),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{ax^2} - \frac{b}{a^2 x} + \frac{b^2}{a^2(a+bx)}\right) dx \\ &= -\frac{1}{ax} - \frac{b \log(x)}{a^2} + \frac{b \log(a+bx)}{a^2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2(a+bx)}\,dx=-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

```
[In] Integrate[1/(x^2*(a + b*x)),x]
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

method	result	size
parallelrisch	$-\frac{b\ln(x)x-\ln(bx+a)xb+a}{a^2x}$	26
default	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
norman	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
risch	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(-bx-a)}{a^2}$	32

```
[In] int(1/x^2/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -(b*ln(x)*x-ln(b*x+a)*x*b+a)/a^2/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2(a+bx)}\,dx=\frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="fricas")
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^2(a+bx)}\,dx = -\frac{1}{ax} + \frac{b\left(-\log\left(x\right) + \log\left(\frac{a}{b} + x\right)\right)}{a^2}$$

```
[In] integrate(1/x**2/(b*x+a),x)
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2(a+bx)}\,dx = \frac{b\log\left(bx+a\right)}{a^2} - \frac{b\log\left(x\right)}{a^2} - \frac{1}{ax}$$

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="maxima")
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^2(a+bx)}\,dx = \frac{b\log\left(|bx+a|\right)}{a^2} - \frac{b\log\left(|x|\right)}{a^2} - \frac{1}{ax}$$

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="giac")
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 69.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2(a+bx)}\,dx = \frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2} - \frac{1}{a\,x}$$

```
[In] int(1/(x^2*(a + b*x)),x)
[Out] (2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

3.48 $\int \frac{1}{x^3(a+bx)} dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{1}{x^3(a+bx)} dx = -\frac{1}{2ax^2} + \frac{b}{a^2x} - \frac{b^2 \log \left(\frac{a+bx}{x}\right)}{a^3}$$

```
[Out] -1/2/a/x^2+b/a^2/x-b^2/a^3*ln((b*x+a)/x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\int \frac{1}{x^3(a+bx)} dx = \frac{b^2 \log(x)}{a^3} - \frac{b^2 \log(a+bx)}{a^3} + \frac{b}{a^2x} - \frac{1}{2ax^2}$$

```
[In] Int[1/(x^3*(a + b*x)),x]
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{ax^3} - \frac{b}{a^2x^2} + \frac{b^2}{a^3x} - \frac{b^3}{a^3(a+bx)}\right) dx \\ &= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{b^2 \log(x)}{a^3} - \frac{b^2 \log(a+bx)}{a^3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^3(a+bx)}\,dx = -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

```
[In] Integrate[1/(x^3*(a + b*x)),x]
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

method	result	size
default	$-\frac{1}{2a\,x^2}+\frac{b^2\ln(x)}{a^3}+\frac{b}{a^2x}-\frac{b^2\ln(bx+a)}{a^3}$	41
norman	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{x^2}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln(bx+a)}{a^3}$	41
risch	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{x^2}-\frac{b^2\ln(bx+a)}{a^3}+\frac{b^2\ln(-x)}{a^3}$	43
parallelrisch	$\frac{2b^2\ln(x)x^2-2\ln(bx+a)x^2b^2+2bax-a^2}{2a^3x^2}$	44

```
[In] int(1/x^3/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/2/a/x^2+b^2/a^3*ln(x)+b/a^2/x-b^2/a^3*ln(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3(a+bx)}\,dx = -\frac{2\,b^2x^2\log\left(bx+a\right)-2\,b^2x^2\log\left(x\right)-2\,abx+a^2}{2\,a^3x^2}$$

```
[In] integrate(1/x^3/(b*x+a),x, algorithm="fricas")
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^3(a+bx)}\,dx = \frac{-a+2bx}{2a^2x^2} + \frac{b^2\left(\log(x) - \log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

```
[In] integrate(1/x**3/(b*x+a),x)
[Out] (-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3(a+bx)}\,dx = -\frac{b^2\log(bx+a)}{a^3} + \frac{b^2\log(x)}{a^3} + \frac{2\,bx-a}{2\,a^2x^2}$$

```
[In] integrate(1/x^3/(b*x+a),x, algorithm="maxima")
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3(a+bx)}\,dx = -\frac{b^2\log(|bx+a|)}{a^3} + \frac{b^2\log(|x|)}{a^3} + \frac{2\,abx-a^2}{2\,a^3x^2}$$

```
[In] integrate(1/x^3/(b*x+a),x, algorithm="giac")
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3(a+bx)}\,dx = -\frac{\frac{a^2}{2} - a\,b\,x}{a^3\,x^2} - \frac{2\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}$$

```
[In] int(1/(x^3*(a + b*x)),x)
[Out] - (a^2/2 - a*b*x)/(a^3*x^2) - (2*b^2*atanh((2*b*x)/a + 1))/a^3
```

3.49 $\int \frac{1}{x^2(a+bx)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 40

$$\int \frac{1}{x^2(a+bx)^2} dx = \frac{-\frac{2b}{a^2} - \frac{1}{ax}}{a+bx} + \frac{2b\log\left(\frac{a+bx}{x}\right)}{a^3}$$

```
[Out] -(1/a/x+2*b/a^2)/(b*x+a)+2*b/a^3*ln((b*x+a)/x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\int \frac{1}{x^2(a+bx)^2} dx = -\frac{2b\log(x)}{a^3} + \frac{2b\log(a+bx)}{a^3} - \frac{b}{a^2(a+bx)} - \frac{1}{a^2x}$$

```
[In] Int[1/(x^2*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^
3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)}\right) dx \\ &= -\frac{1}{a^2x} - \frac{b}{a^2(a+bx)} - \frac{2b\log(x)}{a^3} + \frac{2b\log(a+bx)}{a^3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2(a+bx)^2}\,dx = -\frac{a\left(\frac{1}{x}+\frac{b}{a+bx}\right)+2b\log(x)-2b\log(a+bx)}{a^3}$$

```
[In] Integrate[1/(x^2*(a + b*x)^2),x]
[Out] -((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{b}{a^2(bx+a)}+\frac{2b\ln(bx+a)}{a^3}-\frac{1}{a^2x}-\frac{2b\ln(x)}{a^3}$	43
risch	$\frac{-\frac{2bx}{a^2}-\frac{1}{a}}{x(bx+a)}-\frac{2b\ln(x)}{a^3}+\frac{2b\ln(-bx-a)}{a^3}$	49
norman	$\frac{\frac{2b^2x^2}{a^3}-\frac{1}{a}}{x(bx+a)}-\frac{2b\ln(x)}{a^3}+\frac{2b\ln(bx+a)}{a^3}$	50
parallelrisch	$-\frac{2b^2\ln(x)x^2-2\ln(bx+a)x^2b^2+2\ln(x)xab-2\ln(bx+a)xab-2b^2x^2+a^2}{a^3x(bx+a)}$	70

```
[In] int(1/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -b/a^2/(b*x+a)+2*b/a^3*ln(b*x+a)-1/a^2/x-2*b/a^3*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^2(a+bx)^2}\,dx = -\frac{2\,abx+a^2-2\,(b^2x^2+abx)\log(bx+a)+2\,(b^2x^2+abx)\log(x)}{a^3bx^2+a^4x}$$

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="fricas")
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*lo
g(x))/(a^3*b*x^2 + a^4*x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2(a+bx)^2} \, dx = \frac{-a-2bx}{a^3x+a^2bx^2} + \frac{2b\left(-\log{(x)} + \log{\left(\frac{a}{b}+x\right)}\right)}{a^3}$$

```
[In] integrate(1/x**2/(b*x+a)**2,x)
[Out] (-a - 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2(a+bx)^2} \, dx = -\frac{2\,bx+a}{a^2bx^2+a^3x} + \frac{2\,b\log{(bx+a)}}{a^3} - \frac{2\,b\log{(x)}}{a^3}$$

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="maxima")
[Out] -(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^2(a+bx)^2} \, dx = -\frac{2\,b\log{\left(\left|-\frac{a}{bx+a}+1\right|\right)}}{a^3} - \frac{b}{(bx+a)a^2} + \frac{b}{a^3\left(\frac{a}{bx+a}-1\right)}$$

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="giac")
[Out] -2*b*log(abs(-a/(b*x + a) + 1))/a^3 - b/((b*x + a)*a^2) + b/(a^3*(a/(b*x +
a) - 1))
```

Mupad [B] (verification not implemented)

Time = 18.45 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2(a+bx)^2}\,dx = \frac{2\,b\,\ln\left(\frac{a+b\,x}{x}\right)}{a^3} - \frac{1}{a\,x\,\left(a+b\,x\right)} - \frac{2\,b}{a^2\,\left(a+b\,x\right)}$$

```
[In] int(1/(x^2*(a + b*x)^2),x)
```

```
[Out] (2*b*log((a + b*x)/x))/a^3 - 1/(a*x*(a + b*x)) - (2*b)/(a^2*(a + b*x))
```

3.50 $\int \frac{1}{x^3(a+bx)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 57

$$\int \frac{1}{x^3(a+bx)^2}\,dx = \frac{\frac{3b^2}{a^3}-\frac{1}{2ax^2}+\frac{3b}{2a^2x}}{a+bx}-\frac{3b^2\log\left(\frac{a+bx}{x}\right)}{a^4}$$

```
[Out] (-1/2/a/x^2+3/2*b/a^2/x+3*b^2/a^3)/(b*x+a)-3*b^2/a^4*ln((b*x+a)/x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\int \frac{1}{x^3(a+bx)^2}\,dx = \frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{b^2}{a^3(a+bx)}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

```
[In] Int[1/(x^3*(a + b*x)^2),x]
```

```
[Out] -1/2*1/(a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x])/a^4
 - (3*b^2*Log[a + b*x])/a^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{a^2x^3} - \frac{2b}{a^3x^2} + \frac{3b^2}{a^4x} - \frac{b^3}{a^3(a+bx)^2} - \frac{3b^3}{a^4(a+bx)}\right) dx$$
$$= -\frac{1}{2a^2x^2} + \frac{2b}{a^3x} + \frac{b^2}{a^3(a+bx)} + \frac{3b^2\log(x)}{a^4} - \frac{3b^2\log(a+bx)}{a^4}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3(a+bx)^2}\,dx = \frac{a\left(-\frac{a}{x^2} + \frac{4b}{x} + \frac{2b^2}{a+bx}\right) + 6b^2\log(x) - 6b^2\log(a+bx)}{2a^4}$$

```
[In] Integrate[1/(x^3*(a + b*x)^2),x]
[Out] (a*(-(a/x^2) + (4*b)/x + (2*b^2)/(a + b*x)) + 6*b^2*Log[x] - 6*b^2*Log[a +
b*x])/(2*a^4)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{3b^2\ln(bx+a)}{a^4} + \frac{b^2}{a^3(bx+a)} - \frac{1}{2a^2x^2} + \frac{2b}{a^3x} + \frac{3b^2\ln(x)}{a^4}$	57
norman	$\frac{-\frac{3b^3x^3}{a^4} - \frac{1}{2a} + \frac{3bx}{2a^2}}{x^2(bx+a)} + \frac{3b^2\ln(x)}{a^4} - \frac{3b^2\ln(bx+a)}{a^4}$	61
risch	$\frac{\frac{3b^2x^2}{a^3} + \frac{3bx}{2a^2} - \frac{1}{2a}}{x^2(bx+a)} + \frac{3b^2\ln(-x)}{a^4} - \frac{3b^2\ln(bx+a)}{a^4}$	63
parallelrisch	$\frac{6\ln(x)x^3b^3 - 6\ln(bx+a)x^3b^3 + 6\ln(x)x^2a\,b^2 - 6\ln(bx+a)x^2a\,b^2 - 6b^3x^3 + 3a^2bx - a^3}{2a^4x^2(bx+a)}$	87

```
[In] int(1/x^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -3*b^2/a^4*ln(b*x+a)+b^2/a^3/(b*x+a)-1/2/a^2/x^2+2*b/a^3/x+3*b^2/a^4*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^3(a+bx)^2}\,dx = \frac{6\,ab^2x^2 + 3\,a^2bx - a^3 - 6\,(b^3x^3 + ab^2x^2)\log(bx+a) + 6\,(b^3x^3 + ab^2x^2)\log(x)}{2\,(a^4bx^3 + a^5x^2)}$$

```
[In] integrate(1/x^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*log(b*x + a) +
 6*(b^3*x^3 + a*b^2*x^2)*log(x))/(a^4*b*x^3 + a^5*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3(a+bx)^2}\,dx = \frac{-a^2 + 3abx + 6b^2x^2}{2a^4x^2 + 2a^3bx^3} + \frac{3b^2\left(\log(x) - \log\left(\frac{a}{b} + x\right)\right)}{a^4}$$

```
[In] integrate(1/x**3/(b*x+a)**2,x)
```

```
[Out] (-a**2 + 3*a*b*x + 6*b**2*x**2)/(2*a**4*x**2 + 2*a**3*b*x**3) + 3*b**2*(log
(x) - log(a/b + x))/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^3(a+bx)^2}\,dx = \frac{6\,b^2x^2 + 3\,abx - a^2}{2\,(a^3bx^3 + a^4x^2)} - \frac{3\,b^2\log(bx+a)}{a^4} + \frac{3\,b^2\log(x)}{a^4}$$

```
[In] integrate(1/x^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*log(b*x + a)/
a^4 + 3*b^2*log(x)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^3(a+bx)^2}\,dx = \frac{3\,b^2 \log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^4} + \frac{b^2}{(bx+a)a^3} - \frac{\frac{6\,ab^2}{bx+a} - 5\,b^2}{2\,a^4\left(\frac{a}{bx+a}-1\right)^2}$$

```
[In] integrate(1/x^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3*b^2*log(abs(-a/(b*x + a) + 1))/a^4 + b^2/((b*x + a)*a^3) - 1/2*(6*a*b^2/(
b*x + a) - 5*b^2)/(a^4*(a/(b*x + a) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(a+bx)^2}\,dx = \frac{\frac{3\,b^2\,x^2}{a^3} - \frac{1}{2\,a} + \frac{3\,b\,x}{2\,a^2}}{b\,x^3 + a\,x^2} - \frac{6\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4}$$

```
[In] int(1/(x^3*(a + b*x)^2),x)
```

```
[Out] ((3*b^2*x^2)/a^3 - 1/(2*a) + (3*b*x)/(2*a^2))/(a*x^2 + b*x^3) - (6*b^2*atan
h((2*b*x)/a + 1))/a^4
```

3.51 $\int \frac{1}{x(a+bx)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{1}{x(a+bx)^3} dx = \frac{\frac{3}{2a} + \frac{bx}{a^2}}{(a+bx)^2} - \frac{\log\left(\frac{a+bx}{x}\right)}{a^3}$$

```
[Out] (3/2/a+b*x/a^2)/(b*x+a)^2-1/a^3*ln((b*x+a)/x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\int \frac{1}{x(a+bx)^3} dx = -\frac{\log(a+bx)}{a^3} + \frac{\log(x)}{a^3} + \frac{1}{a^2(a+bx)} + \frac{1}{2a(a+bx)^2}$$

```
[In] Int[1/(x*(a + b*x)^3),x]
[Out] 1/(2*a*(a + b*x)^2) + 1/(a^2*(a + b*x)) + Log[x]/a^3 - Log[a + b*x]/a^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{a^3x} - \frac{b}{a(a+bx)^3} - \frac{b}{a^2(a+bx)^2} - \frac{b}{a^3(a+bx)}\right) dx \\ &= \frac{1}{2a(a+bx)^2} + \frac{1}{a^2(a+bx)} + \frac{\log(x)}{a^3} - \frac{\log(a+bx)}{a^3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{x(a+bx)^3}\,dx = \frac{\frac{a(3a+2bx)}{(a+bx)^2}+2\log(x)-2\log(a+bx)}{2a^3}$$

```
[In] Integrate[1/(x*(a + b*x)^3),x]
[Out] ((a*(3*a + 2*b*x))/(a + b*x)^2 + 2*Log[x] - 2*Log[a + b*x])/(2*a^3)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

method	result	size
risch	$\frac{\frac{3}{2a}+\frac{bx}{a^2}}{(bx+a)^2}+\frac{\ln(-x)}{a^3}-\frac{\ln(bx+a)}{a^3}$	41
default	$-\frac{\ln(bx+a)}{a^3}+\frac{1}{a^2(bx+a)}+\frac{1}{2a(bx+a)^2}+\frac{\ln(x)}{a^3}$	42
norman	$\frac{-\frac{2bx}{a^2}-\frac{3b^2x^2}{2a^3}}{(bx+a)^2}+\frac{\ln(x)}{a^3}-\frac{\ln(bx+a)}{a^3}$	46
parallelrisch	$\frac{2b^2\ln(x)x^2-2\ln(bx+a)x^2b^2+4\ln(x)xab-4\ln(bx+a)xab-3b^2x^2+2\ln(x)a^2-2a^2\ln(bx+a)-4bax}{2a^3(bx+a)^2}$	87

```
[In] int(1/x/(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] (3/2/a+b*x/a^2)/(b*x+a)^2+1/a^3*ln(-x)-1/a^3*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(38) = 76.

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

$$\int \frac{1}{x(a+bx)^3}\,dx = \frac{2\,abx+3\,a^2-2\,(b^2x^2+2\,abx+a^2)\log(bx+a)+2\,(b^2x^2+2\,abx+a^2)\log(x)}{2\,(a^3b^2x^2+2\,a^4bx+a^5)}$$

```
[In] integrate(1/x/(b*x+a)^3,x, algorithm="fricas")
[Out] 1/2*(2*a*b*x + 3*a^2 - 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) + 2*(b^2*x^
2 + 2*a*b*x + a^2)*log(x))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{1}{x(a+bx)^3}\,dx = \frac{3a+2bx}{2a^4+4a^3bx+2a^2b^2x^2} + \frac{\log(x) - \log\left(\frac{a}{b}+x\right)}{a^3}$$

```
[In] integrate(1/x/(b*x+a)**3,x)
```

```
[Out] (3*a + 2*b*x)/(2*a**4 + 4*a**3*b*x + 2*a**2*b**2*x**2) + (log(x) - log(a/b
+ x))/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{1}{x(a+bx)^3}\,dx = \frac{2\,bx+3\,a}{2\,(a^2b^2x^2+2\,a^3bx+a^4)} - \frac{\log(bx+a)}{a^3} + \frac{\log(x)}{a^3}$$

```
[In] integrate(1/x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x + 3*a)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) - log(b*x + a)/a^3 + log(
x)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \frac{1}{x(a+bx)^3}\,dx = -\frac{\log(|bx+a|)}{a^3} + \frac{\log(|x|)}{a^3} + \frac{2\,abx+3\,a^2}{2\,(bx+a)^2a^3}$$

```
[In] integrate(1/x/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^3 + log(abs(x))/a^3 + 1/2*(2*a*b*x + 3*a^2)/((b*x + a)
^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 16.67 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \frac{1}{x(a+bx)^3}\,dx = \frac{\frac{1}{a^2+b\,x\,a}-\frac{\ln\left(\frac{a+b\,x}{x}\right)}{a^2}}{a}+\frac{1}{2\,a\,{\left(a+b\,x\right)}^2}$$

```
[In] int(1/(x*(a + b*x)^3),x)
```

```
[Out] (1/(a^2 + a*b*x) - log((a + b*x)/x)/a^2)/a + 1/(2*a*(a + b*x)^2)
```

3.52 $\int \frac{1}{x^2(a+bx)^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 51

$$\int \frac{1}{x^2(a+bx)^3} \, dx = \frac{-\frac{9b}{2a^2} - \frac{1}{ax} - \frac{3b^2x}{a^3}}{(a+bx)^2} + \frac{3b\log\left(\frac{a+bx}{x}\right)}{a^4}$$

```
[Out] -(1/a/x+9/2*b/a^2+3*b^2*x/a^3)/(b*x+a)^2+3*b/a^4*ln((b*x+a)/x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\int \frac{1}{x^2(a+bx)^3} \, dx = -\frac{3b\log(x)}{a^4} + \frac{3b\log(a+bx)}{a^4} - \frac{2b}{a^3(a+bx)} - \frac{1}{a^3x} - \frac{b}{2a^2(a+bx)^2}$$

```
[In] Int[1/(x^2*(a + b*x)^3),x]
```

```
[Out] -(1/(a^3*x)) - b/(2*a^2*(a + b*x)^2) - (2*b)/(a^3*(a + b*x)) - (3*b*Log[x])
/a^4 + (3*b*Log[a + b*x])/a^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{a^3x^2} - \frac{3b}{a^4x} + \frac{b^2}{a^2(a+bx)^3} + \frac{2b^2}{a^3(a+bx)^2} + \frac{3b^2}{a^4(a+bx)}\right) dx \\ &= -\frac{1}{a^3x} - \frac{b}{2a^2(a+bx)^2} - \frac{2b}{a^3(a+bx)} - \frac{3b\log(x)}{a^4} + \frac{3b\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2(a+bx)^3}\,dx = -\frac{\frac{a(2a^2+9abx+6b^2x^2)}{x(a+bx)^2} + 6b\log(x) - 6b\log(a+bx)}{2a^4}$$

```
[In] Integrate[1/(x^2*(a + b*x)^3),x]
[Out] -1/2*((a*(2*a^2 + 9*a*b*x + 6*b^2*x^2))/(x*(a + b*x)^2) + 6*b*Log[x] - 6*b*
Log[a + b*x])/a^4
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

method	result	si
default	$-\frac{b}{2a^2(bx+a)^2} + \frac{3b\ln(bx+a)}{a^4} - \frac{2b}{a^3(bx+a)} - \frac{1}{a^3x} - \frac{3b\ln(x)}{a^4}$	5
risch	$\frac{-\frac{3b^2x^2}{a^3} - \frac{9bx}{2a^2} - \frac{1}{a}}{x(bx+a)^2} - \frac{3b\ln(x)}{a^4} + \frac{3b\ln(-bx-a)}{a^4}$	6
norman	$\frac{-\frac{1}{a} + \frac{6b^2x^2}{a^3} + \frac{9b^3x^3}{2a^4}}{x(bx+a)^2} - \frac{3b\ln(x)}{a^4} + \frac{3b\ln(bx+a)}{a^4}$	6
parallelrisch	$-\frac{6\ln(x)x^3b^3 - 6\ln(bx+a)x^3b^3 + 12\ln(x)x^2ab^2 - 12\ln(bx+a)x^2ab^2 - 9b^3x^3 + 6\ln(x)xa^2b - 6\ln(bx+a)xa^2b - 12ab^2x^2 + 2a^3}{2a^4x(bx+a)^2}$	1

```
[In] int(1/x^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] -1/2*b/a^2/(b*x+a)^2+3/a^4*b*ln(b*x+a)-2*b/a^3/(b*x+a)-1/a^3/x-3/a^4*b*ln(x
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(50) = 100$.

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.14

$$\int \frac{1}{x^2(a+bx)^3}\,dx = -\frac{6\,ab^2x^2 + 9\,a^2bx + 2\,a^3 - 6\,(b^3x^3 + 2\,ab^2x^2 + a^2bx)\log(bx+a) + 6\,(b^3x^3 + 2\,ab^2x^2 + a^2bx)\log(x)}{2\,(a^4b^2x^3 + 2\,a^5bx^2 + a^6x)}$$

```
[In] integrate(1/x^2/(b*x+a)^3,x, algorithm="fricas")
[Out] -1/2*(6*a*b^2*x^2 + 9*a^2*b*x + 2*a^3 - 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)
*log(b*x + a) + 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*log(x))/(a^4*b^2*x^3 +
2*a^5*b*x^2 + a^6*x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2(a+bx)^3}\,dx = \frac{-2a^2 - 9abx - 6b^2x^2}{2a^5x + 4a^4bx^2 + 2a^3b^2x^3} + \frac{3b\left(-\log(x) + \log\left(\frac{a}{b} + x\right)\right)}{a^4}$$

```
[In] integrate(1/x**2/(b*x+a)**3,x)
[Out] (-2*a**2 - 9*a*b*x - 6*b**2*x**2)/(2*a**5*x + 4*a**4*b*x**2 + 2*a**3*b**2*x
**3) + 3*b*(-log(x) + log(a/b + x))/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^2(a+bx)^3}\,dx = -\frac{6\,b^2x^2 + 9\,abx + 2\,a^2}{2\,(a^3b^2x^3 + 2\,a^4bx^2 + a^5x)} + \frac{3\,b\log(bx+a)}{a^4} - \frac{3\,b\log(x)}{a^4}$$

```
[In] integrate(1/x^2/(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2*(6*b^2*x^2 + 9*a*b*x + 2*a^2)/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x) + 3*
b*log(b*x + a)/a^4 - 3*b*log(x)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^2(a+bx)^3}\,dx = \frac{3\,b\log\left(|bx+a|\right)}{a^4} - \frac{3\,b\log\left(|x|\right)}{a^4} - \frac{6\,ab^2x^2+9\,a^2bx+2\,a^3}{2\,(bx+a)^2a^4x}$$

```
[In] integrate(1/x^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3*b*log(abs(b*x + a))/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(6*a*b^2*x^2 + 9*a^2*
b*x + 2*a^3)/((b*x + a)^2*a^4*x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^2(a+bx)^3}\,dx = \frac{6\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4} - \frac{\frac{1}{a}+\frac{3\,b^2\,x^2}{a^3}+\frac{9\,b\,x}{2\,a^2}}{a^2\,x+2\,a\,b\,x^2+b^2\,x^3}$$

```
[In] int(1/(x^2*(a + b*x)^3),x)
```

```
[Out] (6*b*atanh((2*b*x)/a + 1))/a^4 - (1/a + (3*b^2*x^2)/a^3 + (9*b*x)/(2*a^2))/
(a^2*x + b^2*x^3 + 2*a*b*x^2)
```

3.53 $\int \frac{1}{x^3(a+bx)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 64

$$\int \frac{1}{x^3(a+bx)^3} dx = \frac{\frac{9b^2}{a^3} - \frac{1}{2ax^2} + \frac{2b}{a^2x} + \frac{6b^3x}{a^4}}{(a+bx)^2} - \frac{6b^2 \log\left(\frac{a+bx}{x}\right)}{a^5}$$

```
[Out] (-1/2/a/x^2+2*b/a^2/x+9*b^2/a^3+6*b^3*x/a^4)/(b*x+a)^2-6*b^2/a^5*ln((b*x+a)
/x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$\int \frac{1}{x^3(a+bx)^3} dx = \frac{6b^2 \log(x)}{a^5} - \frac{6b^2 \log(a+bx)}{a^5} + \frac{3b^2}{a^4(a+bx)} + \frac{3b}{a^4x} + \frac{b^2}{2a^3(a+bx)^2} - \frac{1}{2a^3x^2}$$

```
[In] Int[1/(x^3*(a + b*x)^3),x]
```

```
[Out] -1/2*1/(a^3*x^2) + (3*b)/(a^4*x) + b^2/(2*a^3*(a + b*x)^2) + (3*b^2)/(a^4*(
a + b*x)) + (6*b^2*Log[x])/a^5 - (6*b^2*Log[a + b*x])/a^5
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{a^3x^3} - \frac{3b}{a^4x^2} + \frac{6b^2}{a^5x} - \frac{b^3}{a^3(a+bx)^3} - \frac{3b^3}{a^4(a+bx)^2} - \frac{6b^3}{a^5(a+bx)}\right) dx$$
$$= -\frac{1}{2a^3x^2} + \frac{3b}{a^4x} + \frac{b^2}{2a^3(a+bx)^2} + \frac{3b^2}{a^4(a+bx)} + \frac{6b^2\log(x)}{a^5} - \frac{6b^2\log(a+bx)}{a^5}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3(a+bx)^3}\,dx = \frac{\frac{a(-a^3+4a^2bx+18ab^2x^2+12b^3x^3)}{x^2(a+bx)^2} + 12b^2\log(x) - 12b^2\log(a+bx)}{2a^5}$$

```
[In] Integrate[1/(x^3*(a + b*x)^3),x]
```

```
[Out] ((a*(-a^3 + 4*a^2*b*x + 18*a*b^2*x^2 + 12*b^3*x^3))/(x^2*(a + b*x)^2) + 12*
b^2*Log[x] - 12*b^2*Log[a + b*x])/(2*a^5)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

method	result
norman	$\frac{-\frac{9b^4x^4}{a^5}-\frac{1}{2a}+\frac{2bx}{a^2}-\frac{12b^3x^3}{a^4}}{x^2(bx+a)^2}+\frac{6b^2\ln(x)}{a^5}-\frac{6b^2\ln(bx+a)}{a^5}$
default	$-\frac{6b^2\ln(bx+a)}{a^5}+\frac{3b^2}{a^4(bx+a)}+\frac{b^2}{2a^3(bx+a)^2}-\frac{1}{2a^3x^2}+\frac{6b^2\ln(x)}{a^5}+\frac{3b}{a^4x}$
risch	$\frac{\frac{6b^3x^3}{a^4}+\frac{9b^2x^2}{a^3}+\frac{2bx}{a^2}-\frac{1}{2a}}{x^2(bx+a)^2}-\frac{6b^2\ln(bx+a)}{a^5}+\frac{6b^2\ln(-x)}{a^5}$
parallelrisch	$\frac{12\ln(x)x^4b^6-12\ln(bx+a)x^4b^6+24\ln(x)x^3a\,b^5-24\ln(bx+a)x^3a\,b^5+12\ln(x)x^2a^2b^4-12\ln(bx+a)x^2a^2b^4+12x^3a\,b^5+18x^2a^2b^4+}{2a^5b^2x^2(bx+a)^2}$

```
[In] int(1/x^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-9*b^4/a^5*x^4-1/2/a+2*b*x/a^2-12*b^3*x^3/a^4)/x^2/(b*x+a)^2+6/a^5*b^2*ln(
x)-6/a^5*b^2*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(63) = 126.

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.03

$$\int \frac{1}{x^3(a+bx)^3}\,dx$$
$$= \frac{12\,ab^3x^3 + 18\,a^2b^2x^2 + 4\,a^3bx - a^4 - 12\,(b^4x^4 + 2\,ab^3x^3 + a^2b^2x^2)\log(bx+a) + 12\,(b^4x^4 + 2\,ab^3x^3 + a^2b^2}{2\,(a^5b^2x^4 + 2\,a^6bx^3 + a^7x^2)}$$

```
[In] integrate(1/x^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4 - 12*(b^4*x^4 + 2*a*b^
3*x^3 + a^2*b^2*x^2)*log(b*x + a) + 12*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2
)*log(x))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3(a+bx)^3}\,dx = \frac{-a^3 + 4a^2bx + 18ab^2x^2 + 12b^3x^3}{2a^6x^2 + 4a^5bx^3 + 2a^4b^2x^4} + \frac{6b^2\left(\log(x) - \log\left(\frac{a}{b} + x\right)\right)}{a^5}$$

```
[In] integrate(1/x**3/(b*x+a)**3,x)
```

```
[Out] (-a**3 + 4*a**2*b*x + 18*a*b**2*x**2 + 12*b**3*x**3)/(2*a**6*x**2 + 4*a**5*
b*x**3 + 2*a**4*b**2*x**4) + 6*b**2*(log(x) - log(a/b + x))/a**5
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^3(a+bx)^3}\,dx = \frac{12\,b^3x^3 + 18\,ab^2x^2 + 4\,a^2bx - a^3}{2\,(a^4b^2x^4 + 2\,a^5bx^3 + a^6x^2)} - \frac{6\,b^2\log(bx+a)}{a^5} + \frac{6\,b^2\log(x)}{a^5}$$

```
[In] integrate(1/x^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*b^3*x^3 + 18*a*b^2*x^2 + 4*a^2*b*x - a^3)/(a^4*b^2*x^4 + 2*a^5*b*x^
3 + a^6*x^2) - 6*b^2*log(b*x + a)/a^5 + 6*b^2*log(x)/a^5
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^3(a+bx)^3}\,dx = -\frac{6\,b^2\log\left(|bx+a|\right)}{a^5} + \frac{6\,b^2\log\left(|x|\right)}{a^5} + \frac{12\,b^3x^3 + 18\,ab^2x^2 + 4\,a^2bx - a^3}{2\,(bx^2+ax)^2a^4}$$

```
[In] integrate(1/x^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -6*b^2*log(abs(b*x + a))/a^5 + 6*b^2*log(abs(x))/a^5 + 1/2*(12*b^3*x^3 + 18
*a*b^2*x^2 + 4*a^2*b*x - a^3)/((b*x^2 + a*x)^2*a^4)
```

Mupad [B] (verification not implemented)

Time = 16.78 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3(a+bx)^3}\,dx = \frac{\frac{9\,b^2\,x^2}{a^3} - \frac{1}{2\,a} + \frac{6\,b^3\,x^3}{a^4} + \frac{2\,b\,x}{a^2}}{a^2\,x^2 + 2\,a\,b\,x^3 + b^2\,x^4} - \frac{12\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5}$$

```
[In] int(1/(x^3*(a + b*x)^3),x)
```

```
[Out] ((9*b^2*x^2)/a^3 - 1/(2*a) + (6*b^3*x^3)/a^4 + (2*b*x)/a^2)/(a^2*x^2 + b^2*
x^4 + 2*a*b*x^3) - (12*b^2*atanh((2*b*x)/a + 1))/a^5
```

3.54 $\int \frac{1}{x(a+bx)^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 51

$$\int \frac{1}{x(a+bx)^4}\,dx=\frac{\frac{11}{6a}+\frac{5bx}{2a^2}+\frac{b^2x^2}{a^3}}{(a+bx)^3}-\frac{\log\left(\frac{a+bx}{x}\right)}{a^4}$$

```
[Out] (11/6/a+5/2*b*x/a^2+b^2*x^2/a^3)/(b*x+a)^3-1/a^4*ln((b*x+a)/x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\int \frac{1}{x(a+bx)^4}\,dx=-\frac{\log(a+bx)}{a^4}+\frac{\log(x)}{a^4}+\frac{1}{a^3(a+bx)}+\frac{1}{2a^2(a+bx)^2}+\frac{1}{3a(a+bx)^3}$$

```
[In] Int[1/(x*(a + b*x)^4),x]
```

```
[Out] 1/(3*a*(a + b*x)^3) + 1/(2*a^2*(a + b*x)^2) + 1/(a^3*(a + b*x)) + Log[x]/a^
4 - Log[a + b*x]/a^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{a^4x} - \frac{b}{a(a+bx)^4} - \frac{b}{a^2(a+bx)^3} - \frac{b}{a^3(a+bx)^2} - \frac{b}{a^4(a+bx)}\right) dx \\
&= \frac{1}{3a(a+bx)^3} + \frac{1}{2a^2(a+bx)^2} + \frac{1}{a^3(a+bx)} + \frac{\log(x)}{a^4} - \frac{\log(a+bx)}{a^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$
\int \frac{1}{x(a+bx)^4}\,dx = \frac{\frac{a(11a^2+15abx+6b^2x^2)}{(a+bx)^3} + 6\log(x) - 6\log(a+bx)}{6a^4}
$$

```
[In] Integrate[1/(x*(a + b*x)^4),x]
```

```
[Out] ((a*(11*a^2 + 15*a*b*x + 6*b^2*x^2))/(a + b*x)^3 + 6*Log[x] - 6*Log[a + b*x
])/(6*a^4)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

method	result
risch	$\frac{\frac{11}{6a}+\frac{5bx}{2a^2}+\frac{b^2x^2}{a^3}}{(bx+a)^3}-\frac{\ln(bx+a)}{a^4}+\frac{\ln(-x)}{a^4}$
default	$-\frac{\ln(bx+a)}{a^4}+\frac{1}{a^3(bx+a)}+\frac{1}{2a^2(bx+a)^2}+\frac{1}{3a(bx+a)^3}+\frac{\ln(x)}{a^4}$
norman	$\frac{-\frac{3bx}{a^2}-\frac{9b^2x^2}{2a^3}-\frac{11b^3x^3}{6a^4}}{(bx+a)^3}+\frac{\ln(x)}{a^4}-\frac{\ln(bx+a)}{a^4}$
parallelrisch	$\frac{6\ln(x)x^3b^3-6\ln(bx+a)x^3b^3+18\ln(x)x^2a\,b^2-18\ln(bx+a)x^2a\,b^2-11b^3x^3+18\ln(x)x\,a^2b-18\ln(bx+a)x\,a^2b-27a\,b^2x^2+6\ln(x)a}{6a^4(bx+a)^3}$

```
[In] int(1/x/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (11/6/a+5/2*b*x/a^2+b^2*x^2/a^3)/(b*x+a)^3-1/a^4*ln(b*x+a)+1/a^4*ln(-x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(49) = 98.

Time = 0.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.43

$$\int \frac{1}{x(a+bx)^4}\,dx$$
$$= \frac{6\,ab^2x^2 + 15\,a^2bx + 11\,a^3 - 6\,(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3)\log\left(bx + a\right) + 6\,(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a}{6\,(a^4b^3x^3 + 3\,a^5b^2x^2 + 3\,a^6bx + a^7)}$$

```
[In] integrate(1/x/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*b^2*x^2 + 15*a^2*b*x + 11*a^3 - 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b
*x + a^3)*log(b*x + a) + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*log(x)
)/(a^4*b^3*x^3 + 3*a^5*b^2*x^2 + 3*a^6*b*x + a^7)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{1}{x(a+bx)^4}\,dx = \frac{11a^2 + 15abx + 6b^2x^2}{6a^6 + 18a^5bx + 18a^4b^2x^2 + 6a^3b^3x^3} + \frac{\log\left(x\right) - \log\left(\frac{a}{b} + x\right)}{a^4}$$

```
[In] integrate(1/x/(b*x+a)**4,x)
```

```
[Out] (11*a**2 + 15*a*b*x + 6*b**2*x**2)/(6*a**6 + 18*a**5*b*x + 18*a**4*b**2*x**
2 + 6*a**3*b**3*x**3) + (log(x) - log(a/b + x))/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

$$\int \frac{1}{x(a+bx)^4}\,dx = \frac{6\,b^2x^2 + 15\,abx + 11\,a^2}{6\,(a^3b^3x^3 + 3\,a^4b^2x^2 + 3\,a^5bx + a^6)} - \frac{\log\left(bx + a\right)}{a^4} + \frac{\log\left(x\right)}{a^4}$$

```
[In] integrate(1/x/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*b^2*x^2 + 15*a*b*x + 11*a^2)/(a^3*b^3*x^3 + 3*a^4*b^2*x^2 + 3*a^5*b*
x + a^6) - log(b*x + a)/a^4 + log(x)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{1}{x(a+bx)^4}\,dx = -\frac{\log\left(|bx+a|\right)}{a^4} + \frac{\log\left(|x|\right)}{a^4} + \frac{6\,ab^2x^2 + 15\,a^2bx + 11\,a^3}{6\,(bx+a)^3a^4}$$

```
[In] integrate(1/x/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^4 + log(abs(x))/a^4 + 1/6*(6*a*b^2*x^2 + 15*a^2*b*x +
11*a^3)/((b*x + a)^3*a^4)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(a+bx)^4}\,dx = \frac{\frac{\frac{1}{a^2+b\,x\,a} - \frac{\ln\left(\frac{a+b\,x}{x}\right)}{a^2}}{a} + \frac{1}{2\,a\,(a+b\,x)^2}}{a} + \frac{1}{3\,a\,(a+b\,x)^3}$$

```
[In] int(1/(x*(a + b*x)^4),x)
```

```
[Out] ((1/(a^2 + a*b*x) - log((a + b*x)/x)/a^2)/a + 1/(2*a*(a + b*x)^2))/a + 1/(3
*a*(a + b*x)^3)
```

3.55 $\int \frac{1}{x^2(a+bx)^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 62

$$\int \frac{1}{x^2(a+bx)^4}\,dx=\frac{-\frac{22b}{3a^2}-\frac{1}{ax}-\frac{10b^2x}{a^3}-\frac{4b^3x^2}{a^4}}{(a+bx)^3}+\frac{4b\log\left(\frac{a+bx}{x}\right)}{a^5}$$

```
[Out] -(1/a/x+22/3*b/a^2+10*b^2*x/a^3+4*b^3*x^2/a^4)/(b*x+a)^3+4*b/a^5*ln((b*x+a)
/x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\int \frac{1}{x^2(a+bx)^4}\,dx=-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}-\frac{3b}{a^4(a+bx)}-\frac{1}{a^4x}-\frac{b}{a^3(a+bx)^2}-\frac{b}{3a^2(a+bx)^3}$$

```
[In] Int[1/(x^2*(a + b*x)^4),x]
```

```
[Out] -(1/(a^4*x)) - b/(3*a^2*(a + b*x)^3) - b/(a^3*(a + b*x)^2) - (3*b)/(a^4*(a
+ b*x)) - (4*b*Log[x])/a^5 + (4*b*Log[a + b*x])/a^5
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{a^4x^2} - \frac{4b}{a^5x} + \frac{b^2}{a^2(a+bx)^4} + \frac{2b^2}{a^3(a+bx)^3} + \frac{3b^2}{a^4(a+bx)^2} + \frac{4b^2}{a^5(a+bx)}\right) dx \\ &= -\frac{1}{a^4x} - \frac{b}{3a^2(a+bx)^3} - \frac{b}{a^3(a+bx)^2} - \frac{3b}{a^4(a+bx)} - \frac{4b\log(x)}{a^5} + \frac{4b\log(a+bx)}{a^5} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2(a+bx)^4}\,dx = -\frac{\frac{a(3a^3+22a^2bx+30ab^2x^2+12b^3x^3)}{x(a+bx)^3} + 12b\log(x) - 12b\log(a+bx)}{3a^5}$$

```
[In] Integrate[1/(x^2*(a + b*x)^4),x]
```

```
[Out] -1/3*((a*(3*a^3 + 22*a^2*b*x + 30*a*b^2*x^2 + 12*b^3*x^3))/(x*(a + b*x)^3)
+ 12*b*Log[x] - 12*b*Log[a + b*x])/a^5
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

method	result
default	$-\frac{b}{3a^2(bx+a)^3} + \frac{4b\ln(bx+a)}{a^5} - \frac{3b}{a^4(bx+a)} - \frac{b}{a^3(bx+a)^2} - \frac{1}{a^4x} - \frac{4b\ln(x)}{a^5}$
risch	$\frac{-\frac{4b^3x^3}{a^4} - \frac{10b^2x^2}{a^3} - \frac{22bx}{3a^2} - \frac{1}{a}}{x(bx+a)^3} + \frac{4b\ln(-bx-a)}{a^5} - \frac{4b\ln(x)}{a^5}$
norman	$\frac{-\frac{1}{a} + \frac{12b^2x^2}{a^3} + \frac{18b^3x^3}{a^4} + \frac{22b^4x^4}{3a^5}}{x(bx+a)^3} - \frac{4b\ln(x)}{a^5} + \frac{4b\ln(bx+a)}{a^5}$
parallelrisch	$-\frac{12\ln(x)x^4b^4 - 12\ln(bx+a)x^4b^4 + 36\ln(x)x^3a\,b^3 - 36\ln(bx+a)x^3a\,b^3 - 22b^4x^4 + 36\ln(x)x^2a^2b^2 - 36\ln(bx+a)x^2a^2b^2 - 54a\,b^3x^3 +}{3a^5x(bx+a)^3}$

```
[In] int(1/x^2/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b/a^2/(b*x+a)^3+4/a^5*b*ln(b*x+a)-3*b/a^4/(b*x+a)-b/a^3/(b*x+a)^2-1/a^
4/x-4/a^5*b*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(61) = 122$.

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.47

$$\int \frac{1}{x^2(a+bx)^4}\,dx = -\frac{12\,ab^3x^3 + 30\,a^2b^2x^2 + 22\,a^3bx + 3\,a^4 - 12\,(b^4x^4 + 3\,ab^3x^3 + 3\,a^2b^2x^2 + a^3bx)\log(bx+a) + 12\,(b^4x^4 + \cdots}{3\,(a^5b^3x^4 + 3\,a^6b^2x^3 + 3\,a^7bx^2 + a^8x)}$$

```
[In] integrate(1/x^2/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*b^3*x^3 + 30*a^2*b^2*x^2 + 22*a^3*b*x + 3*a^4 - 12*(b^4*x^4 + 3*
a*b^3*x^3 + 3*a^2*b^2*x^2 + a^3*b*x)*log(b*x + a) + 12*(b^4*x^4 + 3*a*b^3*x
^3 + 3*a^2*b^2*x^2 + a^3*b*x)*log(x))/(a^5*b^3*x^4 + 3*a^6*b^2*x^3 + 3*a^7*
b*x^2 + a^8*x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^2(a+bx)^4}\,dx = \frac{-3a^3 - 22a^2bx - 30ab^2x^2 - 12b^3x^3}{3a^7x + 9a^6bx^2 + 9a^5b^2x^3 + 3a^4b^3x^4} + \frac{4b\left(-\log(x) + \log\left(\frac{a}{b} + x\right)\right)}{a^5}$$

```
[In] integrate(1/x**2/(b*x+a)**4,x)
```

```
[Out] (-3*a**3 - 22*a**2*b*x - 30*a*b**2*x**2 - 12*b**3*x**3)/(3*a**7*x + 9*a**6*
b*x**2 + 9*a**5*b**2*x**3 + 3*a**4*b**3*x**4) + 4*b*(-log(x) + log(a/b + x)
)/a**5
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^2(a+bx)^4}\,dx = -\frac{12\,b^3x^3 + 30\,ab^2x^2 + 22\,a^2bx + 3\,a^3}{3\,(a^4b^3x^4 + 3\,a^5b^2x^3 + 3\,a^6bx^2 + a^7x)} + \frac{4\,b\log(bx+a)}{a^5} - \frac{4\,b\log(x)}{a^5}$$

```
[In] integrate(1/x^2/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(12*b^3*x^3 + 30*a*b^2*x^2 + 22*a^2*b*x + 3*a^3)/(a^4*b^3*x^4 + 3*a^5*
b^2*x^3 + 3*a^6*b*x^2 + a^7*x) + 4*b*log(b*x + a)/a^5 - 4*b*log(x)/a^5
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^2(a+bx)^4}\,dx = \frac{4\,b\log\left(|bx+a|\right)}{a^5} - \frac{4\,b\log\left(|x|\right)}{a^5} - \frac{12\,ab^3x^3 + 30\,a^2b^2x^2 + 22\,a^3bx + 3\,a^4}{3\,(bx+a)^3a^5x}$$

```
[In] integrate(1/x^2/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 4*b*log(abs(b*x + a))/a^5 - 4*b*log(abs(x))/a^5 - 1/3*(12*a*b^3*x^3 + 30*a^
2*b^2*x^2 + 22*a^3*b*x + 3*a^4)/((b*x + a)^3*a^5*x)
```

Mupad [B] (verification not implemented)

Time = 16.43 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^2(a+bx)^4}\,dx = \frac{8\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5} - \frac{\frac{1}{a} + \frac{10\,b^2\,x^2}{a^3} + \frac{4\,b^3\,x^3}{a^4} + \frac{22\,b\,x}{3\,a^2}}{a^3\,x + 3\,a^2\,b\,x^2 + 3\,a\,b^2\,x^3 + b^3\,x^4}$$

```
[In] int(1/(x^2*(a + b*x)^4),x)
```

```
[Out] (8*b*atanh((2*b*x)/a + 1))/a^5 - (1/a + (10*b^2*x^2)/a^3 + (4*b^3*x^3)/a^4
+ (22*b*x)/(3*a^2))/(a^3*x + b^3*x^4 + 3*a^2*b*x^2 + 3*a*b^2*x^3)
```

3.56 $\int \frac{1}{x^3(a+bx)^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 79

$$\int \frac{1}{x^3(a+bx)^4}\,dx = \frac{\frac{55b^2}{3a^3}-\frac{1}{2ax^2}+\frac{5b}{2a^2x}+\frac{25b^3x}{a^4}+\frac{10b^4x^2}{a^5}}{(a+bx)^3}-\frac{10b^2\log\left(\frac{a+bx}{x}\right)}{a^6}$$

```
[Out] (-1/2/a/x^2+5/2*b/a^2/x+55/3*b^2/a^3+25*b^3*x/a^4+10*b^4*x^2/a^5)/(b*x+a)^3
-10*b^2/a^6*ln((b*x+a)/x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\int \frac{1}{x^3(a+bx)^4}\,dx = \frac{10b^2\log(x)}{a^6}-\frac{10b^2\log(a+bx)}{a^6}+\frac{6b^2}{a^5(a+bx)}+\frac{4b}{a^5x}+\frac{3b^2}{2a^4(a+bx)^2}-\frac{1}{2a^4x^2}+\frac{b^2}{3a^3(a+bx)^3}$$

```
[In] Int[1/(x^3*(a + b*x)^4),x]
```

```
[Out] -1/2*1/(a^4*x^2) + (4*b)/(a^5*x) + b^2/(3*a^3*(a + b*x)^3) + (3*b^2)/(2*a^4
*(a + b*x)^2) + (6*b^2)/(a^5*(a + b*x)) + (10*b^2*Log[x])/a^6 - (10*b^2*Log
[a + b*x])/a^6
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{a^4x^3} - \frac{4b}{a^5x^2} + \frac{10b^2}{a^6x} - \frac{b^3}{a^3(a+bx)^4} - \frac{3b^3}{a^4(a+bx)^3} - \frac{6b^3}{a^5(a+bx)^2} - \frac{10b^3}{a^6(a+bx)} \right) dx$$
$$= -\frac{1}{2a^4x^2} + \frac{4b}{a^5x} + \frac{b^2}{3a^3(a+bx)^3} + \frac{3b^2}{2a^4(a+bx)^2} + \frac{6b^2}{a^5(a+bx)} + \frac{10b^2\log(x)}{a^6} - \frac{10b^2\log(a+bx)}{a^6}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(a+bx)^4}\,dx = \frac{\frac{a(-3a^4+15a^3bx+110a^2b^2x^2+150ab^3x^3+60b^4x^4)}{x^2(a+bx)^3} + 60b^2\log(x) - 60b^2\log(a+bx)}{6a^6}$$

```
[In] Integrate[1/(x^3*(a + b*x)^4),x]
[Out] ((a*(-3*a^4 + 15*a^3*b*x + 110*a^2*b^2*x^2 + 150*a*b^3*x^3 + 60*b^4*x^4))/(
x^2*(a + b*x)^3) + 60*b^2*Log[x] - 60*b^2*Log[a + b*x])/(6*a^6)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05

method	result
norman	$\frac{-\frac{1}{2a}+\frac{5bx}{2a^2}-\frac{30b^3x^3}{a^4}-\frac{45b^4x^4}{a^5}-\frac{55b^5x^5}{3a^6}}{x^2(bx+a)^3}+\frac{10b^2\ln(x)}{a^6}-\frac{10b^2\ln(bx+a)}{a^6}$
risch	$\frac{\frac{10b^4x^4}{a^5}+\frac{25b^3x^3}{a^4}+\frac{55b^2x^2}{3a^3}+\frac{5bx}{2a^2}-\frac{1}{2a}}{x^2(bx+a)^3}-\frac{10b^2\ln(bx+a)}{a^6}+\frac{10b^2\ln(-x)}{a^6}$
default	$-\frac{10b^2\ln(bx+a)}{a^6}+\frac{6b^2}{a^5(bx+a)}+\frac{3b^2}{2a^4(bx+a)^2}+\frac{b^2}{3a^3(bx+a)^3}-\frac{1}{2a^4x^2}+\frac{10b^2\ln(x)}{a^6}+\frac{4b}{a^5x}$
parallelrisch	$\frac{60\ln(x)x^5b^5-60\ln(bx+a)x^5b^5+180\ln(x)x^4ab^4-180\ln(bx+a)x^4ab^4-110x^5b^5+180\ln(x)x^3a^2b^3-180\ln(bx+a)x^3a^2b^3-270x^4 \cdots}{6a^6x^2(bx+a)^3}$

```
[In] int(1/x^3/(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] (-1/2/a+5/2*b*x/a^2-30*b^3*x^3/a^4-45*b^4/a^5*x^4-55/3*b^5/a^6*x^5)/x^2/(b*
x+a)^3+10/a^6*b^2*ln(x)-10/a^6*b^2*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(74) = 148$.

Time = 0.24 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^3(a+bx)^4}\,dx$$
$$= \frac{60\,ab^4x^4 + 150\,a^2b^3x^3 + 110\,a^3b^2x^2 + 15\,a^4bx - 3\,a^5 - 60\,(b^5x^5 + 3\,ab^4x^4 + 3\,a^2b^3x^3 + a^3b^2x^2)\log\left(bx + a\right.}{6\,(a^6b^3x^5 + 3\,a^7b^2x^4 + 3\,a^8bx^3 + a^9x^2)}$$

```
[In] integrate(1/x^3/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(60*a*b^4*x^4 + 150*a^2*b^3*x^3 + 110*a^3*b^2*x^2 + 15*a^4*b*x - 3*a^5
- 60*(b^5*x^5 + 3*a*b^4*x^4 + 3*a^2*b^3*x^3 + a^3*b^2*x^2)*log(b*x + a) + 6
0*(b^5*x^5 + 3*a*b^4*x^4 + 3*a^2*b^3*x^3 + a^3*b^2*x^2)*log(x))/(a^6*b^3*x^
5 + 3*a^7*b^2*x^4 + 3*a^8*b*x^3 + a^9*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^3(a+bx)^4}\,dx = \frac{-3a^4 + 15a^3bx + 110a^2b^2x^2 + 150ab^3x^3 + 60b^4x^4}{6a^8x^2 + 18a^7bx^3 + 18a^6b^2x^4 + 6a^5b^3x^5} + \frac{10b^2\left(\log\left(x\right) - \log\left(\frac{a}{b} + x\right)\right)}{a^6}$$

```
[In] integrate(1/x**3/(b*x+a)**4,x)
```

```
[Out] (-3*a**4 + 15*a**3*b*x + 110*a**2*b**2*x**2 + 150*a*b**3*x**3 + 60*b**4*x**
4)/(6*a**8*x**2 + 18*a**7*b*x**3 + 18*a**6*b**2*x**4 + 6*a**5*b**3*x**5) +
10*b**2*(log(x) - log(a/b + x))/a**6
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^3(a+bx)^4}\,dx = \frac{60\,b^4x^4 + 150\,ab^3x^3 + 110\,a^2b^2x^2 + 15\,a^3bx - 3\,a^4}{6\,(a^5b^3x^5 + 3\,a^6b^2x^4 + 3\,a^7bx^3 + a^8x^2)} - \frac{10\,b^2\log\left(bx + a\right)}{a^6} + \frac{10\,b^2\log\left(x\right)}{a^6}$$

```
[In] integrate(1/x^3/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(60*b^4*x^4 + 150*a*b^3*x^3 + 110*a^2*b^2*x^2 + 15*a^3*b*x - 3*a^4)/(a^
5*b^3*x^5 + 3*a^6*b^2*x^4 + 3*a^7*b*x^3 + a^8*x^2) - 10*b^2*log(b*x + a)/a^
6 + 10*b^2*log(x)/a^6
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3(a+bx)^4}\,dx = -\frac{10\,b^2\log\left(|bx+a|\right)}{a^6} + \frac{10\,b^2\log\left(|x|\right)}{a^6} + \frac{60\,ab^4x^4 + 150\,a^2b^3x^3 + 110\,a^3b^2x^2 + 15\,a^4bx - 3\,a^5}{6\,(bx+a)^3a^6x^2}$$

```
[In] integrate(1/x^3/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -10*b^2*log(abs(b*x + a))/a^6 + 10*b^2*log(abs(x))/a^6 + 1/6*(60*a*b^4*x^4
+ 150*a^2*b^3*x^3 + 110*a^3*b^2*x^2 + 15*a^4*b*x - 3*a^5)/((b*x + a)^3*a^6*
x^2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^3(a+bx)^4}\,dx = \frac{\frac{55\,b^2\,x^2}{3\,a^3} - \frac{1}{2\,a} + \frac{25\,b^3\,x^3}{a^4} + \frac{10\,b^4\,x^4}{a^5} + \frac{5\,b\,x}{2\,a^2}}{a^3\,x^2 + 3\,a^2\,b\,x^3 + 3\,a\,b^2\,x^4 + b^3\,x^5} - \frac{20\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^6}$$

```
[In] int(1/(x^3*(a + b*x)^4),x)
```

```
[Out] ((55*b^2*x^2)/(3*a^3) - 1/(2*a) + (25*b^3*x^3)/a^4 + (10*b^4*x^4)/a^5 + (5*
b*x)/(2*a^2))/(a^3*x^2 + b^3*x^5 + 3*a^2*b*x^3 + 3*a*b^2*x^4) - (20*b^2*ata
nh((2*b*x)/a + 1))/a^6
```

3.57 $\int \frac{1}{x(a+bx)^5} dx$

Optimal result

Integrand size = 11, antiderivative size = 64

$$\int \frac{1}{x(a+bx)^5} dx = \frac{\frac{25}{12a} + \frac{13bx}{3a^2} + \frac{7b^2x^2}{2a^3} + \frac{b^3x^3}{a^4}}{(a+bx)^4} - \frac{\log\left(\frac{a+bx}{x}\right)}{a^5}$$

```
[Out] (25/12/a+13/3*b*x/a^2+7/2*b^2*x^2/a^3+b^3*x^3/a^4)/(b*x+a)^4-1/a^5*ln((b*x+
a)/x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$\int \frac{1}{x(a+bx)^5} dx = -\frac{\log(a+bx)}{a^5} + \frac{\log(x)}{a^5} + \frac{1}{a^4(a+bx)} + \frac{1}{2a^3(a+bx)^2} + \frac{1}{3a^2(a+bx)^3} + \frac{1}{4a(a+bx)^4}$$

```
[In] Int[1/(x*(a + b*x)^5),x]
```

```
[Out] 1/(4*a*(a + b*x)^4) + 1/(3*a^2*(a + b*x)^3) + 1/(2*a^3*(a + b*x)^2) + 1/(a^
4*(a + b*x)) + Log[x]/a^5 - Log[a + b*x]/a^5
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{a^5x}-\frac{b}{a(a+bx)^5}-\frac{b}{a^2(a+bx)^4}-\frac{b}{a^3(a+bx)^3}-\frac{b}{a^4(a+bx)^2}-\frac{b}{a^5(a+bx)}\right)\,dx\\
&= \frac{1}{4a(a+bx)^4}+\frac{1}{3a^2(a+bx)^3}+\frac{1}{2a^3(a+bx)^2}+\frac{1}{a^4(a+bx)}+\frac{\log(x)}{a^5}-\frac{\log(a+bx)}{a^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$
\int \frac{1}{x(a+bx)^5}\,dx=\frac{\frac{a(25a^3+52a^2bx+42ab^2x^2+12b^3x^3)}{(a+bx)^4}+12\log(x)-12\log(a+bx)}{12a^5}
$$

```
[In] Integrate[1/(x*(a + b*x)^5),x]
```

```
[Out] ((a*(25*a^3 + 52*a^2*b*x + 42*a*b^2*x^2 + 12*b^3*x^3))/(a + b*x)^4 + 12*Log
[x] - 12*Log[a + b*x])/(12*a^5)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

method	result
risch	$\frac{\frac{25}{12a}+\frac{13bx}{3a^2}+\frac{7b^2x^2}{2a^3}+\frac{b^3x^3}{a^4}}{(bx+a)^4}+\frac{\ln(-x)}{a^5}-\frac{\ln(bx+a)}{a^5}$
default	$-\frac{\ln(bx+a)}{a^5}+\frac{1}{a^4(bx+a)}+\frac{1}{2a^3(bx+a)^2}+\frac{1}{3a^2(bx+a)^3}+\frac{1}{4a(bx+a)^4}+\frac{\ln(x)}{a^5}$
norman	$\frac{-\frac{4bx}{a^2}-\frac{9b^2x^2}{a^3}-\frac{22b^3x^3}{3a^4}-\frac{25b^4x^4}{12a^5}}{(bx+a)^4}+\frac{\ln(x)}{a^5}-\frac{\ln(bx+a)}{a^5}$
parallelrisch	$\frac{12\ln(x)x^4b^4-12\ln(bx+a)x^4b^4+48\ln(x)x^3ab^3-48\ln(bx+a)x^3ab^3-25b^4x^4+72\ln(x)x^2a^2b^2-72\ln(bx+a)x^2a^2b^2-88ab^3x^3+48}{12a^5(bx+a)^4}$

```
[In] int(1/x/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (25/12/a+13/3*b*x/a^2+7/2*b^2*x^2/a^3+b^3*x^3/a^4)/(b*x+a)^4+1/a^5*ln(-x)-1
/a^5*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(60) = 120$.

Time = 0.24 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.62

$$\int \frac{1}{x(a+bx)^5}\,dx = \frac{12\,ab^3x^3 + 42\,a^2b^2x^2 + 52\,a^3bx + 25\,a^4 - 12\,(b^4x^4 + 4\,ab^3x^3 + 6\,a^2b^2x^2 + 4\,a^3bx + a^4)\log(bx+a) + 12\,(b}{12\,(a^5b^4x^4 + 4\,a^6b^3x^3 + 6\,a^7b^2x^2 + 4\,a^8bx + a^9)}$$

```
[In] integrate(1/x/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a*b^3*x^3 + 42*a^2*b^2*x^2 + 52*a^3*b*x + 25*a^4 - 12*(b^4*x^4 + 4
*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*log(b*x + a) + 12*(b^4*x^4 +
4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*log(x))/(a^5*b^4*x^4 + 4*a^6
*b^3*x^3 + 6*a^7*b^2*x^2 + 4*a^8*b*x + a^9)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

$$\int \frac{1}{x(a+bx)^5}\,dx = \frac{25a^3 + 52a^2bx + 42ab^2x^2 + 12b^3x^3}{12a^8 + 48a^7bx + 72a^6b^2x^2 + 48a^5b^3x^3 + 12a^4b^4x^4} + \frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a^5}$$

```
[In] integrate(1/x/(b*x+a)**5,x)
```

```
[Out] (25*a**3 + 52*a**2*b*x + 42*a*b**2*x**2 + 12*b**3*x**3)/(12*a**8 + 48*a**7*
b*x + 72*a**6*b**2*x**2 + 48*a**5*b**3*x**3 + 12*a**4*b**4*x**4) + (log(x)
- log(a/b + x))/a**5
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.48

$$\int \frac{1}{x(a+bx)^5}\,dx = \frac{12\,b^3x^3 + 42\,ab^2x^2 + 52\,a^2bx + 25\,a^3}{12\,(a^4b^4x^4 + 4\,a^5b^3x^3 + 6\,a^6b^2x^2 + 4\,a^7bx + a^8)} - \frac{\log(bx+a)}{a^5} + \frac{\log(x)}{a^5}$$

```
[In] integrate(1/x/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(12*b^3*x^3 + 42*a*b^2*x^2 + 52*a^2*b*x + 25*a^3)/(a^4*b^4*x^4 + 4*a^5
*b^3*x^3 + 6*a^6*b^2*x^2 + 4*a^7*b*x + a^8) - log(b*x + a)/a^5 + log(x)/a^5
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.39

$$\int \frac{1}{x(a+bx)^5} dx = \frac{1}{12} b \left(\frac{12 \log \left(\left| -\frac{a}{bx+a} + 1 \right| \right)}{a^5 b} + \frac{\frac{12 b^3}{bx+a} + \frac{6 ab^3}{(bx+a)^2} + \frac{4 a^2 b^3}{(bx+a)^3} + \frac{3 a^3 b^3}{(bx+a)^4}}{a^4 b^4} \right)$$

```
[In] integrate(1/x/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/12*b*(12*log(abs(-a/(b*x + a) + 1))/(a^5*b) + (12*b^3/(b*x + a) + 6*a*b^3
/(b*x + a)^2 + 4*a^2*b^3/(b*x + a)^3 + 3*a^3*b^3/(b*x + a)^4)/(a^4*b^4))
```

Mupad [B] (verification not implemented)

Time = 16.87 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(a+bx)^5} dx = \frac{\frac{\frac{\frac{1}{a^2+b\,x\,a} - \frac{\ln\left(\frac{a+b\,x}{x}\right)}{a^2}}{a} + \frac{1}{2\,a\,(a+b\,x)^2}}{a} + \frac{1}{3\,a\,(a+b\,x)^3}}{a} + \frac{1}{4\,a\,(a+b\,x)^4}$$

```
[In] int(1/(x*(a + b*x)^5),x)
```

```
[Out] (((1/(a^2 + a*b*x) - log((a + b*x)/x)/a^2)/a + 1/(2*a*(a + b*x)^2))/a + 1/(
3*a*(a + b*x)^3))/a + 1/(4*a*(a + b*x)^4)
```

3.58 $\int \frac{1}{x^2(a+bx)^5} dx$

Optimal result

Integrand size = 11, antiderivative size = 77

$$\int \frac{1}{x^2(a+bx)^5} dx = \frac{-\frac{125b}{12a^2} - \frac{1}{ax} - \frac{65b^2x}{3a^3} - \frac{35b^3x^2}{2a^4} - \frac{5b^4x^3}{a^5}}{(a+bx)^4} + \frac{5b\log\left(\frac{a+bx}{x}\right)}{a^6}$$

```
[Out] (-1/a/x-125/12*b/a^2-65/3*b^2*x/a^3-35/2*b^3*x^2/a^4-5*b^4*x^3/a^5)/(b*x+a)
^4+5*b/a^6*ln((b*x+a)/x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$\int \frac{1}{x^2(a+bx)^5} dx = -\frac{5b\log(x)}{a^6} + \frac{5b\log(a+bx)}{a^6} - \frac{4b}{a^5(a+bx)} - \frac{1}{a^5x} - \frac{3b}{2a^4(a+bx)^2} - \frac{2b}{3a^3(a+bx)^3} - \frac{b}{4a^2(a+bx)^4}$$

```
[In] Int[1/(x^2*(a + b*x)^5),x]
```

```
[Out] -(1/(a^5*x)) - b/(4*a^2*(a + b*x)^4) - (2*b)/(3*a^3*(a + b*x)^3) - (3*b)/(2
*a^4*(a + b*x)^2) - (4*b)/(a^5*(a + b*x)) - (5*b*Log[x])/a^6 + (5*b*Log[a +
 b*x])/a^6
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{a^5x^2} - \frac{5b}{a^6x} + \frac{b^2}{a^2(a+bx)^5} + \frac{2b^2}{a^3(a+bx)^4} + \frac{3b^2}{a^4(a+bx)^3} + \frac{4b^2}{a^5(a+bx)^2} + \frac{5b^2}{a^6(a+bx)}\right) dx \\ &= -\frac{1}{a^5x} - \frac{b}{4a^2(a+bx)^4} - \frac{2b}{3a^3(a+bx)^3} - \frac{3b}{2a^4(a+bx)^2} - \frac{4b}{a^5(a+bx)} - \frac{5b\log(x)}{a^6} + \frac{5b\log(a+bx)}{a^6}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2(a+bx)^5}\,dx = -\frac{\frac{a(12a^4+125a^3bx+260a^2b^2x^2+210ab^3x^3+60b^4x^4)}{x(a+bx)^4} + 60b\log(x) - 60b\log(a+bx)}{12a^6}$$

[In] Integrate[1/(x^2*(a + b*x)^5),x]

[Out] -1/12*((a*(12*a^4 + 125*a^3*b*x + 260*a^2*b^2*x^2 + 210*a*b^3*x^3 + 60*b^4*x^4))/(x*(a + b*x)^4) + 60*b*Log[x] - 60*b*Log[a + b*x])/a^6

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06

method	result
default	$-\frac{b}{4a^2(bx+a)^4} + \frac{5b\ln(bx+a)}{a^6} - \frac{4b}{a^5(bx+a)} - \frac{3b}{2a^4(bx+a)^2} - \frac{2b}{3a^3(bx+a)^3} - \frac{1}{a^5x} - \frac{5b\ln(x)}{a^6}$
risch	$\frac{-\frac{5b^4x^4}{a^5} - \frac{35b^3x^3}{2a^4} - \frac{65b^2x^2}{3a^3} - \frac{125bx}{12a^2} - \frac{1}{a}}{x(bx+a)^4} + \frac{5b\ln(-bx-a)}{a^6} - \frac{5b\ln(x)}{a^6}$
norman	$\frac{-\frac{1}{a} + \frac{20b^2x^2}{a^3} + \frac{45b^3x^3}{a^4} + \frac{110b^4x^4}{3a^5} + \frac{125b^5x^5}{12a^6}}{x(bx+a)^4} - \frac{5b\ln(x)}{a^6} + \frac{5b\ln(bx+a)}{a^6}$
parallelrisch	$-\frac{60\ln(x)x^5b^5 - 60\ln(bx+a)x^5b^5 + 240\ln(x)x^4ab^4 - 240\ln(bx+a)x^4ab^4 - 125x^5b^5 + 360\ln(x)x^3a^2b^3 - 360\ln(bx+a)x^3a^2b^3 - 440}{12a^6x(bx+a)^4}$

[In] int(1/x^2/(b*x+a)^5,x,method=_RETURNVERBOSE)

[Out] -1/4*b/a^2/(b*x+a)^4+5/a^6*b*ln(b*x+a)-4*b/a^5/(b*x+a)-3/2/a^4*b/(b*x+a)^2-2/3*b/a^3/(b*x+a)^3-1/a^5/x-5/a^6*b*ln(x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(72) = 144$.

Time = 0.25 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.56

$$\int \frac{1}{x^2(a+bx)^5}\,dx = -\frac{60\,ab^4x^4 + 210\,a^2b^3x^3 + 260\,a^3b^2x^2 + 125\,a^4bx + 12\,a^5 - 60\,(b^5x^5 + 4\,ab^4x^4 + 6\,a^2b^3x^3 + 4\,a^3b^2x^2 + a^4b}{12\,(a^6b^4x^5 + 4\,a^7b^3x^4 + 6\,a^8b^2x^3 + 4\,a^9b}$$

```
[In] integrate(1/x^2/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(60*a*b^4*x^4 + 210*a^2*b^3*x^3 + 260*a^3*b^2*x^2 + 125*a^4*b*x + 12*
a^5 - 60*(b^5*x^5 + 4*a*b^4*x^4 + 6*a^2*b^3*x^3 + 4*a^3*b^2*x^2 + a^4*b*x)*
log(b*x + a) + 60*(b^5*x^5 + 4*a*b^4*x^4 + 6*a^2*b^3*x^3 + 4*a^3*b^2*x^2 +
a^4*b*x)*log(x))/(a^6*b^4*x^5 + 4*a^7*b^3*x^4 + 6*a^8*b^2*x^3 + 4*a^9*b*x^2
 + a^10*x)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^2(a+bx)^5}\,dx = \frac{-12a^4 - 125a^3bx - 260a^2b^2x^2 - 210ab^3x^3 - 60b^4x^4}{12a^9x + 48a^8bx^2 + 72a^7b^2x^3 + 48a^6b^3x^4 + 12a^5b^4x^5} + \frac{5b\left(-\log(x) + \log\left(\frac{a}{b} + x\right)\right)}{a^6}$$

```
[In] integrate(1/x**2/(b*x+a)**5,x)
```

```
[Out] (-12*a**4 - 125*a**3*b*x - 260*a**2*b**2*x**2 - 210*a*b**3*x**3 - 60*b**4*x
**4)/(12*a**9*x + 48*a**8*b*x**2 + 72*a**7*b**2*x**3 + 48*a**6*b**3*x**4 +
12*a**5*b**4*x**5) + 5*b*(-log(x) + log(a/b + x))/a**6
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^2(a+bx)^5}\,dx = -\frac{60\,b^4x^4 + 210\,ab^3x^3 + 260\,a^2b^2x^2 + 125\,a^3bx + 12\,a^4}{12\,(a^5b^4x^5 + 4\,a^6b^3x^4 + 6\,a^7b^2x^3 + 4\,a^8bx^2 + a^9x)} + \frac{5\,b\log(bx+a)}{a^6} - \frac{5\,b\log(x)}{a^6}$$

```
[In] integrate(1/x^2/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(60*b^4*x^4 + 210*a*b^3*x^3 + 260*a^2*b^2*x^2 + 125*a^3*b*x + 12*a^4)
/(a^5*b^4*x^5 + 4*a^6*b^3*x^4 + 6*a^7*b^2*x^3 + 4*a^8*b*x^2 + a^9*x) + 5*b*
log(b*x + a)/a^6 - 5*b*log(x)/a^6
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^2(a+bx)^5}\,dx = -\frac{5\,b\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^6} + \frac{b}{a^6\left(\frac{a}{bx+a}-1\right)} - \frac{\frac{48\,a^3b^9}{bx+a} + \frac{18\,a^4b^9}{(bx+a)^2} + \frac{8\,a^5b^9}{(bx+a)^3} + \frac{3\,a^6b^9}{(bx+a)^4}}{12\,a^8b^8}$$

[In] integrate(1/x^2/(b*x+a)^5,x, algorithm="giac")

[Out] -5*b*log(abs(-a/(b*x + a) + 1))/a^6 + b/(a^6*(a/(b*x + a) - 1)) - 1/12*(48*a^3*b^9/(b*x + a) + 18*a^4*b^9/(b*x + a)^2 + 8*a^5*b^9/(b*x + a)^3 + 3*a^6*b^9/(b*x + a)^4)/(a^8*b^8)

Mupad [B] (verification not implemented)

Time = 16.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^2(a+bx)^5}\,dx = \frac{10\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^6} - \frac{\frac{1}{a} + \frac{65\,b^2\,x^2}{3\,a^3} + \frac{35\,b^3\,x^3}{2\,a^4} + \frac{5\,b^4\,x^4}{a^5} + \frac{125\,b\,x}{12\,a^2}}{a^4\,x + 4\,a^3\,b\,x^2 + 6\,a^2\,b^2\,x^3 + 4\,a\,b^3\,x^4 + b^4\,x^5}$$

[In] int(1/(x^2*(a + b*x)^5),x)

[Out] (10*b*atanh((2*b*x)/a + 1))/a^6 - (1/a + (65*b^2*x^2)/(3*a^3) + (35*b^3*x^3)/(2*a^4) + (5*b^4*x^4)/a^5 + (125*b*x)/(12*a^2))/(a^4*x + b^4*x^5 + 4*a^3*b*x^2 + 4*a*b^3*x^4 + 6*a^2*b^2*x^3)

3.59 $\int \frac{1}{x^3(a+bx)^5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 89

$$\int \frac{1}{x^3(a+bx)^5}\,dx = \frac{\frac{65b^2}{a^4}+\frac{125b^2}{4a^3}-\frac{1}{2ax^2}+\frac{3b}{a^2x}+\frac{105b^4x^2}{2a^5}+\frac{15b^5x^3}{a^6}}{(a+bx)^4}-\frac{15b^2\log\left(\frac{a+bx}{x}\right)}{a^7}$$

```
[Out] (-1/2/a/x^2+3*b/a^2/x+125/4*b^2/a^3+65*b^2/a^4+105/2*b^4*x^2/a^5+15*b^5*x^3
/a^6)/(b*x+a)^4-15*b^2/a^7*ln((b*x+a)/x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\int \frac{1}{x^3(a+bx)^5}\,dx = \frac{15b^2\log(x)}{a^7}-\frac{15b^2\log(a+bx)}{a^7}+\frac{10b^2}{a^6(a+bx)}+\frac{5b}{a^6x}+\frac{3b^2}{a^5(a+bx)^2}-\frac{1}{2a^5x^2}+\frac{b^2}{a^4(a+bx)^3}+\frac{b^2}{4a^3(a+bx)^4}$$

```
[In] Int[1/(x^3*(a + b*x)^5),x]
```

```
[Out] -1/2*1/(a^5*x^2) + (5*b)/(a^6*x) + b^2/(4*a^3*(a + b*x)^4) + b^2/(a^4*(a +
b*x)^3) + (3*b^2)/(a^5*(a + b*x)^2) + (10*b^2)/(a^6*(a + b*x)) + (15*b^2*Lo
g[x])/a^7 - (15*b^2*Log[a + b*x])/a^7
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{a^5x^3} - \frac{5b}{a^6x^2} + \frac{15b^2}{a^7x} - \frac{b^3}{a^3(a+bx)^5} - \frac{3b^3}{a^4(a+bx)^4} - \frac{6b^3}{a^5(a+bx)^3} - \frac{10b^3}{a^6(a+bx)^2} - \frac{15b^3}{a^7(a+bx)}\right) dx \\
&= -\frac{1}{2a^5x^2} + \frac{5b}{a^6x} + \frac{b^2}{4a^3(a+bx)^4} + \frac{b^2}{a^4(a+bx)^3} + \frac{3b^2}{a^5(a+bx)^2} + \frac{10b^2}{a^6(a+bx)} + \frac{15b^2\log(x)}{a^7} - \frac{15b^2\log(a+bx)}{a^7}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$
\int \frac{1}{x^3(a+bx)^5}\,dx = \frac{\frac{a(-2a^5+12a^4bx+125a^3b^2x^2+260a^2b^3x^3+210ab^4x^4+60b^5x^5)}{x^2(a+bx)^4} + 60b^2\log(x) - 60b^2\log(a+bx)}{4a^7}
$$

```
[In] Integrate[1/(x^3*(a + b*x)^5),x]
[Out] ((a*(-2*a^5 + 12*a^4*b*x + 125*a^3*b^2*x^2 + 260*a^2*b^3*x^3 + 210*a*b^4*x^
4 + 60*b^5*x^5))/(x^2*(a + b*x)^4) + 60*b^2*Log[x] - 60*b^2*Log[a + b*x])/(
4*a^7)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

method	result
norman	$\frac{-\frac{1}{2a}+\frac{3bx}{a^2}-\frac{60b^3x^3}{a^4}-\frac{135b^4x^4}{a^5}-\frac{110b^5x^5}{a^6}-\frac{125b^6x^6}{4a^7}}{x^2(bx+a)^4}+\frac{15b^2\ln(x)}{a^7}-\frac{15b^2\ln(bx+a)}{a^7}$
risch	$\frac{\frac{15b^5x^5}{a^6}+\frac{105b^4x^4}{2a^5}+\frac{65b^3x^3}{a^4}+\frac{125b^2x^2}{4a^3}+\frac{3bx}{a^2}-\frac{1}{2a}}{x^2(bx+a)^4}-\frac{15b^2\ln(bx+a)}{a^7}+\frac{15b^2\ln(-x)}{a^7}$
default	$-\frac{15b^2\ln(bx+a)}{a^7}+\frac{10b^2}{a^6(bx+a)}+\frac{3b^2}{a^5(bx+a)^2}+\frac{b^2}{a^4(bx+a)^3}+\frac{b^2}{4a^3(bx+a)^4}-\frac{1}{2a^5x^2}+\frac{15b^2\ln(x)}{a^7}+\frac{5b}{a^6x}$
parallelrisch	$\frac{60\ln(x)x^6b^6-60\ln(bx+a)x^6b^6+240\ln(x)x^5a\,b^5-240\ln(bx+a)x^5a\,b^5-125x^6b^6+360\ln(x)x^4a^2b^4-360\ln(bx+a)x^4a^2b^4-440x^5}{4a^7x^2(bx+}$

```
[In] int(1/x^3/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/a+3*b*x/a^2-60*b^3*x^3/a^4-135*b^4/a^5*x^4-110*b^5/a^6*x^5-125/4*b^6/
a^7*x^6)/x^2/(b*x+a)^4+15/a^7*b^2*ln(x)-15/a^7*b^2*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(84) = 168.

Time = 0.25 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.45

$$\int \frac{1}{x^3(a+bx)^5}\,dx = \frac{60\,ab^5x^5+210\,a^2b^4x^4+260\,a^3b^3x^3+125\,a^4b^2x^2+12\,a^5bx-2\,a^6-60\,(b^6x^6+4\,ab^5x^5+6\,a^2b^4x^4+4\,a^3b^3}{4\,(a^7b^4x^6+4\,a^8b^3x^5+6\,a^9b^2x^4+4}$$

```
[In] integrate(1/x^3/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(60*a*b^5*x^5 + 210*a^2*b^4*x^4 + 260*a^3*b^3*x^3 + 125*a^4*b^2*x^2 + 1
2*a^5*b*x - 2*a^6 - 60*(b^6*x^6 + 4*a*b^5*x^5 + 6*a^2*b^4*x^4 + 4*a^3*b^3*x
^3 + a^4*b^2*x^2)*log(b*x + a) + 60*(b^6*x^6 + 4*a*b^5*x^5 + 6*a^2*b^4*x^4
+ 4*a^3*b^3*x^3 + a^4*b^2*x^2)*log(x))/(a^7*b^4*x^6 + 4*a^8*b^3*x^5 + 6*a^9
*b^2*x^4 + 4*a^10*b*x^3 + a^11*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.44

$$\int \frac{1}{x^3(a+bx)^5}\,dx = \frac{-2a^5+12a^4bx+125a^3b^2x^2+260a^2b^3x^3+210ab^4x^4+60b^5x^5}{4a^{10}x^2+16a^9bx^3+24a^8b^2x^4+16a^7b^3x^5+4a^6b^4x^6} + \frac{15b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^7}$$

```
[In] integrate(1/x**3/(b*x+a)**5,x)
```

```
[Out] (-2*a**5 + 12*a**4*b*x + 125*a**3*b**2*x**2 + 260*a**2*b**3*x**3 + 210*a*b*
*4*x**4 + 60*b**5*x**5)/(4*a**10*x**2 + 16*a**9*b*x**3 + 24*a**8*b**2*x**4
+ 16*a**7*b**3*x**5 + 4*a**6*b**4*x**6) + 15*b**2*(log(x) - log(a/b + x))/a
**7
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^3(a+bx)^5}\,dx = \frac{60\,b^5x^5+210\,ab^4x^4+260\,a^2b^3x^3+125\,a^3b^2x^2+12\,a^4bx-2\,a^5}{4\,(a^6b^4x^6+4\,a^7b^3x^5+6\,a^8b^2x^4+4\,a^9bx^3+a^{10}x^2)} - \frac{15\,b^2\log(bx+a)}{a^7} + \frac{15\,b^2\log(x)}{a^7}$$

```
[In] integrate(1/x^3/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/4*(60*b^5*x^5 + 210*a*b^4*x^4 + 260*a^2*b^3*x^3 + 125*a^3*b^2*x^2 + 12*a^
4*b*x - 2*a^5)/(a^6*b^4*x^6 + 4*a^7*b^3*x^5 + 6*a^8*b^2*x^4 + 4*a^9*b*x^3 +
 a^10*x^2) - 15*b^2*log(b*x + a)/a^7 + 15*b^2*log(x)/a^7
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.44

$$\int \frac{1}{x^3(a+bx)^5}\,dx = \frac{15\,b^2\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^7} - \frac{\frac{12\,ab^2}{bx+a}-11\,b^2}{2\,a^7\left(\frac{a}{bx+a}-1\right)^2} + \frac{\frac{40\,a^6b^{14}}{bx+a}+\frac{12\,a^7b^{14}}{(bx+a)^2}+\frac{4\,a^8b^{14}}{(bx+a)^3}+\frac{a^9b^{14}}{(bx+a)^4}}{4\,a^{12}b^{12}}$$

```
[In] integrate(1/x^3/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 15*b^2*log(abs(-a/(b*x + a) + 1))/a^7 - 1/2*(12*a*b^2/(b*x + a) - 11*b^2)/(
a^7*(a/(b*x + a) - 1)^2) + 1/4*(40*a^6*b^14/(b*x + a) + 12*a^7*b^14/(b*x +
a)^2 + 4*a^8*b^14/(b*x + a)^3 + a^9*b^14/(b*x + a)^4)/(a^12*b^12)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.38

$$\int \frac{1}{x^3(a+bx)^5}\,dx = \frac{\frac{125\,b^2\,x^2}{4\,a^3}-\frac{1}{2\,a}+\frac{65\,b^3\,x^3}{a^4}+\frac{105\,b^4\,x^4}{2\,a^5}+\frac{15\,b^5\,x^5}{a^6}+\frac{3\,b\,x}{a^2}}{a^4\,x^2+4\,a^3\,b\,x^3+6\,a^2\,b^2\,x^4+4\,a\,b^3\,x^5+b^4\,x^6} - \frac{30\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^7}$$

```
[In] int(1/(x^3*(a + b*x)^5),x)
```

```
[Out] ((125*b^2*x^2)/(4*a^3) - 1/(2*a) + (65*b^3*x^3)/a^4 + (105*b^4*x^4)/(2*a^5)
 + (15*b^5*x^5)/a^6 + (3*b*x)/a^2)/(a^4*x^2 + b^4*x^6 + 4*a^3*b*x^3 + 4*a*b
^3*x^5 + 6*a^2*b^2*x^4) - (30*b^2*atanh((2*b*x)/a + 1))/a^7
```

3.60 $\int \frac{1}{a+bx^2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \frac{1}{a+bx^2} \, dx = \frac{\arctan\left(\sqrt{\frac{b}{a}}x\right)}{\sqrt{ab}}$$

```
[Out] 1/(a*b)^(1/2)*arctan(x*(b/a)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {211}

$$\int \frac{1}{a+bx^2} \, dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[In] Int[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\text{integral} = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+bx^2}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[In] Integrate[(a + b*x^2)^(-1),x]
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$-\frac{\ln(bx+\sqrt{-ab})}{2\sqrt{-ab}}+\frac{\ln(-bx+\sqrt{-ab})}{2\sqrt{-ab}}$	41

```
[In] int(1/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.35

$$\int \frac{1}{a+bx^2}\,dx=\left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab},\ \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab}\right]$$

```
[In] integrate(1/(b*x^2+a),x, algorithm="fricas")
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{1}{a+bx^2}\,dx = -\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

```
[In] integrate(1/(b*x**2+a),x)
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{1}{a+bx^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

```
[In] integrate(1/(b*x^2+a),x, algorithm="maxima")
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{1}{a+bx^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

```
[In] integrate(1/(b*x^2+a),x, algorithm="giac")
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{1}{a+bx^2}\,dx=\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[In] int(1/(a + b*x^2),x)
[Out] atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

3.61 $\int x(a + bx^2)^{-m} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int x(a+bx^2)^{-m}\,dx = -\frac{(a+bx^2)^{1-m}}{2b(-1+m)}$$

```
[Out] -1/2/b/(-1+m)/((b*x^2+a)^(-1+m))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{267\}$

$$\int x(a+bx^2)^{-m}\,dx = \frac{(a+bx^2)^{1-m}}{2b(1-m)}$$

```
[In] Int[x/(a + b*x^2)^m,x]
```

```
[Out] (a + b*x^2)^(1 - m)/(2*b*(1 - m))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{(a+bx^2)^{1-m}}{2b(1-m)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int x\left(a+bx^2\right)^{-m}\,dx=\frac{\left(a+bx^2\right)^{1-m}}{2b-2bm}$$

```
[In] Integrate[x/(a + b*x^2)^m,x]
[Out] (a + b*x^2)^(1 - m)/(2*b - 2*b*m)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\left(x^2b+a\right)^{1-m}}{2b(1-m)}$	26
default	$\frac{\left(x^2b+a\right)^{1-m}}{2b(1-m)}$	26
gosper	$-\frac{\left(x^2b+a\right)\left(x^2b+a\right)^{-m}}{2b(-1+m)}$	29
risch	$-\frac{\left(x^2b+a\right)\left(x^2b+a\right)^{-m}}{2b(-1+m)}$	29
norman	$\left(-\frac{x^2}{2(-1+m)}-\frac{a}{2b(-1+m)}\right)\mathrm{e}^{-m\ln\left(x^2b+a\right)}$	37
parallelrisch	$\frac{\left(-x^2ab-a^2\right)\left(x^2b+a\right)^{-m}}{2b(-1+m)a}$	38

```
[In] int(x/((b*x^2+a)^m),x,method=_RETURNVERBOSE)
[Out] 1/2/b*(b*x^2+a)^(1-m)/(1-m)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int x\left(a+bx^2\right)^{-m}\,dx=-\frac{bx^2+a}{2\,(bm-b)(bx^2+a)^m}$$

```
[In] integrate(x/((b*x^2+a)^m),x, algorithm="fricas")
[Out] -1/2*(b*x^2 + a)/((b*m - b)*(b*x^2 + a)^m)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(17) = 34.

Time = 1.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.08

$$\int x(a+bx^2)^{-m}\,dx = \begin{cases} \frac{x^2}{2a} & \text{for } b=0\wedge m=1 \\ \frac{a^{-m}x^2}{2} & \text{for } b=0 \\ \frac{\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b}+\frac{\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b} & \text{for } m=1 \\ -\frac{a}{2bm(a+bx^2)^m-2b(a+bx^2)^m}-\frac{bx^2}{2bm(a+bx^2)^m-2b(a+bx^2)^m} & \text{otherwise} \end{cases}$$

```
[In] integrate(x/((b*x**2+a)**m),x)
[Out] Piecewise((x**2/(2*a), Eq(b, 0) & Eq(m, 1)), (x**2/(2*a**m), Eq(b, 0)), (lo
g(x - sqrt(-a/b))/(2*b) + log(x + sqrt(-a/b))/(2*b), Eq(m, 1)), (-a/(2*b*m*
(a + b*x**2)**m - 2*b*(a + b*x**2)**m) - b*x**2/(2*b*m*(a + b*x**2)**m - 2*
b*(a + b*x**2)**m), True))
```

Maxima [F(-2)]

Exception generated.

$$\int x(a+bx^2)^{-m}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/((b*x^2+a)^m),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-m>0)', see `assume?` for more deta
ils)Is
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int x(a+bx^2)^{-m}\,dx = -\frac{(bx^2+a)^{-m+1}}{2\,b(m-1)}$$

```
[In] integrate(x/((b*x^2+a)^m),x, algorithm="giac")
[Out] -1/2*(b*x^2 + a)^(-m + 1)/(b*(m - 1))
```

Mupad [B] (verification not implemented)

Time = 15.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int x(a+bx^2)^{-m}\,dx=-\frac{(b\,x^2+a)^{1-m}}{2\,b\,(m-1)}$$

```
[In] int(x/(a + b*x^2)^m,x)
[Out] -(a + b*x^2)^(1 - m)/(2*b*(m - 1))
```

3.62 $\int \frac{1}{a+bx^3}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 94

$$\int \frac{1}{a+bx^3}\,dx=\frac{\sqrt[3]{\frac{a}{b}}\left(\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{2\sqrt[3]{\frac{a}{b}}-x}\right)+\frac{1}{2}\log\left(\frac{\left(\sqrt[3]{\frac{a}{b}}+x\right)^2}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\right)\right)}{3a}$$

```
[Out] 1/3*(a/b)^(1/3)/a*(1/2*ln((x+(a/b)^(1/3))^2/(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
)+3^(1/2)*arctan(3^(1/2)*x/(2*(a/b)^(1/3)-x)))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {206, 31, 648, 631, 210, 642}

$$\int \frac{1}{a+bx^3}\,dx=-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

```
[In] Int[(a + b*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*b^(1/3
))) + Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) - Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2]/(6*a^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}} + \frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}} - \frac{\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}\sqrt[3]{b}}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}$$

$$= -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95

$$\int \frac{1}{a+bx^3}\,dx$$

$$= -\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) + \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

```
[In] Integrate[(a + b*x^3)^(-1),x]
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3)
 + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(2/3)*b^
(1/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}}{3b}$	27
default	$\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	91

```
[In] int(1/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/3/b*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 3.18

$$\int \frac{1}{a+bx^3}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-(a^2b)^{\frac{2}{3}}\log\left(abx^2-(a^2 \cdots\right.}{6\,a^2b}\cdots\right.$$

```
[In] integrate(1/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/
3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*
sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^
(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a
^2*b), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*
b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*lo
g(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x
+ (a^2*b)^(2/3)))/(a^2*b)]
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.21

$$\int \frac{1}{a+bx^3}\,dx = \operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)$$

```
[In] integrate(1/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int \frac{1}{a+bx^3}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(1/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(2
/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*log
(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.19

$$\int \frac{1}{a+bx^3}\,dx = -\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a} + \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab} + \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab}$$

```
[In] integrate(1/(b*x^3+a),x, algorithm="giac")
[Out] -1/3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a + 1/3*sqrt(3)*(-a*b^2)^(1/3)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) + 1/6*(-a*b^2)
^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 15.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int \frac{1}{a+bx^3}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)}{3\,a^{2/3}\,b^{1/3}} + \frac{\ln\left(3\,b^2\,x+\frac{3\,a^{1/3}\,b^{5/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}} - \frac{\ln\left(3\,b^2\,x-\frac{3\,a^{1/3}\,b^{5/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}}$$

```
[In] int(1/(a + b*x^3),x)
```

```
[Out] log(b^(1/3)*x + a^(1/3))/(3*a^(2/3)*b^(1/3)) + (log(3*b^2*x + (3*a^(1/3)*b^
(5/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^(2/3)*b^(1/3)) - (log(3*b
^2*x - (3*a^(1/3)*b^(5/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)
*b^(1/3))
```

3.63 $\int \frac{x}{a+bx^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 101

$$\int \frac{x}{a+bx^3} \, dx = -\frac{-\sqrt{3}\arctan\left(\frac{-\sqrt[3]{\frac{a}{b}}+2x}{\sqrt{3}\sqrt[3]{\frac{a}{b}}}\right)+\frac{1}{2}\log\left(\frac{\left(\sqrt[3]{\frac{a}{b}}+x\right)^2}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\right)}{3\sqrt[3]{\frac{a}{b}}b}$$

```
[Out] -1/3/b/(a/b)^(1/3)*(1/2*ln((x+(a/b)^(1/3))^2/(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
))-3^(1/2)*arctan(1/3*(2*x-(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3)))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {298, 31, 648, 631, 210, 642}

$$\int \frac{x}{a+bx^3} \, dx = \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}$$

```
[In] Int[x/(a + b*x^3),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*b^(2/3
))) - Log[a^(1/3) + b^(1/3)*x]/(3*a^(1/3)*b^(2/3)) + Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2]/(6*a^(1/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule Rubi steps

$$\text{integral} = -\frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$= -\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}b^{2/3}} + \frac{\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{b}}$$

$$= -\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}+\frac{\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}$$

$$= -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int\frac{x}{a+bx^3}\,dx$$

$$= \frac{-2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}$$

```
[In] Integrate[x/(a + b*x^3),x]
[Out] (-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) + b
^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2
/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.27

method	result	size
risch	$\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}}{3b}$	27
default	$-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	91

```
[In] int(x/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/3/b*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.01

$$\int \frac{x}{a+bx^3}\,dx$$

$$= \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\,(-ab^2)^{\frac{2}{3}}x^2+(-ab^2)^{\frac{1}{3}}a\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3\,(-ab^2)^{\frac{2}{3}}x}{bx^3+a}\right)+(-ab^2)^{\frac{2}{3}}\log\left(b^2x^2+\right.}{6\,ab^2} \right.$$

```
[In] integrate(x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(
1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/
a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^
2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3))
)/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2
*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*log(b^2*
x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a
*b^2)^(1/3)))/(a*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.24

$$\int \frac{x}{a+bx^3}\,dx = \operatorname{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)$$

```
[In] integrate(x/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

$$\int \frac{x}{a+bx^3}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(1
/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3)) - 1/3*log
(x + (a/b)^(1/3))/(b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.11

$$\int \frac{x}{a+bx^3}\,dx = -\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a} - \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2} + \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^2}$$

```
[In] integrate(x/(b*x^3+a),x, algorithm="giac")
[Out] -1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a - 1/3*sqrt(3)*(-a*b^2)^(2/3)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/6*(-a*b^
2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 15.01 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.10

$$\int \frac{x}{a+bx^3}\,dx = \frac{\ln\left(b^{1/3}x-(-a)^{1/3}\right)}{3(-a)^{1/3}b^{2/3}} + \frac{\ln\left(bx-\frac{(-a)^{1/3}b^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6(-a)^{1/3}b^{2/3}} - \frac{\ln\left(bx-\frac{(-a)^{1/3}b^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6(-a)^{1/3}b^{2/3}}$$

```
[In] int(x/(a + b*x^3),x)
```

```
[Out] log(b^(1/3)*x - (-a)^(1/3))/(3*(-a)^(1/3)*b^(2/3)) + (log(b*x - ((-a)^(1/3)
*b^(2/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(6*(-a)^(1/3)*b^(2/3)) -
(log(b*x - ((-a)^(1/3)*b^(2/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(6*
(-a)^(1/3)*b^(2/3))
```

3.64 $\int \frac{x^2}{a+bx^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x^2}{a+bx^3} dx = \frac{\log(a+bx^3)}{3b}$$

```
[Out] 1/3/b*ln(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {266}

$$\int \frac{x^2}{a+bx^3} dx = \frac{\log(a+bx^3)}{3b}$$

```
[In] Int[x^2/(a + b*x^3),x]
```

```
[Out] Log[a + b*x^3]/(3*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\text{integral} = \frac{\log(a+bx^3)}{3b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+bx^3}\,dx = \frac{\log\left(a+bx^3\right)}{3b}$$

```
[In] Integrate[x^2/(a + b*x^3),x]
[Out] Log[a + b*x^3]/(3*b)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\ln\left(b\,x^{3}+a\right)}{3b}$	14
default	$\frac{\ln\left(b\,x^{3}+a\right)}{3b}$	14
norman	$\frac{\ln\left(b\,x^{3}+a\right)}{3b}$	14
risch	$\frac{\ln\left(b\,x^{3}+a\right)}{3b}$	14
parallelrisch	$\frac{\ln\left(b\,x^{3}+a\right)}{3b}$	14

```
[In] int(x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/3/b*ln(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{a+bx^3}\,dx = \frac{\log\left(bx^3+a\right)}{3\,b}$$

```
[In] integrate(x^2/(b*x^3+a),x, algorithm="fricas")
[Out] 1/3*log(b*x^3 + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{a+bx^3}\,dx = \frac{\log\left(a+bx^3\right)}{3b}$$

```
[In] integrate(x**2/(b*x**3+a),x)
[Out] log(a + b*x**3)/(3*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{a+bx^3}\,dx = \frac{\log\left(bx^3+a\right)}{3\,b}$$

```
[In] integrate(x^2/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*log(b*x^3 + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{a+bx^3}\,dx = \frac{\log\left(|bx^3+a|\right)}{3\,b}$$

```
[In] integrate(x^2/(b*x^3+a),x, algorithm="giac")
[Out] 1/3*log(abs(b*x^3 + a))/b
```

Mupad [B] (verification not implemented)

Time = 15.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{a+bx^3}\,dx = \frac{\ln\left(b\,x^3+a\right)}{3\,b}$$

```
[In] int(x^2/(a + b*x^3),x)
[Out] log(a + b*x^3)/(3*b)
```

3.65 $\int \frac{x^3}{a+bx^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{x^3}{a+bx^3}\,dx=\frac{x}{b}-\frac{\sqrt[3]{\frac{a}{b}}\left(\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{2\sqrt[3]{\frac{a}{b}}-x}\right)+\frac{1}{2}\log\left(\frac{\left(\sqrt[3]{\frac{a}{b}}+x\right)^2}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\right)\right)}{3b}$$

```
[Out] x/b-1/3/b*(a/b)^(1/3)*(1/2*ln((x+(a/b)^(1/3))^2/(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3)))+3^(1/2)*arctan(3^(1/2)*x/(2*(a/b)^(1/3)-x)))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {327, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^3}{a+bx^3}\,dx=\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}+\frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}}+\frac{x}{b}$$

```
[In] Int[x^3/(a + b*x^3),x]
```

```
[Out] x/b + (a^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
b^(4/3)) - (a^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(4/3)) + (a^(1/3)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{x}{b} - \frac{a\int \frac{1}{a+bx^3}\,dx}{b}$$

$$= \frac{x}{b} - \frac{\sqrt[3]{a}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b} - \frac{\sqrt[3]{a}\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b}$$

$$= \frac{x}{b} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}} + \frac{\sqrt[3]{a}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{4/3}} - \frac{a^{2/3}\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b}$$

$$= \frac{x}{b} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}} + \frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}} - \frac{\sqrt[3]{a}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{4/3}}$$

$$= \frac{x}{b} + \frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}} + \frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{a+bx^3}\,dx$$

$$= \frac{6\sqrt[3]{b}x + 2\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) + \sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}$$

```
[In] Integrate[x^3/(a + b*x^3),x]
```

```
[Out] (6*b^(1/3)*x + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]
] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(6*b^(4/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{x}{b}-\frac{a\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}\right)}{3b^{2}}$	34
default	$\frac{x}{b}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b}$	103

```
[In] int(x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] x/b-1/3/b^2*a*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{a+bx^3}\,dx = \frac{2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right)-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.}{6\,b}$$

```
[In] integrate(x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3
)*a)/a) - (-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) + 2*(-a/b)^
(1/3)*log(x - (-a/b)^(1/3)) + 6*x)/b
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.22

$$\int \frac{x^3}{a+bx^3}\,dx = \operatorname{RootSum}\left(27t^3b^4 + a, (t \mapsto t\log(-3tb + x))\right) + \frac{x}{b}$$

```
[In] integrate(x**3/(b*x**3+a),x)
[Out] RootSum(27*_t**3*b**4 + a, Lambda(_t, _t*log(-3*_t*b + x))) + x/b
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{a+bx^3}\,dx = \frac{x}{b} - \frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{a\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{a\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^3/(b*x^3+a),x, algorithm="maxima")
[Out] x/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^
2*(a/b)^(2/3)) + 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2
/3)) - 1/3*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{a+bx^3}\,dx = \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,b} + \frac{x}{b} - \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2}$$

```
[In] integrate(x^3/(b*x^3+a),x, algorithm="giac")
[Out] 1/3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/b + x/b - 1/3*sqrt(3)*(-a*b^2)^
(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^2 - 1/6*(-a*b
^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^2
```

Mupad [B] (verification not implemented)

Time = 14.65 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.14

$$\int \frac{x^3}{a+bx^3}\,dx = \frac{x}{b}+\frac{(-a)^{1/3}\,\ln\left((-a)^{4/3}+a\,b^{1/3}\,x\right)}{3\,b^{4/3}} - \frac{(-a)^{1/3}\,\ln\left(3\,(-a)^{4/3}\,b^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-3\,a\,b\,x\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^{4/3}} + \frac{(-a)^{1/3}\,\ln\left(9\,(-a)^{4/3}\,b^{2/3}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+3\,a\,b\,x\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^{4/3}}$$

```
[In] int(x^3/(a + b*x^3),x)
```

```
[Out] x/b + ((-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*x))/(3*b^(4/3)) - ((-a)^(1/3)*
log(3*(-a)^(4/3)*b^(2/3)*((3^(1/2)*1i)/2 + 1/2) - 3*a*b*x)*((3^(1/2)*1i)/2
+ 1/2))/(3*b^(4/3)) + ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2/3)*((3^(1/2)*1i)/6
- 1/6) + 3*a*b*x)*((3^(1/2)*1i)/6 - 1/6))/b^(4/3)
```

3.66 $\int \frac{x^4}{a+bx^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 113

$$\int \frac{x^4}{a+bx^3}\,dx = \frac{x^2}{2b} + \frac{a\left(-\sqrt{3}\arctan\left(\frac{-\sqrt[3]{\frac{a}{b}}+2x}{\sqrt{3}\sqrt[3]{\frac{a}{b}}}\right)+\frac{1}{2}\log\left(\frac{\left(\sqrt[3]{\frac{a}{b}}+x\right)^2}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\right)\right)}{3\sqrt[3]{\frac{a}{b}}b^2}$$

```
[Out] 1/2*x^2/b+1/3*a/b^2/(a/b)^(1/3)*(1/2*ln((x+(a/b)^(1/3))^2/(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3)))-3^(1/2)*arctan(1/3*(2*x-(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3)))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {327, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^4}{a+bx^3}\,dx = \frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}} + \frac{x^2}{2b}$$

```
[In] Int[x^4/(a + b*x^3),x]
```

```
[Out] x^2/(2*b) + (a^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^(5/3)) + (a^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) - (a^(2/3)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{x^2}{2b} - \frac{a\int \frac{x}{a+bx^3}\,dx}{b}$$

$$= \frac{x^2}{2b} + \frac{a^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{4/3}} - \frac{a^{2/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b^{4/3}}$$

$$= \frac{x^2}{2b} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}} - \frac{a^{2/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{5/3}} - \frac{a\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b^{4/3}}$$

$$= \frac{x^2}{2b} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}} - \frac{a^{2/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{5/3}}$$

$$= \frac{x^2}{2b} + \frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98

$$\int \frac{x^4}{a+bx^3}\,dx$$

$$= \frac{3b^{2/3}x^2 + 2\sqrt{3}a^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) - a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}}$$

```
[In] Integrate[x^4/(a + b*x^3),x]
```

```
[Out] (3*b^(2/3)*x^2 + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]] + 2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x] - a^(2/3)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(6*b^(5/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.33

method	result	size
risch	$\frac{x^2}{2b}-\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{3b^2}$	37
default	$\frac{x^2}{2b}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a}{b}$	106

```
[In] int(x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2/b-1/3/b^2*a*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{x^4}{a+bx^3}\,dx = \frac{3\,x^2-2\,\sqrt{3}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)-\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)+2\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(a\right.}{6\,b}$$

```
[In] integrate(x^4/(b*x^3+a),x, algorithm="fricas")
[Out] 1/6*(3*x^2 - 2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^
(1/3) - sqrt(3)*a)/a) - (a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a
*(a^2/b^2)^(1/3)) + 2*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.28

$$\int \frac{x^4}{a+bx^3}\,dx = \operatorname{RootSum}\left(27t^3b^5 - a^2, \left(t \mapsto t\log\left(\frac{9t^2b^3}{a} + x\right)\right)\right) + \frac{x^2}{2b}$$

[In] integrate(x**4/(b*x**3+a),x)

[Out] RootSum(27*_t**3*b**5 - a**2, Lambda(_t, _t*log(9*_t**2*b**3/a + x))) + x**2/(2*b)

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{a+bx^3}\,dx = \frac{x^2}{2\,b} - \frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{a\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{a\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

[In] integrate(x^4/(b*x^3+a),x, algorithm="maxima")

[Out] 1/2*x^2/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(1/3)) - 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(1/3)) + 1/3*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(1/3))

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01

$$\int \frac{x^4}{a+bx^3}\,dx = \frac{x^2}{2\,b} + \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,b} + \frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3} - \frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3}$$

[In] integrate(x^4/(b*x^3+a),x, algorithm="giac")

[Out] 1/2*x^2/b + 1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/b + 1/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 - 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

$$\int \frac{x^4}{a+bx^3}\,dx = \frac{x^2}{2\,b}+\frac{a^{2/3}\,\ln\left(\frac{a^{7/3}}{b^{4/3}}+\frac{a^2\,x}{b}\right)}{3\,b^{5/3}}-\frac{a^{2/3}\,\ln\left(\frac{a^2\,x}{b}+\frac{a^{7/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^{5/3}}+\frac{a^{2/3}\,\ln\left(\frac{a^2\,x}{b}+\frac{9\,a^{7/3}\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2}{b^{4/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^{5/3}}$$

```
[In] int(x^4/(a + b*x^3),x)
```

```
[Out] x^2/(2*b) + (a^(2/3)*log(a^(7/3)/b^(4/3) + (a^2*x)/b))/(3*b^(5/3)) - (a^(2/
3)*log((a^2*x)/b + (a^(7/3)*((3^(1/2)*1i)/2 + 1/2)^2)/b^(4/3))*((3^(1/2)*1i
)/2 + 1/2))/(3*b^(5/3)) + (a^(2/3)*log((a^2*x)/b + (9*a^(7/3)*((3^(1/2)*1i)
/6 - 1/6)^2)/b^(4/3))*((3^(1/2)*1i)/6 - 1/6))/b^(5/3)
```

3.67 $\int \frac{1}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 112

$$\int \frac{1}{(a+bx^3)^2} dx = \frac{x}{3a(a+bx^3)} + \frac{2\sqrt[3]{\frac{a}{b}}\left(\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{2\sqrt[3]{\frac{a}{b}}-x}\right)+\frac{1}{2}\log\left(\frac{\left(\sqrt[3]{\frac{a}{b}}+x\right)^2}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\right)\right)}{9a^2}$$

```
[Out] 1/3*x/a/(b*x^3+a)+2/9/a^2*(a/b)^(1/3)*(1/2*ln((x+(a/b)^(1/3))^2/(x^2-(a/b)^
(1/3)*x+(a/b)^(2/3)))+3^(1/2)*arctan(3^(1/2)*x/(2*(a/b)^(1/3)-x)))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {205, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{(a+bx^3)^2} dx = -\frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{x}{3a(a+bx^3)}$$

```
[In] Int[(a + b*x^3)^(-2),x]
```

```
[Out] x/(3*a*(a + b*x^3)) - (2*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])
/(3*Sqrt[3]*a^(5/3)*b^(1/3)) + (2*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(1
/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(9*a^(5/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x}{3a\left(a+bx^3\right)} + \frac{2\int \frac{1}{a+bx^3}\,dx}{3a} \\
&= \frac{x}{3a\left(a+bx^3\right)} + \frac{2\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}} + \frac{2\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}} \\
&= \frac{x}{3a\left(a+bx^3\right)} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{4/3}} - \frac{\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}\sqrt[3]{b}} \\
&= \frac{x}{3a\left(a+bx^3\right)} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}\sqrt[3]{b}} \\
&\quad + \frac{2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{5/3}\sqrt[3]{b}} \\
&= \frac{x}{3a\left(a+bx^3\right)} - \frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}\sqrt[3]{b}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}\sqrt[3]{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+bx^3\right)^2}\,dx = \frac{\frac{3a^{2/3}x}{a+bx^3} - \frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}}{9a^{5/3}}$$

```
[In] Integrate[(a + b*x^3)^(-2),x]
[Out] ((3*a^(2/3)*x)/(a + b*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/b^(1/3) + (2*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2]/b^(1/3))/(9*a^(5/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.41

method	result	size
risch	$\frac{x}{3a\left(b\,x^{3}+a\right)}+\frac{2\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}\right)}{9ab}$	46
default	$\frac{x}{3a\left(b\,x^{3}+a\right)}+\frac{\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}$	112

```
[In] int(1/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x/a/(b*x^3+a)+2/9/a/b*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 389, normalized size of antiderivative = 3.47

$$\int \frac{1}{\left(a+bx^{3}\right)^{2}}\,dx$$

$$= \left[\frac{3\,a^{2}bx+3\,\sqrt{\frac{1}{3}}\left(ab^{2}x^{3}+a^{2}b\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}-3\,\left(a^{2}b\right)^{\frac{1}{3}}ax-a^{2}+3\,\sqrt{\frac{1}{3}}\left(2\,abx^{2}+\left(a^{2}b\right)^{\frac{2}{3}}x-\left(a^{2}b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right)-\left(\right.}{9\left(a^{3}b^{2}x^{3}+a^{4}b\right.}\right.$$

```
[In] integrate(1/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(3*a^2*b*x + 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-(a^2*b)^(1/3)/b)*lo
g((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)
^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (b*x^3 +
 a)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b*x
^3 + a)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^2*x^3 + a^4*b), 1/
9*(3*a^2*b*x + 6*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt((a^2*b)^(1/3)/b)*arctan
```

```
(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2)
 - (b*x^3 + a)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*
a) + 2*(b*x^3 + a)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^2*x^3 +
 a^4*b)]
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.35

$$\int \frac{1}{(a+bx^3)^2}\,dx = \frac{x}{3a^2+3abx^3} + \operatorname{RootSum}\left(729t^3a^5b-8,\left(t\mapsto t\log\left(\frac{9ta^2}{2}+x\right)\right)\right)$$

```
[In] integrate(1/(b*x**3+a)**2,x)
```

```
[Out] x/(3*a**2 + 3*a*b*x**3) + RootSum(729*_t**3*a**5*b - 8, Lambda(_t, _t*log(9
*_t*a**2/2 + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.09

$$\int \frac{1}{(a+bx^3)^2}\,dx = \frac{x}{3\,(abx^3+a^2)} + \frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,ab\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(1/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x/(a*b*x^3 + a^2) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/
(a/b)^(1/3))/(a*b*(a/b)^(2/3)) - 1/9*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(a*b*(a/b)^(2/3)) + 2/9*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.13

$$\int \frac{1}{(a+bx^3)^2}\,dx = -\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2}+\frac{x}{3\,(bx^3+a)a}+\frac{2\,\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b}+\frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^2b}$$

```
[In] integrate(1/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -2/9*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*x/((b*x^3 + a)*a) +
2/9*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/(a^2*b) + 1/9*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/(a^2*b)
```

Mupad [B] (verification not implemented)

Time = 16.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+bx^3)^2}\,dx = \frac{x}{3\,a\,\left(b\,x^3+a\right)}+\frac{2\,\ln\left(\frac{2\,b^{5/3}}{a^{2/3}}+\frac{2\,b^2\,x}{a}\right)}{9\,a^{5/3}\,b^{1/3}}+\frac{\ln\left(\frac{2\,b^2\,x}{a}+\frac{b^{5/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{5/3}\,b^{1/3}}-\frac{\ln\left(\frac{2\,b^2\,x}{a}-\frac{b^{5/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{5/3}\,b^{1/3}}$$

```
[In] int(1/(a + b*x^3)^2,x)
```

```
[Out] x/(3*a*(a + b*x^3)) + (2*log((2*b^(5/3))/a^(2/3) + (2*b^2*x)/a))/(9*a^(5/3)
*b^(1/3)) + (log((2*b^2*x)/a + (b^(5/3)*(3^(1/2)*1i - 1))/a^(2/3))*(3^(1/2)
*1i - 1))/(9*a^(5/3)*b^(1/3)) - (log((2*b^2*x)/a - (b^(5/3)*(3^(1/2)*1i + 1
))/a^(2/3))*(3^(1/2)*1i + 1))/(9*a^(5/3)*b^(1/3))
```

3.68 $\int \frac{x}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 124

$$\int \frac{x}{(a+bx^3)^2}\,dx = \frac{x^2}{3a(a+bx^3)} - \frac{-\sqrt{3}\arctan\left(\frac{-\sqrt[3]{\frac{a}{b}}+2x}{\sqrt{3}\sqrt[3]{\frac{a}{b}}}\right) + \frac{1}{2}\log\left(\frac{\left(\sqrt[3]{\frac{a}{b}}+x\right)^2}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\right)}{9a\sqrt[3]{\frac{a}{b}}b}$$

```
[Out] 1/3*x^2/a/(b*x^3+a)-1/9/a/b/(a/b)^(1/3)*(1/2*ln((x+(a/b)^(1/3))^2/(x^2-(a/b
)^(1/3)*x+(a/b)^(2/3)))-3^(1/2)*arctan(1/3*(2*x-(a/b)^(1/3))*3^(1/2)/(a/b)^
(1/3)))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {296, 298, 31, 648, 631, 210, 642}

$$\int \frac{x}{(a+bx^3)^2}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{2/3}} + \frac{x^2}{3a(a+bx^3)}$$

```
[In] Int[x/(a + b*x^3)^2,x]
```

```
[Out] x^2/(3*a*(a + b*x^3)) - ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(
3*Sqrt[3]*a^(4/3)*b^(2/3)) - Log[a^(1/3) + b^(1/3)*x]/(9*a^(4/3)*b^(2/3)) +
 Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(4/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{x^2}{3a(a+bx^3)} + \frac{\int \frac{x}{a+bx^3}\,dx}{3a}$$

$$= \frac{x^2}{3a(a+bx^3)} - \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}} + \frac{\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$= \frac{x^2}{3a(a+bx^3)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{2/3}} + \frac{\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{4/3}b^{2/3}} + \frac{\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a\sqrt[3]{b}}$$

$$= \frac{x^2}{3a(a+bx^3)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{4/3}b^{2/3}}$$

$$= \frac{x^2}{3a(a+bx^3)} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{2/3}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.96

$$\int \frac{x}{(a+bx^3)^2}\,dx = \frac{\frac{6\sqrt[3]{a}x^2}{a+bx^3} - \frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}}{18a^{4/3}}$$

```
[In] Integrate[x/(a + b*x^3)^2,x]
[Out] ((6*a^(1/3)*x^2)/(a + b*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]])/b^(2/3) - (2*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/b^(2/3))/(18*a^(4/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{x^{2}}{3a(b\,x^{3}+a)}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^{3}+a)}\frac{\ln(x-\_R)}{\_R}}{9ab}$	48
default	$\frac{x^{2}}{3a(b\,x^{3}+a)}+\frac{-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{3a}$	114

```
[In] int(x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*x^2/a/(b*x^3+a)+1/9/a/b*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.24

$$\int \frac{x}{(a+bx^3)^2}\,dx$$

$$= \left[\frac{6\,ab^2x^2+3\sqrt{\frac{1}{3}}(ab^2x^3+a^2b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\,(-ab^2)^{\frac{2}{3}}x^2+(-ab^2)^{\frac{1}{3}}a\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3\,(-ab^2)^{\frac{2}{3}}x}{bx^3+a}\right)}{18\,(a^2b^3x^3 -}\right.$$

```
[In] integrate(x/(b*x^3+a)^2,x, algorithm="fricas")
[Out] [1/18*(6*a*b^2*x^2 + 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt((-a*b^2)^(1/3)/a)
*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2
)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (b*x
^3 + a)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) -
 2*(b*x^3 + a)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^3*x^3 + a^3
*b^2), 1/18*(6*a*b^2*x^2 + 6*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-(-a*b^2)^(
```

```
1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b
) + (b*x^3 + a)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^
(2/3)) - 2*(b*x^3 + a)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^3*x
^3 + a^3*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.35

$$\int \frac{x}{\left(a+bx^3\right)^2}\,dx = \frac{x^2}{3a^2+3abx^3} + \operatorname{RootSum}\left(729t^3a^4b^2+1, \left(t \mapsto t\log\left(81t^2a^3b+x\right)\right)\right)$$

```
[In] integrate(x/(b*x**3+a)**2,x)
```

```
[Out] x**2/(3*a**2 + 3*a*b*x**3) + RootSum(729*_t**3*a**4*b**2 + 1, Lambda(_t, _t
*log(81*_t**2*a**3*b + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+bx^3\right)^2}\,dx = \frac{x^2}{3\left(abx^3+a^2\right)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^2/(a*b*x^3 + a^2) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3)
)/(a/b)^(1/3))/(a*b*(a/b)^(1/3)) + 1/18*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/
3))/(a*b*(a/b)^(1/3)) - 1/9*log(x + (a/b)^(1/3))/(a*b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04

$$\int \frac{x}{(a+bx^3)^2}\,dx = \frac{x^2}{3\,(bx^3+a)a} - \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2} - \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2} + \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^2}$$

```
[In] integrate(x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^2/((b*x^3 + a)*a) - 1/9*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2 -
 1/9*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/(a^2*b^2) + 1/18*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3))/(a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 15.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.11

$$\int \frac{x}{(a+bx^3)^2}\,dx = \frac{x^2}{3\,a\,(b\,x^3+a)} + \frac{(-1)^{1/3}\,\ln\left(\frac{(-1)^{2/3}\,b^{2/3}}{9\,a^{5/3}}+\frac{b\,x}{9\,a^2}\right)}{9\,a^{4/3}\,b^{2/3}} - \frac{(-1)^{1/3}\,\ln\left((-1)^{2/3}\,a^{1/3}-2\,b^{1/3}\,x+(-1)^{1/6}\,\sqrt{3}\,a^{1/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{4/3}\,b^{2/3}} + \frac{(-1)^{1/3}\,\ln\left(2\,b^{1/3}\,x-(-1)^{2/3}\,a^{1/3}+(-1)^{1/6}\,\sqrt{3}\,a^{1/3}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{4/3}\,b^{2/3}}$$

```
[In] int(x/(a + b*x^3)^2,x)
```

```
[Out] x^2/(3*a*(a + b*x^3)) + ((-1)^(1/3)*log(((-1)^(2/3)*b^(2/3))/(9*a^(5/3)) +
(b*x)/(9*a^2)))/(9*a^(4/3)*b^(2/3)) - ((-1)^(1/3)*log((-1)^(2/3)*a^(1/3) -
2*b^(1/3)*x + (-1)^(1/6)*3^(1/2)*a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(4/3
)*b^(2/3)) + ((-1)^(1/3)*log(2*b^(1/3)*x - (-1)^(2/3)*a^(1/3) + (-1)^(1/6)*
3^(1/2)*a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(4/3)*b^(2/3))
```

3.69 $\int \frac{x^2}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{x^2}{(a+bx^3)^2} dx = -\frac{1}{3b(a+bx^3)}$$

```
[Out] -1/3/b/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{267\}$

$$\int \frac{x^2}{(a+bx^3)^2} dx = -\frac{1}{3b(a+bx^3)}$$

```
[In] Int[x^2/(a + b*x^3)^2,x]
```

```
[Out] -1/3*1/(b*(a + b*x^3))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{3b(a+bx^3)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^3)^2}\,dx = -\frac{1}{3b\,(a+bx^3)}$$

```
[In] Integrate[x^2/(a + b*x^3)^2,x]
[Out] -1/3*1/(b*(a + b*x^3))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{3b(b\,x^3+a)}$	15
derivativedivides	$-\frac{1}{3b(b\,x^3+a)}$	15
default	$-\frac{1}{3b(b\,x^3+a)}$	15
norman	$-\frac{1}{3b(b\,x^3+a)}$	15
risch	$-\frac{1}{3b(b\,x^3+a)}$	15
parallelrisch	$-\frac{1}{3b(b\,x^3+a)}$	15

```
[In] int(x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/3/b/(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(a+bx^3)^2}\,dx = -\frac{1}{3\,(b^2x^3+ab)}$$

```
[In] integrate(x^2/(b*x^3+a)^2,x, algorithm="fricas")
[Out] -1/3/(b^2*x^3 + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(a+bx^3)^2}\,dx = -\frac{1}{3ab+3b^2x^3}$$

```
[In] integrate(x**2/(b*x**3+a)**2,x)
[Out] -1/(3*a*b + 3*b**2*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(a+bx^3)^2}\,dx = -\frac{1}{3\,(bx^3+a)b}$$

```
[In] integrate(x^2/(b*x^3+a)^2,x, algorithm="maxima")
[Out] -1/3/((b*x^3 + a)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(a+bx^3)^2}\,dx = -\frac{1}{3\,(bx^3+a)b}$$

```
[In] integrate(x^2/(b*x^3+a)^2,x, algorithm="giac")
[Out] -1/3/((b*x^3 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 14.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(a+bx^3)^2}\,dx = -\frac{1}{3\,b\,\left(b\,x^3+a\right)}$$

```
[In] int(x^2/(a + b*x^3)^2,x)
[Out] -1/(3*b*(a + b*x^3))
```

3.70 $\int \frac{x^3}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 115

$$\int \frac{x^3}{(a+bx^3)^2}\,dx = -\frac{x}{3b(a+bx^3)} + \frac{\sqrt[3]{\frac{a}{b}}\left(\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{2\sqrt[3]{\frac{a}{b}}-x}\right)+\frac{1}{2}\log\left(\frac{\left(\sqrt[3]{\frac{a}{b}}+x\right)^2}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\right)\right)}{9ab}$$

```
[Out] -1/3*x/b/(b*x^3+a)+1/9/b*(a/b)^(1/3)/a*(1/2*ln((x+(a/b)^(1/3))^2/(x^2-(a/b)
^(1/3)*x+(a/b)^(2/3)))+3^(1/2)*arctan(3^(1/2)*x/(2*(a/b)^(1/3)-x)))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {294, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^3}{(a+bx^3)^2}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{4/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{4/3}} - \frac{x}{3b(a+bx^3)}$$

```
[In] Int[x^3/(a + b*x^3)^2,x]
```

```
[Out] -1/3*x/(b*(a + b*x^3)) - ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/
(3*Sqrt[3]*a^(2/3)*b^(4/3)) + Log[a^(1/3) + b^(1/3)*x]/(9*a^(2/3)*b^(4/3))
- Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(2/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\frac{x}{3b\left(a+bx^3\right)} + \frac{\int \frac{1}{a+bx^3}\,dx}{3b}$$

$$= -\frac{x}{3b\left(a+bx^3\right)} + \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{2/3}b} + \frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{2/3}b}$$

$$= -\frac{x}{3b\left(a+bx^3\right)} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{4/3}} - \frac{\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{2/3}b^{4/3}} + \frac{\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}b}$$

$$= -\frac{x}{3b\left(a+bx^3\right)} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{4/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{2/3}b^{4/3}}$$

$$= -\frac{x}{3b\left(a+bx^3\right)} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{4/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\int \frac{x^3}{\left(a+bx^3\right)^2}\,dx = \frac{-\frac{6\sqrt[3]{b}x}{a+bx^3} - \frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}}}{18b^{4/3}}$$

```
[In] Integrate[x^3/(a + b*x^3)^2,x]
[Out] ((-6*b^(1/3)*x)/(a + b*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]])/a^(2/3) + (2*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) - Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/a^(2/3))/(18*b^(4/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.37

method	result	size
risch	$-\frac{x}{3b(b\,x^{3}+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}}{9b^{2}}$	43
default	$-\frac{x}{3b(b\,x^{3}+a)}+\frac{\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{3b}$	112

```
[In] int(x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/3*x/b/(b*x^3+a)+1/9/b^2*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.40

$$\int \frac{x^3}{\left(a+bx^3\right)^2}\,dx$$

$$= \left[-\frac{6\,a^{2}bx-3\,\sqrt{\tfrac{1}{3}}(ab^{2}x^{3}+a^{2}b)\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}-3\,(a^{2}b)^{\frac{1}{3}}ax-a^{2}+3\,\sqrt{\tfrac{1}{3}}\left(2\,abx^{2}+(a^{2}b)^{\frac{2}{3}}x-(a^{2}b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right)+}{18\,(a^{2}b^{3}x^{3}+a^{3}b}\right.$$

$$-\frac{6\,a^{2}bx-6\,\sqrt{\tfrac{1}{3}}(ab^{2}x^{3}+a^{2}b)\sqrt{\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\tfrac{1}{3}}\left(2\,(a^{2}b)^{\frac{2}{3}}x-(a^{2}b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^{2}b)^{\frac{1}{3}}}{b}}}{a^{2}}\right)+(bx^{3}+a)(a^{2}b)^{\frac{2}{3}}\log\left(abx^{2}\right.}{18\,(a^{2}b^{3}x^{3}+a^{3}b^{2})}$$

```
[In] integrate(x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(6*a^2*b*x - 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-(a^2*b)^(1/3)/b)*
log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*
b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + (b*x^3
 + a)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*(b
*x^3 + a)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^3*x^3 + a^3*b^2)
, -1/18*(6*a^2*b*x - 6*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt((a^2*b)^(1/3)/b)*
arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b
)/a^2) + (b*x^3 + a)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^
(1/3)*a) - 2*(b*x^3 + a)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^3
*x^3 + a^3*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.34

$$\int \frac{x^3}{(a+bx^3)^2}\,dx = -\frac{x}{3ab+3b^2x^3} + \operatorname{RootSum}\left(729t^3a^2b^4 - 1, \left(t \mapsto t\log\left(9tab+x\right)\right)\right)$$

```
[In] integrate(x**3/(b*x**3+a)**2,x)
```

```
[Out] -x/(3*a*b + 3*b**2*x**3) + RootSum(729*_t**3*a**2*b**4 - 1, Lambda(_t, _t*l
og(9*_t*a*b + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{(a+bx^3)^2}\,dx = -\frac{x}{3\,(b^2x^3+ab)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*x/(b^2*x^3 + a*b) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))
/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) - 1/18*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3
))/(b^2*(a/b)^(2/3)) + 1/9*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.13

$$\int \frac{x^3}{(a+bx^3)^2}\,dx = -\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab} - \frac{x}{3\,(bx^3+a)b} + \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2} + \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2}$$

```
[In] integrate(x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 1/3*x/((b*x^3 + a)*b)
+ 1/9*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
^(1/3))/(a*b^2) + 1/18*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/
3))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 14.82 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{(a+bx^3)^2}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)}{9\,a^{2/3}\,b^{4/3}} - \frac{x}{3\,b\,\left(b\,x^3+a\right)} + \frac{\ln\left(b\,x+\frac{a^{1/3}\,b^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{2/3}\,b^{4/3}} - \frac{\ln\left(b\,x-\frac{a^{1/3}\,b^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{2/3}\,b^{4/3}}$$

```
[In] int(x^3/(a + b*x^3)^2,x)
```

```
[Out] log(b^(1/3)*x + a^(1/3))/(9*a^(2/3)*b^(4/3)) - x/(3*b*(a + b*x^3)) + (log(b
*x + (a^(1/3)*b^(2/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(18*a^(2/3)*b^
(4/3)) - (log(b*x - (a^(1/3)*b^(2/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))
/(18*a^(2/3)*b^(4/3))
```

3.71 $\int \frac{1}{x(a+bx^3)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{1}{x\left(a+bx^3\right)}\,dx=\frac{\log\left(\frac{x^3}{a+bx^3}\right)}{3a}$$

```
[Out] 1/3/a*ln(x^3/(b*x^3+a))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {272, 36, 29, 31}

$$\int \frac{1}{x\left(a+bx^3\right)}\,dx=\frac{\log(x)}{a}-\frac{\log\left(a+bx^3\right)}{3a}$$

```
[In] Int[1/(x*(a + b*x^3)),x]
[Out] Log[x]/a - Log[a + b*x^3]/(3*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, x^3\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^3\right)}{3a} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^3\right)}{3a}\\ &= \frac{\log(x)}{a} - \frac{\log\left(a+bx^3\right)}{3a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a+bx^3\right)}\,dx = \frac{\log(x)}{a} - \frac{\log\left(a+bx^3\right)}{3a}$$

```
[In] Integrate[1/(x*(a + b*x^3)),x]
[Out] Log[x]/a - Log[a + b*x^3]/(3*a)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\ln(b\,x^3+a)}{3a}+\frac{\ln(x)}{a}$	21
norman	$-\frac{\ln(b\,x^3+a)}{3a}+\frac{\ln(x)}{a}$	21
risch	$-\frac{\ln(b\,x^3+a)}{3a}+\frac{\ln(x)}{a}$	21
parallelrisch	$\frac{3\ln(x)-\ln(b\,x^3+a)}{3a}$	21

```
[In] int(1/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a*ln(b*x^3+a)+1/a*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{1}{x(a+bx^3)}\,dx = -\frac{\log(bx^3+a) - 3\,\log(x)}{3\,a}$$

```
[In] integrate(1/x/(b*x^3+a),x, algorithm="fricas")
[Out] -1/3*(log(b*x^3 + a) - 3*log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{x(a+bx^3)}\,dx = \frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b}+x^3\right)}{3a}$$

```
[In] integrate(1/x/(b*x**3+a),x)
[Out] log(x)/a - log(a/b + x**3)/(3*a)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(a+bx^3)}\,dx = -\frac{\log(bx^3+a)}{3\,a} + \frac{\log(x^3)}{3\,a}$$

```
[In] integrate(1/x/(b*x^3+a),x, algorithm="maxima")
[Out] -1/3*log(b*x^3 + a)/a + 1/3*log(x^3)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a+bx^3\right)}\,dx = -\frac{\log\left(|bx^3+a|\right)}{3\,a} + \frac{\log\left(|x|\right)}{a}$$

```
[In] integrate(1/x/(b*x^3+a),x, algorithm="giac")
[Out] -1/3*log(abs(b*x^3 + a))/a + log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a+bx^3\right)}\,dx = -\frac{\ln\left(b\,x^3+a\right)-3\,\ln\left(x\right)}{3\,a}$$

```
[In] int(1/(x*(a + b*x^3)),x)
[Out] -(log(a + b*x^3) - 3*log(x))/(3*a)
```

3.72 $\int \frac{1}{x^2(a+bx^3)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 110

$$\int \frac{1}{x^2(a+bx^3)}\,dx=-\frac{1}{ax}+\frac{-\sqrt{3}\arctan\left(\frac{-\sqrt[3]{\frac{a}{b}}+2x}{\sqrt{3}\sqrt[3]{\frac{a}{b}}}\right)+\frac{1}{2}\log\left(\frac{\left(\sqrt[3]{\frac{a}{b}}+x\right)^2}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\right)}{3a\sqrt[3]{\frac{a}{b}}}$$

```
[Out] -1/a/x+1/3/a/(a/b)^(1/3)*(1/2*ln((x+(a/b)^(1/3))^2/(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3)))-3^(1/2)*arctan(1/3*(2*x-(a/b)^(1/3))*3^(1/2)/(a/b)^(1/3)))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {331, 298, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^2(a+bx^3)}\,dx=\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}}-\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}}+\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}}-\frac{1}{ax}$$

```
[In] Int[1/(x^2*(a + b*x^3)),x]
```

```
[Out] -(1/(a*x)) + (b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(S
qrt[3]*a^(4/3)) + (b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)) - (b^(1/3)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{ax} - \frac{b\int \frac{x}{a+bx^3}\,dx}{a} \\
&= -\frac{1}{ax} + \frac{b^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}} - \frac{b^{2/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{4/3}} \\
&= -\frac{1}{ax} + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}} - \frac{\sqrt[3]{b}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}} - \frac{b^{2/3}\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a} \\
&= -\frac{1}{ax} + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}} - \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}} \\
&\quad - \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{4/3}} \\
&= -\frac{1}{ax} + \frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}} + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}} - \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2(a+bx^3)}\,dx = \frac{-6\sqrt[3]{a}+2\sqrt{3}\sqrt[3]{b}x\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{b}x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\sqrt[3]{b}x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}x}$$

```
[In] Integrate[1/(x^2*(a + b*x^3)),x]
[Out] (-6*a^(1/3) + 2*Sqrt[3]*b^(1/3)*x*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] + 2*b^(1/3)*x*Log[a^(1/3) + b^(1/3)*x] - b^(1/3)*x*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*x)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.48

method	result	size
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{4}\_Z^{3}-b\right)}\_R\ln\left(\left(-4\_R^{3}a^{4}+3b\right)x-a^{3}\_R^{2}\right)\right)}{3}$	53
default	$-\frac{1}{ax}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b}{a}$	106

```
[In] int(1/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] -1/a/x+1/3*sum(_R*ln((-4*_R^3*a^4+3*b)*x-a^3*_R^2),_R=RootOf(_Z^3*a^4-b))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2\left(a+bx^3\right)}\,dx =$$
$$-\frac{2\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right.\right.}{6ax}$$

```
[In] integrate(1/x^2/(b*x^3+a),x, algorithm="fricas")
[Out] -1/6*(2*sqrt(3)*x*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3
)) + x*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 2*x*(b/a)
^(1/3)*log(b*x + a*(b/a)^(2/3)) + 6)/(a*x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^2\,(a+bx^3)}\,dx = \operatorname{RootSum}\left(27t^3a^4 - b, \left(t \mapsto t\log\left(\frac{9t^2a^3}{b} + x\right)\right)\right) - \frac{1}{ax}$$

```
[In] integrate(1/x**2/(b*x**3+a),x)
[Out] RootSum(27*_t**3*a**4 - b, Lambda(_t, _t*log(9*_t**2*a**3/b + x))) - 1/(a*x
)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2\,(a+bx^3)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{1}{ax}$$

```
[In] integrate(1/x^2/(b*x^3+a),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^(
1/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/3)) + 1/3*lo
g(x + (a/b)^(1/3))/(a*(a/b)^(1/3)) - 1/(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2\,(a+bx^3)}\,dx = \frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2} + \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b} - \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b} - \frac{1}{ax}$$

```
[In] integrate(1/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*sqrt(3)*(-a*b^2)^(2
/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - 1/6*(-a
*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\,(a+bx^3)}\,dx = \frac{b^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{3\,a^{4/3}} - \frac{1}{a\,x} - \frac{b^{1/3}\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}+\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{4/3}} + \frac{b^{1/3}\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}-\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{a^{4/3}}$$

```
[In] int(1/(x^2*(a + b*x^3)),x)
```

```
[Out] (b^(1/3)*log(b^(1/3)*x + a^(1/3)))/(3*a^(4/3)) - 1/(a*x) - (b^(1/3)*log(3^(
1/2)*a^(1/3)*2i + 4*b^(1/3)*x - 2*a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(3*a^(4/
3)) + (b^(1/3)*log(4*b^(1/3)*x - 3^(1/2)*a^(1/3)*2i - 2*a^(1/3))*((3^(1/2)*
1i)/6 - 1/6))/a^(4/3)
```

3.73 $\int \frac{1}{x^3(a+bx^3)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 106

$$\int \frac{1}{x^3\left(a+bx^3\right)}\,dx=-\frac{1}{2ax^2}-\frac{\sqrt[3]{\frac{a}{b}}\,b\left(\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{2\sqrt[3]{\frac{a}{b}}-x}\right)+\frac{1}{2}\log\left(\frac{\left(\sqrt[3]{\frac{a}{b}}+x\right)^2}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\right)\right)}{3a^2}$$

```
[Out] -1/2/a/x^2-1/3*b/a^2*(a/b)^(1/3)*(1/2*ln((x+(a/b)^(1/3))^2/(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3)))+3^(1/2)*arctan(3^(1/2)*x/(2*(a/b)^(1/3)-x)))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {331, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^3\left(a+bx^3\right)}\,dx=\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}}-\frac{1}{2ax^2}$$

```
[In] Int[1/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/2*1/(a*x^2) + (b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(5/3)) - (b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)) + (b^(
2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\frac{1}{2ax^2} - \frac{b\int \frac{1}{a+bx^3}\,dx}{a}$$

$$= -\frac{1}{2ax^2} - \frac{b\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{5/3}} - \frac{b\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{5/3}}$$

$$= -\frac{1}{2ax^2} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}} + \frac{b^{2/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{5/3}} - \frac{b\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{4/3}}$$

$$= -\frac{1}{2ax^2} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}} + \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}} - \frac{b^{2/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{5/3}}$$

$$= -\frac{1}{2ax^2} + \frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}} + \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^3(a+bx^3)}\,dx = \frac{-3a^{2/3}+2\sqrt{3}b^{2/3}x^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2b^{2/3}x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+b^{2/3}x^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}x^2}$$

```
[In] Integrate[1/(x^3*(a + b*x^3)),x]

[Out] (-3*a^(2/3) + 2*Sqrt[3]*b^(2/3)*x^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] - 2*b^(2/3)*x^2*Log[a^(1/3) + b^(1/3)*x] + b^(2/3)*x^2*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*x^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.51

method	result	size
risch	$-\frac{1}{2a\,x^2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^5\_Z^3+b^2\right)}\_R\ln\left(\left(-4\_R^3a^5-3b^2\right)x-a^2b\_R\right)\right)}{3}$	54
default	$-\frac{1}{2a\,x^2}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b}{a}$	106

```
[In] int(1/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/x^2+1/3*sum(_R*ln((-4*_R^3*a^5-3*b^2)*x-a^2*b*_R),_R=RootOf(_Z^3*a^5
+b^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^3\left(a+bx^3\right)}\,dx$$
$$=\frac{2\sqrt{3}x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2\,x^2\left(-\right.}{6\,ax^2}$$

```
[In] integrate(1/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*x^2*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2
/3) - sqrt(3)*b)/b) - x^2*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(
1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*x^2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2
)^(1/3)) - 3)/(a*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^3\,(a+bx^3)}\,dx = \operatorname{RootSum}\left(27t^3a^5+b^2,\left(t\mapsto t\log\left(-\frac{3ta^2}{b}+x\right)\right)\right)-\frac{1}{2ax^2}$$

```
[In] integrate(1/x**3/(b*x**3+a),x)
[Out] RootSum(27*_t**3*a**5 + b**2, Lambda(_t, _t*log(-3*_t*a**2/b + x))) - 1/(2*
a*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\,(a+bx^3)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{1}{2\,ax^2}$$

```
[In] integrate(1/x^3/(b*x^3+a),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^(
2/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*lo
g(x + (a/b)^(1/3))/(a*(a/b)^(2/3)) - 1/2/(a*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3\,(a+bx^3)}\,dx = \frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2}-\frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2}-\frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2}-\frac{1}{2\,ax^2}$$

```
[In] integrate(1/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - 1/3*sqrt(3)*(-a*b^2)^(1
/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - 1/6*(-a*b^2
)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 1/2/(a*x^2)
```

Mupad [B] (verification not implemented)

Time = 15.02 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^3 (a+bx^3)} dx = \frac{b^{2/3} \ln \left((-a)^{7/3} - a^2 b^{1/3} x\right)}{3 (-a)^{5/3}} - \frac{1}{2 a x^2} - \frac{b^{2/3} \ln \left(3 a^2 b^3 x + 3 (-a)^{7/3} b^{8/3} \left(\frac{1}{2} + \frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)\right) \left(\frac{1}{2} + \frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)}{3 (-a)^{5/3}} + \frac{b^{2/3} \ln \left(3 a^2 b^3 x - 9 (-a)^{7/3} b^{8/3} \left(-\frac{1}{6} + \frac{\sqrt{3}\, 1\mathrm{i}}{6}\right)\right) \left(-\frac{1}{6} + \frac{\sqrt{3}\, 1\mathrm{i}}{6}\right)}{(-a)^{5/3}}$$

```
[In] int(1/(x^3*(a + b*x^3)),x)
```

```
[Out] (b^(2/3)*log((-a)^(7/3) - a^2*b^(1/3)*x))/(3*(-a)^(5/3)) - 1/(2*a*x^2) - (b
^(2/3)*log(3*a^2*b^3*x + 3*(-a)^(7/3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2))*((3^(
1/2)*1i)/2 + 1/2))/(3*(-a)^(5/3)) + (b^(2/3)*log(3*a^2*b^3*x - 9*(-a)^(7/3)
*b^(8/3)*((3^(1/2)*1i)/6 - 1/6))*((3^(1/2)*1i)/6 - 1/6))/(-a)^(5/3)
```

3.74 $\int \frac{1}{x(a+bx^3)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{1}{x(a+bx^3)^2} dx = \frac{1}{3a(a+bx^3)} + \frac{\log\left(\frac{x^3}{a+bx^3}\right)}{3a^2}$$

```
[Out] 1/3/a/(b*x^3+a)+1/3/a^2*ln(x^3/(b*x^3+a))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 46}

$$\int \frac{1}{x(a+bx^3)^2} dx = -\frac{\log(a+bx^3)}{3a^2} + \frac{\log(x)}{a^2} + \frac{1}{3a(a+bx^3)}$$

```
[In] Int[1/(x*(a + b*x^3)^2),x]
```

```
[Out] 1/(3*a*(a + b*x^3)) + Log[x]/a^2 - Log[a + b*x^3]/(3*a^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)\,dx, x, x^3\right)\\ &= \frac{1}{3a\left(a+bx^3\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^3\right)}{3a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\left(a+bx^3\right)^2}\,dx = \frac{\frac{a}{a+bx^3}+3\log(x)-\log\left(a+bx^3\right)}{3a^2}$$

```
[In] Integrate[1/(x*(a + b*x^3)^2),x]
[Out] (a/(a + b*x^3) + 3*Log[x] - Log[a + b*x^3])/(3*a^2)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{1}{3a\left(b\,x^{3}+a\right)}+\frac{\ln\left(x\right)}{a^{2}}-\frac{\ln\left(b\,x^{3}+a\right)}{3a^{2}}$	35
norman	$-\frac{b\,x^{3}}{3a^{2}\left(b\,x^{3}+a\right)}+\frac{\ln\left(x\right)}{a^{2}}-\frac{\ln\left(b\,x^{3}+a\right)}{3a^{2}}$	39
default	$-\frac{b\left(-\frac{a}{b\left(b\,x^{3}+a\right)}+\frac{\ln\left(b\,x^{3}+a\right)}{b}\right)}{3a^{2}}+\frac{\ln\left(x\right)}{a^{2}}$	42
parallelrisch	$\frac{3\ln\left(x\right)x^{3}b-\ln\left(b\,x^{3}+a\right)x^{3}b-b\,x^{3}+3\ln\left(x\right)a-\ln\left(b\,x^{3}+a\right)a}{3a^{2}\left(b\,x^{3}+a\right)}$	60

```
[In] int(1/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/3/a/(b*x^3+a)+1/a^2*ln(x)-1/3/a^2*ln(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{1}{x\left(a+bx^3\right)^2}\,dx = -\frac{(bx^3+a)\log\left(bx^3+a\right)-3\left(bx^3+a\right)\log\left(x\right)-a}{3\left(a^2bx^3+a^3\right)}$$

```
[In] integrate(1/x/(b*x^3+a)^2,x, algorithm="fricas")
[Out] -1/3*((b*x^3 + a)*log(b*x^3 + a) - 3*(b*x^3 + a)*log(x) - a)/(a^2*b*x^3 + a
^3)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(a+bx^3\right)^2}\,dx = \frac{1}{3a^2+3abx^3}+\frac{\log\left(x\right)}{a^2}-\frac{\log\left(\frac{a}{b}+x^3\right)}{3a^2}$$

```
[In] integrate(1/x/(b*x**3+a)**2,x)
[Out] 1/(3*a**2 + 3*a*b*x**3) + log(x)/a**2 - log(a/b + x**3)/(3*a**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\left(a+bx^3\right)^2}\,dx = \frac{1}{3\left(abx^3+a^2\right)}-\frac{\log\left(bx^3+a\right)}{3\,a^2}+\frac{\log\left(x^3\right)}{3\,a^2}$$

```
[In] integrate(1/x/(b*x^3+a)^2,x, algorithm="maxima")
[Out] 1/3/(a*b*x^3 + a^2) - 1/3*log(b*x^3 + a)/a^2 + 1/3*log(x^3)/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\left(a+bx^3\right)^2}\,dx = -\frac{\log\left(|bx^3+a|\right)}{3\,a^2} + \frac{\log\left(|x|\right)}{a^2} + \frac{bx^3+2\,a}{3\,(bx^3+a)a^2}$$

```
[In] integrate(1/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*log(abs(b*x^3 + a))/a^2 + log(abs(x))/a^2 + 1/3*(b*x^3 + 2*a)/((b*x^3
+ a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 14.73 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(a+bx^3\right)^2}\,dx = \frac{\ln\left(x\right)}{a^2} + \frac{1}{3\,a\,\left(b\,x^3+a\right)} - \frac{\ln\left(b\,x^3+a\right)}{3\,a^2}$$

```
[In] int(1/(x*(a + b*x^3)^2),x)
```

```
[Out] log(x)/a^2 + 1/(3*a*(a + b*x^3)) - log(a + b*x^3)/(3*a^2)
```

3.75 $\int \frac{1}{x^2(a+bx^3)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 132

$$\int \frac{1}{x^2(a+bx^3)^2} dx = \frac{-\frac{1}{ax}-\frac{4bx^2}{3a^2}}{a+bx^3}+\frac{4\left(-\sqrt{3}\arctan\left(\frac{-\sqrt[3]{\frac{a}{b}}+2x}{\sqrt{3}\sqrt[3]{\frac{a}{b}}}\right)+\frac{1}{2}\log\left(\frac{\left(\sqrt[3]{\frac{a}{b}}+x\right)^2}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\right)\right)}{9a^2\sqrt[3]{\frac{a}{b}}}$$

```
[Out] -(1/a/x+4/3*b*x^2/a^2)/(b*x^3+a)+4/9/a^2/(a/b)^(1/3)*(1/2*ln((x+(a/b)^(1/3)
)^2/(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)))-3^(1/2)*arctan(1/3*(2*x-(a/b)^(1/3))*3
^(1/2)/(a/b)^(1/3)))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {296, 331, 298, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^2(a+bx^3)^2} dx = \frac{4\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}}-\frac{2\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{7/3}}+\frac{4\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}}-\frac{4}{3a^2x}+\frac{1}{3ax(a+bx^3)}$$

```
[In] Int[1/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -4/(3*a^2*x) + 1/(3*a*x*(a + b*x^3)) + (4*b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) + (4*b^(1/3)*Log[a^(1/3) + b^
(1/3)*x])/(9*a^(7/3)) - (2*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(9*a^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3ax\left(a+bx^3\right)}+\frac{4\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{3a}\\
&= -\frac{4}{3a^2x}+\frac{1}{3ax\left(a+bx^3\right)}-\frac{(4b)\int \frac{x}{a+bx^3}\,dx}{3a^2}\\
&= -\frac{4}{3a^2x}+\frac{1}{3ax\left(a+bx^3\right)}+\frac{\left(4b^{2/3}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{7/3}}-\frac{\left(4b^{2/3}\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{7/3}}\\
&= -\frac{4}{3a^2x}+\frac{1}{3ax\left(a+bx^3\right)}+\frac{4\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}}\\
&\quad-\frac{\left(2\sqrt[3]{b}\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{7/3}}-\frac{\left(2b^{2/3}\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^2}\\
&= -\frac{4}{3a^2x}+\frac{1}{3ax\left(a+bx^3\right)}+\frac{4\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}}\\
&\quad-\frac{2\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{7/3}}-\frac{\left(4\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{7/3}}\\
&= -\frac{4}{3a^2x}+\frac{1}{3ax\left(a+bx^3\right)}+\frac{4\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}}\\
&\quad+\frac{4\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}}-\frac{2\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{7/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^2 (a+bx^3)^2}\,dx$$

$$= \frac{-\frac{9\sqrt[3]{a}}{x} - \frac{3\sqrt[3]{a}bx^2}{a+bx^3} + 4\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 4\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) - 2\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x\right.}{9a^{7/3}}$$

```
[In] Integrate[1/(x^2*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a^(1/3))/x - (3*a^(1/3)*b*x^2)/(a + b*x^3) + 4*Sqrt[3]*b^(1/3)*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*Log[a^(1/3) + b^(1/3)*x] -
 2*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(7/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.55

method	result	size
risch	$\frac{-\frac{4b\,x^3}{3a^2}-\frac{1}{a}}{x(b\,x^3+a)}+\frac{4\left(\sum_{\_R=\operatorname{RootOf}\left(a^7\_Z^3-b\right)}\_R\ln\left(\left(-4\_R^3a^7+3b\right)x-a^5\_R^2\right)\right)}{9}$	73
default	$-\frac{b\left(\frac{x^2}{3b\,x^3+3a}-\frac{4\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2}-\frac{1}{a^2x}$	120

```
[In] int(1/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-4/3*b/a^2*x^3-1/a)/x/(b*x^3+a)+4/9*sum(_R*ln((-4*_R^3*a^7+3*b)*x-a^5*_R^2
),_R=RootOf(_Z^3*a^7-b))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a+bx^3)^2}\,dx = -\frac{12\,bx^3 + 4\,\sqrt{3}(bx^4+ax)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \frac{1}{3}\,\sqrt{3}\right) + 2\,(bx^4+ax)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2 - ax\left(\frac{b}{a}\right)^{\frac{2}{3}} + a(\right.}{9\,(a^2bx^4 + a^3x)}$$

```
[In] integrate(1/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/9*(12*b*x^3 + 4*sqrt(3)*(b*x^4 + a*x)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(
b/a)^(1/3) - 1/3*sqrt(3)) + 2*(b*x^4 + a*x)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/
a)^(2/3) + a*(b/a)^(1/3)) - 4*(b*x^4 + a*x)*(b/a)^(1/3)*log(b*x + a*(b/a)^(
2/3)) + 9*a)/(a^2*b*x^4 + a^3*x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^2 (a+bx^3)^2}\,dx = \frac{-3a-4bx^3}{3a^3x+3a^2bx^4} + \operatorname{RootSum}\left(729t^3a^7 - 64b, \left(t \mapsto t\log\left(\frac{81t^2a^5}{16b} + x\right)\right)\right)$$

```
[In] integrate(1/x**2/(b*x**3+a)**2,x)
```

```
[Out] (-3*a - 4*b*x**3)/(3*a**3*x + 3*a**2*b*x**4) + RootSum(729*_t**3*a**7 - 64*
b, Lambda(_t, _t*log(81*_t**2*a**5/(16*b) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2 (a+bx^3)^2}\,dx = -\frac{4\,bx^3+3\,a}{3\,(a^2bx^4+a^3x)} - \frac{4\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2\,\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{4\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(1/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(4*b*x^3 + 3*a)/(a^2*b*x^4 + a^3*x) - 4/9*sqrt(3)*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*(a/b)^(1/3)) - 2/9*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3))/(a^2*(a/b)^(1/3)) + 4/9*log(x + (a/b)^(1/3))/(a^2*(a/b)
^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^2 (a+bx^3)^2}\,dx = \frac{4\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3} + \frac{4\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b} - \frac{4\,bx^3+3\,a}{3\,(bx^4+ax)a^2} - \frac{2\,(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^3b}$$

```
[In] integrate(1/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 4/9*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 4/9*sqrt(3)*(-a*b^2)^(2
/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) - 1/3*(4*
b*x^3 + 3*a)/((b*x^4 + a*x)*a^2) - 2/9*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a^3*b)
```

Mupad [B] (verification not implemented)

Time = 14.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2 (a+bx^3)^2}\,dx = \frac{4\,b^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{9\,a^{7/3}} - \frac{\frac{1}{a}+\frac{4\,b\,x^3}{3\,a^2}}{b\,x^4+a\,x} - \frac{4\,b^{1/3}\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}+\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{7/3}} + \frac{b^{1/3}\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}-\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(-\frac{2}{9}+\frac{\sqrt{3}\,2\mathrm{i}}{9}\right)}{a^{7/3}}$$

```
[In] int(1/(x^2*(a + b*x^3)^2),x)
```

```
[Out] (4*b^(1/3)*log(b^(1/3)*x + a^(1/3)))/(9*a^(7/3)) - (1/a + (4*b*x^3)/(3*a^2)
)/(a*x + b*x^4) - (4*b^(1/3)*log(3^(1/2)*a^(1/3)*2i + 4*b^(1/3)*x - 2*a^(1/
3))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(7/3)) + (b^(1/3)*log(4*b^(1/3)*x - 3^(1/2
)*a^(1/3)*2i - 2*a^(1/3))*((3^(1/2)*2i)/9 - 2/9))/a^(7/3)
```

3.76 $\int \frac{1}{x^3(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 126

$$\int \frac{1}{x^3(a+bx^3)^2}\,dx = \frac{-\frac{1}{2ax^2}-\frac{5bx}{6a^2}}{a+bx^3} - \frac{5\sqrt[3]{\frac{a}{b}}\,b\left(\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{2\sqrt[3]{\frac{a}{b}}-x}\right)+\frac{1}{2}\log\left(\frac{\left(\sqrt[3]{\frac{a}{b}}+x\right)^2}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\right)\right)}{9a^3}$$

```
[Out] -(1/2/a/x^2+5/6*b*x/a^2)/(b*x^3+a)-5/9*b/a^3*(a/b)^(1/3)*(1/2*ln((x+(a/b)^(
1/3))^2/(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)))+3^(1/2)*arctan(3^(1/2)*x/(2*(a/b)^
(1/3)-x)))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {296, 331, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^3(a+bx^3)^2}\,dx = \frac{5b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}} + \frac{5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}} - \frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}} - \frac{5}{6a^2x^2} + \frac{1}{3ax^2(a+bx^3)}$$

```
[In] Int[1/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -5/(6*a^2*x^2) + 1/(3*a*x^2*(a + b*x^3)) + (5*b^(2/3)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) - (5*b^(2/3)*Log[a^(1/3)
+ b^(1/3)*x])/(9*a^(8/3)) + (5*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(18*a^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3ax^2(a+bx^3)}+\frac{5\int \frac{1}{x^3(a+bx^3)}\,dx}{3a}\\
&= -\frac{5}{6a^2x^2}+\frac{1}{3ax^2(a+bx^3)}-\frac{(5b)\int \frac{1}{a+bx^3}\,dx}{3a^2}\\
&= -\frac{5}{6a^2x^2}+\frac{1}{3ax^2(a+bx^3)}-\frac{(5b)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{8/3}}-\frac{(5b)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{8/3}}\\
&= -\frac{5}{6a^2x^2}+\frac{1}{3ax^2(a+bx^3)}-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}}\\
&\quad+\frac{\left(5b^{2/3}\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{8/3}}-\frac{(5b)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{7/3}}\\
&= -\frac{5}{6a^2x^2}+\frac{1}{3ax^2(a+bx^3)}-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}}\\
&\quad+\frac{5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}}-\frac{\left(5b^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{8/3}}\\
&= -\frac{5}{6a^2x^2}+\frac{1}{3ax^2(a+bx^3)}+\frac{5b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}}\\
&\quad-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}}+\frac{5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^3 (a+bx^3)^2} \, dx = \frac{-\frac{9a^{2/3}}{x^2} - \frac{6a^{2/3}bx}{a+bx^3} + 10\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) + 5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}}$$

[In] Integrate[1/(x^3*(a + b*x^3)^2),x]

[Out] ((-9*a^(2/3))/x^2 - (6*a^(2/3)*b*x)/(a + b*x^3) + 10*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 10*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] + 5*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(8/3))

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{-\frac{5b\,x^3}{6a^2}-\frac{1}{2a}}{x^2(b\,x^3+a)}+\frac{5\left(\sum_{\_R=\operatorname{RootOf}\left(a^8\_Z^3+b^2\right)}\_R\ln\left(\left(-4\_R^3a^8-3b^2\right)x-a^3b\_R\right)\right)}{9}$	74
default	$-\frac{b\left(\frac{x}{3b\,x^3+3a}+\frac{5\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{a^2}-\frac{1}{2a^2x^2}$	118

[In] int(1/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] (-5/6*b/a^2*x^3-1/2/a)/x^2/(b*x^3+a)+5/9*sum(_R*ln((-4*_R^3*a^8-3*b^2)*x-a^3*b*_R),_R=RootOf(_Z^3*a^8+b^2))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^3 (a+bx^3)^2}\,dx = -\frac{15\,bx^3 - 10\,\sqrt{3}(bx^5+ax^2)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right) + 5\,(bx^5+ax^2)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+ab\right.}{18\,(a^2bx^5+a^3x^2)}$$

```
[In] integrate(1/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(15*b*x^3 - 10*sqrt(3)*(b*x^5 + a*x^2)*(-b^2/a^2)^(1/3)*arctan(1/3*(2
*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 5*(b*x^5 + a*x^2)*(-b^2/a^2
)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 10*(
b*x^5 + a*x^2)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) + 9*a)/(a^2*b
*x^5 + a^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.46

$$\int \frac{1}{x^3 (a+bx^3)^2}\,dx = \frac{-3a-5bx^3}{6a^3x^2+6a^2bx^5} + \operatorname{RootSum}\left(729t^3a^8+125b^2, \left(t\mapsto t\log\left(-\frac{9ta^3}{5b}+x\right)\right)\right)$$

```
[In] integrate(1/x**3/(b*x**3+a)**2,x)
```

```
[Out] (-3*a - 5*b*x**3)/(6*a**3*x**2 + 6*a**2*b*x**5) + RootSum(729*_t**3*a**8 +
125*b**2, Lambda(_t, _t*log(-9*_t*a**3/(5*b) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^3 (a+bx^3)^2}\,dx = -\frac{5\,bx^3+3\,a}{6\,(a^2bx^5+a^3x^2)} - \frac{5\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{5\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(1/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(5*b*x^3 + 3*a)/(a^2*b*x^5 + a^3*x^2) - 5/9*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*(a/b)^(2/3)) + 5/18*log(x^2 - x*(a/b
)^(1/3) + (a/b)^(2/3))/(a^2*(a/b)^(2/3)) - 5/9*log(x + (a/b)^(1/3))/(a^2*(a
/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^3\left(a+bx^3\right)^2}\,dx = \frac{5\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3} - \frac{bx}{3\,(bx^3+a)a^2} - \frac{5\,\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3} - \frac{5\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3} - \frac{1}{2\,a^2x^2}$$

```
[In] integrate(1/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 5/9*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 - 1/3*b*x/((b*x^3 + a)*a^
2) - 5/9*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a
/b)^(1/3))/a^3 - 5/18*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3
))/a^3 - 1/2/(a^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 14.67 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^3\left(a+bx^3\right)^2}\,dx = \frac{5\left(-1\right)^{1/3}b^{2/3}\ln\left(\left(-1\right)^{1/3}a^{1/3}-b^{1/3}\,x\right)}{9\,a^{8/3}} - \frac{\frac{1}{2\,a}+\frac{5\,b\,x^3}{6\,a^2}}{b\,x^5+a\,x^2} - \frac{5\left(-1\right)^{1/3}b^{2/3}\ln\left(\left(-1\right)^{1/3}a^{1/3}+2\,b^{1/3}\,x+\left(-1\right)^{5/6}\sqrt{3}\,a^{1/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{8/3}} + \frac{5\left(-1\right)^{1/3}b^{2/3}\ln\left(\left(-1\right)^{1/3}a^{1/3}+2\,b^{1/3}\,x-\left(-1\right)^{5/6}\sqrt{3}\,a^{1/3}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{8/3}}$$

```
[In] int(1/(x^3*(a + b*x^3)^2),x)
```

```
[Out] (5*(-1)^(1/3)*b^(2/3)*log((-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(9*a^(8/3)) - (1
/(2*a) + (5*b*x^3)/(6*a^2))/(a*x^2 + b*x^5) - (5*(-1)^(1/3)*b^(2/3)*log((-1
)^(1/3)*a^(1/3) + 2*b^(1/3)*x + (-1)^(5/6)*3^(1/2)*a^(1/3))*((3^(1/2)*1i)/2
 + 1/2))/(9*a^(8/3)) + (5*(-1)^(1/3)*b^(2/3)*log((-1)^(1/3)*a^(1/3) + 2*b^(
1/3)*x - (-1)^(5/6)*3^(1/2)*a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(8/3))
```

3.77 $\int \frac{1}{a+bx^4} dx$

Optimal result

Integrand size = 9, antiderivative size = 63

$$\int \frac{1}{a+bx^4} dx = \frac{\sqrt[4]{-\frac{a}{b}}\left(2\arctan\left(\frac{x}{\sqrt[4]{-\frac{a}{b}}}\right)+\log\left(\frac{\sqrt[4]{-\frac{a}{b}}+x}{-\sqrt[4]{-\frac{a}{b}}+x}\right)\right)}{4a}$$

```
[Out] 1/4*(-a/b)^(1/4)/a*(ln((x+(-a/b)^(1/4))/(x-(-a/b)^(1/4)))+2*arctan(x/(-a/b)
^(1/4)))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 185 vs. 2(63) = 126.

Time = 0.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.94, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{a+bx^4} dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

```
[In] Int[(a + b*x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(3/4)*b^(1/4)) + Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - Log[Sqr
```

```
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4))
 + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(4*Sqrt[2]*a^(3/4
)*b^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}} + \frac{\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\sqrt{b}} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\sqrt{b}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}$$
$$+ \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}$$
$$- \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 134 vs. 2(63) = 126.

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.13

$$\int \frac{1}{a+bx^4}\,dx$$
$$= \frac{-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)+\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

```
[In] Integrate[(a + b*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*b^(1/4)
*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(
1/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{4}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}}}{4b}$	27
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}$	102

```
[In] int(1/(b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4/b*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.78

$$\int \frac{1}{a+bx^4}\,dx = \frac{1}{4}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+x\right)+\frac{1}{4}i\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(i\,a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+x\right) - \frac{1}{4}i\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(-i\,a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+x\right) - \frac{1}{4}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+x\right)$$

```
[In] integrate(1/(b*x^4+a),x, algorithm="fricas")
[Out] 1/4*(-1/(a^3*b))^(1/4)*log(a*(-1/(a^3*b))^(1/4) + x) + 1/4*I*(-1/(a^3*b))^(
1/4)*log(I*a*(-1/(a^3*b))^(1/4) + x) - 1/4*I*(-1/(a^3*b))^(1/4)*log(-I*a*(-
1/(a^3*b))^(1/4) + x) - 1/4*(-1/(a^3*b))^(1/4)*log(-a*(-1/(a^3*b))^(1/4) +
x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.32

$$\int \frac{1}{a+bx^4}\,dx = \mathrm{RootSum}\left(256t^4a^3b+1,\left(t\mapsto t\log\left(4ta+x\right)\right)\right)$$

```
[In] integrate(1/(b*x**4+a),x)
[Out] RootSum(256*_t**4*a**3*b + 1, Lambda(_t, _t*log(4*_t*a + x)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(53) = 106.

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.68

$$\int \frac{1}{a+bx^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}b^{\frac{1}{4}}}$$

```
[In] integrate(1/(b*x^4+a),x, algorithm="maxima")
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt
(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 1/4*sqrt(2)*arctan(1/2
*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sq
rt(a)*sqrt(sqrt(a)*sqrt(b))) + 1/8*sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4
)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - 1/8*sqrt(2)*log(sqrt(b)*x^2 - sq
rt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(53) = 106.

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.84

$$\int \frac{1}{a+bx^4}\,dx = \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab} + \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab} + \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab} - \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab}$$

```
[In] integrate(1/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a
/b)^(1/4))/(a*b) + 1/4*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b) + 1/8*sqrt(2)*(a*b^3)^(1/4)*log(x^2 + s
qrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b) - 1/8*sqrt(2)*(a*b^3)^(1/4)*log(x^2
 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.52

$$\int \frac{1}{a+bx^4}\,dx = -\frac{\operatorname{atan}\left(\frac{b^{1/4}\,x}{(-a)^{1/4}}\right)+\operatorname{atanh}\left(\frac{b^{1/4}\,x}{(-a)^{1/4}}\right)}{2\,(-a)^{3/4}\,b^{1/4}}$$

```
[In] int(1/(a + b*x^4),x)
```

```
[Out] -(atan((b^(1/4)*x)/(-a)^(1/4)) + atanh((b^(1/4)*x)/(-a)^(1/4)))/(2*(-a)^(3/
4)*b^(1/4))
```

3.78 $\int \frac{x}{a+bx^4} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{x}{a+bx^4} \, dx = \frac{\arctan\left(\sqrt{\frac{b}{a}}x^2\right)}{2\sqrt{ab}}$$

```
[Out] 1/2/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {281, 211}

$$\int \frac{x}{a+bx^4} \, dx = \frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[In] Int[x/(a + b*x^4),x]
```

```
[Out] ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)$$
$$= \frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{x}{a+bx^4}\,dx = \frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[In] Integrate[x/(a + b*x^4),x]
[Out] ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{\arctan\left(\frac{b\,x^2}{\sqrt{ab}}\right)}{2\sqrt{ab}}$	19
risch	$-\frac{\ln\left(x^2\sqrt{-ab}-a\right)}{4\sqrt{-ab}}+\frac{\ln\left(x^2\sqrt{-ab}+a\right)}{4\sqrt{-ab}}$	46

```
[In] int(x/(b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/2/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.88

$$\int \frac{x}{a+bx^4}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{bx^4-2\sqrt{-ab}x^2-a}{bx^4+a}\right)}{4\,ab}, -\frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{bx^2}\right)}{2\,ab}\right]$$

```
[In] integrate(x/(b*x^4+a),x, algorithm="fricas")
[Out] [-1/4*sqrt(-a*b)*log((b*x^4 - 2*sqrt(-a*b)*x^2 - a)/(b*x^4 + a))/(a*b), -1/
2*sqrt(a*b)*arctan(sqrt(a*b)/(b*x^2))/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(19) = 38$.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.24

$$\int \frac{x}{a+bx^4}\,dx = -\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x^2\right)}{4} + \frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x^2\right)}{4}$$

```
[In] integrate(x/(b*x**4+a),x)
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x**2)/4 + sqrt(-1/(a*b))*log(a*sqrt
(-1/(a*b)) + x**2)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{x}{a+bx^4}\,dx = \frac{\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}}$$

```
[In] integrate(x/(b*x^4+a),x, algorithm="maxima")
[Out] 1/2*arctan(b*x^2/sqrt(a*b))/sqrt(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{x}{a+bx^4}\,dx = \frac{\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}}$$

```
[In] integrate(x/(b*x^4+a),x, algorithm="giac")
[Out] 1/2*arctan(b*x^2/sqrt(a*b))/sqrt(a*b)
```

Mupad [B] (verification not implemented)

Time = 13.95 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{x}{a+bx^4}\,dx=\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

```
[In] int(x/(a + b*x^4),x)
```

```
[Out] atan((b^(1/2)*x^2)/a^(1/2))/(2*a^(1/2)*b^(1/2))
```

3.79 $\int \frac{x^2}{a+bx^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{x^2}{a+bx^4} dx = -\frac{-2\arctan\left(\frac{x}{\sqrt[4]{-\frac{a}{b}}}\right) + \log\left(\frac{\sqrt[4]{-\frac{a}{b}}+x}{-\sqrt[4]{-\frac{a}{b}}+x}\right)}{4\sqrt[4]{-\frac{a}{b}}b}$$

```
[Out] -1/4/b/(-a/b)^(1/4)*(ln((x+(-a/b)^(1/4))/(x-(-a/b)^(1/4)))-2*arctan(x/(-a/b
)^(1/4)))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 185 vs. 2(63) = 126.

Time = 0.12 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.94, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^2}{a+bx^4} dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}} + \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{3/4}} - \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{3/4}}$$

```
[In] Int[x^2/(a + b*x^4),x]
```

```
[Out] -1/2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)) + Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(1/4)*b^(3/4)) + Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(4*Sqrt[2]*a^(1/4)*b^(3/4))
 - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(4*Sqrt[2]*a^(1/4
)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{b}} + \frac{\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{b}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b} + \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}b^{3/4}} + \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}b^{3/4}}$$

$$= \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{3/4}} - \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}}$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}} + \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{3/4}} - \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{3/4}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 134 vs. 2(63) = 126.

Time = 0.02 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.13

$$\int \frac{x^2}{a+bx^4}\,dx = \frac{-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)-\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{3/4}}$$

```
[In] Integrate[x^2/(a + b*x^4),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*b^(1/4)
*x)/a^(1/4)] + Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*b^(
3/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}}{4b}$	27
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	102

```
[In] int(x^2/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.97

$$\begin{aligned}\int \frac{x^2}{a+bx^4}\,dx = {} & \frac{1}{4}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}+x\right) \\ & -\frac{1}{4}i\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(i\,ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}+x\right) \\ & +\frac{1}{4}i\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(-i\,ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}+x\right) \\ & -\frac{1}{4}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(-ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}+x\right)\end{aligned}$$

```
[In] integrate(x^2/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*(-1/(a*b^3))^(1/4)*log(a*b^2*(-1/(a*b^3))^(3/4) + x) - 1/4*I*(-1/(a*b^3
))^(1/4)*log(I*a*b^2*(-1/(a*b^3))^(3/4) + x) + 1/4*I*(-1/(a*b^3))^(1/4)*log
(-I*a*b^2*(-1/(a*b^3))^(3/4) + x) - 1/4*(-1/(a*b^3))^(1/4)*log(-a*b^2*(-1/(
a*b^3))^(3/4) + x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.41

$$\int \frac{x^2}{a+bx^4}\,dx = \text{RootSum}\left(256t^4ab^3+1,\left(t\mapsto t\log\left(64t^3ab^2+x\right)\right)\right)$$

```
[In] integrate(x**2/(b*x**4+a),x)
[Out] RootSum(256*_t**4*a*b**3 + 1, Lambda(_t, _t*log(64*_t**3*a*b**2 + x)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(55) = 110.

Time = 0.30 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.68

$$\int \frac{x^2}{a+bx^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{1}{4}}b^{\frac{3}{4}}}$$

```
[In] integrate(x^2/(b*x^4+a),x, algorithm="maxima")
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt
(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 1/4*sqrt(2)*arctan(1/2
*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sq
rt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/8*sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4
)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + 1/8*sqrt(2)*log(sqrt(b)*x^2 - sq
rt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(55) = 110.

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.84

$$\int \frac{x^2}{a+bx^4}\,dx = \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

```
[In] integrate(x^2/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a
/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) - 1/8*sqrt(2)*(a*b^3)^(3/4)*log(x^2
 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) + 1/8*sqrt(2)*(a*b^3)^(3/4)*l
og(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

$$\int \frac{x^2}{a+bx^4}\,dx = \frac{\operatorname{atan}\left(\frac{b^{1/4}\,x}{(-a)^{1/4}}\right)-\operatorname{atanh}\left(\frac{b^{1/4}\,x}{(-a)^{1/4}}\right)}{2\,(-a)^{1/4}\,b^{3/4}}$$

```
[In] int(x^2/(a + b*x^4),x)
```

```
[Out] (atan((b^(1/4)*x)/(-a)^(1/4)) - atanh((b^(1/4)*x)/(-a)^(1/4)))/(2*(-a)^(1/4
)*b^(3/4))
```

3.80 $\int \frac{x^3}{a+bx^4} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x^3}{a+bx^4} \, dx = \frac{\log(a+bx^4)}{4b}$$

```
[Out] 1/4/b*ln(b*x^4+a)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {266}

$$\int \frac{x^3}{a+bx^4} \, dx = \frac{\log(a+bx^4)}{4b}$$

```
[In] Int[x^3/(a + b*x^4),x]
```

```
[Out] Log[a + b*x^4]/(4*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\text{integral} = \frac{\log(a+bx^4)}{4b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{a+bx^4}\,dx = \frac{\log(a+bx^4)}{4b}$$

```
[In] Integrate[x^3/(a + b*x^4),x]
[Out] Log[a + b*x^4]/(4*b)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\ln(b\,x^4+a)}{4b}$	14
default	$\frac{\ln(b\,x^4+a)}{4b}$	14
norman	$\frac{\ln(b\,x^4+a)}{4b}$	14
risch	$\frac{\ln(b\,x^4+a)}{4b}$	14
parallelrisch	$\frac{\ln(b\,x^4+a)}{4b}$	14

```
[In] int(x^3/(b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4/b*ln(b*x^4+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{a+bx^4}\,dx = \frac{\log(bx^4+a)}{4\,b}$$

```
[In] integrate(x^3/(b*x^4+a),x, algorithm="fricas")
[Out] 1/4*log(b*x^4 + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{a+bx^4}\,dx = \frac{\log(a+bx^4)}{4b}$$

```
[In] integrate(x**3/(b*x**4+a),x)
[Out] log(a + b*x**4)/(4*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{a+bx^4}\,dx = \frac{\log(bx^4+a)}{4\,b}$$

```
[In] integrate(x^3/(b*x^4+a),x, algorithm="maxima")
[Out] 1/4*log(b*x^4 + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{a+bx^4}\,dx = \frac{\log(|bx^4+a|)}{4\,b}$$

```
[In] integrate(x^3/(b*x^4+a),x, algorithm="giac")
[Out] 1/4*log(abs(b*x^4 + a))/b
```

Mupad [B] (verification not implemented)

Time = 14.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{a+bx^4}\,dx = \frac{\ln(b\,x^4+a)}{4\,b}$$

```
[In] int(x^3/(a + b*x^4),x)
[Out] log(a + b*x^4)/(4*b)
```

3.81 $\int \frac{1}{(a+bx^4)^2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 81

$$\int \frac{1}{(a+bx^4)^2} \, dx = \frac{x}{4a(a+bx^4)} + \frac{3\sqrt[4]{-\frac{a}{b}}\left(2\arctan\left(\frac{x}{\sqrt[4]{-\frac{a}{b}}}\right)+\log\left(\frac{\sqrt[4]{-\frac{a}{b}}+x}{-\sqrt[4]{-\frac{a}{b}}+x}\right)\right)}{16a^2}$$

```
[Out] 1/4*x/a/(b*x^4+a)+3/16/a^2*(-a/b)^(1/4)*(ln((x+(-a/b)^(1/4))/(x-(-a/b)^(1/4
)))+2*arctan(x/(-a/b)^(1/4)))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 202 vs. 2(81) = 162.

Time = 0.13 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.49, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {205, 217, 1179, 642, 1176, 631, 210}

$$\begin{aligned}\int \frac{1}{(a+bx^4)^2} \, dx = & -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} \\ & - \frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}} \\ & + \frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{x}{4a(a+bx^4)}\end{aligned}$$

```
[In] Int[(a + b*x^4)^(-2),x]
```

```
[Out] x/(4*a*(a + b*x^4)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*b^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*b^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/(16*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x}{4a\left(a+bx^4\right)}+\frac{3\int\frac{1}{a+bx^4}\,dx}{4a}\\
&= \frac{x}{4a\left(a+bx^4\right)}+\frac{3\int\frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}}+\frac{3\int\frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}}\\
&= \frac{x}{4a\left(a+bx^4\right)}+\frac{3\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}\sqrt{b}}+\frac{3\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}\sqrt{b}}\\
&\quad-\frac{3\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{3\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}\\
&= \frac{x}{4a\left(a+bx^4\right)}-\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}\\
&\quad+\frac{3\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{3\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}\\
&= \frac{x}{4a\left(a+bx^4\right)}-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}\\
&\quad-\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 183 vs. $2(81) = 162$.

Time = 0.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.26

$$\int \frac{1}{(a+bx^4)^2}\,dx = \frac{\frac{8a^{3/4}x}{a+bx^4} - \frac{6\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} + \frac{6\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} - \frac{3\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{\sqrt[4]{b}} + \frac{3\sqrt{2}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{\sqrt[4]{b}}}{32a^{7/4}}$$

```
[In] Integrate[(a + b*x^4)^(-2),x]
```

```
[Out] ((8*a^(3/4)*x)/(a + b*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1
/4)])/b^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(1/4)
 - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/
4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(
1/4))/(32*a^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{x}{4a(b\,x^4+a)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16ab}$	46
default	$\frac{x}{4a(b\,x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}$	118

```
[In] int(1/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x/a/(b*x^4+a)+3/16/a/b*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.26

$$\int \frac{1}{(a+bx^4)^2}\,dx = \frac{3\,(abx^4+a^2)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}+x\right)-3\,(-i\,abx^4-i\,a^2)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}\log\left(i\,a^2\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}+x\right)-3\,(i\,abx^4}{16\,(abx^4+a}$$

```
[In] integrate(1/(b*x^4+a)^2,x, algorithm="fricas")
[Out] 1/16*(3*(a*b*x^4 + a^2)*(-1/(a^7*b))^(1/4)*log(a^2*(-1/(a^7*b))^(1/4) + x)
- 3*(-I*a*b*x^4 - I*a^2)*(-1/(a^7*b))^(1/4)*log(I*a^2*(-1/(a^7*b))^(1/4) +
x) - 3*(I*a*b*x^4 + I*a^2)*(-1/(a^7*b))^(1/4)*log(-I*a^2*(-1/(a^7*b))^(1/4)
 + x) - 3*(a*b*x^4 + a^2)*(-1/(a^7*b))^(1/4)*log(-a^2*(-1/(a^7*b))^(1/4) +
x) + 4*x)/(a*b*x^4 + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.48

$$\int \frac{1}{(a+bx^4)^2}\,dx = \frac{x}{4a^2+4abx^4} + \operatorname{RootSum}\left(65536t^4a^7b+81, \left(t \mapsto t\log\left(\frac{16ta^2}{3}+x\right)\right)\right)$$

```
[In] integrate(1/(b*x**4+a)**2,x)
[Out] x/(4*a**2 + 4*a*b*x**4) + RootSum(65536*_t**4*a**7*b + 81, Lambda(_t, _t*lo
g(16*_t*a**2/3 + x)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(69) = 138.

Time = 0.29 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.33

$$\int \frac{1}{(a+bx^4)^2}\,dx = \frac{x}{4\,(abx^4+a^2)} + \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^2-\right.}{a^{\frac{3}{4}}}\right.}{32\,a}$$

```
[In] integrate(1/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x/(a*b*x^4 + a^2) + 3/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + s
qrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b
))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/
sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(b
)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*lo
g(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(69) = 138$.

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.40

$$\int \frac{1}{(a+bx^4)^2}\,dx = \frac{x}{4\,(bx^4+a)a} + \frac{3\,\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b} + \frac{3\,\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b} + \frac{3\,\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b} - \frac{3\,\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b}$$

```
[In] integrate(1/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x/((b*x^4 + a)*a) + 3/16*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
+ sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b) + 3/16*sqrt(2)*(a*b^3)^(1/4)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b) + 3/32*sq
rt(2)*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b) -
3/32*sqrt(2)*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^
2*b)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int \frac{1}{(a+bx^4)^2}\,dx = \frac{x}{4\,a\,\left(b\,x^4+a\right)}+\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,x}{{(-a)}^{1/4}}\right)}{8\,{(-a)}^{7/4}\,b^{1/4}}+\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,x}{{(-a)}^{1/4}}\right)}{8\,{(-a)}^{7/4}\,b^{1/4}}$$

```
[In] int(1/(a + b*x^4)^2,x)
[Out] x/(4*a*(a + b*x^4)) + (3*atan((b^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*b^(1/4
)) + (3*atanh((b^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*b^(1/4))
```

3.82 $\int \frac{x}{(a+bx^4)^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 48

$$\int \frac{x}{(a+bx^4)^2} \, dx = \frac{x^2}{4a(a+bx^4)} + \frac{\arctan\left(\sqrt{\frac{b}{a}}x^2\right)}{4a\sqrt{ab}}$$

```
[Out] 1/4*x^2/a/(b*x^4+a)+1/4/a/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {281, 205, 211}

$$\int \frac{x}{(a+bx^4)^2} \, dx = \frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{x^2}{4a(a+bx^4)}$$

```
[In] Int[x/(a + b*x^4)^2,x]
```

```
[Out] x^2/(4*a*(a + b*x^4)) + ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(4*a^(3/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, x^2\right)\\ &= \frac{x^2}{4a(a+bx^4)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{4a}\\ &= \frac{x^2}{4a(a+bx^4)} + \frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{x}{(a+bx^4)^2}\,dx = \frac{x^2}{4a(a+bx^4)} + \frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

```
[In] Integrate[x/(a + b*x^4)^2,x]
[Out] x^2/(4*a*(a + b*x^4)) + ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(4*a^(3/2)*Sqrt[b])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x^2}{4a(b\,x^4+a)}+\frac{\arctan\left(\frac{b\,x^2}{\sqrt{ab}}\right)}{4a\sqrt{ab}}$	40
risch	$\frac{x^2}{4a(b\,x^4+a)}-\frac{\ln\left(x^2\sqrt{-ab}-a\right)}{8\sqrt{-ab}\,a}+\frac{\ln\left(x^2\sqrt{-ab}+a\right)}{8\sqrt{-ab}\,a}$	69

```
[In] int(x/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*x^2/a/(b*x^4+a)+1/4/a/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.69

$$\int \frac{x}{\left(a+bx^4\right)^2}\,dx = \left[\frac{2\,abx^2-(bx^4+a)\sqrt{-ab}\log\left(\frac{bx^4-2\,\sqrt{-ab}x^2-a}{bx^4+a}\right)}{8\,(a^2b^2x^4+a^3b)},\ \frac{abx^2-(bx^4+a)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{bx^2}\right)}{4\,(a^2b^2x^4+a^3b)}\right]$$

```
[In] integrate(x/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*b*x^2 - (b*x^4 + a)*sqrt(-a*b)*log((b*x^4 - 2*sqrt(-a*b)*x^2 - a)
/(b*x^4 + a)))/(a^2*b^2*x^4 + a^3*b), 1/4*(a*b*x^2 - (b*x^4 + a)*sqrt(a*b)*
arctan(sqrt(a*b)/(b*x^2)))/(a^2*b^2*x^4 + a^3*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(34) = 68.

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.73

$$\int \frac{x}{\left(a+bx^4\right)^2}\,dx = \frac{x^2}{4a^2+4abx^4} - \frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x^2\right)}{8} + \frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x^2\right)}{8}$$

```
[In] integrate(x/(b*x**4+a)**2,x)
```

```
[Out] x**2/(4*a**2 + 4*a*b*x**4) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b))
+ x**2)/8 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x**2)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{x}{\left(a+bx^4\right)^2}\,dx = \frac{x^2}{4\,(abx^4+a^2)} + \frac{\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a}$$

```
[In] integrate(x/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^2/(a*b*x^4 + a^2) + 1/4*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*a)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{x}{(a+bx^4)^2}\,dx = \frac{x^2}{4\,(bx^4+a)a} + \frac{\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a}$$

```
[In] integrate(x/(b*x^4+a)^2,x, algorithm="giac")
[Out] 1/4*x^2/((b*x^4 + a)*a) + 1/4*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*a)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{x}{(a+bx^4)^2}\,dx = \frac{x^2}{4\,a\,\left(b\,x^4+a\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4\,a^{3/2}\,\sqrt{b}}$$

```
[In] int(x/(a + b*x^4)^2,x)
[Out] x^2/(4*a*(a + b*x^4)) + atan((b^(1/2)*x^2)/a^(1/2))/(4*a^(3/2)*b^(1/2))
```

3.83 $\int \frac{x^2}{(a+bx^4)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 86

$$\int \frac{x^2}{(a+bx^4)^2} dx = \frac{x^3}{4a(a+bx^4)} - \frac{-2\arctan\left(\frac{x}{\sqrt[4]{-\frac{a}{b}}}\right) + \log\left(\frac{\sqrt[4]{-\frac{a}{b}}+x}{-\sqrt[4]{-\frac{a}{b}}+x}\right)}{16a\sqrt[4]{-\frac{a}{b}}b}$$

```
[Out] 1/4*x^3/a/(b*x^4+a)-1/16/a/b/(-a/b)^(1/4)*(ln((x+(-a/b)^(1/4))/(x-(-a/b)^(1
/4)))-2*arctan(x/(-a/b)^(1/4)))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 204 vs. 2(86) = 172.

Time = 0.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.37, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {296, 303, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{x^2}{(a+bx^4)^2} dx = & -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}b^{3/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{5/4}b^{3/4}} \\ & + \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{5/4}b^{3/4}} \\ & - \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{5/4}b^{3/4}} + \frac{x^3}{4a(a+bx^4)}\end{aligned}$$

```
[In] Int[x^2/(a + b*x^4)^2,x]
```

```
[Out] x^3/(4*a*(a + b*x^4)) - ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*
a^(5/4)*b^(3/4)) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(5/
4)*b^(3/4)) + Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(16*Sq
rt[2]*a^(5/4)*b^(3/4)) - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2]/(16*Sqrt[2]*a^(5/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \frac{x^3}{4a\left(a+bx^4\right)} + \frac{\int \frac{x^2}{a+bx^4}\,dx}{4a}$$

$$= \frac{x^3}{4a\left(a+bx^4\right)} - \frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{8a\sqrt{b}} + \frac{\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{8a\sqrt{b}}$$

$$= \frac{x^3}{4a\left(a+bx^4\right)} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16ab} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16ab} + \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{5/4}b^{3/4}} + \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{16\sqrt{2}a^{5/4}b^{3/4}}$$

$$= \frac{x^3}{4a\left(a+bx^4\right)} + \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{5/4}b^{3/4}} - \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{5/4}b^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}b^{3/4}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}b^{3/4}}$$

$$= \frac{x^3}{4a\left(a+bx^4\right)} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}b^{3/4}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}b^{3/4}} + \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{5/4}b^{3/4}} - \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{5/4}b^{3/4}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 184 vs. $2(86) = 172$.

Time = 0.13 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.14

$$\int \frac{x^2}{(a+bx^4)^2}\,dx = \frac{\frac{8\sqrt[4]{a}x^3}{a+bx^4} - \frac{2\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{2\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{b^{3/4}} - \frac{\sqrt{2}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{b^{3/4}}}{32a^{5/4}}$$

```
[In] Integrate[x^2/(a + b*x^4)^2,x]
```

```
[Out] ((8*a^(1/4)*x^3)/(a + b*x^4) - (2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^
(1/4)])/b^(3/4) + (2*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/
4) + (Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/
4) - (Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/
4))/(32*a^(5/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

method	result	size
risch	$\frac{x^3}{4a(b\,x^4+a)}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^4+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}}{16ab}$	48
default	$\frac{x^3}{4a(b\,x^4+a)}+\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	123

```
[In] int(x^2/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^3/a/(b*x^4+a)+1/16/a/b*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.28

$$\int \frac{x^2}{(a+bx^4)^2}\,dx = \frac{4\,x^3 + (abx^4+a^2)\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}\log\left(a^4b^2\left(-\frac{1}{a^5b^3}\right)^{\frac{3}{4}}+x\right) - (i\,abx^4+i\,a^2)\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}\log\left(i\,a^4b^2\left(-\frac{1}{a^5b^3}\right)^{\frac{3}{4}}+x\right) - \cdots}{16\,(abx^4 \cdots}$$

```
[In] integrate(x^2/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*x^3 + (a*b*x^4 + a^2)*(-1/(a^5*b^3))^(1/4)*log(a^4*b^2*(-1/(a^5*b^3
))^(3/4) + x) - (I*a*b*x^4 + I*a^2)*(-1/(a^5*b^3))^(1/4)*log(I*a^4*b^2*(-1/
(a^5*b^3))^(3/4) + x) - (-I*a*b*x^4 - I*a^2)*(-1/(a^5*b^3))^(1/4)*log(-I*a^
4*b^2*(-1/(a^5*b^3))^(3/4) + x) - (a*b*x^4 + a^2)*(-1/(a^5*b^3))^(1/4)*log(
-a^4*b^2*(-1/(a^5*b^3))^(3/4) + x))/(a*b*x^4 + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.53

$$\int \frac{x^2}{(a+bx^4)^2}\,dx = \frac{x^3}{4a^2+4abx^4} + \operatorname{RootSum}\left(65536t^4a^5b^3+1, \left(t\mapsto t\log\left(4096t^3a^4b^2+x\right)\right)\right)$$

```
[In] integrate(x**2/(b*x**4+a)**2,x)
```

```
[Out] x**3/(4*a**2 + 4*a*b*x**4) + RootSum(65536*_t**4*a**5*b**3 + 1, Lambda(_t,
_t*log(4096*_t**3*a**4*b**2 + x)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(76) = 152$.

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.22

$$\int \frac{x^2}{(a+bx^4)^2}\,dx = \frac{x^3}{4\,(abx^4+a^2)} + \frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b \cdots\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}}}{32\,a}$$

```
[In] integrate(x^2/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^3/(a*b*x^4 + a^2) + 1/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x +
 sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqr
t(b)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt
(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*
log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(76) = 152$.

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.28

$$\int \frac{x^2}{(a+bx^4)^2}\,dx = \frac{x^3}{4\,(bx^4+a)a} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

```
[In] integrate(x^2/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^3/((b*x^4 + a)*a) + 1/16*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(a*b^3)^(3/4
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) - 1
/32*sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2
*b^3) + 1/32*sqrt(2)*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a
/b))/(a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{(a+bx^4)^2}\,dx = \frac{\mathrm{atanh}\left(\frac{b^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{5/4}\,b^{3/4}} - \frac{\mathrm{atan}\left(\frac{b^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{5/4}\,b^{3/4}} + \frac{x^3}{4\,a\,\left(b\,x^4+a\right)}$$

```
[In] int(x^2/(a + b*x^4)^2,x)
```

```
[Out] atanh((b^(1/4)*x)/(-a)^(1/4))/(8*(-a)^(5/4)*b^(3/4)) - atan((b^(1/4)*x)/(-a
)^(1/4))/(8*(-a)^(5/4)*b^(3/4)) + x^3/(4*a*(a + b*x^4))
```

3.84 $\int \frac{x^3}{(a+bx^4)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{x^3}{(a+bx^4)^2} \, dx = -\frac{1}{4b(a+bx^4)}$$

```
[Out] -1/4/b/(b*x^4+a)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{267\}$

$$\int \frac{x^3}{(a+bx^4)^2} \, dx = -\frac{1}{4b(a+bx^4)}$$

```
[In] Int[x^3/(a + b*x^4)^2,x]
```

```
[Out] -1/4*1/(b*(a + b*x^4))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{4b(a+bx^4)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a+bx^4)^2}\,dx = -\frac{1}{4b(a+bx^4)}$$

```
[In] Integrate[x^3/(a + b*x^4)^2,x]
[Out] -1/4*1/(b*(a + b*x^4))
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{4b(b\,x^4+a)}$	15
derivativedivides	$-\frac{1}{4b(b\,x^4+a)}$	15
default	$-\frac{1}{4b(b\,x^4+a)}$	15
norman	$-\frac{1}{4b(b\,x^4+a)}$	15
risch	$-\frac{1}{4b(b\,x^4+a)}$	15
parallelrisch	$-\frac{1}{4b(b\,x^4+a)}$	15

```
[In] int(x^3/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/4/b/(b*x^4+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{(a+bx^4)^2}\,dx = -\frac{1}{4(b^2x^4+ab)}$$

```
[In] integrate(x^3/(b*x^4+a)^2,x, algorithm="fricas")
[Out] -1/4/(b^2*x^4 + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{(a+bx^4)^2}\,dx = -\frac{1}{4ab+4b^2x^4}$$

```
[In] integrate(x**3/(b*x**4+a)**2,x)
[Out] -1/(4*a*b + 4*b**2*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a+bx^4)^2}\,dx = -\frac{1}{4\,(bx^4+a)b}$$

```
[In] integrate(x^3/(b*x^4+a)^2,x, algorithm="maxima")
[Out] -1/4/((b*x^4 + a)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a+bx^4)^2}\,dx = -\frac{1}{4\,(bx^4+a)b}$$

```
[In] integrate(x^3/(b*x^4+a)^2,x, algorithm="giac")
[Out] -1/4/((b*x^4 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a+bx^4)^2}\,dx = -\frac{1}{4\,b\,\left(b\,x^4+a\right)}$$

```
[In] int(x^3/(a + b*x^4)^2,x)
[Out] -1/(4*b*(a + b*x^4))
```

3.85 $\int \frac{1}{x(a+bx^4)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{1}{x(a+bx^4)}\,dx = \frac{\log\left(\frac{x^4}{a+bx^4}\right)}{4a}$$

```
[Out] 1/4/a*ln(x^4/(b*x^4+a))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {272, 36, 29, 31}

$$\int \frac{1}{x(a+bx^4)}\,dx = \frac{\log(x)}{a} - \frac{\log(a+bx^4)}{4a}$$

```
[In] Int[1/(x*(a + b*x^4)),x]
```

```
[Out] Log[x]/a - Log[a + b*x^4]/(4*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, x^4\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right)}{4a} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^4\right)}{4a}\\ &= \frac{\log(x)}{a} - \frac{\log\left(a+bx^4\right)}{4a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a+bx^4\right)}\,dx = \frac{\log(x)}{a} - \frac{\log\left(a+bx^4\right)}{4a}$$

```
[In] Integrate[1/(x*(a + b*x^4)),x]
[Out] Log[x]/a - Log[a + b*x^4]/(4*a)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^4+a)}{4a}$	21
norman	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^4+a)}{4a}$	21
risch	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^4+a)}{4a}$	21
parallelrisch	$\frac{4\ln(x)-\ln(b\,x^4+a)}{4a}$	21

```
[In] int(1/x/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*ln(x)-1/4/a*ln(b*x^4+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{1}{x(a+bx^4)}\,dx = -\frac{\log(bx^4+a) - 4\,\log(x)}{4\,a}$$

```
[In] integrate(1/x/(b*x^4+a),x, algorithm="fricas")
[Out] -1/4*(log(b*x^4 + a) - 4*log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{x(a+bx^4)}\,dx = \frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b}+x^4\right)}{4a}$$

```
[In] integrate(1/x/(b*x**4+a),x)
[Out] log(x)/a - log(a/b + x**4)/(4*a)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(a+bx^4)}\,dx = -\frac{\log(bx^4+a)}{4\,a} + \frac{\log(x^4)}{4\,a}$$

```
[In] integrate(1/x/(b*x^4+a),x, algorithm="maxima")
[Out] -1/4*log(b*x^4 + a)/a + 1/4*log(x^4)/a
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+bx^4\right)}\,dx = \frac{\log\left(x^4\right)}{4\,a} - \frac{\log\left(\left|bx^4+a\right|\right)}{4\,a}$$

```
[In] integrate(1/x/(b*x^4+a),x, algorithm="giac")
[Out] 1/4*log(x^4)/a - 1/4*log(abs(b*x^4 + a))/a
```

Mupad [B] (verification not implemented)

Time = 14.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a+bx^4\right)}\,dx = -\frac{\ln\left(b\,x^4+a\right)-4\,\ln\left(x\right)}{4\,a}$$

```
[In] int(1/(x*(a + b*x^4)),x)
[Out] -(log(a + b*x^4) - 4*log(x))/(4*a)
```

3.86 $\int \frac{1}{x^2(a+bx^4)} dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int \frac{1}{x^2 (a+bx^4)} dx = -\frac{1}{ax} + \frac{-2\arctan\left(\frac{x}{\sqrt[4]{-\frac{a}{b}}}\right) + \log\left(\frac{\sqrt[4]{-\frac{a}{b}}+x}{-\sqrt[4]{-\frac{a}{b}}+x}\right)}{4a\sqrt[4]{-\frac{a}{b}}}$$

```
[Out] -1/a/x+1/4/a/(-a/b)^(1/4)*(ln((x+(-a/b)^(1/4))/(x-(-a/b)^(1/4)))-2*arctan(x
/(-a/b)^(1/4)))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 193 vs. 2(72) = 144.

Time = 0.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.68, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {331, 303, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{1}{x^2 (a+bx^4)} dx = & \frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}} - \frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{5/4}} \\ & - \frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}} \\ & + \frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}} - \frac{1}{ax}\end{aligned}$$

```
[In] Int[1/(x^2*(a + b*x^4)),x]
```

```
[Out] -(1/(a*x)) + (b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a
^(5/4)) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(5
/4)) - (b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*
Sqrt[2]*a^(5/4)) + (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/(4*Sqrt[2]*a^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{ax} - \frac{b\int \frac{x^2}{a+bx^4}\,dx}{a} \\
&= -\frac{1}{ax} + \frac{\sqrt{b}\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2a} - \frac{\sqrt{b}\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2a} \\
&= -\frac{1}{ax} - \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4a} - \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4a} \\
&\quad - \frac{\sqrt[4]{b}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{5/4}} - \frac{\sqrt[4]{b}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{5/4}} \\
&= -\frac{1}{ax} - \frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}} + \frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}} \\
&\quad - \frac{\sqrt[4]{b}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}} + \frac{\sqrt[4]{b}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}} \\
&= -\frac{1}{ax} + \frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}} - \frac{\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}} \\
&\quad - \frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}} + \frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 179 vs. $2(72) = 144$.

Time = 0.03 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.49

$$\int \frac{1}{x^2(a+bx^4)}\,dx$$
$$= \frac{-8\sqrt[4]{a}+2\sqrt{2}\sqrt[4]{b}x\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-2\sqrt{2}\sqrt[4]{b}x\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\sqrt{2}\sqrt[4]{b}x\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x\right.}{8a^{5/4}x}$$

```
[In] Integrate[1/(x^2*(a + b*x^4)),x]
```

```
[Out] (-8*a^(1/4) + 2*Sqrt[2]*b^(1/4)*x*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] -
 2*Sqrt[2]*b^(1/4)*x*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*b^(1
/4)*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1
/4)*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(8*a^(5/4)*x)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^4+b\right)}\_R\ln\left(\left(5\_R^4a^5+4b\right)x+\_R^3a^4\right)\right)}{4}$	50
default	$-\frac{1}{ax}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	111

```
[In] int(1/x^2/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/x+1/4*sum(_R*ln((5*_R^4*a^5+4*b)*x+_R^3*a^4),_R=RootOf(_Z^4*a^5+b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.78

$$\int \frac{1}{x^2 (a + bx^4)}\,dx = -\frac{ax\left(-\frac{b}{a^5}\right)^{\frac{1}{4}} \log\left(a^4\left(-\frac{b}{a^5}\right)^{\frac{3}{4}} + bx\right) - i\,ax\left(-\frac{b}{a^5}\right)^{\frac{1}{4}} \log\left(i\,a^4\left(-\frac{b}{a^5}\right)^{\frac{3}{4}} + bx\right) + i\,ax\left(-\frac{b}{a^5}\right)^{\frac{1}{4}} \log\left(-i\,a^4\left(-\frac{b}{a^5}\right)^{\frac{3}{4}}\right.}{4\,ax}$$

```
[In] integrate(1/x^2/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(a*x*(-b/a^5)^(1/4)*log(a^4*(-b/a^5)^(3/4) + b*x) - I*a*x*(-b/a^5)^(1/
4)*log(I*a^4*(-b/a^5)^(3/4) + b*x) + I*a*x*(-b/a^5)^(1/4)*log(-I*a^4*(-b/a^
5)^(3/4) + b*x) - a*x*(-b/a^5)^(1/4)*log(-a^4*(-b/a^5)^(3/4) + b*x) + 4)/(a
*x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^2 (a + bx^4)}\,dx = \operatorname{RootSum}\left(256t^4a^5 + b, \left(t \mapsto t\log\left(-\frac{64t^3a^4}{b} + x\right)\right)\right) - \frac{1}{ax}$$

```
[In] integrate(1/x**2/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**5 + b, Lambda(_t, _t*log(-64*_t**3*a**4/b + x))) - 1/(
a*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(64) = 128.

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.54

$$\int \frac{1}{x^2 (a + bx^4)}\,dx = -\frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{b}x^2-\right.}{a^{\frac{1}{4}}}\right.}{8\,a} - \frac{1}{ax}$$

```
[In] integrate(1/x^2/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/8*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan
(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))
/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4
)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a - 1/(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(64) = 128$.

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.60

$$\int \frac{1}{x^2\left(a+bx^4\right)}\,dx = -\frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,a^2b^2} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,a^2b^2} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,a^2b^2} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,a^2b^2} - \frac{1}{ax}$$

```
[In] integrate(1/x^2/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(
a/b)^(1/4))/(a^2*b^2) - 1/4*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x -
 sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/8*sqrt(2)*(a*b^3)^(3/4)*lo
g(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^2) - 1/8*sqrt(2)*(a*b^3)^
(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^2) - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 15.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^2\left(a+bx^4\right)}\,dx = \frac{(-b)^{1/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\,x}{a^{1/4}}\right)}{2\,a^{5/4}} - \frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\,x}{a^{1/4}}\right)}{2\,a^{5/4}} - \frac{1}{a\,x}$$

```
[In] int(1/(x^2*(a + b*x^4)),x)
```

```
[Out] ((-b)^(1/4)*atanh(((-b)^(1/4)*x)/a^(1/4)))/(2*a^(5/4)) - ((-b)^(1/4)*atan((
(-b)^(1/4)*x)/a^(1/4)))/(2*a^(5/4)) - 1/(a*x)
```

3.87 $\int \frac{1}{1+x} \, dx$

Optimal result

Integrand size = 5, antiderivative size = 4

$$\int \frac{1}{1+x} \, dx = \log(1+x)$$

```
[Out] ln(1+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{31\}$

$$\int \frac{1}{1+x} \, dx = \log(x+1)$$

```
[In] Int[(1 + x)^(-1),x]
```

```
[Out] Log[1 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\text{integral} = \log(1+x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x}\,dx = \log(1+x)$$

```
[In] Integrate[(1 + x)^(-1),x]
[Out] Log[1 + x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$\ln(1+x)$	5
norman	$\ln(1+x)$	5
meijerg	$\ln(1+x)$	5
risch	$\ln(1+x)$	5
parallelrisch	$\ln(1+x)$	5

```
[In] int(1/(1+x),x,method=_RETURNVERBOSE)
[Out] ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x}\,dx = \log(x+1)$$

```
[In] integrate(1/(1+x),x, algorithm="fricas")
[Out] log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+x}\,dx = \log\left(x+1\right)$$

```
[In] integrate(1/(1+x),x)
[Out] log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x}\,dx = \log\left(x+1\right)$$

```
[In] integrate(1/(1+x),x, algorithm="maxima")
[Out] log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \frac{1}{1+x}\,dx = \log\left(|x+1|\right)$$

```
[In] integrate(1/(1+x),x, algorithm="giac")
[Out] log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x}\,dx = \ln\left(x+1\right)$$

```
[In] int(1/(x + 1),x)
[Out] log(x + 1)
```

3.88 $\int \frac{1}{1+x^2} dx$

Optimal result

Integrand size = 7, antiderivative size = 2

$$\int \frac{1}{1+x^2} dx = \arctan(x)$$

```
[Out] arctan(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{209\}$

$$\int \frac{1}{1+x^2} dx = \arctan(x)$$

```
[In] Int[(1 + x^2)^(-1),x]
```

```
[Out] ArcTan[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\text{integral} = \arctan(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x^2}\,dx = \arctan(x)$$

```
[In] Integrate[(1 + x^2)^(-1),x]
[Out] ArcTan[x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\arctan\left(x\right)$	3
meijerg	$\arctan\left(x\right)$	3
risch	$\arctan\left(x\right)$	3
parallelrisch	$\frac{i\ln\left(i+x\right)}{2}-\frac{i\ln\left(x-i\right)}{2}$	18

```
[In] int(1/(x^2+1),x,method=_RETURNVERBOSE)
[Out] arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x^2}\,dx = \arctan\left(x\right)$$

```
[In] integrate(1/(x^2+1),x, algorithm="fricas")
[Out] arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x^2}\,dx = \operatorname{atan}(x)$$

```
[In] integrate(1/(x**2+1),x)
[Out] atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x^2}\,dx = \arctan(x)$$

```
[In] integrate(1/(x^2+1),x, algorithm="maxima")
[Out] arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x^2}\,dx = \arctan(x)$$

```
[In] integrate(1/(x^2+1),x, algorithm="giac")
[Out] arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x^2}\,dx = \operatorname{atan}(x)$$

```
[In] int(1/(x^2 + 1),x)
[Out] atan(x)
```

3.89 $\int \frac{1}{1+x^3}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 43

$$\int \frac{1}{1+x^3}\,dx=\frac{\arctan\left(\frac{\sqrt{3}x}{2-x}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(\frac{1+x}{\sqrt{1-x+x^2}}\right)$$

```
[Out] 1/3*ln((1+x)/(x^2-x+1)^(1/2))+1/3*3^(1/2)*arctan(x*3^(1/2)/(2-x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {206, 31, 648, 632, 210, 642}

$$\int \frac{1}{1+x^3}\,dx=-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log(x+1)$$

```
[In] Int[(1 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x]/3 - Log[1 - x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\int \frac{1}{1+x}\,dx+\frac{1}{3}\int \frac{2-x}{1-x+x^2}\,dx\\ &= \frac{1}{3}\log(1+x)-\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\ &= \frac{1}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{1}{1+x^3}\,dx = \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1+x) - \frac{1}{6}\log\left(1-x+x^2\right)$$

```
[In] Integrate[(1 + x^3)^(-1),x]
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x]/3 - Log[1 - x + x^2]/6
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{\ln(1+x)}{3}-\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	33
default	$\frac{\ln(1+x)}{3}-\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$	35
meijerg	$\frac{x\ln\left(1+\left(x^3\right)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{1}{3}}}-\frac{x\ln\left(1-\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{1}{3}}}+\frac{x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2-\left(x^3\right)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{1}{3}}}$	74

```
[In] int(1/(x^3+1),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(1+x)-1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log\left(x^2-x+1\right) + \frac{1}{3}\log\left(x+1\right)$$

```
[In] integrate(1/(x^3+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) + 1/3*log(
x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{1}{1+x^3}\,dx = \frac{\log(x+1)}{3} - \frac{\log(x^2-x+1)}{6} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate(1/(x**3+1),x)
[Out] log(x + 1)/3 - log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3
)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log\left(x^2-x+1\right) + \frac{1}{3}\log(x+1)$$

```
[In] integrate(1/(x^3+1),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) + 1/3*log(
x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{1}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log\left(x^2-x+1\right) + \frac{1}{3}\log(|x+1|)$$

```
[In] integrate(1/(x^3+1),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) + 1/3*log(
abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{1}{1+x^3}\,dx = \frac{\ln(x+1)}{3} - \frac{\ln\left(\left(x-\frac{1}{2}\right)^2+\frac{3}{4}\right)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,\left(x-\frac{1}{2}\right)}{3}\right)}{3}$$

```
[In] int(1/(x^3 + 1),x)
```

```
[Out] log(x + 1)/3 - log((x - 1/2)^2 + 3/4)/6 + (3^(1/2)*atan((2*3^(1/2)*(x - 1/2
))/3))/3
```

3.90 $\int \frac{1}{1+x^4} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 65

$$\int \frac{1}{1+x^4} \, dx = \frac{\arctan\left(\frac{\sqrt{2}x}{1-x^2}\right)}{2\sqrt{2}} + \frac{\log\left(\frac{1+\sqrt{2}x+x^2}{1-\sqrt{2}x+x^2}\right)}{4\sqrt{2}}$$

```
[Out] 1/8*2^(1/2)*ln((1+x*2^(1/2)+x^2)/(1-x*2^(1/2)+x^2))+1/4*2^(1/2)*arctan(x*2^
(1/2)/(-x^2+1))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{1+x^4} \, dx = -\frac{\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\arctan\left(\sqrt{2}x+1\right)}{2\sqrt{2}} - \frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}} + \frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}$$

```
[In] Int[(1 + x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) - Lo
g[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx+\frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx\\
&= \frac{1}{4}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx-\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{4\sqrt{2}}-\frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{4\sqrt{2}}\\
&= -\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}x\right)}{2\sqrt{2}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}x\right)}{2\sqrt{2}}
\end{aligned}$$

$$= -\frac{\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \frac{1}{1+x^4}\,dx$$
$$= \frac{-2\arctan\left(1-\sqrt{2}x\right)+2\arctan\left(1+\sqrt{2}x\right)-\log\left(1-\sqrt{2}x+x^2\right)+\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

```
[In] Integrate[(1 + x^4)^(-1),x]
[Out] (-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] - Log[1 - Sqrt[2]*x + x
^2] + Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.34

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x\sqrt{2}+x^2}{1-x\sqrt{2}+x^2}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{8}$
meijerg	$-\frac{x\sqrt{2}\,\ln\left(1-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}\right)}{4\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}{2+\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}\right)}{4\left(x^4\right)^{\frac{1}{4}}}$

```
[In] int(1/(x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{1}{1+x^4}\,dx = \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\log\left(2\,x+(i+1)\,\sqrt{2}\right)-\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\log\left(2\,x-(i-1)\,\sqrt{2}\right) +\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\log\left(2\,x+(i-1)\,\sqrt{2}\right)-\left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\log\left(2\,x-(i+1)\,\sqrt{2}\right)$$

```
[In] integrate(1/(x^4+1),x, algorithm="fricas")
```

```
[Out] (1/8*I + 1/8)*sqrt(2)*log(2*x + (I + 1)*sqrt(2)) - (1/8*I - 1/8)*sqrt(2)*lo
g(2*x - (I - 1)*sqrt(2)) + (1/8*I - 1/8)*sqrt(2)*log(2*x + (I - 1)*sqrt(2))
 - (1/8*I + 1/8)*sqrt(2)*log(2*x - (I + 1)*sqrt(2))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12

$$\int \frac{1}{1+x^4}\,dx = -\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8} +\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{4}$$

```
[In] integrate(1/(x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8
+ sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \frac{1}{1+x^4}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) +\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

```
[In] integrate(1/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \frac{1}{1+x^4}\,dx = \frac{1}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) + \frac{1}{8}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

```
[In] integrate(1/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

Mupad [B] (verification not implemented)

Time = 15.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.51

$$\int \frac{1}{1+x^4}\,dx = \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)$$

```
[In] int(1/(x^4 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 + 1i/4) + 2^(1/2)*atan(2^(1/2)*x*
(1/2 + 1i/2))*(1/4 - 1i/4)
```

3.91 $\int \frac{1}{1-x} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \frac{1}{1-x} \, dx = -\log(1-x)$$

```
[Out] -ln(1-x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{31\}$

$$\int \frac{1}{1-x} \, dx = -\log(1-x)$$

```
[In] Int[(1 - x)^(-1),x]
```

```
[Out] -Log[1 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\text{integral} = -\log(1-x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-x}\,dx = -\log(1-x)$$

```
[In] Integrate[(1 - x)^(-1),x]
```

```
[Out] -Log[1 - x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
norman	$-\ln\left(-1+x\right)$	7
risch	$-\ln\left(-1+x\right)$	7
parallelrisch	$-\ln\left(-1+x\right)$	7
default	$-\ln\left(1-x\right)$	9
meijerg	$-\ln\left(1-x\right)$	9

```
[In] int(1/(1-x),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(-1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{1-x}\,dx = -\log\left(x-1\right)$$

```
[In] integrate(1/(1-x),x, algorithm="fricas")
```

```
[Out] -log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1}{1-x}\,dx = -\log(x-1)$$

```
[In] integrate(1/(1-x),x)
[Out] -log(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{1-x}\,dx = -\log(x-1)$$

```
[In] integrate(1/(1-x),x, algorithm="maxima")
[Out] -log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{1-x}\,dx = -\log(|x-1|)$$

```
[In] integrate(1/(1-x),x, algorithm="giac")
[Out] -log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{1-x}\,dx = -\ln(x-1)$$

```
[In] int(-1/(x - 1),x)
[Out] -log(x - 1)
```

3.92 $\int \frac{1}{1-x^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 2

$$\int \frac{1}{1-x^2} dx = \operatorname{arctanh}(x)$$

```
[Out] arctanh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{212\}$

$$\int \frac{1}{1-x^2} dx = \operatorname{arctanh}(x)$$

```
[In] Int[(1 - x^2)^(-1),x]
```

```
[Out] ArcTanh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\text{integral} = \operatorname{arctanh}(x)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 19 vs. $2(2) = 4$.

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 9.50

$$\int \frac{1}{1-x^2}\,dx = -\frac{1}{2}\log(1-x)+\frac{1}{2}\log(1+x)$$

```
[In] Integrate[(1 - x^2)^(-1),x]
[Out] -1/2*Log[1 - x] + Log[1 + x]/2
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\operatorname{arctanh}(x)$	3
meijerg	$\operatorname{arctanh}(x)$	3
norman	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14
risch	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14
parallelrisch	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14

```
[In] int(1/(-x^2+1),x,method=_RETURNVERBOSE)
[Out] arctanh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(2) = 4$.

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{1}{1-x^2}\,dx = \frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

```
[In] integrate(1/(-x^2+1),x, algorithm="fricas")
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(2) = 4$.

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 6.00

$$\int \frac{1}{1-x^2}\,dx = -\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

```
[In] integrate(1/(-x**2+1),x)
[Out] -log(x - 1)/2 + log(x + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(2) = 4$.

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{1}{1-x^2}\,dx = \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

```
[In] integrate(1/(-x^2+1),x, algorithm="maxima")
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(2) = 4$.

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 7.50

$$\int \frac{1}{1-x^2}\,dx = \frac{1}{2}\log(|x+1|) - \frac{1}{2}\log(|x-1|)$$

```
[In] integrate(1/(-x^2+1),x, algorithm="giac")
[Out] 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-x^2} \, dx = \text{atanh}(x)$$

```
[In] int(-1/(x^2 - 1),x)
[Out] atanh(x)
```

3.93 $\int \frac{1}{-1+x^2} dx$

Optimal result

Integrand size = 7, antiderivative size = 4

$$\int \frac{1}{-1+x^2} dx = -\coth^{-1}(x)$$

```
[Out] -arccoth(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {213}

$$\int \frac{1}{-1+x^2} dx = -\text{arctanh}(x)$$

```
[In] Int[(-1 + x^2)^(-1),x]
```

```
[Out] -ArcTanh[x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\text{integral} = -\text{arctanh}(x)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 19 vs. $2(4) = 8$.

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 4.75

$$\int \frac{1}{-1+x^2} \, dx = \frac{1}{2} \log(1-x) - \frac{1}{2} \log(1+x)$$

```
[In] Integrate[(-1 + x^2)^(-1),x]
[Out] Log[1 - x]/2 - Log[1 + x]/2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$-\operatorname{arctanh}(x)$	5
meijerg	$-\operatorname{arctanh}(x)$	5
norman	$\frac{\ln(-1+x)}{2} - \frac{\ln(1+x)}{2}$	14
risch	$\frac{\ln(-1+x)}{2} - \frac{\ln(1+x)}{2}$	14
parallelrisch	$\frac{\ln(-1+x)}{2} - \frac{\ln(1+x)}{2}$	14

```
[In] int(1/(x^2-1),x,method=_RETURNVERBOSE)
[Out] -arctanh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(4) = 8$.

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 3.25

$$\int \frac{1}{-1+x^2} \, dx = -\frac{1}{2} \log(x+1) + \frac{1}{2} \log(x-1)$$

```
[In] integrate(1/(x^2-1),x, algorithm="fricas")
[Out] -1/2*log(x + 1) + 1/2*log(x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(3) = 6$.

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \frac{1}{-1+x^2}\,dx = \frac{\log(x-1)}{2} - \frac{\log(x+1)}{2}$$

```
[In] integrate(1/(x**2-1),x)
[Out] log(x - 1)/2 - log(x + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(4) = 8$.

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 3.25

$$\int \frac{1}{-1+x^2}\,dx = -\frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1)$$

```
[In] integrate(1/(x^2-1),x, algorithm="maxima")
[Out] -1/2*log(x + 1) + 1/2*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(4) = 8$.

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \frac{1}{-1+x^2}\,dx = -\frac{1}{2}\log(|x+1|) + \frac{1}{2}\log(|x-1|)$$

```
[In] integrate(1/(x^2-1),x, algorithm="giac")
[Out] -1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{-1+x^2}\,dx = -\operatorname{atanh}(x)$$

```
[In] int(1/(x^2 - 1),x)
[Out] -atanh(x)
```

3.94 $\int \frac{1}{1-x^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 43

$$\int \frac{1}{1-x^3} dx = \frac{\arctan\left(\frac{\sqrt{3}x}{2+x}\right)}{\sqrt{3}} + \frac{1}{3}\log\left(\frac{\sqrt{1+x+x^2}}{1-x}\right)$$

```
[Out] 1/3*ln((x^2+x+1)^(1/2)/(1-x))+1/3*3^(1/2)*arctan(x*3^(1/2)/(2+x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {206, 31, 648, 632, 210, 642}

$$\int \frac{1}{1-x^3} dx = \frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{6}\log\left(x^2+x+1\right) - \frac{1}{3}\log(1-x)$$

```
[In] Int[(1 - x^3)^(-1),x]
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 - x]/3 + Log[1 + x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\int \frac{1}{1-x}\,dx + \frac{1}{3}\int \frac{2+x}{1+x+x^2}\,dx \\
&= -\frac{1}{3}\log(1-x) + \frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{2}\int \frac{1}{1+x+x^2}\,dx \\
&= -\frac{1}{3}\log(1-x) + \frac{1}{6}\log\left(1+x+x^2\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right) \\
&= \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1-x) + \frac{1}{6}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{1}{1-x^3} \, dx = \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1-x) + \frac{1}{6}\log\left(1+x+x^2\right)$$

[In] Integrate[(1 - x^3)^(-1),x]

[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 - x]/3 + Log[1 + x + x^2]/6

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\ln(x^2+x+1)}{6}+\frac{\sqrt{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)}{3}-\frac{\ln(-1+x)}{3}$	33
risch	$\frac{\ln(4x^2+4x+4)}{6}+\frac{\sqrt{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)}{3}-\frac{\ln(-1+x)}{3}$	37
meijerg	$-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{1}{3}}}$	62

[In] int(1/(-x^3+1),x,method=_RETURNVERBOSE)

[Out] 1/6*ln(x^2+x+1)+1/3*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))-1/3*ln(-1+x)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{1}{1-x^3} \, dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\log\left(x^2+x+1\right) - \frac{1}{3}\log\left(x-1\right)$$

[In] integrate(1/(-x^3+1),x, algorithm="fricas")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*log(x^2 + x + 1) - 1/3*log(
x - 1)

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{1}{1-x^3}\,dx = -\frac{\log(x-1)}{3} + \frac{\log(x^2+x+1)}{6} + \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate(1/(-x**3+1),x)
```

```
[Out] -log(x - 1)/3 + log(x**2 + x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/
3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{1}{1-x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\log\left(x^2+x+1\right) - \frac{1}{3}\log(x-1)$$

```
[In] integrate(1/(-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*log(x^2 + x + 1) - 1/3*log(
x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{1}{1-x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\log\left(x^2+x+1\right) - \frac{1}{3}\log(|x-1|)$$

```
[In] integrate(1/(-x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*log(x^2 + x + 1) - 1/3*log(
abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{1}{1-x^3}\,dx = -\frac{\ln(x-1)}{3} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

```
[In] int(-1/(x^3 - 1),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/6) - log(x - (3^(1/2)*1i)
/2 + 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x - 1)/3
```

3.95 $\int \frac{1}{1-x^4} dx$

Optimal result

Integrand size = 9, antiderivative size = 9

$$\int \frac{1}{1-x^4} dx = \frac{1}{2}(\arctan(x) + \operatorname{arctanh}(x))$$

```
[Out] 1/2*arctanh(x)+1/2*arctan(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {218, 212, 209}

$$\int \frac{1}{1-x^4} dx = \frac{\arctan(x)}{2} + \frac{\operatorname{arctanh}(x)}{2}$$

```
[In] Int[(1 - x^4)^(-1),x]
```

```
[Out] ArcTan[x]/2 + ArcTanh[x]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int \frac{1}{1-x^2}\,dx + \frac{1}{2}\int \frac{1}{1+x^2}\,dx\\ &= \frac{\arctan(x)}{2} + \frac{\text{arctanh}(x)}{2}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. $2(9) = 18$.

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.78

$$\int \frac{1}{1-x^4}\,dx = \frac{\arctan(x)}{2} - \frac{1}{4}\log(1-x) + \frac{1}{4}\log(1+x)$$

```
[In] Integrate[(1 - x^4)^(-1),x]
[Out] ArcTan[x]/2 - Log[1 - x]/4 + Log[1 + x]/4
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\text{arctanh}(x)}{2}+\frac{\arctan(x)}{2}$	10
risch	$-\frac{\ln(-1+x)}{4}+\frac{\arctan(x)}{2}+\frac{\ln(1+x)}{4}$	18
parallelrisch	$-\frac{\ln(-1+x)}{4}+\frac{i\ln(i+x)}{4}-\frac{i\ln(x-i)}{4}+\frac{\ln(1+x)}{4}$	30
meijerg	$-\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}$	38

```
[In] int(1/(-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctanh(x)+1/2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int \frac{1}{1-x^4}\,dx = \frac{1}{2}\arctan(x) + \frac{1}{4}\log(x+1) - \frac{1}{4}\log(x-1)$$

```
[In] integrate(1/(-x^4+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int \frac{1}{1-x^4}\,dx = -\frac{\log(x-1)}{4} + \frac{\log(x+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

```
[In] integrate(1/(-x**4+1),x)
[Out] -log(x - 1)/4 + log(x + 1)/4 + atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int \frac{1}{1-x^4}\,dx = \frac{1}{2}\arctan(x) + \frac{1}{4}\log(x+1) - \frac{1}{4}\log(x-1)$$

```
[In] integrate(1/(-x^4+1),x, algorithm="maxima")
[Out] 1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int \frac{1}{1-x^4}\,dx = \frac{1}{2}\arctan(x) + \frac{1}{4}\log(|x+1|) - \frac{1}{4}\log(|x-1|)$$

```
[In] integrate(1/(-x^4+1),x, algorithm="giac")
[Out] 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 14.57 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-x^4}\,dx = \frac{\mathrm{atan}(x)}{2} + \frac{\mathrm{atanh}(x)}{2}$$

```
[In] int(-1/(x^4 - 1),x)
[Out] atan(x)/2 + atanh(x)/2
```

3.96 $\int \frac{x}{1+x} dx$

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \frac{x}{1+x} \, dx = x - \log(1+x)$$

```
[Out] x-ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = $\{45\}$

$$\int \frac{x}{1+x} \, dx = x - \log(x+1)$$

```
[In] Int[x/(1 + x),x]
```

```
[Out] x - Log[1 + x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(1 + \frac{1}{-1-x}\right) \, dx \\ &= x - \log(1+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x}\,dx = x - \log(1+x)$$

```
[In] Integrate[x/(1 + x),x]
[Out] x - Log[1 + x]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$x - \ln(1+x)$	9
norman	$x - \ln(1+x)$	9
meijerg	$x - \ln(1+x)$	9
risch	$x - \ln(1+x)$	9
parallelrisch	$x - \ln(1+x)$	9

```
[In] int(x/(1+x),x,method=_RETURNVERBOSE)
[Out] x-ln(1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x}\,dx = x - \log(x+1)$$

```
[In] integrate(x/(1+x),x, algorithm="fricas")
[Out] x - log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{x}{1+x}\,dx = x - \log{(x+1)}$$

```
[In] integrate(x/(1+x),x)
[Out] x - log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x}\,dx = x - \log{(x+1)}$$

```
[In] integrate(x/(1+x),x, algorithm="maxima")
[Out] x - log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\int \frac{x}{1+x}\,dx = x - \log{(|x+1|)}$$

```
[In] integrate(x/(1+x),x, algorithm="giac")
[Out] x - log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x}\,dx = x - \ln{(x+1)}$$

```
[In] int(x/(x + 1),x)
[Out] x - log(x + 1)
```

3.97 $\int \frac{x}{1+x^2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{x}{1+x^2} \, dx = \frac{1}{2} \log \left(1+x^2\right)$$

```
[Out] 1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {266}

$$\int \frac{x}{1+x^2} \, dx = \frac{1}{2} \log \left(x^2+1\right)$$

```
[In] Int[x/(1 + x^2),x]
```

```
[Out] Log[1 + x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2} \log \left(1+x^2\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x^2} \, dx = \frac{1}{2} \log \left(1+x^2\right)$$

```
[In] Integrate[x/(1 + x^2),x]
[Out] Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\ln\left(x^{2}+1\right)}{2}$	9
default	$\frac{\ln\left(x^{2}+1\right)}{2}$	9
norman	$\frac{\ln\left(x^{2}+1\right)}{2}$	9
meijerg	$\frac{\ln\left(x^{2}+1\right)}{2}$	9
risch	$\frac{\ln\left(x^{2}+1\right)}{2}$	9
parallelrisch	$\frac{\ln\left(x^{2}+1\right)}{2}$	9

```
[In] int(1/(x^2+1)*x,x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x}{1+x^2} \, dx = \frac{1}{2} \, \log \left(x^{2} + 1\right)$$

```
[In] integrate(x/(x^2+1),x, algorithm="fricas")
[Out] 1/2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{x}{1+x^2}\,dx = \frac{\log(x^2+1)}{2}$$

```
[In] integrate(x/(x**2+1),x)
[Out] log(x**2 + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate(x/(x^2+1),x, algorithm="maxima")
[Out] 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate(x/(x^2+1),x, algorithm="giac")
[Out] 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x}{1+x^2}\,dx = \frac{\ln(x^2+1)}{2}$$

```
[In] int(x/(x^2 + 1),x)
[Out] log(x^2 + 1)/2
```

3.98 $\int \frac{x}{1+x^3} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 40

$$\int \frac{x}{1+x^3} \, dx = \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{6}\log\left(\frac{(1+x)^2}{1-x+x^2}\right)$$

```
[Out] -1/6*ln((1+x)^2/(x^2-x+1))+1/3*3^(1/2)*arctan(1/3*(-1+2*x)*3^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {298, 31, 648, 632, 210, 642}

$$\int \frac{x}{1+x^3} \, dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log(x+1)$$

```
[In] Int[x/(1 + x^3),x]
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{3}\int \frac{1}{1+x}\,dx\right)+\frac{1}{3}\int \frac{1+x}{1-x+x^2}\,dx\\
&= -\frac{1}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\
&= -\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x^3}\,dx=\frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)$$

```
[In] Integrate[x/(1 + x^3),x]
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

method	result	size
risch	$-\frac{\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	33
default	$-\frac{\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$	35
meijerg	$-\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}}+\frac{x^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{2}{3}}}$	80

```
[In] int(x/(x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x}{1+x^3}\,dx=\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)$$

```
[In] integrate(x/(x^3+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{x}{1+x^3}\,dx = -\frac{\log{(x+1)}}{3} + \frac{\log{(x^2-x+1)}}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate(x/(x**3+1),x)
[Out] -log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/
3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x}{1+x^3}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{6}\,\log\left(x^2-x+1\right) - \frac{1}{3}\,\log\left(x+1\right)$$

```
[In] integrate(x/(x^3+1),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{x}{1+x^3}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{6}\,\log\left(x^2-x+1\right) - \frac{1}{3}\,\log\left(|x+1|\right)$$

```
[In] integrate(x/(x^3+1),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 14.69 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{x}{1+x^3}\,dx = -\frac{\ln(x+1)}{3} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

```
[In] int(x/(x^3 + 1),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6) - log(x - (3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x + 1)/3
```

3.99 $\int \frac{x}{1+x^4} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \frac{x}{1+x^4} \, dx = \frac{\arctan(x^2)}{2}$$

```
[Out] 1/2*arctan(x^2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {281, 209}

$$\int \frac{x}{1+x^4} \, dx = \frac{\arctan(x^2)}{2}$$

```
[In] Int[x/(1 + x^4),x]
```

```
[Out] ArcTan[x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right)$$
$$= \frac{\arctan(x^2)}{2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x^4}\,dx = \frac{\arctan(x^2)}{2}$$

```
[In] Integrate[x/(1 + x^4),x]
[Out] ArcTan[x^2]/2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\arctan(x^2)}{2}$	7
meijerg	$\frac{\arctan(x^2)}{2}$	7
risch	$\frac{\arctan(x^2)}{2}$	7
parallelrisch	$\frac{i\ln(x^2+i)}{4}-\frac{i\ln(x^2-i)}{4}$	22

```
[In] int(x/(x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\arctan\left(x^2\right)$$

```
[In] integrate(x/(x^4+1),x, algorithm="fricas")
[Out] 1/2*arctan(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{x}{1+x^4}\,dx = \frac{\operatorname{atan}\left(x^2\right)}{2}$$

```
[In] integrate(x/(x**4+1),x)
[Out] atan(x**2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\arctan\left(x^2\right)$$

```
[In] integrate(x/(x^4+1),x, algorithm="maxima")
[Out] 1/2*arctan(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\arctan\left(x^2\right)$$

```
[In] integrate(x/(x^4+1),x, algorithm="giac")
[Out] 1/2*arctan(x^2)
```

Mupad [B] (verification not implemented)

Time = 14.77 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{1+x^4}\,dx = \frac{\operatorname{atan}(x^2)}{2}$$

```
[In] int(x/(x^4 + 1),x)
[Out] atan(x^2)/2
```

3.100 $\int \frac{x}{1-x} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{x}{1-x} \, dx = -x - \log(1-x)$$

```
[Out] -ln(1-x)-x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{45\}$

$$\int \frac{x}{1-x} \, dx = -x - \log(1-x)$$

```
[In] Int[x/(1 - x),x]
```

```
[Out] -x - Log[1 - x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-1+\frac{1}{1-x}\right) \, dx \\ &= -x-\log(1-x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{1-x}\,dx = -x - \log(1-x)$$

```
[In] Integrate[x/(1 - x),x]
[Out] -x - Log[1 - x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$-x-\ln\left(-1+x\right)$	11
norman	$-x-\ln\left(-1+x\right)$	11
risch	$-x-\ln\left(-1+x\right)$	11
parallelrisch	$-x-\ln\left(-1+x\right)$	11
meijerg	$-\ln\left(1-x\right)-x$	13

```
[In] int(x/(1-x),x,method=_RETURNVERBOSE)
[Out] -x-ln(-1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{1-x}\,dx = -x - \log\left(x-1\right)$$

```
[In] integrate(x/(1-x),x, algorithm="fricas")
[Out] -x - log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{x}{1-x}\,dx = -x - \log{(x-1)}$$

```
[In] integrate(x/(1-x),x)
[Out] -x - log(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{1-x}\,dx = -x - \log{(x-1)}$$

```
[In] integrate(x/(1-x),x, algorithm="maxima")
[Out] -x - log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{x}{1-x}\,dx = -x - \log{(|x-1|)}$$

```
[In] integrate(x/(1-x),x, algorithm="giac")
[Out] -x - log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{1-x}\,dx = -x - \ln{(x-1)}$$

```
[In] int(-x/(x - 1),x)
[Out] - x - log(x - 1)
```

3.101 $\int \frac{x}{1-x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{x}{1-x^2}\,dx = -\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] -1/2*ln(-x^2+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {266}

$$\int \frac{x}{1-x^2}\,dx = -\frac{1}{2}\log\left(1-x^2\right)$$

```
[In] Int[x/(1 - x^2),x]
```

```
[Out] -1/2*Log[1 - x^2]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\text{integral} = -\frac{1}{2}\log\left(1-x^2\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{1-x^2}\,dx = -\frac{1}{2}\log\left(1-x^2\right)$$

```
[In] Integrate[x/(1 - x^2),x]
[Out] -1/2*Log[1 - x^2]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{\ln(x^2-1)}{2}$	9
derivativedivides	$-\frac{\ln(-x^2+1)}{2}$	11
meijerg	$-\frac{\ln(-x^2+1)}{2}$	11
default	$-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$	14
norman	$-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$	14
parallelrisch	$-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$	14

```
[In] int(x/(-x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{1-x^2}\,dx = -\frac{1}{2}\log\left(x^2-1\right)$$

```
[In] integrate(x/(-x^2+1),x, algorithm="fricas")
[Out] -1/2*log(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{1-x^2} \, dx = -\frac{\log(x^2-1)}{2}$$

```
[In] integrate(x/(-x**2+1),x)
[Out] -log(x**2 - 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{1-x^2} \, dx = -\frac{1}{2} \log\left(x^2-1\right)$$

```
[In] integrate(x/(-x^2+1),x, algorithm="maxima")
[Out] -1/2*log(x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{x}{1-x^2} \, dx = -\frac{1}{2} \log\left(\left|x^2-1\right|\right)$$

```
[In] integrate(x/(-x^2+1),x, algorithm="giac")
[Out] -1/2*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{1-x^2} \, dx = -\frac{\ln(x^2-1)}{2}$$

```
[In] int(-x/(x^2 - 1),x)
[Out] -log(x^2 - 1)/2
```

3.102 $\int \frac{x}{1-x^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 41

$$\int \frac{x}{1-x^3} \, dx = -\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{6}\log\left(\frac{(1-x)^2}{1+x+x^2}\right)$$

```
[Out] -1/6*ln((1-x)^2/(x^2+x+1))-1/3*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {298, 31, 648, 632, 210, 642}

$$\int \frac{x}{1-x^3} \, dx = -\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{6}\log\left(x^2+x+1\right) - \frac{1}{3}\log(1-x)$$

```
[In] Int[x/(1 - x^3),x]
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 - x]/3 + Log[1 + x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\int \frac{1}{1-x}\,dx - \frac{1}{3}\int \frac{1-x}{1+x+x^2}\,dx \\
&= -\frac{1}{3}\log(1-x) + \frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx - \frac{1}{2}\int \frac{1}{1+x+x^2}\,dx \\
&= -\frac{1}{3}\log(1-x) + \frac{1}{6}\log\left(1+x+x^2\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right) \\
&= -\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1-x) + \frac{1}{6}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x}{1-x^3} \, dx = -\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1-x) + \frac{1}{6}\log\left(1+x+x^2\right)$$

```
[In] Integrate[x/(1 - x^3),x]
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 - x]/3 + Log[1 + x + x^2]/6
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
risch	$-\frac{\ln(-1+x)}{3}+\frac{\ln(x^2+x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(\frac{1}{2}+x\right)\sqrt{3}}{3}\right)}{3}$	31
default	$\frac{\ln(x^2+x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)}{3}-\frac{\ln(-1+x)}{3}$	33
meijerg	$-\frac{x^2\left(\ln\left(1-\left(x^3\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2+\left(x^3\right)^{\frac{1}{3}}}\right)\right)}{3\left(x^3\right)^{\frac{2}{3}}}$	63

```
[In] int(x/(-x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/3*ln(-1+x)+1/6*ln(x^2+x+1)-1/3*3^(1/2)*arctan(2/3*(1/2+x)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{x}{1-x^3} \, dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\log\left(x^2+x+1\right) - \frac{1}{3}\log\left(x-1\right)$$

```
[In] integrate(x/(-x^3+1),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*log(x^2 + x + 1) - 1/3*log
(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x}{1-x^3}\,dx = -\frac{\log(x-1)}{3} + \frac{\log(x^2+x+1)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate(x/(-x**3+1),x)
```

```
[Out] -log(x - 1)/3 + log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/
3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{x}{1-x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\log\left(x^2+x+1\right) - \frac{1}{3}\log(x-1)$$

```
[In] integrate(x/(-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*log(x^2 + x + 1) - 1/3*log
(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{x}{1-x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\log\left(x^2+x+1\right) - \frac{1}{3}\log(|x-1|)$$

```
[In] integrate(x/(-x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*log(x^2 + x + 1) - 1/3*log
(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 15.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{x}{1-x^3}\,dx = -\frac{\ln(x-1)}{3} + \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

```
[In] int(-x/(x^3 - 1),x)
[Out] log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/6) - log(x - 1)/3 - log(x
 + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/6)
```

3.103 $\int \frac{x}{1-x^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{x}{1-x^4} dx = \frac{1}{4} \log \left(\frac{1+x^2}{1-x^2}\right)$$

```
[Out] 1/4*ln((x^2+1)/(-x^2+1))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.40, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {281, 212}

$$\int \frac{x}{1-x^4} dx = \frac{\operatorname{arctanh}(x^2)}{2}$$

```
[In] Int[x/(1 - x^4),x]
```

```
[Out] ArcTanh[x^2]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, x^2\right)$$
$$= \frac{\operatorname{arctanh}(x^2)}{2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{x}{1-x^4}\,dx = -\frac{1}{4}\log\left(1-x^2\right)+\frac{1}{4}\log\left(1+x^2\right)$$

```
[In] Integrate[x/(1 - x^4),x]
[Out] -1/4*Log[1 - x^2] + Log[1 + x^2]/4
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.35

method	result	size
meijerg	$\frac{\operatorname{arctanh}(x^2)}{2}$	7
risch	$\frac{\ln(x^2+1)}{4}-\frac{\ln(x^2-1)}{4}$	18
default	$-\frac{\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}+\frac{\ln(x^2+1)}{4}$	22
norman	$-\frac{\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}+\frac{\ln(x^2+1)}{4}$	22
parallelrisch	$-\frac{\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}+\frac{\ln(x^2+1)}{4}$	22

```
[In] int(x/(-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctanh(x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{x}{1-x^4}\,dx = \frac{1}{4}\,\log\left(x^2+1\right) - \frac{1}{4}\,\log\left(x^2-1\right)$$

```
[In] integrate(x/(-x^4+1),x, algorithm="fricas")
[Out] 1/4*log(x^2 + 1) - 1/4*log(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{x}{1-x^4}\,dx = -\frac{\log\left(x^2-1\right)}{4} + \frac{\log\left(x^2+1\right)}{4}$$

```
[In] integrate(x/(-x**4+1),x)
[Out] -log(x**2 - 1)/4 + log(x**2 + 1)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{x}{1-x^4}\,dx = \frac{1}{4}\,\log\left(x^2+1\right) - \frac{1}{4}\,\log\left(x^2-1\right)$$

```
[In] integrate(x/(-x^4+1),x, algorithm="maxima")
[Out] 1/4*log(x^2 + 1) - 1/4*log(x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x}{1-x^4}\,dx = \frac{1}{4}\,\log\left(x^2+1\right) - \frac{1}{4}\,\log\left(\left|x^2-1\right|\right)$$

```
[In] integrate(x/(-x^4+1),x, algorithm="giac")
[Out] 1/4*log(x^2 + 1) - 1/4*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.30

$$\int \frac{x}{1-x^4}\,dx = \frac{\operatorname{atanh}(x^2)}{2}$$

```
[In] int(-x/(x^4 - 1),x)
[Out] atanh(x^2)/2
```

3.104 $\int \frac{1}{x(1+x^2)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{1}{x(1+x^2)}\,dx = \log\left(\frac{x}{\sqrt{1+x^2}}\right)$$

```
[Out] ln(x/(x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {272, 36, 29, 31}

$$\int \frac{1}{x(1+x^2)}\,dx = \log(x) - \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] Int[1/(x*(1 + x^2)),x]
```

```
[Out] Log[x] - Log[1 + x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right)\\ &= \log(x) - \frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(1+x^2)}\,dx = \log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[1/(x*(1 + x^2)),x]
[Out] Log[x] - Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

method	result	size
default	$\ln(x) - \frac{\ln(x^2+1)}{2}$	12
norman	$\ln(x) - \frac{\ln(x^2+1)}{2}$	12
meijerg	$\ln(x) - \frac{\ln(x^2+1)}{2}$	12
risch	$\ln(x) - \frac{\ln(x^2+1)}{2}$	12
parallelrisch	$\ln(x) - \frac{\ln(x^2+1)}{2}$	12

```
[In] int(1/x/(x^2+1),x,method=_RETURNVERBOSE)
[Out] ln(x)-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(1+x^2)}\,dx = -\frac{1}{2}\log\left(x^2+1\right)+\log\left(x\right)$$

```
[In] integrate(1/x/(x^2+1),x, algorithm="fricas")
[Out] -1/2*log(x^2 + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(1+x^2)}\,dx = \log\left(x\right)-\frac{\log\left(x^2+1\right)}{2}$$

```
[In] integrate(1/x/(x**2+1),x)
[Out] log(x) - log(x**2 + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{1}{x(1+x^2)}\,dx = -\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x^2\right)$$

```
[In] integrate(1/x/(x^2+1),x, algorithm="maxima")
[Out] -1/2*log(x^2 + 1) + 1/2*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{1}{x(1+x^2)}\,dx = -\frac{1}{2}\log\left(x^2+1\right) + \frac{1}{2}\log\left(x^2\right)$$

```
[In] integrate(1/x/(x^2+1),x, algorithm="giac")
[Out] -1/2*log(x^2 + 1) + 1/2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 14.78 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(1+x^2)}\,dx = \ln(x) - \frac{\ln(x^2+1)}{2}$$

```
[In] int(1/(x*(x^2 + 1)),x)
[Out] log(x) - log(x^2 + 1)/2
```

3.105 $\int \frac{1}{x(1-x^2)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{x(1-x^2)} \, dx = \log \left(\frac{x}{\sqrt{1-x^2}}\right)$$

```
[Out] ln(x/(-x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {272, 36, 31, 29}

$$\int \frac{1}{x(1-x^2)} \, dx = \log(x) - \frac{1}{2} \log \left(1-x^2\right)$$

```
[In] Int[1/(x*(1 - x^2)),x]
```

```
[Out] Log[x] - Log[1 - x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)\\ &= \log(x) - \frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\,(1-x^2)}\,dx = \log(x) - \frac{1}{2}\log\left(1-x^2\right)$$

```
[In] Integrate[1/(x*(1 - x^2)),x]
[Out] Log[x] - Log[1 - x^2]/2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

method	result	size
risch	$\ln(x) - \frac{\ln(x^2-1)}{2}$	12
default	$-\frac{\ln(1+x)}{2} + \ln(x) - \frac{\ln(-1+x)}{2}$	16
norman	$-\frac{\ln(1+x)}{2} + \ln(x) - \frac{\ln(-1+x)}{2}$	16
parallelrisch	$-\frac{\ln(1+x)}{2} + \ln(x) - \frac{\ln(-1+x)}{2}$	16
meijerg	$\ln(x) + \frac{i\pi}{2} - \frac{\ln(-x^2+1)}{2}$	18

```
[In] int(1/x/(-x^2+1),x,method=_RETURNVERBOSE)
[Out] ln(x)-1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1}{x(1-x^2)}\,dx = -\frac{1}{2}\log\left(x^2-1\right)+\log\left(x\right)$$

```
[In] integrate(1/x/(-x^2+1),x, algorithm="fricas")
[Out] -1/2*log(x^2 - 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x(1-x^2)}\,dx = \log\left(x\right)-\frac{\log\left(x^2-1\right)}{2}$$

```
[In] integrate(1/x/(-x**2+1),x)
[Out] log(x) - log(x**2 - 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x(1-x^2)}\,dx = -\frac{1}{2}\log\left(x^2-1\right)+\frac{1}{2}\log\left(x^2\right)$$

```
[In] integrate(1/x/(-x^2+1),x, algorithm="maxima")
[Out] -1/2*log(x^2 - 1) + 1/2*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(1-x^2\right)}\,dx = \frac{1}{2}\log\left(x^2\right) - \frac{1}{2}\log\left(\left|x^2-1\right|\right)$$

```
[In] integrate(1/x/(-x^2+1),x, algorithm="giac")
[Out] 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\left(1-x^2\right)}\,dx = \ln\left(x\right) - \frac{\ln\left(x^2-1\right)}{2}$$

```
[In] int(-1/(x*(x^2 - 1)),x)
[Out] log(x) - log(x^2 - 1)/2
```

3.106 $\int \frac{a+bx}{A+Bx}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{a+bx}{A+Bx}\,dx=\frac{bx}{B}+\frac{(-Ab+aB)\log(A+Bx)}{B^2}$$

```
[Out] b*x/B+(-A*b+B*a)/B^2*ln(B*x+A)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\int \frac{a+bx}{A+Bx}\,dx=\frac{bx}{B}-\frac{(Ab-aB)\log(A+Bx)}{B^2}$$

```
[In] Int[(a + b*x)/(A + B*x),x]
```

```
[Out] (b*x)/B - ((A*b - a*B)*Log[A + B*x])/B^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{b}{B}+\frac{-Ab+aB}{B(A+Bx)}\right)\,dx\\&=\frac{bx}{B}-\frac{(Ab-aB)\log(A+Bx)}{B^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{A+Bx}\,dx=\frac{bx}{B}+\frac{(-Ab+aB)\log(A+Bx)}{B^2}$$

```
[In] Integrate[(a + b*x)/(A + B*x),x]
[Out] (b*x)/B + ((-(A*b) + a*B)*Log[A + B*x])/B^2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{bx}{B}+\frac{(-Ab+Ba)\ln(Bx+A)}{B^2}$	26
norman	$\frac{bx}{B}-\frac{(Ab-Ba)\ln(Bx+A)}{B^2}$	27
parallelrisch	$-\frac{A\ln(Bx+A)b-B\ln(Bx+A)a-xbB}{B^2}$	31
risch	$\frac{bx}{B}-\frac{\ln(Bx+A)Ab}{B^2}+\frac{\ln(Bx+A)a}{B}$	32

```
[In] int((b*x+a)/(B*x+A),x,method=_RETURNVERBOSE)
[Out] b*x/B+(-A*b+B*a)/B^2*ln(B*x+A)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{a+bx}{A+Bx}\,dx=\frac{Bbx+(Ba-Ab)\log(Bx+A)}{B^2}$$

```
[In] integrate((b*x+a)/(B*x+A),x, algorithm="fricas")
[Out] (B*b*x + (B*a - A*b)*log(B*x + A))/B^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{a+bx}{A+Bx}\,dx = \frac{bx}{B} + \frac{(-Ab+Ba)\log(A+Bx)}{B^2}$$

```
[In] integrate((b*x+a)/(B*x+A),x)
[Out] b*x/B + (-A*b + B*a)*log(A + B*x)/B**2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{A+Bx}\,dx = \frac{bx}{B} + \frac{(Ba-Ab)\log(Bx+A)}{B^2}$$

```
[In] integrate((b*x+a)/(B*x+A),x, algorithm="maxima")
[Out] b*x/B + (B*a - A*b)*log(B*x + A)/B^2
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{a+bx}{A+Bx}\,dx = \frac{bx}{B} + \frac{(Ba-Ab)\log(|Bx+A|)}{B^2}$$

```
[In] integrate((b*x+a)/(B*x+A),x, algorithm="giac")
[Out] b*x/B + (B*a - A*b)*log(abs(B*x + A))/B^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{a+bx}{A+Bx}\,dx = \frac{b\,x}{B} - \frac{\ln(A+B\,x)\,(A\,b-B\,a)}{B^2}$$

```
[In] int((a + b*x)/(A + B*x),x)
[Out] (b*x)/B - (log(A + B*x)*(A*b - B*a))/B^2
```

3.107 $\int \frac{1}{(a+bx)(A+Bx)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{1}{(a+bx)(A+Bx)}\,dx = \frac{\log\left(\frac{A+Bx}{a+bx}\right)}{-Ab+aB}$$

```
[Out] 1/(-A*b+B*a)*ln((B*x+A)/(b*x+a))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {36, 31}

$$\int \frac{1}{(a+bx)(A+Bx)}\,dx = \frac{\log(a+bx)}{Ab-aB} - \frac{\log(A+Bx)}{Ab-aB}$$

```
[In] Int[1/((a + b*x)*(A + B*x)),x]
```

```
[Out] Log[a + b*x]/(A*b - a*B) - Log[A + B*x]/(A*b - a*B)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{b\int\frac{1}{a+bx}\,dx}{Ab-aB}-\frac{B\int\frac{1}{A+Bx}\,dx}{Ab-aB}\\&=\frac{\log(a+bx)}{Ab-aB}-\frac{\log(A+Bx)}{Ab-aB}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int\frac{1}{(a+bx)(A+Bx)}\,dx=\frac{\log(a+bx)-\log(A+Bx)}{Ab-aB}$$

```
[In] Integrate[1/((a + b*x)*(A + B*x)),x]
[Out] (Log[a + b*x] - Log[A + B*x])/(A*b - a*B)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
parallelrisch	$\frac{-\ln(Bx+A)+\ln(bx+a)}{Ab-Ba}$	27
default	$\frac{\ln(bx+a)}{Ab-Ba}-\frac{\ln(Bx+A)}{Ab-Ba}$	37
norman	$\frac{\ln(bx+a)}{Ab-Ba}-\frac{\ln(Bx+A)}{Ab-Ba}$	37
risch	$\frac{\ln(-bx-a)}{Ab-Ba}-\frac{\ln(Bx+A)}{Ab-Ba}$	40

```
[In] int(1/(b*x+a)/(B*x+A),x,method=_RETURNVERBOSE)
[Out] (-ln(B*x+A)+ln(b*x+a))/(A*b-B*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{(a+bx)(A+Bx)}\,dx = \frac{\log(Bx+A) - \log(bx+a)}{Ba - Ab}$$

```
[In] integrate(1/(b*x+a)/(B*x+A),x, algorithm="fricas")
[Out] (log(B*x + A) - log(b*x + a))/(B*a - A*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(17) = 34.

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 5.12

$$\int \frac{1}{(a+bx)(A+Bx)}\,dx = \frac{\log\left(x + \frac{-\frac{A^2b^2}{-Ab+Ba} + \frac{2ABab}{-Ab+Ba} + Ab - \frac{B^2a^2}{-Ab+Ba} + Ba}{2Bb}\right)}{-Ab+Ba} - \frac{\log\left(x + \frac{\frac{A^2b^2}{-Ab+Ba} - \frac{2ABab}{-Ab+Ba} + Ab + \frac{B^2a^2}{-Ab+Ba} + Ba}{2Bb}\right)}{-Ab+Ba}$$

```
[In] integrate(1/(b*x+a)/(B*x+A),x)
[Out] log(x + (-A**2*b**2/(-A*b + B*a) + 2*A*B*a*b/(-A*b + B*a) + A*b - B**2*a**2
/(-A*b + B*a) + B*a)/(2*B*b))/(-A*b + B*a) - log(x + (A**2*b**2/(-A*b + B*a
) - 2*A*B*a*b/(-A*b + B*a) + A*b + B**2*a**2/(-A*b + B*a) + B*a)/(2*B*b))/(
-A*b + B*a)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{1}{(a+bx)(A+Bx)}\,dx = \frac{\log(Bx+A)}{Ba - Ab} - \frac{\log(bx+a)}{Ba - Ab}$$

```
[In] integrate(1/(b*x+a)/(B*x+A),x, algorithm="maxima")
[Out] log(B*x + A)/(B*a - A*b) - log(b*x + a)/(B*a - A*b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{1}{(a+bx)(A+Bx)}\,dx = \frac{B\log\left(|Bx+A|\right)}{B^2a-ABb} - \frac{b\log\left(|bx+a|\right)}{Bab-Ab^2}$$

```
[In] integrate(1/(b*x+a)/(B*x+A),x, algorithm="giac")
[Out] B*log(abs(B*x + A))/(B^2*a - A*B*b) - b*log(abs(b*x + a))/(B*a*b - A*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)(A+Bx)}\,dx = \frac{\ln\left(\frac{a+b\,x}{A+B\,x}\right)}{A\,b-B\,a}$$

```
[In] int(1/((A + B*x)*(a + b*x)),x)
[Out] log((a + b*x)/(A + B*x))/(A*b - B*a)
```

3.108 $\int \frac{x}{(a+bx)(A+Bx)} dx$

Optimal result

Integrand size = 16, antiderivative size = 35

$$\int \frac{x}{(a+bx)(A+Bx)} dx = \frac{\frac{a\log(a+bx)}{b} - \frac{A\log(A+Bx)}{B}}{-Ab+aB}$$

```
[Out] 1/(-A*b+B*a)*(a/b*ln(b*x+a)-A/B*ln(B*x+A))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {78}

$$\int \frac{x}{(a+bx)(A+Bx)} dx = \frac{A\log(A+Bx)}{B(Ab-aB)} - \frac{a\log(a+bx)}{b(Ab-aB)}$$

```
[In] Int[x/((a + b*x)*(A + B*x)),x]
```

```
[Out] -((a*Log[a + b*x])/(b*(A*b - a*B))) + (A*Log[A + B*x])/(B*(A*b - a*B))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{a}{(Ab-aB)(a+bx)}+\frac{A}{(Ab-aB)(A+Bx)}\right)dx\\ &= -\frac{a\log(a+bx)}{b(Ab-aB)}+\frac{A\log(A+Bx)}{B(Ab-aB)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{x}{(a+bx)(A+Bx)}\,dx = -\frac{aB\log(a+bx)-Ab\log(A+Bx)}{Ab^2B-abB^2}$$

```
[In] Integrate[x/((a + b*x)*(A + B*x)),x]
[Out] -((a*B*Log[a + b*x] - A*b*Log[A + B*x])/(A*b^2*B - a*b*B^2))
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

method	result	size
parallelrisch	$\frac{A\ln(Bx+A)b-a\ln(bx+a)B}{B(Ab-Ba)b}$	38
default	$-\frac{a\ln(bx+a)}{(Ab-Ba)b}+\frac{A\ln(Bx+A)}{(Ab-Ba)B}$	45
norman	$-\frac{a\ln(bx+a)}{(Ab-Ba)b}+\frac{A\ln(Bx+A)}{(Ab-Ba)B}$	45
risch	$-\frac{a\ln(bx+a)}{(Ab-Ba)b}+\frac{A\ln(-Bx-A)}{(Ab-Ba)B}$	48

```
[In] int(x/(b*x+a)/(B*x+A),x,method=_RETURNVERBOSE)
[Out] (A*ln(B*x+A)*b-a*ln(b*x+a)*B)/B/(A*b-B*a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{x}{(a+bx)(A+Bx)}\,dx = -\frac{Ab\log\left(Bx+A\right)-Ba\log\left(bx+a\right)}{B^2ab-ABb^2}$$

```
[In] integrate(x/(b*x+a)/(B*x+A),x, algorithm="fricas")
[Out] -(A*b*log(B*x + A) - B*a*log(b*x + a))/(B^2*a*b - A*B*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(26) = 52.

Time = 0.36 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.94

$$\int \frac{x}{(a+bx)(A+Bx)}\,dx = -\frac{A\log\left(x+\frac{-\frac{A^3b^2}{B(-Ab+Ba)}+\frac{2A^2ab}{-Ab+Ba}-\frac{ABa^2}{-Ab+Ba}+2Aa}{Ab+Ba}\right)}{B\left(-Ab+Ba\right)} + \frac{a\log\left(x+\frac{\frac{A^2ab}{-Ab+Ba}-\frac{2ABa^2}{-Ab+Ba}+2Aa+\frac{B^2a^3}{b(-Ab+Ba)}}{Ab+Ba}\right)}{b\left(-Ab+Ba\right)}$$

```
[In] integrate(x/(b*x+a)/(B*x+A),x)
[Out] -A*log(x + (-A**3*b**2/(B*(-A*b + B*a)) + 2*A**2*a*b/(-A*b + B*a) - A*B*a**
2/(-A*b + B*a) + 2*A*a)/(A*b + B*a))/(B*(-A*b + B*a)) + a*log(x + (A**2*a*b
/(-A*b + B*a) - 2*A*B*a**2/(-A*b + B*a) + 2*A*a + B**2*a**3/(b*(-A*b + B*a)
))/(A*b + B*a))/(b*(-A*b + B*a))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x}{(a+bx)(A+Bx)}\,dx = -\frac{A\log\left(Bx+A\right)}{B^2a-ABb}+\frac{a\log\left(bx+a\right)}{Bab-Ab^2}$$

```
[In] integrate(x/(b*x+a)/(B*x+A),x, algorithm="maxima")
[Out] -A*log(B*x + A)/(B^2*a - A*B*b) + a*log(b*x + a)/(B*a*b - A*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \frac{x}{(a+bx)(A+Bx)}\,dx = -\frac{A\log\left(|Bx+A|\right)}{B^2a-ABb}+\frac{a\log\left(|bx+a|\right)}{Bab-Ab^2}$$

```
[In] integrate(x/(b*x+a)/(B*x+A),x, algorithm="giac")
[Out] -A*log(abs(B*x + A))/(B^2*a - A*B*b) + a*log(abs(b*x + a))/(B*a*b - A*b^2)
```

Mupad [B] (verification not implemented)

Time = 15.83 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{x}{(a+bx)(A+Bx)}\,dx = \frac{A\,b\,\ln\left(A+B\,x\right)-B\,a\,\ln\left(a+b\,x\right)}{B\,b\,\left(A\,b-B\,a\right)}$$

```
[In] int(x/((A + B*x)*(a + b*x)),x)
[Out] (A*b*log(A + B*x) - B*a*log(a + b*x))/(B*b*(A*b - B*a))
```

3.109 $\int \frac{1}{\sqrt{x}(a+bx)} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{\sqrt{x}(a+bx)}\,dx = \frac{2\arctan\left(\frac{bx}{a}\right)}{\sqrt{ab}}$$

```
[Out] 2/(a*b)^(1/2)*arctan(b*x/a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {65, 211}

$$\int \frac{1}{\sqrt{x}(a+bx)}\,dx = \frac{2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[In] Int[1/(Sqrt[x]*(a + b*x)),x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)$$
$$= \frac{2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{x}(a+bx)}\,dx = \frac{2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)),x]
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19
default	$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19

```
[In] int(1/(b*x+a)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.25

$$\int \frac{1}{\sqrt{x}(a+bx)}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{ab}, -\frac{2\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab}\right]$$

```
[In] integrate(1/(b*x+a)/x^(1/2),x, algorithm="fricas")
[Out] [-sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a))/(a*b), -2*sqrt
(a*b)*arctan(sqrt(a*b)/(b*sqrt(x)))/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(14) = 28.

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 4.56

$$\int \frac{1}{\sqrt{x}(a+bx)}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\ a=0 \wedge b=0 \\ \frac{2\sqrt{x}}{a} & \text{for}\ b=0 \\ -\frac{2}{b\sqrt{x}} & \text{for}\ a=0 \\ \frac{\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}} - \frac{\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(b*x+a)/x**(1/2),x)
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a, Eq(b, 0)), (-2/
(b*sqrt(x)), Eq(a, 0)), (log(sqrt(x) - sqrt(-a/b))/(b*sqrt(-a/b)) - log(sqr
t(x) + sqrt(-a/b))/(b*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{x}(a+bx)}\,dx = \frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

```
[In] integrate(1/(b*x+a)/x^(1/2),x, algorithm="maxima")
[Out] 2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{x}(a+bx)}\,dx = \frac{2\ \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

```
[In] integrate(1/(b*x+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

Mupad [B] (verification not implemented)

Time = 15.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt{x}(a+bx)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[In] int(1/(x^(1/2)*(a + b*x)),x)
```

```
[Out] (2*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(a^(1/2)*b^(1/2))
```

3.110 $\int \frac{\sqrt{x}}{a+bx} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\sqrt{x}}{a+bx} \, dx = \frac{2\sqrt{x}}{b} - \frac{2a \arctan\left(\frac{bx}{a}\right)}{b\sqrt{ab}}$$

```
[Out] 2*x^(1/2)/b-2*a/b/(a*b)^(1/2)*arctan(b*x/a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {52, 65, 211}

$$\int \frac{\sqrt{x}}{a+bx} \, dx = \frac{2\sqrt{x}}{b} - \frac{2\sqrt{a} \arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[In] Int[Sqrt[x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\sqrt{x}}{b} - \frac{a\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{b}\\ &= \frac{2\sqrt{x}}{b} - \frac{(2a)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b}\\ &= \frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{x}}{a+bx}\,dx = \frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[In] Integrate[Sqrt[x]/(a + b*x),x]
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b} - \frac{2a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32
default	$\frac{2\sqrt{x}}{b} - \frac{2a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32
risch	$\frac{2\sqrt{x}}{b} - \frac{2a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32

```
[In] int(x^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 2*x^(1/2)/b-2*a/b/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.74

$$\int \frac{\sqrt{x}}{a+bx}\,dx = \left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,\sqrt{x}}{b}, -\frac{2\left(\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{x}\right)}{b}\right]$$

```
[In] integrate(x^(1/2)/(b*x+a),x, algorithm="fricas")
[Out] [(sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*sqrt(x))
/b, -2*(sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) - sqrt(x))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.84

$$\int \frac{\sqrt{x}}{a+bx}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for } b = 0 \\ \frac{2\sqrt{x}}{b} & \text{for } a = 0 \\ -\frac{a\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b^2\sqrt{-\frac{a}{b}}} + \frac{a\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b^2\sqrt{-\frac{a}{b}}} + \frac{2\sqrt{x}}{b} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(1/2)/(b*x+a),x)
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a), Eq(b, 0)),
 (2*sqrt(x)/b, Eq(a, 0)), (-a*log(sqrt(x) - sqrt(-a/b))/(b**2*sqrt(-a/b)) +
 a*log(sqrt(x) + sqrt(-a/b))/(b**2*sqrt(-a/b)) + 2*sqrt(x)/b, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{a+bx}\,dx = -\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{2\sqrt{x}}{b}$$

```
[In] integrate(x^(1/2)/(b*x+a),x, algorithm="maxima")
[Out] -2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{a+bx}\,dx = -\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{2\sqrt{x}}{b}$$

```
[In] integrate(x^(1/2)/(b*x+a),x, algorithm="giac")
[Out] -2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

Mupad [B] (verification not implemented)

Time = 14.63 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{x}}{a+bx}\,dx = \frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[In] int(x^(1/2)/(a + b*x),x)
[Out] (2*x^(1/2))/b - (2*a^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/b^(3/2)
```

3.111 $\int \frac{x^{3/2}}{a+bx}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \frac{x^{3/2}}{a+bx}\,dx = 2\sqrt{x}\left(-\frac{a}{b^2}+\frac{x}{3b}\right)+\frac{2a^2\arctan\left(\frac{bx}{a}\right)}{b^2\sqrt{ab}}$$

```
[Out] 2*(1/3*x/b-a/b^2)*x^(1/2)+2*a^2/b^2/(a*b)^(1/2)*arctan(b*x/a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {52, 65, 211}

$$\int \frac{x^{3/2}}{a+bx}\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}$$

```
[In] Int[x^(3/2)/(a + b*x),x]
```

```
[Out] (-2*a*Sqrt[x])/b^2 + (2*x^(3/2))/(3*b) + (2*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/b^(5/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2x^{3/2}}{3b} - \frac{a\int \frac{\sqrt{x}}{a+bx}\,dx}{b}\\ &= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} + \frac{a^2\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{b^2}\\ &= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b^2}\\ &= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} + \frac{2a^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int \frac{x^{3/2}}{a+bx}\,dx = \frac{2\sqrt{x}(-3a+bx)}{3b^2} + \frac{2a^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

```
[In] Integrate[x^(3/2)/(a + b*x),x]
[Out] (2*Sqrt[x]*(-3*a + b*x))/(3*b^2) + (2*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/b^(5/2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{2(-bx+3a)\sqrt{x}}{3b^2}+\frac{2a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	42
derivativedivides	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+a\sqrt{x}\right)}{b^2}+\frac{2a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	43
default	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+a\sqrt{x}\right)}{b^2}+\frac{2a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	43

```
[In] int(x^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -2/3*(-b*x+3*a)*x^(1/2)/b^2+2*a^2/b^2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1
/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.29

$$\int \frac{x^{3/2}}{a+bx}\,dx = \left[\frac{3\,a\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(bx-3\,a)\sqrt{x}}{3\,b^2}, \frac{2\left(3\,a\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+(bx-3\,a)\sqrt{}\right.}{3\,b^2}\right.$$

```
[In] integrate(x^(3/2)/(b*x+a),x, algorithm="fricas")
[Out] [1/3*(3*a*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*
(b*x - 3*a)*sqrt(x))/b^2, 2/3*(3*a*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a)
+ (b*x - 3*a)*sqrt(x))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(39) = 78$.

Time = 0.53 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.38

$$\int \frac{x^{3/2}}{a+bx}\,dx = \begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for}\ a=0 \wedge b=0 \\ \frac{2x^{\frac{5}{2}}}{5a} & \text{for}\ b=0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for}\ a=0 \\ \frac{a^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b^3\sqrt{-\frac{a}{b}}}-\frac{a^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b^3\sqrt{-\frac{a}{b}}}-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(3/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a), Eq(b, 0))
, (2*x**(3/2)/(3*b), Eq(a, 0)), (a**2*log(sqrt(x) - sqrt(-a/b))/(b**3*sqrt(
-a/b)) - a**2*log(sqrt(x) + sqrt(-a/b))/(b**3*sqrt(-a/b)) - 2*a*sqrt(x)/b**
2 + 2*x**(3/2)/(3*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{x^{3/2}}{a+bx}\,dx = \frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\left(bx^{\frac{3}{2}} - 3\,a\sqrt{x}\right)}{3\,b^2}$$

```
[In] integrate(x^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b*x^(3/2) - 3*a*sq
rt(x))/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{x^{3/2}}{a+bx}\,dx = \frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\left(b^2x^{\frac{3}{2}} - 3\,ab\sqrt{x}\right)}{3\,b^3}$$

```
[In] integrate(x^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b^2*x^(3/2) - 3*a*
b*sqrt(x))/b^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{x^{3/2}}{a+bx}\,dx = \frac{2\,x^{3/2}}{3\,b} - \frac{2\,a\,\sqrt{x}}{b^2} + \frac{2\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

```
[In] int(x^(3/2)/(a + b*x),x)
```

```
[Out] (2*x^(3/2))/(3*b) - (2*a*x^(1/2))/b^2 + (2*a^(3/2)*atan((b^(1/2)*x^(1/2))/a
^(1/2)))/b^(5/2)
```

3.112 $\int \frac{x^{5/2}}{a+bx}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{x^{5/2}}{a+bx}\,dx = 2\sqrt{x}\left(\frac{a^2}{b^2}-\frac{ax}{3b^2}+\frac{x^2}{5b}\right)-\frac{2a^3\arctan\left(\frac{bx}{a}\right)}{b^3\sqrt{ab}}$$

```
[Out] 2*(1/5*x^2/b-1/3*a*x/b^2+a^2/b^2)*x^(1/2)-2*a^3/b^3/(a*b)^(1/2)*arctan(b*x/
a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {52, 65, 211}

$$\int \frac{x^{5/2}}{a+bx}\,dx = -\frac{2a^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{2a^2\sqrt{x}}{b^3}-\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}$$

```
[In] Int[x^(5/2)/(a + b*x),x]
```

```
[Out] (2*a^2*Sqrt[x])/b^3 - (2*a*x^(3/2))/(3*b^2) + (2*x^(5/2))/(5*b) - (2*a^(5/2
)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x^{5/2}}{5b} - \frac{a\int \frac{x^{3/2}}{a+bx}\,dx}{b} \\
&= -\frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} + \frac{a^2\int \frac{\sqrt{x}}{a+bx}\,dx}{b^2} \\
&= \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{a^3\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{b^3} \\
&= \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b^3} \\
&= \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{2a^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$
\int \frac{x^{5/2}}{a+bx}\,dx = \frac{2\sqrt{x}(15a^2 - 5abx + 3b^2x^2)}{15b^3} - \frac{2a^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
$$

```
[In] Integrate[x^(5/2)/(a + b*x),x]
[Out] (2*Sqrt[x]*(15*a^2 - 5*a*b*x + 3*b^2*x^2))/(15*b^3) - (2*a^(5/2)*ArcTan[(Sq
rt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

method	result	size
risch	$\frac{2(3b^2x^2-5bax+15a^2)\sqrt{x}}{15b^3}-\frac{2a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	53
derivativedivides	$\frac{\frac{2x^{\frac{5}{2}}b^2}{5}-\frac{2ax^{\frac{3}{2}}b}{3}+2a^2\sqrt{x}}{b^3}-\frac{2a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	54
default	$\frac{\frac{2x^{\frac{5}{2}}b^2}{5}-\frac{2ax^{\frac{3}{2}}b}{3}+2a^2\sqrt{x}}{b^3}-\frac{2a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	54

```
[In] int(x^(5/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(3*b^2*x^2-5*a*b*x+15*a^2)*x^(1/2)/b^3-2*a^3/b^3/(a*b)^(1/2)*arctan(b*
x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.32

$$\int \frac{x^{5/2}}{a+bx}\,dx = \left[\frac{15\,a^2\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(3\,b^2x^2-5\,abx+15\,a^2)\sqrt{x}}{15\,b^3},\right.$$
$$\left.-\frac{2\left(15\,a^2\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(3\,b^2x^2-5\,abx+15\,a^2)\sqrt{x}\right)}{15\,b^3}\right]$$

```
[In] integrate(x^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^2*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a))
+ 2*(3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3, -2/15*(15*a^2*sqrt(a/b)*ar
ctan(b*sqrt(x)*sqrt(a/b)/a) - (3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(53) = 106.

Time = 1.80 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.14

$$\int \frac{x^{5/2}}{a+bx}\,dx = \begin{cases} \tilde{\infty} x^{\frac{5}{2}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{7}{2}}}{7a} & \text{for } b = 0 \\ \frac{2x^{\frac{5}{2}}}{5b} & \text{for } a = 0 \\ -\frac{a^3 \log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b^4\sqrt{-\frac{a}{b}}} + \frac{a^3 \log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b^4\sqrt{-\frac{a}{b}}} + \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{\frac{3}{2}}}{3b^2} + \frac{2x^{\frac{5}{2}}}{5b} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(5/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a), Eq(b, 0))
, (2*x**(5/2)/(5*b), Eq(a, 0)), (-a**3*log(sqrt(x) - sqrt(-a/b))/(b**4*sqrt
(-a/b)) + a**3*log(sqrt(x) + sqrt(-a/b))/(b**4*sqrt(-a/b)) + 2*a**2*sqrt(x)
/b**3 - 2*a*x**(3/2)/(3*b**2) + 2*x**(5/2)/(5*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{x^{5/2}}{a+bx}\,dx = -\frac{2\,a^3 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{2\left(3\,b^2x^{\frac{5}{2}} - 5\,abx^{\frac{3}{2}} + 15\,a^2\sqrt{x}\right)}{15\,b^3}$$

```
[In] integrate(x^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^2*x^(5/2) -
5*a*b*x^(3/2) + 15*a^2*sqrt(x))/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{x^{5/2}}{a+bx}\,dx = -\frac{2\,a^3 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{2\left(3\,b^4x^{\frac{5}{2}} - 5\,ab^3x^{\frac{3}{2}} + 15\,a^2b^2\sqrt{x}\right)}{15\,b^5}$$

```
[In] integrate(x^(5/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^4*x^(5/2) -
5*a*b^3*x^(3/2) + 15*a^2*b^2*sqrt(x))/b^5
```

Mupad [B] (verification not implemented)

Time = 15.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{x^{5/2}}{a+bx}\,dx=\frac{2\,x^{5/2}}{5\,b}-\frac{2\,a\,x^{3/2}}{3\,b^2}+\frac{2\,a^2\,\sqrt{x}}{b^3}-\frac{2\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

```
[In] int(x^(5/2)/(a + b*x),x)
[Out] (2*x^(5/2))/(5*b) - (2*a*x^(3/2))/(3*b^2) + (2*a^2*x^(1/2))/b^3 - (2*a^(5/2
)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/b^(7/2)
```

3.113 $\int \frac{1}{\sqrt{x}(a+bx)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{1}{\sqrt{x}(a+bx)^2} dx = \frac{\sqrt{x}\arctan\left(\frac{bx}{a}\right)}{a^2\sqrt{ab}(a+bx)}$$

```
[Out] x^(1/2)/a^2/(b*x+a)/(a*b)^(1/2)*arctan(b*x/a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {44, 65, 211}

$$\int \frac{1}{\sqrt{x}(a+bx)^2} dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}} + \frac{\sqrt{x}}{a(a+bx)}$$

```
[In] Int[1/(Sqrt[x]*(a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{x}}{a(a+bx)} + \frac{\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{2a}\\ &= \frac{\sqrt{x}}{a(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{a}\\ &= \frac{\sqrt{x}}{a(a+bx)} + \frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx = \frac{\sqrt{x}}{a(a+bx)} + \frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^2),x]
[Out] Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{\sqrt{x}}{a(bx+a)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	36
default	$\frac{\sqrt{x}}{a(bx+a)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	36

```
[In] int(1/(b*x+a)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)/a/(b*x+a)+1/a/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.87

$$\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx = \left[\frac{2\,ab\sqrt{x}-\sqrt{-ab}(bx+a)\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{2\,(a^2b^2x+a^3b)}, \frac{ab\sqrt{x}-\sqrt{ab}(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{a^2b^2x+a^3b}\right]$$

```
[In] integrate(1/(b*x+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*sqrt(x) - sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqrt
(x))/(b*x + a)))/(a^2*b^2*x + a^3*b), (a*b*sqrt(x) - sqrt(a*b)*(b*x + a)*ar
ctan(sqrt(a*b)/(b*sqrt(x))))/(a^2*b^2*x + a^3*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(26) = 52$.

Time = 2.19 (sec) , antiderivative size = 277, normalized size of antiderivative = 9.23

$$\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} \\ \frac{2\sqrt{x}}{a^2} \\ -\frac{2}{3b^2x^{\frac{3}{2}}} \\ \frac{a\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} - \frac{a\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} + \frac{2b\sqrt{x}\sqrt{-\frac{a}{b}}}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} + \frac{bx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} - \frac{bx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} \end{cases}$$

```
[In] integrate(1/(b*x+a)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**2, Eq(b, 0)),
(-2/(3*b**2*x**(3/2)), Eq(a, 0)), (a*log(sqrt(x) - sqrt(-a/b))/(2*a**2*b*sq
rt(-a/b) + 2*a*b**2*x*sqrt(-a/b)) - a*log(sqrt(x) + sqrt(-a/b))/(2*a**2*b*s
qrt(-a/b) + 2*a*b**2*x*sqrt(-a/b)) + 2*b*sqrt(x)*sqrt(-a/b)/(2*a**2*b*sqrt(
-a/b) + 2*a*b**2*x*sqrt(-a/b)) + b*x*log(sqrt(x) - sqrt(-a/b))/(2*a**2*b*sq
rt(-a/b) + 2*a*b**2*x*sqrt(-a/b)) - b*x*log(sqrt(x) + sqrt(-a/b))/(2*a**2*b
*sqrt(-a/b) + 2*a*b**2*x*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx = \frac{\sqrt{x}}{abx+a^2} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a}$$

```
[In] integrate(1/(b*x+a)^2/x^(1/2),x, algorithm="maxima")
[Out] sqrt(x)/(a*b*x + a^2) + arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx = \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a} + \frac{\sqrt{x}}{(bx+a)a}$$

```
[In] integrate(1/(b*x+a)^2/x^(1/2),x, algorithm="giac")
[Out] arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) + sqrt(x)/((b*x + a)*a)
```

Mupad [B] (verification not implemented)

Time = 15.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx = \frac{\sqrt{x}}{a\,(a+b\,x)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\,\sqrt{b}}$$

```
[In] int(1/(x^(1/2)*(a + b*x)^2),x)
[Out] x^(1/2)/(a*(a + b*x)) + atan((b^(1/2)*x^(1/2))/a^(1/2))/(a^(3/2)*b^(1/2))
```

3.114 $\int \frac{\sqrt{x}}{(a+bx)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\sqrt{x}}{(a+bx)^2} dx = -\frac{\sqrt{x}\arctan\left(\frac{bx}{a}\right)}{b^2\sqrt{ab}(a+bx)}$$

```
[Out] -x^(1/2)/b^2/(b*x+a)/(a*b)^(1/2)*arctan(b*x/a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {43, 65, 211}

$$\int \frac{\sqrt{x}}{(a+bx)^2} dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}} - \frac{\sqrt{x}}{b(a+bx)}$$

```
[In] Int[Sqrt[x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/
2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{x}}{b(a+bx)} + \frac{\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{2b} \\ &= -\frac{\sqrt{x}}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b} \\ &= -\frac{\sqrt{x}}{b(a+bx)} + \frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{x}}{(a+bx)^2}\,dx = -\frac{\sqrt{x}}{b(a+bx)} + \frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

```
[In] Integrate[Sqrt[x]/(a + b*x)^2,x]
[Out] -(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/
2))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

method	result	size
derivativedivides	$-\frac{\sqrt{x}}{b(bx+a)} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	37
default	$-\frac{\sqrt{x}}{b(bx+a)} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	37

```
[In] int(x^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/b*x^(1/2)/(b*x+a)+1/b/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.71

$$\int \frac{\sqrt{x}}{(a+bx)^2}\,dx = \left[-\frac{2\,ab\sqrt{x}+\sqrt{-ab}(bx+a)\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{2\,(ab^3x+a^2b^2)}, -\frac{ab\sqrt{x}+\sqrt{ab}(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab^3x+a^2b^2}\right]$$

```
[In] integrate(x^(1/2)/(b*x+a)^2,x, algorithm="fricas")
[Out] [-1/2*(2*a*b*sqrt(x) + sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)))/(a*b^3*x + a^2*b^2), -(a*b*sqrt(x) + sqrt(a*b)*(b*x + a)*
arctan(sqrt(a*b)/(b*sqrt(x))))/(a*b^3*x + a^2*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(27) = 54$.

Time = 1.44 (sec) , antiderivative size = 269, normalized size of antiderivative = 8.68

$$\int \frac{\sqrt{x}}{(a+bx)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for} \\ \frac{2x^{\frac{3}{2}}}{3a^2} & \text{for} \\ -\frac{2}{b^2\sqrt{x}} & \text{for} \\ \frac{a\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}} - \frac{a\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}} - \frac{2b\sqrt{x}\sqrt{-\frac{a}{b}}}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}} + \frac{bx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}} - \frac{bx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}} & \text{oth} \end{cases}$$

```
[In] integrate(x**(1/2)/(b*x+a)**2,x)
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**2), Eq(b, 0
)), (-2/(b**2*sqrt(x)), Eq(a, 0)), (a*log(sqrt(x) - sqrt(-a/b))/(2*a*b**2*s
qrt(-a/b) + 2*b**3*x*sqrt(-a/b)) - a*log(sqrt(x) + sqrt(-a/b))/(2*a*b**2*sq
rt(-a/b) + 2*b**3*x*sqrt(-a/b)) - 2*b*sqrt(x)*sqrt(-a/b)/(2*a*b**2*sqrt(-a/
b) + 2*b**3*x*sqrt(-a/b)) + b*x*log(sqrt(x) - sqrt(-a/b))/(2*a*b**2*sqrt(-a
/b) + 2*b**3*x*sqrt(-a/b)) - b*x*log(sqrt(x) + sqrt(-a/b))/(2*a*b**2*sqrt(-
a/b) + 2*b**3*x*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{x}}{(a+bx)^2}\,dx = -\frac{\sqrt{x}}{b^2x+ab} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

```
[In] integrate(x^(1/2)/(b*x+a)^2,x, algorithm="maxima")
[Out] -sqrt(x)/(b^2*x + a*b) + arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{x}}{(a+bx)^2}\,dx = \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b} - \frac{\sqrt{x}}{(bx+a)b}$$

```
[In] integrate(x^(1/2)/(b*x+a)^2,x, algorithm="giac")
[Out] arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) - sqrt(x)/((b*x + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{x}}{(a+bx)^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}} - \frac{\sqrt{x}}{b\,\left(a+b\,x\right)}$$

```
[In] int(x^(1/2)/(a + b*x)^2,x)
[Out] atan((b^(1/2)*x^(1/2))/a^(1/2))/(a^(1/2)*b^(3/2)) - x^(1/2)/(b*(a + b*x))
```

3.115 $\int \frac{x^{3/2}}{(a+bx)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 50

$$\int \frac{x^{3/2}}{(a+bx)^2} dx = \frac{2x^{3/2}}{b(a+bx)} + \frac{3a\sqrt{x}\arctan\left(\frac{bx}{a}\right)}{b^3\sqrt{ab}(a+bx)}$$

```
[Out] 2*x^(3/2)/b/(b*x+a)+3*a/b^3*x^(1/2)/(b*x+a)/(a*b)^(1/2)*arctan(b*x/a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {43, 52, 65, 211}

$$\int \frac{x^{3/2}}{(a+bx)^2} dx = -\frac{3\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}} - \frac{x^{3/2}}{b(a+bx)} + \frac{3\sqrt{x}}{b^2}$$

```
[In] Int[x^(3/2)/(a + b*x)^2,x]
```

```
[Out] (3*Sqrt[x])/b^2 - x^(3/2)/(b*(a + b*x)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/b^(5/2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^{3/2}}{b(a+bx)}+\frac{3\int \frac{\sqrt{x}}{a+bx}\,dx}{2b}\\
&= \frac{3\sqrt{x}}{b^2}-\frac{x^{3/2}}{b(a+bx)}-\frac{(3a)\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{2b^2}\\
&= \frac{3\sqrt{x}}{b^2}-\frac{x^{3/2}}{b(a+bx)}-\frac{(3a)\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= \frac{3\sqrt{x}}{b^2}-\frac{x^{3/2}}{b(a+bx)}-\frac{3\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \frac{x^{3/2}}{(a+bx)^2}\,dx=\frac{\sqrt{x}(3a+2bx)}{b^2(a+bx)}-\frac{3\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

```
[In] Integrate[x^(3/2)/(a + b*x)^2,x]
```

```
[Out] (Sqrt[x]*(3*a + 2*b*x))/(b^2*(a + b*x)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/b^(5/2)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b^{2}}-\frac{2a\left(-\frac{\sqrt{x}}{2(bx+a)}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{2}}$	47
default	$\frac{2\sqrt{x}}{b^{2}}-\frac{2a\left(-\frac{\sqrt{x}}{2(bx+a)}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{2}}$	47
risch	$\frac{2\sqrt{x}}{b^{2}}+\frac{a\sqrt{x}}{b^{2}(bx+a)}-\frac{3a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^{2}\sqrt{ab}}$	47

```
[In] int(x^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 2/b^2*x^(1/2)-2*a/b^2*(-1/2*x^(1/2)/(b*x+a)+3/2/(a*b)^(1/2)*arctan(b*x^(1/2
)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.68

$$\int \frac{x^{3/2}}{(a+bx)^2}\,dx=\left[\frac{3\,(bx+a)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(2\,bx+3\,a)\sqrt{x}}{2\,(b^3x+ab^2)},\right.$$
$$\left.-\frac{3\,(bx+a)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(2\,bx+3\,a)\sqrt{x}}{b^3x+ab^2}\right]$$

```
[In] integrate(x^(3/2)/(b*x+a)^2,x, algorithm="fricas")
[Out] [1/2*(3*(b*x + a)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x +
a)) + 2*(2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2), -(3*(b*x + a)*sqrt(a/b)*arc
tan(b*sqrt(x)*sqrt(a/b)/a) - (2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(42) = 84.

Time = 2.82 (sec) , antiderivative size = 332, normalized size of antiderivative = 6.64

$$\int \frac{x^{3/2}}{(a+bx)^2}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} \\ \frac{2x^{\frac{5}{2}}}{5a^2} \\ \frac{2\sqrt{x}}{b^2} \\ -\frac{3a^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} + \frac{3a^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} + \frac{6ab\sqrt{x}\sqrt{-\frac{a}{b}}}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} - \frac{3abx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} + \frac{3abx\log}{2ab^3\sqrt{-}} \end{cases}$$

```
[In] integrate(x**(3/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**2), Eq(b, 0
)), (2*sqrt(x)/b**2, Eq(a, 0)), (-3*a**2*log(sqrt(x) - sqrt(-a/b))/(2*a*b**
3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 3*a**2*log(sqrt(x) + sqrt(-a/b))/(2*a
*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 6*a*b*sqrt(x)*sqrt(-a/b)/(2*a*b**
3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) - 3*a*b*x*log(sqrt(x) - sqrt(-a/b))/(2*
a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 3*a*b*x*log(sqrt(x) + sqrt(-a/b)
)/(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 4*b**2*x**(3/2)*sqrt(-a/b)/
(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{x^{3/2}}{(a+bx)^2}\,dx = \frac{a\sqrt{x}}{b^3x+ab^2} - \frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\,\sqrt{x}}{b^2}$$

```
[In] integrate(x^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a*sqrt(x)/(b^3*x + a*b^2) - 3*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2)
 + 2*sqrt(x)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{x^{3/2}}{(a+bx)^2}\,dx = -\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{a\sqrt{x}}{(bx+a)b^2} + \frac{2\,\sqrt{x}}{b^2}$$

```
[In] integrate(x^(3/2)/(b*x+a)^2,x, algorithm="giac")
[Out] -3*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + a*sqrt(x)/((b*x + a)*b^2
) + 2*sqrt(x)/b^2
```

Mupad [B] (verification not implemented)

Time = 15.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{x^{3/2}}{(a+bx)^2}\,dx = \frac{2\,\sqrt{x}}{b^2} + \frac{a\,\sqrt{x}}{x\,b^3 + a\,b^2} - \frac{3\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

```
[In] int(x^(3/2)/(a + b*x)^2,x)
[Out] (2*x^(1/2))/b^2 + (a*x^(1/2))/(a*b^2 + b^3*x) - (3*a^(1/2)*atan((b^(1/2)*x^
(1/2))/a^(1/2)))/b^(5/2)
```

3.116 $\int \frac{x^{5/2}}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{x^{5/2}}{(a+bx)^2}\,dx = \frac{2\sqrt{x}\left(-\frac{5ax}{3b^2}+\frac{x^2}{3b}\right)}{a+bx} - \frac{5a^2\sqrt{x}\arctan\left(\frac{bx}{a}\right)}{b^4\sqrt{ab}(a+bx)}$$

```
[Out] 2*(1/3*x^2/b-5/3*a*x/b^2)*x^(1/2)/(b*x+a)-5*a^2/b^4*x^(1/2)/(b*x+a)/(a*b)^(
1/2)*arctan(b*x/a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 52, 65, 211}

$$\int \frac{x^{5/2}}{(a+bx)^2}\,dx = \frac{5a^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}} - \frac{5a\sqrt{x}}{b^3} - \frac{x^{5/2}}{b(a+bx)} + \frac{5x^{3/2}}{3b^2}$$

```
[In] Int[x^(5/2)/(a + b*x)^2,x]
```

```
[Out] (-5*a*Sqrt[x])/b^3 + (5*x^(3/2))/(3*b^2) - x^(5/2)/(b*(a + b*x)) + (5*a^(3/
2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\text{integral} = -\frac{x^{5/2}}{b(a+bx)}+\frac{5\int\frac{x^{3/2}}{a+bx}\,dx}{2b}$$

$$=\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}-\frac{(5a)\int\frac{\sqrt{x}}{a+bx}\,dx}{2b^2}$$

$$=-\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{(5a^2)\int\frac{1}{\sqrt{x}(a+bx)}\,dx}{2b^3}$$

$$=-\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{(5a^2)\,\text{Subst}\left(\int\frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{b^3}$$

$$=-\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{5a^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{x^{5/2}}{(a+bx)^2} dx = \frac{\sqrt{x}(-15a^2 - 10abx + 2b^2x^2)}{3b^3(a+bx)} + \frac{5a^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

```
[In] Integrate[x^(5/2)/(a + b*x)^2,x]
```

```
[Out] (Sqrt[x]*(-15*a^2 - 10*a*b*x + 2*b^2*x^2))/(3*b^3*(a + b*x)) + (5*a^(3/2)*A
rcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{2(-bx+6a)\sqrt{x}}{3b^3}+\frac{a^2\left(-\frac{\sqrt{x}}{bx+a}+\frac{5\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{b^3}$	56
derivativedivides	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+2a\sqrt{x}\right)}{b^3}+\frac{2a^2\left(-\frac{\sqrt{x}}{2(bx+a)}+\frac{5\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^3}$	59
default	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+2a\sqrt{x}\right)}{b^3}+\frac{2a^2\left(-\frac{\sqrt{x}}{2(bx+a)}+\frac{5\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^3}$	59

```
[In] int(x^(5/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-b*x+6*a)*x^(1/2)/b^3+a^2/b^3*(-x^(1/2)/(b*x+a)+5/(a*b)^(1/2)*arctan(
b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.33

$$\int \frac{x^{5/2}}{(a+bx)^2} dx = \left[\frac{15\,(abx+a^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(2\,b^2x^2-10\,abx-15\,a^2)\sqrt{x}}{6\,(b^4x+ab^3)}, \frac{15\,(abx+a^2)}{}\right.$$

```
[In] integrate(x^(5/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b*x + a^2)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b
*x + a)) + 2*(2*b^2*x^2 - 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3), 1/3*
(15*(a*b*x + a^2)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (2*b^2*x^2 - 10
*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(60) = 120$.

Time = 7.33 (sec) , antiderivative size = 389, normalized size of antiderivative = 5.64

$$\int \frac{x^{5/2}}{(a+bx)^2}\,dx = \begin{cases} \tilde{\infty}x^{\frac{3}{2}} \\ \frac{2x^{\frac{7}{2}}}{7a^2} \\ \frac{2x^{\frac{3}{2}}}{3b^2} \\ \frac{15a^3\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{6ab^4\sqrt{-\frac{a}{b}}+6b^5x\sqrt{-\frac{a}{b}}} - \frac{15a^3\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{6ab^4\sqrt{-\frac{a}{b}}+6b^5x\sqrt{-\frac{a}{b}}} - \frac{30a^2b\sqrt{x}\sqrt{-\frac{a}{b}}}{6ab^4\sqrt{-\frac{a}{b}}+6b^5x\sqrt{-\frac{a}{b}}} + \frac{15a^2bx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{6ab^4\sqrt{-\frac{a}{b}}+6b^5x\sqrt{-\frac{a}{b}}} - \frac{15a^2bx}{6ab^4} \end{cases}$$

```
[In] integrate(x**(5/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a**2), Eq(b,
0)), (2*x**(3/2)/(3*b**2), Eq(a, 0)), (15*a**3*log(sqrt(x) - sqrt(-a/b))/(6
*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 15*a**3*log(sqrt(x) + sqrt(-a/b
))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 30*a**2*b*sqrt(x)*sqrt(-a/
b)/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) + 15*a**2*b*x*log(sqrt(x) -
sqrt(-a/b))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 15*a**2*b*x*log(s
qrt(x) + sqrt(-a/b))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 20*a*b**
2*x**(3/2)*sqrt(-a/b)/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) + 4*b**3*
x**(5/2)*sqrt(-a/b)/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{x^{5/2}}{(a+bx)^2}\,dx = -\frac{a^2\sqrt{x}}{b^4x+ab^3} + \frac{5\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{2\left(bx^{\frac{3}{2}}-6\,a\sqrt{x}\right)}{3\,b^3}$$

```
[In] integrate(x^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^2*sqrt(x)/(b^4*x + a*b^3) + 5*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)
*b^3) + 2/3*(b*x^(3/2) - 6*a*sqrt(x))/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{x^{5/2}}{(a+bx)^2}\,dx = \frac{5\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} - \frac{a^2\sqrt{x}}{(bx+a)b^3} + \frac{2\left(b^4x^{\frac{3}{2}} - 6\,ab^3\sqrt{x}\right)}{3\,b^6}$$

```
[In] integrate(x^(5/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) - a^2*sqrt(x)/((b*x + a)*
b^3) + 2/3*(b^4*x^(3/2) - 6*a*b^3*sqrt(x))/b^6
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{x^{5/2}}{(a+bx)^2}\,dx = \frac{2\,x^{3/2}}{3\,b^2} - \frac{4\,a\,\sqrt{x}}{b^3} - \frac{a^2\,\sqrt{x}}{x\,b^4 + a\,b^3} + \frac{5\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

```
[In] int(x^(5/2)/(a + b*x)^2,x)
```

```
[Out] (2*x^(3/2))/(3*b^2) - (4*a*x^(1/2))/b^3 - (a^2*x^(1/2))/(a*b^3 + b^4*x) + (
5*a^(3/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/b^(7/2)
```

3.117 $\int \frac{1}{\sqrt{x}(a+bx)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{1}{\sqrt{x}(a+bx)^3} dx = \sqrt{x}\left(\frac{1}{2a(a+bx)^2} + \frac{1}{4a^2(a+bx)}\right) + \frac{3\arctan\left(\frac{bx}{a}\right)}{4a^2\sqrt{ab}}$$

```
[Out] (1/2/a/(b*x+a)^2+1/4/a^2/(b*x+a))*x^(1/2)+3/4/a^2/(a*b)^(1/2)*arctan(b*x/a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {44, 65, 211}

$$\int \frac{1}{\sqrt{x}(a+bx)^3} dx = \frac{3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}} + \frac{3\sqrt{x}}{4a^2(a+bx)} + \frac{\sqrt{x}}{2a(a+bx)^2}$$

```
[In] Int[1/(Sqrt[x]*(a + b*x)^3),x]
```

```
[Out] Sqrt[x]/(2*a*(a + b*x)^2) + (3*Sqrt[x])/(4*a^2*(a + b*x)) + (3*ArcTan[(Sqrt
[b]*Sqrt[x])/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{x}}{2a(a+bx)^2} + \frac{3\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx}{4a}\\ &= \frac{\sqrt{x}}{2a(a+bx)^2} + \frac{3\sqrt{x}}{4a^2(a+bx)} + \frac{3\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{8a^2}\\ &= \frac{\sqrt{x}}{2a(a+bx)^2} + \frac{3\sqrt{x}}{4a^2(a+bx)} + \frac{3\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{4a^2}\\ &= \frac{\sqrt{x}}{2a(a+bx)^2} + \frac{3\sqrt{x}}{4a^2(a+bx)} + \frac{3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{x}(a+bx)^3}\,dx = \frac{\sqrt{x}(5a+3bx)}{4a^2(a+bx)^2} + \frac{3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}$$

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^3),x]
[Out] (Sqrt[x]*(5*a + 3*b*x))/(4*a^2*(a + b*x)^2) + (3*ArcTan[(Sqrt[b]*Sqrt[x])/S
qrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\sqrt{x}}{2a(bx+a)^2}+\frac{\frac{3\sqrt{x}}{4a(bx+a)}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4a\sqrt{ab}}}{a}$	59
default	$\frac{\sqrt{x}}{2a(bx+a)^2}+\frac{\frac{3\sqrt{x}}{4a(bx+a)}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4a\sqrt{ab}}}{a}$	59

```
[In] int(1/(b*x+a)^3/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^(1/2)/a/(b*x+a)^2+3/2/a*(1/2*x^(1/2)/a/(b*x+a)+1/2/a/(a*b)^(1/2)*arct
an(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.26

$$\int \frac{1}{\sqrt{x}(a+bx)^3}\,dx = \left[-\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{-ab}\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)-2\left(3\,ab^2x+5\,a^2b\right)\sqrt{x}}{8\left(a^3b^3x^2+2\,a^4b^2x+a^5b\right)}, -\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-\left(3\,ab^2x+5\,a^2b\right)\sqrt{x}}{4\left(a^3b^3x^2+2\,a^4b^2x+a^5b\right)}\right]$$

```
[In] integrate(1/(b*x+a)^3/x^(1/2),x, algorithm="fricas")
[Out] [-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*s
qrt(x))/(b*x + a)) - 2*(3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*
b^2*x + a^5*b), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) - (3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*b^2*x
+ a^5*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(48) = 96$.

Time = 7.92 (sec) , antiderivative size = 632, normalized size of antiderivative = 11.09

$$\int \frac{1}{\sqrt{x}(a+bx)^3}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} \\ \frac{2\sqrt{x}}{a^3} \\ -\frac{2}{5b^3x^{\frac{5}{2}}} \\ \frac{3a^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}} - \frac{3a^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}} + \frac{10ab\sqrt{x}\sqrt{-\frac{a}{b}}}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}} + \frac{}{8a} \end{cases}$$

```
[In] integrate(1/(b*x+a)**3/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**3, Eq(b, 0)),
(-2/(5*b**3*x**(5/2)), Eq(a, 0)), (3*a**2*log(sqrt(x) - sqrt(-a/b))/(8*a**4
*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)) -
3*a**2*log(sqrt(x) + sqrt(-a/b))/(8*a**4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt
(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)) + 10*a*b*sqrt(x)*sqrt(-a/b)/(8*a**4*b
*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)) + 6*
a*b*x*log(sqrt(x) - sqrt(-a/b))/(8*a**4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(
-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)) - 6*a*b*x*log(sqrt(x) + sqrt(-a/b))/(8
*a**4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b
)) + 6*b**2*x**(3/2)*sqrt(-a/b)/(8*a**4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(
-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)) + 3*b**2*x**2*log(sqrt(x) - sqrt(-a/b)
)/(8*a**4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(
-a/b)) - 3*b**2*x**2*log(sqrt(x) + sqrt(-a/b))/(8*a**4*b*sqrt(-a/b) + 16*a*
*3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{x}(a+bx)^3}\,dx = \frac{3\,bx^{\frac{3}{2}} + 5\,a\sqrt{x}}{4\,(a^2b^2x^2 + 2\,a^3bx + a^4)} + \frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^2}$$

```
[In] integrate(1/(b*x+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*b*x^(3/2) + 5*a*sqrt(x))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 3/4*arcta
n(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{x}(a+bx)^3}\,dx = \frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^2} + \frac{3\,bx^{\frac{3}{2}} + 5\,a\sqrt{x}}{4\,(bx+a)^2a^2}$$

```
[In] integrate(1/(b*x+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/4*(3*b*x^(3/2) + 5*a*sq
rt(x))/((b*x + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 15.99 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}(a+bx)^3}\,dx = \frac{\frac{5\sqrt{x}}{4\,a} + \frac{3\,b\,x^{3/2}}{4\,a^2}}{a^2 + 2\,a\,b\,x + b^2\,x^2} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{5/2}\,\sqrt{b}}$$

```
[In] int(1/(x^(1/2)*(a + b*x)^3),x)
```

```
[Out] ((5*x^(1/2))/(4*a) + (3*b*x^(3/2))/(4*a^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (3*
atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(5/2)*b^(1/2))
```

3.118 $\int \frac{\sqrt{x}}{(a+bx)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{\sqrt{x}}{(a+bx)^3} dx = \sqrt{x}\left(-\frac{1}{2b(a+bx)^2}+\frac{1}{4ab(a+bx)}\right)+\frac{\arctan\left(\frac{bx}{a}\right)}{4ab\sqrt{ab}}$$

```
[Out] (-1/2/b/(b*x+a)^2+1/4/a/b/(b*x+a))*x^(1/2)+1/4/a/b/(a*b)^(1/2)*arctan(b*x/a
)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {43, 44, 65, 211}

$$\int \frac{\sqrt{x}}{(a+bx)^3} dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(a+bx)}-\frac{\sqrt{x}}{2b(a+bx)^2}$$

```
[In] Int[Sqrt[x]/(a + b*x)^3,x]
```

```
[Out] -1/2*Sqrt[x]/(b*(a + b*x)^2) + Sqrt[x]/(4*a*b*(a + b*x)) + ArcTan[(Sqrt[b]*
Sqrt[x])/Sqrt[a]]/(4*a^(3/2)*b^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx}{4b}\\
&= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\sqrt{x}}{4ab(a+bx)}+\frac{\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{8ab}\\
&= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\sqrt{x}}{4ab(a+bx)}+\frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4ab}\\
&= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\sqrt{x}}{4ab(a+bx)}+\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{x}}{(a+bx)^3}\,dx=-\frac{\sqrt{x}(a-bx)}{4ab(a+bx)^2}+\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$

```
[In] Integrate[Sqrt[x]/(a + b*x)^3,x]
[Out] -1/4*(Sqrt[x]*(a - b*x))/(a*b*(a + b*x)^2) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[
a]]/(4*a^(3/2)*b^(3/2))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\frac{x^{\frac{3}{2}}}{4a}-\frac{\sqrt{x}}{4b}}{(bx+a)^2}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4ba\sqrt{ab}}$	52
default	$\frac{\frac{x^{\frac{3}{2}}}{4a}-\frac{\sqrt{x}}{4b}}{(bx+a)^2}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4ba\sqrt{ab}}$	52

```
[In] int(x^(1/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/8/a*x^(3/2)-1/8*x^(1/2)/b)/(b*x+a)^2+1/4/b/a/(a*b)^(1/2)*arctan(b*x^(1
/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.95

$$\int \frac{\sqrt{x}}{(a+bx)^3}\,dx = \left[-\frac{(b^2x^2+2\,abx+a^2)\sqrt{-ab}\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)-2\,(ab^2x-a^2b)\sqrt{x}}{8\,(a^2b^4x^2+2\,a^3b^3x+a^4b^2)},\ -\frac{(b^2x^2+2\,abx+a^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-(ab^2x-a^2b)\sqrt{x}}{4\,(a^2b^4x^2+2\,a^3b^3x+a^4b^2)}\right]$$

```
[In] integrate(x^(1/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)) - 2*(a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x
+ a^4*b^2), -1/4*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)/(b*s
qrt(x))) - (a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x + a^4*b^2)
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(46) = 92$.

Time = 5.17 (sec) , antiderivative size = 627, normalized size of antiderivative = 9.95

$$\int \frac{\sqrt{x}}{(a+bx)^3}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} \\ \frac{2x^{\frac{3}{2}}}{3a^3} \\ -\frac{2}{3b^3x^{\frac{3}{2}}} \\ \frac{a^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^3b^2\sqrt{-\frac{a}{b}}+16a^2b^3x\sqrt{-\frac{a}{b}}+8ab^4x^2\sqrt{-\frac{a}{b}}} - \frac{a^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^3b^2\sqrt{-\frac{a}{b}}+16a^2b^3x\sqrt{-\frac{a}{b}}+8ab^4x^2\sqrt{-\frac{a}{b}}} - \frac{2ab\sqrt{x}\sqrt{-\frac{a}{b}}}{8a^3b^2\sqrt{-\frac{a}{b}}+16a^2b^3x\sqrt{-\frac{a}{b}}+8ab^4x^2\sqrt{-\frac{a}{b}}} + \end{cases}$$

```
[In] integrate(x**(1/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**3), Eq(b,
0)), (-2/(3*b**3*x**(3/2)), Eq(a, 0)), (a**2*log(sqrt(x) - sqrt(-a/b))/(8*a
**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b))
 - a**2*log(sqrt(x) + sqrt(-a/b))/(8*a**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*
sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)) - 2*a*b*sqrt(x)*sqrt(-a/b)/(8*a**3*b
**2*sqrt(-a/b) + 16*a**2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)) + 2*
a*b*x*log(sqrt(x) - sqrt(-a/b))/(8*a**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sq
rt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)) - 2*a*b*x*log(sqrt(x) + sqrt(-a/b))/(8
*a**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b
)) + 2*b**2*x**(3/2)*sqrt(-a/b)/(8*a**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sq
rt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)) + b**2*x**2*log(sqrt(x) - sqrt(-a/b))/
(8*a**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a
/b)) - b**2*x**2*log(sqrt(x) + sqrt(-a/b))/(8*a**3*b**2*sqrt(-a/b) + 16*a**
2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{x}}{(a+bx)^3}\,dx = \frac{bx^{\frac{3}{2}} - a\sqrt{x}}{4\,(ab^3x^2 + 2\,a^2b^2x + a^3b)} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}ab}$$

```
[In] integrate(x^(1/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^(3/2) - a*sqrt(x))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/4*arctan(
b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{x}}{(a+bx)^3}\,dx = \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}ab} + \frac{bx^{\frac{3}{2}} - a\sqrt{x}}{4\,(bx+a)^2ab}$$

```
[In] integrate(x^(1/2)/(b*x+a)^3,x, algorithm="giac")
[Out] 1/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/4*(b*x^(3/2) - a*sqrt(x
))/((b*x + a)^2*a*b)
```

Mupad [B] (verification not implemented)

Time = 16.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{x}}{(a+bx)^3}\,dx = \frac{\frac{x^{3/2}}{4\,a} - \frac{\sqrt{x}}{4\,b}}{a^2 + 2\,a\,b\,x + b^2\,x^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{3/2}\,b^{3/2}}$$

```
[In] int(x^(1/2)/(a + b*x)^3,x)
[Out] (x^(3/2)/(4*a) - x^(1/2)/(4*b))/(a^2 + b^2*x^2 + 2*a*b*x) + atan((b^(1/2)*x
^(1/2))/a^(1/2))/(4*a^(3/2)*b^(3/2))
```

3.119 $\int \frac{x^{3/2}}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int \frac{x^{3/2}}{(a+bx)^3}\,dx = -\frac{2x^{3/2}}{b(a+bx)^2}+\frac{3a\left(\sqrt{x}\left(-\frac{1}{2b(a+bx)^2}+\frac{1}{4ab(a+bx)}\right)+\frac{\arctan\left(\frac{bx}{a}\right)}{4ab\sqrt{ab}}\right)}{b}$$

```
[Out] -2*x^(3/2)/b/(b*x+a)^2+3*a/b*((-1/2/b/(b*x+a)^2+1/4/a/b/(b*x+a))*x^(1/2)+1/
4/a/b/(a*b)^(1/2)*arctan(b*x/a))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {43, 65, 211}

$$\int \frac{x^{3/2}}{(a+bx)^3}\,dx = \frac{3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}b^{5/2}}-\frac{3\sqrt{x}}{4b^2(a+bx)}-\frac{x^{3/2}}{2b(a+bx)^2}$$

```
[In] Int[x^(3/2)/(a + b*x)^3,x]
```

```
[Out] -1/2*x^(3/2)/(b*(a + b*x)^2) - (3*Sqrt[x])/(4*b^2*(a + b*x)) + (3*ArcTan[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^{3/2}}{2b(a+bx)^2} + \frac{3\int \frac{\sqrt{x}}{(a+bx)^2}\,dx}{4b} \\
&= -\frac{x^{3/2}}{2b(a+bx)^2} - \frac{3\sqrt{x}}{4b^2(a+bx)} + \frac{3\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{8b^2} \\
&= -\frac{x^{3/2}}{2b(a+bx)^2} - \frac{3\sqrt{x}}{4b^2(a+bx)} + \frac{3\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{4b^2} \\
&= -\frac{x^{3/2}}{2b(a+bx)^2} - \frac{3\sqrt{x}}{4b^2(a+bx)} + \frac{3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}b^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.68

$$\int \frac{x^{3/2}}{(a+bx)^3}\,dx = -\frac{\sqrt{x}(3a+5bx)}{4b^2(a+bx)^2} + \frac{3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}b^{5/2}}$$

```
[In] Integrate[x^(3/2)/(a + b*x)^3,x]
[Out] -1/4*(Sqrt[x]*(3*a + 5*b*x))/(b^2*(a + b*x)^2) + (3*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.57

method	result	size
derivativedivides	$\frac{-\frac{5x^{\frac{3}{2}}}{4b}-\frac{3a\sqrt{x}}{4b^2}}{(bx+a)^2}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^2\sqrt{ab}}$	50
default	$\frac{-\frac{5x^{\frac{3}{2}}}{4b}-\frac{3a\sqrt{x}}{4b^2}}{(bx+a)^2}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^2\sqrt{ab}}$	50

```
[In] int(x^(3/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-5/8*x^(3/2)/b-3/8*a/b^2*x^(1/2))/(b*x+a)^2+3/4/b^2/(a*b)^(1/2)*arctan(b
*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.13

$$\int \frac{x^{3/2}}{(a+bx)^3}\,dx = \left[-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{-ab}\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)+2\,(5\,ab^2x+3\,a^2b)\sqrt{x}}{8\,(ab^5x^2+2\,a^2b^4x+a^3b^3)},\right.$$
$$\left.-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+(5\,ab^2x+3\,a^2b)\sqrt{x}}{4\,(ab^5x^2+2\,a^2b^4x+a^3b^3)}\right]$$

```
[In] integrate(x^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*s
qrt(x))/(b*x + a)) + 2*(5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^
4*x + a^3*b^3), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) + (5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^4*x +
a^3*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(66) = 132$.

Time = 9.86 (sec) , antiderivative size = 605, normalized size of antiderivative = 6.95

$$\int \frac{x^{3/2}}{(a+bx)^3}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} \\ \frac{2x^{\frac{5}{2}}}{5a^3} \\ -\frac{2}{b^3\sqrt{x}} \\ \frac{3a^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}} - \frac{3a^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}} - \frac{6ab\sqrt{x}\sqrt{-\frac{a}{b}}}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5} \end{cases}$$

```
[In] integrate(x**(3/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**3), Eq(b, 0
)), (-2/(b**3*sqrt(x)), Eq(a, 0)), (3*a**2*log(sqrt(x) - sqrt(-a/b))/(8*a**
2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt(-a/b)) - 3*a*
*2*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/
b) + 8*b**5*x**2*sqrt(-a/b)) - 6*a*b*sqrt(x)*sqrt(-a/b)/(8*a**2*b**3*sqrt(-
a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt(-a/b)) + 6*a*b*x*log(sqrt(
x) - sqrt(-a/b))/(8*a**2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*
x**2*sqrt(-a/b)) - 6*a*b*x*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b**3*sqrt(-a/b
) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt(-a/b)) - 10*b**2*x**(3/2)*sqr
t(-a/b)/(8*a**2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt
(-a/b)) + 3*b**2*x**2*log(sqrt(x) - sqrt(-a/b))/(8*a**2*b**3*sqrt(-a/b) + 1
6*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt(-a/b)) - 3*b**2*x**2*log(sqrt(x) +
 sqrt(-a/b))/(8*a**2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2
*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int \frac{x^{3/2}}{(a+bx)^3}\,dx = -\frac{5\,bx^{\frac{3}{2}}+3\,a\sqrt{x}}{4\,(b^4x^2+2\,ab^3x+a^2b^2)} + \frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^2}$$

```
[In] integrate(x^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*b*x^(3/2) + 3*a*sqrt(x))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 3/4*arct
an(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.54

$$\int \frac{x^{3/2}}{(a+bx)^3}\,dx = \frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^2} - \frac{5\,bx^{\frac{3}{2}}+3\,a\sqrt{x}}{4\,(bx+a)^2b^2}$$

```
[In] integrate(x^(3/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/4*(5*b*x^(3/2) + 3*a*sq
rt(x))/((b*x + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 15.44 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{x^{3/2}}{(a+bx)^3}\,dx = \frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,\sqrt{a}\,b^{5/2}} - \frac{\frac{5\,x^{3/2}}{4\,b}+\frac{3\,a\,\sqrt{x}}{4\,b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

```
[In] int(x^(3/2)/(a + b*x)^3,x)
```

```
[Out] (3*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(1/2)*b^(5/2)) - ((5*x^(3/2))/(4*b
) + (3*a*x^(1/2))/(4*b^2))/(a^2 + b^2*x^2 + 2*a*b*x)
```

3.120 $\int \frac{x^{5/2}}{(a+bx)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 101

$$\int \frac{x^{5/2}}{(a+bx)^3} \, dx = \frac{2\sqrt{x}\left(\frac{5ax}{b^2}+\frac{x^2}{b}\right)}{(a+bx)^2} - \frac{15a^2\left(\sqrt{x}\left(-\frac{1}{2b(a+bx)^2}+\frac{1}{4ab(a+bx)}\right)+\frac{\arctan\left(\frac{bx}{a}\right)}{4ab\sqrt{ab}}\right)}{b^2}$$

```
[Out] 2*(x^2/b+5*a*x/b^2)*x^(1/2)/(b*x+a)^2-15*a^2/b^2*((-1/2/b/(b*x+a)^2+1/4/a/b
/(b*x+a))*x^(1/2)+1/4/a/b/(a*b)^(1/2)*arctan(b*x/a))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {43, 52, 65, 211}

$$\int \frac{x^{5/2}}{(a+bx)^3} \, dx = -\frac{15\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}} - \frac{5x^{3/2}}{4b^2(a+bx)} - \frac{x^{5/2}}{2b(a+bx)^2} + \frac{15\sqrt{x}}{4b^3}$$

```
[In] Int[x^(5/2)/(a + b*x)^3,x]
```

```
[Out] (15*Sqrt[x])/(4*b^3) - x^(5/2)/(2*b*(a + b*x)^2) - (5*x^(3/2))/(4*b^2*(a +
b*x)) - (15*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^{5/2}}{2b(a+bx)^2}+\frac{5\int \frac{x^{3/2}}{(a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}+\frac{15\int \frac{\sqrt{x}}{a+bx}\,dx}{8b^2}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{(15a)\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{8b^3}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{(15a)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^3}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{15\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

$$\int \frac{x^{5/2}}{(a+bx)^3}\,dx = \frac{\sqrt{x}(15a^2+25abx+8b^2x^2)}{4b^3(a+bx)^2} - \frac{15\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}$$

```
[In] Integrate[x^(5/2)/(a + b*x)^3,x]
```

```
[Out] (Sqrt[x]*(15*a^2 + 25*a*b*x + 8*b^2*x^2))/(4*b^3*(a + b*x)^2) - (15*Sqrt[a]
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b^3} - \frac{2a\left(\frac{-\frac{9b\,x^{\frac{3}{2}}}{8}-\frac{7a\sqrt{x}}{8}}{(bx+a)^2}+\frac{15\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^3}$	56
default	$\frac{2\sqrt{x}}{b^3} - \frac{2a\left(\frac{-\frac{9b\,x^{\frac{3}{2}}}{8}-\frac{7a\sqrt{x}}{8}}{(bx+a)^2}+\frac{15\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^3}$	56
risch	$\frac{2\sqrt{x}}{b^3} - \frac{a\left(\frac{-\frac{9b\,x^{\frac{3}{2}}}{4}-\frac{7a\sqrt{x}}{4}}{(bx+a)^2}+\frac{15\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{b^3}$	57

```
[In] int(x^(5/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*x^(1/2)-2*a/b^3*((-9/8*b*x^(3/2)-7/8*a*x^(1/2))/(b*x+a)^2+15/8/(a*b)^
(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.98

$$\int \frac{x^{5/2}}{(a+bx)^3}\,dx = \left[\frac{15\,(b^2x^2+2\,abx+a^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(8\,b^2x^2+25\,abx+15\,a^2)\sqrt{x}}{8\,(b^5x^2+2\,ab^4x+a^2b^3)}, -\frac{15\,(b^2x^2+2\,abx+a^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(8\,b^2x^2+25\,abx+15\,a^2)\sqrt{x}}{4\,(b^5x^2+2\,ab^4x+a^2b^3)}\right]$$

```
[In] integrate(x^(5/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-
a/b) - a)/(b*x + a)) + 2*(8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
+ 2*a*b^4*x + a^2*b^3), -1/4*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a/b)*arctan
(b*sqrt(x)*sqrt(a/b)/a) - (8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
 + 2*a*b^4*x + a^2*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(82) = 164$.

Time = 16.89 (sec) , antiderivative size = 683, normalized size of antiderivative = 6.76

$$\int \frac{x^{5/2}}{(a+bx)^3}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} \\ \frac{2x^{\frac{7}{2}}}{7a^3} \\ \frac{2\sqrt{x}}{b^3} \\ -\frac{15a^3\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}} + \frac{15a^3\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}} + \frac{30a^2b\sqrt{x}\sqrt{-\frac{a}{b}}}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+\cdots} \end{cases}$$

```
[In] integrate(x**(5/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a**3), Eq(b, 0
)), (2*sqrt(x)/b**3, Eq(a, 0)), (-15*a**3*log(sqrt(x) - sqrt(-a/b))/(8*a**2
*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) + 15*a*
*3*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/
b) + 8*b**6*x**2*sqrt(-a/b)) + 30*a**2*b*sqrt(x)*sqrt(-a/b)/(8*a**2*b**4*sq
rt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) - 30*a**2*b*x*l
```

```
og(sqrt(x) - sqrt(-a/b))/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) +
 8*b**6*x**2*sqrt(-a/b)) + 30*a**2*b*x*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b*
*4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) + 50*a*b**
2*x**(3/2)*sqrt(-a/b)/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*
b**6*x**2*sqrt(-a/b)) - 15*a*b**2*x**2*log(sqrt(x) - sqrt(-a/b))/(8*a**2*b*
*4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) + 15*a*b**
2*x**2*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt
(-a/b) + 8*b**6*x**2*sqrt(-a/b)) + 16*b**3*x**(5/2)*sqrt(-a/b)/(8*a**2*b**4
*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{x^{5/2}}{(a+bx)^3}\,dx = \frac{9\,abx^{\frac{3}{2}}+7\,a^2\sqrt{x}}{4\,(b^5x^2+2\,ab^4x+a^2b^3)} - \frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^3} + \frac{2\,\sqrt{x}}{b^3}$$

```
[In] integrate(x^(5/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(9*a*b*x^(3/2) + 7*a^2*sqrt(x))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3) - 15/4*
a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2*sqrt(x)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

$$\int \frac{x^{5/2}}{(a+bx)^3}\,dx = -\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^3} + \frac{2\,\sqrt{x}}{b^3} + \frac{9\,abx^{\frac{3}{2}}+7\,a^2\sqrt{x}}{4\,(bx+a)^2b^3}$$

```
[In] integrate(x^(5/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -15/4*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2*sqrt(x)/b^3 + 1/4*(
9*a*b*x^(3/2) + 7*a^2*sqrt(x))/((b*x + a)^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.68

$$\int \frac{x^{5/2}}{(a+bx)^3}\,dx = \frac{\frac{7\,a^2\,\sqrt{x}}{4}+\frac{9\,a\,b\,x^{3/2}}{4}}{a^2\,b^3+2\,a\,b^4\,x+b^5\,x^2}+\frac{2\,\sqrt{x}}{b^3}-\frac{15\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,b^{7/2}}$$

```
[In] int(x^(5/2)/(a + b*x)^3,x)
[Out] ((7*a^2*x^(1/2))/4 + (9*a*b*x^(3/2))/4)/(a^2*b^3 + b^5*x^2 + 2*a*b^4*x) + (
2*x^(1/2))/b^3 - (15*a^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*b^(7/2))
```

3.121 $\int \frac{1}{\sqrt{x}(a+bx^2)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx=\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}}{\sqrt{\frac{a}{b}}-x}\right)+\log\left(\frac{\sqrt{\frac{a}{b}}+\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}+x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}\left(\frac{a}{b}\right)^{3/4}b}$$

```
[Out] 1/2/b/(a/b)^(3/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(
b*x^2+a)^(1/2))+arctan((a/b)^(1/4)*2^(1/2)*x^(1/2)/((a/b)^(1/2)-x)))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.94, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx=-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(3/4)*b^(1/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(3/4)*b^(1/4)) -
 Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(3
/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(
2*Sqrt[2]*a^(3/4)*b^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{a}\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{a}\sqrt{b}}$$

$$- \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

$$+ \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

$$- \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx = \frac{-\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (-ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + ArcTanh
[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(3/4)
*b^(1/4))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a}$	106
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a}$	106

```
[In] int(1/x^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(
x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+2*arctan(1/(a/b)^(1/4)*2^(1/2)*
x^(1/2)+1)+2*arctan(1/(a/b)^(1/4)*2^(1/2)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx = \frac{1}{2}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+\sqrt{x}\right) + \frac{1}{2}i\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(i\,a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+\sqrt{x}\right) - \frac{1}{2}i\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(-i\,a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+\sqrt{x}\right) - \frac{1}{2}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+\sqrt{x}\right)$$

```
[In] integrate(1/x^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-1/(a^3*b))^(1/4)*log(a*(-1/(a^3*b))^(1/4) + sqrt(x)) + 1/2*I*(-1/(a^3
*b))^(1/4)*log(I*a*(-1/(a^3*b))^(1/4) + sqrt(x)) - 1/2*I*(-1/(a^3*b))^(1/4)
*log(-I*a*(-1/(a^3*b))^(1/4) + sqrt(x)) - 1/2*(-1/(a^3*b))^(1/4)*log(-a*(-1
/(a^3*b))^(1/4) + sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a=0 \wedge b=0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } a=0 \\ \frac{2\sqrt{x}}{a} & \text{for } b=0 \\ -\frac{\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a} + \frac{\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a} + \frac{\sqrt[4]{-\frac{a}{b}}\,\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x**(1/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b*x**(3/2)), Eq(a, 0)
), (2*sqrt(x)/a, Eq(b, 0)), (-(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(2
*a) + (-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*a) + (-a/b)**(1/4)*atan
(sqrt(x)/(-a/b)**(1/4))/a, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(79) = 158.

Time = 0.29 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\,\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\,\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{4\,a^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{4\,a^{\frac{3}{4}}b^{\frac{1}{4}}}$$

```
[In] integrate(1/x^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 1/2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*
sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 1/4*sqrt(2)*log(sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - 1/4*sqrt(2)*log
(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(79) = 158.

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx = \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab} + \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab} + \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab} - \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab}$$

```
[In] integrate(1/x^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a*b) + 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b) + 1/4*sqrt(2)*(a*b^3)^(1/4
)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b) - 1/4*sqrt(2)*(a*b
^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 17.88 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.37

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx = -\frac{\operatorname{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)+\operatorname{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{(-a)^{3/4}\,b^{1/4}}$$

```
[In] int(1/(x^(1/2)*(a + b*x^2)),x)
```

```
[Out] -(atan((b^(1/4)*x^(1/2))/(-a)^(1/4)) + atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))
/((-a)^(3/4)*b^(1/4))
```

3.122 $\int \frac{\sqrt{x}}{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx=\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}}{\sqrt{\frac{a}{b}}-x}\right)-\log\left(\frac{\sqrt{\frac{a}{b}}+\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}+x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}\sqrt[4]{\frac{a}{b}}b}$$

```
[Out] 1/2/b/(a/b)^(1/4)*2^(1/2)*(-ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/
(b*x^2+a)^(1/2))+arctan((a/b)^(1/4)*2^(1/2)*x^(1/2)/((a/b)^(1/2)-x)))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.90, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx=-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}}+\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}}$$

```
[In] Int[Sqrt[x]/(a + b*x^2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)) +
```

```
 Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1
/4)*b^(3/4)) - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(
2*Sqrt[2]*a^(1/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}}\\
&= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b}\\
&\quad + \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}} + \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}}\\
&= \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}} - \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}}\\
&= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}}\\
&\quad + \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}} - \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx=-\frac{\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}}$$

```
[In] Integrate[Sqrt[x]/(a + b*x^2),x]
```

```
[Out] -((ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + ArcTan
h[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(1/4
)*b^(3/4)))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	106
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	106

```
[In] int(x^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(
x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+2*arctan(1/(a/b)^(1/4)*2^(1/2)*
x^(1/2)+1)+2*arctan(1/(a/b)^(1/4)*2^(1/2)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx = \frac{1}{2}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right) - \frac{1}{2}i\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(i\,ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right) + \frac{1}{2}i\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(-i\,ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right) - \frac{1}{2}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(-ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)$$

```
[In] integrate(x^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-1/(a*b^3))^(1/4)*log(a*b^2*(-1/(a*b^3))^(3/4) + sqrt(x)) - 1/2*I*(-1/
(a*b^3))^(1/4)*log(I*a*b^2*(-1/(a*b^3))^(3/4) + sqrt(x)) + 1/2*I*(-1/(a*b^3
))^(1/4)*log(-I*a*b^2*(-1/(a*b^3))^(3/4) + sqrt(x)) - 1/2*(-1/(a*b^3))^(1/4
)*log(-a*b^2*(-1/(a*b^3))^(3/4) + sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a=0 \wedge b=0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for } b=0 \\ -\frac{2}{b\sqrt{x}} & \text{for } a=0 \\ \frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b\sqrt[4]{-\frac{a}{b}}} - \frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b\sqrt[4]{-\frac{a}{b}}} + \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b\sqrt[4]{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(1/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a), Eq(b, 0)),
 (-2/(b*sqrt(x)), Eq(a, 0)), (log(sqrt(x) - (-a/b)**(1/4))/(2*b*(-a/b)**(1/
4)) - log(sqrt(x) + (-a/b)**(1/4))/(2*b*(-a/b)**(1/4)) + atan(sqrt(x)/(-a/b
)**(1/4))/(b*(-a/b)**(1/4)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(81) = 162.

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{4\,a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{4\,a^{\frac{1}{4}}b^{\frac{3}{4}}}$$

```
[In] integrate(x^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 1/2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*
sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*sqrt(2)*log(sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + 1/4*sqrt(2)*log
(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(81) = 162.

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx = \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^3} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^3} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^3} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^3}$$

```
[In] integrate(x^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a*b^3) + 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(a*b^3)^
(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3) + 1/4*sqrt(2
)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 16.94 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx = \frac{\operatorname{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)-\operatorname{atanh}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{(-a)^{1/4}\,b^{3/4}}$$

```
[In] int(x^(1/2)/(a + b*x^2),x)
[Out] (atan((b^(1/4)*x^(1/2))/(-a)^(1/4)) - atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/
((-a)^(1/4)*b^(3/4))
```

3.123 $\int \frac{x^{3/2}}{a+bx^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 112

$$\int \frac{x^{3/2}}{a+bx^2}\,dx=\frac{2\sqrt{x}}{b}-\frac{a\left(\arctan\left(\frac{\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}}{\sqrt{\frac{a}{b}}-x}\right)+\log\left(\frac{\sqrt{\frac{a}{b}}+\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}+x}{\sqrt{a+bx^2}}\right)\right)}{\sqrt{2}\left(\frac{a}{b}\right)^{3/4}b^2}$$

```
[Out] 2*x^(1/2)/b-1/2*a/b^2/(a/b)^(3/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2
)+(a/b)^(1/2))/(b*x^2+a)^(1/2))+arctan((a/b)^(1/4)*2^(1/2)*x^(1/2)/((a/b)^(
1/2)-x)))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.80, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {327, 335, 217, 1179, 642, 1176, 631, 210}

$$\begin{aligned}\int \frac{x^{3/2}}{a+bx^2}\,dx&=\frac{\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}}-\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{5/4}}\\&\quad+\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}}+\frac{2\sqrt{x}}{b}\end{aligned}$$

```
[In] Int[x^(3/2)/(a + b*x^2),x]
```

```
[Out] (2*Sqrt[x])/b + (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*b^(5/4)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(S
```

```
qrt[2]*b^(5/4)) + (a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)) - (a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{x}}{b} - \frac{a\int \frac{1}{\sqrt{x}(a+bx^2)}\,dx}{b} \\
&= \frac{2\sqrt{x}}{b} - \frac{(2a)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b} \\
&= \frac{2\sqrt{x}}{b} - \frac{\sqrt{a}\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b} - \frac{\sqrt{a}\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b} \\
&= \frac{2\sqrt{x}}{b} - \frac{\sqrt{a}\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}} - \frac{\sqrt{a}\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}} \\
&\quad + \frac{\sqrt[4]{a}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}b^{5/4}} + \frac{\sqrt[4]{a}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}b^{5/4}} \\
&= \frac{2\sqrt{x}}{b} + \frac{\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}} - \frac{\sqrt[4]{a}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}} \\
&\quad - \frac{\sqrt[4]{a}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}} + \frac{\sqrt[4]{a}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}} \\
&= \frac{2\sqrt{x}}{b} + \frac{\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}} - \frac{\sqrt[4]{a}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}} \\
&\quad + \frac{\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}} - \frac{\sqrt[4]{a}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05

$$\int \frac{x^{3/2}}{a+bx^2}\,dx=\frac{4\sqrt[4]{b}\sqrt{x}+\sqrt{2}\sqrt[4]{a}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-\sqrt{2}\sqrt[4]{a}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{2b^{5/4}}$$

```
[In] Integrate[x^(3/2)/(a + b*x^2),x]
```

```
[Out] (4*b^(1/4)*Sqrt[x] + Sqrt[2]*a^(1/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])] - Sqrt[2]*a^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(2*b^(5/4))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b}$	115
default	$\frac{2\sqrt{x}}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b}$	115
risch	$\frac{2\sqrt{x}}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b}$	115

```
[In] int(x^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/b-1/4/b*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a
/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+2*arctan(1/(a/b)^(1
/4)*2^(1/2)*x^(1/2)+1)+2*arctan(1/(a/b)^(1/4)*2^(1/2)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int \frac{x^{3/2}}{a+bx^2}\,dx=$$

$$-\frac{b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)+i\,b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\log\left(i\,b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-i\,b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\log\left(-i\,b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-}{2\,b}$$

```
[In] integrate(x^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(b*(-a/b^5)^(1/4)*log(b*(-a/b^5)^(1/4) + sqrt(x)) + I*b*(-a/b^5)^(1/4)
*log(I*b*(-a/b^5)^(1/4) + sqrt(x)) - I*b*(-a/b^5)^(1/4)*log(-I*b*(-a/b^5)^(
1/4) + sqrt(x)) - b*(-a/b^5)^(1/4)*log(-b*(-a/b^5)^(1/4) + sqrt(x)) - 4*sqr
t(x))/b
```

Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98

$$\int \frac{x^{3/2}}{a+bx^2}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a=0 \wedge b \\ \frac{2x^{\frac{5}{2}}}{5a} & \text{for } b=0 \\ \frac{2\sqrt{x}}{b} & \text{for } a=0 \\ \frac{2\sqrt{x}}{b} + \frac{\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b} - \frac{\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b} - \frac{\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(3/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a), Eq(b, 0)),
 (2*sqrt(x)/b, Eq(a, 0)), (2*sqrt(x)/b + (-a/b)**(1/4)*log(sqrt(x) - (-a/b)
**(1/4))/(2*b) - (-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b) - (-a/b)*
*(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(89) = 178$.

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.65

$$\int \frac{x^{3/2}}{a+bx^2}\,dx = -\frac{\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}\sqrt{a}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}} - \frac{\sqrt{2}a^{\frac{1}{4}}\log(}{}}{4b} + \frac{2\sqrt{x}}{b}$$

```
[In] integrate(x^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqr
t(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt
```

```
(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(-sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))/b + 2*sqrt(x)
/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.59

$$\int \frac{x^{3/2}}{a+bx^2}\,dx = -\frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^2} - \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^2} - \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^2} + \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^2} + \frac{2\sqrt{x}}{b}$$

```
[In] integrate(x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/b^2 - 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^2 - 1/4*sqrt(2)*(a*b^3)^(1/4)*l
og(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^2 + 1/4*sqrt(2)*(a*b^3)^(
1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^2 + 2*sqrt(x)/b
```

Mupad [B] (verification not implemented)

Time = 17.53 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.49

$$\int \frac{x^{3/2}}{a+bx^2}\,dx = \frac{2\sqrt{x}}{b} - \frac{(-a)^{1/4}\,\mathrm{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{b^{5/4}} - \frac{(-a)^{1/4}\,\mathrm{atanh}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{b^{5/4}}$$

```
[In] int(x^(3/2)/(a + b*x^2),x)
```

```
[Out] (2*x^(1/2))/b - ((-a)^(1/4)*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(5/4) - (
(-a)^(1/4)*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(5/4)
```

3.124 $\int \frac{x^{5/2}}{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 113

$$\int \frac{x^{5/2}}{a+bx^2}\,dx=\frac{x^{3/2}}{b}-\frac{a\left(\arctan\left(\frac{\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}}{\sqrt{\frac{a}{b}}-x}\right)-\log\left(\frac{\sqrt{\frac{a}{b}}+\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}+x}{\sqrt{a+bx^2}}\right)\right)}{\sqrt{2}\sqrt[4]{\frac{a}{b}}b^2}$$

```
[Out] x^(3/2)/b-1/2*a/b^2/(a/b)^(1/4)*2^(1/2)*(-ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)
+(a/b)^(1/2))/(b*x^2+a)^(1/2))+arctan((a/b)^(1/4)*2^(1/2)*x^(1/2)/((a/b)^(1
/2)-x)))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.81, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {327, 335, 303, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{x^{5/2}}{a+bx^2}\,dx&=\frac{a^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}}\\&-\frac{a^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{7/4}}-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}}\\&+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}}+\frac{2x^{3/2}}{3b}\end{aligned}$$

```
[In] Int[x^(5/2)/(a + b*x^2),x]
```

```
[Out] (2*x^(3/2))/(3*b) + (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*b^(7/4)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
)/(Sqrt[2]*b^(7/4)) - (a^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(2*Sqrt[2]*b^(7/4)) + (a^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^{3/2}}{3b} - \frac{a\int \frac{\sqrt{x}}{a+bx^2}\,dx}{b}\\
&= \frac{2x^{3/2}}{3b} - \frac{(2a)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b}\\
&= \frac{2x^{3/2}}{3b} + \frac{a\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}} - \frac{a\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}}\\
&= \frac{2x^{3/2}}{3b} - \frac{a\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2} - \frac{a\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2}\\
&\quad - \frac{a^{3/4}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}b^{7/4}} - \frac{a^{3/4}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}b^{7/4}}\\
&= \frac{2x^{3/2}}{3b} - \frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}} + \frac{a^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}}\\
&\quad - \frac{a^{3/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}} + \frac{a^{3/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}}
\end{aligned}$$

$$= \frac{2x^{3/2}}{3b} + \frac{a^{3/4}\arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}} - \frac{a^{3/4}\arctan\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}} - \frac{a^{3/4}\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}x\right)}{2\sqrt{2}b^{7/4}} + \frac{a^{3/4}\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}x\right)}{2\sqrt{2}b^{7/4}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

$$\int \frac{x^{5/2}}{a+bx^2}\,dx = \frac{4b^{3/4}x^{3/2} + 3\sqrt{2}a^{3/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 3\sqrt{2}a^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{6b^{7/4}}$$

```
[In] Integrate[x^(5/2)/(a + b*x^2),x]
```

```
[Out] (4*b^(3/4)*x^(3/2) + 3*Sqrt[2]*a^(3/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x])] + 3*Sqrt[2]*a^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(6*b^(7/4))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3b} - \frac{a\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	116
default	$\frac{2x^{\frac{3}{2}}}{3b} - \frac{a\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	116
risch	$\frac{2x^{\frac{3}{2}}}{3b} - \frac{a\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	116

```
[In] int(x^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(3/2)/b-1/4*a/b^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*2^(1/2)*x^(1
/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+2*arctan(1/(a
/b)^(1/4)*2^(1/2)*x^(1/2)+1)+2*arctan(1/(a/b)^(1/4)*2^(1/2)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.40

$$\int \frac{x^{5/2}}{a+bx^2}\,dx = -\frac{3\,b\left(-\frac{a^3}{b^7}\right)^{\frac{1}{4}}\log\left(b^5\left(-\frac{a^3}{b^7}\right)^{\frac{3}{4}}+a^2\sqrt{x}\right)-3i\,b\left(-\frac{a^3}{b^7}\right)^{\frac{1}{4}}\log\left(i\,b^5\left(-\frac{a^3}{b^7}\right)^{\frac{3}{4}}+a^2\sqrt{x}\right)+3i\,b\left(-\frac{a^3}{b^7}\right)^{\frac{1}{4}}\log\left(-i\,b^5\left(-\right.\right.}{6\,b}$$

```
[In] integrate(x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*(-a^3/b^7)^(1/4)*log(b^5*(-a^3/b^7)^(3/4) + a^2*sqrt(x)) - 3*I*b*
(-a^3/b^7)^(1/4)*log(I*b^5*(-a^3/b^7)^(3/4) + a^2*sqrt(x)) + 3*I*b*(-a^3/b^
7)^(1/4)*log(-I*b^5*(-a^3/b^7)^(3/4) + a^2*sqrt(x)) - 3*b*(-a^3/b^7)^(1/4)*
log(-b^5*(-a^3/b^7)^(3/4) + a^2*sqrt(x)) - 4*x^(3/2))/b
```

Sympy [A] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10

$$\int \frac{x^{5/2}}{a+bx^2}\,dx = \begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } a=0 \wedge b=0 \\ \frac{2x^{\frac{7}{2}}}{7a} & \text{for } b=0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for } a=0 \\ -\frac{a\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b^2\sqrt[4]{-\frac{a}{b}}}+\frac{a\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b^2\sqrt[4]{-\frac{a}{b}}}-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b^2\sqrt[4]{-\frac{a}{b}}}+\frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(5/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a), Eq(b, 0))
, (2*x**(3/2)/(3*b), Eq(a, 0)), (-a*log(sqrt(x) - (-a/b)**(1/4))/(2*b**2*(-
a/b)**(1/4)) + a*log(sqrt(x) + (-a/b)**(1/4))/(2*b**2*(-a/b)**(1/4)) - a*at
an(sqrt(x)/(-a/b)**(1/4))/(b**2*(-a/b)**(1/4)) + 2*x**(3/2)/(3*b), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(90) = 180.

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.65

$$\int \frac{x^{5/2}}{a+bx^2}\,dx = -\frac{a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a\right.}{}\right)}{4\,b} + \frac{2\,x^{\frac{3}{2}}}{3\,b}$$

[In] integrate(x^(5/2)/(b*x^2+a),x, algorithm="maxima")

[Out] -1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt
(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b +
 2/3*x^(3/2)/b

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.58

$$\int \frac{x^{5/2}}{a+bx^2}\,dx = \frac{2\,x^{\frac{3}{2}}}{3\,b} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^4} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^4} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^4} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^4}$$

[In] integrate(x^(5/2)/(b*x^2+a),x, algorithm="giac")

```
[Out] 2/3*x^(3/2)/b - 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)
^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^4 - 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^4 + 1/4*sqrt(2)*
(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4 - 1/4*sq
rt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.48

$$\int \frac{x^{5/2}}{a+bx^2}\,dx = \frac{2\,x^{3/2}}{3\,b} + \frac{{\left(-a\right)}^{3/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{b^{7/4}} - \frac{{\left(-a\right)}^{3/4}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{b^{7/4}}$$

```
[In] int(x^(5/2)/(a + b*x^2),x)
```

```
[Out] (2*x^(3/2))/(3*b) + ((-a)^(3/4)*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(7/4)
 - ((-a)^(3/4)*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(7/4)
```

3.125 $\int \frac{1}{\sqrt{x}(a+bx^2)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 127

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^2}\,dx=\frac{\sqrt{x}}{2a\,(a+bx^2)}+\frac{3\left(\arctan\left(\frac{\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}}{\sqrt{\frac{a}{b}}-x}\right)+\log\left(\frac{\sqrt{\frac{a}{b}}+\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}+x}{\sqrt{a+bx^2}}\right)\right)}{4\sqrt{2}a\left(\frac{a}{b}\right)^{3/4}b}$$

```
[Out] 1/2*x^(1/2)/a/(b*x^2+a)+3/8/a/b/(a/b)^(3/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*2^(1
/2)*x^(1/2)+(a/b)^(1/2))/(b*x^2+a)^(1/2))+arctan((a/b)^(1/4)*2^(1/2)*x^(1/2
)/((a/b)^(1/2)-x)))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.72, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {296, 335, 217, 1179, 642, 1176, 631, 210}

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(a+bx^2)^2}\,dx=&-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}}\\&-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}\\&+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{\sqrt{x}}{2a\,(a+bx^2)}\end{aligned}$$

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] Sqrt[x]/(2*a*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(4*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{x}}{2a(a+bx^2)} + \frac{3\int \frac{1}{\sqrt{x}(a+bx^2)}\,dx}{4a} \\
&= \frac{\sqrt{x}}{2a(a+bx^2)} + \frac{3\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2a} \\
&= \frac{\sqrt{x}}{2a(a+bx^2)} + \frac{3\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a^{3/2}} + \frac{3\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a^{3/2}} \\
&= \frac{\sqrt{x}}{2a(a+bx^2)} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8a^{3/2}\sqrt{b}} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8a^{3/2}\sqrt{b}} - \frac{3\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} \\
&\quad - \frac{3\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} \\
&= \frac{\sqrt{x}}{2a(a+bx^2)} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}} - \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}}
\end{aligned}$$

$$= \frac{\sqrt{x}}{2a(a+bx^2)} - \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.01

$$\int \frac{1}{\sqrt{x}(a+bx^2)^2}\,dx = \frac{\frac{4a^{3/4}\sqrt{x}}{a+bx^2} - \frac{3\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{b}} + \frac{3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt[4]{b}}}{8a^{7/4}}$$

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] ((4*a^(3/4)*Sqrt[x])/(a + b*x^2) - (3*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/
(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(1/4) + (3*Sqrt[2]*ArcTanh[(Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(1/4))/(8*a^(7/4))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{\sqrt{x}}{2a(x^2b+a)} + \frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16a^2}$	124
default	$\frac{\sqrt{x}}{2a(x^2b+a)} + \frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16a^2}$	124

```
[In] int(1/x^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(1/2)/a/(b*x^2+a)+3/16/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*2^(
1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+2*ar
ctan(1/(a/b)^(1/4)*2^(1/2)*x^(1/2)+1)+2*arctan(1/(a/b)^(1/4)*2^(1/2)*x^(1/2
)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.52

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2}\,dx$$
$$= \frac{3\left(abx^2+a^2\right)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}+\sqrt{x}\right)-3\left(-i\,abx^2-i\,a^2\right)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}\log\left(i\,a^2\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}+\sqrt{x}\right)-3\,(i}{8\,(abx^2}$$

```
[In] integrate(1/x^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(a*b*x^2 + a^2)*(-1/(a^7*b))^(1/4)*log(a^2*(-1/(a^7*b))^(1/4) + sqrt
(x)) - 3*(-I*a*b*x^2 - I*a^2)*(-1/(a^7*b))^(1/4)*log(I*a^2*(-1/(a^7*b))^(1/
4) + sqrt(x)) - 3*(I*a*b*x^2 + I*a^2)*(-1/(a^7*b))^(1/4)*log(-I*a^2*(-1/(a^
7*b))^(1/4) + sqrt(x)) - 3*(a*b*x^2 + a^2)*(-1/(a^7*b))^(1/4)*log(-a^2*(-1/
(a^7*b))^(1/4) + sqrt(x)) + 4*sqrt(x))/(a*b*x^2 + a^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(97) = 194$.

Time = 22.76 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.49

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} \\ \frac{2\sqrt{x}}{a^2} \\ -\frac{2}{7b^2x^{\frac{7}{2}}} \\ \frac{4a\sqrt{x}}{8a^3+8a^2bx^2}-\frac{3a\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^3+8a^2bx^2}+\frac{3a\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^3+8a^2bx^2}+\frac{6a\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^3+8a^2bx^2}-\frac{3bx^2\sqrt[4]{-\frac{a}{b}}\log\left(\right.}{8a^3+8a^2} \end{cases}$$

```
[In] integrate(1/x**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**2, Eq(b, 0)),
(-2/(7*b**2*x**(7/2)), Eq(a, 0)), (4*a*sqrt(x)/(8*a**3 + 8*a**2*b*x**2) - 3
*a*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**3 + 8*a**2*b*x**2) + 3*
a*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**3 + 8*a**2*b*x**2) + 6*a
*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**3 + 8*a**2*b*x**2) - 3*b*x
**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**3 + 8*a**2*b*x**2) + 3
*b*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**3 + 8*a**2*b*x**2)
 + 6*b*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**3 + 8*a**2*b*x*
*2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^2}\,dx = \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(-\sqrt{2}a \cdots\right.}{\cdots}\right.}{16\,a} + \frac{\sqrt{x}}{2\,(abx^2+a^2)}$$

```
[In] integrate(1/x^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqr
t(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt
(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a + 1
/2*sqrt(x)/(a*b*x^2 + a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.57

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(a+bx^2)^2}\,dx = {} & \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b} \\ & + \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b} \\ & + \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b} \\ & - \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b} + \frac{\sqrt{x}}{2\,(bx^2+a)a}\end{aligned}$$

```
[In] integrate(1/x^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a^2*b) + 3/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b) + 3/16*sqrt(2)*(a*b^3)
^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b) - 3/16*sqrt
(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b)
 + 1/2*sqrt(x)/((b*x^2 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^2}\,dx = \frac{\sqrt{x}}{2\,a\,\left(b\,x^2+a\right)} + \frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{7/4}\,b^{1/4}} + \frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{7/4}\,b^{1/4}}$$

```
[In] int(1/(x^(1/2)*(a + b*x^2)^2),x)
```

```
[Out] x^(1/2)/(2*a*(a + b*x^2)) + (3*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^
(7/4)*b^(1/4)) + (3*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^(7/4)*b^(1
/4))
```

3.126 $\int \frac{\sqrt{x}}{(a+bx^2)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 129

$$\int \frac{\sqrt{x}}{(a+bx^2)^2} \, dx = \frac{x^{3/2}}{2a(a+bx^2)} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}}{\sqrt{\frac{a}{b}}-x}\right) - \log\left(\frac{\sqrt{\frac{a}{b}}+\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}+x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}a\sqrt[4]{\frac{a}{b}}b}$$

```
[Out] 1/2*x^(3/2)/a/(b*x^2+a)+1/8/a/b/(a/b)^(1/4)*2^(1/2)*(-ln((x+(a/b)^(1/4)*2^(
1/2)*x^(1/2)+(a/b)^(1/2))/(b*x^2+a)^(1/2))+arctan((a/b)^(1/4)*2^(1/2)*x^(1/
2)/((a/b)^(1/2)-x)))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.69, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {296, 335, 303, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{\sqrt{x}}{(a+bx^2)^2} \, dx &= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{5/4}b^{3/4}} \\ &\quad + \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{3/4}} \\ &\quad - \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{3/4}} + \frac{x^{3/2}}{2a(a+bx^2)}\end{aligned}$$

```
[In] Int[Sqrt[x]/(a + b*x^2)^2,x]
```

```
[Out] x^(3/2)/(2*a*(a + b*x^2)) - ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(
4*Sqrt[2]*a^(5/4)*b^(3/4)) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/
(4*Sqrt[2]*a^(5/4)*b^(3/4)) + Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x]/(8*Sqrt[2]*a^(5/4)*b^(3/4)) - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x]/(8*Sqrt[2]*a^(5/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^{3/2}}{2a\left(a+bx^2\right)}+\frac{\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a}\\
&= \frac{x^{3/2}}{2a\left(a+bx^2\right)}+\frac{\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2a}\\
&= \frac{x^{3/2}}{2a\left(a+bx^2\right)}-\frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4a\sqrt{b}}+\frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4a\sqrt{b}}\\
&= \frac{x^{3/2}}{2a\left(a+bx^2\right)}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8ab}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8ab}\\
&\quad+\frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{2}a^{5/4}b^{3/4}}+\frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{2}a^{5/4}b^{3/4}}\\
&= \frac{x^{3/2}}{2a\left(a+bx^2\right)}+\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{3/4}}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}}
\end{aligned}$$

$$= \frac{x^{3/2}}{2a(a+bx^2)} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}} + \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{3/4}} - \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{3/4}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx = \frac{\frac{4\sqrt[4]{a}x^{3/2}}{a+bx^2} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{3/4}} - \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{b^{3/4}}}{8a^{5/4}}$$

```
[In] Integrate[Sqrt[x]/(a + b*x^2)^2,x]
```

```
[Out] ((4*a^(1/4)*x^(3/2))/(a + b*x^2) - (Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(3/4) - (Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(3/4))/(8*a^(5/4))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{x^{\frac{3}{2}}}{2a(x^2b+a)}+\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	127
default	$\frac{x^{\frac{3}{2}}}{2a(x^2b+a)}+\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	127

```
[In] int(x^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(3/2)/a/(b*x^2+a)+1/16/a/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*2^(
1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+2*ar
ctan(1/(a/b)^(1/4)*2^(1/2)*x^(1/2)+1)+2*arctan(1/(a/b)^(1/4)*2^(1/2)*x^(1/2
)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx$$
$$= \frac{(abx^2+a^2)\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}\log\left(a^4b^2\left(-\frac{1}{a^5b^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-(i\,abx^2+i\,a^2)\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}\log\left(i\,a^4b^2\left(-\frac{1}{a^5b^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-(-}{8\,(ab}$$

```
[In] integrate(x^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((a*b*x^2 + a^2)*(-1/(a^5*b^3))^(1/4)*log(a^4*b^2*(-1/(a^5*b^3))^(3/4)
+ sqrt(x)) - (I*a*b*x^2 + I*a^2)*(-1/(a^5*b^3))^(1/4)*log(I*a^4*b^2*(-1/(a^
5*b^3))^(3/4) + sqrt(x)) - (-I*a*b*x^2 - I*a^2)*(-1/(a^5*b^3))^(1/4)*log(-I
*a^4*b^2*(-1/(a^5*b^3))^(3/4) + sqrt(x)) - (a*b*x^2 + a^2)*(-1/(a^5*b^3))^(
1/4)*log(-a^4*b^2*(-1/(a^5*b^3))^(3/4) + sqrt(x)) + 4*x^(3/2))/(a*b*x^2 + a
^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(95) = 190.

Time = 19.03 (sec) , antiderivative size = 400, normalized size of antiderivative = 3.10

$$\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} \\ \frac{2x^{\frac{3}{2}}}{3a^2} \\ -\frac{2}{5b^2x^{\frac{5}{2}}} \\ \frac{a\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b\sqrt[4]{-\frac{a}{b}}+8ab^2x^2\sqrt[4]{-\frac{a}{b}}}-\frac{a\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b\sqrt[4]{-\frac{a}{b}}+8ab^2x^2\sqrt[4]{-\frac{a}{b}}}+\frac{2a\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^2b\sqrt[4]{-\frac{a}{b}}+8ab^2x^2\sqrt[4]{-\frac{a}{b}}}+\frac{4bx^{\frac{3}{2}}\sqrt[4]{-\frac{a}{b}}}{8a^2b\sqrt[4]{-\frac{a}{b}}+8ab^2x^2\sqrt[4]{-\frac{a}{b}}}+\frac{bx^{}}{8a^2b} \end{cases}$$

```
[In] integrate(x**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**2), Eq(b,
0)), (-2/(5*b**2*x**(5/2)), Eq(a, 0)), (a*log(sqrt(x) - (-a/b)**(1/4))/(8*a
**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)) - a*log(sqrt(x) + (-a/b)
**(1/4))/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)) + 2*a*atan(
sqrt(x)/(-a/b)**(1/4))/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4
)) + 4*b*x**(3/2)*(-a/b)**(1/4)/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a
```

```
/b)**(1/4)) + b*x**2*log(sqrt(x) - (-a/b)**(1/4))/(8*a**2*b*(-a/b)**(1/4) +
 8*a*b**2*x**2*(-a/b)**(1/4)) - b*x**2*log(sqrt(x) + (-a/b)**(1/4))/(8*a**2
*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)) + 2*b*x**2*atan(sqrt(x)/(-a
/b)**(1/4))/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx = \frac{x^{\frac{3}{2}}}{2\,(abx^2+a^2)} + \frac{\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{2}a \cdots\right)}{a \cdots}}{16\,a}$$

```
[In] integrate(x^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^(3/2)/(a*b*x^2 + a^2) + 1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a
^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sq
rt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) -
2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) -
 sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4
)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqr
t(a))/(a^(1/4)*b^(3/4)))/a
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.54

$$\begin{aligned}\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx = {} & \frac{x^{\frac{3}{2}}}{2\,(bx^2+a)a} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^3} \\ & + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^3} \\ & - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b^3} \\ & + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b^3}\end{aligned}$$

```
[In] integrate(x^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^(3/2)/((b*x^2 + a)*a) + 1/8*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) + 1/8*sqrt(2)*(a*b
^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4)
)/(a^2*b^3) - 1/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) +
x + sqrt(a/b))/(a^2*b^3) + 1/16*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 16.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx = \frac{x^{3/2}}{2\,a\,\left(b\,x^2+a\right)} - \frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{5/4}\,b^{3/4}} + \frac{\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{5/4}\,b^{3/4}}$$

```
[In] int(x^(1/2)/(a + b*x^2)^2,x)
```

```
[Out] x^(3/2)/(2*a*(a + b*x^2)) - atan((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(5/4
)*b^(3/4)) + atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(5/4)*b^(3/4))
```

3.127 $\int \frac{x^{3/2}}{\left(a+bx^2\right)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 124

$$\int \frac{x^{3/2}}{\left(a+bx^2\right)^2} \, dx = -\frac{\sqrt{x}}{2b\left(a+bx^2\right)} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}}{\sqrt{\frac{a}{b}}-x}\right) + \log\left(\frac{\sqrt{\frac{a}{b}}+\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}+x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}\left(\frac{a}{b}\right)^{3/4}b^2}$$

```
[Out] -1/2*x^(1/2)/b/(b*x^2+a)+1/8/b^2/(a/b)^(3/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*2^(
1/2)*x^(1/2)+(a/b)^(1/2))/(b*x^2+a)^(1/2))+arctan((a/b)^(1/4)*2^(1/2)*x^(1/
2)/((a/b)^(1/2)-x)))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.76, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {294, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^{3/2}}{\left(a+bx^2\right)^2} \, dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}b^{5/4}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}} - \frac{\sqrt{x}}{2b\left(a+bx^2\right)}$$

```
[In] Int[x^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] -1/2*Sqrt[x]/(b*(a + b*x^2)) - ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
]/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
```

```
)]/(4*Sqrt[2]*a^(3/4)*b^(5/4)) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x]/(8*Sqrt[2]*a^(3/4)*b^(5/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(8*Sqrt[2]*a^(3/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{x}}{2b(a+bx^2)} + \frac{\int \frac{1}{\sqrt{x}(a+bx^2)}\,dx}{4b} \\
&= -\frac{\sqrt{x}}{2b(a+bx^2)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b} \\
&= -\frac{\sqrt{x}}{2b(a+bx^2)} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4\sqrt{a}b} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4\sqrt{a}b} \\
&= -\frac{\sqrt{x}}{2b(a+bx^2)} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{a}b^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{a}b^{3/2}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{2}a^{3/4}b^{5/4}} - \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{2}a^{3/4}b^{5/4}} \\
&= -\frac{\sqrt{x}}{2b(a+bx^2)} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}} \\
&= -\frac{\sqrt{x}}{2b(a+bx^2)} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}} \\
&\quad - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx = \frac{-\frac{4\sqrt[4]{b}\sqrt{x}}{a+bx^2} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{3/4}} + \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{3/4}}}{8b^{5/4}}$$

```
[In] Integrate[x^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] ((-4*b^(1/4)*Sqrt[x])/(a + b*x^2) - (Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(3/4) + (Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(3/4))/(8*b^(5/4))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$-\frac{\sqrt{x}}{2b(x^2b+a)}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16ba}$	127
default	$-\frac{\sqrt{x}}{2b(x^2b+a)}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16ba}$	127

```
[In] int(x^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^(1/2)/b/(b*x^2+a)+1/16/b*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*2^
(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+2*a
rctan(1/(a/b)^(1/4)*2^(1/2)*x^(1/2)+1)+2*arctan(1/(a/b)^(1/4)*2^(1/2)*x^(1/
2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.54

$$\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx = \frac{(b^2x^2+ab)\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-(-i\,b^2x^2-i\,ab)\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}\log\left(i\,ab\left(-\frac{1}{a^3b^5}\right)\right.}{}$$

```
[In] integrate(x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((b^2*x^2 + a*b)*(-1/(a^3*b^5))^(1/4)*log(a*b*(-1/(a^3*b^5))^(1/4) + sq
rt(x)) - (-I*b^2*x^2 - I*a*b)*(-1/(a^3*b^5))^(1/4)*log(I*a*b*(-1/(a^3*b^5))
^(1/4) + sqrt(x)) - (I*b^2*x^2 + I*a*b)*(-1/(a^3*b^5))^(1/4)*log(-I*a*b*(-1
/(a^3*b^5))^(1/4) + sqrt(x)) - (b^2*x^2 + a*b)*(-1/(a^3*b^5))^(1/4)*log(-a*
b*(-1/(a^3*b^5))^(1/4) + sqrt(x)) - 4*sqrt(x))/(b^2*x^2 + a*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(95) = 190$.

Time = 34.06 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.60

$$\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} \\ \frac{2x^{\frac{5}{2}}}{5a^2} \\ -\frac{2}{3b^2x^{\frac{3}{2}}} \\ -\frac{4a\sqrt{x}}{8a^2b+8ab^2x^2} - \frac{a\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b+8ab^2x^2} + \frac{a\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b+8ab^2x^2} + \frac{2a\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^2b+8ab^2x^2} \end{cases}$$

```
[In] integrate(x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**2), Eq(b,
0)), (-2/(3*b**2*x**(3/2)), Eq(a, 0)), (-4*a*sqrt(x)/(8*a**2*b + 8*a*b**2*x
**2) - a*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**2*b + 8*a*b**2*x*
*2) + a*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**2*b + 8*a*b**2*x**
2) + 2*a*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**2*b + 8*a*b**2*x**
2) - b*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**2*b + 8*a*b**2
*x**2) + b*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**2*b + 8*a*
b**2*x**2) + 2*b*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**2*b +
 8*a*b**2*x**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(97) = 194$.

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.57

$$\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx = \frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,b} - \frac{\sqrt{x}}{2\,(b^2x^2+ab)}$$

```
[In] integrate(x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqr
t(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt
(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b - 1
/2*sqrt(x)/(b^2*x^2 + a*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(97) = 194$.

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.60

$$\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx = \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^2} + \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^2} + \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^2} - \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^2} - \frac{\sqrt{x}}{2\,(bx^2+a)b}$$

```
[In] integrate(x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a*b^2) + 1/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^2) + 1/16*sqrt(2)*(a*b^3)
^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^2) - 1/16*sqrt
(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^2)
 - 1/2*sqrt(x)/((b*x^2 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 16.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.52

$$\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx = -\frac{\sqrt{x}}{2\,b\,\left(b\,x^2+a\right)} - \frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{3/4}\,b^{5/4}} - \frac{\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{3/4}\,b^{5/4}}$$

```
[In] int(x^(3/2)/(a + b*x^2)^2,x)
```

```
[Out] - x^(1/2)/(2*b*(a + b*x^2)) - atan((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(3
/4)*b^(5/4)) - atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(3/4)*b^(5/4))
```

3.128 $\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx = -\frac{x^{3/2}}{2b(a+bx^2)}+\frac{3\left(\arctan\left(\frac{\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}}{\sqrt{\frac{a}{b}}-x}\right)-\log\left(\frac{\sqrt{\frac{a}{b}}+\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}+x}{\sqrt{a+bx^2}}\right)\right)}{4\sqrt{2}\sqrt[4]{\frac{a}{b}}b^2}$$

```
[Out] -1/2*x^(3/2)/b/(b*x^2+a)+3/8/b^2/(a/b)^(1/4)*2^(1/2)*(-ln((x+(a/b)^(1/4)*2^
(1/2)*x^(1/2)+(a/b)^(1/2))/(b*x^2+a)^(1/2))+arctan((a/b)^(1/4)*2^(1/2)*x^(1
/2)/((a/b)^(1/2)-x)))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.73, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {294, 335, 303, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx &= -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}} \\ &+\frac{3\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}} \\ &-\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{x^{3/2}}{2b(a+bx^2)}\end{aligned}$$

```
[In] Int[x^(5/2)/(a + b*x^2)^2,x]
```

```
[Out] -1/2*x^(3/2)/(b*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(7/4)) - (3*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^{3/2}}{2b\left(a+bx^2\right)}+\frac{3\int\frac{\sqrt{x}}{a+bx^2}\,dx}{4b}\\
&= -\frac{x^{3/2}}{2b\left(a+bx^2\right)}+\frac{3\text{Subst}\left(\int\frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2b}\\
&= -\frac{x^{3/2}}{2b\left(a+bx^2\right)}-\frac{3\text{Subst}\left(\int\frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4b^{3/2}}+\frac{3\text{Subst}\left(\int\frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4b^{3/2}}\\
&= -\frac{x^{3/2}}{2b\left(a+bx^2\right)}+\frac{3\text{Subst}\left(\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^2}\\
&\quad+\frac{3\text{Subst}\left(\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^2}+\frac{3\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}\\
&\quad+\frac{3\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}
\end{aligned}$$

$$= -\frac{x^{3/2}}{2b(a+bx^2)} + \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}} - \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}} + \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}} - \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}$$

$$= -\frac{x^{3/2}}{2b(a+bx^2)} - \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}} + \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}} - \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02

$$\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx = \frac{-\frac{4b^{3/4}x^{3/2}}{a+bx^2} - \frac{3\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{a}} - \frac{3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt[4]{a}}}{8b^{7/4}}$$

```
[In] Integrate[x^(5/2)/(a + b*x^2)^2,x]
```

```
[Out] ((-4*b^(3/4)*x^(3/2))/(a + b*x^2) - (3*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)
/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(1/4) - (3*Sqrt[2]*ArcTanh[(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(1/4))/(8*b^(7/4))
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$-\frac{x^{\frac{3}{2}}}{2b(x^2b+a)}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	124
default	$-\frac{x^{\frac{3}{2}}}{2b(x^2b+a)}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	124

```
[In] int(x^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^(3/2)/b/(b*x^2+a)+3/16/b^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*2^
(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+2*a
rctan(1/(a/b)^(1/4)*2^(1/2)*x^(1/2)+1)+2*arctan(1/(a/b)^(1/4)*2^(1/2)*x^(1/
2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.59

$$\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx = \frac{3\,(b^2x^2+ab)\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}\log\left(ab^5\left(-\frac{1}{ab^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)-3\,(i\,b^2x^2+i\,ab)\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}\log\left(i\,ab^5\left(-\frac{1}{ab^7}\right.\right.}{}$$

```
[In] integrate(x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(b^2*x^2 + a*b)*(-1/(a*b^7))^(1/4)*log(a*b^5*(-1/(a*b^7))^(3/4) + sq
rt(x)) - 3*(I*b^2*x^2 + I*a*b)*(-1/(a*b^7))^(1/4)*log(I*a*b^5*(-1/(a*b^7))^
(3/4) + sqrt(x)) - 3*(-I*b^2*x^2 - I*a*b)*(-1/(a*b^7))^(1/4)*log(-I*a*b^5*(
-1/(a*b^7))^(3/4) + sqrt(x)) - 3*(b^2*x^2 + a*b)*(-1/(a*b^7))^(1/4)*log(-a*
b^5*(-1/(a*b^7))^(3/4) + sqrt(x)) - 4*x^(3/2))/(b^2*x^2 + a*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(97) = 194.

Time = 51.81 (sec) , antiderivative size = 393, normalized size of antiderivative = 3.12

$$\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} \\ \frac{2x^{\frac{7}{2}}}{7a^2} \\ -\frac{2}{b^2\sqrt{x}} \\ \frac{3a\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8ab^2\sqrt[4]{-\frac{a}{b}}+8b^3x^2\sqrt[4]{-\frac{a}{b}}}-\frac{3a\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8ab^2\sqrt[4]{-\frac{a}{b}}+8b^3x^2\sqrt[4]{-\frac{a}{b}}}+\frac{6a\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8ab^2\sqrt[4]{-\frac{a}{b}}+8b^3x^2\sqrt[4]{-\frac{a}{b}}}-\frac{4bx^{\frac{3}{2}}\sqrt[4]{-}}{8ab^2\sqrt[4]{-\frac{a}{b}}+8b^3}\end{cases}$$

```
[In] integrate(x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a**2), Eq(b, 0
)), (-2/(b**2*sqrt(x)), Eq(a, 0)), (3*a*log(sqrt(x) - (-a/b)**(1/4))/(8*a*b
**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)) - 3*a*log(sqrt(x) + (-a/b)**
(1/4))/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)) + 6*a*atan(sqrt
(x)/(-a/b)**(1/4))/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)) - 4
*b*x**(3/2)*(-a/b)**(1/4)/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/
```

```
4)) + 3*b*x**2*log(sqrt(x) - (-a/b)**(1/4))/(8*a*b**2*(-a/b)**(1/4) + 8*b**
3*x**2*(-a/b)**(1/4)) - 3*b*x**2*log(sqrt(x) + (-a/b)**(1/4))/(8*a*b**2*(-a
/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)) + 6*b*x**2*atan(sqrt(x)/(-a/b)**(1/
4))/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.55

$$\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx = -\frac{x^{\frac{3}{2}}}{2\,(b^2x^2+ab)} + \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}\right.}{a}\right.}{16\,b}$$

```
[In] integrate(x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x^(3/2)/(b^2*x^2 + a*b) + 3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*
a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*s
qrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) -
 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b))
- sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/
4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sq
rt(a))/(a^(1/4)*b^(3/4)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(99) = 198.

Time = 0.26 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.58

$$\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx = -\frac{x^{\frac{3}{2}}}{2\,(bx^2+a)b} + \frac{3\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^4} + \frac{3\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^4} - \frac{3\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^4} + \frac{3\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^4}$$

```
[In] integrate(x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^(3/2)/((b*x^2 + a)*b) + 3/8*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) + 3/8*sqrt(2)*(a*b^
3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))
/(a*b^4) - 3/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
 sqrt(a/b))/(a*b^4) + 3/16*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)
^(1/4) + x + sqrt(a/b))/(a*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.51

$$\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx = \frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{1/4}\,b^{7/4}} - \frac{x^{3/2}}{2\,b\,\left(b\,x^2+a\right)} - \frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{1/4}\,b^{7/4}}$$

```
[In] int(x^(5/2)/(a + b*x^2)^2,x)
```

```
[Out] (3*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^(1/4)*b^(7/4)) - x^(3/2)/(2*
b*(a + b*x^2)) - (3*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^(1/4)*b^(7
/4))
```

3.129 $\int \frac{1}{\sqrt{x}(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 145

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx = \sqrt{x}\left(\frac{1}{4a\,(a+bx^2)^2}+\frac{7}{16a^2\,(a+bx^2)}\right) + \frac{21\left(\arctan\left(\frac{\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}}{\sqrt{\frac{a}{b}}-x}\right)+\log\left(\frac{\sqrt{\frac{a}{b}}+\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}+x}{\sqrt{a+bx^2}}\right)\right)}{32\sqrt{2}a^2\left(\frac{a}{b}\right)^{3/4}b}$$

```
[Out] (1/4/a/(b*x^2+a)^2+7/16/a^2/(b*x^2+a))*x^(1/2)+21/64/a^2/b/(a/b)^(3/4)*2^(1
/2)*(ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(b*x^2+a)^(1/2))+arctan
((a/b)^(1/4)*2^(1/2)*x^(1/2)/((a/b)^(1/2)-x)))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.65, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {296, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx = -\frac{21\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{21\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}} - \frac{21\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{7\sqrt{x}}{16a^2\,(a+bx^2)}+\frac{\sqrt{x}}{4a\,(a+bx^2)^2}$$

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] Sqrt[x]/(4*a*(a + b*x^2)^2) + (7*Sqrt[x])/(16*a^2*(a + b*x^2)) - (21*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(1/4)) + (2
1*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(1/
4)) - (21*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*S
qrt[2]*a^(11/4)*b^(1/4)) + (21*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(64*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2}+\frac{7\int \frac{1}{\sqrt{x}(a+bx^2)^2}\,dx}{8a} \\
&= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2}+\frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)}+\frac{21\int \frac{1}{\sqrt{x}(a+bx^2)}\,dx}{32a^2} \\
&= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2}+\frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)}+\frac{21\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{16a^2}
\end{aligned}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2}+\frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)}+\frac{21\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{32a^{5/2}} + \frac{21\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{32a^{5/2}}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2}+\frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)}+\frac{21\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{64a^{5/2}\sqrt{b}} + \frac{21\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{64a^{5/2}\sqrt{b}} - \frac{21\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} - \frac{21\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2}+\frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)}-\frac{21\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{21\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}} - \frac{21\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2}+\frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)}-\frac{21\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{21\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}} - \frac{21\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{21\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx = \frac{\frac{4a^{3/4}\sqrt{x}(11a+7bx^2)}{(a+bx^2)^2} - \frac{21\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{b}} + \frac{21\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt[4]{b}}}{64a^{11/4}}$$

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] ((4*a^(3/4)*Sqrt[x]*(11*a + 7*b*x^2))/(a + b*x^2)^2 - (21*Sqrt[2]*ArcTan[(S
qrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(1/4) + (21*Sqrt[
2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(1/4
))/(64*a^(11/4))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

method	result	size
derivativedivides	$\frac{\sqrt{x}}{4a(x^2b+a)^2}+\frac{\frac{7\sqrt{x}}{16a(x^2b+a)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128a^2}}{a}$	147
default	$\frac{\sqrt{x}}{4a(x^2b+a)^2}+\frac{\frac{7\sqrt{x}}{16a(x^2b+a)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128a^2}}{a}$	147

```
[In] int(1/x^(1/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^(1/2)/a/(b*x^2+a)^2+7/4/a*(1/4*x^(1/2)/a/(b*x^2+a)+3/32/a^2*(a/b)^(1/
4)*2^(1/2)*(ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2
^(1/2)*x^(1/2)+(a/b)^(1/2)))+2*arctan(1/(a/b)^(1/4)*2^(1/2)*x^(1/2)+1)+2*ar
ctan(1/(a/b)^(1/4)*2^(1/2)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.85

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx$$
$$= \frac{21\,(a^2b^2x^4+2\,a^3bx^2+a^4)\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}\log\left(a^3\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}+\sqrt{x}\right)-21\,(-i\,a^2b^2x^4-2i\,a^3bx^2-i\,a^4)\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}\mathrm{lo}}{}$$

```
[In] integrate(1/x^(1/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(21*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^11*b))^(1/4)*log(a^3*(-1/
(a^11*b))^(1/4) + sqrt(x)) - 21*(-I*a^2*b^2*x^4 - 2*I*a^3*b*x^2 - I*a^4)*(-
1/(a^11*b))^(1/4)*log(I*a^3*(-1/(a^11*b))^(1/4) + sqrt(x)) - 21*(I*a^2*b^2*
x^4 + 2*I*a^3*b*x^2 + I*a^4)*(-1/(a^11*b))^(1/4)*log(-I*a^3*(-1/(a^11*b))^(
1/4) + sqrt(x)) - 21*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^11*b))^(1/4)*
log(-a^3*(-1/(a^11*b))^(1/4) + sqrt(x)) + 4*(7*b*x^2 + 11*a)*sqrt(x))/(a^2*
b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. 2(116) = 232.

Time = 141.96 (sec) , antiderivative size = 627, normalized size of antiderivative = 4.32

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}}{x^{\frac{11}{2}}} \\ \frac{2\sqrt{x}}{a^3} \\ -\frac{2}{11b^3x^{\frac{11}{2}}} \\ \frac{44a^2\sqrt{x}}{64a^5+128a^4bx^2+64a^3b^2x^4}-\frac{21a^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{64a^5+128a^4bx^2+64a^3b^2x^4}+\frac{21a^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{64a^5+128a^4bx^2+64a^3b^2x^4}+\frac{42a^2\sqrt[4]{-\frac{a}{b}}\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{64a^5+128a^4bx^2+64a^3b^2x^4} - \end{cases}$$

```
[In] integrate(1/x**(1/2)/(b*x**2+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(11/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**3, Eq(b, 0)),
 (-2/(11*b**3*x**(11/2)), Eq(a, 0)), (44*a**2*sqrt(x)/(64*a**5 + 128*a**4*b
*x**2 + 64*a**3*b**2*x**4) - 21*a**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1
/4))/(64*a**5 + 128*a**4*b*x**2 + 64*a**3*b**2*x**4) + 21*a**2*(-a/b)**(1/4
)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**5 + 128*a**4*b*x**2 + 64*a**3*b**2*x*
```

```
*4) + 42*a**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**5 + 128*a**4
*b*x**2 + 64*a**3*b**2*x**4) + 28*a*b*x**(5/2)/(64*a**5 + 128*a**4*b*x**2 +
 64*a**3*b**2*x**4) - 42*a*b*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4)
)/(64*a**5 + 128*a**4*b*x**2 + 64*a**3*b**2*x**4) + 42*a*b*x**2*(-a/b)**(1/
4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**5 + 128*a**4*b*x**2 + 64*a**3*b**2*x
**4) + 84*a*b*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**5 + 128
*a**4*b*x**2 + 64*a**3*b**2*x**4) - 21*b**2*x**4*(-a/b)**(1/4)*log(sqrt(x)
- (-a/b)**(1/4))/(64*a**5 + 128*a**4*b*x**2 + 64*a**3*b**2*x**4) + 21*b**2*
x**4*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**5 + 128*a**4*b*x**2
+ 64*a**3*b**2*x**4) + 42*b**2*x**4*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4
))/(64*a**5 + 128*a**4*b*x**2 + 64*a**3*b**2*x**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx = \frac{7\,bx^{\frac{5}{2}} + 11\,a\sqrt{x}}{16\,(a^2b^2x^4 + 2\,a^3bx^2 + a^4)} + \frac{21\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(-\right.}{}\right)}{128\,a^2}$$

```
[In] integrate(1/x^(1/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(7*b*x^(5/2) + 11*a*sqrt(x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 21/12
8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1
/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt(2)
*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx = \frac{21\,\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b} + \frac{21\,\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b} + \frac{21\,\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^3b} - \frac{21\,\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^3b} + \frac{7\,bx^{\frac{5}{2}}+11\,a\sqrt{x}}{16\,(bx^2+a)^2a^2}$$

```
[In] integrate(1/x^(1/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 21/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqr
t(x))/(a/b)^(1/4))/(a^3*b) + 21/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) + 21/128*sqrt(2)*(
a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) - 21/
128*sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/(a^3*b) + 1/16*(7*b*x^(5/2) + 11*a*sqrt(x))/((b*x^2 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 17.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx = \frac{\frac{11\,\sqrt{x}}{16\,a}+\frac{7\,b\,x^{5/2}}{16\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} - \frac{21\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{11/4}\,b^{1/4}} - \frac{21\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{11/4}\,b^{1/4}}$$

```
[In] int(1/(x^(1/2)*(a + b*x^2)^3),x)
```

```
[Out] ((11*x^(1/2))/(16*a) + (7*b*x^(5/2))/(16*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)
- (21*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(11/4)*b^(1/4)) - (21*at
anh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(11/4)*b^(1/4))
```

3.130 $\int \frac{\sqrt{x}}{(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 147

$$\int \frac{\sqrt{x}}{(a+bx^2)^3} \, dx = x^{3/2}\left(\frac{1}{4a(a+bx^2)^2}+\frac{5}{16a^2(a+bx^2)}\right) + \frac{5\left(\arctan\left(\frac{\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}}{\sqrt{\frac{a}{b}}-x}\right)-\log\left(\frac{\sqrt{\frac{a}{b}}+\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}+x}{\sqrt{a+bx^2}}\right)\right)}{32\sqrt{2}a^2\sqrt[4]{\frac{a}{b}}b}$$

```
[Out] (1/4/a/(b*x^2+a)^2+5/16/a^2/(b*x^2+a))*x^(3/2)+5/64/a^2/b/(a/b)^(1/4)*2^(1/
2)*(-ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(b*x^2+a)^(1/2))+arctan
((a/b)^(1/4)*2^(1/2)*x^(1/2)/((a/b)^(1/2)-x)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.63, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {296, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx = -\frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}}+\frac{5\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{9/4}b^{3/4}} +\frac{5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}} -\frac{5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}}+\frac{5x^{3/2}}{16a^2(a+bx^2)}+\frac{x^{3/2}}{4a(a+bx^2)^2}$$

```
[In] Int[Sqrt[x]/(a + b*x^2)^3,x]
```

```
[Out] x^(3/2)/(4*a*(a + b*x^2)^2) + (5*x^(3/2))/(16*a^2*(a + b*x^2)) - (5*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(3/4)) + (5*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(3/4))
+ (5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2
]*a^(9/4)*b^(3/4)) - (5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(64*Sqrt[2]*a^(9/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2}+\frac{5\int\frac{\sqrt{x}}{\left(a+bx^2\right)^2}\,dx}{8a}\\
&= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2}+\frac{5x^{3/2}}{16a^2\left(a+bx^2\right)}+\frac{5\int\frac{\sqrt{x}}{a+bx^2}\,dx}{32a^2}\\
&= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2}+\frac{5x^{3/2}}{16a^2\left(a+bx^2\right)}+\frac{5\text{Subst}\left(\int\frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{16a^2}
\end{aligned}$$

$$= \frac{x^{3/2}}{4a(a+bx^2)^2} + \frac{5x^{3/2}}{16a^2(a+bx^2)} - \frac{5\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32a^2\sqrt{b}} + \frac{5\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32a^2\sqrt{b}}$$

$$= \frac{x^{3/2}}{4a(a+bx^2)^2} + \frac{5x^{3/2}}{16a^2(a+bx^2)} + \frac{5\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64a^2b} + \frac{5\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64a^2b} + \frac{5\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{2}a^{9/4}b^{3/4}} + \frac{5\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{2}a^{9/4}b^{3/4}}$$

$$= \frac{x^{3/2}}{4a(a+bx^2)^2} + \frac{5x^{3/2}}{16a^2(a+bx^2)} + \frac{5\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}} - \frac{5\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}} + \frac{5\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}} - \frac{5\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}}$$

$$= \frac{x^{3/2}}{4a(a+bx^2)^2} + \frac{5x^{3/2}}{16a^2(a+bx^2)} - \frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}} + \frac{5\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}} + \frac{5\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}} - \frac{5\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx = \frac{\frac{4\sqrt[4]{a}x^{3/2}(9a+5bx^2)}{(a+bx^2)^2} - \frac{5\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{3/4}} - \frac{5\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{b^{3/4}}}{64a^{9/4}}$$

```
[In] Integrate[Sqrt[x]/(a + b*x^2)^3,x]
```

```
[Out] ((4*a^(1/4)*x^(3/2)*(9*a + 5*b*x^2))/(a + b*x^2)^2 - (5*Sqrt[2]*ArcTan[(Sqr
t[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(3/4) - (5*Sqrt[2]*
ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(3/4))/
(64*a^(9/4))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{x^{\frac{3}{2}}}{4a(x^2b+a)^2}+\frac{\frac{5x^{\frac{3}{2}}}{16a(x^2b+a)}+\frac{5\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{a}$	150
default	$\frac{x^{\frac{3}{2}}}{4a(x^2b+a)^2}+\frac{\frac{5x^{\frac{3}{2}}}{16a(x^2b+a)}+\frac{5\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{a}$	150

```
[In] int(x^(1/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^(3/2)/a/(b*x^2+a)^2+5/4/a*(1/4*x^(3/2)/a/(b*x^2+a)+1/32/a/b/(a/b)^(1/
4)*2^(1/2)*(ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2
^(1/2)*x^(1/2)+(a/b)^(1/2)))+2*arctan(1/(a/b)^(1/4)*2^(1/2)*x^(1/2)+1)+2*ar
ctan(1/(a/b)^(1/4)*2^(1/2)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx = \frac{5\,(a^2b^2x^4+2\,a^3bx^2+a^4)\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}\log\left(a^7b^2\left(-\frac{1}{a^9b^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-5\,(i\,a^2b^2x^4+2i\,a^3bx^2+i\,a^4)\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}\log}{}$$

```
[In] integrate(x^(1/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(5*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^9*b^3))^(1/4)*log(a^7*b^2*
(-1/(a^9*b^3))^(3/4) + sqrt(x)) - 5*(I*a^2*b^2*x^4 + 2*I*a^3*b*x^2 + I*a^4)
*(-1/(a^9*b^3))^(1/4)*log(I*a^7*b^2*(-1/(a^9*b^3))^(3/4) + sqrt(x)) - 5*(-I
*a^2*b^2*x^4 - 2*I*a^3*b*x^2 - I*a^4)*(-1/(a^9*b^3))^(1/4)*log(-I*a^7*b^2*(
-1/(a^9*b^3))^(3/4) + sqrt(x)) - 5*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a
^9*b^3))^(1/4)*log(-a^7*b^2*(-1/(a^9*b^3))^(3/4) + sqrt(x)) + 4*(5*b*x^3 +
9*a*x)*sqrt(x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 887 vs. 2(116) = 232.

Time = 103.85 (sec) , antiderivative size = 887, normalized size of antiderivative = 6.03

$$\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} \\ \frac{2x^{\frac{3}{2}}}{3a^3} \\ -\frac{2}{9b^3x^{\frac{9}{2}}} \\ \frac{5a^2\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{64a^4b\sqrt[4]{-\frac{a}{b}}+128a^3b^2x^2\sqrt[4]{-\frac{a}{b}}+64a^2b^3x^4\sqrt[4]{-\frac{a}{b}}}-\frac{5a^2\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{64a^4b\sqrt[4]{-\frac{a}{b}}+128a^3b^2x^2\sqrt[4]{-\frac{a}{b}}+64a^2b^3x^4\sqrt[4]{-\frac{a}{b}}}+\frac{10a^2\,\mathrm{atan}}{64a^4b\sqrt[4]{-\frac{a}{b}}+128a^3b^2x}\end{cases}$$

```
[In] integrate(x**(1/2)/(b*x**2+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**3), Eq(b,
0)), (-2/(9*b**3*x**(9/2)), Eq(a, 0)), (5*a**2*log(sqrt(x) - (-a/b)**(1/4))
/(64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64*a**2*b**3
*x**4*(-a/b)**(1/4)) - 5*a**2*log(sqrt(x) + (-a/b)**(1/4))/(64*a**4*b*(-a/b
)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64*a**2*b**3*x**4*(-a/b)**(1/
```

```
4)) + 10*a**2*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**4*b*(-a/b)**(1/4) + 128*a*
*3*b**2*x**2*(-a/b)**(1/4) + 64*a**2*b**3*x**4*(-a/b)**(1/4)) + 36*a*b*x**(
3/2)*(-a/b)**(1/4)/(64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1
/4) + 64*a**2*b**3*x**4*(-a/b)**(1/4)) + 10*a*b*x**2*log(sqrt(x) - (-a/b)**
(1/4))/(64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64*a**
2*b**3*x**4*(-a/b)**(1/4)) - 10*a*b*x**2*log(sqrt(x) + (-a/b)**(1/4))/(64*a
**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64*a**2*b**3*x**4*
(-a/b)**(1/4)) + 20*a*b*x**2*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**4*b*(-a/b)*
*(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64*a**2*b**3*x**4*(-a/b)**(1/4)
) + 20*b**2*x**(7/2)*(-a/b)**(1/4)/(64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2
*x**2*(-a/b)**(1/4) + 64*a**2*b**3*x**4*(-a/b)**(1/4)) + 5*b**2*x**4*log(sq
rt(x) - (-a/b)**(1/4))/(64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)
**(1/4) + 64*a**2*b**3*x**4*(-a/b)**(1/4)) - 5*b**2*x**4*log(sqrt(x) + (-a/
b)**(1/4))/(64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64
*a**2*b**3*x**4*(-a/b)**(1/4)) + 10*b**2*x**4*atan(sqrt(x)/(-a/b)**(1/4))/(
64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64*a**2*b**3*x
**4*(-a/b)**(1/4)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx = \frac{5\,bx^{\frac{7}{2}}+9\,ax^{\frac{3}{2}}}{16\,(a^2b^2x^4+2\,a^3bx^2+a^4)} + \frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{\cdots}\right.}{}\cdots\right.}{128\,a^2}$$

```
[In] integrate(x^(1/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(5*b*x^(7/2) + 9*a*x^(3/2))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 5/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqr
t(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4
)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx = \frac{5\,bx^{\frac{7}{2}}+9\,ax^{\frac{3}{2}}}{16\,(bx^2+a)^2a^2} + \frac{5\,\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^3} + \frac{5\,\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^3} - \frac{5\,\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^3b^3} + \frac{5\,\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^3b^3}$$

```
[In] integrate(x^(1/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*(5*b*x^(7/2) + 9*a*x^(3/2))/((b*x^2 + a)^2*a^2) + 5/64*sqrt(2)*(a*b^3)
^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a
^3*b^3) + 5/64*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/
4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^3) - 5/128*sqrt(2)*(a*b^3)^(3/4)*log(sq
rt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3) + 5/128*sqrt(2)*(a*b^3
)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 17.78 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx = \frac{\frac{9\,x^{3/2}}{16\,a}+\frac{5\,b\,x^{7/2}}{16\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} + \frac{5\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{9/4}\,b^{3/4}} - \frac{5\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{9/4}\,b^{3/4}}$$

```
[In] int(x^(1/2)/(a + b*x^2)^3,x)
```

```
[Out] ((9*x^(3/2))/(16*a) + (5*b*x^(7/2))/(16*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) +
 (5*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(9/4)*b^(3/4)) - (5*atanh(
(b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(9/4)*b^(3/4))
```

3.131 $\int \frac{x^{3/2}}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 139

$$\int \frac{x^{3/2}}{(a+bx^2)^3} dx = \frac{\sqrt{x}(-3a+bx^2)}{16ab(a+bx^2)^2} + \frac{3\left(\arctan\left(\frac{\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}}{\sqrt{\frac{a}{b}}-x}\right)+\log\left(\frac{\sqrt{\frac{a}{b}}+\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}+x}{\sqrt{a+bx^2}}\right)\right)}{32\sqrt{2}a\left(\frac{a}{b}\right)^{3/4}b^2}$$

```
[Out] 1/16*(b*x^2-3*a)*x^(1/2)/a/b/(b*x^2+a)^2+3/64/a/b^2/(a/b)^(3/4)*2^(1/2)*(ln
((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(b*x^2+a)^(1/2))+arctan((a/b)^
(1/4)*2^(1/2)*x^(1/2)/((a/b)^(1/2)-x)))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.74, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {294, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$\begin{aligned}\int \frac{x^{3/2}}{(a+bx^2)^3} dx = & -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}} \\ & + \frac{3\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{7/4}b^{5/4}} - \frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}} \\ & + \frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}} + \frac{\sqrt{x}}{16ab(a+bx^2)} - \frac{\sqrt{x}}{4b(a+bx^2)^2}\end{aligned}$$

```
[In] Int[x^(3/2)/(a + b*x^2)^3,x]
```

```
[Out] -1/4*Sqrt[x]/(b*(a + b*x^2)^2) + Sqrt[x]/(16*a*b*(a + b*x^2)) - (3*ArcTan[1
 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(5/4)) + (3*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(5/4)) -
 (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(7/4)*b^(5/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/(64*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}+\frac{\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2}\,dx}{8b}\\
&= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}+\frac{\sqrt{x}}{16ab\left(a+bx^2\right)}+\frac{3\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{32ab}\\
&= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}+\frac{\sqrt{x}}{16ab\left(a+bx^2\right)}+\frac{3\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{16ab}\\
&= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}+\frac{\sqrt{x}}{16ab\left(a+bx^2\right)}+\frac{3\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{32a^{3/2}b}\\
&\quad+\frac{3\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{32a^{3/2}b}
\end{aligned}$$

$$= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2} + \frac{\sqrt{x}}{16ab\left(a+bx^2\right)} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64a^{3/2}b^{3/2}}$$
$$+ \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64a^{3/2}b^{3/2}} - \frac{3\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{2}a^{7/4}b^{5/4}}$$
$$- \frac{3\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{2}a^{7/4}b^{5/4}}$$

$$= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2} + \frac{\sqrt{x}}{16ab\left(a+bx^2\right)} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}}$$
$$+ \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}} + \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}}$$
$$- \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}}$$

$$= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2} + \frac{\sqrt{x}}{16ab\left(a+bx^2\right)} - \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}}$$
$$+ \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}}$$
$$+ \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99

$$\int \frac{x^{3/2}}{(a+bx^2)^3}\,dx = \frac{\frac{4a^{3/4}\sqrt[4]{b}\sqrt{x}(-3a+bx^2)}{(a+bx^2)^2} - 3\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 3\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{64a^{7/4}b^{5/4}}$$

```
[In] Integrate[x^(3/2)/(a + b*x^2)^3,x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*Sqrt[x]*(-3*a + b*x^2))/(a + b*x^2)^2 - 3*Sqrt[2]*ArcTa
n[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 3*Sqrt[2]*ArcT
anh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(64*a^(7/4)*b
^(5/4))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

method	result	size
derivativedivides	$\frac{\frac{x^{\frac{5}{2}}}{16a}-\frac{3\sqrt{x}}{16b}}{(x^2b+a)^2}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128b\,a^2}$	138
default	$\frac{\frac{x^{\frac{5}{2}}}{16a}-\frac{3\sqrt{x}}{16b}}{(x^2b+a)^2}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128b\,a^2}$	138

```
[In] int(x^(3/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32/a*x^(5/2)-3/32*x^(1/2)/b)/(b*x^2+a)^2+3/128/b/a^2*(a/b)^(1/4)*2^(1/
2)*(ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x
^(1/2)+(a/b)^(1/2)))+2*arctan(1/(a/b)^(1/4)*2^(1/2)*x^(1/2)+1)+2*arctan(1/(
a/b)^(1/4)*2^(1/2)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.01

$$\int \frac{x^{3/2}}{(a+bx^2)^3}\,dx = \frac{3\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)\left(-\frac{1}{a^7b^5}\right)^{\frac{1}{4}}\log\left(a^2b\left(-\frac{1}{a^7b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-3\left(-i\,ab^3x^4-2i\,a^2b^2x^2-\right.}{}$$

```
[In] integrate(x^(3/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^7*b^5))^(1/4)*log(a^2*b*
(-1/(a^7*b^5))^(1/4) + sqrt(x)) - 3*(-I*a*b^3*x^4 - 2*I*a^2*b^2*x^2 - I*a^3
*b)*(-1/(a^7*b^5))^(1/4)*log(I*a^2*b*(-1/(a^7*b^5))^(1/4) + sqrt(x)) - 3*(I
*a*b^3*x^4 + 2*I*a^2*b^2*x^2 + I*a^3*b)*(-1/(a^7*b^5))^(1/4)*log(-I*a^2*b*(
-1/(a^7*b^5))^(1/4) + sqrt(x)) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/
(a^7*b^5))^(1/4)*log(-a^2*b*(-1/(a^7*b^5))^(1/4) + sqrt(x)) + 4*(b*x^2 - 3*
a)*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(112) = 224$.

Time = 158.65 (sec) , antiderivative size = 666, normalized size of antiderivative = 4.79

$$\int \frac{x^{3/2}}{(a+bx^2)^3}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} \\ \frac{2x^{\frac{5}{2}}}{5a^3} \\ -\frac{2}{7b^3x^{\frac{7}{2}}} \\ -\frac{12a^2\sqrt{x}}{64a^4b+128a^3b^2x^2+64a^2b^3x^4} - \frac{3a^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{64a^4b+128a^3b^2x^2+64a^2b^3x^4} + \frac{3a^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{64a^4b+128a^3b^2x^2+64a^2b^3x^4} + \frac{6a^2\sqrt[4]{\cdots}}{64a^4b+\cdots} \end{cases}$$

```
[In] integrate(x**(3/2)/(b*x**2+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**3), Eq(b,
0)), (-2/(7*b**3*x**(7/2)), Eq(a, 0)), (-12*a**2*sqrt(x)/(64*a**4*b + 128*a
**3*b**2*x**2 + 64*a**2*b**3*x**4) - 3*a**2*(-a/b)**(1/4)*log(sqrt(x) - (-a
/b)**(1/4))/(64*a**4*b + 128*a**3*b**2*x**2 + 64*a**2*b**3*x**4) + 3*a**2*(
-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**4*b + 128*a**3*b**2*x**2 +
 64*a**2*b**3*x**4) + 6*a**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64*
a**4*b + 128*a**3*b**2*x**2 + 64*a**2*b**3*x**4) + 4*a*b*x**(5/2)/(64*a**4*
b + 128*a**3*b**2*x**2 + 64*a**2*b**3*x**4) - 6*a*b*x**2*(-a/b)**(1/4)*log(
sqrt(x) - (-a/b)**(1/4))/(64*a**4*b + 128*a**3*b**2*x**2 + 64*a**2*b**3*x**
4) + 6*a*b*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**4*b + 128
*a**3*b**2*x**2 + 64*a**2*b**3*x**4) + 12*a*b*x**2*(-a/b)**(1/4)*atan(sqrt(
x)/(-a/b)**(1/4))/(64*a**4*b + 128*a**3*b**2*x**2 + 64*a**2*b**3*x**4) - 3*
b**2*x**4*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(64*a**4*b + 128*a**3*
b**2*x**2 + 64*a**2*b**3*x**4) + 3*b**2*x**4*(-a/b)**(1/4)*log(sqrt(x) + (-
a/b)**(1/4))/(64*a**4*b + 128*a**3*b**2*x**2 + 64*a**2*b**3*x**4) + 6*b**2*
x**4*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**4*b + 128*a**3*b**2*x
**2 + 64*a**2*b**3*x**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.59

$$\int \frac{x^{3/2}}{(a+bx^2)^3}\,dx = \frac{bx^{\frac{5}{2}} - 3\,a\sqrt{x}}{16\,(ab^3x^4 + 2\,a^2b^2x^2 + a^3b)} + \frac{3\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}\right.}{a}\right.}{128\,ab}$$

[In] integrate(x^(3/2)/(b*x^2+a)^3,x, algorithm="maxima")

[Out] 1/16*(b*x^(5/2) - 3*a*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + 3/128*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b)

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.52

$$\int \frac{x^{3/2}}{(a+bx^2)^3}\,dx = \frac{3\,\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^2} + \frac{3\,\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^2} + \frac{3\,\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^2b^2} - \frac{3\,\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^2b^2} + \frac{bx^{\frac{5}{2}} - 3\,a\sqrt{x}}{16\,(bx^2+a)^2ab}$$

[In] integrate(x^(3/2)/(b*x^2+a)^3,x, algorithm="giac")

[Out] 3/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) + 3/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2

```
)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) + 3/128*sqrt(2)*
(a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^2) -
3/128*sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b
))/(a^2*b^2) + 1/16*(b*x^(5/2) - 3*a*sqrt(x))/((b*x^2 + a)^2*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.61

$$\int \frac{x^{3/2}}{(a+bx^2)^3}\,dx = \frac{\frac{x^{5/2}}{16\,a}-\frac{3\,\sqrt{x}}{16\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{7/4}\,b^{5/4}}+\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{7/4}\,b^{5/4}}$$

```
[In] int(x^(3/2)/(a + b*x^2)^3,x)
```

```
[Out] (x^(5/2)/(16*a) - (3*x^(1/2))/(16*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) + (3*atan
((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(7/4)*b^(5/4)) + (3*atanh((b^(1/4)
*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(7/4)*b^(5/4))
```

3.132 $\int \frac{x^{5/2}}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 177

$$\int \frac{x^{5/2}}{(a+bx^2)^3} dx = -\frac{2x^{3/2}}{5b(a+bx^2)^2} + \frac{3a\left(x^{3/2}\left(\frac{1}{4a(a+bx^2)^2}+\frac{5}{16a^2(a+bx^2)}\right)+\frac{5\left(\arctan\left(\frac{\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}}{\sqrt{\frac{a}{b}}-x}\right)-\log\left(\frac{\sqrt{\frac{a}{b}}+\sqrt{2}\sqrt[4]{\frac{a}{b}}\sqrt{x}+x}{\sqrt{a+bx^2}}\right)\right)}{32\sqrt{2}a^2\sqrt[4]{\frac{a}{b}}b}\right)}{5b}$$

```
[Out] -2/5*x^(3/2)/b/(b*x^2+a)^2+3/5*a/b*((1/4/a/(b*x^2+a)^2+5/16/a^2/(b*x^2+a))*
x^(3/2)+5/64/a^2/b/(a/b)^(1/4)*2^(1/2)*(-ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+
(a/b)^(1/2))/(b*x^2+a)^(1/2))+arctan((a/b)^(1/4)*2^(1/2)*x^(1/2)/((a/b)^(1/
2)-x))))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.37, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used

= {294, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^{5/2}}{(a+bx^2)^3}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}} + \frac{3\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{5/4}b^{7/4}} + \frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}} - \frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}} + \frac{3x^{3/2}}{16ab(a+bx^2)} - \frac{x^{3/2}}{4b(a+bx^2)^2}$$

```
[In] Int[x^(5/2)/(a + b*x^2)^3,x]
```

```
[Out] -1/4*x^(3/2)/(b*(a + b*x^2)^2) + (3*x^(3/2))/(16*a*b*(a + b*x^2)) - (3*ArcT
an[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(7/4)) + (
3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(7/4
)) + (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqr
t[2]*a^(5/4)*b^(7/4)) - (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(64*Sqrt[2]*a^(5/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}+\frac{3\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2}\,dx}{8b}\\
&= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}+\frac{3x^{3/2}}{16ab\left(a+bx^2\right)}+\frac{3\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32ab}
\end{aligned}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2} + \frac{3x^{3/2}}{16ab\left(a+bx^2\right)} + \frac{3\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16ab}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2} + \frac{3x^{3/2}}{16ab\left(a+bx^2\right)} - \frac{3\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32ab^{3/2}} + \frac{3\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32ab^{3/2}}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2} + \frac{3x^{3/2}}{16ab\left(a+bx^2\right)} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64ab^2} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64ab^2} + \frac{3\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{2}a^{5/4}b^{7/4}} + \frac{3\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{2}a^{5/4}b^{7/4}}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2} + \frac{3x^{3/2}}{16ab\left(a+bx^2\right)} + \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}} - \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}} + \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}} - \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2} + \frac{3x^{3/2}}{16ab\left(a+bx^2\right)} - \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}} + \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}} - \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.77

$$\int \frac{x^{5/2}}{(a+bx^2)^3}\,dx = \frac{-\frac{4\sqrt[4]{a}b^{3/4}x^{3/2}(a-3bx^2)}{(a+bx^2)^2} - 3\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - 3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{64a^{5/4}b^{7/4}}$$

```
[In] Integrate[x^(5/2)/(a + b*x^2)^3,x]
```

```
[Out] ((-4*a^(1/4)*b^(3/4)*x^(3/2)*(a - 3*b*x^2))/(a + b*x^2)^2 - 3*Sqrt[2]*ArcTa
n[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 3*Sqrt[2]*ArcT
anh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(64*a^(5/4)*b
^(7/4))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{3x^{\frac{7}{2}}}{16a}-\frac{x^{\frac{3}{2}}}{16b}}{(x^2b+a)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128b^2a\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	138
default	$\frac{\frac{3x^{\frac{7}{2}}}{16a}-\frac{x^{\frac{3}{2}}}{16b}}{(x^2b+a)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128b^2a\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	138

```
[In] int(x^(5/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/32/a*x^(7/2)-1/32*x^(3/2)/b)/(b*x^2+a)^2+3/128/b^2/a/(a/b)^(1/4)*2^(1/
2)*(ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x
^(1/2)+(a/b)^(1/2)))+2*arctan(1/(a/b)^(1/4)*2^(1/2)*x^(1/2)+1)+2*arctan(1/(
a/b)^(1/4)*2^(1/2)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.63

$$\int \frac{x^{5/2}}{(a+bx^2)^3}\,dx = \frac{3\,(ab^3x^4+2\,a^2b^2x^2+a^3b)\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}}\log\left(a^4b^5\left(-\frac{1}{a^5b^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)-3\,(i\,ab^3x^4+2i\,a^2b^2x^2+\cdots}{\cdots}$$

```
[In] integrate(x^(5/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^5*b^7))^(1/4)*log(a^4*b^
5*(-1/(a^5*b^7))^(3/4) + sqrt(x)) - 3*(I*a*b^3*x^4 + 2*I*a^2*b^2*x^2 + I*a^
3*b)*(-1/(a^5*b^7))^(1/4)*log(I*a^4*b^5*(-1/(a^5*b^7))^(3/4) + sqrt(x)) - 3
*(-I*a*b^3*x^4 - 2*I*a^2*b^2*x^2 - I*a^3*b)*(-1/(a^5*b^7))^(1/4)*log(-I*a^4
*b^5*(-1/(a^5*b^7))^(3/4) + sqrt(x)) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b
)*(-1/(a^5*b^7))^(1/4)*log(-a^4*b^5*(-1/(a^5*b^7))^(3/4) + sqrt(x)) + 4*(3*
b*x^3 - a*x)*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(a+bx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**(5/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.25

$$\int \frac{x^{5/2}}{(a+bx^2)^3}\,dx = \frac{3\,bx^{\frac{7}{2}}-ax^{\frac{3}{2}}}{16\,(ab^3x^4+2\,a^2b^2x^2+a^3b)} + \frac{3\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a\cdots\right.}{\cdots}\cdots\right)}{128\,ab}$$

```
[In] integrate(x^(5/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*b*x^(7/2) - a*x^(3/2))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + 3/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqr
t(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4
)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.20

$$\int \frac{x^{5/2}}{(a+bx^2)^3}\,dx = \frac{3\,bx^{\frac{7}{2}} - ax^{\frac{3}{2}}}{16\,(bx^2+a)^2ab} + \frac{3\,\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^4} + \frac{3\,\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^4} - \frac{3\,\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^2b^4} + \frac{3\,\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^2b^4}$$

```
[In] integrate(x^(5/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*b*x^(7/2) - a*x^(3/2))/((b*x^2 + a)^2*a*b) + 3/64*sqrt(2)*(a*b^3)^(
3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2
*b^4) + 3/64*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^4) - 3/128*sqrt(2)*(a*b^3)^(3/4)*log(sqrt
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4) + 3/128*sqrt(2)*(a*b^3)^
(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.48

$$\int \frac{x^{5/2}}{(a+bx^2)^3}\,dx = \frac{\frac{3\,x^{7/2}}{16\,a} - \frac{x^{3/2}}{16\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4} - \frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{5/4}\,b^{7/4}} + \frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{5/4}\,b^{7/4}}$$

```
[In] int(x^(5/2)/(a + b*x^2)^3,x)
```

```
[Out] ((3*x^(7/2))/(16*a) - x^(3/2)/(16*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (3*atan
((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(5/4)*b^(7/4)) + (3*atanh((b^(1/4)
*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(5/4)*b^(7/4))
```

3.133 $\int \frac{1}{\sqrt{a+bx}} dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{1}{\sqrt{a+bx}} dx = \frac{2\sqrt{a+bx}}{b}$$

```
[Out] 2/b*(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$\int \frac{1}{\sqrt{a+bx}} dx = \frac{2\sqrt{a+bx}}{b}$$

```
[In] Int[1/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x])/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{a+bx}}{b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}}{b}$$

```
[In] Integrate[1/Sqrt[a + b*x],x]
[Out] (2*Sqrt[a + b*x])/b
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{2\sqrt{bx+a}}{b}$	13
derivativedivides	$\frac{2\sqrt{bx+a}}{b}$	13
default	$\frac{2\sqrt{bx+a}}{b}$	13
trager	$\frac{2\sqrt{bx+a}}{b}$	13
risch	$\frac{2\sqrt{bx+a}}{b}$	13
pseudoelliptic	$\frac{2\sqrt{bx+a}}{b}$	13

```
[In] int(1/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/b*(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\,\sqrt{bx+a}}{b}$$

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}}{b}$$

```
[In] integrate(1/(b*x+a)**(1/2),x)
[Out] 2*sqrt(a + b*x)/b
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}}{b}$$

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(b*x + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}}{b}$$

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+b\,x}}{b}$$

```
[In] int(1/(a + b*x)^(1/2),x)
[Out] (2*(a + b*x)^(1/2))/b
```

3.134 $\int \frac{x}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \frac{x}{\sqrt{a+bx}}\,dx=\frac{2\sqrt{a+bx}\left(-a+\frac{1}{3}(a+bx)\right)}{b^2}$$

```
[Out] 2*(1/3*b*x-2/3*a)*(b*x+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\int \frac{x}{\sqrt{a+bx}}\,dx=\frac{2(a+bx)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx}}{b^2}$$

```
[In] Int[x/Sqrt[a + b*x],x]
[Out] (-2*a*Sqrt[a + b*x])/b^2 + (2*(a + b*x)^(3/2))/(3*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)\,dx\\&=-\frac{2a\sqrt{a+bx}}{b^2}+\frac{2(a+bx)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{a+bx}}\,dx=\frac{2(-2a+bx)\sqrt{a+bx}}{3b^2}$$

```
[In] Integrate[x/Sqrt[a + b*x],x]
[Out] (2*(-2*a + b*x)*Sqrt[a + b*x])/(3*b^2)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^2}$	21
trager	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^2}$	21
risch	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^2}$	21
pseudoelliptic	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^2}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-2a\sqrt{bx+a}}{b^2}$	26
default	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-2a\sqrt{bx+a}}{b^2}$	26

```
[In] int(x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(b*x+a)^(1/2)*(-b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x}{\sqrt{a+bx}}\,dx=\frac{2\,\sqrt{bx+a}(bx-2\,a)}{3\,b^2}$$

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b*x + a)*(b*x - 2*a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(22) = 44$.

Time = 0.63 (sec) , antiderivative size = 162, normalized size of antiderivative = 6.00

$$\int \frac{x}{\sqrt{a+bx}}\,dx = -\frac{4a^{\frac{7}{2}}\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x} + \frac{4a^{\frac{7}{2}}}{3a^2b^2+3ab^3x} - \frac{2a^{\frac{5}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x} + \frac{4a^{\frac{5}{2}}bx}{3a^2b^2+3ab^3x} + \frac{2a^{\frac{3}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}$$

```
[In] integrate(x/(b*x+a)**(1/2),x)
[Out] -4*a**(7/2)*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x) + 4*a**(7/2)/(3*a**2
*b**2 + 3*a*b**3*x) - 2*a**(5/2)*b*x*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**
3*x) + 4*a**(5/2)*b*x/(3*a**2*b**2 + 3*a*b**3*x) + 2*a**(3/2)*b**2*x**2*sqr
t(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x}{\sqrt{a+bx}}\,dx = \frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^2} - \frac{2\,\sqrt{bx+a}a}{b^2}$$

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2/3*(b*x + a)^(3/2)/b^2 - 2*sqrt(b*x + a)*a/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{a+bx}}\,dx = \frac{2\left((bx+a)^{\frac{3}{2}} - 3\,\sqrt{bx+a}a\right)}{3\,b^2}$$

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="giac")
[Out] 2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 18.84 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x}{\sqrt{a+bx}}\,dx = -\frac{6\,a\,\sqrt{a+b\,x}-2\,{\left(a+b\,x\right)}^{3/2}}{3\,b^2}$$

```
[In] int(x/(a + b*x)^(1/2),x)
[Out] -(6*a*(a + b*x)^(1/2) - 2*(a + b*x)^(3/2))/(3*b^2)
```

3.135 $\int \frac{x^2}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}\left(a^2-\frac{2}{3}a(a+bx)+\frac{1}{5}(a+bx)^2\right)}{b^3}$$

```
[Out] 2*(1/5*(b*x+a)^2-2/3*a*(b*x+a)+a^2)*(b*x+a)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{2a^2\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}$$

```
[In] Int[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*Sqrt[a + b*x])/b^3 - (4*a*(a + b*x)^(3/2))/(3*b^3) + (2*(a + b*x)^(5
/2))/(5*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^2}{b^2\sqrt{a+bx}} - \frac{2a\sqrt{a+bx}}{b^2} + \frac{(a+bx)^{3/2}}{b^2} \right) dx$$
$$= \frac{2a^2\sqrt{a+bx}}{b^3} - \frac{4a(a+bx)^{3/2}}{3b^3} + \frac{2(a+bx)^{5/2}}{5b^3}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}(8a^2-4abx+3b^2x^2)}{15b^3}$$

```
[In] Integrate[x^2/Sqrt[a + b*x],x]
[Out] (2*Sqrt[a + b*x]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{2\sqrt{bx+a}\,(3b^2x^2-4bax+8a^2)}{15b^3}$	32
trager	$\frac{2\sqrt{bx+a}\,(3b^2x^2-4bax+8a^2)}{15b^3}$	32
risch	$\frac{2\sqrt{bx+a}\,(3b^2x^2-4bax+8a^2)}{15b^3}$	32
pseudoelliptic	$\frac{2\sqrt{bx+a}\,(3b^2x^2-4bax+8a^2)}{15b^3}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{4a(bx+a)^{\frac{3}{2}}}{3}+2a^2\sqrt{bx+a}}{b^3}$	37
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{4a(bx+a)^{\frac{3}{2}}}{3}+2a^2\sqrt{bx+a}}{b^3}$	37

```
[In] int(x^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(b*x+a)^(1/2)*(3*b^2*x^2-4*a*b*x+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{2\,(3\,b^2x^2-4\,abx+8\,a^2)\sqrt{bx+a}}{15\,b^3}$$

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x + a)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(36) = 72.

Time = 0.96 (sec) , antiderivative size = 600, normalized size of antiderivative = 15.38

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx}}\,dx = &\frac{16a^{\frac{21}{2}}\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} \\ &- \frac{16a^{\frac{21}{2}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} \\ &+ \frac{40a^{\frac{19}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} \\ &- \frac{48a^{\frac{19}{2}}bx}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} \\ &+ \frac{30a^{\frac{17}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} \\ &- \frac{48a^{\frac{17}{2}}b^2x^2}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} \\ &+ \frac{10a^{\frac{15}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} \\ &- \frac{16a^{\frac{15}{2}}b^3x^3}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} \\ &+ \frac{10a^{\frac{13}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} \\ &+ \frac{6a^{\frac{11}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}\end{aligned}$$

```
[In] integrate(x**2/(b*x+a)**(1/2),x)
```

```
[Out] 16*a**(21/2)*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*
x**2 + 15*a**5*b**6*x**3) - 16*a**(21/2)/(15*a**8*b**3 + 45*a**7*b**4*x + 4
5*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 40*a**(19/2)*b*x*sqrt(1 + b*x/a)/(1
5*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*
a**(19/2)*b*x/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*
b**6*x**3) + 30*a**(17/2)*b**2*x**2*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7
*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*a**(17/2)*b**2*x**2/(
15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10
*a**(15/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a*
*6*b**5*x**2 + 15*a**5*b**6*x**3) - 16*a**(15/2)*b**3*x**3/(15*a**8*b**3 +
45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10*a**(13/2)*b**4
*x**4*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 +
15*a**5*b**6*x**3) + 6*a**(11/2)*b**5*x**5*sqrt(1 + b*x/a)/(15*a**8*b**3 +
45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^3} - \frac{4\,(bx+a)^{\frac{3}{2}}a}{3\,b^3} + \frac{2\,\sqrt{bx+a}a^2}{b^3}$$

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^3 - 4/3*(b*x + a)^(3/2)*a/b^3 + 2*sqrt(b*x + a)*a^2/b
^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{2\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}a^2\right)}{15\,b^3}$$

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{6\,(a+b\,x)^{5/2} - 20\,a\,(a+b\,x)^{3/2} + 30\,a^2\,\sqrt{a+b\,x}}{15\,b^3}$$

```
[In] int(x^2/(a + b*x)^(1/2),x)
[Out] (6*(a + b*x)^(5/2) - 20*a*(a + b*x)^(3/2) + 30*a^2*(a + b*x)^(1/2))/(15*b^3
)
```

3.136 $\int \frac{1}{\sqrt{(a+bx)^3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{1}{\sqrt{(a+bx)^3}}\,dx = -\frac{2}{b\sqrt{a+bx}}$$

```
[Out] -2/b/(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {253, 15, 30}

$$\int \frac{1}{\sqrt{(a+bx)^3}}\,dx = -\frac{2(a+bx)}{b\sqrt{(a+bx)^3}}$$

```
[In] Int[1/Sqrt[(a + b*x)^3],x]
```

```
[Out] (-2*(a + b*x))/(b*Sqrt[(a + b*x)^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x^3}}\,dx, x, a+bx\right)}{b}\\ &= \frac{(a+bx)^{3/2}\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, a+bx\right)}{b\sqrt{(a+bx)^3}}\\ &= -\frac{2(a+bx)}{b\sqrt{(a+bx)^3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{(a+bx)^3}}\,dx = -\frac{2(a+bx)}{b\sqrt{(a+bx)^3}}$$

```
[In] Integrate[1/Sqrt[(a + b*x)^3],x]
[Out] (-2*(a + b*x))/(b*Sqrt[(a + b*x)^3])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

method	result	size
gosper	$-\frac{2(bx+a)}{b\sqrt{(bx+a)^3}}$	20
default	$-\frac{2(bx+a)}{b\sqrt{(bx+a)^3}}$	20
trager	$-\frac{2\sqrt{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}}{(bx+a)^2b}$	42

```
[In] int(1/((b*x+a)^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(b*x+a)/b/((b*x+a)^3)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(12) = 24.

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int \frac{1}{\sqrt{(a+bx)^3}}\,dx = -\frac{2\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}}{b^3x^2+2\,ab^2x+a^2b}$$

```
[In] integrate(1/((b*x+a)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)/(b^3*x^2 + 2*a*b^2*x + a^2
*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(12) = 24.

Time = 0.76 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{1}{\sqrt{(a+bx)^3}}\,dx = \begin{cases} -\frac{2\left(\frac{a}{b}+x\right)}{\sqrt{(a+bx)^3}} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{a^3}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/((b*x+a)**3)**(1/2),x)
```

```
[Out] Piecewise((-2*(a/b + x)/sqrt((a + b*x)**3), Ne(b, 0)), (x/sqrt(a**3), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{(a+bx)^3}}\,dx = -\frac{2}{\sqrt{bx+a}b}$$

```
[In] integrate(1/((b*x+a)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*x + a)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{(a+bx)^3}}\,dx = -\frac{2}{\sqrt{bx+a}b}$$

```
[In] integrate(1/((b*x+a)^3)^(1/2),x, algorithm="giac")
[Out] -2/(sqrt(b*x + a)*b)
```

Mupad [B] (verification not implemented)

Time = 18.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{(a+bx)^3}}\,dx = -\frac{2\,\sqrt{{\left(a+b\,x\right)}^3}}{b\,{\left(a+b\,x\right)}^2}$$

```
[In] int(1/((a + b*x)^3)^(1/2),x)
[Out] -(2*((a + b*x)^3)^(1/2))/(b*(a + b*x)^2)
```

3.137 $\int \frac{x}{\sqrt{(a+bx)^3}} dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{x}{\sqrt{(a+bx)^3}}\,dx=\frac{2(2a+bx)}{b^2\sqrt{a+bx}}$$

```
[Out] 2*(b*x+2*a)/b^2/(b*x+a)^(1/2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. $2(21) = 42$.

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.19, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1973, 45}

$$\int \frac{x}{\sqrt{(a+bx)^3}}\,dx=\frac{2(a+bx)^2}{b^2\sqrt{(a+bx)^3}}+\frac{2a(a+bx)}{b^2\sqrt{(a+bx)^3}}$$

```
[In] Int[x/Sqrt[(a + b*x)^3],x]
```

```
[Out] (2*a*(a + b*x))/(b^2*Sqrt[(a + b*x)^3]) + (2*(a + b*x)^2)/(b^2*Sqrt[(a + b*
x)^3])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(1+\frac{bx}{a}\right)^{3/2}\int\frac{x}{\left(1+\frac{bx}{a}\right)^{3/2}}\,dx}{\sqrt{(a+bx)^3}}\\ &= \frac{\left(1+\frac{bx}{a}\right)^{3/2}\int\left(-\frac{a}{b\left(1+\frac{bx}{a}\right)^{3/2}}+\frac{a}{b\sqrt{1+\frac{bx}{a}}}\right)dx}{\sqrt{(a+bx)^3}}\\ &= \frac{2a(a+bx)}{b^2\sqrt{(a+bx)^3}}+\frac{2(a+bx)^2}{b^2\sqrt{(a+bx)^3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int\frac{x}{\sqrt{(a+bx)^3}}\,dx=\frac{2(a+bx)(2a+bx)}{b^2\sqrt{(a+bx)^3}}$$

```
[In] Integrate[x/Sqrt[(a + b*x)^3],x]
[Out] (2*(a + b*x)*(2*a + b*x))/(b^2*Sqrt[(a + b*x)^3])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

method	result	size
gosper	$\frac{2(bx+a)(bx+2a)}{b^2\sqrt{(bx+a)^3}}$	27
default	$\frac{2(bx+a)(bx+2a)}{b^2\sqrt{(bx+a)^3}}$	27
risch	$\frac{2(bx+a)^2}{b^2\sqrt{(bx+a)^3}}+\frac{2a(bx+a)}{b^2\sqrt{(bx+a)^3}}$	43
trager	$\frac{2(bx+2a)\sqrt{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}}{(bx+a)^2b^2}$	49

```
[In] int(x/((b*x+a)^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(b*x+a)*(b*x+2*a)/b^2/((b*x+a)^3)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(19) = 38.

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.95

$$\int \frac{x}{\sqrt{(a+bx)^3}}\,dx = \frac{2\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}(bx+2\,a)}{b^4x^2+2\,ab^3x+a^2b^2}$$

```
[In] integrate(x/((b*x+a)^3)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*(b*x + 2*a)/(b^4*x^2 + 2*a*
b^3*x + a^2*b^2)
```

Sympy [F]

$$\int \frac{x}{\sqrt{(a+bx)^3}}\,dx = \int \frac{x}{\sqrt{(a+bx)^3}}\,dx$$

```
[In] integrate(x/((b*x+a)**3)**(1/2),x)
[Out] Integral(x/sqrt((a + b*x)**3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int \frac{x}{\sqrt{(a+bx)^3}}\,dx = \frac{2\,(b^2x^2+3\,abx+2\,a^2)}{(bx+a)^{\frac{3}{2}}b^2}$$

```
[In] integrate(x/((b*x+a)^3)^(1/2),x, algorithm="maxima")
[Out] 2*(b^2*x^2 + 3*a*b*x + 2*a^2)/((b*x + a)^(3/2)*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{x}{\sqrt{(a+bx)^3}}\,dx = \frac{2\left(\frac{\sqrt{bx+a}}{b} + \frac{a}{\sqrt{bx+ab}}\right)}{b}$$

```
[In] integrate(x/((b*x+a)^3)^(1/2),x, algorithm="giac")
[Out] 2*(sqrt(b*x + a)/b + a/(sqrt(b*x + a)*b))/b
```

Mupad [B] (verification not implemented)

Time = 18.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{x}{\sqrt{(a+bx)^3}}\,dx = \frac{2\,(2\,a+b\,x)\,\sqrt{{(a+b\,x)}^3}}{b^2\,{(a+b\,x)}^2}$$

```
[In] int(x/((a + b*x)^3)^(1/2),x)
[Out] (2*(2*a + b*x)*((a + b*x)^3)^(1/2))/(b^2*(a + b*x)^2)
```

3.138 $\int \frac{x^2}{\sqrt{(a+bx)^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{x^2}{\sqrt{(a+bx)^3}}\,dx=\frac{2\left(-a^2-2a(a+bx)+\frac{1}{3}(a+bx)^2\right)}{b^3\sqrt{a+bx}}$$

```
[Out] 2*(1/3*(b*x+a)^2-2*a*(b*x+a)-a^2)/b^3/(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.72, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1973, 45}

$$\int \frac{x^2}{\sqrt{(a+bx)^3}}\,dx=-\frac{2a^2(a+bx)}{b^3\sqrt{(a+bx)^3}}-\frac{4a(a+bx)^2}{b^3\sqrt{(a+bx)^3}}+\frac{2\sqrt{(a+bx)^3}}{3b^3}$$

```
[In] Int[x^2/Sqrt[(a + b*x)^3],x]
```

```
[Out] (-2*a^2*(a + b*x))/(b^3*Sqrt[(a + b*x)^3]) - (4*a*(a + b*x)^2)/(b^3*Sqrt[(a
 + b*x)^3]) + (2*Sqrt[(a + b*x)^3])/(3*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(1+\frac{bx}{a}\right)^{3/2}\int\frac{x^2}{\left(1+\frac{bx}{a}\right)^{3/2}}\,dx}{\sqrt{(a+bx)^3}}\\
&= \frac{\left(1+\frac{bx}{a}\right)^{3/2}\int\left(\frac{a^2}{b^2\left(1+\frac{bx}{a}\right)^{3/2}}-\frac{2a^2}{b^2\sqrt{1+\frac{bx}{a}}}+\frac{a^2\sqrt{1+\frac{bx}{a}}}{b^2}\right)dx}{\sqrt{(a+bx)^3}}\\
&= -\frac{2a^2(a+bx)}{b^3\sqrt{(a+bx)^3}}-\frac{4a(a+bx)^2}{b^3\sqrt{(a+bx)^3}}+\frac{2\sqrt{(a+bx)^3}}{3b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int\frac{x^2}{\sqrt{(a+bx)^3}}\,dx=\frac{2(a+bx)\left(-8a^2-4abx+b^2x^2\right)}{3b^3\sqrt{(a+bx)^3}}$$

```
[In] Integrate[x^2/Sqrt[(a + b*x)^3],x]
[Out] (2*(a + b*x)*(-8*a^2 - 4*a*b*x + b^2*x^2))/(3*b^3*Sqrt[(a + b*x)^3])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{2(bx+a)\left(-b^2x^2+4bax+8a^2\right)}{3b^3\sqrt{(bx+a)^3}}$	39
default	$-\frac{2(bx+a)\left(-b^2x^2+4bax+8a^2\right)}{3b^3\sqrt{(bx+a)^3}}$	39
risch	$-\frac{2(-bx+5a)(bx+a)^2}{3b^3\sqrt{(bx+a)^3}}-\frac{2a^2(bx+a)}{b^3\sqrt{(bx+a)^3}}$	53
trager	$-\frac{2\left(-b^2x^2+4bax+8a^2\right)\sqrt{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}}{3(bx+a)^2b^3}$	61

```
[In] int(x^2/((b*x+a)^3)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] -2/3*(b*x+a)*(-b^2*x^2+4*a*b*x+8*a^2)/b^3/((b*x+a)^3)^(1/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(33) = 66$.

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.87

$$\int \frac{x^2}{\sqrt{(a+bx)^3}}\,dx = \frac{2\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}\,(b^2x^2-4\,abx-8\,a^2)}{3\,(b^5x^2+2\,ab^4x+a^2b^3)}$$

[In] integrate(x^2/((b*x+a)^3)^(1/2),x, algorithm="fricas")

[Out] 2/3*sqrt(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*(b^2*x^2 - 4*a*b*x - 8*a^2)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)

Sympy [F]

$$\int \frac{x^2}{\sqrt{(a+bx)^3}}\,dx = \int \frac{x^2}{\sqrt{(a+bx)^3}}\,dx$$

[In] integrate(x**2/((b*x+a)**3)**(1/2),x)

[Out] Integral(x**2/sqrt((a + b*x)**3), x)

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{\sqrt{(a+bx)^3}}\,dx = \frac{2\,(b^3x^3-3\,ab^2x^2-12\,a^2bx-8\,a^3)}{3\,(bx+a)^{\frac{3}{2}}b^3}$$

[In] integrate(x^2/((b*x+a)^3)^(1/2),x, algorithm="maxima")

[Out] 2/3*(b^3*x^3 - 3*a*b^2*x^2 - 12*a^2*b*x - 8*a^3)/((b*x + a)^(3/2)*b^3)

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{\sqrt{(a+bx)^3}}\,dx = -\frac{2\,a^2}{\sqrt{bx+a}b^3} + \frac{2\left((bx+a)^{\frac{3}{2}}b^6 - 6\,\sqrt{bx+a}ab^6\right)}{3\,b^9}$$

```
[In] integrate(x^2/((b*x+a)^3)^(1/2),x, algorithm="giac")
[Out] -2*a^2/(sqrt(b*x + a)*b^3) + 2/3*((b*x + a)^(3/2)*b^6 - 6*sqrt(b*x + a)*a*b
^6)/b^9
```

Mupad [B] (verification not implemented)

Time = 18.33 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{x^2}{\sqrt{(a+bx)^3}}\,dx = -\frac{2\,\sqrt{(a+b\,x)^3}\,(8\,a^2+4\,a\,b\,x-b^2\,x^2)}{3\,b^3\,(a+b\,x)^2}$$

```
[In] int(x^2/((a + b*x)^3)^(1/2),x)
[Out] -(2*((a + b*x)^3)^(1/2)*(8*a^2 - b^2*x^2 + 4*a*b*x))/(3*b^3*(a + b*x)^2)
```

3.139 $\int \frac{1}{x\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \frac{1}{x\sqrt{a+bx}}\,dx=\frac{\log\left(\frac{-\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}+\sqrt{a+bx}}\right)}{\sqrt{a}}$$

```
[Out] 1/a^(1/2)*ln(((b*x+a)^(1/2)-a^(1/2))/((b*x+a)^(1/2)+a^(1/2)))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.55, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {65, 214}

$$\int \frac{1}{x\sqrt{a+bx}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Int[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b} \\ &= -\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.55

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Integrate[1/(x*Sqrt[a + b*x]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.43

method	result	size
derivativedivides	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
default	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
pseudoelliptic	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18

```
[In] int(1/x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.50

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \frac{2\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.40

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] int(1/(x*(a + b*x)^(1/2)),x)
[Out] -(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

3.140 $\int \frac{\sqrt{a+bx}}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{\sqrt{a+bx}}{x} dx = 2\sqrt{a+bx} + \sqrt{a}\log\left(\frac{-\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}+\sqrt{a+bx}}\right)$$

```
[Out] 2*(b*x+a)^(1/2)+a^(1/2)*ln(((b*x+a)^(1/2)-a^(1/2))/((b*x+a)^(1/2)+a^(1/2)))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.65, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {52, 65, 214}

$$\int \frac{\sqrt{a+bx}}{x} dx = 2\sqrt{a+bx} - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[In] Int[Sqrt[a + b*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\sqrt{a+bx} + a\int \frac{1}{x\sqrt{a+bx}}\,dx \\ &= 2\sqrt{a+bx} + \frac{(2a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b} \\ &= 2\sqrt{a+bx} - 2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{a+bx}}{x}\,dx = 2\sqrt{a+bx} - 2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[In] Integrate[Sqrt[a + b*x]/x,x]
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.52

method	result	size
derivativedivides	$2\sqrt{bx+a}-2\sqrt{a}\ \text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$	28
default	$2\sqrt{bx+a}-2\sqrt{a}\ \text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$	28
pseudoelliptic	$2\sqrt{bx+a}-2\sqrt{a}\ \text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$	28

```
[In] int((b*x+a)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{a+bx}}{x}\,dx = \left[\sqrt{a}\log\left(\frac{bx - 2\sqrt{bx+a}\sqrt{a} + 2\,a}{x}\right) + 2\sqrt{bx+a}, 2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + 2\sqrt{bx+a}\right]$$

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="fricas")
[Out] [sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a), 2*
sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)]
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{a+bx}}{x}\,dx = -2\sqrt{a}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) + \frac{2a}{\sqrt{b}\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{2\sqrt{b}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

```
[In] integrate((b*x+a)**(1/2)/x,x)
[Out] -2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*a/(sqrt(b)*sqrt(x)*sqrt(a/(
b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(a/(b*x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a+bx}}{x}\,dx = \sqrt{a}\log\left(\frac{\sqrt{bx+a} - \sqrt{a}}{\sqrt{bx+a} + \sqrt{a}}\right) + 2\sqrt{bx+a}$$

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="maxima")
[Out] sqrt(a)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2*sqrt(b
*x + a)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{a+bx}}{x}\,dx = \frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\,\sqrt{bx+a}$$

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="giac")
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)
```

Mupad [B] (verification not implemented)

Time = 17.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{a+bx}}{x}\,dx = 2\,\sqrt{a+b\,x} - 2\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)$$

```
[In] int((a + b*x)^(1/2)/x,x)
[Out] 2*(a + b*x)^(1/2) - 2*a^(1/2)*atanh((a + b*x)^(1/2)/a^(1/2))
```

3.141 $\int \frac{\sqrt{a+bx}}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{\sqrt{a+bx}}{x^2} dx = -\frac{\sqrt{a+bx}}{x} + \frac{b \log \left(\frac{-\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}+\sqrt{a+bx}}\right)}{2\sqrt{a}}$$

```
[Out] -(b*x+a)^(1/2)/x+1/2*b/a^(1/2)*ln(((b*x+a)^(1/2)-a^(1/2))/((b*x+a)^(1/2)+a^
(1/2)))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {43, 65, 214}

$$\int \frac{\sqrt{a+bx}}{x^2} dx = -\frac{b \text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{a+bx}}{x}$$

```
[In] Int[Sqrt[a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{a+bx}}{x}+\frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}\,dx \\
&= -\frac{\sqrt{a+bx}}{x}+\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right) \\
&= -\frac{\sqrt{a+bx}}{x}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = -\frac{\sqrt{a+bx}}{x}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Integrate[Sqrt[a + b*x]/x^2,x]
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.52

method	result	size
risch	$-\frac{\sqrt{bx+a}}{x}-\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	32
pseudoelliptic	$-\frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx+\sqrt{bx+a}\,\sqrt{a}}{x\sqrt{a}}$	36
derivativedivides	$2b\left(-\frac{\sqrt{bx+a}}{2bx}-\frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$	37
default	$2b\left(-\frac{\sqrt{bx+a}}{2bx}-\frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$	37

```
[In] int((b*x+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -(b*x+a)^(1/2)/x-b/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = \left[\frac{\sqrt{a}bx\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\,\sqrt{bx+a}a}{2\,ax}, \frac{\sqrt{-a}bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{bx+a}a}{ax}\right]$$

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="fricas")
[Out] [1/2*(sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(b*x +
 a)*a)/(a*x)]
```

Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}}-\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

```
[In] integrate((b*x+a)**(1/2)/x**2,x)
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/sqrt(x) - b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqr
t(a)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = \frac{b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{bx+a}}{x}$$

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) - sq
rt(b*x + a)/x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = \frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bx+a}b}{x}}{b}$$

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x + a)*b/x)/b
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = -\frac{\sqrt{a+b\,x}}{x} - \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] int((a + b*x)^(1/2)/x^2,x)
```

```
[Out] - (a + b*x)^(1/2)/x - (b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

3.142 $\int \frac{\sqrt{a+bx}}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 90

$$\int \frac{\sqrt{a+bx}}{x^3}\,dx=\frac{b\sqrt{a+bx}}{4ax}-\frac{\sqrt{(a+bx)^3}}{2ax^2}-\frac{b^2\log\left(\frac{-\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}+\sqrt{a+bx}}\right)}{8a^{3/2}}$$

```
[Out] -1/2*((b*x+a)^3)^(1/2)/a/x^2+1/4*b*(b*x+a)^(1/2)/a/x-1/8*b^2/a^(3/2)*ln(((b
*x+a)^(1/2)-a^(1/2))/((b*x+a)^(1/2)+a^(1/2)))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {43, 44, 65, 214}

$$\int \frac{\sqrt{a+bx}}{x^3}\,dx=\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}$$

```
[In] Int[Sqrt[a + b*x]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + b*x]/x^2 - (b*Sqrt[a + b*x])/(4*a*x) + (b^2*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/(4*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{a+bx}}{2x^2}+\frac{1}{4}b\int\frac{1}{x^2\sqrt{a+bx}}\,dx\\
&= -\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}-\frac{b^2\int\frac{1}{x\sqrt{a+bx}}\,dx}{8a}\\
&= -\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}-\frac{b\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{4a}\\
&= -\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}+\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.61

$$\int\frac{\sqrt{a+bx}}{x^3}\,dx=-\frac{\sqrt{a+bx}(2a+bx)}{4ax^2}+\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}$$

```
[In] Integrate[Sqrt[a + b*x]/x^3,x]
[Out] -1/4*(Sqrt[a + b*x]*(2*a + b*x))/(a*x^2) + (b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[
a]])/(4*a^(3/2))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.49

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(bx+2a)}{4x^2a}+\frac{b^2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}}$	44
pseudoelliptic	$\frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^2x^2-\left(2a^{\frac{3}{2}}+\sqrt{a}\,bx\right)\sqrt{bx+a}}{4a^{\frac{3}{2}}x^2}$	50
derivativedivides	$2b^2\left(-\frac{\frac{(bx+a)^{\frac{3}{2}}}{8a}+\frac{\sqrt{bx+a}}{8}}{b^2x^2}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)$	54
default	$2b^2\left(-\frac{\frac{(bx+a)^{\frac{3}{2}}}{8a}+\frac{\sqrt{bx+a}}{8}}{b^2x^2}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)$	54

```
[In] int((b*x+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/4*(b*x+a)^(1/2)*(b*x+2*a)/x^2/a+1/4*b^2/a^(3/2)*arctanh((b*x+a)^(1/2)/a^
(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{a+bx}}{x^3}\,dx = \left[\frac{\sqrt{a}b^2x^2\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\left(abx+2\,a^2\right)\sqrt{bx+a}}{8\,a^2x^2},\ -\frac{\sqrt{-a}b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(abx+2\,a^2\right)\sqrt{bx+a}}{4\,a^2x^2}\right]$$

```
[In] integrate((b*x+a)^(1/2)/x^3,x, algorithm="fricas")
[Out] [1/8*(sqrt(a)*b^2*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(a*b
*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2), -1/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(b*
x + a)*sqrt(-a)/a) + (a*b*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2)]
```

Sympy [A] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+bx}}{x^3}\,dx = -\frac{a}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{3\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{b^{\frac{3}{2}}}{4a\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{3}{2}}}$$

```
[In] integrate((b*x+a)**(1/2)/x**3,x)
```

```
[Out] -a/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) - 3*sqrt(b)/(4*x**(3/2)*sqrt(a/(b
*x) + 1)) - b**(3/2)/(4*a*sqrt(x)*sqrt(a/(b*x) + 1)) + b**2*asinh(sqrt(a)/(
sqrt(b)*sqrt(x)))/(4*a**(3/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+bx}}{x^3}\,dx = -\frac{b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{3}{2}}} - \frac{(bx+a)^{\frac{3}{2}}b^2 + \sqrt{bx+a}ab^2}{4\left((bx+a)^2a - 2\,(bx+a)a^2 + a^3\right)}$$

```
[In] integrate((b*x+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2) -
 1/4*((b*x + a)^(3/2)*b^2 + sqrt(b*x + a)*a*b^2)/((b*x + a)^2*a - 2*(b*x +
a)*a^2 + a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a+bx}}{x^3}\,dx = -\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{(bx+a)^{\frac{3}{2}}b^3 + \sqrt{bx+a}ab^3}{ab^2x^2}}{4\,b}$$

```
[In] integrate((b*x+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*x + a)^(3/2)*b^
3 + sqrt(b*x + a)*a*b^3)/(a*b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a+bx}}{x^3}\,dx=\frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{3/2}}-\frac{(a+b\,x)^{3/2}}{4\,a\,x^2}-\frac{\sqrt{a+b\,x}}{4\,x^2}$$

```
[In] int((a + b*x)^(1/2)/x^3,x)
```

```
[Out] (b^2*atanh((a + b*x)^(1/2)/a^(1/2)))/(4*a^(3/2)) - (a + b*x)^(3/2)/(4*a*x^2
) - (a + b*x)^(1/2)/(4*x^2)
```

3.143 $\int \frac{\sqrt{(a+bx)^3}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \frac{\sqrt{(a+bx)^3}}{x} dx = 2\sqrt{a+bx}\left(a+\frac{1}{3}(a+bx)\right)+a^{3/2}\log\left(\frac{-\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}+\sqrt{a+bx}}\right)$$

```
[Out] 2*(1/3*b*x+4/3*a)*(b*x+a)^(1/2)+a^(3/2)*ln(((b*x+a)^(1/2)-a^(1/2))/((b*x+a)
^(1/2)+a^(1/2)))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1973, 52, 65, 214}

$$\int \frac{\sqrt{(a+bx)^3}}{x} dx = -\frac{2\sqrt{(a+bx)^3}\text{arctanh}\left(\sqrt{\frac{bx}{a}+1}\right)}{\left(\frac{bx}{a}+1\right)^{3/2}}+\frac{2a\sqrt{(a+bx)^3}}{a+bx}+\frac{2}{3}\sqrt{(a+bx)^3}$$

```
[In] Int[Sqrt[(a + b*x)^3]/x,x]
```

```
[Out] (2*Sqrt[(a + b*x)^3])/3 + (2*a*Sqrt[(a + b*x)^3])/(a + b*x) - (2*Sqrt[(a +
b*x)^3]*ArcTanh[Sqrt[1 + (b*x)/a]])/(1 + (b*x)/a)^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{(a+bx)^3}\int \frac{\left(1+\frac{bx}{a}\right)^{3/2}}{x}\,dx}{\left(1+\frac{bx}{a}\right)^{3/2}}$$

$$= \frac{2}{3}\sqrt{(a+bx)^3} + \frac{\sqrt{(a+bx)^3}\int \frac{\sqrt{1+\frac{bx}{a}}}{x}\,dx}{\left(1+\frac{bx}{a}\right)^{3/2}}$$

$$= \frac{2}{3}\sqrt{(a+bx)^3} + \frac{2a\sqrt{(a+bx)^3}}{a+bx} + \frac{\sqrt{(a+bx)^3}\int \frac{1}{x\sqrt{1+\frac{bx}{a}}}\,dx}{\left(1+\frac{bx}{a}\right)^{3/2}}$$

$$= \frac{2}{3}\sqrt{(a+bx)^3} + \frac{2a\sqrt{(a+bx)^3}}{a+bx} + \frac{\left(2a\sqrt{(a+bx)^3}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{ax^2}{b}}\,dx, x, \sqrt{1+\frac{bx}{a}}\right)}{b\left(1+\frac{bx}{a}\right)^{3/2}}$$

$$= \frac{2}{3}\sqrt{(a+bx)^3} + \frac{2a\sqrt{(a+bx)^3}}{a+bx} - \frac{2\sqrt{(a+bx)^3}\,\text{arctanh}\left(\sqrt{1+\frac{bx}{a}}\right)}{\left(1+\frac{bx}{a}\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{(a+bx)^3}}{x}\,dx = \frac{2\sqrt{(a+bx)^3}\left(\sqrt{a+bx}(4a+bx)-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{3(a+bx)^{3/2}}$$

```
[In] Integrate[Sqrt[(a + b*x)^3]/x,x]
```

```
[Out] (2*Sqrt[(a + b*x)^3]*(Sqrt[a + b*x]*(4*a + b*x) - 3*a^(3/2)*ArcTanh[Sqrt[a
+ b*x]/Sqrt[a]]))/(3*(a + b*x)^(3/2))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2\sqrt{(bx+a)^3}\left(-3a^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+(bx+a)^{\frac{3}{2}}+3a\sqrt{bx+a}\right)}{3(bx+a)^{\frac{3}{2}}}$	54

```
[In] int(((b*x+a)^3)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*((b*x+a)^3)^(1/2)*(-3*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)^(3
/2)+3*a*(b*x+a)^(1/2))/(b*x+a)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(49) = 98.

Time = 0.25 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.62

$$\int \frac{\sqrt{(a+bx)^3}}{x}\,dx$$
$$= \left[\frac{3\,(abx+a^2)\sqrt{a}\log\left(\frac{b^2x^2+3\,abx+2\,a^2-2\,\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}\sqrt{a}}{bx^2+ax}\right)+2\,\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}(bx+4\,a)}{3\,(bx+a)}\right.$$

```
[In] integrate(((b*x+a)^3)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a*b*x + a^2)*sqrt(a)*log((b^2*x^2 + 3*a*b*x + 2*a^2 - 2*sqrt(b^3*x
^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(a))/(b*x^2 + a*x)) + 2*sqrt(b^3*x^
3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*(b*x + 4*a))/(b*x + a), 2/3*(3*(a*b*x +
a^2)*sqrt(-a)*arctan(sqrt(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(-a)
/(a*b*x + a^2)) + sqrt(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*(b*x + 4*a)
)/(b*x + a)]
```

Sympy [F]

$$\int \frac{\sqrt{(a+bx)^3}}{x}\,dx = \int \frac{\sqrt{(a+bx)^3}}{x}\,dx$$

```
[In] integrate(((b*x+a)**3)**(1/2)/x,x)
[Out] Integral(sqrt((a + b*x)**3)/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{(a+bx)^3}}{x}\,dx = \int \frac{\sqrt{(bx+a)^3}}{x}\,dx$$

```
[In] integrate(((b*x+a)^3)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt((b*x + a)^3)/x, x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{(a+bx)^3}}{x}\,dx = \frac{2\,a^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2}{3}\,(bx+a)^{\frac{3}{2}} + 2\,\sqrt{bx+a}a$$

```
[In] integrate(((b*x+a)^3)^(1/2)/x,x, algorithm="giac")
[Out] 2*a^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/3*(b*x + a)^(3/2) + 2*sqr
t(b*x + a)*a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{(a+bx)^3}}{x}\,dx = \int \frac{\sqrt{(a+b\,x)^3}}{x}\,dx$$

```
[In] int(((a + b*x)^3)^(1/2)/x,x)
[Out] int(((a + b*x)^3)^(1/2)/x, x)
```

3.144 $\int \frac{\sqrt{(a+bx)^3}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 93

$$\int \frac{\sqrt{(a+bx)^3}}{x^2}\,dx = -\frac{\sqrt{(a+bx)^5}}{ax}+\frac{3b\left(2\sqrt{a+bx}\left(a+\frac{1}{3}(a+bx)\right)+a^{3/2}\log\left(\frac{-\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}+\sqrt{a+bx}}\right)\right)}{2a}$$

```
[Out] -((b*x+a)^5)^(1/2)/a/x+3/2*b/a*(2*(1/3*b*x+4/3*a)*(b*x+a)^(1/2)+a^(3/2)*ln(
((b*x+a)^(1/2)-a^(1/2))/((b*x+a)^(1/2)+a^(1/2))))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1973, 43, 52, 65, 214}

$$\int \frac{\sqrt{(a+bx)^3}}{x^2}\,dx = -\frac{3b\sqrt{(a+bx)^3}\text{arctanh}\left(\sqrt{\frac{bx}{a}+1}\right)}{a\left(\frac{bx}{a}+1\right)^{3/2}}+\frac{3b\sqrt{(a+bx)^3}}{a+bx}-\frac{\sqrt{(a+bx)^3}}{x}$$

```
[In] Int[Sqrt[(a + b*x)^3]/x^2,x]
```

```
[Out] -(Sqrt[(a + b*x)^3]/x) + (3*b*Sqrt[(a + b*x)^3])/(a + b*x) - (3*b*Sqrt[(a +
 b*x)^3]*ArcTanh[Sqrt[1 + (b*x)/a]])/(a*(1 + (b*x)/a)^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{(a+bx)^3}\int \frac{\left(1+\frac{bx}{a}\right)^{3/2}}{x^2}\,dx}{\left(1+\frac{bx}{a}\right)^{3/2}}$$

$$= -\frac{\sqrt{(a+bx)^3}}{x} + \frac{\left(3b\sqrt{(a+bx)^3}\right)\int \frac{\sqrt{1+\frac{bx}{a}}}{x}\,dx}{2a\left(1+\frac{bx}{a}\right)^{3/2}}$$

$$= -\frac{\sqrt{(a+bx)^3}}{x} + \frac{3b\sqrt{(a+bx)^3}}{a+bx} + \frac{\left(3b\sqrt{(a+bx)^3}\right)\int \frac{1}{x\sqrt{1+\frac{bx}{a}}}\,dx}{2a\left(1+\frac{bx}{a}\right)^{3/2}}$$

$$= -\frac{\sqrt{(a+bx)^3}}{x} + \frac{3b\sqrt{(a+bx)^3}}{a+bx} + \frac{\left(3\sqrt{(a+bx)^3}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{ax^2}{b}}\,dx, x, \sqrt{1+\frac{bx}{a}}\right)}{\left(1+\frac{bx}{a}\right)^{3/2}}$$

$$= -\frac{\sqrt{(a+bx)^3}}{x} + \frac{3b\sqrt{(a+bx)^3}}{a+bx} - \frac{3b\sqrt{(a+bx)^3}\text{arctanh}\left(\sqrt{1+\frac{bx}{a}}\right)}{a\left(1+\frac{bx}{a}\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{(a+bx)^3}}{x^2}\,dx = -\frac{\sqrt{(a+bx)^3}\left((a-2bx)\sqrt{a+bx}+3\sqrt{a}bx\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{x(a+bx)^{3/2}}$$

```
[In] Integrate[Sqrt[(a + b*x)^3]/x^2,x]
```

```
[Out] -((Sqrt[(a + b*x)^3]*((a - 2*b*x)*Sqrt[a + b*x] + 3*Sqrt[a]*b*x*ArcTanh[Sqr
t[a + b*x]/Sqrt[a]]))/(x*(a + b*x)^(3/2)))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.73

method	result	size
default	$-\frac{\sqrt{(bx+a)^3}\left(-2bx\sqrt{bx+a}\,\sqrt{a}+3\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abx+\sqrt{bx+a}\,a^{\frac{3}{2}}\right)}{(bx+a)^{\frac{3}{2}}x\sqrt{a}}$	68
risch	$-\frac{a\sqrt{(bx+a)^3}}{(bx+a)x}+\frac{b\left(4\sqrt{bx+a}-6\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)\sqrt{(bx+a)^3}}{2(bx+a)^{\frac{3}{2}}}$	70

```
[In] int(((b*x+a)^3)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -((b*x+a)^3)^(1/2)*(-2*b*x*(b*x+a)^(1/2)*a^(1/2)+3*arctanh((b*x+a)^(1/2)/a^
(1/2))*a*b*x+(b*x+a)^(1/2)*a^(3/2))/(b*x+a)^(3/2)/x/a^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt{(a+bx)^3}}{x^2}\,dx$$

$$= \left[\frac{3\left(b^2x^2+abx\right)\sqrt{a}\log\left(\frac{b^2x^2+3\,abx+2\,a^2-2\,\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}\sqrt{a}}{bx^2+ax}\right)+2\,\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}(2\,bx-}{2\left(bx^2+ax\right)}\right.$$

```
[In] integrate(((b*x+a)^3)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b^2*x^2 + a*b*x)*sqrt(a)*log((b^2*x^2 + 3*a*b*x + 2*a^2 - 2*sqrt(b
^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(a))/(b*x^2 + a*x)) + 2*sqrt(b^
3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*(2*b*x - a))/(b*x^2 + a*x), (3*(b^2*
x^2 + a*b*x)*sqrt(-a)*arctan(sqrt(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*
sqrt(-a)/(a*b*x + a^2)) + sqrt(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*(2*
b*x - a))/(b*x^2 + a*x)]
```

Sympy [F]

$$\int \frac{\sqrt{(a+bx)^3}}{x^2}\,dx = \int \frac{\sqrt{(a+bx)^3}}{x^2}\,dx$$

```
[In] integrate(((b*x+a)**3)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a + b*x)**3)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{(a+bx)^3}}{x^2}\,dx = \int \frac{\sqrt{(bx+a)^3}}{x^2}\,dx$$

```
[In] integrate(((b*x+a)^3)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x + a)^3)/x^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{(a+bx)^3}}{x^2}\,dx = \frac{\frac{3\,ab^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\,\sqrt{bx+a}b^2 - \frac{\sqrt{bx+a}ab}{x}}{b}$$

```
[In] integrate(((b*x+a)^3)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (3*a*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)*b^2 - sq
rt(b*x + a)*a*b/x)/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{(a+bx)^3}}{x^2}\,dx = \int \frac{\sqrt{{(a+b\,x)}^3}}{x^2}\,dx$$

```
[In] int(((a + b*x)^3)^(1/2)/x^2,x)
[Out] int(((a + b*x)^3)^(1/2)/x^2, x)
```

3.145 $\int \frac{\sqrt{(a+bx)^3}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 135

$$\int \frac{\sqrt{(a+bx)^3}}{x^3}\,dx = \left(-\frac{1}{2ax^2}-\frac{b}{4a^2x}\right)\sqrt{(a+bx)^5} + \frac{3b^2\left(\frac{b\sqrt{a+bx}}{4ax}-\frac{\sqrt{(a+bx)^3}}{2ax^2}-\frac{b^2\log\left(\frac{-\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}+\sqrt{a+bx}}\right)}{8a^{3/2}}\right)}{8a^2}$$

```
[Out] -(1/2/a/x^2+1/4*b/a^2/x)*((b*x+a)^5)^(1/2)+3/8*b^2/a^2*(-1/2*((b*x+a)^3)^(1
/2)/a/x^2+1/4*b*(b*x+a)^(1/2)/a/x-1/8*b^2/a^(3/2)*ln(((b*x+a)^(1/2)-a^(1/2)
)/((b*x+a)^(1/2)+a^(1/2))))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1973, 43, 65, 214}

$$\int \frac{\sqrt{(a+bx)^3}}{x^3}\,dx = -\frac{3b^2\sqrt{(a+bx)^3}\text{arctanh}\left(\sqrt{\frac{bx}{a}+1}\right)}{4a^2\left(\frac{bx}{a}+1\right)^{3/2}}-\frac{\sqrt{(a+bx)^3}}{2x^2}-\frac{3b\sqrt{(a+bx)^3}}{4x(a+bx)}$$

```
[In] Int[Sqrt[(a + b*x)^3]/x^3,x]
```

```
[Out] -1/2*Sqrt[(a + b*x)^3]/x^2 - (3*b*Sqrt[(a + b*x)^3])/(4*x*(a + b*x)) - (3*b
^2*Sqrt[(a + b*x)^3]*ArcTanh[Sqrt[1 + (b*x)/a]])/(4*a^2*(1 + (b*x)/a)^(3/2)
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{(a+bx)^3}\int \frac{\left(1+\frac{bx}{a}\right)^{3/2}}{x^3}\,dx}{\left(1+\frac{bx}{a}\right)^{3/2}}$$

$$= -\frac{\sqrt{(a+bx)^3}}{2x^2} + \frac{\left(3b\sqrt{(a+bx)^3}\right)\int \frac{\sqrt{1+\frac{bx}{a}}}{x^2}\,dx}{4a\left(1+\frac{bx}{a}\right)^{3/2}}$$

$$= -\frac{\sqrt{(a+bx)^3}}{2x^2} - \frac{3b\sqrt{(a+bx)^3}}{4x(a+bx)} + \frac{\left(3b^2\sqrt{(a+bx)^3}\right)\int \frac{1}{x\sqrt{1+\frac{bx}{a}}}\,dx}{8a^2\left(1+\frac{bx}{a}\right)^{3/2}}$$

$$= -\frac{\sqrt{(a+bx)^3}}{2x^2} - \frac{3b\sqrt{(a+bx)^3}}{4x(a+bx)} + \frac{\left(3b\sqrt{(a+bx)^3}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{ax^2}{b}}\,dx, x, \sqrt{1+\frac{bx}{a}}\right)}{4a\left(1+\frac{bx}{a}\right)^{3/2}}$$

$$= -\frac{\sqrt{(a+bx)^3}}{2x^2} - \frac{3b\sqrt{(a+bx)^3}}{4x(a+bx)} - \frac{3b^2\sqrt{(a+bx)^3}\,\text{arctanh}\left(\sqrt{1+\frac{bx}{a}}\right)}{4a^2\left(1+\frac{bx}{a}\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{(a+bx)^3}}{x^3}\,dx = -\frac{\sqrt{(a+bx)^3}\left(\sqrt{a}\sqrt{a+bx}(2a+5bx)+3b^2x^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{4\sqrt{a}x^2(a+bx)^{3/2}}$$

```
[In] Integrate[Sqrt[(a + b*x)^3]/x^3,x]
```

```
[Out] -1/4*(Sqrt[(a + b*x)^3]*(Sqrt[a]*Sqrt[a + b*x]*(2*a + 5*b*x) + 3*b^2*x^2*Ar
cTanh[Sqrt[a + b*x]/Sqrt[a]]))/(Sqrt[a]*x^2*(a + b*x)^(3/2))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.50

method	result	size
risch	$-\frac{(5bx+2a)\sqrt{(bx+a)^3}}{4(bx+a)x^2}-\frac{3b^2\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{(bx+a)^3}}{4\sqrt{a}\,(bx+a)^{\frac{3}{2}}}$	67
default	$-\frac{\sqrt{(bx+a)^3}\left(3\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^2x^2+5(bx+a)^{\frac{3}{2}}\sqrt{a}-3\sqrt{bx+a}\,a^{\frac{3}{2}}\right)}{4(bx+a)^{\frac{3}{2}}x^2\sqrt{a}}$	70

```
[In] int(((b*x+a)^3)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(b*x+a)*(5*b*x+2*a)/x^2*((b*x+a)^3)^(1/2)-3/4*b^2/a^(1/2)*arctanh((b*x
+a)^(1/2)/a^(1/2))*((b*x+a)^3)^(1/2)/(b*x+a)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.06

$$\int \frac{\sqrt{(a+bx)^3}}{x^3}\,dx = \left[\frac{3\left(b^3x^3+ab^2x^2\right)\sqrt{a}\log\left(\frac{b^2x^2+3\,abx+2\,a^2-2\,\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}\sqrt{a}}{bx^2+ax}\right)-2\,\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}(5\,a}{8\left(abx^3+a^2x^2\right)}\right.$$

```
[In] integrate(((b*x+a)^3)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^3*x^3 + a*b^2*x^2)*sqrt(a)*log((b^2*x^2 + 3*a*b*x + 2*a^2 - 2*sq
rt(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(a))/(b*x^2 + a*x)) - 2*sqr
t(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*(5*a*b*x + 2*a^2))/(a*b*x^3 + a^
```

```
2*x^2), 1/4*(3*(b^3*x^3 + a*b^2*x^2)*sqrt(-a)*arctan(sqrt(b^3*x^3 + 3*a*b^2
*x^2 + 3*a^2*b*x + a^3)*sqrt(-a)/(a*b*x + a^2)) - sqrt(b^3*x^3 + 3*a*b^2*x^
2 + 3*a^2*b*x + a^3)*(5*a*b*x + 2*a^2))/(a*b*x^3 + a^2*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{(a+bx)^3}}{x^3}\,dx = \int \frac{\sqrt{(a+bx)^3}}{x^3}\,dx$$

```
[In] integrate(((b*x+a)**3)**(1/2)/x**3,x)
[Out] Integral(sqrt((a + b*x)**3)/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{(a+bx)^3}}{x^3}\,dx = \int \frac{\sqrt{(bx+a)^3}}{x^3}\,dx$$

```
[In] integrate(((b*x+a)^3)^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt((b*x + a)^3)/x^3, x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{(a+bx)^3}}{x^3}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{5\,(bx+a)^{\frac{3}{2}}b^3 - 3\,\sqrt{bx+a}ab^3}{b^2x^2}}{4\,b}$$

```
[In] integrate(((b*x+a)^3)^(1/2)/x^3,x, algorithm="giac")
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (5*(b*x + a)^(3/2)*b^3
 - 3*sqrt(b*x + a)*a*b^3)/(b^2*x^2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{(a+bx)^3}}{x^3}\,dx = \int \frac{\sqrt{{\left(a+b\,x\right)}^3}}{x^3}\,dx$$

```
[In] int(((a + b*x)^3)^(1/2)/x^3,x)
[Out] int(((a + b*x)^3)^(1/2)/x^3, x)
```

3.146 $\int \frac{1}{x^2\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = -\frac{\sqrt{a+bx}}{ax} - \frac{b\log\left(\frac{-\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}+\sqrt{a+bx}}\right)}{2\sqrt{a}x}$$

```
[Out] -(b*x+a)^(1/2)/a/x-1/2*b/x/a^(1/2)*ln(((b*x+a)^(1/2)-a^(1/2))/((b*x+a)^(1/2
)+a^(1/2)))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {44, 65, 214}

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\sqrt{a+bx}}{ax}$$

```
[In] Int[1/(x^2*Sqrt[a + b*x]),x]
```

```
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{a+bx}}{ax} - \frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a}\\ &= -\frac{\sqrt{a+bx}}{ax} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{a}\\ &= -\frac{\sqrt{a+bx}}{ax} + \frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = -\frac{\sqrt{a+bx}}{ax} + \frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[In] Integrate[1/(x^2*Sqrt[a + b*x]),x]
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.51

method	result	size
risch	$-\frac{\sqrt{bx+a}}{ax}+\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$	34
pseudoelliptic	$\frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx-\sqrt{bx+a}\,\sqrt{a}}{a^{\frac{3}{2}}x}$	36
derivativedivides	$2b\left(-\frac{\sqrt{bx+a}}{2abx}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	40
default	$2b\left(-\frac{\sqrt{bx+a}}{2abx}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	40

```
[In] int(1/x^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(b*x+a)^(1/2)/a/x+b/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = \left[\frac{\sqrt{a}bx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\sqrt{bx+a}a}{2\,a^2x},\ -\frac{\sqrt{-a}bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\sqrt{bx+a}a}{a^2x}\right]$$

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] [1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a^2*x), -(sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + sqrt(b*
x + a)*a)/(a^2*x)]
```

Sympy [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a\sqrt{x}}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}}$$

```
[In] integrate(1/x**2/(b*x+a)**(1/2),x)
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/(a*sqrt(x)) + b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))
/a**(3/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = -\frac{\sqrt{bx+a}b}{(bx+a)a-a^2} - \frac{b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}$$

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] -sqrt(b*x + a)*b/((b*x + a)*a - a^2) - 1/2*b*log((sqrt(b*x + a) - sqrt(a))/
(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = -\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{\sqrt{bx+a}b}{ax}}{b}$$

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="giac")
[Out] -(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)*b/(a*x))/
b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\sqrt{a+b\,x}}{a\,x}$$

```
[In] int(1/(x^2*(a + b*x)^(1/2)),x)
[Out] (b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(3/2) - (a + b*x)^(1/2)/(a*x)
```

3.147 $\int \frac{1}{x^3\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 81

$$\int \frac{1}{x^3\sqrt{a+bx}}\,dx = \left(-\frac{1}{2ax^2}+\frac{3b}{4a^2x}\right)\sqrt{a+bx}+\frac{3b^2\log\left(\frac{-\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}+\sqrt{a+bx}}\right)}{8a^{5/2}}$$

```
[Out] (-1/2/a/x^2+3/4*b/a^2/x)*(b*x+a)^(1/2)+3/8*b^2/a^(5/2)*ln(((b*x+a)^(1/2)-a^
(1/2))/((b*x+a)^(1/2)+a^(1/2)))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {44, 65, 214}

$$\int \frac{1}{x^3\sqrt{a+bx}}\,dx = -\frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{3b\sqrt{a+bx}}{4a^2x}-\frac{\sqrt{a+bx}}{2ax^2}$$

```
[In] Int[1/(x^3*Sqrt[a + b*x]),x]
```

```
[Out] -1/2*Sqrt[a + b*x]/(a*x^2) + (3*b*Sqrt[a + b*x])/(4*a^2*x) - (3*b^2*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{a+bx}}{2ax^2} - \frac{(3b)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4a}\\ &= -\frac{\sqrt{a+bx}}{2ax^2} + \frac{3b\sqrt{a+bx}}{4a^2x} + \frac{(3b^2)\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a^2}\\ &= -\frac{\sqrt{a+bx}}{2ax^2} + \frac{3b\sqrt{a+bx}}{4a^2x} + \frac{(3b)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{4a^2}\\ &= -\frac{\sqrt{a+bx}}{2ax^2} + \frac{3b\sqrt{a+bx}}{4a^2x} - \frac{3b^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^3\sqrt{a+bx}}\,dx = \frac{\sqrt{a+bx}(-2a+3bx)}{4a^2x^2} - \frac{3b^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}$$

```
[In] Integrate[1/(x^3*Sqrt[a + b*x]),x]
[Out] (Sqrt[a + b*x]*(-2*a + 3*b*x))/(4*a^2*x^2) - (3*b^2*ArcTanh[Sqrt[a + b*x]/S
qrt[a]])/(4*a^(5/2))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.56

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-3bx+2a)}{4a^{2}x^{2}}-\frac{3b^{2}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}}$	45
pseudoelliptic	$\frac{-3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{2}x^{2}-2\sqrt{bx+a}\,a^{\frac{3}{2}}+3bx\sqrt{bx+a}\,\sqrt{a}}{4a^{\frac{5}{2}}x^{2}}$	56
derivativedivides	$2b^{2}\left(-\frac{\sqrt{bx+a}}{4a\,b^{2}x^{2}}-\frac{3\left(-\frac{\sqrt{bx+a}}{2abx}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)$	66
default	$2b^{2}\left(-\frac{\sqrt{bx+a}}{4a\,b^{2}x^{2}}-\frac{3\left(-\frac{\sqrt{bx+a}}{2abx}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)$	66

```
[In] int(1/x^3/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(b*x+a)^(1/2)*(-3*b*x+2*a)/a^2/x^2-3/4*b^2/a^(5/2)*arctanh((b*x+a)^(1/
2)/a^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^3\sqrt{a+bx}}\,dx = \left[\frac{3\sqrt{a}b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\,(3\,abx-2\,a^2)\sqrt{bx+a}}{8\,a^3x^2}, \frac{3\sqrt{-a}b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+(3\,abx-2\,a$$

```
[In] integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] [1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(3
*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sq
rt(b*x + a)*sqrt(-a)/a) + (3*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2)]
```

Sympy [A] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^3\sqrt{a+bx}}\,dx = -\frac{1}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{3b^{\frac{3}{2}}}{4a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}} - \frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{5}{2}}}$$

```
[In] integrate(1/x**3/(b*x+a)**(1/2),x)
```

```
[Out] -1/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) + sqrt(b)/(4*a*x**(3/2)*sqrt(a/(b
*x) + 1)) + 3*b**(3/2)/(4*a**2*sqrt(x)*sqrt(a/(b*x) + 1)) - 3*b**2*asinh(sq
rt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^3\sqrt{a+bx}}\,dx = \frac{3\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}} + \frac{3\,(bx+a)^{\frac{3}{2}}b^2 - 5\,\sqrt{bx+a}ab^2}{4\left((bx+a)^2a^2 - 2\,(bx+a)a^3 + a^4\right)}$$

```
[In] integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2) +
1/4*(3*(b*x + a)^(3/2)*b^2 - 5*sqrt(b*x + a)*a*b^2)/((b*x + a)^2*a^2 - 2*(b
*x + a)*a^3 + a^4)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3\sqrt{a+bx}}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{3\,(bx+a)^{\frac{3}{2}}b^3 - 5\,\sqrt{bx+a}ab^3}{a^2b^2x^2}}{4\,b}$$

```
[In] integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(3/
2)*b^3 - 5*sqrt(b*x + a)*a*b^3)/(a^2*b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^3\sqrt{a+bx}}\,dx = \frac{3\,(a+b\,x)^{3/2}}{4\,a^2\,x^2} - \frac{5\,\sqrt{a+b\,x}}{4\,a\,x^2} - \frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{5/2}}$$

```
[In] int(1/(x^3*(a + b*x)^(1/2)),x)
```

```
[Out] (3*(a + b*x)^(3/2))/(4*a^2*x^2) - (5*(a + b*x)^(1/2))/(4*a*x^2) - (3*b^2*at
anh((a + b*x)^(1/2)/a^(1/2)))/(4*a^(5/2))
```

3.148 $\int \frac{1}{x\sqrt{(a+bx)^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{1}{x\sqrt{(a+bx)^3}}\,dx = \frac{2}{a\sqrt{a+bx}}+\frac{\log\left(\frac{-\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}+\sqrt{a+bx}}\right)}{a^{3/2}}$$

```
[Out] 2/a/(b*x+a)^(1/2)+1/a^(3/2)*ln(((b*x+a)^(1/2)-a^(1/2))/((b*x+a)^(1/2)+a^(1/
2)))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1973, 53, 65, 214}

$$\int \frac{1}{x\sqrt{(a+bx)^3}}\,dx = \frac{2(a+bx)}{a\sqrt{(a+bx)^3}}-\frac{2\left(\frac{bx}{a}+1\right)^{3/2}\text{arctanh}\left(\sqrt{\frac{bx}{a}+1}\right)}{\sqrt{(a+bx)^3}}$$

```
[In] Int[1/(x*Sqrt[(a + b*x)^3]),x]
```

```
[Out] (2*(a + b*x))/(a*Sqrt[(a + b*x)^3]) - (2*(1 + (b*x)/a)^(3/2)*ArcTanh[Sqrt[1
 + (b*x)/a]])/Sqrt[(a + b*x)^3]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(1+\frac{bx}{a}\right)^{3/2}\int \frac{1}{x\left(1+\frac{bx}{a}\right)^{3/2}}\,dx}{\sqrt{(a+bx)^3}}\\
&= \frac{2(a+bx)}{a\sqrt{(a+bx)^3}}+\frac{\left(1+\frac{bx}{a}\right)^{3/2}\int \frac{1}{x\sqrt{1+\frac{bx}{a}}}\,dx}{\sqrt{(a+bx)^3}}\\
&= \frac{2(a+bx)}{a\sqrt{(a+bx)^3}}+\frac{\left(2a\left(1+\frac{bx}{a}\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{ax^2}{b}}\,dx,x,\sqrt{1+\frac{bx}{a}}\right)}{b\sqrt{(a+bx)^3}}\\
&= \frac{2(a+bx)}{a\sqrt{(a+bx)^3}}-\frac{2\left(1+\frac{bx}{a}\right)^{3/2}\text{arctanh}\left(\sqrt{1+\frac{bx}{a}}\right)}{\sqrt{(a+bx)^3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{1}{x\sqrt{(a+bx)^3}}\,dx = \frac{2(a+bx)\left(\sqrt{a}-\sqrt{a+bx}\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{a^{3/2}\sqrt{(a+bx)^3}}$$

```
[In] Integrate[1/(x*Sqrt[(a + b*x)^3]),x]
```

```
[Out] (2*(a + b*x)*(Sqrt[a] - Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(
3/2)*Sqrt[(a + b*x)^3])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{2(bx+a)\left(\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\sqrt{bx+a}-a^{\frac{3}{2}}\right)}{\sqrt{(bx+a)^{3}}\,a^{\frac{5}{2}}}$	47

```
[In] int(1/x/((b*x+a)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*x+a)*(arctanh((b*x+a)^(1/2)/a^(1/2))*a*(b*x+a)^(1/2)-a^(3/2))/((b*x+a
)^3)^(1/2)/a^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(45) = 90$.

Time = 0.26 (sec) , antiderivative size = 268, normalized size of antiderivative = 4.70

$$\int \frac{1}{x\sqrt{(a+bx)^3}}\,dx$$
$$= \left[\frac{(b^2x^2+2\,abx+a^2)\sqrt{a}\log\left(\frac{b^2x^2+3\,abx+2\,a^2-2\,\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}\sqrt{a}}{bx^2+ax}\right)+2\,\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}a}{a^2b^2x^2+2\,a^3bx+a^4}\right.$$

```
[In] integrate(1/x/((b*x+a)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [((b^2*x^2 + 2*a*b*x + a^2)*sqrt(a)*log((b^2*x^2 + 3*a*b*x + 2*a^2 - 2*sqrt
(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(a))/(b*x^2 + a*x)) + 2*sqrt(
b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*a)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4)
, 2*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a)*arctan(sqrt(b^3*x^3 + 3*a*b^2*x^2 +
 3*a^2*b*x + a^3)*sqrt(-a)/(a*b*x + a^2)) + sqrt(b^3*x^3 + 3*a*b^2*x^2 + 3*
a^2*b*x + a^3)*a)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{(a+bx)^3}}\,dx = \int \frac{1}{x\sqrt{(a+bx)^3}}\,dx$$

```
[In] integrate(1/x/((b*x+a)**3)**(1/2),x)
[Out] Integral(1/(x*sqrt((a + b*x)**3)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{(a+bx)^3}}\,dx = \int \frac{1}{\sqrt{(bx+a)^3}x}\,dx$$

```
[In] integrate(1/x/((b*x+a)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt((b*x + a)^3)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.65

$$\int \frac{1}{x\sqrt{(a+bx)^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{2}{\sqrt{bx+a}a}$$

```
[In] integrate(1/x/((b*x+a)^3)^(1/2),x, algorithm="giac")
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + 2/(sqrt(b*x + a)*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{(a+bx)^3}}\,dx = \int \frac{1}{x\,\sqrt{(a+b\,x)^3}}\,dx$$

```
[In] int(1/(x*((a + b*x)^3)^(1/2)),x)
[Out] int(1/(x*((a + b*x)^3)^(1/2)), x)
```

3.149 $\int \frac{1}{x^2\sqrt{(a+bx)^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \frac{1}{x^2\sqrt{(a+bx)^3}}\,dx = \frac{-\frac{3b}{a^2}-\frac{1}{ax}}{\sqrt{a+bx}} - \frac{3b\log\left(\frac{-\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}+\sqrt{a+bx}}\right)}{2a^{5/2}}$$

```
[Out] (-1/a/x-3*b/a^2)/(b*x+a)^(1/2)-3/2*b/a^(5/2)*ln(((b*x+a)^(1/2)-a^(1/2))/((b
*x+a)^(1/2)+a^(1/2)))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1973, 44, 53, 65, 214}

$$\int \frac{1}{x^2\sqrt{(a+bx)^3}}\,dx = -\frac{3b(a+bx)}{a^2\sqrt{(a+bx)^3}} + \frac{3b\left(\frac{bx}{a}+1\right)^{3/2}\text{arctanh}\left(\sqrt{\frac{bx}{a}+1}\right)}{a\sqrt{(a+bx)^3}} - \frac{a+bx}{ax\sqrt{(a+bx)^3}}$$

```
[In] Int[1/(x^2*Sqrt[(a + b*x)^3]),x]
```

```
[Out] (-3*b*(a + b*x))/(a^2*Sqrt[(a + b*x)^3]) - (a + b*x)/(a*x*Sqrt[(a + b*x)^3]
) + (3*b*(1 + (b*x)/a)^(3/2)*ArcTanh[Sqrt[1 + (b*x)/a]])/(a*Sqrt[(a + b*x)^
3])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(1+\frac{bx}{a}\right)^{3/2}\int \frac{1}{x^2\left(1+\frac{bx}{a}\right)^{3/2}}\,dx}{\sqrt{(a+bx)^3}}$$

$$= -\frac{a+bx}{ax\sqrt{(a+bx)^3}} - \frac{\left(3b\left(1+\frac{bx}{a}\right)^{3/2}\right)\int \frac{1}{x\left(1+\frac{bx}{a}\right)^{3/2}}\,dx}{2a\sqrt{(a+bx)^3}}$$

$$= -\frac{3b(a+bx)}{a^2\sqrt{(a+bx)^3}} - \frac{a+bx}{ax\sqrt{(a+bx)^3}} - \frac{\left(3b\left(1+\frac{bx}{a}\right)^{3/2}\right)\int \frac{1}{x\sqrt{1+\frac{bx}{a}}}\,dx}{2a\sqrt{(a+bx)^3}}$$

$$= -\frac{3b(a+bx)}{a^2\sqrt{(a+bx)^3}} - \frac{a+bx}{ax\sqrt{(a+bx)^3}} - \frac{\left(3\left(1+\frac{bx}{a}\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{ax^2}{b}}\,dx, x, \sqrt{1+\frac{bx}{a}}\right)}{\sqrt{(a+bx)^3}}$$

$$= -\frac{3b(a+bx)}{a^2\sqrt{(a+bx)^3}} - \frac{a+bx}{ax\sqrt{(a+bx)^3}} + \frac{3b\left(1+\frac{bx}{a}\right)^{3/2}\operatorname{arctanh}\left(\sqrt{1+\frac{bx}{a}}\right)}{a\sqrt{(a+bx)^3}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2\sqrt{(a+bx)^3}}\,dx = -\frac{(a+bx)\left(\sqrt{a}(a+3bx)-3bx\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{a^{5/2}x\sqrt{(a+bx)^3}}$$

```
[In] Integrate[1/(x^2*Sqrt[(a + b*x)^3]),x]
```

```
[Out] -(((a + b*x)*(Sqrt[a]*(a + 3*b*x) - 3*b*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*
x]/Sqrt[a]]))/(a^(5/2)*x*Sqrt[(a + b*x)^3]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{(bx+a)\left(3\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx-3\sqrt{a}\,bx-a^{\frac{3}{2}}\right)}{\sqrt{(bx+a)^3}\,a^{\frac{5}{2}}\,x}$	58
risch	$-\frac{(bx+a)^2}{a^2x\sqrt{(bx+a)^3}}-\frac{b\left(\frac{4}{\sqrt{bx+a}}-\frac{6\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)(bx+a)^{\frac{3}{2}}}{2a^2\sqrt{(bx+a)^3}}$	75

```
[In] int(1/x^2/((b*x+a)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)*(3*(b*x+a)^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*b*x-3*a^(1/2)*b*x-a
^(3/2))/((b*x+a)^3)^(1/2)/a^(5/2)/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(58) = 116.

Time = 0.26 (sec) , antiderivative size = 309, normalized size of antiderivative = 4.29

$$\int \frac{1}{x^2\sqrt{(a+bx)^3}}\,dx$$
$$= \left[\frac{3\,(b^3x^3+2\,ab^2x^2+a^2bx)\sqrt{a}\log\left(\frac{b^2x^2+3\,abx+2\,a^2+2\,\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}\sqrt{a}}{bx^2+ax}\right)-2\,\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2bx+ \cdots}{2\,(a^3b^2x^3+2\,a^4bx^2+a^5x)}\right.$$
$$-\frac{3\,(b^3x^3+2\,ab^2x^2+a^2bx)\sqrt{-a}\arctan\left(\frac{\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}\sqrt{-a}}{abx+a^2}\right)+\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}(3\,ab \cdots}{a^3b^2x^3+2\,a^4bx^2+a^5x}$$

```
[In] integrate(1/x^2/((b*x+a)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*sqrt(a)*log((b^2*x^2 + 3*a*b*x +
2*a^2 + 2*sqrt(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(a))/(b*x^2 + a
*x)) - 2*sqrt(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*(3*a*b*x + a^2))/(a^
3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x), -(3*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*sqr
t(-a)*arctan(sqrt(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(-a)/(a*b*x
+ a^2)) + sqrt(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*(3*a*b*x + a^2))/(a
^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)]
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{(a+bx)^3}}\,dx = \int \frac{1}{x^2\sqrt{(a+bx)^3}}\,dx$$

```
[In] integrate(1/x**2/((b*x+a)**3)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt((a + b*x)**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{(a+bx)^3}}\,dx = \int \frac{1}{\sqrt{(bx+a)^3}x^2}\,dx$$

```
[In] integrate(1/x^2/((b*x+a)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((b*x + a)^3)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\sqrt{(a+bx)^3}}\,dx = -\frac{3\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} - \frac{3\,(bx+a)b-2\,ab}{\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}a\right)a^2}$$

```
[In] integrate(1/x^2/((b*x+a)^3)^(1/2),x, algorithm="giac")
[Out] -3*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) - (3*(b*x + a)*b - 2*a*b
)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{(a+bx)^3}}\,dx = \int \frac{1}{x^2\,\sqrt{{\left(a+b\,x\right)}^3}}\,dx$$

```
[In] int(1/(x^2*((a + b*x)^3)^(1/2)),x)
[Out] int(1/(x^2*((a + b*x)^3)^(1/2)), x)
```

3.150 $\int \frac{1}{x^3\sqrt{(a+bx)^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \frac{1}{x^3\sqrt{(a+bx)^3}}\,dx = \frac{\frac{15b^2}{4a^3}-\frac{1}{2ax^2}+\frac{5b}{4a^2x}}{\sqrt{a+bx}}+\frac{15b^2\log\left(\frac{-\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}+\sqrt{a+bx}}\right)}{8a^{5/2}}$$

```
[Out] (-1/2/a/x^2+5/4*b/a^2/x+15/4*b^2/a^3)/(b*x+a)^(1/2)+15/8*b^2/a^(5/2)*ln(((b
*x+a)^(1/2)-a^(1/2))/((b*x+a)^(1/2)+a^(1/2)))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.38, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1973, 44, 53, 65, 214}

$$\int \frac{1}{x^3\sqrt{(a+bx)^3}}\,dx = \frac{15b^2(a+bx)}{4a^3\sqrt{(a+bx)^3}}-\frac{15b^2\left(\frac{bx}{a}+1\right)^{3/2}\text{arctanh}\left(\sqrt{\frac{bx}{a}+1}\right)}{4a^2\sqrt{(a+bx)^3}}+\frac{5b(a+bx)}{4a^2x\sqrt{(a+bx)^3}}-\frac{a+bx}{2ax^2\sqrt{(a+bx)^3}}$$

```
[In] Int[1/(x^3*Sqrt[(a + b*x)^3]),x]
```

```
[Out] (15*b^2*(a + b*x))/(4*a^3*Sqrt[(a + b*x)^3]) - (a + b*x)/(2*a*x^2*Sqrt[(a +
 b*x)^3]) + (5*b*(a + b*x))/(4*a^2*x*Sqrt[(a + b*x)^3]) - (15*b^2*(1 + (b*x
)/a)^(3/2)*ArcTanh[Sqrt[1 + (b*x)/a]])/(4*a^2*Sqrt[(a + b*x)^3])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(1+\frac{bx}{a}\right)^{3/2}\int\frac{1}{x^3\left(1+\frac{bx}{a}\right)^{3/2}}\,dx}{\sqrt{(a+bx)^3}}\\
&= -\frac{a+bx}{2ax^2\sqrt{(a+bx)^3}}-\frac{\left(5b\left(1+\frac{bx}{a}\right)^{3/2}\right)\int\frac{1}{x^2\left(1+\frac{bx}{a}\right)^{3/2}}\,dx}{4a\sqrt{(a+bx)^3}}\\
&= -\frac{a+bx}{2ax^2\sqrt{(a+bx)^3}}+\frac{5b(a+bx)}{4a^2x\sqrt{(a+bx)^3}}+\frac{\left(15b^2\left(1+\frac{bx}{a}\right)^{3/2}\right)\int\frac{1}{x\left(1+\frac{bx}{a}\right)^{3/2}}\,dx}{8a^2\sqrt{(a+bx)^3}}
\end{aligned}$$

$$= \frac{15b^2(a+bx)}{4a^3\sqrt{(a+bx)^3}} - \frac{a+bx}{2ax^2\sqrt{(a+bx)^3}} + \frac{5b(a+bx)}{4a^2x\sqrt{(a+bx)^3}} + \frac{\left(15b^2\left(1+\frac{bx}{a}\right)^{3/2}\right)\int\frac{1}{x\sqrt{1+\frac{bx}{a}}}\,dx}{8a^2\sqrt{(a+bx)^3}}$$

$$= \frac{15b^2(a+bx)}{4a^3\sqrt{(a+bx)^3}} - \frac{a+bx}{2ax^2\sqrt{(a+bx)^3}} + \frac{5b(a+bx)}{4a^2x\sqrt{(a+bx)^3}} + \frac{\left(15b\left(1+\frac{bx}{a}\right)^{3/2}\right)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{ax^2}{b}}\,dx, x, \sqrt{1+\frac{bx}{a}}\right)}{4a\sqrt{(a+bx)^3}}$$

$$= \frac{15b^2(a+bx)}{4a^3\sqrt{(a+bx)^3}} - \frac{a+bx}{2ax^2\sqrt{(a+bx)^3}} + \frac{5b(a+bx)}{4a^2x\sqrt{(a+bx)^3}} - \frac{15b^2\left(1+\frac{bx}{a}\right)^{3/2}\text{arctanh}\left(\sqrt{1+\frac{bx}{a}}\right)}{4a^2\sqrt{(a+bx)^3}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int\frac{1}{x^3\sqrt{(a+bx)^3}}\,dx = -\frac{(a+bx)\left(\sqrt{a}(2a^2-5abx-15b^2x^2)+15b^2x^2\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{4a^{7/2}x^2\sqrt{(a+bx)^3}}$$

```
[In] Integrate[1/(x^3*Sqrt[(a + b*x)^3]),x]
```

```
[Out] -1/4*((a + b*x)*(Sqrt[a]*(2*a^2 - 5*a*b*x - 15*b^2*x^2) + 15*b^2*x^2*Sqrt[a
 + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(7/2)*x^2*Sqrt[(a + b*x)^3])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{(bx+a)\left(15\sqrt{bx+a}\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^2x^2-5a^{\frac{3}{2}}bx-15b^2x^2\sqrt{a}+2a^{\frac{5}{2}}\right)}{4\sqrt{(bx+a)^3}\,a^{\frac{7}{2}}x^2}$	74
risch	$-\frac{(bx+a)^2(-7bx+2a)}{4a^3x^2\sqrt{(bx+a)^3}}+\frac{b^2\left(\frac{16}{\sqrt{bx+a}}-\frac{30\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)(bx+a)^{\frac{3}{2}}}{8a^3\sqrt{(bx+a)^3}}$	85

```
[In] int(1/x^3/((b*x+a)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(b*x+a)*(15*(b*x+a)^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*b^2*x^2-5*a^(
3/2)*b*x-15*b^2*x^2*a^(1/2)+2*a^(5/2))/((b*x+a)^3)^(1/2)/a^(7/2)/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(72) = 144.

Time = 0.26 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.81

$$\int \frac{1}{x^3\sqrt{(a+bx)^3}}\,dx = \left[\frac{15\,(b^4x^4+2\,ab^3x^3+a^2b^2x^2)\sqrt{a}\log\left(\frac{b^2x^2+3\,abx+2\,a^2-2\,\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}\sqrt{a}}{bx^2+ax}\right)+2\,\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2}}{8\,(a^4b^2x^4+2\,a^5bx^3+a^6x^2)}\right.$$

```
[In] integrate(1/x^3/((b*x+a)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(a)*log((b^2*x^2 + 3*a*b
*x + 2*a^2 - 2*sqrt(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(a))/(b*x^
2 + a*x)) + 2*sqrt(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*(15*a*b^2*x^2 +
 5*a^2*b*x - 2*a^3))/(a^4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2), 1/4*(15*(b^4*x^
4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-a)*arctan(sqrt(b^3*x^3 + 3*a*b^2*x^2 +
 3*a^2*b*x + a^3)*sqrt(-a)/(a*b*x + a^2)) + sqrt(b^3*x^3 + 3*a*b^2*x^2 + 3*
a^2*b*x + a^3)*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3))/(a^4*b^2*x^4 + 2*a^5*b*x
^3 + a^6*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{(a+bx)^3}}\,dx = \int \frac{1}{x^3\sqrt{(a+bx)^3}}\,dx$$

```
[In] integrate(1/x**3/((b*x+a)**3)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt((a + b*x)**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{(a+bx)^3}}\,dx = \int \frac{1}{\sqrt{(bx+a)^3}x^3}\,dx$$

```
[In] integrate(1/x^3/((b*x+a)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((b*x + a)^3)*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3\sqrt{(a+bx)^3}}\,dx = \frac{15\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^3} + \frac{2\,b^2}{\sqrt{bx+a}a^3} + \frac{7\,(bx+a)^{\frac{3}{2}}b^2 - 9\,\sqrt{bx+a}ab^2}{4\,a^3b^2x^2}$$

```
[In] integrate(1/x^3/((b*x+a)^3)^(1/2),x, algorithm="giac")
[Out] 15/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2*b^2/(sqrt(b*x +
a)*a^3) + 1/4*(7*(b*x + a)^(3/2)*b^2 - 9*sqrt(b*x + a)*a*b^2)/(a^3*b^2*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{(a+bx)^3}}\,dx = \int \frac{1}{x^3\,\sqrt{{(a+b\,x)}^3}}\,dx$$

```
[In] int(1/(x^3*((a + b*x)^3)^(1/2)),x)
[Out] int(1/(x^3*((a + b*x)^3)^(1/2)), x)
```

3.151 $\int \frac{1}{x\sqrt[3]{(a+bx)^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \frac{1}{x\sqrt[3]{(a+bx)^2}}\,dx = \frac{-\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{2\sqrt[3]{a}+\sqrt[3]{a+bx}}\right)+\frac{3}{2}\log\left(\frac{-\sqrt[3]{a}+\sqrt[3]{a+bx}}{\sqrt[3]{x}}\right)}{\sqrt[3]{a^2}}$$

```
[Out] 1/(a^2)^(1/3)*(3/2*ln(((b*x+a)^(1/3)-a^(1/3))/x^(1/3))-3^(1/2)*arctan(3^(1/2)*(b*x+a)^(1/3)/((b*x+a)^(1/3)+2*a^(1/3))))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.61, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1973, 59, 632, 210, 31}

$$\int \frac{1}{x\sqrt[3]{(a+bx)^2}}\,dx = -\frac{\sqrt{3}\left(\frac{bx}{a}+1\right)^{2/3}\arctan\left(\frac{2\sqrt[3]{\frac{bx}{a}+1}+1}{\sqrt{3}}\right)}{\sqrt[3]{(a+bx)^2}} - \frac{\log(x)\left(\frac{bx}{a}+1\right)^{2/3}}{2\sqrt[3]{(a+bx)^2}} + \frac{3\left(\frac{bx}{a}+1\right)^{2/3}\log\left(1-\sqrt[3]{\frac{bx}{a}+1}\right)}{2\sqrt[3]{(a+bx)^2}}$$

```
[In] Int[1/(x*((a + b*x)^2)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*(1 + (b*x)/a)^(2/3)*ArcTan[(1 + 2*(1 + (b*x)/a)^(1/3))/Sqrt[3]])/((a + b*x)^2)^(1/3)) - ((1 + (b*x)/a)^(2/3)*Log[x])/(2*((a + b*x)^2)^(1/3)
```

```
) + (3*(1 + (b*x)/a)^(2/3)*Log[1 - (1 + (b*x)/a)^(1/3)])/(2*((a + b*x)^2)^(
1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(1+\frac{bx}{a}\right)^{2/3} \int \frac{1}{x\left(1+\frac{bx}{a}\right)^{2/3}}\,dx}{\sqrt[3]{(a+bx)^2}} \\
&= -\frac{\left(1+\frac{bx}{a}\right)^{2/3}\log(x)}{2\sqrt[3]{(a+bx)^2}} - \frac{\left(3\left(1+\frac{bx}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+\frac{bx}{a}}\right)}{2\sqrt[3]{(a+bx)^2}} \\
&\quad - \frac{\left(3\left(1+\frac{bx}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1+\frac{bx}{a}}\right)}{2\sqrt[3]{(a+bx)^2}}
\end{aligned}$$

$$= -\frac{\left(1+\frac{bx}{a}\right)^{2/3}\log(x)}{2\sqrt[3]{(a+bx)^2}} + \frac{3\left(1+\frac{bx}{a}\right)^{2/3}\log\left(1-\sqrt[3]{1+\frac{bx}{a}}\right)}{2\sqrt[3]{(a+bx)^2}} + \frac{\left(3\left(1+\frac{bx}{a}\right)^{2/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+\frac{bx}{a}}\right)}{\sqrt[3]{(a+bx)^2}}$$

$$= -\frac{\sqrt{3}\left(1+\frac{bx}{a}\right)^{2/3}\arctan\left(\frac{1+2\sqrt[3]{1+\frac{bx}{a}}}{\sqrt{3}}\right)}{\sqrt[3]{(a+bx)^2}} - \frac{\left(1+\frac{bx}{a}\right)^{2/3}\log(x)}{2\sqrt[3]{(a+bx)^2}} + \frac{3\left(1+\frac{bx}{a}\right)^{2/3}\log\left(1-\sqrt[3]{1+\frac{bx}{a}}\right)}{2\sqrt[3]{(a+bx)^2}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\sqrt[3]{(a+bx)^2}}\,dx = -\frac{(a+bx)^{2/3}\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) + \log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)\right)}{2a^{2/3}\sqrt[3]{(a+bx)^2}}$$

```
[In] Integrate[1/(x*((a + b*x)^2)^(1/3)),x]
```

```
[Out] -1/2*((a + b*x)^(2/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/S
qrt[3]] - 2*Log[a^(1/3) - (a + b*x)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x
)^(1/3) + (a + b*x)^(2/3)]))/(a^(2/3)*((a + b*x)^2)^(1/3))
```

Maple [F]

$$\int \frac{1}{x\left((bx+a)^2\right)^{\frac{1}{3}}}\,dx$$

```
[In] int(1/x/((b*x+a)^2)^(1/3),x)
```

```
[Out] int(1/x/((b*x+a)^2)^(1/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(60) = 120$.

Time = 0.26 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.85

$$\int \frac{1}{x\sqrt[3]{(a+bx)^2}}\,dx = \frac{2\sqrt{3}(a^2)^{\frac{1}{6}}a\arctan\left(\frac{\sqrt{3}(a^2)^{\frac{1}{6}}\left((a^2)^{\frac{1}{3}}(bx+a)+2\,(b^2x^2+2\,abx+a^2)^{\frac{1}{3}}a\right)}{3\,(abx+a^2)}\right) - (a^2)^{\frac{2}{3}}\log\left(\frac{(b^2x^2+2\,abx+a^2)^{\frac{2}{3}}a^2+(b^2x^2+2\,abx+a^2)^{\frac{1}{3}}\cdots}{b^2x^2+2\,a\cdots}\right.\cdots}{2\,a^2}$$

```
[In] integrate(1/x/((b*x+a)^2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*sqrt(3)*(a^2)^(1/6)*((a^2)^(1/3)*(b
*x + a) + 2*(b^2*x^2 + 2*a*b*x + a^2)^(1/3)*a)/(a*b*x + a^2)) - (a^2)^(2/3)
*log(((b^2*x^2 + 2*a*b*x + a^2)^(2/3)*a^2 + (b^2*x^2 + 2*a*b*x + a^2)^(1/3)
*(a*b*x + a^2)*(a^2)^(1/3) + (b^2*x^2 + 2*a*b*x + a^2)*(a^2)^(2/3))/(b^2*x^
2 + 2*a*b*x + a^2)) + 2*(a^2)^(2/3)*log(-((a^2)^(1/3)*(b*x + a) - (b^2*x^2
+ 2*a*b*x + a^2)^(1/3)*a)/(b*x + a)))/a^2
```

Sympy [F]

$$\int \frac{1}{x\sqrt[3]{(a+bx)^2}}\,dx = \int \frac{1}{x\sqrt[3]{\left(a+bx\right)^{2}}}\,dx$$

```
[In] integrate(1/x/((b*x+a)**2)**(1/3),x)
```

```
[Out] Integral(1/(x*((a + b*x)**2)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[3]{(a+bx)^2}}\,dx = \int \frac{1}{\left((bx+a)^2\right)^{\frac{1}{3}}x}\,dx$$

[In] integrate(1/x/((b*x+a)^2)^(1/3),x, algorithm="maxima")

[Out] integrate(1/(((b*x + a)^2)^(1/3)*x), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(60) = 120$.

Time = 3.15 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.77

$$\int \frac{1}{x\sqrt[3]{(a+bx)^2}}\,dx$$
$$= -\frac{\sqrt{3}(a\mathrm{sgn}(bx+a))^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx\mathrm{sgn}(bx+a)+a\mathrm{sgn}(bx+a))^{\frac{1}{3}}+(a\mathrm{sgn}(bx+a))^{\frac{1}{3}}\right)}{3\,(a\mathrm{sgn}(bx+a))^{\frac{1}{3}}}\right)}{a\mathrm{sgn}\,(bx+a)}$$
$$-\frac{(a\mathrm{sgn}(bx+a))^{\frac{1}{3}}\log\left((bx\mathrm{sgn}(bx+a)+a\mathrm{sgn}(bx+a))^{\frac{2}{3}}+(bx\mathrm{sgn}(bx+a)+a\mathrm{sgn}(bx+a))^{\frac{1}{3}}(a\mathrm{sgn}(bx+a\ldots\right.}{2\,a\mathrm{sgn}\,(bx+a)}$$
$$+\frac{(a\mathrm{sgn}(bx+a))^{\frac{1}{3}}\log\left(\left|(bx\mathrm{sgn}(bx+a)+a\mathrm{sgn}(bx+a))^{\frac{1}{3}}-(a\mathrm{sgn}(bx+a))^{\frac{1}{3}}\right|\right)}{a\mathrm{sgn}\,(bx+a)}$$

[In] integrate(1/x/((b*x+a)^2)^(1/3),x, algorithm="giac")

[Out] -sqrt(3)*(a*sgn(b*x + a))^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x*sgn(b*x + a) + a
*sgn(b*x + a))^(1/3) + (a*sgn(b*x + a))^(1/3))/(a*sgn(b*x + a))^(1/3))/(a*s
gn(b*x + a)) - 1/2*(a*sgn(b*x + a))^(1/3)*log((b*x*sgn(b*x + a) + a*sgn(b*x
 + a))^(2/3) + (b*x*sgn(b*x + a) + a*sgn(b*x + a))^(1/3)*(a*sgn(b*x + a))^(
1/3) + (a*sgn(b*x + a))^(2/3))/(a*sgn(b*x + a)) + (a*sgn(b*x + a))^(1/3)*lo
g(abs((b*x*sgn(b*x + a) + a*sgn(b*x + a))^(1/3) - (a*sgn(b*x + a))^(1/3)))/
(a*sgn(b*x + a))

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[3]{(a+bx)^2}}\,dx = \int \frac{1}{x\,\left((a+b\,x)^2\right)^{1/3}}\,dx$$

```
[In] int(1/(x*((a + b*x)^2)^(1/3)),x)
[Out] int(1/(x*((a + b*x)^2)^(1/3)), x)
```

3.152 $\int \frac{\sqrt[3]{a+bx}}{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 92

$$\int \frac{\sqrt[3]{a+bx}}{x}\,dx = 3\sqrt[3]{a+bx}+\frac{a\left(-\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{2\sqrt[3]{a}+\sqrt[3]{a+bx}}\right)+\frac{3}{2}\log\left(\frac{-\sqrt[3]{a}+\sqrt[3]{a+bx}}{\sqrt[3]{x}}\right)\right)}{\sqrt[3]{a^2}}$$

```
[Out] 3*(b*x+a)^(1/3)+a/(a^2)^(1/3)*(3/2*ln(((b*x+a)^(1/3)-a^(1/3))/x^(1/3))-3^(1
/2)*arctan(3^(1/2)*(b*x+a)^(1/3)/((b*x+a)^(1/3)+2*a^(1/3))))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {52, 59, 631, 210, 31}

$$\int \frac{\sqrt[3]{a+bx}}{x}\,dx = -\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)+3\sqrt[3]{a+bx} +\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{1}{2}\sqrt[3]{a}\log(x)$$

```
[In] Int[(a + b*x)^(1/3)/x,x]
```

```
[Out] 3*(a + b*x)^(1/3) - Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(S
qrt[3]*a^(1/3))] - (a^(1/3)*Log[x])/2 + (3*a^(1/3)*Log[a^(1/3) - (a + b*x)^
(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\sqrt[3]{a+bx}+a\int \frac{1}{x(a+bx)^{2/3}}\,dx \\
&= 3\sqrt[3]{a+bx}-\frac{1}{2}\sqrt[3]{a}\log(x)-\frac{1}{2}\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right) \\
&\quad -\frac{1}{2}\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx,x,\sqrt[3]{a+bx}\right) \\
&= 3\sqrt[3]{a+bx}-\frac{1}{2}\sqrt[3]{a}\log(x)+\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) \\
&\quad +\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)
\end{aligned}
$$

$$= 3\sqrt[3]{a+bx} - \sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - \frac{1}{2}\sqrt[3]{a}\log(x) + \frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx}\right)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt[3]{a+bx}}{x}\,dx = 3\sqrt[3]{a+bx} - \sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + \sqrt[3]{a}\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx}\right) - \frac{1}{2}\sqrt[3]{a}\log\left(a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx} + (a+bx)^{2/3}\right)$$

```
[In] Integrate[(a + b*x)^(1/3)/x,x]
[Out] 3*(a + b*x)^(1/3) - Sqrt[3]*a^(1/3)*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3)
)/Sqrt[3]] + a^(1/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - (a^(1/3)*Log[a^(2/3)
+ a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/2
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$3(bx+a)^{\frac{1}{3}} + a^{\frac{1}{3}}\ln\left((bx+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right) - \frac{a^{\frac{1}{3}}\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2} - a^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\right.$
derivativedivides	$3(bx+a)^{\frac{1}{3}} + 3\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}} - \frac{\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}} - \frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{2}{3}}}\right.$
default	$3(bx+a)^{\frac{1}{3}} + 3\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}} - \frac{\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}} - \frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{2}{3}}}\right.$

```
[In] int((b*x+a)^(1/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3*(b*x+a)^(1/3)+a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2*a^(1/3)*ln((b*x+a)^(2
/3)+(b*x+a)^(1/3)*a^(1/3)+a^(2/3))-a^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b
*x+a)^(1/3)+a^(1/3))/a^(1/3))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt[3]{a+bx}}{x}\,dx = -\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right) - \frac{1}{2}a^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + a^{\frac{1}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3(bx+a)^{\frac{1}{3}}$$

```
[In] integrate((b*x+a)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3) + sqrt(3)*a)
/a) - 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
+ a^(1/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(b*x + a)^(1/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.96

$$\int \frac{\sqrt[3]{a+bx}}{x}\,dx = \frac{4\sqrt[3]{a}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{4\sqrt[3]{a}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{4\sqrt[3]{a}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{4\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\Gamma\left(\frac{4}{3}\right)}{\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((b*x+a)**(1/3)/x,x)
```

```
[Out] 4*a**(1/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(3*gamma(
7/3)) + 4*a**(1/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pol
ar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*a**(1/3)*exp(2*I*pi/3)
*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)
/(3*gamma(7/3)) + 4*b**(1/3)*(a/b + x)**(1/3)*gamma(4/3)/gamma(7/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+bx}}{x}\,dx = -\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{2}\,a^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + a^{\frac{1}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\,(bx+a)^{\frac{1}{3}}$$

```
[In] integrate((b*x+a)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(1/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(b*x + a)^(1/3)
```

Giac [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt[3]{a+bx}}{x}\,dx = -\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{2}\,a^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + a^{\frac{1}{3}}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+3\,(bx+a)^{\frac{1}{3}}$$

```
[In] integrate((b*x+a)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(1/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3*(b*x + a)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 18.57 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt[3]{a+bx}}{x}\,dx = a^{1/3}\,\ln\left(9\,a\,(a+b\,x)^{1/3}-9\,a^{4/3}\right)+3\,(a+b\,x)^{1/3}+\frac{a^{1/3}\,\ln\left(9\,a\,(a+b\,x)^{1/3}-\frac{9\,a^{4/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}-\frac{a^{1/3}\,\ln\left(9\,a\,(a+b\,x\right.}{}$$

```
[In] int((a + b*x)^(1/3)/x,x)
```

```
[Out] a^(1/3)*log(9*a*(a + b*x)^(1/3) - 9*a^(4/3)) + 3*(a + b*x)^(1/3) + (a^(1/3)
*log(9*a*(a + b*x)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1)
)/2 - (a^(1/3)*log(9*a*(a + b*x)^(1/3) + (9*a^(4/3)*(3^(1/2)*1i + 1))/2)*(3
^(1/2)*1i + 1))/2
```

3.153 $\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 122

$$\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx=\frac{b\sqrt[3]{a+bx}}{a}+\frac{(-a-bx)\sqrt[3]{a+bx}}{ax}+\frac{b\left(-\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{2\sqrt[3]{a}+\sqrt[3]{a+bx}}\right)+\frac{3}{2}\log\left(\frac{-\sqrt[3]{a}+\sqrt[3]{a+bx}}{\sqrt[3]{x}}\right)\right)}{3\sqrt[3]{a^2}}$$

```
[Out] -(b*x+a)^(4/3)/a/x+b/a*(b*x+a)^(1/3)+1/3*b/(a^2)^(1/3)*(3/2*ln(((b*x+a)^(1/
3)-a^(1/3))/x^(1/3))-3^(1/2)*arctan(3^(1/2)*(b*x+a)^(1/3)/((b*x+a)^(1/3)+2*
a^(1/3))))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {43, 59, 631, 210, 31}

$$\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx=-\frac{b\arctan\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}}-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\sqrt[3]{a+bx}}{x}$$

```
[In] Int[(a + b*x)^(1/3)/x^2,x]
```

```
[Out] -((a + b*x)^(1/3)/x) - (b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a^(2/3)) - (b*Log[x])/(6*a^(2/3)) + (b*Log[a^(1/3) - (a +
b*x)^(1/3)])/(2*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt[3]{a+bx}}{x} + \frac{1}{3}b\int \frac{1}{x(a+bx)^{2/3}}\,dx$$

$$= -\frac{\sqrt[3]{a+bx}}{x} - \frac{b\log(x)}{6a^{2/3}} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{2a^{2/3}} - \frac{b\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}$$

$$= -\frac{\sqrt[3]{a+bx}}{x} - \frac{b\log(x)}{6a^{2/3}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}} + \frac{b\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{2/3}}$$

$$= -\frac{\sqrt[3]{a+bx}}{x} - \frac{b\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}} - \frac{b\log(x)}{6a^{2/3}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx =$$

$$-\frac{6a^{2/3}\sqrt[3]{a+bx} + 2\sqrt{3}bx\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2bx\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) + bx\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+\right.}{6a^{2/3}x}$$

```
[In] Integrate[(a + b*x)^(1/3)/x^2,x]
```

```
[Out] -1/6*(6*a^(2/3)*(a + b*x)^(1/3) + 2*Sqrt[3]*b*x*ArcTan[(1 + (2*(a + b*x)^(1
/3))/a^(1/3))/Sqrt[3]] - 2*b*x*Log[a^(1/3) - (a + b*x)^(1/3)] + b*x*Log[a^(
2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(a^(2/3)*x)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.78

method	result
derivativedivides	$3b\left(-\frac{(bx+a)^{\frac{1}{3}}}{3bx}+\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18a^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{2}{3}}}\right)$
default	$3b\left(-\frac{(bx+a)^{\frac{1}{3}}}{3bx}+\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18a^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{2}{3}}}\right)$
pseudoelliptic	$\frac{-\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)bx+\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)bx-\frac{\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)bx}{2}-3(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}}{3a^{\frac{2}{3}}x}$

```
[In] int((b*x+a)^(1/3)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3*b*(-1/3*(b*x+a)^(1/3)/b/x+1/9/a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/18/a^(2
/3)*ln((b*x+a)^(2/3)+(b*x+a)^(1/3)*a^(1/3)+a^(2/3))-1/9/a^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx = -\frac{2\sqrt{3}(a^2)^{\frac{1}{6}}abx\arctan\left(\frac{(a^2)^{\frac{1}{6}}\left(\sqrt{3}(a^2)^{\frac{1}{3}}a+2\sqrt{3}(a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3a^2}\right)+(a^2)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(a^2)^{\frac{2}{3}}(bx}{6a^2x}$$

```
[In] integrate((b*x+a)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(a^2)^(1/6)*a*b*x*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/
3)*a + 2*sqrt(3)*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + (a^2)^(2/3)*b*x*log((b
*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) - 2*(a^2)^(2
/3)*b*x*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x + a)^(1/3)*a^2)/(a^2*
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 643, normalized size of antiderivative = 5.27

$$\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx = \frac{4a^{\frac{7}{3}}be^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)} + \frac{4a^{\frac{7}{3}}b\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)} + \frac{4a^{\frac{7}{3}}be^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)} - \frac{4a^{\frac{4}{3}}b^2\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)} - \frac{4a^{\frac{4}{3}}b^2\left(\frac{a}{b}+x\right)\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)} - \frac{4a^{\frac{4}{3}}b^2\left(\frac{a}{b}+x\right)e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)} + \frac{12a^2b^{\frac{4}{3}}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((b*x+a)**(1/3)/x**2,x)
```

```
[Out] 4*a**(7/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamm
a(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*
gamma(7/3)) + 4*a**(7/3)*b*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*
pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b
 + x)*exp(2*I*pi/3)*gamma(7/3)) + 4*a**(7/3)*b*exp(-2*I*pi/3)*log(1 - b**(1
/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2
*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(
4/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3
))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I
*pi/3)*gamma(7/3)) - 4*a**(4/3)*b**2*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**
(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(
7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(4/3)*b**2*(a/b
+ x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a
**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*e
xp(2*I*pi/3)*gamma(7/3)) + 12*a**2*b**(4/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*
gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi
/3)*gamma(7/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx = -\frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}} - \frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{2}{3}}} + \frac{b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{2}{3}}} - \frac{(bx+a)^{\frac{1}{3}}}{x}$$

```
[In] integrate((b*x+a)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(2/3) - 1/6*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2
/3) + 1/3*b*log((b*x + a)^(1/3) - a^(1/3))/a^(2/3) - (b*x + a)^(1/3)/x
```

Giac [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx = -\frac{\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{2\,b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}+\frac{6\,(bx+a)^{\frac{1}{3}}b}{x}}{6\,b}$$

```
[In] integrate((b*x+a)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] -1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(2/3) + b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(2/3) - 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3) + 6*(b*x + a)^(1
/3)*b/x)/b
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx = \frac{b\,\ln\left(3\,b\,(a+b\,x)^{1/3}-3\,a^{1/3}\,b\right)}{3\,a^{2/3}}-\frac{(a+b\,x)^{1/3}}{x} - \frac{\ln\left(\frac{3\,a^{1/3}\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{2}+3\,b\,(a+b\,x)^{1/3}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{2/3}} - \frac{\ln\left(\frac{3\,a^{1/3}\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{2}+3\,b\,(a+b\,x)^{1/3}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{2/3}}$$

```
[In] int((a + b*x)^(1/3)/x^2,x)
```

```
[Out] (b*log(3*b*(a + b*x)^(1/3) - 3*a^(1/3)*b))/(3*a^(2/3)) - (a + b*x)^(1/3)/x
- (log((3*a^(1/3)*(b - 3^(1/2)*b*1i))/2 + 3*b*(a + b*x)^(1/3))*(b - 3^(1/2)
*b*1i))/(6*a^(2/3)) - (log((3*a^(1/3)*(b + 3^(1/2)*b*1i))/2 + 3*b*(a + b*x)
^(1/3))*(b + 3^(1/2)*b*1i))/(6*a^(2/3))
```

3.154 $\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 140

$$\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx = -\frac{b^2\sqrt[3]{a+bx}}{3a^2}+\left(-\frac{1}{2ax^2}+\frac{b}{3a^2x}\right)(a+bx)^{4/3} - \frac{b^2\left(-\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{2\sqrt[3]{a}+\sqrt[3]{a+bx}}\right)+\frac{3}{2}\log\left(\frac{-\sqrt[3]{a}+\sqrt[3]{a+bx}}{\sqrt[3]{x}}\right)\right)}{9a\sqrt[3]{a^2}}$$

```
[Out] (-1/2/a/x^2+1/3*b/a^2/x)*(b*x+a)^(4/3)-1/3*b^2/a^2*(b*x+a)^(1/3)-1/9*b^2/a/
(a^2)^(1/3)*(3/2*ln(((b*x+a)^(1/3)-a^(1/3))/x^(1/3))-3^(1/2)*arctan(3^(1/2)
*(b*x+a)^(1/3)/((b*x+a)^(1/3)+2*a^(1/3))))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {43, 44, 59, 631, 210, 31}

$$\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx = \frac{b^2\arctan\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}}+\frac{b^2\log(x)}{18a^{5/3}} - \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}}-\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}$$

```
[In] Int[(a + b*x)^(1/3)/x^3,x]
```

```
[Out] -1/2*(a + b*x)^(1/3)/x^2 - (b*(a + b*x)^(1/3))/(6*a*x) + (b^2*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)) + (b^2*Log[
x])/(18*a^(5/3)) - (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt[3]{a+bx}}{2x^2} + \frac{1}{6}b\int \frac{1}{x^2(a+bx)^{2/3}}\,dx$$

$$= -\frac{\sqrt[3]{a+bx}}{2x^2} - \frac{b\sqrt[3]{a+bx}}{6ax} - \frac{b^2 \int \frac{1}{x(a+bx)^{2/3}} \, dx}{9a}$$

$$= -\frac{\sqrt[3]{a+bx}}{2x^2} - \frac{b\sqrt[3]{a+bx}}{6ax} + \frac{b^2 \log(x)}{18a^{5/3}} + \frac{b^2 \text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x} \, dx, x, \sqrt[3]{a+bx}\right)}{6a^{5/3}} + \frac{b^2 \text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2} \, dx, x, \sqrt[3]{a+bx}\right)}{6a^{4/3}}$$

$$= -\frac{\sqrt[3]{a+bx}}{2x^2} - \frac{b\sqrt[3]{a+bx}}{6ax} + \frac{b^2 \log(x)}{18a^{5/3}} - \frac{b^2 \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}} - \frac{b^2 \text{Subst}\left(\int \frac{1}{-3-x^2} \, dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{3a^{5/3}}$$

$$= -\frac{\sqrt[3]{a+bx}}{2x^2} - \frac{b\sqrt[3]{a+bx}}{6ax} + \frac{b^2 \arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{5/3}} + \frac{b^2 \log(x)}{18a^{5/3}} - \frac{b^2 \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{a+bx}}{x^3} \, dx$$

$$= \frac{-\frac{3a^{2/3}\sqrt[3]{a+bx}(3a+bx)}{x^2} + 2\sqrt{3}b^2 \arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2b^2 \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) + b^2 \log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a}\right.}{18a^{5/3}}$$

```
[In] Integrate[(a + b*x)^(1/3)/x^3,x]
```

```
[Out] ((-3*a^(2/3)*(a + b*x)^(1/3)*(3*a + b*x))/x^2 + 2*Sqrt[3]*b^2*ArcTan[(1 + (
2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*b^2*Log[a^(1/3) - (a + b*x)^(1/3)]
 + b^2*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(18*a^(5/3
))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

method	result
derivativedivides	$3b^2\left(-\frac{-\frac{(bx+a)^{\frac{4}{3}}}{18a}+\frac{(bx+a)^{\frac{1}{3}}}{9}}{b^2x^2}-\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{2}{3}}}}{9a}\right.$
default	$3b^2\left(-\frac{-\frac{(bx+a)^{\frac{4}{3}}}{18a}+\frac{(bx+a)^{\frac{1}{3}}}{9}}{b^2x^2}-\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{2}{3}}}}{9a}\right.$
pseudoelliptic	$\frac{2b^2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)x^2-2b^2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)x^2+b^2\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)x^2-3bx(bx+a}{18a^{\frac{5}{3}}x^2}$

```
[In] int((b*x+a)^(1/3)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*b^2*(-(1/18/a*(b*x+a)^(4/3)+1/9*(b*x+a)^(1/3))/b^2/x^2-1/9/a*(1/3/a^(2/3)
*ln((b*x+a)^(1/3)-a^(1/3))-1/6/a^(2/3)*ln((b*x+a)^(2/3)+(b*x+a)^(1/3)*a^(1/
3)+a^(2/3))-1/3/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)
+1))))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx$$

$$= \frac{2\sqrt{3}ab^2x^2\sqrt{-(-a^2)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}(-a^2)^{\frac{1}{3}}a-2\sqrt{3}(-a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)\sqrt{-(-a^2)^{\frac{1}{3}}}}{3a^2}\right)+(-a^2)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}a\right.}{18\,a}$$

```
[In] integrate((b*x+a)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/18*(2*sqrt(3)*a*b^2*x^2*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(
1/3)*a - 2*sqrt(3)*(-a^2)^(2/3)*(b*x + a)^(1/3))*sqrt(-(-a^2)^(1/3))/a^2) +
 (-a^2)^(2/3)*b^2*x^2*log((b*x + a)^(2/3)*a - (-a^2)^(1/3)*a + (-a^2)^(2/3)
*(b*x + a)^(1/3)) - 2*(-a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (-a^2)^(
2/3)) - 3*(a^2*b*x + 3*a^3)*(b*x + a)^(1/3))/(a^3*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.91 (sec) , antiderivative size = 2266, normalized size of antiderivative = 16.19

$$\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)**(1/3)/x**3,x)
```

```
[Out] -4*a**(16/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))
*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I
*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27
*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(16/3)*b**2*log(1
- b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a*
*7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)
+ 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b +
x)**3*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(16/3)*b**2*exp(-2*I*pi/3)*log(1 - b
**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*
exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 8
1*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)*
*3*exp(2*I*pi/3)*gamma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*exp(2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)
*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(
a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi
/3)*gamma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**(
1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(
7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x
)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gam
ma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b
 + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3
)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*
(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*p
i/3)*gamma(7/3)) - 12*a**(10/3)*b**4*(a/b + x)**2*exp(2*I*pi/3)*log(1 - b**
(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/
3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)*
*2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma
(7/3)) - 12*a**(10/3)*b**4*(a/b + x)**2*log(1 - b**(1/3)*(a/b + x)**(1/3)*e
```

```
xp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) -
 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*e
xp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3
)) - 12*a**(10/3)*b**4*(a/b + x)**2*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b +
x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*g
amma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/
b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3
)*gamma(7/3)) + 4*a**(7/3)*b**5*(a/b + x)**3*exp(2*I*pi/3)*log(1 - b**(1/3)
*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) -
81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*ex
p(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)
) + 4*a**(7/3)*b**5*(a/b + x)**3*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pola
r(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**
6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*
pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) + 4*
a**(7/3)*b**5*(a/b + x)**3*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)
*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3)
 - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2
*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7
/3)) - 12*a**5*b**(7/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(4/3)/(27*a**7*
exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 8
1*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)*
*3*exp(2*I*pi/3)*gamma(7/3)) + 6*a**4*b**(10/3)*(a/b + x)**(4/3)*exp(2*I*pi
/3)*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(
2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) -
 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) + 6*a**3*b**(13/3)*(a/
b + x)**(7/3)*exp(2*I*pi/3)*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) -
81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*ex
p(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx = \frac{\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{5}{3}}} + \frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{5}{3}}} - \frac{b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{5}{3}}} - \frac{(bx+a)^{\frac{4}{3}}b^2+2\,(bx+a)^{\frac{1}{3}}ab^2}{6\left((bx+a)^2a-2\,(bx+a)a^2+a^3\right)}$$

```
[In] integrate((b*x+a)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(5/3) + 1/18*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(5/3) - 1/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(5/3) - 1/6*((b*x + a)^(
4/3)*b^2 + 2*(b*x + a)^(1/3)*a*b^2)/((b*x + a)^2*a - 2*(b*x + a)*a^2 + a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx = \frac{\frac{2\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2\,b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\frac{3\left((bx+a)^{\frac{4}{3}}b^3+2\,(bx+a)^{\frac{1}{3}}ab^3\right)}{ab^2x^2}}{18\,b}$$

```
[In] integrate((b*x+a)^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] 1/18*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(5/3) + b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(5/3) - 2*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*((b*x + a)^(
4/3)*b^3 + 2*(b*x + a)^(1/3)*a*b^3)/(a*b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 20.99 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx = \frac{b^2\,\ln\left(\frac{b^2}{(-a)^{2/3}}-\frac{b^2\,(a+b\,x)^{1/3}}{a}\right)}{9\,(-a)^{5/3}}-\frac{\ln\left(\frac{b^2+\sqrt{3}\,b^2\,1\mathrm{i}}{2\,(-a)^{2/3}}+\frac{b^2\,(a+b\,x)^{1/3}}{a}\right)\,\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)}{18\,(-a)^{5/3}} - \frac{\frac{b^2\,(a+b\,x)^{1/3}}{3}+\frac{b^2\,(a+b\,x)^{4/3}}{6\,a}}{(a+b\,x)^2-2\,a\,(a+b\,x)+a^2} + \frac{b^2\,\ln\left(\frac{b^2\,(a+b\,x)^{1/3}}{a}-\frac{b^2\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{(-a)^{2/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,(-a)^{5/3}}$$

```
[In] int((a + b*x)^(1/3)/x^3,x)
```

```
[Out] (b^2*log(b^2/(-a)^(2/3) - (b^2*(a + b*x)^(1/3))/a))/(9*(-a)^(5/3)) - (log((
3^(1/2)*b^2*1i + b^2)/(2*(-a)^(2/3)) + (b^2*(a + b*x)^(1/3))/a)*(3^(1/2)*b^
2*1i + b^2))/(18*(-a)^(5/3)) - ((b^2*(a + b*x)^(1/3))/3 + (b^2*(a + b*x)^(4
/3))/(6*a))/((a + b*x)^2 - 2*a*(a + b*x) + a^2) + (b^2*log((b^2*(a + b*x)^(
1/3))/a - (b^2*((3^(1/2)*1i)/2 - 1/2))/(-a)^(2/3))*((3^(1/2)*1i)/2 - 1/2))/
(9*(-a)^(5/3))
```

3.155 $\int \frac{1}{x^2\sqrt[3]{(a+bx)^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{1}{x^2\sqrt[3]{(a+bx)^2}}\,dx = -\frac{\sqrt[3]{a+bx}}{ax} - \frac{2b\left(-\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{2\sqrt[3]{a}+\sqrt[3]{a+bx}}\right)+\frac{3}{2}\log\left(\frac{-\sqrt[3]{a}+\sqrt[3]{a+bx}}{\sqrt[3]{x}}\right)\right)}{3a\sqrt[3]{a^2}}$$

```
[Out] -(b*x+a)^(1/3)/a/x-2/3*b/a/(a^2)^(1/3)*(3/2*ln(((b*x+a)^(1/3)-a^(1/3))/x^(1
/3))-3^(1/2)*arctan(3^(1/2)*(b*x+a)^(1/3)/((b*x+a)^(1/3)+2*a^(1/3))))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.55, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1973, 44, 59, 632, 210, 31}

$$\int \frac{1}{x^2\sqrt[3]{(a+bx)^2}}\,dx = \frac{2b\left(\frac{bx}{a}+1\right)^{2/3}\arctan\left(\frac{2\sqrt[3]{\frac{bx}{a}+1}+1}{\sqrt{3}}\right)}{\sqrt{3}a\sqrt[3]{(a+bx)^2}} - \frac{a+bx}{ax\sqrt[3]{(a+bx)^2}} + \frac{b\log(x)\left(\frac{bx}{a}+1\right)^{2/3}}{3a\sqrt[3]{(a+bx)^2}} - \frac{b\left(\frac{bx}{a}+1\right)^{2/3}\log\left(1-\sqrt[3]{\frac{bx}{a}+1}\right)}{a\sqrt[3]{(a+bx)^2}}$$

```
[In] Int[1/(x^2*((a + b*x)^2)^(1/3)),x]
```

```
[Out] -((a + b*x)/(a*x*((a + b*x)^2)^(1/3))) + (2*b*(1 + (b*x)/a)^(2/3)*ArcTan[(1
 + 2*(1 + (b*x)/a)^(1/3))/Sqrt[3]])/(Sqrt[3]*a*((a + b*x)^2)^(1/3)) + (b*(1
 + (b*x)/a)^(2/3)*Log[x])/(3*a*((a + b*x)^2)^(1/3)) - (b*(1 + (b*x)/a)^(2/3
)*Log[1 - (1 + (b*x)/a)^(1/3)])/(a*((a + b*x)^2)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(1+\frac{bx}{a}\right)^{2/3}\int \frac{1}{x^2\left(1+\frac{bx}{a}\right)^{2/3}}\,dx}{\sqrt[3]{(a+bx)^2}}$$

$$= -\frac{a+bx}{ax\sqrt[3]{(a+bx)^2}} - \frac{\left(2b\left(1+\frac{bx}{a}\right)^{2/3}\right)\int \frac{1}{x\left(1+\frac{bx}{a}\right)^{2/3}}\,dx}{3a\sqrt[3]{(a+bx)^2}}$$

$$= -\frac{a+bx}{ax\sqrt[3]{(a+bx)^2}} + \frac{b\left(1+\frac{bx}{a}\right)^{2/3}\log(x)}{3a\sqrt[3]{(a+bx)^2}} + \frac{\left(b\left(1+\frac{bx}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+\frac{bx}{a}}\right)}{a\sqrt[3]{(a+bx)^2}} + \frac{\left(b\left(1+\frac{bx}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1+\frac{bx}{a}}\right)}{a\sqrt[3]{(a+bx)^2}}$$

$$= -\frac{a+bx}{ax\sqrt[3]{(a+bx)^2}} + \frac{b\left(1+\frac{bx}{a}\right)^{2/3}\log(x)}{3a\sqrt[3]{(a+bx)^2}} - \frac{b\left(1+\frac{bx}{a}\right)^{2/3}\log\left(1-\sqrt[3]{1+\frac{bx}{a}}\right)}{a\sqrt[3]{(a+bx)^2}} - \frac{\left(2b\left(1+\frac{bx}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+\frac{bx}{a}}\right)}{a\sqrt[3]{(a+bx)^2}}$$

$$= -\frac{a+bx}{ax\sqrt[3]{(a+bx)^2}} + \frac{2b\left(1+\frac{bx}{a}\right)^{2/3}\arctan\left(\frac{1+2\sqrt[3]{1+\frac{bx}{a}}}{\sqrt{3}}\right)}{\sqrt{3}a\sqrt[3]{(a+bx)^2}} + \frac{b\left(1+\frac{bx}{a}\right)^{2/3}\log(x)}{3a\sqrt[3]{(a+bx)^2}} - \frac{b\left(1+\frac{bx}{a}\right)^{2/3}\log\left(1-\sqrt[3]{1+\frac{bx}{a}}\right)}{a\sqrt[3]{(a+bx)^2}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^2\sqrt[3]{(a+bx)^2}}\,dx = \frac{-3a^{5/3}-3a^{2/3}bx+2\sqrt{3}bx(a+bx)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2bx(a+bx)^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+bx}{3a^{5/3}x\sqrt[3]{(a+bx)^2}}$$

```
[In] Integrate[1/(x^2*((a + b*x)^2)^(1/3)),x]
```

```
[Out] (-3*a^(5/3) - 3*a^(2/3)*b*x + 2*Sqrt[3]*b*x*(a + b*x)^(2/3)*ArcTan[(1 + (2*
(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*b*x*(a + b*x)^(2/3)*Log[a^(1/3) - (a
 + b*x)^(1/3)] + b*x*(a + b*x)^(2/3)*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3)
+ (a + b*x)^(2/3)])/(3*a^(5/3)*x*((a + b*x)^2)^(1/3))
```

Maple [F]

$$\int \frac{1}{x^2\left((bx+a)^2\right)^{\frac{1}{3}}}\,dx$$

```
[In] int(1/x^2/((b*x+a)^2)^(1/3),x)
```

```
[Out] int(1/x^2/((b*x+a)^2)^(1/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(80) = 160.

Time = 0.25 (sec) , antiderivative size = 325, normalized size of antiderivative = 3.12

$$\int \frac{1}{x^2\sqrt[3]{(a+bx)^2}}\,dx = -\frac{2\sqrt{3}(ab^2x^2+a^2bx)\sqrt{-(-a^2)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}(-a^2)^{\frac{1}{3}}(bx+a)-2\sqrt{3}(b^2x^2+2abx+a^2)^{\frac{1}{3}}a\right)\sqrt{-(-a^2)^{\frac{1}{3}}}}{3(abx+a^2)}\right)+3\left(b^2x^2+2\right.}{}$$

```
[In] integrate(1/x^2/((b*x+a)^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(3)*(a*b^2*x^2 + a^2*b*x)*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt
(3)*(-a^2)^(1/3)*(b*x + a) - 2*sqrt(3)*(b^2*x^2 + 2*a*b*x + a^2)^(1/3)*a)*s
qrt(-(-a^2)^(1/3))/(a*b*x + a^2)) + 3*(b^2*x^2 + 2*a*b*x + a^2)^(2/3)*a^2 -
```

```
(b^2*x^2 + a*b*x)*(-a^2)^(2/3)*log(((b^2*x^2 + 2*a*b*x + a^2)^(2/3)*a^2 -
(b^2*x^2 + 2*a*b*x + a^2)^(1/3)*(a*b*x + a^2)*(-a^2)^(1/3) + (b^2*x^2 + 2*a
*b*x + a^2)*(-a^2)^(2/3))/(b^2*x^2 + 2*a*b*x + a^2)) + 2*(b^2*x^2 + a*b*x)*
(-a^2)^(2/3)*log(((-a^2)^(1/3)*(b*x + a) + (b^2*x^2 + 2*a*b*x + a^2)^(1/3)*
a)/(b*x + a)))/(a^3*b*x^2 + a^4*x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt[3]{(a+bx)^2}}\,dx = \int \frac{1}{x^2\sqrt[3]{(a+bx)^2}}\,dx$$

```
[In] integrate(1/x**2/((b*x+a)**2)**(1/3),x)
[Out] Integral(1/(x**2*((a + b*x)**2)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[3]{(a+bx)^2}}\,dx = \int \frac{1}{\left((bx+a)^2\right)^{\frac{1}{3}}x^2}\,dx$$

```
[In] integrate(1/x^2/((b*x+a)^2)^(1/3),x, algorithm="maxima")
[Out] integrate(1/(((b*x + a)^2)^(1/3)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(80) = 160$.

Time = 3.13 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.37

$$\int \frac{1}{x^2\sqrt[3]{(a+bx)^2}}\,dx = \frac{\frac{2\sqrt{3}(a\mathrm{sgn}(bx+a))^{\frac{1}{3}}b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx\mathrm{sgn}(bx+a)+a\mathrm{sgn}(bx+a)\right)^{\frac{1}{3}}+(a\mathrm{sgn}(bx+a))^{\frac{1}{3}}\right)}{3\left(a\mathrm{sgn}(bx+a)\right)^{\frac{1}{3}}}\right)}{a^2} + \frac{(a\mathrm{sgn}(bx+a))^{\frac{1}{3}}b^2\log\left(\left(bx\mathrm{sgn}(bx+a)+a\mathrm{sgn}(bx+a)\right)^{\frac{2}{3}}+(b\right.}{}}{}$$

```
[In] integrate(1/x^2/((b*x+a)^2)^(1/3),x, algorithm="giac")
[Out] 1/3*(2*sqrt(3)*(a*sgn(b*x + a))^(1/3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x*sgn(b*
x + a) + a*sgn(b*x + a))^(1/3) + (a*sgn(b*x + a))^(1/3))/(a*sgn(b*x + a))^(
1/3))/a^2 + (a*sgn(b*x + a))^(1/3)*b^2*log((b*x*sgn(b*x + a) + a*sgn(b*x +
a))^(2/3) + (b*x*sgn(b*x + a) + a*sgn(b*x + a))^(1/3)*(a*sgn(b*x + a))^(1/3
) + (a*sgn(b*x + a))^(2/3))/a^2 - 2*(a*sgn(b*x + a))^(1/3)*b^2*log(abs((b*x
*sgn(b*x + a) + a*sgn(b*x + a))^(1/3) - (a*sgn(b*x + a))^(1/3)))/a^2 - 3*(b
*x*sgn(b*x + a) + a*sgn(b*x + a))^(1/3)*b/(a*x))/(b*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \sqrt[3]{(a+bx)^2}}\, dx = \int \frac{1}{x^2 \left((a+b\,x)^2\right)^{1/3}}\, dx$$

```
[In] int(1/(x^2*((a + b*x)^2)^(1/3)),x)
[Out] int(1/(x^2*((a + b*x)^2)^(1/3)), x)
```

3.156 $\int \frac{1}{x^3 \sqrt[3]{(a+bx)^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 118

$$\int \frac{1}{x^3 \sqrt[3]{(a+bx)^2}} \, dx = \left(-\frac{1}{2ax^2} + \frac{5b}{6a^2x}\right) \sqrt[3]{a+bx} + \frac{5b^2\left(-\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{2\sqrt[3]{a}+\sqrt[3]{a+bx}}\right) + \frac{3}{2}\log\left(\frac{-\sqrt[3]{a}+\sqrt[3]{a+bx}}{\sqrt[3]{x}}\right)\right)}{9\left(a^2\right)^{4/3}}$$

```
[Out] (-1/2/a/x^2+5/6*b/a^2/x)*(b*x+a)^(1/3)+5/9*b^2/a^2/(a^2)^(1/3)*(3/2*ln(((b*
x+a)^(1/3)-a^(1/3))/x^(1/3))-3^(1/2)*arctan(3^(1/2)*(b*x+a)^(1/3)/((b*x+a)^
(1/3)+2*a^(1/3))))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.69, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1973, 44, 59, 632, 210, 31}

$$\int \frac{1}{x^3\sqrt[3]{(a+bx)^2}}\,dx = -\frac{5b^2\left(\frac{bx}{a}+1\right)^{2/3}\arctan\left(\frac{2\sqrt[3]{\frac{bx}{a}+1}+1}{\sqrt{3}}\right)}{3\sqrt{3}a^2\sqrt[3]{(a+bx)^2}} - \frac{5b^2\log(x)\left(\frac{bx}{a}+1\right)^{2/3}}{18a^2\sqrt[3]{(a+bx)^2}} + \frac{5b^2\left(\frac{bx}{a}+1\right)^{2/3}\log\left(1-\sqrt[3]{\frac{bx}{a}+1}\right)}{6a^2\sqrt[3]{(a+bx)^2}} + \frac{5b(a+bx)}{6a^2x\sqrt[3]{(a+bx)^2}} - \frac{a+bx}{2ax^2\sqrt[3]{(a+bx)^2}}$$

```
[In] Int[1/(x^3*((a + b*x)^2)^(1/3)),x]
```

```
[Out] -1/2*(a + b*x)/(a*x^2*((a + b*x)^2)^(1/3)) + (5*b*(a + b*x))/(6*a^2*x*((a +
 b*x)^2)^(1/3)) - (5*b^2*(1 + (b*x)/a)^(2/3)*ArcTan[(1 + 2*(1 + (b*x)/a)^(1
/3))/Sqrt[3]])/(3*Sqrt[3]*a^2*((a + b*x)^2)^(1/3)) - (5*b^2*(1 + (b*x)/a)^(
2/3)*Log[x])/(18*a^2*((a + b*x)^2)^(1/3)) + (5*b^2*(1 + (b*x)/a)^(2/3)*Log[
1 - (1 + (b*x)/a)^(1/3)])/(6*a^2*((a + b*x)^2)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(1+\frac{bx}{a}\right)^{2/3}\int \frac{1}{x^3\left(1+\frac{bx}{a}\right)^{2/3}}\,dx}{\sqrt[3]{(a+bx)^2}}\\
&= -\frac{a+bx}{2ax^2\sqrt[3]{(a+bx)^2}}-\frac{\left(5b\left(1+\frac{bx}{a}\right)^{2/3}\right)\int \frac{1}{x^2\left(1+\frac{bx}{a}\right)^{2/3}}\,dx}{6a\sqrt[3]{(a+bx)^2}}\\
&= -\frac{a+bx}{2ax^2\sqrt[3]{(a+bx)^2}}+\frac{5b(a+bx)}{6a^2x\sqrt[3]{(a+bx)^2}}+\frac{\left(5b^2\left(1+\frac{bx}{a}\right)^{2/3}\right)\int \frac{1}{x\left(1+\frac{bx}{a}\right)^{2/3}}\,dx}{9a^2\sqrt[3]{(a+bx)^2}}\\
&= -\frac{a+bx}{2ax^2\sqrt[3]{(a+bx)^2}}+\frac{5b(a+bx)}{6a^2x\sqrt[3]{(a+bx)^2}}-\frac{5b^2\left(1+\frac{bx}{a}\right)^{2/3}\log(x)}{18a^2\sqrt[3]{(a+bx)^2}}\\
&\quad-\frac{\left(5b^2\left(1+\frac{bx}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1+\frac{bx}{a}}\right)}{6a^2\sqrt[3]{(a+bx)^2}}\\
&\quad-\frac{\left(5b^2\left(1+\frac{bx}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,\sqrt[3]{1+\frac{bx}{a}}\right)}{6a^2\sqrt[3]{(a+bx)^2}}
\end{aligned}
$$

$$= -\frac{a+bx}{2ax^2\sqrt[3]{(a+bx)^2}} + \frac{5b(a+bx)}{6a^2x\sqrt[3]{(a+bx)^2}} - \frac{5b^2\left(1+\frac{bx}{a}\right)^{2/3}\log(x)}{18a^2\sqrt[3]{(a+bx)^2}} + \frac{5b^2\left(1+\frac{bx}{a}\right)^{2/3}\log\left(1-\sqrt[3]{1+\frac{bx}{a}}\right)}{6a^2\sqrt[3]{(a+bx)^2}} + \frac{\left(5b^2\left(1+\frac{bx}{a}\right)^{2/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{1+\frac{bx}{a}}\right)}{3a^2\sqrt[3]{(a+bx)^2}}$$

$$= -\frac{a+bx}{2ax^2\sqrt[3]{(a+bx)^2}} + \frac{5b(a+bx)}{6a^2x\sqrt[3]{(a+bx)^2}} - \frac{5b^2\left(1+\frac{bx}{a}\right)^{2/3}\arctan\left(\frac{1+2\sqrt[3]{1+\frac{bx}{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^2\sqrt[3]{(a+bx)^2}} - \frac{5b^2\left(1+\frac{bx}{a}\right)^{2/3}\log(x)}{18a^2\sqrt[3]{(a+bx)^2}} + \frac{5b^2\left(1+\frac{bx}{a}\right)^{2/3}\log\left(1-\sqrt[3]{1+\frac{bx}{a}}\right)}{6a^2\sqrt[3]{(a+bx)^2}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.55

$$\int\frac{1}{x^3\sqrt[3]{(a+bx)^2}}\,dx$$

$$= \frac{-9a^{8/3}+6a^{5/3}bx+15a^{2/3}b^2x^2-10\sqrt{3}b^2x^2(a+bx)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+10b^2x^2(a+bx)^{2/3}\log\left(\sqrt[3]{\right.}}{18a^{8/3}x^2\sqrt[3]{(a+bx)^2}}$$

```
[In] Integrate[1/(x^3*((a + b*x)^2)^(1/3)),x]
```

```
[Out] (-9*a^(8/3) + 6*a^(5/3)*b*x + 15*a^(2/3)*b^2*x^2 - 10*Sqrt[3]*b^2*x^2*(a +
b*x)^(2/3)*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 10*b^2*x^2*(
a + b*x)^(2/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - 5*b^2*x^2*(a + b*x)^(2/3)*L
og[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(18*a^(8/3)*x^2*((
a + b*x)^2)^(1/3))
```

Maple [F]

$$\int \frac{1}{x^3 \left((bx+a)^2\right)^{\frac{1}{3}}}dx$$

```
[In] int(1/x^3/((b*x+a)^2)^(1/3),x)
```

```
[Out] int(1/x^3/((b*x+a)^2)^(1/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(94) = 188.

Time = 0.27 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.76

$$\int \frac{1}{x^3\sqrt[3]{(a+bx)^2}}\,dx = \frac{10\sqrt{3}(ab^3x^3+a^2b^2x^2)(a^2)^{\frac{1}{6}}\arctan\left(\frac{(a^2)^{\frac{1}{6}}\left(\sqrt{3}(a^2)^{\frac{1}{3}}(bx+a)+2\sqrt{3}(b^2x^2+2abx+a^2)^{\frac{1}{3}}a\right)}{3(abx+a^2)}\right)-5(b^3x^3+ab^2x^2)(a^2)^{\frac{2}{3}}\mathrm{lo}}{}$$

```
[In] integrate(1/x^3/((b*x+a)^2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/18*(10*sqrt(3)*(a*b^3*x^3 + a^2*b^2*x^2)*(a^2)^(1/6)*arctan(1/3*(a^2)^(1/
6)*(sqrt(3)*(a^2)^(1/3)*(b*x + a) + 2*sqrt(3)*(b^2*x^2 + 2*a*b*x + a^2)^(1/
3)*a)/(a*b*x + a^2)) - 5*(b^3*x^3 + a*b^2*x^2)*(a^2)^(2/3)*log(((b^2*x^2 +
2*a*b*x + a^2)^(2/3)*a^2 + (b^2*x^2 + 2*a*b*x + a^2)^(1/3)*(a*b*x + a^2)*(a
^2)^(1/3) + (b^2*x^2 + 2*a*b*x + a^2)*(a^2)^(2/3))/(b^2*x^2 + 2*a*b*x + a^2
)) + 10*(b^3*x^3 + a*b^2*x^2)*(a^2)^(2/3)*log(-((a^2)^(1/3)*(b*x + a) - (b^
2*x^2 + 2*a*b*x + a^2)^(1/3)*a)/(b*x + a)) + 3*(5*a^2*b*x - 3*a^3)*(b^2*x^2
 + 2*a*b*x + a^2)^(2/3))/(a^4*b*x^3 + a^5*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt[3]{(a+bx)^2}}\,dx = \int \frac{1}{x^3\sqrt[3]{(a+bx)^2}}\,dx$$

```
[In] integrate(1/x**3/((b*x+a)**2)**(1/3),x)
```

```
[Out] Integral(1/(x**3*((a + b*x)**2)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt[3]{(a+bx)^2}}\,dx=\int \frac{1}{\left((bx+a)^2\right)^{\frac{1}{3}}x^3}\,dx$$

```
[In] integrate(1/x^3/((b*x+a)^2)^(1/3),x, algorithm="maxima")
[Out] integrate(1/(((b*x + a)^2)^(1/3)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(94) = 188$.

Time = 3.15 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.50

$$\int \frac{1}{x^3\sqrt[3]{(a+bx)^2}}\,dx = -\frac{\frac{10\sqrt{3}(a\operatorname{sgn}(bx+a))^{\frac{1}{3}}b^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx\operatorname{sgn}(bx+a)+a\operatorname{sgn}(bx+a))^{\frac{1}{3}}+(a\operatorname{sgn}(bx+a))^{\frac{1}{3}}\right)}{3\,(a\operatorname{sgn}(bx+a))^{\frac{1}{3}}}\right)}{a^3}+\frac{5\,(a\operatorname{sgn}(bx+a))^{\frac{1}{3}}b^3\log\left((bx\operatorname{sgn}(bx+a)+a\operatorname{sgn}(bx+a))^{\frac{2}{3}}\right.}{}}{}$$

```
[In] integrate(1/x^3/((b*x+a)^2)^(1/3),x, algorithm="giac")
[Out] -1/18*(10*sqrt(3)*(a*sgn(b*x + a))^(1/3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x*sgn
(b*x + a) + a*sgn(b*x + a))^(1/3) + (a*sgn(b*x + a))^(1/3))/(a*sgn(b*x + a)
)^(1/3))/a^3 + 5*(a*sgn(b*x + a))^(1/3)*b^3*log((b*x*sgn(b*x + a) + a*sgn(b
*x + a))^(2/3) + (b*x*sgn(b*x + a) + a*sgn(b*x + a))^(1/3)*(a*sgn(b*x + a))
^(1/3) + (a*sgn(b*x + a))^(2/3))/a^3 - 10*(a*sgn(b*x + a))^(1/3)*b^3*log(ab
s((b*x*sgn(b*x + a) + a*sgn(b*x + a))^(1/3) - (a*sgn(b*x + a))^(1/3)))/a^3
- 3*(5*(b*x*sgn(b*x + a) + a*sgn(b*x + a))^(4/3)*b^3*sgn(b*x + a) - 8*(b*x*
sgn(b*x + a) + a*sgn(b*x + a))^(1/3)*a*b^3)/(a^2*b^2*x^2*sgn(b*x + a)^2))/(
b*sgn(b*x + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt[3]{(a+bx)^2}}\,dx=\int \frac{1}{x^3\,{\left({\left(a+b\,x\right)}^2\right)}^{1/3}}\,dx$$

```
[In] int(1/(x^3*((a + b*x)^2)^(1/3)),x)
[Out] int(1/(x^3*((a + b*x)^2)^(1/3)), x)
```

3.157 $\int \frac{1}{x\sqrt[3]{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 78

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx=\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{2\sqrt[3]{a}+\sqrt[3]{a+bx}}\right)+\frac{3}{2}\log\left(\frac{-\sqrt[3]{a}+\sqrt[3]{a+bx}}{\sqrt[3]{x}}\right)}{\sqrt[3]{a^2}}$$

```
[Out] 1/(a^2)^(1/3)*(3/2*ln(((b*x+a)^(1/3)-a^(1/3))/x^(1/3))+3^(1/2)*arctan(3^(1/
2)*(b*x+a)^(1/3)/((b*x+a)^(1/3)+2*a^(1/3))))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {57, 631, 210, 31}

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx=\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

```
[In] Int[1/(x*(a + b*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(1/3) -
 Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = -\frac{\log(x)}{2\sqrt[3]{a}} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx}\right) - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}} - \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

$$= \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} - \frac{\log(x)}{2\sqrt[3]{a}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{2\sqrt[3]{a}}$$

```
[In] Integrate[1/(x*(a + b*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3
) - (a + b*x)^(1/3)] - Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2
/3)])/(2*a^(1/3))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}$	75
default	$\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}$	75
pseudoelliptic	$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)+2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}$	75

```
[In] int(1/x/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2/a^(1/3)*ln((b*x+a)^(2/3)+(b*x+a)^(1
/3)*a^(1/3)+a^(2/3))+3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^
(1/3)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.73

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx = \left[\frac{\sqrt{3}a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx+\sqrt{3}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x}\right)-a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a\right.}{2\,a}\right.$$

[In] integrate(1/x/(b*x+a)^(1/3),x, algorithm="fricas")

[Out] [1/2*(sqrt(3)*a*sqrt(-1/a^(2/3))*log((2*b*x + sqrt(3)*(2*(b*x + a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x + a)^(1/3)*a^(2/3) + 3*a)/x) - a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3)))/a, 1/2*(2*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3)))/a]

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.99

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx = \frac{2\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}+\frac{2e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}+\frac{2e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}$$

[In] integrate(1/x/(b*x+a)**(1/3),x)

[Out] 2*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) + 2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/

```
3)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) + 2*exp(-2*I*pi/3)*log(1 -
b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(3*a**(1
/3)*gamma(5/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}} - \frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{1}{3}}} + \frac{\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}$$

```
[In] integrate(1/x/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) + log
((b*x + a)^(1/3) - a^(1/3))/a^(1/3)
```

Giac [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}} - \frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{1}{3}}} + \frac{\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{1}{3}}}$$

```
[In] integrate(1/x/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) + log
(abs((b*x + a)^(1/3) - a^(1/3)))/a^(1/3)
```

Mupad [B] (verification not implemented)

Time = 16.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx = \frac{\ln\left(9\,(a+b\,x)^{1/3}-9\,a^{1/3}\right)}{a^{1/3}} + \frac{\ln\left(9\,(a+b\,x)^{1/3}-\frac{9\,a^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{1/3}} - \frac{\ln\left(9\,(a+b\,x)^{1/3}-\frac{9\,a^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{1/3}}$$

```
[In] int(1/(x*(a + b*x)^(1/3)),x)
```

```
[Out] log(9*(a + b*x)^(1/3) - 9*a^(1/3))/a^(1/3) + (log(9*(a + b*x)^(1/3) - (9*a^
(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*a^(1/3)) - (log(9*(a + b*
x)^(1/3) - (9*a^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(2*a^(1/3))
```

3.158 $\int \frac{\sqrt[3]{(a+bx)^2}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int \frac{\sqrt[3]{(a+bx)^2}}{x} \, dx = \frac{3}{2}\sqrt[3]{(a+bx)^2} + \frac{a\left(\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{2\sqrt[3]{a}+\sqrt[3]{a+bx}}\right)+\frac{3}{2}\log\left(\frac{-\sqrt[3]{a}+\sqrt[3]{a+bx}}{\sqrt[3]{x}}\right)\right)}{\sqrt[3]{a^2}}$$

```
[Out] 3/2*((b*x+a)^2)^(1/3)+a/(a^2)^(1/3)*(3/2*ln(((b*x+a)^(1/3)-a^(1/3))/x^(1/3)
)+3^(1/2)*arctan(3^(1/2)*(b*x+a)^(1/3)/((b*x+a)^(1/3)+2*a^(1/3))))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.48, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1973, 52, 57, 632, 210, 31}

$$\int \frac{\sqrt[3]{(a+bx)^2}}{x} \, dx = \frac{\sqrt{3}\sqrt[3]{(a+bx)^2}\arctan\left(\frac{2\sqrt[3]{\frac{bx}{a}+1}+1}{\sqrt{3}}\right)}{\left(\frac{bx}{a}+1\right)^{2/3}} + \frac{3}{2}\sqrt[3]{(a+bx)^2} - \frac{\log(x)\sqrt[3]{(a+bx)^2}}{2\left(\frac{bx}{a}+1\right)^{2/3}} + \frac{3\sqrt[3]{(a+bx)^2}\log\left(1-\sqrt[3]{\frac{bx}{a}+1}\right)}{2\left(\frac{bx}{a}+1\right)^{2/3}}$$

```
[In] Int[((a + b*x)^2)^(1/3)/x,x]
```

```
[Out] (3*((a + b*x)^2)^(1/3))/2 + (Sqrt[3]*((a + b*x)^2)^(1/3)*ArcTan[(1 + 2*(1 +
 (b*x)/a)^(1/3))/Sqrt[3]])/(1 + (b*x)/a)^(2/3) - (((a + b*x)^2)^(1/3)*Log[x
])/(2*(1 + (b*x)/a)^(2/3)) + (3*((a + b*x)^2)^(1/3)*Log[1 - (1 + (b*x)/a)^(
1/3)])/(2*(1 + (b*x)/a)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{(a+bx)^2}\int \frac{\left(1+\frac{bx}{a}\right)^{2/3}}{x}\,dx}{\left(1+\frac{bx}{a}\right)^{2/3}}$$

$$= \frac{3}{2}\sqrt[3]{(a+bx)^2} + \frac{\sqrt[3]{(a+bx)^2}\int \frac{1}{x\sqrt[3]{1+\frac{bx}{a}}}\,dx}{\left(1+\frac{bx}{a}\right)^{2/3}}$$

$$= \frac{3}{2}\sqrt[3]{(a+bx)^2} - \frac{\sqrt[3]{(a+bx)^2}\log(x)}{2\left(1+\frac{bx}{a}\right)^{2/3}} - \frac{\left(3\sqrt[3]{(a+bx)^2}\right)\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+\frac{bx}{a}}\right)}{2\left(1+\frac{bx}{a}\right)^{2/3}} + \frac{\left(3\sqrt[3]{(a+bx)^2}\right)\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1+\frac{bx}{a}}\right)}{2\left(1+\frac{bx}{a}\right)^{2/3}}$$

$$= \frac{3}{2}\sqrt[3]{(a+bx)^2} - \frac{\sqrt[3]{(a+bx)^2}\log(x)}{2\left(1+\frac{bx}{a}\right)^{2/3}} + \frac{3\sqrt[3]{(a+bx)^2}\log\left(1-\sqrt[3]{1+\frac{bx}{a}}\right)}{2\left(1+\frac{bx}{a}\right)^{2/3}} - \frac{\left(3\sqrt[3]{(a+bx)^2}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+\frac{bx}{a}}\right)}{\left(1+\frac{bx}{a}\right)^{2/3}}$$

$$= \frac{3}{2}\sqrt[3]{(a+bx)^2} + \frac{\sqrt{3}\sqrt[3]{(a+bx)^2}\arctan\left(\frac{1+2\sqrt[3]{1+\frac{bx}{a}}}{\sqrt{3}}\right)}{\left(1+\frac{bx}{a}\right)^{2/3}} - \frac{\sqrt[3]{(a+bx)^2}\log(x)}{2\left(1+\frac{bx}{a}\right)^{2/3}} + \frac{3\sqrt[3]{(a+bx)^2}\log\left(1-\sqrt[3]{1+\frac{bx}{a}}\right)}{2\left(1+\frac{bx}{a}\right)^{2/3}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt[3]{(a+bx)^2}}{x}\,dx = \frac{\sqrt[3]{(a+bx)^2}\left(3(a+bx)^{2/3}+2\sqrt{3}a^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-a^{2/3}\log\left(a^{2/3}\right.\right.}{2(a+bx)^{2/3}}$$

```
[In] Integrate[((a + b*x)^2)^(1/3)/x,x]
[Out] (((a + b*x)^2)^(1/3)*(3*(a + b*x)^(2/3) + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*
(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 2*a^(2/3)*Log[a^(1/3) - (a + b*x)^(1/3
)] - a^(2/3)*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)]))/(2*
(a + b*x)^(2/3))
```

Maple [F]

$$\int \frac{\left((bx+a)^2\right)^{\frac{1}{3}}}{x}dx$$

```
[In] int(((b*x+a)^2)^(1/3)/x,x)
[Out] int(((b*x+a)^2)^(1/3)/x,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(73) = 146.

Time = 0.25 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.51

$$\int \frac{\sqrt[3]{(a+bx)^2}}{x}\,dx = -\sqrt{3}\left(a^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}(abx+a^2)+2\sqrt{3}(b^2x^2+2\,abx+a^2)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}}{3\,(abx+a^2)}\right)$$
$$-\frac{1}{2}\left(a^2\right)^{\frac{1}{3}}\log\left(\frac{(b^2x^2+2\,abx+a^2)^{\frac{2}{3}}a^2+(b^2x^2+2\,abx+a^2)^{\frac{1}{3}}(abx+a^2)(a^2)^{\frac{1}{3}}+(b^2x^2+2\,abx+a^2)(a^2)^{\frac{2}{3}}}{b^2x^2+2\,abx+a^2}\right.$$
$$+\left(a^2\right)^{\frac{1}{3}}\log\left(-\frac{(a^2)^{\frac{1}{3}}(bx+a)-(b^2x^2+2\,abx+a^2)^{\frac{1}{3}}a}{bx+a}\right)+\frac{3}{2}\left(b^2x^2+2\,abx+a^2\right)^{\frac{1}{3}}$$

```
[In] integrate(((b*x+a)^2)^(1/3)/x,x, algorithm="fricas")
[Out] -sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt(3)*(a*b*x + a^2) + 2*sqrt(3)*(b^2*x^2
 + 2*a*b*x + a^2)^(1/3)*(a^2)^(2/3))/(a*b*x + a^2)) - 1/2*(a^2)^(1/3)*log((
```

```
(b^2*x^2 + 2*a*b*x + a^2)^(2/3)*a^2 + (b^2*x^2 + 2*a*b*x + a^2)^(1/3)*(a*b*
x + a^2)*(a^2)^(1/3) + (b^2*x^2 + 2*a*b*x + a^2)*(a^2)^(2/3))/(b^2*x^2 + 2*
a*b*x + a^2)) + (a^2)^(1/3)*log(-((a^2)^(1/3)*(b*x + a) - (b^2*x^2 + 2*a*b*
x + a^2)^(1/3)*a)/(b*x + a)) + 3/2*(b^2*x^2 + 2*a*b*x + a^2)^(1/3)
```

Sympy [F]

$$\int \frac{\sqrt[3]{(a+bx)^2}}{x}\,dx = \int \frac{\sqrt[3]{(a+bx)^2}}{x}\,dx$$

```
[In] integrate(((b*x+a)**2)**(1/3)/x,x)
[Out] Integral(((a + b*x)**2)**(1/3)/x, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{(a+bx)^2}}{x}\,dx = \int \frac{\left((bx+a)^2\right)^{\frac{1}{3}}}{x}\,dx$$

```
[In] integrate(((b*x+a)^2)^(1/3)/x,x, algorithm="maxima")
[Out] integrate(((b*x + a)^2)^(1/3)/x, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(73) = 146$.

Time = 3.16 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.62

$$\int \frac{\sqrt[3]{(a+bx)^2}}{x}\,dx = \frac{1}{2}\left(\frac{2\sqrt{3}(a\text{sgn}(bx+a))^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx\text{sgn}(bx+a)+a\text{sgn}(bx+a))^{\frac{1}{3}}+(a\text{sgn}(bx+a))^{\frac{1}{3}}\right)}{3\,(a\text{sgn}(bx+a))^{\frac{1}{3}}}\right)}{\text{sgn}\,(bx+a)} - \frac{(a\text{sgn}(bx+a))^{\frac{2}{3}}\log\left((bx\right.}{} \right. + a)$$

```
[In] integrate(((b*x+a)^2)^(1/3)/x,x, algorithm="giac")
[Out] 1/2*(2*sqrt(3)*(a*sgn(b*x + a))^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x*sgn(b*x +
a) + a*sgn(b*x + a))^(1/3) + (a*sgn(b*x + a))^(1/3))/(a*sgn(b*x + a))^(1/3)
```

```
)/sgn(b*x + a) - (a*sgn(b*x + a))^(2/3)*log((b*x*sgn(b*x + a) + a*sgn(b*x +
 a))^(2/3) + (b*x*sgn(b*x + a) + a*sgn(b*x + a))^(1/3)*(a*sgn(b*x + a))^(1/
3) + (a*sgn(b*x + a))^(2/3))/sgn(b*x + a) + 2*(a*sgn(b*x + a))^(2/3)*log(ab
s((b*x*sgn(b*x + a) + a*sgn(b*x + a))^(1/3) - (a*sgn(b*x + a))^(1/3)))/sgn(
b*x + a) + 3*(b*x*sgn(b*x + a) + a*sgn(b*x + a))^(2/3)/sgn(b*x + a))*sgn(b*
x + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{(a+bx)^2}}{x}\,dx=\int \frac{{\left({\left(a+b\,x\right)}^2\right)}^{1/3}}{x} \,d x$$

```
[In] int(((a + b*x)^2)^(1/3)/x,x)
[Out] int(((a + b*x)^2)^(1/3)/x, x)
```

3.159 $\int \frac{\sqrt[3]{(a+bx)^2}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 115

$$\int \frac{\sqrt[3]{(a+bx)^2}}{x^2}\,dx = \frac{b\sqrt[3]{(a+bx)^2}}{a} - \frac{\sqrt[3]{(a+bx)^5}}{ax} + \frac{b\left(\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{2\sqrt[3]{a}+\sqrt[3]{a+bx}}\right)+\frac{3}{2}\log\left(\frac{-\sqrt[3]{a}+\sqrt[3]{a+bx}}{\sqrt[3]{x}}\right)\right)}{\sqrt[3]{a^2}}$$

```
[Out] -((b*x+a)^5)^(1/3)/a/x+b/a*((b*x+a)^2)^(1/3)+b/(a^2)^(1/3)*(3/2*ln(((b*x+a)
^(1/3)-a^(1/3))/x^(1/3))+3^(1/2)*arctan(3^(1/2)*(b*x+a)^(1/3)/((b*x+a)^(1/3
)+2*a^(1/3))))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1973, 43, 57, 632, 210, 31}

$$\int \frac{\sqrt[3]{(a+bx)^2}}{x^2}\,dx = \frac{2b\sqrt[3]{(a+bx)^2}\arctan\left(\frac{2\sqrt[3]{\frac{bx}{a}+1}+1}{\sqrt{3}}\right)}{\sqrt{3}a\left(\frac{bx}{a}+1\right)^{2/3}} - \frac{\sqrt[3]{(a+bx)^2}}{x} - \frac{b\log(x)\sqrt[3]{(a+bx)^2}}{3a\left(\frac{bx}{a}+1\right)^{2/3}} + \frac{b\sqrt[3]{(a+bx)^2}\log\left(1-\sqrt[3]{\frac{bx}{a}+1}\right)}{a\left(\frac{bx}{a}+1\right)^{2/3}}$$

```
[In] Int[((a + b*x)^2)^(1/3)/x^2,x]
```

```
[Out] -(((a + b*x)^2)^(1/3)/x) + (2*b*((a + b*x)^2)^(1/3)*ArcTan[(1 + 2*(1 + (b*x
)/a)^(1/3))/Sqrt[3]])/(Sqrt[3]*a*(1 + (b*x)/a)^(2/3)) - (b*((a + b*x)^2)^(1
/3)*Log[x])/(3*a*(1 + (b*x)/a)^(2/3)) + (b*((a + b*x)^2)^(1/3)*Log[1 - (1 +
 (b*x)/a)^(1/3)])/(a*(1 + (b*x)/a)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{(a+bx)^2}\int \frac{\left(1+\frac{bx}{a}\right)^{2/3}}{x^2}\,dx}{\left(1+\frac{bx}{a}\right)^{2/3}}$$

$$= -\frac{\sqrt[3]{(a+bx)^2}}{x} + \frac{\left(2b\sqrt[3]{(a+bx)^2}\right)\int \frac{1}{x\sqrt[3]{1+\frac{bx}{a}}}\,dx}{3a\left(1+\frac{bx}{a}\right)^{2/3}}$$

$$= -\frac{\sqrt[3]{(a+bx)^2}}{x} - \frac{b\sqrt[3]{(a+bx)^2}\log(x)}{3a\left(1+\frac{bx}{a}\right)^{2/3}} - \frac{\left(b\sqrt[3]{(a+bx)^2}\right)\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+\frac{bx}{a}}\right)}{a\left(1+\frac{bx}{a}\right)^{2/3}} + \frac{\left(b\sqrt[3]{(a+bx)^2}\right)\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1+\frac{bx}{a}}\right)}{a\left(1+\frac{bx}{a}\right)^{2/3}}$$

$$= -\frac{\sqrt[3]{(a+bx)^2}}{x} - \frac{b\sqrt[3]{(a+bx)^2}\log(x)}{3a\left(1+\frac{bx}{a}\right)^{2/3}} + \frac{b\sqrt[3]{(a+bx)^2}\log\left(1-\sqrt[3]{1+\frac{bx}{a}}\right)}{a\left(1+\frac{bx}{a}\right)^{2/3}} - \frac{\left(2b\sqrt[3]{(a+bx)^2}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+\frac{bx}{a}}\right)}{a\left(1+\frac{bx}{a}\right)^{2/3}}$$

$$= -\frac{\sqrt[3]{(a+bx)^2}}{x} + \frac{2b\sqrt[3]{(a+bx)^2}\arctan\left(\frac{1+2\sqrt[3]{1+\frac{bx}{a}}}{\sqrt{3}}\right)}{\sqrt{3}a\left(1+\frac{bx}{a}\right)^{2/3}} - \frac{b\sqrt[3]{(a+bx)^2}\log(x)}{3a\left(1+\frac{bx}{a}\right)^{2/3}} + \frac{b\sqrt[3]{(a+bx)^2}\log\left(1-\sqrt[3]{1+\frac{bx}{a}}\right)}{a\left(1+\frac{bx}{a}\right)^{2/3}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt[3]{(a+bx)^2}}{x^2}\,dx = -\frac{\sqrt[3]{(a+bx)^2}\left(3\sqrt[3]{a}(a+bx)^{2/3}-2\sqrt{3}bx\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2bx\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+bx\log\left(a^2\right.\right.}{3\sqrt[3]{a}x(a+bx)^{2/3}}$$

[In] Integrate[((a + b*x)^2)^(1/3)/x^2,x]

[Out] -1/3*(((a + b*x)^2)^(1/3)*(3*a^(1/3)*(a + b*x)^(2/3) - 2*Sqrt[3]*b*x*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*b*x*Log[a^(1/3) - (a + b*x)^(1/3)] + b*x*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)]))/(a^(1/3)*x*(a + b*x)^(2/3))

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{\left((bx+a)^2\right)^{\frac{1}{3}}}{x}+\frac{2b\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}\right)\left((bx+a)^2\right)^{\frac{1}{3}}}{3(bx+a)^{\frac{2}{3}}}$	109

[In] int(((b*x+a)^2)^(1/3)/x^2,x,method=_RETURNVERBOSE)

[Out] -((b*x+a)^2)^(1/3)/x+2/3*b*(1/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2/a^(1/3)*ln((b*x+a)^(2/3)+(b*x+a)^(1/3)*a^(1/3)+a^(2/3))+3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1)))/(b*x+a)^(2/3)*((b*x+a)^2)^(1/3)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(93) = 186$.

Time = 0.26 (sec) , antiderivative size = 560, normalized size of antiderivative = 4.87

$$\int \frac{\sqrt[3]{(a+bx)^2}}{x^2}\,dx$$
$$= \left[\frac{3\sqrt{\frac{1}{3}}abx\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(-\frac{b^2x^2+4\,abx+3\,a^2+3\,\sqrt{\frac{1}{3}}\left((b^2x^2+2\,abx+a^2)^{\frac{1}{3}}(bx+a)a^{\frac{2}{3}}-2\,(b^2x^2+2\,abx+a^2)^{\frac{2}{3}}a+(b^2x^2+2\,abx+a^2)a^{\frac{1}{3}}\right)\sqrt{-\cdots}}{bx^2+ax}\right.\cdots}{\cdots} \right.$$
$$-\frac{6\sqrt{\frac{1}{3}}a^{\frac{2}{3}}bx\arctan\left(\frac{\sqrt{\frac{1}{3}}\left((bx+a)a^{\frac{1}{3}}+2\,(b^2x^2+2\,abx+a^2)^{\frac{1}{3}}a^{\frac{2}{3}}\right)}{(bx+a)a^{\frac{1}{3}}}\right)+a^{\frac{2}{3}}bx\log\left(\frac{(b^2x^2+2\,abx+a^2)^{\frac{1}{3}}(bx+a)a^{\frac{2}{3}}+(b^2x^2+2\,abx+a\cdots}{b^2x^2+2\,abx+a^2}\right.\cdots}{3\,ax}$$

```
[In] integrate(((b*x+a)^2)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(1/3)*a*b*x*sqrt(-1/a^(2/3))*log(-(b^2*x^2 + 4*a*b*x + 3*a^2 +
3*sqrt(1/3)*((b^2*x^2 + 2*a*b*x + a^2)^(1/3)*(b*x + a)*a^(2/3) - 2*(b^2*x^2
 + 2*a*b*x + a^2)^(2/3)*a + (b^2*x^2 + 2*a*b*x + a^2)*a^(1/3))*sqrt(-1/a^(2
/3)) - 3*(b^2*x^2 + 2*a*b*x + a^2)^(1/3)*(b*x + a)*a^(1/3))/(b*x^2 + a*x))
- a^(2/3)*b*x*log(((b^2*x^2 + 2*a*b*x + a^2)^(1/3)*(b*x + a)*a^(2/3) + (b^2
*x^2 + 2*a*b*x + a^2)^(2/3)*a + (b^2*x^2 + 2*a*b*x + a^2)*a^(1/3))/(b^2*x^2
 + 2*a*b*x + a^2)) + 2*a^(2/3)*b*x*log(-((b*x + a)*a^(2/3) - (b^2*x^2 + 2*a
*b*x + a^2)^(1/3)*a)/(b*x + a)) - 3*(b^2*x^2 + 2*a*b*x + a^2)^(1/3)*a)/(a*x
), -1/3*(6*sqrt(1/3)*a^(2/3)*b*x*arctan(sqrt(1/3)*((b*x + a)*a^(1/3) + 2*(b
^2*x^2 + 2*a*b*x + a^2)^(1/3)*a^(2/3))/((b*x + a)*a^(1/3))) + a^(2/3)*b*x*l
og(((b^2*x^2 + 2*a*b*x + a^2)^(1/3)*(b*x + a)*a^(2/3) + (b^2*x^2 + 2*a*b*x
+ a^2)^(2/3)*a + (b^2*x^2 + 2*a*b*x + a^2)*a^(1/3))/(b^2*x^2 + 2*a*b*x + a^
2)) - 2*a^(2/3)*b*x*log(-((b*x + a)*a^(2/3) - (b^2*x^2 + 2*a*b*x + a^2)^(1/
3)*a)/(b*x + a)) + 3*(b^2*x^2 + 2*a*b*x + a^2)^(1/3)*a)/(a*x)]
```

Sympy [F]

$$\int \frac{\sqrt[3]{(a+bx)^2}}{x^2}\,dx = \int \frac{\sqrt[3]{\left(a+bx\right)^2}}{x^2}\,dx$$

```
[In] integrate(((b*x+a)**2)**(1/3)/x**2,x)
[Out] Integral(((a + b*x)**2)**(1/3)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{(a+bx)^2}}{x^2}\,dx = \int \frac{\left((bx+a)^2\right)^{\frac{1}{3}}}{x^2}\,dx$$

```
[In] integrate(((b*x+a)^2)^(1/3)/x^2,x, algorithm="maxima")
[Out] integrate(((b*x + a)^2)^(1/3)/x^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(93) = 186.

Time = 3.04 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt[3]{(a+bx)^2}}{x^2}\,dx = \frac{\frac{2\sqrt{3}(a\mathrm{sgn}(bx+a))^{\frac{2}{3}}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx\mathrm{sgn}(bx+a)+a\mathrm{sgn}(bx+a))^{\frac{1}{3}}+(a\mathrm{sgn}(bx+a))^{\frac{1}{3}}\right)}{3\,(a\mathrm{sgn}(bx+a))^{\frac{1}{3}}}\right)}{a} - \frac{(a\mathrm{sgn}(bx+a))^{\frac{2}{3}}b^2\log\left((bx\mathrm{sgn}(bx+a)+a\mathrm{sgn}(bx+a))^{\frac{2}{3}}+(bx\mathrm{s}\right.}{}}{}$$

```
[In] integrate(((b*x+a)^2)^(1/3)/x^2,x, algorithm="giac")
[Out] 1/3*(2*sqrt(3)*(a*sgn(b*x + a))^(2/3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x*sgn(b*
x + a) + a*sgn(b*x + a))^(1/3) + (a*sgn(b*x + a))^(1/3))/(a*sgn(b*x + a))^(
1/3))/a - (a*sgn(b*x + a))^(2/3)*b^2*log((b*x*sgn(b*x + a) + a*sgn(b*x + a)
)^(2/3) + (b*x*sgn(b*x + a) + a*sgn(b*x + a))^(1/3)*(a*sgn(b*x + a))^(1/3)
+ (a*sgn(b*x + a))^(2/3))/a + 2*(a*sgn(b*x + a))^(2/3)*b^2*log(abs((b*x*sgn
(b*x + a) + a*sgn(b*x + a))^(1/3) - (a*sgn(b*x + a))^(1/3)))/a - 3*(b*x*sgn
(b*x + a) + a*sgn(b*x + a))^(2/3)*b/x)/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{(a+bx)^2}}{x^2}\,dx = \int \frac{\left((a+b\,x)^2\right)^{1/3}}{x^2}\,dx$$

```
[In] int(((a + b*x)^2)^(1/3)/x^2,x)
[Out] int(((a + b*x)^2)^(1/3)/x^2, x)
```

3.160 $\int \frac{\sqrt[3]{(a+bx)^3}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 138

$$\int \frac{\sqrt[3]{(a+bx)^3}}{x^2}\,dx = \left(\frac{b}{6a^3}-\frac{1}{2ax^2}\right)(a+bx)^{5/3}-\frac{b^2\sqrt[3]{(a+bx)^2}}{6a^2} - \frac{b^2\left(\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{2\sqrt[3]{a}+\sqrt[3]{a+bx}}\right)+\frac{3}{2}\log\left(\frac{-\sqrt[3]{a}+\sqrt[3]{a+bx}}{\sqrt[3]{x}}\right)\right)}{9a\sqrt[3]{a^2}}$$

```
[Out] (-1/2/a/x^2+1/6*b/a^3)*(b*x+a)^(5/3)-1/6*b^2/a^2*((b*x+a)^2)^(1/3)-1/9*b^2/
a/(a^2)^(1/3)*(3/2*ln(((b*x+a)^(1/3)-a^(1/3))/x^(1/3))+3^(1/2)*arctan(3^(1/
2)*(b*x+a)^(1/3)/((b*x+a)^(1/3)+2*a^(1/3))))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.34, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1973, 45}

$$\int \frac{\sqrt[3]{(a+bx)^3}}{x^2}\,dx = \frac{b\log(x)\sqrt[3]{(a+bx)^3}}{a+bx}-\frac{a\sqrt[3]{(a+bx)^3}}{x(a+bx)}$$

```
[In] Int[((a + b*x)^3)^(1/3)/x^2,x]
```

```
[Out] -((a*((a + b*x)^3)^(1/3))/(x*(a + b*x))) + (b*((a + b*x)^3)^(1/3)*Log[x])/(
a + b*x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt[3]{(a+bx)^3}\int \frac{1+\frac{bx}{a}}{x^2}\,dx}{1+\frac{bx}{a}}\\ &= \frac{\sqrt[3]{(a+bx)^3}\int \left(\frac{1}{x^2}+\frac{b}{ax}\right)\,dx}{1+\frac{bx}{a}}\\ &= -\frac{a\sqrt[3]{(a+bx)^3}}{x(a+bx)}+\frac{b\sqrt[3]{(a+bx)^3}\log(x)}{a+bx}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt[3]{(a+bx)^3}}{x^2}\,dx=\frac{\sqrt[3]{(a+bx)^3}(-a+bx\log(x))}{x(a+bx)}$$

```
[In] Integrate[((a + b*x)^3)^(1/3)/x^2,x]
[Out] (((a + b*x)^3)^(1/3)*(-a + b*x*Log[x]))/(x*(a + b*x))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.32

method	result	size
risch	$-\frac{\left((bx+a)^3\right)^{\frac{1}{3}}a}{(bx+a)x}+\frac{\left((bx+a)^3\right)^{\frac{1}{3}}b\ln(x)}{bx+a}$	44

```
[In] int(((b*x+a)^3)^(1/3)/x^2,x,method=_RETURNVERBOSE)
[Out] -((b*x+a)^3)^(1/3)/(b*x+a)*a/x+((b*x+a)^3)^(1/3)/(b*x+a)*b*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt[3]{(a+bx)^3}}{x^2} dx = \frac{bx \log(x) - a}{x}$$

```
[In] integrate(((b*x+a)^3)^(1/3)/x^2,x, algorithm="fricas")
[Out] (b*x*log(x) - a)/x
```

Sympy [F]

$$\int \frac{\sqrt[3]{(a+bx)^3}}{x^2} dx = \int \frac{\sqrt[3]{(a+bx)^3}}{x^2} dx$$

```
[In] integrate(((b*x+a)**3)**(1/3)/x**2,x)
[Out] Integral(((a + b*x)**3)**(1/3)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.08

$$\int \frac{\sqrt[3]{(a+bx)^3}}{x^2} dx = b \log(x) - \frac{a}{x}$$

```
[In] integrate(((b*x+a)^3)^(1/3)/x^2,x, algorithm="maxima")
[Out] b*log(x) - a/x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt[3]{(a+bx)^3}}{x^2} dx = b \log(|x|) - \frac{a}{x}$$

```
[In] integrate(((b*x+a)^3)^(1/3)/x^2,x, algorithm="giac")
[Out] b*log(abs(x)) - a/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{(a+bx)^3}}{x^2}\,dx = \int \frac{\left((a+b\,x)^3\right)^{1/3}}{x^2}\,dx$$

```
[In] int(((a + b*x)^3)^(1/3)/x^2,x)
[Out] int(((a + b*x)^3)^(1/3)/x^2, x)
```

3.161 $\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 105

$$\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx = -\frac{\sqrt[3]{(a+bx)^2}}{ax} - \frac{b\left(\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{2\sqrt[3]{a}+\sqrt[3]{a+bx}}\right)+\frac{3}{2}\log\left(\frac{-\sqrt[3]{a}+\sqrt[3]{a+bx}}{\sqrt[3]{x}}\right)\right)}{3a\sqrt[3]{a^2}}$$

```
[Out] -((b*x+a)^2)^(1/3)/a/x-1/3*b/a/(a^2)^(1/3)*(3/2*ln(((b*x+a)^(1/3)-a^(1/3))/
x^(1/3))+3^(1/2)*arctan(3^(1/2)*(b*x+a)^(1/3)/((b*x+a)^(1/3)+2*a^(1/3))))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {44, 57, 631, 210, 31}

$$\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx = -\frac{b\arctan\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}} - \frac{(a+bx)^{2/3}}{ax}$$

```
[In] Int[1/(x^2*(a + b*x)^(1/3)),x]
```

```
[Out] -((a + b*x)^(2/3)/(a*x)) - (b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a^(1/3) - (
a + b*x)^(1/3)])/(2*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+bx)^{2/3}}{ax} - \frac{b\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{3a} \\
&= -\frac{(a+bx)^{2/3}}{ax} + \frac{b\log(x)}{6a^{4/3}} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{2a^{4/3}} \\
&\quad - \frac{b\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx}\right)}{2a}
\end{aligned}$$

$$= -\frac{(a+bx)^{2/3}}{ax} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}} + \frac{b\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{4/3}}$$

$$= -\frac{(a+bx)^{2/3}}{ax} - \frac{b\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx =$$

$$-\frac{6\sqrt[3]{a}(a+bx)^{2/3} + 2\sqrt{3}bx\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2bx\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) - bx\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+}\right.}{6a^{4/3}x}$$

```
[In] Integrate[1/(x^2*(a + b*x)^(1/3)),x]
```

```
[Out] -1/6*(6*a^(1/3)*(a + b*x)^(2/3) + 2*Sqrt[3]*b*x*ArcTan[(1 + (2*(a + b*x)^(1
/3))/a^(1/3))/Sqrt[3]] + 2*b*x*Log[a^(1/3) - (a + b*x)^(1/3)] - b*x*Log[a^(
2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(a^(4/3)*x)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}}{ax}-\frac{b\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{4}{3}}}+\frac{b\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{4}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{4}{3}}}$
pseudoelliptic	$\frac{-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)bx-6a^{\frac{1}{3}}(bx+a)^{\frac{2}{3}}-2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)bx+\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)bx}{6a^{\frac{4}{3}}x}$
derivativedivides	$3b\left(-\frac{(bx+a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}}{3a}\right)$
default	$3b\left(-\frac{(bx+a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}}{3a}\right)$

```
[In] int(1/x^2/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*(b*x+a)^(2/3)/x-1/3*b/a^(4/3)*ln((b*x+a)^(1/3)-a^(1/3))+1/6*b/a^(4/3)*
ln((b*x+a)^(2/3)+(b*x+a)^(1/3)*a^(1/3)+a^(2/3))-1/3*b/a^(4/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.91

$$\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}abx\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx-3\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3\,a}{x}\right)+(-a)^{\frac{2}{3}}\,bx\log\left(\cdots\right.}{6\,a^2x}\right.$$

$$-\frac{6\sqrt{\frac{1}{3}}abx\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-(-a)^{\frac{2}{3}}\,bx\log\left((bx+a)^{\frac{2}{3}}-(bx+a\cdots\right.}{6\,a^2x}$$

```
[In] integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*x*sqrt((-a)^(1/3)/a)*log((2*b*x - 3*sqrt(1/3)*(2*(b*x
 + a)^(2/3)*(-a)^(2/3) - (b*x + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/
a) - 3*(b*x + a)^(1/3)*(-a)^(2/3) + 3*a)/x) + (-a)^(2/3)*b*x*log((b*x + a)^
(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log((b*
x + a)^(1/3) + (-a)^(1/3)) - 6*(b*x + a)^(2/3)*a)/(a^2*x), -1/6*(6*sqrt(1/3
)*a*b*x*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x + a)^(1/3) - (-a)^(1/3
))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)*b*x*log((b*x + a)^(2/3) - (b*x + a)^(1
/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(-a)^(2/3)*b*x*log((b*x + a)^(1/3) + (-a)^
(1/3)) + 6*(b*x + a)^(2/3)*a)/(a^2*x)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 831, normalized size of antiderivative = 7.91

$$\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x**2/(b*x+a)**(1/3),x)
```

```
[Out] -2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b +
 x)**(1/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*p
```

```
i/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)
) - 2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a
/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a
/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*
exp(2*I*pi/3)*gamma(5/3)) - 2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*log(1 - b*
*(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b*
*(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x
)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3)*b**(10/3)*(a/b + x)**(7/3)*
exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(9*a**
3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b
 + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3)*b**(10/3)*(a/b + x)**(7
/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a*
*(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/
3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3
)*b**(10/3)*(a/b + x)**(7/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*
I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3
)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) +
 6*a*b**3*(a/b + x)**2*exp(2*I*pi/3)*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)*
*(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*
pi/3)*gamma(5/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx = -\frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{4}{3}}} - \frac{(bx+a)^{\frac{2}{3}}b}{(bx+a)a-a^2} + \frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{4}{3}}} - \frac{b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{4}{3}}}$$

```
[In] integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(4/3) - (b*x + a)^(2/3)*b/((b*x + a)*a - a^2) + 1/6*b*log((b*x + a)^(2/3) +
 (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) - 1/3*b*log((b*x + a)^(1/3) - a
^(1/3))/a^(4/3)
```

Giac [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx = -\frac{\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}} - \frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}} + \frac{2b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}} + \frac{6(bx+a)^{\frac{2}{3}}b}{ax}}{6b}$$

```
[In] integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] -1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(4/3) - b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(4/3) + 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 6*(b*x + a)^(2
/3)*b/(a*x))/b
```

Mupad [B] (verification not implemented)

Time = 15.94 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx = -\frac{(a+b\,x)^{2/3}}{a\,x} + \frac{\ln\left(\frac{\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}} - \frac{b^2\,(a+b\,x)^{1/3}}{a^2}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}} + \frac{\ln\left(\frac{\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}} - \frac{b^2\,(a+b\,x)^{1/3}}{a^2}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}} - \frac{b\,\ln\left((a+b\,x)^{1/3}-a^{1/3}\right)}{3\,a^{4/3}}$$

```
[In] int(1/(x^2*(a + b*x)^(1/3)),x)
```

```
[Out] (log((b - 3^(1/2)*b*1i)^2/(4*a^(5/3)) - (b^2*(a + b*x)^(1/3))/a^2)*(b - 3^(
1/2)*b*1i))/(6*a^(4/3)) - (a + b*x)^(2/3)/(a*x) + (log((b + 3^(1/2)*b*1i)^2
/(4*a^(5/3)) - (b^2*(a + b*x)^(1/3))/a^2)*(b + 3^(1/2)*b*1i))/(6*a^(4/3)) -
 (b*log((a + b*x)^(1/3) - a^(1/3)))/(3*a^(4/3))
```

3.162 $\int \frac{1}{x^3 \sqrt[3]{a+bx}} dx$

Optimal result

Integrand size = 13, antiderivative size = 117

$$\int \frac{1}{x^3 \sqrt[3]{a+bx}} dx = \left(-\frac{1}{2ax^2} + \frac{2b}{3a^2x}\right) \sqrt[3]{a+bx} + \frac{2b^2 \left(\sqrt{3} \arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{2\sqrt[3]{a}+\sqrt[3]{a+bx}}\right) + \frac{3}{2} \log\left(\frac{-\sqrt[3]{a}+\sqrt[3]{a+bx}}{\sqrt[3]{x}}\right)\right)}{9 (a^2)^{4/3}}$$

```
[Out] (-1/2/a/x^2+2/3*b/a^2/x)*(b*x+a)^(1/3)+2/9*b^2/a^2/(a^2)^(1/3)*(3/2*ln(((b*
x+a)^(1/3)-a^(1/3))/x^(1/3))+3^(1/2)*arctan(3^(1/2)*(b*x+a)^(1/3)/((b*x+a)^
(1/3)+2*a^(1/3))))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {44, 57, 631, 210, 31}

$$\int \frac{1}{x^3 \sqrt[3]{a+bx}} dx = \frac{2b^2 \arctan\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}} - \frac{b^2 \log(x)}{9a^{7/3}} + \frac{b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx}\right)}{3a^{7/3}} + \frac{2b(a+bx)^{2/3}}{3a^2x} - \frac{(a+bx)^{2/3}}{2ax^2}$$

```
[In] Int[1/(x^3*(a + b*x)^(1/3)),x]
```

```
[Out] -1/2*(a + b*x)^(2/3)/(a*x^2) + (2*b*(a + b*x)^(2/3))/(3*a^2*x) + (2*b^2*Arc
Tan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) -
```

```
(b^2*Log[x])/(9*a^(7/3)) + (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(7/3)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = -\frac{(a+bx)^{2/3}}{2ax^2} - \frac{(2b)\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx}{3a}$$

$$= -\frac{(a+bx)^{2/3}}{2ax^2} + \frac{2b(a+bx)^{2/3}}{3a^2x} + \frac{(2b^2)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{9a^2}$$

$$= -\frac{(a+bx)^{2/3}}{2ax^2} + \frac{2b(a+bx)^{2/3}}{3a^2x} - \frac{b^2\log(x)}{9a^{7/3}} - \frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{3a^{7/3}} + \frac{b^2\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx}\right)}{3a^2}$$

$$= -\frac{(a+bx)^{2/3}}{2ax^2} + \frac{2b(a+bx)^{2/3}}{3a^2x} - \frac{b^2\log(x)}{9a^{7/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}} - \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{3a^{7/3}}$$

$$= -\frac{(a+bx)^{2/3}}{2ax^2} + \frac{2b(a+bx)^{2/3}}{3a^2x} + \frac{2b^2\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{7/3}} - \frac{b^2\log(x)}{9a^{7/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx = -\frac{(a+bx)^{2/3}(7a-4(a+bx))}{6a^2x^2} + \frac{2b^2\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}} + \frac{2b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{7/3}} - \frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{9a^{7/3}}$$

```
[In] Integrate[1/(x^3*(a + b*x)^(1/3)),x]
```

```
[Out] -1/6*((a + b*x)^(2/3)*(7*a - 4*(a + b*x)))/(a^2*x^2) + (2*b^2*ArcTan[1/Sqrt
[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) + (2*b^2*
Log[a^(1/3) - (a + b*x)^(1/3)])/(9*a^(7/3)) - (b^2*Log[a^(2/3) + a^(1/3)*(a
 + b*x)^(1/3) + (a + b*x)^(2/3)])/(9*a^(7/3))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}(-4bx+3a)}{6a^2x^2}+\frac{2b^2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{7}{3}}}-\frac{b^2\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{2b^2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+}{a^{}}\right.}{3}\right.}{9a^{\frac{7}{3}}}$
pseudoelliptic	$\frac{4b^2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)x^2+4b^2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)x^2-2b^2\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)x^2+12bx(bx+a}{18a^{\frac{7}{3}}x^2}$
derivativedivides	$3b^2\left(-\frac{(bx+a)^{\frac{2}{3}}}{6a\,b^2x^2}-\frac{2\left(-\frac{(bx+a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+\right.}{3}\right.}{3a^{\frac{1}{3}}}}{3a}\right.}{3a}\right.$
default	$3b^2\left(-\frac{(bx+a)^{\frac{2}{3}}}{6a\,b^2x^2}-\frac{2\left(-\frac{(bx+a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+\right.}{3}\right.}{3a^{\frac{1}{3}}}}{3a}\right.}{3a}\right.$

```
[In] int(1/x^3/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(b*x+a)^(2/3)*(-4*b*x+3*a)/a^2/x^2+2/9*b^2/a^(7/3)*ln((b*x+a)^(1/3)-a^
(1/3))-1/9*b^2/a^(7/3)*ln((b*x+a)^(2/3)+(b*x+a)^(1/3)*a^(1/3)+a^(2/3))+2/9*
b^2/a^(7/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.53

$$\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx = \left[\frac{6\sqrt{\frac{1}{3}}ab^2x^2\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx+3\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x}\right)-2\,a^{\frac{2}{3}}b^2x^2\log\left((bx+a\right.}{18\,a^3x^2}\right.$$

```
[In] integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt(-1/a^(2/3))*log((2*b*x + 3*sqrt(1/3)*(2*(
b*x + a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*
(b*x + a)^(1/3)*a^(2/3) + 3*a)/x) - 2*a^(2/3)*b^2*x^2*log((b*x + a)^(2/3) +
 (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 4*a^(2/3)*b^2*x^2*log((b*x + a)^(1/3)
 - a^(1/3)) + 3*(4*a*b*x - 3*a^2)*(b*x + a)^(2/3))/(a^3*x^2), 1/18*(12*sqrt
(1/3)*a^(2/3)*b^2*x^2*arctan(sqrt(1/3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
)) - 2*a^(2/3)*b^2*x^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2
/3)) + 4*a^(2/3)*b^2*x^2*log((b*x + a)^(1/3) - a^(1/3)) + 3*(4*a*b*x - 3*a^
2)*(b*x + a)^(2/3))/(a^3*x^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.37 (sec) , antiderivative size = 2730, normalized size of antiderivative = 23.33

$$\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x**3/(b*x+a)**(1/3),x)
```

```
[Out] 4*a**(14/3)*b**(10/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
+ x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I
```

*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5
/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**
4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 4*a**(14/3)*b**(1
0/3)*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_
polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp
(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gam
ma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27
*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 4*a**(14/3)*b
(10/3)*(a/b + x)(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*
gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 8
1*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(1
3/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(
a/b + x)**(7/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*g
amma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*
a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)
*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**
(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(a/b + x)**(7/3)*
exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/
3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3)
- 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(
10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b +
 x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(a/b + x)**(
7/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(
2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*
b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b
 + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3
)*exp(2*I*pi/3)*gamma(5/3)) + 12*a**(8/3)*b**(16/3)*(a/b + x)**(10/3)*exp(2
*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b*
(4/3)(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b +
x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*ex
p(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*
gamma(5/3)) + 12*a**(8/3)*b**(16/3)*(a/b + x)**(10/3)*exp(-2*I*pi/3)*log(1
- b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a*
*7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a
/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/
3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*p
i/3)*gamma(5/3)) + 12*a**(8/3)*b**(16/3)*(a/b + x)**(10/3)*log(1 - b**(1/3)
*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3
)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(
7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I
*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma
(5/3)) - 4*a**(5/3)*b**(19/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*log(1 - b**(1
/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3
)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3

```
)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3)
 - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(5/
3)*b**(19/3)*(a/b + x)**(13/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(
1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**
(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*
pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(
5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a*
*(5/3)*b**(19/3)*(a/b + x)**(13/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_po
lar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2
*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma
(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a
**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 21*a**4*b**4*(a
/b + x)**2*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(
2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamm
a(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*
a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 33*a**3*b**5*(
a/b + x)**3*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp
(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gam
ma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27
*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**2*b**6*
(a/b + x)**4*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*ex
p(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*ga
mma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 2
7*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx = \frac{2\,\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{7}{3}}} - \frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{7}{3}}} + \frac{2\,b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{7}{3}}} + \frac{4\,(bx+a)^{\frac{5}{3}}b^2-7\,(bx+a)^{\frac{2}{3}}ab^2}{6\left((bx+a)^2a^2-2\,(bx+a)a^3+a^4\right)}$$

```
[In] integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(7/3) - 1/9*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(7/3) + 2/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(7/3) + 1/6*(4*(b*x + a)^
```

```
(5/3)*b^2 - 7*(b*x + a)^(2/3)*a*b^2)/((b*x + a)^2*a^2 - 2*(b*x + a)*a^3 + a
^4)
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \sqrt[3]{a+bx}}\,dx = \frac{\frac{4\sqrt{3}b^3 \arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}} - \frac{2b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{7}{3}}} + \frac{4b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{7}{3}}} + \frac{3\left(4(bx+a)^{\frac{5}{3}}b^3-7(bx+a)^{\frac{2}{3}}ab\right.}{a^2b^2x^2}}{18b}$$

```
[In] integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/18*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(7/3) - 2*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(7/3) + 4*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(7/3) + 3*(4*(b*x +
a)^(5/3)*b^3 - 7*(b*x + a)^(2/3)*a*b^3)/(a^2*b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 16.75 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.56

$$\int \frac{1}{x^3 \sqrt[3]{a+bx}}\,dx = \frac{2b^2 \ln\left((a+bx)^{1/3}-a^{1/3}\right)}{9a^{7/3}} - \frac{\frac{7b^2(a+bx)^{2/3}}{6a}-\frac{2b^2(a+bx)^{5/3}}{3a^2}}{(a+bx)^2-2a(a+bx)+a^2} - \frac{\ln\left(\frac{4b^4(a+bx)^{1/3}}{9a^4}-\frac{\left(b^2+\sqrt{3}b^2\,1\mathrm{i}\right)^2}{9a^{11/3}}\right)\left(b^2+\sqrt{3}b^2\,1\mathrm{i}\right)}{9a^{7/3}} + \frac{b^2\ln\left(\frac{4b^4(a+bx)^{1/3}}{9a^4}-\frac{9b^4\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2}{a^{11/3}}\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)}{a^{7/3}}$$

```
[In] int(1/(x^3*(a + b*x)^(1/3)),x)
```

```
[Out] (2*b^2*log((a + b*x)^(1/3) - a^(1/3)))/(9*a^(7/3)) - ((7*b^2*(a + b*x)^(2/3
))/(6*a) - (2*b^2*(a + b*x)^(5/3))/(3*a^2))/((a + b*x)^2 - 2*a*(a + b*x) +
a^2) - (log((4*b^4*(a + b*x)^(1/3))/(9*a^4) - (3^(1/2)*b^2*1i + b^2)^2/(9*a
^(11/3)))*(3^(1/2)*b^2*1i + b^2))/(9*a^(7/3)) + (b^2*log((4*b^4*(a + b*x)^(
1/3))/(9*a^4) - (9*b^4*((3^(1/2)*1i)/9 - 1/9)^2)/a^(11/3))*((3^(1/2)*1i)/9
- 1/9))/a^(7/3)
```

3.163 $\int \frac{A+Bx}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{A+Bx}{\sqrt{a+bx}}\,dx = \frac{2A\sqrt{a+bx}}{b} + \frac{2B\sqrt{a+bx}\left(-a+\frac{1}{3}(a+bx)\right)}{b^2}$$

```
[Out] 2*alpha*(b*x+a)^(1/2)/b+2*beta*(1/3*b*x-2/3*a)*(b*x+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\int \frac{A+Bx}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}(Ab-aB)}{b^2} + \frac{2B(a+bx)^{3/2}}{3b^2}$$

```
[In] Int[(A + B*x)/Sqrt[a + b*x],x]
[Out] (2*(A*b - a*B)*Sqrt[a + b*x])/b^2 + (2*B*(a + b*x)^(3/2))/(3*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{Ab-aB}{b\sqrt{a+bx}} + \frac{B\sqrt{a+bx}}{b}\right) dx \\
&= \frac{2(Ab-aB)\sqrt{a+bx}}{b^2} + \frac{2B(a+bx)^{3/2}}{3b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}(3Ab-2aB+bBx)}{3b^2}$$

```
[In] Integrate[(A + B*x)/Sqrt[a + b*x],x]
[Out] (2*Sqrt[a + b*x]*(3*A*b - 2*a*B + b*B*x))/(3*b^2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(-b\beta x+2a\beta-3\alpha b)}{3b^2}$	27
trager	$-\frac{2\sqrt{bx+a}\,(-b\beta x+2a\beta-3\alpha b)}{3b^2}$	27
risch	$-\frac{2\sqrt{bx+a}\,(-b\beta x+2a\beta-3\alpha b)}{3b^2}$	27
pseudoelliptic	$-\frac{2\sqrt{bx+a}\,(-b\beta x+2a\beta-3\alpha b)}{3b^2}$	27
derivativedivides	$\frac{\frac{2\beta(bx+a)^{\frac{3}{2}}}{3}-2a\beta\sqrt{bx+a}+2\alpha b\sqrt{bx+a}}{b^2}$	38
default	$\frac{\frac{2\beta(bx+a)^{\frac{3}{2}}}{3}-2a\beta\sqrt{bx+a}+2\alpha b\sqrt{bx+a}}{b^2}$	38

```
[In] int((beta*x+alpha)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(b*x+a)^(1/2)*(-b*beta*x+2*a*beta-3*alpha*b)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.57

$$\int \frac{A+Bx}{\sqrt{a+bx}}\,dx = \frac{2\,(b\beta x+3\,\alpha b-2\,a\beta)\sqrt{bx+a}}{3\,b^2}$$

```
[In] integrate((beta*x+alpha)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*beta*x + 3*alpha*b - 2*a*beta)*sqrt(b*x + a)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx}{\sqrt{a+bx}}\,dx = \begin{cases} \frac{2A\sqrt{a+bx}+\frac{2B\left(-a\sqrt{a+bx}+\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b}}{b} & \text{for } b\neq 0 \\ \frac{Ax+\frac{Bx^2}{2}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

```
[In] integrate((B*x+A)/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise(((2*A*sqrt(a + b*x) + 2*B*(-a*sqrt(a + b*x) + (a + b*x)**(3/2)/3)
/b)/b, Ne(b, 0)), ((A*x + B*x**2/2)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{\sqrt{a+bx}}\,dx = \frac{2\left(3\sqrt{bx+a}\alpha+\frac{\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)\beta}{b}\right)}{3b}$$

```
[In] integrate((beta*x+alpha)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(b*x + a)*alpha + ((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*beta/b)/
b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{\sqrt{a+bx}}\,dx = \frac{2\left(3\sqrt{bx+a}\alpha+\frac{\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)\beta}{b}\right)}{3b}$$

```
[In] integrate((beta*x+alpha)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*sqrt(b*x + a)*alpha + ((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*beta/b)/
b
```

Mupad [B] (verification not implemented)

Time = 16.96 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+b\,x}\,\left(3\,\alpha\,b+(a+b\,x)\ \beta-3\,a\,\beta\right)}{3\,b^2}$$

```
[In] int((alpha + x*beta)/(a + b*x)^(1/2),x)
[Out] (2*(a + b*x)^(1/2)*(3*alpha*b + (a + b*x)*beta - 3*a*beta))/(3*b^2)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```