Computer Algebra Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.3-Miscellaneous/51-1.3.1-Rational-functions

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [494]. This is test number [51].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (494)	0.00 (0)
Rubi	99.39 (491)	0.61 (3)
Maple	98.99 (489)	1.01 (5)
Mupad	98.18 (485)	1.82 (9)
Fricas	92.31 (456)	7.69 (38)
Sympy	88.26 (436)	11.74 (58)
Giac	86.23 (426)	13.77 (68)
Maxima	82.59 (408)	17.41 (86)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

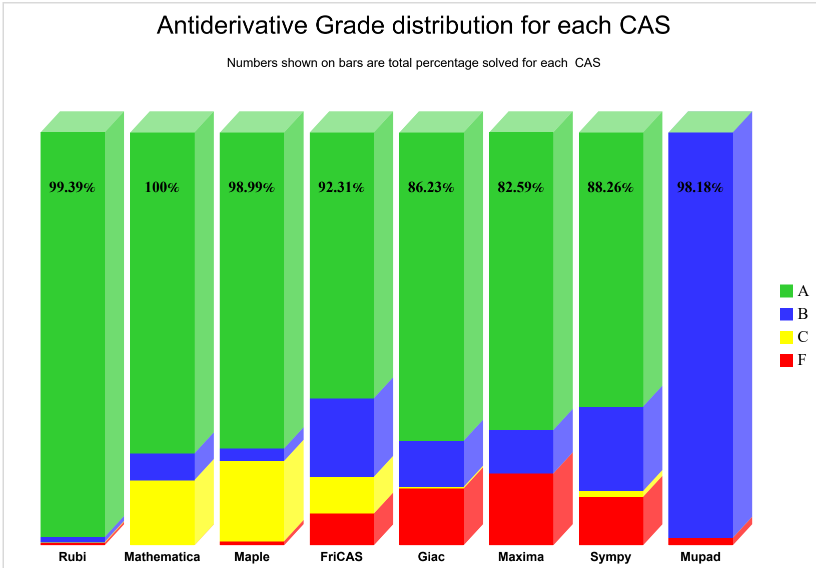
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

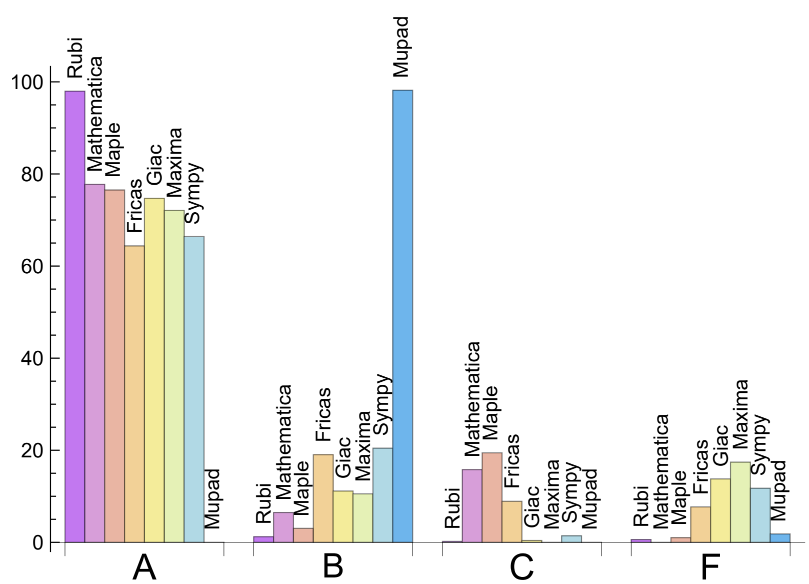
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.976	1.215	0.202	0.607
Mathematica	77.733	6.478	15.789	0.000
Maple	76.518	3.036	19.433	1.012
Giac	74.696	11.134	0.405	13.765
Maxima	72.065	10.526	0.000	17.409
Sympy	66.397	20.445	1.417	11.741
Fricas	64.372	19.028	8.907	7.692
Mupad	0.000	98.178	0.000	1.822

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	3	100.00	0.00	0.00
Maple	5	100.00	0.00	0.00
Mupad	9	0.00	100.00	0.00
Fricas	38	10.53	76.32	13.16
Sympy	58	8.62	91.38	0.00
Giac	68	98.53	0.00	1.47
Maxima	86	96.51	0.00	3.49

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Mathematica	0.05
Rubi	0.14
Maxima	0.23
Giac	0.47
Maple	0.49
Fricas	1.36
Sympy	1.47
Mupad	5.01

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	63.27	0.99	28.00	0.86
Mathematica	81.37	1.26	38.00	1.00
Maxima	116.03	1.79	28.00	0.88
Rubi	119.38	1.04	33.00	1.00
Giac	184.22	1.78	29.00	0.92
Sympy	198.72	2.51	39.00	0.91
Mupad	450.36	2.59	41.00	0.95
Fricas	14149.51	68.45	39.00	1.14

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

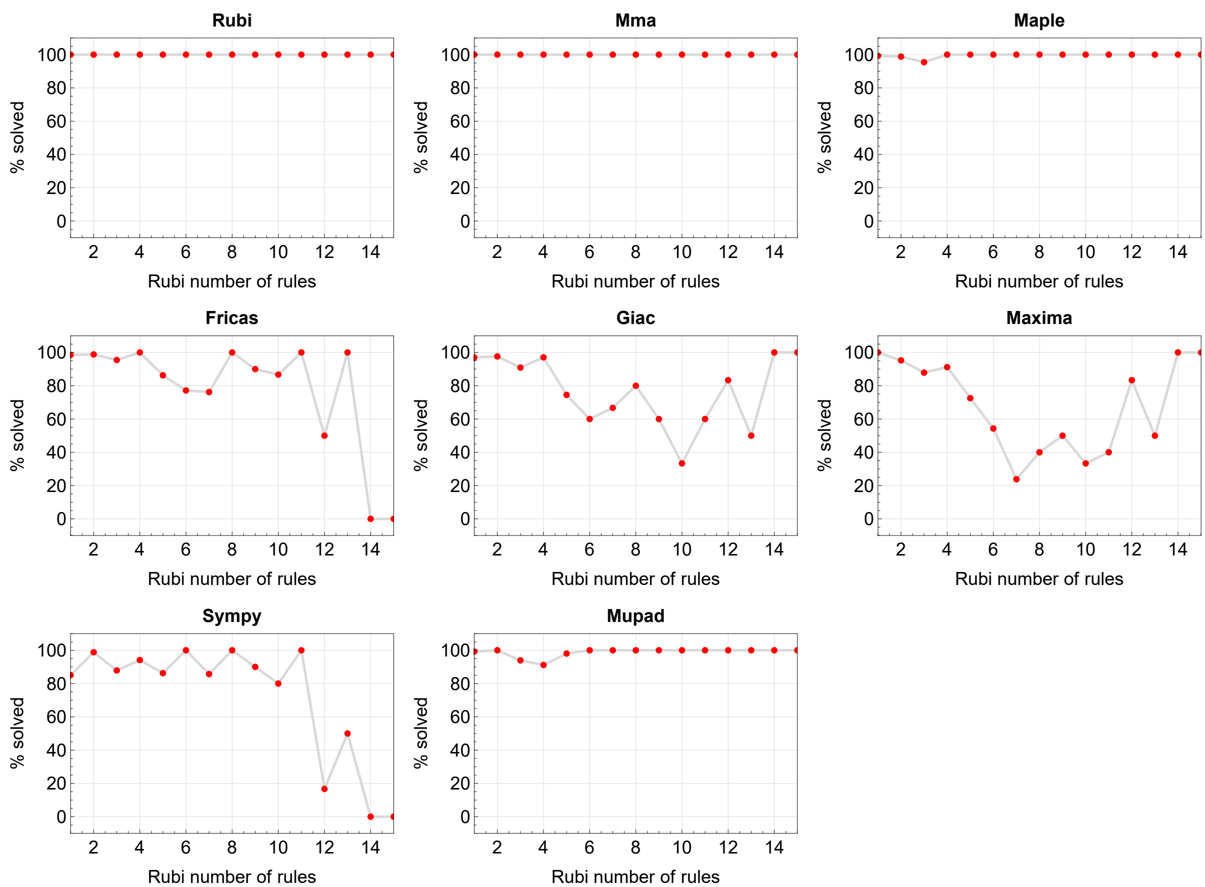


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

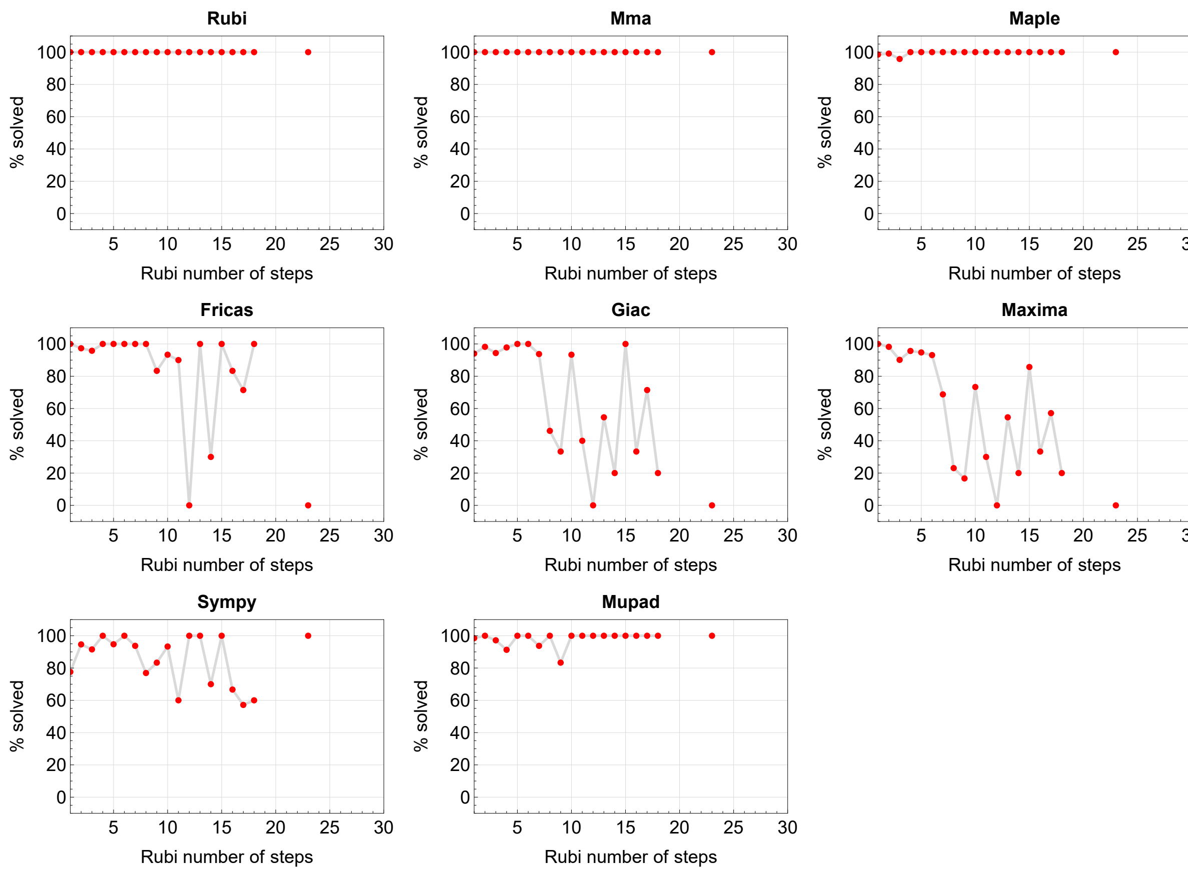


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

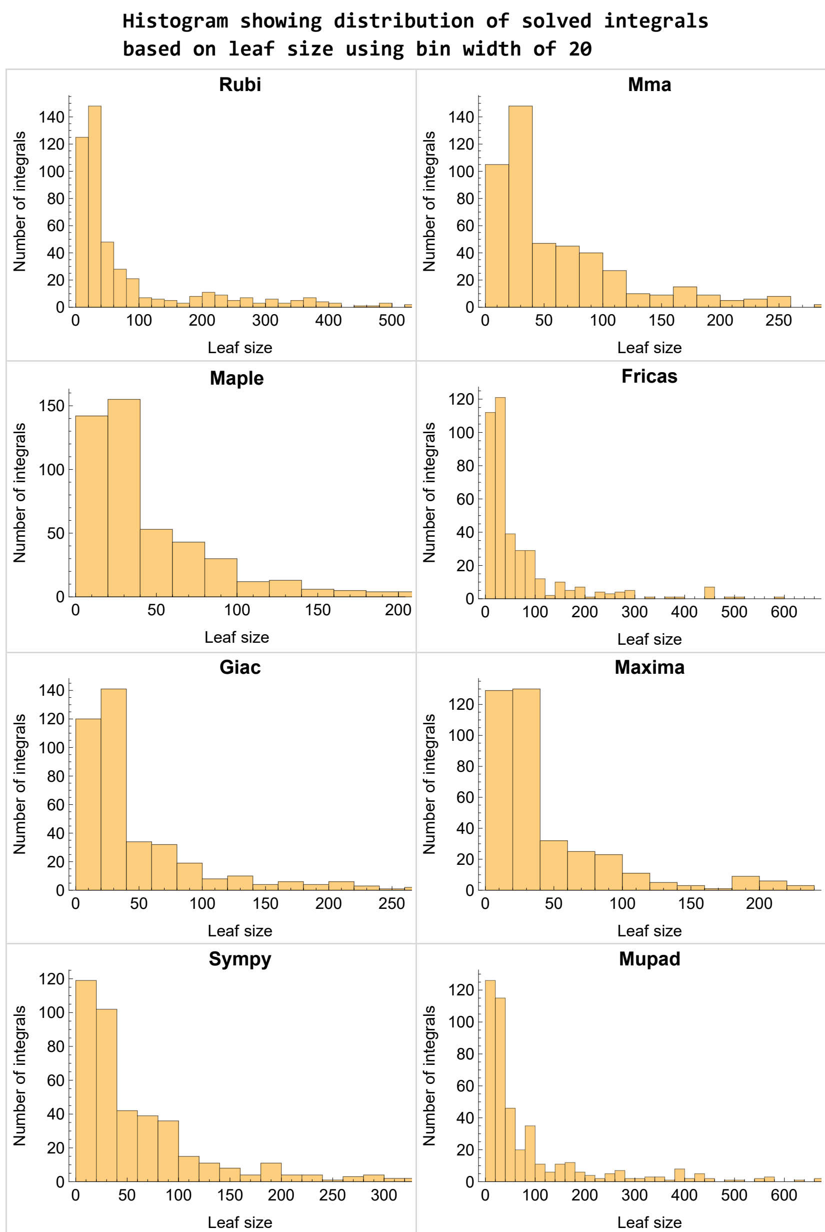


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

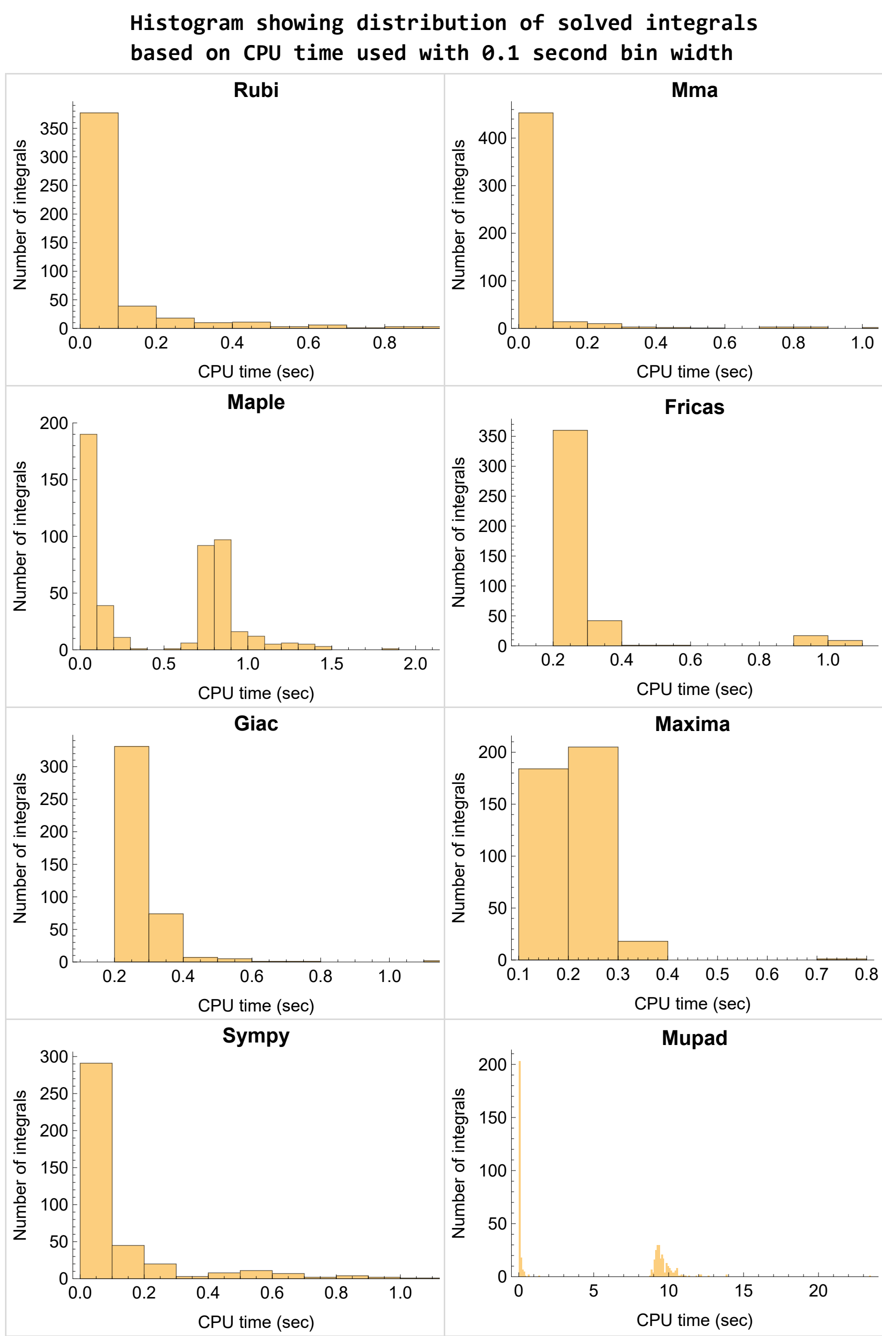


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

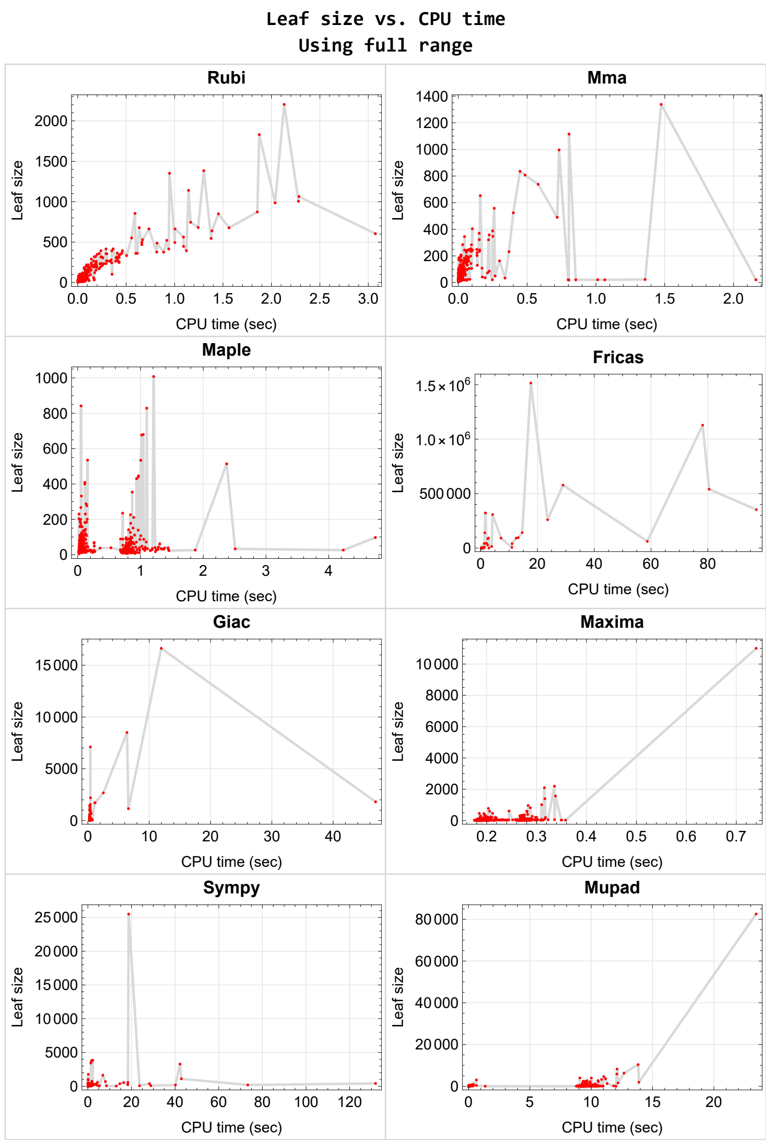


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{154, 155, 156, 157\}$

Mathematica $\{32\}$

Maple $\{1, 174\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

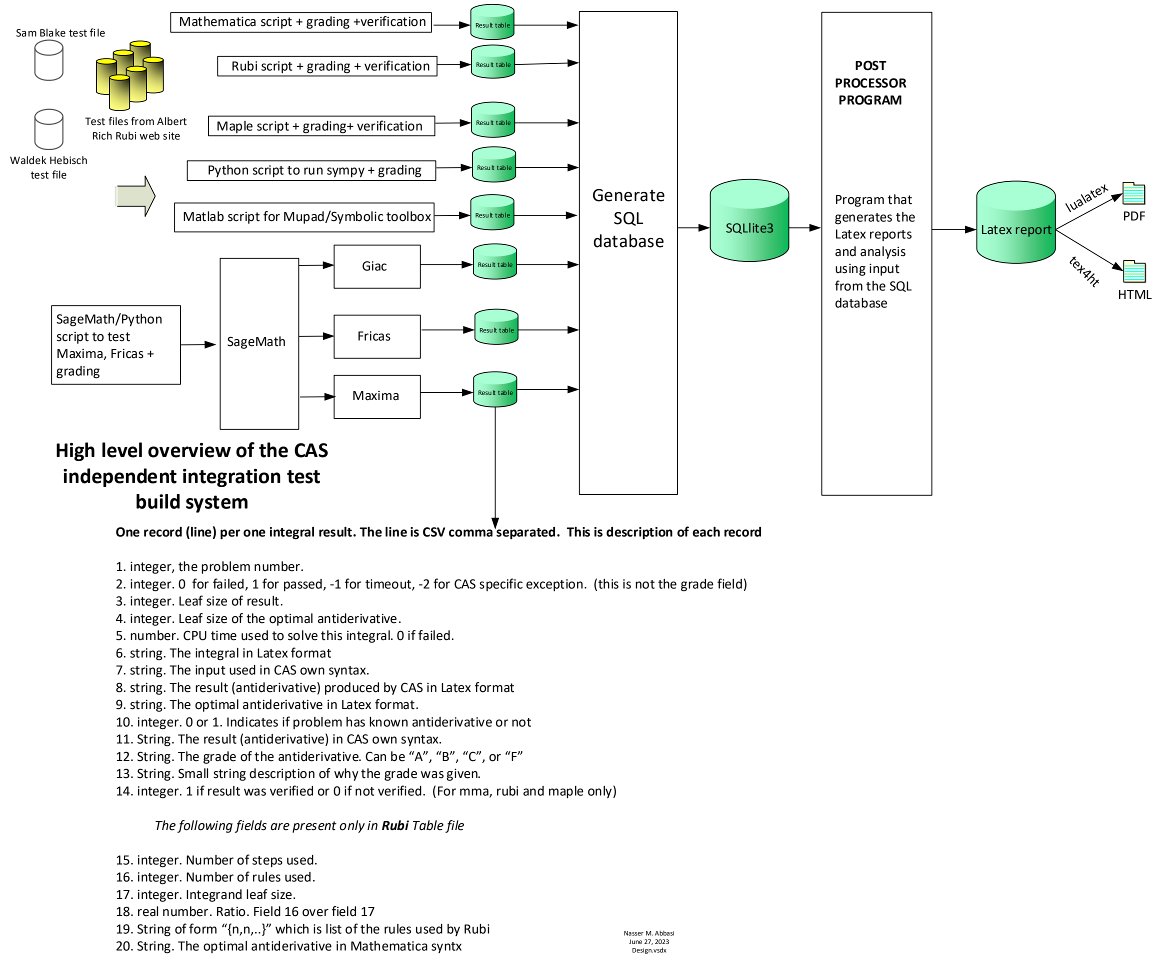
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 22
2.2 Detailed conclusion table per each integral for all CAS systems 29
2.3 Detailed conclusion table specific for Rubi results 129

2.1 List of integrals sorted by grade for each CAS

Rubi 22
Mma 23
Maple 23
Fricas 24
Maxima 25
Giac 26
Mupad 26
Sympy 27

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492 }

B grade { 65, 77, 221, 222, 233, 424 }

C grade { 174 }

F normal fail { 393, 493, 494 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 102, 106, 113, 116, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 177, 178, 179, 180, 181, 182, 183, 186, 187, 188, 189, 190, 191, 193, 194, 208, 216, 217, 218, 219, 220, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 492, 493, 494 }

B grade { 65, 88, 92, 95, 101, 160, 161, 162, 192, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 221, 222, 421 }

C grade { 12, 13, 14, 32, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 173, 174, 175, 176, 184, 185, 227, 250, 251, 252, 253, 254, 255, 256, 257, 387, 388, 389, 390, 391, 392, 393, 491 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 2, 5, 6, 7, 8, 10, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 101, 102, 116, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 172, 173, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 200, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 339, 340, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 399, 400, 405, 406, 407, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 492, 493, 494 }

B grade { 3, 4, 9, 15, 63, 64, 92, 95, 163, 171, 194, 197, 201, 205, 222 }

C grade { 1, 12, 13, 14, 27, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 174, 250, 251, 252, 253, 254, 255, 256, 257, 334, 337, 338, 341, 342, 367, 368, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 401, 402, 403, 404, 408, 409, 410, 411, 455, 491 }

F normal fail { 29, 30, 31, 32, 176 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 5, 6, 9, 10, 11, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 69, 70, 71, 72, 73, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 96, 98, 99, 100, 101, 102, 106, 116, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 164, 165, 166, 167, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 208, 209, 216, 217, 218, 219, 220, 224, 225, 226, 228, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 339, 340, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 415, 416, 417, 418, 419, 420, 422, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 489, 490, 492 }

B grade { 3, 4, 7, 8, 12, 13, 14, 15, 37, 38, 43, 44, 63, 64, 65, 66, 67, 68, 74, 75, 76, 77, 87, 91, 92, 93, 94, 95, 97, 120, 121, 122, 146, 154, 160, 161, 162, 163, 168, 169, 170, 171, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 221, 222, 223, 229, 233, 250, 251, 252, 253, 254, 255, 256, 257, 268, 277, 343, 344, 387, 388, 389, 390, 421, 423, 424, 453, 455, 477, 484, 493, 494 }

C grade { 49, 50, 55, 56, 61, 62, 103, 104, 105, 107, 108, 109, 111, 112, 113, 114, 115, 127, 134, 138, 334, 337, 338, 341, 342, 367, 368, 391, 392, 393, 394, 395, 396, 397, 398, 401, 402, 403, 404, 408, 409, 410, 411, 491 }

F normal fail { 29, 30, 31, 32 }

F(-1) timedout fail { 19, 20, 110, 128, 129, 135, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 227, 399, 400, 405, 406, 407, 412, 413, 414 }

F(-2) exception fail { 136, 137, 139, 140, 141 }

Maxima

A grade { 2, 5, 6, 10, 11, 15, 16, 17, 18, 21, 22, 23, 24, 25, 27, 28, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 90, 93, 96, 97, 98, 99, 100, 116, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 160, 164, 165, 166, 168, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 208, 209, 216, 217, 218, 219, 220, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 492 }

B grade { 3, 4, 7, 8, 9, 19, 20, 63, 64, 65, 66, 67, 68, 74, 87, 88, 91, 92, 94, 95, 101, 102, 161, 162, 163, 167, 169, 170, 171, 194, 197, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 221, 222, 421, 424, 477, 493, 494 }

C grade { }

F normal fail { 1, 12, 13, 14, 29, 30, 31, 32, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 227, 228, 250, 251, 252, 253, 254, 255, 256, 257, 335, 336, 337, 338, 387, 388, 389, 390, 391, 392, 393, 491 }

F(-1) timedout fail { }

F(-2) exception fail { 26, 489, 490 }

Giac

A grade { 1, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 101, 102, 106, 113, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 160, 164, 165, 166, 168, 169, 170, 172, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 189, 190, 191, 193, 194, 196, 197, 199, 200, 201, 206, 208, 209, 210, 212, 214, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 486, 487, 488, 489, 490, 491, 492 }

B grade { 2, 3, 4, 9, 15, 19, 20, 43, 44, 63, 64, 65, 74, 92, 95, 116, 120, 121, 122, 159, 161, 162, 163, 173, 174, 175, 182, 188, 192, 195, 198, 202, 203, 204, 205, 207, 211, 213, 215, 234, 235, 236, 237, 238, 268, 282, 324, 337, 338, 359, 421, 424, 485, 493, 494 }

C grade { 55, 56 }

F normal fail { 29, 30, 31, 32, 49, 50, 61, 62, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 115, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 167, 171, 176, 239, 240, 241, 250, 251, 252, 253, 254, 255, 256, 257, 387, 388, 389, 390, 391, 392, 393 }

F(-1) timedout fail { }

F(-2) exception fail { 227 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 172, 173, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494 }

C grade { }

F normal fail { }

F(-1) timedout fail { 31, 100, 101, 102, 167, 171, 174, 176, 227 }

F(-2) exception fail { }

Sympy

A grade { 1, 5, 10, 11, 12, 13, 14, 16, 17, 21, 22, 23, 24, 25, 27, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 51, 52, 53, 54, 57, 58, 59, 60, 61, 69, 70, 71, 72, 73, 75, 76, 93, 96, 97, 98, 99, 100, 103, 104, 105, 106, 110, 111, 112, 113, 116, 117, 118, 119, 120, 123, 124, 125, 126, 127, 130, 131, 132, 133, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 164, 165, 166, 177, 178, 219, 220, 224, 225, 226, 228, 229, 230, 231, 232, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 402, 403, 404, 408, 409, 410, 411, 415, 416, 417, 418, 419, 420, 422, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 491, 492 }

B grade { 3, 4, 6, 7, 8, 9, 15, 26, 28, 50, 55, 56, 62, 63, 64, 65, 66, 67, 68, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 92, 94, 95, 101, 102, 121, 122, 128, 129, 134, 135, 160, 161, 162, 167, 168, 169, 170, 172, 173, 179, 184, 185, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 215, 216, 217, 221, 222, 223, 233, 251, 255, 268, 339, 421, 423, 424, 459, 477, 489, 490, 493, 494 }

C grade { 32, 89, 90, 91, 257, 279, 458 }

F normal fail { 2, 29, 30, 31, 174 }

F(-1) timedout fail { 18, 19, 20, 38, 44, 107, 108, 109, 114, 115, 136, 137, 138, 139, 140, 141, 142, 163, 171, 175, 176, 180, 181, 182, 183, 186, 190, 209, 218, 227, 234, 235, 236, 237, 238, 239, 240, 241, 337, 338, 340, 341, 342, 398, 399, 400, 401, 405, 406, 407, 412, 413, 414 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	77	77	143	43	0	76	60	55	51
N.S.	1	1.00	1.86	0.56	0.00	0.99	0.78	0.71	0.66
time (sec)	N/A	0.047	0.035	0.113	0.000	0.246	0.152	0.300	0.118

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	25	26	25	43	0	73	52
N.S.	1	1.00	0.83	0.87	0.83	1.43	0.00	2.43	1.73
time (sec)	N/A	0.013	0.013	0.051	0.192	0.234	0.000	0.282	9.335

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	98	216	97	107	97	97
N.S.	1	1.00	1.00	7.00	15.43	6.93	7.64	6.93	6.93
time (sec)	N/A	0.005	0.001	0.034	0.197	0.258	0.033	0.290	9.169

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	65	99	64	66	64	64
N.S.	1	1.00	1.00	4.64	7.07	4.57	4.71	4.57	4.57
time (sec)	N/A	0.006	0.002	0.032	0.198	0.247	0.027	0.338	0.017

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	13	31	31	32	31	31
N.S.	1	1.00	1.00	0.37	0.89	0.89	0.91	0.89	0.89
time (sec)	N/A	0.005	0.000	0.026	0.191	0.230	0.018	0.286	0.022

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	24	24	26	12	26
N.S.	1	1.00	1.00	0.93	1.71	1.71	1.86	0.86	1.86
time (sec)	N/A	0.006	0.003	0.035	0.200	0.237	0.078	0.298	0.021

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	57	57	61	12	59
N.S.	1	1.00	1.00	0.93	4.07	4.07	4.36	0.86	4.21
time (sec)	N/A	0.006	0.005	0.075	0.202	0.255	0.171	0.296	9.285

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	90	90	97	12	92
N.S.	1	1.00	1.00	0.93	6.43	6.43	6.93	0.86	6.57
time (sec)	N/A	0.007	0.002	0.107	0.201	0.240	0.251	0.277	9.019

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	159	159	204	156	175	166	149
N.S.	1	1.00	1.89	1.89	2.43	1.86	2.08	1.98	1.77
time (sec)	N/A	0.106	0.016	0.035	0.211	0.250	0.044	0.276	9.303

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	82	82	93	80	87	83	79
N.S.	1	1.00	1.46	1.46	1.66	1.43	1.55	1.48	1.41
time (sec)	N/A	0.056	0.007	0.029	0.209	0.254	0.030	0.297	0.026

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	28	28	31	28	28
N.S.	1	1.00	1.00	0.91	0.88	0.88	0.97	0.88	0.88
time (sec)	N/A	0.004	0.000	0.021	0.195	0.252	0.020	0.283	0.022

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	63	57	0	387	53	212	174
N.S.	1	1.00	0.34	0.30	0.00	2.06	0.28	1.13	0.93
time (sec)	N/A	0.231	0.014	0.048	0.000	0.288	0.204	0.278	0.321

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	112	134	0	704	192	289	247
N.S.	1	1.00	0.46	0.55	0.00	2.87	0.78	1.18	1.01
time (sec)	N/A	0.189	0.044	0.085	0.000	0.275	0.664	0.321	10.171

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	149	277	0	1268	474	366	483
N.S.	1	1.00	0.49	0.91	0.00	4.16	1.55	1.20	1.58
time (sec)	N/A	0.259	0.058	0.146	0.000	0.283	1.390	0.278	10.188

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	653	842	461	727	1018	987	787
N.S.	1	1.00	1.81	2.33	1.28	2.01	2.82	2.73	2.18
time (sec)	N/A	0.454	0.160	0.053	0.212	0.251	0.087	0.286	9.629

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	241	188	180	269	345	346	270
N.S.	1	1.00	1.25	0.97	0.93	1.39	1.79	1.79	1.40
time (sec)	N/A	0.170	0.056	0.037	0.206	0.231	0.043	0.351	0.048

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	76	53	50	52	63	50	54
N.S.	1	1.00	1.36	0.95	0.89	0.93	1.12	0.89	0.96
time (sec)	N/A	0.013	0.000	0.027	0.201	0.238	0.019	0.262	0.026

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	80	87	112	112	0	131	106
N.S.	1	1.00	0.93	1.01	1.30	1.30	0.00	1.52	1.23
time (sec)	N/A	0.059	0.048	0.126	0.217	2.906	0.000	0.285	9.712

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	232	235	2096	0	0	1435	1940
N.S.	1	1.00	0.99	1.00	8.96	0.00	0.00	6.13	8.29
time (sec)	N/A	0.293	0.368	0.716	0.316	0.000	0.000	0.282	13.890

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	495	495	490	514	11005	0	0	7111	82532
N.S.	1	1.00	0.99	1.04	22.23	0.00	0.00	14.37	166.73
time (sec)	N/A	1.003	0.718	2.373	0.740	0.000	0.000	0.404	23.447

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	20	25
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00
time (sec)	N/A	0.012	0.006	0.056	0.286	0.313	0.057	0.278	0.030

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	25	25	24	26	25
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.77	0.84	0.81
time (sec)	N/A	0.015	0.006	0.063	0.288	0.332	0.065	0.321	0.034

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.001	0.001	0.016	0.200	0.329	0.019	0.284	0.019

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	26	28	26	19	30	25
N.S.	1	1.00	1.00	0.93	1.00	0.93	0.68	1.07	0.89
time (sec)	N/A	0.012	0.005	0.691	0.198	0.310	0.082	0.296	0.038

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	18	15	24	18
N.S.	1	1.00	1.00	0.95	0.91	0.82	0.68	1.09	0.82
time (sec)	N/A	0.010	0.005	0.683	0.193	0.311	0.098	0.276	9.264

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	61	61	0	211	564	62	213
N.S.	1	1.00	0.98	0.98	0.00	3.40	9.10	1.00	3.44
time (sec)	N/A	0.044	0.052	0.089	0.000	0.323	4.405	0.333	0.283

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	89	27	98	299	20	112	99
N.S.	1	1.00	0.77	0.23	0.85	2.60	0.17	0.97	0.86
time (sec)	N/A	0.048	0.021	0.680	0.282	0.320	0.071	0.277	0.143

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	17	16	29	16	16
N.S.	1	1.00	1.00	1.06	1.06	1.00	1.81	1.00	1.00
time (sec)	N/A	0.003	0.002	0.027	0.202	0.298	0.277	0.266	9.845

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	53	0	0	0	0	0	56
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	1.02
time (sec)	N/A	0.016	0.027	0.000	0.000	0.000	0.000	0.000	9.640

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	59	61	0	0	0	0	0	56
N.S.	1	1.11	1.15	0.00	0.00	0.00	0.00	0.00	1.06
time (sec)	N/A	0.020	0.030	0.000	0.000	0.000	0.000	0.000	10.146

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	129	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.119	0.138	0.000	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	35	44	203	0	0	0	34	0	41
N.S.	1	1.26	5.80	0.00	0.00	0.00	0.97	0.00	1.17
time (sec)	N/A	0.008	0.136	0.000	0.000	0.000	5.126	0.000	10.478

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	285	267	372	271	299	277	261
N.S.	1	1.00	1.06	0.99	1.38	1.00	1.11	1.03	0.97
time (sec)	N/A	0.415	0.032	0.050	0.196	0.271	0.048	0.279	10.568

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	171	163	205	163	180	166	160
N.S.	1	1.00	1.00	0.95	1.20	0.95	1.05	0.97	0.94
time (sec)	N/A	0.067	0.015	0.047	0.218	0.263	0.037	0.281	10.577

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	83	94	82	95	83	82
N.S.	1	1.00	1.00	0.90	1.02	0.89	1.03	0.90	0.89
time (sec)	N/A	0.030	0.010	0.038	0.190	0.284	0.028	0.269	0.023

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	28	28	31	28	28
N.S.	1	1.00	1.00	0.91	0.88	0.88	0.97	0.88	0.88
time (sec)	N/A	0.005	0.000	0.029	0.195	0.279	0.020	0.266	0.021

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	529	529	71	64	0	905	88	603	1551
N.S.	1	1.00	0.13	0.12	0.00	1.71	0.17	1.14	2.93
time (sec)	N/A	0.669	0.020	0.137	0.000	0.293	0.595	0.290	12.187

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	746	746	182	230	0	3222	0	1057	5844
N.S.	1	1.00	0.24	0.31	0.00	4.32	0.00	1.42	7.83
time (sec)	N/A	1.166	0.071	0.122	0.000	0.373	0.000	0.389	12.090

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	345	332	383	332	366	353	331
N.S.	1	1.00	1.17	1.13	1.30	1.13	1.24	1.20	1.12
time (sec)	N/A	0.406	0.045	0.059	0.195	0.286	0.051	0.295	10.542

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	207	200	214	198	218	205	201
N.S.	1	1.00	1.02	0.99	1.05	0.98	1.07	1.01	0.99
time (sec)	N/A	0.089	0.019	0.048	0.202	0.274	0.038	0.295	0.100

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	109	99	101	98	112	99	98
N.S.	1	1.00	1.02	0.93	0.94	0.92	1.05	0.93	0.92
time (sec)	N/A	0.038	0.011	0.038	0.198	0.272	0.026	0.293	10.174

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	33	33	36	33	33
N.S.	1	1.00	1.00	0.92	0.89	0.89	0.97	0.89	0.89
time (sec)	N/A	0.006	0.000	0.030	0.191	0.258	0.020	0.275	0.023

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	71	67	0	1115	122	577	1264
N.S.	1	1.00	0.46	0.44	0.00	7.29	0.80	3.77	8.26
time (sec)	N/A	0.188	0.018	0.261	0.000	0.310	0.991	0.284	11.302

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	234	288	0	4285	0	1115	10351
N.S.	1	1.00	0.68	0.84	0.00	12.53	0.00	3.26	30.27
time (sec)	N/A	0.427	0.096	0.133	0.000	0.482	0.000	0.317	13.831

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	85	84	84	94	84	84
N.S.	1	1.00	1.00	0.89	0.88	0.88	0.98	0.88	0.88
time (sec)	N/A	0.021	0.003	0.030	0.193	0.281	0.028	0.294	0.197

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	65	64	64	71	64	64
N.S.	1	1.00	1.00	0.88	0.86	0.86	0.96	0.86	0.86
time (sec)	N/A	0.015	0.001	0.028	0.187	0.289	0.027	0.279	0.066

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	49	44	44
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.011	0.002	0.022	0.187	0.308	0.022	0.336	0.020

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.83	0.83	0.83
time (sec)	N/A	0.002	0.000	0.022	0.194	0.281	0.026	0.291	0.017

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	45	41	0	1015	41	0	123
N.S.	1	1.00	0.17	0.15	0.00	3.79	0.15	0.00	0.46
time (sec)	N/A	0.305	0.008	0.047	0.000	1.086	0.540	0.000	10.581

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	113	83	0	1201	3834	0	176
N.S.	1	1.00	0.32	0.23	0.00	3.36	10.74	0.00	0.49
time (sec)	N/A	0.292	0.014	0.055	0.000	1.049	1.777	0.000	0.136

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	78	77	77	94	77	77
N.S.	1	1.00	1.00	0.80	0.79	0.79	0.97	0.79	0.79
time (sec)	N/A	0.018	0.002	0.029	0.199	0.256	0.029	0.303	0.118

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83
time (sec)	N/A	0.014	0.002	0.026	0.196	0.270	0.026	0.277	0.063

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	37	37	42	37	37
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.93	0.82	0.82
time (sec)	N/A	0.010	0.002	0.024	0.199	0.292	0.025	0.306	0.018

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	19	17	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.90	0.81	0.81
time (sec)	N/A	0.002	0.000	0.021	0.190	0.242	0.018	0.304	0.016

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	47	41	0	499	3432	265	87
N.S.	1	1.00	0.20	0.18	0.00	2.13	14.67	1.13	0.37
time (sec)	N/A	0.240	0.018	0.059	0.000	0.989	1.343	0.309	10.965

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	108	79	0	704	3834	315	174
N.S.	1	1.00	0.34	0.25	0.00	2.22	12.09	0.99	0.55
time (sec)	N/A	0.249	0.019	0.079	0.000	0.975	1.936	0.380	10.660

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	104	85	84	84	100	84	84
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.96	0.81	0.81
time (sec)	N/A	0.022	0.002	0.034	0.208	0.257	0.031	0.300	10.543

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	65	64	64	73	64	64
N.S.	1	1.00	1.00	0.86	0.84	0.84	0.96	0.84	0.84
time (sec)	N/A	0.016	0.002	0.029	0.196	0.259	0.027	0.293	0.065

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	45	44	44	49	44	44
N.S.	1	1.00	1.00	0.87	0.85	0.85	0.94	0.85	0.85
time (sec)	N/A	0.012	0.002	0.025	0.194	0.242	0.024	0.285	0.020

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	27	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.90	0.80	0.80
time (sec)	N/A	0.003	0.000	0.021	0.200	0.267	0.018	0.290	0.012

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	55	49	0	1297	41	0	123
N.S.	1	1.00	0.21	0.19	0.00	4.93	0.16	0.00	0.47
time (sec)	N/A	0.391	0.009	0.045	0.000	0.977	1.271	0.000	0.267

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	128	96	0	1540	3839	0	181
N.S.	1	1.00	0.35	0.26	0.00	4.21	10.49	0.00	0.49
time (sec)	N/A	0.408	0.014	0.055	0.000	1.099	2.232	0.000	0.150

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	164	592	163	185	163	163
N.S.	1	1.00	1.00	11.71	42.29	11.64	13.21	11.64	11.64
time (sec)	N/A	0.011	0.002	0.052	0.206	0.276	0.040	0.271	0.127

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	109	228	108	114	108	108
N.S.	1	1.00	1.00	7.79	16.29	7.71	8.14	7.71	7.71
time (sec)	N/A	0.011	0.002	0.040	0.207	0.300	0.031	0.279	0.055

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	61	61	13	53	53	60	53	53
N.S.	1	4.36	4.36	0.93	3.79	3.79	4.29	3.79	3.79
time (sec)	N/A	0.009	0.000	0.032	0.203	0.275	0.021	0.295	0.016

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	46	46	49	12	48
N.S.	1	1.00	1.00	0.93	3.29	3.29	3.50	0.86	3.43
time (sec)	N/A	0.012	0.004	0.058	0.196	0.290	0.134	0.294	0.033

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	101	101	109	12	103
N.S.	1	1.00	1.00	0.93	7.21	7.21	7.79	0.86	7.36
time (sec)	N/A	0.012	0.003	0.130	0.203	0.266	0.282	0.280	10.735

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	156	156	168	12	158
N.S.	1	1.00	1.00	0.93	11.14	11.14	12.00	0.86	11.29
time (sec)	N/A	0.012	0.003	0.244	0.219	0.282	0.426	0.281	11.831

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	30	30	29	31	36
N.S.	1	1.00	1.00	0.82	0.79	0.79	0.76	0.82	0.95
time (sec)	N/A	0.019	0.006	0.081	0.281	0.261	0.073	0.268	0.030

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	84	65	64	64	80	64	64
N.S.	1	1.00	1.00	0.77	0.76	0.76	0.95	0.76	0.76
time (sec)	N/A	0.054	0.003	0.039	0.197	0.257	0.033	0.272	0.077

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	50	49	49	60	49	49
N.S.	1	1.00	1.00	0.79	0.78	0.78	0.95	0.78	0.78
time (sec)	N/A	0.051	0.003	0.033	0.193	0.245	0.028	0.281	0.053

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	35	34	34	41	34	34
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.93	0.77	0.77
time (sec)	N/A	0.046	0.001	0.028	0.190	0.272	0.026	0.276	0.015

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	22	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.88	0.76	0.76
time (sec)	N/A	0.002	0.000	0.046	0.189	0.254	0.019	0.287	0.020

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	62	42	54	56	63	62	27
N.S.	1	1.00	2.00	1.35	1.74	1.81	2.03	2.00	0.87
time (sec)	N/A	0.017	0.012	0.069	0.288	0.288	0.072	0.290	10.332

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	103	84	89	177	104	97	64
N.S.	1	1.00	1.16	0.94	1.00	1.99	1.17	1.09	0.72
time (sec)	N/A	0.043	0.039	0.117	0.273	0.264	0.817	0.296	0.047

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	137	108	119	282	134	112	93
N.S.	1	1.00	0.85	0.67	0.74	1.75	0.83	0.70	0.58
time (sec)	N/A	0.089	0.061	0.140	0.278	0.298	0.877	0.301	0.049

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	205	132	116	114	223	296	134	126
N.S.	1	2.25	1.45	1.27	1.25	2.45	3.25	1.47	1.38
time (sec)	N/A	0.099	0.065	0.131	0.286	0.277	0.806	0.298	10.083

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	73	111	81	198	209	77	87
N.S.	1	1.00	0.94	1.42	1.04	2.54	2.68	0.99	1.12
time (sec)	N/A	0.046	0.035	0.906	0.284	0.283	0.395	0.273	9.963

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	54	70	61	157	153	54	206
N.S.	1	1.00	1.08	1.40	1.22	3.14	3.06	1.08	4.12
time (sec)	N/A	0.029	0.021	0.882	0.278	0.285	0.257	0.285	0.057

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	38	54	50	136	124	43	46
N.S.	1	1.00	0.93	1.32	1.22	3.32	3.02	1.05	1.12
time (sec)	N/A	0.017	0.012	0.845	0.284	0.282	0.116	0.281	9.987

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	28	24	83	54	17	17
N.S.	1	1.00	1.00	1.33	1.14	3.95	2.57	0.81	0.81
time (sec)	N/A	0.006	0.007	0.826	0.277	0.272	0.085	0.291	0.026

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	48	74	68	154	738	62	173
N.S.	1	1.00	0.81	1.25	1.15	2.61	12.51	1.05	2.93
time (sec)	N/A	0.026	0.028	0.843	0.292	0.284	1.845	0.295	10.576

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	81	96	123	229	1620	117	425
N.S.	1	1.00	1.03	1.22	1.56	2.90	20.51	1.48	5.38
time (sec)	N/A	0.065	0.031	0.865	0.288	0.295	6.884	0.295	10.100

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	106	151	197	371	3284	195	573
N.S.	1	1.00	0.88	1.25	1.63	3.07	27.14	1.61	4.74
time (sec)	N/A	0.100	0.097	0.882	0.298	0.288	42.252	0.291	10.425

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	34	30	109	61	24	27
N.S.	1	1.00	1.00	1.10	0.97	3.52	1.97	0.77	0.87
time (sec)	N/A	0.012	0.009	1.112	0.281	0.270	0.089	0.301	0.037

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	86	75	253	117	65	76
N.S.	1	1.00	0.95	1.37	1.19	4.02	1.86	1.03	1.21
time (sec)	N/A	0.022	0.021	0.990	0.288	0.282	0.294	0.289	9.907

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	75	139	184	595	257	103	181
N.S.	1	1.00	0.82	1.53	2.02	6.54	2.82	1.13	1.99
time (sec)	N/A	0.031	0.044	0.963	0.298	0.282	0.619	0.305	10.264

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	198	42	66	279	92	30	31
N.S.	1	1.00	5.66	1.20	1.89	7.97	2.63	0.86	0.89
time (sec)	N/A	0.024	0.089	1.077	0.277	0.273	0.098	0.280	0.073

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	18	10	24	10	10
N.S.	1	1.00	1.00	1.10	1.80	1.00	2.40	1.00	1.00
time (sec)	N/A	0.002	0.005	0.981	0.273	0.250	0.078	0.294	0.025

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	31	43	51	55	56	41	42
N.S.	1	1.00	0.84	1.16	1.38	1.49	1.51	1.11	1.14
time (sec)	N/A	0.007	0.011	0.924	0.301	0.272	0.225	0.304	10.887

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	52	71	115	153	146	73	111
N.S.	1	1.00	0.87	1.18	1.92	2.55	2.43	1.22	1.85
time (sec)	N/A	0.011	0.014	0.935	0.282	0.253	0.480	0.282	0.077

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	32	23	25	22	22	27	10
N.S.	1	1.00	3.20	2.30	2.50	2.20	2.20	2.70	1.00
time (sec)	N/A	0.002	0.005	0.875	0.189	0.285	0.080	0.299	10.474

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	45	52	56	85	54	56	43
N.S.	1	1.00	1.15	1.33	1.44	2.18	1.38	1.44	1.10
time (sec)	N/A	0.010	0.019	0.822	0.198	0.288	0.225	0.278	10.358

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	65	78	122	220	141	88	114
N.S.	1	1.00	1.02	1.22	1.91	3.44	2.20	1.38	1.78
time (sec)	N/A	0.017	0.023	0.844	0.196	0.258	0.513	0.288	10.060

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	15	12	11	11	10	13	4
N.S.	1	1.00	3.75	3.00	2.75	2.75	2.50	3.25	1.00
time (sec)	N/A	0.002	0.003	0.708	0.202	0.264	0.044	0.297	0.110

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	25	39	24	27	23
N.S.	1	1.00	0.96	0.89	0.93	1.44	0.89	1.00	0.85
time (sec)	N/A	0.005	0.013	0.760	0.192	0.268	0.052	0.302	0.046

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	37	36	44	71	44	39	36
N.S.	1	1.00	0.82	0.80	0.98	1.58	0.98	0.87	0.80
time (sec)	N/A	0.010	0.014	0.731	0.188	0.254	0.067	0.284	10.257

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	64	61	54	54	58	62	55
N.S.	1	1.00	1.08	1.03	0.92	0.92	0.98	1.05	0.93
time (sec)	N/A	0.041	0.018	0.899	0.186	0.259	0.076	0.284	0.033

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	11	12	11	11	10	11	11
N.S.	1	1.00	1.10	1.20	1.10	1.10	1.00	1.10	1.10
time (sec)	N/A	0.009	0.007	0.892	0.187	0.266	0.038	0.285	10.372

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	55	25	36	35	26	23	0
N.S.	1	1.00	1.25	0.57	0.82	0.80	0.59	0.52	0.00
time (sec)	N/A	0.019	0.050	0.987	0.268	0.294	0.441	0.286	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	163	107	139	92	231	64	0
N.S.	1	1.00	2.43	1.60	2.07	1.37	3.45	0.96	0.00
time (sec)	N/A	0.037	0.299	1.005	0.276	0.295	0.751	0.307	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	86	87	135	70	228	86	0
N.S.	1	1.00	1.37	1.38	2.14	1.11	3.62	1.37	0.00
time (sec)	N/A	0.029	0.225	1.052	0.195	0.297	0.643	0.302	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	132	108	0	6315	238	0	374
N.S.	1	1.00	0.56	0.46	0.00	26.99	1.02	0.00	1.60
time (sec)	N/A	0.270	0.037	0.071	0.000	10.969	1.641	0.000	9.623

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	81	74	0	4759	158	0	437
N.S.	1	1.00	0.39	0.35	0.00	22.66	0.75	0.00	2.08
time (sec)	N/A	0.167	0.021	0.056	0.000	0.945	0.542	0.000	9.864

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	79	72	0	1950	83	0	145
N.S.	1	1.00	0.44	0.40	0.00	10.83	0.46	0.00	0.81
time (sec)	N/A	0.119	0.017	0.059	0.000	0.964	0.368	0.000	9.747

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	116	71	0	442	26	160	144
N.S.	1	1.00	0.83	0.51	0.00	3.16	0.19	1.14	1.03
time (sec)	N/A	0.079	0.024	0.047	0.000	0.285	0.118	0.302	10.075

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	238	119	105	0	4370	0	0	553
N.S.	1	1.06	0.53	0.47	0.00	19.51	0.00	0.00	2.47
time (sec)	N/A	0.237	0.037	0.092	0.000	0.950	0.000	0.000	0.075

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	312	173	143	0	8919	0	0	1588
N.S.	1	0.99	0.55	0.46	0.00	28.40	0.00	0.00	5.06
time (sec)	N/A	0.428	0.062	0.122	0.000	1.334	0.000	0.000	9.900

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	244	200	0	14765	0	0	1328
N.S.	1	1.00	0.62	0.51	0.00	37.57	0.00	0.00	3.38
time (sec)	N/A	0.461	0.103	0.151	0.000	3.883	0.000	0.000	9.265

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	106	97	0	0	374	0	1003
N.S.	1	1.00	0.30	0.27	0.00	0.00	1.05	0.00	2.82
time (sec)	N/A	0.301	0.028	0.097	0.000	0.000	2.238	0.000	9.617

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	106	97	0	61993	274	0	625
N.S.	1	1.00	0.33	0.31	0.00	194.95	0.86	0.00	1.97
time (sec)	N/A	0.240	0.024	0.078	0.000	58.699	1.600	0.000	9.271

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	104	95	0	40785	131	0	205
N.S.	1	1.00	0.40	0.36	0.00	156.26	0.50	0.00	0.79
time (sec)	N/A	0.192	0.019	0.074	0.000	11.051	0.478	0.000	9.128

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	161	94	0	152	26	103	60
N.S.	1	1.00	0.73	0.43	0.00	0.69	0.12	0.47	0.27
time (sec)	N/A	0.141	0.059	0.062	0.000	0.298	0.132	0.311	0.073

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	163	139	0	307773	0	0	882
N.S.	1	1.00	0.41	0.35	0.00	783.14	0.00	0.00	2.24
time (sec)	N/A	0.352	0.045	0.110	0.000	4.194	0.000	0.000	9.294

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	496	496	238	184	0	1128605	0	0	2440
N.S.	1	1.00	0.48	0.37	0.00	2275.41	0.00	0.00	4.92
time (sec)	N/A	0.662	0.088	0.142	0.000	78.189	0.000	0.000	9.527

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	195	179	192	179	199	219	175
N.S.	1	1.00	1.59	1.46	1.56	1.46	1.62	1.78	1.42
time (sec)	N/A	0.175	0.020	0.046	0.197	0.241	0.047	0.277	0.205

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	114	110	119	107	114	128	108
N.S.	1	1.00	0.95	0.92	0.99	0.89	0.95	1.07	0.90
time (sec)	N/A	0.050	0.012	0.033	0.199	0.253	0.031	0.296	0.081

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	66	60	65	56	65	65	61
N.S.	1	1.00	0.92	0.83	0.90	0.78	0.90	0.90	0.85
time (sec)	N/A	0.022	0.007	0.030	0.209	0.249	0.030	0.285	0.023

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	22	22	22	22
N.S.	1	1.00	1.00	0.88	0.85	0.85	0.85	0.85	0.85
time (sec)	N/A	0.003	0.000	0.023	0.204	0.248	0.020	0.294	0.011

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	57	51	0	457	66	2669	571
N.S.	1	1.00	0.64	0.57	0.00	5.13	0.74	29.99	6.42
time (sec)	N/A	0.067	0.018	0.067	0.000	0.249	0.543	2.518	0.323

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	150	158	0	1948	294	8503	4591
N.S.	1	1.00	0.89	0.93	0.00	11.53	1.74	50.31	27.17
time (sec)	N/A	0.220	0.047	0.069	0.000	0.269	3.339	6.370	11.032

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	254	400	0	3971	697	16632	8242
N.S.	1	1.00	1.01	1.59	0.00	15.76	2.77	66.00	32.71
time (sec)	N/A	0.427	0.091	0.112	0.000	0.310	8.015	11.962	12.115

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	204	182	182	182	212	222	178
N.S.	1	1.00	0.97	0.87	0.87	0.87	1.01	1.06	0.85
time (sec)	N/A	0.169	0.021	0.112	0.204	0.250	0.044	0.258	0.206

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	130	115	113	113	128	133	113
N.S.	1	1.00	0.97	0.86	0.84	0.84	0.96	0.99	0.84
time (sec)	N/A	0.120	0.013	0.109	0.206	0.237	0.033	0.272	9.638

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	75	63	59	59	70	68	64
N.S.	1	1.00	0.95	0.80	0.75	0.75	0.89	0.86	0.81
time (sec)	N/A	0.056	0.007	0.097	0.192	0.251	0.023	0.259	0.024

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	29	27	27
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.83	0.77	0.77
time (sec)	N/A	0.007	0.003	0.022	0.203	0.253	0.017	0.265	0.013

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	59	52	0	140500	155	0	275
N.S.	1	1.00	0.51	0.45	0.00	1211.21	1.34	0.00	2.37
time (sec)	N/A	0.075	0.016	0.043	0.000	1.452	2.478	0.000	9.580

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	166	158	0	0	539	0	1167
N.S.	1	1.00	0.72	0.68	0.00	0.00	2.33	0.00	5.05
time (sec)	N/A	0.229	0.049	0.060	0.000	0.000	16.396	0.000	10.332

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	284	409	0	0	1102	0	2200
N.S.	1	1.00	0.81	1.17	0.00	0.00	3.16	0.00	6.30
time (sec)	N/A	0.440	0.096	0.114	0.000	0.000	42.836	0.000	9.931

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	204	182	182	182	219	222	178
N.S.	1	1.00	0.97	0.87	0.87	0.87	1.04	1.06	0.85
time (sec)	N/A	0.162	0.020	0.122	0.193	0.239	0.042	0.290	9.273

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	132	115	113	113	134	133	113
N.S.	1	1.00	0.96	0.83	0.82	0.82	0.97	0.96	0.82
time (sec)	N/A	0.113	0.012	0.126	0.192	0.242	0.032	0.284	9.107

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	73	63	59	59	73	68	64
N.S.	1	1.00	0.92	0.80	0.75	0.75	0.92	0.86	0.81
time (sec)	N/A	0.065	0.007	0.107	0.189	0.239	0.024	0.293	0.026

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	29	27	27
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.83	0.77	0.77
time (sec)	N/A	0.007	0.002	0.022	0.189	0.238	0.018	0.277	0.012

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	61	54	0	1515766	172	0	878
N.S.	1	1.00	0.62	0.55	0.00	15310.77	1.74	0.00	8.87
time (sec)	N/A	0.075	0.016	0.040	0.000	17.658	4.082	0.000	0.399

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	182	160	0	0	561	0	1218
N.S.	1	1.00	0.81	0.71	0.00	0.00	2.49	0.00	5.41
time (sec)	N/A	0.181	0.051	0.064	0.000	0.000	18.357	0.000	9.444

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	545	545	99	93	0	0	0	0	1563
N.S.	1	1.00	0.18	0.17	0.00	0.00	0.00	0.00	2.87
time (sec)	N/A	1.377	0.040	0.078	0.000	0.000	0.000	0.000	9.522

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	487	487	99	93	0	0	0	0	1354
N.S.	1	1.00	0.20	0.19	0.00	0.00	0.00	0.00	2.78
time (sec)	N/A	0.819	0.031	0.078	0.000	0.000	0.000	0.000	9.569

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	97	93	0	27094	0	0	825
N.S.	1	1.00	0.29	0.28	0.00	81.12	0.00	0.00	2.47
time (sec)	N/A	0.505	0.026	0.082	0.000	2.428	0.000	0.000	9.515

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	469	469	95	91	0	0	0	0	1057
N.S.	1	1.00	0.20	0.19	0.00	0.00	0.00	0.00	2.25
time (sec)	N/A	0.662	0.027	0.081	0.000	0.000	0.000	0.000	9.612

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	522	99	90	0	0	0	0	1394
N.S.	1	1.00	0.19	0.17	0.00	0.00	0.00	0.00	2.67
time (sec)	N/A	0.920	0.036	0.072	0.000	0.000	0.000	0.000	9.501

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	563	563	157	134	0	0	0	0	4002
N.S.	1	1.00	0.28	0.24	0.00	0.00	0.00	0.00	7.11
time (sec)	N/A	1.092	0.071	0.109	0.000	0.000	0.000	0.000	9.073

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	645	640	163	133	0	0	0	0	2663
N.S.	1	0.99	0.25	0.21	0.00	0.00	0.00	0.00	4.13
time (sec)	N/A	1.386	0.076	0.111	0.000	0.000	0.000	0.000	9.683

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	61	56	0	0	70	0	427
N.S.	1	1.00	0.15	0.14	0.00	0.00	0.18	0.00	1.08
time (sec)	N/A	1.122	0.013	0.056	0.000	0.000	0.121	0.000	0.375

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	61	56	0	0	65	0	390
N.S.	1	1.00	0.16	0.15	0.00	0.00	0.17	0.00	1.03
time (sec)	N/A	0.887	0.010	0.045	0.000	0.000	0.135	0.000	9.234

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	61	56	0	0	61	0	276
N.S.	1	1.00	0.17	0.16	0.00	0.00	0.17	0.00	0.76
time (sec)	N/A	0.600	0.010	0.044	0.000	0.000	0.111	0.000	0.304

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	59	56	0	1277	48	0	247
N.S.	1	1.00	0.24	0.23	0.00	5.15	0.19	0.00	1.00
time (sec)	N/A	0.425	0.009	0.046	0.000	0.929	0.083	0.000	9.526

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	57	54	0	0	61	0	176
N.S.	1	1.00	0.16	0.15	0.00	0.00	0.17	0.00	0.49
time (sec)	N/A	0.615	0.010	0.045	0.000	0.000	0.112	0.000	9.097

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	62	53	0	0	65	0	306
N.S.	1	1.00	0.16	0.14	0.00	0.00	0.17	0.00	0.81
time (sec)	N/A	0.816	0.009	0.043	0.000	0.000	0.132	0.000	9.356

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	415	415	103	73	0	0	82	0	432
N.S.	1	1.00	0.25	0.18	0.00	0.00	0.20	0.00	1.04
time (sec)	N/A	0.940	0.013	0.066	0.000	0.000	0.561	0.000	9.438

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	448	109	69	0	0	70	0	340
N.S.	1	1.00	0.24	0.15	0.00	0.00	0.16	0.00	0.76
time (sec)	N/A	1.093	0.014	0.066	0.000	0.000	0.191	0.000	9.038

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1064	1064	167	122	0	0	112	0	388
N.S.	1	1.00	0.16	0.11	0.00	0.00	0.11	0.00	0.36
time (sec)	N/A	2.284	0.025	0.064	0.000	0.000	0.234	0.000	9.153

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1005	1005	167	122	0	0	112	0	387
N.S.	1	1.00	0.17	0.12	0.00	0.00	0.11	0.00	0.39
time (sec)	N/A	2.279	0.019	0.065	0.000	0.000	0.281	0.000	9.322

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	677	677	167	122	0	0	112	0	388
N.S.	1	1.00	0.25	0.18	0.00	0.00	0.17	0.00	0.57
time (sec)	N/A	1.562	0.025	0.060	0.000	0.000	0.248	0.000	9.045

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	682	682	167	122	0	1445	104	0	299
N.S.	1	1.00	0.24	0.18	0.00	2.12	0.15	0.00	0.44
time (sec)	N/A	1.243	0.018	0.064	0.000	0.959	0.120	0.000	0.185

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	850	850	167	122	0	0	112	0	388
N.S.	1	1.00	0.20	0.14	0.00	0.00	0.13	0.00	0.46
time (sec)	N/A	1.453	0.024	0.063	0.000	0.000	0.246	0.000	9.104

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	873	873	167	122	0	0	112	0	387
N.S.	1	1.00	0.19	0.14	0.00	0.00	0.13	0.00	0.44
time (sec)	N/A	1.854	0.018	0.062	0.000	0.000	0.282	0.000	9.314

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	986	986	167	122	0	0	112	0	388
N.S.	1	1.00	0.17	0.12	0.00	0.00	0.11	0.00	0.39
time (sec)	N/A	2.038	0.027	0.059	0.000	0.000	0.235	0.000	9.090

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.035	0.003	0.713	0.181	0.238	0.026	0.276	0.021

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	79	96	105	105	88	365	106
N.S.	1	1.00	0.84	1.02	1.12	1.12	0.94	3.88	1.13
time (sec)	N/A	0.086	0.024	0.771	0.182	0.259	0.144	0.282	0.033

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	172	13	13	154	175	13	154
N.S.	1	1.00	11.47	0.87	0.87	10.27	11.67	0.87	10.27
time (sec)	N/A	0.013	0.004	0.764	0.188	0.267	0.054	0.306	8.985

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	182	15	156	156	182	156	156
N.S.	1	1.00	11.38	0.94	9.75	9.75	11.38	9.75	9.75
time (sec)	N/A	0.036	0.004	0.781	0.187	0.236	0.049	0.282	8.856

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	186	15	156	156	185	156	156
N.S.	1	1.00	11.62	0.94	9.75	9.75	11.56	9.75	9.75
time (sec)	N/A	0.031	0.005	0.808	0.200	0.226	0.052	0.286	9.110

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	229	262	0	189	229
N.S.	1	1.00	1.00	10.95	10.90	12.48	0.00	9.00	10.90
time (sec)	N/A	0.020	0.012	0.021	0.201	0.252	0.000	0.500	10.208

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	9	9	10	10	8	11	8
N.S.	1	1.00	0.90	0.90	1.00	1.00	0.80	1.10	0.80
time (sec)	N/A	0.003	0.004	0.724	0.192	0.261	0.056	0.276	9.154

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	18	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	1.20	0.87
time (sec)	N/A	0.017	0.004	0.735	0.194	0.256	0.084	0.266	8.969

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	15	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	1.00	0.87
time (sec)	N/A	0.017	0.005	0.733	0.207	0.249	0.093	0.268	8.925

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	19	18	47	23	29	0	0
N.S.	1	1.00	1.27	1.20	3.13	1.53	1.93	0.00	0.00
time (sec)	N/A	0.016	0.006	0.817	0.199	0.274	0.537	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	14	13	13	81	87	13	12
N.S.	1	1.00	0.93	0.87	0.87	5.40	5.80	0.87	0.80
time (sec)	N/A	0.003	0.015	0.717	0.199	0.260	0.459	0.268	10.133

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	81	81	87	15	14
N.S.	1	1.00	1.00	0.94	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.015	0.020	0.761	0.208	0.256	0.693	0.287	1.366

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	81	81	87	15	14
N.S.	1	1.00	1.00	0.94	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.017	0.024	0.856	0.217	0.265	0.926	0.299	12.023

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	203	612	143	0	0	0
N.S.	1	1.00	1.00	9.67	29.14	6.81	0.00	0.00	0.00
time (sec)	N/A	0.021	0.009	0.020	0.245	0.387	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	17	20	19	26	46	19	23
N.S.	1	1.00	0.89	1.05	1.00	1.37	2.42	1.00	1.21
time (sec)	N/A	0.003	0.006	0.726	0.203	0.253	0.272	0.263	9.105

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	97	31	35	32	80	54	45
N.S.	1	1.00	3.59	1.15	1.30	1.19	2.96	2.00	1.67
time (sec)	N/A	0.015	0.063	1.445	0.226	0.262	28.849	0.370	9.528

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	C	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	27	116	97	97	35	33	0	54	0
N.S.	1	4.30	3.59	3.59	1.30	1.22	0.00	2.00	0.00
time (sec)	N/A	0.073	0.002	4.743	0.236	0.271	0.000	0.311	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	99	33	35	34	0	58	49
N.S.	1	1.00	3.41	1.14	1.21	1.17	0.00	2.00	1.69
time (sec)	N/A	0.015	0.061	2.509	0.239	0.285	0.000	0.278	9.288

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	108	0	39	42	0	0	0
N.S.	1	1.00	3.00	0.00	1.08	1.17	0.00	0.00	0.00
time (sec)	N/A	0.054	0.172	0.000	0.276	0.296	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	26	26	29	26	26
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.021	0.002	0.688	0.196	0.269	0.028	0.286	0.027

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.031	0.002	0.706	0.203	0.268	0.028	0.289	0.011

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	32	31	98	124	31	32
N.S.	1	1.00	1.00	0.76	0.74	2.33	2.95	0.74	0.76
time (sec)	N/A	0.032	0.014	0.784	0.284	0.257	0.134	0.265	9.211

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	25	38	0	25	54
N.S.	1	1.00	0.92	1.04	1.00	1.52	0.00	1.00	2.16
time (sec)	N/A	0.015	0.067	0.144	0.215	0.258	0.000	0.274	9.245

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	21	25	24	36	0	24	46
N.S.	1	1.00	0.88	1.04	1.00	1.50	0.00	1.00	1.92
time (sec)	N/A	0.011	0.247	0.114	0.199	0.263	0.000	0.290	9.315

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	26	39	35	0	65	51
N.S.	1	1.00	0.96	1.04	1.56	1.40	0.00	2.60	2.04
time (sec)	N/A	0.014	0.084	1.873	0.252	0.271	0.000	0.304	9.114

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	28	0	20	39
N.S.	1	1.00	1.00	1.05	1.00	1.40	0.00	1.00	1.95
time (sec)	N/A	0.006	0.028	0.099	0.193	0.254	0.000	0.263	9.124

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	106	20	19	26	63	19	25
N.S.	1	1.00	5.58	1.05	1.00	1.37	3.32	1.00	1.32
time (sec)	N/A	0.006	0.079	0.795	0.203	0.260	5.275	0.271	9.206

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	108	23	31	27	66	39	26
N.S.	1	1.00	4.91	1.05	1.41	1.23	3.00	1.77	1.18
time (sec)	N/A	0.005	0.029	1.453	0.259	0.255	23.671	0.443	9.022

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	22	32	0	22	43
N.S.	1	1.00	0.95	1.05	1.00	1.45	0.00	1.00	1.95
time (sec)	N/A	0.012	0.052	0.094	0.219	0.256	0.000	0.274	9.017

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	22	21	30	53	21	27
N.S.	1	1.00	0.90	1.05	1.00	1.43	2.52	1.00	1.29
time (sec)	N/A	0.008	0.041	0.754	0.194	0.259	0.431	0.262	9.135

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	28	32	31	56	51	28
N.S.	1	1.00	0.92	1.17	1.33	1.29	2.33	2.12	1.17
time (sec)	N/A	0.014	0.013	1.184	0.244	0.248	1.549	0.401	9.519

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	32	29	53	41	26
N.S.	1	1.00	1.00	1.05	1.45	1.32	2.41	1.86	1.18
time (sec)	N/A	0.006	0.011	1.158	0.229	0.257	1.528	0.272	9.138

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	32	32	0	22	43
N.S.	1	1.00	0.95	1.05	1.45	1.45	0.00	1.00	1.95
time (sec)	N/A	0.009	0.002	0.097	0.238	0.291	0.000	0.300	9.013

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	26	32	30	53	21	27
N.S.	1	1.00	0.90	1.24	1.52	1.43	2.52	1.00	1.29
time (sec)	N/A	0.008	0.002	0.761	0.227	0.242	0.423	0.287	9.175

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	143	20	19	1528	1771	160	1576
N.S.	1	1.00	6.81	0.95	0.90	72.76	84.33	7.62	75.05
time (sec)	N/A	0.081	0.093	0.158	0.195	0.258	0.183	0.304	9.903

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	18	18	18	441	469	18	418
N.S.	1	1.00	0.90	0.90	0.90	22.05	23.45	0.90	20.90
time (sec)	N/A	0.037	0.017	0.273	0.191	0.267	0.084	0.297	9.396

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	439	441	441	469	18	418
N.S.	1	1.00	0.95	23.11	23.21	23.21	24.68	0.95	22.00
time (sec)	N/A	0.049	0.007	0.964	0.194	0.250	0.075	0.281	9.143

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	127	15	14	456	483	120	438
N.S.	1	1.00	7.94	0.94	0.88	28.50	30.19	7.50	27.38
time (sec)	N/A	0.016	0.037	0.220	0.198	0.237	0.082	0.272	9.690

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	98	14	13	88	97	13	88
N.S.	1	1.00	6.53	0.93	0.87	5.87	6.47	0.87	5.87
time (sec)	N/A	0.008	0.004	0.749	0.199	0.237	0.042	0.277	0.031

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	98	89	88	88	97	13	88
N.S.	1	1.00	6.12	5.56	5.50	5.50	6.06	0.81	5.50
time (sec)	N/A	0.018	0.002	0.768	0.200	0.247	0.034	0.331	0.023

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	115	17	16	458	484	136	440
N.S.	1	1.00	6.39	0.94	0.89	25.44	26.89	7.56	24.44
time (sec)	N/A	0.031	0.037	0.099	0.202	0.250	0.078	0.291	9.575

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	98	13	15	88	97	15	88
N.S.	1	1.00	5.76	0.76	0.88	5.18	5.71	0.88	5.18
time (sec)	N/A	0.019	0.006	0.725	0.200	0.229	0.041	0.378	9.325

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	98	13	88	88	97	15	88
N.S.	1	1.00	7.00	0.93	6.29	6.29	6.93	1.07	6.29
time (sec)	N/A	0.047	0.002	0.694	0.180	0.227	0.035	0.280	0.026

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	98	88	88	88	97	15	88
N.S.	1	1.00	7.00	6.29	6.29	6.29	6.93	1.07	6.29
time (sec)	N/A	0.007	0.002	0.718	0.195	0.235	0.035	0.281	0.023

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	115	17	458	458	484	488	440
N.S.	1	1.00	6.39	0.94	25.44	25.44	26.89	27.11	24.44
time (sec)	N/A	0.040	0.007	0.068	0.185	0.246	0.074	0.290	0.434

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	98	13	88	88	97	88	88
N.S.	1	1.00	7.00	0.93	6.29	6.29	6.93	6.29	6.29
time (sec)	N/A	0.017	0.002	0.722	0.186	0.241	0.035	0.266	0.024

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	98	13	88	88	97	88	88
N.S.	1	1.00	5.44	0.72	4.89	4.89	5.39	4.89	4.89
time (sec)	N/A	0.015	0.002	0.734	0.190	0.256	0.036	0.294	0.025

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	88	88	97	88	88
N.S.	1	1.00	7.00	6.36	6.29	6.29	6.93	6.29	6.29
time (sec)	N/A	0.001	0.002	0.683	0.192	0.250	0.034	0.274	0.024

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	80	25	66	66	70	24	66
N.S.	1	1.00	2.86	0.89	2.36	2.36	2.50	0.86	2.36
time (sec)	N/A	0.018	0.004	0.724	0.186	0.244	0.034	0.293	0.030

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	108	27	187	187	194	88	180
N.S.	1	1.00	3.48	0.87	6.03	6.03	6.26	2.84	5.81
time (sec)	N/A	0.020	0.025	0.882	0.186	0.264	0.048	0.281	0.077

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	31	52	48	228	30	31
N.S.	1	1.00	1.00	0.91	1.53	1.41	6.71	0.88	0.91
time (sec)	N/A	0.006	0.340	0.818	0.300	0.262	18.289	0.267	9.074

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	73	33	54	52	0	32	58
N.S.	1	1.00	2.09	0.94	1.54	1.49	0.00	0.91	1.66
time (sec)	N/A	0.006	0.212	1.041	0.336	0.269	0.000	0.286	9.051

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	93	25	77	77	87	24	77
N.S.	1	1.00	3.10	0.83	2.57	2.57	2.90	0.80	2.57
time (sec)	N/A	0.015	0.006	0.801	0.191	0.248	0.036	0.276	9.206

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	140	27	280	280	314	105	266
N.S.	1	1.00	4.52	0.87	9.03	9.03	10.13	3.39	8.58
time (sec)	N/A	0.027	0.030	0.196	0.188	0.256	0.060	0.267	9.490

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	98	31	80	80	90	30	80
N.S.	1	1.00	2.88	0.91	2.35	2.35	2.65	0.88	2.35
time (sec)	N/A	0.023	0.006	0.732	0.195	0.266	0.038	0.281	0.040

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	146	33	289	289	321	126	273
N.S.	1	1.00	3.56	0.80	7.05	7.05	7.83	3.07	6.66
time (sec)	N/A	0.031	0.035	0.082	0.198	0.269	0.062	0.279	9.412

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	244	37	289	289	323	37	270
N.S.	1	1.00	5.30	0.80	6.28	6.28	7.02	0.80	5.87
time (sec)	N/A	0.032	0.045	0.356	0.188	0.383	0.068	0.283	9.318

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	248	39	773	773	930	153	753
N.S.	1	1.00	5.28	0.83	16.45	16.45	19.79	3.26	16.02
time (sec)	N/A	0.060	0.076	0.530	0.203	0.252	0.116	0.436	9.340

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	36	31	54	48	190	30	33
N.S.	1	1.00	1.06	0.91	1.59	1.41	5.59	0.88	0.97
time (sec)	N/A	0.006	0.195	0.803	0.303	0.274	40.139	0.288	9.163

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	42	37	71	57	189	36	37
N.S.	1	1.00	0.95	0.84	1.61	1.30	4.30	0.82	0.84
time (sec)	N/A	0.006	0.174	0.817	0.310	0.257	73.303	0.340	9.485

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	49	43	83	72	0	42	73
N.S.	1	1.00	0.98	0.86	1.66	1.44	0.00	0.84	1.46
time (sec)	N/A	0.006	0.267	0.162	0.316	0.259	0.000	0.271	9.204

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	33	18	17	29	29	30	29
N.S.	1	1.00	1.74	0.95	0.89	1.53	1.53	1.58	1.53
time (sec)	N/A	0.006	0.002	0.062	0.191	0.245	0.019	0.276	0.014

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	21	15	14	17	17	22	17
N.S.	1	1.00	1.31	0.94	0.88	1.06	1.06	1.38	1.06
time (sec)	N/A	0.004	0.002	0.106	0.189	0.239	0.018	0.273	0.019

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	96	96	29	86	86	94	28	86
N.S.	1	2.91	2.91	0.88	2.61	2.61	2.85	0.85	2.61
time (sec)	N/A	0.132	0.006	0.783	0.185	0.254	0.032	0.293	9.264

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	96	96	86	86	86	94	28	86
N.S.	1	2.91	2.91	2.61	2.61	2.61	2.85	0.85	2.61
time (sec)	N/A	0.097	0.004	0.786	0.191	0.255	0.035	0.313	0.192

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	57	56	12	12
N.S.	1	1.00	1.00	0.93	0.86	4.07	4.00	0.86	0.86
time (sec)	N/A	0.005	0.005	0.049	0.192	0.234	0.080	0.291	9.369

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	0.93	0.87
time (sec)	N/A	0.006	0.005	0.031	0.184	0.234	0.037	0.289	0.030

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	20	14	15	15	14	18	13
N.S.	1	1.00	1.18	0.82	0.88	0.88	0.82	1.06	0.76
time (sec)	N/A	0.006	0.005	0.035	0.187	0.231	0.041	0.311	0.043

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	23	24	23	23	22	23	23
N.S.	1	1.00	0.58	0.60	0.58	0.58	0.55	0.58	0.58
time (sec)	N/A	0.061	0.040	0.067	0.196	0.267	12.989	0.328	0.064

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	605	605	98	535	0	0	0	0	0
N.S.	1	1.00	0.16	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.073	0.040	0.157	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	55	53	0	83	58	58	75
N.S.	1	1.00	0.87	0.84	0.00	1.32	0.92	0.92	1.19
time (sec)	N/A	0.045	0.022	0.091	0.000	0.317	0.056	0.287	0.111

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	22	38	20	13	12
N.S.	1	1.00	1.00	0.93	1.57	2.71	1.43	0.93	0.86
time (sec)	N/A	0.033	0.006	0.039	0.191	0.305	0.036	0.301	0.023

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	24	22	32	46	29	23	21
N.S.	1	1.00	0.86	0.79	1.14	1.64	1.04	0.82	0.75
time (sec)	N/A	0.018	0.011	0.036	0.193	0.294	0.044	0.291	0.022

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	28	29	38	38	36	28	28
N.S.	1	1.00	0.47	0.49	0.64	0.64	0.61	0.47	0.47
time (sec)	N/A	0.061	0.009	0.050	0.215	0.323	0.061	0.569	0.029

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	11	11
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	1.00	1.00
time (sec)	N/A	0.005	0.005	0.054	0.188	0.300	0.043	0.285	9.457

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	205	132	116	114	223	272	134	124
N.S.	1	2.25	1.45	1.27	1.25	2.45	2.99	1.47	1.36
time (sec)	N/A	0.107	0.065	0.113	0.278	0.340	0.804	0.290	9.235

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	44	40	0	99	49
N.S.	1	1.00	0.92	1.04	1.76	1.60	0.00	3.96	1.96
time (sec)	N/A	0.017	1.358	4.232	0.244	0.509	0.000	0.543	9.146

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	39	39	0	89	39
N.S.	1	1.00	0.91	1.04	1.70	1.70	0.00	3.87	1.70
time (sec)	N/A	0.057	2.163	0.260	0.229	0.327	0.000	0.531	9.106

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	39	39	0	89	39
N.S.	1	1.00	0.91	1.04	1.70	1.70	0.00	3.87	1.70
time (sec)	N/A	0.040	1.065	0.218	0.224	0.294	0.000	0.394	9.269

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	22	37	37	0	87	37
N.S.	1	1.00	0.90	1.05	1.76	1.76	0.00	4.14	1.76
time (sec)	N/A	0.031	0.803	0.174	0.221	0.313	0.000	0.400	9.399

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	17	20	33	33	0	52	19
N.S.	1	1.00	0.89	1.05	1.74	1.74	0.00	2.74	1.00
time (sec)	N/A	0.027	0.062	0.151	0.230	0.306	0.000	0.270	9.108

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	23
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00
time (sec)	N/A	0.023	0.798	0.203	0.221	0.343	0.000	0.000	9.651

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	23
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00
time (sec)	N/A	0.024	0.853	0.205	0.266	0.341	0.000	0.000	10.067

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	23
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00
time (sec)	N/A	0.023	1.013	0.207	0.233	0.331	0.000	0.000	9.856

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	83	74	73	79	97	73	97
N.S.	1	1.00	0.86	0.76	0.75	0.81	1.00	0.75	1.00
time (sec)	N/A	0.090	0.025	0.092	0.272	0.332	0.117	0.299	0.120

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	78	69	68	74	92	68	92
N.S.	1	1.00	0.87	0.77	0.76	0.82	1.02	0.76	1.02
time (sec)	N/A	0.080	0.017	0.073	0.270	0.336	0.121	0.277	0.102

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	72	62	61	67	78	61	85
N.S.	1	1.00	0.94	0.81	0.79	0.87	1.01	0.79	1.10
time (sec)	N/A	0.074	0.021	0.072	0.273	0.274	0.113	0.308	9.259

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	69	57	56	62	75	56	80
N.S.	1	1.00	0.96	0.79	0.78	0.86	1.04	0.78	1.11
time (sec)	N/A	0.061	0.018	0.063	0.291	0.303	0.128	0.282	9.195

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	65	56	55	61	75	55	79
N.S.	1	1.00	0.92	0.79	0.77	0.86	1.06	0.77	1.11
time (sec)	N/A	0.051	0.013	0.059	0.275	0.274	0.116	0.307	0.084

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	69	60	59	65	78	60	83
N.S.	1	1.00	0.92	0.80	0.79	0.87	1.04	0.80	1.11
time (sec)	N/A	0.086	0.016	0.080	0.286	0.293	0.142	0.337	9.296

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	78	65	64	76	87	65	88
N.S.	1	1.00	0.93	0.77	0.76	0.90	1.04	0.77	1.05
time (sec)	N/A	0.095	0.023	0.091	0.283	0.298	0.160	0.291	9.087

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	82	70	69	89	94	70	92
N.S.	1	1.00	0.90	0.77	0.76	0.98	1.03	0.77	1.01
time (sec)	N/A	0.101	0.037	0.082	0.271	0.275	0.164	0.280	0.091

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	109	74	0	1202	61	0	128
N.S.	1	1.00	0.36	0.24	0.00	3.92	0.20	0.00	0.42
time (sec)	N/A	0.397	0.014	0.054	0.000	1.058	0.580	0.000	9.856

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	101	67	0	1145	3662	0	188
N.S.	1	1.00	0.38	0.25	0.00	4.26	13.61	0.00	0.70
time (sec)	N/A	0.299	0.012	0.046	0.000	1.024	1.587	0.000	0.078

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	94	62	0	1190	48	0	183
N.S.	1	1.00	0.41	0.27	0.00	5.17	0.21	0.00	0.80
time (sec)	N/A	0.263	0.013	0.041	0.000	0.996	0.519	0.000	9.478

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	90	58	0	1189	46	0	181
N.S.	1	1.00	0.45	0.29	0.00	6.01	0.23	0.00	0.91
time (sec)	N/A	0.153	0.010	0.037	0.000	0.998	0.529	0.000	9.827

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	101	51	0	1143	60	0	237
N.S.	1	1.00	0.41	0.21	0.00	4.67	0.24	0.00	0.97
time (sec)	N/A	0.350	0.013	0.058	0.000	1.022	8.492	0.000	9.698

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	109	58	0	1245	25507	0	242
N.S.	1	1.00	0.39	0.21	0.00	4.43	90.77	0.00	0.86
time (sec)	N/A	0.345	0.013	0.066	0.000	1.011	18.687	0.000	9.668

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	116	62	0	1274	70	0	246
N.S.	1	1.00	0.37	0.20	0.00	4.02	0.22	0.00	0.78
time (sec)	N/A	0.403	0.014	0.064	0.000	0.977	1.748	0.000	9.596

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	87	49	0	83	44	0	252
N.S.	1	1.00	4.58	2.58	0.00	4.37	2.32	0.00	13.26
time (sec)	N/A	0.070	0.031	0.265	0.000	0.258	0.609	0.000	9.547

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	57	33	33	47	37	34	38
N.S.	1	1.00	1.33	0.77	0.77	1.09	0.86	0.79	0.88
time (sec)	N/A	0.041	0.018	0.842	0.267	0.267	0.081	0.309	0.034

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	18	21
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	1.06	1.24
time (sec)	N/A	0.025	0.005	0.788	0.240	0.249	0.060	0.289	10.429

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	20	24	30
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.80	0.96	1.20
time (sec)	N/A	0.024	0.005	0.800	0.191	0.253	0.058	0.295	9.754

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	20	19	18	24	14	28	18
N.S.	1	1.00	0.91	0.86	0.82	1.09	0.64	1.27	0.82
time (sec)	N/A	0.011	0.007	0.790	0.183	0.247	0.048	0.318	0.024

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	38	22	21	21	26	22	25
N.S.	1	1.00	1.41	0.81	0.78	0.78	0.96	0.81	0.93
time (sec)	N/A	0.025	0.010	0.810	0.281	0.257	0.068	0.274	0.036

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	17	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.81	0.81
time (sec)	N/A	0.010	0.004	0.783	0.275	0.251	0.037	0.292	0.018

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	23	22	23	23
N.S.	1	1.00	1.00	0.89	0.85	0.85	0.81	0.85	0.85
time (sec)	N/A	0.017	0.006	1.069	0.268	0.244	0.052	0.313	9.439

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	26	25	25	26	29	25
N.S.	1	1.00	1.00	0.67	0.64	0.64	0.67	0.74	0.64
time (sec)	N/A	0.013	0.005	0.057	0.191	0.261	0.127	0.297	9.283

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	23	19	18	18	17	19	18
N.S.	1	1.00	1.05	0.86	0.82	0.82	0.77	0.86	0.82
time (sec)	N/A	0.009	0.003	0.738	0.195	0.254	0.030	0.282	0.015

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.019	0.005	0.793	0.269	0.245	0.037	0.278	9.350

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	24	21	20	26	17	22	10
N.S.	1	1.00	2.00	1.75	1.67	2.17	1.42	1.83	0.83
time (sec)	N/A	0.014	0.010	0.038	0.186	0.262	0.044	0.294	0.044

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	22	19	23	19
N.S.	1	1.00	1.00	0.88	0.84	0.88	0.76	0.92	0.76
time (sec)	N/A	0.023	0.006	0.834	0.189	0.263	0.047	0.296	9.328

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85
time (sec)	N/A	0.018	0.006	0.069	0.264	0.263	0.050	0.318	0.023

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	31	35	55	36	30	35
N.S.	1	1.00	1.00	0.89	1.00	1.57	1.03	0.86	1.00
time (sec)	N/A	0.026	0.012	0.817	0.284	0.251	0.061	0.303	9.521

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	20	17
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.87	0.74
time (sec)	N/A	0.024	0.005	0.046	0.190	0.251	0.085	0.290	9.588

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	19	20	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.83	0.87	0.83
time (sec)	N/A	0.037	0.007	0.046	0.197	0.238	0.051	0.273	0.035

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	23	32	20	23	25
N.S.	1	1.00	0.86	0.83	0.79	1.10	0.69	0.79	0.86
time (sec)	N/A	0.010	0.008	0.833	0.272	0.250	0.054	0.295	0.019

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	42	41	58	49	43	55
N.S.	1	1.00	1.00	0.95	0.93	1.32	1.11	0.98	1.25
time (sec)	N/A	0.169	0.019	0.798	0.278	0.265	0.097	0.294	0.077

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	38	51	38	88
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.11	0.83	1.91
time (sec)	N/A	0.088	0.016	1.439	0.275	0.260	0.102	0.274	0.099

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	34	44	72	41	44	56
N.S.	1	1.00	1.00	1.03	1.33	2.18	1.24	1.33	1.70
time (sec)	N/A	0.112	0.014	1.328	0.190	0.280	0.073	0.309	9.390

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	11	11	10	11	11
N.S.	1	1.00	1.00	0.71	0.65	0.65	0.59	0.65	0.65
time (sec)	N/A	0.004	0.005	0.809	0.262	0.277	0.064	0.301	0.019

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	20	20	34	20	20
N.S.	1	1.00	0.92	0.88	0.83	0.83	1.42	0.83	0.83
time (sec)	N/A	0.013	0.008	0.822	0.287	0.277	0.081	0.292	9.343

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	23	18	17	17	17	20	19
N.S.	1	1.00	1.53	1.20	1.13	1.13	1.13	1.33	1.27
time (sec)	N/A	0.038	0.008	0.788	0.268	0.265	0.069	0.283	0.037

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	17	17	19	17	17
N.S.	1	1.00	1.00	0.90	0.85	0.85	0.95	0.85	0.85
time (sec)	N/A	0.007	0.008	0.774	0.262	0.272	0.070	0.275	0.031

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	27	44	29	60	35
N.S.	1	1.00	0.89	0.76	0.73	1.19	0.78	1.62	0.95
time (sec)	N/A	0.026	0.018	0.838	0.280	0.276	0.077	0.280	9.419

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	22	21	21	26	21	21
N.S.	1	1.00	1.00	0.85	0.81	0.81	1.00	0.81	0.81
time (sec)	N/A	0.008	0.008	0.766	0.279	0.288	0.039	0.281	0.019

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	16	10	11	10
N.S.	1	1.00	1.00	0.92	0.83	1.33	0.83	0.92	0.83
time (sec)	N/A	0.021	0.004	0.790	0.177	0.274	0.037	0.284	9.458

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	20	21
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.95	1.00
time (sec)	N/A	0.018	0.005	0.043	0.182	0.271	0.066	0.271	0.032

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	34	17	20	20
N.S.	1	1.00	1.00	0.95	0.91	1.55	0.77	0.91	0.91
time (sec)	N/A	0.010	0.007	0.063	0.267	0.265	0.051	0.288	0.020

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	32	19	20	22
N.S.	1	1.00	1.00	0.88	0.83	1.33	0.79	0.83	0.92
time (sec)	N/A	0.011	0.008	0.057	0.265	0.259	0.049	0.268	0.017

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	33	33	39	33	56
N.S.	1	1.00	1.00	0.94	0.92	0.92	1.08	0.92	1.56
time (sec)	N/A	0.017	0.012	0.813	0.276	0.295	0.099	0.265	0.065

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	28	27	27	29	27	37
N.S.	1	1.00	1.00	0.76	0.73	0.73	0.78	0.73	1.00
time (sec)	N/A	0.017	0.007	0.819	0.272	0.267	0.084	0.290	9.555

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	21	21	22	23	21
N.S.	1	1.00	1.00	0.76	0.72	0.72	0.76	0.79	0.72
time (sec)	N/A	0.011	0.005	0.873	0.176	0.255	0.056	0.270	9.809

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	14	16	15
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.74	0.84	0.79
time (sec)	N/A	0.009	0.004	0.802	0.185	0.287	0.031	0.270	9.674

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	35	34	34	46	34	36
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.12	0.83	0.88
time (sec)	N/A	0.019	0.010	1.190	0.261	0.250	0.052	0.278	0.026

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	39	32	31	31	34	31	31
N.S.	1	1.00	0.95	0.78	0.76	0.76	0.83	0.76	0.76
time (sec)	N/A	0.018	0.006	1.073	0.267	0.245	0.060	0.269	9.570

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	24	21	20	20	24	23	20
N.S.	1	1.00	0.80	0.70	0.67	0.67	0.80	0.77	0.67
time (sec)	N/A	0.035	0.012	0.849	0.188	0.278	0.069	0.272	9.509

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	31	30	27
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.89	0.86	0.77
time (sec)	N/A	0.024	0.006	0.055	0.178	0.277	0.071	0.275	0.027

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	32	25	24	34	27	34	26
N.S.	1	1.00	0.94	0.74	0.71	1.00	0.79	1.00	0.76
time (sec)	N/A	0.031	0.010	0.787	0.182	0.246	0.065	0.258	9.382

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	35	33	44	36	33	39
N.S.	1	1.00	1.00	0.83	0.79	1.05	0.86	0.79	0.93
time (sec)	N/A	0.013	0.015	0.763	0.265	0.256	0.058	0.272	9.210

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	38	37	37	46	37	41
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.94	0.76	0.84
time (sec)	N/A	0.104	0.010	1.382	0.268	0.266	0.104	0.301	0.045

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	20	19	19	24	22	19
N.S.	1	1.00	1.00	0.69	0.66	0.66	0.83	0.76	0.66
time (sec)	N/A	0.032	0.007	0.852	0.190	0.255	0.064	0.266	9.623

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	34	33	33	41	34	50
N.S.	1	1.00	0.93	0.74	0.72	0.72	0.89	0.74	1.09
time (sec)	N/A	0.029	0.014	0.799	0.262	0.253	0.068	0.281	0.053

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	14	15	14	17	10	15	14
N.S.	1	1.00	0.88	0.94	0.88	1.06	0.62	0.94	0.88
time (sec)	N/A	0.016	0.007	0.042	0.204	0.259	0.041	0.328	9.329

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	14	12	17	17	14	12	12
N.S.	1	1.00	0.67	0.57	0.81	0.81	0.67	0.57	0.57
time (sec)	N/A	0.013	0.003	0.027	0.189	0.237	0.039	0.261	9.283

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	23	23	17	30	15
N.S.	1	1.00	1.00	1.07	1.53	1.53	1.13	2.00	1.00
time (sec)	N/A	0.018	0.009	0.785	0.185	0.248	0.048	0.287	9.220

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	28	28	3	29	57
N.S.	1	1.00	1.00	0.94	0.90	0.90	0.10	0.94	1.84
time (sec)	N/A	0.030	0.009	0.051	0.268	0.262	0.062	0.275	0.069

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	19	19	19	22	19
N.S.	1	1.00	1.00	0.72	0.76	0.76	0.76	0.88	0.76
time (sec)	N/A	0.029	0.005	0.045	0.188	0.283	0.065	0.277	0.038

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	29	25	24	36	20	26	22
N.S.	1	1.00	0.97	0.83	0.80	1.20	0.67	0.87	0.73
time (sec)	N/A	0.020	0.012	0.049	0.186	0.248	0.042	0.261	9.441

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	18	21
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.78	0.91
time (sec)	N/A	0.026	0.004	0.788	0.268	0.264	0.069	0.271	9.807

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	93	72	77	136	88	74	96
N.S.	1	1.00	0.90	0.70	0.75	1.32	0.85	0.72	0.93
time (sec)	N/A	0.354	0.033	1.092	0.274	0.298	0.292	0.282	9.342

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	30	27	25	36	24	25	32
N.S.	1	1.00	0.91	0.82	0.76	1.09	0.73	0.76	0.97
time (sec)	N/A	0.015	0.009	0.793	0.258	0.290	0.055	0.264	0.021

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	29	44	27	30	33
N.S.	1	1.00	1.00	0.85	0.88	1.33	0.82	0.91	1.00
time (sec)	N/A	0.027	0.013	0.822	0.261	0.295	0.065	0.283	9.594

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	23	19	17	26	19
N.S.	1	1.00	1.00	0.80	0.92	0.76	0.68	1.04	0.76
time (sec)	N/A	0.029	0.005	0.845	0.189	0.279	0.044	0.269	0.019

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	31	31	36	31	56
N.S.	1	1.00	1.00	0.89	0.86	0.86	1.00	0.86	1.56
time (sec)	N/A	0.084	0.011	0.793	0.262	0.291	0.094	0.259	0.062

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	21	33	22	21	23
N.S.	1	1.00	1.00	0.76	0.72	1.14	0.76	0.72	0.79
time (sec)	N/A	0.073	0.012	0.829	0.266	0.311	0.083	0.283	0.026

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	36	35	39	46	36	49
N.S.	1	1.00	1.00	0.78	0.76	0.85	1.00	0.78	1.07
time (sec)	N/A	0.041	0.018	0.063	0.270	0.325	0.083	0.269	9.426

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	26	26	19	18	18	17	20	14
N.S.	1	1.18	1.18	0.86	0.82	0.82	0.77	0.91	0.64
time (sec)	N/A	0.011	0.004	0.835	0.180	0.340	0.040	0.296	0.024

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	18	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	1.06	0.88
time (sec)	N/A	0.027	0.005	0.042	0.185	0.339	0.064	0.262	0.042

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	18	14	16	14
N.S.	1	1.00	1.00	1.07	1.00	1.29	1.00	1.14	1.00
time (sec)	N/A	0.016	0.004	0.773	0.193	0.325	0.052	0.289	9.487

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	25	18	17	17	17	20	17
N.S.	1	1.00	1.32	0.95	0.89	0.89	0.89	1.05	0.89
time (sec)	N/A	0.018	0.005	0.039	0.184	0.340	0.069	0.276	9.620

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	19	17	33
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.83	0.74	1.43
time (sec)	N/A	0.019	0.009	0.072	0.268	0.316	0.090	0.265	0.039

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	57	51	50	50	68	53	58
N.S.	1	1.00	0.90	0.81	0.79	0.79	1.08	0.84	0.92
time (sec)	N/A	0.056	0.020	0.082	0.263	0.327	0.216	0.336	0.093

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	86	67	54	59	103	65	59	71
N.S.	1	1.25	0.97	0.78	0.86	1.49	0.94	0.86	1.03
time (sec)	N/A	0.063	0.032	0.075	0.283	0.298	0.114	0.289	0.083

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	19	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	1.12	0.71	0.76	0.76
time (sec)	N/A	0.009	0.006	0.790	0.270	0.247	0.049	0.265	0.020

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	15	17	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	0.79	0.89	0.89
time (sec)	N/A	0.017	0.005	0.724	0.260	0.260	0.046	0.266	9.578

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	7	28	19
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.78	3.11	2.11
time (sec)	N/A	0.046	0.005	0.798	0.266	0.298	0.072	0.297	9.263

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	12	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.010	0.005	0.936	0.181	0.315	0.036	0.298	0.017

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	58	41	51	66	99	55	48
N.S.	1	1.00	0.89	0.63	0.78	1.02	1.52	0.85	0.74
time (sec)	N/A	0.042	0.025	0.875	0.323	0.309	0.245	0.266	0.063

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	46	24	28	28
N.S.	1	1.00	1.00	1.04	1.00	1.64	0.86	1.00	1.00
time (sec)	N/A	0.016	0.009	0.946	0.350	0.260	0.058	0.266	9.431

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	26	26	29	26	26
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.028	0.023	0.829	0.349	0.265	0.048	0.310	0.014

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	41	39	24	23	23	26	27	23
N.S.	1	1.32	1.26	0.77	0.74	0.74	0.84	0.87	0.74
time (sec)	N/A	0.032	0.006	0.052	0.226	0.291	0.106	0.300	0.029

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	26	19	23	30
N.S.	1	1.00	1.00	0.96	0.92	1.08	0.79	0.96	1.25
time (sec)	N/A	0.117	0.007	0.831	0.358	0.276	0.068	0.276	9.211

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	35	20	19	23
N.S.	1	1.00	1.00	0.83	1.04	1.52	0.87	0.83	1.00
time (sec)	N/A	0.017	0.009	0.796	0.313	0.246	0.056	0.348	9.253

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	35	20	19	23
N.S.	1	1.00	1.00	0.83	1.04	1.52	0.87	0.83	1.00
time (sec)	N/A	0.026	0.004	0.074	0.301	0.259	0.059	0.273	0.017

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	15	10	13	13
N.S.	1	1.00	1.00	1.08	1.00	1.15	0.77	1.00	1.00
time (sec)	N/A	0.031	0.005	0.037	0.216	0.265	0.042	0.284	9.257

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	193	68	199	4545	138	208	357
N.S.	1	1.00	0.94	0.33	0.97	22.06	0.67	1.01	1.73
time (sec)	N/A	0.195	0.071	0.777	0.318	0.943	0.732	0.272	9.359

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	0	47	44	38	42
N.S.	1	1.00	1.00	0.84	0.00	1.04	0.98	0.84	0.93
time (sec)	N/A	0.045	0.019	0.056	0.000	0.273	0.078	0.418	9.337

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	44	41	0	75	46	42	47
N.S.	1	1.00	0.75	0.69	0.00	1.27	0.78	0.71	0.80
time (sec)	N/A	0.044	0.017	0.117	0.000	0.248	0.085	0.739	0.028

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	234	48	0	578003	0	1714	3942
N.S.	1	1.00	1.12	0.23	0.00	2765.56	0.00	8.20	18.86
time (sec)	N/A	0.253	0.154	0.151	0.000	29.017	0.000	1.121	10.016

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	245	54	0	540080	0	1724	3046
N.S.	1	1.00	1.09	0.24	0.00	2411.07	0.00	7.70	13.60
time (sec)	N/A	0.191	0.155	0.720	0.000	80.465	0.000	1.190	0.649

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	57	60	65	190	62	61
N.S.	1	1.00	1.00	1.02	1.07	1.16	3.39	1.11	1.09
time (sec)	N/A	0.027	0.018	0.809	0.184	0.275	0.612	0.270	0.108

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	73	75	84	162	0	85	347
N.S.	1	1.00	0.76	0.78	0.88	1.69	0.00	0.89	3.61
time (sec)	N/A	0.073	0.025	0.849	0.278	0.287	0.000	0.279	0.652

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	228	177	279	5975	0	320	570
N.S.	1	1.00	0.86	0.67	1.06	22.63	0.00	1.21	2.16
time (sec)	N/A	0.331	0.059	0.845	0.275	1.052	0.000	0.290	9.578

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	417	370	226	349	259898	0	406	823
N.S.	1	1.00	0.89	0.54	0.84	623.26	0.00	0.97	1.97
time (sec)	N/A	0.366	0.152	0.841	0.274	23.563	0.000	0.285	10.072

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	24	21	20	26	19	21	12
N.S.	1	1.00	1.50	1.31	1.25	1.62	1.19	1.31	0.75
time (sec)	N/A	0.006	0.007	0.746	0.190	0.249	0.044	0.276	0.029

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	27	24	23	34	20	30	17
N.S.	1	1.00	1.42	1.26	1.21	1.79	1.05	1.58	0.89
time (sec)	N/A	0.008	0.009	0.826	0.179	0.249	0.054	0.262	9.908

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	85	72	75	106	92	77	103
N.S.	1	1.00	0.88	0.74	0.77	1.09	0.95	0.79	1.06
time (sec)	N/A	0.049	0.036	0.822	0.268	0.269	0.184	0.291	0.111

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	0.87	0.87
time (sec)	N/A	0.069	0.006	0.801	0.269	0.262	0.051	0.266	10.154

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85
time (sec)	N/A	0.063	0.006	0.785	0.263	0.273	0.054	0.285	0.022

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	25	24	24	29	24	51
N.S.	1	1.00	1.00	0.86	0.83	0.83	1.00	0.83	1.76
time (sec)	N/A	0.080	0.011	0.764	0.269	0.291	0.091	0.333	9.894

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	17	10	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.21	0.71	1.00	1.00
time (sec)	N/A	0.008	0.007	0.912	0.264	0.293	0.059	0.298	0.025

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	12	7	11	10
N.S.	1	1.00	1.00	0.92	0.83	1.00	0.58	0.92	0.83
time (sec)	N/A	0.008	0.003	0.791	0.203	0.244	0.038	0.299	0.028

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	10	17
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.91	1.55
time (sec)	N/A	0.022	0.005	0.783	0.263	0.252	0.059	0.282	0.026

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	13	8	11	10
N.S.	1	1.00	1.00	0.92	0.83	1.08	0.67	0.92	0.83
time (sec)	N/A	0.014	0.003	0.779	0.183	0.271	0.035	0.306	0.028

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	12	14	12
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.00	1.17	1.00
time (sec)	N/A	0.012	0.004	0.719	0.182	0.264	0.052	0.283	0.029

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	16	23
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	0.94	1.35
time (sec)	N/A	0.023	0.007	0.783	0.308	0.290	0.060	0.309	9.919

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	10	14	19
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.77	1.08	1.46
time (sec)	N/A	0.021	0.020	0.810	0.264	0.271	0.063	0.274	10.457

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	22	21	20	26	20	22	20
N.S.	1	1.00	0.79	0.75	0.71	0.93	0.71	0.79	0.71
time (sec)	N/A	0.007	0.013	0.815	0.192	0.252	0.053	0.274	0.046

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	25	26	37	26	43	22
N.S.	1	1.00	1.00	0.78	0.81	1.16	0.81	1.34	0.69
time (sec)	N/A	0.016	0.013	0.784	0.179	0.254	0.061	0.264	9.963

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	28	20	19	19	19	20	25
N.S.	1	1.00	1.22	0.87	0.83	0.83	0.83	0.87	1.09
time (sec)	N/A	0.022	0.006	0.765	0.265	0.279	0.063	0.270	0.030

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	20	36	20	47	28
N.S.	1	1.00	0.92	0.88	0.83	1.50	0.83	1.96	1.17
time (sec)	N/A	0.023	0.010	0.807	0.290	0.268	0.075	0.385	9.875

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	40	39	39	44	39	41
N.S.	1	1.00	1.00	0.82	0.80	0.80	0.90	0.80	0.84
time (sec)	N/A	0.094	0.011	1.234	0.278	0.274	0.105	0.277	9.705

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	19	20	19	19	19	22	19
N.S.	1	1.00	0.76	0.80	0.76	0.76	0.76	0.88	0.76
time (sec)	N/A	0.033	0.007	0.813	0.192	0.282	0.064	0.307	0.029

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	48	47	60	63	60	61
N.S.	1	1.00	0.90	0.80	0.78	1.00	1.05	1.00	1.02
time (sec)	N/A	0.163	0.037	1.022	0.271	0.269	0.125	0.274	0.076

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	7	9	9	5	9	6
N.S.	1	1.00	1.00	0.64	0.82	0.82	0.45	0.82	0.55
time (sec)	N/A	0.004	0.001	0.913	0.185	0.247	0.021	0.283	0.009

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	21	21	22	23	21
N.S.	1	1.00	1.00	0.76	0.72	0.72	0.76	0.79	0.72
time (sec)	N/A	0.010	0.004	0.821	0.194	0.256	0.057	0.340	9.255

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	37	37	52	37	37	49
N.S.	1	1.00	1.00	0.82	0.82	1.16	0.82	0.82	1.09
time (sec)	N/A	0.016	0.009	1.072	0.271	0.287	0.062	0.288	0.026

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	28	52	29	29	44
N.S.	1	1.00	1.00	0.91	0.88	1.62	0.91	0.91	1.38
time (sec)	N/A	0.171	0.015	0.847	0.268	0.271	0.119	0.292	9.284

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	159	44	132	106	146	122	124
N.S.	1	1.00	1.07	0.30	0.89	0.72	0.99	0.82	0.84
time (sec)	N/A	0.091	0.053	0.127	0.272	0.260	0.213	0.276	9.105

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	101	54	99	515	61	92	170
N.S.	1	1.00	0.90	0.48	0.88	4.60	0.54	0.82	1.52
time (sec)	N/A	0.073	0.034	0.805	0.277	0.954	0.526	0.272	9.350

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	14	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	1.00	0.86
time (sec)	N/A	0.011	0.003	0.788	0.183	0.248	0.043	0.322	0.030

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	12	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	1.00	0.83
time (sec)	N/A	0.018	0.004	0.864	0.209	0.255	0.047	0.281	9.144

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	21	14	17	15
N.S.	1	1.00	1.00	0.94	0.88	1.24	0.82	1.00	0.88
time (sec)	N/A	0.019	0.003	0.798	0.186	0.230	0.047	0.266	0.016

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	17	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	1.00	0.88
time (sec)	N/A	0.020	0.004	0.791	0.214	0.247	0.044	0.275	0.019

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	25	15	23	16
N.S.	1	1.00	1.00	0.94	0.89	1.39	0.83	1.28	0.89
time (sec)	N/A	0.021	0.004	0.768	0.269	0.232	0.046	0.271	0.033

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	14	7	11	10
N.S.	1	1.00	1.00	1.10	1.00	1.40	0.70	1.10	1.00
time (sec)	N/A	0.013	0.004	0.034	0.195	0.264	0.035	0.284	9.397

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	31	30	30	36	33	30
N.S.	1	1.00	1.00	0.74	0.71	0.71	0.86	0.79	0.71
time (sec)	N/A	0.026	0.006	0.052	0.185	0.242	0.073	0.277	0.032

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	38	51	38	88
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.11	0.83	1.91
time (sec)	N/A	0.091	0.013	0.984	0.297	0.248	0.103	0.285	0.001

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	15	15	14	17	8
N.S.	1	1.00	1.00	0.74	0.79	0.79	0.74	0.89	0.42
time (sec)	N/A	0.042	0.003	0.762	0.185	0.240	0.048	0.260	9.434

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	27	30	30	34	32	26
N.S.	1	1.00	1.00	0.68	0.75	0.75	0.85	0.80	0.65
time (sec)	N/A	0.014	0.004	0.035	0.192	0.248	0.053	0.267	9.305

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	22	25	25	27	27	21
N.S.	1	1.00	1.00	0.67	0.76	0.76	0.82	0.82	0.64
time (sec)	N/A	0.014	0.003	0.037	0.180	0.252	0.054	0.293	0.019

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	17	20	20	20	22	16
N.S.	1	1.00	1.00	0.65	0.77	0.77	0.77	0.85	0.62
time (sec)	N/A	0.010	0.005	0.034	0.181	0.232	0.054	0.303	0.042

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	17	17	17	19	13
N.S.	1	1.00	1.00	0.67	0.81	0.81	0.81	0.90	0.62
time (sec)	N/A	0.005	0.003	0.033	0.188	0.234	0.049	0.268	9.420

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	17	17	15	19	8
N.S.	1	1.00	1.00	0.67	0.81	0.81	0.71	0.90	0.38
time (sec)	N/A	0.007	0.002	0.045	0.181	0.253	0.055	0.289	0.046

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	18	21	21	24	24	17
N.S.	1	1.00	1.00	0.67	0.78	0.78	0.89	0.89	0.63
time (sec)	N/A	0.012	0.003	0.045	0.186	0.275	0.073	0.255	0.057

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	26	30	31	29	22
N.S.	1	1.00	1.00	0.79	0.76	0.88	0.91	0.85	0.65
time (sec)	N/A	0.019	0.003	0.049	0.186	0.278	0.081	0.255	0.022

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	31	31	39	36	34	26
N.S.	1	1.00	1.00	0.76	0.76	0.95	0.88	0.83	0.63
time (sec)	N/A	0.023	0.004	0.053	0.181	0.285	0.089	0.285	9.327

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	36	36	44	41	39	32
N.S.	1	1.00	1.00	0.75	0.75	0.92	0.85	0.81	0.67
time (sec)	N/A	0.026	0.004	0.049	0.183	0.285	0.092	0.259	0.022

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	61	54	0	1506	41	0	144
N.S.	1	1.00	0.39	0.34	0.00	9.59	0.26	0.00	0.92
time (sec)	N/A	0.139	0.015	0.071	0.000	1.019	0.115	0.000	9.908

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	61	56	0	1546	41	0	142
N.S.	1	1.00	0.39	0.36	0.00	9.85	0.26	0.00	0.90
time (sec)	N/A	0.107	0.015	0.075	0.000	0.987	0.117	0.000	9.943

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	61	54	0	2271	39	0	142
N.S.	1	1.00	0.32	0.29	0.00	12.08	0.21	0.00	0.76
time (sec)	N/A	0.196	0.014	0.056	0.000	0.976	0.097	0.000	10.041

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	61	56	0	2259	39	0	142
N.S.	1	1.00	0.32	0.30	0.00	12.02	0.21	0.00	0.76
time (sec)	N/A	0.112	0.012	0.058	0.000	0.992	0.099	0.000	10.268

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	663	663	63	69	0	322185	133	0	328
N.S.	1	1.00	0.10	0.10	0.00	485.95	0.20	0.00	0.49
time (sec)	N/A	1.005	0.027	0.261	0.000	1.659	2.158	0.000	9.828

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	663	663	63	69	0	322185	133	0	328
N.S.	1	1.00	0.10	0.10	0.00	485.95	0.20	0.00	0.49
time (sec)	N/A	0.735	0.023	0.075	0.000	1.656	2.139	0.000	0.001

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	C	F	C	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	0	95	67	0	5653	42	0	504
N.S.	1	0.00	0.57	0.40	0.00	33.65	0.25	0.00	3.00
time (sec)	N/A	0.000	0.042	0.822	0.000	0.922	1.041	0.000	10.476

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	322	54	310	141845	384	314	894
N.S.	1	1.00	1.01	0.17	0.97	443.27	1.20	0.98	2.79
time (sec)	N/A	0.171	0.153	0.769	0.266	14.634	28.210	0.276	9.674

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	243	43	275	86139	277	287	556
N.S.	1	1.00	0.84	0.15	0.95	296.01	0.95	0.99	1.91
time (sec)	N/A	0.140	0.067	0.813	0.272	2.481	1.685	0.291	9.198

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	184	32	207	41851	124	213	160
N.S.	1	1.00	0.84	0.15	0.95	191.10	0.57	0.97	0.73
time (sec)	N/A	0.109	0.044	0.795	0.300	1.227	0.442	0.286	9.161

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	134	27	169	112	20	179	33
N.S.	1	1.00	0.72	0.15	0.91	0.61	0.11	0.97	0.18
time (sec)	N/A	0.067	0.012	0.797	0.273	0.267	0.076	0.267	0.048

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	404	211	345	352864	0	377	874
N.S.	1	1.00	0.97	0.51	0.83	848.23	0.00	0.91	2.10
time (sec)	N/A	0.292	0.102	0.892	0.272	97.112	0.000	0.293	9.348

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	552	552	524	354	561	0	0	668	2436
N.S.	1	1.00	0.95	0.64	1.02	0.00	0.00	1.21	4.41
time (sec)	N/A	0.556	0.400	0.869	0.281	0.000	0.000	0.550	9.345

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	680	680	738	446	817	0	0	941	1955
N.S.	1	1.00	1.09	0.66	1.20	0.00	0.00	1.38	2.88
time (sec)	N/A	0.634	0.580	0.968	0.287	0.000	0.000	0.367	9.892

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	347	100	332	91191	0	345	670
N.S.	1	1.00	0.99	0.29	0.95	261.29	0.00	0.99	1.92
time (sec)	N/A	0.201	0.251	0.849	0.271	12.487	0.000	0.278	0.261

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	321	90	318	90963	318	325	391
N.S.	1	1.00	1.00	0.28	0.99	282.49	0.99	1.01	1.21
time (sec)	N/A	0.184	0.219	0.819	0.268	2.742	3.727	0.304	9.255

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	224	66	238	43065	155	238	282
N.S.	1	1.00	0.93	0.27	0.99	178.69	0.64	0.99	1.17
time (sec)	N/A	0.140	0.137	0.866	0.272	1.356	0.610	0.300	0.157

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	183	46	189	183	39	194	58
N.S.	1	1.00	0.91	0.23	0.94	0.91	0.19	0.96	0.29
time (sec)	N/A	0.088	0.074	0.762	0.272	0.259	0.139	0.276	0.051

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	855	855	558	430	601	0	0	795	1591
N.S.	1	1.00	0.65	0.50	0.70	0.00	0.00	0.93	1.86
time (sec)	N/A	0.591	0.261	0.938	0.280	0.000	0.000	0.459	9.659

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1141	1141	807	534	961	0	0	1145	2246
N.S.	1	1.00	0.71	0.47	0.84	0.00	0.00	1.00	1.97
time (sec)	N/A	1.144	0.486	1.005	0.283	0.000	0.000	6.610	10.589

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1384	1384	996	680	1394	0	0	1557	3256
N.S.	1	1.00	0.72	0.49	1.01	0.00	0.00	1.12	2.35
time (sec)	N/A	1.302	0.732	1.044	0.317	0.000	0.000	0.387	10.849

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	388	141	392	95566	413	392	721
N.S.	1	1.00	0.98	0.36	0.99	242.55	1.05	0.99	1.83
time (sec)	N/A	0.240	0.249	0.810	0.282	13.227	131.883	0.319	0.275

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	358	126	364	91420	374	358	676
N.S.	1	1.00	0.99	0.35	1.01	253.94	1.04	0.99	1.88
time (sec)	N/A	0.221	0.225	0.838	0.275	7.152	14.652	0.282	0.268

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	249	86	269	43180	192	256	315
N.S.	1	1.00	0.94	0.32	1.01	162.33	0.72	0.96	1.18
time (sec)	N/A	0.164	0.135	0.841	0.291	2.106	1.193	0.280	0.175

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	200	57	212	255	63	204	80
N.S.	1	1.00	0.91	0.26	0.97	1.16	0.29	0.93	0.37
time (sec)	N/A	0.092	0.058	0.802	0.267	0.316	0.248	0.274	0.053

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1352	1352	835	677	1015	0	0	1311	2720
N.S.	1	1.00	0.62	0.50	0.75	0.00	0.00	0.97	2.01
time (sec)	N/A	0.948	0.448	1.019	0.310	0.000	0.000	0.345	10.551

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1830	1830	1115	829	1564	0	0	1809	3572
N.S.	1	1.00	0.61	0.45	0.85	0.00	0.00	0.99	1.95
time (sec)	N/A	1.874	0.804	1.100	0.338	0.000	0.000	46.908	11.176

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2204	2204	1338	1008	2198	0	0	2200	6280
N.S.	1	1.00	0.61	0.46	1.00	0.00	0.00	1.00	2.85
time (sec)	N/A	2.132	1.474	1.210	0.336	0.000	0.000	0.436	12.674

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	33	29	28	28	34	28	30
N.S.	1	1.00	1.03	0.91	0.88	0.88	1.06	0.88	0.94
time (sec)	N/A	0.016	0.007	1.277	0.280	0.319	0.051	0.277	0.025

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	33	29	28	28	34	28	30
N.S.	1	1.00	1.03	0.91	0.88	0.88	1.06	0.88	0.94
time (sec)	N/A	0.025	0.005	0.743	0.276	0.274	0.057	0.287	0.020

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	27	26	26	36	26	30
N.S.	1	1.00	0.97	0.84	0.81	0.81	1.12	0.81	0.94
time (sec)	N/A	0.011	0.008	1.253	0.296	0.297	0.052	0.274	9.175

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	27	26	26	36	26	30
N.S.	1	1.00	0.97	0.84	0.81	0.81	1.12	0.81	0.94
time (sec)	N/A	0.022	0.004	0.868	0.273	0.268	0.056	0.336	0.020

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	29	35	45	39	44	34
N.S.	1	1.00	0.93	0.64	0.78	1.00	0.87	0.98	0.76
time (sec)	N/A	0.010	0.015	0.887	0.282	0.307	0.048	0.280	0.084

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	29	35	45	39	44	34
N.S.	1	1.00	0.93	0.64	0.78	1.00	0.87	0.98	0.76
time (sec)	N/A	0.014	0.004	0.031	0.271	0.335	0.055	0.284	0.026

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	23	7	17	17	15	15	6
N.S.	1	1.00	3.83	1.17	2.83	2.83	2.50	2.50	1.00
time (sec)	N/A	0.002	0.003	0.770	0.175	0.334	0.046	0.297	9.091

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	23	14	17	17	15	19	6
N.S.	1	1.00	1.10	0.67	0.81	0.81	0.71	0.90	0.29
time (sec)	N/A	0.002	0.002	0.798	0.179	0.347	0.043	0.303	0.083

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	9	24	22	9	9
N.S.	1	1.00	0.85	0.77	0.69	1.85	1.69	0.69	0.69
time (sec)	N/A	0.001	0.002	0.797	0.188	0.258	0.044	0.267	9.355

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	81	11	13	57	24	22	57	9
N.S.	1	6.23	0.85	1.00	4.38	1.85	1.69	4.38	0.69
time (sec)	N/A	0.007	0.004	0.941	0.186	0.252	0.137	0.275	0.015

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	78	67	66	66	85	68	94
N.S.	1	1.00	1.13	0.97	0.96	0.96	1.23	0.99	1.36
time (sec)	N/A	0.081	0.012	0.826	0.261	0.259	0.126	0.296	0.057

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	78	63	66	66	85	68	94
N.S.	1	1.00	1.13	0.91	0.96	0.96	1.23	0.99	1.36
time (sec)	N/A	0.090	0.004	0.058	0.257	0.264	0.129	0.287	0.021

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	31	21	20	20	20	21	20
N.S.	1	1.00	1.29	0.88	0.83	0.83	0.83	0.88	0.83
time (sec)	N/A	0.014	0.005	0.783	0.179	0.268	0.031	0.275	0.022

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	21	20	20	19	21	20
N.S.	1	1.00	0.96	0.81	0.77	0.77	0.73	0.81	0.77
time (sec)	N/A	0.011	0.004	0.786	0.184	0.270	0.029	0.290	0.019

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	22	16	15	18	15	15	17
N.S.	1	1.00	1.29	0.94	0.88	1.06	0.88	0.88	1.00
time (sec)	N/A	0.004	0.000	0.014	0.183	0.235	0.018	0.266	0.014

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	16	20	18	15	20	17
N.S.	1	1.00	0.92	0.67	0.83	0.75	0.62	0.83	0.71
time (sec)	N/A	0.003	0.001	0.020	0.178	0.222	0.019	0.279	0.012

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	24	23	22	25	20	23	28
N.S.	1	1.00	1.09	1.05	1.00	1.14	0.91	1.05	1.27
time (sec)	N/A	0.013	0.005	0.773	0.273	0.262	0.064	0.295	0.025

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	27	16	15	15	15	18	15
N.S.	1	1.00	1.59	0.94	0.88	0.88	0.88	1.06	0.88
time (sec)	N/A	0.009	0.004	0.806	0.193	0.250	0.059	0.294	9.175

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	13	12	16	13
N.S.	1	1.00	1.00	0.74	0.68	0.68	0.63	0.84	0.68
time (sec)	N/A	0.014	0.005	0.806	0.192	0.253	0.040	0.284	0.045

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	17	11	10	10	8	13	10
N.S.	1	1.00	1.42	0.92	0.83	0.83	0.67	1.08	0.83
time (sec)	N/A	0.005	0.006	0.814	0.182	0.255	0.036	0.274	0.028

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	11	11	10	10	8	11	10
N.S.	1	1.00	1.10	1.10	1.00	1.00	0.80	1.10	1.00
time (sec)	N/A	0.006	0.006	0.791	0.188	0.257	0.063	0.280	0.033

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	18	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	1.06	0.88
time (sec)	N/A	0.012	0.005	0.810	0.194	0.251	0.058	0.279	9.603

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	18	21
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.78	0.91
time (sec)	N/A	0.013	0.005	0.798	0.261	0.256	0.061	0.269	9.448

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.87	0.87
time (sec)	N/A	0.006	0.004	0.031	0.186	0.236	0.038	0.271	0.025

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	20	20	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.95	0.95	0.81
time (sec)	N/A	0.014	0.006	0.043	0.181	0.240	0.062	0.271	9.553

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	9	11	9	9	7	9	10
N.S.	1	1.00	0.90	1.10	0.90	0.90	0.70	0.90	1.00
time (sec)	N/A	0.007	0.005	0.033	0.179	0.238	0.032	0.276	0.026

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	17	17	20	20	17
N.S.	1	1.00	1.00	0.72	0.68	0.68	0.80	0.80	0.68
time (sec)	N/A	0.015	0.006	0.041	0.184	0.235	0.062	0.290	0.070

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	12	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.00	0.86	1.00	1.00
time (sec)	N/A	0.016	0.004	0.806	0.264	0.229	0.047	0.299	0.019

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	14	13	11	16	17	11	20
N.S.	1	1.00	1.08	1.00	0.85	1.23	1.31	0.85	1.54
time (sec)	N/A	0.005	0.006	0.829	0.185	0.230	0.055	0.280	9.884

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	26	26	25	20	27	25
N.S.	1	1.00	0.96	1.00	1.00	0.96	0.77	1.04	0.96
time (sec)	N/A	0.028	0.006	0.812	0.184	0.252	0.082	0.275	9.334

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	4	5	4	4	3	5	4
N.S.	1	1.00	0.67	0.83	0.67	0.67	0.50	0.83	0.67
time (sec)	N/A	0.005	0.001	0.023	0.191	0.247	0.020	0.310	0.010

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	20	25	19	27	18
N.S.	1	1.00	1.00	0.95	1.00	1.25	0.95	1.35	0.90
time (sec)	N/A	0.009	0.005	0.810	0.197	0.271	0.045	0.289	0.022

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	40	28	27	27	31	28	44
N.S.	1	1.00	1.05	0.74	0.71	0.71	0.82	0.74	1.16
time (sec)	N/A	0.016	0.008	0.770	0.280	0.252	0.060	0.292	0.095

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	18	16	15	15	14	16	15
N.S.	1	1.00	1.06	0.94	0.88	0.88	0.82	0.94	0.88
time (sec)	N/A	0.006	0.003	0.808	0.190	0.234	0.029	0.263	0.018

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	36	22	21	21	22	22	25
N.S.	1	1.00	1.16	0.71	0.68	0.68	0.71	0.71	0.81
time (sec)	N/A	0.007	0.005	0.835	0.277	0.252	0.066	0.297	9.316

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	17	15	17	17	13
N.S.	1	1.00	1.00	0.74	0.89	0.79	0.89	0.89	0.68
time (sec)	N/A	0.010	0.006	0.835	0.192	0.244	0.050	0.291	0.061

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	9	9	9	8	11	9
N.S.	1	1.00	0.82	0.82	0.82	0.82	0.73	1.00	0.82
time (sec)	N/A	0.004	0.003	0.796	0.203	0.252	0.035	0.303	9.010

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	12	12	14	12	12
N.S.	1	1.00	1.00	0.72	0.67	0.67	0.78	0.67	0.67
time (sec)	N/A	0.009	0.005	0.078	0.281	0.260	0.070	0.293	8.844

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	39	46	83	46	52	45
N.S.	1	1.00	0.96	0.85	1.00	1.80	1.00	1.13	0.98
time (sec)	N/A	0.015	0.012	0.779	0.195	0.257	0.094	0.274	8.875

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	10	9	8	8	7	9	8
N.S.	1	1.00	0.83	0.75	0.67	0.67	0.58	0.75	0.67
time (sec)	N/A	0.002	0.002	0.793	0.190	0.237	0.032	0.304	9.268

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	27	23	23	34	20	25	17
N.S.	1	1.00	1.29	1.10	1.10	1.62	0.95	1.19	0.81
time (sec)	N/A	0.002	0.006	0.812	0.196	0.268	0.049	0.309	0.017

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	21	12	15	17
N.S.	1	1.00	1.00	0.84	0.79	1.11	0.63	0.79	0.89
time (sec)	N/A	0.002	0.006	0.839	0.285	0.256	0.052	0.268	0.016

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	8	7	9	6
N.S.	1	1.00	1.00	0.70	0.80	0.80	0.70	0.90	0.60
time (sec)	N/A	0.001	0.002	0.795	0.189	0.248	0.018	0.294	0.041

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	20	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	2.00	1.00	1.00
time (sec)	N/A	0.002	0.002	0.842	0.282	0.239	0.052	0.287	0.023

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.003	0.003	0.802	0.268	0.265	0.060	0.284	8.861

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	16
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84
time (sec)	N/A	0.008	0.004	1.238	0.275	0.241	0.049	0.275	8.792

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	17	16	17
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.77	0.73	0.77
time (sec)	N/A	0.004	0.001	0.823	0.188	0.232	0.016	0.281	0.019

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	15	16	17
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.68	0.73	0.77
time (sec)	N/A	0.003	0.001	0.806	0.187	0.239	0.016	0.292	0.017

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	10	14	15
N.S.	1	1.00	1.00	0.94	0.88	0.94	0.62	0.88	0.94
time (sec)	N/A	0.003	0.001	0.025	0.200	0.252	0.026	0.262	0.015

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	30	39	30	30
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.05	0.81	0.81
time (sec)	N/A	0.014	0.008	1.357	0.273	0.263	0.053	0.275	0.026

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	12	13
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.93
time (sec)	N/A	0.003	0.000	0.790	0.196	0.240	0.017	0.272	0.013

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	7	9	9	7	9	6
N.S.	1	1.00	1.00	0.64	0.82	0.82	0.64	0.82	0.55
time (sec)	N/A	0.002	0.000	0.773	0.186	0.243	0.018	0.271	0.010

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	20	20	25
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.80	0.80	1.00
time (sec)	N/A	0.006	0.007	0.807	0.280	0.263	0.067	0.286	9.095

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	20	25
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00
time (sec)	N/A	0.006	0.004	0.795	0.271	0.282	0.061	0.316	0.025

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	20	25
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00
time (sec)	N/A	0.011	0.005	0.811	0.276	0.248	0.055	0.287	8.869

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	7	7	8	7	12	5	8	7
N.S.	1	0.78	0.78	0.89	0.78	1.33	0.56	0.89	0.78
time (sec)	N/A	0.004	0.003	0.793	0.214	0.245	0.032	0.300	0.012

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.007	0.010	0.072	0.279	0.244	0.071	0.291	0.033

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	30	44	30	30
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.19	0.81	0.81
time (sec)	N/A	0.015	0.011	1.368	0.284	0.235	0.058	0.287	0.027

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	15	19	14	16	13
N.S.	1	1.00	1.00	0.93	1.00	1.27	0.93	1.07	0.87
time (sec)	N/A	0.003	0.002	0.031	0.204	0.241	0.037	0.276	8.823

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	15	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	1.00	0.67	0.93	0.87
time (sec)	N/A	0.002	0.001	0.028	0.191	0.235	0.030	0.269	0.013

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	13	11	8	13	11
N.S.	1	1.00	1.00	1.09	1.18	1.00	0.73	1.18	1.00
time (sec)	N/A	0.008	0.003	0.838	0.197	0.250	0.040	0.270	0.033

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	19	18	17	17	17	17	17
N.S.	1	1.00	0.86	0.82	0.77	0.77	0.77	0.77	0.77
time (sec)	N/A	0.003	0.001	0.062	0.190	0.231	0.020	0.283	0.017

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	19	19	19	9	9
N.S.	1	1.00	1.00	0.91	1.73	1.73	1.73	0.82	0.82
time (sec)	N/A	0.001	0.001	0.800	0.185	0.234	0.018	0.261	0.081

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	16	12	13	13	12	14	11
N.S.	1	1.00	1.23	0.92	1.00	1.00	0.92	1.08	0.85
time (sec)	N/A	0.006	0.003	0.811	0.202	0.259	0.029	0.297	0.020

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	18	14	16	12
N.S.	1	1.00	1.00	0.94	0.88	1.12	0.88	1.00	0.75
time (sec)	N/A	0.004	0.003	0.827	0.192	0.267	0.043	0.282	0.021

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	25	39	24	27	23
N.S.	1	1.00	0.96	0.89	0.93	1.44	0.89	1.00	0.85
time (sec)	N/A	0.007	0.012	0.042	0.195	0.252	0.048	0.284	8.809

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	18	16	15	15	12	16	15
N.S.	1	1.00	1.06	0.94	0.88	0.88	0.71	0.94	0.88
time (sec)	N/A	0.006	0.003	0.816	0.186	0.276	0.029	0.267	0.015

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	20	15	17	16
N.S.	1	1.00	1.00	0.94	0.89	1.11	0.83	0.94	0.89
time (sec)	N/A	0.004	0.001	0.032	0.188	0.249	0.029	0.264	0.015

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	14	14	13
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.78	0.78	0.72
time (sec)	N/A	0.007	0.001	0.049	0.266	0.257	0.018	0.291	0.013

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	34	46	31	25	23
N.S.	1	1.00	1.00	0.96	1.48	2.00	1.35	1.09	1.00
time (sec)	N/A	0.011	0.007	0.789	0.192	0.275	0.048	0.277	0.019

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	12	13	12	15	10	32	12
N.S.	1	1.00	0.75	0.81	0.75	0.94	0.62	2.00	0.75
time (sec)	N/A	0.008	0.007	0.851	0.270	0.288	0.052	0.271	9.022

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	30	26	25	25	24	26	25
N.S.	1	1.00	1.03	0.90	0.86	0.86	0.83	0.90	0.86
time (sec)	N/A	0.013	0.007	0.804	0.192	0.249	0.030	0.281	0.015

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	12	13
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.81
time (sec)	N/A	0.004	0.000	0.793	0.197	0.243	0.020	0.279	0.012

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	13	17	25	14	12	17
N.S.	1	1.00	1.00	0.76	1.00	1.47	0.82	0.71	1.00
time (sec)	N/A	0.009	0.010	0.859	0.279	0.251	0.065	0.280	0.018

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	43	0	190	151	46	42
N.S.	1	1.00	1.00	0.91	0.00	4.04	3.21	0.98	0.89
time (sec)	N/A	0.042	0.019	1.286	0.000	0.272	0.163	0.280	0.055

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	61	61	0	240	294	60	82
N.S.	1	1.00	1.07	1.07	0.00	4.21	5.16	1.05	1.44
time (sec)	N/A	0.051	0.018	1.307	0.000	0.264	0.184	0.274	0.037

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	39	36	0	53	24	147	101
N.S.	1	1.00	0.21	0.19	0.00	0.28	0.13	0.78	0.54
time (sec)	N/A	0.126	0.032	0.108	0.000	0.288	0.304	0.653	0.057

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	27	28	65	65	60	30	27
N.S.	1	1.00	0.45	0.47	1.08	1.08	1.00	0.50	0.45
time (sec)	N/A	0.087	0.010	0.054	0.195	0.278	0.090	0.290	8.996

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	B	B	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	0	27	28	65	65	60	197	27
N.S.	1	0.00	1.00	1.04	2.41	2.41	2.22	7.30	1.00
time (sec)	N/A	0.000	0.006	0.110	0.232	0.273	0.149	0.285	0.025

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	B	B	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	0	27	28	65	65	60	111	27
N.S.	1	0.00	1.00	1.04	2.41	2.41	2.22	4.11	1.00
time (sec)	N/A	0.000	0.005	0.089	0.192	0.267	0.112	0.309	0.023

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [412] had the largest ratio of [.8823999999999999962]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	24	0.083
2	A	3	3	1.00	29	0.103
3	A	2	2	1.00	29	0.069
4	A	2	2	1.00	29	0.069
5	A	1	0	1.00	27	0.000
6	A	2	2	1.00	29	0.069
7	A	2	2	1.00	29	0.069
8	A	2	2	1.00	29	0.069
9	A	3	2	1.00	27	0.074
10	A	3	2	1.00	27	0.074
11	A	1	0	1.00	25	0.000
12	A	7	7	1.00	27	0.259
13	A	8	8	1.00	27	0.296
14	A	9	8	1.00	27	0.296
15	A	3	2	1.00	46	0.043
16	A	3	2	1.00	46	0.043
17	A	1	0	1.00	44	0.000
18	A	2	1	1.00	46	0.022
19	A	2	1	1.00	46	0.022
20	A	2	1	1.00	46	0.022
21	A	5	4	1.00	11	0.364
22	A	5	4	1.00	17	0.235
23	A	2	2	1.00	7	0.286
24	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	5	5	1.00	11	0.454
26	A	7	7	1.00	16	0.438
27	A	6	6	1.00	9	0.667
28	A	2	2	1.00	7	0.286
29	A	3	3	1.00	13	0.231
30	A	3	3	1.11	11	0.273
31	A	3	3	1.00	16	0.188
32	A	2	2	1.26	9	0.222
33	A	3	2	1.00	29	0.069
34	A	2	1	1.00	29	0.034
35	A	2	1	1.00	29	0.034
36	A	1	0	1.00	27	0.000
37	A	10	6	1.00	29	0.207
38	A	11	7	1.00	29	0.241
39	A	3	2	1.00	32	0.062
40	A	2	1	1.00	32	0.031
41	A	2	1	1.00	32	0.031
42	A	1	0	1.00	30	0.000
43	A	4	3	1.00	32	0.094
44	A	5	4	1.00	32	0.125
45	A	2	1	1.00	17	0.059
46	A	2	1	1.00	17	0.059
47	A	2	1	1.00	17	0.059
48	A	1	0	1.00	15	0.000
49	A	16	9	1.00	17	0.529
50	A	18	11	1.00	17	0.647
51	A	2	1	1.00	17	0.059
52	A	2	1	1.00	17	0.059
53	A	2	1	1.00	17	0.059
54	A	1	0	1.00	15	0.000
55	A	15	9	1.00	17	0.529
56	A	17	11	1.00	17	0.647
57	A	2	1	1.00	22	0.045
58	A	2	1	1.00	22	0.045
59	A	2	1	1.00	22	0.045

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	1	0	1.00	20	0.000
61	A	16	9	1.00	22	0.409
62	A	18	11	1.00	22	0.500
63	A	2	2	1.00	51	0.039
64	A	2	2	1.00	51	0.039
65	B	1	0	4.36	49	0.000
66	A	2	2	1.00	51	0.039
67	A	2	2	1.00	51	0.039
68	A	2	2	1.00	51	0.039
69	A	6	5	1.00	13	0.385
70	A	5	3	1.00	19	0.158
71	A	5	3	1.00	19	0.158
72	A	5	3	1.00	19	0.158
73	A	1	0	1.00	17	0.000
74	A	5	2	1.00	19	0.105
75	A	7	3	1.00	19	0.158
76	A	10	3	1.00	19	0.158
77	B	15	7	2.25	17	0.412
78	A	6	5	1.00	15	0.333
79	A	6	5	1.00	15	0.333
80	A	4	4	1.00	13	0.308
81	A	2	2	1.00	11	0.182
82	A	6	6	1.00	15	0.400
83	A	7	6	1.00	15	0.400
84	A	7	6	1.00	15	0.400
85	A	2	2	1.00	13	0.154
86	A	3	3	1.00	13	0.231
87	A	4	3	1.00	13	0.231
88	A	2	2	1.00	19	0.105
89	A	2	2	1.00	11	0.182
90	A	3	3	1.00	11	0.273
91	A	4	3	1.00	11	0.273
92	A	2	2	1.00	13	0.154
93	A	3	3	1.00	13	0.231
94	A	4	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	2	2	1.00	11	0.182
96	A	3	3	1.00	11	0.273
97	A	4	3	1.00	11	0.273
98	A	3	2	1.00	15	0.133
99	A	4	3	1.00	13	0.231
100	A	4	4	1.00	17	0.235
101	A	4	4	1.00	19	0.210
102	A	4	4	1.00	17	0.235
103	A	11	10	1.00	17	0.588
104	A	9	9	1.00	17	0.529
105	A	7	7	1.00	15	0.467
106	A	7	7	1.00	13	0.538
107	A	11	10	1.06	17	0.588
108	A	11	10	0.99	17	0.588
109	A	11	10	1.00	17	0.588
110	A	16	12	1.00	17	0.706
111	A	14	10	1.00	17	0.588
112	A	14	10	1.00	15	0.667
113	A	10	7	1.00	13	0.538
114	A	18	13	1.00	17	0.765
115	A	18	13	1.00	17	0.765
116	A	3	2	1.00	22	0.091
117	A	2	1	1.00	22	0.045
118	A	2	1	1.00	22	0.045
119	A	1	0	1.00	20	0.000
120	A	4	3	1.00	22	0.136
121	A	5	4	1.00	22	0.182
122	A	6	5	1.00	22	0.227
123	A	2	1	1.00	24	0.042
124	A	2	1	1.00	24	0.042
125	A	2	1	1.00	24	0.042
126	A	2	1	1.00	22	0.045
127	A	8	7	1.00	24	0.292
128	A	10	9	1.00	24	0.375
129	A	12	10	1.00	24	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	2	1	1.00	26	0.038
131	A	2	1	1.00	26	0.038
132	A	2	1	1.00	26	0.038
133	A	2	1	1.00	24	0.042
134	A	9	8	1.00	26	0.308
135	A	11	10	1.00	26	0.385
136	A	14	5	1.00	46	0.109
137	A	14	5	1.00	46	0.109
138	A	8	3	1.00	46	0.065
139	A	14	5	1.00	44	0.114
140	A	14	5	1.00	42	0.119
141	A	14	5	1.00	46	0.109
142	A	14	5	0.99	46	0.109
143	A	14	6	1.00	26	0.231
144	A	14	6	1.00	26	0.231
145	A	14	6	1.00	26	0.231
146	A	8	4	1.00	26	0.154
147	A	14	6	1.00	24	0.250
148	A	14	6	1.00	22	0.273
149	A	14	6	1.00	26	0.231
150	A	14	6	1.00	26	0.231
151	A	23	7	1.00	26	0.269
152	A	23	7	1.00	26	0.269
153	A	14	6	1.00	26	0.231
154	A	17	5	1.00	26	0.192
155	A	23	7	1.00	26	0.269
156	A	23	7	1.00	26	0.269
157	A	23	7	1.00	26	0.269
158	A	2	1	1.00	52	0.019
159	A	4	3	1.00	52	0.058
160	A	1	1	1.00	18	0.056
161	A	3	3	1.00	23	0.130
162	A	3	3	1.00	23	0.130
163	A	3	3	1.00	29	0.103
164	A	1	1	1.00	18	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	4	3	1.00	20	0.150
166	A	4	3	1.00	20	0.150
167	A	4	3	1.00	22	0.136
168	A	1	1	1.00	18	0.056
169	A	3	3	1.00	23	0.130
170	A	3	3	1.00	23	0.130
171	A	3	3	1.00	29	0.103
172	A	1	1	1.00	18	0.056
173	A	1	1	1.00	25	0.040
174	C	7	3	4.30	38	0.079
175	A	1	1	1.00	27	0.037
176	A	1	1	1.00	31	0.032
177	A	2	1	1.00	54	0.019
178	A	3	1	1.00	54	0.019
179	A	5	4	1.00	54	0.074
180	A	1	1	1.00	30	0.033
181	A	1	1	1.00	29	0.034
182	A	1	1	1.00	28	0.036
183	A	1	1	1.00	21	0.048
184	A	1	1	1.00	20	0.050
185	A	1	1	1.00	21	0.048
186	A	1	1	1.00	26	0.038
187	A	1	1	1.00	25	0.040
188	A	2	2	1.00	26	0.077
189	A	1	1	1.00	24	0.042
190	A	1	1	1.00	24	0.042
191	A	1	1	1.00	23	0.043
192	A	1	1	1.00	30	0.033
193	A	1	1	1.00	29	0.034
194	A	1	1	1.00	28	0.036
195	A	1	1	1.00	21	0.048
196	A	1	1	1.00	20	0.050
197	A	2	2	1.00	21	0.095
198	A	1	1	1.00	26	0.038
199	A	1	1	1.00	25	0.040

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	2	2	1.00	26	0.077
201	A	1	1	1.00	22	0.045
202	A	1	1	1.00	24	0.042
203	A	2	2	1.00	23	0.087
204	A	1	1	1.00	23	0.043
205	A	1	1	1.00	19	0.053
206	A	2	1	1.00	22	0.045
207	A	2	1	1.00	23	0.043
208	A	2	1	1.00	22	0.045
209	A	2	1	1.00	23	0.043
210	A	2	1	1.00	24	0.042
211	A	2	1	1.00	25	0.040
212	A	2	1	1.00	31	0.032
213	A	2	1	1.00	32	0.031
214	A	2	1	1.00	35	0.029
215	A	2	1	1.00	36	0.028
216	A	2	1	1.00	24	0.042
217	A	2	1	1.00	31	0.032
218	A	2	1	1.00	35	0.029
219	A	1	1	1.00	22	0.045
220	A	1	1	1.00	18	0.056
221	B	3	2	2.91	26	0.077
222	B	2	1	2.91	28	0.036
223	A	1	1	1.00	18	0.056
224	A	1	1	1.00	20	0.050
225	A	1	1	1.00	21	0.048
226	A	3	3	1.00	52	0.058
227	A	9	5	1.00	38	0.132
228	A	3	2	1.00	32	0.062
229	A	4	3	1.00	33	0.091
230	A	3	2	1.00	34	0.059
231	A	6	6	1.00	43	0.140
232	A	1	1	1.00	16	0.062
233	B	15	7	2.25	25	0.280
234	A	1	1	1.00	56	0.018

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	1	1	1.00	51	0.020
236	A	1	1	1.00	49	0.020
237	A	1	1	1.00	46	0.022
238	A	2	2	1.00	48	0.042
239	A	1	1	1.00	49	0.020
240	A	1	1	1.00	48	0.021
241	A	1	1	1.00	48	0.021
242	A	10	5	1.00	35	0.143
243	A	10	5	1.00	35	0.143
244	A	10	5	1.00	35	0.143
245	A	10	5	1.00	33	0.152
246	A	10	5	1.00	32	0.156
247	A	13	6	1.00	35	0.171
248	A	13	6	1.00	35	0.171
249	A	13	6	1.00	35	0.171
250	A	13	6	1.00	35	0.171
251	A	13	6	1.00	35	0.171
252	A	11	6	1.00	33	0.182
253	A	9	5	1.00	32	0.156
254	A	13	6	1.00	35	0.171
255	A	13	6	1.00	35	0.171
256	A	13	6	1.00	35	0.171
257	A	2	2	1.00	40	0.050
258	A	6	5	1.00	20	0.250
259	A	3	2	1.00	20	0.100
260	A	3	2	1.00	20	0.100
261	A	2	1	1.00	16	0.062
262	A	5	4	1.00	22	0.182
263	A	3	2	1.00	21	0.095
264	A	6	5	1.00	26	0.192
265	A	2	1	1.00	20	0.050
266	A	2	1	1.00	11	0.091
267	A	4	3	1.00	22	0.136
268	A	3	2	1.00	21	0.095
269	A	3	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	6	6	1.00	22	0.273
271	A	5	5	1.00	31	0.161
272	A	3	2	1.00	21	0.095
273	A	4	3	1.00	33	0.091
274	A	4	4	1.00	14	0.286
275	A	7	6	1.00	33	0.182
276	A	7	6	1.00	29	0.207
277	A	6	5	1.00	44	0.114
278	A	3	2	1.00	15	0.133
279	A	5	4	1.00	15	0.267
280	A	4	3	1.00	18	0.167
281	A	3	2	1.00	20	0.100
282	A	5	4	1.00	26	0.154
283	A	3	2	1.00	13	0.154
284	A	3	2	1.00	18	0.111
285	A	2	1	1.00	26	0.038
286	A	5	5	1.00	19	0.263
287	A	5	5	1.00	24	0.208
288	A	8	6	1.00	20	0.300
289	A	8	6	1.00	18	0.333
290	A	5	3	1.00	19	0.158
291	A	4	3	1.00	13	0.231
292	A	6	5	1.00	22	0.227
293	A	6	5	1.00	24	0.208
294	A	2	1	1.00	29	0.034
295	A	2	1	1.00	30	0.033
296	A	2	1	1.00	19	0.053
297	A	4	4	1.00	16	0.250
298	A	10	5	1.00	36	0.139
299	A	2	1	1.00	21	0.048
300	A	5	4	1.00	16	0.250
301	A	2	1	1.00	24	0.042
302	A	2	1	1.00	21	0.048
303	A	2	1	1.00	24	0.042
304	A	6	5	1.00	26	0.192

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	3	2	1.00	25	0.080
306	A	2	1	1.00	29	0.034
307	A	6	5	1.00	20	0.250
308	A	14	10	1.00	32	0.312
309	A	4	4	1.00	23	0.174
310	A	6	5	1.00	26	0.192
311	A	4	3	1.00	26	0.115
312	A	8	4	1.00	25	0.160
313	A	6	3	1.00	23	0.130
314	A	7	6	1.00	23	0.261
315	A	5	3	1.18	20	0.150
316	A	3	2	1.00	25	0.080
317	A	3	2	1.00	22	0.091
318	A	2	1	1.00	24	0.042
319	A	7	6	1.00	24	0.250
320	A	6	5	1.00	43	0.116
321	A	7	5	1.25	50	0.100
322	A	3	2	1.00	16	0.125
323	A	6	5	1.00	15	0.333
324	A	5	4	1.00	20	0.200
325	A	3	2	1.00	24	0.083
326	A	5	3	1.00	27	0.111
327	A	5	5	1.00	26	0.192
328	A	3	2	1.00	16	0.125
329	A	11	8	1.32	22	0.364
330	A	5	4	1.00	24	0.167
331	A	4	3	1.00	26	0.115
332	A	5	3	1.00	36	0.083
333	A	4	3	1.00	26	0.115
334	A	10	9	1.00	20	0.450
335	A	6	6	1.00	27	0.222
336	A	7	7	1.00	20	0.350
337	A	8	7	1.00	25	0.280
338	A	8	7	1.00	22	0.318
339	A	2	1	1.00	18	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	5	4	1.00	20	0.200
341	A	10	9	1.00	20	0.450
342	A	16	12	1.00	20	0.600
343	A	3	2	1.00	14	0.143
344	A	2	2	1.00	20	0.100
345	A	14	8	1.00	20	0.400
346	A	4	3	1.00	26	0.115
347	A	4	3	1.00	24	0.125
348	A	6	4	1.00	30	0.133
349	A	4	4	1.00	21	0.190
350	A	2	1	1.00	15	0.067
351	A	4	3	1.00	18	0.167
352	A	3	2	1.00	22	0.091
353	A	3	2	1.00	16	0.125
354	A	6	5	1.00	16	0.312
355	A	3	2	1.00	25	0.080
356	A	3	2	1.00	19	0.105
357	A	2	1	1.00	23	0.043
358	A	5	4	1.00	23	0.174
359	A	5	4	1.00	21	0.190
360	A	10	6	1.00	28	0.214
361	A	2	1	1.00	24	0.042
362	A	6	5	1.00	26	0.192
363	A	2	1	1.00	14	0.071
364	A	5	3	1.00	16	0.188
365	A	5	5	1.00	16	0.312
366	A	7	5	1.00	43	0.116
367	A	17	13	1.00	26	0.500
368	A	18	13	1.00	16	0.812
369	A	3	2	1.00	15	0.133
370	A	3	2	1.00	15	0.133
371	A	3	2	1.00	17	0.118
372	A	3	2	1.00	15	0.133
373	A	4	3	1.00	15	0.200
374	A	4	3	1.00	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	3	2	1.00	20	0.100
376	A	7	6	1.00	29	0.207
377	A	4	3	1.00	22	0.136
378	A	6	4	1.00	16	0.250
379	A	6	4	1.00	16	0.250
380	A	5	4	1.00	14	0.286
381	A	4	3	1.00	12	0.250
382	A	4	3	1.00	16	0.188
383	A	6	5	1.00	16	0.312
384	A	4	3	1.00	16	0.188
385	A	4	3	1.00	16	0.188
386	A	4	3	1.00	16	0.188
387	A	8	5	1.00	17	0.294
388	A	8	5	1.00	19	0.263
389	A	8	5	1.00	15	0.333
390	A	8	5	1.00	17	0.294
391	A	16	10	1.00	23	0.435
392	A	17	10	1.00	21	0.476
393	F	0	0	N/A	0.000	N/A
394	A	15	11	1.00	17	0.647
395	A	13	9	1.00	17	0.529
396	A	13	9	1.00	15	0.600
397	A	9	6	1.00	9	0.667
398	A	17	12	1.00	17	0.706
399	A	17	12	1.00	17	0.706
400	A	17	12	1.00	17	0.706
401	A	16	12	1.00	17	0.706
402	A	14	10	1.00	17	0.588
403	A	14	10	1.00	15	0.667
404	A	10	7	1.00	9	0.778
405	A	31	14	1.00	17	0.824
406	A	31	14	1.00	17	0.824
407	A	31	14	1.00	17	0.824
408	A	15	11	1.00	17	0.647
409	A	15	10	1.00	17	0.588

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
410	A	15	10	1.00	15	0.667
411	A	11	7	1.00	9	0.778
412	A	46	15	1.00	17	0.882
413	A	46	15	1.00	17	0.882
414	A	46	15	1.00	17	0.882
415	A	4	4	1.00	14	0.286
416	A	5	5	1.00	13	0.385
417	A	4	4	1.00	16	0.250
418	A	5	5	1.00	18	0.278
419	A	3	2	1.00	14	0.143
420	A	4	3	1.00	16	0.188
421	A	2	2	1.00	11	0.182
422	A	1	0	1.00	17	0.000
423	A	1	1	1.00	11	0.091
424	B	1	0	6.23	73	0.000
425	A	11	7	1.00	13	0.538
426	A	13	9	1.00	19	0.474
427	A	3	2	1.00	15	0.133
428	A	3	2	1.00	11	0.182
429	A	1	0	1.00	11	0.000
430	A	1	0	1.00	11	0.000
431	A	5	4	1.00	16	0.250
432	A	2	1	1.00	16	0.062
433	A	4	3	1.00	15	0.200
434	A	1	1	1.00	15	0.067
435	A	1	1	1.00	20	0.050
436	A	3	2	1.00	15	0.133
437	A	6	5	1.00	13	0.385
438	A	1	1	1.00	22	0.045
439	A	3	2	1.00	18	0.111
440	A	3	3	1.00	20	0.150
441	A	3	2	1.00	16	0.125
442	A	6	6	1.00	17	0.353
443	A	1	1	1.00	17	0.059
444	A	4	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
445	A	2	2	1.00	20	0.100
446	A	3	2	1.00	18	0.111
447	A	6	5	1.00	15	0.333
448	A	2	1	1.00	11	0.091
449	A	5	5	1.00	13	0.385
450	A	3	2	1.00	20	0.100
451	A	2	1	1.00	16	0.062
452	A	4	3	1.00	16	0.188
453	A	2	1	1.00	16	0.062
454	A	1	1	1.00	9	0.111
455	A	2	2	1.00	7	0.286
456	A	2	2	1.00	11	0.182
457	A	1	1	1.00	7	0.143
458	A	1	1	1.00	9	0.111
459	A	1	1	1.00	9	0.111
460	A	2	2	1.00	10	0.200
461	A	2	1	1.00	13	0.077
462	A	2	1	1.00	11	0.091
463	A	2	1	1.00	14	0.071
464	A	4	4	1.00	16	0.250
465	A	2	1	1.00	7	0.143
466	A	2	1	1.00	11	0.091
467	A	5	5	1.00	13	0.385
468	A	5	5	1.00	13	0.385
469	A	5	4	1.00	14	0.286
470	A	2	1	0.78	13	0.077
471	A	3	2	1.00	20	0.100
472	A	4	4	1.00	18	0.222
473	A	2	1	1.00	12	0.083
474	A	2	1	1.00	10	0.100
475	A	3	2	1.00	16	0.125
476	A	2	1	1.00	11	0.091
477	A	1	1	1.00	7	0.143
478	A	2	1	1.00	15	0.067
479	A	2	1	1.00	12	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
480	A	3	3	1.00	16	0.188
481	A	2	1	1.00	11	0.091
482	A	2	1	1.00	17	0.059
483	A	2	1	1.00	29	0.034
484	A	2	1	1.00	18	0.056
485	A	3	2	1.00	14	0.143
486	A	2	1	1.00	24	0.042
487	A	2	1	1.00	11	0.091
488	A	3	2	1.00	18	0.111
489	A	3	3	1.00	15	0.200
490	A	3	3	1.00	16	0.188
491	A	10	7	1.00	15	0.467
492	A	5	2	1.00	50	0.040
493	F	0	0	N/A	0.000	N/A
494	F	0	0	N/A	0.000	N/A

CHAPTER 3

LISTING OF INTEGRALS

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3.293 $\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx$ . . . 1884
3.294 $\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx$ . . . 1888
3.295 $\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx$ . . . 1891
3.296 $\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx$ . . . 1894
3.297 $\int \frac{3+x^2+x^3}{(2+x^2)^2}\,dx$ . . . 1897
3.298 $\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx$ . . . 1901
3.299 $\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx$ . . . 1906
3.300 $\int \frac{x^4}{(-1+x)(2+x^2)}\,dx$ . . . 1909
3.301 $\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3}\,dx$ . . . 1913
3.302 $\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx$ . . . 1916
3.303 $\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx$ . . . 1919
3.304 $\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx$ . . . 1922
3.305 $\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx$ . . . 1927
3.306 $\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx$ . . . 1931
3.307 $\int \frac{4-x+2x^2}{4x+x^3}\,dx$ . . . 1934
3.308 $\int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)}\,dx$ . . . 1938
3.309 $\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2}\,dx$ . . . 1944
3.310 $\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2}\,dx$ . . . 1948
3.311 $\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx$ . . . 1952
3.312 $\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx$ . . . 1956
3.313 $\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx$ . . . 1960
3.314 $\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx$ . . . 1964
3.315 $\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx$ . . . 1969
3.316 $\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx$ . . . 1973
3.317 $\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx$ . . . 1977
3.318 $\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx$ . . . 1981
3.319 $\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx$ . . . 1984
3.320 $\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx$ . . . 1988

3.321 $\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx$ 1994
3.322 $\int \frac{9+x^4}{x^2(9+x^2)}\,dx$ 2000
3.323 $\int \frac{2x+x^4}{1+x^2}\,dx$ 2004
3.324 $\int \frac{-x+x^3}{(-1+x)^2(1+x^2)}\,dx$ 2008
3.325 $\int \frac{2+5x+3x^2+2x^3}{1+x+x^2}\,dx$ 2012
3.326 $\int \frac{3-4x-5x^2+3x^3}{x^3(-1+x+x^2)}\,dx$ 2016
3.327 $\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2}\,dx$ 2021
3.328 $\int \frac{(-1+x)^4x^4}{1+x^2}\,dx$ 2025
3.329 $\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx$ 2029
3.330 $\int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)}\,dx$ 2034
3.331 $\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3}\,dx$ 2038
3.332 $\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx$ 2042
3.333 $\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx$ 2046
3.334 $\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx$ 2050
3.335 $\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx$ 2060
3.336 $\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx$ 2064
3.337 $\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx$ 2069
3.338 $\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx$ 2077
3.339 $\int \frac{x^2}{(a+bx)(c+dx)}\,dx$ 2085
3.340 $\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx$ 2089
3.341 $\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx$ 2094
3.342 $\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx$ 2102
3.343 $\int \frac{x}{(1-x)(1+x)^2}\,dx$ 2111
3.344 $\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx$ 2115
3.345 $\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx$ 2119
3.346 $\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx$ 2125
3.347 $\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx$ 2129
3.348 $\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx$ 2133
3.349 $\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx$ 2137
3.350 $\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx$ 2141
3.351 $\int \frac{1+x+4x^2}{x+4x^3}\,dx$ 2145
3.352 $\int \frac{1-x+3x^2}{-x^2+x^3}\,dx$ 2149
3.353 $\int \frac{4+3x+x^2}{x+x^2}\,dx$ 2153
3.354 $\int \frac{4+x+3x^2}{x+x^3}\,dx$ 2157
3.355 $\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx$ 2161

3.356 $\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx$ 2165

3.357 $\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx$ 2169

3.358 $\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx$ 2173

3.359 $\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx$ 2177

3.360 $\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx$ 2181

3.361 $\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx$ 2186

3.362 $\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)}\,dx$ 2189

3.363 $\int \frac{-1+x^3}{1+x+x^2}\,dx$ 2194

3.364 $\int \frac{-3+x^3}{-7-6x+x^2}\,dx$ 2197

3.365 $\int \frac{1+x^3}{(13+4x+x^2)^2}\,dx$ 2201

3.366 $\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2}\,dx$ 2206

3.367 $\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx$ 2211

3.368 $\int \frac{1+x^3+x^6}{x+x^5}\,dx$ 2219

3.369 $\int \frac{1+x^2}{-x+x^2}\,dx$ 2226

3.370 $\int \frac{1+x^3}{-x+x^3}\,dx$ 2230

3.371 $\int \frac{1+x^3}{-x^2+x^3}\,dx$ 2234

3.372 $\int \frac{-1+x^5}{-x+x^3}\,dx$ 2238

3.373 $\int \frac{1+x^4}{x^3+x^5}\,dx$ 2242

3.374 $\int \frac{1+x^2}{x+2x^2+x^3}\,dx$ 2246

3.375 $\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx$ 2250

3.376 $\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx$ 2254

3.377 $\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx$ 2259

3.378 $\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx$ 2263

3.379 $\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx$ 2267

3.380 $\int \frac{x}{13+\frac{2}{x}+15x}\,dx$ 2271

3.381 $\int \frac{1}{13+\frac{2}{x}+15x}\,dx$ 2275

3.382 $\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx$ 2279

3.383 $\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx$ 2283

3.384 $\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx$ 2287

3.385 $\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx$ 2291

3.386 $\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx$ 2295

3.387 $\int \frac{x^2}{2-(1+x^2)^4}\,dx$ 2299

3.388 $\int \frac{x^2}{2-(1-x^2)^4}\,dx$ 2305

3.389 $\int \frac{x^2}{2+(1+x^2)^4}\,dx$ 2311

3.390 $\int \frac{x^2}{2+(1-x^2)^4}\,dx$ 2319

3.391 $\int \frac{1-x^2}{a+b(1-x^2)^4}\,dx$ . . . 2327

3.392 $\int \frac{1-x^2}{a+b(-1+x^2)^4}\,dx$ . . . 2336

3.393 $\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx$ . . . 2345

3.394 $\int \frac{(d+ex)^3}{a+cx^4}\,dx$ . . . 2350

3.395 $\int \frac{(d+ex)^2}{a+cx^4}\,dx$ . . . 2359

3.396 $\int \frac{d+ex}{a+cx^4}\,dx$ . . . 2367

3.397 $\int \frac{1}{a+cx^4}\,dx$ . . . 2374

3.398 $\int \frac{1}{(d+ex)(a+cx^4)}\,dx$ . . . 2380

3.399 $\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx$ . . . 2389

3.400 $\int \frac{1}{(d+ex)^3(a+cx^4)}\,dx$ . . . 2400

3.401 $\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx$ . . . 2412

3.402 $\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx$ . . . 2421

3.403 $\int \frac{d+ex}{(a+cx^4)^2}\,dx$ . . . 2430

3.404 $\int \frac{1}{(a+cx^4)^2}\,dx$ . . . 2438

3.405 $\int \frac{1}{(d+ex)(a+cx^4)^2}\,dx$ . . . 2445

3.406 $\int \frac{1}{(d+ex)^2(a+cx^4)^2}\,dx$ . . . 2460

3.407 $\int \frac{1}{(d+ex)^3(a+cx^4)^2}\,dx$ . . . 2478

3.408 $\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx$ . . . 2495

3.409 $\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx$ . . . 2505

3.410 $\int \frac{d+ex}{(a+cx^4)^3}\,dx$ . . . 2514

3.411 $\int \frac{1}{(a+cx^4)^3}\,dx$ . . . 2523

3.412 $\int \frac{1}{(d+ex)(a+cx^4)^3}\,dx$ . . . 2530

3.413 $\int \frac{1}{(d+ex)^2(a+cx^4)^3}\,dx$ . . . 2546

3.414 $\int \frac{1}{(d+ex)^3(a+cx^4)^3}\,dx$ . . . 2564

3.415 $\int \frac{-1+x}{1-x+x^2}\,dx$ . . . 2584

3.416 $\int \frac{-1+x^2}{1+x^3}\,dx$ . . . 2588

3.417 $\int \frac{-4+3x}{4-2x+x^2}\,dx$ . . . 2592

3.418 $\int \frac{-8+2x+3x^2}{8+x^3}\,dx$ . . . 2596

3.419 $\int \frac{2+x}{-1+2x+x^2}\,dx$ . . . 2600

3.420 $\int \frac{-4+x^2}{2-5x+x^3}\,dx$ . . . 2604

3.421 $\int \frac{2}{-1+4x^2}\,dx$ . . . 2608

3.422 $\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx$ . . . 2612

3.423 $\int \frac{x}{(1-x^2)^5}\,dx$ . . . 2615

3.424 $\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right)\,dx$ 26

3.425 $\int \frac{1+x^6}{-1+x^6}\,dx$ . . . 2623

3.426 $\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx$ 2629
3.427 $\int \frac{-x+x^3}{6+2x}\,dx$ 2635
3.428 $\int \frac{x+x^3}{-1+x}\,dx$ 2639
3.429 $\int (ac+(bc+d)x)\,dx$ 2643
3.430 $\int (dx+c(a+bx))\,dx$ 2646
3.431 $\int \frac{4+4x}{x^2(1+x^2)}\,dx$ 2649
3.432 $\int \frac{24+8x}{x(-4+x^2)}\,dx$ 2653
3.433 $\int \frac{-1+x^2}{-2x+x^3}\,dx$ 2656
3.434 $\int \frac{1+x^2}{3x+x^3}\,dx$ 2660
3.435 $\int \frac{a+3bx^2}{ax+bx^3}\,dx$ 2663
3.436 $\int \frac{-2+4x}{-x+x^3}\,dx$ 2666
3.437 $\int \frac{4+x}{4x+x^3}\,dx$ 2670
3.438 $\int \frac{-x+2x^3}{1-x^2+x^4}\,dx$ 2674
3.439 $\int \frac{-3+x}{2x+3x^2+x^3}\,dx$ 2677
3.440 $\int \frac{2+4x}{x^2+2x^3+x^4}\,dx$ 2681
3.441 $\int \frac{1+x}{-6x+x^2+x^3}\,dx$ 2685
3.442 $\int \frac{4x^2+x^3}{x+x^3}\,dx$ 2689
3.443 $\int \frac{x+2x^3}{(x^2+x^4)^3}\,dx$ 2693
3.444 $\int \frac{ax^2+bx^3}{cx^2+dx^3}\,dx$ 2696
3.445 $\int \frac{x+x^2}{-2x-x^2+x^3}\,dx$ 2700
3.446 $\int \frac{1-5x^2}{x^3(1+x^2)}\,dx$ 2703
3.447 $\int \frac{2x}{(-1+x)(5+x^2)}\,dx$ 2707
3.448 $\int \frac{2+x^2}{2+x}\,dx$ 2711
3.449 $\int \frac{1}{(-3+x)(4+x^2)}\,dx$ 2714
3.450 $\int \frac{-2+3x^6}{x(5+2x^6)}\,dx$ 2718
3.451 $\int \frac{3+2x}{(-2+x)(5+x)}\,dx$ 2722
3.452 $\int \frac{x^4}{4+5x^2+x^4}\,dx$ 2725
3.453 $\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx$ 2729
3.454 $\int \frac{x}{-1+x^2}\,dx$ 2733
3.455 $\int \frac{1}{(-1+x^2)^2}\,dx$ 2736
3.456 $\int \frac{x^2}{(1+x^2)^2}\,dx$ 2740
3.457 $\int \frac{1}{2+3x}\,dx$ 2744
3.458 $\int \frac{1}{a^2+x^2}\,dx$ 2747
3.459 $\int \frac{1}{a+bx^2}\,dx$ 2751
3.460 $\int \frac{1}{2-x+x^2}\,dx$ 2755
3.461 $\int x^2(4-x^2)^2\,dx$ 2759
3.462 $\int x(1-x^3)^2\,dx$ 2762
3.463 $\int \frac{-4+5x^2+x^3}{x^2}\,dx$ 2765

3.464 $\int \frac{-1+x}{3-4x+3x^2}\,dx$ . . . 2768
3.465 $\int \left(2+x^3\right)^2\,dx$ . . . 2772
3.466 $\int \frac{-4+x^2}{2+x}\,dx$ . . . 2775
3.467 $\int \frac{1}{(2+x)(1+x^2)}\,dx$ . . . 2778
3.468 $\int \frac{1}{(1+x)(1+x^2)}\,dx$ . . . 2782
3.469 $\int \frac{x}{(1+x)(1+x^2)}\,dx$ . . . 2786
3.470 $\int \frac{2x+x^2}{(1+x)^2}\,dx$ . . . 2790
3.471 $\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx$ . . . 2793
3.472 $\int \frac{31+5x}{11-4x+3x^2}\,dx$ . . . 2797
3.473 $\int \frac{-2+x^2+x^3}{x^4}\,dx$ . . . 2801
3.474 $\int \frac{1+x+x^3}{x^2}\,dx$ . . . 2804
3.475 $\int \frac{-2+x^2}{x(2+x^2)}\,dx$ . . . 2807
3.476 $\int (-3+x)\left(-7+4x^2\right)\,dx$ . . . 2811
3.477 $\int (-2+7x)^3\,dx$ . . . 2814
3.478 $\int \frac{-7+4x^2}{3+2x}\,dx$ . . . 2817
3.479 $\int \frac{1+x}{(-1+x)x^2}\,dx$ . . . 2820
3.480 $\int \frac{1}{4x^2+4x^3+x^4}\,dx$ . . . 2824
3.481 $\int \frac{1+x^2}{1+x}\,dx$ . . . 2828
3.482 $\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx$ . . . 2831
3.483 $\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx$ . . . 2834
3.484 $\int \frac{4+3x^2+2x^3}{(1+x)^4}\,dx$ . . . 2837
3.485 $\int \frac{x}{(1+x)^2(1+x^2)}\,dx$ . . . 2840
3.486 $\int \frac{7-2x+3x^2-x^3+x^4}{2+x}\,dx$ . . . 2844
3.487 $\int \frac{-1+x^3}{-1+x}\,dx$ . . . 2847
3.488 $\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx$ . . . 2850
3.489 $\int \frac{1}{bx+c(d+ex)^2}\,dx$ . . . 2854
3.490 $\int \frac{1}{a+bx+c(d+ex)^2}\,dx$ . . . 2858
3.491 $\int \frac{x^2}{1+(-1+x^2)^2}\,dx$ . . . 2862
3.492 $\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{\left(3+x+x^4\right)^4}\,dx$ . . . 2869
3.493 $\int \left(\frac{3\left(-47+228x+120x^2+19x^3\right)}{\left(3+x+x^4\right)^4}+\frac{42-320x-75x^2-8x^3}{\left(3+x+x^4\right)^3}+\frac{30x}{\left(3+x+x^4\right)^2}\right)\,dx$ . . . 2874
3.494 $\int \left(\frac{-3+10x+4x^3-30x^5}{\left(3+x+x^4\right)^3}-\frac{3\left(1+4x^3\right)\left(2-3x+5x^2+x^4-5x^6\right)}{\left(3+x+x^4\right)^4}\right)\,dx$ . . . 2879

3.1 $\int \frac{1}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx$

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Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{1}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx=\frac{1}{3\sqrt{3}\sqrt{b}\left(\sqrt{3}\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}x\right)}{27b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}x\right)}{27b}$$

```
[Out] -1/27*ln(-x*3^(1/2)+b^(1/2))/b+1/27*ln(x*3^(1/2)+2*b^(1/2))/b+1/9*3^(1/2)/b
^(1/2)/(-3*x+3^(1/2)*b^(1/2))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2088, 46}

$$\int \frac{1}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx=\frac{1}{3\sqrt{3}\sqrt{b}\left(\sqrt{3}\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}x\right)}{27b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}x\right)}{27b}$$

```
[In] Int[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1),x]
```

```
[Out] 1/(3*Sqrt[3]*Sqrt[b]*(Sqrt[3]*Sqrt[b] - 3*x)) - Log[Sqrt[b] - Sqrt[3]*x]/(2
7*b) + Log[2*Sqrt[b] + Sqrt[3]*x]/(27*b)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rule 2088

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[1/(3^(3*p)*
a^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d},
 x] && EqQ[4*b^3 + 27*a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (324b^3)\int \frac{1}{\left(6\sqrt{3}b^{3/2}-18bx\right)^2\left(6\sqrt{3}b^{3/2}+9bx\right)}\,dx \\
&= (324b^3)\int \left(\frac{1}{324\sqrt{3}b^{7/2}\left(\sqrt{3}\sqrt{b}-3x\right)^2}+\frac{1}{2916b^4\left(\sqrt{3}\sqrt{b}-3x\right)}+\frac{1}{2916b^4\left(2\sqrt{3}\sqrt{b}+3x\right)}\right)dx \\
&= \frac{1}{3\sqrt{3}\sqrt{b}\left(\sqrt{3}\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}x\right)}{27b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}x\right)}{27b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.86

$$\int \frac{1}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx = -\frac{\left(-\sqrt{3}\sqrt{b}+3x\right)\left(2\sqrt{3}\sqrt{b}+3x\right)\left(3\sqrt{3}\sqrt{b}+\left(-\sqrt{3}\sqrt{b}+3x\right)\log\left(-\sqrt{3}\sqrt{b}+3x\right)+\left(\sqrt{3}\sqrt{b}-3x\right)\log\left(2\sqrt{3}\sqrt{b}+3x\right)\right)}{81b\left(2\sqrt{3}b^{3/2}-9bx+9x^3\right)}$$

```
[In] Integrate[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1),x]
```

```
[Out] -1/81*((-(Sqrt[3]*Sqrt[b]) + 3*x)*(2*Sqrt[3]*Sqrt[b] + 3*x)*(3*Sqrt[3]*Sqrt
[b] + (-(Sqrt[3]*Sqrt[b]) + 3*x)*Log[-(Sqrt[3]*Sqrt[b]) + 3*x] + (Sqrt[3]*S
qrt[b] - 3*x)*Log[2*Sqrt[3]*Sqrt[b] + 3*x]))/(b*(2*Sqrt[3]*b^(3/2) - 9*b*x
+ 9*x^3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(-9b\_Z+9\_Z^{3}+2b^{\frac{3}{2}}\sqrt{3}\right)}\frac{\ln\left(x-\_R\right)}{3\_R^{2}-b}}{9}$	43

```
[In] int(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/9*sum(1/(3*_R^2-b)*ln(x-_R),_R=RootOf(-9*b*_Z+9*_Z^3+2*b^(3/2)*3^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{1}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx = -\frac{3\sqrt{3}\sqrt{b}x-(3x^2-b)\log\left(2\sqrt{3}\sqrt{b}+3x\right)+(3x^2-b)\log\left(-\sqrt{3}\sqrt{b}+3x\right)+3b}{27(3bx^2-b^2)}$$

```
[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="fricas")
[Out] -1/27*(3*sqrt(3)*sqrt(b)*x - (3*x^2 - b)*log(2*sqrt(3)*sqrt(b) + 3*x) + (3*
x^2 - b)*log(-sqrt(3)*sqrt(b) + 3*x) + 3*b)/(3*b*x^2 - b^2)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

$$\int \frac{1}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx = -\frac{3\sqrt{3}}{81\sqrt{b}x-27\sqrt{3}b}+\frac{-\frac{\log\left(-\frac{\sqrt{3}\sqrt{b}}{3}+x\right)}{27}+\frac{\log\left(\frac{2\sqrt{3}\sqrt{b}}{3}+x\right)}{27}}{b}$$

```
[In] integrate(1/(-9*b*x+9*x**3+2*b**(3/2)*3**(1/2)),x)
[Out] -3*sqrt(3)/(81*sqrt(b)*x - 27*sqrt(3)*b) + (-log(-sqrt(3)*sqrt(b)/3 + x)/27
 + log(2*sqrt(3)*sqrt(b)/3 + x)/27)/b
```

Maxima [F]

$$\int \frac{1}{2\sqrt{3}b^{3/2} - 9bx + 9x^3} \, dx = \int \frac{1}{9\,x^3 + 2\,\sqrt{3}b^{\frac{3}{2}} - 9\,bx} \, dx$$

```
[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(9*x^3 + 2*sqrt(3)*b^(3/2) - 9*b*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{1}{2\sqrt{3}b^{3/2} - 9bx + 9x^3} \, dx = \frac{\log\left(\left|9\,\sqrt{3}x + 18\,\sqrt{b}\right|\right)}{27\,b} - \frac{\log\left(\left|-\sqrt{3}x + \sqrt{b}\right|\right)}{27\,b} - \frac{1}{9\,\left(\sqrt{3}x - \sqrt{b}\right)\sqrt{b}}$$

```
[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="giac")
[Out] 1/27*log(abs(9*sqrt(3)*x + 18*sqrt(b)))/b - 1/27*log(abs(-sqrt(3)*x + sqrt(
b)))/b - 1/9/((sqrt(3)*x - sqrt(b))*sqrt(b))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\int \frac{1}{2\sqrt{3}b^{3/2} - 9bx + 9x^3} \, dx = \frac{2\,\sqrt{3}\,\sqrt{27}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,\sqrt{27}}{27} + \frac{2\,\sqrt{27}\,x}{9\,\sqrt{b}}\right)}{243\,b} - \frac{\sqrt{3}}{27\,\sqrt{b}\,\left(x - \frac{\sqrt{3}\,\sqrt{b}}{3}\right)}$$

```
[In] int(1/(2*3^(1/2)*b^(3/2) - 9*b*x + 9*x^3),x)
[Out] (2*3^(1/2)*27^(1/2)*atanh((3^(1/2)*27^(1/2))/27 + (2*27^(1/2)*x)/(9*b^(1/2)
)))/(243*b) - 3^(1/2)/(27*b^(1/2)*(x - (3^(1/2)*b^(1/2))/3))
```

3.2 $\int (a^3+3a^2bx+3ab^2x^2+b^3x^3)^p\,dx$

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Optimal result

Integrand size = 29, antiderivative size = 30

$$\int (a^3+3a^2bx+3ab^2x^2+b^3x^3)^p\,dx=\frac{\left(\frac{a}{b}+x\right)\left(b^3\left(\frac{a}{b}+x\right)^3\right)^p}{1+3p}$$

```
[Out] (a/b+x)*(b^3*(a/b+x)^3)^p/(1+3*p)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2092, 15, 30}

$$\int (a^3+3a^2bx+3ab^2x^2+b^3x^3)^p\,dx=\frac{\left(\frac{a}{b}+x\right)\left(b^3\left(\frac{a}{b}+x\right)^3\right)^p}{3p+1}$$

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]
```

```
[Out] ((a/b + x)*(b^3*(a/b + x)^3)^p)/(1 + 3*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \left(b^3x^3\right)^p\,dx, x, \frac{a}{b}+x\right)\\ &= \left(\left(\frac{a}{b}+x\right)^{-3p}\left(b^3\left(\frac{a}{b}+x\right)^3\right)^p\right)\text{Subst}\left(\int x^{3p}\,dx, x, \frac{a}{b}+x\right)\\ &= \frac{(a+bx)\left((a+bx)^3\right)^p}{b(1+3p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx = \frac{(a+bx)\left((a+bx)^3\right)^p}{b(1+3p)}$$

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]
[Out] ((a + b*x)*((a + b*x)^3)^p)/(b*(1 + 3*p))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

method	result	size
risch	$\frac{(bx+a)\left((bx+a)^3\right)^p}{b(1+3p)}$	26
gosper	$\frac{(bx+a)\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)^p}{b(1+3p)}$	46
parallelrisch	$\frac{x\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)^p ab+\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)^p a^2}{(1+3p)ab}$	82
norman	$\frac{x\,\mathrm{e}^{p\ln\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)}}{1+3p}+\frac{a\,\mathrm{e}^{p\ln\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)}}{b(1+3p)}$	85

```
[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x,method=_RETURNVERBOSE)
[Out] (b*x+a)/b/(1+3*p)*((b*x+a)^3)^p
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx=\frac{(bx+a)(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)^p}{3\,bp+b}$$

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="fricas")
[Out] (b*x + a)*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p/(3*b*p + b)
```

Sympy [F]

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx$$
$$=\begin{cases} \frac{x}{\sqrt[3]{a^3}} & \text{for } b=0\wedge p=-\frac{1}{3} \\ x(a^3)^p & \text{for } b=0 \\ \int \frac{1}{\sqrt[3]{a^3+3a^2bx+3ab^2x^2+b^3x^3}}\,dx & \text{for } p=-\frac{1}{3} \\ \frac{a(a^3+3a^2bx+3ab^2x^2+b^3x^3)^p}{3bp+b}+\frac{bx(a^3+3a^2bx+3ab^2x^2+b^3x^3)^p}{3bp+b} & \text{otherwise} \end{cases}$$

```
[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**p,x)
[Out] Piecewise((x/(a**3)**(1/3), Eq(b, 0) & Eq(p, -1/3)), (x*(a**3)**p, Eq(b, 0)
), (Integral((a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**(-1/3), x), E
q(p, -1/3)), (a*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(3*b*p +
 b) + b*x*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(3*b*p + b), T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx=\frac{(bx+a)(bx+a)^{3\,p}}{b(3\,p+1)}$$

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="maxima")
[Out] (b*x + a)*(b*x + a)^(3*p)/(b*(3*p + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.43

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx = \frac{(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)^pbx+(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)^pa}{3\,bp+b}$$

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="giac")
```

```
[Out] ((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*b*x + (b^3*x^3 + 3*a*b^2*x^2 +
 3*a^2*b*x + a^3)^p*a)/(3*b*p + b)
```

Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx = \left(\frac{x}{3\,p+1}+\frac{a}{b\,\left(3\,p+1\right)}\right)\,\left(a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3\right)^p$$

```
[In] int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^p,x)
```

```
[Out] (x/(3*p + 1) + a/(b*(3*p + 1)))*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^p
```

3.3 $\int (a^3+3a^2bx+3ab^2x^2+b^3x^3)^3\,dx$

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Optimal result

Integrand size = 29, antiderivative size = 14

$$\int (a^3+3a^2bx+3ab^2x^2+b^3x^3)^3\,dx=\frac{(a+bx)^{10}}{10b}$$

```
[Out] 1/10*(b*x+a)^10/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2084, 32}

$$\int (a^3+3a^2bx+3ab^2x^2+b^3x^3)^3\,dx=\frac{(a+bx)^{10}}{10b}$$

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]
```

```
[Out] (a + b*x)^10/(10*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2084

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int (a+bx)^9\,dx$$
$$= \frac{(a+bx)^{10}}{10b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3\,dx = \frac{(a+bx)^{10}}{10b}$$

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]
[Out] (a + b*x)^10/(10*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(12) = 24.

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 7.00

method	result
default	$\frac{1}{10}b^9x^{10}+a\,b^8x^9+\frac{9}{2}a^2b^7x^8+12a^3b^6x^7+21a^4b^5x^6+\frac{126}{5}a^5b^4x^5+21a^6b^3x^4+12a^7b^2x^3+\frac{9}{2}a^{8}$
norman	$\frac{1}{10}b^9x^{10}+a\,b^8x^9+\frac{9}{2}a^2b^7x^8+12a^3b^6x^7+21a^4b^5x^6+\frac{126}{5}a^5b^4x^5+21a^6b^3x^4+12a^7b^2x^3+\frac{9}{2}a^{8}$
risch	$\frac{1}{10}b^9x^{10}+a\,b^8x^9+\frac{9}{2}a^2b^7x^8+12a^3b^6x^7+21a^4b^5x^6+\frac{126}{5}a^5b^4x^5+21a^6b^3x^4+12a^7b^2x^3+\frac{9}{2}a^{8}$
parallelrisch	$\frac{1}{10}b^9x^{10}+a\,b^8x^9+\frac{9}{2}a^2b^7x^8+12a^3b^6x^7+21a^4b^5x^6+\frac{126}{5}a^5b^4x^5+21a^6b^3x^4+12a^7b^2x^3+\frac{9}{2}a^{8}$
gosper	$\frac{x\left(b^9x^9+10a\,b^8x^8+45a^2b^7x^7+120a^3b^6x^6+210a^4b^5x^5+252a^5b^4x^4+210a^6b^3x^3+120a^7b^2x^2+45a^8bx+10a^9\right)}{10}$

```
[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x,method=_RETURNVERBOSE)
[Out] 1/10*b^9*x^10+a*b^8*x^9+9/2*a^2*b^7*x^8+12*a^3*b^6*x^7+21*a^4*b^5*x^6+126/5
*a^5*b^4*x^5+21*a^6*b^3*x^4+12*a^7*b^2*x^3+9/2*a^8*b*x^2+a^9*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.93

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3\,dx = \frac{1}{10}\,b^9x^{10}+ab^8x^9+\frac{9}{2}\,a^2b^7x^8+12\,a^3b^6x^7+21\,a^4b^5x^6+\frac{126}{5}\,a^5b^4x^5+21\,a^6b^3x^4+12\,a^7b^2x^3+\frac{9}{2}\,a^8bx^2+a^9x$$

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="fricas")
```

```
[Out] 1/10*b^9*x^10 + a*b^8*x^9 + 9/2*a^2*b^7*x^8 + 12*a^3*b^6*x^7 + 21*a^4*b^5*x
^6 + 126/5*a^5*b^4*x^5 + 21*a^6*b^3*x^4 + 12*a^7*b^2*x^3 + 9/2*a^8*b*x^2 +
a^9*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(8) = 16$.

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 7.64

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3\,dx = a^9x+\frac{9a^8bx^2}{2}+12a^7b^2x^3+21a^6b^3x^4+\frac{126a^5b^4x^5}{5}+21a^4b^5x^6+12a^3b^6x^7+\frac{9a^2b^7x^8}{2}+ab^8x^9+\frac{b^9x^{10}}{10}$$

```
[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)
```

```
[Out] a**9*x + 9*a**8*b*x**2/2 + 12*a**7*b**2*x**3 + 21*a**6*b**3*x**4 + 126*a**5
*b**4*x**5/5 + 21*a**4*b**5*x**6 + 12*a**3*b**6*x**7 + 9*a**2*b**7*x**8/2 +
 a*b**8*x**9 + b**9*x**10/10
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(12) = 24$.

Time = 0.20 (sec) , antiderivative size = 216, normalized size of antiderivative = 15.43

$$\begin{aligned}\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3\,dx &= \frac{1}{10}\,b^9x^{10}+ab^8x^9+\frac{27}{8}\,a^2b^7x^8+\frac{27}{7}\,a^3b^6x^7+\frac{27}{4}\,a^6b^3x^4+a^9x\\ &+\frac{3}{4}\left(b^3x^4+4\,ab^2x^3+6\,a^2bx^2\right)a^6+\frac{9}{10}\left(5\,b^3x^6+18\,ab^2x^5\right)a^4b^2\\ &+\frac{3}{70}\left(10\,b^6x^7+70\,ab^5x^6+126\,a^2b^4x^5+210\,a^4b^2x^3+21\left(4\,b^3x^5+15\,ab^2x^4\right)a^2b\right)a^3\\ &+\frac{9}{56}\left(7\,b^6x^8+48\,ab^5x^7+84\,a^2b^4x^6\right)a^2b\end{aligned}$$

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^9*x^10 + a*b^8*x^9 + 27/8*a^2*b^7*x^8 + 27/7*a^3*b^6*x^7 + 27/4*a^6*
b^3*x^4 + a^9*x + 3/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2)*a^6 + 9/10*(5*b
^3*x^6 + 18*a*b^2*x^5)*a^4*b^2 + 3/70*(10*b^6*x^7 + 70*a*b^5*x^6 + 126*a^2*
b^4*x^5 + 210*a^4*b^2*x^3 + 21*(4*b^3*x^5 + 15*a*b^2*x^4)*a^2*b)*a^3 + 9/56
*(7*b^6*x^8 + 48*a*b^5*x^7 + 84*a^2*b^4*x^6)*a^2*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(12) = 24$.

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.93

$$\int \left(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3\right)^3 \, dx = \frac{1}{10} b^9x^{10} + ab^8x^9 + \frac{9}{2} a^2b^7x^8 + 12 a^3b^6x^7 + 21 a^4b^5x^6 + \frac{126}{5} a^5b^4x^5 + 21 a^6b^3x^4 + 12 a^7b^2x^3 + \frac{9}{2} a^8bx^2 + a^9x$$

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="giac")
```

```
[Out] 1/10*b^9*x^10 + a*b^8*x^9 + 9/2*a^2*b^7*x^8 + 12*a^3*b^6*x^7 + 21*a^4*b^5*x
^6 + 126/5*a^5*b^4*x^5 + 21*a^6*b^3*x^4 + 12*a^7*b^2*x^3 + 9/2*a^8*b*x^2 +
a^9*x
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.93

$$\int \left(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3\right)^3 \, dx = a^9\, x + \frac{9\, a^8\, b\, x^2}{2} + 12\, a^7\, b^2\, x^3 + 21\, a^6\, b^3\, x^4 + \frac{126\, a^5\, b^4\, x^5}{5} + 21\, a^4\, b^5\, x^6 + 12\, a^3\, b^6\, x^7 + \frac{9\, a^2\, b^7\, x^8}{2} + a\, b^8\, x^9 + \frac{b^9\, x^{10}}{10}$$

```
[In] int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^3,x)
```

```
[Out] a^9*x + (b^9*x^10)/10 + (9*a^8*b*x^2)/2 + a*b^8*x^9 + 12*a^7*b^2*x^3 + 21*a
^6*b^3*x^4 + (126*a^5*b^4*x^5)/5 + 21*a^4*b^5*x^6 + 12*a^3*b^6*x^7 + (9*a^2
*b^7*x^8)/2
```

3.4 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx$

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Mathematica [A] (verified) 172
Maple [B] (verified) 172
Fricas [B] (verification not implemented) 173
Sympy [B] (verification not implemented) 173
Maxima [B] (verification not implemented) 173
Giac [B] (verification not implemented) 174
Mupad [B] (verification not implemented) 174

Optimal result

Integrand size = 29, antiderivative size = 14

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx=\frac{(a+bx)^7}{7b}$$

```
[Out] 1/7*(b*x+a)^7/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2084, 32}

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx=\frac{(a+bx)^7}{7b}$$

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]
```

```
[Out] (a + b*x)^7/(7*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2084

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int (a+bx)^6 \, dx$$
$$= \frac{(a+bx)^7}{7b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2 \, dx = \frac{(a+bx)^7}{7b}$$

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]
[Out] (a + b*x)^7/(7*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(12) = 24.

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 4.64

method	result	size
default	$\frac{1}{7}b^6x^7+a\,b^5x^6+3a^2b^4x^5+5a^3b^3x^4+5a^4b^2x^3+3a^5b\,x^2+a^6x$	65
norman	$\frac{1}{7}b^6x^7+a\,b^5x^6+3a^2b^4x^5+5a^3b^3x^4+5a^4b^2x^3+3a^5b\,x^2+a^6x$	65
risch	$\frac{1}{7}b^6x^7+a\,b^5x^6+3a^2b^4x^5+5a^3b^3x^4+5a^4b^2x^3+3a^5b\,x^2+a^6x$	65
parallelrisch	$\frac{1}{7}b^6x^7+a\,b^5x^6+3a^2b^4x^5+5a^3b^3x^4+5a^4b^2x^3+3a^5b\,x^2+a^6x$	65
gosper	$\frac{x\left(b^6x^6+7a\,b^5x^5+21a^2b^4x^4+35a^3b^3x^3+35a^4b^2x^2+21a^5bx+7a^6\right)}{7}$	66

```
[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x,method=_RETURNVERBOSE)
[Out] 1/7*b^6*x^7+a*b^5*x^6+3*a^2*b^4*x^5+5*a^3*b^3*x^4+5*a^4*b^2*x^3+3*a^5*b*x^2
+a^6*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 4.57

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx = \frac{1}{7}\,b^6x^7+ab^5x^6+3\,a^2b^4x^5+5\,a^3b^3x^4+5\,a^4b^2x^3+3\,a^5bx^2+a^6x$$

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="fricas")
[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 3*a^2*b^4*x^5 + 5*a^3*b^3*x^4 + 5*a^4*b^2*x^3 + 3
*a^5*b*x^2 + a^6*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(8) = 16$.

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 4.71

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx = a^6x+3a^5bx^2+5a^4b^2x^3+5a^3b^3x^4+3a^2b^4x^5+ab^5x^6+\frac{b^6x^7}{7}$$

```
[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)
[Out] a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 + 3*a**2*b**4*
x**5 + a*b**5*x**6 + b**6*x**7/7
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(12) = 24$.

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 7.07

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx = \frac{1}{7}\,b^6x^7+ab^5x^6+\frac{9}{5}\,a^2b^4x^5+3\,a^4b^2x^3+a^6x+\frac{1}{2}\,\left(b^3x^4+4\,ab^2x^3+6\,a^2bx^2\right)a^3+\frac{3}{10}\,\left(4\,b^3x^5+15\,ab^2x^4\right)a^2b$$

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="maxima")
[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 9/5*a^2*b^4*x^5 + 3*a^4*b^2*x^3 + a^6*x + 1/2*(b^
3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2)*a^3 + 3/10*(4*b^3*x^5 + 15*a*b^2*x^4)*a^
2*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(12) = 24.

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 4.57

$$\int \left(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3\right)^2 \, dx = \frac{1}{7}\,b^6x^7 + ab^5x^6 + 3\,a^2b^4x^5 + 5\,a^3b^3x^4 + 5\,a^4b^2x^3 + 3\,a^5bx^2 + a^6x$$

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="giac")
[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 3*a^2*b^4*x^5 + 5*a^3*b^3*x^4 + 5*a^4*b^2*x^3 + 3
*a^5*b*x^2 + a^6*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 4.57

$$\int \left(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3\right)^2 \, dx = a^6\,x + 3\,a^5\,b\,x^2 + 5\,a^4\,b^2\,x^3 + 5\,a^3\,b^3\,x^4 + 3\,a^2\,b^4\,x^5 + a\,b^5\,x^6 + \frac{b^6\,x^7}{7}$$

```
[In] int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^2,x)
[Out] a^6*x + (b^6*x^7)/7 + 3*a^5*b*x^2 + a*b^5*x^6 + 5*a^4*b^2*x^3 + 5*a^3*b^3*x
^4 + 3*a^2*b^4*x^5
```

3.5 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx$

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Rubi [A] (verified) 175
Mathematica [A] (verified) 176
Maple [A] (verified) 176
Fricas [A] (verification not implemented) 176
Sympy [A] (verification not implemented) 177
Maxima [A] (verification not implemented) 177
Giac [A] (verification not implemented) 177
Mupad [B] (verification not implemented) 177

Optimal result

Integrand size = 27, antiderivative size = 35

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx=a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

```
[Out] a^3*x+3/2*a^2*b*x^2+a*b^2*x^3+1/4*b^3*x^4
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx=a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

```
[In] Int[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3,x]
```

```
[Out] a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4
```

Rubi steps

$$\text{integral}=a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx = a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

```
[In] Integrate[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3,x]
[Out] a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.37

method	result	size
default	$\frac{(bx+a)^4}{4b}$	13
norman	$a^3x+\frac{3}{2}a^2b\,x^2+a\,b^2x^3+\frac{1}{4}b^3x^4$	32
risch	$a^3x+\frac{3}{2}a^2b\,x^2+a\,b^2x^3+\frac{1}{4}b^3x^4$	32
parallelrisch	$a^3x+\frac{3}{2}a^2b\,x^2+a\,b^2x^3+\frac{1}{4}b^3x^4$	32
parts	$a^3x+\frac{3}{2}a^2b\,x^2+a\,b^2x^3+\frac{1}{4}b^3x^4$	32
gosper	$\frac{x\left(b^3x^3+4a\,b^2x^2+6a^2bx+4a^3\right)}{4}$	33

```
[In] int(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x,method=_RETURNVERBOSE)
[Out] 1/4*(b*x+a)^4/b
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx = \frac{1}{4}\,b^3x^4+ab^2x^3+\frac{3}{2}\,a^2bx^2+a^3x$$

```
[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="fricas")
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx=a^3x+\frac{3a^2bx^2}{2}+ab^2x^3+\frac{b^3x^4}{4}$$

```
[In] integrate(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3,x)
[Out] a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx=\frac{1}{4}\,b^3x^4+ab^2x^3+\frac{3}{2}\,a^2bx^2+a^3x$$

```
[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="maxima")
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx=\frac{1}{4}\,b^3x^4+ab^2x^3+\frac{3}{2}\,a^2bx^2+a^3x$$

```
[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="giac")
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx=a^3\,x+\frac{3\,a^2\,b\,x^2}{2}+a\,b^2\,x^3+\frac{b^3\,x^4}{4}$$

```
[In] int(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x,x)
[Out] a^3*x + (b^3*x^4)/4 + (3*a^2*b*x^2)/2 + a*b^2*x^3
```

3.6 $\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx$

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Mathematica [A] (verified) 179
Maple [A] (verified) 179
Fricas [A] (verification not implemented) 180
Sympy [B] (verification not implemented) 180
Maxima [A] (verification not implemented) 180
Giac [A] (verification not implemented) 180
Mupad [B] (verification not implemented) 181

Optimal result

Integrand size = 29, antiderivative size = 14

$$\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx = -\frac{1}{2b(a+bx)^2}$$

```
[Out] -1/2/b/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2083, 32}

$$\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx = -\frac{1}{2b(a+bx)^2}$$

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1),x]
```

```
[Out] -1/2*1/(b*(a + b*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{(a+bx)^3}\,dx \\
&= -\frac{1}{2b(a+bx)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx = -\frac{1}{2b(a+bx)^2}$$

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1),x]
[Out] -1/2*1/(b*(a + b*x)^2)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{2b(bx+a)^2}$	13
norman	$-\frac{1}{2b(bx+a)^2}$	13
gosper	$-\frac{1}{2b(b^2x^2+2abx+a^2)}$	24
risch	$-\frac{1}{2b(b^2x^2+2abx+a^2)}$	24
parallelrisch	$-\frac{1}{2b(b^2x^2+2abx+a^2)}$	24

```
[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x,method=_RETURNVERBOSE)
[Out] -1/2/b/(b*x+a)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx=-\frac{1}{2\,(b^3x^2+2\,ab^2x+a^2b)}$$

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="fricas")
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12)=24$.

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx=-\frac{1}{2a^2b+4ab^2x+2b^3x^2}$$

```
[In] integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3),x)
[Out] -1/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx=-\frac{1}{2\,(b^3x^2+2\,ab^2x+a^2b)}$$

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="maxima")
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx=-\frac{1}{2\,(bx+a)^2b}$$

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="giac")
[Out] -1/2/((b*x + a)^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx=-\frac{1}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2}$$

```
[In] int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x),x)
[Out] -1/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x)
```

3.7 $\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2}\,dx$

Optimal result 182
Rubi [A] (verified) 182
Mathematica [A] (verified) 183
Maple [A] (verified) 183
Fricas [B] (verification not implemented) 184
Sympy [B] (verification not implemented) 184
Maxima [B] (verification not implemented) 184
Giac [A] (verification not implemented) 185
Mupad [B] (verification not implemented) 185

Optimal result

Integrand size = 29, antiderivative size = 14

$$\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2}\,dx=-\frac{1}{5b(a+bx)^5}$$

```
[Out] -1/5/b/(b*x+a)^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2083, 32}

$$\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2}\,dx=-\frac{1}{5b(a+bx)^5}$$

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2),x]
[Out] -1/5*1/(b*(a + b*x)^5)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{(a+bx)^6}\,dx \\ &= -\frac{1}{5b(a+bx)^5} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2}\,dx = -\frac{1}{5b(a+bx)^5}$$

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2),x]
[Out] -1/5*1/(b*(a + b*x)^5)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{5b(bx+a)^5}$	13
norman	$-\frac{1}{5b(bx+a)^5}$	13
risch	$-\frac{1}{5b(b^2x^2+2abx+a^2)^2(bx+a)}$	31
gosper	$-\frac{1}{5(b^2x^2+2abx+a^2)(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)b}$	53
parallelrisch	$-\frac{1}{5(b^2x^2+2abx+a^2)(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)b}$	53

```
[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x,method=_RETURNVERBOSE)
[Out] -1/5/b/(b*x+a)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.07

$$\int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^2}\,dx = -\frac{1}{5\,(b^6x^5 + 5\,ab^5x^4 + 10\,a^2b^4x^3 + 10\,a^3b^3x^2 + 5\,a^4b^2x + a^5b)}$$

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="fricas")
[Out] -1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x
 + a^5*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(12) = 24$.

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 4.36

$$\int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^2}\,dx = -\frac{1}{5a^5b + 25a^4b^2x + 50a^3b^3x^2 + 50a^2b^4x^3 + 25ab^5x^4 + 5b^6x^5}$$

```
[In] integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)
[Out] -1/(5*a**5*b + 25*a**4*b**2*x + 50*a**3*b**3*x**2 + 50*a**2*b**4*x**3 + 25*
a*b**5*x**4 + 5*b**6*x**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(12) = 24$.

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.07

$$\int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^2}\,dx = -\frac{1}{5\,(b^6x^5 + 5\,ab^5x^4 + 10\,a^2b^4x^3 + 10\,a^3b^3x^2 + 5\,a^4b^2x + a^5b)}$$

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="maxima")
[Out] -1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x
 + a^5*b)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2}\,dx=-\frac{1}{5\,(bx+a)^5b}$$

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="giac")
```

```
[Out] -1/5/((b*x + a)^5*b)
```

Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 4.21

$$\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2}\,dx$$
$$=-\frac{1}{5\,a^5\,b+25\,a^4\,b^2\,x+50\,a^3\,b^3\,x^2+50\,a^2\,b^4\,x^3+25\,a\,b^5\,x^4+5\,b^6\,x^5}$$

```
[In] int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^2,x)
```

```
[Out] -1/(5*a^5*b + 5*b^6*x^5 + 25*a^4*b^2*x + 25*a*b^5*x^4 + 50*a^3*b^3*x^2 + 50
*a^2*b^4*x^3)
```

3.8 $\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^3}\,dx$

Optimal result 186
Rubi [A] (verified) 186
Mathematica [A] (verified) 187
Maple [A] (verified) 187
Fricas [B] (verification not implemented) 188
Sympy [B] (verification not implemented) 188
Maxima [B] (verification not implemented) 188
Giac [A] (verification not implemented) 189
Mupad [B] (verification not implemented) 189

Optimal result

Integrand size = 29, antiderivative size = 14

$$\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^3}\,dx = -\frac{1}{8b(a+bx)^8}$$

```
[Out] -1/8/b/(b*x+a)^8
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2083, 32}

$$\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^3}\,dx = -\frac{1}{8b(a+bx)^8}$$

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3),x]
```

```
[Out] -1/8*1/(b*(a + b*x)^8)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{(a+bx)^9}\,dx\\
&= -\frac{1}{8b(a+bx)^8}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^3}\,dx = -\frac{1}{8b(a+bx)^8}$$

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3),x]
[Out] -1/8*1/(b*(a + b*x)^8)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{8b(bx+a)^8}$	13
norman	$-\frac{1}{8b(bx+a)^8}$	13
risch	$-\frac{1}{8b(b^2x^2+2abx+a^2)^3(bx+a)^2}$	31
gosper	$-\frac{1}{8(b^2x^2+2abx+a^2)(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)^2b}$	53
parallelrisch	$-\frac{1}{8(b^2x^2+2abx+a^2)(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)^2b}$	53

```
[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x,method=_RETURNVERBOSE)
[Out] -1/8/b/(b*x+a)^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(12) = 24$.

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 6.43

$$\int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3} \, dx = -\frac{1}{8\left(b^9x^8 + 8\,ab^8x^7 + 28\,a^2b^7x^6 + 56\,a^3b^6x^5 + 70\,a^4b^5x^4 + 56\,a^5b^4x^3 + 28\,a^6b^3x^2 + 8\,a^7b^2x + a^8b\right)}$$

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="fricas")
[Out] -1/8/(b^9*x^8 + 8*a*b^8*x^7 + 28*a^2*b^7*x^6 + 56*a^3*b^6*x^5 + 70*a^4*b^5*
x^4 + 56*a^5*b^4*x^3 + 28*a^6*b^3*x^2 + 8*a^7*b^2*x + a^8*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.93

$$\int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3} \, dx = -\frac{1}{8a^8b + 64a^7b^2x + 224a^6b^3x^2 + 448a^5b^4x^3 + 560a^4b^5x^4 + 448a^3b^6x^5 + 224a^2b^7x^6 + 64ab^8x^7 + 8b^9x^8}$$

```
[In] integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)
[Out] -1/(8*a**8*b + 64*a**7*b**2*x + 224*a**6*b**3*x**2 + 448*a**5*b**4*x**3 + 5
60*a**4*b**5*x**4 + 448*a**3*b**6*x**5 + 224*a**2*b**7*x**6 + 64*a*b**8*x**
7 + 8*b**9*x**8)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(12) = 24$.

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 6.43

$$\int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3} \, dx = -\frac{1}{8\left(b^9x^8 + 8\,ab^8x^7 + 28\,a^2b^7x^6 + 56\,a^3b^6x^5 + 70\,a^4b^5x^4 + 56\,a^5b^4x^3 + 28\,a^6b^3x^2 + 8\,a^7b^2x + a^8b\right)}$$

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="maxima")
[Out] -1/8/(b^9*x^8 + 8*a*b^8*x^7 + 28*a^2*b^7*x^6 + 56*a^3*b^6*x^5 + 70*a^4*b^5*
x^4 + 56*a^5*b^4*x^3 + 28*a^6*b^3*x^2 + 8*a^7*b^2*x + a^8*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^3}\,dx = -\frac{1}{8\,(bx+a)^8b}$$

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="giac")
```

```
[Out] -1/8/((b*x + a)^8*b)
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 6.57

$$\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^3}\,dx = -\frac{1}{8\,a^8\,b+64\,a^7\,b^2\,x+224\,a^6\,b^3\,x^2+448\,a^5\,b^4\,x^3+560\,a^4\,b^5\,x^4+448\,a^3\,b^6\,x^5+224\,a^2\,b^7\,x^6+64\,a\,b^8\,x^7+}$$

```
[In] int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^3,x)
```

```
[Out] -1/(8*a^8*b + 8*b^9*x^8 + 64*a^7*b^2*x + 64*a*b^8*x^7 + 224*a^6*b^3*x^2 + 4
48*a^5*b^4*x^3 + 560*a^4*b^5*x^4 + 448*a^3*b^6*x^5 + 224*a^2*b^7*x^6)
```

3.9 $\int (3ab+3b^2x+3bcx^2+c^2x^3)^3\,dx$

Optimal result . . . 190
Rubi [A] (verified) . . . 190
Mathematica [A] (verified) . . . 191
Maple [B] (verified) . . . 192
Fricas [A] (verification not implemented) . . . 192
Sympy [B] (verification not implemented) . . . 193
Maxima [B] (verification not implemented) . . . 193
Giac [B] (verification not implemented) . . . 194
Mupad [B] (verification not implemented) . . . 194

Optimal result

Integrand size = 27, antiderivative size = 84

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx=-\frac{b^3(b^2-3ac)^3\,x}{c^3}+\frac{3b^2(b^2-3ac)^2\,(b+cx)^4}{4c^4}-\frac{3b(b^2-3ac)\,(b+cx)^7}{7c^4}+\frac{(b+cx)^{10}}{10c^4}$$

```
[Out] -b^3*(-3*a*c+b^2)^3*x/c^3+3/4*b^2*(-3*a*c+b^2)^2*(c*x+b)^4/c^4-3/7*b*(-3*a*
c+b^2)*(c*x+b)^7/c^4+1/10*(c*x+b)^10/c^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2085, 200}

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx=-\frac{3b(b^2-3ac)\,(b+cx)^7}{7c^4}+\frac{3b^2(b^2-3ac)^2\,(b+cx)^4}{4c^4}-\frac{b^3x(b^2-3ac)^3}{c^3}+\frac{(b+cx)^{10}}{10c^4}$$

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

```
[Out] -((b^3*(b^2 - 3*a*c)^3*x)/c^3) + (3*b^2*(b^2 - 3*a*c)^2*(b + c*x)^4)/(4*c^4
) - (3*b*(b^2 - 3*a*c)*(b + c*x)^7)/(7*c^4) + (b + c*x)^10/(10*c^4)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2085

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3)^(p_), x_Symbol] :> D
ist[1/3^p, Subst[Int[Simp[(3*a*c - b^2)/c + c^2*(x^3/b), x]^p, x], x, c/(3*
d) + x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0] && EqQ[c^2 - 3*b*d, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{27}\text{Subst}\left(\int \left(3b\left(3a-\frac{b^2}{c}\right)+3c^2x^3\right)^3 dx, x, \frac{b}{c}+x\right)$$

$$= \frac{1}{27}\text{Subst}\left(\int \left(\frac{27(-b^3+3abc)^3}{c^3}+81\left(b^3-3abc\right)^2 x^3-81bc^3\left(b^2-3ac\right)x^6 + 27c^6x^9\right) dx, x, \frac{b}{c}+x\right)$$

$$= -\frac{b^3(b^2-3ac)^3 x}{c^3}+\frac{3b^2(b^2-3ac)^2 (b+cx)^4}{4c^4}-\frac{3b(b^2-3ac)(b+cx)^7}{7c^4}+\frac{(b+cx)^{10}}{10c^4}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.89

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3 dx = 27a^3b^3x+\frac{81}{2}a^2b^4x^2+27ab^3\left(b^2+ac\right)x^3 + \frac{27}{4}b^2\left(b^4+6ab^2c+a^2c^2\right)x^4 + \frac{27}{5}b^3c\left(3b^2+5ac\right)x^5+9b^2c^2\left(2b^2+ac\right)x^6 + \frac{9}{7}bc^3\left(9b^2+ac\right)x^7+\frac{9}{2}b^2c^4x^8+bc^5x^9+\frac{c^6x^{10}}{10}$$

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

```
[Out] 27*a^3*b^3*x + (81*a^2*b^4*x^2)/2 + 27*a*b^3*(b^2 + a*c)*x^3 + (27*b^2*(b^4
 + 6*a*b^2*c + a^2*c^2)*x^4)/4 + (27*b^3*c*(3*b^2 + 5*a*c)*x^5)/5 + 9*b^2*c
^2*(2*b^2 + a*c)*x^6 + (9*b*c^3*(9*b^2 + a*c)*x^7)/7 + (9*b^2*c^4*x^8)/2 +
b*c^5*x^9 + (c^6*x^10)/10
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(78) = 156.

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.89

method	result
norman	$\frac{c^6x^{10}}{10}+b\,c^5x^9+\frac{9b^2c^4x^8}{2}+\left(\frac{9}{7}ab\,c^4+\frac{81}{7}b^3c^3\right)x^7+\left(9a\,b^2c^3+18b^4c^2\right)x^6+\left(27a\,b^3c^2+\frac{81}{5}b^5c\right)x^5$
gosper	$\frac{1}{10}c^6x^{10}+b\,c^5x^9+\frac{9}{2}b^2c^4x^8+\frac{9}{7}x^7ab\,c^4+\frac{81}{7}x^7b^3c^3+9a\,b^2c^3x^6+18b^4c^2x^6+27x^5a\,b^3c^2+\frac{81}{5}x^5$
risch	$\frac{1}{10}c^6x^{10}+b\,c^5x^9+\frac{9}{2}b^2c^4x^8+\frac{9}{7}x^7ab\,c^4+\frac{81}{7}x^7b^3c^3+9a\,b^2c^3x^6+18b^4c^2x^6+27x^5a\,b^3c^2+\frac{81}{5}x^5$
parallelrisch	$\frac{1}{10}c^6x^{10}+b\,c^5x^9+\frac{9}{2}b^2c^4x^8+\frac{9}{7}x^7ab\,c^4+\frac{81}{7}x^7b^3c^3+9a\,b^2c^3x^6+18b^4c^2x^6+27x^5a\,b^3c^2+\frac{81}{5}x^5$
default	$\frac{c^6x^{10}}{10}+b\,c^5x^9+\frac{9b^2c^4x^8}{2}+\frac{\left(3ab\,c^4+63b^3c^3+c^2\left(6ab\,c^2+18b^3c\right)\right)x^7}{7}+\frac{\left(18a\,b^2c^3+45b^4c^2+3bc\left(6ab\,c^2+18b^3c\right)+c^2\left(18a\,b^2\right.\right.}{6}$

```
[In] int((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*c^6*x^10+b*c^5*x^9+9/2*b^2*c^4*x^8+(9/7*a*b*c^4+81/7*b^3*c^3)*x^7+(9*a
*b^2*c^3+18*b^4*c^2)*x^6+(27*a*b^3*c^2+81/5*b^5*c)*x^5+(27/4*a^2*b^2*c^2+81
/2*a*b^4*c+27/4*b^6)*x^4+(27*a^2*b^3*c+27*a*b^5)*x^3+81/2*a^2*b^4*x^2+27*a^
3*b^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.86

$$\begin{aligned}\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx &= \frac{1}{10}\,c^6x^{10}+bc^5x^9+\frac{9}{2}\,b^2c^4x^8+\frac{81}{2}\,a^2b^4x^2\\&\quad+\frac{9}{7}\left(9\,b^3c^3+abc^4\right)x^7+27\,a^3b^3x\\&\quad+9\left(2\,b^4c^2+ab^2c^3\right)x^6+\frac{27}{5}\left(3\,b^5c+5\,ab^3c^2\right)x^5\\&\quad+\frac{27}{4}\left(b^6+6\,ab^4c+a^2b^2c^2\right)x^4+27\left(ab^5+a^2b^3c\right)x^3\end{aligned}$$

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="fricas")
```

```
[Out] 1/10*c^6*x^10 + b*c^5*x^9 + 9/2*b^2*c^4*x^8 + 81/2*a^2*b^4*x^2 + 9/7*(9*b^3
*c^3 + a*b*c^4)*x^7 + 27*a^3*b^3*x + 9*(2*b^4*c^2 + a*b^2*c^3)*x^6 + 27/5*(
3*b^5*c + 5*a*b^3*c^2)*x^5 + 27/4*(b^6 + 6*a*b^4*c + a^2*b^2*c^2)*x^4 + 27*
(a*b^5 + a^2*b^3*c)*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(80) = 160$.

Time = 0.04 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.08

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3 \, dx = 27a^3b^3x + \frac{81a^2b^4x^2}{2} + \frac{9b^2c^4x^8}{2} + bc^5x^9 + \frac{c^6x^{10}}{10} + x^7 \cdot \left(\frac{9abc^4}{7} + \frac{81b^3c^3}{7}\right) + x^6 \cdot \left(9ab^2c^3 + 18b^4c^2\right) + x^5 \cdot \left(27ab^3c^2 + \frac{81b^5c}{5}\right) + x^4 \cdot \left(\frac{27a^2b^2c^2}{4} + \frac{81ab^4c}{2} + \frac{27b^6}{4}\right) + x^3 \cdot \left(27a^2b^3c + 27ab^5\right)$$

```
[In] integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

```
[Out] 27*a**3*b**3*x + 81*a**2*b**4*x**2/2 + 9*b**2*c**4*x**8/2 + b*c**5*x**9 + c
**6*x**10/10 + x**7*(9*a*b*c**4/7 + 81*b**3*c**3/7) + x**6*(9*a*b**2*c**3 +
 18*b**4*c**2) + x**5*(27*a*b**3*c**2 + 81*b**5*c/5) + x**4*(27*a**2*b**2*c
**2/4 + 81*a*b**4*c/2 + 27*b**6/4) + x**3*(27*a**2*b**3*c + 27*a*b**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(78) = 156$.

Time = 0.21 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.43

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3 \, dx = \frac{1}{10}\,c^6x^{10} + bc^5x^9 + \frac{27}{8}\,b^2c^4x^8 + \frac{27}{7}\,b^3c^3x^7 + \frac{27}{4}\,b^6x^4 + 27\,a^3b^3x + \frac{27}{4}\,\left(c^2x^4 + 4\,bcx^3 + 6\,b^2x^2\right)a^2b^2 + \frac{9}{10}\,\left(5\,c^2x^6 + 18\,bcx^5\right)b^4 + \frac{9}{70}\,\left(10\,c^4x^7 + 70\,bc^3x^6 + 126\,b^2c^2x^5 + 210\,b^4x^3 + 21\,\left(4\,c^2x^5 + 15\,bcx^4\right)b^2\right)ab + \frac{9}{56}\,\left(7\,c^4x^8 + 48\,bc^3x^7 + 84\,b^2c^2x^6\right)b^2$$

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="maxima")
```

```
[Out] 1/10*c^6*x^10 + b*c^5*x^9 + 27/8*b^2*c^4*x^8 + 27/7*b^3*c^3*x^7 + 27/4*b^6*
x^4 + 27*a^3*b^3*x + 27/4*(c^2*x^4 + 4*b*c*x^3 + 6*b^2*x^2)*a^2*b^2 + 9/10*
(5*c^2*x^6 + 18*b*c*x^5)*b^4 + 9/70*(10*c^4*x^7 + 70*b*c^3*x^6 + 126*b^2*c^
2*x^5 + 210*b^4*x^3 + 21*(4*c^2*x^5 + 15*b*c*x^4)*b^2)*a*b + 9/56*(7*c^4*x^
8 + 48*b*c^3*x^7 + 84*b^2*c^2*x^6)*b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(78) = 156$.

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.98

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx = \frac{1}{10}\,c^6x^{10}+bc^5x^9+\frac{9}{2}\,b^2c^4x^8+\frac{81}{7}\,b^3c^3x^7+\frac{9}{7}\,abc^4x^7+18\,b^4c^2x^6+9\,ab^2c^3x^6+\frac{81}{5}\,b^5cx^5+27\,ab^3c^2x^5+\frac{27}{4}\,b^6x^4+\frac{81}{2}\,ab^4cx^4+\frac{27}{4}\,a^2b^2c^2x^4+27\,ab^5x^3+27\,a^2b^3cx^3+\frac{81}{2}\,a^2b^4x^2+27\,a^3b^3x$$

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="giac")
```

```
[Out] 1/10*c^6*x^10 + b*c^5*x^9 + 9/2*b^2*c^4*x^8 + 81/7*b^3*c^3*x^7 + 9/7*a*b*c^
4*x^7 + 18*b^4*c^2*x^6 + 9*a*b^2*c^3*x^6 + 81/5*b^5*c*x^5 + 27*a*b^3*c^2*x^
5 + 27/4*b^6*x^4 + 81/2*a*b^4*c*x^4 + 27/4*a^2*b^2*c^2*x^4 + 27*a*b^5*x^3 +
 27*a^2*b^3*c*x^3 + 81/2*a^2*b^4*x^2 + 27*a^3*b^3*x
```

Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.77

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx = x^4\left(\frac{27\,a^2\,b^2\,c^2}{4}+\frac{81\,a\,b^4\,c}{2}+\frac{27\,b^6}{4}\right)+\frac{c^6\,x^{10}}{10}+27\,a^3\,b^3\,x+b\,c^5\,x^9+\frac{81\,a^2\,b^4\,x^2}{2}+\frac{9\,b^2\,c^4\,x^8}{2}+9\,b^2\,c^2\,x^6\left(2\,b^2+a\,c\right)+27\,a\,b^3\,x^3\left(b^2+a\,c\right)+\frac{27\,b^3\,c\,x^5\,\left(3\,b^2+5\,a\,c\right)}{5}+\frac{9\,b\,c^3\,x^7\,\left(9\,b^2+a\,c\right)}{7}$$

```
[In] int((3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^3,x)
```

```
[Out] x^4*((27*b^6)/4 + (27*a^2*b^2*c^2)/4 + (81*a*b^4*c)/2) + (c^6*x^10)/10 + 27
*a^3*b^3*x + b*c^5*x^9 + (81*a^2*b^4*x^2)/2 + (9*b^2*c^4*x^8)/2 + 9*b^2*c^2
*x^6*(a*c + 2*b^2) + 27*a*b^3*x^3*(a*c + b^2) + (27*b^3*c*x^5*(5*a*c + 3*b^
2))/5 + (9*b*c^3*x^7*(a*c + 9*b^2))/7
```

3.10 $\int (3ab+3b^2x+3bcx^2+c^2x^3)^2\, dx$

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Optimal result

Integrand size = 27, antiderivative size = 56

$$\int (3ab+3b^2x+3bcx^2+c^2x^3)^2\, dx = \frac{b^2(b^2-3ac)^2 x}{c^2} - \frac{b(b^2-3ac)(b+cx)^4}{2c^3} + \frac{(b+cx)^7}{7c^3}$$

```
[Out] b^2*(-3*a*c+b^2)^2*x/c^2-1/2*b*(-3*a*c+b^2)*(c*x+b)^4/c^3+1/7*(c*x+b)^7/c^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2085, 200}

$$\int (3ab+3b^2x+3bcx^2+c^2x^3)^2\, dx = -\frac{b(b^2-3ac)(b+cx)^4}{2c^3} + \frac{b^2x(b^2-3ac)^2}{c^2} + \frac{(b+cx)^7}{7c^3}$$

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
```

```
[Out] (b^2*(b^2 - 3*a*c)^2*x)/c^2 - (b*(b^2 - 3*a*c)*(b + c*x)^4)/(2*c^3) + (b +
c*x)^7/(7*c^3)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2085

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3)^(p_), x_Symbol] :> D
ist[1/3^p, Subst[Int[Simp[(3*a*c - b^2)/c + c^2*(x^3/b), x]^p, x], x, c/(3*
```

```
d) + x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0] && EqQ[c^2 - 3*b*d, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{9}\text{Subst}\left(\int \left(3b\left(3a-\frac{b^2}{c}\right)+3c^2x^3\right)^2 dx, x, \frac{b}{c}+x\right) \\ &= \frac{1}{9}\text{Subst}\left(\int \left(\frac{9(-b^3+3abc)^2}{c^2}-18bc(b^2-3ac)x^3+9c^4x^6\right) dx, x, \frac{b}{c}+x\right) \\ &= \frac{b^2(b^2-3ac)^2x}{c^2}-\frac{b(b^2-3ac)(b+cx)^4}{2c^3}+\frac{(b+cx)^7}{7c^3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.46

$$\int (3ab+3b^2x+3bcx^2+c^2x^3)^2 dx = 9a^2b^2x+9ab^3x^2+3b^2(b^2+2ac)x^3 + \frac{3}{2}bc(3b^2+ac)x^4+3b^2c^2x^5+bc^3x^6+\frac{c^4x^7}{7}$$

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
[Out] 9*a^2*b^2*x + 9*a*b^3*x^2 + 3*b^2*(b^2 + 2*a*c)*x^3 + (3*b*c*(3*b^2 + a*c)*
x^4)/2 + 3*b^2*c^2*x^5 + b*c^3*x^6 + (c^4*x^7)/7
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.46

method	result	size
norman	$\frac{c^4x^7}{7}+c^3x^6b+3b^2c^2x^5+\left(\frac{3}{2}ab\,c^2+\frac{9}{2}b^3c\right)x^4+\left(6a\,b^2c+3b^4\right)x^3+9a\,b^3x^2+9a^2b^2x$	82
gosper	$\frac{1}{7}c^4x^7+c^3x^6b+3b^2c^2x^5+\frac{3}{2}x^4ab\,c^2+\frac{9}{2}x^4b^3c+6a\,b^2c\,x^3+3b^4x^3+9a\,b^3x^2+9a^2b^2x$	84
default	$\frac{c^4x^7}{7}+c^3x^6b+3b^2c^2x^5+\frac{(6ab\,c^2+18b^3c)x^4}{4}+\frac{(18a\,b^2c+9b^4)x^3}{3}+9a\,b^3x^2+9a^2b^2x$	84
risch	$\frac{1}{7}c^4x^7+c^3x^6b+3b^2c^2x^5+\frac{3}{2}x^4ab\,c^2+\frac{9}{2}x^4b^3c+6a\,b^2c\,x^3+3b^4x^3+9a\,b^3x^2+9a^2b^2x$	84
parallelrisch	$\frac{1}{7}c^4x^7+c^3x^6b+3b^2c^2x^5+\frac{3}{2}x^4ab\,c^2+\frac{9}{2}x^4b^3c+6a\,b^2c\,x^3+3b^4x^3+9a\,b^3x^2+9a^2b^2x$	84

```
[In] int((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x,method=_RETURNVERBOSE)
[Out] 1/7*c^4*x^7+c^3*x^6*b+3*b^2*c^2*x^5+(3/2*a*b*c^2+9/2*b^3*c)*x^4+(6*a*b^2*c+
3*b^4)*x^3+9*a*b^3*x^2+9*a^2*b^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.43

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2\,dx=\frac{1}{7}\,c^4x^7+bc^3x^6+3\,b^2c^2x^5+9\,ab^3x^2+9\,a^2b^2x+\frac{3}{2}\,\left(3\,b^3c+abc^2\right)x^4+3\,\left(b^4+2\,ab^2c\right)x^3$$

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="fricas")
[Out] 1/7*c^4*x^7 + b*c^3*x^6 + 3*b^2*c^2*x^5 + 9*a*b^3*x^2 + 9*a^2*b^2*x + 3/2*(
3*b^3*c + a*b*c^2)*x^4 + 3*(b^4 + 2*a*b^2*c)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2\,dx=9a^2b^2x+9ab^3x^2+3b^2c^2x^5+bc^3x^6+\frac{c^4x^7}{7}+x^4\cdot\left(\frac{3abc^2}{2}+\frac{9b^3c}{2}\right)+x^3\cdot\left(6ab^2c+3b^4\right)$$

```
[In] integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
[Out] 9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**2*c**2*x**5 + b*c**3*x**6 + c**4*x**7/
7 + x**4*(3*a*b*c**2/2 + 9*b**3*c/2) + x**3*(6*a*b**2*c + 3*b**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.66

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2\,dx=\frac{1}{7}\,c^4x^7+bc^3x^6+\frac{9}{5}\,b^2c^2x^5+3\,b^4x^3+9\,a^2b^2x+\frac{3}{2}\,\left(c^2x^4+4\,bcx^3+6\,b^2x^2\right)ab+\frac{3}{10}\,\left(4\,c^2x^5+15\,bcx^4\right)b^2$$

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="maxima")
[Out] 1/7*c^4*x^7 + b*c^3*x^6 + 9/5*b^2*c^2*x^5 + 3*b^4*x^3 + 9*a^2*b^2*x + 3/2*(
c^2*x^4 + 4*b*c*x^3 + 6*b^2*x^2)*a*b + 3/10*(4*c^2*x^5 + 15*b*c*x^4)*b^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.48

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2 \, dx = \frac{1}{7}\,c^4x^7+bc^3x^6+3\,b^2c^2x^5+\frac{9}{2}\,b^3cx^4+\frac{3}{2}\,abc^2x^4 + 3\,b^4x^3+6\,ab^2cx^3+9\,ab^3x^2+9\,a^2b^2x$$

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="giac")
```

```
[Out] 1/7*c^4*x^7 + b*c^3*x^6 + 3*b^2*c^2*x^5 + 9/2*b^3*c*x^4 + 3/2*a*b*c^2*x^4 +
 3*b^4*x^3 + 6*a*b^2*c*x^3 + 9*a*b^3*x^2 + 9*a^2*b^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2 \, dx = x^3\,\left(3\,b^4+6\,a\,c\,b^2\right)+\frac{c^4\,x^7}{7}+9\,a^2\,b^2\,x+9\,a\,b^3\,x^2 + b\,c^3\,x^6+3\,b^2\,c^2\,x^5+\frac{3\,b\,c\,x^4\,\left(3\,b^2+a\,c\right)}{2}$$

```
[In] int((3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^2,x)
```

```
[Out] x^3*(3*b^4 + 6*a*b^2*c) + (c^4*x^7)/7 + 9*a^2*b^2*x + 9*a*b^3*x^2 + b*c^3*x
^6 + 3*b^2*c^2*x^5 + (3*b*c*x^4*(a*c + 3*b^2))/2
```

3.11 $\int (3ab+3b^2x+3bcx^2+c^2x^3)\ dx$

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Mathematica [A] (verified) . . . 200
Maple [A] (verified) . . . 200
Fricas [A] (verification not implemented) . . . 200
Sympy [A] (verification not implemented) . . . 201
Maxima [A] (verification not implemented) . . . 201
Giac [A] (verification not implemented) . . . 201
Mupad [B] (verification not implemented) . . . 201

Optimal result

Integrand size = 25, antiderivative size = 32

$$\int (3ab+3b^2x+3bcx^2+c^2x^3)\ dx = 3abx+\frac{3b^2x^2}{2}+bcx^3+\frac{c^2x^4}{4}$$

```
[Out] 3*a*b*x+3/2*b^2*x^2+b*c*x^3+1/4*c^2*x^4
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (3ab+3b^2x+3bcx^2+c^2x^3)\ dx = 3abx+\frac{3b^2x^2}{2}+bcx^3+\frac{c^2x^4}{4}$$

```
[In] Int[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3,x]
[Out] 3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4
```

Rubi steps

$$\text{integral} = 3abx+\frac{3b^2x^2}{2}+bcx^3+\frac{c^2x^4}{4}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \left(3ab + 3b^2x + 3bcx^2 + c^2x^3\right) \, dx = 3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

```
[In] Integrate[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3,x]
[Out] 3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
gosper	$3abx + \frac{3}{2}b^2x^2 + bc\,x^3 + \frac{1}{4}c^2x^4$	29
default	$3abx + \frac{3}{2}b^2x^2 + bc\,x^3 + \frac{1}{4}c^2x^4$	29
norman	$3abx + \frac{3}{2}b^2x^2 + bc\,x^3 + \frac{1}{4}c^2x^4$	29
risch	$3abx + \frac{3}{2}b^2x^2 + bc\,x^3 + \frac{1}{4}c^2x^4$	29
parallelrisch	$3abx + \frac{3}{2}b^2x^2 + bc\,x^3 + \frac{1}{4}c^2x^4$	29
parts	$3abx + \frac{3}{2}b^2x^2 + bc\,x^3 + \frac{1}{4}c^2x^4$	29

```
[In] int(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x,method=_RETURNVERBOSE)
[Out] 3*a*b*x+3/2*b^2*x^2+b*c*x^3+1/4*c^2*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(3ab + 3b^2x + 3bcx^2 + c^2x^3\right) \, dx = \frac{1}{4}\,c^2x^4 + bcx^3 + \frac{3}{2}\,b^2x^2 + 3\,abx$$

```
[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="fricas")
[Out] 1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \left(3ab + 3b^2x + 3bcx^2 + c^2x^3\right) dx = 3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

```
[In] integrate(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b,x)
[Out] 3*a*b*x + 3*b**2*x**2/2 + b*c*x**3 + c**2*x**4/4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(3ab + 3b^2x + 3bcx^2 + c^2x^3\right) dx = \frac{1}{4}c^2x^4 + bcx^3 + \frac{3}{2}b^2x^2 + 3\,abx$$

```
[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="maxima")
[Out] 1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(3ab + 3b^2x + 3bcx^2 + c^2x^3\right) dx = \frac{1}{4}c^2x^4 + bcx^3 + \frac{3}{2}b^2x^2 + 3\,abx$$

```
[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="giac")
[Out] 1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(3ab + 3b^2x + 3bcx^2 + c^2x^3\right) dx = \frac{3\,b^2\,x^2}{2} + b\,c\,x^3 + 3\,a\,b\,x + \frac{c^2\,x^4}{4}$$

```
[In] int(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2,x)
[Out] (3*b^2*x^2)/2 + (c^2*x^4)/4 + 3*a*b*x + b*c*x^3
```

3.12 $\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx$

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Optimal result

Integrand size = 27, antiderivative size = 188

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{b}+\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}b^{2/3}\left(b^2-3ac\right)^{2/3}}+\frac{\log\left(b-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{\log\left(b^{2/3}(b^2-3ac)^{2/3}+\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+c^2\left(\frac{b}{c}+x\right)^2\right)}{6b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

```
[Out] 1/3*ln(b-b^(1/3)*(-3*a*c+b^2)^(1/3)+c*x)/b^(2/3)/(-3*a*c+b^2)^(2/3)-1/6*ln(
b^(2/3)*(-3*a*c+b^2)^(2/3)+b^(1/3)*c*(-3*a*c+b^2)^(1/3)*(b/c+x)+c^2*(b/c+x)
^2)/b^(2/3)/(-3*a*c+b^2)^(2/3)-1/3*arctan(1/3*(b^(1/3)+2*(c*x+b)/(-3*a*c+b^
2)^(1/3))/b^(1/3)*3^(1/2))/b^(2/3)/(-3*a*c+b^2)^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used

= {2092, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = -\frac{\arctan\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}b^{2/3}\left(b^2-3ac\right)^{2/3}} - \frac{\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}(b^2-3ac)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{6b^{2/3}\left(b^2-3ac\right)^{2/3}} + \frac{\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(Sqrt[3]*b^(1/3))]/(
Sqrt[3]*b^(2/3)*(b^2 - 3*a*c)^(2/3))) + Log[b - b^(1/3)*(b^2 - 3*a*c)^(1/3)
 + c*x]/(3*b^(2/3)*(b^2 - 3*a*c)^(2/3)) - Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) +
 b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^2]/(6*b^(2/3)*(b^2
 - 3*a*c)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{b\left(3a-\frac{b^2}{c}\right)+c^2x^3}\,dx, x, \frac{b}{c}+x\right)\\
&= \frac{c^{2/3}\text{Subst}\left(\int \frac{1}{-\frac{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}+c^{2/3}x}\,dx, x, \frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}\\
&\quad+\frac{c^{2/3}\text{Subst}\left(\int \frac{-\frac{2\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}-c^{2/3}x}{\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}+\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}\,x+c^{4/3}x^2}\,dx, x, \frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}
\end{aligned}$$

$$= \frac{\log\left(\sqrt[3]{b}\left(b^{2/3} - \sqrt[3]{b^2 - 3ac}\right) + cx\right)}{3b^{2/3}\left(b^2 - 3ac\right)^{2/3}} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}+2c^{4/3}x}{\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}+\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}x+c^{4/3}x^2}dx, x, \frac{b}{c}+x\right)}{6b^{2/3}\left(b^2 - 3ac\right)^{2/3}} - \frac{\sqrt[3]{c}\text{Subst}\left(\int \frac{1}{\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}+\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}x+c^{4/3}x^2}dx, x, \frac{b}{c}+x\right)}{2\sqrt[3]{b}\sqrt[3]{b^2-3ac}}$$

$$= \frac{\log\left(\sqrt[3]{b}\left(b^{2/3} - \sqrt[3]{b^2 - 3ac}\right) + cx\right)}{3b^{2/3}\left(b^2 - 3ac\right)^{2/3}} - \frac{\log\left(b^{2/3}(b^2-3ac)^{2/3} + \sqrt[3]{b}\sqrt[3]{b^2-3ac}(b+cx) + (b+cx)^2\right)}{6b^{2/3}\left(b^2 - 3ac\right)^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1 + \frac{2c\left(\frac{b}{c}+x\right)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}\right)}{b^{2/3}\left(b^2 - 3ac\right)^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2(b+cx)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}\left(b^2 - 3ac\right)^{2/3}} + \frac{\log\left(\sqrt[3]{b}\left(b^{2/3} - \sqrt[3]{b^2 - 3ac}\right) + cx\right)}{3b^{2/3}\left(b^2 - 3ac\right)^{2/3}} - \frac{\log\left(b^{2/3}(b^2-3ac)^{2/3} + \sqrt[3]{b}\sqrt[3]{b^2-3ac}(b+cx) + (b+cx)^2\right)}{6b^{2/3}\left(b^2 - 3ac\right)^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.34

$$\int \frac{1}{3ab + 3b^2x + 3bcx^2 + c^2x^3}\,dx = \frac{1}{3}\text{RootSum}\left[3ab + 3b^2\#1 + 3bc\#1^2 + c^2\#1^3\&, \frac{\log(x - \#1)}{b^2 + 2bc\#1 + c^2\#1^2}\&\right]$$

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1),x]
[Out] RootSum[3*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b
*c*#1 + c^2*#1^2) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.30

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c^{2}\_Z^{3}+3b\_Z^{2}c+3b^{2}\_Z+3ab\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}c^{2}+2\_Rbc+b^{2}}\right)}{3}$	57
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c^{2}\_Z^{3}+3b\_Z^{2}c+3b^{2}\_Z+3ab\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}c^{2}+2\_Rbc+b^{2}}\right)}{3}$	57

```
[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*sum(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(x-_R),_R=RootOf(_Z^3*c^2+3*_Z^2*b*c+3*
_Z*b^2+3*a*b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(151) = 302.

Time = 0.29 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.06

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = -\frac{2\sqrt{3}(b^6-6ab^4c+9a^2b^2c^2)^{\frac{1}{6}}(b^3-3abc)\arctan\left(\frac{2\sqrt{3}(b^6-6ab^4c+9a^2b^2c^2)^{\frac{2}{3}}(cx+b)+\sqrt{3}(b^6-6ab^4c+9a^2b^2c^2)^{\frac{1}{3}}(b^3-3ab}{3(b^6-6ab^4c+9a^2b^2c^2)^{\frac{5}{6}}}\right.}{}$$

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/6)*(b^3 - 3*a*b*c)*arc
tan(1/3*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sqrt
(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 - 6*a*b^4
*c + 9*a^2*b^2*c^2)^(5/6)) + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b
^5 + 3*a*b^3*c - (b^3*c^2 - 3*a*b*c^3)*x^2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b
^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) - (b^6 - 6*a*b^4*c + 9*a^2*
b^2*c^2)^(1/3)*(b^3 - 3*a*b*c)) - 2*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)
*log(-b^4 + 3*a*b^2*c - (b^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a^2*b^
2*c^2)^(2/3)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.28

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = \text{RootSum}\left(t^3\cdot\left(243a^2b^2c^2-162ab^4c+27b^6\right)-1,\left(t\mapsto t\log\left(x+\frac{9tabc-3tb^3+b}{c}\right)\right)\right)$$

```
[In] integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b),x)
[Out] RootSum(_t**3*(243*a**2*b**2*c**2 - 162*a*b**4*c + 27*b**6) - 1, Lambda(_t,
 _t*log(x + (9*_t*a*b*c - 3*_t*b**3 + b)/c)))
```

Maxima [F]

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = \int \frac{1}{c^2x^3+3\,bcx^2+3\,b^2x+3\,ab}\,dx$$

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="maxima")
[Out] integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.13

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3\,abc\right)^{\frac{1}{3}}}{cx+b+\left(-b^3+3\,abc\right)^{\frac{1}{3}}}\right)}{3\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)^{\frac{1}{3}}} - \frac{\log\left(4\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3\,abc\right)^{\frac{1}{3}}\right)^2+4\left(cx+b+\left(-b^3+3\,abc\right)^{\frac{1}{3}}\right)^2\right)}{6\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)^{\frac{1}{3}}} + \frac{\log\left(\left|cx+b+\left(-b^3+3\,abc\right)^{\frac{1}{3}}\right|\right)}{3\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)^{\frac{1}{3}}}$$

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan((sqrt(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3
))/(c*x + b + (-b^3 + 3*a*b*c)^(1/3)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1
/3) - 1/6*log(4*(sqrt(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))^
2 + 4*(c*x + b + (-b^3 + 3*a*b*c)^(1/3))^2)/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^
2)^(1/3) + 1/3*log(abs(c*x + b + (-b^3 + 3*a*b*c)^(1/3)))/(b^6 - 6*a*b^4*c
+ 9*a^2*b^2*c^2)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.93

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = \frac{\ln\left(b+b^{1/3}\,(3\,a\,c-b^2)^{1/3}+c\,x\right)}{3\,b^{2/3}\,(3\,a\,c-b^2)^{2/3}} + \frac{\ln\left(3\,b\,c^3+3\,c^4\,x+\frac{3\,b^{1/3}\,c^3\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,(3\,a\,c-b^2)^{1/3}}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,b^{2/3}\,(3\,a\,c-b^2)^{2/3}} - \frac{\ln\left(3\,b\,c^3+3\,c^4\,x-\frac{3\,b^{1/3}\,c^3\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,(3\,a\,c-b^2)^{1/3}}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,b^{2/3}\,(3\,a\,c-b^2)^{2/3}}$$

```
[In] int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2),x)
```

```
[Out] log(b + b^(1/3)*(3*a*c - b^2)^(1/3) + c*x)/(3*b^(2/3)*(3*a*c - b^2)^(2/3))
+ (log(3*b*c^3 + 3*c^4*x + (3*b^(1/3)*c^3*(3^(1/2)*1i - 1)*(3*a*c - b^2)^(1
/3))/2)*(3^(1/2)*1i - 1))/(6*b^(2/3)*(3*a*c - b^2)^(2/3)) - (log(3*b*c^3 +
3*c^4*x - (3*b^(1/3)*c^3*(3^(1/2)*1i + 1)*(3*a*c - b^2)^(1/3))/2)*(3^(1/2)*
1i + 1))/(6*b^(2/3)*(3*a*c - b^2)^(2/3))
```

3.13 $\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx$

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Optimal result

Integrand size = 27, antiderivative size = 245

$$\begin{aligned}\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx &= -\frac{c\left(\frac{b}{c}+x\right)}{3b(b^2-3ac)(3ab+3b^2x+3bcx^2+c^2x^3)} \\ &\quad+\frac{2c\arctan\left(\frac{\sqrt[3]{b}+\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3}b^{5/3}(b^2-3ac)^{5/3}}-\frac{2c\log\left(b-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+cx\right)}{9b^{5/3}(b^2-3ac)^{5/3}} \\ &\quad+\frac{c\log\left(b^{2/3}(b^2-3ac)^{2/3}+\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+c^2\left(\frac{b}{c}+x\right)^2\right)}{9b^{5/3}(b^2-3ac)^{5/3}}\end{aligned}$$

```
[Out] -1/3*c*(b/c+x)/b/(-3*a*c+b^2)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)-2/9*c*ln(b-
b^(1/3)*(-3*a*c+b^2)^(1/3)+c*x)/b^(5/3)/(-3*a*c+b^2)^(5/3)+1/9*c*ln(b^(2/3)
*(-3*a*c+b^2)^(2/3)+b^(1/3)*c*(-3*a*c+b^2)^(1/3)*(b/c+x)+c^2*(b/c+x)^2)/b^(
5/3)/(-3*a*c+b^2)^(5/3)+2/9*c*arctan(1/3*(b^(1/3)+2*(c*x+b)/(-3*a*c+b^2)^(1
/3))/b^(1/3)*3^(1/2))/b^(5/3)/(-3*a*c+b^2)^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2092, 205, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx = \frac{2c\arctan\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3}b^{5/3}(b^2-3ac)^{5/3}}-\frac{c\left(\frac{b}{c}+x\right)}{3b(b^2-3ac)(3ab+3b^2x+3bcx^2+c^2x^3)}+\frac{c\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}(b^2-3ac)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{9b^{5/3}(b^2-3ac)^{5/3}}-\frac{2c\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{9b^{5/3}(b^2-3ac)^{5/3}}$$

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2),x]
```

```
[Out] -1/3*(c*(b/c + x))/(b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)
) + (2*c*ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(Sqrt[3]*b^(1
/3))])/(3*Sqrt[3]*b^(5/3)*(b^2 - 3*a*c)^(5/3)) - (2*c*Log[b - b^(1/3)*(b^2
- 3*a*c)^(1/3) + c*x]])/(9*b^(5/3)*(b^2 - 3*a*c)^(5/3)) + (c*Log[b^(2/3)*(b^
2 - 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^
2])/(9*b^(5/3)*(b^2 - 3*a*c)^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\left(b\left(3a-\frac{b^2}{c}\right)+c^2x^3\right)^2}\,dx, x, \frac{b}{c}+x\right)$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{(2c)\text{Subst}\left(\int \frac{1}{b\left(3a-\frac{b^2}{c}\right)+c^2x^3}\,dx, x, \frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)}$$

$$
= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{\left(2c^{5/3}\right)\text{Subst}\left(\int \frac{1}{-\frac{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}+c^{2/3}x}dx, x, \frac{b}{c}+x\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}} - \frac{\left(2c^{5/3}\right)\text{Subst}\left(\int \frac{-\frac{2\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}-c^{2/3}x}{\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}+\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}x+c^{4/3}x^2}dx, x, \frac{b}{c}+x\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}
$$

$$
= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{2c\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}} + \frac{c\text{Subst}\left(\int \frac{\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}+2c^{4/3}x}{\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}+\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}x+c^{4/3}x^2}dx, x, \frac{b}{c}+x\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}} + \frac{c^{4/3}\text{Subst}\left(\int \frac{1}{\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}+\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}x+c^{4/3}x^2}dx, x, \frac{b}{c}+x\right)}{3b^{4/3}\left(b^2-3ac\right)^{4/3}}
$$

$$
= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{2c\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}} + \frac{c\log\left(b^{2/3}(b^2-3ac)^{2/3}+\sqrt[3]{b}\sqrt[3]{b^2-3ac}(b+cx)+(b+cx)^2\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}} - \frac{(2c)\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1+\frac{2c\left(\frac{b}{c}+x\right)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}\right)}{3b^{5/3}\left(b^2-3ac\right)^{5/3}}
$$

$$
= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} + \frac{2c\tan^{-1}\left(\frac{1+\frac{2(b+cx)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}\left(b^2-3ac\right)^{5/3}} - \frac{2c\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}} + \frac{c\log\left(b^{2/3}(b^2-3ac)^{2/3}+\sqrt[3]{b}\sqrt[3]{b^2-3ac}(b+cx)+(b+cx)^2\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.46

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx = -\frac{\frac{3(b+cx)}{3ab+x(3b^2+3bcx+c^2x^2)}+2c\text{RootSum}\left[3ab+3b^2\#1+3bc\#1^2+c^2\#1^3\&,\frac{\log(x-\#1)}{b^2+2bc\#1+c^2\#1^2}\&\right]}{9(b^3-3abc)}$$

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2),x]
```

```
[Out] -1/9*((3*(b + c*x))/(3*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2)) + 2*c*RootSum[3
*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b*c*#1 + c
^2*#1^2) & ])/(b^3 - 3*a*b*c)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\frac{cx}{9b(3ac-b^2)}+\frac{1}{27ac-9b^2}}{\frac{1}{3}c^2x^3+bc\,x^2+b^2x+ab}+\frac{2c\left(\sum_{\_R=\text{RootOf}\left(c^2\_Z^3+3b\_Z^2c+3b^2\_Z+3ab\right)}\frac{\ln\left(x-\_R\right)}{\_R^2c^2+2\_Rbc+b^2}\right)}{9b(3ac-b^2)}$	134
risch	$\frac{\frac{cx}{9b(3ac-b^2)}+\frac{1}{27ac-9b^2}}{\frac{1}{3}c^2x^3+bc\,x^2+b^2x+ab}+\frac{2c\left(\sum_{\_R=\text{RootOf}\left(c^2\_Z^3+3b\_Z^2c+3b^2\_Z+3ab\right)}\frac{\ln\left(x-\_R\right)}{(3ac-b^2)\left(\_R^2c^2+2\_Rbc+b^2\right)}\right)}{9b}$	134

```
[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/9*c/b/(3*a*c-b^2)*x+1/9/(3*a*c-b^2))/(1/3*c^2*x^3+b*c*x^2+b^2*x+a*b)+2/9
*c/b/(3*a*c-b^2)*sum(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(x-_R),_R=RootOf(_Z^3*c^2+
3*_Z^2*b*c+3*_Z*b^2+3*a*b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 704 vs. 2(204) = 408.

Time = 0.28 (sec) , antiderivative size = 704, normalized size of antiderivative = 2.87

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx =$$

$$-\frac{3\,b^7-18\,ab^5c+27\,a^2b^3c^2-2\,\sqrt{3}(b^6-6\,ab^4c+9\,a^2b^2c^2)^{\frac{1}{6}}(3\,ab^4c-9\,a^2b^2c^2+(b^3c^3-3\,abc^4)x^3+3\,(b^4c^2}{}$$

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="fricas")
```

```
[Out] -1/9*(3*b^7 - 18*a*b^5*c + 27*a^2*b^3*c^2 - 2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*
a^2*b^2*c^2)^(1/6)*(3*a*b^4*c - 9*a^2*b^2*c^2 + (b^3*c^3 - 3*a*b*c^4)*x^3 +
 3*(b^4*c^2 - 3*a*b^2*c^3)*x^2 + 3*(b^5*c - 3*a*b^3*c^2)*x)*arctan(1/3*(2*s
qrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sqrt(3)*(b^6 - 6
*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 - 6*a*b^4*c + 9*a^2*b
^2*c^2)^(5/6)) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c^3*x^3 + 3*b*c^2
*x^2 + 3*b^2*c*x + 3*a*b*c)*log(-b^5 + 3*a*b^3*c - (b^3*c^2 - 3*a*b*c^3)*x^
2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*
x + b) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c)) + 2*(b^6
- 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + 3*a
*b*c)*log(-b^4 + 3*a*b^2*c - (b^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a
^2*b^2*c^2)^(2/3)) + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3)*x)/(3*a*b^10 -
 27*a^2*b^8*c + 81*a^3*b^6*c^2 - 81*a^4*b^4*c^3 + (b^9*c^2 - 9*a*b^7*c^3 +
27*a^2*b^5*c^4 - 27*a^3*b^3*c^5)*x^3 + 3*(b^10*c - 9*a*b^8*c^2 + 27*a^2*b^6
*c^3 - 27*a^3*b^4*c^4)*x^2 + 3*(b^11 - 9*a*b^9*c + 27*a^2*b^7*c^2 - 27*a^3*
b^5*c^3)*x)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.78

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx$$

$$= \frac{b+cx}{27a^2b^2c-9ab^4+x^3\cdot(9abc^3-3b^3c^2)+x^2\cdot(27ab^2c^2-9b^4c)+x\,(27ab^3c-9b^5)}$$

$$+\operatorname{RootSum}\left(t^3\cdot\left(177147a^5b^5c^5-295245a^4b^7c^4+196830a^3b^9c^3-65610a^2b^{11}c^2+10935ab^{13}c-729b^{15}\right)-\right.$$

```
[In] integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
```

```
[Out] (b + c*x)/(27*a**2*b**2*c - 9*a*b**4 + x**3*(9*a*b*c**3 - 3*b**3*c**2) + x*
*2*(27*a*b**2*c**2 - 9*b**4*c) + x*(27*a*b**3*c - 9*b**5)) + RootSum(_t**3*
```

```
(177147*a**5*b**5*c**5 - 295245*a**4*b**7*c**4 + 196830*a**3*b**9*c**3 - 65
610*a**2*b**11*c**2 + 10935*a*b**13*c - 729*b**15) - 8*c**3, Lambda(_t, _t*
log(x + (81*_t*a**2*b**2*c**2 - 54*_t*a*b**4*c + 9*_t*b**6 + 2*b*c)/(2*c**2
))))
```

Maxima [F]

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx = \int \frac{1}{(c^2x^3+3\,bcx^2+3\,b^2x+3\,ab)^2}\,dx$$

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="maxima")
```

```
[Out] -2/3*c*integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)/(b^3 - 3*a*b
*c) - 1/3*(c*x + b)/(3*a*b^4 - 9*a^2*b^2*c + (b^3*c^2 - 3*a*b*c^3)*x^3 + 3*
(b^4*c - 3*a*b^2*c^2)*x^2 + 3*(b^5 - 3*a*b^3*c)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.18

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx = -\frac{2\sqrt{3}\left(\frac{c^3}{b^6-6\,ab^4c+9\,a^2b^2c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}(-b^3+3\,abc)^{\frac{1}{3}}}{cx+b+(-b^3+3\,abc)^{\frac{1}{3}}}\right)-\left(\frac{c^3}{b^6-6\,ab^4c+9\,a^2b^2c^2}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}cx+\sqrt{3}b\cdots\right.\right.}{9\cdots} - \frac{cx+b}{3\,(c^2x^3+3\,bcx^2+3\,b^2x+3\,ab)(b^3-3\,abc)}$$

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*sqrt(3)*(c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*arctan((sqrt(
3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))/(c*x + b + (-b^3 + 3*a
*b*c)^(1/3))) - (c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(4*(sqrt(3
)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))^2 + 4*(c*x + b + (-b^3
+ 3*a*b*c)^(1/3))^2) + 2*(c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(
abs(c*x + b + (-b^3 + 3*a*b*c)^(1/3))))/(b^3 - 3*a*b*c) - 1/3*(c*x + b)/((c
^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)*(b^3 - 3*a*b*c))
```

Mupad [B] (verification not implemented)

Time = 10.17 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.01

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx$$
$$= \frac{\frac{1}{3\,(3\,a\,c-b^2)}+\frac{c\,x}{3\,b\,(3\,a\,c-b^2)}}{3\,b^2\,x+3\,b\,c\,x^2+3\,a\,b+c^2\,x^3}+\frac{2\,c\,\ln\left(b+b^{1/3}\,{\left(3\,a\,c-b^2\right)}^{1/3}+c\,x\right)}{9\,b^{5/3}\,{\left(3\,a\,c-b^2\right)}^{5/3}}$$
$$-\frac{\ln\left(2\,b-b^{1/3}\,{\left(3\,a\,c-b^2\right)}^{1/3}+2\,c\,x-\sqrt{3}\,b^{1/3}\,{\left(3\,a\,c-b^2\right)}^{1/3}\,1\mathrm{i}\right)\,\left(c+\sqrt{3}\,c\,1\mathrm{i}\right)}{9\,b^{5/3}\,{\left(3\,a\,c-b^2\right)}^{5/3}}$$
$$-\frac{\ln\left(2\,b-b^{1/3}\,{\left(3\,a\,c-b^2\right)}^{1/3}+2\,c\,x+\sqrt{3}\,b^{1/3}\,{\left(3\,a\,c-b^2\right)}^{1/3}\,1\mathrm{i}\right)\,\left(c-\sqrt{3}\,c\,1\mathrm{i}\right)}{9\,b^{5/3}\,{\left(3\,a\,c-b^2\right)}^{5/3}}$$

```
[In] int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^2,x)
```

```
[Out] (1/(3*(3*a*c - b^2)) + (c*x)/(3*b*(3*a*c - b^2)))/(3*a*b + 3*b^2*x + c^2*x^
3 + 3*b*c*x^2) + (2*c*log(b + b^(1/3)*(3*a*c - b^2)^(1/3) + c*x))/(9*b^(5/3
)*(3*a*c - b^2)^(5/3)) - (log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3) + 2*c*x - 3
^(1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*(c + 3^(1/2)*c*1i))/(9*b^(5/3)*(3*a*
c - b^2)^(5/3)) - (log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3) + 2*c*x + 3^(1/2)*
b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*(c - 3^(1/2)*c*1i))/(9*b^(5/3)*(3*a*c - b^2
)^(5/3))
```

3.14 $\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$

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Optimal result

Integrand size = 27, antiderivative size = 305

$$\begin{aligned}
&\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx \\
&= -\frac{c\left(\frac{b}{c}+x\right)}{6b\,(b^2-3ac)\,(3ab+3b^2x+3bcx^2+c^2x^3)^2} \\
&\quad+\frac{5c^2\left(\frac{b}{c}+x\right)}{18b^2\,(b^2-3ac)^2\,(3ab+3b^2x+3bcx^2+c^2x^3)} \\
&\quad-\frac{5c^2\arctan\left(\frac{\sqrt[3]{b}+\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}}{\sqrt{3}\sqrt[3]{b}}\right)}{9\sqrt{3}b^{8/3}\,(b^2-3ac)^{8/3}}+\frac{5c^2\log\left(b-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+cx\right)}{27b^{8/3}\,(b^2-3ac)^{8/3}} \\
&\quad-\frac{5c^2\log\left(b^{2/3}(b^2-3ac)^{2/3}+\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+c^2\left(\frac{b}{c}+x\right)^2\right)}{54b^{8/3}\,(b^2-3ac)^{8/3}}
\end{aligned}$$

```
[Out] -1/6*c*(b/c+x)/b/(-3*a*c+b^2)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2+5/18*c^2*
(b/c+x)/b^2/(-3*a*c+b^2)^2/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)+5/27*c^2*ln(b-
b^(1/3)*(-3*a*c+b^2)^(1/3)+c*x)/b^(8/3)/(-3*a*c+b^2)^(8/3)-5/54*c^2*ln(b^(2
/3)*(-3*a*c+b^2)^(2/3)+b^(1/3)*c*(-3*a*c+b^2)^(1/3)*(b/c+x)+c^2*(b/c+x)^2)/
b^(8/3)/(-3*a*c+b^2)^(8/3)-5/27*c^2*arctan(1/3*(b^(1/3)+2*(c*x+b)/(-3*a*c+b
^2)^(1/3))/b^(1/3)*3^(1/2))/b^(8/3)/(-3*a*c+b^2)^(8/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2092, 205, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$$
$$= -\frac{5c^2\arctan\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{9\sqrt{3}b^{8/3}(b^2-3ac)^{8/3}}+\frac{5c^2\left(\frac{b}{c}+x\right)}{18b^2(b^2-3ac)^2(3ab+3b^2x+3bcx^2+c^2x^3)}$$
$$-\frac{c\left(\frac{b}{c}+x\right)}{6b(b^2-3ac)(3ab+3b^2x+3bcx^2+c^2x^3)^2}+\frac{5c^2\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{27b^{8/3}(b^2-3ac)^{8/3}}$$
$$-\frac{5c^2\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}(b^2-3ac)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{54b^{8/3}(b^2-3ac)^{8/3}}$$

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3),x]
```

```
[Out] -1/6*(c*(b/c + x))/(b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)
^2) + (5*c^2*(b/c + x))/(18*b^2*(b^2 - 3*a*c)^2*(3*a*b + 3*b^2*x + 3*b*c*x^
2 + c^2*x^3)) - (5*c^2*ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))
/(Sqrt[3]*b^(1/3))])/(9*Sqrt[3]*b^(8/3)*(b^2 - 3*a*c)^(8/3)) + (5*c^2*Log[b
 - b^(1/3)*(b^2 - 3*a*c)^(1/3) + c*x]])/(27*b^(8/3)*(b^2 - 3*a*c)^(8/3)) - (
5*c^2*Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c
+ x) + c^2*(b/c + x)^2]])/(54*b^(8/3)*(b^2 - 3*a*c)^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\left(b\left(3a-\frac{b^2}{c}\right)+c^2x^3\right)^3}\,dx, x, \frac{b}{c}+x\right)$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} - \frac{(5c)\text{Subst}\left(\int \frac{1}{\left(b\left(3a-\frac{b^2}{c}\right)+c^2x^3\right)^2}\,dx, x, \frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b+cx)}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} + \frac{\left(5c^2\right)\text{Subst}\left(\int \frac{1}{b\left(3a-\frac{b^2}{c}\right)+c^2x^3}\,dx, x, \frac{b}{c}+x\right)}{9b^2\left(b^2-3ac\right)^2}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b+cx)}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} + \frac{\left(5c^{8/3}\right)\text{Subst}\left(\int \frac{1}{-\frac{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}+c^{2/3}x}\,dx, x, \frac{b}{c}+x\right)}{27b^{8/3}\left(b^2-3ac\right)^{8/3}} + \frac{\left(5c^{8/3}\right)\text{Subst}\left(\int \frac{-\frac{2\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}-c^{2/3}x}{\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}+\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}x+c^{4/3}x^2}\,dx, x, \frac{b}{c}+x\right)}{27b^{8/3}\left(b^2-3ac\right)^{8/3}}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b+cx)}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} + \frac{5c^2\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{27b^{8/3}\left(b^2-3ac\right)^{8/3}} - \frac{\left(5c^2\right)\text{Subst}\left(\int \frac{\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}+2c^{4/3}x}{\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}+\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}x+c^{4/3}x^2}\,dx, x, \frac{b}{c}+x\right)}{54b^{8/3}\left(b^2-3ac\right)^{8/3}} - \frac{\left(5c^{7/3}\right)\text{Subst}\left(\int \frac{1}{\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}+\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}x+c^{4/3}x^2}\,dx, x, \frac{b}{c}+x\right)}{18b^{7/3}\left(b^2-3ac\right)^{7/3}}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b+cx)}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} + \frac{5c^2\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{27b^{8/3}\left(b^2-3ac\right)^{8/3}} - \frac{5c^2\log\left(b^{2/3}(b^2-3ac)^{2/3}+\sqrt[3]{b}\sqrt[3]{b^2-3ac}(b+cx)+(b+cx)^2\right)}{54b^{8/3}\left(b^2-3ac\right)^{8/3}} + \frac{(5c^2)\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2c\left(\frac{b}{c}+x\right)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}\right)}{9b^{8/3}\left(b^2-3ac\right)^{8/3}}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b+cx)}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{5c^2\tan^{-1}\left(\frac{1+\frac{2(b+cx)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{8/3}\left(b^2-3ac\right)^{8/3}} + \frac{5c^2\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{27b^{8/3}\left(b^2-3ac\right)^{8/3}} - \frac{5c^2\log\left(b^{2/3}(b^2-3ac)^{2/3}+\sqrt[3]{b}\sqrt[3]{b^2-3ac}(b+cx)+(b+cx)^2\right)}{54b^{8/3}\left(b^2-3ac\right)^{8/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.49

$$\int\frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3}\,dx = \frac{-\frac{3(b+cx)\left(3b^3-15b^2cx-5c^3x^3-3bc(8a+5cx^2)\right)}{\left(3ab+x(3b^2+3bcx+c^2x^2)\right)^2}+10c^2\text{RootSum}\left[3ab+3b^2\#1+3bc\#1^2+c^2\#1^3\&,\frac{\log(x-\#1)}{b^2+2bc\#1+c^2\#1^2}\&\right]}{54\left(b^3-3abc\right)^2}$$

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3),x]
```

```
[Out] ((-3*(b + c*x)*(3*b^3 - 15*b^2*c*x - 5*c^3*x^3 - 3*b*c*(8*a + 5*c*x^2)))/(3
*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2))^2 + 10*c^2*RootSum[3*a*b + 3*b^2*#1 +
 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b*c*#1 + c^2*#1^2) & ])/(54
*(b^3 - 3*a*b*c)^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.91

method	result
risch	$\frac{\frac{5c^4x^4}{18\left(9a^2c^2-6a\,b^2c+b^4\right)b^2}+\frac{10c^3x^3}{9b\left(9a^2c^2-6a\,b^2c+b^4\right)}+\frac{5c^2x^2}{3\left(9a^2c^2-6a\,b^2c+b^4\right)}+\frac{2\left(2ac+b^2\right)cx}{3b\left(9a^2c^2-6a\,b^2c+b^4\right)}+\frac{9\left(8ac-b^2\right)}{486a^2c^2-324a\,b^2c+54b^4}}{\left(c^2x^3+3bc\,x^2+3b^2x+3ab\right)^2}+\frac{5c^2\left(\_R=\text{Roo}\right.}{}$
default	$\frac{\frac{5c^4x^4}{18\left(9a^2c^2-6a\,b^2c+b^4\right)b^2}+\frac{10c^3x^3}{9b\left(9a^2c^2-6a\,b^2c+b^4\right)}+\frac{5c^2x^2}{3\left(9a^2c^2-6a\,b^2c+b^4\right)}+\frac{2\left(2ac+b^2\right)cx}{3b\left(9a^2c^2-6a\,b^2c+b^4\right)}+\frac{9\left(8ac-b^2\right)}{486a^2c^2-324a\,b^2c+54b^4}}{\left(c^2x^3+3bc\,x^2+3b^2x+3ab\right)^2}+\frac{5c^2\left(\_R=\text{Roo}\right.}{}$

```
[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 9*(5/162*c^4/(9*a^2*c^2-6*a*b^2*c+b^4)/b^2*x^4+10/81/b*c^3/(9*a^2*c^2-6*a*b
^2*c+b^4)*x^3+5/27*c^2/(9*a^2*c^2-6*a*b^2*c+b^4)*x^2+2/27*(2*a*c+b^2)*c/b/(
9*a^2*c^2-6*a*b^2*c+b^4)*x+1/54*(8*a*c-b^2)/(9*a^2*c^2-6*a*b^2*c+b^4))/(c^2
*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2+5/27*c^2/b^2*sum(1/(9*a^2*c^2-6*a*b^2*c+b^4
)/(_R^2*c^2+2*_R*b*c+b^2)*ln(x-_R),_R=RootOf(_Z^3*c^2+3*_Z^2*b*c+3*_Z*b^2+3
*a*b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. 2(262) = 524.

Time = 0.28 (sec) , antiderivative size = 1268, normalized size of antiderivative = 4.16

$$\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="fricas")
```

```
[Out] -1/54*(9*b^10 - 126*a*b^8*c + 513*a^2*b^6*c^2 - 648*a^3*b^4*c^3 - 15*(b^6*c
^4 - 6*a*b^4*c^5 + 9*a^2*b^2*c^6)*x^4 - 60*(b^7*c^3 - 6*a*b^5*c^4 + 9*a^2*b
^3*c^5)*x^3 - 90*(b^8*c^2 - 6*a*b^6*c^3 + 9*a^2*b^4*c^4)*x^2 + 10*sqrt(3)*(
9*a^2*b^5*c^2 - 27*a^3*b^3*c^3 + (b^3*c^6 - 3*a*b*c^7)*x^6 + 6*(b^4*c^5 - 3
*a*b^2*c^6)*x^5 + 15*(b^5*c^4 - 3*a*b^3*c^5)*x^4 + 6*(3*b^6*c^3 - 8*a*b^4*c
^4 - 3*a^2*b^2*c^5)*x^3 + 9*(b^7*c^2 - a*b^5*c^3 - 6*a^2*b^3*c^4)*x^2 + 18*
(a*b^6*c^2 - 3*a^2*b^4*c^3)*x)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/6)*arct
an(1/3*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sqrt(
3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 - 6*a*b^4*
c + 9*a^2*b^2*c^2)^(5/6)) + 5*(c^6*x^6 + 6*b*c^5*x^5 + 15*b^2*c^4*x^4 + 18*
a*b^3*c^2*x + 9*a^2*b^2*c^2 + 6*(3*b^3*c^3 + a*b*c^4)*x^3 + 9*(b^4*c^2 + 2*
a*b^2*c^3)*x^2)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b^5 + 3*a*b^3*
```

```
c - (b^3*c^2 - 3*a*b*c^3)*x^2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b^6 - 6*a*b^4*
c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3
)*(b^3 - 3*a*b*c)) - 10*(c^6*x^6 + 6*b*c^5*x^5 + 15*b^2*c^4*x^4 + 18*a*b^3*
c^2*x + 9*a^2*b^2*c^2 + 6*(3*b^3*c^3 + a*b*c^4)*x^3 + 9*(b^4*c^2 + 2*a*b^2*
c^3)*x^2)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b^4 + 3*a*b^2*c - (b
^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)) - 36*(b^9*c
- 4*a*b^7*c^2 - 3*a^2*b^5*c^3 + 18*a^3*b^3*c^4)*x)/(9*a^2*b^14 - 108*a^3*b^
12*c + 486*a^4*b^10*c^2 - 972*a^5*b^8*c^3 + 729*a^6*b^6*c^4 + (b^12*c^4 - 1
2*a*b^10*c^5 + 54*a^2*b^8*c^6 - 108*a^3*b^6*c^7 + 81*a^4*b^4*c^8)*x^6 + 6*(
b^13*c^3 - 12*a*b^11*c^4 + 54*a^2*b^9*c^5 - 108*a^3*b^7*c^6 + 81*a^4*b^5*c^
7)*x^5 + 15*(b^14*c^2 - 12*a*b^12*c^3 + 54*a^2*b^10*c^4 - 108*a^3*b^8*c^5 +
 81*a^4*b^6*c^6)*x^4 + 6*(3*b^15*c - 35*a*b^13*c^2 + 150*a^2*b^11*c^3 - 270
*a^3*b^9*c^4 + 135*a^4*b^7*c^5 + 81*a^5*b^5*c^6)*x^3 + 9*(b^16 - 10*a*b^14*
c + 30*a^2*b^12*c^2 - 135*a^4*b^8*c^4 + 162*a^5*b^6*c^5)*x^2 + 18*(a*b^15 -
 12*a^2*b^13*c + 54*a^3*b^11*c^2 - 108*a^4*b^9*c^3 + 81*a^5*b^7*c^4)*x)
```

Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.55

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$$

$$= \frac{}{1458a^4b^4c^2-972a^3b^6c+162a^2b^8+x^6\cdot(162a^2b^2c^6-108ab^4c^5+18b^6c^4)+x^5\cdot(972a^2b^3c^5-648ab^5c^4+1}$$
$$+\mathrm{RootSum}\left(t^3\cdot\left(129140163a^8b^8c^8-344373768a^7b^{10}c^7+401769396a^6b^{12}c^6-267846264a^5b^{14}c^5+11160\right.\right.$$

```
[In] integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

```
[Out] (24*a*b**2*c - 3*b**4 + 30*b**2*c**2*x**2 + 20*b*c**3*x**3 + 5*c**4*x**4 +
x*(24*a*b*c**2 + 12*b**3*c))/(1458*a**4*b**4*c**2 - 972*a**3*b**6*c + 162*a
**2*b**8 + x**6*(162*a**2*b**2*c**6 - 108*a*b**4*c**5 + 18*b**6*c**4) + x**
5*(972*a**2*b**3*c**5 - 648*a*b**5*c**4 + 108*b**7*c**3) + x**4*(2430*a**2*
b**4*c**4 - 1620*a*b**6*c**3 + 270*b**8*c**2) + x**3*(972*a**3*b**3*c**4 +
2268*a**2*b**5*c**3 - 1836*a*b**7*c**2 + 324*b**9*c) + x**2*(2916*a**3*b**4
*c**3 - 486*a**2*b**6*c**2 - 648*a*b**8*c + 162*b**10) + x*(2916*a**3*b**5*
c**2 - 1944*a**2*b**7*c + 324*a*b**9)) + RootSum(_t**3*(129140163*a**8*b**8
*c**8 - 344373768*a**7*b**10*c**7 + 401769396*a**6*b**12*c**6 - 267846264*a
**5*b**14*c**5 + 111602610*a**4*b**16*c**4 - 29760696*a**3*b**18*c**3 + 496
0116*a**2*b**20*c**2 - 472392*a*b**22*c + 19683*b**24) - 125*c**6, Lambda(_
t, _t*log(x + (729*_t*a**3*b**3*c**3 - 729*_t*a**2*b**5*c**2 + 243*_t*a*b**
7*c - 27*_t*b**9 + 5*b*c**2)/(5*c**3))))
```

Maxima [F]

$$\int \frac{1}{(3ab + 3b^2x + 3bcx^2 + c^2x^3)^3} \, dx = \int \frac{1}{(c^2x^3 + 3\,bcx^2 + 3\,b^2x + 3\,ab)^3} \, dx$$

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="maxima")
```

```
[Out] 5/9*c^2*integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)/(b^6 - 6*a*
b^4*c + 9*a^2*b^2*c^2) + 1/18*(5*c^4*x^4 + 20*b*c^3*x^3 + 30*b^2*c^2*x^2 -
3*b^4 + 24*a*b^2*c + 12*(b^3*c + 2*a*b*c^2)*x)/(9*a^2*b^8 - 54*a^3*b^6*c +
81*a^4*b^4*c^2 + (b^6*c^4 - 6*a*b^4*c^5 + 9*a^2*b^2*c^6)*x^6 + 6*(b^7*c^3 -
 6*a*b^5*c^4 + 9*a^2*b^3*c^5)*x^5 + 15*(b^8*c^2 - 6*a*b^6*c^3 + 9*a^2*b^4*c
^4)*x^4 + 6*(3*b^9*c - 17*a*b^7*c^2 + 21*a^2*b^5*c^3 + 9*a^3*b^3*c^4)*x^3 +
 9*(b^10 - 4*a*b^8*c - 3*a^2*b^6*c^2 + 18*a^3*b^4*c^3)*x^2 + 18*(a*b^9 - 6*
a^2*b^7*c + 9*a^3*b^5*c^2)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.20

$$\int \frac{1}{(3ab + 3b^2x + 3bcx^2 + c^2x^3)^3} \, dx = \frac{5\left(2\sqrt{3}\left(\frac{c^6}{b^6 - 6\,ab^4c + 9\,a^2b^2c^2}\right)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}cx + \sqrt{3}b - \sqrt{3}(-b^3 + 3\,abc)^{\frac{1}{3}}}{cx + b + (-b^3 + 3\,abc)^{\frac{1}{3}}}\right) - \left(\frac{c^6}{b^6 - 6\,ab^4c + 9\,a^2b^2c^2}\right)^{\frac{1}{3}} \log\left(4\left(\sqrt{3}cx + \sqrt{3}b - \cdots\right.\right.\right.}{54\,(b^6 - 6 \cdots} + \frac{5\,c^4x^4 + 20\,bc^3x^3 + 30\,b^2c^2x^2 + 12\,b^3cx + 24\,abc^2x - 3\,b^4 + 24\,ab^2c}{18\,(b^6 - 6\,ab^4c + 9\,a^2b^2c^2)(c^2x^3 + 3\,bcx^2 + 3\,b^2x + 3\,ab)^2}$$

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="giac")
```

```
[Out] 5/54*(2*sqrt(3)*(c^6/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*arctan((sqrt(
3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))/(c*x + b + (-b^3 + 3*a
*b*c)^(1/3))) - (c^6/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(4*(sqrt(3
)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))^2 + 4*(c*x + b + (-b^3
+ 3*a*b*c)^(1/3))^2) + 2*(c^6/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(
abs(c*x + b + (-b^3 + 3*a*b*c)^(1/3))))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2) +
 1/18*(5*c^4*x^4 + 20*b*c^3*x^3 + 30*b^2*c^2*x^2 + 12*b^3*c*x + 24*a*b*c^2*
x - 3*b^4 + 24*a*b^2*c)/((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)*(c^2*x^3 + 3*b*c
*x^2 + 3*b^2*x + 3*a*b)^2)
```

Mupad [B] (verification not implemented)

Time = 10.19 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.58

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$$
$$= \frac{\frac{8\,a\,c-b^2}{6\,(9\,a^2\,c^2-6\,a\,b^2\,c+b^4)}+\frac{5\,c^2\,x^2}{3\,(9\,a^2\,c^2-6\,a\,b^2\,c+b^4)}+\frac{10\,c^3\,x^3}{9\,b\,(9\,a^2\,c^2-6\,a\,b^2\,c+b^4)}+\frac{5\,c^4\,x^4}{18\,b^2\,(9\,a^2\,c^2-6\,a\,b^2\,c+b^4)}+\frac{2\,c\,x\,(b^2+2\,a\,c)}{3\,b\,(9\,a^2\,c^2-6\,a\,b^2\,c+b^4)}}{x^2\,(9\,b^4+18\,a\,c\,b^2)+9\,a^2\,b^2+c^4\,x^6+x^3\,(18\,b^3\,c+6\,a\,b\,c^2)+6\,b\,c^3\,x^5+15\,b^2\,c^2\,x^4+18\,a\,b^3\,x}$$
$$+\frac{5\,c^2\,\ln\left(b\,(3\,a\,c-b^2)^{8/3}-b^{19/3}+c\,x\,(3\,a\,c-b^2)^{8/3}+27\,a^3\,b^{1/3}\,c^3-27\,a^2\,b^{7/3}\,c^2+9\,a\,b^{13/3}\,c\right)}{27\,b^{8/3}\,(3\,a\,c-b^2)^{8/3}}$$
$$-\frac{5\,c^2\,\ln\left(2\,b-b^{1/3}\,(3\,a\,c-b^2)^{1/3}+2\,c\,x-\sqrt{3}\,b^{1/3}\,(3\,a\,c-b^2)^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27\,b^{8/3}\,(3\,a\,c-b^2)^{8/3}}$$
$$+\frac{5\,c^2\,\ln\left(2\,b-b^{1/3}\,(3\,a\,c-b^2)^{1/3}+2\,c\,x+\sqrt{3}\,b^{1/3}\,(3\,a\,c-b^2)^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27\,b^{8/3}\,(3\,a\,c-b^2)^{8/3}}$$

```
[In] int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^3,x)
```

```
[Out] ((8*a*c - b^2)/(6*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)) + (5*c^2*x^2)/(3*(b^4 + 9*
a^2*c^2 - 6*a*b^2*c)) + (10*c^3*x^3)/(9*b*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)) +
(5*c^4*x^4)/(18*b^2*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)) + (2*c*x*(2*a*c + b^2))/
(3*b*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)))/(x^2*(9*b^4 + 18*a*b^2*c) + 9*a^2*b^2
+ c^4*x^6 + x^3*(18*b^3*c + 6*a*b*c^2) + 6*b*c^3*x^5 + 15*b^2*c^2*x^4 + 18*
a*b^3*x) + (5*c^2*log(b*(3*a*c - b^2)^(8/3) - b^(19/3) + c*x*(3*a*c - b^2)^
(8/3) + 27*a^3*b^(1/3)*c^3 - 27*a^2*b^(7/3)*c^2 + 9*a*b^(13/3)*c))/(27*b^(8
/3)*(3*a*c - b^2)^(8/3)) - (5*c^2*log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3) + 2
*c*x - 3^(1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2))/(27*
b^(8/3)*(3*a*c - b^2)^(8/3)) + (5*c^2*log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3)
 + 2*c*x + 3^(1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2))/
(27*b^(8/3)*(3*a*c - b^2)^(8/3))
```

3.15 $\int (ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+$

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Optimal result

Integrand size = 46, antiderivative size = 361

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3\,dx = \frac{(bc-ad)^3(be-af)^3(a+bx)^4}{4b^7}+\frac{3(bc-ad)^2(be-af)^2(bde+bcf-2adf)(a+bx)^5}{5b^7} + \frac{(bc-ad)(be-af)\left(5a^2d^2f^2-5abdf(de+cf)+b^2(d^2e^2+3cdef+c^2f^2)\right)(a+bx)^6}{2b^7} + \frac{(bde+bcf-2adf)\left(10a^2d^2f^2-10abdf(de+cf)+b^2(d^2e^2+8cdef+c^2f^2)\right)(a+bx)^7}{7b^7} + \frac{3df\left(5a^2d^2f^2-5abdf(de+cf)+b^2(d^2e^2+3cdef+c^2f^2)\right)(a+bx)^8}{8b^7} + \frac{d^2f^2(bde+bcf-2adf)(a+bx)^9}{3b^7}+\frac{d^3f^3(a+bx)^{10}}{10b^7}$$

```
[Out] 1/4*(-a*d+b*c)^3*(-a*f+b*e)^3*(b*x+a)^4/b^7+3/5*(-a*d+b*c)^2*(-a*f+b*e)^2*(
-2*a*d*f+b*c*f+b*d*e)*(b*x+a)^5/b^7+1/2*(-a*d+b*c)*(-a*f+b*e)*(5*a^2*d^2*f^
2-5*a*b*d*f*(c*f+d*e)+b^2*(c^2*f^2+3*c*d*e*f+d^2*e^2))*(b*x+a)^6/b^7+1/7*(-
2*a*d*f+b*c*f+b*d*e)*(10*a^2*d^2*f^2-10*a*b*d*f*(c*f+d*e)+b^2*(c^2*f^2+8*c*
d*e*f+d^2*e^2))*(b*x+a)^7/b^7+3/8*d*f*(5*a^2*d^2*f^2-5*a*b*d*f*(c*f+d*e)+b^
2*(c^2*f^2+3*c*d*e*f+d^2*e^2))*(b*x+a)^8/b^7+1/3*d^2*f^2*(-2*a*d*f+b*c*f+b*
d*e)*(b*x+a)^9/b^7+1/10*d^3*f^3*(b*x+a)^10/b^7
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2084, 90}

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3\,dx$$
$$=\frac{3df(a+bx)^8\left(5a^2d^2f^2-5abdf(cf+de)+b^2(c^2f^2+3cdef+d^2e^2)\right)}{8b^7}$$
$$+\frac{(a+bx)^7(-2adf+bcf+bde)\left(10a^2d^2f^2-10abdf(cf+de)+b^2(c^2f^2+8cdef+d^2e^2)\right)}{7b^7}$$
$$+\frac{(a+bx)^6(bc-ad)(be-af)\left(5a^2d^2f^2-5abdf(cf+de)+b^2(c^2f^2+3cdef+d^2e^2)\right)}{2b^7}$$
$$+\frac{d^2f^2(a+bx)^9(-2adf+bcf+bde)}{3b^7}$$
$$+\frac{3(a+bx)^5(bc-ad)^2(be-af)^2(-2adf+bcf+bde)}{5b^7}$$
$$+\frac{(a+bx)^4(bc-ad)^3(be-af)^3}{4b^7}+\frac{d^3f^3(a+bx)^{10}}{10b^7}$$

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^3,x]
```

```
[Out] ((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^4)/(4*b^7) + (3*(b*c - a*d)^2*(b*e -
 a*f)^2*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^5)/(5*b^7) + ((b*c - a*d)*(b*e
- a*f)*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 3*c*d*e*f +
c^2*f^2))*(a + b*x)^6)/(2*b^7) + ((b*d*e + b*c*f - 2*a*d*f)*(10*a^2*d^2*f^2
 - 10*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 8*c*d*e*f + c^2*f^2))*(a + b*x)^
7)/(7*b^7) + (3*d*f*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2))*(a + b*x)^8)/(8*b^7) + (d^2*f^2*(b*d*e + b*c*f - 2*a
*d*f)*(a + b*x)^9)/(3*b^7) + (d^3*f^3*(a + b*x)^10)/(10*b^7)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2084

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (a+bx)^3(c+dx)^3(e+fx)^3\,dx\\
&= \int \left(\frac{(bc-ad)^3(be-af)^3(a+bx)^3}{b^6} + \frac{3(bc-ad)^2(be-af)^2(bde+bcf-2adf)(a+bx)^4}{b^6}\right.\\
&\quad + \frac{3(bc-ad)(be-af)\left(5a^2d^2f^2-5abdf(de+cf)+b^2(d^2e^2+3cdef+c^2f^2)\right)(a+bx)^5}{b^6}\\
&\quad + \frac{(bde+bcf-2adf)\left(b^2d^2e^2+8b^2cdef-10abd^2ef+b^2c^2f^2-10abcdf^2+10a^2d^2f^2\right)(a+bx)^6}{b^6}\\
&\quad + \frac{3df\left(5a^2d^2f^2-5abdf(de+cf)+b^2(d^2e^2+3cdef+c^2f^2)\right)(a+bx)^7}{b^6}\\
&\quad \left. + \frac{3d^2f^2(bde+bcf-2adf)(a+bx)^8}{b^6} + \frac{d^3f^3(a+bx)^9}{b^6}\right)dx\\
&= \frac{(bc-ad)^3(be-af)^3(a+bx)^4}{4b^7} + \frac{3(bc-ad)^2(be-af)^2(bde+bcf-2adf)(a+bx)^5}{5b^7}\\
&\quad + \frac{(bc-ad)(be-af)\left(5a^2d^2f^2-5abdf(de+cf)+b^2(d^2e^2+3cdef+c^2f^2)\right)(a+bx)^6}{2b^7}\\
&\quad + \frac{(bde+bcf-2adf)\left(10a^2d^2f^2-10abdf(de+cf)+b^2(d^2e^2+8cdef+c^2f^2)\right)(a+bx)^7}{7b^7}\\
&\quad + \frac{3df\left(5a^2d^2f^2-5abdf(de+cf)+b^2(d^2e^2+3cdef+c^2f^2)\right)(a+bx)^8}{8b^7}\\
&\quad + \frac{d^2f^2(bde+bcf-2adf)(a+bx)^9}{3b^7} + \frac{d^3f^3(a+bx)^{10}}{10b^7}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.81

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3 \, dx$$
$$= a^3c^3e^3x+\frac{3}{2}a^2c^2e^2(bce+ade+acf)x^2 + ace\left(b^2c^2e^2+3abce(de+cf)+a^2\left(d^2e^2+3cdef+c^2f^2\right)\right)x^3+\frac{1}{4}\left(b^3c^3e^3+9ab^2c^2e^2(de+cf) + 9a^2bce\left(d^2e^2+3cdef+c^2f^2\right)+a^3\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^4 + \frac{3}{5}\left(b^3c^2e^2(de+cf)+3ab^2ce\left(d^2e^2+3cdef+c^2f^2\right)+a^3df\left(d^2e^2+3cdef+c^2f^2\right) + a^2b\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^5 + \frac{1}{2}\left(a^3d^2f^2(de+cf)+b^3ce\left(d^2e^2+3cdef+c^2f^2\right)+3a^2bdf\left(d^2e^2+3cdef+c^2f^2\right) + ab^2\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^6+\frac{1}{7}\left(a^3d^3f^3+9a^2bd^2f^2(de+cf) + 9ab^2df\left(d^2e^2+3cdef+c^2f^2\right)+b^3\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^7 + \frac{3}{8}bdf\left(a^2d^2f^2+3abdf(de+cf)+b^2\left(d^2e^2+3cdef+c^2f^2\right)\right)x^8 + \frac{1}{3}b^2d^2f^2(bde+bcf+adf)x^9+\frac{1}{10}b^3d^3f^3x^{10}$$

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^3,x]
```

```
[Out] a^3*c^3*e^3*x + (3*a^2*c^2*e^2*(b*c*e + a*d*e + a*c*f)*x^2)/2 + a*c*e*(b^2*
c^2*e^2 + 3*a*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^3
+ ((b^3*c^3*e^3 + 9*a*b^2*c^2*e^2*(d*e + c*f) + 9*a^2*b*c*e*(d^2*e^2 + 3*c*
d*e*f + c^2*f^2) + a^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))
*x^4)/4 + (3*(b^3*c^2*e^2*(d*e + c*f) + 3*a*b^2*c*e*(d^2*e^2 + 3*c*d*e*f +
c^2*f^2) + a^3*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a^2*b*(d^3*e^3 + 9*c*d
^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^5)/5 + ((a^3*d^2*f^2*(d*e + c*f) + b
^3*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + 3*a^2*b*d*f*(d^2*e^2 + 3*c*d*e*f +
 c^2*f^2) + a*b^2*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^6)
/2 + ((a^3*d^3*f^3 + 9*a^2*b*d^2*f^2*(d*e + c*f) + 9*a*b^2*d*f*(d^2*e^2 + 3
*c*d*e*f + c^2*f^2) + b^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^
3))*x^7)/7 + (3*b*d*f*(a^2*d^2*f^2 + 3*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2))*x^8)/8 + (b^2*d^2*f^2*(b*d*e + b*c*f + a*d*f)*x^9)/3
 + (b^3*d^3*f^3*x^10)/10
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 841 vs. $2(347) = 694$.

Time = 0.05 (sec) , antiderivative size = 842, normalized size of antiderivative = 2.33

method	result
norman	$\frac{d^{3} f^{3} b^{3} x^{10}}{10}+\left(\frac{1}{3} d^{3} f^{3} b^{2} a+\frac{1}{3} b^{3} c\, d^{2} f^{3}+\frac{1}{3} b^{3} d^{3} e\, f^{2}\right) x^{9}+\left(\frac{3}{8} d^{3} f^{3} a^{2} b+\frac{9}{8} a\, b^{2} c\, d^{2} f^{3}+\frac{9}{8} a\, b^{2} d^{3} e\, f^{2}+\right.$
default	$\frac{d^{3} f^{3} b^{3} x^{10}}{10}+\frac{(adf+fbc+bde) b^{2} d^{2} f^{2} x^{9}}{3}+\frac{\left((acf+eda+ebc) b^{2} d^{2} f^{2}+2(adf+fbc+bde)^{2} bdf+bdf\left(2(acf+eda+ebc) bdf+(adf+\right.\right.}{8}$
gosper	$\frac{x\left(84 d^{3} f^{3} b^{3} x^{9}+280 x^{8} d^{3} f^{3} b^{2} a+280 x^{8} b^{3} c\, d^{2} f^{3}+280 x^{8} b^{3} d^{3} e\, f^{2}+315 x^{7} d^{3} f^{3} a^{2} b+945 x^{7} a\, b^{2} c\, d^{2} f^{3}+945 x^{7} a\, b^{2} d^{3} e\, f^{2}+315 x^{7} b^{3} c^{2}\right.}{}$
risch	$\frac{9}{8} x^{8} a\, b^{2} c\, d^{2} f^{3}+\frac{9}{8} x^{8} a\, b^{2} d^{3} e\, f^{2}+\frac{9}{8} x^{8} b^{3} c\, d^{2} e\, f^{2}+\frac{9}{7} x^{7} a^{2} b c\, d^{2} f^{3}+\frac{9}{7} x^{7} a^{2} b\, d^{3} e\, f^{2}+\frac{9}{7} x^{7} b^{3} c\, d^{2} e^{2} f$
parallelrisch	$\frac{9}{8} x^{8} a\, b^{2} c\, d^{2} f^{3}+\frac{9}{8} x^{8} a\, b^{2} d^{3} e\, f^{2}+\frac{9}{8} x^{8} b^{3} c\, d^{2} e\, f^{2}+\frac{9}{7} x^{7} a^{2} b c\, d^{2} f^{3}+\frac{9}{7} x^{7} a^{2} b\, d^{3} e\, f^{2}+\frac{9}{7} x^{7} b^{3} c\, d^{2} e^{2} f$

```
[In] int((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x,met
hod=_RETURNVERBOSE)
```

```
[Out] 1/10*d^3*f^3*b^3*x^10+(1/3*d^3*f^3*b^2*a+1/3*b^3*c*d^2*f^3+1/3*b^3*d^3*e*f^
2)*x^9+(3/8*d^3*f^3*a^2*b+9/8*a*b^2*c*d^2*f^3+9/8*a*b^2*d^3*e*f^2+3/8*b^3*c
^2*d*f^3+9/8*b^3*c*d^2*e*f^2+3/8*b^3*d^3*e^2*f)*x^8+(1/7*d^3*f^3*a^3+9/7*a^
2*b*c*d^2*f^3+9/7*a^2*b*d^3*e*f^2+9/7*a*b^2*c^2*d*f^3+27/7*a*b^2*c*d^2*e*f^
2+9/7*a*b^2*d^3*e^2*f+1/7*b^3*c^3*f^3+9/7*b^3*c^2*d*e*f^2+9/7*b^3*c*d^2*e^2
*f+1/7*b^3*d^3*e^3)*x^7+(1/2*a^3*c*d^2*f^3+1/2*a^3*d^3*e*f^2+3/2*a^2*b*c^2*
d*f^3+9/2*a^2*b*c*d^2*e*f^2+3/2*a^2*b*d^3*e^2*f+1/2*a*b^2*c^3*f^3+9/2*a*b^2
*c^2*d*e*f^2+9/2*a*b^2*c*d^2*e^2*f+1/2*a*b^2*d^3*e^3+1/2*b^3*c^3*e*f^2+3/2*
b^3*c^2*d*e^2*f+1/2*b^3*c*d^2*e^3)*x^6+(3/5*a^3*c^2*d*f^3+9/5*a^3*c*d^2*e*f
^2+3/5*a^3*d^3*e^2*f+3/5*a^2*b*c^3*f^3+27/5*a^2*b*c^2*d*e*f^2+27/5*a^2*b*c*
d^2*e^2*f+3/5*a^2*b*d^3*e^3+9/5*a*b^2*c^3*e*f^2+27/5*a*b^2*c^2*d*e^2*f+9/5*
a*b^2*c*d^2*e^3+3/5*b^3*c^3*e^2*f+3/5*b^3*c^2*d*e^3)*x^5+(1/4*a^3*c^3*f^3+9
/4*a^3*c^2*d*e*f^2+9/4*a^3*c*d^2*e^2*f+1/4*a^3*d^3*e^3+9/4*a^2*b*c^3*e*f^2+
27/4*a^2*b*c^2*d*e^2*f+9/4*a^2*b*c*d^2*e^3+9/4*a*b^2*c^3*e^2*f+9/4*a*b^2*c^
2*d*e^3+1/4*c^3*e^3*b^3)*x^4+(a^3*c^3*e*f^2+3*a^3*c^2*d*e^2*f+a^3*c*d^2*e^3
+3*a^2*b*c^3*e^2*f+3*a^2*b*c^2*d*e^3+a*b^2*c^3*e^3)*x^3+(3/2*a^3*c^3*e^2*f+
3/2*a^3*c^2*d*e^3+3/2*c^3*e^3*a^2*b)*x^2+c^3*e^3*a^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. 2(347) = 694.

Time = 0.25 (sec) , antiderivative size = 727, normalized size of antiderivative = 2.01

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3\,dx$$
$$=\frac{1}{10}b^3d^3f^3x^{10}+a^3c^3e^3x+\frac{1}{3}\left(b^3d^3ef^2+\left(b^3cd^2+ab^2d^3\right)f^3\right)x^9$$
$$+\frac{3}{8}\left(b^3d^3e^2f+3\left(b^3cd^2+ab^2d^3\right)ef^2+\left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)f^3\right)x^8$$
$$+\frac{1}{7}\left(b^3d^3e^3+9\left(b^3cd^2+ab^2d^3\right)e^2f+9\left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)ef^2+\left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d\right.\right.$$
$$+\frac{1}{2}\left(\left(b^3cd^2+ab^2d^3\right)e^3+3\left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)e^2f+\left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)ef^2+\left(ab\right.\right.$$
$$+\frac{3}{5}\left(\left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)e^3+\left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)e^2f+3\left(ab^2c^3+3\,a^2bc^2d+a^3cd^2\right.\right.$$
$$+\frac{1}{4}\left(a^3c^3f^3+\left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)e^3+9\left(ab^2c^3+3\,a^2bc^2d+a^3cd^2\right)e^2f+9\left(a^2bc^3+a^3c^2d\right.\right.$$
$$+\left(a^3c^3ef^2+\left(ab^2c^3+3\,a^2bc^2d+a^3cd^2\right)e^3+3\left(a^2bc^3+a^3c^2d\right)e^2f\right)x^3$$
$$+\frac{3}{2}\left(a^3c^3e^2f+\left(a^2bc^3+a^3c^2d\right)e^3\right)x^2$$

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3
,x, algorithm="fricas")
```

```
[Out] 1/10*b^3*d^3*f^3*x^10 + a^3*c^3*e^3*x + 1/3*(b^3*d^3*e*f^2 + (b^3*c*d^2 + a
*b^2*d^3)*f^3)*x^9 + 3/8*(b^3*d^3*e^2*f + 3*(b^3*c*d^2 + a*b^2*d^3)*e*f^2 +
 (b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^3)*x^8 + 1/7*(b^3*d^3*e^3 + 9*(b
^3*c*d^2 + a*b^2*d^3)*e^2*f + 9*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e*f
^2 + (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3)*x^7 + 1/2*((b
^3*c*d^2 + a*b^2*d^3)*e^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^2*f
 + (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e*f^2 + (a*b^2*c^3 +
 3*a^2*b*c^2*d + a^3*c*d^2)*f^3)*x^6 + 3/5*((b^3*c^2*d + 3*a*b^2*c*d^2 + a^
2*b*d^3)*e^3 + (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^2*f +
3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e*f^2 + (a^2*b*c^3 + a^3*c^2*d)*f
^3)*x^5 + 1/4*(a^3*c^3*f^3 + (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3
*d^3)*e^3 + 9*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e^2*f + 9*(a^2*b*c^3
+ a^3*c^2*d)*e*f^2)*x^4 + (a^3*c^3*e*f^2 + (a*b^2*c^3 + 3*a^2*b*c^2*d + a^3
*c*d^2)*e^3 + 3*(a^2*b*c^3 + a^3*c^2*d)*e^2*f)*x^3 + 3/2*(a^3*c^3*e^2*f + (
a^2*b*c^3 + a^3*c^2*d)*e^3)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. 2(364) = 728.

Time = 0.09 (sec) , antiderivative size = 1018, normalized size of antiderivative = 2.82

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3\,dx$$
$$=a^3c^3e^3x+\frac{b^3d^3f^3x^{10}}{10}+x^9\left(\frac{ab^2d^3f^3}{3}+\frac{b^3cd^2f^3}{3}+\frac{b^3d^3ef^2}{3}\right)+x^8\cdot\left(\frac{3a^2bd^3f^3}{8}+\frac{9ab^2cd^2f^3}{8}+\frac{9ab^2d^3ef^2}{8}+\frac{3b^3c^2df^3}{8}+\frac{9b^3cd^2ef^2}{8}+\frac{3b^3d^3e^2f}{8}\right)$$
$$+x^7\left(\frac{a^3d^3f^3}{7}+\frac{9a^2bcd^2f^3}{7}+\frac{9a^2bd^3ef^2}{7}+\frac{9ab^2c^2df^3}{7}+\frac{27ab^2cd^2ef^2}{7}+\frac{9ab^2d^3e^2f}{7}+\frac{b^3c^3f^3}{7}+\frac{9b^3c^2def^2}{7}+\frac{9b^3cd^2e^2f}{7}+\frac{b^3d^3e^3}{7}\right)$$
$$+x^6\left(\frac{a^3cd^2f^3}{2}+\frac{a^3d^3ef^2}{2}+\frac{3a^2bc^2df^3}{2}+\frac{9a^2bcd^2ef^2}{2}+\frac{3a^2bd^3e^2f}{2}+\frac{ab^2c^3f^3}{2}+\frac{9ab^2c^2def^2}{2}+\frac{9ab^2cd^2e^2f}{2}+\frac{ab^2d^3e^3}{2}+\frac{b^3c^3ef^2}{2}+\frac{3b^3c^2de^2f}{2}+\frac{b^3cd^2e^3}{2}\right)+x^5$$
$$\cdot\left(\frac{3a^3c^2df^3}{5}+\frac{9a^3cd^2ef^2}{5}+\frac{3a^3d^3e^2f}{5}+\frac{3a^2bc^3f^3}{5}+\frac{27a^2bc^2def^2}{5}+\frac{27a^2bcd^2e^2f}{5}+\frac{3a^2bd^3e^3}{5}+\frac{9ab^2c^3ef^2}{5}+\frac{27ab^2c^2de^2f}{5}+\frac{9ab^2cd^2e^3}{5}+\frac{3b^3c^3e^2f}{5}+\frac{3b^3c^2de^3}{5}\right)$$
$$+x^4\left(\frac{a^3c^3f^3}{4}+\frac{9a^3c^2def^2}{4}+\frac{9a^3cd^2e^2f}{4}+\frac{a^3d^3e^3}{4}+\frac{9a^2bc^3ef^2}{4}+\frac{27a^2bc^2de^2f}{4}+\frac{9a^2bcd^2e^3}{4}+\frac{9ab^2c^3e^2f}{4}+\frac{9ab^2c^2de^3}{4}+\frac{b^3c^3e^3}{4}\right)$$
$$+x^3\left(a^3c^3ef^2+3a^3c^2de^2f+a^3cd^2e^3+3a^2bc^3e^2f+3a^2bc^2de^3+ab^2c^3e^3\right)+x^2\cdot\left(\frac{3a^3c^3e^2f}{2}+\frac{3a^3c^2de^3}{2}+\frac{3a^2bc^3e^3}{2}\right)$$

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)
**3,x)
```

```
[Out] a**3*c**3*e**3*x + b**3*d**3*f**3*x**10/10 + x**9*(a*b**2*d**3*f**3/3 + b**
3*c*d**2*f**3/3 + b**3*d**3*e*f**2/3) + x**8*(3*a**2*b*d**3*f**3/8 + 9*a*b*
*2*c*d**2*f**3/8 + 9*a*b**2*d**3*e*f**2/8 + 3*b**3*c**2*d*f**3/8 + 9*b**3*c
*d**2*e*f**2/8 + 3*b**3*d**3*e**2*f/8) + x**7*(a**3*d**3*f**3/7 + 9*a**2*b*
c*d**2*f**3/7 + 9*a**2*b*d**3*e*f**2/7 + 9*a*b**2*c**2*d*f**3/7 + 27*a*b**2
*c*d**2*e*f**2/7 + 9*a*b**2*d**3*e**2*f/7 + b**3*c**3*f**3/7 + 9*b**3*c**2*
d*e*f**2/7 + 9*b**3*c*d**2*e**2*f/7 + b**3*d**3*e**3/7) + x**6*(a**3*c*d**2
*f**3/2 + a**3*d**3*e*f**2/2 + 3*a**2*b*c**2*d*f**3/2 + 9*a**2*b*c*d**2*e*f
```

```
**2/2 + 3*a**2*b*d**3*e**2*f/2 + a*b**2*c**3*f**3/2 + 9*a*b**2*c**2*d*e*f**
2/2 + 9*a*b**2*c*d**2*e**2*f/2 + a*b**2*d**3*e**3/2 + b**3*c**3*e*f**2/2 +
3*b**3*c**2*d*e**2*f/2 + b**3*c*d**2*e**3/2) + x**5*(3*a**3*c**2*d*f**3/5 +
 9*a**3*c*d**2*e*f**2/5 + 3*a**3*d**3*e**2*f/5 + 3*a**2*b*c**3*f**3/5 + 27*
a**2*b*c**2*d*e*f**2/5 + 27*a**2*b*c*d**2*e**2*f/5 + 3*a**2*b*d**3*e**3/5 +
 9*a*b**2*c**3*e*f**2/5 + 27*a*b**2*c**2*d*e**2*f/5 + 9*a*b**2*c*d**2*e**3/
5 + 3*b**3*c**3*e**2*f/5 + 3*b**3*c**2*d*e**3/5) + x**4*(a**3*c**3*f**3/4 +
 9*a**3*c**2*d*e*f**2/4 + 9*a**3*c*d**2*e**2*f/4 + a**3*d**3*e**3/4 + 9*a**
2*b*c**3*e*f**2/4 + 27*a**2*b*c**2*d*e**2*f/4 + 9*a**2*b*c*d**2*e**3/4 + 9*
a*b**2*c**3*e**2*f/4 + 9*a*b**2*c**2*d*e**3/4 + b**3*c**3*e**3/4) + x**3*(a
**3*c**3*e*f**2 + 3*a**3*c**2*d*e**2*f + a**3*c*d**2*e**3 + 3*a**2*b*c**3*e
**2*f + 3*a**2*b*c**2*d*e**3 + a*b**2*c**3*e**3) + x**2*(3*a**3*c**3*e**2*f
/2 + 3*a**3*c**2*d*e**3/2 + 3*a**2*b*c**3*e**3/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.28

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3\,dx$$
$$= \frac{1}{10}b^3d^3f^3x^{10}+\frac{1}{3}(bde+bcf+adf)b^2d^2f^2x^9$$
$$+\frac{3}{8}(bde+bcf+adf)^2bdfx^8+a^3c^3e^3x+\frac{1}{7}(bde+bcf+adf)^3x^7$$
$$+\frac{1}{4}\left(3\,bdfx^4+4\,(bde+bcf+adf)x^3+6\,(bce+ade+acf)x^2\right)a^2c^2e^2$$
$$+\frac{1}{4}(bce+ade+acf)^3x^4$$
$$+\frac{1}{70}\left(30\,b^2d^2f^2x^7+70\,(bde+bcf+adf)bdfx^6+42\,(bde+bcf+adf)^2x^5+70\,(bce+ade+acf)^2x^3+2\right.$$
$$+\frac{1}{10}\left(5\,bdfx^6+6\,(bde+(bc+ad)f)x^5\right)(bce+ade+acf)^2$$
$$+\frac{1}{56}\left(21\,b^2d^2f^2x^8+48\,\left(b^2d^2ef+\left(b^2cd+abd^2\right)f^2\right)x^7+28\,\left(b^2d^2e^2+2\,\left(b^2cd+abd^2\right)ef+\left(b^2c^2+2\,abcd\right.\right.\right.$$

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3
,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*d^3*f^3*x^10 + 1/3*(b*d*e + b*c*f + a*d*f)*b^2*d^2*f^2*x^9 + 3/8*(
b*d*e + b*c*f + a*d*f)^2*b*d*f*x^8 + a^3*c^3*e^3*x + 1/7*(b*d*e + b*c*f + a
*d*f)^3*x^7 + 1/4*(3*b*d*f*x^4 + 4*(b*d*e + b*c*f + a*d*f)*x^3 + 6*(b*c*e +
 a*d*e + a*c*f)*x^2)*a^2*c^2*e^2 + 1/4*(b*c*e + a*d*e + a*c*f)^3*x^4 + 1/70
*(30*b^2*d^2*f^2*x^7 + 70*(b*d*e + b*c*f + a*d*f)*b*d*f*x^6 + 42*(b*d*e + b
*c*f + a*d*f)^2*x^5 + 70*(b*c*e + a*d*e + a*c*f)^2*x^3 + 21*(4*b*d*f*x^5 +
5*(b*d*e + (b*c + a*d)*f)*x^4)*(b*c*e + a*d*e + a*c*f))*a*c*e + 1/10*(5*b*d
```

```
*f*x^6 + 6*(b*d*e + (b*c + a*d)*f)*x^5)*(b*c*e + a*d*e + a*c*f)^2 + 1/56*(2
1*b^2*d^2*f^2*x^8 + 48*(b^2*d^2*e*f + (b^2*c*d + a*b*d^2)*f^2)*x^7 + 28*(b^
2*d^2*e^2 + 2*(b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 + 2*a*b*c*d + a^2*d^2)*f^2
)*x^6)*(b*c*e + a*d*e + a*c*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 987 vs. $2(347) = 694$.

Time = 0.29 (sec) , antiderivative size = 987, normalized size of antiderivative = 2.73

$$\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^3 \, dx$$
$$\begin{aligned}
&= \frac{1}{10}b^3d^3f^3x^{10} + \frac{1}{3}b^3d^3ef^2x^9 + \frac{1}{3}b^3cd^2f^3x^9 + \frac{1}{3}ab^2d^3f^3x^9 + \frac{3}{8}b^3d^3e^2fx^8 + \frac{9}{8}b^3cd^2ef^2x^8 \\
&+ \frac{9}{8}ab^2d^3ef^2x^8 + \frac{3}{8}b^3c^2df^3x^8 + \frac{9}{8}ab^2cd^2f^3x^8 + \frac{3}{8}a^2bd^3f^3x^8 + \frac{1}{7}b^3d^3e^3x^7 \\
&+ \frac{9}{7}b^3cd^2e^2fx^7 + \frac{9}{7}ab^2d^3e^2fx^7 + \frac{9}{7}b^3c^2def^2x^7 + \frac{27}{7}ab^2cd^2ef^2x^7 + \frac{9}{7}a^2bd^3ef^2x^7 \\
&+ \frac{1}{7}b^3c^3f^3x^7 + \frac{9}{7}ab^2c^2df^3x^7 + \frac{9}{7}a^2bcd^2f^3x^7 + \frac{1}{7}a^3d^3f^3x^7 + \frac{1}{2}b^3cd^2e^3x^6 + \frac{1}{2}ab^2d^3e^3x^6 \\
&+ \frac{3}{2}b^3c^2de^2fx^6 + \frac{9}{2}ab^2cd^2e^2fx^6 + \frac{3}{2}a^2bd^3e^2fx^6 + \frac{1}{2}b^3c^3ef^2x^6 + \frac{9}{2}ab^2c^2def^2x^6 \\
&+ \frac{9}{2}a^2bcd^2ef^2x^6 + \frac{1}{2}a^3d^3ef^2x^6 + \frac{1}{2}ab^2c^3f^3x^6 + \frac{3}{2}a^2bc^2df^3x^6 + \frac{1}{2}a^3cd^2f^3x^6 \\
&+ \frac{3}{5}b^3c^2de^3x^5 + \frac{9}{5}ab^2cd^2e^3x^5 + \frac{3}{5}a^2bd^3e^3x^5 + \frac{3}{5}b^3c^3e^2fx^5 + \frac{27}{5}ab^2c^2de^2fx^5 \\
&+ \frac{27}{5}a^2bcd^2e^2fx^5 + \frac{3}{5}a^3d^3e^2fx^5 + \frac{9}{5}ab^2c^3ef^2x^5 + \frac{27}{5}a^2bc^2def^2x^5 + \frac{9}{5}a^3cd^2ef^2x^5 \\
&+ \frac{3}{5}a^2bc^3f^3x^5 + \frac{3}{5}a^3c^2df^3x^5 + \frac{1}{4}b^3c^3e^3x^4 + \frac{9}{4}ab^2c^2de^3x^4 + \frac{9}{4}a^2bcd^2e^3x^4 \\
&+ \frac{1}{4}a^3d^3e^3x^4 + \frac{9}{4}ab^2c^3e^2fx^4 + \frac{27}{4}a^2bc^2de^2fx^4 + \frac{9}{4}a^3cd^2e^2fx^4 + \frac{9}{4}a^2bc^3ef^2x^4 \\
&+ \frac{9}{4}a^3c^2def^2x^4 + \frac{1}{4}a^3c^3f^3x^4 + ab^2c^3e^3x^3 + 3a^2bc^2de^3x^3 + a^3cd^2e^3x^3 + 3a^2bc^3e^2fx^3 \\
&+ 3a^3c^2de^2fx^3 + a^3c^3ef^2x^3 + \frac{3}{2}a^2bc^3e^3x^2 + \frac{3}{2}a^3c^2de^3x^2 + \frac{3}{2}a^3c^3e^2fx^2 + a^3c^3e^3x
\end{aligned}$$

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3
,x, algorithm="giac")
```

```
[Out] 1/10*b^3*d^3*f^3*x^10 + 1/3*b^3*d^3*e*f^2*x^9 + 1/3*b^3*c*d^2*f^3*x^9 + 1/3
*a*b^2*d^3*f^3*x^9 + 3/8*b^3*d^3*e^2*f*x^8 + 9/8*b^3*c*d^2*e*f^2*x^8 + 9/8*
a*b^2*d^3*e*f^2*x^8 + 3/8*b^3*c^2*d*f^3*x^8 + 9/8*a*b^2*c*d^2*f^3*x^8 + 3/8
*a^2*b*d^3*f^3*x^8 + 1/7*b^3*d^3*e^3*x^7 + 9/7*b^3*c*d^2*e^2*f*x^7 + 9/7*a*
b^2*d^3*e^2*f*x^7 + 9/7*b^3*c^2*d*e*f^2*x^7 + 27/7*a*b^2*c*d^2*e*f^2*x^7 +
9/7*a^2*b*d^3*e*f^2*x^7 + 1/7*b^3*c^3*f^3*x^7 + 9/7*a*b^2*c^2*d*f^3*x^7 + 9
```

```
/7*a^2*b*c*d^2*f^3*x^7 + 1/7*a^3*d^3*f^3*x^7 + 1/2*b^3*c*d^2*e^3*x^6 + 1/2*
a*b^2*d^3*e^3*x^6 + 3/2*b^3*c^2*d*e^2*f*x^6 + 9/2*a*b^2*c*d^2*e^2*f*x^6 + 3
/2*a^2*b*d^3*e^2*f*x^6 + 1/2*b^3*c^3*e*f^2*x^6 + 9/2*a*b^2*c^2*d*e*f^2*x^6
+ 9/2*a^2*b*c*d^2*e*f^2*x^6 + 1/2*a^3*d^3*e*f^2*x^6 + 1/2*a*b^2*c^3*f^3*x^6
 + 3/2*a^2*b*c^2*d*f^3*x^6 + 1/2*a^3*c*d^2*f^3*x^6 + 3/5*b^3*c^2*d*e^3*x^5
+ 9/5*a*b^2*c*d^2*e^3*x^5 + 3/5*a^2*b*d^3*e^3*x^5 + 3/5*b^3*c^3*e^2*f*x^5 +
 27/5*a*b^2*c^2*d*e^2*f*x^5 + 27/5*a^2*b*c*d^2*e^2*f*x^5 + 3/5*a^3*d^3*e^2*
f*x^5 + 9/5*a*b^2*c^3*e*f^2*x^5 + 27/5*a^2*b*c^2*d*e*f^2*x^5 + 9/5*a^3*c*d^
2*e*f^2*x^5 + 3/5*a^2*b*c^3*f^3*x^5 + 3/5*a^3*c^2*d*f^3*x^5 + 1/4*b^3*c^3*e
^3*x^4 + 9/4*a*b^2*c^2*d*e^3*x^4 + 9/4*a^2*b*c*d^2*e^3*x^4 + 1/4*a^3*d^3*e^
3*x^4 + 9/4*a*b^2*c^3*e^2*f*x^4 + 27/4*a^2*b*c^2*d*e^2*f*x^4 + 9/4*a^3*c*d^
2*e^2*f*x^4 + 9/4*a^2*b*c^3*e*f^2*x^4 + 9/4*a^3*c^2*d*e*f^2*x^4 + 1/4*a^3*c
^3*f^3*x^4 + a*b^2*c^3*e^3*x^3 + 3*a^2*b*c^2*d*e^3*x^3 + a^3*c*d^2*e^3*x^3
+ 3*a^2*b*c^3*e^2*f*x^3 + 3*a^3*c^2*d*e^2*f*x^3 + a^3*c^3*e*f^2*x^3 + 3/2*a
^2*b*c^3*e^3*x^2 + 3/2*a^3*c^2*d*e^3*x^2 + 3/2*a^3*c^3*e^2*f*x^2 + a^3*c^3*
e^3*x
```

Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 787, normalized size of antiderivative = 2.18

$$
\begin{aligned}
&\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3\,dx \\
&= x^7\left(\frac{a^3d^3f^3}{7}+\frac{9a^2bcd^2f^3}{7}+\frac{9a^2bd^3ef^2}{7}+\frac{9ab^2c^2df^3}{7}+\frac{27ab^2cd^2ef^2}{7}\right. \\
&\quad\left.+\frac{9ab^2d^3e^2f}{7}+\frac{b^3c^3f^3}{7}+\frac{9b^3c^2def^2}{7}+\frac{9b^3cd^2e^2f}{7}+\frac{b^3d^3e^3}{7}\right)+x^5\left(\frac{3a^3c^2df^3}{5}\right. \\
&\quad+\frac{9a^3cd^2ef^2}{5}+\frac{3a^3d^3e^2f}{5}+\frac{3a^2bc^3f^3}{5}+\frac{27a^2bc^2def^2}{5}+\frac{27a^2bcd^2e^2f}{5} \\
&\quad\left.+\frac{3a^2bd^3e^3}{5}+\frac{9ab^2c^3ef^2}{5}+\frac{27ab^2c^2de^2f}{5}+\frac{9ab^2cd^2e^3}{5}+\frac{3b^3c^3e^2f}{5}+\frac{3b^3c^2de^3}{5}\right) \\
&\quad+x^6\left(\frac{a^3cd^2f^3}{2}+\frac{a^3d^3ef^2}{2}+\frac{3a^2bc^2df^3}{2}+\frac{9a^2bcd^2ef^2}{2}+\frac{3a^2bd^3e^2f}{2}+\frac{ab^2c^3f^3}{2}\right. \\
&\quad\left.+\frac{9ab^2c^2def^2}{2}+\frac{9ab^2cd^2e^2f}{2}+\frac{ab^2d^3e^3}{2}+\frac{b^3c^3ef^2}{2}+\frac{3b^3c^2de^2f}{2}+\frac{b^3cd^2e^3}{2}\right) \\
&\quad+x^4\left(\frac{a^3c^3f^3}{4}+\frac{9a^3c^2def^2}{4}+\frac{9a^3cd^2e^2f}{4}+\frac{a^3d^3e^3}{4}+\frac{9a^2bc^3ef^2}{4}+\frac{27a^2bc^2de^2f}{4}\right. \\
&\quad\left.+\frac{9a^2bcd^2e^3}{4}+\frac{9ab^2c^3e^2f}{4}+\frac{9ab^2c^2de^3}{4}+\frac{b^3c^3e^3}{4}\right)+a^3c^3e^3x \\
&\quad+\frac{b^3d^3f^3x^{10}}{10}+\frac{3a^2c^2e^2x^2\,(acf+ade+bce)}{2}+\frac{b^2d^2f^2x^9\,(adf+bcf+bde)}{3} \\
&\quad+ace\,x^3\left(a^2c^2f^2+3a^2cdef+a^2d^2e^2+3abc^2ef+3abcde^2+b^2c^2e^2\right) \\
&\quad+\frac{3bdfx^8\left(a^2d^2f^2+3abcdf^2+3abd^2ef+b^2c^2f^2+3b^2cdef+b^2d^2e^2\right)}{8}
\end{aligned}
$$

```
[In] int((x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*
f*x^3)^3,x)
```

```
[Out] x^7*((a^3*d^3*f^3)/7 + (b^3*c^3*f^3)/7 + (b^3*d^3*e^3)/7 + (9*a*b^2*c^2*d*f
^3)/7 + (9*a^2*b*c*d^2*f^3)/7 + (9*a*b^2*d^3*e^2*f)/7 + (9*a^2*b*d^3*e*f^2)
/7 + (9*b^3*c*d^2*e^2*f)/7 + (9*b^3*c^2*d*e*f^2)/7 + (27*a*b^2*c*d^2*e*f^2)
/7) + x^5*((3*a^2*b*c^3*f^3)/5 + (3*a^2*b*d^3*e^3)/5 + (3*a^3*c^2*d*f^3)/5
+ (3*b^3*c^2*d*e^3)/5 + (3*a^3*d^3*e^2*f)/5 + (3*b^3*c^3*e^2*f)/5 + (9*a*b^
2*c*d^2*e^3)/5 + (9*a*b^2*c^3*e*f^2)/5 + (9*a^3*c*d^2*e*f^2)/5 + (27*a*b^2*
c^2*d*e^2*f)/5 + (27*a^2*b*c*d^2*e^2*f)/5 + (27*a^2*b*c^2*d*e*f^2)/5) + x^6
*((a*b^2*c^3*f^3)/2 + (a*b^2*d^3*e^3)/2 + (a^3*c*d^2*f^3)/2 + (b^3*c*d^2*e^
3)/2 + (a^3*d^3*e*f^2)/2 + (b^3*c^3*e*f^2)/2 + (3*a^2*b*c^2*d*f^3)/2 + (3*a
^2*b*d^3*e^2*f)/2 + (3*b^3*c^2*d*e^2*f)/2 + (9*a*b^2*c*d^2*e^2*f)/2 + (9*a*
b^2*c^2*d*e*f^2)/2 + (9*a^2*b*c*d^2*e*f^2)/2) + x^4*((a^3*c^3*f^3)/4 + (a^3
*d^3*e^3)/4 + (b^3*c^3*e^3)/4 + (9*a*b^2*c^2*d*e^3)/4 + (9*a^2*b*c*d^2*e^3)
/4 + (9*a*b^2*c^3*e^2*f)/4 + (9*a^2*b*c^3*e*f^2)/4 + (9*a^3*c*d^2*e^2*f)/4
+ (9*a^3*c^2*d*e*f^2)/4 + (27*a^2*b*c^2*d*e^2*f)/4) + a^3*c^3*e^3*x + (b^3*
d^3*f^3*x^10)/10 + (3*a^2*c^2*e^2*x^2*(a*c*f + a*d*e + b*c*e))/2 + (b^2*d^2
*f^2*x^9*(a*d*f + b*c*f + b*d*e))/3 + a*c*e*x^3*(a^2*c^2*f^2 + a^2*d^2*e^2
+ b^2*c^2*e^2 + 3*a*b*c*d*e^2 + 3*a*b*c^2*e*f + 3*a^2*c*d*e*f) + (3*b*d*f*x
^8*(a^2*d^2*f^2 + b^2*c^2*f^2 + b^2*d^2*e^2 + 3*a*b*c*d*f^2 + 3*a*b*d^2*e*f
 + 3*b^2*c*d*e*f))/8
```

3.16 $\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2$ -

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Optimal result

Integrand size = 46, antiderivative size = 193

$$\begin{aligned}
&\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2\,dx\\
&=\frac{(bc-ad)^2(be-af)^2(a+bx)^3}{3b^5}+\frac{(bc-ad)(be-af)(bde+bcf-2adf)(a+bx)^4}{2b^5}\\
&\quad+\frac{\left(6a^2d^2f^2-6abdf(de+cf)+b^2(d^2e^2+4cdef+c^2f^2)\right)(a+bx)^5}{5b^5}\\
&\quad+\frac{df(bde+bcf-2adf)(a+bx)^6}{3b^5}+\frac{d^2f^2(a+bx)^7}{7b^5}
\end{aligned}$$

```
[Out] 1/3*(-a*d+b*c)^2*(-a*f+b*e)^2*(b*x+a)^3/b^5+1/2*(-a*d+b*c)*(-a*f+b*e)*(-2*a
*d*f+b*c*f+b*d*e)*(b*x+a)^4/b^5+1/5*(6*a^2*d^2*f^2-6*a*b*d*f*(c*f+d*e)+b^2*
(c^2*f^2+4*c*d*e*f+d^2*e^2))*(b*x+a)^5/b^5+1/3*d*f*(-2*a*d*f+b*c*f+b*d*e)*(
b*x+a)^6/b^5+1/7*d^2*f^2*(b*x+a)^7/b^5
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2084, 90}

$$\begin{aligned}
&\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2\,dx\\
&=\frac{(a+bx)^5\left(6a^2d^2f^2-6abdf(cf+de)+b^2(c^2f^2+4cdef+d^2e^2)\right)}{5b^5}\\
&\quad+\frac{df(a+bx)^6(-2adf+bcf+bde)}{3b^5}+\frac{(a+bx)^4(bc-ad)(be-af)(-2adf+bcf+bde)}{2b^5}\\
&\quad+\frac{(a+bx)^3(bc-ad)^2(be-af)^2}{3b^5}+\frac{d^2f^2(a+bx)^7}{7b^5}
\end{aligned}$$

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^2,x]
```

```
[Out] ((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^3)/(3*b^5) + ((b*c - a*d)*(b*e - a*f
)*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^4)/(2*b^5) + ((6*a^2*d^2*f^2 - 6*a*b*
d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*(a + b*x)^5)/(5*b^5)
 + (d*f*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^6)/(3*b^5) + (d^2*f^2*(a + b*x)
^7)/(7*b^5)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2084

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (a+bx)^2(c+dx)^2(e+fx)^2\,dx \\
&= \int \left(\frac{(bc-ad)^2(be-af)^2(a+bx)^2}{b^4} + \frac{2(bc-ad)(be-af)(bde+bcf-2adf)(a+bx)^3}{b^4} + \frac{\left(6a^2d^2f^2-6abdf(de+cf)+b^2\left(d^2e^2+4cdef+c^2f^2\right)\right)(a+bx)^4}{b^4} + \frac{2df(bde+bcf-2adf)(a+bx)^5}{b^4} + \frac{d^2f^2(a+bx)^6}{b^4}\right) dx \\
&= \frac{(bc-ad)^2(be-af)^2(a+bx)^3}{3b^5} + \frac{(bc-ad)(be-af)(bde+bcf-2adf)(a+bx)^4}{2b^5} + \frac{\left(6a^2d^2f^2-6abdf(de+cf)+b^2\left(d^2e^2+4cdef+c^2f^2\right)\right)(a+bx)^5}{5b^5} + \frac{df(bde+bcf-2adf)(a+bx)^6}{3b^5} + \frac{d^2f^2(a+bx)^7}{7b^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.25

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2\,dx$$
$$=a^2c^2e^2x+ace(bce+ade+acf)x^2+\frac{1}{3}\left(b^2c^2e^2+4abce(de+cf)+a^2\left(d^2e^2+4cdef+c^2f^2\right)\right)x^3$$
$$+\frac{1}{2}\left(b^2ce(de+cf)+a^2df(de+cf)+ab\left(d^2e^2+4cdef+c^2f^2\right)\right)x^4$$
$$+\frac{1}{5}\left(a^2d^2f^2+4abdf(de+cf)+b^2\left(d^2e^2+4cdef+c^2f^2\right)\right)x^5$$
$$+\frac{1}{3}bdf(bde+bcf+adf)x^6+\frac{1}{7}b^2d^2f^2x^7$$

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
 + b*d*f*x^3)^2,x]
```

```
[Out] a^2*c^2*e^2*x + a*c*e*(b*c*e + a*d*e + a*c*f)*x^2 + ((b^2*c^2*e^2 + 4*a*b*c
*e*(d*e + c*f) + a^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^3)/3 + ((b^2*c*e*(d
*e + c*f) + a^2*d*f*(d*e + c*f) + a*b*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^4)
/2 + ((a^2*d^2*f^2 + 4*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f + c^2
*f^2))*x^5)/5 + (b*d*f*(b*d*e + b*c*f + a*d*f)*x^6)/3 + (b^2*d^2*f^2*x^7)/7
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.97

method	result
default	$\frac{d^2f^2b^2x^7}{7}+\frac{(adf+fbc+bde)bdf\,x^6}{3}+\frac{\left(2(acf+eda+ebc)bdf+(adf+fbc+bde)^2\right)x^5}{5}+\frac{(2acebdf+2(acf+eda+ebc)(adf+fb}{4}$
norman	$\frac{d^2f^2b^2x^7}{7}+\left(\frac{1}{3}d^2f^2ab+\frac{1}{3}b^2cd\,f^2+\frac{1}{3}b^2d^2ef\right)x^6+\left(\frac{1}{5}a^2d^2f^2+\frac{4}{5}abcd\,f^2+\frac{4}{5}ab\,d^2ef+\frac{1}{5}b^2c^2f^2\right.$
gosper	$\frac{x\left(30d^2f^2b^2x^6+70x^5d^2f^2ab+70x^5b^2cd\,f^2+70x^5b^2d^2ef+42x^4a^2d^2f^2+168x^4abcd\,f^2+168x^4ab\,d^2ef+42x^4b^2c^2f^2+168x^4b^2cd\right.}{}$
risch	$\frac{4}{5}x^5abcd\,f^2+\frac{4}{5}x^5ab\,d^2ef+\frac{4}{5}x^5b^2cdef+\frac{4}{3}x^3a^2cdef+\frac{4}{3}x^3ab\,c^2ef+\frac{4}{3}x^3abcd\,e^2+a^2c^2e^2x+$
parallelrisch	$\frac{4}{5}x^5abcd\,f^2+\frac{4}{5}x^5ab\,d^2ef+\frac{4}{5}x^5b^2cdef+\frac{4}{3}x^3a^2cdef+\frac{4}{3}x^3ab\,c^2ef+\frac{4}{3}x^3abcd\,e^2+a^2c^2e^2x+$

```
[In] int((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x,met
hod=_RETURNVERBOSE)
```

```
[Out] 1/7*d^2*f^2*b^2*x^7+1/3*(a*d*f+b*c*f+b*d*e)*b*d*f*x^6+1/5*(2*(a*c*f+a*d*e+b
*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)*x^5+1/4*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*
c*e)*(a*d*f+b*c*f+b*d*e))*x^4+1/3*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e
+b*c*e)^2)*x^3+a*c*e*(a*c*f+a*d*e+b*c*e)*x^2+a^2*c^2*e^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.39

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2\,dx$$
$$=\frac{1}{7}b^2d^2f^2x^7+a^2c^2e^2x+\frac{1}{3}\left(b^2d^2ef+\left(b^2cd+abd^2\right)f^2\right)x^6$$
$$+\frac{1}{5}\left(b^2d^2e^2+4\left(b^2cd+abd^2\right)ef+\left(b^2c^2+4\,abcd+a^2d^2\right)f^2\right)x^5$$
$$+\frac{1}{2}\left(\left(b^2cd+abd^2\right)e^2+\left(b^2c^2+4\,abcd+a^2d^2\right)ef+\left(abc^2+a^2cd\right)f^2\right)x^4$$
$$+\frac{1}{3}\left(a^2c^2f^2+\left(b^2c^2+4\,abcd+a^2d^2\right)e^2+4\left(abc^2+a^2cd\right)ef\right)x^3$$
$$+\left(a^2c^2ef+\left(abc^2+a^2cd\right)e^2\right)x^2$$

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2
,x, algorithm="fricas")
```

```
[Out] 1/7*b^2*d^2*f^2*x^7 + a^2*c^2*e^2*x + 1/3*(b^2*d^2*e*f + (b^2*c*d + a*b*d^2
)*f^2)*x^6 + 1/5*(b^2*d^2*e^2 + 4*(b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*f^2)*x^5 + 1/2*((b^2*c*d + a*b*d^2)*e^2 + (b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*e*f + (a*b*c^2 + a^2*c*d)*f^2)*x^4 + 1/3*(a^2*c^2*f^2 + (b^
2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2 + 4*(a*b*c^2 + a^2*c*d)*e*f)*x^3 + (a^2*c^
2*e*f + (a*b*c^2 + a^2*c*d)*e^2)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.79

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2\,dx$$
$$=a^2c^2e^2x+\frac{b^2d^2f^2x^7}{7}+x^6\left(\frac{abd^2f^2}{3}+\frac{b^2cdf^2}{3}+\frac{b^2d^2ef}{3}\right)$$
$$+x^5\left(\frac{a^2d^2f^2}{5}+\frac{4abcdf^2}{5}+\frac{4abd^2ef}{5}+\frac{b^2c^2f^2}{5}+\frac{4b^2cdef}{5}+\frac{b^2d^2e^2}{5}\right)$$
$$+x^4\left(\frac{a^2cdf^2}{2}+\frac{a^2d^2ef}{2}+\frac{abc^2f^2}{2}+2abcdef+\frac{abd^2e^2}{2}+\frac{b^2c^2ef}{2}+\frac{b^2cde^2}{2}\right)$$
$$+x^3\left(\frac{a^2c^2f^2}{3}+\frac{4a^2cdef}{3}+\frac{a^2d^2e^2}{3}+\frac{4abc^2ef}{3}+\frac{4abcde^2}{3}+\frac{b^2c^2e^2}{3}\right)$$
$$+x^2\left(a^2c^2ef+a^2cde^2+abc^2e^2\right)$$

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)
**2,x)
```

```
[Out] a**2*c**2*e**2*x + b**2*d**2*f**2*x**7/7 + x**6*(a*b*d**2*f**2/3 + b**2*c*d
*f**2/3 + b**2*d**2*e*f/3) + x**5*(a**2*d**2*f**2/5 + 4*a*b*c*d*f**2/5 + 4*
a*b*d**2*e*f/5 + b**2*c**2*f**2/5 + 4*b**2*c*d*e*f/5 + b**2*d**2*e**2/5) +
x**4*(a**2*c*d*f**2/2 + a**2*d**2*e*f/2 + a*b*c**2*f**2/2 + 2*a*b*c*d*e*f +
 a*b*d**2*e**2/2 + b**2*c**2*e*f/2 + b**2*c*d*e**2/2) + x**3*(a**2*c**2*f**
2/3 + 4*a**2*c*d*e*f/3 + a**2*d**2*e**2/3 + 4*a*b*c**2*e*f/3 + 4*a*b*c*d*e*
*2/3 + b**2*c**2*e**2/3) + x**2*(a**2*c**2*e*f + a**2*c*d*e**2 + a*b*c**2*e
**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.93

$$\begin{aligned}
&\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right)^2 \, dx \\
&= \frac{1}{7}\, b^2d^2f^2x^7 + \frac{1}{3}\,(bde + bcf + adf)bdfx^6 + a^2c^2e^2x \\
&\quad + \frac{1}{5}\,(bde + bcf + adf)^2x^5 + \frac{1}{3}\,(bce + ade + acf)^2x^3 \\
&\quad + \frac{1}{6}\,\left(3\,bdfx^4 + 4\,(bde + bcf + adf)x^3 + 6\,(bce + ade + acf)x^2\right)ace \\
&\quad + \frac{1}{10}\,\left(4\,bdfx^5 + 5\,(bde + (bc + ad)f)x^4\right)(bce + ade + acf)
\end{aligned}$$

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2
,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d^2*f^2*x^7 + 1/3*(b*d*e + b*c*f + a*d*f)*b*d*f*x^6 + a^2*c^2*e^2*x
 + 1/5*(b*d*e + b*c*f + a*d*f)^2*x^5 + 1/3*(b*c*e + a*d*e + a*c*f)^2*x^3 +
1/6*(3*b*d*f*x^4 + 4*(b*d*e + b*c*f + a*d*f)*x^3 + 6*(b*c*e + a*d*e + a*c*f
)*x^2)*a*c*e + 1/10*(4*b*d*f*x^5 + 5*(b*d*e + (b*c + a*d)*f)*x^4)*(b*c*e +
a*d*e + a*c*f)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.79

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2 \,dx$$
$$= \frac{1}{7}b^2d^2f^2x^7+\frac{1}{3}b^2d^2efx^6+\frac{1}{3}b^2cdf^2x^6+\frac{1}{3}abd^2f^2x^6+\frac{1}{5}b^2d^2e^2x^5+\frac{4}{5}b^2cdefx^5$$
$$+\frac{4}{5}abd^2efx^5+\frac{1}{5}b^2c^2f^2x^5+\frac{4}{5}abcdf^2x^5+\frac{1}{5}a^2d^2f^2x^5+\frac{1}{2}b^2cde^2x^4$$
$$+\frac{1}{2}abd^2e^2x^4+\frac{1}{2}b^2c^2efx^4+2\,abcdefx^4+\frac{1}{2}a^2d^2efx^4+\frac{1}{2}abc^2f^2x^4$$
$$+\frac{1}{2}a^2cdf^2x^4+\frac{1}{3}b^2c^2e^2x^3+\frac{4}{3}abcde^2x^3+\frac{1}{3}a^2d^2e^2x^3+\frac{4}{3}abc^2efx^3$$
$$+\frac{4}{3}a^2cdefx^3+\frac{1}{3}a^2c^2f^2x^3+abc^2e^2x^2+a^2cde^2x^2+a^2c^2efx^2+a^2c^2e^2x$$

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2
,x, algorithm="giac")
```

```
[Out] 1/7*b^2*d^2*f^2*x^7 + 1/3*b^2*d^2*e*f*x^6 + 1/3*b^2*c*d*f^2*x^6 + 1/3*a*b*d
^2*f^2*x^6 + 1/5*b^2*d^2*e^2*x^5 + 4/5*b^2*c*d*e*f*x^5 + 4/5*a*b*d^2*e*f*x^
5 + 1/5*b^2*c^2*f^2*x^5 + 4/5*a*b*c*d*f^2*x^5 + 1/5*a^2*d^2*f^2*x^5 + 1/2*b
^2*c*d*e^2*x^4 + 1/2*a*b*d^2*e^2*x^4 + 1/2*b^2*c^2*e*f*x^4 + 2*a*b*c*d*e*f*
x^4 + 1/2*a^2*d^2*e*f*x^4 + 1/2*a*b*c^2*f^2*x^4 + 1/2*a^2*c*d*f^2*x^4 + 1/3
*b^2*c^2*e^2*x^3 + 4/3*a*b*c*d*e^2*x^3 + 1/3*a^2*d^2*e^2*x^3 + 4/3*a*b*c^2*
e*f*x^3 + 4/3*a^2*c*d*e*f*x^3 + 1/3*a^2*c^2*f^2*x^3 + a*b*c^2*e^2*x^2 + a^2
*c*d*e^2*x^2 + a^2*c^2*e*f*x^2 + a^2*c^2*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.40

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2 \,dx$$
$$= x^4\left(\frac{a^2\,c\,d\,f^2}{2}+\frac{a^2\,d^2\,e\,f}{2}+\frac{a\,b\,c^2\,f^2}{2}+2\,a\,b\,c\,d\,e\,f+\frac{a\,b\,d^2\,e^2}{2}+\frac{b^2\,c^2\,e\,f}{2}+\frac{b^2\,c\,d\,e^2}{2}\right)$$
$$+x^3\left(\frac{a^2\,c^2\,f^2}{3}+\frac{4\,a^2\,c\,d\,e\,f}{3}+\frac{a^2\,d^2\,e^2}{3}+\frac{4\,a\,b\,c^2\,e\,f}{3}+\frac{4\,a\,b\,c\,d\,e^2}{3}+\frac{b^2\,c^2\,e^2}{3}\right)$$
$$+x^5\left(\frac{a^2\,d^2\,f^2}{5}+\frac{4\,a\,b\,c\,d\,f^2}{5}+\frac{4\,a\,b\,d^2\,e\,f}{5}+\frac{b^2\,c^2\,f^2}{5}+\frac{4\,b^2\,c\,d\,e\,f}{5}+\frac{b^2\,d^2\,e^2}{5}\right)$$
$$+a^2\,c^2\,e^2\,x+\frac{b^2\,d^2\,f^2\,x^7}{7}+a\,c\,e\,x^2\,(a\,c\,f+a\,d\,e+b\,c\,e)+\frac{b\,d\,f\,x^6\,(a\,d\,f+b\,c\,f+b\,d\,e)}{3}$$

```
[In] int((x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*
f*x^3)^2,x)
```

```
[Out] x^4*((a*b*c^2*f^2)/2 + (a*b*d^2*e^2)/2 + (a^2*c*d*f^2)/2 + (b^2*c*d*e^2)/2
+ (a^2*d^2*e*f)/2 + (b^2*c^2*e*f)/2 + 2*a*b*c*d*e*f) + x^3*((a^2*c^2*f^2)/3
 + (a^2*d^2*e^2)/3 + (b^2*c^2*e^2)/3 + (4*a*b*c*d*e^2)/3 + (4*a*b*c^2*e*f)/
3 + (4*a^2*c*d*e*f)/3) + x^5*((a^2*d^2*f^2)/5 + (b^2*c^2*f^2)/5 + (b^2*d^2*
e^2)/5 + (4*a*b*c*d*f^2)/5 + (4*a*b*d^2*e*f)/5 + (4*b^2*c*d*e*f)/5) + a^2*c
^2*e^2*x + (b^2*d^2*f^2*x^7)/7 + a*c*e*x^2*(a*c*f + a*d*e + b*c*e) + (b*d*f
*x^6*(a*d*f + b*c*f + b*d*e))/3
```

3.17 $\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 +$

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Mathematica [A] (verified) 245
Maple [A] (verified) 245
Fricas [A] (verification not implemented) 246
Sympy [A] (verification not implemented) 246
Maxima [A] (verification not implemented) 246
Giac [A] (verification not implemented) 247
Mupad [B] (verification not implemented) 247

Optimal result

Integrand size = 44, antiderivative size = 56

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right) dx = acex + \frac{1}{2}(bce + ade + acf)x^2 + \frac{1}{3}(bde + bcf + adf)x^3 + \frac{1}{4}bdfx^4$$

```
[Out] a*c*e*x+1/2*(a*c*f+a*d*e+b*c*e)*x^2+1/3*(a*d*f+b*c*f+b*d*e)*x^3+1/4*b*d*f*x
^4
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right) dx = \frac{1}{3}x^3(adf + bcf + bde) + \frac{1}{2}x^2(acf + ade + bce) + acex + \frac{1}{4}bdfx^4$$

```
[In] Int[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f
*x^3,x]
```

```
[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^2)/2 + ((b*d*e + b*c*f + a*d*f)*x^3)/3
 + (b*d*f*x^4)/4
```

Rubi steps

$$\text{integral} = acex + \frac{1}{2}(bce + ade + acf)x^2 + \frac{1}{3}(bde + bcf + adf)x^3 + \frac{1}{4}bdfx^4$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)\,dx$$
$$=acex+\frac{1}{2}bcex^2+\frac{1}{2}adex^2+\frac{1}{2}acfx^2+\frac{1}{3}bdex^3+\frac{1}{3}bcfx^3+\frac{1}{3}adfx^3+\frac{1}{4}bdfx^4$$

```
[In] Integrate[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 +
 b*d*f*x^3,x]
```

```
[Out] a*c*e*x + (b*c*e*x^2)/2 + (a*d*e*x^2)/2 + (a*c*f*x^2)/2 + (b*d*e*x^3)/3 + (
b*c*f*x^3)/3 + (a*d*f*x^3)/3 + (b*d*f*x^4)/4
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{bdf\,x^4}{4}+\frac{((cf+ed)b+adf)x^3}{3}+\frac{(ebc+(cf+ed)a)x^2}{2}+acex$	53
norman	$\frac{bdf\,x^4}{4}+\left(\frac{1}{3}adf+\frac{1}{3}fbc+\frac{1}{3}bde\right)x^3+\left(\frac{1}{2}acf+\frac{1}{2}eda+\frac{1}{2}ebc\right)x^2+acex$	55
gosper	$\frac{x\left(3bdf\,x^3+4adf\,x^2+4bcf\,x^2+4bde\,x^2+6acfx+6edax+6bcex+12ace\right)}{12}$	60
risch	$acex+\frac{1}{2}acf\,x^2+\frac{1}{2}ade\,x^2+\frac{1}{2}bce\,x^2+\frac{1}{3}adf\,x^3+\frac{1}{3}bcf\,x^3+\frac{1}{3}eb\,x^3d+\frac{1}{4}bdf\,x^4$	63
parallelrisch	$acex+\frac{1}{2}acf\,x^2+\frac{1}{2}ade\,x^2+\frac{1}{2}bce\,x^2+\frac{1}{3}adf\,x^3+\frac{1}{3}bcf\,x^3+\frac{1}{3}eb\,x^3d+\frac{1}{4}bdf\,x^4$	63
parts	$acex+\frac{1}{2}acf\,x^2+\frac{1}{2}ade\,x^2+\frac{1}{2}bce\,x^2+\frac{1}{3}adf\,x^3+\frac{1}{3}bcf\,x^3+\frac{1}{3}eb\,x^3d+\frac{1}{4}bdf\,x^4$	63

```
[In] int(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,method=
_RETURNVERBOSE)
```

```
[Out] 1/4*b*d*f*x^4+1/3*((c*f+d*e)*b+a*d*f)*x^3+1/2*(e*b*c+(c*f+d*e)*a)*x^2+a*c*e
*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right) dx = \frac{1}{4}bdfx^4 + acex + \frac{1}{3}(bde + (bc + ad)f)x^3 + \frac{1}{2}(acf + (bc + ad)e)x^2$$

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
algorithm="fricas")
```

```
[Out] 1/4*b*d*f*x^4 + a*c*e*x + 1/3*(b*d*e + (b*c + a*d)*f)*x^3 + 1/2*(a*c*f + (b
*c + a*d)*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right) dx = acex + \frac{bdfx^4}{4} + x^3\left(\frac{adf}{3} + \frac{bcf}{3} + \frac{bde}{3}\right) + x^2\left(\frac{acf}{2} + \frac{ade}{2} + \frac{bce}{2}\right)$$

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3,x
)
```

```
[Out] a*c*e*x + b*d*f*x**4/4 + x**3*(a*d*f/3 + b*c*f/3 + b*d*e/3) + x**2*(a*c*f/2
 + a*d*e/2 + b*c*e/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right) dx = \frac{1}{4}bdfx^4 + acex + \frac{1}{3}(bde + bcf + adf)x^3 + \frac{1}{2}(bce + ade + acf)x^2$$

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
algorithm="maxima")
```

```
[Out] 1/4*b*d*f*x^4 + a*c*e*x + 1/3*(b*d*e + b*c*f + a*d*f)*x^3 + 1/2*(b*c*e + a*
d*e + a*c*f)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)\,dx$$
$$=\frac{1}{4}\,bdfx^4+acex+\frac{1}{3}\,(bde+bcf+adf)x^3+\frac{1}{2}\,(bce+ade+acf)x^2$$

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
algorithm="giac")
```

```
[Out] 1/4*b*d*f*x^4 + a*c*e*x + 1/3*(b*d*e + b*c*f + a*d*f)*x^3 + 1/2*(b*c*e + a*
d*e + a*c*f)*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)\,dx$$
$$=\frac{b\,d\,f\,x^4}{4}+\left(\frac{a\,d\,f}{3}+\frac{b\,c\,f}{3}+\frac{b\,d\,e}{3}\right)\,x^3+\left(\frac{a\,c\,f}{2}+\frac{a\,d\,e}{2}+\frac{b\,c\,e}{2}\right)\,x^2+a\,c\,e\,x$$

```
[In] int(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f
*x^3,x)
```

```
[Out] x^2*((a*c*f)/2 + (a*d*e)/2 + (b*c*e)/2) + x^3*((a*d*f)/3 + (b*c*f)/3 + (b*d
*e)/3) + a*c*e*x + (b*d*f*x^4)/4
```

3.18 $\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx$

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Rubi [A] (verified) 248
Mathematica [A] (verified) 249
Maple [A] (verified) 249
Fricas [A] (verification not implemented) 250
Sympy [F(-1)] 250
Maxima [A] (verification not implemented) 250
Giac [A] (verification not implemented) 251
Mupad [B] (verification not implemented) 251

Optimal result

Integrand size = 46, antiderivative size = 86

$$\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx = \frac{b\log(a+bx)}{(bc-ad)(be-af)}-\frac{d\log(c+dx)}{(bc-ad)(de-cf)}+\frac{f\log(e+fx)}{(be-af)(de-cf)}$$

```
[Out] b*ln(b*x+a)/(-a*d+b*c)/(-a*f+b*e)-d*ln(d*x+c)/(-a*d+b*c)/(-c*f+d*e)+f*ln(f*
x+e)/(-a*f+b*e)/(-c*f+d*e)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {2083}

$$\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx = \frac{b\log(a+bx)}{(bc-ad)(be-af)}-\frac{d\log(c+dx)}{(bc-ad)(de-cf)}+\frac{f\log(e+fx)}{(be-af)(de-cf)}$$

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^(-1),x]
```

```
[Out] (b*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)) - (d*Log[c + d*x])/((b*c - a*d)*
(d*e - c*f)) + (f*Log[e + f*x])/((b*e - a*f)*(d*e - c*f))
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{b^2}{(bc-ad)(be-af)(a+bx)} + \frac{d^2}{(bc-ad)(-de+cf)(c+dx)} + \frac{f^2}{(be-af)(de-cf)(e+fx)}\right) dx$$

$$= \frac{b\log(a+bx)}{(bc-ad)(be-af)} - \frac{d\log(c+dx)}{(bc-ad)(de-cf)} + \frac{f\log(e+fx)}{(be-af)(de-cf)}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx$$
$$= \frac{b(-de+cf)\log(a+bx)+d(be-af)\log(c+dx)+(-bc+ad)f\log(e+fx)}{(bc-ad)(be-af)(-de+cf)}$$

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^(-1),x]
```

```
[Out] (b*(-(d*e) + c*f)*Log[a + b*x] + d*(b*e - a*f)*Log[c + d*x] + (-(b*c) + a*d
)*f*Log[e + f*x])/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{f\ln(fx+e)}{(cf-ed)(af-be)} - \frac{d\ln(dx+c)}{(cf-ed)(da-bc)} + \frac{b\ln(bx+a)}{(af-be)(da-bc)}$	87
norman	$\frac{f\ln(fx+e)}{ac\,f^2-adef-bcef+d\,e^2b} + \frac{b\ln(bx+a)}{(af-be)(da-bc)} - \frac{d\ln(dx+c)}{(cf-ed)(da-bc)}$	94
parallelrisch	$\frac{\ln(bx+a)bcf-\ln(bx+a)bde-\ln(dx+c)adf+\ln(dx+c)bde+\ln(fx+e)adf-\ln(fx+e)bcf}{(ac\,f^2-adef-bcef+d\,e^2b)(da-bc)}$	103
risch	$-\frac{d\ln(dx+c)}{acdf-a\,d^2e-b\,c^2f+bcde} + \frac{f\ln(-fx-e)}{ac\,f^2-adef-bcef+d\,e^2b} + \frac{b\ln(bx+a)}{a^2df-abcf-abde+b^2ce}$	111

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3),x,met
hod=_RETURNVERBOSE)
```

```
[Out] f/(c*f-d*e)/(a*f-b*e)*ln(f*x+e)-d/(c*f-d*e)/(a*d-b*c)*ln(d*x+c)+b/(a*f-b*e)
/(a*d-b*c)*ln(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 2.91 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

$$\int \frac{1}{ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3} dx = \frac{(bc - ad)f \log(fx + e) + (bde - bcf) \log(bx + a) - (bde - adf) \log(dx + c)}{(b^2cd - abd^2)e^2 - (b^2c^2 - a^2d^2)ef + (abc^2 - a^2cd)f^2}$$

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
,x, algorithm="fricas")
```

```
[Out] ((b*c - a*d)*f*log(f*x + e) + (b*d*e - b*c*f)*log(b*x + a) - (b*d*e - a*d*f
)*log(d*x + c))/((b^2*c*d - a*b*d^2)*e^2 - (b^2*c^2 - a^2*d^2)*e*f + (a*b*c
^2 - a^2*c*d)*f^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3} dx = \text{Timed out}$$

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**
3),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

$$\int \frac{1}{ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3} dx = \frac{b \log(bx + a)}{(b^2c - abd)e - (abc - a^2d)f} - \frac{d \log(dx + c)}{(bcd - ad^2)e - (bc^2 - acd)f} + \frac{f \log(fx + e)}{bde^2 + acf^2 - (bc + ad)ef}$$

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
,x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/((b^2*c - a*b*d)*e - (a*b*c - a^2*d)*f) - d*log(d*x + c)/((b
*c*d - a*d^2)*e - (b*c^2 - a*c*d)*f) + f*log(f*x + e)/(b*d*e^2 + a*c*f^2 -
(b*c + a*d)*e*f)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.52

$$\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx = \frac{b^2\log(|bx+a|)}{b^3ce-ab^2de-ab^2cf+a^2bdf}-\frac{d^2\log(|dx+c|)}{bcd^2e-ad^3e-bc^2df+acd^2f} + \frac{f^2\log(|fx+e|)}{bde^2f-bcef^2-adef^2+acf^3}$$

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
,x, algorithm="giac")
```

```
[Out] b^2*log(abs(b*x + a))/(b^3*c*e - a*b^2*d*e - a*b^2*c*f + a^2*b*d*f) - d^2*l
og(abs(d*x + c))/(b*c*d^2*e - a*d^3*e - b*c^2*d*f + a*c*d^2*f) + f^2*log(ab
s(f*x + e))/(b*d*e^2*f - b*c*e*f^2 - a*d*e*f^2 + a*c*f^3)
```

Mupad [B] (verification not implemented)

Time = 9.71 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.23

$$\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx = \frac{b\,\ln(a+b\,x)}{b^2\,c\,e+a^2\,d\,f-a\,b\,c\,f-a\,b\,d\,e}+\frac{d\,\ln(c+d\,x)}{a\,d^2\,e+b\,c^2\,f-a\,c\,d\,f-b\,c\,d\,e} + \frac{f\,\ln(e+f\,x)}{a\,c\,f^2+b\,d\,e^2-a\,d\,e\,f-b\,c\,e\,f}$$

```
[In] int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*
d*f*x^3),x)
```

```
[Out] (b*log(a + b*x))/(b^2*c*e + a^2*d*f - a*b*c*f - a*b*d*e) + (d*log(c + d*x))
/(a*d^2*e + b*c^2*f - a*c*d*f - b*c*d*e) + (f*log(e + f*x))/(a*c*f^2 + b*d*
e^2 - a*d*e*f - b*c*e*f)
```

3.19 $\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^2}\,dx$

Optimal result 252
Rubi [A] (verified) 252
Mathematica [A] (verified) 254
Maple [A] (verified) 254
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Optimal result

Integrand size = 46, antiderivative size = 234

$$\begin{aligned}
&\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^2}\,dx \\
&= -\frac{b^3}{(bc-ad)^2(be-af)^2(a+bx)} - \frac{d^3}{(bc-ad)^2(de-cf)^2(c+dx)} \\
&\quad - \frac{f^3}{(be-af)^2(de-cf)^2(e+fx)} - \frac{2b^3(bde+bcf-2adf)\log(a+bx)}{(bc-ad)^3(be-af)^3} \\
&\quad + \frac{2d^3(bde-2bcf+adf)\log(c+dx)}{(bc-ad)^3(de-cf)^3} + \frac{2f^3(2bde-bcf-adf)\log(e+fx)}{(be-af)^3(de-cf)^3}
\end{aligned}$$

```
[Out] -b^3/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)-d^3/(-a*d+b*c)^2/(-c*f+d*e)^2/(d*x+c
)-f^3/(-a*f+b*e)^2/(-c*f+d*e)^2/(f*x+e)-2*b^3*(-2*a*d*f+b*c*f+b*d*e)*ln(b*x
+a)/(-a*d+b*c)^3/(-a*f+b*e)^3+2*d^3*(a*d*f-2*b*c*f+b*d*e)*ln(d*x+c)/(-a*d+b
*c)^3/(-c*f+d*e)^3+2*f^3*(-a*d*f-b*c*f+2*b*d*e)*ln(f*x+e)/(-a*f+b*e)^3/(-c*
f+d*e)^3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used

$= \{2083\}$

$$\int \frac{1}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2}\,dx$$
$$= -\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2} - \frac{2b^3\log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3} - \frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2} + \frac{2d^3\log(c+dx)(adf-2bcf+bde)}{(bc-ad)^3(de-cf)^3} - \frac{f^3}{(e+fx)(be-af)^2(de-cf)^2} + \frac{2f^3\log(e+fx)(-adf-bcf+2bde)}{(be-af)^3(de-cf)^3}$$

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^(-2),x]
```

```
[Out] -(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)^2*(d*e -
c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) - (2*b^3*(b
*d*e + b*c*f - 2*a*d*f)*Log[a + b*x])/((b*c - a*d)^3*(b*e - a*f)^3) + (2*d^
3*(b*d*e - 2*b*c*f + a*d*f)*Log[c + d*x])/((b*c - a*d)^3*(d*e - c*f)^3) + (
2*f^3*(2*b*d*e - b*c*f - a*d*f)*Log[e + f*x])/((b*e - a*f)^3*(d*e - c*f)^3)
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{b^4}{(bc-ad)^2(be-af)^2(a+bx)^2} - \frac{2b^4(bde+bcf-2adf)}{(bc-ad)^3(be-af)^3(a+bx)} + \frac{d^4}{(bc-ad)^2(-de+cf)^2(c+dx)^2} - \frac{2d^4(bde-2bcf+adf)}{(bc-ad)^3(-de+cf)^3(c+dx)} + \frac{f^4}{(be-af)^2(de-cf)^2(e+fx)^2} - \frac{2f^4(-2bde+bcf+adf)}{(be-af)^3(de-cf)^3(e+fx)}\right) dx$$
$$= -\frac{b^3}{(bc-ad)^2(be-af)^2(a+bx)} - \frac{d^3}{(bc-ad)^2(de-cf)^2(c+dx)} - \frac{f^3}{(be-af)^2(de-cf)^2(e+fx)} - \frac{2b^3(bde+bcf-2adf)\log(a+bx)}{(bc-ad)^3(be-af)^3} + \frac{2d^3(bde-2bcf+adf)\log(c+dx)}{(bc-ad)^3(de-cf)^3} + \frac{2f^3(2bde-bcf-adf)\log(e+fx)}{(be-af)^3(de-cf)^3}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.99

$$\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^2}\,dx$$
$$= -\frac{b^3}{(bc-ad)^2(be-af)^2(a+bx)} - \frac{d^3}{(bc-ad)^2(de-cf)^2(c+dx)} - \frac{f^3}{(be-af)^2(de-cf)^2(e+fx)} - \frac{2b^3(bde+bcf-2adf)\log(a+bx)}{(bc-ad)^3(be-af)^3} - \frac{2d^3(bde-2bcf+adf)\log(c+dx)}{(bc-ad)^3(-de+cf)^3} - \frac{2f^3(-2bde+bcf+adf)\log(e+fx)}{(be-af)^3(de-cf)^3}$$

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^(-2),x]
```

```
[Out] -(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)^2*(d*e -
c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) - (2*b^3*(b
*d*e + b*c*f - 2*a*d*f)*Log[a + b*x])/((b*c - a*d)^3*(b*e - a*f)^3) - (2*d^
3*(b*d*e - 2*b*c*f + a*d*f)*Log[c + d*x])/((b*c - a*d)^3*(-(d*e) + c*f)^3)
- (2*f^3*(-2*b*d*e + b*c*f + a*d*f)*Log[e + f*x])/((b*e - a*f)^3*(d*e - c*f
)^3)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00

method	result
default	$-\frac{f^3}{(cf-ed)^2(af-be)^2(fx+e)} - \frac{2f^3(adf+fbc-2bde)\ln(fx+e)}{(cf-ed)^3(af-be)^3} - \frac{d^3}{(cf-ed)^2(da-bc)^2(dx+c)} + \frac{2d^3(adf-2fbc+bde)\ln(dx+}{(cf-ed)^3(da-bc)^3}$
norman	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -f^3/(c*f-d*e)^2/(a*f-b*e)^2/(f*x+e)-2*f^3*(a*d*f+b*c*f-2*b*d*e)/(c*f-d*e)^
3/(a*f-b*e)^3*ln(f*x+e)-d^3/(c*f-d*e)^2/(a*d-b*c)^2/(d*x+c)+2*d^3*(a*d*f-2*
b*c*f+b*d*e)/(c*f-d*e)^3/(a*d-b*c)^3*ln(d*x+c)-b^3/(a*f-b*e)^2/(a*d-b*c)^2/
(b*x+a)+2*b^3*(2*a*d*f-b*c*f-b*d*e)/(a*f-b*e)^3/(a*d-b*c)^3*ln(b*x+a)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2} \, dx = \text{Timed out}$$

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2} \, dx = \text{Timed out}$$

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**
3)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2096 vs. 2(234) = 468.

Time = 0.32 (sec) , antiderivative size = 2096, normalized size of antiderivative = 8.96

$$\int \frac{1}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^2,x, algorithm="maxima")
```

```
[Out] -2*(b^4*d*e + (b^4*c - 2*a*b^3*d)*f)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*d
 + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*e^3 - 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*
a^3*b^3*c*d^2 - a^4*b^2*d^3)*e^2*f + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a
^4*b^2*c*d^2 - a^5*b*d^3)*e*f^2 - (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*
c*d^2 - a^6*d^3)*f^3) + 2*(b*d^4*e - (2*b*c*d^3 - a*d^4)*f)*log(d*x + c)/((
b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d^6)*e^3 - 3*(b^3*c^4*d
^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^5)*e^2*f + 3*(b^3*c^5*d -
3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4)*e*f^2 - (b^3*c^6 - 3*a*b^2
*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*f^3) + 2*(2*b*d*e*f^3 - (b*c + a*d)
*f^4)*log(f*x + e)/(b^3*d^3*e^6 + a^3*c^3*f^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*e
```

```
^5*f + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^4*f^2 - (b^3*c^3 + 9*a*b
^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^3*f^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d
+ a^3*c*d^2)*e^2*f^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*e*f^5) - ((b^3*c*d^2 + a*b
^2*d^3)*e^3 - 2*(b^3*c^2*d + a^2*b*d^3)*e^2*f + (b^3*c^3 + a^3*d^3)*e*f^2 +
 (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*f^3 + 2*(b^3*d^3*e^2*f - (b^3*c*d^
2 + a*b^2*d^3)*e*f^2 + (b^3*c^2*d - a*b^2*c*d^2 + a^2*b*d^3)*f^3)*x^2 + (2*
b^3*d^3*e^3 - (b^3*c*d^2 + a*b^2*d^3)*e^2*f - (b^3*c^2*d + a^2*b*d^3)*e*f^2
 + (2*b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + 2*a^3*d^3)*f^3)*x)/((a*b^4*c^3*
d^2 - 2*a^2*b^3*c^2*d^3 + a^3*b^2*c*d^4)*e^5 - 2*(a*b^4*c^4*d - a^2*b^3*c^3
*d^2 - a^3*b^2*c^2*d^3 + a^4*b*c*d^4)*e^4*f + (a*b^4*c^5 + 2*a^2*b^3*c^4*d
- 6*a^3*b^2*c^3*d^2 + 2*a^4*b*c^2*d^3 + a^5*c*d^4)*e^3*f^2 - 2*(a^2*b^3*c^5
 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*e^2*f^3 + (a^3*b^2*c^5 - 2*
a^4*b*c^4*d + a^5*c^3*d^2)*e*f^4 + ((b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*
d^5)*e^4*f - 2*(b^5*c^3*d^2 - a*b^4*c^2*d^3 - a^2*b^3*c*d^4 + a^3*b^2*d^5)*
e^3*f^2 + (b^5*c^4*d + 2*a*b^4*c^3*d^2 - 6*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^
4 + a^4*b*d^5)*e^2*f^3 - 2*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3
 + a^4*b*c*d^4)*e*f^4 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)
*f^5)*x^3 + ((b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5)*e^5 - (b^5*c^3*d^2
 - a*b^4*c^2*d^3 - a^2*b^3*c*d^4 + a^3*b^2*d^5)*e^4*f - (b^5*c^4*d - 2*a*b^
4*c^3*d^2 + 2*a^2*b^3*c^2*d^3 - 2*a^3*b^2*c*d^4 + a^4*b*d^5)*e^3*f^2 + (b^5
*c^5 + a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2 - 2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 +
a^5*d^5)*e^2*f^3 - (2*a*b^4*c^5 - a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 - a^4*b
*c^2*d^3 + 2*a^5*c*d^4)*e*f^4 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^
2 + a^5*c^2*d^3)*f^5)*x^2 + ((b^5*c^3*d^2 - a*b^4*c^2*d^3 - a^2*b^3*c*d^4 +
 a^3*b^2*d^5)*e^5 - (2*b^5*c^4*d - a*b^4*c^3*d^2 - 2*a^2*b^3*c^2*d^3 - a^3*
b^2*c*d^4 + 2*a^4*b*d^5)*e^4*f + (b^5*c^5 + a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2
 - 2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 + a^5*d^5)*e^3*f^2 - (a*b^4*c^5 - 2*a^2*
b^3*c^4*d + 2*a^3*b^2*c^3*d^2 - 2*a^4*b*c^2*d^3 + a^5*c*d^4)*e^2*f^3 - (a^2
*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*e*f^4 + (a^3*b^2*c^
5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*f^5)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1435 vs. $2(234) = 468$.

Time = 0.28 (sec) , antiderivative size = 1435, normalized size of antiderivative = 6.13

$$\int \frac{1}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^2,x, algorithm="giac")
```

```
[Out] -2*(b^5*d*e + b^5*c*f - 2*a*b^4*d*f)*log(abs(b*x + a))/(b^7*c^3*e^3 - 3*a*b
^6*c^2*d*e^3 + 3*a^2*b^5*c*d^2*e^3 - a^3*b^4*d^3*e^3 - 3*a*b^6*c^3*e^2*f +
```

```
9*a^2*b^5*c^2*d*e^2*f - 9*a^3*b^4*c*d^2*e^2*f + 3*a^4*b^3*d^3*e^2*f + 3*a^2
*b^5*c^3*e*f^2 - 9*a^3*b^4*c^2*d*e*f^2 + 9*a^4*b^3*c*d^2*e*f^2 - 3*a^5*b^2*
d^3*e*f^2 - a^3*b^4*c^3*f^3 + 3*a^4*b^3*c^2*d*f^3 - 3*a^5*b^2*c*d^2*f^3 + a
^6*b*d^3*f^3) + 2*(b*d^5*e - 2*b*c*d^4*f + a*d^5*f)*log(abs(d*x + c))/(b^3*
c^3*d^4*e^3 - 3*a*b^2*c^2*d^5*e^3 + 3*a^2*b*c*d^6*e^3 - a^3*d^7*e^3 - 3*b^3
*c^4*d^3*e^2*f + 9*a*b^2*c^3*d^4*e^2*f - 9*a^2*b*c^2*d^5*e^2*f + 3*a^3*c*d^
6*e^2*f + 3*b^3*c^5*d^2*e*f^2 - 9*a*b^2*c^4*d^3*e*f^2 + 9*a^2*b*c^3*d^4*e*f
^2 - 3*a^3*c^2*d^5*e*f^2 - b^3*c^6*d*f^3 + 3*a*b^2*c^5*d^2*f^3 - 3*a^2*b*c^
4*d^3*f^3 + a^3*c^3*d^4*f^3) + 2*(2*b*d*e*f^4 - b*c*f^5 - a*d*f^5)*log(abs(
f*x + e))/(b^3*d^3*e^6*f - 3*b^3*c*d^2*e^5*f^2 - 3*a*b^2*d^3*e^5*f^2 + 3*b^
3*c^2*d*e^4*f^3 + 9*a*b^2*c*d^2*e^4*f^3 + 3*a^2*b*d^3*e^4*f^3 - b^3*c^3*e^3
*f^4 - 9*a*b^2*c^2*d*e^3*f^4 - 9*a^2*b*c*d^2*e^3*f^4 - a^3*d^3*e^3*f^4 + 3*
a*b^2*c^3*e^2*f^5 + 9*a^2*b*c^2*d*e^2*f^5 + 3*a^3*c*d^2*e^2*f^5 - 3*a^2*b*c
^3*e*f^6 - 3*a^3*c^2*d*e*f^6 + a^3*c^3*f^7) - (2*b^3*d^3*e^2*f*x^2 - 2*b^3*
c*d^2*e*f^2*x^2 - 2*a*b^2*d^3*e*f^2*x^2 + 2*b^3*c^2*d*f^3*x^2 - 2*a*b^2*c*d
^2*f^3*x^2 + 2*a^2*b*d^3*f^3*x^2 + 2*b^3*d^3*e^3*x - b^3*c*d^2*e^2*f*x - a*
b^2*d^3*e^2*f*x - b^3*c^2*d*e*f^2*x - a^2*b*d^3*e*f^2*x + 2*b^3*c^3*f^3*x -
 a*b^2*c^2*d*f^3*x - a^2*b*c*d^2*f^3*x + 2*a^3*d^3*f^3*x + b^3*c*d^2*e^3 +
a*b^2*d^3*e^3 - 2*b^3*c^2*d*e^2*f - 2*a^2*b*d^3*e^2*f + b^3*c^3*e*f^2 + a^3
*d^3*e*f^2 + a*b^2*c^3*f^3 - 2*a^2*b*c^2*d*f^3 + a^3*c*d^2*f^3)/((b^4*c^2*d
^2*e^4 - 2*a*b^3*c*d^3*e^4 + a^2*b^2*d^4*e^4 - 2*b^4*c^3*d*e^3*f + 2*a*b^3*
c^2*d^2*e^3*f + 2*a^2*b^2*c*d^3*e^3*f - 2*a^3*b*d^4*e^3*f + b^4*c^4*e^2*f^2
 + 2*a*b^3*c^3*d*e^2*f^2 - 6*a^2*b^2*c^2*d^2*e^2*f^2 + 2*a^3*b*c*d^3*e^2*f^
2 + a^4*d^4*e^2*f^2 - 2*a*b^3*c^4*e*f^3 + 2*a^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c
^2*d^2*e*f^3 - 2*a^4*c*d^3*e*f^3 + a^2*b^2*c^4*f^4 - 2*a^3*b*c^3*d*f^4 + a^
4*c^2*d^2*f^4)*(b*d*f*x^3 + b*d*e*x^2 + b*c*f*x^2 + a*d*f*x^2 + b*c*e*x + a
*d*e*x + a*c*f*x + a*c*e))
```

Mupad [B] (verification not implemented)

Time = 13.89 (sec) , antiderivative size = 1940, normalized size of antiderivative = 8.29

$$\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*
d*f*x^3)^2,x)
```

```
[Out] - ((a*b^2*c^3*f^3 + a*b^2*d^3*e^3 + a^3*c*d^2*f^3 + b^3*c*d^2*e^3 + a^3*d^3
*e*f^2 + b^3*c^3*e*f^2 - 2*a^2*b*c^2*d*f^3 - 2*a^2*b*d^3*e^2*f - 2*b^3*c^2*
d*e^2*f)/(a^2*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c^2*d^2*f^4 + b^4*c^2*d^2
*e^4 + a^4*d^4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^3*b*c^3*
d*f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a^4*c*d^3*e*f^3 - 2*b^4*c
^3*d*e^3*f + 2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*e^2*f^2 + 2*a^2*b^2*c*d^
3*e^3*f + 2*a^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2*f^2 + 2*a^3*b*c^2*d^2*e
```

```
*f^3 - 6*a^2*b^2*c^2*d^2*e^2*f^2) + (2*x^2*(a^2*b*d^3*f^3 + b^3*c^2*d*f^3 +
 b^3*d^3*e^2*f - a*b^2*c*d^2*f^3 - a*b^2*d^3*e*f^2 - b^3*c*d^2*e*f^2))/(a^2
*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c^2*d^2*f^4 + b^4*c^2*d^2*e^4 + a^4*d^
4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^3*b*c^3*d*f^4 - 2*a*b
^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a^4*c*d^3*e*f^3 - 2*b^4*c^3*d*e^3*f +
2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*e^2*f^2 + 2*a^2*b^2*c*d^3*e^3*f + 2*a
^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2*f^2 + 2*a^3*b*c^2*d^2*e*f^3 - 6*a^2*
b^2*c^2*d^2*e^2*f^2) - (x*(a*b^2*c^2*d*f^3 - 2*b^3*c^3*f^3 - 2*b^3*d^3*e^3
- 2*a^3*d^3*f^3 + a^2*b*c*d^2*f^3 + a*b^2*d^3*e^2*f + a^2*b*d^3*e*f^2 + b^3
*c*d^2*e^2*f + b^3*c^2*d*e*f^2))/(a^2*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c
^2*d^2*f^4 + b^4*c^2*d^2*e^4 + a^4*d^4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*
c*d^3*e^4 - 2*a^3*b*c^3*d*f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a
^4*c*d^3*e*f^3 - 2*b^4*c^3*d*e^3*f + 2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*
e^2*f^2 + 2*a^2*b^2*c*d^3*e^3*f + 2*a^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2
*f^2 + 2*a^3*b*c^2*d^2*e*f^3 - 6*a^2*b^2*c^2*d^2*e^2*f^2))/(x^2*(a*d*f + b*
c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f*x^3) - (log(a + b*
x)*(b^4*(2*c*f + 2*d*e) - 4*a*b^3*d*f))/(b^6*c^3*e^3 + a^6*d^3*f^3 - a^3*b^
3*c^3*f^3 - a^3*b^3*d^3*e^3 - 3*a*b^5*c^2*d*e^3 - 3*a^5*b*c*d^2*f^3 - 3*a*b
^5*c^3*e^2*f - 3*a^5*b*d^3*e*f^2 + 3*a^2*b^4*c*d^2*e^3 + 3*a^4*b^2*c^2*d*f^
3 + 3*a^2*b^4*c^3*e*f^2 + 3*a^4*b^2*d^3*e^2*f + 9*a^2*b^4*c^2*d*e^2*f - 9*a
^3*b^3*c*d^2*e^2*f - 9*a^3*b^3*c^2*d*e*f^2 + 9*a^4*b^2*c*d^2*e*f^2) - (log(
c + d*x)*(d^4*(2*a*f + 2*b*e) - 4*b*c*d^3*f))/(a^3*d^6*e^3 + b^3*c^6*f^3 -
a^3*c^3*d^3*f^3 - b^3*c^3*d^3*e^3 - 3*a^2*b*c*d^5*e^3 - 3*a*b^2*c^5*d*f^3 -
 3*a^3*c*d^5*e^2*f - 3*b^3*c^5*d*e*f^2 + 3*a*b^2*c^2*d^4*e^3 + 3*a^2*b*c^4*
d^2*f^3 + 3*a^3*c^2*d^4*e*f^2 + 3*b^3*c^4*d^2*e^2*f - 9*a*b^2*c^3*d^3*e^2*f
 + 9*a*b^2*c^4*d^2*e*f^2 + 9*a^2*b*c^2*d^4*e^2*f - 9*a^2*b*c^3*d^3*e*f^2) -
 (log(e + f*x)*(f^4*(2*a*d + 2*b*c) - 4*b*d*e*f^3))/(a^3*c^3*f^6 + b^3*d^3*
e^6 - a^3*d^3*e^3*f^3 - b^3*c^3*e^3*f^3 - 3*a^2*b*c^3*e*f^5 - 3*a*b^2*d^3*e
^5*f - 3*a^3*c^2*d*e*f^5 - 3*b^3*c*d^2*e^5*f + 3*a*b^2*c^3*e^2*f^4 + 3*a^2*
b*d^3*e^4*f^2 + 3*a^3*c*d^2*e^2*f^4 + 3*b^3*c^2*d*e^4*f^2 + 9*a*b^2*c*d^2*e
^4*f^2 - 9*a*b^2*c^2*d*e^3*f^3 - 9*a^2*b*c*d^2*e^3*f^3 + 9*a^2*b*c^2*d*e^2*
f^4)
```

3.20 $\int \frac{1}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3}\,dx$

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Optimal result

Integrand size = 46, antiderivative size = 495

$$\begin{aligned}
&\int \frac{1}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3}\,dx \\
&= -\frac{b^5}{2(bc-ad)^3(be-af)^3(a+bx)^2}+\frac{3b^5(bde+bcf-2adf)}{(bc-ad)^4(be-af)^4(a+bx)} \\
&\quad+\frac{d^5}{2(bc-ad)^3(de-cf)^3(c+dx)^2}+\frac{3d^5(bde-2bcf+adf)}{(bc-ad)^4(de-cf)^4(c+dx)} \\
&\quad-\frac{f^5}{2(be-af)^3(de-cf)^3(e+fx)^2}-\frac{3f^5(2bde-bcf-adf)}{(be-af)^4(de-cf)^4(e+fx)} \\
&\quad+\frac{3b^5\left(7a^2d^2f^2-7abdf(de+cf)+b^2(2d^2e^2+3cdef+2c^2f^2)\right)\log(a+bx)}{(bc-ad)^5(be-af)^5} \\
&\quad-\frac{3d^5\left(2a^2d^2f^2+abdf(3de-7cf)+b^2(2d^2e^2-7cdef+7c^2f^2)\right)\log(c+dx)}{(bc-ad)^5(de-cf)^5} \\
&\quad+\frac{3f^5\left(2a^2d^2f^2-abdf(7de-3cf)+b^2(7d^2e^2-7cdef+2c^2f^2)\right)\log(e+fx)}{(be-af)^5(de-cf)^5}
\end{aligned}$$

```
[Out] -1/2*b^5/(-a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)^2+3*b^5*(-2*a*d*f+b*c*f+b*d*e)/(
-a*d+b*c)^4/(-a*f+b*e)^4/(b*x+a)+1/2*d^5/(-a*d+b*c)^3/(-c*f+d*e)^3/(d*x+c)^
2+3*d^5*(a*d*f-2*b*c*f+b*d*e)/(-a*d+b*c)^4/(-c*f+d*e)^4/(d*x+c)-1/2*f^5/(-a
*f+b*e)^3/(-c*f+d*e)^3/(f*x+e)^2-3*f^5*(-a*d*f-b*c*f+2*b*d*e)/(-a*f+b*e)^4/
(-c*f+d*e)^4/(f*x+e)+3*b^5*(7*a^2*d^2*f^2-7*a*b*d*f*(c*f+d*e)+b^2*(2*c^2*f^
2+3*c*d*e*f+2*d^2*e^2))*ln(b*x+a)/(-a*d+b*c)^5/(-a*f+b*e)^5-3*d^5*(2*a^2*d^
2*f^2+a*b*d*f*(-7*c*f+3*d*e)+b^2*(7*c^2*f^2-7*c*d*e*f+2*d^2*e^2))*ln(d*x+c)
/(-a*d+b*c)^5/(-c*f+d*e)^5+3*f^5*(2*a^2*d^2*f^2-a*b*d*f*(-3*c*f+7*d*e)+b^2*
(2*c^2*f^2-7*c*d*e*f+7*d^2*e^2))*ln(f*x+e)/(-a*f+b*e)^5/(-c*f+d*e)^5
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {2083}

$$\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx$$
$$= \frac{3f^5\log(e+fx)\left(2a^2d^2f^2-abdf(7de-3cf)+b^2(2c^2f^2-7cdef+7d^2e^2)\right)}{(be-af)^5(de-cf)^5}$$
$$-\frac{3d^5\log(c+dx)\left(2a^2d^2f^2+abdf(3de-7cf)+b^2(7c^2f^2-7cdef+2d^2e^2)\right)}{(bc-ad)^5(de-cf)^5}$$
$$+\frac{3b^5\log(a+bx)\left(7a^2d^2f^2-7abdf(cf+de)+b^2(2c^2f^2+3cdef+2d^2e^2)\right)}{(bc-ad)^5(be-af)^5}$$
$$+\frac{3b^5(-2adf+bcf+bde)}{(a+bx)(bc-ad)^4(be-af)^4}-\frac{b^5}{2(a+bx)^2(bc-ad)^3(be-af)^3}$$
$$+\frac{3d^5(adf-2bcf+bde)}{(c+dx)(bc-ad)^4(de-cf)^4}+\frac{d^5}{2(c+dx)^2(bc-ad)^3(de-cf)^3}$$
$$-\frac{3f^5(-adf-bcf+2bde)}{(e+fx)(be-af)^4(de-cf)^4}-\frac{f^5}{2(e+fx)^2(be-af)^3(de-cf)^3}$$

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^(-3),x]
```

```
[Out] -1/2*b^5/((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2) + (3*b^5*(b*d*e + b*c*f
- 2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) + d^5/(2*(b*c - a*d)^3*
(d*e - c*f)^3*(c + d*x)^2) + (3*d^5*(b*d*e - 2*b*c*f + a*d*f))/((b*c - a*d)
^4*(d*e - c*f)^4*(c + d*x)) - f^5/(2*(b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^
2) - (3*f^5*(2*b*d*e - b*c*f - a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e + f*
x)) + (3*b^5*(7*a^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 + 3*c*
d*e*f + 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) - (3*d^5*(2
*a^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*d*e*f + 7*c^2
*f^2))*Log[c + d*x])/((b*c - a*d)^5*(d*e - c*f)^5) + (3*f^5*(2*a^2*d^2*f^2
- a*b*d*f*(7*d*e - 3*c*f) + b^2*(7*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2))*Log[e
+ f*x])/((b*e - a*f)^5*(d*e - c*f)^5)
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{b^6}{(bc-ad)^3(be-af)^3(a+bx)^3}-\frac{3b^6(bde+bcf-2adf)}{(bc-ad)^4(be-af)^4(a+bx)^2}\right. \\
&\quad +\frac{3b^6(7a^2d^2f^2-7abdf(de+cf)+b^2(2d^2e^2+3cdef+2c^2f^2))}{(bc-ad)^5(be-af)^5(a+bx)} \\
&\quad +\frac{d^6}{(bc-ad)^3(-de+cf)^3(c+dx)^3}-\frac{3d^6(bde-2bcf+adf)}{(bc-ad)^4(-de+cf)^4(c+dx)^2} \\
&\quad +\frac{3d^6(-2a^2d^2f^2-abdf(3de-7cf)-b^2(2d^2e^2-7cdef+7c^2f^2))}{(bc-ad)^5(de-cf)^5(c+dx)} \\
&\quad +\frac{f^6}{(be-af)^3(de-cf)^3(e+fx)^3}-\frac{3f^6(-2bde+bcf+adf)}{(be-af)^4(de-cf)^4(e+fx)^2} \\
&\quad \left.+\frac{3f^6(2a^2d^2f^2-abdf(7de-3cf)+b^2(7d^2e^2-7cdef+2c^2f^2))}{(be-af)^5(de-cf)^5(e+fx)}\right)dx \\
&= -\frac{b^5}{2(bc-ad)^3(be-af)^3(a+bx)^2}+\frac{3b^5(bde+bcf-2adf)}{(bc-ad)^4(be-af)^4(a+bx)} \\
&\quad +\frac{d^5}{2(bc-ad)^3(de-cf)^3(c+dx)^2}+\frac{3d^5(bde-2bcf+adf)}{(bc-ad)^4(de-cf)^4(c+dx)} \\
&\quad -\frac{f^5}{2(be-af)^3(de-cf)^3(e+fx)^2}-\frac{3f^5(2bde-bcf-adf)}{(be-af)^4(de-cf)^4(e+fx)} \\
&\quad +\frac{3b^5(7a^2d^2f^2-7abdf(de+cf)+b^2(2d^2e^2+3cdef+2c^2f^2))\log(a+bx)}{(bc-ad)^5(be-af)^5} \\
&\quad -\frac{3d^5(2a^2d^2f^2+abdf(3de-7cf)+b^2(2d^2e^2-7cdef+7c^2f^2))\log(c+dx)}{(bc-ad)^5(de-cf)^5} \\
&\quad +\frac{3f^5(2a^2d^2f^2-abdf(7de-3cf)+b^2(7d^2e^2-7cdef+2c^2f^2))\log(e+fx)}{(be-af)^5(de-cf)^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 490, normalized size of antiderivative = 0.99

$$\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx$$
$$=\frac{1}{2}\left(-\frac{b^5}{(bc-ad)^3(be-af)^3(a+bx)^2}+\frac{6b^5(bde+bcf-2adf)}{(bc-ad)^4(be-af)^4(a+bx)}\right.$$
$$-\frac{d^5}{(bc-ad)^3(-de+cf)^3(c+dx)^2}+\frac{6d^5(bde-2bcf+adf)}{(bc-ad)^4(de-cf)^4(c+dx)}$$
$$-\frac{f^5}{(be-af)^3(de-cf)^3(e+fx)^2}+\frac{6f^5(-2bde+bcf+adf)}{(be-af)^4(de-cf)^4(e+fx)}$$
$$+\frac{6b^5(7a^2d^2f^2-7abdf(de+cf)+b^2(2d^2e^2+3cdef+2c^2f^2))\log(a+bx)}{(bc-ad)^5(be-af)^5}$$
$$+\frac{6d^5(2a^2d^2f^2+abdf(3de-7cf)+b^2(2d^2e^2-7cdef+7c^2f^2))\log(c+dx)}{(bc-ad)^5(-de+cf)^5}$$
$$\left.+\frac{6f^5(2a^2d^2f^2+abdf(-7de+3cf)+b^2(7d^2e^2-7cdef+2c^2f^2))\log(e+fx)}{(be-af)^5(de-cf)^5}\right)$$

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^(-3),x]
```

```
[Out] (-(b^5/((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2)) + (6*b^5*(b*d*e + b*c*f -
 2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) - d^5/((b*c - a*d)^3*(-(
d*e) + c*f)^3*(c + d*x)^2) + (6*d^5*(b*d*e - 2*b*c*f + a*d*f))/((b*c - a*d)
^4*(d*e - c*f)^4*(c + d*x)) - f^5/((b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^2)
 + (6*f^5*(-2*b*d*e + b*c*f + a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e + f*x
)) + (6*b^5*(7*a^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 + 3*c*d
*e*f + 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) + (6*d^5*(2*
a^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*d*e*f + 7*c^2*
f^2))*Log[c + d*x])/((b*c - a*d)^5*(-(d*e) + c*f)^5) + (6*f^5*(2*a^2*d^2*f^
2 + a*b*d*f*(-7*d*e + 3*c*f) + b^2*(7*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2))*Log
[e + f*x])/((b*e - a*f)^5*(d*e - c*f)^5))/2
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.04

method	result
default	$-\frac{f^5}{2(cf-ed)^3(af-be)^3(fx+e)^2}+\frac{3f^5(adf+fbc-2bde)}{(cf-ed)^4(af-be)^4(fx+e)}+\frac{3f^5\left(2a^2d^2f^2+3abcd\,f^2-7ab\,d^2ef+2b^2c^2f^2-7b^2cdef+7b\right.}{(cf-ed)^5(af-be)^5}$
norman	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/2*f^5/(c*f-d*e)^3/(a*f-b*e)^3/(f*x+e)^2+3*f^5*(a*d*f+b*c*f-2*b*d*e)/(c*f
-d*e)^4/(a*f-b*e)^4/(f*x+e)+3*f^5*(2*a^2*d^2*f^2+3*a*b*c*d*f^2-7*a*b*d^2*e*
f+2*b^2*c^2*f^2-7*b^2*c*d*e*f+7*b^2*d^2*e^2)/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x
+e)+1/2*d^5/(c*f-d*e)^3/(a*d-b*c)^3/(d*x+c)^2+3*d^5*(a*d*f-2*b*c*f+b*d*e)/(
c*f-d*e)^4/(a*d-b*c)^4/(d*x+c)-3*d^5*(2*a^2*d^2*f^2-7*a*b*c*d*f^2+3*a*b*d^2
*e*f+7*b^2*c^2*f^2-7*b^2*c*d*e*f+2*b^2*d^2*e^2)/(c*f-d*e)^5/(a*d-b*c)^5*ln(
d*x+c)-1/2*b^5/(a*f-b*e)^3/(a*d-b*c)^3/(b*x+a)^2-3*b^5*(2*a*d*f-b*c*f-b*d*e
)/(a*f-b*e)^4/(a*d-b*c)^4/(b*x+a)+3*b^5*(7*a^2*d^2*f^2-7*a*b*c*d*f^2-7*a*b*
d^2*e*f+2*b^2*c^2*f^2+3*b^2*c*d*e*f+2*b^2*d^2*e^2)/(a*f-b*e)^5/(a*d-b*c)^5*
ln(b*x+a)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**
3)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11005 vs. 2(489) = 978.

Time = 0.74 (sec) , antiderivative size = 11005, normalized size of antiderivative = 22.23

$$\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^3,x, algorithm="maxima")
```

```
[Out] 3*(2*b^7*d^2*e^2 + (3*b^7*c*d - 7*a*b^6*d^2)*e*f + (2*b^7*c^2 - 7*a*b^6*c*d
 + 7*a^2*b^5*d^2)*f^2)*log(b*x + a)/((b^10*c^5 - 5*a*b^9*c^4*d + 10*a^2*b^8
*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5*b^5*d^5)*e^5 - 5*(a*b
^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5*a^5*
b^5*c*d^4 - a^6*b^4*d^5)*e^4*f + 10*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 10*a^4
*b^6*c^3*d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*e^3*f^2
- 10*(a^3*b^7*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^4*c^2*d
^3 + 5*a^7*b^3*c*d^4 - a^8*b^2*d^5)*e^2*f^3 + 5*(a^4*b^6*c^5 - 5*a^5*b^5*c^
4*d + 10*a^6*b^4*c^3*d^2 - 10*a^7*b^3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5
)*e*f^4 - (a^5*b^5*c^5 - 5*a^6*b^4*c^4*d + 10*a^7*b^3*c^3*d^2 - 10*a^8*b^2*
c^2*d^3 + 5*a^9*b*c*d^4 - a^10*d^5)*f^5) - 3*(2*b^2*d^7*e^2 - (7*b^2*c*d^6
- 3*a*b*d^7)*e*f + (7*b^2*c^2*d^5 - 7*a*b*c*d^6 + 2*a^2*d^7)*f^2)*log(d*x +
 c)/((b^5*c^5*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*a^3*b^2*c^2*d
^8 + 5*a^4*b*c*d^9 - a^5*d^10)*e^5 - 5*(b^5*c^6*d^4 - 5*a*b^4*c^5*d^5 + 10*
a^2*b^3*c^4*d^6 - 10*a^3*b^2*c^3*d^7 + 5*a^4*b*c^2*d^8 - a^5*c*d^9)*e^4*f +
 10*(b^5*c^7*d^3 - 5*a*b^4*c^6*d^4 + 10*a^2*b^3*c^5*d^5 - 10*a^3*b^2*c^4*d^
6 + 5*a^4*b*c^3*d^7 - a^5*c^2*d^8)*e^3*f^2 - 10*(b^5*c^8*d^2 - 5*a*b^4*c^7*
d^3 + 10*a^2*b^3*c^6*d^4 - 10*a^3*b^2*c^5*d^5 + 5*a^4*b*c^4*d^6 - a^5*c^3*d
^7)*e^2*f^3 + 5*(b^5*c^9*d - 5*a*b^4*c^8*d^2 + 10*a^2*b^3*c^7*d^3 - 10*a^3*
b^2*c^6*d^4 + 5*a^4*b*c^5*d^5 - a^5*c^4*d^6)*e*f^4 - (b^5*c^10 - 5*a*b^4*c^
9*d + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 + 5*a^4*b*c^6*d^4 - a^5*c^5*d
^5)*f^5) + 3*(7*b^2*d^2*e^2*f^5 - 7*(b^2*c*d + a*b*d^2)*e*f^6 + (2*b^2*c^2
+ 3*a*b*c*d + 2*a^2*d^2)*f^7)*log(f*x + e)/(b^5*d^5*e^10 + a^5*c^5*f^10 - 5
*(b^5*c*d^4 + a*b^4*d^5)*e^9*f + 5*(2*b^5*c^2*d^3 + 5*a*b^4*c*d^4 + 2*a^2*b
^3*d^5)*e^8*f^2 - 10*(b^5*c^3*d^2 + 5*a*b^4*c^2*d^3 + 5*a^2*b^3*c*d^4 + a^3
*b^2*d^5)*e^7*f^3 + 5*(b^5*c^4*d + 10*a*b^4*c^3*d^2 + 20*a^2*b^3*c^2*d^3 +
10*a^3*b^2*c*d^4 + a^4*b*d^5)*e^6*f^4 - (b^5*c^5 + 25*a*b^4*c^4*d + 100*a^2
*b^3*c^3*d^2 + 100*a^3*b^2*c^2*d^3 + 25*a^4*b*c*d^4 + a^5*d^5)*e^5*f^5 + 5*
(a*b^4*c^5 + 10*a^2*b^3*c^4*d + 20*a^3*b^2*c^3*d^2 + 10*a^4*b*c^2*d^3 + a^5
*c*d^4)*e^4*f^6 - 10*(a^2*b^3*c^5 + 5*a^3*b^2*c^4*d + 5*a^4*b*c^3*d^2 + a^5
*c^2*d^3)*e^3*f^7 + 5*(2*a^3*b^2*c^5 + 5*a^4*b*c^4*d + 2*a^5*c^3*d^2)*e^2*f
^8 - 5*(a^4*b*c^5 + a^5*c^4*d)*e*f^9) - 1/2*((b^7*c^3*d^4 - 7*a*b^6*c^2*d^5
 - 7*a^2*b^5*c*d^6 + a^3*b^4*d^7)*e^7 - (4*b^7*c^4*d^3 - 21*a*b^6*c^3*d^4 -
 26*a^2*b^5*c^2*d^5 - 21*a^3*b^4*c*d^6 + 4*a^4*b^3*d^7)*e^6*f + 2*(3*b^7*c^
```

5*d^2 - 7*a*b^6*c^4*d^3 - 26*a^2*b^5*c^3*d^4 - 26*a^3*b^4*c^2*d^5 - 7*a^4*b
^3*c*d^6 + 3*a^5*b^2*d^7)*e^5*f^2 - 2*(2*b^7*c^6*d + 7*a*b^6*c^5*d^2 - 39*a
^2*b^5*c^4*d^3 - 39*a^4*b^3*c^2*d^5 + 7*a^5*b^2*c*d^6 + 2*a^6*b*d^7)*e^4*f^
3 + (b^7*c^7 + 21*a*b^6*c^6*d - 52*a^2*b^5*c^5*d^2 - 52*a^5*b^2*c^2*d^5 + 2
1*a^6*b*c*d^6 + a^7*d^7)*e^3*f^4 - (7*a*b^6*c^7 - 26*a^2*b^5*c^6*d + 52*a^3
*b^4*c^5*d^2 - 78*a^4*b^3*c^4*d^3 + 52*a^5*b^2*c^3*d^4 - 26*a^6*b*c^2*d^5 +
 7*a^7*c*d^6)*e^2*f^5 - 7*(a^2*b^5*c^7 - 3*a^3*b^4*c^6*d + 2*a^4*b^3*c^5*d^
2 + 2*a^5*b^2*c^4*d^3 - 3*a^6*b*c^3*d^4 + a^7*c^2*d^5)*e*f^6 + (a^3*b^4*c^7
 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4*a^6*b*c^4*d^3 + a^7*c^3*d^4)*f^7
 - 6*(2*b^7*d^7*e^5*f^2 - 5*(b^7*c*d^6 + a*b^6*d^7)*e^4*f^3 + 2*(b^7*c^2*d^
5 + 8*a*b^6*c*d^6 + a^2*b^5*d^7)*e^3*f^4 + 2*(b^7*c^3*d^4 - 6*a*b^6*c^2*d^5
 - 6*a^2*b^5*c*d^6 + a^3*b^4*d^7)*e^2*f^5 - (5*b^7*c^4*d^3 - 16*a*b^6*c^3*d
^4 + 12*a^2*b^5*c^2*d^5 - 16*a^3*b^4*c*d^6 + 5*a^4*b^3*d^7)*e*f^6 + (2*b^7*
c^5*d^2 - 5*a*b^6*c^4*d^3 + 2*a^2*b^5*c^3*d^4 + 2*a^3*b^4*c^2*d^5 - 5*a^4*b
^3*c*d^6 + 2*a^5*b^2*d^7)*f^7)*x^5 - 3*(8*b^7*d^7*e^6*f - 14*(b^7*c*d^6 + a
*b^6*d^7)*e^5*f^2 - (7*b^7*c^2*d^5 - 34*a*b^6*c*d^6 + 7*a^2*b^5*d^7)*e^4*f^
3 + 2*(7*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 + 7*a^3*b^4*d^7)*e
^3*f^4 - (7*b^7*c^4*d^3 - 6*a*b^6*c^3*d^4 + 78*a^2*b^5*c^2*d^5 - 6*a^3*b^4*
c*d^6 + 7*a^4*b^3*d^7)*e^2*f^5 - 2*(7*b^7*c^5*d^2 - 17*a*b^6*c^4*d^3 - 3*a^
2*b^5*c^3*d^4 - 3*a^3*b^4*c^2*d^5 - 17*a^4*b^3*c*d^6 + 7*a^5*b^2*d^7)*e*f^6
 + (8*b^7*c^6*d - 14*a*b^6*c^5*d^2 - 7*a^2*b^5*c^4*d^3 + 14*a^3*b^4*c^3*d^4
 - 7*a^4*b^3*c^2*d^5 - 14*a^5*b^2*c*d^6 + 8*a^6*b*d^7)*f^7)*x^4 - 2*(6*b^7*
d^7*e^7 + 3*(b^7*c*d^6 + a*b^6*d^7)*e^6*f - (37*b^7*c^2*d^5 + 28*a*b^6*c*d^
6 + 37*a^2*b^5*d^7)*e^5*f^2 + (19*b^7*c^3*d^4 + 86*a*b^6*c^2*d^5 + 86*a^2*b
^5*c*d^6 + 19*a^3*b^4*d^7)*e^4*f^3 + (19*b^7*c^4*d^3 - 68*a*b^6*c^3*d^4 - 5
2*a^2*b^5*c^2*d^5 - 68*a^3*b^4*c*d^6 + 19*a^4*b^3*d^7)*e^3*f^4 - (37*b^7*c^
5*d^2 - 86*a*b^6*c^4*d^3 + 52*a^2*b^5*c^3*d^4 + 52*a^3*b^4*c^2*d^5 - 86*a^4
*b^3*c*d^6 + 37*a^5*b^2*d^7)*e^2*f^5 + (3*b^7*c^6*d - 28*a*b^6*c^5*d^2 + 86
*a^2*b^5*c^4*d^3 - 68*a^3*b^4*c^3*d^4 + 86*a^4*b^3*c^2*d^5 - 28*a^5*b^2*c*d
^6 + 3*a^6*b*d^7)*e*f^6 + (6*b^7*c^7 + 3*a*b^6*c^6*d - 37*a^2*b^5*c^5*d^2 +
 19*a^3*b^4*c^4*d^3 + 19*a^4*b^3*c^3*d^4 - 37*a^5*b^2*c^2*d^5 + 3*a^6*b*c*d
^6 + 6*a^7*d^7)*f^7)*x^3 - (18*(b^7*c*d^6 + a*b^6*d^7)*e^7 - (37*b^7*c^2*d^
5 + 34*a*b^6*c*d^6 + 37*a^2*b^5*d^7)*e^6*f - 3*(b^7*c^3*d^4 - 3*a*b^6*c^2*d
^5 - 3*a^2*b^5*c*d^6 + a^3*b^4*d^7)*e^5*f^2 + (32*b^7*c^4*d^3 + a*b^6*c^3*d
^4 + 234*a^2*b^5*c^2*d^5 + a^3*b^4*c*d^6 + 32*a^4*b^3*d^7)*e^4*f^3 - (3*b^7
*c^5*d^2 - a*b^6*c^4*d^3 + 208*a^2*b^5*c^3*d^4 + 208*a^3*b^4*c^2*d^5 - a^4*
b^3*c*d^6 + 3*a^5*b^2*d^7)*e^3*f^4 - (37*b^7*c^6*d - 9*a*b^6*c^5*d^2 - 234*
a^2*b^5*c^4*d^3 + 208*a^3*b^4*c^3*d^4 - 234*a^4*b^3*c^2*d^5 - 9*a^5*b^2*c*d
^6 + 37*a^6*b*d^7)*e^2*f^5 + (18*b^7*c^7 - 34*a*b^6*c^6*d + 9*a^2*b^5*c^5*d
^2 + a^3*b^4*c^4*d^3 + a^4*b^3*c^3*d^4 + 9*a^5*b^2*c^2*d^5 - 34*a^6*b*c*d^6
 + 18*a^7*d^7)*e*f^6 + (18*a*b^6*c^7 - 37*a^2*b^5*c^6*d - 3*a^3*b^4*c^5*d^2
 + 32*a^4*b^3*c^4*d^3 - 3*a^5*b^2*c^3*d^4 - 37*a^6*b*c^2*d^5 + 18*a^7*c*d^6
)*f^7)*x^2 - 2*(2*(b^7*c^2*d^5 + 7*a*b^6*c*d^6 + a^2*b^5*d^7)*e^7 - 3*(2*b^
7*c^3*d^4 + 11*a*b^6*c^2*d^5 + 11*a^2*b^5*c*d^6 + 2*a^3*b^4*d^7)*e^6*f + (4
*b^7*c^4*d^3 + 17*a*b^6*c^3*d^4 + 78*a^2*b^5*c^2*d^5 + 17*a^3*b^4*c*d^6 + 4

*a^4*b^3*d^7)*e^5*f^2 + 2*(2*b^7*c^5*d^2 - 4*a*b^6*c^4*d^3 - 13*a^2*b^5*c^3
*d^4 - 13*a^3*b^4*c^2*d^5 - 4*a^4*b^3*c*d^6 + 2*a^5*b^2*d^7)*e^4*f^3 - (6*b
^7*c^6*d - 17*a*b^6*c^5*d^2 + 26*a^2*b^5*c^4*d^3 + 26*a^4*b^3*c^2*d^5 - 17*
a^5*b^2*c*d^6 + 6*a^6*b*d^7)*e^3*f^4 + (2*b^7*c^7 - 33*a*b^6*c^6*d + 78*a^2
*b^5*c^5*d^2 - 26*a^3*b^4*c^4*d^3 - 26*a^4*b^3*c^3*d^4 + 78*a^5*b^2*c^2*d^5
 - 33*a^6*b*c*d^6 + 2*a^7*d^7)*e^2*f^5 + (14*a*b^6*c^7 - 33*a^2*b^5*c^6*d +
 17*a^3*b^4*c^5*d^2 - 8*a^4*b^3*c^4*d^3 + 17*a^5*b^2*c^3*d^4 - 33*a^6*b*c^2
*d^5 + 14*a^7*c*d^6)*e*f^6 + 2*(a^2*b^5*c^7 - 3*a^3*b^4*c^6*d + 2*a^4*b^3*c
^5*d^2 + 2*a^5*b^2*c^4*d^3 - 3*a^6*b*c^3*d^4 + a^7*c^2*d^5)*f^7)*x)/((a^2*b
^8*c^6*d^4 - 4*a^3*b^7*c^5*d^5 + 6*a^4*b^6*c^4*d^6 - 4*a^5*b^5*c^3*d^7 + a^
6*b^4*c^2*d^8)*e^10 - 4*(a^2*b^8*c^7*d^3 - 3*a^3*b^7*c^6*d^4 + 2*a^4*b^6*c^
5*d^5 + 2*a^5*b^5*c^4*d^6 - 3*a^6*b^4*c^3*d^7 + a^7*b^3*c^2*d^8)*e^9*f + 2*
(3*a^2*b^8*c^8*d^2 - 4*a^3*b^7*c^7*d^3 - 11*a^4*b^6*c^6*d^4 + 24*a^5*b^5*c^
5*d^5 - 11*a^6*b^4*c^4*d^6 - 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8)*e^8*f^2
 - 4*(a^2*b^8*c^9*d + 2*a^3*b^7*c^8*d^2 - 12*a^4*b^6*c^7*d^3 + 9*a^5*b^5*c^
6*d^4 + 9*a^6*b^4*c^5*d^5 - 12*a^7*b^3*c^4*d^6 + 2*a^8*b^2*c^3*d^7 + a^9*b*
c^2*d^8)*e^7*f^3 + (a^2*b^8*c^10 + 12*a^3*b^7*c^9*d - 22*a^4*b^6*c^8*d^2 -
36*a^5*b^5*c^7*d^3 + 90*a^6*b^4*c^6*d^4 - 36*a^7*b^3*c^5*d^5 - 22*a^8*b^2*c
^4*d^6 + 12*a^9*b*c^3*d^7 + a^10*c^2*d^8)*e^6*f^4 - 4*(a^3*b^7*c^10 + 2*a^4
*b^6*c^9*d - 12*a^5*b^5*c^8*d^2 + 9*a^6*b^4*c^7*d^3 + 9*a^7*b^3*c^6*d^4 - 1
2*a^8*b^2*c^5*d^5 + 2*a^9*b*c^4*d^6 + a^10*c^3*d^7)*e^5*f^5 + 2*(3*a^4*b^6*
c^10 - 4*a^5*b^5*c^9*d - 11*a^6*b^4*c^8*d^2 + 24*a^7*b^3*c^7*d^3 - 11*a^8*b
^2*c^6*d^4 - 4*a^9*b*c^5*d^5 + 3*a^10*c^4*d^6)*e^4*f^6 - 4*(a^5*b^5*c^10 -
3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 +
 a^10*c^5*d^5)*e^3*f^7 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^
2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*e^2*f^8 + ((b^10*c^4*d^6 - 4*a*b^9*c^3*
d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*e^8*f^2 - 4*(b^10
*c^5*d^5 - 3*a*b^9*c^4*d^6 + 2*a^2*b^8*c^3*d^7 + 2*a^3*b^7*c^2*d^8 - 3*a^4*
b^6*c*d^9 + a^5*b^5*d^10)*e^7*f^3 + 2*(3*b^10*c^6*d^4 - 4*a*b^9*c^5*d^5 - 1
1*a^2*b^8*c^4*d^6 + 24*a^3*b^7*c^3*d^7 - 11*a^4*b^6*c^2*d^8 - 4*a^5*b^5*c*d
^9 + 3*a^6*b^4*d^10)*e^6*f^4 - 4*(b^10*c^7*d^3 + 2*a*b^9*c^6*d^4 - 12*a^2*b
^8*c^5*d^5 + 9*a^3*b^7*c^4*d^6 + 9*a^4*b^6*c^3*d^7 - 12*a^5*b^5*c^2*d^8 + 2
*a^6*b^4*c*d^9 + a^7*b^3*d^10)*e^5*f^5 + (b^10*c^8*d^2 + 12*a*b^9*c^7*d^3 -
 22*a^2*b^8*c^6*d^4 - 36*a^3*b^7*c^5*d^5 + 90*a^4*b^6*c^4*d^6 - 36*a^5*b^5*
c^3*d^7 - 22*a^6*b^4*c^2*d^8 + 12*a^7*b^3*c*d^9 + a^8*b^2*d^10)*e^4*f^6 - 4
*(a*b^9*c^8*d^2 + 2*a^2*b^8*c^7*d^3 - 12*a^3*b^7*c^6*d^4 + 9*a^4*b^6*c^5*d^
5 + 9*a^5*b^5*c^4*d^6 - 12*a^6*b^4*c^3*d^7 + 2*a^7*b^3*c^2*d^8 + a^8*b^2*c*
d^9)*e^3*f^7 + 2*(3*a^2*b^8*c^8*d^2 - 4*a^3*b^7*c^7*d^3 - 11*a^4*b^6*c^6*d^
4 + 24*a^5*b^5*c^5*d^5 - 11*a^6*b^4*c^4*d^6 - 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2
*c^2*d^8)*e^2*f^8 - 4*(a^3*b^7*c^8*d^2 - 3*a^4*b^6*c^7*d^3 + 2*a^5*b^5*c^6*
d^4 + 2*a^6*b^4*c^5*d^5 - 3*a^7*b^3*c^4*d^6 + a^8*b^2*c^3*d^7)*e*f^9 + (a^4
*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6*d^4 - 4*a^7*b^3*c^5*d^5 +
a^8*b^2*c^4*d^6)*f^10)*x^6 + 2*((b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8
*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*e^9*f - 3*(b^10*c^5*d^5 - 3*a*b^
9*c^4*d^6 + 2*a^2*b^8*c^3*d^7 + 2*a^3*b^7*c^2*d^8 - 3*a^4*b^6*c*d^9 + a^5*b

^5*d^10)*e^8*f^2 + 2*(b^10*c^6*d^4 - 9*a^2*b^8*c^4*d^6 + 16*a^3*b^7*c^3*d^7 - 9*a^4*b^6*c^2*d^8 + a^6*b^4*d^10)*e^7*f^3 + 2*(b^10*c^7*d^3 - 5*a*b^9*c^6*d^4 + 9*a^2*b^8*c^5*d^5 - 5*a^3*b^7*c^4*d^6 - 5*a^4*b^6*c^3*d^7 + 9*a^5*b^5*c^2*d^8 - 5*a^6*b^4*c*d^9 + a^7*b^3*d^10)*e^6*f^4 - 3*(b^10*c^8*d^2 - 6*a^2*b^8*c^6*d^4 + 8*a^3*b^7*c^5*d^5 - 6*a^4*b^6*c^4*d^6 + 8*a^5*b^5*c^3*d^7 - 6*a^6*b^4*c^2*d^8 + a^8*b^2*d^10)*e^5*f^5 + (b^10*c^9*d + 9*a*b^9*c^8*d^2 - 18*a^2*b^8*c^7*d^3 - 10*a^3*b^7*c^6*d^4 + 18*a^4*b^6*c^5*d^5 + 18*a^5*b^5*c^4*d^6 - 10*a^6*b^4*c^3*d^7 - 18*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 + a^9*b*d^10)*e^4*f^6 - 2*(2*a*b^9*c^9*d + 3*a^2*b^8*c^8*d^2 - 16*a^3*b^7*c^7*d^3 + 5*a^4*b^6*c^6*d^4 + 12*a^5*b^5*c^5*d^5 + 5*a^6*b^4*c^4*d^6 - 16*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8 + 2*a^9*b*c*d^9)*e^3*f^7 + 6*(a^2*b^8*c^9*d - a^3*b^7*c^8*d^2 - 3*a^4*b^6*c^7*d^3 + 3*a^5*b^5*c^6*d^4 + 3*a^6*b^4*c^5*d^5 - 3*a^7*b^3*c^4*d^6 - a^8*b^2*c^3*d^7 + a^9*b*c^2*d^8)*e^2*f^8 - (4*a^3*b^7*c^9*d - 9*a^4*b^6*c^8*d^2 + 10*a^6*b^4*c^6*d^4 - 9*a^8*b^2*c^4*d^6 + 4*a^9*b*c^3*d^7)*e*f^9 + (a^4*b^6*c^9*d - 3*a^5*b^5*c^8*d^2 + 2*a^6*b^4*c^7*d^3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5 + a^9*b*c^4*d^6)*f^10)*x^5 + ((b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*e^10 - 3*(3*b^10*c^6*d^4 - 8*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 5*a^4*b^6*c^2*d^8 - 8*a^5*b^5*c*d^9 + 3*a^6*b^4*d^10)*e^8*f^2 + 4*(4*b^10*c^7*d^3 - 5*a*b^9*c^6*d^4 - 9*a^2*b^8*c^5*d^5 + 10*a^3*b^7*c^4*d^6 + 10*a^4*b^6*c^3*d^7 - 9*a^5*b^5*c^2*d^8 - 5*a^6*b^4*c*d^9 + 4*a^7*b^3*d^10)*e^7*f^3 - (9*b^10*c^8*d^2 + 20*a*b^9*c^7*d^3 - 90*a^2*b^8*c^6*d^4 + 36*a^3*b^7*c^5*d^5 + 50*a^4*b^6*c^4*d^6 + 36*a^5*b^5*c^3*d^7 - 90*a^6*b^4*c^2*d^8 + 20*a^7*b^3*c*d^9 + 9*a^8*b^2*d^10)*e^6*f^4 + 12*(2*a*b^9*c^8*d^2 - 3*a^2*b^8*c^7*d^3 - 3*a^3*b^7*c^6*d^4 + 4*a^4*b^6*c^5*d^5 + 4*a^5*b^5*c^4*d^6 - 3*a^6*b^4*c^3*d^7 - 3*a^7*b^3*c^2*d^8 + 2*a^8*b^2*c*d^9)*e^5*f^5 + (b^10*c^10 - 15*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 - 50*a^4*b^6*c^6*d^4 + 48*a^5*b^5*c^5*d^5 - 50*a^6*b^4*c^4*d^6 + 40*a^7*b^3*c^3*d^7 - 15*a^8*b^2*c^2*d^8 + a^10*d^10)*e^4*f^6 - 4*(a*b^9*c^10 - 10*a^4*b^6*c^7*d^3 + 9*a^5*b^5*c^6*d^4 + 9*a^6*b^4*c^5*d^5 - 10*a^7*b^3*c^4*d^6 + a^10*c*d^9)*e^3*f^7 + 3*(2*a^2*b^8*c^10 - 5*a^4*b^6*c^8*d^2 - 12*a^5*b^5*c^7*d^3 + 30*a^6*b^4*c^6*d^4 - 12*a^7*b^3*c^5*d^5 - 5*a^8*b^2*c^4*d^6 + 2*a^10*c^2*d^8)*e^2*f^8 - 4*(a^3*b^7*c^10 - 6*a^5*b^5*c^8*d^2 + 5*a^6*b^4*c^7*d^3 + 5*a^7*b^3*c^6*d^4 - 6*a^8*b^2*c^5*d^5 + a^10*c^3*d^7)*e*f^9 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*f^10)*x^4 + 2*((b^10*c^5*d^5 - 3*a*b^9*c^4*d^6 + 2*a^2*b^8*c^3*d^7 + 2*a^3*b^7*c^2*d^8 - 3*a^4*b^6*c*d^9 + a^5*b^5*d^10)*e^10 - (3*b^10*c^6*d^4 - 8*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 5*a^4*b^6*c^2*d^8 - 8*a^5*b^5*c*d^9 + 3*a^6*b^4*d^10)*e^9*f + (2*b^10*c^7*d^3 - 5*a*b^9*c^6*d^4 + 3*a^2*b^8*c^5*d^5 + 3*a^5*b^5*c^2*d^8 - 5*a^6*b^4*c*d^9 + 2*a^7*b^3*d^10)*e^8*f^2 + 2*(b^10*c^8*d^2 - 16*a^3*b^7*c^5*d^5 + 30*a^4*b^6*c^4*d^6 - 16*a^5*b^5*c^3*d^7 + a^8*b^2*d^10)*e^7*f^3 - (3*b^10*c^9*d + 5*a*b^9*c^8*d^2 - 60*a^3*b^7*c^6*d^4 + 52*a^4*b^6*c^5*d^5 + 52*a^5*b^5*c^4*d^6 - 60*a^6*b^4*c^3*d^7 + 5*a^8*b^2*c*d^9 + 3*a^9*b*d^10)*e^6*f^4 + (b^10*c^10 + 8*a*b^9*c^9*d + 3*a^2*b^8*c^8*d^2 - 32*a^3*b^7*c^7*d^3 - 52*a^4*b^6*c^6*d^4 + 144*a^5*b^5*c^5*d^5 - 52*a^6*b^4*c^4*d^6 - 32*a^7*b^3*c^3*d^

```
7 + 3*a^8*b^2*c^2*d^8 + 8*a^9*b*c*d^9 + a^10*d^10)*e^5*f^5 - (3*a*b^9*c^10
+ 5*a^2*b^8*c^9*d - 60*a^4*b^6*c^7*d^3 + 52*a^5*b^5*c^6*d^4 + 52*a^6*b^4*c^
5*d^5 - 60*a^7*b^3*c^4*d^6 + 5*a^9*b*c^2*d^8 + 3*a^10*c*d^9)*e^4*f^6 + 2*(a
^2*b^8*c^10 - 16*a^5*b^5*c^7*d^3 + 30*a^6*b^4*c^6*d^4 - 16*a^7*b^3*c^5*d^5
+ a^10*c^2*d^8)*e^3*f^7 + (2*a^3*b^7*c^10 - 5*a^4*b^6*c^9*d + 3*a^5*b^5*c^8
*d^2 + 3*a^8*b^2*c^5*d^5 - 5*a^9*b*c^4*d^6 + 2*a^10*c^3*d^7)*e^2*f^8 - (3*a
^4*b^6*c^10 - 8*a^5*b^5*c^9*d + 5*a^6*b^4*c^8*d^2 + 5*a^8*b^2*c^6*d^4 - 8*a
^9*b*c^5*d^5 + 3*a^10*c^4*d^6)*e*f^9 + (a^5*b^5*c^10 - 3*a^6*b^4*c^9*d + 2*
a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*f^10)
*x^3 + ((b^10*c^6*d^4 - 9*a^2*b^8*c^4*d^6 + 16*a^3*b^7*c^3*d^7 - 9*a^4*b^6*
c^2*d^8 + a^6*b^4*d^10)*e^10 - 4*(b^10*c^7*d^3 - 6*a^2*b^8*c^5*d^5 + 5*a^3*
b^7*c^4*d^6 + 5*a^4*b^6*c^3*d^7 - 6*a^5*b^5*c^2*d^8 + a^7*b^3*d^10)*e^9*f +
 3*(2*b^10*c^8*d^2 - 5*a^2*b^8*c^6*d^4 - 12*a^3*b^7*c^5*d^5 + 30*a^4*b^6*c^
4*d^6 - 12*a^5*b^5*c^3*d^7 - 5*a^6*b^4*c^2*d^8 + 2*a^8*b^2*d^10)*e^8*f^2 -
4*(b^10*c^9*d - 10*a^3*b^7*c^6*d^4 + 9*a^4*b^6*c^5*d^5 + 9*a^5*b^5*c^4*d^6
- 10*a^6*b^4*c^3*d^7 + a^9*b*d^10)*e^7*f^3 + (b^10*c^10 - 15*a^2*b^8*c^8*d^
2 + 40*a^3*b^7*c^7*d^3 - 50*a^4*b^6*c^6*d^4 + 48*a^5*b^5*c^5*d^5 - 50*a^6*b
^4*c^4*d^6 + 40*a^7*b^3*c^3*d^7 - 15*a^8*b^2*c^2*d^8 + a^10*d^10)*e^6*f^4 +
 12*(2*a^2*b^8*c^9*d - 3*a^3*b^7*c^8*d^2 - 3*a^4*b^6*c^7*d^3 + 4*a^5*b^5*c^
6*d^4 + 4*a^6*b^4*c^5*d^5 - 3*a^7*b^3*c^4*d^6 - 3*a^8*b^2*c^3*d^7 + 2*a^9*b
*c^2*d^8)*e^5*f^5 - (9*a^2*b^8*c^10 + 20*a^3*b^7*c^9*d - 90*a^4*b^6*c^8*d^2
 + 36*a^5*b^5*c^7*d^3 + 50*a^6*b^4*c^6*d^4 + 36*a^7*b^3*c^5*d^5 - 90*a^8*b^
2*c^4*d^6 + 20*a^9*b*c^3*d^7 + 9*a^10*c^2*d^8)*e^4*f^6 + 4*(4*a^3*b^7*c^10
- 5*a^4*b^6*c^9*d - 9*a^5*b^5*c^8*d^2 + 10*a^6*b^4*c^7*d^3 + 10*a^7*b^3*c^6
*d^4 - 9*a^8*b^2*c^5*d^5 - 5*a^9*b*c^4*d^6 + 4*a^10*c^3*d^7)*e^3*f^7 - 3*(3
*a^4*b^6*c^10 - 8*a^5*b^5*c^9*d + 5*a^6*b^4*c^8*d^2 + 5*a^8*b^2*c^6*d^4 - 8
*a^9*b*c^5*d^5 + 3*a^10*c^4*d^6)*e^2*f^8 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d
+ 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*f^10)*x^2 + 2*((a*b^9
*c^6*d^4 - 3*a^2*b^8*c^5*d^5 + 2*a^3*b^7*c^4*d^6 + 2*a^4*b^6*c^3*d^7 - 3*a^
5*b^5*c^2*d^8 + a^6*b^4*c*d^9)*e^10 - (4*a*b^9*c^7*d^3 - 9*a^2*b^8*c^6*d^4
+ 10*a^4*b^6*c^4*d^6 - 9*a^6*b^4*c^2*d^8 + 4*a^7*b^3*c*d^9)*e^9*f + 6*(a*b^
9*c^8*d^2 - a^2*b^8*c^7*d^3 - 3*a^3*b^7*c^6*d^4 + 3*a^4*b^6*c^5*d^5 + 3*a^5
*b^5*c^4*d^6 - 3*a^6*b^4*c^3*d^7 - a^7*b^3*c^2*d^8 + a^8*b^2*c*d^9)*e^8*f^2
 - 2*(2*a*b^9*c^9*d + 3*a^2*b^8*c^8*d^2 - 16*a^3*b^7*c^7*d^3 + 5*a^4*b^6*c^
6*d^4 + 12*a^5*b^5*c^5*d^5 + 5*a^6*b^4*c^4*d^6 - 16*a^7*b^3*c^3*d^7 + 3*a^8
*b^2*c^2*d^8 + 2*a^9*b*c*d^9)*e^7*f^3 + (a*b^9*c^10 + 9*a^2*b^8*c^9*d - 18*
a^3*b^7*c^8*d^2 - 10*a^4*b^6*c^7*d^3 + 18*a^5*b^5*c^6*d^4 + 18*a^6*b^4*c^5*
d^5 - 10*a^7*b^3*c^4*d^6 - 18*a^8*b^2*c^3*d^7 + 9*a^9*b*c^2*d^8 + a^10*c*d^
9)*e^6*f^4 - 3*(a^2*b^8*c^10 - 6*a^4*b^6*c^8*d^2 + 8*a^5*b^5*c^7*d^3 - 6*a^
6*b^4*c^6*d^4 + 8*a^7*b^3*c^5*d^5 - 6*a^8*b^2*c^4*d^6 + a^10*c^2*d^8)*e^5*f
^5 + 2*(a^3*b^7*c^10 - 5*a^4*b^6*c^9*d + 9*a^5*b^5*c^8*d^2 - 5*a^6*b^4*c^7*
d^3 - 5*a^7*b^3*c^6*d^4 + 9*a^8*b^2*c^5*d^5 - 5*a^9*b*c^4*d^6 + a^10*c^3*d^
7)*e^4*f^6 + 2*(a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^7*d^3 - 9*a
^8*b^2*c^6*d^4 + a^10*c^4*d^6)*e^3*f^7 - 3*(a^5*b^5*c^10 - 3*a^6*b^4*c^9*d
+ 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*e
```

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^2*f^8 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*
d^3 + a^10*c^6*d^4)*e*f^9)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7111 vs. $2(489) = 978$.

Time = 0.40 (sec) , antiderivative size = 7111, normalized size of antiderivative = 14.37

$$\int \frac{1}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^3,x, algorithm="giac")
```

```
[Out] 3*(2*b^8*d^2*e^2 + 3*b^8*c*d*e*f - 7*a*b^7*d^2*e*f + 2*b^8*c^2*f^2 - 7*a*b^
7*c*d*f^2 + 7*a^2*b^6*d^2*f^2)*log(abs(b*x + a))/(b^11*c^5*e^5 - 5*a*b^10*c
^4*d*e^5 + 10*a^2*b^9*c^3*d^2*e^5 - 10*a^3*b^8*c^2*d^3*e^5 + 5*a^4*b^7*c*d^
4*e^5 - a^5*b^6*d^5*e^5 - 5*a*b^10*c^5*e^4*f + 25*a^2*b^9*c^4*d*e^4*f - 50*
a^3*b^8*c^3*d^2*e^4*f + 50*a^4*b^7*c^2*d^3*e^4*f - 25*a^5*b^6*c*d^4*e^4*f +
 5*a^6*b^5*d^5*e^4*f + 10*a^2*b^9*c^5*e^3*f^2 - 50*a^3*b^8*c^4*d*e^3*f^2 +
100*a^4*b^7*c^3*d^2*e^3*f^2 - 100*a^5*b^6*c^2*d^3*e^3*f^2 + 50*a^6*b^5*c*d^
4*e^3*f^2 - 10*a^7*b^4*d^5*e^3*f^2 - 10*a^3*b^8*c^5*e^2*f^3 + 50*a^4*b^7*c^
4*d*e^2*f^3 - 100*a^5*b^6*c^3*d^2*e^2*f^3 + 100*a^6*b^5*c^2*d^3*e^2*f^3 - 5
0*a^7*b^4*c*d^4*e^2*f^3 + 10*a^8*b^3*d^5*e^2*f^3 + 5*a^4*b^7*c^5*e*f^4 - 25
*a^5*b^6*c^4*d*e*f^4 + 50*a^6*b^5*c^3*d^2*e*f^4 - 50*a^7*b^4*c^2*d^3*e*f^4
+ 25*a^8*b^3*c*d^4*e*f^4 - 5*a^9*b^2*d^5*e*f^4 - a^5*b^6*c^5*f^5 + 5*a^6*b^
5*c^4*d*f^5 - 10*a^7*b^4*c^3*d^2*f^5 + 10*a^8*b^3*c^2*d^3*f^5 - 5*a^9*b^2*c
*d^4*f^5 + a^10*b*d^5*f^5) - 3*(2*b^2*d^8*e^2 - 7*b^2*c*d^7*e*f + 3*a*b*d^8
*e*f + 7*b^2*c^2*d^6*f^2 - 7*a*b*c*d^7*f^2 + 2*a^2*d^8*f^2)*log(abs(d*x + c
))/(b^5*c^5*d^6*e^5 - 5*a*b^4*c^4*d^7*e^5 + 10*a^2*b^3*c^3*d^8*e^5 - 10*a^3
*b^2*c^2*d^9*e^5 + 5*a^4*b*c*d^10*e^5 - a^5*d^11*e^5 - 5*b^5*c^6*d^5*e^4*f
+ 25*a*b^4*c^5*d^6*e^4*f - 50*a^2*b^3*c^4*d^7*e^4*f + 50*a^3*b^2*c^3*d^8*e^
4*f - 25*a^4*b*c^2*d^9*e^4*f + 5*a^5*c*d^10*e^4*f + 10*b^5*c^7*d^4*e^3*f^2
- 50*a*b^4*c^6*d^5*e^3*f^2 + 100*a^2*b^3*c^5*d^6*e^3*f^2 - 100*a^3*b^2*c^4*
d^7*e^3*f^2 + 50*a^4*b*c^3*d^8*e^3*f^2 - 10*a^5*c^2*d^9*e^3*f^2 - 10*b^5*c^
8*d^3*e^2*f^3 + 50*a*b^4*c^7*d^4*e^2*f^3 - 100*a^2*b^3*c^6*d^5*e^2*f^3 + 10
0*a^3*b^2*c^5*d^6*e^2*f^3 - 50*a^4*b*c^4*d^7*e^2*f^3 + 10*a^5*c^3*d^8*e^2*f
^3 + 5*b^5*c^9*d^2*e*f^4 - 25*a*b^4*c^8*d^3*e*f^4 + 50*a^2*b^3*c^7*d^4*e*f^
4 - 50*a^3*b^2*c^6*d^5*e*f^4 + 25*a^4*b*c^5*d^6*e*f^4 - 5*a^5*c^4*d^7*e*f^4
 - b^5*c^10*d*f^5 + 5*a*b^4*c^9*d^2*f^5 - 10*a^2*b^3*c^8*d^3*f^5 + 10*a^3*b
^2*c^7*d^4*f^5 - 5*a^4*b*c^6*d^5*f^5 + a^5*c^5*d^6*f^5) + 3*(7*b^2*d^2*e^2*
f^6 - 7*b^2*c*d*e*f^7 - 7*a*b*d^2*e*f^7 + 2*b^2*c^2*f^8 + 3*a*b*c*d*f^8 + 2
*a^2*d^2*f^8)*log(abs(f*x + e))/(b^5*d^5*e^10*f - 5*b^5*c*d^4*e^9*f^2 - 5*a
*b^4*d^5*e^9*f^2 + 10*b^5*c^2*d^3*e^8*f^3 + 25*a*b^4*c*d^4*e^8*f^3 + 10*a^2
```

*b^3*d^5*e^8*f^3 - 10*b^5*c^3*d^2*e^7*f^4 - 50*a*b^4*c^2*d^3*e^7*f^4 - 50*a
^2*b^3*c*d^4*e^7*f^4 - 10*a^3*b^2*d^5*e^7*f^4 + 5*b^5*c^4*d*e^6*f^5 + 50*a*
b^4*c^3*d^2*e^6*f^5 + 100*a^2*b^3*c^2*d^3*e^6*f^5 + 50*a^3*b^2*c*d^4*e^6*f^
5 + 5*a^4*b*d^5*e^6*f^5 - b^5*c^5*e^5*f^6 - 25*a*b^4*c^4*d*e^5*f^6 - 100*a^
2*b^3*c^3*d^2*e^5*f^6 - 100*a^3*b^2*c^2*d^3*e^5*f^6 - 25*a^4*b*c*d^4*e^5*f^
6 - a^5*d^5*e^5*f^6 + 5*a*b^4*c^5*e^4*f^7 + 50*a^2*b^3*c^4*d*e^4*f^7 + 100*
a^3*b^2*c^3*d^2*e^4*f^7 + 50*a^4*b*c^2*d^3*e^4*f^7 + 5*a^5*c*d^4*e^4*f^7 -
10*a^2*b^3*c^5*e^3*f^8 - 50*a^3*b^2*c^4*d*e^3*f^8 - 50*a^4*b*c^3*d^2*e^3*f^
8 - 10*a^5*c^2*d^3*e^3*f^8 + 10*a^3*b^2*c^5*e^2*f^9 + 25*a^4*b*c^4*d*e^2*f^
9 + 10*a^5*c^3*d^2*e^2*f^9 - 5*a^4*b*c^5*e*f^10 - 5*a^5*c^4*d*e*f^10 + a^5*
c^5*f^11) + 1/2*(12*b^7*d^7*e^5*f^2*x^5 - 30*b^7*c*d^6*e^4*f^3*x^5 - 30*a*b
^6*d^7*e^4*f^3*x^5 + 12*b^7*c^2*d^5*e^3*f^4*x^5 + 96*a*b^6*c*d^6*e^3*f^4*x^
5 + 12*a^2*b^5*d^7*e^3*f^4*x^5 + 12*b^7*c^3*d^4*e^2*f^5*x^5 - 72*a*b^6*c^2*
d^5*e^2*f^5*x^5 - 72*a^2*b^5*c*d^6*e^2*f^5*x^5 + 12*a^3*b^4*d^7*e^2*f^5*x^5
 - 30*b^7*c^4*d^3*e*f^6*x^5 + 96*a*b^6*c^3*d^4*e*f^6*x^5 - 72*a^2*b^5*c^2*d
^5*e*f^6*x^5 + 96*a^3*b^4*c*d^6*e*f^6*x^5 - 30*a^4*b^3*d^7*e*f^6*x^5 + 12*b
^7*c^5*d^2*f^7*x^5 - 30*a*b^6*c^4*d^3*f^7*x^5 + 12*a^2*b^5*c^3*d^4*f^7*x^5
+ 12*a^3*b^4*c^2*d^5*f^7*x^5 - 30*a^4*b^3*c*d^6*f^7*x^5 + 12*a^5*b^2*d^7*f^
7*x^5 + 24*b^7*d^7*e^6*f*x^4 - 42*b^7*c*d^6*e^5*f^2*x^4 - 42*a*b^6*d^7*e^5*
f^2*x^4 - 21*b^7*c^2*d^5*e^4*f^3*x^4 + 102*a*b^6*c*d^6*e^4*f^3*x^4 - 21*a^2
*b^5*d^7*e^4*f^3*x^4 + 42*b^7*c^3*d^4*e^3*f^4*x^4 + 18*a*b^6*c^2*d^5*e^3*f^
4*x^4 + 18*a^2*b^5*c*d^6*e^3*f^4*x^4 + 42*a^3*b^4*d^7*e^3*f^4*x^4 - 21*b^7*
c^4*d^3*e^2*f^5*x^4 + 18*a*b^6*c^3*d^4*e^2*f^5*x^4 - 234*a^2*b^5*c^2*d^5*e^
2*f^5*x^4 + 18*a^3*b^4*c*d^6*e^2*f^5*x^4 - 21*a^4*b^3*d^7*e^2*f^5*x^4 - 42*
b^7*c^5*d^2*e*f^6*x^4 + 102*a*b^6*c^4*d^3*e*f^6*x^4 + 18*a^2*b^5*c^3*d^4*e*
f^6*x^4 + 18*a^3*b^4*c^2*d^5*e*f^6*x^4 + 102*a^4*b^3*c*d^6*e*f^6*x^4 - 42*a
^5*b^2*d^7*e*f^6*x^4 + 24*b^7*c^6*d*f^7*x^4 - 42*a*b^6*c^5*d^2*f^7*x^4 - 21
*a^2*b^5*c^4*d^3*f^7*x^4 + 42*a^3*b^4*c^3*d^4*f^7*x^4 - 21*a^4*b^3*c^2*d^5*
f^7*x^4 - 42*a^5*b^2*c*d^6*f^7*x^4 + 24*a^6*b*d^7*f^7*x^4 + 12*b^7*d^7*e^7*
x^3 + 6*b^7*c*d^6*e^6*f*x^3 + 6*a*b^6*d^7*e^6*f*x^3 - 74*b^7*c^2*d^5*e^5*f^
2*x^3 - 56*a*b^6*c*d^6*e^5*f^2*x^3 - 74*a^2*b^5*d^7*e^5*f^2*x^3 + 38*b^7*c^
3*d^4*e^4*f^3*x^3 + 172*a*b^6*c^2*d^5*e^4*f^3*x^3 + 172*a^2*b^5*c*d^6*e^4*f
^3*x^3 + 38*a^3*b^4*d^7*e^4*f^3*x^3 + 38*b^7*c^4*d^3*e^3*f^4*x^3 - 136*a*b^
6*c^3*d^4*e^3*f^4*x^3 - 104*a^2*b^5*c^2*d^5*e^3*f^4*x^3 - 136*a^3*b^4*c*d^6
*e^3*f^4*x^3 + 38*a^4*b^3*d^7*e^3*f^4*x^3 - 74*b^7*c^5*d^2*e^2*f^5*x^3 + 17
2*a*b^6*c^4*d^3*e^2*f^5*x^3 - 104*a^2*b^5*c^3*d^4*e^2*f^5*x^3 - 104*a^3*b^4
*c^2*d^5*e^2*f^5*x^3 + 172*a^4*b^3*c*d^6*e^2*f^5*x^3 - 74*a^5*b^2*d^7*e^2*f
^5*x^3 + 6*b^7*c^6*d*e*f^6*x^3 - 56*a*b^6*c^5*d^2*e*f^6*x^3 + 172*a^2*b^5*c
^4*d^3*e*f^6*x^3 - 136*a^3*b^4*c^3*d^4*e*f^6*x^3 + 172*a^4*b^3*c^2*d^5*e*f^
6*x^3 - 56*a^5*b^2*c*d^6*e*f^6*x^3 + 6*a^6*b*d^7*e*f^6*x^3 + 12*b^7*c^7*f^7
*x^3 + 6*a*b^6*c^6*d*f^7*x^3 - 74*a^2*b^5*c^5*d^2*f^7*x^3 + 38*a^3*b^4*c^4*
d^3*f^7*x^3 + 38*a^4*b^3*c^3*d^4*f^7*x^3 - 74*a^5*b^2*c^2*d^5*f^7*x^3 + 6*a
^6*b*c*d^6*f^7*x^3 + 12*a^7*d^7*f^7*x^3 + 18*b^7*c*d^6*e^7*x^2 + 18*a*b^6*d
^7*e^7*x^2 - 37*b^7*c^2*d^5*e^6*f*x^2 - 34*a*b^6*c*d^6*e^6*f*x^2 - 37*a^2*b
^5*d^7*e^6*f*x^2 - 3*b^7*c^3*d^4*e^5*f^2*x^2 + 9*a*b^6*c^2*d^5*e^5*f^2*x^2

+ 9*a^2*b^5*c*d^6*e^5*f^2*x^2 - 3*a^3*b^4*d^7*e^5*f^2*x^2 + 32*b^7*c^4*d^3*
e^4*f^3*x^2 + a*b^6*c^3*d^4*e^4*f^3*x^2 + 234*a^2*b^5*c^2*d^5*e^4*f^3*x^2 +
 a^3*b^4*c*d^6*e^4*f^3*x^2 + 32*a^4*b^3*d^7*e^4*f^3*x^2 - 3*b^7*c^5*d^2*e^3
*f^4*x^2 + a*b^6*c^4*d^3*e^3*f^4*x^2 - 208*a^2*b^5*c^3*d^4*e^3*f^4*x^2 - 20
8*a^3*b^4*c^2*d^5*e^3*f^4*x^2 + a^4*b^3*c*d^6*e^3*f^4*x^2 - 3*a^5*b^2*d^7*e
^3*f^4*x^2 - 37*b^7*c^6*d*e^2*f^5*x^2 + 9*a*b^6*c^5*d^2*e^2*f^5*x^2 + 234*a
^2*b^5*c^4*d^3*e^2*f^5*x^2 - 208*a^3*b^4*c^3*d^4*e^2*f^5*x^2 + 234*a^4*b^3*
c^2*d^5*e^2*f^5*x^2 + 9*a^5*b^2*c*d^6*e^2*f^5*x^2 - 37*a^6*b*d^7*e^2*f^5*x^
2 + 18*b^7*c^7*e*f^6*x^2 - 34*a*b^6*c^6*d*e*f^6*x^2 + 9*a^2*b^5*c^5*d^2*e*f
^6*x^2 + a^3*b^4*c^4*d^3*e*f^6*x^2 + a^4*b^3*c^3*d^4*e*f^6*x^2 + 9*a^5*b^2*
c^2*d^5*e*f^6*x^2 - 34*a^6*b*c*d^6*e*f^6*x^2 + 18*a^7*d^7*e*f^6*x^2 + 18*a*
b^6*c^7*f^7*x^2 - 37*a^2*b^5*c^6*d*f^7*x^2 - 3*a^3*b^4*c^5*d^2*f^7*x^2 + 32
*a^4*b^3*c^4*d^3*f^7*x^2 - 3*a^5*b^2*c^3*d^4*f^7*x^2 - 37*a^6*b*c^2*d^5*f^7
*x^2 + 18*a^7*c*d^6*f^7*x^2 + 4*b^7*c^2*d^5*e^7*x + 28*a*b^6*c*d^6*e^7*x +
4*a^2*b^5*d^7*e^7*x - 12*b^7*c^3*d^4*e^6*f*x - 66*a*b^6*c^2*d^5*e^6*f*x - 6
6*a^2*b^5*c*d^6*e^6*f*x - 12*a^3*b^4*d^7*e^6*f*x + 8*b^7*c^4*d^3*e^5*f^2*x
+ 34*a*b^6*c^3*d^4*e^5*f^2*x + 156*a^2*b^5*c^2*d^5*e^5*f^2*x + 34*a^3*b^4*c
*d^6*e^5*f^2*x + 8*a^4*b^3*d^7*e^5*f^2*x + 8*b^7*c^5*d^2*e^4*f^3*x - 16*a*b
^6*c^4*d^3*e^4*f^3*x - 52*a^2*b^5*c^3*d^4*e^4*f^3*x - 52*a^3*b^4*c^2*d^5*e^
4*f^3*x - 16*a^4*b^3*c*d^6*e^4*f^3*x + 8*a^5*b^2*d^7*e^4*f^3*x - 12*b^7*c^6
*d*e^3*f^4*x + 34*a*b^6*c^5*d^2*e^3*f^4*x - 52*a^2*b^5*c^4*d^3*e^3*f^4*x -
52*a^4*b^3*c^2*d^5*e^3*f^4*x + 34*a^5*b^2*c*d^6*e^3*f^4*x - 12*a^6*b*d^7*e^
3*f^4*x + 4*b^7*c^7*e^2*f^5*x - 66*a*b^6*c^6*d*e^2*f^5*x + 156*a^2*b^5*c^5*
d^2*e^2*f^5*x - 52*a^3*b^4*c^4*d^3*e^2*f^5*x - 52*a^4*b^3*c^3*d^4*e^2*f^5*x
 + 156*a^5*b^2*c^2*d^5*e^2*f^5*x - 66*a^6*b*c*d^6*e^2*f^5*x + 4*a^7*d^7*e^2
*f^5*x + 28*a*b^6*c^7*e*f^6*x - 66*a^2*b^5*c^6*d*e*f^6*x + 34*a^3*b^4*c^5*d
^2*e*f^6*x - 16*a^4*b^3*c^4*d^3*e*f^6*x + 34*a^5*b^2*c^3*d^4*e*f^6*x - 66*a
^6*b*c^2*d^5*e*f^6*x + 28*a^7*c*d^6*e*f^6*x + 4*a^2*b^5*c^7*f^7*x - 12*a^3*
b^4*c^6*d*f^7*x + 8*a^4*b^3*c^5*d^2*f^7*x + 8*a^5*b^2*c^4*d^3*f^7*x - 12*a^
6*b*c^3*d^4*f^7*x + 4*a^7*c^2*d^5*f^7*x - b^7*c^3*d^4*e^7 + 7*a*b^6*c^2*d^5
*e^7 + 7*a^2*b^5*c*d^6*e^7 - a^3*b^4*d^7*e^7 + 4*b^7*c^4*d^3*e^6*f - 21*a*b
^6*c^3*d^4*e^6*f - 26*a^2*b^5*c^2*d^5*e^6*f - 21*a^3*b^4*c*d^6*e^6*f + 4*a^
4*b^3*d^7*e^6*f - 6*b^7*c^5*d^2*e^5*f^2 + 14*a*b^6*c^4*d^3*e^5*f^2 + 52*a^2
*b^5*c^3*d^4*e^5*f^2 + 52*a^3*b^4*c^2*d^5*e^5*f^2 + 14*a^4*b^3*c*d^6*e^5*f^
2 - 6*a^5*b^2*d^7*e^5*f^2 + 4*b^7*c^6*d*e^4*f^3 + 14*a*b^6*c^5*d^2*e^4*f^3
- 78*a^2*b^5*c^4*d^3*e^4*f^3 - 78*a^4*b^3*c^2*d^5*e^4*f^3 + 14*a^5*b^2*c*d^
6*e^4*f^3 + 4*a^6*b*d^7*e^4*f^3 - b^7*c^7*e^3*f^4 - 21*a*b^6*c^6*d*e^3*f^4
+ 52*a^2*b^5*c^5*d^2*e^3*f^4 + 52*a^5*b^2*c^2*d^5*e^3*f^4 - 21*a^6*b*c*d^6*
e^3*f^4 - a^7*d^7*e^3*f^4 + 7*a*b^6*c^7*e^2*f^5 - 26*a^2*b^5*c^6*d*e^2*f^5
+ 52*a^3*b^4*c^5*d^2*e^2*f^5 - 78*a^4*b^3*c^4*d^3*e^2*f^5 + 52*a^5*b^2*c^3*
d^4*e^2*f^5 - 26*a^6*b*c^2*d^5*e^2*f^5 + 7*a^7*c*d^6*e^2*f^5 + 7*a^2*b^5*c^
7*e*f^6 - 21*a^3*b^4*c^6*d*e*f^6 + 14*a^4*b^3*c^5*d^2*e*f^6 + 14*a^5*b^2*c^
4*d^3*e*f^6 - 21*a^6*b*c^3*d^4*e*f^6 + 7*a^7*c^2*d^5*e*f^6 - a^3*b^4*c^7*f^
7 + 4*a^4*b^3*c^6*d*f^7 - 6*a^5*b^2*c^5*d^2*f^7 + 4*a^6*b*c^4*d^3*f^7 - a^7
*c^3*d^4*f^7)/((b^8*c^4*d^4*e^8 - 4*a*b^7*c^3*d^5*e^8 + 6*a^2*b^6*c^2*d^6*e

```
^8 - 4*a^3*b^5*c*d^7*e^8 + a^4*b^4*d^8*e^8 - 4*b^8*c^5*d^3*e^7*f + 12*a*b^7
*c^4*d^4*e^7*f - 8*a^2*b^6*c^3*d^5*e^7*f - 8*a^3*b^5*c^2*d^6*e^7*f + 12*a^4
*b^4*c*d^7*e^7*f - 4*a^5*b^3*d^8*e^7*f + 6*b^8*c^6*d^2*e^6*f^2 - 8*a*b^7*c^
5*d^3*e^6*f^2 - 22*a^2*b^6*c^4*d^4*e^6*f^2 + 48*a^3*b^5*c^3*d^5*e^6*f^2 - 2
2*a^4*b^4*c^2*d^6*e^6*f^2 - 8*a^5*b^3*c*d^7*e^6*f^2 + 6*a^6*b^2*d^8*e^6*f^2
 - 4*b^8*c^7*d*e^5*f^3 - 8*a*b^7*c^6*d^2*e^5*f^3 + 48*a^2*b^6*c^5*d^3*e^5*f
^3 - 36*a^3*b^5*c^4*d^4*e^5*f^3 - 36*a^4*b^4*c^3*d^5*e^5*f^3 + 48*a^5*b^3*c
^2*d^6*e^5*f^3 - 8*a^6*b^2*c*d^7*e^5*f^3 - 4*a^7*b*d^8*e^5*f^3 + b^8*c^8*e^
4*f^4 + 12*a*b^7*c^7*d*e^4*f^4 - 22*a^2*b^6*c^6*d^2*e^4*f^4 - 36*a^3*b^5*c^
5*d^3*e^4*f^4 + 90*a^4*b^4*c^4*d^4*e^4*f^4 - 36*a^5*b^3*c^3*d^5*e^4*f^4 - 2
2*a^6*b^2*c^2*d^6*e^4*f^4 + 12*a^7*b*c*d^7*e^4*f^4 + a^8*d^8*e^4*f^4 - 4*a*
b^7*c^8*e^3*f^5 - 8*a^2*b^6*c^7*d*e^3*f^5 + 48*a^3*b^5*c^6*d^2*e^3*f^5 - 36
*a^4*b^4*c^5*d^3*e^3*f^5 - 36*a^5*b^3*c^4*d^4*e^3*f^5 + 48*a^6*b^2*c^3*d^5*
e^3*f^5 - 8*a^7*b*c^2*d^6*e^3*f^5 - 4*a^8*c*d^7*e^3*f^5 + 6*a^2*b^6*c^8*e^2
*f^6 - 8*a^3*b^5*c^7*d*e^2*f^6 - 22*a^4*b^4*c^6*d^2*e^2*f^6 + 48*a^5*b^3*c^
5*d^3*e^2*f^6 - 22*a^6*b^2*c^4*d^4*e^2*f^6 - 8*a^7*b*c^3*d^5*e^2*f^6 + 6*a^
8*c^2*d^6*e^2*f^6 - 4*a^3*b^5*c^8*e*f^7 + 12*a^4*b^4*c^7*d*e*f^7 - 8*a^5*b^
3*c^6*d^2*e*f^7 - 8*a^6*b^2*c^5*d^3*e*f^7 + 12*a^7*b*c^4*d^4*e*f^7 - 4*a^8*
c^3*d^5*e*f^7 + a^4*b^4*c^8*f^8 - 4*a^5*b^3*c^7*d*f^8 + 6*a^6*b^2*c^6*d^2*f
^8 - 4*a^7*b*c^5*d^3*f^8 + a^8*c^4*d^4*f^8)*(b*d*f*x^3 + b*d*e*x^2 + b*c*f*
x^2 + a*d*f*x^2 + b*c*e*x + a*d*e*x + a*c*f*x + a*c*e)^2)
```

Mupad [B] (verification not implemented)

Time = 23.45 (sec) , antiderivative size = 82532, normalized size of antiderivative = 166.73

$$\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*
d*f*x^3)^3,x)
```

```
[Out] symsum(log(root(756756*a^10*b^10*c^10*d^10*e^10*f^10*z^3 + 573300*a^12*b^8*
c^9*d^11*e^9*f^11*z^3 + 573300*a^11*b^9*c^11*d^9*e^8*f^12*z^3 + 573300*a^11
*b^9*c^8*d^12*e^11*f^9*z^3 + 573300*a^9*b^11*c^12*d^8*e^9*f^11*z^3 + 573300
*a^9*b^11*c^9*d^11*e^12*f^8*z^3 + 573300*a^8*b^12*c^11*d^9*e^11*f^9*z^3 - 3
43980*a^11*b^9*c^10*d^10*e^9*f^11*z^3 - 343980*a^11*b^9*c^9*d^11*e^10*f^10*
z^3 - 343980*a^10*b^10*c^11*d^9*e^9*f^11*z^3 - 343980*a^10*b^10*c^9*d^11*e^
11*f^9*z^3 - 343980*a^9*b^11*c^11*d^9*e^10*f^10*z^3 - 343980*a^9*b^11*c^10*
d^10*e^11*f^9*z^3 + 326340*a^13*b^7*c^10*d^10*e^7*f^13*z^3 + 326340*a^13*b^
7*c^7*d^13*e^10*f^10*z^3 + 326340*a^10*b^10*c^13*d^7*e^7*f^13*z^3 + 326340*
a^10*b^10*c^7*d^13*e^13*f^7*z^3 + 326340*a^7*b^13*c^13*d^7*e^10*f^10*z^3 +
326340*a^7*b^13*c^10*d^10*e^13*f^7*z^3 - 267540*a^12*b^8*c^10*d^10*e^8*f^12
*z^3 - 267540*a^12*b^8*c^8*d^12*e^10*f^10*z^3 - 267540*a^10*b^10*c^12*d^8*e
^8*f^12*z^3 - 267540*a^10*b^10*c^8*d^12*e^12*f^8*z^3 - 267540*a^8*b^12*c^12
```

*d^8*e^10*f^10*z^3 - 267540*a^8*b^12*c^10*d^10*e^12*f^8*z^3 + 245700*a^14*b
^6*c^8*d^12*e^8*f^12*z^3 + 245700*a^12*b^8*c^12*d^8*e^6*f^14*z^3 + 245700*a
^12*b^8*c^6*d^14*e^12*f^8*z^3 + 245700*a^8*b^12*c^14*d^6*e^8*f^12*z^3 + 245
700*a^8*b^12*c^8*d^12*e^14*f^6*z^3 + 245700*a^6*b^14*c^12*d^8*e^12*f^8*z^3
- 191100*a^13*b^7*c^9*d^11*e^8*f^12*z^3 - 191100*a^13*b^7*c^8*d^12*e^9*f^11
*z^3 - 191100*a^12*b^8*c^11*d^9*e^7*f^13*z^3 - 191100*a^12*b^8*c^7*d^13*e^1
1*f^9*z^3 - 191100*a^11*b^9*c^12*d^8*e^7*f^13*z^3 - 191100*a^11*b^9*c^7*d^1
3*e^12*f^8*z^3 - 191100*a^9*b^11*c^13*d^7*e^8*f^12*z^3 - 191100*a^9*b^11*c^
8*d^12*e^13*f^7*z^3 - 191100*a^8*b^12*c^13*d^7*e^9*f^11*z^3 - 191100*a^8*b^
12*c^9*d^11*e^13*f^7*z^3 - 191100*a^7*b^13*c^12*d^8*e^11*f^9*z^3 - 191100*a
^7*b^13*c^11*d^9*e^12*f^8*z^3 - 123900*a^14*b^6*c^9*d^11*e^7*f^13*z^3 - 123
900*a^14*b^6*c^7*d^13*e^9*f^11*z^3 - 123900*a^13*b^7*c^11*d^9*e^6*f^14*z^3
- 123900*a^13*b^7*c^6*d^14*e^11*f^9*z^3 - 123900*a^11*b^9*c^13*d^7*e^6*f^14
*z^3 - 123900*a^11*b^9*c^6*d^14*e^13*f^7*z^3 - 123900*a^9*b^11*c^14*d^6*e^7
*f^13*z^3 - 123900*a^9*b^11*c^7*d^13*e^14*f^6*z^3 - 123900*a^7*b^13*c^14*d^
6*e^9*f^11*z^3 - 123900*a^7*b^13*c^9*d^11*e^14*f^6*z^3 - 123900*a^6*b^14*c^
13*d^7*e^11*f^9*z^3 - 123900*a^6*b^14*c^11*d^9*e^13*f^7*z^3 + 101700*a^15*b
^5*c^9*d^11*e^6*f^14*z^3 + 101700*a^15*b^5*c^6*d^14*e^9*f^11*z^3 + 101700*a
^14*b^6*c^11*d^9*e^5*f^15*z^3 + 101700*a^14*b^6*c^5*d^15*e^11*f^9*z^3 + 101
700*a^11*b^9*c^14*d^6*e^5*f^15*z^3 + 101700*a^11*b^9*c^5*d^15*e^14*f^6*z^3
+ 101700*a^9*b^11*c^15*d^5*e^6*f^14*z^3 + 101700*a^9*b^11*c^6*d^14*e^15*f^5
*z^3 + 101700*a^6*b^14*c^15*d^5*e^9*f^11*z^3 + 101700*a^6*b^14*c^9*d^11*e^1
5*f^5*z^3 + 101700*a^5*b^15*c^14*d^6*e^11*f^9*z^3 + 101700*a^5*b^15*c^11*d^
9*e^14*f^6*z^3 - 65820*a^14*b^6*c^10*d^10*e^6*f^14*z^3 - 65820*a^14*b^6*c^6
*d^14*e^10*f^10*z^3 - 65820*a^10*b^10*c^14*d^6*e^6*f^14*z^3 - 65820*a^10*b^
10*c^6*d^14*e^14*f^6*z^3 - 65820*a^6*b^14*c^14*d^6*e^10*f^10*z^3 - 65820*a^
6*b^14*c^10*d^10*e^14*f^6*z^3 + 56700*a^16*b^4*c^7*d^13*e^7*f^13*z^3 - 5670
0*a^15*b^5*c^8*d^12*e^7*f^13*z^3 - 56700*a^15*b^5*c^7*d^13*e^8*f^12*z^3 + 5
6700*a^13*b^7*c^13*d^7*e^4*f^16*z^3 - 56700*a^13*b^7*c^12*d^8*e^5*f^15*z^3
- 56700*a^13*b^7*c^5*d^15*e^12*f^8*z^3 + 56700*a^13*b^7*c^4*d^16*e^13*f^7*z
^3 - 56700*a^12*b^8*c^13*d^7*e^5*f^15*z^3 - 56700*a^12*b^8*c^5*d^15*e^13*f^
7*z^3 - 56700*a^8*b^12*c^15*d^5*e^7*f^13*z^3 - 56700*a^8*b^12*c^7*d^13*e^15
*f^5*z^3 + 56700*a^7*b^13*c^16*d^4*e^7*f^13*z^3 - 56700*a^7*b^13*c^15*d^5*e
^8*f^12*z^3 - 56700*a^7*b^13*c^8*d^12*e^15*f^5*z^3 + 56700*a^7*b^13*c^7*d^1
3*e^16*f^4*z^3 - 56700*a^5*b^15*c^13*d^7*e^12*f^8*z^3 - 56700*a^5*b^15*c^12
*d^8*e^13*f^7*z^3 + 56700*a^4*b^16*c^13*d^7*e^13*f^7*z^3 - 48252*a^15*b^5*c
^10*d^10*e^5*f^15*z^3 - 48252*a^15*b^5*c^5*d^15*e^10*f^10*z^3 - 48252*a^10*
b^10*c^15*d^5*e^5*f^15*z^3 - 48252*a^10*b^10*c^5*d^15*e^15*f^5*z^3 - 48252*
a^5*b^15*c^15*d^5*e^10*f^10*z^3 - 48252*a^5*b^15*c^10*d^10*e^15*f^5*z^3 - 3
2400*a^16*b^4*c^8*d^12*e^6*f^14*z^3 - 32400*a^16*b^4*c^6*d^14*e^8*f^12*z^3
- 32400*a^14*b^6*c^12*d^8*e^4*f^16*z^3 - 32400*a^14*b^6*c^4*d^16*e^12*f^8*z
^3 - 32400*a^12*b^8*c^14*d^6*e^4*f^16*z^3 - 32400*a^12*b^8*c^4*d^16*e^14*f^
6*z^3 - 32400*a^8*b^12*c^16*d^4*e^6*f^14*z^3 - 32400*a^8*b^12*c^6*d^14*e^16
*f^4*z^3 - 32400*a^6*b^14*c^16*d^4*e^8*f^12*z^3 - 32400*a^6*b^14*c^8*d^12*e
^16*f^4*z^3 - 32400*a^4*b^16*c^14*d^6*e^12*f^8*z^3 - 32400*a^4*b^16*c^12*d^

8*e^14*f^6*z^3 + 20565*a^16*b^4*c^10*d^10*e^4*f^16*z^3 + 20565*a^16*b^4*c^4
*d^16*e^10*f^10*z^3 + 20565*a^10*b^10*c^16*d^4*e^4*f^16*z^3 + 20565*a^10*b^
10*c^4*d^16*e^16*f^4*z^3 + 20565*a^4*b^16*c^16*d^4*e^10*f^10*z^3 + 20565*a^
4*b^16*c^10*d^10*e^16*f^4*z^3 + 15660*a^17*b^3*c^8*d^12*e^5*f^15*z^3 + 1566
0*a^17*b^3*c^5*d^15*e^8*f^12*z^3 + 15660*a^15*b^5*c^12*d^8*e^3*f^17*z^3 + 1
5660*a^15*b^5*c^3*d^17*e^12*f^8*z^3 + 15660*a^12*b^8*c^15*d^5*e^3*f^17*z^3
+ 15660*a^12*b^8*c^3*d^17*e^15*f^5*z^3 + 15660*a^8*b^12*c^17*d^3*e^5*f^15*z
^3 + 15660*a^8*b^12*c^5*d^15*e^17*f^3*z^3 + 15660*a^5*b^15*c^17*d^3*e^8*f^1
2*z^3 + 15660*a^5*b^15*c^8*d^12*e^17*f^3*z^3 + 15660*a^3*b^17*c^15*d^5*e^12
*f^8*z^3 + 15660*a^3*b^17*c^12*d^8*e^15*f^5*z^3 - 9750*a^17*b^3*c^9*d^11*e^
4*f^16*z^3 - 9750*a^17*b^3*c^4*d^16*e^9*f^11*z^3 - 9750*a^16*b^4*c^11*d^9*e
^3*f^17*z^3 - 9750*a^16*b^4*c^3*d^17*e^11*f^9*z^3 - 9750*a^11*b^9*c^16*d^4*
e^3*f^17*z^3 - 9750*a^11*b^9*c^3*d^17*e^16*f^4*z^3 - 9750*a^9*b^11*c^17*d^3
*e^4*f^16*z^3 - 9750*a^9*b^11*c^4*d^16*e^17*f^3*z^3 - 9750*a^4*b^16*c^17*d^
3*e^9*f^11*z^3 - 9750*a^4*b^16*c^9*d^11*e^17*f^3*z^3 - 9750*a^3*b^17*c^16*d
^4*e^11*f^9*z^3 - 9750*a^3*b^17*c^11*d^9*e^16*f^4*z^3 - 8100*a^17*b^3*c^7*d
^13*e^6*f^14*z^3 - 8100*a^17*b^3*c^6*d^14*e^7*f^13*z^3 - 8100*a^14*b^6*c^13
*d^7*e^3*f^17*z^3 - 8100*a^14*b^6*c^3*d^17*e^13*f^7*z^3 - 8100*a^13*b^7*c^1
4*d^6*e^3*f^17*z^3 - 8100*a^13*b^7*c^3*d^17*e^14*f^6*z^3 - 8100*a^7*b^13*c^
17*d^3*e^6*f^14*z^3 - 8100*a^7*b^13*c^6*d^14*e^17*f^3*z^3 - 8100*a^6*b^14*c
^17*d^3*e^7*f^13*z^3 - 8100*a^6*b^14*c^7*d^13*e^17*f^3*z^3 - 8100*a^3*b^17*
c^14*d^6*e^13*f^7*z^3 - 8100*a^3*b^17*c^13*d^7*e^14*f^6*z^3 - 7980*a^16*b^4
*c^9*d^11*e^5*f^15*z^3 - 7980*a^16*b^4*c^5*d^15*e^9*f^11*z^3 - 7980*a^15*b^
5*c^11*d^9*e^4*f^16*z^3 - 7980*a^15*b^5*c^4*d^16*e^11*f^9*z^3 - 7980*a^11*b
^9*c^15*d^5*e^4*f^16*z^3 - 7980*a^11*b^9*c^4*d^16*e^15*f^5*z^3 - 7980*a^9*b
^11*c^16*d^4*e^5*f^15*z^3 - 7980*a^9*b^11*c^5*d^15*e^16*f^4*z^3 - 7980*a^5*
b^15*c^16*d^4*e^9*f^11*z^3 - 7980*a^5*b^15*c^9*d^11*e^16*f^4*z^3 - 7980*a^4
*b^16*c^15*d^5*e^11*f^9*z^3 - 7980*a^4*b^16*c^11*d^9*e^15*f^5*z^3 + 6300*a^
18*b^2*c^6*d^14*e^6*f^14*z^3 + 6300*a^14*b^6*c^14*d^6*e^2*f^18*z^3 + 6300*a
^14*b^6*c^2*d^18*e^14*f^6*z^3 + 6300*a^6*b^14*c^18*d^2*e^6*f^14*z^3 + 6300*
a^6*b^14*c^6*d^14*e^18*f^2*z^3 + 6300*a^2*b^18*c^14*d^6*e^14*f^6*z^3 - 4260
*a^18*b^2*c^7*d^13*e^5*f^15*z^3 - 4260*a^18*b^2*c^5*d^15*e^7*f^13*z^3 - 426
0*a^15*b^5*c^13*d^7*e^2*f^18*z^3 - 4260*a^15*b^5*c^2*d^18*e^13*f^7*z^3 - 42
60*a^13*b^7*c^15*d^5*e^2*f^18*z^3 - 4260*a^13*b^7*c^2*d^18*e^15*f^5*z^3 - 4
260*a^7*b^13*c^18*d^2*e^5*f^15*z^3 - 4260*a^7*b^13*c^5*d^15*e^18*f^2*z^3 -
4260*a^5*b^15*c^18*d^2*e^7*f^13*z^3 - 4260*a^5*b^15*c^7*d^13*e^18*f^2*z^3 -
4260*a^2*b^18*c^15*d^5*e^13*f^7*z^3 - 4260*a^2*b^18*c^13*d^7*e^15*f^5*z^3
+ 1470*a^17*b^3*c^10*d^10*e^3*f^17*z^3 + 1470*a^17*b^3*c^3*d^17*e^10*f^10*z
^3 + 1470*a^10*b^10*c^17*d^3*e^3*f^17*z^3 + 1470*a^10*b^10*c^3*d^17*e^17*f^
3*z^3 + 1470*a^3*b^17*c^17*d^3*e^10*f^10*z^3 + 1470*a^3*b^17*c^10*d^10*e^17
*f^3*z^3 + 1350*a^18*b^2*c^9*d^11*e^3*f^17*z^3 + 1350*a^18*b^2*c^3*d^17*e^9
*f^11*z^3 + 1350*a^17*b^3*c^11*d^9*e^2*f^18*z^3 + 1350*a^17*b^3*c^2*d^18*e^
11*f^9*z^3 + 1350*a^11*b^9*c^17*d^3*e^2*f^18*z^3 + 1350*a^11*b^9*c^2*d^18*e
^17*f^3*z^3 + 1350*a^9*b^11*c^18*d^2*e^3*f^17*z^3 + 1350*a^9*b^11*c^3*d^17*
e^18*f^2*z^3 + 1350*a^3*b^17*c^18*d^2*e^9*f^11*z^3 + 1350*a^3*b^17*c^9*d^11

*e^18*f^2*z^3 + 1350*a^2*b^18*c^17*d^3*e^11*f^9*z^3 + 1350*a^2*b^18*c^11*d^
9*e^17*f^3*z^3 - 1070*a^18*b^2*c^10*d^10*e^2*f^18*z^3 - 1070*a^18*b^2*c^2*d
^18*e^10*f^10*z^3 - 1070*a^10*b^10*c^18*d^2*e^2*f^18*z^3 - 1070*a^10*b^10*c
^2*d^18*e^18*f^2*z^3 - 1070*a^2*b^18*c^18*d^2*e^10*f^10*z^3 - 1070*a^2*b^18
*c^10*d^10*e^18*f^2*z^3 + 525*a^18*b^2*c^8*d^12*e^4*f^16*z^3 + 525*a^18*b^2
*c^4*d^16*e^8*f^12*z^3 + 525*a^16*b^4*c^12*d^8*e^2*f^18*z^3 + 525*a^16*b^4*
c^2*d^18*e^12*f^8*z^3 + 525*a^12*b^8*c^16*d^4*e^2*f^18*z^3 + 525*a^12*b^8*c
^2*d^18*e^16*f^4*z^3 + 525*a^8*b^12*c^18*d^2*e^4*f^16*z^3 + 525*a^8*b^12*c^
4*d^16*e^18*f^2*z^3 + 525*a^4*b^16*c^18*d^2*e^8*f^12*z^3 + 525*a^4*b^16*c^8
*d^12*e^18*f^2*z^3 + 525*a^2*b^18*c^16*d^4*e^12*f^8*z^3 + 525*a^2*b^18*c^12
*d^8*e^16*f^4*z^3 + 900*a^19*b*c^7*d^13*e^4*f^16*z^3 + 900*a^19*b*c^4*d^16*
e^7*f^13*z^3 + 900*a^16*b^4*c^13*d^7*e*f^19*z^3 + 900*a^16*b^4*c*d^19*e^13*
f^7*z^3 + 900*a^13*b^7*c^16*d^4*e*f^19*z^3 + 900*a^13*b^7*c*d^19*e^16*f^4*z
^3 + 900*a^7*b^13*c^19*d*e^4*f^16*z^3 + 900*a^7*b^13*c^4*d^16*e^19*f*z^3 +
900*a^4*b^16*c^19*d*e^7*f^13*z^3 + 900*a^4*b^16*c^7*d^13*e^19*f*z^3 + 900*a
*b^19*c^16*d^4*e^13*f^7*z^3 + 900*a*b^19*c^13*d^7*e^16*f^4*z^3 - 750*a^19*b
*c^8*d^12*e^3*f^17*z^3 - 750*a^19*b*c^3*d^17*e^8*f^12*z^3 - 750*a^17*b^3*c^
12*d^8*e*f^19*z^3 - 750*a^17*b^3*c*d^19*e^12*f^8*z^3 - 750*a^12*b^8*c^17*d^
3*e*f^19*z^3 - 750*a^12*b^8*c*d^19*e^17*f^3*z^3 - 750*a^8*b^12*c^19*d*e^3*f
^17*z^3 - 750*a^8*b^12*c^3*d^17*e^19*f*z^3 - 750*a^3*b^17*c^19*d*e^8*f^12*z
^3 - 750*a^3*b^17*c^8*d^12*e^19*f*z^3 - 750*a*b^19*c^17*d^3*e^12*f^8*z^3 -
750*a*b^19*c^12*d^8*e^17*f^3*z^3 - 420*a^19*b*c^6*d^14*e^5*f^15*z^3 - 420*a
^19*b*c^5*d^15*e^6*f^14*z^3 - 420*a^15*b^5*c^14*d^6*e*f^19*z^3 - 420*a^15*b
^5*c*d^19*e^14*f^6*z^3 - 420*a^14*b^6*c^15*d^5*e*f^19*z^3 - 420*a^14*b^6*c*
d^19*e^15*f^5*z^3 - 420*a^6*b^14*c^19*d*e^5*f^15*z^3 - 420*a^6*b^14*c^5*d^1
5*e^19*f*z^3 - 420*a^5*b^15*c^19*d*e^6*f^14*z^3 - 420*a^5*b^15*c^6*d^14*e^1
9*f*z^3 - 420*a*b^19*c^15*d^5*e^14*f^6*z^3 - 420*a*b^19*c^14*d^6*e^15*f^5*z
^3 + 350*a^19*b*c^9*d^11*e^2*f^18*z^3 + 350*a^19*b*c^2*d^18*e^9*f^11*z^3 +
350*a^18*b^2*c^11*d^9*e*f^19*z^3 + 350*a^18*b^2*c*d^19*e^11*f^9*z^3 + 350*a
^11*b^9*c^18*d^2*e*f^19*z^3 + 350*a^11*b^9*c*d^19*e^18*f^2*z^3 + 350*a^9*b^
11*c^19*d*e^2*f^18*z^3 + 350*a^9*b^11*c^2*d^18*e^19*f*z^3 + 350*a^2*b^18*c^
19*d*e^9*f^11*z^3 + 350*a^2*b^18*c^9*d^11*e^19*f*z^3 + 350*a*b^19*c^18*d^2*
e^11*f^9*z^3 + 350*a*b^19*c^11*d^9*e^18*f^2*z^3 - 90*a^19*b*c^10*d^10*e*f^1
9*z^3 - 90*a^19*b*c*d^19*e^10*f^10*z^3 - 90*a^10*b^10*c^19*d*e*f^19*z^3 - 9
0*a^10*b^10*c*d^19*e^19*f*z^3 - 90*a*b^19*c^19*d*e^10*f^10*z^3 - 90*a*b^19*
c^10*d^10*e^19*f*z^3 + 10*b^20*c^19*d*e^11*f^9*z^3 + 10*b^20*c^11*d^9*e^19*
f*z^3 + 10*a^20*c^9*d^11*e*f^19*z^3 + 10*a^20*c*d^19*e^9*f^11*z^3 + 10*a^19
*b*d^20*e^11*f^9*z^3 + 10*a^11*b^9*d^20*e^19*f*z^3 + 10*a^9*b^11*c^20*e*f^1
9*z^3 + 10*a*b^19*c^20*e^9*f^11*z^3 + 10*a^19*b*c^11*d^9*f^20*z^3 + 10*a^11
*b^9*c^19*d*f^20*z^3 + 10*a^9*b^11*c*d^19*e^20*z^3 + 10*a*b^19*c^9*d^11*e^2
0*z^3 + 252*b^20*c^15*d^5*e^15*f^5*z^3 - 210*b^20*c^16*d^4*e^14*f^6*z^3 - 2
10*b^20*c^14*d^6*e^16*f^4*z^3 + 120*b^20*c^17*d^3*e^13*f^7*z^3 + 120*b^20*c
^13*d^7*e^17*f^3*z^3 - 45*b^20*c^18*d^2*e^12*f^8*z^3 - 45*b^20*c^12*d^8*e^1
8*f^2*z^3 + 252*a^20*c^5*d^15*e^5*f^15*z^3 - 210*a^20*c^6*d^14*e^4*f^16*z^3
 - 210*a^20*c^4*d^16*e^6*f^14*z^3 + 120*a^20*c^7*d^13*e^3*f^17*z^3 + 120*a^

20*c^3*d^17*e^7*f^13*z^3 - 45*a^20*c^8*d^12*e^2*f^18*z^3 - 45*a^20*c^2*d^18
*e^8*f^12*z^3 + 252*a^15*b^5*d^20*e^15*f^5*z^3 - 210*a^16*b^4*d^20*e^14*f^6
*z^3 - 210*a^14*b^6*d^20*e^16*f^4*z^3 + 120*a^17*b^3*d^20*e^13*f^7*z^3 + 12
0*a^13*b^7*d^20*e^17*f^3*z^3 - 45*a^18*b^2*d^20*e^12*f^8*z^3 - 45*a^12*b^8*
d^20*e^18*f^2*z^3 + 252*a^5*b^15*c^20*e^5*f^15*z^3 - 210*a^6*b^14*c^20*e^4*
f^16*z^3 - 210*a^4*b^16*c^20*e^6*f^14*z^3 + 120*a^7*b^13*c^20*e^3*f^17*z^3
+ 120*a^3*b^17*c^20*e^7*f^13*z^3 - 45*a^8*b^12*c^20*e^2*f^18*z^3 - 45*a^2*b
^18*c^20*e^8*f^12*z^3 + 252*a^15*b^5*c^15*d^5*f^20*z^3 - 210*a^16*b^4*c^14*
d^6*f^20*z^3 - 210*a^14*b^6*c^16*d^4*f^20*z^3 + 120*a^17*b^3*c^13*d^7*f^20*
z^3 + 120*a^13*b^7*c^17*d^3*f^20*z^3 - 45*a^18*b^2*c^12*d^8*f^20*z^3 - 45*a
^12*b^8*c^18*d^2*f^20*z^3 + 252*a^5*b^15*c^5*d^15*e^20*z^3 - 210*a^6*b^14*c
^4*d^16*e^20*z^3 - 210*a^4*b^16*c^6*d^14*e^20*z^3 + 120*a^7*b^13*c^3*d^17*e
^20*z^3 + 120*a^3*b^17*c^7*d^13*e^20*z^3 - 45*a^8*b^12*c^2*d^18*e^20*z^3 -
45*a^2*b^18*c^8*d^12*e^20*z^3 - b^20*c^20*e^10*f^10*z^3 - a^20*d^20*e^10*f^
10*z^3 - b^20*c^10*d^10*e^20*z^3 - a^20*c^10*d^10*f^20*z^3 - a^10*b^10*d^20
*e^20*z^3 - a^10*b^10*c^20*f^20*z^3 + 1890*a^12*b^2*c*d^13*e*f^13*z + 1890*
a*b^13*c^12*d^2*e*f^13*z + 1890*a*b^13*c*d^13*e^12*f^2*z + 92610*a^6*b^8*c^
4*d^10*e^4*f^10*z + 92610*a^4*b^10*c^6*d^8*e^4*f^10*z + 92610*a^4*b^10*c^4*
d^10*e^6*f^8*z + 66150*a^8*b^6*c^3*d^11*e^3*f^11*z - 66150*a^7*b^7*c^4*d^10
*e^3*f^11*z - 66150*a^7*b^7*c^3*d^11*e^4*f^10*z - 66150*a^4*b^10*c^7*d^7*e^
3*f^11*z - 66150*a^4*b^10*c^3*d^11*e^7*f^7*z + 66150*a^3*b^11*c^8*d^6*e^3*f
^11*z - 66150*a^3*b^11*c^7*d^7*e^4*f^10*z - 66150*a^3*b^11*c^4*d^10*e^7*f^7
*z + 66150*a^3*b^11*c^3*d^11*e^8*f^6*z - 55566*a^5*b^9*c^5*d^9*e^4*f^10*z -
 55566*a^5*b^9*c^4*d^10*e^5*f^9*z - 55566*a^4*b^10*c^5*d^9*e^5*f^9*z - 3213
0*a^9*b^5*c^3*d^11*e^2*f^12*z - 32130*a^9*b^5*c^2*d^12*e^3*f^11*z - 32130*a
^3*b^11*c^9*d^5*e^2*f^12*z - 32130*a^3*b^11*c^2*d^12*e^9*f^5*z - 32130*a^2*
b^12*c^9*d^5*e^3*f^11*z - 32130*a^2*b^12*c^3*d^11*e^9*f^5*z + 22680*a^8*b^6
*c^4*d^10*e^2*f^12*z + 22680*a^8*b^6*c^2*d^12*e^4*f^10*z + 22680*a^4*b^10*c
^8*d^6*e^2*f^12*z + 22680*a^4*b^10*c^2*d^12*e^8*f^6*z + 22680*a^2*b^12*c^8*
d^6*e^4*f^10*z + 22680*a^2*b^12*c^4*d^10*e^8*f^6*z + 19278*a^10*b^4*c^2*d^1
2*e^2*f^12*z + 19278*a^2*b^12*c^10*d^4*e^2*f^12*z + 19278*a^2*b^12*c^2*d^12
*e^10*f^4*z + 18522*a^6*b^8*c^5*d^9*e^3*f^11*z + 18522*a^6*b^8*c^3*d^11*e^5
*f^9*z + 18522*a^5*b^9*c^6*d^8*e^3*f^11*z + 18522*a^5*b^9*c^3*d^11*e^6*f^8*
z + 18522*a^3*b^11*c^6*d^8*e^5*f^9*z + 18522*a^3*b^11*c^5*d^9*e^6*f^8*z - 1
3230*a^6*b^8*c^6*d^8*e^2*f^12*z - 13230*a^6*b^8*c^2*d^12*e^6*f^8*z - 13230*
a^2*b^12*c^6*d^8*e^6*f^8*z + 3402*a^7*b^7*c^5*d^9*e^2*f^12*z + 3402*a^7*b^7
*c^2*d^12*e^5*f^9*z + 3402*a^5*b^9*c^7*d^7*e^2*f^12*z + 3402*a^5*b^9*c^2*d^
12*e^7*f^7*z + 3402*a^2*b^12*c^7*d^7*e^5*f^9*z + 3402*a^2*b^12*c^5*d^9*e^7*
f^7*z + 7938*a^10*b^4*c^3*d^11*e*f^13*z + 7938*a^10*b^4*c*d^13*e^3*f^11*z +
 7938*a^3*b^11*c^10*d^4*e*f^13*z + 7938*a^3*b^11*c*d^13*e^10*f^4*z + 7938*a
*b^13*c^10*d^4*e^3*f^11*z + 7938*a*b^13*c^3*d^11*e^10*f^4*z - 5670*a^11*b^3
*c^2*d^12*e*f^13*z - 5670*a^11*b^3*c*d^13*e^2*f^12*z - 5670*a^2*b^12*c^11*d
^3*e*f^13*z - 5670*a^2*b^12*c*d^13*e^11*f^3*z - 5670*a*b^13*c^11*d^3*e^2*f^
12*z - 5670*a*b^13*c^2*d^12*e^11*f^3*z - 3780*a^9*b^5*c^4*d^10*e*f^13*z - 3
780*a^9*b^5*c*d^13*e^4*f^10*z - 3780*a^4*b^10*c^9*d^5*e*f^13*z - 3780*a^4*b

^10*c*d^13*e^9*f^5*z - 3780*a*b^13*c^9*d^5*e^4*f^10*z - 3780*a*b^13*c^4*d^1
0*e^9*f^5*z - 2268*a^8*b^6*c^5*d^9*e*f^13*z - 2268*a^8*b^6*c*d^13*e^5*f^9*z
 - 2268*a^5*b^9*c^8*d^6*e*f^13*z - 2268*a^5*b^9*c*d^13*e^8*f^6*z - 2268*a*b
^13*c^8*d^6*e^5*f^9*z - 2268*a*b^13*c^5*d^9*e^8*f^6*z + 1890*a^7*b^7*c^6*d^
8*e*f^13*z + 1890*a^7*b^7*c*d^13*e^6*f^8*z + 1890*a^6*b^8*c^7*d^7*e*f^13*z
 + 1890*a^6*b^8*c*d^13*e^7*f^7*z + 1890*a*b^13*c^7*d^7*e^6*f^8*z + 1890*a*b^
13*c^6*d^8*e^7*f^7*z - 252*b^14*c^13*d*e*f^13*z - 252*b^14*c*d^13*e^13*f*z
 - 252*a^13*b*d^14*e*f^13*z - 252*a*b^13*d^14*e^13*f*z - 252*a^13*b*c*d^13*f
^14*z - 252*a*b^13*c^13*d*f^14*z - 918*b^14*c^7*d^7*e^7*f^7*z - 882*b^14*c^
11*d^3*e^3*f^11*z - 882*b^14*c^3*d^11*e^11*f^3*z + 693*b^14*c^12*d^2*e^2*f^
12*z + 693*b^14*c^2*d^12*e^12*f^2*z + 567*b^14*c^8*d^6*e^6*f^8*z + 567*b^14
*c^6*d^8*e^8*f^6*z + 441*b^14*c^10*d^4*e^4*f^10*z + 441*b^14*c^4*d^10*e^10*
f^4*z - 126*b^14*c^9*d^5*e^5*f^9*z - 126*b^14*c^5*d^9*e^9*f^5*z - 918*a^7*b
^7*d^14*e^7*f^7*z - 882*a^11*b^3*d^14*e^3*f^11*z - 882*a^3*b^11*d^14*e^11*f
^3*z + 693*a^12*b^2*d^14*e^2*f^12*z + 693*a^2*b^12*d^14*e^12*f^2*z + 567*a^
8*b^6*d^14*e^6*f^8*z + 567*a^6*b^8*d^14*e^8*f^6*z + 441*a^10*b^4*d^14*e^4*f
^10*z + 441*a^4*b^10*d^14*e^10*f^4*z - 126*a^9*b^5*d^14*e^5*f^9*z - 126*a^5
*b^9*d^14*e^9*f^5*z - 918*a^7*b^7*c^7*d^7*f^14*z - 882*a^11*b^3*c^3*d^11*f^
14*z - 882*a^3*b^11*c^11*d^3*f^14*z + 693*a^12*b^2*c^2*d^12*f^14*z + 693*a^
2*b^12*c^12*d^2*f^14*z + 567*a^8*b^6*c^6*d^8*f^14*z + 567*a^6*b^8*c^8*d^6*f
^14*z + 441*a^10*b^4*c^4*d^10*f^14*z + 441*a^4*b^10*c^10*d^4*f^14*z - 126*a
^9*b^5*c^5*d^9*f^14*z - 126*a^5*b^9*c^9*d^5*f^14*z + 36*b^14*d^14*e^14*z +
36*b^14*c^14*f^14*z + 36*a^14*d^14*f^14*z - 27054*a^2*b^9*c^2*d^9*e^2*f^9 +
 9018*a^3*b^8*c^2*d^9*e*f^10 + 9018*a^3*b^8*c*d^10*e^2*f^9 + 9018*a^2*b^9*c
^3*d^8*e*f^10 + 9018*a^2*b^9*c*d^10*e^3*f^8 + 9018*a*b^10*c^3*d^8*e^2*f^9 +
 9018*a*b^10*c^2*d^9*e^3*f^8 - 9018*a^4*b^7*c*d^10*e*f^10 - 9018*a*b^10*c^4
*d^7*e*f^10 - 9018*a*b^10*c*d^10*e^4*f^7 + 2268*b^11*c^5*d^6*e*f^10 + 2268*
b^11*c*d^10*e^5*f^6 + 2268*a^5*b^6*d^11*e*f^10 + 2268*a*b^10*d^11*e^5*f^6 +
 2268*a^5*b^6*c*d^10*f^11 + 2268*a*b^10*c^5*d^6*f^11 - 1458*b^11*c^3*d^8*e^
3*f^8 - 1161*b^11*c^4*d^7*e^2*f^9 - 1161*b^11*c^2*d^9*e^4*f^7 - 1458*a^3*b^
8*d^11*e^3*f^8 - 1161*a^4*b^7*d^11*e^2*f^9 - 1161*a^2*b^9*d^11*e^4*f^7 - 14
58*a^3*b^8*c^3*d^8*f^11 - 1161*a^4*b^7*c^2*d^9*f^11 - 1161*a^2*b^9*c^4*d^7*
f^11 - 756*b^11*d^11*e^6*f^5 - 756*b^11*c^6*d^5*f^11 - 756*a^6*b^5*d^11*f^1
1, z, k)*((39*a^5*b^11*c^14*d^2*f^16 - 102*a^6*b^10*c^13*d^3*f^16 + 132*a^7
*b^9*c^12*d^4*f^16 - 84*a^8*b^8*c^11*d^5*f^16 + 21*a^9*b^7*c^10*d^6*f^16 +
21*a^10*b^6*c^9*d^7*f^16 - 84*a^11*b^5*c^8*d^8*f^16 + 132*a^12*b^4*c^7*d^9*
f^16 - 102*a^13*b^3*c^6*d^10*f^16 + 39*a^14*b^2*c^5*d^11*f^16 + 39*a^5*b^11
*d^16*e^14*f^2 - 102*a^6*b^10*d^16*e^13*f^3 + 132*a^7*b^9*d^16*e^12*f^4 - 8
4*a^8*b^8*d^16*e^11*f^5 + 21*a^9*b^7*d^16*e^10*f^6 + 21*a^10*b^6*d^16*e^9*f
^7 - 84*a^11*b^5*d^16*e^8*f^8 + 132*a^12*b^4*d^16*e^7*f^9 - 102*a^13*b^3*d^
16*e^6*f^10 + 39*a^14*b^2*d^16*e^5*f^11 + 39*b^16*c^5*d^11*e^14*f^2 - 102*b
^16*c^6*d^10*e^13*f^3 + 132*b^16*c^7*d^9*e^12*f^4 - 84*b^16*c^8*d^8*e^11*f^
5 + 21*b^16*c^9*d^7*e^10*f^6 + 21*b^16*c^10*d^6*e^9*f^7 - 84*b^16*c^11*d^5*
e^8*f^8 + 132*b^16*c^12*d^4*e^7*f^9 - 102*b^16*c^13*d^3*e^6*f^10 + 39*b^16*
c^14*d^2*e^5*f^11 - 6*a^4*b^12*c^15*d*f^16 - 6*a^15*b*c^4*d^12*f^16 - 6*a^4

*b^12*d^16*e^15*f - 6*a^15*b*d^16*e^4*f^12 - 6*b^16*c^4*d^12*e^15*f - 6*b^1
6*c^15*d*e^4*f^12 + 24*a*b^15*c^3*d^13*e^15*f + 24*a*b^15*c^15*d*e^3*f^13 +
 24*a^3*b^13*c*d^15*e^15*f + 24*a^3*b^13*c^15*d*e*f^15 + 24*a^15*b*c*d^15*e
^3*f^13 + 24*a^15*b*c^3*d^13*e*f^15 - 117*a*b^15*c^4*d^12*e^14*f^2 + 150*a*
b^15*c^5*d^11*e^13*f^3 + 159*a*b^15*c^6*d^10*e^12*f^4 - 546*a*b^15*c^7*d^9*
e^11*f^5 + 414*a*b^15*c^8*d^8*e^10*f^6 - 168*a*b^15*c^9*d^7*e^9*f^7 + 414*a
*b^15*c^10*d^6*e^8*f^8 - 546*a*b^15*c^11*d^5*e^7*f^9 + 159*a*b^15*c^12*d^4*
e^6*f^10 + 150*a*b^15*c^13*d^3*e^5*f^11 - 117*a*b^15*c^14*d^2*e^4*f^12 - 36
*a^2*b^14*c^2*d^14*e^15*f - 36*a^2*b^14*c^15*d*e^2*f^14 - 117*a^4*b^12*c*d^
15*e^14*f^2 - 117*a^4*b^12*c^14*d^2*e*f^15 + 150*a^5*b^11*c*d^15*e^13*f^3 +
 150*a^5*b^11*c^13*d^3*e*f^15 + 159*a^6*b^10*c*d^15*e^12*f^4 + 159*a^6*b^10
*c^12*d^4*e*f^15 - 546*a^7*b^9*c*d^15*e^11*f^5 - 546*a^7*b^9*c^11*d^5*e*f^1
5 + 414*a^8*b^8*c*d^15*e^10*f^6 + 414*a^8*b^8*c^10*d^6*e*f^15 - 168*a^9*b^7
*c*d^15*e^9*f^7 - 168*a^9*b^7*c^9*d^7*e*f^15 + 414*a^10*b^6*c*d^15*e^8*f^8
 + 414*a^10*b^6*c^8*d^8*e*f^15 - 546*a^11*b^5*c*d^15*e^7*f^9 - 546*a^11*b^5*
c^7*d^9*e*f^15 + 159*a^12*b^4*c*d^15*e^6*f^10 + 159*a^12*b^4*c^6*d^10*e*f^1
5 + 150*a^13*b^3*c*d^15*e^5*f^11 + 150*a^13*b^3*c^5*d^11*e*f^15 - 117*a^14*
b^2*c*d^15*e^4*f^12 - 117*a^14*b^2*c^4*d^12*e*f^15 - 36*a^15*b*c^2*d^14*e^2
*f^14 + 78*a^2*b^14*c^3*d^13*e^14*f^2 + 318*a^2*b^14*c^4*d^12*e^13*f^3 - 12
69*a^2*b^14*c^5*d^11*e^12*f^4 + 1134*a^2*b^14*c^6*d^10*e^11*f^5 + 618*a^2*b
^14*c^7*d^9*e^10*f^6 - 843*a^2*b^14*c^8*d^8*e^9*f^7 - 843*a^2*b^14*c^9*d^7*
e^8*f^8 + 618*a^2*b^14*c^10*d^6*e^7*f^9 + 1134*a^2*b^14*c^11*d^5*e^6*f^10 -
 1269*a^2*b^14*c^12*d^4*e^5*f^11 + 318*a^2*b^14*c^13*d^3*e^4*f^12 + 78*a^2*
b^14*c^14*d^2*e^3*f^13 + 78*a^3*b^13*c^2*d^14*e^14*f^2 - 732*a^3*b^13*c^3*d
^13*e^13*f^3 + 978*a^3*b^13*c^4*d^12*e^12*f^4 + 1722*a^3*b^13*c^5*d^11*e^11
*f^5 - 4548*a^3*b^13*c^6*d^10*e^10*f^6 + 1362*a^3*b^13*c^7*d^9*e^9*f^7 + 22
32*a^3*b^13*c^8*d^8*e^8*f^8 + 1362*a^3*b^13*c^9*d^7*e^7*f^9 - 4548*a^3*b^13
*c^10*d^6*e^6*f^10 + 1722*a^3*b^13*c^11*d^5*e^5*f^11 + 978*a^3*b^13*c^12*d^
4*e^4*f^12 - 732*a^3*b^13*c^13*d^3*e^3*f^13 + 78*a^3*b^13*c^14*d^2*e^2*f^14
 + 318*a^4*b^12*c^2*d^14*e^13*f^3 + 978*a^4*b^12*c^3*d^13*e^12*f^4 - 4452*a
^4*b^12*c^4*d^12*e^11*f^5 + 3495*a^4*b^12*c^5*d^11*e^10*f^6 + 4302*a^4*b^12
*c^6*d^10*e^9*f^7 - 4518*a^4*b^12*c^7*d^9*e^8*f^8 - 4518*a^4*b^12*c^8*d^8*e
^7*f^9 + 4302*a^4*b^12*c^9*d^7*e^6*f^10 + 3495*a^4*b^12*c^10*d^6*e^5*f^11 -
 4452*a^4*b^12*c^11*d^5*e^4*f^12 + 978*a^4*b^12*c^12*d^4*e^3*f^13 + 318*a^4
*b^12*c^13*d^3*e^2*f^14 - 1269*a^5*b^11*c^2*d^14*e^12*f^4 + 1722*a^5*b^11*c
^3*d^13*e^11*f^5 + 3495*a^5*b^11*c^4*d^12*e^10*f^6 - 9348*a^5*b^11*c^5*d^11
*e^9*f^7 + 2799*a^5*b^11*c^6*d^10*e^8*f^8 + 4824*a^5*b^11*c^7*d^9*e^7*f^9 +
 2799*a^5*b^11*c^8*d^8*e^6*f^10 - 9348*a^5*b^11*c^9*d^7*e^5*f^11 + 3495*a^5
*b^11*c^10*d^6*e^4*f^12 + 1722*a^5*b^11*c^11*d^5*e^3*f^13 - 1269*a^5*b^11*c
^12*d^4*e^2*f^14 + 1134*a^6*b^10*c^2*d^14*e^11*f^5 - 4548*a^6*b^10*c^3*d^13
*e^10*f^6 + 4302*a^6*b^10*c^4*d^12*e^9*f^7 + 2799*a^6*b^10*c^5*d^11*e^8*f^8
 - 3744*a^6*b^10*c^6*d^10*e^7*f^9 - 3744*a^6*b^10*c^7*d^9*e^6*f^10 + 2799*a
^6*b^10*c^8*d^8*e^5*f^11 + 4302*a^6*b^10*c^9*d^7*e^4*f^12 - 4548*a^6*b^10*c
^10*d^6*e^3*f^13 + 1134*a^6*b^10*c^11*d^5*e^2*f^14 + 618*a^7*b^9*c^2*d^14*e
^10*f^6 + 1362*a^7*b^9*c^3*d^13*e^9*f^7 - 4518*a^7*b^9*c^4*d^12*e^8*f^8 + 4

824*a^7*b^9*c^5*d^11*e^7*f^9 - 3744*a^7*b^9*c^6*d^10*e^6*f^10 + 4824*a^7*b^
9*c^7*d^9*e^5*f^11 - 4518*a^7*b^9*c^8*d^8*e^4*f^12 + 1362*a^7*b^9*c^9*d^7*e
^3*f^13 + 618*a^7*b^9*c^10*d^6*e^2*f^14 - 843*a^8*b^8*c^2*d^14*e^9*f^7 + 22
32*a^8*b^8*c^3*d^13*e^8*f^8 - 4518*a^8*b^8*c^4*d^12*e^7*f^9 + 2799*a^8*b^8*
c^5*d^11*e^6*f^10 + 2799*a^8*b^8*c^6*d^10*e^5*f^11 - 4518*a^8*b^8*c^7*d^9*e
^4*f^12 + 2232*a^8*b^8*c^8*d^8*e^3*f^13 - 843*a^8*b^8*c^9*d^7*e^2*f^14 - 84
3*a^9*b^7*c^2*d^14*e^8*f^8 + 1362*a^9*b^7*c^3*d^13*e^7*f^9 + 4302*a^9*b^7*c
^4*d^12*e^6*f^10 - 9348*a^9*b^7*c^5*d^11*e^5*f^11 + 4302*a^9*b^7*c^6*d^10*e
^4*f^12 + 1362*a^9*b^7*c^7*d^9*e^3*f^13 - 843*a^9*b^7*c^8*d^8*e^2*f^14 + 61
8*a^10*b^6*c^2*d^14*e^7*f^9 - 4548*a^10*b^6*c^3*d^13*e^6*f^10 + 3495*a^10*b
^6*c^4*d^12*e^5*f^11 + 3495*a^10*b^6*c^5*d^11*e^4*f^12 - 4548*a^10*b^6*c^6*
d^10*e^3*f^13 + 618*a^10*b^6*c^7*d^9*e^2*f^14 + 1134*a^11*b^5*c^2*d^14*e^6*
f^10 + 1722*a^11*b^5*c^3*d^13*e^5*f^11 - 4452*a^11*b^5*c^4*d^12*e^4*f^12 +
1722*a^11*b^5*c^5*d^11*e^3*f^13 + 1134*a^11*b^5*c^6*d^10*e^2*f^14 - 1269*a^
12*b^4*c^2*d^14*e^5*f^11 + 978*a^12*b^4*c^3*d^13*e^4*f^12 + 978*a^12*b^4*c^
4*d^12*e^3*f^13 - 1269*a^12*b^4*c^5*d^11*e^2*f^14 + 318*a^13*b^3*c^2*d^14*e
^4*f^12 - 732*a^13*b^3*c^3*d^13*e^3*f^13 + 318*a^13*b^3*c^4*d^12*e^2*f^14 +
78*a^14*b^2*c^2*d^14*e^3*f^13 + 78*a^14*b^2*c^3*d^13*e^2*f^14)/(56*a^3*b^1
3*c^5*d^11*e^16 - a^8*b^8*d^16*e^16 - a^16*c^8*d^8*f^16 - b^16*c^8*d^8*e^16
- a^16*d^16*e^8*f^8 - b^16*c^16*e^8*f^8 - 28*a^2*b^14*c^6*d^10*e^16 - a^8*
b^8*c^16*f^16 - 70*a^4*b^12*c^4*d^12*e^16 + 56*a^5*b^11*c^3*d^13*e^16 - 28*
a^6*b^10*c^2*d^14*e^16 - 28*a^10*b^6*c^14*d^2*f^16 + 56*a^11*b^5*c^13*d^3*f
^16 - 70*a^12*b^4*c^12*d^4*f^16 + 56*a^13*b^3*c^11*d^5*f^16 - 28*a^14*b^2*c
^10*d^6*f^16 - 28*a^2*b^14*c^16*e^6*f^10 + 56*a^3*b^13*c^16*e^5*f^11 - 70*a
^4*b^12*c^16*e^4*f^12 + 56*a^5*b^11*c^16*e^3*f^13 - 28*a^6*b^10*c^16*e^2*f^
14 - 28*a^10*b^6*d^16*e^14*f^2 + 56*a^11*b^5*d^16*e^13*f^3 - 70*a^12*b^4*d^
16*e^12*f^4 + 56*a^13*b^3*d^16*e^11*f^5 - 28*a^14*b^2*d^16*e^10*f^6 - 28*a^
16*c^2*d^14*e^6*f^10 + 56*a^16*c^3*d^13*e^5*f^11 - 70*a^16*c^4*d^12*e^4*f^1
2 + 56*a^16*c^5*d^11*e^3*f^13 - 28*a^16*c^6*d^10*e^2*f^14 - 28*b^16*c^10*d^
6*e^14*f^2 + 56*b^16*c^11*d^5*e^13*f^3 - 70*b^16*c^12*d^4*e^12*f^4 + 56*b^1
6*c^13*d^3*e^11*f^5 - 28*b^16*c^14*d^2*e^10*f^6 + 8*a*b^15*c^7*d^9*e^16 + 8
*a^7*b^9*c*d^15*e^16 + 8*a^9*b^7*c^15*d*f^16 + 8*a^15*b*c^9*d^7*f^16 + 8*a*
b^15*c^16*e^7*f^9 + 8*a^7*b^9*c^16*e*f^15 + 8*a^9*b^7*d^16*e^15*f + 8*a^15*
b*d^16*e^9*f^7 + 8*a^16*c*d^15*e^7*f^9 + 8*a^16*c^7*d^9*e*f^15 + 8*b^16*c^9
*d^7*e^15*f + 8*b^16*c^15*d*e^9*f^7 - 56*a*b^15*c^8*d^8*e^15*f - 56*a*b^15*
c^15*d*e^8*f^8 - 56*a^8*b^8*c*d^15*e^15*f - 56*a^8*b^8*c^15*d*e*f^15 - 56*a
^15*b*c*d^15*e^8*f^8 - 56*a^15*b*c^8*d^8*e*f^15 + 160*a*b^15*c^9*d^7*e^14*f
^2 - 224*a*b^15*c^10*d^6*e^13*f^3 + 112*a*b^15*c^11*d^5*e^12*f^4 + 112*a*b^
15*c^12*d^4*e^11*f^5 - 224*a*b^15*c^13*d^3*e^10*f^6 + 160*a*b^15*c^14*d^2*e
^9*f^7 + 160*a^2*b^14*c^7*d^9*e^15*f + 160*a^2*b^14*c^15*d*e^7*f^9 - 224*a^
3*b^13*c^6*d^10*e^15*f - 224*a^3*b^13*c^15*d*e^6*f^10 + 112*a^4*b^12*c^5*d^
11*e^15*f + 112*a^4*b^12*c^15*d*e^5*f^11 + 112*a^5*b^11*c^4*d^12*e^15*f + 1
12*a^5*b^11*c^15*d*e^4*f^12 - 224*a^6*b^10*c^3*d^13*e^15*f - 224*a^6*b^10*c
^15*d*e^3*f^13 + 160*a^7*b^9*c^2*d^14*e^15*f + 160*a^7*b^9*c^15*d*e^2*f^14
+ 160*a^9*b^7*c*d^15*e^14*f^2 + 160*a^9*b^7*c^14*d^2*e*f^15 - 224*a^10*b^6*

c*d^15*e^13*f^3 - 224*a^10*b^6*c^13*d^3*e*f^15 + 112*a^11*b^5*c*d^15*e^12*f
^4 + 112*a^11*b^5*c^12*d^4*e*f^15 + 112*a^12*b^4*c*d^15*e^11*f^5 + 112*a^12
*b^4*c^11*d^5*e*f^15 - 224*a^13*b^3*c*d^15*e^10*f^6 - 224*a^13*b^3*c^10*d^6
*e*f^15 + 160*a^14*b^2*c*d^15*e^9*f^7 + 160*a^14*b^2*c^9*d^7*e*f^15 + 160*a
^15*b*c^2*d^14*e^7*f^9 - 224*a^15*b*c^3*d^13*e^6*f^10 + 112*a^15*b*c^4*d^12
*e^5*f^11 + 112*a^15*b*c^5*d^11*e^4*f^12 - 224*a^15*b*c^6*d^10*e^3*f^13 + 1
60*a^15*b*c^7*d^9*e^2*f^14 - 300*a^2*b^14*c^8*d^8*e^14*f^2 + 840*a^2*b^14*c
^10*d^6*e^12*f^4 - 1344*a^2*b^14*c^11*d^5*e^11*f^5 + 840*a^2*b^14*c^12*d^4*
e^10*f^6 - 300*a^2*b^14*c^14*d^2*e^8*f^8 + 1400*a^3*b^13*c^8*d^8*e^13*f^3 -
 2800*a^3*b^13*c^9*d^7*e^12*f^4 + 1568*a^3*b^13*c^10*d^6*e^11*f^5 + 1568*a^
3*b^13*c^11*d^5*e^10*f^6 - 2800*a^3*b^13*c^12*d^4*e^9*f^7 + 1400*a^3*b^13*c
^13*d^3*e^8*f^8 + 840*a^4*b^12*c^6*d^10*e^14*f^2 - 2800*a^4*b^12*c^7*d^9*e^
13*f^3 + 1750*a^4*b^12*c^8*d^8*e^12*f^4 + 4480*a^4*b^12*c^9*d^7*e^11*f^5 -
8624*a^4*b^12*c^10*d^6*e^10*f^6 + 4480*a^4*b^12*c^11*d^5*e^9*f^7 + 1750*a^4
*b^12*c^12*d^4*e^8*f^8 - 2800*a^4*b^12*c^13*d^3*e^7*f^9 + 840*a^4*b^12*c^14
*d^2*e^6*f^10 - 1344*a^5*b^11*c^5*d^11*e^14*f^2 + 1568*a^5*b^11*c^6*d^10*e^
13*f^3 + 4480*a^5*b^11*c^7*d^9*e^12*f^4 - 12264*a^5*b^11*c^8*d^8*e^11*f^5 +
 7392*a^5*b^11*c^9*d^7*e^10*f^6 + 7392*a^5*b^11*c^10*d^6*e^9*f^7 - 12264*a^
5*b^11*c^11*d^5*e^8*f^8 + 4480*a^5*b^11*c^12*d^4*e^7*f^9 + 1568*a^5*b^11*c^
13*d^3*e^6*f^10 - 1344*a^5*b^11*c^14*d^2*e^5*f^11 + 840*a^6*b^10*c^4*d^12*e
^14*f^2 + 1568*a^6*b^10*c^5*d^11*e^13*f^3 - 8624*a^6*b^10*c^6*d^10*e^12*f^4
 + 7392*a^6*b^10*c^7*d^9*e^11*f^5 + 11396*a^6*b^10*c^8*d^8*e^10*f^6 - 24640
*a^6*b^10*c^9*d^7*e^9*f^7 + 11396*a^6*b^10*c^10*d^6*e^8*f^8 + 7392*a^6*b^10
*c^11*d^5*e^7*f^9 - 8624*a^6*b^10*c^12*d^4*e^6*f^10 + 1568*a^6*b^10*c^13*d^
3*e^5*f^11 + 840*a^6*b^10*c^14*d^2*e^4*f^12 - 2800*a^7*b^9*c^4*d^12*e^13*f^
3 + 4480*a^7*b^9*c^5*d^11*e^12*f^4 + 7392*a^7*b^9*c^6*d^10*e^11*f^5 - 24640
*a^7*b^9*c^7*d^9*e^10*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9*c
^9*d^7*e^8*f^8 - 24640*a^7*b^9*c^10*d^6*e^7*f^9 + 7392*a^7*b^9*c^11*d^5*e^6
*f^10 + 4480*a^7*b^9*c^12*d^4*e^5*f^11 - 2800*a^7*b^9*c^13*d^3*e^4*f^12 - 3
00*a^8*b^8*c^2*d^14*e^14*f^2 + 1400*a^8*b^8*c^3*d^13*e^13*f^3 + 1750*a^8*b^
8*c^4*d^12*e^12*f^4 - 12264*a^8*b^8*c^5*d^11*e^11*f^5 + 11396*a^8*b^8*c^6*d
^10*e^10*f^6 + 15400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^8*d^8*e^8*f^
8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^10*d^6*e^6*f^10 - 12264
*a^8*b^8*c^11*d^5*e^5*f^11 + 1750*a^8*b^8*c^12*d^4*e^4*f^12 + 1400*a^8*b^8*
c^13*d^3*e^3*f^13 - 300*a^8*b^8*c^14*d^2*e^2*f^14 - 2800*a^9*b^7*c^3*d^13*e
^12*f^4 + 4480*a^9*b^7*c^4*d^12*e^11*f^5 + 7392*a^9*b^7*c^5*d^11*e^10*f^6 -
 24640*a^9*b^7*c^6*d^10*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^9
*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^10 + 7392*a^9*b^7*c^10*d
^6*e^5*f^11 + 4480*a^9*b^7*c^11*d^5*e^4*f^12 - 2800*a^9*b^7*c^12*d^4*e^3*f^
13 + 840*a^10*b^6*c^2*d^14*e^12*f^4 + 1568*a^10*b^6*c^3*d^13*e^11*f^5 - 862
4*a^10*b^6*c^4*d^12*e^10*f^6 + 7392*a^10*b^6*c^5*d^11*e^9*f^7 + 11396*a^10*
b^6*c^6*d^10*e^8*f^8 - 24640*a^10*b^6*c^7*d^9*e^7*f^9 + 11396*a^10*b^6*c^8*
d^8*e^6*f^10 + 7392*a^10*b^6*c^9*d^7*e^5*f^11 - 8624*a^10*b^6*c^10*d^6*e^4*
f^12 + 1568*a^10*b^6*c^11*d^5*e^3*f^13 + 840*a^10*b^6*c^12*d^4*e^2*f^14 - 1
344*a^11*b^5*c^2*d^14*e^11*f^5 + 1568*a^11*b^5*c^3*d^13*e^10*f^6 + 4480*a^1

1*b^5*c^4*d^12*e^9*f^7 - 12264*a^11*b^5*c^5*d^11*e^8*f^8 + 7392*a^11*b^5*c^6*d^10*e^7*f^9 + 7392*a^11*b^5*c^7*d^9*e^6*f^10 - 12264*a^11*b^5*c^8*d^8*e^5*f^11 + 4480*a^11*b^5*c^9*d^7*e^4*f^12 + 1568*a^11*b^5*c^10*d^6*e^3*f^13 - 1344*a^11*b^5*c^11*d^5*e^2*f^14 + 840*a^12*b^4*c^2*d^14*e^10*f^6 - 2800*a^12*b^4*c^3*d^13*e^9*f^7 + 1750*a^12*b^4*c^4*d^12*e^8*f^8 + 4480*a^12*b^4*c^5*d^11*e^7*f^9 - 8624*a^12*b^4*c^6*d^10*e^6*f^10 + 4480*a^12*b^4*c^7*d^9*e^5*f^11 + 1750*a^12*b^4*c^8*d^8*e^4*f^12 - 2800*a^12*b^4*c^9*d^7*e^3*f^13 + 840*a^12*b^4*c^10*d^6*e^2*f^14 + 1400*a^13*b^3*c^3*d^13*e^8*f^8 - 2800*a^13*b^3*c^4*d^12*e^7*f^9 + 1568*a^13*b^3*c^5*d^11*e^6*f^10 + 1568*a^13*b^3*c^6*d^10*e^5*f^11 - 2800*a^13*b^3*c^7*d^9*e^4*f^12 + 1400*a^13*b^3*c^8*d^8*e^3*f^13 - 300*a^14*b^2*c^2*d^14*e^8*f^8 + 840*a^14*b^2*c^4*d^12*e^6*f^10 - 1344*a^14*b^2*c^5*d^11*e^5*f^11 + 840*a^14*b^2*c^6*d^10*e^4*f^12 - 300*a^14*b^2*c^8*d^8*e^2*f^14) + root(756756*a^10*b^10*c^10*d^10*e^10*f^10*z^3 + 573300*a^12*b^8*c^9*d^11*e^9*f^11*z^3 + 573300*a^11*b^9*c^11*d^9*e^8*f^12*z^3 + 573300*a^11*b^9*c^8*d^12*e^11*f^9*z^3 + 573300*a^9*b^11*c^12*d^8*e^9*f^11*z^3 + 573300*a^9*b^11*c^9*d^11*e^12*f^8*z^3 + 573300*a^8*b^12*c^11*d^9*e^11*f^9*z^3 - 343980*a^11*b^9*c^10*d^10*e^9*f^11*z^3 - 343980*a^11*b^9*c^9*d^11*e^10*f^10*z^3 - 343980*a^10*b^10*c^11*d^9*e^9*f^11*z^3 - 343980*a^10*b^10*c^9*d^11*e^11*f^9*z^3 - 343980*a^9*b^11*c^11*d^9*e^10*f^10*z^3 - 343980*a^9*b^11*c^10*d^10*e^11*f^9*z^3 + 326340*a^13*b^7*c^10*d^10*e^7*f^13*z^3 + 326340*a^13*b^7*c^7*d^13*e^10*f^10*z^3 + 326340*a^10*b^10*c^13*d^7*e^7*f^13*z^3 + 326340*a^10*b^10*c^7*d^13*e^13*f^7*z^3 + 326340*a^7*b^13*c^13*d^7*e^10*f^10*z^3 + 326340*a^7*b^13*c^10*d^10*e^13*f^7*z^3 - 267540*a^12*b^8*c^10*d^10*e^8*f^12*z^3 - 267540*a^12*b^8*c^8*d^12*e^10*f^10*z^3 - 267540*a^10*b^10*c^12*d^8*e^8*f^12*z^3 - 267540*a^10*b^10*c^8*d^12*e^12*f^8*z^3 - 267540*a^8*b^12*c^12*d^8*e^10*f^10*z^3 - 267540*a^8*b^12*c^10*d^10*e^12*f^8*z^3 + 245700*a^14*b^6*c^8*d^12*e^8*f^12*z^3 + 245700*a^12*b^8*c^12*d^8*e^6*f^14*z^3 + 245700*a^12*b^8*c^6*d^14*e^12*f^8*z^3 + 245700*a^8*b^12*c^14*d^6*e^8*f^12*z^3 + 245700*a^8*b^12*c^8*d^12*e^14*f^6*z^3 + 245700*a^6*b^14*c^12*d^8*e^12*f^8*z^3 - 191100*a^13*b^7*c^9*d^11*e^8*f^12*z^3 - 191100*a^13*b^7*c^8*d^12*e^9*f^11*z^3 - 191100*a^12*b^8*c^11*d^9*e^7*f^13*z^3 - 191100*a^12*b^8*c^7*d^13*e^11*f^9*z^3 - 191100*a^11*b^9*c^12*d^8*e^7*f^13*z^3 - 191100*a^11*b^9*c^7*d^13*e^12*f^8*z^3 - 191100*a^9*b^11*c^13*d^7*e^8*f^12*z^3 - 191100*a^9*b^11*c^8*d^12*e^13*f^7*z^3 - 191100*a^8*b^12*c^13*d^7*e^9*f^11*z^3 - 191100*a^8*b^12*c^9*d^11*e^13*f^7*z^3 - 191100*a^7*b^13*c^12*d^8*e^11*f^9*z^3 - 191100*a^7*b^13*c^11*d^9*e^12*f^8*z^3 - 123900*a^14*b^6*c^9*d^11*e^7*f^13*z^3 - 123900*a^14*b^6*c^7*d^13*e^9*f^11*z^3 - 123900*a^13*b^7*c^11*d^9*e^6*f^14*z^3 - 123900*a^13*b^7*c^6*d^14*e^11*f^9*z^3 - 123900*a^11*b^9*c^13*d^7*e^6*f^14*z^3 - 123900*a^11*b^9*c^6*d^14*e^13*f^7*z^3 - 123900*a^9*b^11*c^14*d^6*e^7*f^13*z^3 - 123900*a^9*b^11*c^7*d^13*e^14*f^6*z^3 - 123900*a^7*b^13*c^14*d^6*e^9*f^11*z^3 - 123900*a^7*b^13*c^9*d^11*e^14*f^6*z^3 - 123900*a^6*b^14*c^13*d^7*e^11*f^9*z^3 - 123900*a^6*b^14*c^11*d^9*e^13*f^7*z^3 + 101700*a^15*b^5*c^9*d^11*e^6*f^14*z^3 + 101700*a^15*b^5*c^6*d^14*e^9*f^11*z^3 + 101700*a^14*b^6*c^11*d^9*e^5*f^15*z^3 + 101700*a^14*b^6*c^5*d^15*e^11*f^9*z^3 + 101700*a^11*b^9*c^14*d^6*e^5*f^15*z^3 + 101700*a^11*b^9*c^5*d^15*e

^14*f^6*z^3 + 101700*a^9*b^11*c^15*d^5*e^6*f^14*z^3 + 101700*a^9*b^11*c^6*d
^14*e^15*f^5*z^3 + 101700*a^6*b^14*c^15*d^5*e^9*f^11*z^3 + 101700*a^6*b^14*
c^9*d^11*e^15*f^5*z^3 + 101700*a^5*b^15*c^14*d^6*e^11*f^9*z^3 + 101700*a^5*
b^15*c^11*d^9*e^14*f^6*z^3 - 65820*a^14*b^6*c^10*d^10*e^6*f^14*z^3 - 65820*
a^14*b^6*c^6*d^14*e^10*f^10*z^3 - 65820*a^10*b^10*c^14*d^6*e^6*f^14*z^3 - 6
5820*a^10*b^10*c^6*d^14*e^14*f^6*z^3 - 65820*a^6*b^14*c^14*d^6*e^10*f^10*z^
3 - 65820*a^6*b^14*c^10*d^10*e^14*f^6*z^3 + 56700*a^16*b^4*c^7*d^13*e^7*f^1
3*z^3 - 56700*a^15*b^5*c^8*d^12*e^7*f^13*z^3 - 56700*a^15*b^5*c^7*d^13*e^8*
f^12*z^3 + 56700*a^13*b^7*c^13*d^7*e^4*f^16*z^3 - 56700*a^13*b^7*c^12*d^8*e
^5*f^15*z^3 - 56700*a^13*b^7*c^5*d^15*e^12*f^8*z^3 + 56700*a^13*b^7*c^4*d^1
6*e^13*f^7*z^3 - 56700*a^12*b^8*c^13*d^7*e^5*f^15*z^3 - 56700*a^12*b^8*c^5*
d^15*e^13*f^7*z^3 - 56700*a^8*b^12*c^15*d^5*e^7*f^13*z^3 - 56700*a^8*b^12*c
^7*d^13*e^15*f^5*z^3 + 56700*a^7*b^13*c^16*d^4*e^7*f^13*z^3 - 56700*a^7*b^1
3*c^15*d^5*e^8*f^12*z^3 - 56700*a^7*b^13*c^8*d^12*e^15*f^5*z^3 + 56700*a^7*
b^13*c^7*d^13*e^16*f^4*z^3 - 56700*a^5*b^15*c^13*d^7*e^12*f^8*z^3 - 56700*a
^5*b^15*c^12*d^8*e^13*f^7*z^3 + 56700*a^4*b^16*c^13*d^7*e^13*f^7*z^3 - 4825
2*a^15*b^5*c^10*d^10*e^5*f^15*z^3 - 48252*a^15*b^5*c^5*d^15*e^10*f^10*z^3 -
 48252*a^10*b^10*c^15*d^5*e^5*f^15*z^3 - 48252*a^10*b^10*c^5*d^15*e^15*f^5*
z^3 - 48252*a^5*b^15*c^15*d^5*e^10*f^10*z^3 - 48252*a^5*b^15*c^10*d^10*e^15
*f^5*z^3 - 32400*a^16*b^4*c^8*d^12*e^6*f^14*z^3 - 32400*a^16*b^4*c^6*d^14*e
^8*f^12*z^3 - 32400*a^14*b^6*c^12*d^8*e^4*f^16*z^3 - 32400*a^14*b^6*c^4*d^1
6*e^12*f^8*z^3 - 32400*a^12*b^8*c^14*d^6*e^4*f^16*z^3 - 32400*a^12*b^8*c^4*
d^16*e^14*f^6*z^3 - 32400*a^8*b^12*c^16*d^4*e^6*f^14*z^3 - 32400*a^8*b^12*c
^6*d^14*e^16*f^4*z^3 - 32400*a^6*b^14*c^16*d^4*e^8*f^12*z^3 - 32400*a^6*b^1
4*c^8*d^12*e^16*f^4*z^3 - 32400*a^4*b^16*c^14*d^6*e^12*f^8*z^3 - 32400*a^4*
b^16*c^12*d^8*e^14*f^6*z^3 + 20565*a^16*b^4*c^10*d^10*e^4*f^16*z^3 + 20565*
a^16*b^4*c^4*d^16*e^10*f^10*z^3 + 20565*a^10*b^10*c^16*d^4*e^4*f^16*z^3 + 2
0565*a^10*b^10*c^4*d^16*e^16*f^4*z^3 + 20565*a^4*b^16*c^16*d^4*e^10*f^10*z^
3 + 20565*a^4*b^16*c^10*d^10*e^16*f^4*z^3 + 15660*a^17*b^3*c^8*d^12*e^5*f^1
5*z^3 + 15660*a^17*b^3*c^5*d^15*e^8*f^12*z^3 + 15660*a^15*b^5*c^12*d^8*e^3*
f^17*z^3 + 15660*a^15*b^5*c^3*d^17*e^12*f^8*z^3 + 15660*a^12*b^8*c^15*d^5*e
^3*f^17*z^3 + 15660*a^12*b^8*c^3*d^17*e^15*f^5*z^3 + 15660*a^8*b^12*c^17*d^
3*e^5*f^15*z^3 + 15660*a^8*b^12*c^5*d^15*e^17*f^3*z^3 + 15660*a^5*b^15*c^17
*d^3*e^8*f^12*z^3 + 15660*a^5*b^15*c^8*d^12*e^17*f^3*z^3 + 15660*a^3*b^17*c
^15*d^5*e^12*f^8*z^3 + 15660*a^3*b^17*c^12*d^8*e^15*f^5*z^3 - 9750*a^17*b^3
*c^9*d^11*e^4*f^16*z^3 - 9750*a^17*b^3*c^4*d^16*e^9*f^11*z^3 - 9750*a^16*b^
4*c^11*d^9*e^3*f^17*z^3 - 9750*a^16*b^4*c^3*d^17*e^11*f^9*z^3 - 9750*a^11*b
^9*c^16*d^4*e^3*f^17*z^3 - 9750*a^11*b^9*c^3*d^17*e^16*f^4*z^3 - 9750*a^9*b
^11*c^17*d^3*e^4*f^16*z^3 - 9750*a^9*b^11*c^4*d^16*e^17*f^3*z^3 - 9750*a^4*
b^16*c^17*d^3*e^9*f^11*z^3 - 9750*a^4*b^16*c^9*d^11*e^17*f^3*z^3 - 9750*a^3
*b^17*c^16*d^4*e^11*f^9*z^3 - 9750*a^3*b^17*c^11*d^9*e^16*f^4*z^3 - 8100*a^
17*b^3*c^7*d^13*e^6*f^14*z^3 - 8100*a^17*b^3*c^6*d^14*e^7*f^13*z^3 - 8100*a
^14*b^6*c^13*d^7*e^3*f^17*z^3 - 8100*a^14*b^6*c^3*d^17*e^13*f^7*z^3 - 8100*
a^13*b^7*c^14*d^6*e^3*f^17*z^3 - 8100*a^13*b^7*c^3*d^17*e^14*f^6*z^3 - 8100
*a^7*b^13*c^17*d^3*e^6*f^14*z^3 - 8100*a^7*b^13*c^6*d^14*e^17*f^3*z^3 - 810

0*a^6*b^14*c^17*d^3*e^7*f^13*z^3 - 8100*a^6*b^14*c^7*d^13*e^17*f^3*z^3 - 81
00*a^3*b^17*c^14*d^6*e^13*f^7*z^3 - 8100*a^3*b^17*c^13*d^7*e^14*f^6*z^3 - 7
980*a^16*b^4*c^9*d^11*e^5*f^15*z^3 - 7980*a^16*b^4*c^5*d^15*e^9*f^11*z^3 -
7980*a^15*b^5*c^11*d^9*e^4*f^16*z^3 - 7980*a^15*b^5*c^4*d^16*e^11*f^9*z^3 -
 7980*a^11*b^9*c^15*d^5*e^4*f^16*z^3 - 7980*a^11*b^9*c^4*d^16*e^15*f^5*z^3
- 7980*a^9*b^11*c^16*d^4*e^5*f^15*z^3 - 7980*a^9*b^11*c^5*d^15*e^16*f^4*z^3
 - 7980*a^5*b^15*c^16*d^4*e^9*f^11*z^3 - 7980*a^5*b^15*c^9*d^11*e^16*f^4*z^
3 - 7980*a^4*b^16*c^15*d^5*e^11*f^9*z^3 - 7980*a^4*b^16*c^11*d^9*e^15*f^5*z
^3 + 6300*a^18*b^2*c^6*d^14*e^6*f^14*z^3 + 6300*a^14*b^6*c^14*d^6*e^2*f^18*
z^3 + 6300*a^14*b^6*c^2*d^18*e^14*f^6*z^3 + 6300*a^6*b^14*c^18*d^2*e^6*f^14
*z^3 + 6300*a^6*b^14*c^6*d^14*e^18*f^2*z^3 + 6300*a^2*b^18*c^14*d^6*e^14*f^
6*z^3 - 4260*a^18*b^2*c^7*d^13*e^5*f^15*z^3 - 4260*a^18*b^2*c^5*d^15*e^7*f^
13*z^3 - 4260*a^15*b^5*c^13*d^7*e^2*f^18*z^3 - 4260*a^15*b^5*c^2*d^18*e^13*
f^7*z^3 - 4260*a^13*b^7*c^15*d^5*e^2*f^18*z^3 - 4260*a^13*b^7*c^2*d^18*e^15
*f^5*z^3 - 4260*a^7*b^13*c^18*d^2*e^5*f^15*z^3 - 4260*a^7*b^13*c^5*d^15*e^1
8*f^2*z^3 - 4260*a^5*b^15*c^18*d^2*e^7*f^13*z^3 - 4260*a^5*b^15*c^7*d^13*e^
18*f^2*z^3 - 4260*a^2*b^18*c^15*d^5*e^13*f^7*z^3 - 4260*a^2*b^18*c^13*d^7*e
^15*f^5*z^3 + 1470*a^17*b^3*c^10*d^10*e^3*f^17*z^3 + 1470*a^17*b^3*c^3*d^17
*e^10*f^10*z^3 + 1470*a^10*b^10*c^17*d^3*e^3*f^17*z^3 + 1470*a^10*b^10*c^3*
d^17*e^17*f^3*z^3 + 1470*a^3*b^17*c^17*d^3*e^10*f^10*z^3 + 1470*a^3*b^17*c^
10*d^10*e^17*f^3*z^3 + 1350*a^18*b^2*c^9*d^11*e^3*f^17*z^3 + 1350*a^18*b^2*
c^3*d^17*e^9*f^11*z^3 + 1350*a^17*b^3*c^11*d^9*e^2*f^18*z^3 + 1350*a^17*b^3
*c^2*d^18*e^11*f^9*z^3 + 1350*a^11*b^9*c^17*d^3*e^2*f^18*z^3 + 1350*a^11*b^
9*c^2*d^18*e^17*f^3*z^3 + 1350*a^9*b^11*c^18*d^2*e^3*f^17*z^3 + 1350*a^9*b^
11*c^3*d^17*e^18*f^2*z^3 + 1350*a^3*b^17*c^18*d^2*e^9*f^11*z^3 + 1350*a^3*b
^17*c^9*d^11*e^18*f^2*z^3 + 1350*a^2*b^18*c^17*d^3*e^11*f^9*z^3 + 1350*a^2*
b^18*c^11*d^9*e^17*f^3*z^3 - 1070*a^18*b^2*c^10*d^10*e^2*f^18*z^3 - 1070*a^
18*b^2*c^2*d^18*e^10*f^10*z^3 - 1070*a^10*b^10*c^18*d^2*e^2*f^18*z^3 - 1070
*a^10*b^10*c^2*d^18*e^18*f^2*z^3 - 1070*a^2*b^18*c^18*d^2*e^10*f^10*z^3 - 1
070*a^2*b^18*c^10*d^10*e^18*f^2*z^3 + 525*a^18*b^2*c^8*d^12*e^4*f^16*z^3 +
525*a^18*b^2*c^4*d^16*e^8*f^12*z^3 + 525*a^16*b^4*c^12*d^8*e^2*f^18*z^3 + 5
25*a^16*b^4*c^2*d^18*e^12*f^8*z^3 + 525*a^12*b^8*c^16*d^4*e^2*f^18*z^3 + 52
5*a^12*b^8*c^2*d^18*e^16*f^4*z^3 + 525*a^8*b^12*c^18*d^2*e^4*f^16*z^3 + 525
*a^8*b^12*c^4*d^16*e^18*f^2*z^3 + 525*a^4*b^16*c^18*d^2*e^8*f^12*z^3 + 525*
a^4*b^16*c^8*d^12*e^18*f^2*z^3 + 525*a^2*b^18*c^16*d^4*e^12*f^8*z^3 + 525*a
^2*b^18*c^12*d^8*e^16*f^4*z^3 + 900*a^19*b*c^7*d^13*e^4*f^16*z^3 + 900*a^19
*b*c^4*d^16*e^7*f^13*z^3 + 900*a^16*b^4*c^13*d^7*e*f^19*z^3 + 900*a^16*b^4*
c*d^19*e^13*f^7*z^3 + 900*a^13*b^7*c^16*d^4*e*f^19*z^3 + 900*a^13*b^7*c*d^1
9*e^16*f^4*z^3 + 900*a^7*b^13*c^19*d*e^4*f^16*z^3 + 900*a^7*b^13*c^4*d^16*e
^19*f*z^3 + 900*a^4*b^16*c^19*d*e^7*f^13*z^3 + 900*a^4*b^16*c^7*d^13*e^19*f
*z^3 + 900*a*b^19*c^16*d^4*e^13*f^7*z^3 + 900*a*b^19*c^13*d^7*e^16*f^4*z^3
- 750*a^19*b*c^8*d^12*e^3*f^17*z^3 - 750*a^19*b*c^3*d^17*e^8*f^12*z^3 - 750
*a^17*b^3*c^12*d^8*e*f^19*z^3 - 750*a^17*b^3*c*d^19*e^12*f^8*z^3 - 750*a^12
*b^8*c^17*d^3*e*f^19*z^3 - 750*a^12*b^8*c*d^19*e^17*f^3*z^3 - 750*a^8*b^12*
c^19*d*e^3*f^17*z^3 - 750*a^8*b^12*c^3*d^17*e^19*f*z^3 - 750*a^3*b^17*c^19*

d*e^8*f^12*z^3 - 750*a^3*b^17*c^8*d^12*e^19*f*z^3 - 750*a*b^19*c^17*d^3*e^1
2*f^8*z^3 - 750*a*b^19*c^12*d^8*e^17*f^3*z^3 - 420*a^19*b*c^6*d^14*e^5*f^15
*z^3 - 420*a^19*b*c^5*d^15*e^6*f^14*z^3 - 420*a^15*b^5*c^14*d^6*e*f^19*z^3
- 420*a^15*b^5*c*d^19*e^14*f^6*z^3 - 420*a^14*b^6*c^15*d^5*e*f^19*z^3 - 420
*a^14*b^6*c*d^19*e^15*f^5*z^3 - 420*a^6*b^14*c^19*d*e^5*f^15*z^3 - 420*a^6*
b^14*c^5*d^15*e^19*f*z^3 - 420*a^5*b^15*c^19*d*e^6*f^14*z^3 - 420*a^5*b^15*
c^6*d^14*e^19*f*z^3 - 420*a*b^19*c^15*d^5*e^14*f^6*z^3 - 420*a*b^19*c^14*d^
6*e^15*f^5*z^3 + 350*a^19*b*c^9*d^11*e^2*f^18*z^3 + 350*a^19*b*c^2*d^18*e^9
*f^11*z^3 + 350*a^18*b^2*c^11*d^9*e*f^19*z^3 + 350*a^18*b^2*c*d^19*e^11*f^9
*z^3 + 350*a^11*b^9*c^18*d^2*e*f^19*z^3 + 350*a^11*b^9*c*d^19*e^18*f^2*z^3
+ 350*a^9*b^11*c^19*d*e^2*f^18*z^3 + 350*a^9*b^11*c^2*d^18*e^19*f*z^3 + 350
*a^2*b^18*c^19*d*e^9*f^11*z^3 + 350*a^2*b^18*c^9*d^11*e^19*f*z^3 + 350*a*b^
19*c^18*d^2*e^11*f^9*z^3 + 350*a*b^19*c^11*d^9*e^18*f^2*z^3 - 90*a^19*b*c^1
0*d^10*e*f^19*z^3 - 90*a^19*b*c*d^19*e^10*f^10*z^3 - 90*a^10*b^10*c^19*d*e*
f^19*z^3 - 90*a^10*b^10*c*d^19*e^19*f*z^3 - 90*a*b^19*c^19*d*e^10*f^10*z^3
- 90*a*b^19*c^10*d^10*e^19*f*z^3 + 10*b^20*c^19*d*e^11*f^9*z^3 + 10*b^20*c^
11*d^9*e^19*f*z^3 + 10*a^20*c^9*d^11*e*f^19*z^3 + 10*a^20*c*d^19*e^9*f^11*z
^3 + 10*a^19*b*d^20*e^11*f^9*z^3 + 10*a^11*b^9*d^20*e^19*f*z^3 + 10*a^9*b^1
1*c^20*e*f^19*z^3 + 10*a*b^19*c^20*e^9*f^11*z^3 + 10*a^19*b*c^11*d^9*f^20*z
^3 + 10*a^11*b^9*c^19*d*f^20*z^3 + 10*a^9*b^11*c*d^19*e^20*z^3 + 10*a*b^19*
c^9*d^11*e^20*z^3 + 252*b^20*c^15*d^5*e^15*f^5*z^3 - 210*b^20*c^16*d^4*e^14
*f^6*z^3 - 210*b^20*c^14*d^6*e^16*f^4*z^3 + 120*b^20*c^17*d^3*e^13*f^7*z^3
+ 120*b^20*c^13*d^7*e^17*f^3*z^3 - 45*b^20*c^18*d^2*e^12*f^8*z^3 - 45*b^20*
c^12*d^8*e^18*f^2*z^3 + 252*a^20*c^5*d^15*e^5*f^15*z^3 - 210*a^20*c^6*d^14*
e^4*f^16*z^3 - 210*a^20*c^4*d^16*e^6*f^14*z^3 + 120*a^20*c^7*d^13*e^3*f^17*
z^3 + 120*a^20*c^3*d^17*e^7*f^13*z^3 - 45*a^20*c^8*d^12*e^2*f^18*z^3 - 45*a
^20*c^2*d^18*e^8*f^12*z^3 + 252*a^15*b^5*d^20*e^15*f^5*z^3 - 210*a^16*b^4*d
^20*e^14*f^6*z^3 - 210*a^14*b^6*d^20*e^16*f^4*z^3 + 120*a^17*b^3*d^20*e^13*
f^7*z^3 + 120*a^13*b^7*d^20*e^17*f^3*z^3 - 45*a^18*b^2*d^20*e^12*f^8*z^3 -
45*a^12*b^8*d^20*e^18*f^2*z^3 + 252*a^5*b^15*c^20*e^5*f^15*z^3 - 210*a^6*b^
14*c^20*e^4*f^16*z^3 - 210*a^4*b^16*c^20*e^6*f^14*z^3 + 120*a^7*b^13*c^20*e
^3*f^17*z^3 + 120*a^3*b^17*c^20*e^7*f^13*z^3 - 45*a^8*b^12*c^20*e^2*f^18*z^
3 - 45*a^2*b^18*c^20*e^8*f^12*z^3 + 252*a^15*b^5*c^15*d^5*f^20*z^3 - 210*a^
16*b^4*c^14*d^6*f^20*z^3 - 210*a^14*b^6*c^16*d^4*f^20*z^3 + 120*a^17*b^3*c^
13*d^7*f^20*z^3 + 120*a^13*b^7*c^17*d^3*f^20*z^3 - 45*a^18*b^2*c^12*d^8*f^2
0*z^3 - 45*a^12*b^8*c^18*d^2*f^20*z^3 + 252*a^5*b^15*c^5*d^15*e^20*z^3 - 21
0*a^6*b^14*c^4*d^16*e^20*z^3 - 210*a^4*b^16*c^6*d^14*e^20*z^3 + 120*a^7*b^1
3*c^3*d^17*e^20*z^3 + 120*a^3*b^17*c^7*d^13*e^20*z^3 - 45*a^8*b^12*c^2*d^18
*e^20*z^3 - 45*a^2*b^18*c^8*d^12*e^20*z^3 - b^20*c^20*e^10*f^10*z^3 - a^20*
d^20*e^10*f^10*z^3 - b^20*c^10*d^10*e^20*z^3 - a^20*c^10*d^10*f^20*z^3 - a^
10*b^10*d^20*e^20*z^3 - a^10*b^10*c^20*f^20*z^3 + 1890*a^12*b^2*c*d^13*e*f^
13*z + 1890*a*b^13*c^12*d^2*e*f^13*z + 1890*a*b^13*c*d^13*e^12*f^2*z + 9261
0*a^6*b^8*c^4*d^10*e^4*f^10*z + 92610*a^4*b^10*c^6*d^8*e^4*f^10*z + 92610*a
^4*b^10*c^4*d^10*e^6*f^8*z + 66150*a^8*b^6*c^3*d^11*e^3*f^11*z - 66150*a^7*
b^7*c^4*d^10*e^3*f^11*z - 66150*a^7*b^7*c^3*d^11*e^4*f^10*z - 66150*a^4*b^1

0*c^7*d^7*e^3*f^11*z - 66150*a^4*b^10*c^3*d^11*e^7*f^7*z + 66150*a^3*b^11*c
^8*d^6*e^3*f^11*z - 66150*a^3*b^11*c^7*d^7*e^4*f^10*z - 66150*a^3*b^11*c^4*
d^10*e^7*f^7*z + 66150*a^3*b^11*c^3*d^11*e^8*f^6*z - 55566*a^5*b^9*c^5*d^9*
e^4*f^10*z - 55566*a^5*b^9*c^4*d^10*e^5*f^9*z - 55566*a^4*b^10*c^5*d^9*e^5*
f^9*z - 32130*a^9*b^5*c^3*d^11*e^2*f^12*z - 32130*a^9*b^5*c^2*d^12*e^3*f^11
*z - 32130*a^3*b^11*c^9*d^5*e^2*f^12*z - 32130*a^3*b^11*c^2*d^12*e^9*f^5*z
- 32130*a^2*b^12*c^9*d^5*e^3*f^11*z - 32130*a^2*b^12*c^3*d^11*e^9*f^5*z + 2
2680*a^8*b^6*c^4*d^10*e^2*f^12*z + 22680*a^8*b^6*c^2*d^12*e^4*f^10*z + 2268
0*a^4*b^10*c^8*d^6*e^2*f^12*z + 22680*a^4*b^10*c^2*d^12*e^8*f^6*z + 22680*a
^2*b^12*c^8*d^6*e^4*f^10*z + 22680*a^2*b^12*c^4*d^10*e^8*f^6*z + 19278*a^10
*b^4*c^2*d^12*e^2*f^12*z + 19278*a^2*b^12*c^10*d^4*e^2*f^12*z + 19278*a^2*b
^12*c^2*d^12*e^10*f^4*z + 18522*a^6*b^8*c^5*d^9*e^3*f^11*z + 18522*a^6*b^8*
c^3*d^11*e^5*f^9*z + 18522*a^5*b^9*c^6*d^8*e^3*f^11*z + 18522*a^5*b^9*c^3*d
^11*e^6*f^8*z + 18522*a^3*b^11*c^6*d^8*e^5*f^9*z + 18522*a^3*b^11*c^5*d^9*e
^6*f^8*z - 13230*a^6*b^8*c^6*d^8*e^2*f^12*z - 13230*a^6*b^8*c^2*d^12*e^6*f^
8*z - 13230*a^2*b^12*c^6*d^8*e^6*f^8*z + 3402*a^7*b^7*c^5*d^9*e^2*f^12*z +
3402*a^7*b^7*c^2*d^12*e^5*f^9*z + 3402*a^5*b^9*c^7*d^7*e^2*f^12*z + 3402*a^
5*b^9*c^2*d^12*e^7*f^7*z + 3402*a^2*b^12*c^7*d^7*e^5*f^9*z + 3402*a^2*b^12*
c^5*d^9*e^7*f^7*z + 7938*a^10*b^4*c^3*d^11*e*f^13*z + 7938*a^10*b^4*c*d^13*
e^3*f^11*z + 7938*a^3*b^11*c^10*d^4*e*f^13*z + 7938*a^3*b^11*c*d^13*e^10*f^
4*z + 7938*a*b^13*c^10*d^4*e^3*f^11*z + 7938*a*b^13*c^3*d^11*e^10*f^4*z - 5
670*a^11*b^3*c^2*d^12*e*f^13*z - 5670*a^11*b^3*c*d^13*e^2*f^12*z - 5670*a^2
*b^12*c^11*d^3*e*f^13*z - 5670*a^2*b^12*c*d^13*e^11*f^3*z - 5670*a*b^13*c^1
1*d^3*e^2*f^12*z - 5670*a*b^13*c^2*d^12*e^11*f^3*z - 3780*a^9*b^5*c^4*d^10*
e*f^13*z - 3780*a^9*b^5*c*d^13*e^4*f^10*z - 3780*a^4*b^10*c^9*d^5*e*f^13*z
- 3780*a^4*b^10*c*d^13*e^9*f^5*z - 3780*a*b^13*c^9*d^5*e^4*f^10*z - 3780*a*
b^13*c^4*d^10*e^9*f^5*z - 2268*a^8*b^6*c^5*d^9*e*f^13*z - 2268*a^8*b^6*c*d^
13*e^5*f^9*z - 2268*a^5*b^9*c^8*d^6*e*f^13*z - 2268*a^5*b^9*c*d^13*e^8*f^6*
z - 2268*a*b^13*c^8*d^6*e^5*f^9*z - 2268*a*b^13*c^5*d^9*e^8*f^6*z + 1890*a^
7*b^7*c^6*d^8*e*f^13*z + 1890*a^7*b^7*c*d^13*e^6*f^8*z + 1890*a^6*b^8*c^7*d
^7*e*f^13*z + 1890*a^6*b^8*c*d^13*e^7*f^7*z + 1890*a*b^13*c^7*d^7*e^6*f^8*z
 + 1890*a*b^13*c^6*d^8*e^7*f^7*z - 252*b^14*c^13*d*e*f^13*z - 252*b^14*c*d^
13*e^13*f*z - 252*a^13*b*d^14*e*f^13*z - 252*a*b^13*d^14*e^13*f*z - 252*a^1
3*b*c*d^13*f^14*z - 252*a*b^13*c^13*d*f^14*z - 918*b^14*c^7*d^7*e^7*f^7*z -
 882*b^14*c^11*d^3*e^3*f^11*z - 882*b^14*c^3*d^11*e^11*f^3*z + 693*b^14*c^1
2*d^2*e^2*f^12*z + 693*b^14*c^2*d^12*e^12*f^2*z + 567*b^14*c^8*d^6*e^6*f^8*
z + 567*b^14*c^6*d^8*e^8*f^6*z + 441*b^14*c^10*d^4*e^4*f^10*z + 441*b^14*c^
4*d^10*e^10*f^4*z - 126*b^14*c^9*d^5*e^5*f^9*z - 126*b^14*c^5*d^9*e^9*f^5*z
 - 918*a^7*b^7*d^14*e^7*f^7*z - 882*a^11*b^3*d^14*e^3*f^11*z - 882*a^3*b^11
*d^14*e^11*f^3*z + 693*a^12*b^2*d^14*e^2*f^12*z + 693*a^2*b^12*d^14*e^12*f^
2*z + 567*a^8*b^6*d^14*e^6*f^8*z + 567*a^6*b^8*d^14*e^8*f^6*z + 441*a^10*b^
4*d^14*e^4*f^10*z + 441*a^4*b^10*d^14*e^10*f^4*z - 126*a^9*b^5*d^14*e^5*f^9
*z - 126*a^5*b^9*d^14*e^9*f^5*z - 918*a^7*b^7*c^7*d^7*f^14*z - 882*a^11*b^3
*c^3*d^11*f^14*z - 882*a^3*b^11*c^11*d^3*f^14*z + 693*a^12*b^2*c^2*d^12*f^1
4*z + 693*a^2*b^12*c^12*d^2*f^14*z + 567*a^8*b^6*c^6*d^8*f^14*z + 567*a^6*b

^8*c^8*d^6*f^14*z + 441*a^10*b^4*c^4*d^10*f^14*z + 441*a^4*b^10*c^10*d^4*f^
14*z - 126*a^9*b^5*c^5*d^9*f^14*z - 126*a^5*b^9*c^9*d^5*f^14*z + 36*b^14*d^
14*e^14*z + 36*b^14*c^14*f^14*z + 36*a^14*d^14*f^14*z - 27054*a^2*b^9*c^2*d
^9*e^2*f^9 + 9018*a^3*b^8*c^2*d^9*e*f^10 + 9018*a^3*b^8*c*d^10*e^2*f^9 + 90
18*a^2*b^9*c^3*d^8*e*f^10 + 9018*a^2*b^9*c*d^10*e^3*f^8 + 9018*a*b^10*c^3*d
^8*e^2*f^9 + 9018*a*b^10*c^2*d^9*e^3*f^8 - 9018*a^4*b^7*c*d^10*e*f^10 - 901
8*a*b^10*c^4*d^7*e*f^10 - 9018*a*b^10*c*d^10*e^4*f^7 + 2268*b^11*c^5*d^6*e*
f^10 + 2268*b^11*c*d^10*e^5*f^6 + 2268*a^5*b^6*d^11*e*f^10 + 2268*a*b^10*d^
11*e^5*f^6 + 2268*a^5*b^6*c*d^10*f^11 + 2268*a*b^10*c^5*d^6*f^11 - 1458*b^1
1*c^3*d^8*e^3*f^8 - 1161*b^11*c^4*d^7*e^2*f^9 - 1161*b^11*c^2*d^9*e^4*f^7 -
 1458*a^3*b^8*d^11*e^3*f^8 - 1161*a^4*b^7*d^11*e^2*f^9 - 1161*a^2*b^9*d^11*
e^4*f^7 - 1458*a^3*b^8*c^3*d^8*f^11 - 1161*a^4*b^7*c^2*d^9*f^11 - 1161*a^2*
b^9*c^4*d^7*f^11 - 756*b^11*d^11*e^6*f^5 - 756*b^11*c^6*d^5*f^11 - 756*a^6*
b^5*d^11*f^11, z, k)*((20*a^11*b^8*c^16*d^3*f^19 - 7*a^10*b^9*c^17*d^2*f^19
 - 28*a^12*b^7*c^15*d^4*f^19 + 14*a^13*b^6*c^14*d^5*f^19 + 14*a^14*b^5*c^13
*d^6*f^19 - 28*a^15*b^4*c^12*d^7*f^19 + 20*a^16*b^3*c^11*d^8*f^19 - 7*a^17*
b^2*c^10*d^9*f^19 - 7*a^10*b^9*d^19*e^17*f^2 + 20*a^11*b^8*d^19*e^16*f^3 -
28*a^12*b^7*d^19*e^15*f^4 + 14*a^13*b^6*d^19*e^14*f^5 + 14*a^14*b^5*d^19*e^
13*f^6 - 28*a^15*b^4*d^19*e^12*f^7 + 20*a^16*b^3*d^19*e^11*f^8 - 7*a^17*b^2
*d^19*e^10*f^9 - 7*b^19*c^10*d^9*e^17*f^2 + 20*b^19*c^11*d^8*e^16*f^3 - 28*
b^19*c^12*d^7*e^15*f^4 + 14*b^19*c^13*d^6*e^14*f^5 + 14*b^19*c^14*d^5*e^13*
f^6 - 28*b^19*c^15*d^4*e^12*f^7 + 20*b^19*c^16*d^3*e^11*f^8 - 7*b^19*c^17*d
^2*e^10*f^9 + a^9*b^10*c^18*d*f^19 + a^18*b*c^9*d^10*f^19 + a^9*b^10*d^19*e
^18*f + a^18*b*d^19*e^9*f^10 + b^19*c^9*d^10*e^18*f + b^19*c^18*d*e^9*f^10
- 7*a*b^18*c^8*d^11*e^18*f - 7*a*b^18*c^18*d*e^8*f^11 - 7*a^8*b^11*c*d^18*e
^18*f - 7*a^8*b^11*c^18*d*e*f^18 - 7*a^18*b*c*d^18*e^8*f^11 - 7*a^18*b*c^8*
d^11*e*f^18 + 34*a*b^18*c^9*d^10*e^17*f^2 - 27*a*b^18*c^10*d^9*e^16*f^3 - 1
68*a*b^18*c^11*d^8*e^15*f^4 + 546*a*b^18*c^12*d^7*e^14*f^5 - 756*a*b^18*c^1
3*d^6*e^13*f^6 + 546*a*b^18*c^14*d^5*e^12*f^7 - 168*a*b^18*c^15*d^4*e^11*f^
8 - 27*a*b^18*c^16*d^3*e^10*f^9 + 34*a*b^18*c^17*d^2*e^9*f^10 + 20*a^2*b^17
*c^7*d^12*e^18*f + 20*a^2*b^17*c^18*d*e^7*f^12 - 28*a^3*b^16*c^6*d^13*e^18*
f - 28*a^3*b^16*c^18*d*e^6*f^13 + 14*a^4*b^15*c^5*d^14*e^18*f + 14*a^4*b^15
*c^18*d*e^5*f^14 + 14*a^5*b^14*c^4*d^15*e^18*f + 14*a^5*b^14*c^18*d*e^4*f^1
5 - 28*a^6*b^13*c^3*d^16*e^18*f - 28*a^6*b^13*c^18*d*e^3*f^16 + 20*a^7*b^12
*c^2*d^17*e^18*f + 20*a^7*b^12*c^18*d*e^2*f^17 + 34*a^9*b^10*c*d^18*e^17*f^
2 + 34*a^9*b^10*c^17*d^2*e*f^18 - 27*a^10*b^9*c*d^18*e^16*f^3 - 27*a^10*b^9
*c^16*d^3*e*f^18 - 168*a^11*b^8*c*d^18*e^15*f^4 - 168*a^11*b^8*c^15*d^4*e*f
^18 + 546*a^12*b^7*c*d^18*e^14*f^5 + 546*a^12*b^7*c^14*d^5*e*f^18 - 756*a^1
3*b^6*c*d^18*e^13*f^6 - 756*a^13*b^6*c^13*d^6*e*f^18 + 546*a^14*b^5*c*d^18*
e^12*f^7 + 546*a^14*b^5*c^12*d^7*e*f^18 - 168*a^15*b^4*c*d^18*e^11*f^8 - 16
8*a^15*b^4*c^11*d^8*e*f^18 - 27*a^16*b^3*c*d^18*e^10*f^9 - 27*a^16*b^3*c^10
*d^9*e*f^18 + 34*a^17*b^2*c*d^18*e^9*f^10 + 34*a^17*b^2*c^9*d^10*e*f^18 + 2
0*a^18*b*c^2*d^17*e^7*f^12 - 28*a^18*b*c^3*d^16*e^6*f^13 + 14*a^18*b*c^4*d^
15*e^5*f^14 + 14*a^18*b*c^5*d^14*e^4*f^15 - 28*a^18*b*c^6*d^13*e^3*f^16 + 2
0*a^18*b*c^7*d^12*e^2*f^17 - 27*a^2*b^17*c^8*d^11*e^17*f^2 - 371*a^2*b^17*c

^9*d^10*e^16*f^3 + 1560*a^2*b^17*c^10*d^9*e^15*f^4 - 2484*a^2*b^17*c^11*d^8
*e^14*f^5 + 1302*a^2*b^17*c^12*d^7*e^13*f^6 + 1302*a^2*b^17*c^13*d^6*e^12*f
^7 - 2484*a^2*b^17*c^14*d^5*e^11*f^8 + 1560*a^2*b^17*c^15*d^4*e^10*f^9 - 37
1*a^2*b^17*c^16*d^3*e^9*f^10 - 27*a^2*b^17*c^17*d^2*e^8*f^11 - 168*a^3*b^16
*c^7*d^12*e^17*f^2 + 1560*a^3*b^16*c^8*d^11*e^16*f^3 - 3464*a^3*b^16*c^9*d^
10*e^15*f^4 + 924*a^3*b^16*c^10*d^9*e^14*f^5 + 7728*a^3*b^16*c^11*d^8*e^13*
f^6 - 13104*a^3*b^16*c^12*d^7*e^12*f^7 + 7728*a^3*b^16*c^13*d^6*e^11*f^8 +
924*a^3*b^16*c^14*d^5*e^10*f^9 - 3464*a^3*b^16*c^15*d^4*e^9*f^10 + 1560*a^3
*b^16*c^16*d^3*e^8*f^11 - 168*a^3*b^16*c^17*d^2*e^7*f^12 + 546*a^4*b^15*c^6
*d^13*e^17*f^2 - 2484*a^4*b^15*c^7*d^12*e^16*f^3 + 924*a^4*b^15*c^8*d^11*e^
15*f^4 + 12550*a^4*b^15*c^9*d^10*e^14*f^5 - 26838*a^4*b^15*c^10*d^9*e^13*f^
6 + 15288*a^4*b^15*c^11*d^8*e^12*f^7 + 15288*a^4*b^15*c^12*d^7*e^11*f^8 - 2
6838*a^4*b^15*c^13*d^6*e^10*f^9 + 12550*a^4*b^15*c^14*d^5*e^9*f^10 + 924*a^
4*b^15*c^15*d^4*e^8*f^11 - 2484*a^4*b^15*c^16*d^3*e^7*f^12 + 546*a^4*b^15*c
^17*d^2*e^6*f^13 - 756*a^5*b^14*c^5*d^14*e^17*f^2 + 1302*a^5*b^14*c^6*d^13*
e^16*f^3 + 7728*a^5*b^14*c^7*d^12*e^15*f^4 - 26838*a^5*b^14*c^8*d^11*e^14*f
^5 + 18004*a^5*b^14*c^9*d^10*e^13*f^6 + 39858*a^5*b^14*c^10*d^9*e^12*f^7 -
78624*a^5*b^14*c^11*d^8*e^11*f^8 + 39858*a^5*b^14*c^12*d^7*e^10*f^9 + 18004
*a^5*b^14*c^13*d^6*e^9*f^10 - 26838*a^5*b^14*c^14*d^5*e^8*f^11 + 7728*a^5*b
^14*c^15*d^4*e^7*f^12 + 1302*a^5*b^14*c^16*d^3*e^6*f^13 - 756*a^5*b^14*c^17
*d^2*e^5*f^14 + 546*a^6*b^13*c^4*d^15*e^17*f^2 + 1302*a^6*b^13*c^5*d^14*e^1
6*f^3 - 13104*a^6*b^13*c^6*d^13*e^15*f^4 + 15288*a^6*b^13*c^7*d^12*e^14*f^5
 + 39858*a^6*b^13*c^8*d^11*e^13*f^6 - 110474*a^6*b^13*c^9*d^10*e^12*f^7 + 6
6612*a^6*b^13*c^10*d^9*e^11*f^8 + 66612*a^6*b^13*c^11*d^8*e^10*f^9 - 110474
*a^6*b^13*c^12*d^7*e^9*f^10 + 39858*a^6*b^13*c^13*d^6*e^8*f^11 + 15288*a^6*
b^13*c^14*d^5*e^7*f^12 - 13104*a^6*b^13*c^15*d^4*e^6*f^13 + 1302*a^6*b^13*c
^16*d^3*e^5*f^14 + 546*a^6*b^13*c^17*d^2*e^4*f^15 - 168*a^7*b^12*c^3*d^16*e
^17*f^2 - 2484*a^7*b^12*c^4*d^15*e^16*f^3 + 7728*a^7*b^12*c^5*d^14*e^15*f^4
 + 15288*a^7*b^12*c^6*d^13*e^14*f^5 - 78624*a^7*b^12*c^7*d^12*e^13*f^6 + 66
612*a^7*b^12*c^8*d^11*e^12*f^7 + 99736*a^7*b^12*c^9*d^10*e^11*f^8 - 216216*
a^7*b^12*c^10*d^9*e^10*f^9 + 99736*a^7*b^12*c^11*d^8*e^9*f^10 + 66612*a^7*b
^12*c^12*d^7*e^8*f^11 - 78624*a^7*b^12*c^13*d^6*e^7*f^12 + 15288*a^7*b^12*c
^14*d^5*e^6*f^13 + 7728*a^7*b^12*c^15*d^4*e^5*f^14 - 2484*a^7*b^12*c^16*d^3
*e^4*f^15 - 168*a^7*b^12*c^17*d^2*e^3*f^16 - 27*a^8*b^11*c^2*d^17*e^17*f^2
+ 1560*a^8*b^11*c^3*d^16*e^16*f^3 + 924*a^8*b^11*c^4*d^15*e^15*f^4 - 26838*
a^8*b^11*c^5*d^14*e^14*f^5 + 39858*a^8*b^11*c^6*d^13*e^13*f^6 + 66612*a^8*b
^11*c^7*d^12*e^12*f^7 - 216216*a^8*b^11*c^8*d^11*e^11*f^8 + 134134*a^8*b^11
*c^9*d^10*e^10*f^9 + 134134*a^8*b^11*c^10*d^9*e^9*f^10 - 216216*a^8*b^11*c^
11*d^8*e^8*f^11 + 66612*a^8*b^11*c^12*d^7*e^7*f^12 + 39858*a^8*b^11*c^13*d^
6*e^6*f^13 - 26838*a^8*b^11*c^14*d^5*e^5*f^14 + 924*a^8*b^11*c^15*d^4*e^4*f
^15 + 1560*a^8*b^11*c^16*d^3*e^3*f^16 - 27*a^8*b^11*c^17*d^2*e^2*f^17 - 371
*a^9*b^10*c^2*d^17*e^16*f^3 - 3464*a^9*b^10*c^3*d^16*e^15*f^4 + 12550*a^9*b
^10*c^4*d^15*e^14*f^5 + 18004*a^9*b^10*c^5*d^14*e^13*f^6 - 110474*a^9*b^10*
c^6*d^13*e^12*f^7 + 99736*a^9*b^10*c^7*d^12*e^11*f^8 + 134134*a^9*b^10*c^8*
d^11*e^10*f^9 - 300300*a^9*b^10*c^9*d^10*e^9*f^10 + 134134*a^9*b^10*c^10*d^

9*e^8*f^11 + 99736*a^9*b^10*c^11*d^8*e^7*f^12 - 110474*a^9*b^10*c^12*d^7*e^
6*f^13 + 18004*a^9*b^10*c^13*d^6*e^5*f^14 + 12550*a^9*b^10*c^14*d^5*e^4*f^1
5 - 3464*a^9*b^10*c^15*d^4*e^3*f^16 - 371*a^9*b^10*c^16*d^3*e^2*f^17 + 1560
*a^10*b^9*c^2*d^17*e^15*f^4 + 924*a^10*b^9*c^3*d^16*e^14*f^5 - 26838*a^10*b
^9*c^4*d^15*e^13*f^6 + 39858*a^10*b^9*c^5*d^14*e^12*f^7 + 66612*a^10*b^9*c^
6*d^13*e^11*f^8 - 216216*a^10*b^9*c^7*d^12*e^10*f^9 + 134134*a^10*b^9*c^8*d
^11*e^9*f^10 + 134134*a^10*b^9*c^9*d^10*e^8*f^11 - 216216*a^10*b^9*c^10*d^9
*e^7*f^12 + 66612*a^10*b^9*c^11*d^8*e^6*f^13 + 39858*a^10*b^9*c^12*d^7*e^5*
f^14 - 26838*a^10*b^9*c^13*d^6*e^4*f^15 + 924*a^10*b^9*c^14*d^5*e^3*f^16 +
1560*a^10*b^9*c^15*d^4*e^2*f^17 - 2484*a^11*b^8*c^2*d^17*e^14*f^5 + 7728*a^
11*b^8*c^3*d^16*e^13*f^6 + 15288*a^11*b^8*c^4*d^15*e^12*f^7 - 78624*a^11*b^
8*c^5*d^14*e^11*f^8 + 66612*a^11*b^8*c^6*d^13*e^10*f^9 + 99736*a^11*b^8*c^7
*d^12*e^9*f^10 - 216216*a^11*b^8*c^8*d^11*e^8*f^11 + 99736*a^11*b^8*c^9*d^1
0*e^7*f^12 + 66612*a^11*b^8*c^10*d^9*e^6*f^13 - 78624*a^11*b^8*c^11*d^8*e^5
*f^14 + 15288*a^11*b^8*c^12*d^7*e^4*f^15 + 7728*a^11*b^8*c^13*d^6*e^3*f^16
- 2484*a^11*b^8*c^14*d^5*e^2*f^17 + 1302*a^12*b^7*c^2*d^17*e^13*f^6 - 13104
*a^12*b^7*c^3*d^16*e^12*f^7 + 15288*a^12*b^7*c^4*d^15*e^11*f^8 + 39858*a^12
*b^7*c^5*d^14*e^10*f^9 - 110474*a^12*b^7*c^6*d^13*e^9*f^10 + 66612*a^12*b^7
*c^7*d^12*e^8*f^11 + 66612*a^12*b^7*c^8*d^11*e^7*f^12 - 110474*a^12*b^7*c^9
*d^10*e^6*f^13 + 39858*a^12*b^7*c^10*d^9*e^5*f^14 + 15288*a^12*b^7*c^11*d^8
*e^4*f^15 - 13104*a^12*b^7*c^12*d^7*e^3*f^16 + 1302*a^12*b^7*c^13*d^6*e^2*f
^17 + 1302*a^13*b^6*c^2*d^17*e^12*f^7 + 7728*a^13*b^6*c^3*d^16*e^11*f^8 - 2
6838*a^13*b^6*c^4*d^15*e^10*f^9 + 18004*a^13*b^6*c^5*d^14*e^9*f^10 + 39858*
a^13*b^6*c^6*d^13*e^8*f^11 - 78624*a^13*b^6*c^7*d^12*e^7*f^12 + 39858*a^13*
b^6*c^8*d^11*e^6*f^13 + 18004*a^13*b^6*c^9*d^10*e^5*f^14 - 26838*a^13*b^6*c
^10*d^9*e^4*f^15 + 7728*a^13*b^6*c^11*d^8*e^3*f^16 + 1302*a^13*b^6*c^12*d^7
*e^2*f^17 - 2484*a^14*b^5*c^2*d^17*e^11*f^8 + 924*a^14*b^5*c^3*d^16*e^10*f^
9 + 12550*a^14*b^5*c^4*d^15*e^9*f^10 - 26838*a^14*b^5*c^5*d^14*e^8*f^11 + 1
5288*a^14*b^5*c^6*d^13*e^7*f^12 + 15288*a^14*b^5*c^7*d^12*e^6*f^13 - 26838*
a^14*b^5*c^8*d^11*e^5*f^14 + 12550*a^14*b^5*c^9*d^10*e^4*f^15 + 924*a^14*b^
5*c^10*d^9*e^3*f^16 - 2484*a^14*b^5*c^11*d^8*e^2*f^17 + 1560*a^15*b^4*c^2*d
^17*e^10*f^9 - 3464*a^15*b^4*c^3*d^16*e^9*f^10 + 924*a^15*b^4*c^4*d^15*e^8*
f^11 + 7728*a^15*b^4*c^5*d^14*e^7*f^12 - 13104*a^15*b^4*c^6*d^13*e^6*f^13 +
 7728*a^15*b^4*c^7*d^12*e^5*f^14 + 924*a^15*b^4*c^8*d^11*e^4*f^15 - 3464*a^
15*b^4*c^9*d^10*e^3*f^16 + 1560*a^15*b^4*c^10*d^9*e^2*f^17 - 371*a^16*b^3*c
^2*d^17*e^9*f^10 + 1560*a^16*b^3*c^3*d^16*e^8*f^11 - 2484*a^16*b^3*c^4*d^15
*e^7*f^12 + 1302*a^16*b^3*c^5*d^14*e^6*f^13 + 1302*a^16*b^3*c^6*d^13*e^5*f^
14 - 2484*a^16*b^3*c^7*d^12*e^4*f^15 + 1560*a^16*b^3*c^8*d^11*e^3*f^16 - 37
1*a^16*b^3*c^9*d^10*e^2*f^17 - 27*a^17*b^2*c^2*d^17*e^8*f^11 - 168*a^17*b^2
*c^3*d^16*e^7*f^12 + 546*a^17*b^2*c^4*d^15*e^6*f^13 - 756*a^17*b^2*c^5*d^14
*e^5*f^14 + 546*a^17*b^2*c^6*d^13*e^4*f^15 - 168*a^17*b^2*c^7*d^12*e^3*f^16
 - 27*a^17*b^2*c^8*d^11*e^2*f^17)/(56*a^3*b^13*c^5*d^11*e^16 - a^8*b^8*d^16
*e^16 - a^16*c^8*d^8*f^16 - b^16*c^8*d^8*e^16 - a^16*d^16*e^8*f^8 - b^16*c^
16*e^8*f^8 - 28*a^2*b^14*c^6*d^10*e^16 - a^8*b^8*c^16*f^16 - 70*a^4*b^12*c^
4*d^12*e^16 + 56*a^5*b^11*c^3*d^13*e^16 - 28*a^6*b^10*c^2*d^14*e^16 - 28*a^

10*b^6*c^14*d^2*f^16 + 56*a^11*b^5*c^13*d^3*f^16 - 70*a^12*b^4*c^12*d^4*f^1
6 + 56*a^13*b^3*c^11*d^5*f^16 - 28*a^14*b^2*c^10*d^6*f^16 - 28*a^2*b^14*c^1
6*e^6*f^10 + 56*a^3*b^13*c^16*e^5*f^11 - 70*a^4*b^12*c^16*e^4*f^12 + 56*a^5
*b^11*c^16*e^3*f^13 - 28*a^6*b^10*c^16*e^2*f^14 - 28*a^10*b^6*d^16*e^14*f^2
 + 56*a^11*b^5*d^16*e^13*f^3 - 70*a^12*b^4*d^16*e^12*f^4 + 56*a^13*b^3*d^16
*e^11*f^5 - 28*a^14*b^2*d^16*e^10*f^6 - 28*a^16*c^2*d^14*e^6*f^10 + 56*a^16
*c^3*d^13*e^5*f^11 - 70*a^16*c^4*d^12*e^4*f^12 + 56*a^16*c^5*d^11*e^3*f^13
- 28*a^16*c^6*d^10*e^2*f^14 - 28*b^16*c^10*d^6*e^14*f^2 + 56*b^16*c^11*d^5*
e^13*f^3 - 70*b^16*c^12*d^4*e^12*f^4 + 56*b^16*c^13*d^3*e^11*f^5 - 28*b^16*
c^14*d^2*e^10*f^6 + 8*a*b^15*c^7*d^9*e^16 + 8*a^7*b^9*c*d^15*e^16 + 8*a^9*b
^7*c^15*d*f^16 + 8*a^15*b*c^9*d^7*f^16 + 8*a*b^15*c^16*e^7*f^9 + 8*a^7*b^9*
c^16*e*f^15 + 8*a^9*b^7*d^16*e^15*f + 8*a^15*b*d^16*e^9*f^7 + 8*a^16*c*d^15
*e^7*f^9 + 8*a^16*c^7*d^9*e*f^15 + 8*b^16*c^9*d^7*e^15*f + 8*b^16*c^15*d*e^
9*f^7 - 56*a*b^15*c^8*d^8*e^15*f - 56*a*b^15*c^15*d*e^8*f^8 - 56*a^8*b^8*c*
d^15*e^15*f - 56*a^8*b^8*c^15*d*e*f^15 - 56*a^15*b*c*d^15*e^8*f^8 - 56*a^15
*b*c^8*d^8*e*f^15 + 160*a*b^15*c^9*d^7*e^14*f^2 - 224*a*b^15*c^10*d^6*e^13*
f^3 + 112*a*b^15*c^11*d^5*e^12*f^4 + 112*a*b^15*c^12*d^4*e^11*f^5 - 224*a*b
^15*c^13*d^3*e^10*f^6 + 160*a*b^15*c^14*d^2*e^9*f^7 + 160*a^2*b^14*c^7*d^9*
e^15*f + 160*a^2*b^14*c^15*d*e^7*f^9 - 224*a^3*b^13*c^6*d^10*e^15*f - 224*a
^3*b^13*c^15*d*e^6*f^10 + 112*a^4*b^12*c^5*d^11*e^15*f + 112*a^4*b^12*c^15*
d*e^5*f^11 + 112*a^5*b^11*c^4*d^12*e^15*f + 112*a^5*b^11*c^15*d*e^4*f^12 -
224*a^6*b^10*c^3*d^13*e^15*f - 224*a^6*b^10*c^15*d*e^3*f^13 + 160*a^7*b^9*c
^2*d^14*e^15*f + 160*a^7*b^9*c^15*d*e^2*f^14 + 160*a^9*b^7*c*d^15*e^14*f^2
+ 160*a^9*b^7*c^14*d^2*e*f^15 - 224*a^10*b^6*c*d^15*e^13*f^3 - 224*a^10*b^6
*c^13*d^3*e*f^15 + 112*a^11*b^5*c*d^15*e^12*f^4 + 112*a^11*b^5*c^12*d^4*e*f
^15 + 112*a^12*b^4*c*d^15*e^11*f^5 + 112*a^12*b^4*c^11*d^5*e*f^15 - 224*a^1
3*b^3*c*d^15*e^10*f^6 - 224*a^13*b^3*c^10*d^6*e*f^15 + 160*a^14*b^2*c*d^15*
e^9*f^7 + 160*a^14*b^2*c^9*d^7*e*f^15 + 160*a^15*b*c^2*d^14*e^7*f^9 - 224*a
^15*b*c^3*d^13*e^6*f^10 + 112*a^15*b*c^4*d^12*e^5*f^11 + 112*a^15*b*c^5*d^1
1*e^4*f^12 - 224*a^15*b*c^6*d^10*e^3*f^13 + 160*a^15*b*c^7*d^9*e^2*f^14 - 3
00*a^2*b^14*c^8*d^8*e^14*f^2 + 840*a^2*b^14*c^10*d^6*e^12*f^4 - 1344*a^2*b^
14*c^11*d^5*e^11*f^5 + 840*a^2*b^14*c^12*d^4*e^10*f^6 - 300*a^2*b^14*c^14*d
^2*e^8*f^8 + 1400*a^3*b^13*c^8*d^8*e^13*f^3 - 2800*a^3*b^13*c^9*d^7*e^12*f^
4 + 1568*a^3*b^13*c^10*d^6*e^11*f^5 + 1568*a^3*b^13*c^11*d^5*e^10*f^6 - 280
0*a^3*b^13*c^12*d^4*e^9*f^7 + 1400*a^3*b^13*c^13*d^3*e^8*f^8 + 840*a^4*b^12
*c^6*d^10*e^14*f^2 - 2800*a^4*b^12*c^7*d^9*e^13*f^3 + 1750*a^4*b^12*c^8*d^8
*e^12*f^4 + 4480*a^4*b^12*c^9*d^7*e^11*f^5 - 8624*a^4*b^12*c^10*d^6*e^10*f^
6 + 4480*a^4*b^12*c^11*d^5*e^9*f^7 + 1750*a^4*b^12*c^12*d^4*e^8*f^8 - 2800*
a^4*b^12*c^13*d^3*e^7*f^9 + 840*a^4*b^12*c^14*d^2*e^6*f^10 - 1344*a^5*b^11*
c^5*d^11*e^14*f^2 + 1568*a^5*b^11*c^6*d^10*e^13*f^3 + 4480*a^5*b^11*c^7*d^9
*e^12*f^4 - 12264*a^5*b^11*c^8*d^8*e^11*f^5 + 7392*a^5*b^11*c^9*d^7*e^10*f^
6 + 7392*a^5*b^11*c^10*d^6*e^9*f^7 - 12264*a^5*b^11*c^11*d^5*e^8*f^8 + 4480
*a^5*b^11*c^12*d^4*e^7*f^9 + 1568*a^5*b^11*c^13*d^3*e^6*f^10 - 1344*a^5*b^1
1*c^14*d^2*e^5*f^11 + 840*a^6*b^10*c^4*d^12*e^14*f^2 + 1568*a^6*b^10*c^5*d^
11*e^13*f^3 - 8624*a^6*b^10*c^6*d^10*e^12*f^4 + 7392*a^6*b^10*c^7*d^9*e^11*

f^5 + 11396*a^6*b^10*c^8*d^8*e^10*f^6 - 24640*a^6*b^10*c^9*d^7*e^9*f^7 + 11
396*a^6*b^10*c^10*d^6*e^8*f^8 + 7392*a^6*b^10*c^11*d^5*e^7*f^9 - 8624*a^6*b
^10*c^12*d^4*e^6*f^10 + 1568*a^6*b^10*c^13*d^3*e^5*f^11 + 840*a^6*b^10*c^14
*d^2*e^4*f^12 - 2800*a^7*b^9*c^4*d^12*e^13*f^3 + 4480*a^7*b^9*c^5*d^11*e^12
*f^4 + 7392*a^7*b^9*c^6*d^10*e^11*f^5 - 24640*a^7*b^9*c^7*d^9*e^10*f^6 + 15
400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9*c^9*d^7*e^8*f^8 - 24640*a^7*b^9
*c^10*d^6*e^7*f^9 + 7392*a^7*b^9*c^11*d^5*e^6*f^10 + 4480*a^7*b^9*c^12*d^4*
e^5*f^11 - 2800*a^7*b^9*c^13*d^3*e^4*f^12 - 300*a^8*b^8*c^2*d^14*e^14*f^2 +
1400*a^8*b^8*c^3*d^13*e^13*f^3 + 1750*a^8*b^8*c^4*d^12*e^12*f^4 - 12264*a^
8*b^8*c^5*d^11*e^11*f^5 + 11396*a^8*b^8*c^6*d^10*e^10*f^6 + 15400*a^8*b^8*c
^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^8*d^8*e^8*f^8 + 15400*a^8*b^8*c^9*d^7*e^7*
f^9 + 11396*a^8*b^8*c^10*d^6*e^6*f^10 - 12264*a^8*b^8*c^11*d^5*e^5*f^11 + 1
750*a^8*b^8*c^12*d^4*e^4*f^12 + 1400*a^8*b^8*c^13*d^3*e^3*f^13 - 300*a^8*b^
8*c^14*d^2*e^2*f^14 - 2800*a^9*b^7*c^3*d^13*e^12*f^4 + 4480*a^9*b^7*c^4*d^1
2*e^11*f^5 + 7392*a^9*b^7*c^5*d^11*e^10*f^6 - 24640*a^9*b^7*c^6*d^10*e^9*f^
7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^9*b^7*c^8*d^8*e^7*f^9 - 24640*a
^9*b^7*c^9*d^7*e^6*f^10 + 7392*a^9*b^7*c^10*d^6*e^5*f^11 + 4480*a^9*b^7*c^1
1*d^5*e^4*f^12 - 2800*a^9*b^7*c^12*d^4*e^3*f^13 + 840*a^10*b^6*c^2*d^14*e^1
2*f^4 + 1568*a^10*b^6*c^3*d^13*e^11*f^5 - 8624*a^10*b^6*c^4*d^12*e^10*f^6 +
7392*a^10*b^6*c^5*d^11*e^9*f^7 + 11396*a^10*b^6*c^6*d^10*e^8*f^8 - 24640*a
^10*b^6*c^7*d^9*e^7*f^9 + 11396*a^10*b^6*c^8*d^8*e^6*f^10 + 7392*a^10*b^6*c
^9*d^7*e^5*f^11 - 8624*a^10*b^6*c^10*d^6*e^4*f^12 + 1568*a^10*b^6*c^11*d^5*
e^3*f^13 + 840*a^10*b^6*c^12*d^4*e^2*f^14 - 1344*a^11*b^5*c^2*d^14*e^11*f^5
+ 1568*a^11*b^5*c^3*d^13*e^10*f^6 + 4480*a^11*b^5*c^4*d^12*e^9*f^7 - 12264
*a^11*b^5*c^5*d^11*e^8*f^8 + 7392*a^11*b^5*c^6*d^10*e^7*f^9 + 7392*a^11*b^5
*c^7*d^9*e^6*f^10 - 12264*a^11*b^5*c^8*d^8*e^5*f^11 + 4480*a^11*b^5*c^9*d^7
*e^4*f^12 + 1568*a^11*b^5*c^10*d^6*e^3*f^13 - 1344*a^11*b^5*c^11*d^5*e^2*f^
14 + 840*a^12*b^4*c^2*d^14*e^10*f^6 - 2800*a^12*b^4*c^3*d^13*e^9*f^7 + 1750
*a^12*b^4*c^4*d^12*e^8*f^8 + 4480*a^12*b^4*c^5*d^11*e^7*f^9 - 8624*a^12*b^4
*c^6*d^10*e^6*f^10 + 4480*a^12*b^4*c^7*d^9*e^5*f^11 + 1750*a^12*b^4*c^8*d^8
*e^4*f^12 - 2800*a^12*b^4*c^9*d^7*e^3*f^13 + 840*a^12*b^4*c^10*d^6*e^2*f^14
+ 1400*a^13*b^3*c^3*d^13*e^8*f^8 - 2800*a^13*b^3*c^4*d^12*e^7*f^9 + 1568*a
^13*b^3*c^5*d^11*e^6*f^10 + 1568*a^13*b^3*c^6*d^10*e^5*f^11 - 2800*a^13*b^3
*c^7*d^9*e^4*f^12 + 1400*a^13*b^3*c^8*d^8*e^3*f^13 - 300*a^14*b^2*c^2*d^14*
e^8*f^8 + 840*a^14*b^2*c^4*d^12*e^6*f^10 - 1344*a^14*b^2*c^5*d^11*e^5*f^11
+ 840*a^14*b^2*c^6*d^10*e^4*f^12 - 300*a^14*b^2*c^8*d^8*e^2*f^14) - (x*(18*
a^9*b^10*c^17*d^2*f^19 - 74*a^10*b^9*c^16*d^3*f^19 + 184*a^11*b^8*c^15*d^4*
f^19 - 308*a^12*b^7*c^14*d^5*f^19 + 364*a^13*b^6*c^13*d^6*f^19 - 308*a^14*b
^5*c^12*d^7*f^19 + 184*a^15*b^4*c^11*d^8*f^19 - 74*a^16*b^3*c^10*d^9*f^19 +
18*a^17*b^2*c^9*d^10*f^19 + 18*a^9*b^10*d^19*e^17*f^2 - 74*a^10*b^9*d^19*e
^16*f^3 + 184*a^11*b^8*d^19*e^15*f^4 - 308*a^12*b^7*d^19*e^14*f^5 + 364*a^1
3*b^6*d^19*e^13*f^6 - 308*a^14*b^5*d^19*e^12*f^7 + 184*a^15*b^4*d^19*e^11*f
^8 - 74*a^16*b^3*d^19*e^10*f^9 + 18*a^17*b^2*d^19*e^9*f^10 + 18*b^19*c^9*d^
10*e^17*f^2 - 74*b^19*c^10*d^9*e^16*f^3 + 184*b^19*c^11*d^8*e^15*f^4 - 308*
b^19*c^12*d^7*e^14*f^5 + 364*b^19*c^13*d^6*e^13*f^6 - 308*b^19*c^14*d^5*e^1

2*f^7 + 184*b^19*c^15*d^4*e^11*f^8 - 74*b^19*c^16*d^3*e^10*f^9 + 18*b^19*c^
17*d^2*e^9*f^10 - 2*a^8*b^11*c^18*d*f^19 - 2*a^18*b*c^8*d^11*f^19 - 2*a^8*b
^11*d^19*e^18*f - 2*a^18*b*d^19*e^8*f^11 - 2*b^19*c^8*d^11*e^18*f - 2*b^19*
c^18*d*e^8*f^11 + 16*a*b^18*c^7*d^12*e^18*f + 16*a*b^18*c^18*d*e^7*f^12 + 1
6*a^7*b^12*c*d^18*e^18*f + 16*a^7*b^12*c^18*d*e*f^18 + 16*a^18*b*c*d^18*e^7
*f^12 + 16*a^18*b*c^7*d^12*e*f^18 - 126*a*b^18*c^8*d^11*e^17*f^2 + 434*a*b^
18*c^9*d^10*e^16*f^3 - 840*a*b^18*c^10*d^9*e^15*f^4 + 936*a*b^18*c^11*d^8*e
^14*f^5 - 420*a*b^18*c^12*d^7*e^13*f^6 - 420*a*b^18*c^13*d^6*e^12*f^7 + 936
*a*b^18*c^14*d^5*e^11*f^8 - 840*a*b^18*c^15*d^4*e^10*f^9 + 434*a*b^18*c^16*
d^3*e^9*f^10 - 126*a*b^18*c^17*d^2*e^8*f^11 - 56*a^2*b^17*c^6*d^13*e^18*f -
 56*a^2*b^17*c^18*d*e^6*f^13 + 112*a^3*b^16*c^5*d^14*e^18*f + 112*a^3*b^16*
c^18*d*e^5*f^14 - 140*a^4*b^15*c^4*d^15*e^18*f - 140*a^4*b^15*c^18*d*e^4*f^
15 + 112*a^5*b^14*c^3*d^16*e^18*f + 112*a^5*b^14*c^18*d*e^3*f^16 - 56*a^6*b
^13*c^2*d^17*e^18*f - 56*a^6*b^13*c^18*d*e^2*f^17 - 126*a^8*b^11*c*d^18*e^1
7*f^2 - 126*a^8*b^11*c^17*d^2*e*f^18 + 434*a^9*b^10*c*d^18*e^16*f^3 + 434*a
^9*b^10*c^16*d^3*e*f^18 - 840*a^10*b^9*c*d^18*e^15*f^4 - 840*a^10*b^9*c^15*
d^4*e*f^18 + 936*a^11*b^8*c*d^18*e^14*f^5 + 936*a^11*b^8*c^14*d^5*e*f^18 -
420*a^12*b^7*c*d^18*e^13*f^6 - 420*a^12*b^7*c^13*d^6*e*f^18 - 420*a^13*b^6*
c*d^18*e^12*f^7 - 420*a^13*b^6*c^12*d^7*e*f^18 + 936*a^14*b^5*c*d^18*e^11*f
^8 + 936*a^14*b^5*c^11*d^8*e*f^18 - 840*a^15*b^4*c*d^18*e^10*f^9 - 840*a^15
*b^4*c^10*d^9*e*f^18 + 434*a^16*b^3*c*d^18*e^9*f^10 + 434*a^16*b^3*c^9*d^10
*e*f^18 - 126*a^17*b^2*c*d^18*e^8*f^11 - 126*a^17*b^2*c^8*d^11*e*f^18 - 56*
a^18*b*c^2*d^17*e^6*f^13 + 112*a^18*b*c^3*d^16*e^5*f^14 - 140*a^18*b*c^4*d^
15*e^4*f^15 + 112*a^18*b*c^5*d^14*e^3*f^16 - 56*a^18*b*c^6*d^13*e^2*f^17 +
360*a^2*b^17*c^7*d^12*e^17*f^2 - 882*a^2*b^17*c^8*d^11*e^16*f^3 + 728*a^2*b
^17*c^9*d^10*e^15*f^4 + 1152*a^2*b^17*c^10*d^9*e^14*f^5 - 4032*a^2*b^17*c^1
1*d^8*e^13*f^6 + 5460*a^2*b^17*c^12*d^7*e^12*f^7 - 4032*a^2*b^17*c^13*d^6*e
^11*f^8 + 1152*a^2*b^17*c^14*d^5*e^10*f^9 + 728*a^2*b^17*c^15*d^4*e^9*f^10
- 882*a^2*b^17*c^16*d^3*e^8*f^11 + 360*a^2*b^17*c^17*d^2*e^7*f^12 - 504*a^3
*b^16*c^6*d^13*e^17*f^2 + 312*a^3*b^16*c^7*d^12*e^16*f^3 + 2520*a^3*b^16*c^
8*d^11*e^15*f^4 - 7480*a^3*b^16*c^9*d^10*e^14*f^5 + 9408*a^3*b^16*c^10*d^9*
e^13*f^6 - 4368*a^3*b^16*c^11*d^8*e^12*f^7 - 4368*a^3*b^16*c^12*d^7*e^11*f^
8 + 9408*a^3*b^16*c^13*d^6*e^10*f^9 - 7480*a^3*b^16*c^14*d^5*e^9*f^10 + 252
0*a^3*b^16*c^15*d^4*e^8*f^11 + 312*a^3*b^16*c^16*d^3*e^7*f^12 - 504*a^3*b^1
6*c^17*d^2*e^6*f^13 + 252*a^4*b^15*c^5*d^14*e^17*f^2 + 1596*a^4*b^15*c^6*d^
13*e^16*f^3 - 6288*a^4*b^15*c^7*d^12*e^15*f^4 + 7380*a^4*b^15*c^8*d^11*e^14
*f^5 + 2660*a^4*b^15*c^9*d^10*e^13*f^6 - 18564*a^4*b^15*c^10*d^9*e^12*f^7 +
 26208*a^4*b^15*c^11*d^8*e^11*f^8 - 18564*a^4*b^15*c^12*d^7*e^10*f^9 + 2660
*a^4*b^15*c^13*d^6*e^9*f^10 + 7380*a^4*b^15*c^14*d^5*e^8*f^11 - 6288*a^4*b^
15*c^15*d^4*e^7*f^12 + 1596*a^4*b^15*c^16*d^3*e^6*f^13 + 252*a^4*b^15*c^17*
d^2*e^5*f^14 + 252*a^5*b^14*c^4*d^15*e^17*f^2 - 2772*a^5*b^14*c^5*d^14*e^16
*f^3 + 3696*a^5*b^14*c^6*d^13*e^15*f^4 + 7056*a^5*b^14*c^7*d^12*e^14*f^5 -
25452*a^5*b^14*c^8*d^11*e^13*f^6 + 30212*a^5*b^14*c^9*d^10*e^12*f^7 - 13104
*a^5*b^14*c^10*d^9*e^11*f^8 - 13104*a^5*b^14*c^11*d^8*e^10*f^9 + 30212*a^5*
b^14*c^12*d^7*e^9*f^10 - 25452*a^5*b^14*c^13*d^6*e^8*f^11 + 7056*a^5*b^14*c

^14*d^5*e^7*f^12 + 3696*a^5*b^14*c^15*d^4*e^6*f^13 - 2772*a^5*b^14*c^16*d^3
*e^5*f^14 + 252*a^5*b^14*c^17*d^2*e^4*f^15 - 504*a^6*b^13*c^3*d^16*e^17*f^2
 + 1596*a^6*b^13*c^4*d^15*e^16*f^3 + 3696*a^6*b^13*c^5*d^14*e^15*f^4 - 1747
2*a^6*b^13*c^6*d^13*e^14*f^5 + 17472*a^6*b^13*c^7*d^12*e^13*f^6 + 9828*a^6*
b^13*c^8*d^11*e^12*f^7 - 38584*a^6*b^13*c^9*d^10*e^11*f^8 + 48048*a^6*b^13*
c^10*d^9*e^10*f^9 - 38584*a^6*b^13*c^11*d^8*e^9*f^10 + 9828*a^6*b^13*c^12*d
^7*e^8*f^11 + 17472*a^6*b^13*c^13*d^6*e^7*f^12 - 17472*a^6*b^13*c^14*d^5*e^
6*f^13 + 3696*a^6*b^13*c^15*d^4*e^5*f^14 + 1596*a^6*b^13*c^16*d^3*e^4*f^15
 - 504*a^6*b^13*c^17*d^2*e^3*f^16 + 360*a^7*b^12*c^2*d^17*e^17*f^2 + 312*a^7
*b^12*c^3*d^16*e^16*f^3 - 6288*a^7*b^12*c^4*d^15*e^15*f^4 + 7056*a^7*b^12*c
^5*d^14*e^14*f^5 + 17472*a^7*b^12*c^6*d^13*e^13*f^6 - 43680*a^7*b^12*c^7*d^
12*e^12*f^7 + 32760*a^7*b^12*c^8*d^11*e^11*f^8 - 8008*a^7*b^12*c^9*d^10*e^1
0*f^9 - 8008*a^7*b^12*c^10*d^9*e^9*f^10 + 32760*a^7*b^12*c^11*d^8*e^8*f^11
 - 43680*a^7*b^12*c^12*d^7*e^7*f^12 + 17472*a^7*b^12*c^13*d^6*e^6*f^13 + 705
6*a^7*b^12*c^14*d^5*e^5*f^14 - 6288*a^7*b^12*c^15*d^4*e^4*f^15 + 312*a^7*b^
12*c^16*d^3*e^3*f^16 + 360*a^7*b^12*c^17*d^2*e^2*f^17 - 882*a^8*b^11*c^2*d^
17*e^16*f^3 + 2520*a^8*b^11*c^3*d^16*e^15*f^4 + 7380*a^8*b^11*c^4*d^15*e^14
*f^5 - 25452*a^8*b^11*c^5*d^14*e^13*f^6 + 9828*a^8*b^11*c^6*d^13*e^12*f^7 +
 32760*a^8*b^11*c^7*d^12*e^11*f^8 - 36036*a^8*b^11*c^8*d^11*e^10*f^9 + 2002
0*a^8*b^11*c^9*d^10*e^9*f^10 - 36036*a^8*b^11*c^10*d^9*e^8*f^11 + 32760*a^8
*b^11*c^11*d^8*e^7*f^12 + 9828*a^8*b^11*c^12*d^7*e^6*f^13 - 25452*a^8*b^11*
c^13*d^6*e^5*f^14 + 7380*a^8*b^11*c^14*d^5*e^4*f^15 + 2520*a^8*b^11*c^15*d^
4*e^3*f^16 - 882*a^8*b^11*c^16*d^3*e^2*f^17 + 728*a^9*b^10*c^2*d^17*e^15*f^
4 - 7480*a^9*b^10*c^3*d^16*e^14*f^5 + 2660*a^9*b^10*c^4*d^15*e^13*f^6 + 302
12*a^9*b^10*c^5*d^14*e^12*f^7 - 38584*a^9*b^10*c^6*d^13*e^11*f^8 - 8008*a^9
*b^10*c^7*d^12*e^10*f^9 + 20020*a^9*b^10*c^8*d^11*e^9*f^10 + 20020*a^9*b^10
*c^9*d^10*e^8*f^11 - 8008*a^9*b^10*c^10*d^9*e^7*f^12 - 38584*a^9*b^10*c^11*
d^8*e^6*f^13 + 30212*a^9*b^10*c^12*d^7*e^5*f^14 + 2660*a^9*b^10*c^13*d^6*e^
4*f^15 - 7480*a^9*b^10*c^14*d^5*e^3*f^16 + 728*a^9*b^10*c^15*d^4*e^2*f^17 +
 1152*a^10*b^9*c^2*d^17*e^14*f^5 + 9408*a^10*b^9*c^3*d^16*e^13*f^6 - 18564*
a^10*b^9*c^4*d^15*e^12*f^7 - 13104*a^10*b^9*c^5*d^14*e^11*f^8 + 48048*a^10*
b^9*c^6*d^13*e^10*f^9 - 8008*a^10*b^9*c^7*d^12*e^9*f^10 - 36036*a^10*b^9*c^
8*d^11*e^8*f^11 - 8008*a^10*b^9*c^9*d^10*e^7*f^12 + 48048*a^10*b^9*c^10*d^9
*e^6*f^13 - 13104*a^10*b^9*c^11*d^8*e^5*f^14 - 18564*a^10*b^9*c^12*d^7*e^4*
f^15 + 9408*a^10*b^9*c^13*d^6*e^3*f^16 + 1152*a^10*b^9*c^14*d^5*e^2*f^17 -
4032*a^11*b^8*c^2*d^17*e^13*f^6 - 4368*a^11*b^8*c^3*d^16*e^12*f^7 + 26208*a
^11*b^8*c^4*d^15*e^11*f^8 - 13104*a^11*b^8*c^5*d^14*e^10*f^9 - 38584*a^11*b
^8*c^6*d^13*e^9*f^10 + 32760*a^11*b^8*c^7*d^12*e^8*f^11 + 32760*a^11*b^8*c^
8*d^11*e^7*f^12 - 38584*a^11*b^8*c^9*d^10*e^6*f^13 - 13104*a^11*b^8*c^10*d^
9*e^5*f^14 + 26208*a^11*b^8*c^11*d^8*e^4*f^15 - 4368*a^11*b^8*c^12*d^7*e^3*
f^16 - 4032*a^11*b^8*c^13*d^6*e^2*f^17 + 5460*a^12*b^7*c^2*d^17*e^12*f^7 -
4368*a^12*b^7*c^3*d^16*e^11*f^8 - 18564*a^12*b^7*c^4*d^15*e^10*f^9 + 30212*
a^12*b^7*c^5*d^14*e^9*f^10 + 9828*a^12*b^7*c^6*d^13*e^8*f^11 - 43680*a^12*b
^7*c^7*d^12*e^7*f^12 + 9828*a^12*b^7*c^8*d^11*e^6*f^13 + 30212*a^12*b^7*c^9
*d^10*e^5*f^14 - 18564*a^12*b^7*c^10*d^9*e^4*f^15 - 4368*a^12*b^7*c^11*d^8*

e^3*f^16 + 5460*a^12*b^7*c^12*d^7*e^2*f^17 - 4032*a^13*b^6*c^2*d^17*e^11*f^
8 + 9408*a^13*b^6*c^3*d^16*e^10*f^9 + 2660*a^13*b^6*c^4*d^15*e^9*f^10 - 254
52*a^13*b^6*c^5*d^14*e^8*f^11 + 17472*a^13*b^6*c^6*d^13*e^7*f^12 + 17472*a^
13*b^6*c^7*d^12*e^6*f^13 - 25452*a^13*b^6*c^8*d^11*e^5*f^14 + 2660*a^13*b^6
*c^9*d^10*e^4*f^15 + 9408*a^13*b^6*c^10*d^9*e^3*f^16 - 4032*a^13*b^6*c^11*d
^8*e^2*f^17 + 1152*a^14*b^5*c^2*d^17*e^10*f^9 - 7480*a^14*b^5*c^3*d^16*e^9*
f^10 + 7380*a^14*b^5*c^4*d^15*e^8*f^11 + 7056*a^14*b^5*c^5*d^14*e^7*f^12 -
17472*a^14*b^5*c^6*d^13*e^6*f^13 + 7056*a^14*b^5*c^7*d^12*e^5*f^14 + 7380*a
^14*b^5*c^8*d^11*e^4*f^15 - 7480*a^14*b^5*c^9*d^10*e^3*f^16 + 1152*a^14*b^5
*c^10*d^9*e^2*f^17 + 728*a^15*b^4*c^2*d^17*e^9*f^10 + 2520*a^15*b^4*c^3*d^1
6*e^8*f^11 - 6288*a^15*b^4*c^4*d^15*e^7*f^12 + 3696*a^15*b^4*c^5*d^14*e^6*f
^13 + 3696*a^15*b^4*c^6*d^13*e^5*f^14 - 6288*a^15*b^4*c^7*d^12*e^4*f^15 + 2
520*a^15*b^4*c^8*d^11*e^3*f^16 + 728*a^15*b^4*c^9*d^10*e^2*f^17 - 882*a^16*
b^3*c^2*d^17*e^8*f^11 + 312*a^16*b^3*c^3*d^16*e^7*f^12 + 1596*a^16*b^3*c^4*
d^15*e^6*f^13 - 2772*a^16*b^3*c^5*d^14*e^5*f^14 + 1596*a^16*b^3*c^6*d^13*e^
4*f^15 + 312*a^16*b^3*c^7*d^12*e^3*f^16 - 882*a^16*b^3*c^8*d^11*e^2*f^17 +
360*a^17*b^2*c^2*d^17*e^7*f^12 - 504*a^17*b^2*c^3*d^16*e^6*f^13 + 252*a^17*
b^2*c^4*d^15*e^5*f^14 + 252*a^17*b^2*c^5*d^14*e^4*f^15 - 504*a^17*b^2*c^6*d
^13*e^3*f^16 + 360*a^17*b^2*c^7*d^12*e^2*f^17))/(56*a^3*b^13*c^5*d^11*e^16
- a^8*b^8*d^16*e^16 - a^16*c^8*d^8*f^16 - b^16*c^8*d^8*e^16 - a^16*d^16*e^8
*f^8 - b^16*c^16*e^8*f^8 - 28*a^2*b^14*c^6*d^10*e^16 - a^8*b^8*c^16*f^16 -
70*a^4*b^12*c^4*d^12*e^16 + 56*a^5*b^11*c^3*d^13*e^16 - 28*a^6*b^10*c^2*d^1
4*e^16 - 28*a^10*b^6*c^14*d^2*f^16 + 56*a^11*b^5*c^13*d^3*f^16 - 70*a^12*b^
4*c^12*d^4*f^16 + 56*a^13*b^3*c^11*d^5*f^16 - 28*a^14*b^2*c^10*d^6*f^16 - 2
8*a^2*b^14*c^16*e^6*f^10 + 56*a^3*b^13*c^16*e^5*f^11 - 70*a^4*b^12*c^16*e^4
*f^12 + 56*a^5*b^11*c^16*e^3*f^13 - 28*a^6*b^10*c^16*e^2*f^14 - 28*a^10*b^6
*d^16*e^14*f^2 + 56*a^11*b^5*d^16*e^13*f^3 - 70*a^12*b^4*d^16*e^12*f^4 + 56
*a^13*b^3*d^16*e^11*f^5 - 28*a^14*b^2*d^16*e^10*f^6 - 28*a^16*c^2*d^14*e^6*
f^10 + 56*a^16*c^3*d^13*e^5*f^11 - 70*a^16*c^4*d^12*e^4*f^12 + 56*a^16*c^5*
d^11*e^3*f^13 - 28*a^16*c^6*d^10*e^2*f^14 - 28*b^16*c^10*d^6*e^14*f^2 + 56*
b^16*c^11*d^5*e^13*f^3 - 70*b^16*c^12*d^4*e^12*f^4 + 56*b^16*c^13*d^3*e^11*
f^5 - 28*b^16*c^14*d^2*e^10*f^6 + 8*a*b^15*c^7*d^9*e^16 + 8*a^7*b^9*c*d^15*
e^16 + 8*a^9*b^7*c^15*d*f^16 + 8*a^15*b*c^9*d^7*f^16 + 8*a*b^15*c^16*e^7*f^
9 + 8*a^7*b^9*c^16*e*f^15 + 8*a^9*b^7*d^16*e^15*f + 8*a^15*b*d^16*e^9*f^7 +
8*a^16*c*d^15*e^7*f^9 + 8*a^16*c^7*d^9*e*f^15 + 8*b^16*c^9*d^7*e^15*f + 8*
b^16*c^15*d*e^9*f^7 - 56*a*b^15*c^8*d^8*e^15*f - 56*a*b^15*c^15*d*e^8*f^8 -
56*a^8*b^8*c*d^15*e^15*f - 56*a^8*b^8*c^15*d*e*f^15 - 56*a^15*b*c*d^15*e^8
*f^8 - 56*a^15*b*c^8*d^8*e*f^15 + 160*a*b^15*c^9*d^7*e^14*f^2 - 224*a*b^15*
c^10*d^6*e^13*f^3 + 112*a*b^15*c^11*d^5*e^12*f^4 + 112*a*b^15*c^12*d^4*e^11
*f^5 - 224*a*b^15*c^13*d^3*e^10*f^6 + 160*a*b^15*c^14*d^2*e^9*f^7 + 160*a^2
*b^14*c^7*d^9*e^15*f + 160*a^2*b^14*c^15*d*e^7*f^9 - 224*a^3*b^13*c^6*d^10*
e^15*f - 224*a^3*b^13*c^15*d*e^6*f^10 + 112*a^4*b^12*c^5*d^11*e^15*f + 112*
a^4*b^12*c^15*d*e^5*f^11 + 112*a^5*b^11*c^4*d^12*e^15*f + 112*a^5*b^11*c^15
*d*e^4*f^12 - 224*a^6*b^10*c^3*d^13*e^15*f - 224*a^6*b^10*c^15*d*e^3*f^13 +
160*a^7*b^9*c^2*d^14*e^15*f + 160*a^7*b^9*c^15*d*e^2*f^14 + 160*a^9*b^7*c*

d^15*e^14*f^2 + 160*a^9*b^7*c^14*d^2*e*f^15 - 224*a^10*b^6*c*d^15*e^13*f^3
- 224*a^10*b^6*c^13*d^3*e*f^15 + 112*a^11*b^5*c*d^15*e^12*f^4 + 112*a^11*b^
5*c^12*d^4*e*f^15 + 112*a^12*b^4*c*d^15*e^11*f^5 + 112*a^12*b^4*c^11*d^5*e*
f^15 - 224*a^13*b^3*c*d^15*e^10*f^6 - 224*a^13*b^3*c^10*d^6*e*f^15 + 160*a^
14*b^2*c*d^15*e^9*f^7 + 160*a^14*b^2*c^9*d^7*e*f^15 + 160*a^15*b*c^2*d^14*e
^7*f^9 - 224*a^15*b*c^3*d^13*e^6*f^10 + 112*a^15*b*c^4*d^12*e^5*f^11 + 112*
a^15*b*c^5*d^11*e^4*f^12 - 224*a^15*b*c^6*d^10*e^3*f^13 + 160*a^15*b*c^7*d^
9*e^2*f^14 - 300*a^2*b^14*c^8*d^8*e^14*f^2 + 840*a^2*b^14*c^10*d^6*e^12*f^4
 - 1344*a^2*b^14*c^11*d^5*e^11*f^5 + 840*a^2*b^14*c^12*d^4*e^10*f^6 - 300*a
^2*b^14*c^14*d^2*e^8*f^8 + 1400*a^3*b^13*c^8*d^8*e^13*f^3 - 2800*a^3*b^13*c
^9*d^7*e^12*f^4 + 1568*a^3*b^13*c^10*d^6*e^11*f^5 + 1568*a^3*b^13*c^11*d^5*
e^10*f^6 - 2800*a^3*b^13*c^12*d^4*e^9*f^7 + 1400*a^3*b^13*c^13*d^3*e^8*f^8
+ 840*a^4*b^12*c^6*d^10*e^14*f^2 - 2800*a^4*b^12*c^7*d^9*e^13*f^3 + 1750*a^
4*b^12*c^8*d^8*e^12*f^4 + 4480*a^4*b^12*c^9*d^7*e^11*f^5 - 8624*a^4*b^12*c^
10*d^6*e^10*f^6 + 4480*a^4*b^12*c^11*d^5*e^9*f^7 + 1750*a^4*b^12*c^12*d^4*e
^8*f^8 - 2800*a^4*b^12*c^13*d^3*e^7*f^9 + 840*a^4*b^12*c^14*d^2*e^6*f^10 -
1344*a^5*b^11*c^5*d^11*e^14*f^2 + 1568*a^5*b^11*c^6*d^10*e^13*f^3 + 4480*a^
5*b^11*c^7*d^9*e^12*f^4 - 12264*a^5*b^11*c^8*d^8*e^11*f^5 + 7392*a^5*b^11*c
^9*d^7*e^10*f^6 + 7392*a^5*b^11*c^10*d^6*e^9*f^7 - 12264*a^5*b^11*c^11*d^5*
e^8*f^8 + 4480*a^5*b^11*c^12*d^4*e^7*f^9 + 1568*a^5*b^11*c^13*d^3*e^6*f^10
- 1344*a^5*b^11*c^14*d^2*e^5*f^11 + 840*a^6*b^10*c^4*d^12*e^14*f^2 + 1568*a
^6*b^10*c^5*d^11*e^13*f^3 - 8624*a^6*b^10*c^6*d^10*e^12*f^4 + 7392*a^6*b^10
*c^7*d^9*e^11*f^5 + 11396*a^6*b^10*c^8*d^8*e^10*f^6 - 24640*a^6*b^10*c^9*d^
7*e^9*f^7 + 11396*a^6*b^10*c^10*d^6*e^8*f^8 + 7392*a^6*b^10*c^11*d^5*e^7*f^
9 - 8624*a^6*b^10*c^12*d^4*e^6*f^10 + 1568*a^6*b^10*c^13*d^3*e^5*f^11 + 840
*a^6*b^10*c^14*d^2*e^4*f^12 - 2800*a^7*b^9*c^4*d^12*e^13*f^3 + 4480*a^7*b^9
*c^5*d^11*e^12*f^4 + 7392*a^7*b^9*c^6*d^10*e^11*f^5 - 24640*a^7*b^9*c^7*d^9
*e^10*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9*c^9*d^7*e^8*f^8 -
 24640*a^7*b^9*c^10*d^6*e^7*f^9 + 7392*a^7*b^9*c^11*d^5*e^6*f^10 + 4480*a^7
*b^9*c^12*d^4*e^5*f^11 - 2800*a^7*b^9*c^13*d^3*e^4*f^12 - 300*a^8*b^8*c^2*d
^14*e^14*f^2 + 1400*a^8*b^8*c^3*d^13*e^13*f^3 + 1750*a^8*b^8*c^4*d^12*e^12*
f^4 - 12264*a^8*b^8*c^5*d^11*e^11*f^5 + 11396*a^8*b^8*c^6*d^10*e^10*f^6 + 1
5400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^8*d^8*e^8*f^8 + 15400*a^8*b^
8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^10*d^6*e^6*f^10 - 12264*a^8*b^8*c^11*d^
5*e^5*f^11 + 1750*a^8*b^8*c^12*d^4*e^4*f^12 + 1400*a^8*b^8*c^13*d^3*e^3*f^1
3 - 300*a^8*b^8*c^14*d^2*e^2*f^14 - 2800*a^9*b^7*c^3*d^13*e^12*f^4 + 4480*a
^9*b^7*c^4*d^12*e^11*f^5 + 7392*a^9*b^7*c^5*d^11*e^10*f^6 - 24640*a^9*b^7*c
^6*d^10*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^9*b^7*c^8*d^8*e^7
*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^10 + 7392*a^9*b^7*c^10*d^6*e^5*f^11 + 44
80*a^9*b^7*c^11*d^5*e^4*f^12 - 2800*a^9*b^7*c^12*d^4*e^3*f^13 + 840*a^10*b^
6*c^2*d^14*e^12*f^4 + 1568*a^10*b^6*c^3*d^13*e^11*f^5 - 8624*a^10*b^6*c^4*d
^12*e^10*f^6 + 7392*a^10*b^6*c^5*d^11*e^9*f^7 + 11396*a^10*b^6*c^6*d^10*e^8
*f^8 - 24640*a^10*b^6*c^7*d^9*e^7*f^9 + 11396*a^10*b^6*c^8*d^8*e^6*f^10 + 7
392*a^10*b^6*c^9*d^7*e^5*f^11 - 8624*a^10*b^6*c^10*d^6*e^4*f^12 + 1568*a^10
*b^6*c^11*d^5*e^3*f^13 + 840*a^10*b^6*c^12*d^4*e^2*f^14 - 1344*a^11*b^5*c^2

*d^14*e^11*f^5 + 1568*a^11*b^5*c^3*d^13*e^10*f^6 + 4480*a^11*b^5*c^4*d^12*e
^9*f^7 - 12264*a^11*b^5*c^5*d^11*e^8*f^8 + 7392*a^11*b^5*c^6*d^10*e^7*f^9 +
 7392*a^11*b^5*c^7*d^9*e^6*f^10 - 12264*a^11*b^5*c^8*d^8*e^5*f^11 + 4480*a^
11*b^5*c^9*d^7*e^4*f^12 + 1568*a^11*b^5*c^10*d^6*e^3*f^13 - 1344*a^11*b^5*c
^11*d^5*e^2*f^14 + 840*a^12*b^4*c^2*d^14*e^10*f^6 - 2800*a^12*b^4*c^3*d^13*
e^9*f^7 + 1750*a^12*b^4*c^4*d^12*e^8*f^8 + 4480*a^12*b^4*c^5*d^11*e^7*f^9 -
 8624*a^12*b^4*c^6*d^10*e^6*f^10 + 4480*a^12*b^4*c^7*d^9*e^5*f^11 + 1750*a^
12*b^4*c^8*d^8*e^4*f^12 - 2800*a^12*b^4*c^9*d^7*e^3*f^13 + 840*a^12*b^4*c^1
0*d^6*e^2*f^14 + 1400*a^13*b^3*c^3*d^13*e^8*f^8 - 2800*a^13*b^3*c^4*d^12*e^
7*f^9 + 1568*a^13*b^3*c^5*d^11*e^6*f^10 + 1568*a^13*b^3*c^6*d^10*e^5*f^11 -
 2800*a^13*b^3*c^7*d^9*e^4*f^12 + 1400*a^13*b^3*c^8*d^8*e^3*f^13 - 300*a^14
*b^2*c^2*d^14*e^8*f^8 + 840*a^14*b^2*c^4*d^12*e^6*f^10 - 1344*a^14*b^2*c^5*
d^11*e^5*f^11 + 840*a^14*b^2*c^6*d^10*e^4*f^12 - 300*a^14*b^2*c^8*d^8*e^2*f
^14)) + (x*(84*a^5*b^11*c^13*d^3*f^16 - 12*a^4*b^12*c^14*d^2*f^16 - 243*a^6
*b^10*c^12*d^4*f^16 + 366*a^7*b^9*c^11*d^5*f^16 - 321*a^8*b^8*c^10*d^6*f^16
 + 252*a^9*b^7*c^9*d^7*f^16 - 321*a^10*b^6*c^8*d^8*f^16 + 366*a^11*b^5*c^7*
d^9*f^16 - 243*a^12*b^4*c^6*d^10*f^16 + 84*a^13*b^3*c^5*d^11*f^16 - 12*a^14
*b^2*c^4*d^12*f^16 - 12*a^4*b^12*d^16*e^14*f^2 + 84*a^5*b^11*d^16*e^13*f^3
- 243*a^6*b^10*d^16*e^12*f^4 + 366*a^7*b^9*d^16*e^11*f^5 - 321*a^8*b^8*d^16
*e^10*f^6 + 252*a^9*b^7*d^16*e^9*f^7 - 321*a^10*b^6*d^16*e^8*f^8 + 366*a^11
*b^5*d^16*e^7*f^9 - 243*a^12*b^4*d^16*e^6*f^10 + 84*a^13*b^3*d^16*e^5*f^11
- 12*a^14*b^2*d^16*e^4*f^12 - 12*b^16*c^4*d^12*e^14*f^2 + 84*b^16*c^5*d^11*
e^13*f^3 - 243*b^16*c^6*d^10*e^12*f^4 + 366*b^16*c^7*d^9*e^11*f^5 - 321*b^1
6*c^8*d^8*e^10*f^6 + 252*b^16*c^9*d^7*e^9*f^7 - 321*b^16*c^10*d^6*e^8*f^8 +
 366*b^16*c^11*d^5*e^7*f^9 - 243*b^16*c^12*d^4*e^6*f^10 + 84*b^16*c^13*d^3*
e^5*f^11 - 12*b^16*c^14*d^2*e^4*f^12 + 48*a*b^15*c^3*d^13*e^14*f^2 - 252*a*
b^15*c^4*d^12*e^13*f^3 + 366*a*b^15*c^5*d^11*e^12*f^4 + 354*a*b^15*c^6*d^10
*e^11*f^5 - 1458*a*b^15*c^7*d^9*e^10*f^6 + 942*a*b^15*c^8*d^8*e^9*f^7 + 942
*a*b^15*c^9*d^7*e^8*f^8 - 1458*a*b^15*c^10*d^6*e^7*f^9 + 354*a*b^15*c^11*d^
5*e^6*f^10 + 366*a*b^15*c^12*d^4*e^5*f^11 - 252*a*b^15*c^13*d^3*e^4*f^12 +
48*a*b^15*c^14*d^2*e^3*f^13 + 48*a^3*b^13*c*d^15*e^14*f^2 + 48*a^3*b^13*c^1
4*d^2*e*f^15 - 252*a^4*b^12*c*d^15*e^13*f^3 - 252*a^4*b^12*c^13*d^3*e*f^15
+ 366*a^5*b^11*c*d^15*e^12*f^4 + 366*a^5*b^11*c^12*d^4*e*f^15 + 354*a^6*b^1
0*c*d^15*e^11*f^5 + 354*a^6*b^10*c^11*d^5*e*f^15 - 1458*a^7*b^9*c*d^15*e^10
*f^6 - 1458*a^7*b^9*c^10*d^6*e*f^15 + 942*a^8*b^8*c*d^15*e^9*f^7 + 942*a^8*
b^8*c^9*d^7*e*f^15 + 942*a^9*b^7*c*d^15*e^8*f^8 + 942*a^9*b^7*c^8*d^8*e*f^1
5 - 1458*a^10*b^6*c*d^15*e^7*f^9 - 1458*a^10*b^6*c^7*d^9*e*f^15 + 354*a^11*
b^5*c*d^15*e^6*f^10 + 354*a^11*b^5*c^6*d^10*e*f^15 + 366*a^12*b^4*c*d^15*e^
5*f^11 + 366*a^12*b^4*c^5*d^11*e*f^15 - 252*a^13*b^3*c*d^15*e^4*f^12 - 252*
a^13*b^3*c^4*d^12*e*f^15 + 48*a^14*b^2*c*d^15*e^3*f^13 + 48*a^14*b^2*c^3*d^
13*e*f^15 - 72*a^2*b^14*c^2*d^14*e^14*f^2 + 168*a^2*b^14*c^3*d^13*e^13*f^3
+ 723*a^2*b^14*c^4*d^12*e^12*f^4 - 3258*a^2*b^14*c^5*d^11*e^11*f^5 + 3156*a
^2*b^14*c^6*d^10*e^10*f^6 + 3522*a^2*b^14*c^7*d^9*e^9*f^7 - 8478*a^2*b^14*c
^8*d^8*e^8*f^8 + 3522*a^2*b^14*c^9*d^7*e^7*f^9 + 3156*a^2*b^14*c^10*d^6*e^6
*f^10 - 3258*a^2*b^14*c^11*d^5*e^5*f^11 + 723*a^2*b^14*c^12*d^4*e^4*f^12 +

168*a^2*b^14*c^13*d^3*e^3*f^13 - 72*a^2*b^14*c^14*d^2*e^2*f^14 + 168*a^3*b^
13*c^2*d^14*e^13*f^3 - 1692*a^3*b^13*c^3*d^13*e^12*f^4 + 2538*a^3*b^13*c^4*
d^12*e^11*f^5 + 5634*a^3*b^13*c^5*d^11*e^10*f^6 - 18738*a^3*b^13*c^6*d^10*e
^9*f^7 + 12042*a^3*b^13*c^7*d^9*e^8*f^8 + 12042*a^3*b^13*c^8*d^8*e^7*f^9 -
18738*a^3*b^13*c^9*d^7*e^6*f^10 + 5634*a^3*b^13*c^10*d^6*e^5*f^11 + 2538*a^
3*b^13*c^11*d^5*e^4*f^12 - 1692*a^3*b^13*c^12*d^4*e^3*f^13 + 168*a^3*b^13*c
^13*d^3*e^2*f^14 + 723*a^4*b^12*c^2*d^14*e^12*f^4 + 2538*a^4*b^12*c^3*d^13*
e^11*f^5 - 14022*a^4*b^12*c^4*d^12*e^10*f^6 + 14022*a^4*b^12*c^5*d^11*e^9*f
^7 + 21087*a^4*b^12*c^6*d^10*e^8*f^8 - 48168*a^4*b^12*c^7*d^9*e^7*f^9 + 210
87*a^4*b^12*c^8*d^8*e^6*f^10 + 14022*a^4*b^12*c^9*d^7*e^5*f^11 - 14022*a^4*
b^12*c^10*d^6*e^4*f^12 + 2538*a^4*b^12*c^11*d^5*e^3*f^13 + 723*a^4*b^12*c^1
2*d^4*e^2*f^14 - 3258*a^5*b^11*c^2*d^14*e^11*f^5 + 5634*a^5*b^11*c^3*d^13*e
^10*f^6 + 14022*a^5*b^11*c^4*d^12*e^9*f^7 - 50544*a^5*b^11*c^5*d^11*e^8*f^8
 + 33696*a^5*b^11*c^6*d^10*e^7*f^9 + 33696*a^5*b^11*c^7*d^9*e^6*f^10 - 5054
4*a^5*b^11*c^8*d^8*e^5*f^11 + 14022*a^5*b^11*c^9*d^7*e^4*f^12 + 5634*a^5*b^
11*c^10*d^6*e^3*f^13 - 3258*a^5*b^11*c^11*d^5*e^2*f^14 + 3156*a^6*b^10*c^2*
d^14*e^10*f^6 - 18738*a^6*b^10*c^3*d^13*e^9*f^7 + 21087*a^6*b^10*c^4*d^12*e
^8*f^8 + 33696*a^6*b^10*c^5*d^11*e^7*f^9 - 78624*a^6*b^10*c^6*d^10*e^6*f^10
 + 33696*a^6*b^10*c^7*d^9*e^5*f^11 + 21087*a^6*b^10*c^8*d^8*e^4*f^12 - 1873
8*a^6*b^10*c^9*d^7*e^3*f^13 + 3156*a^6*b^10*c^10*d^6*e^2*f^14 + 3522*a^7*b^
9*c^2*d^14*e^9*f^7 + 12042*a^7*b^9*c^3*d^13*e^8*f^8 - 48168*a^7*b^9*c^4*d^1
2*e^7*f^9 + 33696*a^7*b^9*c^5*d^11*e^6*f^10 + 33696*a^7*b^9*c^6*d^10*e^5*f^
11 - 48168*a^7*b^9*c^7*d^9*e^4*f^12 + 12042*a^7*b^9*c^8*d^8*e^3*f^13 + 3522
*a^7*b^9*c^9*d^7*e^2*f^14 - 8478*a^8*b^8*c^2*d^14*e^8*f^8 + 12042*a^8*b^8*c
^3*d^13*e^7*f^9 + 21087*a^8*b^8*c^4*d^12*e^6*f^10 - 50544*a^8*b^8*c^5*d^11*
e^5*f^11 + 21087*a^8*b^8*c^6*d^10*e^4*f^12 + 12042*a^8*b^8*c^7*d^9*e^3*f^13
 - 8478*a^8*b^8*c^8*d^8*e^2*f^14 + 3522*a^9*b^7*c^2*d^14*e^7*f^9 - 18738*a^
9*b^7*c^3*d^13*e^6*f^10 + 14022*a^9*b^7*c^4*d^12*e^5*f^11 + 14022*a^9*b^7*c
^5*d^11*e^4*f^12 - 18738*a^9*b^7*c^6*d^10*e^3*f^13 + 3522*a^9*b^7*c^7*d^9*e
^2*f^14 + 3156*a^10*b^6*c^2*d^14*e^6*f^10 + 5634*a^10*b^6*c^3*d^13*e^5*f^11
 - 14022*a^10*b^6*c^4*d^12*e^4*f^12 + 5634*a^10*b^6*c^5*d^11*e^3*f^13 + 315
6*a^10*b^6*c^6*d^10*e^2*f^14 - 3258*a^11*b^5*c^2*d^14*e^5*f^11 + 2538*a^11*
b^5*c^3*d^13*e^4*f^12 + 2538*a^11*b^5*c^4*d^12*e^3*f^13 - 3258*a^11*b^5*c^5
*d^11*e^2*f^14 + 723*a^12*b^4*c^2*d^14*e^4*f^12 - 1692*a^12*b^4*c^3*d^13*e^
3*f^13 + 723*a^12*b^4*c^4*d^12*e^2*f^14 + 168*a^13*b^3*c^2*d^14*e^3*f^13 +
168*a^13*b^3*c^3*d^13*e^2*f^14 - 72*a^14*b^2*c^2*d^14*e^2*f^14))/(56*a^3*b^
13*c^5*d^11*e^16 - a^8*b^8*d^16*e^16 - a^16*c^8*d^8*f^16 - b^16*c^8*d^8*e^1
6 - a^16*d^16*e^8*f^8 - b^16*c^16*e^8*f^8 - 28*a^2*b^14*c^6*d^10*e^16 - a^8
*b^8*c^16*f^16 - 70*a^4*b^12*c^4*d^12*e^16 + 56*a^5*b^11*c^3*d^13*e^16 - 28
*a^6*b^10*c^2*d^14*e^16 - 28*a^10*b^6*c^14*d^2*f^16 + 56*a^11*b^5*c^13*d^3*
f^16 - 70*a^12*b^4*c^12*d^4*f^16 + 56*a^13*b^3*c^11*d^5*f^16 - 28*a^14*b^2*
c^10*d^6*f^16 - 28*a^2*b^14*c^16*e^6*f^10 + 56*a^3*b^13*c^16*e^5*f^11 - 70*
a^4*b^12*c^16*e^4*f^12 + 56*a^5*b^11*c^16*e^3*f^13 - 28*a^6*b^10*c^16*e^2*f
^14 - 28*a^10*b^6*d^16*e^14*f^2 + 56*a^11*b^5*d^16*e^13*f^3 - 70*a^12*b^4*d
^16*e^12*f^4 + 56*a^13*b^3*d^16*e^11*f^5 - 28*a^14*b^2*d^16*e^10*f^6 - 28*a

^16*c^2*d^14*e^6*f^10 + 56*a^16*c^3*d^13*e^5*f^11 - 70*a^16*c^4*d^12*e^4*f^
12 + 56*a^16*c^5*d^11*e^3*f^13 - 28*a^16*c^6*d^10*e^2*f^14 - 28*b^16*c^10*d
^6*e^14*f^2 + 56*b^16*c^11*d^5*e^13*f^3 - 70*b^16*c^12*d^4*e^12*f^4 + 56*b^
16*c^13*d^3*e^11*f^5 - 28*b^16*c^14*d^2*e^10*f^6 + 8*a*b^15*c^7*d^9*e^16 +
8*a^7*b^9*c*d^15*e^16 + 8*a^9*b^7*c^15*d*f^16 + 8*a^15*b*c^9*d^7*f^16 + 8*a
*b^15*c^16*e^7*f^9 + 8*a^7*b^9*c^16*e*f^15 + 8*a^9*b^7*d^16*e^15*f + 8*a^15
*b*d^16*e^9*f^7 + 8*a^16*c*d^15*e^7*f^9 + 8*a^16*c^7*d^9*e*f^15 + 8*b^16*c^
9*d^7*e^15*f + 8*b^16*c^15*d*e^9*f^7 - 56*a*b^15*c^8*d^8*e^15*f - 56*a*b^15
*c^15*d*e^8*f^8 - 56*a^8*b^8*c*d^15*e^15*f - 56*a^8*b^8*c^15*d*e*f^15 - 56*
a^15*b*c*d^15*e^8*f^8 - 56*a^15*b*c^8*d^8*e*f^15 + 160*a*b^15*c^9*d^7*e^14*
f^2 - 224*a*b^15*c^10*d^6*e^13*f^3 + 112*a*b^15*c^11*d^5*e^12*f^4 + 112*a*b
^15*c^12*d^4*e^11*f^5 - 224*a*b^15*c^13*d^3*e^10*f^6 + 160*a*b^15*c^14*d^2*
e^9*f^7 + 160*a^2*b^14*c^7*d^9*e^15*f + 160*a^2*b^14*c^15*d*e^7*f^9 - 224*a
^3*b^13*c^6*d^10*e^15*f - 224*a^3*b^13*c^15*d*e^6*f^10 + 112*a^4*b^12*c^5*d
^11*e^15*f + 112*a^4*b^12*c^15*d*e^5*f^11 + 112*a^5*b^11*c^4*d^12*e^15*f +
112*a^5*b^11*c^15*d*e^4*f^12 - 224*a^6*b^10*c^3*d^13*e^15*f - 224*a^6*b^10*
c^15*d*e^3*f^13 + 160*a^7*b^9*c^2*d^14*e^15*f + 160*a^7*b^9*c^15*d*e^2*f^14
 + 160*a^9*b^7*c*d^15*e^14*f^2 + 160*a^9*b^7*c^14*d^2*e*f^15 - 224*a^10*b^6
*c*d^15*e^13*f^3 - 224*a^10*b^6*c^13*d^3*e*f^15 + 112*a^11*b^5*c*d^15*e^12*
f^4 + 112*a^11*b^5*c^12*d^4*e*f^15 + 112*a^12*b^4*c*d^15*e^11*f^5 + 112*a^1
2*b^4*c^11*d^5*e*f^15 - 224*a^13*b^3*c*d^15*e^10*f^6 - 224*a^13*b^3*c^10*d^
6*e*f^15 + 160*a^14*b^2*c*d^15*e^9*f^7 + 160*a^14*b^2*c^9*d^7*e*f^15 + 160*
a^15*b*c^2*d^14*e^7*f^9 - 224*a^15*b*c^3*d^13*e^6*f^10 + 112*a^15*b*c^4*d^1
2*e^5*f^11 + 112*a^15*b*c^5*d^11*e^4*f^12 - 224*a^15*b*c^6*d^10*e^3*f^13 +
160*a^15*b*c^7*d^9*e^2*f^14 - 300*a^2*b^14*c^8*d^8*e^14*f^2 + 840*a^2*b^14*
c^10*d^6*e^12*f^4 - 1344*a^2*b^14*c^11*d^5*e^11*f^5 + 840*a^2*b^14*c^12*d^4
*e^10*f^6 - 300*a^2*b^14*c^14*d^2*e^8*f^8 + 1400*a^3*b^13*c^8*d^8*e^13*f^3
- 2800*a^3*b^13*c^9*d^7*e^12*f^4 + 1568*a^3*b^13*c^10*d^6*e^11*f^5 + 1568*a
^3*b^13*c^11*d^5*e^10*f^6 - 2800*a^3*b^13*c^12*d^4*e^9*f^7 + 1400*a^3*b^13*
c^13*d^3*e^8*f^8 + 840*a^4*b^12*c^6*d^10*e^14*f^2 - 2800*a^4*b^12*c^7*d^9*e
^13*f^3 + 1750*a^4*b^12*c^8*d^8*e^12*f^4 + 4480*a^4*b^12*c^9*d^7*e^11*f^5 -
 8624*a^4*b^12*c^10*d^6*e^10*f^6 + 4480*a^4*b^12*c^11*d^5*e^9*f^7 + 1750*a^
4*b^12*c^12*d^4*e^8*f^8 - 2800*a^4*b^12*c^13*d^3*e^7*f^9 + 840*a^4*b^12*c^1
4*d^2*e^6*f^10 - 1344*a^5*b^11*c^5*d^11*e^14*f^2 + 1568*a^5*b^11*c^6*d^10*e
^13*f^3 + 4480*a^5*b^11*c^7*d^9*e^12*f^4 - 12264*a^5*b^11*c^8*d^8*e^11*f^5
+ 7392*a^5*b^11*c^9*d^7*e^10*f^6 + 7392*a^5*b^11*c^10*d^6*e^9*f^7 - 12264*a
^5*b^11*c^11*d^5*e^8*f^8 + 4480*a^5*b^11*c^12*d^4*e^7*f^9 + 1568*a^5*b^11*c
^13*d^3*e^6*f^10 - 1344*a^5*b^11*c^14*d^2*e^5*f^11 + 840*a^6*b^10*c^4*d^12*
e^14*f^2 + 1568*a^6*b^10*c^5*d^11*e^13*f^3 - 8624*a^6*b^10*c^6*d^10*e^12*f^
4 + 7392*a^6*b^10*c^7*d^9*e^11*f^5 + 11396*a^6*b^10*c^8*d^8*e^10*f^6 - 2464
0*a^6*b^10*c^9*d^7*e^9*f^7 + 11396*a^6*b^10*c^10*d^6*e^8*f^8 + 7392*a^6*b^1
0*c^11*d^5*e^7*f^9 - 8624*a^6*b^10*c^12*d^4*e^6*f^10 + 1568*a^6*b^10*c^13*d
^3*e^5*f^11 + 840*a^6*b^10*c^14*d^2*e^4*f^12 - 2800*a^7*b^9*c^4*d^12*e^13*f
^3 + 4480*a^7*b^9*c^5*d^11*e^12*f^4 + 7392*a^7*b^9*c^6*d^10*e^11*f^5 - 2464
0*a^7*b^9*c^7*d^9*e^10*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9*

c^9*d^7*e^8*f^8 - 24640*a^7*b^9*c^10*d^6*e^7*f^9 + 7392*a^7*b^9*c^11*d^5*e^
6*f^10 + 4480*a^7*b^9*c^12*d^4*e^5*f^11 - 2800*a^7*b^9*c^13*d^3*e^4*f^12 -
300*a^8*b^8*c^2*d^14*e^14*f^2 + 1400*a^8*b^8*c^3*d^13*e^13*f^3 + 1750*a^8*b
^8*c^4*d^12*e^12*f^4 - 12264*a^8*b^8*c^5*d^11*e^11*f^5 + 11396*a^8*b^8*c^6*
d^10*e^10*f^6 + 15400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^8*d^8*e^8*f
^8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^10*d^6*e^6*f^10 - 1226
4*a^8*b^8*c^11*d^5*e^5*f^11 + 1750*a^8*b^8*c^12*d^4*e^4*f^12 + 1400*a^8*b^8
*c^13*d^3*e^3*f^13 - 300*a^8*b^8*c^14*d^2*e^2*f^14 - 2800*a^9*b^7*c^3*d^13*
e^12*f^4 + 4480*a^9*b^7*c^4*d^12*e^11*f^5 + 7392*a^9*b^7*c^5*d^11*e^10*f^6
- 24640*a^9*b^7*c^6*d^10*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^
9*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^10 + 7392*a^9*b^7*c^10*
d^6*e^5*f^11 + 4480*a^9*b^7*c^11*d^5*e^4*f^12 - 2800*a^9*b^7*c^12*d^4*e^3*f
^13 + 840*a^10*b^6*c^2*d^14*e^12*f^4 + 1568*a^10*b^6*c^3*d^13*e^11*f^5 - 86
24*a^10*b^6*c^4*d^12*e^10*f^6 + 7392*a^10*b^6*c^5*d^11*e^9*f^7 + 11396*a^10
*b^6*c^6*d^10*e^8*f^8 - 24640*a^10*b^6*c^7*d^9*e^7*f^9 + 11396*a^10*b^6*c^8
*d^8*e^6*f^10 + 7392*a^10*b^6*c^9*d^7*e^5*f^11 - 8624*a^10*b^6*c^10*d^6*e^4
*f^12 + 1568*a^10*b^6*c^11*d^5*e^3*f^13 + 840*a^10*b^6*c^12*d^4*e^2*f^14 -
1344*a^11*b^5*c^2*d^14*e^11*f^5 + 1568*a^11*b^5*c^3*d^13*e^10*f^6 + 4480*a^
11*b^5*c^4*d^12*e^9*f^7 - 12264*a^11*b^5*c^5*d^11*e^8*f^8 + 7392*a^11*b^5*c
^6*d^10*e^7*f^9 + 7392*a^11*b^5*c^7*d^9*e^6*f^10 - 12264*a^11*b^5*c^8*d^8*e
^5*f^11 + 4480*a^11*b^5*c^9*d^7*e^4*f^12 + 1568*a^11*b^5*c^10*d^6*e^3*f^13
- 1344*a^11*b^5*c^11*d^5*e^2*f^14 + 840*a^12*b^4*c^2*d^14*e^10*f^6 - 2800*a
^12*b^4*c^3*d^13*e^9*f^7 + 1750*a^12*b^4*c^4*d^12*e^8*f^8 + 4480*a^12*b^4*c
^5*d^11*e^7*f^9 - 8624*a^12*b^4*c^6*d^10*e^6*f^10 + 4480*a^12*b^4*c^7*d^9*e
^5*f^11 + 1750*a^12*b^4*c^8*d^8*e^4*f^12 - 2800*a^12*b^4*c^9*d^7*e^3*f^13 +
840*a^12*b^4*c^10*d^6*e^2*f^14 + 1400*a^13*b^3*c^3*d^13*e^8*f^8 - 2800*a^1
3*b^3*c^4*d^12*e^7*f^9 + 1568*a^13*b^3*c^5*d^11*e^6*f^10 + 1568*a^13*b^3*c^
6*d^10*e^5*f^11 - 2800*a^13*b^3*c^7*d^9*e^4*f^12 + 1400*a^13*b^3*c^8*d^8*e^
3*f^13 - 300*a^14*b^2*c^2*d^14*e^8*f^8 + 840*a^14*b^2*c^4*d^12*e^6*f^10 - 1
344*a^14*b^2*c^5*d^11*e^5*f^11 + 840*a^14*b^2*c^6*d^10*e^4*f^12 - 300*a^14*
b^2*c^8*d^8*e^2*f^14)) - (36*a^11*b^2*d^13*f^13 + 36*b^13*c^11*d^2*f^13 + 3
6*b^13*d^13*e^11*f^2 + 297*a^2*b^11*c^9*d^4*f^13 - 108*a^3*b^10*c^8*d^5*f^1
3 - 198*a^4*b^9*c^7*d^6*f^13 + 153*a^5*b^8*c^6*d^7*f^13 + 153*a^6*b^7*c^5*d
^8*f^13 - 198*a^7*b^6*c^4*d^9*f^13 - 108*a^8*b^5*c^3*d^10*f^13 + 297*a^9*b^
4*c^2*d^11*f^13 + 297*a^2*b^11*d^13*e^9*f^4 - 108*a^3*b^10*d^13*e^8*f^5 - 1
98*a^4*b^9*d^13*e^7*f^6 + 153*a^5*b^8*d^13*e^6*f^7 + 153*a^6*b^7*d^13*e^5*f
^8 - 198*a^7*b^6*d^13*e^4*f^9 - 108*a^8*b^5*d^13*e^3*f^10 + 297*a^9*b^4*d^1
3*e^2*f^11 + 297*b^13*c^2*d^11*e^9*f^4 - 108*b^13*c^3*d^10*e^8*f^5 - 198*b^
13*c^4*d^9*e^7*f^6 + 153*b^13*c^5*d^8*e^6*f^7 + 153*b^13*c^6*d^7*e^5*f^8 -
198*b^13*c^7*d^6*e^4*f^9 - 108*b^13*c^8*d^5*e^3*f^10 + 297*b^13*c^9*d^4*e^2
*f^11 - 180*a*b^12*c^10*d^3*f^13 - 180*a^10*b^3*c*d^12*f^13 - 180*a*b^12*d^
13*e^10*f^3 - 180*a^10*b^3*d^13*e*f^12 - 180*b^13*c*d^12*e^10*f^3 - 180*b^1
3*c^10*d^3*e*f^12 + 1026*a*b^12*c*d^12*e^9*f^4 + 1026*a*b^12*c^9*d^4*e*f^12
+ 1026*a^9*b^4*c*d^12*e*f^12 - 2052*a*b^12*c^2*d^11*e^8*f^5 + 1548*a*b^12*
c^3*d^10*e^7*f^6 + 297*a*b^12*c^4*d^9*e^6*f^7 - 1242*a*b^12*c^5*d^8*e^5*f^8

+ 297*a*b^12*c^6*d^7*e^4*f^9 + 1548*a*b^12*c^7*d^6*e^3*f^10 - 2052*a*b^12*
c^8*d^5*e^2*f^11 - 2052*a^2*b^11*c*d^12*e^8*f^5 - 2052*a^2*b^11*c^8*d^5*e*f
^12 + 1548*a^3*b^10*c*d^12*e^7*f^6 + 1548*a^3*b^10*c^7*d^6*e*f^12 + 297*a^4
*b^9*c*d^12*e^6*f^7 + 297*a^4*b^9*c^6*d^7*e*f^12 - 1242*a^5*b^8*c*d^12*e^5*
f^8 - 1242*a^5*b^8*c^5*d^8*e*f^12 + 297*a^6*b^7*c*d^12*e^4*f^9 + 297*a^6*b^
7*c^4*d^9*e*f^12 + 1548*a^7*b^6*c*d^12*e^3*f^10 + 1548*a^7*b^6*c^3*d^10*e*f
^12 - 2052*a^8*b^5*c*d^12*e^2*f^11 - 2052*a^8*b^5*c^2*d^11*e*f^12 + 4860*a^
2*b^11*c^2*d^11*e^7*f^6 - 4986*a^2*b^11*c^3*d^10*e^6*f^7 + 1701*a^2*b^11*c^
4*d^9*e^5*f^8 + 1701*a^2*b^11*c^5*d^8*e^4*f^9 - 4986*a^2*b^11*c^6*d^7*e^3*f
^10 + 4860*a^2*b^11*c^7*d^6*e^2*f^11 - 4986*a^3*b^10*c^2*d^11*e^6*f^7 + 633
6*a^3*b^10*c^3*d^10*e^5*f^8 - 3960*a^3*b^10*c^4*d^9*e^4*f^9 + 6336*a^3*b^10
*c^5*d^8*e^3*f^10 - 4986*a^3*b^10*c^6*d^7*e^2*f^11 + 1701*a^4*b^9*c^2*d^11*
e^5*f^8 - 3960*a^4*b^9*c^3*d^10*e^4*f^9 - 3960*a^4*b^9*c^4*d^9*e^3*f^10 + 1
701*a^4*b^9*c^5*d^8*e^2*f^11 + 1701*a^5*b^8*c^2*d^11*e^4*f^9 + 6336*a^5*b^8
*c^3*d^10*e^3*f^10 + 1701*a^5*b^8*c^4*d^9*e^2*f^11 - 4986*a^6*b^7*c^2*d^11*
e^3*f^10 - 4986*a^6*b^7*c^3*d^10*e^2*f^11 + 4860*a^7*b^6*c^2*d^11*e^2*f^11)
/(56*a^3*b^13*c^5*d^11*e^16 - a^8*b^8*d^16*e^16 - a^16*c^8*d^8*f^16 - b^16*
c^8*d^8*e^16 - a^16*d^16*e^8*f^8 - b^16*c^16*e^8*f^8 - 28*a^2*b^14*c^6*d^10
*e^16 - a^8*b^8*c^16*f^16 - 70*a^4*b^12*c^4*d^12*e^16 + 56*a^5*b^11*c^3*d^1
3*e^16 - 28*a^6*b^10*c^2*d^14*e^16 - 28*a^10*b^6*c^14*d^2*f^16 + 56*a^11*b^
5*c^13*d^3*f^16 - 70*a^12*b^4*c^12*d^4*f^16 + 56*a^13*b^3*c^11*d^5*f^16 - 2
8*a^14*b^2*c^10*d^6*f^16 - 28*a^2*b^14*c^16*e^6*f^10 + 56*a^3*b^13*c^16*e^5
*f^11 - 70*a^4*b^12*c^16*e^4*f^12 + 56*a^5*b^11*c^16*e^3*f^13 - 28*a^6*b^10
*c^16*e^2*f^14 - 28*a^10*b^6*d^16*e^14*f^2 + 56*a^11*b^5*d^16*e^13*f^3 - 70
*a^12*b^4*d^16*e^12*f^4 + 56*a^13*b^3*d^16*e^11*f^5 - 28*a^14*b^2*d^16*e^10
*f^6 - 28*a^16*c^2*d^14*e^6*f^10 + 56*a^16*c^3*d^13*e^5*f^11 - 70*a^16*c^4*
d^12*e^4*f^12 + 56*a^16*c^5*d^11*e^3*f^13 - 28*a^16*c^6*d^10*e^2*f^14 - 28*
b^16*c^10*d^6*e^14*f^2 + 56*b^16*c^11*d^5*e^13*f^3 - 70*b^16*c^12*d^4*e^12*
f^4 + 56*b^16*c^13*d^3*e^11*f^5 - 28*b^16*c^14*d^2*e^10*f^6 + 8*a*b^15*c^7*
d^9*e^16 + 8*a^7*b^9*c*d^15*e^16 + 8*a^9*b^7*c^15*d*f^16 + 8*a^15*b*c^9*d^7
*f^16 + 8*a*b^15*c^16*e^7*f^9 + 8*a^7*b^9*c^16*e*f^15 + 8*a^9*b^7*d^16*e^15
*f + 8*a^15*b*d^16*e^9*f^7 + 8*a^16*c*d^15*e^7*f^9 + 8*a^16*c^7*d^9*e*f^15
+ 8*b^16*c^9*d^7*e^15*f + 8*b^16*c^15*d*e^9*f^7 - 56*a*b^15*c^8*d^8*e^15*f
- 56*a*b^15*c^15*d*e^8*f^8 - 56*a^8*b^8*c*d^15*e^15*f - 56*a^8*b^8*c^15*d*e
*f^15 - 56*a^15*b*c*d^15*e^8*f^8 - 56*a^15*b*c^8*d^8*e*f^15 + 160*a*b^15*c^
9*d^7*e^14*f^2 - 224*a*b^15*c^10*d^6*e^13*f^3 + 112*a*b^15*c^11*d^5*e^12*f^
4 + 112*a*b^15*c^12*d^4*e^11*f^5 - 224*a*b^15*c^13*d^3*e^10*f^6 + 160*a*b^1
5*c^14*d^2*e^9*f^7 + 160*a^2*b^14*c^7*d^9*e^15*f + 160*a^2*b^14*c^15*d*e^7*
f^9 - 224*a^3*b^13*c^6*d^10*e^15*f - 224*a^3*b^13*c^15*d*e^6*f^10 + 112*a^4
*b^12*c^5*d^11*e^15*f + 112*a^4*b^12*c^15*d*e^5*f^11 + 112*a^5*b^11*c^4*d^1
2*e^15*f + 112*a^5*b^11*c^15*d*e^4*f^12 - 224*a^6*b^10*c^3*d^13*e^15*f - 22
4*a^6*b^10*c^15*d*e^3*f^13 + 160*a^7*b^9*c^2*d^14*e^15*f + 160*a^7*b^9*c^15
*d*e^2*f^14 + 160*a^9*b^7*c*d^15*e^14*f^2 + 160*a^9*b^7*c^14*d^2*e*f^15 - 2
24*a^10*b^6*c*d^15*e^13*f^3 - 224*a^10*b^6*c^13*d^3*e*f^15 + 112*a^11*b^5*c
*d^15*e^12*f^4 + 112*a^11*b^5*c^12*d^4*e*f^15 + 112*a^12*b^4*c*d^15*e^11*f^

5 + 112*a^12*b^4*c^11*d^5*e*f^15 - 224*a^13*b^3*c*d^15*e^10*f^6 - 224*a^13*
b^3*c^10*d^6*e*f^15 + 160*a^14*b^2*c*d^15*e^9*f^7 + 160*a^14*b^2*c^9*d^7*e*
f^15 + 160*a^15*b*c^2*d^14*e^7*f^9 - 224*a^15*b*c^3*d^13*e^6*f^10 + 112*a^1
5*b*c^4*d^12*e^5*f^11 + 112*a^15*b*c^5*d^11*e^4*f^12 - 224*a^15*b*c^6*d^10*
e^3*f^13 + 160*a^15*b*c^7*d^9*e^2*f^14 - 300*a^2*b^14*c^8*d^8*e^14*f^2 + 84
0*a^2*b^14*c^10*d^6*e^12*f^4 - 1344*a^2*b^14*c^11*d^5*e^11*f^5 + 840*a^2*b^
14*c^12*d^4*e^10*f^6 - 300*a^2*b^14*c^14*d^2*e^8*f^8 + 1400*a^3*b^13*c^8*d^
8*e^13*f^3 - 2800*a^3*b^13*c^9*d^7*e^12*f^4 + 1568*a^3*b^13*c^10*d^6*e^11*f
^5 + 1568*a^3*b^13*c^11*d^5*e^10*f^6 - 2800*a^3*b^13*c^12*d^4*e^9*f^7 + 140
0*a^3*b^13*c^13*d^3*e^8*f^8 + 840*a^4*b^12*c^6*d^10*e^14*f^2 - 2800*a^4*b^1
2*c^7*d^9*e^13*f^3 + 1750*a^4*b^12*c^8*d^8*e^12*f^4 + 4480*a^4*b^12*c^9*d^7
*e^11*f^5 - 8624*a^4*b^12*c^10*d^6*e^10*f^6 + 4480*a^4*b^12*c^11*d^5*e^9*f^
7 + 1750*a^4*b^12*c^12*d^4*e^8*f^8 - 2800*a^4*b^12*c^13*d^3*e^7*f^9 + 840*a
^4*b^12*c^14*d^2*e^6*f^10 - 1344*a^5*b^11*c^5*d^11*e^14*f^2 + 1568*a^5*b^11
*c^6*d^10*e^13*f^3 + 4480*a^5*b^11*c^7*d^9*e^12*f^4 - 12264*a^5*b^11*c^8*d^
8*e^11*f^5 + 7392*a^5*b^11*c^9*d^7*e^10*f^6 + 7392*a^5*b^11*c^10*d^6*e^9*f^
7 - 12264*a^5*b^11*c^11*d^5*e^8*f^8 + 4480*a^5*b^11*c^12*d^4*e^7*f^9 + 1568
*a^5*b^11*c^13*d^3*e^6*f^10 - 1344*a^5*b^11*c^14*d^2*e^5*f^11 + 840*a^6*b^1
0*c^4*d^12*e^14*f^2 + 1568*a^6*b^10*c^5*d^11*e^13*f^3 - 8624*a^6*b^10*c^6*d
^10*e^12*f^4 + 7392*a^6*b^10*c^7*d^9*e^11*f^5 + 11396*a^6*b^10*c^8*d^8*e^10
*f^6 - 24640*a^6*b^10*c^9*d^7*e^9*f^7 + 11396*a^6*b^10*c^10*d^6*e^8*f^8 + 7
392*a^6*b^10*c^11*d^5*e^7*f^9 - 8624*a^6*b^10*c^12*d^4*e^6*f^10 + 1568*a^6*
b^10*c^13*d^3*e^5*f^11 + 840*a^6*b^10*c^14*d^2*e^4*f^12 - 2800*a^7*b^9*c^4*
d^12*e^13*f^3 + 4480*a^7*b^9*c^5*d^11*e^12*f^4 + 7392*a^7*b^9*c^6*d^10*e^11
*f^5 - 24640*a^7*b^9*c^7*d^9*e^10*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 154
00*a^7*b^9*c^9*d^7*e^8*f^8 - 24640*a^7*b^9*c^10*d^6*e^7*f^9 + 7392*a^7*b^9*
c^11*d^5*e^6*f^10 + 4480*a^7*b^9*c^12*d^4*e^5*f^11 - 2800*a^7*b^9*c^13*d^3*
e^4*f^12 - 300*a^8*b^8*c^2*d^14*e^14*f^2 + 1400*a^8*b^8*c^3*d^13*e^13*f^3 +
 1750*a^8*b^8*c^4*d^12*e^12*f^4 - 12264*a^8*b^8*c^5*d^11*e^11*f^5 + 11396*a
^8*b^8*c^6*d^10*e^10*f^6 + 15400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^
8*d^8*e^8*f^8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^10*d^6*e^6*
f^10 - 12264*a^8*b^8*c^11*d^5*e^5*f^11 + 1750*a^8*b^8*c^12*d^4*e^4*f^12 + 1
400*a^8*b^8*c^13*d^3*e^3*f^13 - 300*a^8*b^8*c^14*d^2*e^2*f^14 - 2800*a^9*b^
7*c^3*d^13*e^12*f^4 + 4480*a^9*b^7*c^4*d^12*e^11*f^5 + 7392*a^9*b^7*c^5*d^1
1*e^10*f^6 - 24640*a^9*b^7*c^6*d^10*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8
 + 15400*a^9*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^10 + 7392*a^
9*b^7*c^10*d^6*e^5*f^11 + 4480*a^9*b^7*c^11*d^5*e^4*f^12 - 2800*a^9*b^7*c^1
2*d^4*e^3*f^13 + 840*a^10*b^6*c^2*d^14*e^12*f^4 + 1568*a^10*b^6*c^3*d^13*e^
11*f^5 - 8624*a^10*b^6*c^4*d^12*e^10*f^6 + 7392*a^10*b^6*c^5*d^11*e^9*f^7 +
 11396*a^10*b^6*c^6*d^10*e^8*f^8 - 24640*a^10*b^6*c^7*d^9*e^7*f^9 + 11396*a
^10*b^6*c^8*d^8*e^6*f^10 + 7392*a^10*b^6*c^9*d^7*e^5*f^11 - 8624*a^10*b^6*c
^10*d^6*e^4*f^12 + 1568*a^10*b^6*c^11*d^5*e^3*f^13 + 840*a^10*b^6*c^12*d^4*
e^2*f^14 - 1344*a^11*b^5*c^2*d^14*e^11*f^5 + 1568*a^11*b^5*c^3*d^13*e^10*f^
6 + 4480*a^11*b^5*c^4*d^12*e^9*f^7 - 12264*a^11*b^5*c^5*d^11*e^8*f^8 + 7392
*a^11*b^5*c^6*d^10*e^7*f^9 + 7392*a^11*b^5*c^7*d^9*e^6*f^10 - 12264*a^11*b^

5*c^8*d^8*e^5*f^11 + 4480*a^11*b^5*c^9*d^7*e^4*f^12 + 1568*a^11*b^5*c^10*d^6*e^3*f^13 - 1344*a^11*b^5*c^11*d^5*e^2*f^14 + 840*a^12*b^4*c^2*d^14*e^10*f^6 - 2800*a^12*b^4*c^3*d^13*e^9*f^7 + 1750*a^12*b^4*c^4*d^12*e^8*f^8 + 4480*a^12*b^4*c^5*d^11*e^7*f^9 - 8624*a^12*b^4*c^6*d^10*e^6*f^10 + 4480*a^12*b^4*c^7*d^9*e^5*f^11 + 1750*a^12*b^4*c^8*d^8*e^4*f^12 - 2800*a^12*b^4*c^9*d^7*e^3*f^13 + 840*a^12*b^4*c^10*d^6*e^2*f^14 + 1400*a^13*b^3*c^3*d^13*e^8*f^8 - 2800*a^13*b^3*c^4*d^12*e^7*f^9 + 1568*a^13*b^3*c^5*d^11*e^6*f^10 + 1568*a^13*b^3*c^6*d^10*e^5*f^11 - 2800*a^13*b^3*c^7*d^9*e^4*f^12 + 1400*a^13*b^3*c^8*d^8*e^3*f^13 - 300*a^14*b^2*c^2*d^14*e^8*f^8 + 840*a^14*b^2*c^4*d^12*e^6*f^10 - 1344*a^14*b^2*c^5*d^11*e^5*f^11 + 840*a^14*b^2*c^6*d^10*e^4*f^12 - 300*a^14*b^2*c^8*d^8*e^2*f^14) + (x*(108*a^3*b^10*c^7*d^6*f^13 - 36*b^13*c^10*d^3*f^13 - 36*b^13*d^13*e^10*f^3 - 297*a^2*b^11*c^8*d^5*f^13 - 36*a^10*b^3*d^13*f^13 + 324*a^4*b^9*c^6*d^7*f^13 - 594*a^5*b^8*c^5*d^8*f^13 + 324*a^6*b^7*c^4*d^9*f^13 + 108*a^7*b^6*c^3*d^10*f^13 - 297*a^8*b^5*c^2*d^11*f^13 - 297*a^2*b^11*d^13*e^8*f^5 + 108*a^3*b^10*d^13*e^7*f^6 + 324*a^4*b^9*d^13*e^6*f^7 - 594*a^5*b^8*d^13*e^5*f^8 + 324*a^6*b^7*d^13*e^4*f^9 + 108*a^7*b^6*d^13*e^3*f^10 - 297*a^8*b^5*d^13*e^2*f^11 - 297*b^13*c^2*d^11*e^8*f^5 + 108*b^13*c^3*d^10*e^7*f^6 + 324*b^13*c^4*d^9*e^6*f^7 - 594*b^13*c^5*d^8*e^5*f^8 + 324*b^13*c^6*d^7*e^4*f^9 + 108*b^13*c^7*d^6*e^3*f^10 - 297*b^13*c^8*d^5*e^2*f^11 + 180*a*b^12*c^9*d^4*f^13 + 180*a^9*b^4*c*d^12*f^13 + 180*a*b^12*d^13*e^9*f^4 + 180*a^9*b^4*d^13*e*f^12 + 180*b^13*c*d^12*e^9*f^4 + 180*b^13*c^9*d^4*e*f^12 - 1026*a*b^12*c*d^12*e^8*f^5 - 1026*a*b^12*c^8*d^5*e*f^12 - 1026*a^8*b^5*c*d^12*e*f^12 + 2052*a*b^12*c^2*d^11*e^7*f^6 - 2052*a*b^12*c^3*d^10*e^6*f^7 + 1026*a*b^12*c^4*d^9*e^5*f^8 + 1026*a*b^12*c^5*d^8*e^4*f^9 - 2052*a*b^12*c^6*d^7*e^3*f^10 + 2052*a*b^12*c^7*d^6*e^2*f^11 + 2052*a^2*b^11*c*d^12*e^7*f^6 + 2052*a^2*b^11*c^7*d^6*e*f^12 - 2052*a^3*b^10*c*d^12*e^6*f^7 - 2052*a^3*b^10*c^6*d^7*e*f^12 + 1026*a^4*b^9*c*d^12*e^5*f^8 + 1026*a^4*b^9*c^5*d^8*e*f^12 + 1026*a^5*b^8*c*d^12*e^4*f^9 + 1026*a^5*b^8*c^4*d^9*e*f^12 - 2052*a^6*b^7*c*d^12*e^3*f^10 - 2052*a^6*b^7*c^3*d^10*e*f^12 + 2052*a^7*b^6*c*d^12*e^2*f^11 + 2052*a^7*b^6*c^2*d^11*e*f^12 - 4104*a^2*b^11*c^2*d^11*e^6*f^7 + 4104*a^2*b^11*c^3*d^10*e^5*f^8 - 5130*a^2*b^11*c^4*d^9*e^4*f^9 + 4104*a^2*b^11*c^5*d^8*e^3*f^10 - 4104*a^2*b^11*c^6*d^7*e^2*f^11 + 4104*a^3*b^10*c^2*d^11*e^5*f^8 + 4104*a^3*b^10*c^5*d^8*e^2*f^11 - 5130*a^4*b^9*c^2*d^11*e^4*f^9 - 5130*a^4*b^9*c^4*d^9*e^2*f^11 + 4104*a^5*b^8*c^2*d^11*e^3*f^10 + 4104*a^5*b^8*c^3*d^10*e^2*f^11 - 4104*a^6*b^7*c^2*d^11*e^2*f^11))/(56*a^3*b^13*c^5*d^11*e^16 - a^8*b^8*d^16*e^16 - a^16*c^8*d^8*f^16 - b^16*c^8*d^8*e^16 - a^16*d^16*e^8*f^8 - b^16*c^16*e^8*f^8 - 28*a^2*b^14*c^6*d^10*e^16 - a^8*b^8*c^16*f^16 - 70*a^4*b^12*c^4*d^12*e^16 + 56*a^5*b^11*c^3*d^13*e^16 - 28*a^6*b^10*c^2*d^14*e^16 - 28*a^10*b^6*c^14*d^2*f^16 + 56*a^11*b^5*c^13*d^3*f^16 - 70*a^12*b^4*c^12*d^4*f^16 + 56*a^13*b^3*c^11*d^5*f^16 - 28*a^14*b^2*c^10*d^6*f^16 - 28*a^2*b^14*c^16*e^6*f^10 + 56*a^3*b^13*c^16*e^5*f^11 - 70*a^4*b^12*c^16*e^4*f^12 + 56*a^5*b^11*c^16*e^3*f^13 - 28*a^6*b^10*c^16*e^2*f^14 - 28*a^10*b^6*d^16*e^14*f^2 + 56*a^11*b^5*d^16*e^13*f^3 - 70*a^12*b^4*d^16*e^12*f^4 + 56*a^13*b^3*d^16*e^11*f^5 - 28*a^14*b^2*d^16*e^10*f^6 - 28*a^16*c^2*d^14*e^6*f^10 + 56*a^16*c^3*d^13*e^5*f^11 - 70*a^16*c^

4*d^12*e^4*f^12 + 56*a^16*c^5*d^11*e^3*f^13 - 28*a^16*c^6*d^10*e^2*f^14 - 2
8*b^16*c^10*d^6*e^14*f^2 + 56*b^16*c^11*d^5*e^13*f^3 - 70*b^16*c^12*d^4*e^1
2*f^4 + 56*b^16*c^13*d^3*e^11*f^5 - 28*b^16*c^14*d^2*e^10*f^6 + 8*a*b^15*c^
7*d^9*e^16 + 8*a^7*b^9*c*d^15*e^16 + 8*a^9*b^7*c^15*d*f^16 + 8*a^15*b*c^9*d
^7*f^16 + 8*a*b^15*c^16*e^7*f^9 + 8*a^7*b^9*c^16*e*f^15 + 8*a^9*b^7*d^16*e^
15*f + 8*a^15*b*d^16*e^9*f^7 + 8*a^16*c*d^15*e^7*f^9 + 8*a^16*c^7*d^9*e*f^1
5 + 8*b^16*c^9*d^7*e^15*f + 8*b^16*c^15*d*e^9*f^7 - 56*a*b^15*c^8*d^8*e^15*
f - 56*a*b^15*c^15*d*e^8*f^8 - 56*a^8*b^8*c*d^15*e^15*f - 56*a^8*b^8*c^15*d
*e*f^15 - 56*a^15*b*c*d^15*e^8*f^8 - 56*a^15*b*c^8*d^8*e*f^15 + 160*a*b^15*
c^9*d^7*e^14*f^2 - 224*a*b^15*c^10*d^6*e^13*f^3 + 112*a*b^15*c^11*d^5*e^12*
f^4 + 112*a*b^15*c^12*d^4*e^11*f^5 - 224*a*b^15*c^13*d^3*e^10*f^6 + 160*a*b
^15*c^14*d^2*e^9*f^7 + 160*a^2*b^14*c^7*d^9*e^15*f + 160*a^2*b^14*c^15*d*e^
7*f^9 - 224*a^3*b^13*c^6*d^10*e^15*f - 224*a^3*b^13*c^15*d*e^6*f^10 + 112*a
^4*b^12*c^5*d^11*e^15*f + 112*a^4*b^12*c^15*d*e^5*f^11 + 112*a^5*b^11*c^4*d
^12*e^15*f + 112*a^5*b^11*c^15*d*e^4*f^12 - 224*a^6*b^10*c^3*d^13*e^15*f -
224*a^6*b^10*c^15*d*e^3*f^13 + 160*a^7*b^9*c^2*d^14*e^15*f + 160*a^7*b^9*c^
15*d*e^2*f^14 + 160*a^9*b^7*c*d^15*e^14*f^2 + 160*a^9*b^7*c^14*d^2*e*f^15 -
224*a^10*b^6*c*d^15*e^13*f^3 - 224*a^10*b^6*c^13*d^3*e*f^15 + 112*a^11*b^5
*c*d^15*e^12*f^4 + 112*a^11*b^5*c^12*d^4*e*f^15 + 112*a^12*b^4*c*d^15*e^11*
f^5 + 112*a^12*b^4*c^11*d^5*e*f^15 - 224*a^13*b^3*c*d^15*e^10*f^6 - 224*a^1
3*b^3*c^10*d^6*e*f^15 + 160*a^14*b^2*c*d^15*e^9*f^7 + 160*a^14*b^2*c^9*d^7*
e*f^15 + 160*a^15*b*c^2*d^14*e^7*f^9 - 224*a^15*b*c^3*d^13*e^6*f^10 + 112*a
^15*b*c^4*d^12*e^5*f^11 + 112*a^15*b*c^5*d^11*e^4*f^12 - 224*a^15*b*c^6*d^1
0*e^3*f^13 + 160*a^15*b*c^7*d^9*e^2*f^14 - 300*a^2*b^14*c^8*d^8*e^14*f^2 +
840*a^2*b^14*c^10*d^6*e^12*f^4 - 1344*a^2*b^14*c^11*d^5*e^11*f^5 + 840*a^2*
b^14*c^12*d^4*e^10*f^6 - 300*a^2*b^14*c^14*d^2*e^8*f^8 + 1400*a^3*b^13*c^8*
d^8*e^13*f^3 - 2800*a^3*b^13*c^9*d^7*e^12*f^4 + 1568*a^3*b^13*c^10*d^6*e^11
*f^5 + 1568*a^3*b^13*c^11*d^5*e^10*f^6 - 2800*a^3*b^13*c^12*d^4*e^9*f^7 + 1
400*a^3*b^13*c^13*d^3*e^8*f^8 + 840*a^4*b^12*c^6*d^10*e^14*f^2 - 2800*a^4*b
^12*c^7*d^9*e^13*f^3 + 1750*a^4*b^12*c^8*d^8*e^12*f^4 + 4480*a^4*b^12*c^9*d
^7*e^11*f^5 - 8624*a^4*b^12*c^10*d^6*e^10*f^6 + 4480*a^4*b^12*c^11*d^5*e^9*
f^7 + 1750*a^4*b^12*c^12*d^4*e^8*f^8 - 2800*a^4*b^12*c^13*d^3*e^7*f^9 + 840
*a^4*b^12*c^14*d^2*e^6*f^10 - 1344*a^5*b^11*c^5*d^11*e^14*f^2 + 1568*a^5*b^
11*c^6*d^10*e^13*f^3 + 4480*a^5*b^11*c^7*d^9*e^12*f^4 - 12264*a^5*b^11*c^8*
d^8*e^11*f^5 + 7392*a^5*b^11*c^9*d^7*e^10*f^6 + 7392*a^5*b^11*c^10*d^6*e^9*
f^7 - 12264*a^5*b^11*c^11*d^5*e^8*f^8 + 4480*a^5*b^11*c^12*d^4*e^7*f^9 + 15
68*a^5*b^11*c^13*d^3*e^6*f^10 - 1344*a^5*b^11*c^14*d^2*e^5*f^11 + 840*a^6*b
^10*c^4*d^12*e^14*f^2 + 1568*a^6*b^10*c^5*d^11*e^13*f^3 - 8624*a^6*b^10*c^6
*d^10*e^12*f^4 + 7392*a^6*b^10*c^7*d^9*e^11*f^5 + 11396*a^6*b^10*c^8*d^8*e^
10*f^6 - 24640*a^6*b^10*c^9*d^7*e^9*f^7 + 11396*a^6*b^10*c^10*d^6*e^8*f^8 +
7392*a^6*b^10*c^11*d^5*e^7*f^9 - 8624*a^6*b^10*c^12*d^4*e^6*f^10 + 1568*a^
6*b^10*c^13*d^3*e^5*f^11 + 840*a^6*b^10*c^14*d^2*e^4*f^12 - 2800*a^7*b^9*c^
4*d^12*e^13*f^3 + 4480*a^7*b^9*c^5*d^11*e^12*f^4 + 7392*a^7*b^9*c^6*d^10*e^
11*f^5 - 24640*a^7*b^9*c^7*d^9*e^10*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 1
5400*a^7*b^9*c^9*d^7*e^8*f^8 - 24640*a^7*b^9*c^10*d^6*e^7*f^9 + 7392*a^7*b^

9*c^11*d^5*e^6*f^10 + 4480*a^7*b^9*c^12*d^4*e^5*f^11 - 2800*a^7*b^9*c^13*d^
3*e^4*f^12 - 300*a^8*b^8*c^2*d^14*e^14*f^2 + 1400*a^8*b^8*c^3*d^13*e^13*f^3
 + 1750*a^8*b^8*c^4*d^12*e^12*f^4 - 12264*a^8*b^8*c^5*d^11*e^11*f^5 + 11396
*a^8*b^8*c^6*d^10*e^10*f^6 + 15400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*
c^8*d^8*e^8*f^8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^10*d^6*e^
6*f^10 - 12264*a^8*b^8*c^11*d^5*e^5*f^11 + 1750*a^8*b^8*c^12*d^4*e^4*f^12 +
 1400*a^8*b^8*c^13*d^3*e^3*f^13 - 300*a^8*b^8*c^14*d^2*e^2*f^14 - 2800*a^9*
b^7*c^3*d^13*e^12*f^4 + 4480*a^9*b^7*c^4*d^12*e^11*f^5 + 7392*a^9*b^7*c^5*d
^11*e^10*f^6 - 24640*a^9*b^7*c^6*d^10*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f
^8 + 15400*a^9*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^10 + 7392*
a^9*b^7*c^10*d^6*e^5*f^11 + 4480*a^9*b^7*c^11*d^5*e^4*f^12 - 2800*a^9*b^7*c
^12*d^4*e^3*f^13 + 840*a^10*b^6*c^2*d^14*e^12*f^4 + 1568*a^10*b^6*c^3*d^13*
e^11*f^5 - 8624*a^10*b^6*c^4*d^12*e^10*f^6 + 7392*a^10*b^6*c^5*d^11*e^9*f^7
 + 11396*a^10*b^6*c^6*d^10*e^8*f^8 - 24640*a^10*b^6*c^7*d^9*e^7*f^9 + 11396
*a^10*b^6*c^8*d^8*e^6*f^10 + 7392*a^10*b^6*c^9*d^7*e^5*f^11 - 8624*a^10*b^6
*c^10*d^6*e^4*f^12 + 1568*a^10*b^6*c^11*d^5*e^3*f^13 + 840*a^10*b^6*c^12*d^
4*e^2*f^14 - 1344*a^11*b^5*c^2*d^14*e^11*f^5 + 1568*a^11*b^5*c^3*d^13*e^10*
f^6 + 4480*a^11*b^5*c^4*d^12*e^9*f^7 - 12264*a^11*b^5*c^5*d^11*e^8*f^8 + 73
92*a^11*b^5*c^6*d^10*e^7*f^9 + 7392*a^11*b^5*c^7*d^9*e^6*f^10 - 12264*a^11*
b^5*c^8*d^8*e^5*f^11 + 4480*a^11*b^5*c^9*d^7*e^4*f^12 + 1568*a^11*b^5*c^10*
d^6*e^3*f^13 - 1344*a^11*b^5*c^11*d^5*e^2*f^14 + 840*a^12*b^4*c^2*d^14*e^10
*f^6 - 2800*a^12*b^4*c^3*d^13*e^9*f^7 + 1750*a^12*b^4*c^4*d^12*e^8*f^8 + 44
80*a^12*b^4*c^5*d^11*e^7*f^9 - 8624*a^12*b^4*c^6*d^10*e^6*f^10 + 4480*a^12*
b^4*c^7*d^9*e^5*f^11 + 1750*a^12*b^4*c^8*d^8*e^4*f^12 - 2800*a^12*b^4*c^9*d
^7*e^3*f^13 + 840*a^12*b^4*c^10*d^6*e^2*f^14 + 1400*a^13*b^3*c^3*d^13*e^8*f
^8 - 2800*a^13*b^3*c^4*d^12*e^7*f^9 + 1568*a^13*b^3*c^5*d^11*e^6*f^10 + 156
8*a^13*b^3*c^6*d^10*e^5*f^11 - 2800*a^13*b^3*c^7*d^9*e^4*f^12 + 1400*a^13*b
^3*c^8*d^8*e^3*f^13 - 300*a^14*b^2*c^2*d^14*e^8*f^8 + 840*a^14*b^2*c^4*d^12
*e^6*f^10 - 1344*a^14*b^2*c^5*d^11*e^5*f^11 + 840*a^14*b^2*c^6*d^10*e^4*f^1
2 - 300*a^14*b^2*c^8*d^8*e^2*f^14))*root(756756*a^10*b^10*c^10*d^10*e^10*f^
10*z^3 + 573300*a^12*b^8*c^9*d^11*e^9*f^11*z^3 + 573300*a^11*b^9*c^11*d^9*e
^8*f^12*z^3 + 573300*a^11*b^9*c^8*d^12*e^11*f^9*z^3 + 573300*a^9*b^11*c^12*
d^8*e^9*f^11*z^3 + 573300*a^9*b^11*c^9*d^11*e^12*f^8*z^3 + 573300*a^8*b^12*
c^11*d^9*e^11*f^9*z^3 - 343980*a^11*b^9*c^10*d^10*e^9*f^11*z^3 - 343980*a^1
1*b^9*c^9*d^11*e^10*f^10*z^3 - 343980*a^10*b^10*c^11*d^9*e^9*f^11*z^3 - 343
980*a^10*b^10*c^9*d^11*e^11*f^9*z^3 - 343980*a^9*b^11*c^11*d^9*e^10*f^10*z^
3 - 343980*a^9*b^11*c^10*d^10*e^11*f^9*z^3 + 326340*a^13*b^7*c^10*d^10*e^7*
f^13*z^3 + 326340*a^13*b^7*c^7*d^13*e^10*f^10*z^3 + 326340*a^10*b^10*c^13*d
^7*e^7*f^13*z^3 + 326340*a^10*b^10*c^7*d^13*e^13*f^7*z^3 + 326340*a^7*b^13*
c^13*d^7*e^10*f^10*z^3 + 326340*a^7*b^13*c^10*d^10*e^13*f^7*z^3 - 267540*a^
12*b^8*c^10*d^10*e^8*f^12*z^3 - 267540*a^12*b^8*c^8*d^12*e^10*f^10*z^3 - 26
7540*a^10*b^10*c^12*d^8*e^8*f^12*z^3 - 267540*a^10*b^10*c^8*d^12*e^12*f^8*z
^3 - 267540*a^8*b^12*c^12*d^8*e^10*f^10*z^3 - 267540*a^8*b^12*c^10*d^10*e^1
2*f^8*z^3 + 245700*a^14*b^6*c^8*d^12*e^8*f^12*z^3 + 245700*a^12*b^8*c^12*d^
8*e^6*f^14*z^3 + 245700*a^12*b^8*c^6*d^14*e^12*f^8*z^3 + 245700*a^8*b^12*c^

14*d^6*e^8*f^12*z^3 + 245700*a^8*b^12*c^8*d^12*e^14*f^6*z^3 + 245700*a^6*b^
14*c^12*d^8*e^12*f^8*z^3 - 191100*a^13*b^7*c^9*d^11*e^8*f^12*z^3 - 191100*a
^13*b^7*c^8*d^12*e^9*f^11*z^3 - 191100*a^12*b^8*c^11*d^9*e^7*f^13*z^3 - 191
100*a^12*b^8*c^7*d^13*e^11*f^9*z^3 - 191100*a^11*b^9*c^12*d^8*e^7*f^13*z^3
- 191100*a^11*b^9*c^7*d^13*e^12*f^8*z^3 - 191100*a^9*b^11*c^13*d^7*e^8*f^12
*z^3 - 191100*a^9*b^11*c^8*d^12*e^13*f^7*z^3 - 191100*a^8*b^12*c^13*d^7*e^9
*f^11*z^3 - 191100*a^8*b^12*c^9*d^11*e^13*f^7*z^3 - 191100*a^7*b^13*c^12*d^
8*e^11*f^9*z^3 - 191100*a^7*b^13*c^11*d^9*e^12*f^8*z^3 - 123900*a^14*b^6*c^
9*d^11*e^7*f^13*z^3 - 123900*a^14*b^6*c^7*d^13*e^9*f^11*z^3 - 123900*a^13*b
^7*c^11*d^9*e^6*f^14*z^3 - 123900*a^13*b^7*c^6*d^14*e^11*f^9*z^3 - 123900*a
^11*b^9*c^13*d^7*e^6*f^14*z^3 - 123900*a^11*b^9*c^6*d^14*e^13*f^7*z^3 - 123
900*a^9*b^11*c^14*d^6*e^7*f^13*z^3 - 123900*a^9*b^11*c^7*d^13*e^14*f^6*z^3
- 123900*a^7*b^13*c^14*d^6*e^9*f^11*z^3 - 123900*a^7*b^13*c^9*d^11*e^14*f^6
*z^3 - 123900*a^6*b^14*c^13*d^7*e^11*f^9*z^3 - 123900*a^6*b^14*c^11*d^9*e^1
3*f^7*z^3 + 101700*a^15*b^5*c^9*d^11*e^6*f^14*z^3 + 101700*a^15*b^5*c^6*d^1
4*e^9*f^11*z^3 + 101700*a^14*b^6*c^11*d^9*e^5*f^15*z^3 + 101700*a^14*b^6*c^
5*d^15*e^11*f^9*z^3 + 101700*a^11*b^9*c^14*d^6*e^5*f^15*z^3 + 101700*a^11*b
^9*c^5*d^15*e^14*f^6*z^3 + 101700*a^9*b^11*c^15*d^5*e^6*f^14*z^3 + 101700*a
^9*b^11*c^6*d^14*e^15*f^5*z^3 + 101700*a^6*b^14*c^15*d^5*e^9*f^11*z^3 + 101
700*a^6*b^14*c^9*d^11*e^15*f^5*z^3 + 101700*a^5*b^15*c^14*d^6*e^11*f^9*z^3
+ 101700*a^5*b^15*c^11*d^9*e^14*f^6*z^3 - 65820*a^14*b^6*c^10*d^10*e^6*f^14
*z^3 - 65820*a^14*b^6*c^6*d^14*e^10*f^10*z^3 - 65820*a^10*b^10*c^14*d^6*e^6
*f^14*z^3 - 65820*a^10*b^10*c^6*d^14*e^14*f^6*z^3 - 65820*a^6*b^14*c^14*d^6
*e^10*f^10*z^3 - 65820*a^6*b^14*c^10*d^10*e^14*f^6*z^3 + 56700*a^16*b^4*c^7
*d^13*e^7*f^13*z^3 - 56700*a^15*b^5*c^8*d^12*e^7*f^13*z^3 - 56700*a^15*b^5*
c^7*d^13*e^8*f^12*z^3 + 56700*a^13*b^7*c^13*d^7*e^4*f^16*z^3 - 56700*a^13*b
^7*c^12*d^8*e^5*f^15*z^3 - 56700*a^13*b^7*c^5*d^15*e^12*f^8*z^3 + 56700*a^1
3*b^7*c^4*d^16*e^13*f^7*z^3 - 56700*a^12*b^8*c^13*d^7*e^5*f^15*z^3 - 56700*
a^12*b^8*c^5*d^15*e^13*f^7*z^3 - 56700*a^8*b^12*c^15*d^5*e^7*f^13*z^3 - 567
00*a^8*b^12*c^7*d^13*e^15*f^5*z^3 + 56700*a^7*b^13*c^16*d^4*e^7*f^13*z^3 -
56700*a^7*b^13*c^15*d^5*e^8*f^12*z^3 - 56700*a^7*b^13*c^8*d^12*e^15*f^5*z^3
 + 56700*a^7*b^13*c^7*d^13*e^16*f^4*z^3 - 56700*a^5*b^15*c^13*d^7*e^12*f^8*
z^3 - 56700*a^5*b^15*c^12*d^8*e^13*f^7*z^3 + 56700*a^4*b^16*c^13*d^7*e^13*f
^7*z^3 - 48252*a^15*b^5*c^10*d^10*e^5*f^15*z^3 - 48252*a^15*b^5*c^5*d^15*e^
10*f^10*z^3 - 48252*a^10*b^10*c^15*d^5*e^5*f^15*z^3 - 48252*a^10*b^10*c^5*d
^15*e^15*f^5*z^3 - 48252*a^5*b^15*c^15*d^5*e^10*f^10*z^3 - 48252*a^5*b^15*c
^10*d^10*e^15*f^5*z^3 - 32400*a^16*b^4*c^8*d^12*e^6*f^14*z^3 - 32400*a^16*b
^4*c^6*d^14*e^8*f^12*z^3 - 32400*a^14*b^6*c^12*d^8*e^4*f^16*z^3 - 32400*a^1
4*b^6*c^4*d^16*e^12*f^8*z^3 - 32400*a^12*b^8*c^14*d^6*e^4*f^16*z^3 - 32400*
a^12*b^8*c^4*d^16*e^14*f^6*z^3 - 32400*a^8*b^12*c^16*d^4*e^6*f^14*z^3 - 324
00*a^8*b^12*c^6*d^14*e^16*f^4*z^3 - 32400*a^6*b^14*c^16*d^4*e^8*f^12*z^3 -
32400*a^6*b^14*c^8*d^12*e^16*f^4*z^3 - 32400*a^4*b^16*c^14*d^6*e^12*f^8*z^3
 - 32400*a^4*b^16*c^12*d^8*e^14*f^6*z^3 + 20565*a^16*b^4*c^10*d^10*e^4*f^16
*z^3 + 20565*a^16*b^4*c^4*d^16*e^10*f^10*z^3 + 20565*a^10*b^10*c^16*d^4*e^4
*f^16*z^3 + 20565*a^10*b^10*c^4*d^16*e^16*f^4*z^3 + 20565*a^4*b^16*c^16*d^4

*e^10*f^10*z^3 + 20565*a^4*b^16*c^10*d^10*e^16*f^4*z^3 + 15660*a^17*b^3*c^8
*d^12*e^5*f^15*z^3 + 15660*a^17*b^3*c^5*d^15*e^8*f^12*z^3 + 15660*a^15*b^5*
c^12*d^8*e^3*f^17*z^3 + 15660*a^15*b^5*c^3*d^17*e^12*f^8*z^3 + 15660*a^12*b
^8*c^15*d^5*e^3*f^17*z^3 + 15660*a^12*b^8*c^3*d^17*e^15*f^5*z^3 + 15660*a^8
*b^12*c^17*d^3*e^5*f^15*z^3 + 15660*a^8*b^12*c^5*d^15*e^17*f^3*z^3 + 15660*
a^5*b^15*c^17*d^3*e^8*f^12*z^3 + 15660*a^5*b^15*c^8*d^12*e^17*f^3*z^3 + 156
60*a^3*b^17*c^15*d^5*e^12*f^8*z^3 + 15660*a^3*b^17*c^12*d^8*e^15*f^5*z^3 -
9750*a^17*b^3*c^9*d^11*e^4*f^16*z^3 - 9750*a^17*b^3*c^4*d^16*e^9*f^11*z^3 -
 9750*a^16*b^4*c^11*d^9*e^3*f^17*z^3 - 9750*a^16*b^4*c^3*d^17*e^11*f^9*z^3
 - 9750*a^11*b^9*c^16*d^4*e^3*f^17*z^3 - 9750*a^11*b^9*c^3*d^17*e^16*f^4*z^3
 - 9750*a^9*b^11*c^17*d^3*e^4*f^16*z^3 - 9750*a^9*b^11*c^4*d^16*e^17*f^3*z^
3 - 9750*a^4*b^16*c^17*d^3*e^9*f^11*z^3 - 9750*a^4*b^16*c^9*d^11*e^17*f^3*z
^3 - 9750*a^3*b^17*c^16*d^4*e^11*f^9*z^3 - 9750*a^3*b^17*c^11*d^9*e^16*f^4*
z^3 - 8100*a^17*b^3*c^7*d^13*e^6*f^14*z^3 - 8100*a^17*b^3*c^6*d^14*e^7*f^13
*z^3 - 8100*a^14*b^6*c^13*d^7*e^3*f^17*z^3 - 8100*a^14*b^6*c^3*d^17*e^13*f^
7*z^3 - 8100*a^13*b^7*c^14*d^6*e^3*f^17*z^3 - 8100*a^13*b^7*c^3*d^17*e^14*f
^6*z^3 - 8100*a^7*b^13*c^17*d^3*e^6*f^14*z^3 - 8100*a^7*b^13*c^6*d^14*e^17*
f^3*z^3 - 8100*a^6*b^14*c^17*d^3*e^7*f^13*z^3 - 8100*a^6*b^14*c^7*d^13*e^17
*f^3*z^3 - 8100*a^3*b^17*c^14*d^6*e^13*f^7*z^3 - 8100*a^3*b^17*c^13*d^7*e^1
4*f^6*z^3 - 7980*a^16*b^4*c^9*d^11*e^5*f^15*z^3 - 7980*a^16*b^4*c^5*d^15*e^
9*f^11*z^3 - 7980*a^15*b^5*c^11*d^9*e^4*f^16*z^3 - 7980*a^15*b^5*c^4*d^16*e
^11*f^9*z^3 - 7980*a^11*b^9*c^15*d^5*e^4*f^16*z^3 - 7980*a^11*b^9*c^4*d^16*
e^15*f^5*z^3 - 7980*a^9*b^11*c^16*d^4*e^5*f^15*z^3 - 7980*a^9*b^11*c^5*d^15
*e^16*f^4*z^3 - 7980*a^5*b^15*c^16*d^4*e^9*f^11*z^3 - 7980*a^5*b^15*c^9*d^1
1*e^16*f^4*z^3 - 7980*a^4*b^16*c^15*d^5*e^11*f^9*z^3 - 7980*a^4*b^16*c^11*d
^9*e^15*f^5*z^3 + 6300*a^18*b^2*c^6*d^14*e^6*f^14*z^3 + 6300*a^14*b^6*c^14*
d^6*e^2*f^18*z^3 + 6300*a^14*b^6*c^2*d^18*e^14*f^6*z^3 + 6300*a^6*b^14*c^18
*d^2*e^6*f^14*z^3 + 6300*a^6*b^14*c^6*d^14*e^18*f^2*z^3 + 6300*a^2*b^18*c^1
4*d^6*e^14*f^6*z^3 - 4260*a^18*b^2*c^7*d^13*e^5*f^15*z^3 - 4260*a^18*b^2*c^
5*d^15*e^7*f^13*z^3 - 4260*a^15*b^5*c^13*d^7*e^2*f^18*z^3 - 4260*a^15*b^5*c
^2*d^18*e^13*f^7*z^3 - 4260*a^13*b^7*c^15*d^5*e^2*f^18*z^3 - 4260*a^13*b^7*
c^2*d^18*e^15*f^5*z^3 - 4260*a^7*b^13*c^18*d^2*e^5*f^15*z^3 - 4260*a^7*b^13
*c^5*d^15*e^18*f^2*z^3 - 4260*a^5*b^15*c^18*d^2*e^7*f^13*z^3 - 4260*a^5*b^1
5*c^7*d^13*e^18*f^2*z^3 - 4260*a^2*b^18*c^15*d^5*e^13*f^7*z^3 - 4260*a^2*b^
18*c^13*d^7*e^15*f^5*z^3 + 1470*a^17*b^3*c^10*d^10*e^3*f^17*z^3 + 1470*a^17
*b^3*c^3*d^17*e^10*f^10*z^3 + 1470*a^10*b^10*c^17*d^3*e^3*f^17*z^3 + 1470*a
^10*b^10*c^3*d^17*e^17*f^3*z^3 + 1470*a^3*b^17*c^17*d^3*e^10*f^10*z^3 + 147
0*a^3*b^17*c^10*d^10*e^17*f^3*z^3 + 1350*a^18*b^2*c^9*d^11*e^3*f^17*z^3 + 1
350*a^18*b^2*c^3*d^17*e^9*f^11*z^3 + 1350*a^17*b^3*c^11*d^9*e^2*f^18*z^3 +
1350*a^17*b^3*c^2*d^18*e^11*f^9*z^3 + 1350*a^11*b^9*c^17*d^3*e^2*f^18*z^3 +
 1350*a^11*b^9*c^2*d^18*e^17*f^3*z^3 + 1350*a^9*b^11*c^18*d^2*e^3*f^17*z^3
 + 1350*a^9*b^11*c^3*d^17*e^18*f^2*z^3 + 1350*a^3*b^17*c^18*d^2*e^9*f^11*z^3
 + 1350*a^3*b^17*c^9*d^11*e^18*f^2*z^3 + 1350*a^2*b^18*c^17*d^3*e^11*f^9*z^
3 + 1350*a^2*b^18*c^11*d^9*e^17*f^3*z^3 - 1070*a^18*b^2*c^10*d^10*e^2*f^18*
z^3 - 1070*a^18*b^2*c^2*d^18*e^10*f^10*z^3 - 1070*a^10*b^10*c^18*d^2*e^2*f^

18*z^3 - 1070*a^10*b^10*c^2*d^18*e^18*f^2*z^3 - 1070*a^2*b^18*c^18*d^2*e^10
*f^10*z^3 - 1070*a^2*b^18*c^10*d^10*e^18*f^2*z^3 + 525*a^18*b^2*c^8*d^12*e^
4*f^16*z^3 + 525*a^18*b^2*c^4*d^16*e^8*f^12*z^3 + 525*a^16*b^4*c^12*d^8*e^2
*f^18*z^3 + 525*a^16*b^4*c^2*d^18*e^12*f^8*z^3 + 525*a^12*b^8*c^16*d^4*e^2*
f^18*z^3 + 525*a^12*b^8*c^2*d^18*e^16*f^4*z^3 + 525*a^8*b^12*c^18*d^2*e^4*f
^16*z^3 + 525*a^8*b^12*c^4*d^16*e^18*f^2*z^3 + 525*a^4*b^16*c^18*d^2*e^8*f^
12*z^3 + 525*a^4*b^16*c^8*d^12*e^18*f^2*z^3 + 525*a^2*b^18*c^16*d^4*e^12*f^
8*z^3 + 525*a^2*b^18*c^12*d^8*e^16*f^4*z^3 + 900*a^19*b*c^7*d^13*e^4*f^16*z
^3 + 900*a^19*b*c^4*d^16*e^7*f^13*z^3 + 900*a^16*b^4*c^13*d^7*e*f^19*z^3 +
900*a^16*b^4*c*d^19*e^13*f^7*z^3 + 900*a^13*b^7*c^16*d^4*e*f^19*z^3 + 900*a
^13*b^7*c*d^19*e^16*f^4*z^3 + 900*a^7*b^13*c^19*d*e^4*f^16*z^3 + 900*a^7*b^
13*c^4*d^16*e^19*f*z^3 + 900*a^4*b^16*c^19*d*e^7*f^13*z^3 + 900*a^4*b^16*c^
7*d^13*e^19*f*z^3 + 900*a*b^19*c^16*d^4*e^13*f^7*z^3 + 900*a*b^19*c^13*d^7*
e^16*f^4*z^3 - 750*a^19*b*c^8*d^12*e^3*f^17*z^3 - 750*a^19*b*c^3*d^17*e^8*f
^12*z^3 - 750*a^17*b^3*c^12*d^8*e*f^19*z^3 - 750*a^17*b^3*c*d^19*e^12*f^8*z
^3 - 750*a^12*b^8*c^17*d^3*e*f^19*z^3 - 750*a^12*b^8*c*d^19*e^17*f^3*z^3 -
750*a^8*b^12*c^19*d*e^3*f^17*z^3 - 750*a^8*b^12*c^3*d^17*e^19*f*z^3 - 750*a
^3*b^17*c^19*d*e^8*f^12*z^3 - 750*a^3*b^17*c^8*d^12*e^19*f*z^3 - 750*a*b^19
*c^17*d^3*e^12*f^8*z^3 - 750*a*b^19*c^12*d^8*e^17*f^3*z^3 - 420*a^19*b*c^6*
d^14*e^5*f^15*z^3 - 420*a^19*b*c^5*d^15*e^6*f^14*z^3 - 420*a^15*b^5*c^14*d^
6*e*f^19*z^3 - 420*a^15*b^5*c*d^19*e^14*f^6*z^3 - 420*a^14*b^6*c^15*d^5*e*f
^19*z^3 - 420*a^14*b^6*c*d^19*e^15*f^5*z^3 - 420*a^6*b^14*c^19*d*e^5*f^15*z
^3 - 420*a^6*b^14*c^5*d^15*e^19*f*z^3 - 420*a^5*b^15*c^19*d*e^6*f^14*z^3 -
420*a^5*b^15*c^6*d^14*e^19*f*z^3 - 420*a*b^19*c^15*d^5*e^14*f^6*z^3 - 420*a
*b^19*c^14*d^6*e^15*f^5*z^3 + 350*a^19*b*c^9*d^11*e^2*f^18*z^3 + 350*a^19*b
*c^2*d^18*e^9*f^11*z^3 + 350*a^18*b^2*c^11*d^9*e*f^19*z^3 + 350*a^18*b^2*c*
d^19*e^11*f^9*z^3 + 350*a^11*b^9*c^18*d^2*e*f^19*z^3 + 350*a^11*b^9*c*d^19*
e^18*f^2*z^3 + 350*a^9*b^11*c^19*d*e^2*f^18*z^3 + 350*a^9*b^11*c^2*d^18*e^1
9*f*z^3 + 350*a^2*b^18*c^19*d*e^9*f^11*z^3 + 350*a^2*b^18*c^9*d^11*e^19*f*z
^3 + 350*a*b^19*c^18*d^2*e^11*f^9*z^3 + 350*a*b^19*c^11*d^9*e^18*f^2*z^3 -
90*a^19*b*c^10*d^10*e*f^19*z^3 - 90*a^19*b*c*d^19*e^10*f^10*z^3 - 90*a^10*b
^10*c^19*d*e*f^19*z^3 - 90*a^10*b^10*c*d^19*e^19*f*z^3 - 90*a*b^19*c^19*d*e
^10*f^10*z^3 - 90*a*b^19*c^10*d^10*e^19*f*z^3 + 10*b^20*c^19*d*e^11*f^9*z^3
 + 10*b^20*c^11*d^9*e^19*f*z^3 + 10*a^20*c^9*d^11*e*f^19*z^3 + 10*a^20*c*d^
19*e^9*f^11*z^3 + 10*a^19*b*d^20*e^11*f^9*z^3 + 10*a^11*b^9*d^20*e^19*f*z^3
 + 10*a^9*b^11*c^20*e*f^19*z^3 + 10*a*b^19*c^20*e^9*f^11*z^3 + 10*a^19*b*c^
11*d^9*f^20*z^3 + 10*a^11*b^9*c^19*d*f^20*z^3 + 10*a^9*b^11*c*d^19*e^20*z^3
 + 10*a*b^19*c^9*d^11*e^20*z^3 + 252*b^20*c^15*d^5*e^15*f^5*z^3 - 210*b^20*
c^16*d^4*e^14*f^6*z^3 - 210*b^20*c^14*d^6*e^16*f^4*z^3 + 120*b^20*c^17*d^3*
e^13*f^7*z^3 + 120*b^20*c^13*d^7*e^17*f^3*z^3 - 45*b^20*c^18*d^2*e^12*f^8*z
^3 - 45*b^20*c^12*d^8*e^18*f^2*z^3 + 252*a^20*c^5*d^15*e^5*f^15*z^3 - 210*a
^20*c^6*d^14*e^4*f^16*z^3 - 210*a^20*c^4*d^16*e^6*f^14*z^3 + 120*a^20*c^7*d
^13*e^3*f^17*z^3 + 120*a^20*c^3*d^17*e^7*f^13*z^3 - 45*a^20*c^8*d^12*e^2*f^
18*z^3 - 45*a^20*c^2*d^18*e^8*f^12*z^3 + 252*a^15*b^5*d^20*e^15*f^5*z^3 - 2
10*a^16*b^4*d^20*e^14*f^6*z^3 - 210*a^14*b^6*d^20*e^16*f^4*z^3 + 120*a^17*b

^3*d^20*e^13*f^7*z^3 + 120*a^13*b^7*d^20*e^17*f^3*z^3 - 45*a^18*b^2*d^20*e^
12*f^8*z^3 - 45*a^12*b^8*d^20*e^18*f^2*z^3 + 252*a^5*b^15*c^20*e^5*f^15*z^3
 - 210*a^6*b^14*c^20*e^4*f^16*z^3 - 210*a^4*b^16*c^20*e^6*f^14*z^3 + 120*a^
7*b^13*c^20*e^3*f^17*z^3 + 120*a^3*b^17*c^20*e^7*f^13*z^3 - 45*a^8*b^12*c^2
0*e^2*f^18*z^3 - 45*a^2*b^18*c^20*e^8*f^12*z^3 + 252*a^15*b^5*c^15*d^5*f^20
*z^3 - 210*a^16*b^4*c^14*d^6*f^20*z^3 - 210*a^14*b^6*c^16*d^4*f^20*z^3 + 12
0*a^17*b^3*c^13*d^7*f^20*z^3 + 120*a^13*b^7*c^17*d^3*f^20*z^3 - 45*a^18*b^2
*c^12*d^8*f^20*z^3 - 45*a^12*b^8*c^18*d^2*f^20*z^3 + 252*a^5*b^15*c^5*d^15*
e^20*z^3 - 210*a^6*b^14*c^4*d^16*e^20*z^3 - 210*a^4*b^16*c^6*d^14*e^20*z^3
 + 120*a^7*b^13*c^3*d^17*e^20*z^3 + 120*a^3*b^17*c^7*d^13*e^20*z^3 - 45*a^8*
b^12*c^2*d^18*e^20*z^3 - 45*a^2*b^18*c^8*d^12*e^20*z^3 - b^20*c^20*e^10*f^1
0*z^3 - a^20*d^20*e^10*f^10*z^3 - b^20*c^10*d^10*e^20*z^3 - a^20*c^10*d^10*
f^20*z^3 - a^10*b^10*d^20*e^20*z^3 - a^10*b^10*c^20*f^20*z^3 + 1890*a^12*b^
2*c*d^13*e*f^13*z + 1890*a*b^13*c^12*d^2*e*f^13*z + 1890*a*b^13*c*d^13*e^12
*f^2*z + 92610*a^6*b^8*c^4*d^10*e^4*f^10*z + 92610*a^4*b^10*c^6*d^8*e^4*f^1
0*z + 92610*a^4*b^10*c^4*d^10*e^6*f^8*z + 66150*a^8*b^6*c^3*d^11*e^3*f^11*z
 - 66150*a^7*b^7*c^4*d^10*e^3*f^11*z - 66150*a^7*b^7*c^3*d^11*e^4*f^10*z -
66150*a^4*b^10*c^7*d^7*e^3*f^11*z - 66150*a^4*b^10*c^3*d^11*e^7*f^7*z + 661
50*a^3*b^11*c^8*d^6*e^3*f^11*z - 66150*a^3*b^11*c^7*d^7*e^4*f^10*z - 66150*
a^3*b^11*c^4*d^10*e^7*f^7*z + 66150*a^3*b^11*c^3*d^11*e^8*f^6*z - 55566*a^5
*b^9*c^5*d^9*e^4*f^10*z - 55566*a^5*b^9*c^4*d^10*e^5*f^9*z - 55566*a^4*b^10
*c^5*d^9*e^5*f^9*z - 32130*a^9*b^5*c^3*d^11*e^2*f^12*z - 32130*a^9*b^5*c^2*
d^12*e^3*f^11*z - 32130*a^3*b^11*c^9*d^5*e^2*f^12*z - 32130*a^3*b^11*c^2*d^
12*e^9*f^5*z - 32130*a^2*b^12*c^9*d^5*e^3*f^11*z - 32130*a^2*b^12*c^3*d^11*
e^9*f^5*z + 22680*a^8*b^6*c^4*d^10*e^2*f^12*z + 22680*a^8*b^6*c^2*d^12*e^4*
f^10*z + 22680*a^4*b^10*c^8*d^6*e^2*f^12*z + 22680*a^4*b^10*c^2*d^12*e^8*f^
6*z + 22680*a^2*b^12*c^8*d^6*e^4*f^10*z + 22680*a^2*b^12*c^4*d^10*e^8*f^6*z
 + 19278*a^10*b^4*c^2*d^12*e^2*f^12*z + 19278*a^2*b^12*c^10*d^4*e^2*f^12*z
 + 19278*a^2*b^12*c^2*d^12*e^10*f^4*z + 18522*a^6*b^8*c^5*d^9*e^3*f^11*z + 1
8522*a^6*b^8*c^3*d^11*e^5*f^9*z + 18522*a^5*b^9*c^6*d^8*e^3*f^11*z + 18522*
a^5*b^9*c^3*d^11*e^6*f^8*z + 18522*a^3*b^11*c^6*d^8*e^5*f^9*z + 18522*a^3*b
^11*c^5*d^9*e^6*f^8*z - 13230*a^6*b^8*c^6*d^8*e^2*f^12*z - 13230*a^6*b^8*c^
2*d^12*e^6*f^8*z - 13230*a^2*b^12*c^6*d^8*e^6*f^8*z + 3402*a^7*b^7*c^5*d^9*
e^2*f^12*z + 3402*a^7*b^7*c^2*d^12*e^5*f^9*z + 3402*a^5*b^9*c^7*d^7*e^2*f^1
2*z + 3402*a^5*b^9*c^2*d^12*e^7*f^7*z + 3402*a^2*b^12*c^7*d^7*e^5*f^9*z + 3
402*a^2*b^12*c^5*d^9*e^7*f^7*z + 7938*a^10*b^4*c^3*d^11*e*f^13*z + 7938*a^1
0*b^4*c*d^13*e^3*f^11*z + 7938*a^3*b^11*c^10*d^4*e*f^13*z + 7938*a^3*b^11*c
*d^13*e^10*f^4*z + 7938*a*b^13*c^10*d^4*e^3*f^11*z + 7938*a*b^13*c^3*d^11*e
^10*f^4*z - 5670*a^11*b^3*c^2*d^12*e*f^13*z - 5670*a^11*b^3*c*d^13*e^2*f^12
*z - 5670*a^2*b^12*c^11*d^3*e*f^13*z - 5670*a^2*b^12*c*d^13*e^11*f^3*z - 56
70*a*b^13*c^11*d^3*e^2*f^12*z - 5670*a*b^13*c^2*d^12*e^11*f^3*z - 3780*a^9*
b^5*c^4*d^10*e*f^13*z - 3780*a^9*b^5*c*d^13*e^4*f^10*z - 3780*a^4*b^10*c^9*
d^5*e*f^13*z - 3780*a^4*b^10*c*d^13*e^9*f^5*z - 3780*a*b^13*c^9*d^5*e^4*f^1
0*z - 3780*a*b^13*c^4*d^10*e^9*f^5*z - 2268*a^8*b^6*c^5*d^9*e*f^13*z - 2268
*a^8*b^6*c*d^13*e^5*f^9*z - 2268*a^5*b^9*c^8*d^6*e*f^13*z - 2268*a^5*b^9*c*

d^13*e^8*f^6*z - 2268*a*b^13*c^8*d^6*e^5*f^9*z - 2268*a*b^13*c^5*d^9*e^8*f^
6*z + 1890*a^7*b^7*c^6*d^8*e*f^13*z + 1890*a^7*b^7*c*d^13*e^6*f^8*z + 1890*
a^6*b^8*c^7*d^7*e*f^13*z + 1890*a^6*b^8*c*d^13*e^7*f^7*z + 1890*a*b^13*c^7*
d^7*e^6*f^8*z + 1890*a*b^13*c^6*d^8*e^7*f^7*z - 252*b^14*c^13*d*e*f^13*z -
252*b^14*c*d^13*e^13*f*z - 252*a^13*b*d^14*e*f^13*z - 252*a*b^13*d^14*e^13*
f*z - 252*a^13*b*c*d^13*f^14*z - 252*a*b^13*c^13*d*f^14*z - 918*b^14*c^7*d^
7*e^7*f^7*z - 882*b^14*c^11*d^3*e^3*f^11*z - 882*b^14*c^3*d^11*e^11*f^3*z +
693*b^14*c^12*d^2*e^2*f^12*z + 693*b^14*c^2*d^12*e^12*f^2*z + 567*b^14*c^8
*d^6*e^6*f^8*z + 567*b^14*c^6*d^8*e^8*f^6*z + 441*b^14*c^10*d^4*e^4*f^10*z
+ 441*b^14*c^4*d^10*e^10*f^4*z - 126*b^14*c^9*d^5*e^5*f^9*z - 126*b^14*c^5*
d^9*e^9*f^5*z - 918*a^7*b^7*d^14*e^7*f^7*z - 882*a^11*b^3*d^14*e^3*f^11*z -
882*a^3*b^11*d^14*e^11*f^3*z + 693*a^12*b^2*d^14*e^2*f^12*z + 693*a^2*b^12
*d^14*e^12*f^2*z + 567*a^8*b^6*d^14*e^6*f^8*z + 567*a^6*b^8*d^14*e^8*f^6*z
+ 441*a^10*b^4*d^14*e^4*f^10*z + 441*a^4*b^10*d^14*e^10*f^4*z - 126*a^9*b^5
*d^14*e^5*f^9*z - 126*a^5*b^9*d^14*e^9*f^5*z - 918*a^7*b^7*c^7*d^7*f^14*z -
882*a^11*b^3*c^3*d^11*f^14*z - 882*a^3*b^11*c^11*d^3*f^14*z + 693*a^12*b^2
*c^2*d^12*f^14*z + 693*a^2*b^12*c^12*d^2*f^14*z + 567*a^8*b^6*c^6*d^8*f^14*
z + 567*a^6*b^8*c^8*d^6*f^14*z + 441*a^10*b^4*c^4*d^10*f^14*z + 441*a^4*b^1
0*c^10*d^4*f^14*z - 126*a^9*b^5*c^5*d^9*f^14*z - 126*a^5*b^9*c^9*d^5*f^14*z
+ 36*b^14*d^14*e^14*z + 36*b^14*c^14*f^14*z + 36*a^14*d^14*f^14*z - 27054*
a^2*b^9*c^2*d^9*e^2*f^9 + 9018*a^3*b^8*c^2*d^9*e*f^10 + 9018*a^3*b^8*c*d^10
*e^2*f^9 + 9018*a^2*b^9*c^3*d^8*e*f^10 + 9018*a^2*b^9*c*d^10*e^3*f^8 + 9018
*a*b^10*c^3*d^8*e^2*f^9 + 9018*a*b^10*c^2*d^9*e^3*f^8 - 9018*a^4*b^7*c*d^10
*e*f^10 - 9018*a*b^10*c^4*d^7*e*f^10 - 9018*a*b^10*c*d^10*e^4*f^7 + 2268*b^
11*c^5*d^6*e*f^10 + 2268*b^11*c*d^10*e^5*f^6 + 2268*a^5*b^6*d^11*e*f^10 + 2
268*a*b^10*d^11*e^5*f^6 + 2268*a^5*b^6*c*d^10*f^11 + 2268*a*b^10*c^5*d^6*f^
11 - 1458*b^11*c^3*d^8*e^3*f^8 - 1161*b^11*c^4*d^7*e^2*f^9 - 1161*b^11*c^2*
d^9*e^4*f^7 - 1458*a^3*b^8*d^11*e^3*f^8 - 1161*a^4*b^7*d^11*e^2*f^9 - 1161*
a^2*b^9*d^11*e^4*f^7 - 1458*a^3*b^8*c^3*d^8*f^11 - 1161*a^4*b^7*c^2*d^9*f^1
1 - 1161*a^2*b^9*c^4*d^7*f^11 - 756*b^11*d^11*e^6*f^5 - 756*b^11*c^6*d^5*f^
11 - 756*a^6*b^5*d^11*f^11, z, k), k, 1, 3) - ((7*a*b^6*c^2*d^5*e^7 - a^3*b
^4*d^7*e^7 - a^7*c^3*d^4*f^7 - b^7*c^3*d^4*e^7 - a^7*d^7*e^3*f^4 - b^7*c^7*
e^3*f^4 - 6*a^5*b^2*c^5*d^2*f^7 - 6*a^5*b^2*d^7*e^5*f^2 - 6*b^7*c^5*d^2*e^5
*f^2 - a^3*b^4*c^7*f^7 + 7*a^2*b^5*c*d^6*e^7 + 4*a^4*b^3*c^6*d*f^7 + 4*a^6*
b*c^4*d^3*f^7 + 7*a*b^6*c^7*e^2*f^5 + 7*a^2*b^5*c^7*e*f^6 + 4*a^4*b^3*d^7*e
^6*f + 4*a^6*b*d^7*e^4*f^3 + 7*a^7*c*d^6*e^2*f^5 + 7*a^7*c^2*d^5*e*f^6 + 4*
b^7*c^4*d^3*e^6*f + 4*b^7*c^6*d*e^4*f^3 - 21*a*b^6*c^3*d^4*e^6*f - 21*a*b^6
*c^6*d*e^3*f^4 - 21*a^3*b^4*c*d^6*e^6*f - 21*a^3*b^4*c^6*d*e*f^6 - 21*a^6*b
*c*d^6*e^3*f^4 - 21*a^6*b*c^3*d^4*e*f^6 + 14*a*b^6*c^4*d^3*e^5*f^2 + 14*a*b
^6*c^5*d^2*e^4*f^3 - 26*a^2*b^5*c^2*d^5*e^6*f - 26*a^2*b^5*c^6*d*e^2*f^5 +
14*a^4*b^3*c*d^6*e^5*f^2 + 14*a^4*b^3*c^5*d^2*e*f^6 + 14*a^5*b^2*c*d^6*e^4*
f^3 + 14*a^5*b^2*c^4*d^3*e*f^6 - 26*a^6*b*c^2*d^5*e^2*f^5 + 52*a^2*b^5*c^3*
d^4*e^5*f^2 - 78*a^2*b^5*c^4*d^3*e^4*f^3 + 52*a^2*b^5*c^5*d^2*e^3*f^4 + 52*
a^3*b^4*c^2*d^5*e^5*f^2 + 52*a^3*b^4*c^5*d^2*e^2*f^5 - 78*a^4*b^3*c^2*d^5*e
^4*f^3 - 78*a^4*b^3*c^4*d^3*e^2*f^5 + 52*a^5*b^2*c^2*d^5*e^3*f^4 + 52*a^5*b

^2*c^3*d^4*e^2*f^5)/(2*(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*c^4*d^4
*f^8 - b^8*c^4*d^4*e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b^6*c^2*
d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*d^8*e^6
*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f^8 + 4*
a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*b^7*c^8
*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e^5*f^3
+ 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f + 4*b^8*c
^7*d*e^5*f^3 - 12*a*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*a^4*b^4
*c*d^7*e^7*f - 12*a^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*a^7*b*c
^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8*a^2*b^
6*c^3*d^5*e^7*f + 8*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f + 8*a^3
*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f^7 + 8*
a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^3*f^5 +
 8*a^7*b*c^3*d^5*e^2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c^5*d^3*
e^5*f^3 + 22*a^2*b^6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 + 36*a^3*
b^5*c^4*d^4*e^5*f^3 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^2*e^3*f
^5 + 22*a^4*b^4*c^2*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^4*b^4*c
^4*d^4*e^4*f^4 + 36*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2*f^6 -
48*a^5*b^3*c^2*d^6*e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3*c^4*d^
4*e^3*f^5 - 48*a^5*b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4 - 48*a^
6*b^2*c^3*d^5*e^3*f^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6)) + (3*x^5*(2*a^5*b^2*d^
7*f^7 + 2*b^7*c^5*d^2*f^7 + 2*b^7*d^7*e^5*f^2 + 2*a^2*b^5*c^3*d^4*f^7 + 2*a
^3*b^4*c^2*d^5*f^7 + 2*a^2*b^5*d^7*e^3*f^4 + 2*a^3*b^4*d^7*e^2*f^5 + 2*b^7*
c^2*d^5*e^3*f^4 + 2*b^7*c^3*d^4*e^2*f^5 - 5*a*b^6*c^4*d^3*f^7 - 5*a^4*b^3*c
*d^6*f^7 - 5*a*b^6*d^7*e^4*f^3 - 5*a^4*b^3*d^7*e*f^6 - 5*b^7*c*d^6*e^4*f^3
- 5*b^7*c^4*d^3*e*f^6 + 16*a*b^6*c*d^6*e^3*f^4 + 16*a*b^6*c^3*d^4*e*f^6 + 1
6*a^3*b^4*c*d^6*e*f^6 - 12*a*b^6*c^2*d^5*e^2*f^5 - 12*a^2*b^5*c*d^6*e^2*f^5
 - 12*a^2*b^5*c^2*d^5*e*f^6))/(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*
c^4*d^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b
^6*c^2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*
d^8*e^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f
^8 + 4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*
b^7*c^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e
^5*f^3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f +
4*b^8*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*
a^4*b^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*
a^7*b*c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8
*a^2*b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f
+ 8*a^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f
^7 + 8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^
3*f^5 + 8*a^7*b*c^3*d^5*e^2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c
^5*d^3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 +
36*a^3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^
2*e^3*f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^
4*b^4*c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2

*f^6 - 48*a^5*b^3*c^2*d^6*e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3
*c^4*d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4
- 48*a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6) + (3*x^4*(8*a^6*
b*d^7*f^7 + 8*b^7*c^6*d*f^7 + 8*b^7*d^7*e^6*f - 7*a^2*b^5*c^4*d^3*f^7 + 14*
a^3*b^4*c^3*d^4*f^7 - 7*a^4*b^3*c^2*d^5*f^7 - 7*a^2*b^5*d^7*e^4*f^3 + 14*a^
3*b^4*d^7*e^3*f^4 - 7*a^4*b^3*d^7*e^2*f^5 - 7*b^7*c^2*d^5*e^4*f^3 + 14*b^7*
c^3*d^4*e^3*f^4 - 7*b^7*c^4*d^3*e^2*f^5 - 14*a*b^6*c^5*d^2*f^7 - 14*a^5*b^2
*c*d^6*f^7 - 14*a*b^6*d^7*e^5*f^2 - 14*a^5*b^2*d^7*e*f^6 - 14*b^7*c*d^6*e^5
*f^2 - 14*b^7*c^5*d^2*e*f^6 + 34*a*b^6*c*d^6*e^4*f^3 + 34*a*b^6*c^4*d^3*e*f
^6 + 34*a^4*b^3*c*d^6*e*f^6 + 6*a*b^6*c^2*d^5*e^3*f^4 + 6*a*b^6*c^3*d^4*e^2
*f^5 + 6*a^2*b^5*c*d^6*e^3*f^4 + 6*a^2*b^5*c^3*d^4*e*f^6 + 6*a^3*b^4*c*d^6*
e^2*f^5 + 6*a^3*b^4*c^2*d^5*e*f^6 - 78*a^2*b^5*c^2*d^5*e^2*f^5))/(2*(4*a*b^
7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*c^4*d^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d
^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b^6*c^2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^
8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*d^8*e^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 -
6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f^8 + 4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*
c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*b^7*c^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7
+ 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e^5*f^3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*
c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f + 4*b^8*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^
4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*a^4*b^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*
d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*a^7*b*c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3
*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8*a^2*b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^
7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f + 8*a^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3
*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f^7 + 8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*
b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^3*f^5 + 8*a^7*b*c^3*d^5*e^2*f^6 + 22*
a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c^5*d^3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e
^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 + 36*a^3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b
^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^2*e^3*f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^
2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^4*b^4*c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^
5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2*f^6 - 48*a^5*b^3*c^2*d^6*e^5*f^3 + 3
6*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3*c^4*d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3
*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4 - 48*a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6
*b^2*c^4*d^4*e^2*f^6)) + (x^2*(18*a*b^6*c^7*f^7 + 18*a*b^6*d^7*e^7 + 18*a^7
*c*d^6*f^7 + 18*b^7*c*d^6*e^7 + 18*a^7*d^7*e*f^6 + 18*b^7*c^7*e*f^6 - 3*a^3
*b^4*c^5*d^2*f^7 + 32*a^4*b^3*c^4*d^3*f^7 - 3*a^5*b^2*c^3*d^4*f^7 - 3*a^3*b
^4*d^7*e^5*f^2 + 32*a^4*b^3*d^7*e^4*f^3 - 3*a^5*b^2*d^7*e^3*f^4 - 3*b^7*c^3
*d^4*e^5*f^2 + 32*b^7*c^4*d^3*e^4*f^3 - 3*b^7*c^5*d^2*e^3*f^4 - 37*a^2*b^5*
c^6*d*f^7 - 37*a^6*b*c^2*d^5*f^7 - 37*a^2*b^5*d^7*e^6*f - 37*a^6*b*d^7*e^2*
f^5 - 37*b^7*c^2*d^5*e^6*f - 37*b^7*c^6*d*e^2*f^5 + 9*a*b^6*c^2*d^5*e^5*f^2
+ a*b^6*c^3*d^4*e^4*f^3 + a*b^6*c^4*d^3*e^3*f^4 + 9*a*b^6*c^5*d^2*e^2*f^5
+ 9*a^2*b^5*c*d^6*e^5*f^2 + 9*a^2*b^5*c^5*d^2*e*f^6 + a^3*b^4*c*d^6*e^4*f^3
+ a^3*b^4*c^4*d^3*e*f^6 + a^4*b^3*c*d^6*e^3*f^4 + a^4*b^3*c^3*d^4*e*f^6 +
9*a^5*b^2*c*d^6*e^2*f^5 + 9*a^5*b^2*c^2*d^5*e*f^6 - 34*a*b^6*c*d^6*e^6*f -
34*a*b^6*c^6*d*e*f^6 - 34*a^6*b*c*d^6*e*f^6 + 234*a^2*b^5*c^2*d^5*e^4*f^3 -
208*a^2*b^5*c^3*d^4*e^3*f^4 + 234*a^2*b^5*c^4*d^3*e^2*f^5 - 208*a^3*b^4*c^

2*d^5*e^3*f^4 - 208*a^3*b^4*c^3*d^4*e^2*f^5 + 234*a^4*b^3*c^2*d^5*e^2*f^5))
/(2*(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*c^4*d^4*f^8 - b^8*c^4*d^4*
e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b^6*c^2*d^6*e^8 - 6*a^6*b^2
*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*d^8*e^6*f^2 - 6*a^8*c^2*d^
6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f^8 + 4*a^3*b^5*c*d^7*e^8 +
 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*b^7*c^8*e^3*f^5 + 4*a^3*b^
5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e^5*f^3 + 4*a^8*c*d^7*e^3*f
^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f + 4*b^8*c^7*d*e^5*f^3 - 12*a
*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*a^4*b^4*c*d^7*e^7*f - 12*a
^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*a^7*b*c^4*d^4*e*f^7 + 8*a*
b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8*a^2*b^6*c^3*d^5*e^7*f + 8
*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f + 8*a^3*b^5*c^7*d*e^2*f^6
 + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f^7 + 8*a^6*b^2*c*d^7*e^5*f
^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^3*f^5 + 8*a^7*b*c^3*d^5*e^
2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c^5*d^3*e^5*f^3 + 22*a^2*b^
6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 + 36*a^3*b^5*c^4*d^4*e^5*f^3
 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^2*e^3*f^5 + 22*a^4*b^4*c^2
*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^4*b^4*c^4*d^4*e^4*f^4 + 36
*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2*f^6 - 48*a^5*b^3*c^2*d^6*
e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3*c^4*d^4*e^3*f^5 - 48*a^5*
b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4 - 48*a^6*b^2*c^3*d^5*e^3*f
^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6)) + (x*(2*a^2*b^5*c^7*f^7 + 2*a^2*b^5*d^7*e
^7 + 2*a^7*c^2*d^5*f^7 + 2*b^7*c^2*d^5*e^7 + 2*a^7*d^7*e^2*f^5 + 2*b^7*c^7*
e^2*f^5 + 4*a^4*b^3*c^5*d^2*f^7 + 4*a^5*b^2*c^4*d^3*f^7 + 4*a^4*b^3*d^7*e^5
*f^2 + 4*a^5*b^2*d^7*e^4*f^3 + 4*b^7*c^4*d^3*e^5*f^2 + 4*b^7*c^5*d^2*e^4*f^
3 + 14*a*b^6*c*d^6*e^7 + 14*a*b^6*c^7*e*f^6 + 14*a^7*c*d^6*e*f^6 - 6*a^3*b^
4*c^6*d*f^7 - 6*a^6*b*c^3*d^4*f^7 - 6*a^3*b^4*d^7*e^6*f - 6*a^6*b*d^7*e^3*f
^4 - 6*b^7*c^3*d^4*e^6*f - 6*b^7*c^6*d*e^3*f^4 - 33*a*b^6*c^2*d^5*e^6*f - 3
3*a*b^6*c^6*d*e^2*f^5 - 33*a^2*b^5*c*d^6*e^6*f - 33*a^2*b^5*c^6*d*e*f^6 - 3
3*a^6*b*c*d^6*e^2*f^5 - 33*a^6*b*c^2*d^5*e*f^6 + 17*a*b^6*c^3*d^4*e^5*f^2 -
 8*a*b^6*c^4*d^3*e^4*f^3 + 17*a*b^6*c^5*d^2*e^3*f^4 + 17*a^3*b^4*c*d^6*e^5*
f^2 + 17*a^3*b^4*c^5*d^2*e*f^6 - 8*a^4*b^3*c*d^6*e^4*f^3 - 8*a^4*b^3*c^4*d^
3*e*f^6 + 17*a^5*b^2*c*d^6*e^3*f^4 + 17*a^5*b^2*c^3*d^4*e*f^6 + 78*a^2*b^5*
c^2*d^5*e^5*f^2 - 26*a^2*b^5*c^3*d^4*e^4*f^3 - 26*a^2*b^5*c^4*d^3*e^3*f^4 +
 78*a^2*b^5*c^5*d^2*e^2*f^5 - 26*a^3*b^4*c^2*d^5*e^4*f^3 - 26*a^3*b^4*c^4*d
^3*e^2*f^5 - 26*a^4*b^3*c^2*d^5*e^3*f^4 - 26*a^4*b^3*c^3*d^4*e^2*f^5 + 78*a
^5*b^2*c^2*d^5*e^2*f^5))/(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*c^4*d
^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b^6*c^
2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*d^8*e
^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f^8 +
4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*b^7*c
^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e^5*f^
3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f + 4*b^8
*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*a^4*b
^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*a^7*b

*c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8*a^2*
b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f + 8*a
^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f^7 +
8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^3*f^5
 + 8*a^7*b*c^3*d^5*e^2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c^5*d^
3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 + 36*a^
3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^2*e^3
*f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^4*b^4
*c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2*f^6
- 48*a^5*b^3*c^2*d^6*e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3*c^4*
d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4 - 48*
a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6) + (x^3*(6*a^7*d^7*f^7
 + 6*b^7*c^7*f^7 + 6*b^7*d^7*e^7 - 37*a^2*b^5*c^5*d^2*f^7 + 19*a^3*b^4*c^4*
d^3*f^7 + 19*a^4*b^3*c^3*d^4*f^7 - 37*a^5*b^2*c^2*d^5*f^7 - 37*a^2*b^5*d^7*
e^5*f^2 + 19*a^3*b^4*d^7*e^4*f^3 + 19*a^4*b^3*d^7*e^3*f^4 - 37*a^5*b^2*d^7*
e^2*f^5 - 37*b^7*c^2*d^5*e^5*f^2 + 19*b^7*c^3*d^4*e^4*f^3 + 19*b^7*c^4*d^3*
e^3*f^4 - 37*b^7*c^5*d^2*e^2*f^5 + 3*a*b^6*c^6*d*f^7 + 3*a^6*b*c*d^6*f^7 +
3*a*b^6*d^7*e^6*f + 3*a^6*b*d^7*e*f^6 + 3*b^7*c*d^6*e^6*f + 3*b^7*c^6*d*e*f
^6 - 28*a*b^6*c*d^6*e^5*f^2 - 28*a*b^6*c^5*d^2*e*f^6 - 28*a^5*b^2*c*d^6*e*f
^6 + 86*a*b^6*c^2*d^5*e^4*f^3 - 68*a*b^6*c^3*d^4*e^3*f^4 + 86*a*b^6*c^4*d^3
*e^2*f^5 + 86*a^2*b^5*c*d^6*e^4*f^3 + 86*a^2*b^5*c^4*d^3*e*f^6 - 68*a^3*b^4
*c*d^6*e^3*f^4 - 68*a^3*b^4*c^3*d^4*e*f^6 + 86*a^4*b^3*c*d^6*e^2*f^5 + 86*a
^4*b^3*c^2*d^5*e*f^6 - 52*a^2*b^5*c^2*d^5*e^3*f^4 - 52*a^2*b^5*c^3*d^4*e^2*
f^5 - 52*a^3*b^4*c^2*d^5*e^2*f^5))/(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 -
 a^8*c^4*d^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*
a^2*b^6*c^2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6
*b^2*d^8*e^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*
c^8*f^8 + 4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 +
 4*a*b^7*c^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*
d^8*e^5*f^3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7
*f + 4*b^8*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4
- 12*a^4*b^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4
- 12*a^7*b*c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^
3 + 8*a^2*b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e
^7*f + 8*a^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^
2*e*f^7 + 8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d
^6*e^3*f^5 + 8*a^7*b*c^3*d^5*e^2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*
b^6*c^5*d^3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f
^2 + 36*a^3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c
^6*d^2*e^3*f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 -
90*a^4*b^4*c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^
2*e^2*f^6 - 48*a^5*b^3*c^2*d^6*e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^
5*b^3*c^4*d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4
*f^4 - 48*a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6))/(x*(2*a*b*
c^2*e^2 + 2*a^2*c*d*e^2 + 2*a^2*c^2*e*f) + x^3*(2*a*b*c^2*f^2 + 2*a*b*d^2*e

```
^2 + 2*a^2*c*d*f^2 + 2*b^2*c*d*e^2 + 2*a^2*d^2*e*f + 2*b^2*c^2*e*f + 8*a*b*
c*d*e*f) + x^2*(a^2*c^2*f^2 + a^2*d^2*e^2 + b^2*c^2*e^2 + 4*a*b*c*d*e^2 + 4
*a*b*c^2*e*f + 4*a^2*c*d*e*f) + x^5*(2*a*b*d^2*f^2 + 2*b^2*c*d*f^2 + 2*b^2*
d^2*e*f) + x^4*(a^2*d^2*f^2 + b^2*c^2*f^2 + b^2*d^2*e^2 + 4*a*b*c*d*f^2 + 4
*a*b*d^2*e*f + 4*b^2*c*d*e*f) + a^2*c^2*e^2 + b^2*d^2*f^2*x^6)
```

3.21 $\int \frac{1}{1+x+x^2+x^3}\,dx$

Optimal result 314
Rubi [A] (verified) 314
Mathematica [A] (verified) 315
Maple [A] (verified) 315
Fricas [A] (verification not implemented) 316
Sympy [A] (verification not implemented) 316
Maxima [A] (verification not implemented) 316
Giac [A] (verification not implemented) 317
Mupad [B] (verification not implemented) 317

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{1}{1+x+x^2+x^3}\,dx = \frac{\arctan(x)}{2}+\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)$$

```
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2083, 649, 209, 266}

$$\int \frac{1}{1+x+x^2+x^3}\,dx = \frac{\arctan(x)}{2}-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)$$

```
[In] Int[(1 + x + x^2 + x^3)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2(1+x)}+\frac{1-x}{2\left(1+x^2\right)}\right) dx \\
&= \frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1-x}{1+x^2}\,dx \\
&= \frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx-\frac{1}{2}\int \frac{x}{1+x^2}\,dx \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x+x^2+x^3}\,dx=\frac{\arctan(x)}{2}+\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)$$

```
[In] Integrate[(1 + x + x^2 + x^3)^(-1),x]
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{2}-\frac{\ln(x^2+1)}{4}$	20
risch	$\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{2}-\frac{\ln(x^2+1)}{4}$	20
parallelrisch	$\frac{\ln(x+1)}{2}-\frac{\ln(x-i)}{4}-\frac{i\ln(x-i)}{4}-\frac{\ln(x+i)}{4}+\frac{i\ln(x+i)}{4}$	38

```
[In] int(1/(x^3+x^2+x+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x)+1/2*ln(x+1)-1/4*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+x+x^2+x^3}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1)$$

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+x+x^2+x^3}\,dx = \frac{\log(x+1)}{2} - \frac{\log(x^2+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

```
[In] integrate(1/(x**3+x**2+x+1),x)
[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+x+x^2+x^3}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1)$$

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="maxima")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{1+x+x^2+x^3}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log\left(|x+1|\right)$$

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="giac")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x+x^2+x^3}\,dx = \frac{\ln(x+1)}{2} + \ln(x-\mathrm{i})\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

```
[In] int(1/(x + x^2 + x^3 + 1),x)
[Out] log(x + 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)
```

3.22 $\int \frac{1}{-1+4x-4x^2+16x^3} \, dx$

Optimal result 318
Rubi [A] (verified) 318
Mathematica [A] (verified) 319
Maple [A] (verified) 319
Fricas [A] (verification not implemented) 320
Sympy [A] (verification not implemented) 320
Maxima [A] (verification not implemented) 320
Giac [A] (verification not implemented) 321
Mupad [B] (verification not implemented) 321

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{1}{-1+4x-4x^2+16x^3} \, dx = -\frac{1}{10}\arctan(2x)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\log\left(1+4x^2\right)$$

```
[Out] -1/10*arctan(2*x)+1/5*ln(1-4*x)-1/10*ln(4*x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2083, 649, 209, 266}

$$\int \frac{1}{-1+4x-4x^2+16x^3} \, dx = -\frac{1}{10}\arctan(2x)-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log(1-4x)$$

```
[In] Int[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1),x]
```

```
[Out] -1/10*ArcTan[2*x] + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{4}{5(-1+4x)}+\frac{-1-4x}{5\left(1+4x^2\right)}\right)\,dx\\ &= \frac{1}{5}\log(1-4x)+\frac{1}{5}\int \frac{-1-4x}{1+4x^2}\,dx\\ &= \frac{1}{5}\log(1-4x)-\frac{1}{5}\int \frac{1}{1+4x^2}\,dx-\frac{4}{5}\int \frac{x}{1+4x^2}\,dx\\ &= -\frac{1}{10}\tan^{-1}(2x)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\log\left(1+4x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{-1+4x-4x^2+16x^3}\,dx=-\frac{1}{10}\arctan(2x)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\log\left(1+4x^2\right)$$

```
[In] Integrate[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1),x]
[Out] -1/10*ArcTan[2*x] + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\ln(4x^2+1)}{10}-\frac{\arctan(2x)}{10}+\frac{\ln(-1+4x)}{5}$	26
risch	$-\frac{\ln(4x^2+1)}{10}-\frac{\arctan(2x)}{10}+\frac{\ln(-1+4x)}{5}$	26
parallelrisch	$-\frac{\ln(x-\frac{i}{2})}{10}+\frac{i\ln(x-\frac{i}{2})}{20}-\frac{\ln(x+\frac{i}{2})}{10}-\frac{i\ln(x+\frac{i}{2})}{20}+\frac{\ln(x-\frac{1}{4})}{5}$	38

```
[In] int(1/(16*x^3-4*x^2+4*x-1),x,method=_RETURNVERBOSE)
[Out] -1/10*ln(4*x^2+1)-1/10*arctan(2*x)+1/5*ln(-1+4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{-1+4x-4x^2+16x^3}\,dx = -\frac{1}{10}\,\arctan\left(2\,x\right) - \frac{1}{10}\,\log\left(4\,x^2+1\right) + \frac{1}{5}\,\log\left(4\,x-1\right)$$

```
[In] integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="fricas")
[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(4*x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{-1+4x-4x^2+16x^3}\,dx = \frac{\log\left(x-\frac{1}{4}\right)}{5} - \frac{\log\left(x^2+\frac{1}{4}\right)}{10} - \frac{\operatorname{atan}\left(2x\right)}{10}$$

```
[In] integrate(1/(16*x**3-4*x**2+4*x-1),x)
[Out] log(x - 1/4)/5 - log(x**2 + 1/4)/10 - atan(2*x)/10
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{-1+4x-4x^2+16x^3}\,dx = -\frac{1}{10}\,\arctan\left(2\,x\right) - \frac{1}{10}\,\log\left(4\,x^2+1\right) + \frac{1}{5}\,\log\left(4\,x-1\right)$$

```
[In] integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="maxima")
[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(4*x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{-1+4x-4x^2+16x^3}\,dx = -\frac{1}{10}\,\arctan\left(2\,x\right) - \frac{1}{10}\,\log\left(4\,x^2+1\right) + \frac{1}{5}\,\log\left(\left|4\,x-1\right|\right)$$

```
[In] integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="giac")
[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(abs(4*x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{-1+4x-4x^2+16x^3}\,dx = \frac{\ln\left(x-\frac{1}{4}\right)}{5} + \ln\left(x-\frac{1}{2}\mathrm{i}\right)\,\left(-\frac{1}{10}+\frac{1}{20}\mathrm{i}\right) + \ln\left(x+\frac{1}{2}\mathrm{i}\right)\,\left(-\frac{1}{10}-\frac{1}{20}\mathrm{i}\right)$$

```
[In] int(1/(4*x - 4*x^2 + 16*x^3 - 1),x)
[Out] log(x - 1/4)/5 - log(x - 1i/2)*(1/10 - 1i/20) - log(x + 1i/2)*(1/10 + 1i/20
)
```

3.23 $\int \frac{1}{dx^3} \, dx$

Optimal result 322
Rubi [A] (verified) 322
Mathematica [A] (verified) 323
Maple [A] (verified) 323
Fricas [A] (verification not implemented) 324
Sympy [A] (verification not implemented) 324
Maxima [A] (verification not implemented) 324
Giac [A] (verification not implemented) 324
Mupad [B] (verification not implemented) 325

Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{dx^3} \, dx = -\frac{1}{2dx^2}$$

```
[Out] -1/2/d/x^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {12, 30}

$$\int \frac{1}{dx^3} \, dx = -\frac{1}{2dx^2}$$

```
[In] Int[1/(d*x^3),x]
```

```
[Out] -1/2*1/(d*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int \frac{1}{x^3}\,dx}{d} \\ &= -\frac{1}{2dx^2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{dx^3}\,dx = -\frac{1}{2dx^2}$$

```
[In] Integrate[1/(d*x^3),x]
[Out] -1/2*1/(d*x^2)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
gosper	$-\frac{1}{2d\,x^2}$	9
default	$-\frac{1}{2d\,x^2}$	9
norman	$-\frac{1}{2d\,x^2}$	9
risch	$-\frac{1}{2d\,x^2}$	9
parallelrisch	$-\frac{1}{2d\,x^2}$	9

```
[In] int(1/d/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2/d/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{dx^3}\,dx = -\frac{1}{2\,dx^2}$$

```
[In] integrate(1/d/x^3,x, algorithm="fricas")
[Out] -1/2/(d*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{dx^3}\,dx = -\frac{1}{2dx^2}$$

```
[In] integrate(1/d/x**3,x)
[Out] -1/(2*d*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{dx^3}\,dx = -\frac{1}{2\,dx^2}$$

```
[In] integrate(1/d/x^3,x, algorithm="maxima")
[Out] -1/2/(d*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{dx^3}\,dx = -\frac{1}{2\,dx^2}$$

```
[In] integrate(1/d/x^3,x, algorithm="giac")
[Out] -1/2/(d*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{dx^3}\,dx = -\frac{1}{2\,d\,x^2}$$

```
[In] int(1/(d*x^3),x)
[Out] -1/(2*d*x^2)
```

3.24 $\int \frac{1}{cx^2+dx^3}\,dx$

Optimal result 326
Rubi [A] (verified) 326
Mathematica [A] (verified) 327
Maple [A] (verified) 327
Fricas [A] (verification not implemented) 328
Sympy [A] (verification not implemented) 328
Maxima [A] (verification not implemented) 328
Giac [A] (verification not implemented) 328
Mupad [B] (verification not implemented) 329

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{cx^2+dx^3}\,dx=-\frac{1}{cx}-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}$$

```
[Out] -1/c/x-d*ln(x)/c^2+d*ln(d*x+c)/c^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1607, 46}

$$\int \frac{1}{cx^2+dx^3}\,dx=-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}-\frac{1}{cx}$$

```
[In] Int[(c*x^2 + d*x^3)^(-1),x]
```

```
[Out] -(1/(c*x)) - (d*Log[x])/c^2 + (d*Log[c + d*x])/c^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^2(c+dx)}\,dx \\
&= \int \left(\frac{1}{cx^2}-\frac{d}{c^2x}+\frac{d^2}{c^2(c+dx)}\right)dx \\
&= -\frac{1}{cx}-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{cx^2+dx^3}\,dx=-\frac{1}{cx}-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}$$

```
[In] Integrate[(c*x^2 + d*x^3)^(-1),x]
[Out] -(1/(c*x)) - (d*Log[x])/c^2 + (d*Log[c + d*x])/c^2
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$-\frac{d\ln(x)x-d\ln(dx+c)x+c}{c^2x}$	26
default	$-\frac{1}{cx}-\frac{d\ln(x)}{c^2}+\frac{d\ln(dx+c)}{c^2}$	29
norman	$-\frac{1}{cx}-\frac{d\ln(x)}{c^2}+\frac{d\ln(dx+c)}{c^2}$	29
risch	$-\frac{1}{cx}+\frac{d\ln(-dx-c)}{c^2}-\frac{d\ln(x)}{c^2}$	32

```
[In] int(1/(d*x^3+c*x^2),x,method=_RETURNVERBOSE)
[Out] -(d*ln(x)*x-d*ln(d*x+c)*x+c)/c^2/x
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{cx^2 + dx^3}\, dx = \frac{dx \log(dx + c) - dx \log(x) - c}{c^2 x}$$

```
[In] integrate(1/(d*x^3+c*x^2),x, algorithm="fricas")
[Out] (d*x*log(d*x + c) - d*x*log(x) - c)/(c^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{cx^2 + dx^3}\, dx = -\frac{1}{cx} + \frac{d\left(-\log(x) + \log\left(\frac{c}{d} + x\right)\right)}{c^2}$$

```
[In] integrate(1/(d*x**3+c*x**2),x)
[Out] -1/(c*x) + d*(-log(x) + log(c/d + x))/c**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{cx^2 + dx^3}\, dx = \frac{d \log(dx + c)}{c^2} - \frac{d \log(x)}{c^2} - \frac{1}{cx}$$

```
[In] integrate(1/(d*x^3+c*x^2),x, algorithm="maxima")
[Out] d*log(d*x + c)/c^2 - d*log(x)/c^2 - 1/(c*x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{cx^2 + dx^3}\, dx = \frac{d \log(|dx + c|)}{c^2} - \frac{d \log(|x|)}{c^2} - \frac{1}{cx}$$

```
[In] integrate(1/(d*x^3+c*x^2),x, algorithm="giac")
[Out] d*log(abs(d*x + c))/c^2 - d*log(abs(x))/c^2 - 1/(c*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{cx^2 + dx^3}\,dx = \frac{2\,d\,\mathrm{atanh}\left(\frac{2\,d\,x}{c} + 1\right)}{c^2} - \frac{1}{c\,x}$$

```
[In] int(1/(c*x^2 + d*x^3),x)
[Out] (2*d*atanh((2*d*x)/c + 1))/c^2 - 1/(c*x)
```

3.25 $\int \frac{1}{bx+dx^3}\,dx$

Optimal result 330
Rubi [A] (verified) 330
Mathematica [A] (verified) 331
Maple [A] (verified) 332
Fricas [A] (verification not implemented) 332
Sympy [A] (verification not implemented) 332
Maxima [A] (verification not implemented) 333
Giac [A] (verification not implemented) 333
Mupad [B] (verification not implemented) 333

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{1}{bx+dx^3}\,dx=\frac{\log(x)}{b}-\frac{\log\left(b+dx^2\right)}{2b}$$

```
[Out] ln(x)/b-1/2*ln(d*x^2+b)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {1607, 272, 36, 29, 31}

$$\int \frac{1}{bx+dx^3}\,dx=\frac{\log(x)}{b}-\frac{\log\left(b+dx^2\right)}{2b}$$

```
[In] Int[(b*x + d*x^3)^(-1),x]
```

```
[Out] Log[x]/b - Log[b + d*x^2]/(2*b)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x\left(b+dx^2\right)}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(b+dx)}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2b}-\frac{d\text{Subst}\left(\int \frac{1}{b+dx}\,dx,x,x^2\right)}{2b}\\
&= \frac{\log(x)}{b}-\frac{\log\left(b+dx^2\right)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{bx+dx^3}\,dx=\frac{\log(x)}{b}-\frac{\log\left(b+dx^2\right)}{2b}$$

```
[In] Integrate[(b*x + d*x^3)^(-1),x]
[Out] Log[x]/b - Log[b + d*x^2]/(2*b)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\ln(x)}{b}-\frac{\ln(d\,x^{2}+b)}{2b}$	21
norman	$\frac{\ln(x)}{b}-\frac{\ln(d\,x^{2}+b)}{2b}$	21
risch	$\frac{\ln(x)}{b}-\frac{\ln(d\,x^{2}+b)}{2b}$	21
parallelrisch	$\frac{2\ln(x)-\ln(d\,x^{2}+b)}{2b}$	21

```
[In] int(1/(d*x^3+b*x),x,method=_RETURNVERBOSE)
[Out] ln(x)/b-1/2*ln(d*x^2+b)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{bx+dx^3}\,dx=-\frac{\log\left(dx^2+b\right)-2\,\log\left(x\right)}{2\,b}$$

```
[In] integrate(1/(d*x^3+b*x),x, algorithm="fricas")
[Out] -1/2*(log(d*x^2 + b) - 2*log(x))/b
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{1}{bx+dx^3}\,dx=\frac{\log\left(x\right)}{b}-\frac{\log\left(\frac{b}{d}+x^2\right)}{2b}$$

```
[In] integrate(1/(d*x**3+b*x),x)
[Out] log(x)/b - log(b/d + x**2)/(2*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{bx + dx^3} \, dx = -\frac{\log (dx^2 + b)}{2\,b} + \frac{\log (x)}{b}$$

```
[In] integrate(1/(d*x^3+b*x),x, algorithm="maxima")
[Out] -1/2*log(d*x^2 + b)/b + log(x)/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{bx + dx^3} \, dx = \frac{\log (x^2)}{2\,b} - \frac{\log (|dx^2 + b|)}{2\,b}$$

```
[In] integrate(1/(d*x^3+b*x),x, algorithm="giac")
[Out] 1/2*log(x^2)/b - 1/2*log(abs(d*x^2 + b))/b
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{bx + dx^3} \, dx = -\frac{\ln (d\,x^2 + b) - 2\,\ln (x)}{2\,b}$$

```
[In] int(1/(b*x + d*x^3),x)
[Out] -(log(b + d*x^2) - 2*log(x))/(2*b)
```

3.26 $\int \frac{1}{bx+cx^2+dx^3} dx$

Optimal result . . . 334
Rubi [A] (verified) . . . 334
Mathematica [A] (verified) . . . 336
Maple [A] (verified) . . . 336
Fricas [A] (verification not implemented) . . . 336
Sympy [B] (verification not implemented) . . . 337
Maxima [F(-2)] . . . 338
Giac [A] (verification not implemented) . . . 338
Mupad [B] (verification not implemented) . . . 338

Optimal result

Integrand size = 16, antiderivative size = 62

$$\int \frac{1}{bx+cx^2+dx^3} dx = \frac{c\,\mathrm{arctanh}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}} + \frac{\log(x)}{b} - \frac{\log(b+cx+dx^2)}{2b}$$

```
[Out] ln(x)/b-1/2*ln(d*x^2+c*x+b)/b+c*arctanh((2*d*x+c)/(-4*b*d+c^2)^(1/2))/b/(-4
*b*d+c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1608, 719, 29, 648, 632, 212, 642}

$$\int \frac{1}{bx+cx^2+dx^3} dx = \frac{c\,\mathrm{arctanh}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}} - \frac{\log(b+cx+dx^2)}{2b} + \frac{\log(x)}{b}$$

```
[In] Int[(b*x + c*x^2 + d*x^3)^(-1),x]
```

```
[Out] (c*ArcTanh[(c + 2*d*x)/Sqrt[c^2 - 4*b*d]])/(b*Sqrt[c^2 - 4*b*d]) + Log[x]/b
 - Log[b + c*x + d*x^2]/(2*b)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x\left(b+cx+dx^2\right)}\,dx \\
&= \frac{\int \frac{1}{x}\,dx}{b}+\frac{\int \frac{-c-dx}{b+cx+dx^2}\,dx}{b} \\
&= \frac{\log(x)}{b}-\frac{\int \frac{c+2dx}{b+cx+dx^2}\,dx}{2b}-\frac{c\int \frac{1}{b+cx+dx^2}\,dx}{2b} \\
&= \frac{\log(x)}{b}-\frac{\log\left(b+cx+dx^2\right)}{2b}+\frac{c\text{Subst}\left(\int \frac{1}{c^2-4bd-x^2}\,dx,x,c+2dx\right)}{b}
\end{aligned}
$$

$$= \frac{c\tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}} + \frac{\log(x)}{b} - \frac{\log\left(b+cx+dx^2\right)}{2b}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{1}{bx+cx^2+dx^3}\,dx = -\frac{\frac{2c\arctan\left(\frac{c+2dx}{\sqrt{-c^2+4bd}}\right)}{\sqrt{-c^2+4bd}} - 2\log(x) + \log(b+x(c+dx))}{2b}$$

```
[In] Integrate[(b*x + c*x^2 + d*x^3)^(-1),x]
```

```
[Out] -1/2*((2*c*ArcTan[(c + 2*d*x)/Sqrt[-c^2 + 4*b*d]])/Sqrt[-c^2 + 4*b*d] - 2*L
og[x] + Log[b + x*(c + d*x)])/b
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result
default	$\frac{\ln(x)}{b}+\frac{-\frac{\ln\left(d\,x^2+cx+b\right)}{2}-\frac{c\arctan\left(\frac{2dx+c}{\sqrt{4bd-c^2}}\right)}{\sqrt{4bd-c^2}}}{b}$
risch	$-\frac{2\ln\left(\left(8b\,c^2d-2c^4+6\sqrt{-c^2(4bd-c^2)}\,bd-2\sqrt{-c^2(4bd-c^2)}\,c^2\right)x+12b^2cd-3b\,c^3-\sqrt{-c^2(4bd-c^2)}\,bc\right)d}{4bd-c^2}+\frac{\ln\left(\left(8b\,c^2d-2c^4+6\sqrt{-c^2(\right.\right.}{}$

```
[In] int(1/(d*x^3+c*x^2+b*x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/b+1/b*(-1/2*ln(d*x^2+c*x+b)-c/(4*b*d-c^2)^(1/2)*arctan((2*d*x+c)/(4*b
*d-c^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.40

$$\int \frac{1}{bx+cx^2+dx^3}\,dx$$
$$= \left[\frac{\sqrt{c^2-4\,bd}c\log\left(\frac{2\,d^2x^2+2\,cdx+c^2-2\,bd+\sqrt{c^2-4\,bd}(2\,dx+c)}{dx^2+cx+b}\right) - \left(c^2-4\,bd\right)\log\left(dx^2+cx+b\right) + 2\left(c^2-4\,bd\right)\log\left(x\right)}{2\left(bc^2-4\,b^2d\right)}\right.$$

```
[In] integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c^2 - 4*b*d)*c*log((2*d^2*x^2 + 2*c*d*x + c^2 - 2*b*d + sqrt(c^2
 - 4*b*d)*(2*d*x + c))/(d*x^2 + c*x + b)) - (c^2 - 4*b*d)*log(d*x^2 + c*x +
 b) + 2*(c^2 - 4*b*d)*log(x))/(b*c^2 - 4*b^2*d), 1/2*(2*sqrt(-c^2 + 4*b*d)*
c*arctan(-sqrt(-c^2 + 4*b*d)*(2*d*x + c)/(c^2 - 4*b*d)) - (c^2 - 4*b*d)*log
(d*x^2 + c*x + b) + 2*(c^2 - 4*b*d)*log(x))/(b*c^2 - 4*b^2*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(54) = 108$.

Time = 4.40 (sec) , antiderivative size = 564, normalized size of antiderivative = 9.10

$$\int \frac{1}{bx+cx^2+dx^3}\,dx = \left(-\frac{c\sqrt{-4bd+c^2}}{2b\,(4bd-c^2)} - \frac{1}{2b}\right)\log\left(x + \frac{24b^4d^2\left(-\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)^2 - 14b^3c^2d\left(-\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)^2 - 12b^3d^2\left(-\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right) + \cdots}{9bcd^2-2c\cdots}\right. + \left(\frac{c\sqrt{-4bd+c^2}}{2b\,(4bd-c^2)} - \frac{1}{2b}\right)\log\left(x + \frac{24b^4d^2\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)^2 - 14b^3c^2d\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)^2 - 12b^3d^2\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right) + 2b^2c^{\cdots}}{9bcd^2-2c^3d}\right. + \frac{\log(x)}{b}$$

```
[In] integrate(1/(d*x**3+c*x**2+b*x),x)
```

```
[Out] (-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))*log(x + (24*b**4*d*
*2*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 - 14*b**3*c**
2*d*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 - 12*b**3*d*
*2*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b)) + 2*b**2*c**4*(-
c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 + 3*b**2*c**2*d*(-
c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b)) - 12*b**2*d**2 + 11*b
*c**2*d - 2*c**4)/(9*b*c*d**2 - 2*c**3*d)) + (c*sqrt(-4*b*d + c**2)/(2*b*(4
*b*d - c**2)) - 1/(2*b))*log(x + (24*b**4*d**2*(c*sqrt(-4*b*d + c**2)/(2*b*
(4*b*d - c**2)) - 1/(2*b))**2 - 14*b**3*c**2*d*(c*sqrt(-4*b*d + c**2)/(2*b*
(4*b*d - c**2)) - 1/(2*b))**2 - 12*b**3*d**2*(c*sqrt(-4*b*d + c**2)/(2*b*(4
*b*d - c**2)) - 1/(2*b)) + 2*b**2*c**4*(c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d -
 c**2)) - 1/(2*b))**2 + 3*b**2*c**2*d*(c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d -
c**2)) - 1/(2*b)) - 12*b**2*d**2 + 11*b*c**2*d - 2*c**4)/(9*b*c*d**2 - 2*c*
*3*d)) + log(x)/b
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{bx + cx^2 + dx^3} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b*d-c^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{1}{bx + cx^2 + dx^3} \, dx = -\frac{c \arctan\left(\frac{2\,dx+c}{\sqrt{-c^2+4\,bd}}\right)}{\sqrt{-c^2+4\,bd}b} - \frac{\log\left(dx^2+cx+b\right)}{2\,b} + \frac{\log\left(|x|\right)}{b}$$

```
[In] integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="giac")
```

```
[Out] -c*arctan((2*d*x + c)/sqrt(-c^2 + 4*b*d))/(sqrt(-c^2 + 4*b*d)*b) - 1/2*log(
d*x^2 + c*x + b)/b + log(abs(x))/b
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.44

$$\int \frac{1}{bx + cx^2 + dx^3} \, dx = \frac{\ln(x)}{b} - \ln\left(\left(x\left(6\,b\,d^2 - 2\,c^2\,d\right) - b\,c\,d\right)\left(\frac{1}{2\,b} - \frac{c\sqrt{c^2-4\,b\,d}}{2\,\left(b\,c^2-4\,b^2\,d\right)}\right) - c\,d - 3\,d^2\,x\right)\left(\frac{1}{2\,b} - \frac{c\sqrt{c^2-4\,b\,d}}{2\,\left(b\,c^2-4\,b^2\,d\right)}\right) - \ln\left(\left(x\left(6\,b\,d^2 - 2\,c^2\,d\right) - b\,c\,d\right)\left(\frac{1}{2\,b} + \frac{c\sqrt{c^2-4\,b\,d}}{2\,\left(b\,c^2-4\,b^2\,d\right)}\right) - c\,d - 3\,d^2\,x\right)\left(\frac{1}{2\,b} + \frac{c\sqrt{c^2-4\,b\,d}}{2\,\left(b\,c^2-4\,b^2\,d\right)}\right)$$

```
[In] int(1/(b*x + c*x^2 + d*x^3),x)
```

```
[Out] log(x)/b - log((x*(6*b*d^2 - 2*c^2*d) - b*c*d)*(1/(2*b) - (c*(c^2 - 4*b*d)^
(1/2))/(2*(b*c^2 - 4*b^2*d))) - c*d - 3*d^2*x)*(1/(2*b) - (c*(c^2 - 4*b*d)^
(1/2))/(2*(b*c^2 - 4*b^2*d))) - log((x*(6*b*d^2 - 2*c^2*d) - b*c*d)*(1/(2*b
) + (c*(c^2 - 4*b*d)^(1/2))/(2*(b*c^2 - 4*b^2*d))) - c*d - 3*d^2*x)*(1/(2*b
) + (c*(c^2 - 4*b*d)^(1/2))/(2*(b*c^2 - 4*b^2*d)))
```

3.27 $\int \frac{1}{a+dx^3} dx$

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Optimal result

Integrand size = 9, antiderivative size = 115

$$\int \frac{1}{a+dx^3} dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{d}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}$$

```
[Out] 1/3*ln(a^(1/3)+d^(1/3)*x)/a^(2/3)/d^(1/3)-1/6*ln(a^(2/3)-a^(1/3)*d^(1/3)*x+
d^(2/3)*x^2)/a^(2/3)/d^(1/3)-1/3*arctan(1/3*(a^(1/3)-2*d^(1/3)*x)/a^(1/3)*3
^(1/2))/a^(2/3)/d^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {206, 31, 648, 631, 210, 642}

$$\int \frac{1}{a+dx^3} dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{d}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}}$$

```
[In] Int[(a + d*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*d^(1/3
))) + Log[a^(1/3) + d^(1/3)*x]/(3*a^(2/3)*d^(1/3)) - Log[a^(2/3) - a^(1/3)*
d^(1/3)*x + d^(2/3)*x^2]/(6*a^(2/3)*d^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{d}x}\,dx}{3a^{2/3}}+\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{d}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3a^{2/3}}\\
&= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}}+\frac{\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{2\sqrt[3]{a}}-\frac{\int \frac{-\sqrt[3]{a}\sqrt[3]{d}+2d^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6a^{2/3}\sqrt[3]{d}}\\
&= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{d}}
\end{aligned}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{d}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{1}{a+dx^3}\,dx$$
$$= -\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right) + \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}$$

```
[In] Integrate[(a + d*x^3)^(-1),x]
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3)
 + d^(1/3)*x] + Log[a^(2/3) - a^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(a^(2/3)*d^
(1/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.23

method	result	size
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(d\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}}{3d}$	27
default	$\frac{\ln\left(x+\left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{a}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{d}\right)^{\frac{1}{3}}x+\left(\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{a}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{a}{d}\right)^{\frac{2}{3}}}$	91

```
[In] int(1/(d*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/3/d*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.60

$$\int \frac{1}{a+dx^3}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}ad\sqrt{-\frac{(a^2d)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,adx^3-3\,(a^2d)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,adx^2+(a^2d)^{\frac{2}{3}}x-(a^2d)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2d)^{\frac{1}{3}}}{d}}}{dx^3+a}\right)-(a^2d)^{\frac{2}{3}}\log\left(adx^2-\right.}{6\,a^2d}\right.$$

```
[In] integrate(1/(d*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*d*sqrt(-(a^2*d)^(1/3)/d)*log((2*a*d*x^3 - 3*(a^2*d)^(1/
3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*d*x^2 + (a^2*d)^(2/3)*x - (a^2*d)^(1/3)*a)*
sqrt(-(a^2*d)^(1/3)/d))/(d*x^3 + a)) - (a^2*d)^(2/3)*log(a*d*x^2 - (a^2*d)^
(2/3)*x + (a^2*d)^(1/3)*a) + 2*(a^2*d)^(2/3)*log(a*d*x + (a^2*d)^(2/3)))/(a
^2*d), 1/6*(6*sqrt(1/3)*a*d*sqrt((a^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(a^2*
d)^(2/3)*x - (a^2*d)^(1/3)*a)*sqrt((a^2*d)^(1/3)/d)/a^2) - (a^2*d)^(2/3)*lo
g(a*d*x^2 - (a^2*d)^(2/3)*x + (a^2*d)^(1/3)*a) + 2*(a^2*d)^(2/3)*log(a*d*x
+ (a^2*d)^(2/3)))/(a^2*d)]
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.17

$$\int \frac{1}{a+dx^3}\,dx = \operatorname{RootSum}\left(27t^3a^2d-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)$$

```
[In] integrate(1/(d*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*d - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85

$$\int \frac{1}{a+dx^3}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{d}\right)^{\frac{1}{3}}}\right)}{3\,d\left(\frac{a}{d}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{d}\right)^{\frac{1}{3}}+\left(\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6\,d\left(\frac{a}{d}\right)^{\frac{2}{3}}} + \frac{\log\left(x+\left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\,d\left(\frac{a}{d}\right)^{\frac{2}{3}}}$$

```
[In] integrate(1/(d*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/d)^(1/3))/(a/d)^(1/3))/(d*(a/d)^(2
/3)) - 1/6*log(x^2 - x*(a/d)^(1/3) + (a/d)^(2/3))/(d*(a/d)^(2/3)) + 1/3*log
(x + (a/d)^(1/3))/(d*(a/d)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{1}{a+dx^3}\,dx = -\frac{\left(-\frac{a}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,a} + \frac{\sqrt{3}(-ad^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{d}\right)^{\frac{1}{3}}}\right)}{3\,ad} + \frac{(-ad^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{d}\right)^{\frac{1}{3}}+\left(-\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6\,ad}$$

```
[In] integrate(1/(d*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(-a/d)^(1/3)*log(abs(x - (-a/d)^(1/3)))/a + 1/3*sqrt(3)*(-a*d^2)^(1/3)
*arctan(1/3*sqrt(3)*(2*x + (-a/d)^(1/3))/(-a/d)^(1/3))/(a*d) + 1/6*(-a*d^2)
^(1/3)*log(x^2 + x*(-a/d)^(1/3) + (-a/d)^(2/3))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int \frac{1}{a+dx^3}\,dx = \frac{\ln\left(d^{1/3}\,x+a^{1/3}\right)}{3\,a^{2/3}\,d^{1/3}} + \frac{\ln\left(3\,d^2\,x+\frac{3\,a^{1/3}\,d^{5/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,d^{1/3}} - \frac{\ln\left(3\,d^2\,x-\frac{3\,a^{1/3}\,d^{5/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,d^{1/3}}$$

```
[In] int(1/(a + d*x^3),x)
```

```
[Out] log(d^(1/3)*x + a^(1/3))/(3*a^(2/3)*d^(1/3)) + (log(3*d^2*x + (3*a^(1/3)*d^
(5/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^(2/3)*d^(1/3)) - (log(3*d
^2*x - (3*a^(1/3)*d^(5/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)
*d^(1/3))
```

3.28 $\int (dx^3)^n \, dx$

Optimal result 346
Rubi [A] (verified) 346
Mathematica [A] (verified) 347
Maple [A] (verified) 347
Fricas [A] (verification not implemented) 348
Sympy [B] (verification not implemented) 348
Maxima [A] (verification not implemented) 348
Giac [A] (verification not implemented) 349
Mupad [B] (verification not implemented) 349

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int (dx^3)^n \, dx = \frac{x(dx^3)^n}{1+3n}$$

```
[Out] x*(d*x^3)^n/(1+3*n)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {15, 30}

$$\int (dx^3)^n \, dx = \frac{x(dx^3)^n}{3n+1}$$

```
[In] Int[(d*x^3)^n,x]
```

```
[Out] (x*(d*x^3)^n)/(1 + 3*n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\text{integral} = \left(x^{-3n}\left(dx^3\right)^n\right)\int x^{3n}\,dx$$
$$= \frac{x(dx^3)^n}{1+3n}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(dx^3\right)^n\,dx = \frac{x(dx^3)^n}{1+3n}$$

```
[In] Integrate[(d*x^3)^n,x]
[Out] (x*(d*x^3)^n)/(1 + 3*n)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{x\left(x^3 d\right)^n}{1+3n}$	17
risch	$\frac{x\left(x^3 d\right)^n}{1+3n}$	17
parallelrisch	$\frac{x\left(x^3 d\right)^n}{1+3n}$	17
norman	$\frac{x\,\mathrm{e}^{n\ln\left(x^3 d\right)}}{1+3n}$	19

```
[In] int((x^3*d)^n,x,method=_RETURNVERBOSE)
[Out] x*(x^3*d)^n/(1+3*n)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx^3)^n \, dx = \frac{(dx^3)^n x}{3n+1}$$

```
[In] integrate((d*x^3)^n,x, algorithm="fricas")
[Out] (d*x^3)^n*x/(3*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int (dx^3)^n \, dx = \begin{cases} \frac{x(dx^3)^n}{3n+1} & \text{for } n \neq -\frac{1}{3} \\ \frac{x \log(x)}{\sqrt[3]{dx^3}} & \text{otherwise} \end{cases}$$

```
[In] integrate((d*x**3)**n,x)
[Out] Piecewise((x*(d*x**3)**n/(3*n + 1), Ne(n, -1/3)), (x*log(x)/(d*x**3)**(1/3)
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (dx^3)^n \, dx = \frac{d^n x x^{3n}}{3n+1}$$

```
[In] integrate((d*x^3)^n,x, algorithm="maxima")
[Out] d^n*x*x^(3*n)/(3*n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx^3)^n\,dx = \frac{(dx^3)^n\,x}{3\,n+1}$$

```
[In] integrate((d*x^3)^n,x, algorithm="giac")
[Out] (d*x^3)^n*x/(3*n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.84 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx^3)^n\,dx = \frac{x\,(d\,x^3)^n}{3\,n+1}$$

```
[In] int((d*x^3)^n,x)
[Out] (x*(d*x^3)^n)/(3*n + 1)
```

3.29 $\int (cx^2 + dx^3)^n \, dx$

Optimal result 350
Rubi [A] (verified) 350
Mathematica [A] (verified) 351
Maple [F] 352
Fricas [F] 352
Sympy [F] 352
Maxima [F] 352
Giac [F] 353
Mupad [B] (verification not implemented) 353

Optimal result

Integrand size = 13, antiderivative size = 55

$$\int (cx^2+dx^3)^n \, dx = \frac{x\left(1+\frac{dx}{c}\right)^{-n}(cx^2+dx^3)^n \text{Hypergeometric2F1}\left(-n, 1+2n, 2(1+n), -\frac{dx}{c}\right)}{1+2n}$$

```
[Out] x*(d*x^3+c*x^2)^n*hypergeom([-n, 1+2*n],[2+2*n],-d*x/c)/(1+2*n)/((1+d*x/c)^
n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2036, 68, 66}

$$\int (cx^2+dx^3)^n \, dx = \frac{x\left(\frac{dx}{c}+1\right)^{-n}(cx^2+dx^3)^n \text{Hypergeometric2F1}\left(-n, 2n+1, 2(n+1), -\frac{dx}{c}\right)}{2n+1}$$

```
[In] Int[(c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(c*x^2 + d*x^3)^n*Hypergeometric2F1[-n, 1 + 2*n, 2*(1 + n), -((d*x)/c)])
/((1 + 2*n)*(1 + (d*x)/c)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(x^{-2n}(c+dx)^{-n}\left(cx^2+dx^3\right)^n\right)\int x^{2n}(c+dx)^n\,dx\\ &= \left(x^{-2n}\left(1+\frac{dx}{c}\right)^{-n}\left(cx^2+dx^3\right)^n\right)\int x^{2n}\left(1+\frac{dx}{c}\right)^n\,dx\\ &= \frac{x\left(1+\frac{dx}{c}\right)^{-n}\left(cx^2+dx^3\right)^n\,{}_2F_1\left(-n,1+2n;2(1+n);-\frac{dx}{c}\right)}{1+2n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int\left(cx^2+dx^3\right)^n\,dx = \frac{x\left(x^2(c+dx)\right)^n\left(1+\frac{dx}{c}\right)^{-n}\text{Hypergeometric2F1}\left(-n,1+2n,2+2n,-\frac{dx}{c}\right)}{1+2n}$$

```
[In] Integrate[(c*x^2 + d*x^3)^n,x]
[Out] (x*(x^2*(c + d*x))^n*Hypergeometric2F1[-n, 1 + 2*n, 2 + 2*n, -((d*x)/c)])/(
(1 + 2*n)*(1 + (d*x)/c)^n)
```

Maple [F]

$$\int \left(x^3 d + c\,x^2\right)^n dx$$

```
[In] int((d*x^3+c*x^2)^n,x)
[Out] int((d*x^3+c*x^2)^n,x)
```

Fricas [F]

$$\int \left(cx^2 + dx^3\right)^n\,dx = \int \left(dx^3 + cx^2\right)^n dx$$

```
[In] integrate((d*x^3+c*x^2)^n,x, algorithm="fricas")
[Out] integral((d*x^3 + c*x^2)^n, x)
```

Sympy [F]

$$\int \left(cx^2 + dx^3\right)^n\,dx = \int \left(cx^2 + dx^3\right)^n\,dx$$

```
[In] integrate((d*x**3+c*x**2)**n,x)
[Out] Integral((c*x**2 + d*x**3)**n, x)
```

Maxima [F]

$$\int \left(cx^2 + dx^3\right)^n\,dx = \int \left(dx^3 + cx^2\right)^n dx$$

```
[In] integrate((d*x^3+c*x^2)^n,x, algorithm="maxima")
[Out] integrate((d*x^3 + c*x^2)^n, x)
```

Giac [F]

$$\int \left(cx^2+dx^3\right)^n\,dx=\int \left(dx^3+cx^2\right)^n\,dx$$

```
[In] integrate((d*x^3+c*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c*x^2)^n, x)
```

Mupad [B] (verification not implemented)

Time = 9.64 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \left(cx^2+dx^3\right)^n\,dx=\frac{x\,\left(d\,x^3+c\,x^2\right)^n\,{}_2\mathrm{F}_1\left(2\,n+1,-n;\ 2\,n+2;\ -\frac{d\,x}{c}\right)}{\left(2\,n+1\right)\,{\left(\frac{d\,x}{c}+1\right)}^n}$$

```
[In] int((c*x^2 + d*x^3)^n,x)
```

```
[Out] (x*(c*x^2 + d*x^3)^n*hypergeom([2*n + 1, -n], 2*n + 2, -(d*x)/c))/((2*n + 1
)*((d*x)/c + 1)^n)
```

3.30 $\int (bx + dx^3)^n \, dx$

Optimal result . . . 354
Rubi [A] (verified) . . . 354
Mathematica [A] (verified) . . . 355
Maple [F] . . . 356
Fricas [F] . . . 356
Sympy [F] . . . 356
Maxima [F] . . . 356
Giac [F] . . . 357
Mupad [B] (verification not implemented) . . . 357

Optimal result

Integrand size = 11, antiderivative size = 53

$$\int (bx + dx^3)^n \, dx = \frac{x(b + dx^2)(bx + dx^3)^n \operatorname{Hypergeometric2F1}\left(1, \frac{3(1+n)}{2}, \frac{3+n}{2}, -\frac{dx^2}{b}\right)}{b(1+n)}$$

```
[Out] x*(d*x^2+b)*(d*x^3+b*x)^n*hypergeom([1, 3/2+3/2*n],[3/2+1/2*n],-d*x^2/b)/b/
(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2036, 372, 371}

$$\int (bx + dx^3)^n \, dx = \frac{x\left(\frac{dx^2}{b} + 1\right)^{-n} (bx + dx^3)^n \operatorname{Hypergeometric2F1}\left(-n, \frac{n+1}{2}, \frac{n+3}{2}, -\frac{dx^2}{b}\right)}{n+1}$$

```
[In] Int[(b*x + d*x^3)^n,x]
```

```
[Out] (x*(b*x + d*x^3)^n*Hypergeometric2F1[-n, (1 + n)/2, (3 + n)/2, -((d*x^2)/b)
])/((1 + n)*(1 + (d*x^2)/b)^n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(x^{-n}\left(b+dx^2\right)^{-n}\left(bx+dx^3\right)^n\right)\int x^n\left(b+dx^2\right)^n\,dx\\ &= \left(x^{-n}\left(1+\frac{dx^2}{b}\right)^{-n}\left(bx+dx^3\right)^n\right)\int x^n\left(1+\frac{dx^2}{b}\right)^n\,dx\\ &= \frac{x\left(1+\frac{dx^2}{b}\right)^{-n}\left(bx+dx^3\right)^n\,{}_2F_1\left(-n,\frac{1+n}{2};\frac{3+n}{2};-\frac{dx^2}{b}\right)}{1+n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int \left(bx+dx^3\right)^n\,dx = \frac{x\left(x\left(b+dx^2\right)\right)^n\left(1+\frac{dx^2}{b}\right)^{-n}\text{Hypergeometric2F1}\left(-n,\frac{1+n}{2},1+\frac{1+n}{2},-\frac{dx^2}{b}\right)}{1+n}$$

```
[In] Integrate[(b*x + d*x^3)^n,x]
[Out] (x*(x*(b + d*x^2))^n*Hypergeometric2F1[-n, (1 + n)/2, 1 + (1 + n)/2, -((d*x
^2)/b)])/((1 + n)*(1 + (d*x^2)/b)^n)
```

Maple [F]

$$\int \left(x^3d + bx\right)^n dx$$

```
[In] int((d*x^3+b*x)^n,x)
[Out] int((d*x^3+b*x)^n,x)
```

Fricas [F]

$$\int (bx + dx^3)^n \, dx = \int \left(dx^3 + bx\right)^n dx$$

```
[In] integrate((d*x^3+b*x)^n,x, algorithm="fricas")
[Out] integral((d*x^3 + b*x)^n, x)
```

Sympy [F]

$$\int (bx + dx^3)^n \, dx = \int \left(bx + dx^3\right)^n \, dx$$

```
[In] integrate((d*x**3+b*x)**n,x)
[Out] Integral((b*x + d*x**3)**n, x)
```

Maxima [F]

$$\int (bx + dx^3)^n \, dx = \int \left(dx^3 + bx\right)^n dx$$

```
[In] integrate((d*x^3+b*x)^n,x, algorithm="maxima")
[Out] integrate((d*x^3 + b*x)^n, x)
```

Giac [F]

$$\int (bx + dx^3)^n \, dx = \int (dx^3 + bx)^n \, dx$$

```
[In] integrate((d*x^3+b*x)^n,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + b*x)^n, x)
```

Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int (bx + dx^3)^n \, dx = \frac{x \, (d \, x^3 + b \, x)^n \, {}_2\mathrm{F}_1\left(\frac{n}{2} + \frac{1}{2}, -n; \, \frac{n}{2} + \frac{3}{2}; \, -\frac{d \, x^2}{b}\right)}{\left(\frac{d \, x^2}{b} + 1\right)^n \, (n + 1)}$$

```
[In] int((b*x + d*x^3)^n,x)
```

```
[Out] (x*(b*x + d*x^3)^n*hypergeom([n/2 + 1/2, -n], n/2 + 3/2, -(d*x^2)/b))/(((d*
x^2)/b + 1)^n*(n + 1))
```

3.31 $\int (bx + cx^2 + dx^3)^n \, dx$

Optimal result 358
Rubi [A] (verified) 358
Mathematica [A] (verified) 360
Maple [F] 360
Fricas [F] 360
Sympy [F] 360
Maxima [F] 361
Giac [F] 361
Mupad [F(-1)] 361

Optimal result

Integrand size = 16, antiderivative size = 132

$$\int (bx + cx^2 + dx^3)^n \, dx = \frac{x\left(1+\frac{2dx}{c-\sqrt{c^2-4bd}}\right)^{-n}\left(1+\frac{2dx}{c+\sqrt{c^2-4bd}}\right)^{-n}(bx+cx^2+dx^3)^n \operatorname{AppellF1}\left(1+n,-n,-n,2+n,-\frac{2dx}{c-\sqrt{c^2-4bd}},-\frac{2dx}{c+\sqrt{c^2-4bd}}\right)}{1+n}$$

```
[Out] x*(d*x^3+c*x^2+b*x)^n*AppellF1(1+n,-n,-n,2+n,-2*d*x/(c-(-4*b*d+c^2)^(1/2)),
-2*d*x/(c+(-4*b*d+c^2)^(1/2)))/(1+n)/((1+2*d*x/(c-(-4*b*d+c^2)^(1/2)))^n)/(
(1+2*d*x/(c+(-4*b*d+c^2)^(1/2)))^n)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1922, 773, 138}

$$\int (bx + cx^2 + dx^3)^n \, dx = \frac{x\left(\frac{2dx}{c-\sqrt{c^2-4bd}}+1\right)^{-n}\left(\frac{2dx}{\sqrt{c^2-4bd}+c}+1\right)^{-n}(bx+cx^2+dx^3)^n \operatorname{AppellF1}\left(n+1,-n,-n,n+2,-\frac{2dx}{c-\sqrt{c^2-4bd}},-\frac{2dx}{c+\sqrt{c^2-4bd}}\right)}{n+1}$$

```
[In] Int[(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(b*x + c*x^2 + d*x^3)^n*AppellF1[1 + n, -n, -n, 2 + n, (-2*d*x)/(c - Sqr
t[c^2 - 4*b*d]), (-2*d*x)/(c + Sqrt[c^2 - 4*b*d])])/((1 + n)*(1 + (2*d*x)/(
c - Sqrt[c^2 - 4*b*d]))^n*(1 + (2*d*x)/(c + Sqrt[c^2 - 4*b*d]))^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 1922

```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Dist[(a*x^q + b*x^n + c*x^(2*n - q))^p/(x^(p*q)*(a + b*x^(n - q) + c*x
^(2*(n - q)))^p), Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Int
egerQ[p]
```

Rubi steps

$$\text{integral} = \left(x^{-n}\left(b+cx+dx^2\right)^{-n}\left(bx+cx^2+dx^3\right)^n\right)\int x^n\left(b+cx+dx^2\right)^n\,dx$$

$$= \left(x^{-n}\left(1+\frac{2dx}{c-\sqrt{c^2-4bd}}\right)^{-n}\left(1+\frac{2dx}{c+\sqrt{c^2-4bd}}\right)^{-n}\left(bx+cx^2+dx^3\right)^n\right)\text{Subst}\left(\int x^n\left(1+\frac{2dx}{c-\sqrt{c^2-4bd}}\right)^n\left(1+\frac{2dx}{c+\sqrt{c^2-4bd}}\right)^n dx,x,x\right)$$

$$= \frac{x\left(1+\frac{2dx}{c-\sqrt{c^2-4bd}}\right)^{-n}\left(1+\frac{2dx}{c+\sqrt{c^2-4bd}}\right)^{-n}\left(bx+cx^2+dx^3\right)^n F_1\left(1+n;-n,-n;2+n;-\frac{2dx}{c-\sqrt{c^2-4bd}},-\right.}{1+n}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

$$\int \left(bx+cx^2+dx^3\right)^n \, dx$$
$$=\frac{x\left(1+\frac{2dx}{c-\sqrt{c^2-4bd}}\right)^{-n}\left(1+\frac{2dx}{c+\sqrt{c^2-4bd}}\right)^{-n}(x(b+x(c+dx)))^n \operatorname{AppellF1}\left(1+n,-n,-n,2+n,-\frac{2dx}{c+\sqrt{c^2-4bd}},-\right.}{1+n}$$

```
[In] Integrate[(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(x*(b + x*(c + d*x)))^n*AppellF1[1 + n, -n, -n, 2 + n, (-2*d*x)/(c + Sqr
t[c^2 - 4*b*d]), (2*d*x)/(-c + Sqrt[c^2 - 4*b*d])])/((1 + n)*(1 + (2*d*x)/(
c - Sqrt[c^2 - 4*b*d]))^n*(1 + (2*d*x)/(c + Sqrt[c^2 - 4*b*d]))^n)
```

Maple [F]

$$\int \left(x^3 d+c\,x^2+bx\right)^n dx$$

```
[In] int((d*x^3+c*x^2+b*x)^n,x)
```

```
[Out] int((d*x^3+c*x^2+b*x)^n,x)
```

Fricas [F]

$$\int \left(bx+cx^2+dx^3\right)^n \, dx=\int \left(dx^3+cx^2+bx\right)^n dx$$

```
[In] integrate((d*x^3+c*x^2+b*x)^n,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c*x^2 + b*x)^n, x)
```

Sympy [F]

$$\int \left(bx+cx^2+dx^3\right)^n \, dx=\int \left(bx+cx^2+dx^3\right)^n \, dx$$

```
[In] integrate((d*x**3+c*x**2+b*x)**n,x)
```

```
[Out] Integral((b*x + c*x**2 + d*x**3)**n, x)
```

Maxima [F]

$$\int \left(bx + cx^2 + dx^3\right)^n \, dx = \int \left(dx^3 + cx^2 + bx\right)^n \, dx$$

```
[In] integrate((d*x^3+c*x^2+b*x)^n,x, algorithm="maxima")
[Out] integrate((d*x^3 + c*x^2 + b*x)^n, x)
```

Giac [F]

$$\int \left(bx + cx^2 + dx^3\right)^n \, dx = \int \left(dx^3 + cx^2 + bx\right)^n \, dx$$

```
[In] integrate((d*x^3+c*x^2+b*x)^n,x, algorithm="giac")
[Out] integrate((d*x^3 + c*x^2 + b*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(bx + cx^2 + dx^3\right)^n \, dx = \int \left(d\,x^3 + c\,x^2 + b\,x\right)^n \, dx$$

```
[In] int((b*x + c*x^2 + d*x^3)^n,x)
[Out] int((b*x + c*x^2 + d*x^3)^n, x)
```

3.32 $\int (a+dx^3)^n \, dx$

Optimal result 362
Rubi [A] (verified) 362
Mathematica [C] (warning: unable to verify) 363
Maple [F] 363
Fricas [F] 364
Sympy [C] (verification not implemented) 364
Maxima [F] 364
Giac [F] 364
Mupad [B] (verification not implemented) 365

Optimal result

Integrand size = 9, antiderivative size = 35

$$\int (a+dx^3)^n \, dx = \frac{x(a+dx^3)^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{4}{3}+n,\frac{4}{3},-\frac{dx^3}{a}\right)}{a}$$

```
[Out] x*(d*x^3+a)^(1+n)*hypergeom([1, 4/3+n],[4/3],-d*x^3/a)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {252, 251}

$$\int (a+dx^3)^n \, dx = x(a+dx^3)^n \left(\frac{dx^3}{a}+1\right)^{-n} \text{Hypergeometric2F1}\left(\frac{1}{3},-n,\frac{4}{3},-\frac{dx^3}{a}\right)$$

```
[In] Int[(a + d*x^3)^n,x]
```

```
[Out] (x*(a + d*x^3)^n*Hypergeometric2F1[1/3, -n, 4/3, -((d*x^3)/a)])/(1 + (d*x^3
)/a)^n
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((a+dx^3)^n\left(1+\frac{dx^3}{a}\right)^{-n}\right)\int\left(1+\frac{dx^3}{a}\right)^n\,dx\\ &= x(a+dx^3)^n\left(1+\frac{dx^3}{a}\right)^{-n}{}_2F_1\left(\frac{1}{3},-n;\frac{4}{3};-\frac{dx^3}{a}\right)\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 5.80

$$\int (a+dx^3)^n\,dx$$

$$= \frac{2^{-n}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{d}x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{d}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-n}\left(\frac{i\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}\right)^{-n}(a+dx^3)^n\,\text{AppellF1}\left(1+n,-n,-n,2\right.}{\sqrt[3]{d}(1+n)}$$

```
[In] Integrate[(a + d*x^3)^n,x]
```

```
[Out] (((-1)^(2/3)*a^(1/3) + d^(1/3)*x)*(a + d*x^3)^n*AppellF1[1 + n, -n, -n, 2 +
 n, -(((-1)^(2/3)*((-1)^(2/3)*a^(1/3) + d^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/
3))), (I + Sqrt[3] - ((2*I)*d^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])])/(2^n*d^(1
/3)*(1 + n)*((a^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)))^n
*((I*(1 + (d^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3]))^n)
```

Maple [F]

$$\int (x^3d+a)^n\,dx$$

```
[In] int((d*x^3+a)^n,x)
```

```
[Out] int((d*x^3+a)^n,x)
```

Fricas [F]

$$\int \left(a+dx^3\right)^n\,dx=\int \left(dx^3+a\right)^n\,dx$$

```
[In] integrate((d*x^3+a)^n,x, algorithm="fricas")
[Out] integral((d*x^3 + a)^n, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \left(a+dx^3\right)^n\,dx=\frac{a^n x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},-n\\ \frac{4}{3}\end{matrix}\right|\frac{dx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((d*x**3+a)**n,x)
[Out] a**n*x*gamma(1/3)*hyper((1/3, -n), (4/3,), d*x**3*exp_polar(I*pi)/a)/(3*gam
ma(4/3))
```

Maxima [F]

$$\int \left(a+dx^3\right)^n\,dx=\int \left(dx^3+a\right)^n\,dx$$

```
[In] integrate((d*x^3+a)^n,x, algorithm="maxima")
[Out] integrate((d*x^3 + a)^n, x)
```

Giac [F]

$$\int \left(a+dx^3\right)^n\,dx=\int \left(dx^3+a\right)^n\,dx$$

```
[In] integrate((d*x^3+a)^n,x, algorithm="giac")
[Out] integrate((d*x^3 + a)^n, x)
```

Mupad [B] (verification not implemented)

Time = 10.48 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \left(a+dx^3\right)^n\,dx=\frac{x\,{\left(d\,x^3+a\right)}^n\,{}_2\mathrm{F}_1\left(\frac{1}{3},-n;\ \frac{4}{3};\ -\frac{d\,x^3}{a}\right)}{{\left(\frac{d\,x^3}{a}+1\right)}^n}$$

```
[In] int((a + d*x^3)^n,x)
[Out] (x*(a + d*x^3)^n*hypergeom([1/3, -n], 4/3, -(d*x^3)/a))/((d*x^3)/a + 1)^n
```

3.33 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4 \, dx$

Optimal result . . . 366
Rubi [A] (verified) . . . 367
Mathematica [A] (verified) . . . 368
Maple [A] (verified) . . . 369
Fricas [A] (verification not implemented) . . . 370
Sympy [A] (verification not implemented) . . . 371
Maxima [A] (verification not implemented) . . . 371
Giac [A] (verification not implemented) . . . 373
Mupad [B] (verification not implemented) . . . 374

Optimal result

Integrand size = 29, antiderivative size = 270

$$\begin{aligned}\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4 \, dx &= \frac{c^4(c^3+4ad^2)^4 x}{d^8} - \frac{8c^5(c^3+4ad^2)^3 \left(\frac{c}{d}+x\right)^3}{3d^6} \\ &+ \frac{4c^3(c^3+4ad^2)^2 (7c^3+4ad^2) \left(\frac{c}{d}+x\right)^5}{5d^4} \\ &- \frac{8c^4(c^3+4ad^2)(7c^3+12ad^2) \left(\frac{c}{d}+x\right)^7}{7d^2} \\ &+ \frac{2}{9}c^2\left(35c^6+120ac^3d^2+48a^2d^4\right)\left(\frac{c}{d}+x\right)^9 \\ &- \frac{8}{11}c^3d^2\left(7c^3+12ad^2\right)\left(\frac{c}{d}+x\right)^{11} \\ &+ \frac{4}{13}cd^4\left(7c^3+4ad^2\right)\left(\frac{c}{d}+x\right)^{13} \\ &- \frac{8}{15}c^2d^6\left(\frac{c}{d}+x\right)^{15} + \frac{1}{17}d^8\left(\frac{c}{d}+x\right)^{17}\end{aligned}$$

```
[Out] c^4*(4*a*d^2+c^3)^4*x/d^8-8/3*c^5*(4*a*d^2+c^3)^3*(c/d+x)^3/d^6+4/5*c^3*(4*
a*d^2+c^3)^2*(4*a*d^2+7*c^3)*(c/d+x)^5/d^4-8/7*c^4*(4*a*d^2+c^3)*(12*a*d^2+
7*c^3)*(c/d+x)^7/d^2+2/9*c^2*(48*a^2*d^4+120*a*c^3*d^2+35*c^6)*(c/d+x)^9-8/
11*c^3*d^2*(12*a*d^2+7*c^3)*(c/d+x)^11+4/13*c*d^4*(4*a*d^2+7*c^3)*(c/d+x)^1
3-8/15*c^2*d^6*(c/d+x)^15+1/17*d^8*(c/d+x)^17
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1120, 1104}

$$\begin{aligned}\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx = &\frac{2}{9}c^2\left(48a^2d^4+120ac^3d^2+35c^6\right)\left(\frac{c}{d}+x\right)^9\\ &-\frac{8}{11}c^3d^2\left(12ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{11}\\ &+\frac{4}{13}cd^4\left(4ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{13}\\ &+\frac{4c^3(4ad^2+c^3)^2\left(4ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^5}{5d^4}\\ &-\frac{8c^5(4ad^2+c^3)^3\left(\frac{c}{d}+x\right)^3}{3d^6}\\ &-\frac{8c^4(4ad^2+c^3)\left(12ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^7}{7d^2}\\ &+\frac{c^4x(4ad^2+c^3)^4}{d^8}\\ &-\frac{8}{15}c^2d^6\left(\frac{c}{d}+x\right)^{15}+\frac{1}{17}d^8\left(\frac{c}{d}+x\right)^{17}\end{aligned}$$

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]
```

```
[Out] (c^4*(c^3 + 4*a*d^2)^4*x)/d^8 - (8*c^5*(c^3 + 4*a*d^2)^3*(c/d + x)^3)/(3*d^
6) + (4*c^3*(c^3 + 4*a*d^2)^2*(7*c^3 + 4*a*d^2)*(c/d + x)^5)/(5*d^4) - (8*c
^4*(c^3 + 4*a*d^2)*(7*c^3 + 12*a*d^2)*(c/d + x)^7)/(7*d^2) + (2*c^2*(35*c^6
 + 120*a*c^3*d^2 + 48*a^2*d^4)*(c/d + x)^9)/9 - (8*c^3*d^2*(7*c^3 + 12*a*d^
2)*(c/d + x)^11)/11 + (4*c*d^4*(7*c^3 + 4*a*d^2)*(c/d + x)^13)/13 - (8*c^2*
d^6*(c/d + x)^15)/15 + (d^8*(c/d + x)^17)/17
```

Rule 1104

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4\right)^4\,dx, x, \frac{c}{d}+x\right)$$

$$= \text{Subst}\left(\int \left(\frac{(c^4+4acd^2)^4}{d^8}-\frac{8c^5(c^3+4ad^2)^3x^2}{d^6}+\frac{24c^6(c^3+4ad^2)^2\left(\frac{7}{6}+\frac{2ad^2}{3c^3}\right)x^4}{d^4} - \frac{32c^7(c^3+4ad^2)\left(\frac{7}{4}+\frac{3ad^2}{c^3}\right)x^6}{d^2}+16c^8\left(\frac{35}{8}+\frac{15ad^2}{c^3}+\frac{6a^2d^4}{c^6}\right)x^8 - 32c^6d^2\left(\frac{7}{4}+\frac{3ad^2}{c^3}\right)x^{10}+24c^4d^4\left(\frac{7}{6}+\frac{2ad^2}{3c^3}\right)x^{12}-8c^2d^6x^{14}+d^8x^{16}\right)dx, x, \frac{c}{d}+x\right)$$

$$= \frac{c^4(c^3+4ad^2)^4x}{d^8}-\frac{8c^5(c^3+4ad^2)^3\left(\frac{c}{d}+x\right)^3}{3d^6} + \frac{4c^3(c^3+4ad^2)^2(7c^3+4ad^2)\left(\frac{c}{d}+x\right)^5}{5d^4}-\frac{8c^4(c^3+4ad^2)(7c^3+12ad^2)\left(\frac{c}{d}+x\right)^7}{7d^2} + \frac{2}{9}c^2\left(35c^6+120ac^3d^2+48a^2d^4\right)\left(\frac{c}{d}+x\right)^9-\frac{8}{11}c^3d^2(7c^3+12ad^2)\left(\frac{c}{d}+x\right)^{11} + \frac{4}{13}cd^4(7c^3+4ad^2)\left(\frac{c}{d}+x\right)^{13}-\frac{8}{15}c^2d^6\left(\frac{c}{d}+x\right)^{15}+\frac{1}{17}d^8\left(\frac{c}{d}+x\right)^{17}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.06

$$\int (4ac+4c^2x^2+4cdx^3+d^2x^4)^4\,dx = 256a^4c^4x+\frac{1024}{3}a^3c^5x^3+256a^3c^4dx^4 + \frac{256}{5}a^2c^3(6c^3+ad^2)x^5+512a^2c^5dx^6 + \frac{256}{7}ac^4(4c^3+9ad^2)x^7+96ac^3d(4c^3+ad^2)x^8 + \frac{32}{9}c^2(8c^6+120ac^3d^2+3a^2d^4)x^9 + \frac{256}{5}c^4d(2c^3+5ad^2)x^{10}+\frac{64}{11}c^3d^2(28c^3+15ad^2)x^{11} + \frac{16}{3}c^2d^3(28c^3+3ad^2)x^{12}+\frac{16}{13}cd^4(70c^3+ad^2)x^{13} + 32c^3d^5x^{14}+\frac{112}{15}c^2d^6x^{15}+cd^7x^{16}+\frac{d^8x^{17}}{17}$$

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]
```

```
[Out] 256*a^4*c^4*x + (1024*a^3*c^5*x^3)/3 + 256*a^3*c^4*d*x^4 + (256*a^2*c^3*(6*
c^3 + a*d^2)*x^5)/5 + 512*a^2*c^5*d*x^6 + (256*a*c^4*(4*c^3 + 9*a*d^2)*x^7)
/7 + 96*a*c^3*d*(4*c^3 + a*d^2)*x^8 + (32*c^2*(8*c^6 + 120*a*c^3*d^2 + 3*a^
2*d^4)*x^9)/9 + (256*c^4*d*(2*c^3 + 5*a*d^2)*x^10)/5 + (64*c^3*d^2*(28*c^3
+ 15*a*d^2)*x^11)/11 + (16*c^2*d^3*(28*c^3 + 3*a*d^2)*x^12)/3 + (16*c*d^4*(
70*c^3 + a*d^2)*x^13)/13 + 32*c^3*d^5*x^14 + (112*c^2*d^6*x^15)/15 + c*d^7*
x^16 + (d^8*x^17)/17
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.99

method	result
norman	$256a^4c^4x + \frac{1024a^3c^5x^3}{3} + 256a^3c^4d\,x^4 + \left(\frac{256}{5}a^3c^3d^2 + \frac{1536}{5}a^2c^6\right)x^5 + 512a^2c^5d\,x^6 + \left(\frac{2304}{7}a^2c^4d\right.$
gosper	$384a\,c^6d\,x^8 + 96a^2c^3d^3x^8 + \frac{960}{11}x^{11}a\,c^3d^4 + 16x^{12}a\,c^2d^5 + \frac{16}{13}x^{13}ac\,d^6 + \frac{32}{3}x^9a^2c^2d^4 + \frac{1280}{3}x^9a$
risch	$384a\,c^6d\,x^8 + 96a^2c^3d^3x^8 + \frac{960}{11}x^{11}a\,c^3d^4 + 16x^{12}a\,c^2d^5 + \frac{16}{13}x^{13}ac\,d^6 + \frac{32}{3}x^9a^2c^2d^4 + \frac{1280}{3}x^9a$
parallelrisch	$384a\,c^6d\,x^8 + 96a^2c^3d^3x^8 + \frac{960}{11}x^{11}a\,c^3d^4 + 16x^{12}a\,c^2d^5 + \frac{16}{13}x^{13}ac\,d^6 + \frac{32}{3}x^9a^2c^2d^4 + \frac{1280}{3}x^9a$
default	$\frac{d^8x^{17}}{17} + d^7c\,x^{16} + \frac{112c^2d^6x^{15}}{15} + 32c^3d^5x^{14} + \frac{(2(8d^2ac+16c^4)d^4+1088c^4d^4)x^{13}}{13} + \frac{(64a\,c^2d^5+16(8d^2ac+16c^4)c}{12}$

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 256*a^4*c^4*x+1024/3*a^3*c^5*x^3+256*a^3*c^4*d*x^4+(256/5*a^3*c^3*d^2+1536/
5*a^2*c^6)*x^5+512*a^2*c^5*d*x^6+(2304/7*a^2*c^4*d^2+1024/7*a*c^7)*x^7+(96*
a^2*c^3*d^3+384*a*c^6*d)*x^8+(32/3*a^2*c^2*d^4+1280/3*a*c^5*d^2+256/9*c^8)*
x^9+(256*a*c^4*d^3+512/5*c^7*d)*x^10+(960/11*a*c^3*d^4+1792/11*c^6*d^2)*x^1
1+(16*a*c^2*d^5+448/3*c^5*d^3)*x^12+(16/13*a*c*d^6+1120/13*c^4*d^4)*x^13+32
*c^3*d^5*x^14+112/15*c^2*d^6*x^15+d^7*c*x^16+1/17*d^8*x^17
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4 \, dx = \frac{1}{17}\,d^8x^{17}+cd^7x^{16}+\frac{112}{15}\,c^2d^6x^{15}+32\,c^3d^5x^{14}+512\,a^2c^5dx^6+\frac{16}{13}\,\left(70\,c^4d^4+acd^6\right)x^{13}+\frac{16}{3}\,\left(28\,c^5d^3+3\,ac^2d^5\right)x^{12}+256\,a^3c^4dx^4+\frac{64}{11}\,\left(28\,c^6d^2+15\,ac^3d^4\right)x^{11}+\frac{1024}{3}\,a^3c^5x^3+\frac{256}{5}\,\left(2\,c^7d+5\,ac^4d^3\right)x^{10}+\frac{32}{9}\,\left(8\,c^8+120\,ac^5d^2+3\,a^2c^2d^4\right)x^9+256\,a^4c^4x+96\,\left(4\,ac^6d+a^2c^3d^3\right)x^8+\frac{256}{7}\,\left(4\,ac^7+9\,a^2c^4d^2\right)x^7+\frac{256}{5}\,\left(6\,a^2c^6+a^3c^3d^2\right)x^5$$

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="fricas")
```

```
[Out] 1/17*d^8*x^17 + c*d^7*x^16 + 112/15*c^2*d^6*x^15 + 32*c^3*d^5*x^14 + 512*a^
2*c^5*d*x^6 + 16/13*(70*c^4*d^4 + a*c*d^6)*x^13 + 16/3*(28*c^5*d^3 + 3*a*c^
2*d^5)*x^12 + 256*a^3*c^4*d*x^4 + 64/11*(28*c^6*d^2 + 15*a*c^3*d^4)*x^11 +
1024/3*a^3*c^5*x^3 + 256/5*(2*c^7*d + 5*a*c^4*d^3)*x^10 + 32/9*(8*c^8 + 120
*a*c^5*d^2 + 3*a^2*c^2*d^4)*x^9 + 256*a^4*c^4*x + 96*(4*a*c^6*d + a^2*c^3*d
^3)*x^8 + 256/7*(4*a*c^7 + 9*a^2*c^4*d^2)*x^7 + 256/5*(6*a^2*c^6 + a^3*c^3*
d^2)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.11

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx = 256a^4c^4x+\frac{1024a^3c^5x^3}{3}+256a^3c^4dx^4+512a^2c^5dx^6+32c^3d^5x^{14}+\frac{112c^2d^6x^{15}}{15}+cd^7x^{16}+\frac{d^8x^{17}}{17}+x^{13}\cdot\left(\frac{16acd^6}{13}+\frac{1120c^4d^4}{13}\right)+x^{12}\cdot\left(16ac^2d^5+\frac{448c^5d^3}{3}\right)+x^{11}\cdot\left(\frac{960ac^3d^4}{11}+\frac{1792c^6d^2}{11}\right)+x^{10}\cdot\left(256ac^4d^3+\frac{512c^7d}{5}\right)+x^9\cdot\left(\frac{32a^2c^2d^4}{3}+\frac{1280ac^5d^2}{3}+\frac{256c^8}{9}\right)+x^8\cdot\left(96a^2c^3d^3+384ac^6d\right)+x^7\cdot\left(\frac{2304a^2c^4d^2}{7}+\frac{1024ac^7}{7}\right)+x^5\cdot\left(\frac{256a^3c^3d^2}{5}+\frac{1536a^2c^6}{5}\right)$$

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**4,x)
```

```
[Out] 256*a**4*c**4*x + 1024*a**3*c**5*x**3/3 + 256*a**3*c**4*d*x**4 + 512*a**2*c
**5*d*x**6 + 32*c**3*d**5*x**14 + 112*c**2*d**6*x**15/15 + c*d**7*x**16 + d
**8*x**17/17 + x**13*(16*a*c*d**6/13 + 1120*c**4*d**4/13) + x**12*(16*a*c**
2*d**5 + 448*c**5*d**3/3) + x**11*(960*a*c**3*d**4/11 + 1792*c**6*d**2/11)
+ x**10*(256*a*c**4*d**3 + 512*c**7*d/5) + x**9*(32*a**2*c**2*d**4/3 + 1280
*a*c**5*d**2/3 + 256*c**8/9) + x**8*(96*a**2*c**3*d**3 + 384*a*c**6*d) + x*
*7*(2304*a**2*c**4*d**2/7 + 1024*a*c**7/7) + x**5*(256*a**3*c**3*d**2/5 + 1
536*a**2*c**6/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.38

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4 \, dx$$
$$= \frac{1}{17}\,d^8x^{17}+cd^7x^{16}+\frac{32}{5}\,c^2d^6x^{15}+\frac{128}{7}\,c^3d^5x^{14}+\frac{256}{13}\,c^4d^4x^{13}+\frac{256}{9}\,c^8x^9$$
$$+256\,a^4c^4x+\frac{256}{15}\,\left(3\,d^2x^5+15\,cdx^4+20\,c^2x^3\right)a^3c^3+\frac{256}{55}\,\left(5\,d^2x^{11}+22\,cdx^{10}\right)c^6$$
$$+\frac{32}{105}\,\left(35\,d^4x^9+315\,cd^3x^8+720\,c^2d^2x^7+1008\,c^4x^5+120\,\left(3\,d^2x^7+14\,cdx^6\right)c^2\right)a^2c^2$$
$$+\frac{32}{143}\,\left(33\,d^4x^{13}+286\,cd^3x^{12}+624\,c^2d^2x^{11}\right)c^4$$
$$+\frac{16}{15015}\,\left(1155\,d^6x^{13}+15015\,cd^5x^{12}+65520\,c^2d^4x^{11}+96096\,c^3d^3x^{10}+137280\,c^6x^7+40040\,\left(2\,d^2x^9+9\,cd\right.\right.$$
$$+\frac{16}{1365}\,\left(91\,d^6x^{15}+1170\,cd^5x^{14}+5040\,c^2d^4x^{13}+7280\,c^3d^3x^{12}\right)c^2$$

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="maxima")
```

```
[Out] 1/17*d^8*x^17 + c*d^7*x^16 + 32/5*c^2*d^6*x^15 + 128/7*c^3*d^5*x^14 + 256/1
3*c^4*d^4*x^13 + 256/9*c^8*x^9 + 256*a^4*c^4*x + 256/15*(3*d^2*x^5 + 15*c*d
*x^4 + 20*c^2*x^3)*a^3*c^3 + 256/55*(5*d^2*x^11 + 22*c*d*x^10)*c^6 + 32/105
*(35*d^4*x^9 + 315*c*d^3*x^8 + 720*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*d^2*
x^7 + 14*c*d*x^6)*c^2)*a^2*c^2 + 32/143*(33*d^4*x^13 + 286*c*d^3*x^12 + 624
*c^2*d^2*x^11)*c^4 + 16/15015*(1155*d^6*x^13 + 15015*c*d^5*x^12 + 65520*c^2
*d^4*x^11 + 96096*c^3*d^3*x^10 + 137280*c^6*x^7 + 40040*(2*d^2*x^9 + 9*c*d*
x^8)*c^4 + 364*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x^9)*c^2)*a*c +
16/1365*(91*d^6*x^15 + 1170*c*d^5*x^14 + 5040*c^2*d^4*x^13 + 7280*c^3*d^3*x
^12)*c^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.03

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx=\frac{1}{17}\,d^8x^{17}+cd^7x^{16}+\frac{112}{15}\,c^2d^6x^{15}+32\,c^3d^5x^{14}+\frac{1120}{13}\,c^4d^4x^{13}+\frac{16}{13}\,acd^6x^{13}+\frac{448}{3}\,c^5d^3x^{12}+16\,ac^2d^5x^{12}+\frac{1792}{11}\,c^6d^2x^{11}+\frac{960}{11}\,ac^3d^4x^{11}+\frac{512}{5}\,c^7dx^{10}+256\,ac^4d^3x^{10}+\frac{256}{9}\,c^8x^9+\frac{1280}{3}\,ac^5d^2x^9+\frac{32}{3}\,a^2c^2d^4x^9+384\,ac^6dx^8+96\,a^2c^3d^3x^8+\frac{1024}{7}\,ac^7x^7+\frac{2304}{7}\,a^2c^4d^2x^7+512\,a^2c^5dx^6+\frac{1536}{5}\,a^2c^6x^5+\frac{256}{5}\,a^3c^3d^2x^5+256\,a^3c^4dx^4+\frac{1024}{3}\,a^3c^5x^3+256\,a^4c^4x$$

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="giac")
```

```
[Out] 1/17*d^8*x^17 + c*d^7*x^16 + 112/15*c^2*d^6*x^15 + 32*c^3*d^5*x^14 + 1120/1
3*c^4*d^4*x^13 + 16/13*a*c*d^6*x^13 + 448/3*c^5*d^3*x^12 + 16*a*c^2*d^5*x^1
2 + 1792/11*c^6*d^2*x^11 + 960/11*a*c^3*d^4*x^11 + 512/5*c^7*d*x^10 + 256*a
*c^4*d^3*x^10 + 256/9*c^8*x^9 + 1280/3*a*c^5*d^2*x^9 + 32/3*a^2*c^2*d^4*x^9
 + 384*a*c^6*d*x^8 + 96*a^2*c^3*d^3*x^8 + 1024/7*a*c^7*x^7 + 2304/7*a^2*c^4
*d^2*x^7 + 512*a^2*c^5*d*x^6 + 1536/5*a^2*c^6*x^5 + 256/5*a^3*c^3*d^2*x^5 +
 256*a^3*c^4*d*x^4 + 1024/3*a^3*c^5*x^3 + 256*a^4*c^4*x
```

Mupad [B] (verification not implemented)

Time = 10.57 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.97

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx = x^{10}\left(\frac{512\,c^7\,d}{5}+256\,a\,c^4\,d^3\right) + x^{13}\left(\frac{1120\,c^4\,d^4}{13}+\frac{16\,a\,c\,d^6}{13}\right) + x^9\left(\frac{32\,a^2\,c^2\,d^4}{3}+\frac{1280\,a\,c^5\,d^2}{3}+\frac{256\,c^8}{9}\right) + x^{12}\left(\frac{448\,c^5\,d^3}{3}+16\,a\,c^2\,d^5\right) + x^{11}\left(\frac{1792\,c^6\,d^2}{11}+\frac{960\,a\,c^3\,d^4}{11}\right)+\frac{d^8\,x^{17}}{17} + 256\,a^4\,c^4\,x+c\,d^7\,x^{16}+\frac{1024\,a^3\,c^5\,x^3}{3} + 32\,c^3\,d^5\,x^{14}+\frac{112\,c^2\,d^6\,x^{15}}{15}+256\,a^3\,c^4\,d\,x^4 + 512\,a^2\,c^5\,d\,x^6+\frac{256\,a\,c^4\,x^7\,(4\,c^3+9\,a\,d^2)}{7} + \frac{256\,a^2\,c^3\,x^5\,(6\,c^3+a\,d^2)}{5}+96\,a\,c^3\,d\,x^8\,\left(4\,c^3+a\,d^2\right)$$

```
[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^4,x)
```

```
[Out] x^10*((512*c^7*d)/5 + 256*a*c^4*d^3) + x^13*((1120*c^4*d^4)/13 + (16*a*c*d^
6)/13) + x^9*((256*c^8)/9 + (1280*a*c^5*d^2)/3 + (32*a^2*c^2*d^4)/3) + x^12
*((448*c^5*d^3)/3 + 16*a*c^2*d^5) + x^11*((1792*c^6*d^2)/11 + (960*a*c^3*d^
4)/11) + (d^8*x^17)/17 + 256*a^4*c^4*x + c*d^7*x^16 + (1024*a^3*c^5*x^3)/3
+ 32*c^3*d^5*x^14 + (112*c^2*d^6*x^15)/15 + 256*a^3*c^4*d*x^4 + 512*a^2*c^5
*d*x^6 + (256*a*c^4*x^7*(9*a*d^2 + 4*c^3))/7 + (256*a^2*c^3*x^5*(a*d^2 + 6*
c^3))/5 + 96*a*c^3*d*x^8*(a*d^2 + 4*c^3)
```

3.34 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx$

Optimal result 375
Rubi [A] (verified) 375
Mathematica [A] (verified) 376
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Maxima [A] (verification not implemented) 378
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Optimal result

Integrand size = 29, antiderivative size = 171

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx = 64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+\frac{48}{5}ac^2\left(4c^3+ad^2\right)x^5 + 64ac^4dx^6+\frac{32}{7}c^3\left(2c^3+9ad^2\right)x^7 + 12c^2d\left(2c^3+ad^2\right)x^8+\frac{4}{3}cd^2\left(20c^3+ad^2\right)x^9 + 16c^3d^3x^{10}+\frac{60}{11}c^2d^4x^{11}+cd^5x^{12}+\frac{d^6x^{13}}{13}$$

```
[Out] 64*a^3*c^3*x+64*a^2*c^4*x^3+48*a^2*c^3*d*x^4+48/5*a*c^2*(a*d^2+4*c^3)*x^5+6
4*a*c^4*d*x^6+32/7*c^3*(9*a*d^2+2*c^3)*x^7+12*c^2*d*(a*d^2+2*c^3)*x^8+4/3*c
*d^2*(a*d^2+20*c^3)*x^9+16*c^3*d^3*x^10+60/11*c^2*d^4*x^11+c*d^5*x^12+1/13*
d^6*x^13
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2086}

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx = 64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+64ac^4dx^6 + \frac{4}{3}cd^2x^9\left(ad^2+20c^3\right)+\frac{32}{7}c^3x^7\left(9ad^2+2c^3\right) + 12c^2dx^8\left(ad^2+2c^3\right)+\frac{48}{5}ac^2x^5\left(ad^2+4c^3\right) + 16c^3d^3x^{10}+\frac{60}{11}c^2d^4x^{11}+cd^5x^{12}+\frac{d^6x^{13}}{13}$$

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]
```

```
[Out] 64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c^3 + a*d^2
)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x^7)/7 + 12*c^2*d*(2*
c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*x^9)/3 + 16*c^3*d^3*x^10 + (60
*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d^6*x^13)/13
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(64a^3c^3 + 192a^2c^4x^2 + 192a^2c^3dx^3 + 48ac^2\left(4c^3 + ad^2\right)x^4 + 384ac^4dx^5 \right. \\ &\quad + 32c^3\left(2c^3 + 9ad^2\right)x^6 + 96c^2d\left(2c^3 + ad^2\right)x^7 + 12cd^2\left(20c^3 + ad^2\right)x^8 + 160c^3d^3x^9 \\ &\quad \left. + 60c^2d^4x^{10} + 12cd^5x^{11} + d^6x^{12}\right) dx \\ &= 64a^3c^3x + 64a^2c^4x^3 + 48a^2c^3dx^4 + \frac{48}{5}ac^2\left(4c^3 + ad^2\right)x^5 + 64ac^4dx^6 + \frac{32}{7}c^3\left(2c^3 + 9ad^2\right)x^7 \\ &\quad + 12c^2d\left(2c^3 + ad^2\right)x^8 + \frac{4}{3}cd^2\left(20c^3 + ad^2\right)x^9 + 16c^3d^3x^{10} + \frac{60}{11}c^2d^4x^{11} + cd^5x^{12} + \frac{d^6x^{13}}{13}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^3 dx &= 64a^3c^3x + 64a^2c^4x^3 + 48a^2c^3dx^4 + \frac{48}{5}ac^2\left(4c^3 + ad^2\right)x^5 \\ &\quad + 64ac^4dx^6 + \frac{32}{7}c^3\left(2c^3 + 9ad^2\right)x^7 \\ &\quad + 12c^2d\left(2c^3 + ad^2\right)x^8 + \frac{4}{3}cd^2\left(20c^3 + ad^2\right)x^9 \\ &\quad + 16c^3d^3x^{10} + \frac{60}{11}c^2d^4x^{11} + cd^5x^{12} + \frac{d^6x^{13}}{13}\end{aligned}$$

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]
```

```
[Out] 64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c^3 + a*d^2
)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x^7)/7 + 12*c^2*d*(2*
c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*x^9)/3 + 16*c^3*d^3*x^10 + (60
*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d^6*x^13)/13
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95

method	result
norman	$\frac{d^6x^{13}}{13}+c\,d^5x^{12}+\frac{60c^2d^4x^{11}}{11}+16c^3d^3x^{10}+\left(\frac{4}{3}ac\,d^4+\frac{80}{3}d^2c^4\right)x^9+\left(12a\,c^2d^3+24c^5d\right)x^8+\left(\frac{28}{7}\right.$
gosper	$\frac{1}{13}d^6x^{13}+c\,d^5x^{12}+\frac{60}{11}c^2d^4x^{11}+16c^3d^3x^{10}+\frac{4}{3}x^9ac\,d^4+\frac{80}{3}x^9d^2c^4+12a\,c^2d^3x^8+24c^5d\,x^8+$
risch	$\frac{1}{13}d^6x^{13}+c\,d^5x^{12}+\frac{60}{11}c^2d^4x^{11}+16c^3d^3x^{10}+\frac{4}{3}x^9ac\,d^4+\frac{80}{3}x^9d^2c^4+12a\,c^2d^3x^8+24c^5d\,x^8+$
parallelrisch	$\frac{1}{13}d^6x^{13}+c\,d^5x^{12}+\frac{60}{11}c^2d^4x^{11}+16c^3d^3x^{10}+\frac{4}{3}x^9ac\,d^4+\frac{80}{3}x^9d^2c^4+12a\,c^2d^3x^8+24c^5d\,x^8+$
default	$\frac{d^6x^{13}}{13}+c\,d^5x^{12}+\frac{60c^2d^4x^{11}}{11}+16c^3d^3x^{10}+\frac{\left(4ac\,d^4+224d^2c^4+d^2\left(8d^2ac+16c^4\right)\right)x^9}{9}+\frac{\left(64a\,c^2d^3+128c^5d+4cd(8\right.}{8}$

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*d^6*x^13+c*d^5*x^12+60/11*c^2*d^4*x^11+16*c^3*d^3*x^10+(4/3*a*c*d^4+80
/3*d^2*c^4)*x^9+(12*a*c^2*d^3+24*c^5*d)*x^8+(288/7*a*c^3*d^2+64/7*c^6)*x^7+
64*a*c^4*d*x^6+(48/5*a^2*c^2*d^2+192/5*a*c^5)*x^5+48*a^2*c^3*d*x^4+64*a^2*c
^4*x^3+64*a^3*c^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx &= \frac{1}{13}\,d^6x^{13}+cd^5x^{12}+\frac{60}{11}\,c^2d^4x^{11}+16\,c^3d^3x^{10}+64\,ac^4dx^6\\&\quad+48\,a^2c^3dx^4+\frac{4}{3}\left(20\,c^4d^2+acd^4\right)x^9+64\,a^2c^4x^3\\&\quad+12\left(2\,c^5d+ac^2d^3\right)x^8+\frac{32}{7}\left(2\,c^6+9\,ac^3d^2\right)x^7\\&\quad+64\,a^3c^3x+\frac{48}{5}\left(4\,ac^5+a^2c^2d^2\right)x^5\end{aligned}$$

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*d^6*x^13 + c*d^5*x^12 + 60/11*c^2*d^4*x^11 + 16*c^3*d^3*x^10 + 64*a*c^
4*d*x^6 + 48*a^2*c^3*d*x^4 + 4/3*(20*c^4*d^2 + a*c*d^4)*x^9 + 64*a^2*c^4*x^
3 + 12*(2*c^5*d + a*c^2*d^3)*x^8 + 32/7*(2*c^6 + 9*a*c^3*d^2)*x^7 + 64*a^3*
c^3*x + 48/5*(4*a*c^5 + a^2*c^2*d^2)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.05

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx = 64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+64ac^4dx^6 + 16c^3d^3x^{10}+\frac{60c^2d^4x^{11}}{11}+cd^5x^{12}+\frac{d^6x^{13}}{13}+x^9 \cdot\left(\frac{4acd^4}{3}+\frac{80c^4d^2}{3}\right)+x^8\cdot\left(12ac^2d^3+24c^5d\right)+x^7 \cdot\left(\frac{288ac^3d^2}{7}+\frac{64c^6}{7}\right)+x^5\cdot\left(\frac{48a^2c^2d^2}{5}+\frac{192ac^5}{5}\right)$$

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**3,x)
```

```
[Out] 64*a**3*c**3*x + 64*a**2*c**4*x**3 + 48*a**2*c**3*d*x**4 + 64*a*c**4*d*x**6
 + 16*c**3*d**3*x**10 + 60*c**2*d**4*x**11/11 + c*d**5*x**12 + d**6*x**13/1
3 + x**9*(4*a*c*d**4/3 + 80*c**4*d**2/3) + x**8*(12*a*c**2*d**3 + 24*c**5*d
) + x**7*(288*a*c**3*d**2/7 + 64*c**6/7) + x**5*(48*a**2*c**2*d**2/5 + 192*
a*c**5/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.20

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx = \frac{1}{13}\,d^6x^{13}+cd^5x^{12}+\frac{48}{11}\,c^2d^4x^{11}+\frac{32}{5}\,c^3d^3x^{10}+\frac{64}{7}\,c^6x^7+64\,a^3c^3x + \frac{16}{5}\left(3\,d^2x^5+15\,cdx^4+20\,c^2x^3\right)a^2c^2+\frac{8}{3}\left(2\,d^2x^9+9\,cdx^8\right)c^4 + \frac{4}{105}\left(35\,d^4x^9+315\,cd^3x^8+720\,c^2d^2x^7+1008\,c^4x^5+120\left(3\,d^2x^7+14\,cdx^6\right)c^2\right)ac + \frac{4}{165}\left(45\,d^4x^{11}+396\,cd^3x^{10}+880\,c^2d^2x^9\right)c^2$$

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*d^6*x^13 + c*d^5*x^12 + 48/11*c^2*d^4*x^11 + 32/5*c^3*d^3*x^10 + 64/7*
c^6*x^7 + 64*a^3*c^3*x + 16/5*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a^2*c^2
 + 8/3*(2*d^2*x^9 + 9*c*d*x^8)*c^4 + 4/105*(35*d^4*x^9 + 315*c*d^3*x^8 + 72
0*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*d^2*x^7 + 14*c*d*x^6)*c^2)*a*c + 4/16
5*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x^9)*c^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.97

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx=\frac{1}{13}\,d^6x^{13}+cd^5x^{12}+\frac{60}{11}\,c^2d^4x^{11}+16\,c^3d^3x^{10}+\frac{80}{3}\,c^4d^2x^9+\frac{4}{3}\,acd^4x^9+24\,c^5dx^8+12\,ac^2d^3x^8+\frac{64}{7}\,c^6x^7+\frac{288}{7}\,ac^3d^2x^7+64\,ac^4dx^6+\frac{192}{5}\,ac^5x^5+\frac{48}{5}\,a^2c^2d^2x^5+48\,a^2c^3dx^4+64\,a^2c^4x^3+64\,a^3c^3x$$

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="giac")
```

```
[Out] 1/13*d^6*x^13 + c*d^5*x^12 + 60/11*c^2*d^4*x^11 + 16*c^3*d^3*x^10 + 80/3*c^
4*d^2*x^9 + 4/3*a*c*d^4*x^9 + 24*c^5*d*x^8 + 12*a*c^2*d^3*x^8 + 64/7*c^6*x^
7 + 288/7*a*c^3*d^2*x^7 + 64*a*c^4*d*x^6 + 192/5*a*c^5*x^5 + 48/5*a^2*c^2*d
^2*x^5 + 48*a^2*c^3*d*x^4 + 64*a^2*c^4*x^3 + 64*a^3*c^3*x
```

Mupad [B] (verification not implemented)

Time = 10.58 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.94

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx=x^8\,\left(24\,c^5\,d+12\,a\,c^2\,d^3\right)+x^9\,\left(\frac{80\,c^4\,d^2}{3}+\frac{4\,a\,c\,d^4}{3}\right)+\frac{d^6\,x^{13}}{13}+x^7\,\left(\frac{64\,c^6}{7}+\frac{288\,a\,c^3\,d^2}{7}\right)+64\,a^3\,c^3\,x+c\,d^5\,x^{12}+64\,a^2\,c^4\,x^3+16\,c^3\,d^3\,x^{10}+\frac{60\,c^2\,d^4\,x^{11}}{11}+48\,a^2\,c^3\,d\,x^4+\frac{48\,a\,c^2\,x^5\,\left(4\,c^3+a\,d^2\right)}{5}+64\,a\,c^4\,d\,x^6$$

```
[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^3,x)
```

```
[Out] x^8*(24*c^5*d + 12*a*c^2*d^3) + x^9*((80*c^4*d^2)/3 + (4*a*c*d^4)/3) + (d^6
*x^13)/13 + x^7*((64*c^6)/7 + (288*a*c^3*d^2)/7) + 64*a^3*c^3*x + c*d^5*x^1
2 + 64*a^2*c^4*x^3 + 16*c^3*d^3*x^10 + (60*c^2*d^4*x^11)/11 + 48*a^2*c^3*d*
x^4 + (48*a*c^2*x^5*(a*d^2 + 4*c^3))/5 + 64*a*c^4*d*x^6
```

3.35 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2\,dx$

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Optimal result

Integrand size = 29, antiderivative size = 92

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2\,dx=16a^2c^2x+\frac{32}{3}ac^3x^3+8ac^2dx^4+\frac{8}{5}c\left(2c^3+ad^2\right)x^5+\frac{16}{3}c^3dx^6+\frac{24}{7}c^2d^2x^7+cd^3x^8+\frac{d^4x^9}{9}$$

```
[Out] 16*a^2*c^2*x+32/3*a*c^3*x^3+8*a*c^2*d*x^4+8/5*c*(a*d^2+2*c^3)*x^5+16/3*c^3*
d*x^6+24/7*c^2*d^2*x^7+c*d^3*x^8+1/9*d^4*x^9
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2086}

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2\,dx=16a^2c^2x+\frac{8}{5}cx^5\left(ad^2+2c^3\right)+\frac{32}{3}ac^3x^3+8ac^2dx^4+\frac{16}{3}c^3dx^6+\frac{24}{7}c^2d^2x^7+cd^3x^8+\frac{d^4x^9}{9}$$

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]
```

```
[Out] 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a*d^2)*x^5)
/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8 + (d^4*x^9)/9
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(16a^2c^2 + 32ac^3x^2 + 32ac^2dx^3 + 8c\left(2c^3 + ad^2\right)x^4 + 32c^3dx^5 + 24c^2d^2x^6 + 8cd^3x^7 + d^4x^8\right) dx$$
$$= 16a^2c^2x + \frac{32}{3}ac^3x^3 + 8ac^2dx^4 + \frac{8}{5}c\left(2c^3 + ad^2\right)x^5 + \frac{16}{3}c^3dx^6 + \frac{24}{7}c^2d^2x^7 + cd^3x^8 + \frac{d^4x^9}{9}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^2 dx = 16a^2c^2x + \frac{32}{3}ac^3x^3 + 8ac^2dx^4 + \frac{8}{5}c\left(2c^3 + ad^2\right)x^5 + \frac{16}{3}c^3dx^6 + \frac{24}{7}c^2d^2x^7 + cd^3x^8 + \frac{d^4x^9}{9}$$

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]
```

```
[Out] 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a*d^2)*x^5)
/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8 + (d^4*x^9)/9
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.90

method	result
norman	$\frac{d^4x^9}{9} + c\,d^3x^8 + \frac{24c^2d^2x^7}{7} + \frac{16c^3d\,x^6}{3} + \left(\frac{8}{5}d^2ac + \frac{16}{5}c^4\right)x^5 + 8a\,c^2d\,x^4 + \frac{32a\,c^3x^3}{3} + 16a^2c^2x$
gosper	$\frac{1}{9}d^4x^9 + c\,d^3x^8 + \frac{24}{7}c^2d^2x^7 + \frac{16}{3}c^3d\,x^6 + \frac{8}{5}x^5d^2ac + \frac{16}{5}x^5c^4 + 8a\,c^2d\,x^4 + \frac{32}{3}a\,c^3x^3 + 16a^2c^2x$
default	$\frac{d^4x^9}{9} + c\,d^3x^8 + \frac{24c^2d^2x^7}{7} + \frac{16c^3d\,x^6}{3} + \frac{(8d^2ac+16c^4)x^5}{5} + 8a\,c^2d\,x^4 + \frac{32a\,c^3x^3}{3} + 16a^2c^2x$
risch	$\frac{1}{9}d^4x^9 + c\,d^3x^8 + \frac{24}{7}c^2d^2x^7 + \frac{16}{3}c^3d\,x^6 + \frac{8}{5}x^5d^2ac + \frac{16}{5}x^5c^4 + 8a\,c^2d\,x^4 + \frac{32}{3}a\,c^3x^3 + 16a^2c^2x$
parallelrisch	$\frac{1}{9}d^4x^9 + c\,d^3x^8 + \frac{24}{7}c^2d^2x^7 + \frac{16}{3}c^3d\,x^6 + \frac{8}{5}x^5d^2ac + \frac{16}{5}x^5c^4 + 8a\,c^2d\,x^4 + \frac{32}{3}a\,c^3x^3 + 16a^2c^2x$

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*d^4*x^9+c*d^3*x^8+24/7*c^2*d^2*x^7+16/3*c^3*d*x^6+(8/5*d^2*a*c+16/5*c^4
)*x^5+8*a*c^2*d*x^4+32/3*a*c^3*x^3+16*a^2*c^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2\,dx = \frac{1}{9}\,d^4x^9+cd^3x^8+\frac{24}{7}\,c^2d^2x^7+\frac{16}{3}\,c^3dx^6+8\,ac^2dx^4 + \frac{32}{3}\,ac^3x^3+\frac{8}{5}\,\left(2\,c^4+acd^2\right)x^5+16\,a^2c^2x$$

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="fricas")
[Out] 1/9*d^4*x^9 + c*d^3*x^8 + 24/7*c^2*d^2*x^7 + 16/3*c^3*d*x^6 + 8*a*c^2*d*x^4
 + 32/3*a*c^3*x^3 + 8/5*(2*c^4 + a*c*d^2)*x^5 + 16*a^2*c^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2\,dx = 16a^2c^2x+\frac{32ac^3x^3}{3}+8ac^2dx^4+\frac{16c^3dx^6}{3} + \frac{24c^2d^2x^7}{7}+cd^3x^8+\frac{d^4x^9}{9}+x^5\cdot\left(\frac{8acd^2}{5}+\frac{16c^4}{5}\right)$$

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
[Out] 16*a**2*c**2*x + 32*a*c**3*x**3/3 + 8*a*c**2*d*x**4 + 16*c**3*d*x**6/3 + 24
*c**2*d**2*x**7/7 + c*d**3*x**8 + d**4*x**9/9 + x**5*(8*a*c*d**2/5 + 16*c**
4/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.02

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2\,dx = \frac{1}{9}\,d^4x^9+cd^3x^8+\frac{16}{7}\,c^2d^2x^7+\frac{16}{5}\,c^4x^5 + 16\,a^2c^2x+\frac{8}{15}\,\left(3\,d^2x^5+15\,cdx^4+20\,c^2x^3\right)ac + \frac{8}{21}\,\left(3\,d^2x^7+14\,cdx^6\right)c^2$$

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="maxima")
[Out] 1/9*d^4*x^9 + c*d^3*x^8 + 16/7*c^2*d^2*x^7 + 16/5*c^4*x^5 + 16*a^2*c^2*x +
8/15*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a*c + 8/21*(3*d^2*x^7 + 14*c*d*x
^6)*c^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.90

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2 \, dx = \frac{1}{9}\,d^4x^9 + cd^3x^8 + \frac{24}{7}\,c^2d^2x^7 + \frac{16}{3}\,c^3dx^6 + \frac{16}{5}\,c^4x^5 + \frac{8}{5}\,acd^2x^5 + 8\,ac^2dx^4 + \frac{32}{3}\,ac^3x^3 + 16\,a^2c^2x$$

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="giac")
```

```
[Out] 1/9*d^4*x^9 + c*d^3*x^8 + 24/7*c^2*d^2*x^7 + 16/3*c^3*d*x^6 + 16/5*c^4*x^5
+ 8/5*a*c*d^2*x^5 + 8*a*c^2*d*x^4 + 32/3*a*c^3*x^3 + 16*a^2*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2 \, dx = x^5\left(\frac{16\,c^4}{5}+\frac{8\,a\,c\,d^2}{5}\right)+\frac{d^4\,x^9}{9}+16\,a^2\,c^2\,x+\frac{32\,a\,c^3\,x^3}{3} + \frac{16\,c^3\,d\,x^6}{3}+c\,d^3\,x^8+\frac{24\,c^2\,d^2\,x^7}{7}+8\,a\,c^2\,d\,x^4$$

```
[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^2,x)
```

```
[Out] x^5*((16*c^4)/5 + (8*a*c*d^2)/5) + (d^4*x^9)/9 + 16*a^2*c^2*x + (32*a*c^3*x
^3)/3 + (16*c^3*d*x^6)/3 + c*d^3*x^8 + (24*c^2*d^2*x^7)/7 + 8*a*c^2*d*x^4
```

3.36 $\int (4ac+4c^2x^2+4cdx^3+d^2x^4)\,dx$

Optimal result 384
Rubi [A] (verified) 384
Mathematica [A] (verified) 385
Maple [A] (verified) 385
Fricas [A] (verification not implemented) 385
Sympy [A] (verification not implemented) 386
Maxima [A] (verification not implemented) 386
Giac [A] (verification not implemented) 386
Mupad [B] (verification not implemented) 386

Optimal result

Integrand size = 27, antiderivative size = 32

$$\int (4ac+4c^2x^2+4cdx^3+d^2x^4)\,dx=4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

```
[Out] 4*a*c*x+4/3*c^2*x^3+c*d*x^4+1/5*d^2*x^5
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (4ac+4c^2x^2+4cdx^3+d^2x^4)\,dx=4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

```
[In] Int[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]
```

```
[Out] 4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5
```

Rubi steps

$$\text{integral}=4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)\,dx=4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

```
[In] Integrate[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]
[Out] 4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
gosper	$4acx+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}d^2x^5$	29
default	$4acx+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}d^2x^5$	29
norman	$4acx+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}d^2x^5$	29
risch	$4acx+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}d^2x^5$	29
parallelrisch	$4acx+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}d^2x^5$	29
parts	$4acx+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}d^2x^5$	29

```
[In] int(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x,method=_RETURNVERBOSE)
[Out] 4*a*c*x+4/3*c^2*x^3+c*d*x^4+1/5*d^2*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)\,dx=\frac{1}{5}\,d^2x^5+cdx^4+\frac{4}{3}\,c^2x^3+4\,acx$$

```
[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="fricas")
[Out] 1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)\,dx=4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

```
[In] integrate(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c,x)
[Out] 4*a*c*x + 4*c**2*x**3/3 + c*d*x**4 + d**2*x**5/5
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)\,dx=\frac{1}{5}\,d^2x^5+cdx^4+\frac{4}{3}\,c^2x^3+4\,acx$$

```
[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="maxima")
[Out] 1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)\,dx=\frac{1}{5}\,d^2x^5+cdx^4+\frac{4}{3}\,c^2x^3+4\,acx$$

```
[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="giac")
[Out] 1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)\,dx=\frac{4\,c^2\,x^3}{3}+c\,d\,x^4+4\,a\,c\,x+\frac{d^2\,x^5}{5}$$

```
[In] int(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3,x)
[Out] (4*c^2*x^3)/3 + (d^2*x^5)/5 + 4*a*c*x + c*d*x^4
```

3.37 $\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

Optimal result 387
Rubi [A] (verified) 388
Mathematica [C] (verified) 391
Maple [C] (verified) 391
Fricas [B] (verification not implemented) 391
Sympy [A] (verification not implemented) 393
Maxima [F] 393
Giac [A] (verification not implemented) 394
Mupad [B] (verification not implemented) 395

Optimal result

Integrand size = 29, antiderivative size = 529

$$\begin{aligned}\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx &= -\frac{d\operatorname{arctanh}\left(\frac{\sqrt{2}c+\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}+\sqrt{2}dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}(c+dx)}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\\ &\quad-\frac{d\log\left(\sqrt{c}\sqrt{c^3+4ad^2}-\sqrt{2}\sqrt[4]{c}d\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{4\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}\\ &\quad+\frac{d\log\left(\sqrt{c}\sqrt{c^3+4ad^2}+\sqrt{2}\sqrt[4]{c}d\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{4\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}\end{aligned}$$

```
[Out] -1/4*d*arctanh((c*2^(1/2)+d*x*2^(1/2)+c^(1/4)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))
^(1/2))/c^(1/4)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(4*a*d
^2+c^3)^(1/2)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2)+1/4*d*arctanh((-(d*x+c)*2
^(1/2)+c^(1/4)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(1/4)/(c^(3/2)-(4*a*d
^2+c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(4*a*d^2+c^3)^(1/2)/(c^(3/2)-(4*a*d^2
+c^3)^(1/2))^(1/2)-1/8*d*ln(d^2*(c/d+x)^2+c^(1/2)*(4*a*d^2+c^3)^(1/2)-c^(1/
4)*d*(c/d+x)*2^(1/2)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(
4*a*d^2+c^3)^(1/2)/(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)+1/8*d*ln(d^2*(c/d+x)
^2+c^(1/2)*(4*a*d^2+c^3)^(1/2)+c^(1/4)*d*(c/d+x)*2^(1/2)*(c^(3/2)+(4*a*d^2+
c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(4*a*d^2+c^3)^(1/2)/(c^(3/2)+(4*a*d^2+c^
3)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1120, 1108, 648, 632, 212, 642}

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$$
$$= -\frac{d\,\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}+\sqrt{2}c+\sqrt{2}dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}-\sqrt{2}(c+dx)}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}$$
$$-\frac{d\log\left(\sqrt{c}\sqrt{4ad^2+c^3}-\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{4\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$
$$+\frac{d\log\left(\sqrt{c}\sqrt{4ad^2+c^3}+\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{4\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]
```

```
[Out] -1/2*(d*ArcTanh[(Sqrt[2]*c + c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] +
Sqrt[2]*d*x)/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(Sqrt[2]*c^(3/
4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) + (d*ArcTanh[(c
^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sq
rt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(2*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]
*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2]
- Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/
d + x)^2])/(4*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 +
 4*a*d^2]]) + (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt[2]*c^(1/4)*d*Sqrt[c
^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2])/(4*Sqrt[2]*c^(3
/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}\,dx, x, \frac{c}{d}+x\right)$$

$$= \frac{d\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{d}-x}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}x}{d}+x^2}\,dx, x, \frac{c}{d}+x\right)}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}} + \frac{d\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{d}+x}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}x}{d}+x^2}\,dx, x, \frac{c}{d}+x\right)}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}x}{d}+x^2}dx,x,\frac{c}{d}+x\right)}{4\sqrt{c}\sqrt{c^3+4ad^2}}$$
$$+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}x}{d}+x^2}dx,x,\frac{c}{d}+x\right)}{4\sqrt{c}\sqrt{c^3+4ad^2}}$$
$$-\frac{d\text{Subst}\left(\int \frac{-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{d}+2x}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}x}{d}+x^2}dx,x,\frac{c}{d}+x\right)}{4\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$
$$+\frac{d\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{d}+2x}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}x}{d}+x^2}dx,x,\frac{c}{d}+x\right)}{4\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$

$$= -\frac{d\log\left(\sqrt{c}\sqrt{c^3+4ad^2}-\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}(c+dx)+(c+dx)^2\right)}{4\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$
$$+\frac{d\log\left(\sqrt{c}\sqrt{c^3+4ad^2}+\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}(c+dx)+(c+dx)^2\right)}{4\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$
$$-\frac{\text{Subst}\left(\int \frac{1}{\frac{2\sqrt{c}\left(c^{3/2}-\sqrt{c^3+4ad^2}\right)}{d^2}-x^2}dx,x,-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{d}+2\left(\frac{c}{d}+x\right)\right)}{2\sqrt{c}\sqrt{c^3+4ad^2}}$$
$$-\frac{\text{Subst}\left(\int \frac{1}{\frac{2\sqrt{c}\left(c^{3/2}-\sqrt{c^3+4ad^2}\right)}{d^2}-x^2}dx,x,\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{d}+2\left(\frac{c}{d}+x\right)\right)}{2\sqrt{c}\sqrt{c^3+4ad^2}}$$

$$= -\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\sqrt{2}c^{3/4}-\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\right)+\sqrt{2}dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}$$
$$-\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\sqrt{2}c^{3/4}+\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\right)+\sqrt{2}dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}$$
$$-\frac{d\log\left(\sqrt{c}\sqrt{c^3+4ad^2}-\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}(c+dx)+(c+dx)^2\right)}{4\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$
$$+\frac{d\log\left(\sqrt{c}\sqrt{c^3+4ad^2}+\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}(c+dx)+(c+dx)^2\right)}{4\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.13

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \frac{1}{4}\text{RootSum}\left[4ac+4c^2\#1^2+4cd\#1^3+d^2\#1^4\&,\frac{\log(x-\#1)}{2c^2\#1+3cd\#1^2+d^2\#1^3}\&\right]$$

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]
[Out] RootSum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 & , Log[x - #1]/(2*c^2*#
1 + 3*c*d*#1^2 + d^2*#1^3) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.12

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(d^2\_Z^4+4cd\_Z^3+4c^2\_Z^2+4ac\right)}\frac{\ln\left(x-\_R\right)}{\_R^3d^2+3\_R^2cd+2\_Rc^2}\right)}{4}$	64
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(d^2\_Z^4+4cd\_Z^3+4c^2\_Z^2+4ac\right)}\frac{\ln\left(x-\_R\right)}{\_R^3d^2+3\_R^2cd+2\_Rc^2}\right)}{4}$	64

```
[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(1/(_R^3*d^2+3*_R^2*c*d+2*_R*c^2)*ln(x-_R),_R=RootOf(_Z^4*d^2+4*_Z^3
*c*d+4*_Z^2*c^2+4*a*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 905 vs. 2(408) = 816.

Time = 0.29 (sec) , antiderivative size = 905, normalized size of antiderivative = 1.71

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$$

$$= \frac{1}{8}\sqrt{-\frac{2\left(ac^3+4\,a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}+1}{ac^3+4\,a^2d^2}}\log\Bigg(d^2x+cd$$

$$+\left(2\,acd^2+\left(ac^7+4\,a^2c^4d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}\right)\sqrt{-\frac{2\left(ac^3+4\,a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}}{ac^3+4\,a^2d^2}}$$

$$-\frac{1}{8}\sqrt{-\frac{2\left(ac^3+4\,a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}+1}{ac^3+4\,a^2d^2}}\log\Bigg(d^2x+cd$$

$$-\left(2\,acd^2+\left(ac^7+4\,a^2c^4d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}\right)\sqrt{-\frac{2\left(ac^3+4\,a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}}{ac^3+4\,a^2d^2}}$$

$$+\frac{1}{8}\sqrt{\frac{2\left(ac^3+4\,a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}-1}{ac^3+4\,a^2d^2}}\log\Bigg(d^2x+cd$$

$$+\left(2\,acd^2-\left(ac^7+4\,a^2c^4d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}\right)\sqrt{\frac{2\left(ac^3+4\,a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}-}{ac^3+4\,a^2d^2}}$$

$$-\frac{1}{8}\sqrt{\frac{2\left(ac^3+4\,a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}-1}{ac^3+4\,a^2d^2}}\log\Bigg(d^2x+cd$$

$$-\left(2\,acd^2-\left(ac^7+4\,a^2c^4d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}\right)\sqrt{\frac{2\left(ac^3+4\,a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}-}{ac^3+4\,a^2d^2}}$$

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*
c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d + (2*a*c*d^2 + (a*c^7 +
 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt(-
(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4))
```

```
+ 1)/(a*c^3 + 4*a^2*d^2))) - 1/8*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*
c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x
+ c*d - (2*a*c*d^2 + (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d
^2 + 16*a^3*c^3*d^4)))*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^
2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))) + 1/8*sqrt((2*(a*c^
3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*
c^3 + 4*a^2*d^2))*log(d^2*x + c*d + (2*a*c*d^2 - (a*c^7 + 4*a^2*c^4*d^2)*sq
rt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt((2*(a*c^3 + 4*a^2*d
^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2
*d^2))) - 1/8*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2
+ 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d - (2*a*c*d^2 -
 (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)
))*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^
3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2)))
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.17

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$$
$$= \operatorname{RootSum}\left(t^4\cdot\left(16384a^3c^3d^2+4096a^2c^6\right)+128t^2ac^3+1, \left(t\mapsto t\log\left(x+\frac{-1024t^3a^2c^4d^2-256t^3ac^7+1}{d^2}\right.\right.\right.$$

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c),x)
```

```
[Out] RootSum(_t**4*(16384*a**3*c**3*d**2 + 4096*a**2*c**6) + 128*_t**2*a*c**3 +
1, Lambda(_t, _t*log(x + (-1024*_t**3*a**2*c**4*d**2 - 256*_t**3*a*c**7 + 1
6*_t*a*c*d**2 - 4*_t*c**4 + c*d)/d**2)))
```

Maxima [F]

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \int \frac{1}{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}\,dx$$

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="maxima")
```

```
[Out] integrate(1/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.14

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = -\frac{\log\left(x+\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}{4\left(d^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^3-3\,cd\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^2+2\,c^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)\right)} + \frac{\log\left(x-\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}{4\left(d^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)^3+3\,cd\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)^2+2\,c^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)\right)} - \frac{\log\left(x+\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}{4\left(d^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^3-3\,cd\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^2+2\,c^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)\right)} + \frac{\log\left(x-\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}{4\left(d^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)^3+3\,cd\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)^2+2\,c^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)\right)}$$

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="giac")
```

```
[Out] -1/4*log(x + sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*
d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)
*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d))
 + 1/4*log(x - sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^
2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*
c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d
)) - 1/4*log(x + sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((
c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 - 2*sqrt(-
a*c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c
/d)) + 1/4*log(x - sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt
((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 - 2*sqrt
(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) -
 c/d))
```

Mupad [B] (verification not implemented)

Time = 12.19 (sec) , antiderivative size = 1551, normalized size of antiderivative = 2.93

$$\int \frac{1}{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}\, dx = \text{Too large to display}$$

```
[In] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3),x)
```

```
[Out] atan(((-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2
)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*
(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) - 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a
*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)*1i + (-(2*
d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*
c^4*d^5 + 256*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 +
4*a^3*c^3*d^2)))^(1/2) + 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(
a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)*1i)/((-(2*d*(-a^3*c^3
)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 2
56*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d
^2)))^(1/2) - 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4
*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x) - (-(2*d*(-a^3*c^3)^(1/2) + a*c^
3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x
)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) +
64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2))
)^(1/2) + 4*c*d^5 + 4*d^6*x)))*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^
6 + 4*a^3*c^3*d^2)))^(1/2)*2i + atan((((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(
a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*((2*d*
(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) - 64*a*c*d^
6)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) +
4*c*d^5 + 4*d^6*x)*1i + ((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^
3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*((2*d*(-a^3*c^3)^(1/
2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 64*a*c*d^6)*((2*d*(-a^3
*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^
6*x)*1i)/(((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(
1/2)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(6
4*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) - 64*a*c*d^6)*((2*d*(-a^3*c^3)^(1/2) -
a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x) - ((2*d*(
-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4
*d^5 + 256*a*c^3*d^6*x)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^
3*c^3*d^2)))^(1/2) + 64*a*c*d^6)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c
^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)))*((2*d*(-a^3*c^3)^(1/2) -
 a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*2i
```

3.38 $\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx$

Optimal result 396
Rubi [A] (verified) 397
Mathematica [C] (verified) 401
Maple [C] (verified) 402
Fricas [B] (verification not implemented) 402
Sympy [F(-1)] 404
Maxima [F] 404
Giac [A] (verification not implemented) 405
Mupad [B] (verification not implemented) 406

Optimal result

Integrand size = 29, antiderivative size = 746

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx = -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\,(c^3+4ad^2)\,(4ac+4c^2x^2+4cdx^3+d^2x^4)} - \frac{d\left(c^3+12ad^2+c^{3/2}\sqrt{c^3+4ad^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}c+\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}+\sqrt{2}dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{32\sqrt{2}ac^{7/4}\,(c^3+4ad^2)^{3/2}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}} + \frac{d\left(c^3+12ad^2+c^{3/2}\sqrt{c^3+4ad^2}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}(c+dx)}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{32\sqrt{2}ac^{7/4}\,(c^3+4ad^2)^{3/2}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}} - \frac{d\left(c^3+12ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\log\left(\sqrt{c}\sqrt{c^3+4ad^2}-\sqrt{2}\sqrt[4]{c}d\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{64\sqrt{2}ac^{7/4}\,(c^3+4ad^2)^{3/2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}} + \frac{d\left(c^3+12ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\log\left(\sqrt{c}\sqrt{c^3+4ad^2}+\sqrt{2}\sqrt[4]{c}d\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{64\sqrt{2}ac^{7/4}\,(c^3+4ad^2)^{3/2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$

```
[Out] -1/16*(c/d+x)*(c^3-4*a*d^2-c*d^2*(c/d+x)^2)/a/c/(4*a*d^2+c^3)/(d^2*x^4+4*c*
d*x^3+4*c^2*x^2+4*a*c)-1/64*d*arctanh((c*2^(1/2)+d*x*2^(1/2)+c^(1/4)*(c^(3/
2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(1/4)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2))
*(c^3+12*a*d^2+c^(3/2)*(4*a*d^2+c^3)^(1/2))/a/c^(7/4)/(4*a*d^2+c^3)^(3/2)*2
^(1/2)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2)+1/64*d*arctanh((-(d*x+c)*2^(1/2)
+c^(1/4)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(1/4)/(c^(3/2)-(4*a*d^2+c^3
)^(1/2))^(1/2))*(c^3+12*a*d^2+c^(3/2)*(4*a*d^2+c^3)^(1/2))/a/c^(7/4)/(4*a*d
^2+c^3)^(3/2)*2^(1/2)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2)-1/128*d*ln(d^2*(c
/d+x)^2+c^(1/2)*(4*a*d^2+c^3)^(1/2)-c^(1/4)*d*(c/d+x)*2^(1/2)*(c^(3/2)+(4*a
*d^2+c^3)^(1/2))^(1/2))*(c^3+12*a*d^2-c^(3/2)*(4*a*d^2+c^3)^(1/2))/a/c^(7/4
```

```
)/(4*a*d^2+c^3)^(3/2)*2^(1/2)/(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)+1/128*d*l
n(d^2*(c/d+x)^2+c^(1/2)*(4*a*d^2+c^3)^(1/2)+c^(1/4)*d*(c/d+x)*2^(1/2)*(c^(3
/2)+(4*a*d^2+c^3)^(1/2))^(1/2))*(c^3+12*a*d^2-c^(3/2)*(4*a*d^2+c^3)^(1/2))/
a/c^(7/4)/(4*a*d^2+c^3)^(3/2)*2^(1/2)/(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1120, 1106, 1183, 648, 632, 212, 642}

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx$$
$$= -\frac{d\left(c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}+\sqrt{2}c+\sqrt{2}dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{32\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}$$
$$+\frac{d\left(c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}-\sqrt{2}(c+dx)}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{32\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}$$
$$-\frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(\sqrt{c}\sqrt{4ad^2+c^3}-\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+\right.\right.}{64\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$
$$+\frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(\sqrt{c}\sqrt{4ad^2+c^3}+\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+\right.\right.}{64\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$
$$-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(4ad^2+c^3\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}$$

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2),x]
```

```
[Out] -1/16*((c/d + x)*(c^3 - 4*a*d^2 - c*d^2*(c/d + x)^2))/(a*c*(c^3 + 4*a*d^2)*
(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)) - (d*(c^3 + 12*a*d^2 + c^(3/2)*S
qrt[c^3 + 4*a*d^2])*ArcTanh[(Sqrt[2]*c + c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 +
4*a*d^2]] + Sqrt[2]*d*x)/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(3
2*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2
]]) + (d*(c^3 + 12*a*d^2 + c^(3/2)*Sqrt[c^3 + 4*a*d^2])*ArcTanh[(c^(1/4)*Sq
rt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sqrt[c^(3/2
) - Sqrt[c^3 + 4*a*d^2]])])/(32*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqr
t[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*(c^3 + 12*a*d^2 - c^(3/2)*Sqrt[c^3 +
 4*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] - Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2)
 + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2])/(64*Sqrt[2]*a*c^(7/4)
*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]) + (d*(c^3 + 12*
a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt
```

```
[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)
^2])/(64*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) + Sqrt[c^3 +
4*a*d^2]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4\right)^2}\,dx, x, \frac{c}{d}+x\right)$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} + \frac{\text{Subst}\left(\int \frac{4c^4-2c\left(4a+\frac{c^3}{d^2}\right)d^2-2\left(4c^4-4c\left(4a+\frac{c^3}{d^2}\right)d^2\right)+2c^2d^2x^2}{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}\,dx, x, \frac{c}{d}+x\right)}{32ac^2\left(c^3+4ad^2\right)}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} + \frac{d\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(4c^4-2c\left(4a+\frac{c^3}{d^2}\right)d^2-2\left(4c^4-4c\left(4a+\frac{c^3}{d^2}\right)d^2\right)\right)}{d}-\left(4c^4-2c\left(4a+\frac{c^3}{d^2}\right)d^2-2c^{5/2}\sqrt{c^3+4ad^2}-2\left(\cdots\right.\right.}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}x}{d}+x^2}\cdots\right.}{64\sqrt{2}ac^{11/4}\left(c^3+4ad^2\right)^{3/2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}} + \frac{d\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(4c^4-2c\left(4a+\frac{c^3}{d^2}\right)d^2-2\left(4c^4-4c\left(4a+\frac{c^3}{d^2}\right)d^2\right)\right)}{d}+\left(4c^4-2c\left(4a+\frac{c^3}{d^2}\right)d^2-2c^{5/2}\sqrt{c^3+4ad^2}-2\left(\cdots\right.\right.}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}x}{d}+x^2}\cdots\right.}{64\sqrt{2}ac^{11/4}\left(c^3+4ad^2\right)^{3/2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} - \frac{\left(d\left(c^3+12ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\right)\text{Subst}\left(\int \frac{-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{d}+2x}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}x}{d}+x^2}dx,x,\frac{c}{d}+x\right)}{64\sqrt{2}ac^{7/4}\left(c^3+4ad^2\right)^{3/2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}} + \frac{\left(d\left(c^3+12ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{d}+2x}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}x}{d}+x^2}dx,x,\frac{c}{d}+x\right)}{64\sqrt{2}ac^{7/4}\left(c^3+4ad^2\right)^{3/2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}} + \frac{\left(c^3+12ad^2+c^{3/2}\sqrt{c^3+4ad^2}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}x}{d}+x^2}dx,x,\frac{c}{d}+x\right)}{64ac^{3/2}\left(c^3+4ad^2\right)^{3/2}} + \frac{\left(c^3+12ad^2+c^{3/2}\sqrt{c^3+4ad^2}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}x}{d}+x^2}dx,x,\frac{c}{d}+x\right)}{64ac^{3/2}\left(c^3+4ad^2\right)^{3/2}}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} - \frac{d\left(c^3+12ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\log\left(\sqrt{c}\sqrt{c^3+4ad^2}-\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}(c+dx)+(c-\right.}{64\sqrt{2}ac^{7/4}\left(c^3+4ad^2\right)^{3/2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}} + \frac{d\left(c^3+12ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\log\left(\sqrt{c}\sqrt{c^3+4ad^2}+\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}(c+dx)+(c+\right.}{64\sqrt{2}ac^{7/4}\left(c^3+4ad^2\right)^{3/2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}} - \frac{\left(c^3+12ad^2+c^{3/2}\sqrt{c^3+4ad^2}\right)\text{Subst}\left(\int \frac{1}{\frac{2\sqrt{c}\left(c^{3/2}-\sqrt{c^3+4ad^2}\right)}{d^2}-x^2}dx,x,-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{d}+2\left(\frac{c}{d}\right.\right.}{32ac^{3/2}\left(c^3+4ad^2\right)^{3/2}} - \frac{\left(c^3+12ad^2+c^{3/2}\sqrt{c^3+4ad^2}\right)\text{Subst}\left(\int \frac{1}{\frac{2\sqrt{c}\left(c^{3/2}-\sqrt{c^3+4ad^2}\right)}{d^2}-x^2}dx,x,\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{d}+2\left(\frac{c}{d}+\right.\right.}{32ac^{3/2}\left(c^3+4ad^2\right)^{3/2}}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}$$
$$-\frac{d\left(c^3+12ad^2+c^{3/2}\sqrt{c^3+4ad^2}\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\sqrt{2}c^{3/4}-\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\right)+\sqrt{2}dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{32\sqrt{2}ac^{7/4}\left(c^3+4ad^2\right)^{3/2}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}$$
$$-\frac{d\left(c^3+12ad^2+c^{3/2}\sqrt{c^3+4ad^2}\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\sqrt{2}c^{3/4}+\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\right)+\sqrt{2}dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{32\sqrt{2}ac^{7/4}\left(c^3+4ad^2\right)^{3/2}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}$$
$$-\frac{d\left(c^3+12ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\log\left(\sqrt{c}\sqrt{c^3+4ad^2}-\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}(c+dx)+(c\right.}{64\sqrt{2}ac^{7/4}\left(c^3+4ad^2\right)^{3/2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$
$$+\frac{d\left(c^3+12ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\log\left(\sqrt{c}\sqrt{c^3+4ad^2}+\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}(c+dx)+(c\right.}{64\sqrt{2}ac^{7/4}\left(c^3+4ad^2\right)^{3/2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.24

$$\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2}\,dx$$
$$= \frac{\frac{4(c+dx)(4ad+cx(2c+dx))}{4ac+x^2(2c+dx)^2}+\text{RootSum}\left[4ac+4c^2\#1^2+4cd\#1^3+d^2\#1^4\&,\frac{2c^3\log(x-\#1)+12ad^2\log(x-\#1)+2c^2d\log(x}{2c^2\#1+3cd\#1^2+c}\right.}{64ac\left(c^3+4ad^2\right)}$$

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2),x]
```

```
[Out] ((4*(c + d*x)*(4*a*d + c*x*(2*c + d*x)))/(4*a*c + x^2*(2*c + d*x)^2) + Root
Sum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 & , (2*c^3*Log[x - #1] + 12*
a*d^2*Log[x - #1] + 2*c^2*d*Log[x - #1]*#1 + c*d^2*Log[x - #1]*#1^2)/(2*c^2
*#1 + 3*c*d*#1^2 + d^2*#1^3) & ])/(64*a*c*(c^3 + 4*a*d^2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.31

method	result
default	$\frac{\frac{d^2x^3}{64a\left(4a\,d^2+c^3\right)}+\frac{3dc\,x^2}{64a\left(4a\,d^2+c^3\right)}+\frac{\left(2a\,d^2+c^3\right)x}{32c\left(4a\,d^2+c^3\right)a}+\frac{d}{64a\,d^2+16c^3}}{\frac{1}{4}d^2x^4+cd\,x^3+c^2x^2+ac}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d^2\_Z^4+4cd\_Z^3+4c^2\_Z^2+4ac\right)}\frac{\left(\_R^2c\,d^2+2\_R\,c^2d+\right.}{\_R^3d^2+3\_}}{64ac(4a\,d^2+c^3)}$
risch	$\frac{\frac{d^2x^3}{64a\left(4a\,d^2+c^3\right)}+\frac{3dc\,x^2}{64a\left(4a\,d^2+c^3\right)}+\frac{\left(2a\,d^2+c^3\right)x}{32c\left(4a\,d^2+c^3\right)a}+\frac{d}{64a\,d^2+16c^3}}{\frac{1}{4}d^2x^4+cd\,x^3+c^2x^2+ac}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d^2\_Z^4+4cd\_Z^3+4c^2\_Z^2+4ac\right)}\frac{\left(\frac{d^2\_R^2}{4a\,d^2+c^3}+\frac{2cd\_R}{4a\,d^2+c^3}+\right.}{\_R^3d^2+3\_}}{64a}$

```
[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/64*d^2/a/(4*a*d^2+c^3)*x^3+3/64*d/a*c/(4*a*d^2+c^3)*x^2+1/32/c*(2*a*d^2+
c^3)/(4*a*d^2+c^3)/a*x+1/16*d/(4*a*d^2+c^3))/(1/4*d^2*x^4+c*d*x^3+c^2*x^2+a
*c)+1/64/a/c/(4*a*d^2+c^3)*sum((_R^2*c*d^2+2*_R*c^2*d+12*a*d^2+2*c^3)/(_R^3
*d^2+3*_R^2*c*d+2*_R*c^2)*ln(x-_R),_R=RootOf(_Z^4*d^2+4*_Z^3*c*d+4*_Z^2*c^2
+4*a*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3222 vs. 2(608) = 1216.

Time = 0.37 (sec) , antiderivative size = 3222, normalized size of antiderivative = 4.32

$$\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(4*c*d^2*x^3 + 12*c^2*d*x^2 + 16*a*c*d + (4*a^2*c^5 + 16*a^3*c^2*d^2 +
 (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6
 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11
 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 36
0*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4
 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^
7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))*lo
g(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 32
4*a^2*d^8)*x + (5*a^2*c^8*d^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 + (a^3*c^1
9 + 20*a^4*c^16*d^2 + 144*a^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8
)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^2
2*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8
```

```
*c^10*d^10 + 4096*a^9*c^7*d^12)))*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 +
2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c
^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a
^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 +
4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c
^2*d^6))) - (4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4
*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6
+ 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4
 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*
c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^
13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d
^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))*log(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a
^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 324*a^2*d^8)*x - (5*a^2*c^8*d^4 + 96
*a^3*c^5*d^6 + 432*a^4*c^2*d^8 + (a^3*c^19 + 20*a^4*c^16*d^2 + 144*a^5*c^13
*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^
8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^
6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))*
sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*
a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*
d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 +
3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 1
2*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))) + (4*a^2*c^5 + 16*a^3*c^
2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4
*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a
^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d
^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c
^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096
*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d
^6))*log(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d
^6 + 324*a^2*d^8)*x + (5*a^2*c^8*d^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 - (
a^3*c^19 + 20*a^4*c^16*d^2 + 144*a^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*
c^7*d^8)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*
a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6
144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2
*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt
(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2
+ 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*
d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 6
4*a^6*c^2*d^6))) - (4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*
x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt
(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*
c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10
)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840
*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^
4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))*log(5*c^7*d^3 + 81*a*c^4*d^5
```

```
+ 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 324*a^2*d^8)*x - (5*a^2*c^8*d
^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 - (a^3*c^19 + 20*a^4*c^16*d^2 + 144*a
^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8)*sqrt(-(25*c^6*d^6 + 360*a
*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 +
1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d
^12))*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^
2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 12
96*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16
*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c
^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))) + 8*(c^3 + 2*a*d^
2)*x)/(4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^
5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx=\int \frac{1}{(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac)^2}\,dx$$

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(c*d^2*x^3 + 3*c^2*d*x^2 + 4*a*c*d + 2*(c^3 + 2*a*d^2)*x)/(4*a^2*c^5 +
 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^
3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2) + 1/16*integrate((c*d^2*x^2 + 2*c^2
*d*x + 2*c^3 + 12*a*d^2)/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)/(a*c
^4 + 4*a^2*c*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 1057, normalized size of antiderivative = 1.42

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx =$$

$$-\frac{\frac{\left(cd^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^2-2\,c^2d\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)+2\,c^3+12\,ad^2\right)\log\left(x+\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}{d^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^3-3\,cd\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^2+2\,c^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}-\frac{\left(cd^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}-\right.\right.}{d^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-\right.}}}$$

$$+\frac{cd^2x^3+3\,c^2dx^2+2\,c^3x+4\,ad^2x+4\,acd}{16\,(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac)(ac^4+4\,a^2cd^2)}$$

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="giac")
```

```
[Out] -1/64*((c*d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 - 2*c^2*d*(s
qrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d) + 2*c^3 + 12*a*d^2)*log(x + sq
rt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 + 2*sqrt(-a
*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/
d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)) - (c*d^2*(sqrt
((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*d*(sqrt((c^2*d^2 + 2*sq
rt(-a*c)*d^3)/d^4) - c/d) + 2*c^3 + 12*a*d^2)*log(x - sqrt((c^2*d^2 + 2*sqr
t(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d
)^3 + 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt(
(c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)) + (c*d^2*(sqrt((c^2*d^2 - 2*sqrt(
-a*c)*d^3)/d^4) + c/d)^2 - 2*c^2*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4)
 + c/d) + 2*c^3 + 12*a*d^2)*log(x + sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) +
 c/d)/(d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((
c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-
a*c)*d^3)/d^4) + c/d)) - (c*d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c
/d)^2 + 2*c^2*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d) + 2*c^3 + 12
*a*d^2)*log(x - sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c
^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 - 2*sqrt(-a
*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/
d)))/(a*c^4 + 4*a^2*c*d^2) + 1/16*(c*d^2*x^3 + 3*c^2*d*x^2 + 2*c^3*x + 4*a*
d^2*x + 4*a*c*d)/((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)*(a*c^4 + 4*a^2*
c*d^2))
```

Mupad [B] (verification not implemented)

Time = 12.09 (sec) , antiderivative size = 5844, normalized size of antiderivative = 7.83

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^2,x)
```

```
[Out] (d/(4*(4*a*d^2 + c^3)) + (d^2*x^3)/(16*a*(4*a*d^2 + c^3)) + (x*(2*a*d^2 + c
^3))/(8*a*c*(4*a*d^2 + c^3)) + (3*c*d*x^2)/(16*a*(4*a*d^2 + c^3)))/(4*a*c +
 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3) - atan(((-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^
7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(40
96*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*
((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*
(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768
*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c
^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a
^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 +
 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) - (4096*a^3*c^8*d^6 + 65536*a^4*
c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*
d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*
c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48
*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 +
704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36
*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*
c^2*d^4)))*1i + (-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2
+ 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13
*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*((((262144*a^4*c^12*d^5 +
2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 +
 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c
^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 + 10
*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^
9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*
c^7*d^6)))^(1/2) + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d
^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 + 10*c^
3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c
^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7
*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^
3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a
*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*1i)/((9*a*d^8 +
 c^3*d^6)/(512*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) - (-(a^3*c^11 +
10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-
a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^
9*c^7*d^6)))^(1/2)*((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*
```

a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) - (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4))) + (-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4))))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*2i - atan(((-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) - (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*1i + (-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7

*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*((((262144*a^4*c^12*d
^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*
d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*
a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11
 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5
*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64
*a^9*c^7*d^6)))^(1/2) + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*
c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 -
10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-
a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^
9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(102
4*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 +
 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*1i)/((9*a*
d^8 + c^3*d^6)/(512*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) - (-(a^3*c^
11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d
^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 +
64*a^9*c^7*d^6)))^(1/2)*((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 419
4304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4
096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 +
8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2)
 + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*
c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) - (4096*
a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a
^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) +
15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^1
6 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7
*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 1
6*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 +
 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4))) + (-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1
/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a
^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*((((2
62144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*
c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*
c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^
4)))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^
5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a
^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d
^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4))
)*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d
^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*
c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a
^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*
d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d
^4)))))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5
*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 4

```
8*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*2i
```

3.39 $\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^4 \, dx$

Optimal result 410
Rubi [A] (verified) 411
Mathematica [A] (verified) 413
Maple [A] (verified) 414
Fricas [A] (verification not implemented) 414
Sympy [A] (verification not implemented) 416
Maxima [A] (verification not implemented) 417
Giac [A] (verification not implemented) 418
Mupad [B] (verification not implemented) 419

Optimal result

Integrand size = 32, antiderivative size = 295

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^4 dx = \frac{(5d^4+256ae^3)^4 x}{1048576e^4} - \frac{d^2(5d^4+256ae^3)^3\left(\frac{d}{4e}+x\right)^3}{8192e^2} + \frac{(5d^4+256ae^3)^2(59d^4+256ae^3)\left(\frac{d}{4e}+x\right)^5}{5120} - \frac{9}{224}d^2e^2(5d^4+256ae^3)(17d^4+256ae^3)\left(\frac{d}{4e}+x\right)^7 + \frac{1}{24}e^4(601d^8+20992ad^4e^3+65536a^2e^6)\left(\frac{d}{4e}+x\right)^9 - \frac{72}{11}d^2e^6(17d^4+256ae^3)\left(\frac{d}{4e}+x\right)^{11} + \frac{64}{13}e^8(59d^4+256ae^3)\left(\frac{d}{4e}+x\right)^{13} - \frac{2048}{5}d^2e^{10}\left(\frac{d}{4e}+x\right)^{15} + \frac{4096}{17}e^{12}\left(\frac{d}{4e}+x\right)^{17}$$

```
[Out] 1/1048576*(256*a*e^3+5*d^4)^4*x/e^4-1/8192*d^2*(256*a*e^3+5*d^4)^3*(1/4*d/e
+x)^3/e^2+1/5120*(256*a*e^3+5*d^4)^2*(256*a*e^3+59*d^4)*(1/4*d/e+x)^5-9/224
*d^2*e^2*(256*a*e^3+5*d^4)*(256*a*e^3+17*d^4)*(1/4*d/e+x)^7+1/24*e^4*(65536
*a^2*e^6+20992*a*d^4*e^3+601*d^8)*(1/4*d/e+x)^9-72/11*d^2*e^6*(256*a*e^3+17
*d^4)*(1/4*d/e+x)^11+64/13*e^8*(256*a*e^3+59*d^4)*(1/4*d/e+x)^13-2048/5*d^2
*e^10*(1/4*d/e+x)^15+4096/17*e^12*(1/4*d/e+x)^17
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1120, 1104}

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4\,dx = \frac{1}{24}e^4\left(65536a^2e^6+20992ad^4e^3+601d^8\right)\left(\frac{d}{4e}+x\right)^9 + \frac{\left(256ae^3+5d^4\right)^2\left(256ae^3+59d^4\right)\left(\frac{d}{4e}+x\right)^5}{5120} + \frac{64}{13}e^8\left(256ae^3+59d^4\right)\left(\frac{d}{4e}+x\right)^{13} + \frac{x\left(256ae^3+5d^4\right)^4}{1048576e^4} - \frac{72}{11}d^2e^6\left(256ae^3+17d^4\right)\left(\frac{d}{4e}+x\right)^{11} - \frac{9}{224}d^2e^2\left(256ae^3+5d^4\right)\left(256ae^3+17d^4\right)\left(\frac{d}{4e}+x\right)^7 - \frac{d^2\left(256ae^3+5d^4\right)^3\left(\frac{d}{4e}+x\right)^3}{8192e^2} - \frac{2048}{5}d^2e^{10}\left(\frac{d}{4e}+x\right)^{15} + \frac{4096}{17}e^{12}\left(\frac{d}{4e}+x\right)^{17}$$

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]
```

```
[Out] ((5*d^4 + 256*a*e^3)^4*x)/(1048576*e^4) - (d^2*(5*d^4 + 256*a*e^3)^3*(d/(4*
e) + x)^3)/(8192*e^2) + ((5*d^4 + 256*a*e^3)^2*(59*d^4 + 256*a*e^3)*(d/(4*e
) + x)^5)/5120 - (9*d^2*e^2*(5*d^4 + 256*a*e^3)*(17*d^4 + 256*a*e^3)*(d/(4*
e) + x)^7)/224 + (e^4*(601*d^8 + 20992*a*d^4*e^3 + 65536*a^2*e^6)*(d/(4*e)
+ x)^9)/24 - (72*d^2*e^6*(17*d^4 + 256*a*e^3)*(d/(4*e) + x)^11)/11 + (64*e^
8*(59*d^4 + 256*a*e^3)*(d/(4*e) + x)^13)/13 - (2048*d^2*e^10*(d/(4*e) + x)^
15)/5 + (4096*e^12*(d/(4*e) + x)^17)/17
```

Rule 1104

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
```

```
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \left(\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4\right)^4 dx, x, \frac{d}{4e}+x\right)$$

$$= \text{Subst}\left(\int \left(\frac{(5d^4+256ae^3)^4}{1048576e^4}-\frac{3d^2(5d^4+256ae^3)^3x^2}{8192e^2} + \frac{27}{512}d^4(5d^4+256ae^3)^2\left(1+\frac{1}{54}\left(5+\frac{256ae^3}{d^4}\right)\right)x^4 - \frac{27}{8}d^6e^2(5d^4+256ae^3)\left(\frac{17}{12}+\frac{64ae^3}{3d^4}\right)x^6 + 81d^8e^4\left(1+\frac{(5d^4+256ae^3)(77d^4+256ae^3)}{216d^8}\right)x^8 - 864d^6e^6\left(\frac{17}{12}+\frac{64ae^3}{3d^4}\right)x^{10} + 3456d^4e^8\left(1+\frac{1}{54}\left(5+\frac{256ae^3}{d^4}\right)\right)x^{12} - 6144d^2e^{10}x^{14} + 4096e^{12}x^{16}\right) dx, x, \frac{d}{4e} + x\right)$$

$$= \frac{(5d^4+256ae^3)^4x}{1048576e^4} - \frac{d^2(5d^4+256ae^3)^3\left(\frac{d}{4e}+x\right)^3}{8192e^2} + \frac{(5d^4+256ae^3)^2(59d^4+256ae^3)\left(\frac{d}{4e}+x\right)^5}{5120} - \frac{9}{224}d^2e^2(5d^4+256ae^3)(17d^4+256ae^3)\left(\frac{d}{4e}+x\right)^7 + \frac{1}{24}e^4(601d^8+20992ad^4e^3+65536a^2e^6)\left(\frac{d}{4e}+x\right)^9 - \frac{72}{11}d^2e^6(17d^4+256ae^3)\left(\frac{d}{4e}+x\right)^{11} + \frac{64}{13}e^8(59d^4+256ae^3)\left(\frac{d}{4e}+x\right)^{13} - \frac{2048}{5}d^2e^{10}\left(\frac{d}{4e}+x\right)^{15} + \frac{4096}{17}e^{12}\left(\frac{d}{4e}+x\right)^{17}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.17

$$\int (8ae^2-d^3x+8de^2x^3+8e^3x^4)^4\,dx = 4096a^4e^8x-1024a^3d^3e^6x^2+128a^2d^6e^4x^3 + 8ade^2(-d^8+512a^2e^6)x^4 + \frac{1}{5}(d^{12}-6144a^2d^4e^6+16384a^3e^9)x^5 - 128ad^3e^4(-d^4+8ae^3)x^6 - \frac{32}{7}d^2e^2(d^8-24ad^4e^3-768a^2e^6)x^7 - 4de^3(d^8+192ad^4e^3-1536a^2e^6)x^8 + \frac{128}{3}e^4(d^8-32ad^4e^3+64a^2e^6)x^9 + \frac{128}{5}d^3e^5(3d^4+40ae^3)x^{10} + \frac{128}{11}d^2e^6(-13d^4+384ae^3)x^{11} - 512de^7(d^4-8ae^3)x^{12} + \frac{2048}{13}e^8(-d^4+8ae^3)x^{13} + 1024d^3e^9x^{14} + \frac{8192}{5}d^2e^{10}x^{15} + 1024de^{11}x^{16} + \frac{4096e^{12}x^{17}}{17}$$

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]
```

```
[Out] 4096*a^4*e^8*x - 1024*a^3*d^3*e^6*x^2 + 128*a^2*d^6*e^4*x^3 + 8*a*d*e^2*(-d
^8 + 512*a^2*e^6)*x^4 + ((d^12 - 6144*a^2*d^4*e^6 + 16384*a^3*e^9)*x^5)/5 -
 128*a*d^3*e^4*(-d^4 + 8*a*e^3)*x^6 - (32*d^2*e^2*(d^8 - 24*a*d^4*e^3 - 768
*a^2*e^6)*x^7)/7 - 4*d*e^3*(d^8 + 192*a*d^4*e^3 - 1536*a^2*e^6)*x^8 + (128*
e^4*(d^8 - 32*a*d^4*e^3 + 64*a^2*e^6)*x^9)/3 + (128*d^3*e^5*(3*d^4 + 40*a*e
^3)*x^10)/5 + (128*d^2*e^6*(-13*d^4 + 384*a*e^3)*x^11)/11 - 512*d*e^7*(d^4
- 8*a*e^3)*x^12 + (2048*e^8*(-d^4 + 8*a*e^3)*x^13)/13 + 1024*d^3*e^9*x^14 +
 (8192*d^2*e^10*x^15)/5 + 1024*d*e^11*x^16 + (4096*e^12*x^17)/17
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.13

method	result
norman	$4096a^4e^8x - 1024a^3e^6d^3x^2 + 128a^2e^4d^6x^3 + (4096a^3e^8d - 8a\,d^9e^2)\,x^4 + \left(\frac{16384}{5}a^3e^9 - \frac{6144}{5}a^2d\right.$
gosper	$-768a\,d^5e^6x^8 + 4096ad\,e^{10}x^{12} - 1024a^2d^3e^7x^6 + 128a\,d^7e^4x^6 + 6144a^2d\,e^9x^8 - \frac{4096}{3}x^9a\,d^4e^7$
risch	$-768a\,d^5e^6x^8 + 4096ad\,e^{10}x^{12} - 1024a^2d^3e^7x^6 + 128a\,d^7e^4x^6 + 6144a^2d\,e^9x^8 - \frac{4096}{3}x^9a\,d^4e^7$
parallelrisch	$-768a\,d^5e^6x^8 + 4096ad\,e^{10}x^{12} - 1024a^2d^3e^7x^6 + 128a\,d^7e^4x^6 + 6144a^2d\,e^9x^8 - \frac{4096}{3}x^9a\,d^4e^7$
default	$\frac{4096e^{12}x^{17}}{17} + 1024d\,e^{11}x^{16} + \frac{8192d^2e^{10}x^{15}}{5} + 1024d^3e^9x^{14} + \frac{128(128a\,e^5-16d^4e^2)e^6x^{13}}{13} + \frac{(16384ad\,e^{10}+256}{}$

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 4096*a^4*e^8*x-1024*a^3*e^6*d^3*x^2+128*a^2*e^4*d^6*x^3+(4096*a^3*d*e^8-8*a
*d^9*e^2)*x^4+(16384/5*a^3*e^9-6144/5*a^2*d^4*e^6+1/5*d^12)*x^5+(-1024*a^2*
d^3*e^7+128*a*d^7*e^4)*x^6+(24576/7*a^2*d^2*e^8+768/7*a*d^6*e^5-32/7*d^10*e
^2)*x^7+(6144*a^2*d*e^9-768*a*d^5*e^6-4*d^9*e^3)*x^8+(8192/3*a^2*e^10-4096/
3*a*d^4*e^7+128/3*d^8*e^4)*x^9+(1024*a*d^3*e^8+384/5*d^7*e^5)*x^10+(49152/1
1*a*d^2*e^9-1664/11*d^6*e^6)*x^11+(4096*a*d*e^10-512*d^5*e^7)*x^12+(16384/1
3*a*e^11-2048/13*d^4*e^8)*x^13+1024*d^3*e^9*x^14+8192/5*d^2*e^10*x^15+1024*
d*e^11*x^16+4096/17*e^12*x^17
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.13

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4\,dx=\frac{4096}{17}e^{12}x^{17}+1024\,de^{11}x^{16}+\frac{8192}{5}d^2e^{10}x^{15}+1024\,d^3e^9x^{14}+128\,a^2d^6e^4x^3-1024\,a^3d^3e^6x^2-\frac{2048}{13}\left(d^4e^8-8\,ae^{11}\right)x^{13}+4096\,a^4e^8x-512\left(d^5e^7-8\,ade^{10}\right)x^{12}-\frac{128}{11}\left(13\,d^6e^6-384\,ad^2e^9\right)x^{11}+\frac{128}{5}\left(3\,d^7e^5+40\,ad^3e^8\right)x^{10}+\frac{128}{3}\left(d^8e^4-32\,ad^4e^7+64\,a^2e^{10}\right)x^9-4\left(d^9e^3+192\,ad^5e^6-1536\,a^2de^9\right)x^8-\frac{32}{7}\left(d^{10}e^2-24\,ad^6e^5-768\,a^2d^2e^8\right)x^7+128\left(ad^7e^4-8\,a^2d^3e^7\right)x^6+\frac{1}{5}\left(d^{12}-6144\,a^2d^4e^6+16384\,a^3e^9\right)x^5-8\left(ad^9e^2-512\,a^3de^8\right)x^4$$

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="fricas")
```

```
[Out] 4096/17*e^12*x^17 + 1024*d*e^11*x^16 + 8192/5*d^2*e^10*x^15 + 1024*d^3*e^9*
x^14 + 128*a^2*d^6*e^4*x^3 - 1024*a^3*d^3*e^6*x^2 - 2048/13*(d^4*e^8 - 8*a*
e^11)*x^13 + 4096*a^4*e^8*x - 512*(d^5*e^7 - 8*a*d*e^10)*x^12 - 128/11*(13*
d^6*e^6 - 384*a*d^2*e^9)*x^11 + 128/5*(3*d^7*e^5 + 40*a*d^3*e^8)*x^10 + 128
/3*(d^8*e^4 - 32*a*d^4*e^7 + 64*a^2*e^10)*x^9 - 4*(d^9*e^3 + 192*a*d^5*e^6
- 1536*a^2*d*e^9)*x^8 - 32/7*(d^10*e^2 - 24*a*d^6*e^5 - 768*a^2*d^2*e^8)*x^
7 + 128*(a*d^7*e^4 - 8*a^2*d^3*e^7)*x^6 + 1/5*(d^12 - 6144*a^2*d^4*e^6 + 16
384*a^3*e^9)*x^5 - 8*(a*d^9*e^2 - 512*a^3*d*e^8)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.24

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4\,dx = 4096a^4e^8x-1024a^3d^3e^6x^2+128a^2d^6e^4x^3 + 1024d^3e^9x^{14}+\frac{8192d^2e^{10}x^{15}}{5}+1024de^{11}x^{16} + \frac{4096e^{12}x^{17}}{17}+x^{13}\cdot\left(\frac{16384ae^{11}}{13}-\frac{2048d^4e^8}{13}\right) + x^{12}\cdot\left(4096ade^{10}-512d^5e^7\right)+x^{11} \cdot\left(\frac{49152ad^2e^9}{11}-\frac{1664d^6e^6}{11}\right)+x^{10} \cdot\left(1024ad^3e^8+\frac{384d^7e^5}{5}\right)+x^9 \cdot\left(\frac{8192a^2e^{10}}{3}-\frac{4096ad^4e^7}{3}+\frac{128d^8e^4}{3}\right) + x^8\cdot\left(6144a^2de^9-768ad^5e^6-4d^9e^3\right)+x^7 \cdot\left(\frac{24576a^2d^2e^8}{7}+\frac{768ad^6e^5}{7}-\frac{32d^{10}e^2}{7}\right) + x^6\left(-1024a^2d^3e^7+128ad^7e^4\right)+x^5 \cdot\left(\frac{16384a^3e^9}{5}-\frac{6144a^2d^4e^6}{5}+\frac{d^{12}}{5}\right) + x^4\cdot\left(4096a^3de^8-8ad^9e^2\right)$$

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**4,x)
```

```
[Out] 4096*a**4*e**8*x - 1024*a**3*d**3*e**6*x**2 + 128*a**2*d**6*e**4*x**3 + 102
4*d**3*e**9*x**14 + 8192*d**2*e**10*x**15/5 + 1024*d*e**11*x**16 + 4096*e**
12*x**17/17 + x**13*(16384*a*e**11/13 - 2048*d**4*e**8/13) + x**12*(4096*a*
d*e**10 - 512*d**5*e**7) + x**11*(49152*a*d**2*e**9/11 - 1664*d**6*e**6/11)
 + x**10*(1024*a*d**3*e**8 + 384*d**7*e**5/5) + x**9*(8192*a**2*e**10/3 - 4
096*a*d**4*e**7/3 + 128*d**8*e**4/3) + x**8*(6144*a**2*d*e**9 - 768*a*d**5*
e**6 - 4*d**9*e**3) + x**7*(24576*a**2*d**2*e**8/7 + 768*a*d**6*e**5/7 - 32
*d**10*e**2/7) + x**6*(-1024*a**2*d**3*e**7 + 128*a*d**7*e**4) + x**5*(1638
4*a**3*e**9/5 - 6144*a**2*d**4*e**6/5 + d**12/5) + x**4*(4096*a**3*d*e**8 -
 8*a*d**9*e**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.30

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^4 \, dx = \frac{4096}{17}\, e^{12}x^{17} + 1024\, de^{11}x^{16} + \frac{8192}{5}\, d^2e^{10}x^{15} + \frac{8192}{7}\, d^3e^9x^{14} + \frac{4096}{13}\, d^4e^8x^{13} + \frac{1}{5}\, d^{12}x^5 + 4096\, a^4e^8x - \frac{4}{7}\left(7\, e^3x^8 + 8\, de^2x^7\right)d^9 + \frac{1024}{5}\left(16\, e^3x^5 + 20\, de^2x^4 - 5\, d^3x^2\right)a^3e^6 + \frac{128}{165}\left(45\, e^6x^{11} + 99\, de^5x^{10} + 55\, d^2e^4x^9\right)d^6 + \frac{128}{105}\left(2240\, e^6x^9 + 5040\, de^5x^8 + 2880\, d^2e^4x^7 + 105\, d^6x^3 - 168\left(5\, e^3x^6 + 6\, de^2x^5\right)d^3\right)a^2e^4 - \frac{512}{1001}\left(286\, e^9x^{14} + 924\, de^8x^{13} + 1001\, d^2e^7x^{12} + 364\, d^3e^6x^{11}\right)d^3 + \frac{8}{15015}\left(2365440\, e^9x^{13} + 7687680\, de^8x^{12} + 8386560\, d^2e^7x^{11} + 3075072\, d^3e^6x^{10} - 15015\, d^9x^4 + 34320\, (6\right.$$

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="maxima")
```

```
[Out] 4096/17*e^12*x^17 + 1024*d*e^11*x^16 + 8192/5*d^2*e^10*x^15 + 8192/7*d^3*e^
9*x^14 + 4096/13*d^4*e^8*x^13 + 1/5*d^12*x^5 + 4096*a^4*e^8*x - 4/7*(7*e^3*
x^8 + 8*d*e^2*x^7)*d^9 + 1024/5*(16*e^3*x^5 + 20*d*e^2*x^4 - 5*d^3*x^2)*a^3
*e^6 + 128/165*(45*e^6*x^11 + 99*d*e^5*x^10 + 55*d^2*e^4*x^9)*d^6 + 128/105
*(2240*e^6*x^9 + 5040*d*e^5*x^8 + 2880*d^2*e^4*x^7 + 105*d^6*x^3 - 168*(5*e
^3*x^6 + 6*d*e^2*x^5)*d^3)*a^2*e^4 - 512/1001*(286*e^9*x^14 + 924*d*e^8*x^1
3 + 1001*d^2*e^7*x^12 + 364*d^3*e^6*x^11)*d^3 + 8/15015*(2365440*e^9*x^13 +
 7687680*d*e^8*x^12 + 8386560*d^2*e^7*x^11 + 3075072*d^3*e^6*x^10 - 15015*d
^9*x^4 + 34320*(6*e^3*x^7 + 7*d*e^2*x^6)*d^6 - 32032*(36*e^6*x^10 + 80*d*e^
5*x^9 + 45*d^2*e^4*x^8)*d^3)*a*e^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.20

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4\,dx=\frac{4096}{17}\,e^{12}x^{17}+1024\,de^{11}x^{16}+\frac{8192}{5}\,d^2e^{10}x^{15}+1024\,d^3e^9x^{14}-\frac{2048}{13}\,d^4e^8x^{13}+\frac{16384}{13}\,ae^{11}x^{13}-512\,d^5e^7x^{12}+4096\,ade^{10}x^{12}-\frac{1664}{11}\,d^6e^6x^{11}+\frac{49152}{11}\,ad^2e^9x^{11}+\frac{384}{5}\,d^7e^5x^{10}+1024\,ad^3e^8x^{10}+\frac{128}{3}\,d^8e^4x^9-\frac{4096}{3}\,ad^4e^7x^9+\frac{8192}{3}\,a^2e^{10}x^9-4\,d^9e^3x^8-768\,ad^5e^6x^8+6144\,a^2de^9x^8-\frac{32}{7}\,d^{10}e^2x^7+\frac{768}{7}\,ad^6e^5x^7+\frac{24576}{7}\,a^2d^2e^8x^7+128\,ad^7e^4x^6-1024\,a^2d^3e^7x^6+\frac{1}{5}\,d^{12}x^5-\frac{6144}{5}\,a^2d^4e^6x^5+\frac{16384}{5}\,a^3e^9x^5-8\,ad^9e^2x^4+4096\,a^3de^8x^4+128\,a^2d^6e^4x^3-1024\,a^3d^3e^6x^2+4096\,a^4e^8x$$

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="giac")
```

```
[Out] 4096/17*e^12*x^17 + 1024*d*e^11*x^16 + 8192/5*d^2*e^10*x^15 + 1024*d^3*e^9*
x^14 - 2048/13*d^4*e^8*x^13 + 16384/13*a*e^11*x^13 - 512*d^5*e^7*x^12 + 409
6*a*d*e^10*x^12 - 1664/11*d^6*e^6*x^11 + 49152/11*a*d^2*e^9*x^11 + 384/5*d^
7*e^5*x^10 + 1024*a*d^3*e^8*x^10 + 128/3*d^8*e^4*x^9 - 4096/3*a*d^4*e^7*x^9
 + 8192/3*a^2*e^10*x^9 - 4*d^9*e^3*x^8 - 768*a*d^5*e^6*x^8 + 6144*a^2*d*e^9
*x^8 - 32/7*d^10*e^2*x^7 + 768/7*a*d^6*e^5*x^7 + 24576/7*a^2*d^2*e^8*x^7 +
128*a*d^7*e^4*x^6 - 1024*a^2*d^3*e^7*x^6 + 1/5*d^12*x^5 - 6144/5*a^2*d^4*e^
6*x^5 + 16384/5*a^3*e^9*x^5 - 8*a*d^9*e^2*x^4 + 4096*a^3*d*e^8*x^4 + 128*a^
2*d^6*e^4*x^3 - 1024*a^3*d^3*e^6*x^2 + 4096*a^4*e^8*x
```

Mupad [B] (verification not implemented)

Time = 10.54 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.12

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4\,dx = x^5\left(\frac{16384\,a^3\,e^9}{5}-\frac{6144\,a^2\,d^4\,e^6}{5}+\frac{d^{12}}{5}\right)+x^{10}\left(\frac{384\,d^7\,e^5}{5}+1024\,a\,d^3\,e^8\right)-x^{11}\left(\frac{1664\,d^6\,e^6}{11}-\frac{49152\,a\,d^2\,e^9}{11}\right)+\frac{4096\,e^{12}\,x^{17}}{17}+\frac{2048\,e^8\,x^{13}\,(8\,a\,e^3-d^4)}{13}+\frac{128\,e^4\,x^9\,(64\,a^2\,e^6-32\,a\,d^4\,e^3+d^8)}{3}+4096\,a^4\,e^8\,x+1024\,d\,e^{11}\,x^{16}+1024\,d^3\,e^9\,x^{14}+\frac{8192\,d^2\,e^{10}\,x^{15}}{5}+512\,d\,e^7\,x^{12}\left(8\,a\,e^3-d^4\right)+\frac{32\,d^2\,e^2\,x^7\,(768\,a^2\,e^6+24\,a\,d^4\,e^3-d^8)}{7}-1024\,a^3\,d^3\,e^6\,x^2+128\,a^2\,d^6\,e^4\,x^3-4\,d\,e^3\,x^8\left(-1536\,a^2\,e^6+192\,a\,d^4\,e^3+d^8\right)-128\,a\,d^3\,e^4\,x^6\left(8\,a\,e^3-d^4\right)-8\,a\,d\,e^2\,x^4\left(d^8-512\,a^2\,e^6\right)$$

```
[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^4,x)
```

```
[Out] x^5*(d^12/5 + (16384*a^3*e^9)/5 - (6144*a^2*d^4*e^6)/5) + x^10*((384*d^7*e^
5)/5 + 1024*a*d^3*e^8) - x^11*((1664*d^6*e^6)/11 - (49152*a*d^2*e^9)/11) +
(4096*e^12*x^17)/17 + (2048*e^8*x^13*(8*a*e^3 - d^4))/13 + (128*e^4*x^9*(d^
8 + 64*a^2*e^6 - 32*a*d^4*e^3))/3 + 4096*a^4*e^8*x + 1024*d*e^11*x^16 + 102
4*d^3*e^9*x^14 + (8192*d^2*e^10*x^15)/5 + 512*d*e^7*x^12*(8*a*e^3 - d^4) +
(32*d^2*e^2*x^7*(768*a^2*e^6 - d^8 + 24*a*d^4*e^3))/7 - 1024*a^3*d^3*e^6*x^
2 + 128*a^2*d^6*e^4*x^3 - 4*d*e^3*x^8*(d^8 - 1536*a^2*e^6 + 192*a*d^4*e^3)
- 128*a*d^3*e^4*x^6*(8*a*e^3 - d^4) - 8*a*d*e^2*x^4*(d^8 - 512*a^2*e^6)
```

3.40 $\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3 \, dx$

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Optimal result

Integrand size = 32, antiderivative size = 203

$$\begin{aligned}\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3 \, dx &= 512a^3e^6x-96a^2d^3e^4x^2+8ad^6e^2x^3\\ &\quad-\frac{1}{4}d\left(d^8-1536a^2e^6\right)x^4-\frac{384}{5}ae^4\left(d^4-4ae^3\right)x^5\\ &\quad+4d^3e^2\left(d^4-16ae^3\right)x^6+\frac{24}{7}d^2e^3\left(d^4+64ae^3\right)x^7\\ &\quad-24de^4\left(d^4-16ae^3\right)x^8-\frac{128}{3}e^5\left(d^4-4ae^3\right)x^9\\ &\quad+32d^3e^6x^{10}+\frac{1536}{11}d^2e^7x^{11}+128de^8x^{12}+\frac{512e^9x^{13}}{13}\end{aligned}$$

```
[Out] 512*a^3*e^6*x-96*a^2*d^3*e^4*x^2+8*a*d^6*e^2*x^3-1/4*d*(-1536*a^2*e^6+d^8)*
x^4-384/5*a*e^4*(-4*a*e^3+d^4)*x^5+4*d^3*e^2*(-16*a*e^3+d^4)*x^6+24/7*d^2*e
^3*(64*a*e^3+d^4)*x^7-24*d*e^4*(-16*a*e^3+d^4)*x^8-128/3*e^5*(-4*a*e^3+d^4)
*x^9+32*d^3*e^6*x^10+1536/11*d^2*e^7*x^11+128*d*e^8*x^12+512/13*e^9*x^13
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used

$= \{2086\}$

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^3 \, dx = 512a^3e^6x - \frac{1}{4}dx^4\left(d^8 - 1536a^2e^6\right) - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 - \frac{128}{3}e^5x^9\left(d^4 - 4ae^3\right) - 24de^4x^8\left(d^4 - 16ae^3\right) - \frac{384}{5}ae^4x^5\left(d^4 - 4ae^3\right) + 4d^3e^2x^6\left(d^4 - 16ae^3\right) + \frac{24}{7}d^2e^3x^7\left(64ae^3 + d^4\right) + 32d^3e^6x^{10} + \frac{1536}{11}d^2e^7x^{11} + 128de^8x^{12} + \frac{512e^9x^{13}}{13}$$

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3,x]
```

```
[Out] 512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 - 1536*a^2*e
^6)*x^4)/4 - (384*a*e^4*(d^4 - 4*a*e^3)*x^5)/5 + 4*d^3*e^2*(d^4 - 16*a*e^3)
*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 - 24*d*e^4*(d^4 - 16*a*e^3)*x^8
- (128*e^5*(d^4 - 4*a*e^3)*x^9)/3 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x^11)/1
1 + 128*d*e^8*x^12 + (512*e^9*x^13)/13
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(512a^3e^6 - 192a^2d^3e^4x + 24ad^6e^2x^2 - d\left(d^8 - 1536a^2e^6\right)x^3 - 384ae^4\left(d^4 - 4ae^3\right)x^4 + 24d^3e^2\left(d^4 - 16ae^3\right)x^5 + 24d^2e^3\left(d^4 + 64ae^3\right)x^6 - 192de^4\left(d^4 - 16ae^3\right)x^7 - 384e^5\left(d^4 - 4ae^3\right)x^8 + 320d^3e^6x^9 + 1536d^2e^7x^{10} + 1536de^8x^{11} + 512e^9x^{12}\right) dx$$

$$= 512a^3e^6x - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 - \frac{1}{4}d\left(d^8 - 1536a^2e^6\right)x^4 - \frac{384}{5}ae^4\left(d^4 - 4ae^3\right)x^5 + 4d^3e^2\left(d^4 - 16ae^3\right)x^6 + \frac{24}{7}d^2e^3\left(d^4 + 64ae^3\right)x^7 - 24de^4\left(d^4 - 16ae^3\right)x^8 - \frac{128}{3}e^5\left(d^4 - 4ae^3\right)x^9 + 32d^3e^6x^{10} + \frac{1536}{11}d^2e^7x^{11} + 128de^8x^{12} + \frac{512e^9x^{13}}{13}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.02

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3 \, dx = 512a^3e^6x-96a^2d^3e^4x^2+8ad^6e^2x^3 -\frac{1}{4}d\left(d^8-1536a^2e^6\right)x^4+\frac{384}{5}ae^4\left(-d^4+4ae^3\right)x^5 +4d^3e^2\left(d^4-16ae^3\right)x^6+\frac{24}{7}d^2e^3\left(d^4+64ae^3\right)x^7 -24de^4\left(d^4-16ae^3\right)x^8+\frac{128}{3}e^5\left(-d^4+4ae^3\right)x^9 +32d^3e^6x^{10}+\frac{1536}{11}d^2e^7x^{11}+128de^8x^{12}+\frac{512e^9x^{13}}{13}$$

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3,x]
```

```
[Out] 512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 - 1536*a^2*e
^6)*x^4)/4 + (384*a*e^4*(-d^4 + 4*a*e^3)*x^5)/5 + 4*d^3*e^2*(d^4 - 16*a*e^3
)*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 - 24*d*e^4*(d^4 - 16*a*e^3)*x^8
 + (128*e^5*(-d^4 + 4*a*e^3)*x^9)/3 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x^11)
/11 + 128*d*e^8*x^12 + (512*e^9*x^13)/13
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.99

method	result
norman	$\frac{512e^9x^{13}}{13}+128d\,e^8x^{12}+\frac{1536d^2e^7x^{11}}{11}+32d^3e^6x^{10}+\left(\frac{512}{3}a\,e^8-\frac{128}{3}d^4e^5\right)x^9+(384a\,e^7d-24d^5e^4$
gosper	$\frac{512}{13}e^9x^{13}+128d\,e^8x^{12}+\frac{1536}{11}d^2e^7x^{11}+32d^3e^6x^{10}+\frac{512}{3}x^9a\,e^8-\frac{128}{3}x^9d^4e^5+384ad\,e^7x^8-24d$
risch	$\frac{512}{13}e^9x^{13}+128d\,e^8x^{12}+\frac{1536}{11}d^2e^7x^{11}+32d^3e^6x^{10}+\frac{512}{3}x^9a\,e^8-\frac{128}{3}x^9d^4e^5+384ad\,e^7x^8-24d$
parallelrisch	$\frac{512}{13}e^9x^{13}+128d\,e^8x^{12}+\frac{1536}{11}d^2e^7x^{11}+32d^3e^6x^{10}+\frac{512}{3}x^9a\,e^8-\frac{128}{3}x^9d^4e^5+384ad\,e^7x^8-24d$
default	$\frac{512e^9x^{13}}{13}+128d\,e^8x^{12}+\frac{1536d^2e^7x^{11}}{11}+32d^3e^6x^{10}+\frac{\left(512a\,e^8-256d^4e^5+8e^3\left(128a\,e^5-16d^4e^2\right)\right)x^9}{9}+\frac{(2048a\,e^7}{}$

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 512/13*e^9*x^13+128*d*e^8*x^12+1536/11*d^2*e^7*x^11+32*d^3*e^6*x^10+(512/3*
a*e^8-128/3*d^4*e^5)*x^9+(384*a*d*e^7-24*d^5*e^4)*x^8+(1536/7*a*e^6*d^2+24/
7*d^6*e^3)*x^7+(-64*a*d^3*e^5+4*d^7*e^2)*x^6+(1536/5*a^2*e^7-384/5*a*d^4*e^
4)*x^5+(384*a^2*e^6*d-1/4*d^9)*x^4+8*a*d^6*e^2*x^3-96*a^2*d^3*e^4*x^2+512*a
^3*e^6*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.98

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3\,dx=\frac{512}{13}\,e^9x^{13}+128\,de^8x^{12}+\frac{1536}{11}\,d^2e^7x^{11}+32\,d^3e^6x^{10}+8\,ad^6e^2x^3-96\,a^2d^3e^4x^2+512\,a^3e^6x-\frac{128}{3}\left(d^4e^5-4\,ae^8\right)x^9-24\left(d^5e^4-16\,ade^7\right)x^8+\frac{24}{7}\left(d^6e^3+64\,ad^2e^6\right)x^7+4\left(d^7e^2-16\,ad^3e^5\right)x^6-\frac{384}{5}\left(ad^4e^4-4\,a^2e^7\right)x^5-\frac{1}{4}\left(d^9-1536\,a^2de^6\right)x^4$$

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="fricas")
```

```
[Out] 512/13*e^9*x^13 + 128*d*e^8*x^12 + 1536/11*d^2*e^7*x^11 + 32*d^3*e^6*x^10 +
 8*a*d^6*e^2*x^3 - 96*a^2*d^3*e^4*x^2 + 512*a^3*e^6*x - 128/3*(d^4*e^5 - 4*
a*e^8)*x^9 - 24*(d^5*e^4 - 16*a*d*e^7)*x^8 + 24/7*(d^6*e^3 + 64*a*d^2*e^6)*
x^7 + 4*(d^7*e^2 - 16*a*d^3*e^5)*x^6 - 384/5*(a*d^4*e^4 - 4*a^2*e^7)*x^5 -
1/4*(d^9 - 1536*a^2*d*e^6)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.07

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3\,dx=512a^3e^6x-96a^2d^3e^4x^2+8ad^6e^2x^3+32d^3e^6x^{10}+\frac{1536d^2e^7x^{11}}{11}+128de^8x^{12}+\frac{512e^9x^{13}}{13}+x^9\cdot\left(\frac{512ae^8}{3}-\frac{128d^4e^5}{3}\right)+x^8\cdot\left(384ade^7-24d^5e^4\right)+x^7\cdot\left(\frac{1536ad^2e^6}{7}+\frac{24d^6e^3}{7}\right)+x^6\left(-64ad^3e^5+4d^7e^2\right)+x^5\cdot\left(\frac{1536a^2e^7}{5}-\frac{384ad^4e^4}{5}\right)+x^4\cdot\left(384a^2de^6-\frac{d^9}{4}\right)$$

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**3,x)
```

```
[Out] 512*a**3*e**6*x - 96*a**2*d**3*e**4*x**2 + 8*a*d**6*e**2*x**3 + 32*d**3*e**
6*x**10 + 1536*d**2*e**7*x**11/11 + 128*d*e**8*x**12 + 512*e**9*x**13/13 +
x**9*(512*a*e**8/3 - 128*d**4*e**5/3) + x**8*(384*a*d*e**7 - 24*d**5*e**4)
+ x**7*(1536*a*d**2*e**6/7 + 24*d**6*e**3/7) + x**6*(-64*a*d**3*e**5 + 4*d*
*7*e**2) + x**5*(1536*a**2*e**7/5 - 384*a*d**4*e**4/5) + x**4*(384*a**2*d*e
**6 - d**9/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.05

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3\,dx$$
$$=\frac{512}{13}\,e^9x^{13}+128\,de^8x^{12}+\frac{1536}{11}\,d^2e^7x^{11}+\frac{256}{5}\,d^3e^6x^{10}$$
$$-\frac{1}{4}\,d^9x^4+512\,a^3e^6x+\frac{4}{7}\left(6\,e^3x^7+7\,de^2x^6\right)d^6$$
$$+\frac{96}{5}\left(16\,e^3x^5+20\,de^2x^4-5\,d^3x^2\right)a^2e^4-\frac{8}{15}\left(36\,e^6x^{10}+80\,de^5x^9+45\,d^2e^4x^8\right)d^3$$
$$+\frac{8}{105}\left(2240\,e^6x^9+5040\,de^5x^8+2880\,d^2e^4x^7+105\,d^6x^3-168\left(5\,e^3x^6+6\,de^2x^5\right)d^3\right)ae^2$$

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="maxima")
```

```
[Out] 512/13*e^9*x^13 + 128*d*e^8*x^12 + 1536/11*d^2*e^7*x^11 + 256/5*d^3*e^6*x^1
0 - 1/4*d^9*x^4 + 512*a^3*e^6*x + 4/7*(6*e^3*x^7 + 7*d*e^2*x^6)*d^6 + 96/5*
(16*e^3*x^5 + 20*d*e^2*x^4 - 5*d^3*x^2)*a^2*e^4 - 8/15*(36*e^6*x^10 + 80*d*
e^5*x^9 + 45*d^2*e^4*x^8)*d^3 + 8/105*(2240*e^6*x^9 + 5040*d*e^5*x^8 + 2880
*d^2*e^4*x^7 + 105*d^6*x^3 - 168*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3)*a*e^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.01

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3\,dx=\frac{512}{13}\,e^9x^{13}+128\,de^8x^{12}+\frac{1536}{11}\,d^2e^7x^{11}+32\,d^3e^6x^{10}$$
$$-\frac{128}{3}\,d^4e^5x^9+\frac{512}{3}\,ae^8x^9-24\,d^5e^4x^8+384\,ade^7x^8$$
$$+\frac{24}{7}\,d^6e^3x^7+\frac{1536}{7}\,ad^2e^6x^7+4\,d^7e^2x^6$$
$$-64\,ad^3e^5x^6-\frac{384}{5}\,ad^4e^4x^5+\frac{1536}{5}\,a^2e^7x^5-\frac{1}{4}\,d^9x^4$$
$$+384\,a^2de^6x^4+8\,ad^6e^2x^3-96\,a^2d^3e^4x^2+512\,a^3e^6x$$

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="giac")
```

```
[Out] 512/13*e^9*x^13 + 128*d*e^8*x^12 + 1536/11*d^2*e^7*x^11 + 32*d^3*e^6*x^10 -
 128/3*d^4*e^5*x^9 + 512/3*a*e^8*x^9 - 24*d^5*e^4*x^8 + 384*a*d*e^7*x^8 + 2
4/7*d^6*e^3*x^7 + 1536/7*a*d^2*e^6*x^7 + 4*d^7*e^2*x^6 - 64*a*d^3*e^5*x^6 -
 384/5*a*d^4*e^4*x^5 + 1536/5*a^2*e^7*x^5 - 1/4*d^9*x^4 + 384*a^2*d*e^6*x^4
 + 8*a*d^6*e^2*x^3 - 96*a^2*d^3*e^4*x^2 + 512*a^3*e^6*x
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.99

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3\,dx=\frac{512\,e^9\,x^{13}}{13}-x^4\,\left(\frac{d^9}{4}-384\,a^2\,d\,e^6\right)+\frac{128\,e^5\,x^9\,\left(4\,a\,e^3-d^4\right)}{3}+512\,a^3\,e^6\,x+128\,d\,e^8\,x^{12}+32\,d^3\,e^6\,x^{10}+\frac{1536\,d^2\,e^7\,x^{11}}{11}+8\,a\,d^6\,e^2\,x^3+\frac{384\,a\,e^4\,x^5\,\left(4\,a\,e^3-d^4\right)}{5}+24\,d\,e^4\,x^8\,\left(16\,a\,e^3-d^4\right)+\frac{24\,d^2\,e^3\,x^7\,\left(d^4+64\,a\,e^3\right)}{7}-96\,a^2\,d^3\,e^4\,x^2-4\,d^3\,e^2\,x^6\,\left(16\,a\,e^3-d^4\right)$$

```
[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^3,x)
```

```
[Out] (512*e^9*x^13)/13 - x^4*(d^9/4 - 384*a^2*d*e^6) + (128*e^5*x^9*(4*a*e^3 - d
^4))/3 + 512*a^3*e^6*x + 128*d*e^8*x^12 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x
^11)/11 + 8*a*d^6*e^2*x^3 + (384*a*e^4*x^5*(4*a*e^3 - d^4))/5 + 24*d*e^4*x^
8*(16*a*e^3 - d^4) + (24*d^2*e^3*x^7*(64*a*e^3 + d^4))/7 - 96*a^2*d^3*e^4*x
^2 - 4*d^3*e^2*x^6*(16*a*e^3 - d^4)
```

3.41 $\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2 \, dx$

Optimal result . . . 426
Rubi [A] (verified) . . . 426
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Fricas [A] (verification not implemented) . . . 428
Sympy [A] (verification not implemented) . . . 428
Maxima [A] (verification not implemented) . . . 429
Giac [A] (verification not implemented) . . . 429
Mupad [B] (verification not implemented) . . . 429

Optimal result

Integrand size = 32, antiderivative size = 107

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2 \, dx = 64a^2e^4x - 8ad^3e^2x^2 + \frac{d^6x^3}{3} + 32ade^4x^4 - \frac{16}{5}e^2(d^4 - 8ae^3)x^5 - \frac{8}{3}d^3e^3x^6 + \frac{64}{7}d^2e^4x^7 + 16de^5x^8 + \frac{64e^6x^9}{9}$$

```
[Out] 64*a^2*e^4*x-8*a*d^3*e^2*x^2+1/3*d^6*x^3+32*a*d*e^4*x^4-16/5*e^2*(-8*a*e^3+
d^4)*x^5-8/3*d^3*e^3*x^6+64/7*d^2*e^4*x^7+16*d*e^5*x^8+64/9*e^6*x^9
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {2086}

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2 \, dx = 64a^2e^4x - \frac{16}{5}e^2x^5(d^4 - 8ae^3) - 8ad^3e^2x^2 + 32ade^4x^4 + \frac{d^6x^3}{3} - \frac{8}{3}d^3e^3x^6 + \frac{64}{7}d^2e^4x^7 + 16de^5x^8 + \frac{64e^6x^9}{9}$$

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2,x]
```

```
[Out] 64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 - (16*e^2*(d^
4 - 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 + 16*d*e^5*x^8
 + (64*e^6*x^9)/9
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(64a^2e^4 - 16ad^3e^2x + d^6x^2 + 128ade^4x^3 - 16e^2\left(d^4 - 8ae^3\right)x^4 - 16d^3e^3x^5 + 64d^2e^4x^6 + 128de^5x^7 + 64e^6x^8\right)\,dx \\ &= 64a^2e^4x - 8ad^3e^2x^2 + \frac{d^6x^3}{3} + 32ade^4x^4 - \frac{16}{5}e^2\left(d^4 - 8ae^3\right)x^5 - \frac{8}{3}d^3e^3x^6 + \frac{64}{7}d^2e^4x^7 + 16de^5x^8 + \frac{64e^6x^9}{9}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.02

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^2\,dx = 64a^2e^4x - 8ad^3e^2x^2 + \frac{d^6x^3}{3} + 32ade^4x^4 + \frac{16}{5}e^2\left(-d^4 + 8ae^3\right)x^5 - \frac{8}{3}d^3e^3x^6 + \frac{64}{7}d^2e^4x^7 + 16de^5x^8 + \frac{64e^6x^9}{9}$$

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2,x]
```

```
[Out] 64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 + (16*e^2*(-d
^4 + 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 + 16*d*e^5*x^
8 + (64*e^6*x^9)/9
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

method	result
norman	$\frac{64e^6x^9}{9} + 16d\,e^5x^8 + \frac{64d^2e^4x^7}{7} - \frac{8d^3e^3x^6}{3} + \left(\frac{128}{5}a\,e^5 - \frac{16}{5}d^4e^2\right)x^5 + 32ad\,e^4x^4 + \frac{d^6x^3}{3} - 8a\,d^3e^2x$
gosper	$\frac{64}{9}e^6x^9 + 16d\,e^5x^8 + \frac{64}{7}d^2e^4x^7 - \frac{8}{3}d^3e^3x^6 + \frac{128}{5}x^5a\,e^5 - \frac{16}{5}x^5d^4e^2 + 32ad\,e^4x^4 + \frac{1}{3}d^6x^3 - 8a$
default	$\frac{64e^6x^9}{9} + 16d\,e^5x^8 + \frac{64d^2e^4x^7}{7} - \frac{8d^3e^3x^6}{3} + \frac{\left(128a\,e^5 - 16d^4e^2\right)x^5}{5} + 32ad\,e^4x^4 + \frac{d^6x^3}{3} - 8a\,d^3e^2x^2 + 6$
risch	$\frac{64}{9}e^6x^9 + 16d\,e^5x^8 + \frac{64}{7}d^2e^4x^7 - \frac{8}{3}d^3e^3x^6 + \frac{128}{5}x^5a\,e^5 - \frac{16}{5}x^5d^4e^2 + 32ad\,e^4x^4 + \frac{1}{3}d^6x^3 - 8a$
parallelrisch	$\frac{64}{9}e^6x^9 + 16d\,e^5x^8 + \frac{64}{7}d^2e^4x^7 - \frac{8}{3}d^3e^3x^6 + \frac{128}{5}x^5a\,e^5 - \frac{16}{5}x^5d^4e^2 + 32ad\,e^4x^4 + \frac{1}{3}d^6x^3 - 8a$

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 64/9*e^6*x^9+16*d*e^5*x^8+64/7*d^2*e^4*x^7-8/3*d^3*e^3*x^6+(128/5*a*e^5-16/
5*d^4*e^2)*x^5+32*a*d*e^4*x^4+1/3*d^6*x^3-8*a*d^3*e^2*x^2+64*a^2*e^4*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2\,dx = \frac{64}{9}\,e^6x^9+16\,de^5x^8+\frac{64}{7}\,d^2e^4x^7-\frac{8}{3}\,d^3e^3x^6 + 32\,ade^4x^4+\frac{1}{3}\,d^6x^3-8\,ad^3e^2x^2 + 64\,a^2e^4x-\frac{16}{5}\,\left(d^4e^2-8\,ae^5\right)x^5$$

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="fricas")
```

```
[Out] 64/9*e^6*x^9 + 16*d*e^5*x^8 + 64/7*d^2*e^4*x^7 - 8/3*d^3*e^3*x^6 + 32*a*d*e
^4*x^4 + 1/3*d^6*x^3 - 8*a*d^3*e^2*x^2 + 64*a^2*e^4*x - 16/5*(d^4*e^2 - 8*a
*e^5)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2\,dx = 64a^2e^4x-8ad^3e^2x^2+32ade^4x^4+\frac{d^6x^3}{3} - \frac{8d^3e^3x^6}{3}+\frac{64d^2e^4x^7}{7}+16de^5x^8 + \frac{64e^6x^9}{9}+x^5\cdot\left(\frac{128ae^5}{5}-\frac{16d^4e^2}{5}\right)$$

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)
```

```
[Out] 64*a**2*e**4*x - 8*a*d**3*e**2*x**2 + 32*a*d*e**4*x**4 + d**6*x**3/3 - 8*d*
*3*e**3*x**6/3 + 64*d**2*e**4*x**7/7 + 16*d*e**5*x**8 + 64*e**6*x**9/9 + x*
*5*(128*a*e**5/5 - 16*d**4*e**2/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2\,dx=\frac{64}{9}\,e^6x^9+16\,de^5x^8+\frac{64}{7}\,d^2e^4x^7+\frac{1}{3}\,d^6x^3+64\,a^2e^4x-\frac{8}{15}\,\left(5\,e^3x^6+6\,de^2x^5\right)d^3+\frac{8}{5}\,\left(16\,e^3x^5+20\,de^2x^4-5\,d^3x^2\right)ae^2$$

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="maxima")
[Out] 64/9*e^6*x^9 + 16*d*e^5*x^8 + 64/7*d^2*e^4*x^7 + 1/3*d^6*x^3 + 64*a^2*e^4*x
 - 8/15*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3 + 8/5*(16*e^3*x^5 + 20*d*e^2*x^4 - 5*
d^3*x^2)*a*e^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2\,dx=\frac{64}{9}\,e^6x^9+16\,de^5x^8+\frac{64}{7}\,d^2e^4x^7-\frac{8}{3}\,d^3e^3x^6-\frac{16}{5}\,d^4e^2x^5+\frac{128}{5}\,ae^5x^5+32\,ade^4x^4+\frac{1}{3}\,d^6x^3-8\,ad^3e^2x^2+64\,a^2e^4x$$

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")
[Out] 64/9*e^6*x^9 + 16*d*e^5*x^8 + 64/7*d^2*e^4*x^7 - 8/3*d^3*e^3*x^6 - 16/5*d^4
*e^2*x^5 + 128/5*a*e^5*x^5 + 32*a*d*e^4*x^4 + 1/3*d^6*x^3 - 8*a*d^3*e^2*x^2
 + 64*a^2*e^4*x
```

Mupad [B] (verification not implemented)

Time = 10.17 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2\,dx=x^5\,\left(\frac{128\,a\,e^5}{5}-\frac{16\,d^4\,e^2}{5}\right)+\frac{d^6\,x^3}{3}+\frac{64\,e^6\,x^9}{9}+64\,a^2\,e^4\,x+16\,d\,e^5\,x^8-\frac{8\,d^3\,e^3\,x^6}{3}+\frac{64\,d^2\,e^4\,x^7}{7}-8\,a\,d^3\,e^2\,x^2+32\,a\,d\,e^4\,x^4$$

```
[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^2,x)
```

```
[Out] x^5*((128*a*e^5)/5 - (16*d^4*e^2)/5) + (d^6*x^3)/3 + (64*e^6*x^9)/9 + 64*a^
2*e^4*x + 16*d*e^5*x^8 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 - 8*a*d^3*e
^2*x^2 + 32*a*d*e^4*x^4
```

3.42 $\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4) \, dx$

Optimal result 431
Rubi [A] (verified) 431
Mathematica [A] (verified) 432
Maple [A] (verified) 432
Fricas [A] (verification not implemented) 432
Sympy [A] (verification not implemented) 433
Maxima [A] (verification not implemented) 433
Giac [A] (verification not implemented) 433
Mupad [B] (verification not implemented) 433

Optimal result

Integrand size = 30, antiderivative size = 37

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4) \, dx = 8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

```
[Out] 8*a*e^2*x-1/2*d^3*x^2+2*d*e^2*x^4+8/5*e^3*x^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4) \, dx = 8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

```
[In] Int[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4,x]
```

```
[Out] 8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5
```

Rubi steps

$$\text{integral} = 8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)\,dx=8ae^2x-\frac{d^3x^2}{2}+2de^2x^4+\frac{8e^3x^5}{5}$$

```
[In] Integrate[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4,x]
[Out] 8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
gosper	$8a\,e^2x-\frac{1}{2}d^3x^2+2d\,e^2x^4+\frac{8}{5}e^3x^5$	34
default	$8a\,e^2x-\frac{1}{2}d^3x^2+2d\,e^2x^4+\frac{8}{5}e^3x^5$	34
norman	$8a\,e^2x-\frac{1}{2}d^3x^2+2d\,e^2x^4+\frac{8}{5}e^3x^5$	34
risch	$8a\,e^2x-\frac{1}{2}d^3x^2+2d\,e^2x^4+\frac{8}{5}e^3x^5$	34
parallelrisch	$8a\,e^2x-\frac{1}{2}d^3x^2+2d\,e^2x^4+\frac{8}{5}e^3x^5$	34
parts	$8a\,e^2x-\frac{1}{2}d^3x^2+2d\,e^2x^4+\frac{8}{5}e^3x^5$	34

```
[In] int(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x,method=_RETURNVERBOSE)
[Out] 8*a*e^2*x-1/2*d^3*x^2+2*d*e^2*x^4+8/5*e^3*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)\,dx=\frac{8}{5}\,e^3x^5+2\,de^2x^4-\frac{1}{2}\,d^3x^2+8\,ae^2x$$

```
[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="fricas")
[Out] 8/5*e^3*x^5 + 2*d*e^2*x^4 - 1/2*d^3*x^2 + 8*a*e^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)\,dx=8ae^2x-\frac{d^3x^2}{2}+2de^2x^4+\frac{8e^3x^5}{5}$$

```
[In] integrate(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2,x)
[Out] 8*a*e**2*x - d**3*x**2/2 + 2*d*e**2*x**4 + 8*e**3*x**5/5
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)\,dx=\frac{8}{5}\,e^3x^5+2\,de^2x^4-\frac{1}{2}\,d^3x^2+8\,ae^2x$$

```
[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="maxima")
[Out] 8/5*e^3*x^5 + 2*d*e^2*x^4 - 1/2*d^3*x^2 + 8*a*e^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)\,dx=\frac{8}{5}\,e^3x^5+2\,de^2x^4-\frac{1}{2}\,d^3x^2+8\,ae^2x$$

```
[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="giac")
[Out] 8/5*e^3*x^5 + 2*d*e^2*x^4 - 1/2*d^3*x^2 + 8*a*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)\,dx=-\frac{d^3\,x^2}{2}+2\,d\,e^2\,x^4+\frac{8\,e^3\,x^5}{5}+8\,a\,e^2\,x$$

```
[In] int(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3,x)
[Out] (8*e^3*x^5)/5 - (d^3*x^2)/2 + 2*d*e^2*x^4 + 8*a*e^2*x
```

3.43 $\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$

Optimal result . . . 434
Rubi [A] (verified) . . . 434
Mathematica [C] (verified) . . . 436
Maple [C] (verified) . . . 436
Fricas [B] (verification not implemented) . . . 436
Sympy [A] (verification not implemented) . . . 437
Maxima [F] . . . 438
Giac [B] (verification not implemented) . . . 438
Mupad [B] (verification not implemented) . . . 439

Optimal result

Integrand size = 32, antiderivative size = 153

$$\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx=\frac{2\text{arctanh}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}-\frac{2\text{arctanh}\left(\frac{d+4ex}{\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}$$

```
[Out] 2*arctanh((4*e*x+d)/(3*d^2-2*(-64*a*e^3+d^4)^(1/2))^(1/2))/(-64*a*e^3+d^4)^
(1/2)/(3*d^2-2*(-64*a*e^3+d^4)^(1/2))^(1/2)-2*arctanh((4*e*x+d)/(3*d^2+2*(-
64*a*e^3+d^4)^(1/2))^(1/2))/(-64*a*e^3+d^4)^(1/2)/(3*d^2+2*(-64*a*e^3+d^4)^
(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1120, 1107, 214}

$$\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx=\frac{2\text{arctanh}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}-\frac{2\text{arctanh}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{\sqrt{d^4-64ae^3}\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1),x]
```

```
[Out] (2*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqrt[d^4 - 6
4*a*e^3]*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]) - (2*ArcTanh[(d + 4*e*x)/Sqr
```

```
t[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqrt[d^4 - 64*a*e^3]*Sqrt[3*d^2 + 2*Sq
rt[d^4 - 64*a*e^3]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4}\,dx, x, \frac{d}{4e}+x\right)$$

$$= \frac{(4e^2)\,\text{Subst}\left(\int \frac{1}{-\frac{3d^2e}{2}-e\sqrt{d^4-64ae^3}+8e^3x^2}\,dx, x, \frac{d}{4e}+x\right)}{\sqrt{d^4-64ae^3}} - \frac{(4e^2)\,\text{Subst}\left(\int \frac{1}{-\frac{3d^2e}{2}+e\sqrt{d^4-64ae^3}+8e^3x^2}\,dx, x, \frac{d}{4e}+x\right)}{\sqrt{d^4-64ae^3}}$$

$$= \frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}} - \frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.46

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} \, dx = -\text{RootSum}\left[8ae^2 - d^3\#1 + 8de^2\#1^3 + 8e^3\#1^4\&, \frac{\log(x - \#1)}{d^3 - 24de^2\#1^2 - 32e^3\#1^3}\&\right]$$

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1),x]
[Out] -RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 & , Log[x - #1]/(d^3
 - 24*d*e^2*#1^2 - 32*e^3*#1^3) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.44

method	result	size
default	$\sum_{\_R=\text{RootOf}\left(8e^3\_Z^4+8d\,e^2\_Z^3-d^3\_Z+8a\,e^2\right)} \frac{\ln\left(x-\_R\right)}{32\_R^3e^3+24\_R^2d\,e^2-d^3}$	67
risch	$\sum_{\_R=\text{RootOf}\left(8e^3\_Z^4+8d\,e^2\_Z^3-d^3\_Z+8a\,e^2\right)} \frac{\ln\left(x-\_R\right)}{32\_R^3e^3+24\_R^2d\,e^2-d^3}$	67

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x,method=_RETURNVERBOSE)
[Out] sum(1/(32*_R^3*e^3+24*_R^2*d*e^2-d^3)*ln(x-_R),_R=RootOf(8*_Z^4*e^3+8*_Z^3*
d*e^2-_Z*d^3+8*a*e^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1115 vs. 2(133) = 266.

Time = 0.31 (sec) , antiderivative size = 1115, normalized size of antiderivative = 7.29

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="fricas")
[Out] -sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*
a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 1
```

```
6384*a^2*e^6))*log(8*e*x + 2*(2*d^4 - 128*a*e^3 - 3*(5*d^10 - 64*a*d^6*e^3
- 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 419
4304*a^3*e^9))*sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(
25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64
*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) + sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3
 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 419430
4*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x - 2*(2*d^4 -
128*a*e^3 - 3*(5*d^10 - 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 96
0*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2 + 2*(5*d^8
- 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^
4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) -
sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a
*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16
384*a^2*e^6))*log(8*e*x + 2*(2*d^4 - 128*a*e^3 + 3*(5*d^10 - 64*a*d^6*e^3 -
 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194
304*a^3*e^9))*sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(2
5*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*
a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) + sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3
- 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304
*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x - 2*(2*d^4 - 1
28*a*e^3 + 3*(5*d^10 - 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960
*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2 - 2*(5*d^8 -
 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4
*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d)
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.80

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\,dx$$
$$= \text{RootSum}\left(t^4 \cdot \left(1048576a^3e^9 - 12288a^2d^4e^6 - 384ad^8e^3 + 5d^{12}\right) + t^2 \cdot \left(384ad^2e^3 - 6d^6\right) + 1, \left(t \mapsto t \log \right.\right.$$

```
[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2),x)
```

```
[Out] RootSum(_t**4*(1048576*a**3*e**9 - 12288*a**2*d**4*e**6 - 384*a*d**8*e**3 +
 5*d**12) + _t**2*(384*a*d**2*e**3 - 6*d**6) + 1, Lambda(_t, _t*log(x + (-4
9152*_t**3*a**2*d**2*e**6 - 192*_t**3*a*d**6*e**3 + 15*_t**3*d**10 + 256*_t
*a*e**3 - 13*_t*d**4 + 2*d)/(8*e))))
```

Maxima [F]

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\,dx = \int \frac{1}{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}\,dx$$

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="maxima")
[Out] integrate(1/(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(133) = 266.

Time = 0.28 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.77

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\,dx$$
$$= -\frac{2\,\log\left(x+\frac{1}{4}\sqrt{\frac{3\,d^2e^2+2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}+\frac{d}{4\,e}\right)}{e^3\left(\sqrt{\frac{3\,d^2e^2+2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}+\frac{d}{e}\right)^3-3\,de^2\left(\sqrt{\frac{3\,d^2e^2+2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}+\frac{d}{e}\right)^2+2\,d^3}$$
$$+\frac{2\,\log\left(x-\frac{1}{4}\sqrt{\frac{3\,d^2e^2+2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}+\frac{d}{4\,e}\right)}{e^3\left(\sqrt{\frac{3\,d^2e^2+2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}-\frac{d}{e}\right)^3+3\,de^2\left(\sqrt{\frac{3\,d^2e^2+2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}-\frac{d}{e}\right)^2-2\,d^3}$$
$$-\frac{2\,\log\left(x+\frac{1}{4}\sqrt{\frac{3\,d^2e^2-2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}+\frac{d}{4\,e}\right)}{e^3\left(\sqrt{\frac{3\,d^2e^2-2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}+\frac{d}{e}\right)^3-3\,de^2\left(\sqrt{\frac{3\,d^2e^2-2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}+\frac{d}{e}\right)^2+2\,d^3}$$
$$+\frac{2\,\log\left(x-\frac{1}{4}\sqrt{\frac{3\,d^2e^2-2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}+\frac{d}{4\,e}\right)}{e^3\left(\sqrt{\frac{3\,d^2e^2-2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}-\frac{d}{e}\right)^3+3\,de^2\left(\sqrt{\frac{3\,d^2e^2-2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}-\frac{d}{e}\right)^2-2\,d^3}$$

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="giac")
[Out] -2*log(x + 1/4*sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + 1/4*d/e
)/(e^3*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^3 - 3*d*e
^2*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^2 + 2*d^3) +
2*log(x - 1/4*sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + 1/4*d/e)
/(e^3*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e)^3 + 3*d*e^
2*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e)^2 - 2*d^3) - 2
*log(x + 1/4*sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + 1/4*d/e)/
```

```
(e^3*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^3 - 3*d*e^2
*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^2 + 2*d^3) + 2*
log(x - 1/4*sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + 1/4*d/e)/(
e^3*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e)^3 + 3*d*e^2*
(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e)^2 - 2*d^3)
```

Mupad [B] (verification not implemented)

Time = 11.30 (sec) , antiderivative size = 1264, normalized size of antiderivative = 8.26

$$\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx =$$

$$-\mathrm{atan}\left(\frac{d^3\sqrt{-262144\,a^3\,e^9+12288\,a^2\,d^4\,e^6-192\,a\,d^8\,e^3+d^{12}}\,3\mathrm{i}+d^9\,2\cdots}{5\,d^{12}\sqrt{\frac{2\sqrt{-262144\,a^3\,e^9+12288\,a^2\,d^4\,e^6-192\,a\,d^8\,e^3+d^{12}}+3\,d^6-192\,a\,d^2\,e^3}{1048576\,a^3\,e^9-12288\,a^2\,d^4\,e^6-384\,a\,d^8\,e^3+5\,d^{12}}}+1048576\,a^3\,e^9\sqrt{\frac{2\sqrt{-262144\,a^3\,e^9+12288\cdots}}{1048576\,a^3\,e^9-\cdots}}\cdots}\right.$$

$$+\mathrm{atan}\left(\frac{d^3\sqrt{-262144\,a^3\,e^9+12288\,a^2\,d^4\,e^6-192\,a\,d^8\,e^3+d^{12}}\,3\mathrm{i}-\cdots}{5\,d^{12}\sqrt{-\frac{2\sqrt{-262144\,a^3\,e^9+12288\,a^2\,d^4\,e^6-192\,a\,d^8\,e^3+d^{12}}-3\,d^6+192\,a\,d^2\,e^3}{1048576\,a^3\,e^9-12288\,a^2\,d^4\,e^6-384\,a\,d^8\,e^3+5\,d^{12}}}+1048576\,a^3\,e^9\sqrt{-\frac{2\sqrt{-262144\,a^3\,e^9+\cdots}}{1048576\,a\cdots}}\cdots}\right.$$

```
[In] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3),x)
```

```
[Out] atan((d^3*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)
*3i - d^9*2i + a*d^5*e^3*256i - a^2*d*e^6*8192i - a^2*e^7*x*32768i - d^8*e*
x*8i + d^2*e*x*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^
(1/2)*12i + a*d^4*e^4*x*1024i)/(5*d^12*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*
d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 10485
76*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) + 1048576*a^3*e^9*(-
(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^
6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^
4*e^6))^(1/2) - 384*a*d^8*e^3*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 +
 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^
9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) - 12288*a^2*d^4*e^6*(-(2*(d^1
2 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192
*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))
^(1/2)))*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(
1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 1
2288*a^2*d^4*e^6))^(1/2)*2i - atan((d^3*(d^12 - 262144*a^3*e^9 - 192*a*d^8*
e^3 + 12288*a^2*d^4*e^6)^(1/2)*3i + d^9*2i - a*d^5*e^3*256i + a^2*d*e^6*819
2i + a^2*e^7*x*32768i + d^8*e*x*8i + d^2*e*x*(d^12 - 262144*a^3*e^9 - 192*a
*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)*12i - a*d^4*e^4*x*1024i)/(5*d^12*((2*(d
^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) + 3*d^6 - 1
92*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6
))^(1/2) + 1048576*a^3*e^9*((2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 122
88*a^2*d^4*e^6)^(1/2) + 3*d^6 - 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 -
```

```
384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) - 384*a*d^8*e^3*((2*(d^12 - 26214
4*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) + 3*d^6 - 192*a*d^2*e^
3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) -
12288*a^2*d^4*e^6*((2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^
4*e^6)^(1/2) + 3*d^6 - 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8
*e^3 - 12288*a^2*d^4*e^6))^(1/2)))*((2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e
^3 + 12288*a^2*d^4*e^6)^(1/2) + 3*d^6 - 192*a*d^2*e^3)/(5*d^12 + 1048576*a^
3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2)*2i
```

3.44 $\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^2}\,dx$

Optimal result 441
Rubi [A] (verified) 442
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Optimal result

Integrand size = 32, antiderivative size = 342

$$\begin{aligned}\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^2}\,dx &= \frac{2e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(5d^8-64ad^4e^3-16384a^2e^6)\,(8ae^2-d^3x+8de^2x^3+8e^3x^4)} \\ &\quad - \frac{24e\left(d^4+128ae^3-d^2\sqrt{d^4-64ae^3}\right)\operatorname{arctanh}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{(d^4-64ae^3)^{3/2}\,(5d^4+256ae^3)\,\sqrt{3d^2-2\sqrt{d^4-64ae^3}}} \\ &\quad + \frac{24e\left(d^4+128ae^3+d^2\sqrt{d^4-64ae^3}\right)\operatorname{arctanh}\left(\frac{d+4ex}{\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}\right)}{(d^4-64ae^3)^{3/2}\,(5d^4+256ae^3)\,\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}\end{aligned}$$

```
[Out] 2*e*(1/4*d/e+x)*(13*d^4-256*a*e^3-48*d^2*e^2*(1/4*d/e+x)^2)/(-16384*a^2*e^6
-64*a*d^4*e^3+5*d^8)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)-24*e*arctanh((4*
e*x+d)/(3*d^2-2*(-64*a*e^3+d^4)^(1/2))^(1/2))*(d^4+128*a*e^3-d^2*(-64*a*e^3
+d^4)^(1/2))/(-64*a*e^3+d^4)^(3/2)/(256*a*e^3+5*d^4)/(3*d^2-2*(-64*a*e^3+d^
4)^(1/2))^(1/2)+24*e*arctanh((4*e*x+d)/(3*d^2+2*(-64*a*e^3+d^4)^(1/2))^(1/2
))*(d^4+128*a*e^3+d^2*(-64*a*e^3+d^4)^(1/2))/(-64*a*e^3+d^4)^(3/2)/(256*a*e
^3+5*d^4)/(3*d^2+2*(-64*a*e^3+d^4)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1120, 1106, 1180, 214}

$$\begin{aligned}&\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^2}\,dx\\&=\frac{2e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(-16384a^2e^6-64ad^4e^3+5d^8)\,(8ae^2-d^3x+8de^2x^3+8e^3x^4)}\\&\quad-\frac{24e\left(-d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\operatorname{arctanh}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{(d^4-64ae^3)^{3/2}\,(256ae^3+5d^4)\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\\&\quad+\frac{24e\left(d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\operatorname{arctanh}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{(d^4-64ae^3)^{3/2}\,(256ae^3+5d^4)\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\end{aligned}$$

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]
```

```
[Out] (2*e*(d/(4*e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4*e) + x)^2))/((5*d
^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4)) - (24*e*(d^4 + 128*a*e^3 - d^2*Sqrt[d^4 - 64*a*e^3])*ArcTanh[(d + 4*e*
x)/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]])/((d^4 - 64*a*e^3)^(3/2)*(5*d^4 +
256*a*e^3)*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]) + (24*e*(d^4 + 128*a*e^3 +
 d^2*Sqrt[d^4 - 64*a*e^3])*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 + 2*Sqrt[d^4 - 64
*a*e^3]]])/((d^4 - 64*a*e^3)^(3/2)*(5*d^4 + 256*a*e^3)*Sqrt[3*d^2 + 2*Sqrt[
d^4 - 64*a*e^3]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
```

```
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4\right)^2}\,dx, x, \frac{d}{4e}+x\right)$$

$$= \frac{2e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)} - \frac{4\text{Subst}\left(\int \frac{9d^4e^2-\frac{1}{2}e^3\left(\frac{5d^4}{e}+256ae^2\right)-2\left(9d^4e^2-e^3\left(\frac{5d^4}{e}+256ae^2\right)\right)+24d^2e^4x^2}{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4}\,dx, x, \frac{d}{4e}+x\right)}{e\left(5d^8-64ad^4e^3-16384a^2e^6\right)}$$

$$= \frac{2e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)} + \frac{\left(48e^3\left(d^4+128ae^3-d^2\sqrt{d^4-64ae^3}\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{3d^2e}{2}+e\sqrt{d^4-64ae^3}+8e^3x^2}\,dx, x, \frac{d}{4e}+x\right)}{\left(d^4-64ae^3\right)^{3/2}\left(5d^4+256ae^3\right)} - \frac{\left(48e^3\left(d^4+128ae^3+d^2\sqrt{d^4-64ae^3}\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{3d^2e}{2}-e\sqrt{d^4-64ae^3}+8e^3x^2}\,dx, x, \frac{d}{4e}+x\right)}{\left(d^4-64ae^3\right)^{3/2}\left(5d^4+256ae^3\right)}$$

$$= \frac{2e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)} - \frac{24e\left(d^4+128ae^3-d^2\sqrt{d^4-64ae^3}\right)\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\left(d^4-64ae^3\right)^{3/2}\left(5d^4+256ae^3\right)\sqrt{3d^2-2\sqrt{d^4-64ae^3}}} + \frac{24e\left(d^4+128ae^3+d^2\sqrt{d^4-64ae^3}\right)\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}\right)}{\left(d^4-64ae^3\right)^{3/2}\left(5d^4+256ae^3\right)\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.68

$$\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^2}\,dx$$
$$=\frac{(d+4ex)\,(5d^4-128ae^3-12d^3ex-24d^2e^2x^2)}{(d^4-64ae^3)\,(5d^4+256ae^3)\,(8ae^2-d^3x+8de^2x^3+8e^3x^4)}$$
$$+\frac{48e^2\text{RootSum}\left[8ae^2-d^3\#1+8de^2\#1^3+8e^3\#1^4\&,\frac{32ae^2\log(x-\#1)+d^3\log(x-\#1)\#1+2d^2e\log(x-\#1)\#1^2}{-d^3+24de^2\#1^2+32e^3\#1^3}\&\right]}{-5d^8+64ad^4e^3+16384a^2e^6}$$

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]
```

```
[Out] ((d + 4*e*x)*(5*d^4 - 128*a*e^3 - 12*d^3*e*x - 24*d^2*e^2*x^2))/((d^4 - 64*
a*e^3)*(5*d^4 + 256*a*e^3)*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)) + (
48*e^2*RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 & , (32*a*e^2*L
og[x - #1] + d^3*Log[x - #1]*#1 + 2*d^2*e*Log[x - #1]*#1^2)/(-d^3 + 24*d*e^
2*#1^2 + 32*e^3*#1^3) & ])/(-5*d^8 + 64*a*d^4*e^3 + 16384*a^2*e^6)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.84

method	result
default	$\frac{\frac{12d^2e^3x^3}{(256a\,e^3+5d^4)(64a\,e^3-d^4)}+\frac{9d^3e^2x^2}{(256a\,e^3+5d^4)(64a\,e^3-d^4)}+\frac{ex}{256a\,e^3+5d^4}+\frac{d(128a\,e^3-5d^4)}{131072a^2e^6+512a\,d^4e^3-40d^8}}{e^3x^4+d\,e^2x^3-\frac{1}{8}d^3x+a\,e^2}+\frac{384e^2\left(\sum_{\_R=\text{RootOf}(8e^3\_Z^4+8d\right.}}{}$
risch	$\frac{\frac{12d^2e^3x^3}{(256a\,e^3+5d^4)(64a\,e^3-d^4)}+\frac{9d^3e^2x^2}{(256a\,e^3+5d^4)(64a\,e^3-d^4)}+\frac{ex}{256a\,e^3+5d^4}+\frac{d(128a\,e^3-5d^4)}{131072a^2e^6+512a\,d^4e^3-40d^8}}{e^3x^4+d\,e^2x^3-\frac{1}{8}d^3x+a\,e^2}+48e^2\left(\sum_{\_R=\text{RootOf}(8e^3\_Z^4+\right.$

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (12*d^2*e^3/(256*a*e^3+5*d^4)/(64*a*e^3-d^4)*x^3+9*d^3*e^2/(256*a*e^3+5*d^4
)/(64*a*e^3-d^4)*x^2+e/(256*a*e^3+5*d^4)*x+1/8*d*(128*a*e^3-5*d^4)/(16384*a
^2*e^6+64*a*d^4*e^3-5*d^8))/(e^3*x^4+d*e^2*x^3-1/8*d^3*x+a*e^2)+384*e^2/(20
48*a*e^3+40*d^4)/(64*a*e^3-d^4)*sum((2*_R^2*d^2*e+_R*d^3+32*a*e^2)/(32*_R^3
*e^3+24*_R^2*d*e^2-d^3)*ln(x-_R),_R=RootOf(8*_Z^4*e^3+8*_Z^3*d*e^2-_Z*d^3+8
*a*e^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4285 vs. 2(316) = 632.

Time = 0.48 (sec) , antiderivative size = 4285, normalized size of antiderivative = 12.53

$$\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="fricas")
[Out] -(96*d^2*e^3*x^3 + 72*d^3*e^2*x^2 - 5*d^5 + 128*a*d*e^3 + 12*sqrt(2)*(40*a*
d^8*e^2 - 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 -
16384*a^2*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (
5*d^11 - 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^
5 + 40960*a^2*d^2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6
+ 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^1
5 - 4398046511104*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)
/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^
3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 27
44381022928896*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 51881467707
30811392*a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a
*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d
^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*
d^5*e^6 + 226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x + 1382
4*sqrt(2)*(d^16*e^2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*
e^11 - 268435456*a^4*e^14 - (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22
*e^6 - 566493184*a^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a
^5*d^10*e^15 - 30786325577728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)
*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^
32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a
^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^1
8 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 737
86976294838206464*a^9*e^27)))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^
2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d
^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 439804651110
4*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1
800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150
994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a
^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4
*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167
360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 515396
07552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))) - 12*sqrt(2)*(40*a*d^8*e^2 -
 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2
*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 -
64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960
```

*a^2*d^2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 3119513
6*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 43980
46511104*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d
^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^
9 - 150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 27443810229
28896*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*
a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3
 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 -
 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*d^5*e^6 +
 226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x - 13824*sqrt(2)
*(d^16*e^2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^11 - 26
8435456*a^4*e^14 - (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22*e^6 - 56
6493184*a^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a^5*d^10*e
^15 - 30786325577728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)*sqrt((d^
8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 -
115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e
^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 70931
694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 737869762948
38206464*a^9*e^27)))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 + (
125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 +
 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^1
8)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*
d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000
*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e
^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 7
3786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d
^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5
*d^4*e^15 - 4398046511104*a^6*e^18))) + 12*sqrt(2)*(40*a*d^8*e^2 - 512*a^2*
d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4
 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a*d^7*
e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*
e^8 - (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^1
2*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*
a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 180
0000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 15099
4944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6
*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e
^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 116736
0*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607
552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*d^5*e^6 + 22649241
6*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x + 13824*sqrt(2)*(d^16*e^
2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^11 - 268435456*a
^4*e^14 + (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22*e^6 - 566493184*a
^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a^5*d^10*e^15 - 307
86325577728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)*sqrt((d^8*e^4 + 5

12*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000
*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 780
82505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 70931694131085
312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 73786976294838206464*
a^9*e^27)))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 - (125*d^24
- 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 38252052
48*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18)*sqrt((
d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3
- 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20
*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 709
31694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 7378697629
4838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 +
31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15
- 4398046511104*a^6*e^18))) - 12*sqrt(2)*(40*a*d^8*e^2 - 512*a^2*d^4*e^5 -
131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4 + 8*(5*d
^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a*d^7*e^3 - 163
84*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 - (12
5*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3
825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18)
*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^
32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a
^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^1
8 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 737
86976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^1
6*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d
^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*d^5*e^6 + 226492416*a^2*d*e
^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x - 13824*sqrt(2)*(d^16*e^2 - 128*a
*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^11 - 268435456*a^4*e^14 +
(125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22*e^6 - 566493184*a^3*d^18*e
^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a^5*d^10*e^15 - 307863255777
28*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)*sqrt((d^8*e^4 + 512*a*d^4*
e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28
*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 780825054412
80*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 70931694131085312*a^7*d
^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)
))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 - (125*d^24 - 4800*a*
d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^
8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18)*sqrt((d^8*e^4 +
512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 1152000
00*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 7
8082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 709316941310
85312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 7378697629483820646
4*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136
*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 439804
6511104*a^6*e^18))) - 8*(d^4*e - 64*a*e^4)*x)/(40*a*d^8*e^2 - 512*a^2*d^4*e

```
^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4 + 8*
(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a*d^7*e^3 -
 16384*a^2*d^3*e^6)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2}\,dx = \int \frac{1}{(8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2)^2}\,dx$$

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="maxima")
[Out] -48*e^2*integrate((2*d^2*e*x^2 + d^3*x + 32*a*e^2)/(8*e^3*x^4 + 8*d*e^2*x^3
 - d^3*x + 8*a*e^2), x)/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6) - (96*d^2*e^
3*x^3 + 72*d^3*e^2*x^2 - 5*d^5 + 128*a*d*e^3 - 8*(d^4*e - 64*a*e^4)*x)/(40*
a*d^8*e^2 - 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6
- 16384*a^2*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 -
 (5*d^11 - 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1115 vs. $2(316) = 632$.

Time = 0.32 (sec) , antiderivative size = 1115, normalized size of antiderivative = 3.26

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")
[Out] 12*((d^2*e^3*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^2 -
 2*d^3*e^2*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e) + 256
*a*e^4)*log(x + 1/4*sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + 1/
4*d/e)/(e^3*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^3 -
3*d*e^2*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^2 + 2*d^
```

```
3) - (d^2*e^3*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e)^2
+ 2*d^3*e^2*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e) + 25
6*a*e^4)*log(x - 1/4*sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + 1
/4*d/e)/(e^3*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e)^3 +
 3*d*e^2*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e)^2 - 2*d
^3) + (d^2*e^3*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^2
 - 2*d^3*e^2*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e) + 2
56*a*e^4)*log(x + 1/4*sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) +
1/4*d/e)/(e^3*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^3
- 3*d*e^2*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^2 + 2*
d^3) - (d^2*e^3*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e)^
2 + 2*d^3*e^2*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e) +
256*a*e^4)*log(x - 1/4*sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) +
 1/4*d/e)/(e^3*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e)^3
 + 3*d*e^2*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e)^2 - 2
*d^3))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6) - (96*d^2*e^3*x^3 + 72*d^3*e^
2*x^2 - 8*d^4*e*x + 512*a*e^4*x - 5*d^5 + 128*a*d*e^3)/((5*d^8 - 64*a*d^4*e
^3 - 16384*a^2*e^6)*(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2))
```

Mupad [B] (verification not implemented)

Time = 13.83 (sec) , antiderivative size = 10351, normalized size of antiderivative = 30.27

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2}\, dx = \text{Too large to display}$$

```
[In] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^2,x)
```

```
[Out] ((8*e*x)/(256*a*e^3 + 5*d^4) - (5*d^5 - 128*a*d*e^3)/((64*a*e^3 - d^4)*(256
*a*e^3 + 5*d^4)) + (72*d^3*e^2*x^2)/((64*a*e^3 - d^4)*(256*a*e^3 + 5*d^4))
+ (96*d^2*e^3*x^3)/((64*a*e^3 - d^4)*(256*a*e^3 + 5*d^4)))/(8*a*e^2 - d^3*x
 + 8*e^3*x^4 + 8*d*e^2*x^3) + atan((((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 -
 d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11
 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^
3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32
*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20
*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 79164837
1998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2)*(((1536*(6871
9476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12*e^15 -
5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17179869184*a^5
*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 1342
17728*a^4*d^4*e^12) - ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 24576*a^2*d^
19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 25769803776*a^
5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^5*e^15 -
 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a
```

^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2*d^14*e^
15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*a^5*d^2*
e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6
+ 2097152*a^3*d^4*e^9))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/
2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440
*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^
(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 12902
40*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 9663
6764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*
d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*((288*(d^22*e^2 + d^4*e^
2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 616038
4*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a
e^5(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27
 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250
489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*
e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2
) + (1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 196608*a^
2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2
*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(78643
2*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(25*d^16 + 268435456*a^4*e^12
- 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*1i + ((288*(d
^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d
^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*
d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 115292150460
6846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*
d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062
494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8
*d^4*e^24))^(1/2)*((1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e
^19 + 196608*a^2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e
^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) -
 ((1536*(68719476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2
*d^12*e^15 - 5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17
179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d
^8*e^9 + 134217728*a^4*d^4*e^12) + ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 +
 24576*a^2*d^19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 2
5769803776*a^5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 171798691
84*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9
+ 134217728*a^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 11878
4*a^2*d^14*e^15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 1717986
9184*a^5*d^2*e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744
*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3
- d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^1
1 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e
^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^3
2*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^2

0*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 7916483
71998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*((288*(d^22
*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14
*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2
*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 115292150460684
6976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^2
4*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494
720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^
4*e^24))^(1/2) + (6144*x*(786432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))
/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097
152*a^3*d^4*e^9))*1i)/(((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2
) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*
a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(
1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 129024
0*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636
764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d
^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2)*(((1536*(68719476736*a^5*e
^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12*e^15 - 5242880*a^3*d
^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17179869184*a^5*e^15 - 2240*
a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4
e^12) - ((1536(25*d^27*e^8 - 3840*a*d^23*e^11 + 24576*a^2*d^19*e^14 + 199
22944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 25769803776*a^5*d^7*e^23 +
1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*
e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12)
+ (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2*d^14*e^15 + 12320768
*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*a^5*d^2*e^24))/(25*d^
16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3
*d^4*e^9))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^1
8*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14
 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*
d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e
^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d
^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40
532396646334464*a^8*d^4*e^24))^(1/2))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3
 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^
11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*
e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^
32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^
20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648
371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2) + (1536*(96
*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 196608*a^2*d^5*e^16))/
(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 1
2320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(786432*a^2*e^17 +
96*d^8*e^11 + 9216*a*d^4*e^14))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*
e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9)) - ((288*(d^22*e^2 + d^4*e^

2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 616038
4*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a
e^5(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27
 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250
489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*
e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2
)*((1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 196608*a^2
*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*
d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) - ((1536*(6871947
6736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12*e^15 - 524
2880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17179869184*a^5*e^
15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 1342177
28*a^4*d^4*e^12) + ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 24576*a^2*d^19*
e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 25769803776*a^5*d
^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^5*e^15 - 22
40*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*
d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2*d^14*e^15
 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*a^5*d^2*e^2
4))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2
097152*a^3*d^4*e^9))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2)
 - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^
4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/
2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*
a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 9663676
4160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8
*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*((288*(d^22*e^2 + d^4*e^2*(
-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a
^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^
5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 -
28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489
344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^1
8 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2) +
 (6144*x*(786432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(25*d^16 + 2684
35456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9)
) + (113246208*a*d^2*e^14)/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^
3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12)))*
((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 225
28*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418
240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152
921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577
856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 +
44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 4053239664633
4464*a^8*d^4*e^24))^(1/2)*2i + atan(((-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^
(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^
11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*

e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2)*(((1536*(68719476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12*e^15 - 5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) - ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 24576*a^2*d^19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 25769803776*a^5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2*d^14*e^15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*a^5*d^2*e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2) + (1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 196608*a^2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(786432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*1i + (-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2)*((1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 196608*a^2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) - ((1536*(68719476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12*e^15 - 5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + ((1536*(25*d^27*e^8 - 3840*a*d^23*e

^11 + 24576*a^2*d^19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^2
0 - 25769803776*a^5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 1717
9869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8
*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 -
118784*a^2*d^14*e^15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17
179869184*a^5*d^2*e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 1
59744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4
)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^
10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(
64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800
*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a
^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 7
91648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*(-(28
8*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a
^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*
a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 11529215
04606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*
a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 4432
4062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464
*a^8*d^4*e^24))^(1/2) + (6144*x*(786432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4
*e^14))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6
+ 2097152*a^3*d^4*e^9))*1i)/((-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) -
d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 46
1373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d
^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3
+ 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12
- 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 7916483719987
20*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2)*(((1536*(687194767
36*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12*e^15 - 52428
80*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17179869184*a^5*e^15
- 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728
*a^4*d^4*e^12) - ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 24576*a^2*d^19*e^
14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 25769803776*a^5*d^7
*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^5*e^15 - 2240
*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^
4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2*d^14*e^15 +
12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*a^5*d^2*e^24)
)/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 209
7152*a^3*d^4*e^9))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 +
32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4
*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2
)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a
^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764
160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*
e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*(-(288*(d^4*e^2*(-(64*a*e^3

- d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a
^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^
5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 -
28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489
344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^1
8 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2) +
(1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 196608*a^2*d
^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^
12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(786432*a
^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(25*d^16 + 268435456*a^4*e^12 - 6
40*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9)) - (-(288*(d^4*e^
2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e
^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e
^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846
976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*
e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 4432406249472
0*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*
e^24))^(1/2)*((1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 +
196608*a^2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 -
118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) - ((15
36*(68719476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12
*e^15 - 5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 1717986
9184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^
9 + 134217728*a^4*d^4*e^12) + ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 2457
6*a^2*d^19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 257698
03776*a^5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^
5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134
217728*a^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2
*d^14*e^15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*
a^5*d^2*e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*
d^8*e^6 + 2097152*a^3*d^4*e^9))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2)
- d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 -
461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 -
d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^
3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^
12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 79164837199
8720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*(-(288*(d^4*e^2
*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^
8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^
17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 115292150460684697
6*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e
^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720
*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e
^24))^(1/2) + (6144*x*(786432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(2
5*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152

```
*a^3*d^4*e^9)) + (113246208*a*d^2*e^14)/(25*d^20 - 17179869184*a^5*e^15 - 2
240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4
*d^4*e^12)))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*
d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e
^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(1
25*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^2
8*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^
5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 -
 40532396646334464*a^8*d^4*e^24))^(1/2)*2i
```

3.45 $\int \left(8+8x-x^3+8x^4\right)^4 \, dx$

Optimal result . . . 457
Rubi [A] (verified) . . . 457
Mathematica [A] (verified) . . . 458
Maple [A] (verified) . . . 458
Fricas [A] (verification not implemented) . . . 459
Sympy [A] (verification not implemented) . . . 459
Maxima [A] (verification not implemented) . . . 460
Giac [A] (verification not implemented) . . . 460
Mupad [B] (verification not implemented) . . . 461

Optimal result

Integrand size = 17, antiderivative size = 96

$$\int \left(8+8x-x^3+8x^4\right)^4 \, dx = 4096x+8192x^2+8192x^3+3584x^4+\frac{14336x^5}{5}+7168x^6+6784x^7+1376x^8+1408x^9+\frac{21488x^{10}}{5}+\frac{25312x^{11}}{11}-448x^{12}+\frac{10241x^{13}}{13}+1168x^{14}+\frac{128x^{15}}{5}-128x^{16}+\frac{4096x^{17}}{17}$$

```
[Out] 4096*x+8192*x^2+8192*x^3+3584*x^4+14336/5*x^5+7168*x^6+6784*x^7+1376*x^8+14
08*x^9+21488/5*x^10+25312/11*x^11-448*x^12+10241/13*x^13+1168*x^14+128/5*x^
15-128*x^16+4096/17*x^17
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2086}

$$\int \left(8+8x-x^3+8x^4\right)^4 \, dx = \frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+7168x^6+\frac{14336x^5}{5}+3584x^4+8192x^3+8192x^2+4096x$$

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x
^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (1
0241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(4096 + 16384x + 24576x^2 + 14336x^3 + 14336x^4 + 43008x^5 + 47488x^6 \right. \\ &\quad + 11008x^7 + 12672x^8 + 42976x^9 + 25312x^{10} - 5376x^{11} + 10241x^{12} + 16352x^{13} \\ &\quad \left. + 384x^{14} - 2048x^{15} + 4096x^{16}\right) dx \\ &= 4096x + 8192x^2 + 8192x^3 + 3584x^4 + \frac{14336x^5}{5} + 7168x^6 + 6784x^7 + 1376x^8 + 1408x^9 \\ &\quad + \frac{21488x^{10}}{5} + \frac{25312x^{11}}{11} - 448x^{12} + \frac{10241x^{13}}{13} + 1168x^{14} + \frac{128x^{15}}{5} - 128x^{16} + \frac{4096x^{17}}{17}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \left(8 + 8x - x^3 + 8x^4\right)^4 dx &= 4096x + 8192x^2 + 8192x^3 + 3584x^4 + \frac{14336x^5}{5} + 7168x^6 \\ &\quad + 6784x^7 + 1376x^8 + 1408x^9 + \frac{21488x^{10}}{5} + \frac{25312x^{11}}{11} \\ &\quad - 448x^{12} + \frac{10241x^{13}}{13} + 1168x^{14} + \frac{128x^{15}}{5} - 128x^{16} + \frac{4096x^{17}}{17}\end{aligned}$$

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^4,x]
[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x
^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (1
0241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

method	result
gosper	$4096x+8192x^2+8192x^3+3584x^4+\frac{14336}{5}x^5+7168x^6+6784x^7+1376x^8+1408x^9+\frac{214}{5}$
default	$4096x+8192x^2+8192x^3+3584x^4+\frac{14336}{5}x^5+7168x^6+6784x^7+1376x^8+1408x^9+\frac{214}{5}$
norman	$4096x+8192x^2+8192x^3+3584x^4+\frac{14336}{5}x^5+7168x^6+6784x^7+1376x^8+1408x^9+\frac{214}{5}$
risch	$4096x+8192x^2+8192x^3+3584x^4+\frac{14336}{5}x^5+7168x^6+6784x^7+1376x^8+1408x^9+\frac{214}{5}$
parallelrisch	$4096x+8192x^2+8192x^3+3584x^4+\frac{14336}{5}x^5+7168x^6+6784x^7+1376x^8+1408x^9+\frac{214}{5}$

```
[In] int((8*x^4-x^3+8*x+8)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 4096*x+8192*x^2+8192*x^3+3584*x^4+14336/5*x^5+7168*x^6+6784*x^7+1376*x^8+14
08*x^9+21488/5*x^10+25312/11*x^11-448*x^12+10241/13*x^13+1168*x^14+128/5*x^
15-128*x^16+4096/17*x^17
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int (8+8x-x^3+8x^4)^4\,dx=\frac{4096}{17}\,x^{17}-128\,x^{16}+\frac{128}{5}\,x^{15}+1168\,x^{14}+\frac{10241}{13}\,x^{13}-448\,x^{12}+\frac{25312}{11}\,x^{11}+\frac{21488}{5}\,x^{10}+1408\,x^9+1376\,x^8+6784\,x^7+7168\,x^6+\frac{14336}{5}\,x^5+3584\,x^4+8192\,x^3+8192\,x^2+4096\,x$$

```
[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="fricas")
```

```
[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^12
 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x^6
 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98

$$\int (8+8x-x^3+8x^4)^4\,dx=\frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+7168x^6+\frac{14336x^5}{5}+3584x^4+8192x^3+8192x^2+4096x$$

```
[In] integrate((8*x**4-x**3+8*x+8)**4,x)
```

```
[Out] 4096*x**17/17 - 128*x**16 + 128*x**15/5 + 1168*x**14 + 10241*x**13/13 - 448
*x**12 + 25312*x**11/11 + 21488*x**10/5 + 1408*x**9 + 1376*x**8 + 6784*x**7
 + 7168*x**6 + 14336*x**5/5 + 3584*x**4 + 8192*x**3 + 8192*x**2 + 4096*x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \left(8+8x-x^3+8x^4\right)^4\,dx = \frac{4096}{17}\,x^{17} - 128\,x^{16} + \frac{128}{5}\,x^{15} + 1168\,x^{14} + \frac{10241}{13}\,x^{13} - 448\,x^{12} + \frac{25312}{11}\,x^{11} + \frac{21488}{5}\,x^{10} + 1408\,x^9 + 1376\,x^8 + 6784\,x^7 + 7168\,x^6 + \frac{14336}{5}\,x^5 + 3584\,x^4 + 8192\,x^3 + 8192\,x^2 + 4096\,x$$

```
[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="maxima")
```

```
[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^12
 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x^6
 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \left(8+8x-x^3+8x^4\right)^4\,dx = \frac{4096}{17}\,x^{17} - 128\,x^{16} + \frac{128}{5}\,x^{15} + 1168\,x^{14} + \frac{10241}{13}\,x^{13} - 448\,x^{12} + \frac{25312}{11}\,x^{11} + \frac{21488}{5}\,x^{10} + 1408\,x^9 + 1376\,x^8 + 6784\,x^7 + 7168\,x^6 + \frac{14336}{5}\,x^5 + 3584\,x^4 + 8192\,x^3 + 8192\,x^2 + 4096\,x$$

```
[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="giac")
```

```
[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^12
 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x^6
 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \left(8+8x-x^3+8x^4\right)^4\,dx=\frac{4096\,x^{17}}{17}-128\,x^{16}+\frac{128\,x^{15}}{5}+1168\,x^{14}+\frac{10241\,x^{13}}{13}-448\,x^{12}+\frac{25312\,x^{11}}{11}+\frac{21488\,x^{10}}{5}+1408\,x^9+1376\,x^8+6784\,x^7+7168\,x^6+\frac{14336\,x^5}{5}+3584\,x^4+8192\,x^3+8192\,x^2+4096\,x$$

```
[In] int((8*x - x^3 + 8*x^4 + 8)^4,x)
```

```
[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x
^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (1
0241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

3.46 $\int (8+8x-x^3+8x^4)^3 \, dx$

Optimal result 462
Rubi [A] (verified) 462
Mathematica [A] (verified) 463
Maple [A] (verified) 463
Fricas [A] (verification not implemented) 464
Sympy [A] (verification not implemented) 464
Maxima [A] (verification not implemented) 464
Giac [A] (verification not implemented) 465
Mupad [B] (verification not implemented) 465

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int (8+8x-x^3+8x^4)^3 \, dx = 512x+768x^2+512x^3+80x^4+\frac{1152x^5}{5}+480x^6+\frac{1560x^7}{7} -45x^8+128x^9+\frac{307x^{10}}{2}+\frac{24x^{11}}{11}-16x^{12}+\frac{512x^{13}}{13}$$

```
[Out] 512*x+768*x^2+512*x^3+80*x^4+1152/5*x^5+480*x^6+1560/7*x^7-45*x^8+128*x^9+3
07/2*x^10+24/11*x^11-16*x^12+512/13*x^13
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2086}

$$\int (8+8x-x^3+8x^4)^3 \, dx = \frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7} +480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
- 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(512+1536x+1536x^2+320x^3+1152x^4+2880x^5+1560x^6-360x^7 \right.\\ &\qquad \left. +1152x^8+1535x^9+24x^{10}-192x^{11}+512x^{12}\right) dx\\ &= 512x+768x^2+512x^3+80x^4+\frac{1152x^5}{5}+480x^6+\frac{1560x^7}{7}\\ &\quad -45x^8+128x^9+\frac{307x^{10}}{2}+\frac{24x^{11}}{11}-16x^{12}+\frac{512x^{13}}{13}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \left(8+8x-x^3+8x^4\right)^3 dx = 512x+768x^2+512x^3+80x^4+\frac{1152x^5}{5}+480x^6+\frac{1560x^7}{7} -45x^8+128x^9+\frac{307x^{10}}{2}+\frac{24x^{11}}{11}-16x^{12}+\frac{512x^{13}}{13}$$

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
- 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

method	result
gosper	$512x+768x^2+512x^3+80x^4+\frac{1152}{5}x^5+480x^6+\frac{1560}{7}x^7-45x^8+128x^9+\frac{307}{2}x^{10}+\frac{24}{11}x^{11}$
default	$512x+768x^2+512x^3+80x^4+\frac{1152}{5}x^5+480x^6+\frac{1560}{7}x^7-45x^8+128x^9+\frac{307}{2}x^{10}+\frac{24}{11}x^{11}$
norman	$512x+768x^2+512x^3+80x^4+\frac{1152}{5}x^5+480x^6+\frac{1560}{7}x^7-45x^8+128x^9+\frac{307}{2}x^{10}+\frac{24}{11}x^{11}$
risch	$512x+768x^2+512x^3+80x^4+\frac{1152}{5}x^5+480x^6+\frac{1560}{7}x^7-45x^8+128x^9+\frac{307}{2}x^{10}+\frac{24}{11}x^{11}$
parallelrisch	$512x+768x^2+512x^3+80x^4+\frac{1152}{5}x^5+480x^6+\frac{1560}{7}x^7-45x^8+128x^9+\frac{307}{2}x^{10}+\frac{24}{11}x^{11}$

```
[In] int((8*x^4-x^3+8*x+8)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 512*x+768*x^2+512*x^3+80*x^4+1152/5*x^5+480*x^6+1560/7*x^7-45*x^8+128*x^9+3
07/2*x^10+24/11*x^11-16*x^12+512/13*x^13
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \left(8+8x-x^3+8x^4\right)^3\,dx=\frac{512}{13}\,x^{13}-16\,x^{12}+\frac{24}{11}\,x^{11}+\frac{307}{2}\,x^{10}+128\,x^9-45\,x^8+\frac{1560}{7}\,x^7+480\,x^6+\frac{1152}{5}\,x^5+80\,x^4+512\,x^3+768\,x^2+512\,x$$

```
[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="fricas")
[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7
*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \left(8+8x-x^3+8x^4\right)^3\,dx=\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

```
[In] integrate((8*x**4-x**3+8*x+8)**3,x)
[Out] 512*x**13/13 - 16*x**12 + 24*x**11/11 + 307*x**10/2 + 128*x**9 - 45*x**8 +
1560*x**7/7 + 480*x**6 + 1152*x**5/5 + 80*x**4 + 512*x**3 + 768*x**2 + 512*
x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \left(8+8x-x^3+8x^4\right)^3\,dx=\frac{512}{13}\,x^{13}-16\,x^{12}+\frac{24}{11}\,x^{11}+\frac{307}{2}\,x^{10}+128\,x^9-45\,x^8+\frac{1560}{7}\,x^7+480\,x^6+\frac{1152}{5}\,x^5+80\,x^4+512\,x^3+768\,x^2+512\,x$$

```
[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="maxima")
[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7
*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \left(8+8x-x^3+8x^4\right)^3\,dx=\frac{512}{13}\,x^{13}-16\,x^{12}+\frac{24}{11}\,x^{11}+\frac{307}{2}\,x^{10}+128\,x^9-45\,x^8+\frac{1560}{7}\,x^7+480\,x^6+\frac{1152}{5}\,x^5+80\,x^4+512\,x^3+768\,x^2+512\,x$$

```
[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="giac")
[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7
*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \left(8+8x-x^3+8x^4\right)^3\,dx=\frac{512\,x^{13}}{13}-16\,x^{12}+\frac{24\,x^{11}}{11}+\frac{307\,x^{10}}{2}+128\,x^9-45\,x^8+\frac{1560\,x^7}{7}+480\,x^6+\frac{1152\,x^5}{5}+80\,x^4+512\,x^3+768\,x^2+512\,x$$

```
[In] int((8*x - x^3 + 8*x^4 + 8)^3,x)
[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
- 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13
```

3.47 $\int (8+8x-x^3+8x^4)^2\,dx$

Optimal result 466
Rubi [A] (verified) 466
Mathematica [A] (verified) 467
Maple [A] (verified) 467
Fricas [A] (verification not implemented) 467
Sympy [A] (verification not implemented) 468
Maxima [A] (verification not implemented) 468
Giac [A] (verification not implemented) 468
Mupad [B] (verification not implemented) 469

Optimal result

Integrand size = 17, antiderivative size = 54

$$\int (8+8x-x^3+8x^4)^2\,dx = 64x+64x^2+\frac{64x^3}{3}-4x^4+\frac{112x^5}{5}+\frac{64x^6}{3}+\frac{x^7}{7}-2x^8+\frac{64x^9}{9}$$

```
[Out] 64*x+64*x^2+64/3*x^3-4*x^4+112/5*x^5+64/3*x^6+1/7*x^7-2*x^8+64/9*x^9
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2086}

$$\int (8+8x-x^3+8x^4)^2\,dx = \frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x
^8 + (64*x^9)/9
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int (64+128x+64x^2-16x^3+112x^4+128x^5+x^6-16x^7+64x^8)\,dx \\ &= 64x+64x^2+\frac{64x^3}{3}-4x^4+\frac{112x^5}{5}+\frac{64x^6}{3}+\frac{x^7}{7}-2x^8+\frac{64x^9}{9} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \left(8+8x-x^3+8x^4\right)^2\,dx=64x+64x^2+\frac{64x^3}{3}-4x^4+\frac{112x^5}{5}+\frac{64x^6}{3}+\frac{x^7}{7}-2x^8+\frac{64x^9}{9}$$

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x
^8 + (64*x^9)/9
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
gosper	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45
default	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45
norman	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45
risch	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45
parallelrisch	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45

```
[In] int((8*x^4-x^3+8*x+8)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 64*x+64*x^2+64/3*x^3-4*x^4+112/5*x^5+64/3*x^6+1/7*x^7-2*x^8+64/9*x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(8+8x-x^3+8x^4\right)^2\,dx=\frac{64}{9}\,x^9-2\,x^8+\frac{1}{7}\,x^7+\frac{64}{3}\,x^6+\frac{112}{5}\,x^5-4\,x^4+\frac{64}{3}\,x^3+64\,x^2+64\,x$$

```
[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="fricas")
```

```
[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x
^2 + 64*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \left(8+8x-x^3+8x^4\right)^2 dx = \frac{64x^9}{9} - 2x^8 + \frac{x^7}{7} + \frac{64x^6}{3} + \frac{112x^5}{5} - 4x^4 + \frac{64x^3}{3} + 64x^2 + 64x$$

```
[In] integrate((8*x**4-x**3+8*x+8)**2,x)
[Out] 64*x**9/9 - 2*x**8 + x**7/7 + 64*x**6/3 + 112*x**5/5 - 4*x**4 + 64*x**3/3 +
 64*x**2 + 64*x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(8+8x-x^3+8x^4\right)^2 dx = \frac{64}{9}\,x^9 - 2\,x^8 + \frac{1}{7}\,x^7 + \frac{64}{3}\,x^6 + \frac{112}{5}\,x^5 - 4\,x^4 + \frac{64}{3}\,x^3 + 64\,x^2 + 64\,x$$

```
[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="maxima")
[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x
^2 + 64*x
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(8+8x-x^3+8x^4\right)^2 dx = \frac{64}{9}\,x^9 - 2\,x^8 + \frac{1}{7}\,x^7 + \frac{64}{3}\,x^6 + \frac{112}{5}\,x^5 - 4\,x^4 + \frac{64}{3}\,x^3 + 64\,x^2 + 64\,x$$

```
[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="giac")
[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x
^2 + 64*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(8+8x-x^3+8x^4\right)^2\,dx=\frac{64\,x^9}{9}-2\,x^8+\frac{x^7}{7}+\frac{64\,x^6}{3}+\frac{112\,x^5}{5}-4\,x^4+\frac{64\,x^3}{3}+64\,x^2+64\,x$$

```
[In] int((8*x - x^3 + 8*x^4 + 8)^2,x)
```

```
[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x
^8 + (64*x^9)/9
```

3.48 $\int (8+8x-x^3+8x^4)\,dx$

Optimal result 470
Rubi [A] (verified) 470
Mathematica [A] (verified) 471
Maple [A] (verified) 471
Fricas [A] (verification not implemented) 471
Sympy [A] (verification not implemented) 472
Maxima [A] (verification not implemented) 472
Giac [A] (verification not implemented) 472
Mupad [B] (verification not implemented) 472

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int (8+8x-x^3+8x^4)\,dx = 8x+4x^2-\frac{x^4}{4}+\frac{8x^5}{5}$$

```
[Out] 8*x+4*x^2-1/4*x^4+8/5*x^5
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (8+8x-x^3+8x^4)\,dx = \frac{8x^5}{5}-\frac{x^4}{4}+4x^2+8x$$

```
[In] Int[8 + 8*x - x^3 + 8*x^4,x]
[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

Rubi steps

$$\text{integral} = 8x+4x^2-\frac{x^4}{4}+\frac{8x^5}{5}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(8+8x-x^3+8x^4\right) \, dx = 8x+4x^2-\frac{x^4}{4}+\frac{8x^5}{5}$$

```
[In] Integrate[8 + 8*x - x^3 + 8*x^4,x]
[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
gosper	$8x+4x^2-\frac{1}{4}x^4+\frac{8}{5}x^5$	20
default	$8x+4x^2-\frac{1}{4}x^4+\frac{8}{5}x^5$	20
norman	$8x+4x^2-\frac{1}{4}x^4+\frac{8}{5}x^5$	20
risch	$8x+4x^2-\frac{1}{4}x^4+\frac{8}{5}x^5$	20
parallelrisch	$8x+4x^2-\frac{1}{4}x^4+\frac{8}{5}x^5$	20
parts	$8x+4x^2-\frac{1}{4}x^4+\frac{8}{5}x^5$	20

```
[In] int(8*x^4-x^3+8*x+8,x,method=_RETURNVERBOSE)
[Out] 8*x+4*x^2-1/4*x^4+8/5*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(8+8x-x^3+8x^4\right) \, dx = \frac{8}{5}\,x^5-\frac{1}{4}\,x^4+4\,x^2+8\,x$$

```
[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="fricas")
[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(8+8x-x^3+8x^4\right)\,dx = \frac{8x^5}{5} - \frac{x^4}{4} + 4x^2 + 8x$$

```
[In] integrate(8*x**4-x**3+8*x+8,x)
[Out] 8*x**5/5 - x**4/4 + 4*x**2 + 8*x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(8+8x-x^3+8x^4\right)\,dx = \frac{8}{5}\,x^5 - \frac{1}{4}\,x^4 + 4\,x^2 + 8\,x$$

```
[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="maxima")
[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(8+8x-x^3+8x^4\right)\,dx = \frac{8}{5}\,x^5 - \frac{1}{4}\,x^4 + 4\,x^2 + 8\,x$$

```
[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="giac")
[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(8+8x-x^3+8x^4\right)\,dx = \frac{8\,x^5}{5} - \frac{x^4}{4} + 4\,x^2 + 8\,x$$

```
[In] int(8*x - x^3 + 8*x^4 + 8,x)
[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

3.49 $\int \frac{1}{8+8x-x^3+8x^4} \, dx$

Optimal result 473
Rubi [A] (verified) 474
Mathematica [C] (verified) 477
Maple [C] (verified) 478
Fricas [C] (verification not implemented) 478
Sympy [A] (verification not implemented) 479
Maxima [F] 480
Giac [F] 480
Mupad [B] (verification not implemented) 480

Optimal result

Integrand size = 17, antiderivative size = 268

$$\int \frac{1}{8+8x-x^3+8x^4} \, dx = -\frac{\arctan\left(\frac{3-\left(1+\frac{4}{x}\right)^2}{6\sqrt{7}}\right)}{12\sqrt{7}} - \frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\arctan\left(\frac{2-\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right) - \frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\arctan\left(\frac{2+\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right) - \frac{1}{24}\sqrt{\frac{-109+67\sqrt{29}}{1218}}\log\left(3\sqrt{29}-\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right) + \frac{1}{24}\sqrt{\frac{-109+67\sqrt{29}}{1218}}\log\left(3\sqrt{29}+\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right)$$

```
[Out] -1/84*arctan(1/42*(3-(1+4/x)^2)*7^(1/2))*7^(1/2)-1/29232*ln((1+4/x)^2+3*29^
(1/2)-(1+4/x)*(6+6*29^(1/2))^(1/2))*(-132762+81606*29^(1/2))^(1/2)+1/29232*
ln((1+4/x)^2+3*29^(1/2)+(1+4/x)*(6+6*29^(1/2))^(1/2))*(-132762+81606*29^(1/
2))^(1/2)-1/14616*arctan((2+8/x-(6+6*29^(1/2))^(1/2))/(-6+6*29^(1/2))^(1/2)
)*(132762+81606*29^(1/2))^(1/2)-1/14616*arctan((2+8/x+(6+6*29^(1/2))^(1/2))
/(-6+6*29^(1/2))^(1/2))*(132762+81606*29^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {2094, 12, 1687, 1183, 648, 632, 210, 642, 1121}

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx = -\frac{\arctan\left(\frac{3-\left(\frac{4}{x}+1\right)^2}{6\sqrt{7}}\right)}{12\sqrt{7}} - \frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\arctan\left(\frac{\frac{8}{x}-\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right) - \frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\arctan\left(\frac{\frac{8}{x}+\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right) - \frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2-\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right) + \frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2+\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)$$

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^(-1),x]
```

```
[Out] -1/12*ArcTan[(3 - (1 + 4/x)^2)/(6*Sqrt[7])]/Sqrt[7] - (Sqrt[(109 + 67*Sqrt[
29])/1218]*ArcTan[(2 - Sqrt[6*(1 + Sqrt[29])] + 8/x)/Sqrt[6*(-1 + Sqrt[29])
]])/12 - (Sqrt[(109 + 67*Sqrt[29])/1218]*ArcTan[(2 + Sqrt[6*(1 + Sqrt[29])]
 + 8/x)/Sqrt[6*(-1 + Sqrt[29])]])/12 - (Sqrt[(-109 + 67*Sqrt[29])/1218]*Log
[3*Sqrt[29] - Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x) + (1 + 4/x)^2])/24 + (Sqrt[(
-109 + 67*Sqrt[29])/1218]*Log[3*Sqrt[29] + Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x)
 + (1 + 4/x)^2])/24
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
```

```
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\left(1024\text{Subst}\left(\int \frac{(8-32x)^2}{8\,(1069056-393216x^2+1048576x^4)}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(128\text{Subst}\left(\int \frac{(8-32x)^2}{1069056-393216x^2+1048576x^4}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(128\text{Subst}\left(\int -\frac{512x}{1069056-393216x^2+1048576x^4}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right) - 128\text{Subst}\left(\int \frac{64+1024x^2}{1069056-393216x^2+1048576x^4}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)$$

$$= 65536\text{Subst}\left(\int \frac{x}{1069056-393216x^2+1048576x^4}\,dx, x, \frac{1}{4}+\frac{1}{x}\right) - \frac{\text{Subst}\left(\int \frac{16\sqrt{6\left(1+\sqrt{29}\right)}-\left(64-192\sqrt{29}\right)x}{\frac{3\sqrt{29}}{16}-\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}x+x^2}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)}{768\sqrt{174\left(1+\sqrt{29}\right)}} - \frac{\text{Subst}\left(\int \frac{16\sqrt{6\left(1+\sqrt{29}\right)}+\left(64-192\sqrt{29}\right)x}{\frac{3\sqrt{29}}{16}+\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}x+x^2}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)}{768\sqrt{174\left(1+\sqrt{29}\right)}}$$

$$= 32768\text{Subst}\left(\int \frac{1}{1069056-393216x+1048576x^2}\,dx, x, \left(\frac{1}{4}+\frac{1}{x}\right)^2\right) - \frac{\left(87+\sqrt{29}\right)\text{Subst}\left(\int \frac{1}{\frac{3\sqrt{29}}{16}-\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}x+x^2}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)}{2784} - \frac{\left(87+\sqrt{29}\right)\text{Subst}\left(\int \frac{1}{\frac{3\sqrt{29}}{16}+\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}x+x^2}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)}{2784} - \frac{1}{24}\sqrt{\frac{-109+67\sqrt{29}}{1218}}\text{Subst}\left(\int \frac{-\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}+2x}{\frac{3\sqrt{29}}{16}-\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}x+x^2}\,dx, x, \frac{1}{4}+\frac{1}{x}\right) + \frac{1}{24}\sqrt{\frac{-109+67\sqrt{29}}{1218}}\text{Subst}\left(\int \frac{\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}+2x}{\frac{3\sqrt{29}}{16}+\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}x+x^2}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)$$

$$= -\frac{1}{24}\sqrt{\frac{-109+67\sqrt{29}}{1218}}\log\left(3\sqrt{29}-\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right)$$
$$+\frac{1}{24}\sqrt{\frac{-109+67\sqrt{29}}{1218}}\log\left(3\sqrt{29}+\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right)$$
$$-65536\text{Subst}\left(\int\frac{1}{-4329327034368-x^2}\,dx,x,-393216+2097152\left(\frac{1}{4}+\frac{1}{x}\right)^2\right)$$
$$+\frac{\left(87+\sqrt{29}\right)\text{Subst}\left(\int\frac{1}{\frac{3}{8}\left(1-\sqrt{29}\right)-x^2}\,dx,x,-\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}+2\left(\frac{1}{4}+\frac{1}{x}\right)\right)}{1392}$$
$$+\frac{\left(87+\sqrt{29}\right)\text{Subst}\left(\int\frac{1}{\frac{3}{8}\left(1-\sqrt{29}\right)-x^2}\,dx,x,\frac{1}{4}\left(2+\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}\right)\right)}{1392}$$

$$= -\frac{\tan^{-1}\left(\frac{3-\left(1+\frac{4}{x}\right)^2}{6\sqrt{7}}\right)}{12\sqrt{7}}-\frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\tan^{-1}\left(\frac{2+\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right)$$
$$-\frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\tan^{-1}\left(\frac{8+\left(2-\sqrt{6\left(1+\sqrt{29}\right)}\right)x}{\sqrt{6\left(-1+\sqrt{29}\right)}x}\right)$$
$$-\frac{1}{24}\sqrt{\frac{-109+67\sqrt{29}}{1218}}\log\left(3\sqrt{29}-\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right)$$
$$+\frac{1}{24}\sqrt{\frac{-109+67\sqrt{29}}{1218}}\log\left(3\sqrt{29}+\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.17

$$\int\frac{1}{8+8x-x^3+8x^4}\,dx=\text{RootSum}\left[8+8\#1-\#1^3+8\#1^4\&,\frac{\log(x-\#1)}{8-3\#1^2+32\#1^3}\&\right]$$

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^(-1),x]
[Out] RootSum[8 + 8*#1 - #1^3 + 8*#1^4 & , Log[x - #1]/(8 - 3*#1^2 + 32*#1^3) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.15

method	result	size
default	$\sum_{\_R=\operatorname{RootOf}\left(8\_Z^4-\_Z^3+8\_Z+8\right)}\frac{\ln\left(x-\_R\right)}{32\_R^3-3\_R^2+8}$	41
risch	$\sum_{\_R=\operatorname{RootOf}\left(8\_Z^4-\_Z^3+8\_Z+8\right)}\frac{\ln\left(x-\_R\right)}{32\_R^3-3\_R^2+8}$	41

```
[In] int(1/(8*x^4-x^3+8*x+8),x,method=_RETURNVERBOSE)
[Out] sum(1/(32*_R^3-3*_R^2+8)*ln(x-_R),_R=RootOf(8*_Z^4-_Z^3+8*_Z+8))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 1015, normalized size of antiderivative = 3.79

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="fricas")
[Out] -1/168*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696))*log(287314195
392*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^3 - 120389
06880*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^2 + 1687
8104*x + 4897683*I*sqrt(7) - 411405372*sqrt(65/43848*I*sqrt(7) - 109/87696)
 + 6055613) - 1/168*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/87696))*
log(-35914274424*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696
))^3 + 16443*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I*sqrt(7) - 109/87696))
^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/43848*I*sqrt(7) - 109/87696) - 91520
) + 609*(351027*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696)
)^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/87696)) + 2109763
*x - 1911147/8*I*sqrt(7) + 40134087/2*sqrt(65/43848*I*sqrt(7) - 109/87696)
- 1461344) + 1/1044*(sqrt(174)*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*sqrt(65/43
848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848
*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7
) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696)) - 7)
 + 261*sqrt(65/43848*I*sqrt(7) - 109/87696) + 261*sqrt(-65/43848*I*sqrt(7)
- 109/87696))*log(-16443/2*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I*sqrt(7)
 - 109/87696))^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/43848*I*sqrt(7) - 109/
```

```
87696) - 91520) - 609/2*(351027*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt
(7) - 109/87696))^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/8
7696)) + 752431680*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/876
96))^2 + 1/32*(3*(13001*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7)
- 109/87696)) - 91520*sqrt(174))*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) -
 109/87696)) - 274560*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) -
109/87696)) + 1922368*sqrt(174))*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*sqrt(65/
43848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/438
48*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt
(7) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696)) -
7) + 2109763*x - 373317/2*I*sqrt(7) + 15679314*sqrt(65/43848*I*sqrt(7) - 10
9/87696) + 220336) - 1/1044*(sqrt(174)*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*sq
rt(65/43848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt(-
65/43848*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/43848*
I*sqrt(7) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/8769
6)) - 7) - 261*sqrt(65/43848*I*sqrt(7) - 109/87696) - 261*sqrt(-65/43848*I*
sqrt(7) - 109/87696))*log(-16443/2*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I
*sqrt(7) - 109/87696))^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/43848*I*sqrt(7
) - 109/87696) - 91520) - 609/2*(351027*(1/168*I*sqrt(7) - 1/2*sqrt(65/4384
8*I*sqrt(7) - 109/87696))^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7)
 - 109/87696)) + 752431680*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) -
 109/87696))^2 - 1/32*(3*(13001*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*
sqrt(7) - 109/87696)) - 91520*sqrt(174))*(I*sqrt(7) + 84*sqrt(-65/43848*I*s
qrt(7) - 109/87696)) - 274560*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*sq
rt(7) - 109/87696)) + 1922368*sqrt(174))*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*
sqrt(65/43848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt
(-65/43848*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/4384
8*I*sqrt(7) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87
696)) - 7) + 2109763*x - 373317/2*I*sqrt(7) + 15679314*sqrt(65/43848*I*sqrt
(7) - 109/87696) + 220336)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.15

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx$$
$$=\text{RootSum}\left(66298176t^4+74088t^2+4095t+64,\left(t\mapsto t\log\left(\frac{35914274424t^3}{2109763}-\frac{1504863360t^2}{2109763}+\frac{1028513}{210976}\right.\right.\right.$$

```
[In] integrate(1/(8*x**4-x**3+8*x+8),x)
```

```
[Out] RootSum(66298176*_t**4 + 74088*_t**2 + 4095*_t + 64, Lambda(_t, _t*log(3591
4274424*_t**3/2109763 - 1504863360*_t**2/2109763 + 102851343*_t/2109763 + x
 + 6055613/16878104)))
```

Maxima [F]

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx = \int \frac{1}{8\,x^4-x^3+8\,x+8}\,dx$$

[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="maxima")

[Out] integrate(1/(8*x^4 - x^3 + 8*x + 8), x)

Giac [F]

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx = \int \frac{1}{8\,x^4-x^3+8\,x+8}\,dx$$

[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="giac")

[Out] integrate(1/(8*x^4 - x^3 + 8*x + 8), x)

Mupad [B] (verification not implemented)

Time = 10.58 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.46

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx = \sum_{k=1}^{4} \ln\left(-\frac{\mathrm{root}\left(z^4+\frac{7\,z^2}{6264}+\frac{65\,z}{1052352}+\frac{1}{1035909},z,k\right)\,\left(8064\,\mathrm{root}\left(z^4+\frac{7\,z^2}{6264}+\frac{65\,z}{1052352}+\frac{1}{1035909},z,k\right)+256\,x-\right.}{} \cdots + \frac{7\,z^2}{6264}+\frac{65\,z}{1052352}+\frac{1}{1035909},z,k\right)$$

[In] int(1/(8*x - x^3 + 8*x^4 + 8),x)

[Out] symsum(log(-(root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)*(8064*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k) + 256*x + 12285*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)*x + 148176*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)^2*x + 198072*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)^2 - 8))/4096)*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k), k, 1, 4)

3.50 $\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx$

Optimal result 481
Rubi [A] (verified) 482
Mathematica [C] (verified) 487
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Mupad [B] (verification not implemented) 496

Optimal result

Integrand size = 17, antiderivative size = 357

$$\begin{aligned}
&\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx \\
&= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}+\frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} \\
&-\frac{17\arctan\left(\frac{3-\left(1+\frac{4}{x}\right)^2}{6\sqrt{7}}\right)}{1008\sqrt{7}}-\frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\arctan\left(\frac{2-\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right)}{87696} \\
&-\frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\arctan\left(\frac{2+\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right)}{87696} \\
&-\frac{\sqrt{\frac{-180983329+45923327\sqrt{29}}{1218}}\log\left(3\sqrt{29}-\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right)}{175392} \\
&+\frac{\sqrt{\frac{-180983329+45923327\sqrt{29}}{1218}}\log\left(3\sqrt{29}+\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right)}{175392}
\end{aligned}$$

```
[Out] 1/336*(-207-29*(1+4/x)^2)/(261-6*(1+4/x)^2+(1+4/x)^4)+5/87696*(5157+199*(1+
4/x)^2)*(1+4/x)/(261-6*(1+4/x)^2+(1+4/x)^4)-17/7056*arctan(1/42*(3-(1+4/x)^
2)*7^(1/2))*7^(1/2)-1/213627456*ln((1+4/x)^2+3*29^(1/2)-(1+4/x)*(6+6*29^(1/
2))^(1/2))*(-220437694722+55934612286*29^(1/2))^(1/2)+1/213627456*ln((1+4/x
)^2+3*29^(1/2)+(1+4/x)*(6+6*29^(1/2))^(1/2))*(-220437694722+55934612286*29^
(1/2))^(1/2)-1/106813728*arctan((2+8/x-(6+6*29^(1/2))^(1/2))/(-6+6*29^(1/2)
```

```
)^(1/2))*(220437694722+55934612286*29^(1/2))^(1/2)-1/106813728*arctan((2+8/
x+(6+6*29^(1/2))^(1/2))/(-6+6*29^(1/2))^(1/2))*(220437694722+55934612286*29
^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {2094, 12, 1687, 1692, 1183, 648, 632, 210, 642, 1677, 1674}

$$\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx$$

$$= -\frac{17\arctan\left(\frac{3-\left(\frac{4}{x}+1\right)^2}{6\sqrt{7}}\right)}{1008\sqrt{7}} - \frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\arctan\left(\frac{\frac{8}{x}-\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right)}{87696}$$
$$-\frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\arctan\left(\frac{\frac{8}{x}+\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right)}{87696}$$
$$-\frac{29\left(\frac{4}{x}+1\right)^2+207}{336\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)} + \frac{5\left(199\left(\frac{4}{x}+1\right)^2+5157\right)\left(\frac{4}{x}+1\right)}{87696\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}$$
$$-\frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2-\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)}{175392}$$
$$+\frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2+\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)}{175392}$$

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^(-2),x]
```

```
[Out] -1/336*(207 + 29*(1 + 4/x)^2)/(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4) + (5*(515
7 + 199*(1 + 4/x)^2)*(1 + 4/x))/(87696*(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4))
 - (17*ArcTan[(3 - (1 + 4/x)^2)/(6*Sqrt[7])])/(1008*Sqrt[7]) - (Sqrt[(18098
3329 + 45923327*Sqrt[29])/1218]*ArcTan[(2 - Sqrt[6*(1 + Sqrt[29])] + 8/x)/S
qrt[6*(-1 + Sqrt[29])]])/87696 - (Sqrt[(180983329 + 45923327*Sqrt[29])/1218
]*ArcTan[(2 + Sqrt[6*(1 + Sqrt[29])] + 8/x)/Sqrt[6*(-1 + Sqrt[29])]])/87696
 - (Sqrt[(-180983329 + 45923327*Sqrt[29])/1218]*Log[3*Sqrt[29] - Sqrt[6*(1
+ Sqrt[29])]*(1 + 4/x) + (1 + 4/x)^2])/175392 + (Sqrt[(-180983329 + 4592332
7*Sqrt[29])/1218]*Log[3*Sqrt[29] + Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x) + (1 +
4/x)^2])/175392
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\left(1024\text{Subst}\left(\int \frac{(8-32x)^6}{64\,(1069056-393216x^2+1048576x^4)^2}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(16\text{Subst}\left(\int \frac{(8-32x)^6}{(1069056-393216x^2+1048576x^4)^2}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(16\text{Subst}\left(\int \frac{x(-6291456-335544320x^2-1610612736x^4)}{(1069056-393216x^2+1048576x^4)^2}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$
$$-16\text{Subst}\left(\int \frac{262144+62914560x^2+1006632960x^4+1073741824x^6}{(1069056-393216x^2+1048576x^4)^2}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)$$

$$= \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}$$
$$-\frac{\text{Subst}\left(\int \frac{278927740761415247462 4+77580088044994733015 04x^2}{1069056-393216x^2+1048576x^4}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)}{578536630256664576}$$
$$-8\text{Subst}\left(\int \frac{-6291456-335544320x-1610612736x^2}{(1069056-393216x+1048576x^2)^2}\,dx, x, \left(\frac{1}{4}+\frac{1}{x}\right)^2\right)$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}+\frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}$$
$$-\frac{\text{Subst}\left(\int -\frac{3588805953060864}{1069056-393216x+1048576x^2}\,dx, x, \left(\frac{1}{4}+\frac{1}{x}\right)^2\right)}{541165879296}$$
$$-\frac{\text{Subst}\left(\int \frac{697319351903538118656\sqrt{6\left(1+\sqrt{29}\right)}-\left(2789277407614152474624-1454626650843651244032\sqrt{29}\right)x}{\frac{3\sqrt{29}}{16}-\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}x+x^2}\,dx, x, \frac{1}{4}+\right.}{568724649007511544791 04\sqrt{174\left(1+\sqrt{29}\right)}}$$
$$-\frac{\text{Subst}\left(\int \frac{697319351903538118656\sqrt{6\left(1+\sqrt{29}\right)}+\left(2789277407614152474624-1454626650843651244032\sqrt{29}\right)x}{\frac{3\sqrt{29}}{16}+\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}x+x^2}\,dx, x, \frac{1}{4}+\right.}{568724649007511544791 04\sqrt{174\left(1+\sqrt{29}\right)}}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}+\frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}$$

$$+\frac{139264}{21}\text{Subst}\left(\int\frac{1}{1069056-393216x+1048576x^2}\,dx,x,\left(\frac{1}{4}+\frac{1}{x}\right)^2\right)$$

$$-\frac{\sqrt{\frac{1}{58}\left(82199511+9647143\sqrt{29}\right)}\text{Subst}\left(\int\frac{1}{\frac{3\sqrt{29}}{16}-\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}x+x^2}\,dx,x,\frac{1}{4}+\frac{1}{x}\right)}{350784}$$

$$-\frac{\sqrt{\frac{1}{58}\left(82199511+9647143\sqrt{29}\right)}\text{Subst}\left(\int\frac{1}{\frac{3\sqrt{29}}{16}+\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}x+x^2}\,dx,x,\frac{1}{4}+\frac{1}{x}\right)}{350784}$$

$$-\frac{\sqrt{\frac{-180983329+45923327\sqrt{29}}{1218}}\text{Subst}\left(\int\frac{-\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}+2x}{\frac{3\sqrt{29}}{16}-\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}x+x^2}\,dx,x,\frac{1}{4}+\frac{1}{x}\right)}{175392}$$

$$+\frac{\sqrt{\frac{-180983329+45923327\sqrt{29}}{1218}}\text{Subst}\left(\int\frac{\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}+2x}{\frac{3\sqrt{29}}{16}+\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}x+x^2}\,dx,x,\frac{1}{4}+\frac{1}{x}\right)}{175392}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}+\frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}$$

$$-\frac{\sqrt{\frac{-180983329+45923327\sqrt{29}}{1218}}\log\left(3\sqrt{29}-\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right)}{175392}$$

$$+\frac{\sqrt{\frac{-180983329+45923327\sqrt{29}}{1218}}\log\left(3\sqrt{29}+\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right)}{175392}$$

$$-\frac{278528}{21}\text{Subst}\left(\int\frac{1}{-4329327034368-x^2}\,dx,x,-393216+2097152\left(\frac{1}{4}+\frac{1}{x}\right)^2\right)$$

$$+\frac{\sqrt{\frac{1}{58}\left(82199511+9647143\sqrt{29}\right)}\text{Subst}\left(\int\frac{1}{\frac{3}{8}\left(1-\sqrt{29}\right)-x^2}\,dx,x,-\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}+2\left(\frac{1}{4}+\frac{1}{x}\right)\right)}{175392}$$

$$+\frac{\sqrt{\frac{1}{58}\left(82199511+9647143\sqrt{29}\right)}\text{Subst}\left(\int\frac{1}{\frac{3}{8}\left(1-\sqrt{29}\right)-x^2}\,dx,x,\frac{1}{4}\left(2+\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}\right)\right)}{175392}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}$$

$$- \frac{17\tan^{-1}\left(\frac{3-\left(1+\frac{4}{x}\right)^2}{6\sqrt{7}}\right)}{1008\sqrt{7}} - \frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\tan^{-1}\left(\frac{2+\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right)}{87696}$$

$$- \frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\tan^{-1}\left(\frac{8+\left(2-\sqrt{6\left(1+\sqrt{29}\right)}\right)x}{\sqrt{6\left(-1+\sqrt{29}\right)}x}\right)}{87696}$$

$$- \frac{\sqrt{\frac{-180983329+45923327\sqrt{29}}{1218}}\log\left(3\sqrt{29}-\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right)}{175392}$$

$$+ \frac{\sqrt{\frac{-180983329+45923327\sqrt{29}}{1218}}\log\left(3\sqrt{29}+\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right)}{175392}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.32

$$\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx = \frac{544+1539x-1146x^2+784x^3}{43848\left(8+8x-x^3+8x^4\right)} + \frac{\text{RootSum}\left[8+8\#1-\#1^3+8\#1^4\&, \frac{2243\log(x-\#1)-1097\log(x-\#1)\#1+392\log(x-\#1)\#1^2}{8-3\#1^2+32\#1^3}\&\right]}{21924}$$

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^(-2),x]
```

```
[Out] (544 + 1539*x - 1146*x^2 + 784*x^3)/(43848*(8 + 8*x - x^3 + 8*x^4)) + RootS
um[8 + 8*#1 - #1^3 + 8*#1^4 & , (2243*Log[x - #1] - 1097*Log[x - #1]*#1 + 3
92*Log[x - #1]*#1^2)/(8 - 3*#1^2 + 32*#1^3) & ]/21924
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.23

method	result	size
default	$\frac{\frac{7}{3132}x^3-\frac{191}{58464}x^2+\frac{57}{12992}x+\frac{17}{10962}}{x^4-\frac{1}{8}x^3+x+1}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(8\_Z^4-\_Z^3+8\_Z+8\right)}\frac{\left(392\_R^2-1097\_R+2243\right)\ln\left(x-\_R\right)}{32\_R^3-3\_R^2+8}\right)}{21924}$	83
risch	$\frac{\frac{7}{3132}x^3-\frac{191}{58464}x^2+\frac{57}{12992}x+\frac{17}{10962}}{x^4-\frac{1}{8}x^3+x+1}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(8\_Z^4-\_Z^3+8\_Z+8\right)}\frac{\left(392\_R^2-1097\_R+2243\right)\ln\left(x-\_R\right)}{32\_R^3-3\_R^2+8}\right)}{21924}$	83

```
[In] int(1/(8*x^4-x^3+8*x+8)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (7/3132*x^3-191/58464*x^2+57/12992*x+17/10962)/(x^4-1/8*x^3+x+1)+1/21924*su
m((392*_R^2-1097*_R+2243)/(32*_R^3-3*_R^2+8)*ln(x-_R),_R=RootOf(8*_Z^4-_Z^3
+8*_Z+8))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 1201, normalized size of antiderivative = 3.36

$$\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="fricas")
```

```
[Out] 1/213627456*(3819648*x^3 - 15138*(8*x^4 - x^3 + 8*x + 8)*(-17*I*sqrt(7) + 7
056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))*log(621
7850567873065654359973859328*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/3345405
96096*I*sqrt(7) - 180983329/4683568345344))^3 - 10028767243179717478632775 6
80*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 18098332
9/4683568345344))^2 + 67481665655469287031416*x + 320944207138750561964778*
I*sqrt(7) - 133210725033589645013145504*sqrt(4550065/334540596096*I*sqrt(7)
 - 180983329/4683568345344) + 333979081113202533090737) - 15138*(8*x^4 - x^
3 + 8*x + 8)*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 18
0983329/4683568345344))*log(-777231320984133206794996732416*(17/14112*I*sqr
t(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^
3 + 878169064752*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596096*I*sq
rt(7) - 180983329/4683568345344))^2*(-1066184864424603*I*sqrt(7) + 44252943
5492941104*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344) -
```

1427510892508480) + 7569*(7276511507810430573072*(17/14112*I*sqrt(7) - 1/2
*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 233594
23554371543)*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 18
0983329/4683568345344)) + 8435208206933660878927*x - 148449195141328682772633/4*I*sqrt(7) + 154037870723119880241720036*sqrt(4550065/334540596096*I*sqr
t(7) - 180983329/4683568345344) - 4739360660696595067616) - 5583312*x^2 +
(56*sqrt(87)*(8*x^4 - x^3 + 8*x + 8)*sqrt(-125452723536*(17/14112*I*sqrt(7)
 - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 -
125452723536*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596096*I*sqrt(7
) - 180983329/4683568345344))^2 - 658503/1568*(17*I*sqrt(7) + 7056*sqrt(-45
50065/334540596096*I*sqrt(7) - 180983329/4683568345344))*(-17*I*sqrt(7) + 7
056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 66301
91) + 7569*(8*x^4 - x^3 + 8*x + 8)*(17*I*sqrt(7) + 7056*sqrt(-4550065/33454
0596096*I*sqrt(7) - 180983329/4683568345344)) + 7569*(8*x^4 - x^3 + 8*x + 8
)*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/468
3568345344)))*log(-439084532376*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/33
4540596096*I*sqrt(7) - 180983329/4683568345344))^2*(-1066184864424603*I*sqr
t(7) + 44252943549294110 4*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4
683568345344) - 1427510892508480) - 7569/2*(7276511507810430573072*(17/1411
2*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/468356834
5344))^2 - 23359423554371543)*(17*I*sqrt(7) + 7056*sqrt(-4550065/3345405960
96*I*sqrt(7) - 180983329/4683568345344)) + 626797952698732342414548480*(17/
14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/46835
68345344))^2 + 1/16*(261*(62716756730859*sqrt(87)*(-17*I*sqrt(7) + 7056*sqr
t(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 142751089250
8480*sqrt(87))*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) -
180983329/4683568345344)) - 372580342944713280*sqrt(87)*(-17*I*sqrt(7) + 70
56*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) + 104650
21752358451264*sqrt(87))*sqrt(-125452723536*(17/14112*I*sqrt(7) - 1/2*sqrt(
4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 125452723536
*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596096*I*sqrt(7) - 18098332
9/4683568345344))^2 - 658503/1568*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540
596096*I*sqrt(7) - 180983329/4683568345344))*(-17*I*sqrt(7) + 7056*sqrt(455
0065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 6630191) + 843520
8206933660878927*x - 300572710701164955 2439/2*I*sqrt(7) + 62377677844335880
1235576*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344) + 22
959102208397854107 04) - (56*sqrt(87)*(8*x^4 - x^3 + 8*x + 8)*sqrt(-12545272
3536*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983
329/4683568345344))^2 - 125452723536*(-17/14112*I*sqrt(7) - 1/2*sqrt(-45500
65/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 658503/1568*(17*I
*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345
344))*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329
/4683568345344)) - 6630191) - 7569*(8*x^4 - x^3 + 8*x + 8)*(17*I*sqrt(7) +
7056*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 756
9*(8*x^4 - x^3 + 8*x + 8)*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I

```
*sqrt(7) - 180983329/4683568345344)))*log(-439084532376*(-17/14112*I*sqrt(7
) - 1/2*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2*
(-1066184864424603*I*sqrt(7) + 442529435492941104*sqrt(4550065/334540596096
*I*sqrt(7) - 180983329/4683568345344) - 1427510892508480) - 7569/2*(7276511
507810430573072*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(
7) - 180983329/4683568345344))^2 - 23359423554371543)*(17*I*sqrt(7) + 7056*
sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) + 62679795
2698732342414548480*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*s
qrt(7) - 180983329/4683568345344))^2 - 1/16*(261*(62716756730859*sqrt(87)*(
-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/468356
8345344)) - 1427510892508480*sqrt(87))*(17*I*sqrt(7) + 7056*sqrt(-4550065/3
34540596096*I*sqrt(7) - 180983329/4683568345344)) - 372580342944713280*sqrt
(87)*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/
4683568345344)) + 10465021752358451264*sqrt(87))*sqrt(-125452723536*(17/141
12*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/46835683
45344))^2 - 125452723536*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596
096*I*sqrt(7) - 180983329/4683568345344))^2 - 658503/1568*(17*I*sqrt(7) + 7
056*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))*(-17*I
*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/46835683453
44)) - 6630191) + 8435208206933660878927*x - 3005727107011649552439/2*I*sqr
t(7) + 623776778443358801235576*sqrt(4550065/334540596096*I*sqrt(7) - 18098
3329/4683568345344) + 2295910220839785410704) + 7498008*x + 2650368)/(8*x^4
 - x^3 + 8*x + 8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3834 vs. $2(274) = 548$.

Time = 1.78 (sec) , antiderivative size = 3834, normalized size of antiderivative = 10.74

$$\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(8*x**4-x**3+8*x+8)**2,x)
```

```
[Out] (784*x**3 - 1146*x**2 + 1539*x + 544)/(350784*x**4 - 43848*x**3 + 350784*x
+ 350784) - sqrt(-180983329/37468546762752 + 1583563*sqrt(29)/1292018853888
)*log(x**2 + x*(-62716756730859*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(
29))*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329
 + 45923327*sqrt(29)) + 40699873480352667)/227008323264998681573683424 - 26
7658292345340*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-
180983329 + 45923327*sqrt(29)) + 40699873480352667)/8435208206933660878927
- 2157374520970352866823*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/11
3504161632499340786841712 + 3881045239007430*sqrt(29)/5326727264361229 + 43
5853770857118353330297/33740832827734643515708 + 20905585576953*sqrt(42)*sq
```

rt(-180983329 + 45923327*sqrt(29))/85227636229779664) - 2942814074101429415
0840305101822042500675556953*sqrt(214095423017213*sqrt(29) + 47106822945*sqr
t(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/888496186
7514852012539664011390752874524165340066272 - 142576256328563148358311429727
65102609010539559351093/27765505835983912539186450035596102732888016687696
- 75184631502818837388875900060881355871*sqrt(1218)*sqrt(-180983329 + 45923
327*sqrt(29))*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-
180983329 + 45923327*sqrt(29)) + 40699873480352667)/30637799543154662112205
737970312940946635052896768 - 963314181796141259748858766106570487809406229
9*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/3063779954315466211220573
7970312940946635052896768 - 1398888334001652366855237255*sqrt(42)*sqrt(-180
983329 + 45923327*sqrt(29))*sqrt(214095423017213*sqrt(29) + 47106822945*sqr
t(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/359456428
291497016547944746810895370264 + 91245981690030498967778233214015591679*sqr
t(42)*sqrt(-180983329 + 45923327*sqrt(29))/23005211410655809059068463795897
303696896 + 10304175351841941260676745569701505519*sqrt(29)*sqrt(2140954230
17213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)
) + 40699873480352667)/19347382796361535418676578052349632409089536 + 63911
1088489748962499984017403917984374085485*sqrt(29)/4836845699090383854669144
513087408102272384) + sqrt(-180983329/37468546762752 + 1583563*sqrt(29)/129
2018853888)*log(x**2 + x*(-62716756730859*sqrt(1218)*sqrt(-180983329 + 4592
3327*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt
(29)) + 214095423017213*sqrt(29) + 40699873480352667)/227008323264998681573
683424 - 20905585576953*sqrt(42)*sqrt(-180983329 + 45923327*sqrt(29))/85227
636229779664 + 3881045239007430*sqrt(29)/5326727264361229 + 267658292345340
*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095
423017213*sqrt(29) + 40699873480352667)/8435208206933660878927 + 2157374520
970352866823*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/11350416163249
9340786841712 + 435853770857118353330297/33740832827734643515708) - 1425762
5632856314835831142972765102609010539559351093/2776550583598391253918645003
5596102732888016687696 - 10304175351841941260676745569701505519*sqrt(29)*sq
rt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423
017213*sqrt(29) + 40699873480352667)/19347382796361535418676578052349632409
089536 - 91245981690030498967778233214015591679*sqrt(42)*sqrt(-180983329 +
45923327*sqrt(29))/23005211410655809059068463795897303696896 - 751846315028
18837388875900060881355871*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))*
sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 2140954
23017213*sqrt(29) + 40699873480352667)/306377995431546621122057379703129409
46635052896768 - 1398888334001652366855237255*sqrt(42)*sqrt(-180983329 + 45
923327*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sq
rt(29)) + 214095423017213*sqrt(29) + 40699873480352667)/3594564282914970165
47944746810895370264 + 9633141817961412597488587661065704878094062299*sqrt(
1218)*sqrt(-180983329 + 45923327*sqrt(29))/30637799543154662112205737970312
940946635052896768 + 2942814074101429415084030510182204250067556953*sqrt(-4
7106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 21409542301721

3*sqrt(29) + 40699873480352667)/888496186751485201253966401139075287452416534006272 + 639111088489748962499984017403917984374085485*sqrt(29)/483684569909038385466914451308740810227 2384) - 2*sqrt(199631405/37468546762752 + sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)/9367136690688 + 1583563*sqrt(29)/430672951296)*atan(454016646529997363147366848*x/(-450967351627246742986 0*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29)) + 3601609981798895040*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)) + 29324241703266922812062382 16/(-4509673516272467429860*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29)) + 3601609981798895040*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)) + 4314749041940705733646*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/(-4509673516272467429860*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29)) + 3601609981798895040*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)) + 7203219963597790080*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)/(-4509673516272467429860*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29)) + 3601609981798895040*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29))*sqrt(-47106822945*sq

rt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) +
40699873480352667)) + 165397912920614705160598080*sqrt(29)/(-45096735162724
67429860*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-1
80983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 4069987348035266
7) + 137769981*sqrt(29)) + 3601609981798895040*sqrt(-180983329 + 45923327*s
qrt(29))*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 +
45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769
981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822
945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(
29) + 40699873480352667) + 137769981*sqrt(29))*sqrt(-47106822945*sqrt(1218)
*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873
480352667)) - 55683134469459984392598*sqrt(42)*sqrt(-180983329 + 45923327*s
qrt(29))/(-4509673516272467429860*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106
822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sq
rt(29) + 40699873480352667) + 137769981*sqrt(29)) + 3601609981798895040*sqr
t(-180983329 + 45923327*sqrt(29))*sqrt(199631405 + 4*sqrt(-47106822945*sqrt
(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40
699873480352667) + 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(199
631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)
) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29))*sqr
t(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 2140954230
17213*sqrt(29) + 40699873480352667)) - 62716756730859*sqrt(1218)*sqrt(-1809
83329 + 45923327*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 4
5923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)/(-4509673
516272467429860*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*
sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 406998734
80352667) + 137769981*sqrt(29)) + 3601609981798895040*sqrt(-180983329 + 459
23327*sqrt(29))*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-18098
3329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) +
 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(199631405 + 4*sqrt(-4
7106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 21409542301721
3*sqrt(29) + 40699873480352667) + 137769981*sqrt(29))*sqrt(-47106822945*sqr
t(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 4
0699873480352667))) + 2*sqrt(-sqrt(214095423017213*sqrt(29) + 47106822945*s
qrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/9367136
690688 + 199631405/37468546762752 + 1583563*sqrt(29)/430672951296)*atan(454
016646529997363147366848*x/(3601609981798895040*sqrt(-180983329 + 45923327*
sqrt(29))*sqrt(-4*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sq
rt(-180983329 + 45923327*sqrt(29)) + 40699873480352667) + 199631405 + 13776
9981*sqrt(29)) + 4509673516272467429860*sqrt(1218)*sqrt(-4*sqrt(21409542301
7213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))
 + 40699873480352667) + 199631405 + 137769981*sqrt(29)) + 20905585576953*sq
rt(1218)*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-18098
3329 + 45923327*sqrt(29)) + 40699873480352667)*sqrt(-4*sqrt(214095423017213
*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 4

0699873480352667) + 199631405 + 137769981*sqrt(29))) - 62716756730859*sqrt(
1218)*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(214095423017213*sqrt(29) +
47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 4069987348035
2667)/(3601609981798895040*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(-4*sqr
t(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923
327*sqrt(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29)) + 4509
673516272467429860*sqrt(1218)*sqrt(-4*sqrt(214095423017213*sqrt(29) + 47106
822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)
 + 199631405 + 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(2140954
23017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(
29)) + 40699873480352667)*sqrt(-4*sqrt(214095423017213*sqrt(29) + 471068229
45*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667) + 1
99631405 + 137769981*sqrt(29))) - 7203219963597790080*sqrt(214095423017213*
sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40
699873480352667)/(3601609981798895040*sqrt(-180983329 + 45923327*sqrt(29))*
sqrt(-4*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983
329 + 45923327*sqrt(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(
29)) + 4509673516272467429860*sqrt(1218)*sqrt(-4*sqrt(214095423017213*sqrt(
29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 4069987
3480352667) + 199631405 + 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*s
qrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 459
23327*sqrt(29)) + 40699873480352667)*sqrt(-4*sqrt(214095423017213*sqrt(29)
+ 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480
352667) + 199631405 + 137769981*sqrt(29))) - 4314749041940705733646*sqrt(12
18)*sqrt(-180983329 + 45923327*sqrt(29))/(3601609981798895040*sqrt(-1809833
29 + 45923327*sqrt(29))*sqrt(-4*sqrt(214095423017213*sqrt(29) + 47106822945
*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667) + 199
631405 + 137769981*sqrt(29)) + 4509673516272467429860*sqrt(1218)*sqrt(-4*sq
rt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 4592
3327*sqrt(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29)) + 209
05585576953*sqrt(1218)*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(121
8)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)*sqrt(-4*sqrt(2
14095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327
*sqrt(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29))) + 165397
912920614705160598080*sqrt(29)/(3601609981798895040*sqrt(-180983329 + 45923
327*sqrt(29))*sqrt(-4*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218
)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667) + 199631405 + 1
37769981*sqrt(29)) + 4509673516272467429860*sqrt(1218)*sqrt(-4*sqrt(2140954
23017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(
29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29)) + 2090558557695
3*sqrt(1218)*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-1
80983329 + 45923327*sqrt(29)) + 40699873480352667)*sqrt(-4*sqrt(21409542301
7213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))
 + 40699873480352667) + 199631405 + 137769981*sqrt(29))) + 2932424170326692
281206238216/(3601609981798895040*sqrt(-180983329 + 45923327*sqrt(29))*sqrt

```
(-4*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329
+ 45923327*sqrt(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29))
 + 4509673516272467429860*sqrt(1218)*sqrt(-4*sqrt(214095423017213*sqrt(29)
+ 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480
352667) + 199631405 + 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(
214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 4592332
7*sqrt(29)) + 40699873480352667)*sqrt(-4*sqrt(214095423017213*sqrt(29) + 47
106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 406998734803526
67) + 199631405 + 137769981*sqrt(29))) + 5568313446945998439259 8*sqrt(42)*s
qrt(-180983329 + 45923327*sqrt(29))/(3601609981798895040*sqrt(-180983329 +
45923327*sqrt(29))*sqrt(-4*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt
(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667) + 19963140
5 + 137769981*sqrt(29)) + 4509673516272467429860*sqrt(1218)*sqrt(-4*sqrt(21
4095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*
sqrt(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29)) + 20905585
576953*sqrt(1218)*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sq
rt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)*sqrt(-4*sqrt(214095
423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt
(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29))))
```

Maxima [F]

$$\int \frac{1}{\left(8+8x-x^3+8x^4\right)^2}\,dx=\int \frac{1}{\left(8\,x^4-x^3+8\,x+8\right)^2}\,dx$$

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="maxima")
```

```
[Out] 1/43848*(784*x^3 - 1146*x^2 + 1539*x + 544)/(8*x^4 - x^3 + 8*x + 8) + 1/219
24*integrate((392*x^2 - 1097*x + 2243)/(8*x^4 - x^3 + 8*x + 8), x)
```

Giac [F]

$$\int \frac{1}{\left(8+8x-x^3+8x^4\right)^2}\,dx=\int \frac{1}{\left(8\,x^4-x^3+8\,x+8\right)^2}\,dx$$

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="giac")
```

```
[Out] integrate((8*x^4 - x^3 + 8*x + 8)^(-2), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.49

$$\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx = \left(\sum_{k=1}^{4}\ln\left(\frac{2615257\,\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)}{72918171648}+\frac{4225\,x}{40375589184}-\frac{\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)\,x\,34885379}{72918171648}-\frac{\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^2\,x\,191555}{475136}-\frac{\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^3\,x\,9261}{256}-\frac{11205\,\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^2}{59392}-\frac{24759\,\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^3}{512}+\frac{10901}{107668237824}\right)\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)\right)+\frac{\frac{7\,x^3}{3132}-\frac{191\,x^2}{58464}+\frac{57\,x}{12992}+\frac{17}{10962}}{x^4-\frac{x^3}{8}+x+1}$$

```
[In] int(1/(8*x - x^3 + 8*x^4 + 8)^2,x)
```

```
[Out] symsum(log((2615257*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/67
4433841729536 + 1114096/13723971258377709, z, k))/72918171648 + (4225*x)/40
375589184 - (34885379*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/
674433841729536 + 1114096/13723971258377709, z, k)*x)/72918171648 - (191555
*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 111
4096/13723971258377709, z, k)^2*x)/475136 - (9261*root(z^4 + (6630191*z^2)/
167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709, z,
 k)^3*x)/256 - (11205*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/
674433841729536 + 1114096/13723971258377709, z, k)^2)/59392 - (24759*root(z
^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13
```

```
723971258377709, z, k)^3)/512 + 10901/107668237824)*root(z^4 + (6630191*z^2
)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709,
z, k), k, 1, 4) + ((57*x)/12992 - (191*x^2)/58464 + (7*x^3)/3132 + 17/10962
)/(x - x^3/8 + x^4 + 1)
```

3.51 $\int (1+4x+4x^2+4x^4)^4\,dx$

Optimal result 498
Rubi [A] (verified) 498
Mathematica [A] (verified) 499
Maple [A] (verified) 499
Fricas [A] (verification not implemented) 500
Sympy [A] (verification not implemented) 500
Maxima [A] (verification not implemented) 501
Giac [A] (verification not implemented) 501
Mupad [B] (verification not implemented) 501

Optimal result

Integrand size = 17, antiderivative size = 97

$$\int (1+4x+4x^2+4x^4)^4\,dx = x+8x^2+\frac{112x^3}{3}+112x^4+\frac{1136x^5}{5}+\frac{992x^6}{3}+\frac{2752x^7}{7}+448x^8+\frac{4192x^9}{9}+384x^{10}+\frac{3328x^{11}}{11}+256x^{12}+\frac{1792x^{13}}{13}+\frac{512x^{14}}{7}+\frac{1024x^{15}}{15}+\frac{256x^{17}}{17}$$

```
[Out] x+8*x^2+112/3*x^3+112*x^4+1136/5*x^5+992/3*x^6+2752/7*x^7+448*x^8+4192/9*x^
9+384*x^10+3328/11*x^11+256*x^12+1792/13*x^13+512/7*x^14+1024/15*x^15+256/1
7*x^17
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2086}

$$\int (1+4x+4x^2+4x^4)^4\,dx = \frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10}+\frac{4192x^9}{9}+448x^8+\frac{2752x^7}{7}+\frac{992x^6}{3}+\frac{1136x^5}{5}+112x^4+\frac{112x^3}{3}+8x^2+x$$

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^4,x]
```

```
[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)
/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (1792*
x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(1+16x+112x^2+448x^3+1136x^4+1984x^5+2752x^6+3584x^7+4192x^8 \right. \\ &\quad \left. +3840x^9+3328x^{10}+3072x^{11}+1792x^{12}+1024x^{13}+1024x^{14}+256x^{16}\right) dx \\ &= x+8x^2+\frac{112x^3}{3}+112x^4+\frac{1136x^5}{5}+\frac{992x^6}{3}+\frac{2752x^7}{7}+448x^8+\frac{4192x^9}{9} \\ &\quad +384x^{10}+\frac{3328x^{11}}{11}+256x^{12}+\frac{1792x^{13}}{13}+\frac{512x^{14}}{7}+\frac{1024x^{15}}{15}+\frac{256x^{17}}{17}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \left(1+4x+4x^2+4x^4\right)^4 dx &= x+8x^2+\frac{112x^3}{3}+112x^4+\frac{1136x^5}{5}+\frac{992x^6}{3} \\ &\quad +\frac{2752x^7}{7}+448x^8+\frac{4192x^9}{9}+384x^{10}+\frac{3328x^{11}}{11} \\ &\quad +256x^{12}+\frac{1792x^{13}}{13}+\frac{512x^{14}}{7}+\frac{1024x^{15}}{15}+\frac{256x^{17}}{17}\end{aligned}$$

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^4,x]
```

```
[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)
/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (1792*
x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

method	result
gosper	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}+$
default	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}+$
norman	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}+$
risch	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}+$
parallelrisch	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}+$

```
[In] int((4*x^4+4*x^2+4*x+1)^4,x,method=_RETURNVERBOSE)
```

```
[Out] x+8*x^2+112/3*x^3+112*x^4+1136/5*x^5+992/3*x^6+2752/7*x^7+448*x^8+4192/9*x^
9+384*x^10+3328/11*x^11+256*x^12+1792/13*x^13+512/7*x^14+1024/15*x^15+256/1
7*x^17
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int \left(1+4x+4x^2+4x^4\right)^4\,dx=\frac{256}{17}\,x^{17}+\frac{1024}{15}\,x^{15}+\frac{512}{7}\,x^{14}+\frac{1792}{13}\,x^{13}+256\,x^{12}+\frac{3328}{11}\,x^{11}+384\,x^{10}+\frac{4192}{9}\,x^{9}+448\,x^{8}+\frac{2752}{7}\,x^{7}+\frac{992}{3}\,x^{6}+\frac{1136}{5}\,x^{5}+112\,x^{4}+\frac{112}{3}\,x^{3}+8\,x^{2}+x$$

```
[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="fricas")
```

```
[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/11
*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x
^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int \left(1+4x+4x^2+4x^4\right)^4\,dx=\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10}+\frac{4192x^{9}}{9}+448x^{8}+\frac{2752x^{7}}{7}+\frac{992x^{6}}{3}+\frac{1136x^{5}}{5}+112x^{4}+\frac{112x^{3}}{3}+8x^{2}+x$$

```
[In] integrate((4*x**4+4*x**2+4*x+1)**4,x)
```

```
[Out] 256*x**17/17 + 1024*x**15/15 + 512*x**14/7 + 1792*x**13/13 + 256*x**12 + 33
28*x**11/11 + 384*x**10 + 4192*x**9/9 + 448*x**8 + 2752*x**7/7 + 992*x**6/3
 + 1136*x**5/5 + 112*x**4 + 112*x**3/3 + 8*x**2 + x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int (1+4x+4x^2+4x^4)^4\,dx = \frac{256}{17}x^{17}+\frac{1024}{15}x^{15}+\frac{512}{7}x^{14}+\frac{1792}{13}x^{13}+256\,x^{12} + \frac{3328}{11}x^{11}+384\,x^{10}+\frac{4192}{9}x^{9}+448\,x^{8}+\frac{2752}{7}x^{7} + \frac{992}{3}x^{6}+\frac{1136}{5}x^{5}+112\,x^{4}+\frac{112}{3}x^{3}+8\,x^{2}+x$$

```
[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="maxima")
[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/11
*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x
^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int (1+4x+4x^2+4x^4)^4\,dx = \frac{256}{17}x^{17}+\frac{1024}{15}x^{15}+\frac{512}{7}x^{14}+\frac{1792}{13}x^{13}+256\,x^{12} + \frac{3328}{11}x^{11}+384\,x^{10}+\frac{4192}{9}x^{9}+448\,x^{8}+\frac{2752}{7}x^{7} + \frac{992}{3}x^{6}+\frac{1136}{5}x^{5}+112\,x^{4}+\frac{112}{3}x^{3}+8\,x^{2}+x$$

```
[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="giac")
[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/11
*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x
^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int (1+4x+4x^2+4x^4)^4\,dx = \frac{256\,x^{17}}{17}+\frac{1024\,x^{15}}{15}+\frac{512\,x^{14}}{7}+\frac{1792\,x^{13}}{13}+256\,x^{12} + \frac{3328\,x^{11}}{11}+384\,x^{10}+\frac{4192\,x^{9}}{9}+448\,x^{8}+\frac{2752\,x^{7}}{7} + \frac{992\,x^{6}}{3}+\frac{1136\,x^{5}}{5}+112\,x^{4}+\frac{112\,x^{3}}{3}+8\,x^{2}+x$$

```
[In] int((4*x + 4*x^2 + 4*x^4 + 1)^4,x)

[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)
/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (1792*
x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

3.52 $\int (1+4x+4x^2+4x^4)^3 \, dx$

Optimal result 503
Rubi [A] (verified) 503
Mathematica [A] (verified) 504
Maple [A] (verified) 504
Fricas [A] (verification not implemented) 505
Sympy [A] (verification not implemented) 505
Maxima [A] (verification not implemented) 505
Giac [A] (verification not implemented) 506
Mupad [B] (verification not implemented) 506

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int (1+4x+4x^2+4x^4)^3 \, dx = x+6x^2+20x^3+40x^4+\frac{252x^5}{5}+48x^6+\frac{352x^7}{7}+48x^8+\frac{80x^9}{3}+\frac{96x^{10}}{5}+\frac{192x^{11}}{11}+\frac{64x^{13}}{13}$$

```
[Out] x+6*x^2+20*x^3+40*x^4+252/5*x^5+48*x^6+352/7*x^7+48*x^8+80/3*x^9+96/5*x^10+
192/11*x^11+64/13*x^13
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2086}

$$\int (1+4x+4x^2+4x^4)^3 \, dx = \frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^3,x]
```

```
[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 +
 (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(1+12x+60x^2+160x^3+252x^4+288x^5+352x^6+384x^7+240x^8+192x^9+192x^{10}+64x^{12}\right) dx$$
$$= x+6x^2+20x^3+40x^4+\frac{252x^5}{5}+48x^6+\frac{352x^7}{7}+48x^8+\frac{80x^9}{3}+\frac{96x^{10}}{5}+\frac{192x^{11}}{11}+\frac{64x^{13}}{13}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \left(1+4x+4x^2+4x^4\right)^3 dx = x+6x^2+20x^3+40x^4+\frac{252x^5}{5}+48x^6+\frac{352x^7}{7}+48x^8+\frac{80x^9}{3}+\frac{96x^{10}}{5}+\frac{192x^{11}}{11}+\frac{64x^{13}}{13}$$

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^3,x]
[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 +
 (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
gosper	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$	58
default	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$	58
norman	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$	58
risch	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$	58
parallelrisch	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$	58

```
[In] int((4*x^4+4*x^2+4*x+1)^3,x,method=_RETURNVERBOSE)
[Out] x+6*x^2+20*x^3+40*x^4+252/5*x^5+48*x^6+352/7*x^7+48*x^8+80/3*x^9+96/5*x^10+
192/11*x^11+64/13*x^13
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \left(1+4x+4x^2+4x^4\right)^3 \, dx = \frac{64}{13}\, x^{13} + \frac{192}{11}\, x^{11} + \frac{96}{5}\, x^{10} + \frac{80}{3}\, x^9 + 48\, x^8 + \frac{352}{7}\, x^7 + 48\, x^6 + \frac{252}{5}\, x^5 + 40\, x^4 + 20\, x^3 + 6\, x^2 + x$$

```
[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="fricas")
```

```
[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x
^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \left(1+4x+4x^2+4x^4\right)^3 \, dx = \frac{64x^{13}}{13} + \frac{192x^{11}}{11} + \frac{96x^{10}}{5} + \frac{80x^9}{3} + 48x^8 + \frac{352x^7}{7} + 48x^6 + \frac{252x^5}{5} + 40x^4 + 20x^3 + 6x^2 + x$$

```
[In] integrate((4*x**4+4*x**2+4*x+1)**3,x)
```

```
[Out] 64*x**13/13 + 192*x**11/11 + 96*x**10/5 + 80*x**9/3 + 48*x**8 + 352*x**7/7
+ 48*x**6 + 252*x**5/5 + 40*x**4 + 20*x**3 + 6*x**2 + x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \left(1+4x+4x^2+4x^4\right)^3 \, dx = \frac{64}{13}\, x^{13} + \frac{192}{11}\, x^{11} + \frac{96}{5}\, x^{10} + \frac{80}{3}\, x^9 + 48\, x^8 + \frac{352}{7}\, x^7 + 48\, x^6 + \frac{252}{5}\, x^5 + 40\, x^4 + 20\, x^3 + 6\, x^2 + x$$

```
[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="maxima")
```

```
[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x
^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \left(1+4x+4x^2+4x^4\right)^3 \, dx = \frac{64}{13}\,x^{13}+\frac{192}{11}\,x^{11}+\frac{96}{5}\,x^{10}+\frac{80}{3}\,x^{9}+48\,x^{8}+\frac{352}{7}\,x^{7}+48\,x^{6}+\frac{252}{5}\,x^{5}+40\,x^{4}+20\,x^{3}+6\,x^{2}+x$$

```
[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="giac")
```

```
[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x
^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \left(1+4x+4x^2+4x^4\right)^3 \, dx = \frac{64\,x^{13}}{13}+\frac{192\,x^{11}}{11}+\frac{96\,x^{10}}{5}+\frac{80\,x^{9}}{3}+48\,x^{8}+\frac{352\,x^{7}}{7}+48\,x^{6}+\frac{252\,x^{5}}{5}+40\,x^{4}+20\,x^{3}+6\,x^{2}+x$$

```
[In] int((4*x + 4*x^2 + 4*x^4 + 1)^3,x)
```

```
[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 +
 (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

3.53 $\int (1+4x+4x^2+4x^4)^2\,dx$

Optimal result 507
Rubi [A] (verified) 507
Mathematica [A] (verified) 508
Maple [A] (verified) 508
Fricas [A] (verification not implemented) 508
Sympy [A] (verification not implemented) 509
Maxima [A] (verification not implemented) 509
Giac [A] (verification not implemented) 509
Mupad [B] (verification not implemented) 509

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int (1+4x+4x^2+4x^4)^2\,dx = x+4x^2+8x^3+8x^4+\frac{24x^5}{5}+\frac{16x^6}{3}+\frac{32x^7}{7}+\frac{16x^9}{9}$$

```
[Out] x+4*x^2+8*x^3+8*x^4+24/5*x^5+16/3*x^6+32/7*x^7+16/9*x^9
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2086}

$$\int (1+4x+4x^2+4x^4)^2\,dx = \frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^2,x]
```

```
[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)
/9
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int (1+8x+24x^2+32x^3+24x^4+32x^5+32x^6+16x^8)\,dx \\ &= x+4x^2+8x^3+8x^4+\frac{24x^5}{5}+\frac{16x^6}{3}+\frac{32x^7}{7}+\frac{16x^9}{9} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \left(1+4x+4x^2+4x^4\right)^2\,dx=x+4x^2+8x^3+8x^4+\frac{24x^5}{5}+\frac{16x^6}{3}+\frac{32x^7}{7}+\frac{16x^9}{9}$$

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^2,x]
```

```
[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)
/9
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

method	result	size
gosper	$x+4x^2+8x^3+8x^4+\frac{24}{5}x^5+\frac{16}{3}x^6+\frac{32}{7}x^7+\frac{16}{9}x^9$	38
default	$x+4x^2+8x^3+8x^4+\frac{24}{5}x^5+\frac{16}{3}x^6+\frac{32}{7}x^7+\frac{16}{9}x^9$	38
norman	$x+4x^2+8x^3+8x^4+\frac{24}{5}x^5+\frac{16}{3}x^6+\frac{32}{7}x^7+\frac{16}{9}x^9$	38
risch	$x+4x^2+8x^3+8x^4+\frac{24}{5}x^5+\frac{16}{3}x^6+\frac{32}{7}x^7+\frac{16}{9}x^9$	38
parallelrisch	$x+4x^2+8x^3+8x^4+\frac{24}{5}x^5+\frac{16}{3}x^6+\frac{32}{7}x^7+\frac{16}{9}x^9$	38

```
[In] int((4*x^4+4*x^2+4*x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x+4*x^2+8*x^3+8*x^4+24/5*x^5+16/3*x^6+32/7*x^7+16/9*x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \left(1+4x+4x^2+4x^4\right)^2\,dx=\frac{16}{9}\,x^9+\frac{32}{7}\,x^7+\frac{16}{3}\,x^6+\frac{24}{5}\,x^5+8\,x^4+8\,x^3+4\,x^2+x$$

```
[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="fricas")
```

```
[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \left(1+4x+4x^2+4x^4\right)^2\,dx=\frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

```
[In] integrate((4*x**4+4*x**2+4*x+1)**2,x)
[Out] 16*x**9/9 + 32*x**7/7 + 16*x**6/3 + 24*x**5/5 + 8*x**4 + 8*x**3 + 4*x**2 +
x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \left(1+4x+4x^2+4x^4\right)^2\,dx=\frac{16}{9}\,x^9+\frac{32}{7}\,x^7+\frac{16}{3}\,x^6+\frac{24}{5}\,x^5+8\,x^4+8\,x^3+4\,x^2+x$$

```
[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="maxima")
[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \left(1+4x+4x^2+4x^4\right)^2\,dx=\frac{16}{9}\,x^9+\frac{32}{7}\,x^7+\frac{16}{3}\,x^6+\frac{24}{5}\,x^5+8\,x^4+8\,x^3+4\,x^2+x$$

```
[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="giac")
[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \left(1+4x+4x^2+4x^4\right)^2\,dx=\frac{16\,x^9}{9}+\frac{32\,x^7}{7}+\frac{16\,x^6}{3}+\frac{24\,x^5}{5}+8\,x^4+8\,x^3+4\,x^2+x$$

```
[In] int((4*x + 4*x^2 + 4*x^4 + 1)^2,x)
[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)
/9
```

3.54 $\int (1+4x+4x^2+4x^4)\, dx$

Optimal result 510
Rubi [A] (verified) 510
Mathematica [A] (verified) 511
Maple [A] (verified) 511
Fricas [A] (verification not implemented) 511
Sympy [A] (verification not implemented) 512
Maxima [A] (verification not implemented) 512
Giac [A] (verification not implemented) 512
Mupad [B] (verification not implemented) 512

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int (1+4x+4x^2+4x^4)\, dx = x+2x^2+\frac{4x^3}{3}+\frac{4x^5}{5}$$

```
[Out] x+2*x^2+4/3*x^3+4/5*x^5
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (1+4x+4x^2+4x^4)\, dx = \frac{4x^5}{5}+\frac{4x^3}{3}+2x^2+x$$

```
[In] Int[1 + 4*x + 4*x^2 + 4*x^4,x]
[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

Rubi steps

$$\text{integral} = x+2x^2+\frac{4x^3}{3}+\frac{4x^5}{5}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (1+4x+4x^2+4x^4)\ dx = x+2x^2+\frac{4x^3}{3}+\frac{4x^5}{5}$$

```
[In] Integrate[1 + 4*x + 4*x^2 + 4*x^4,x]
[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18
default	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18
norman	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18
risch	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18
parallelrisch	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18
parts	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18

```
[In] int(4*x^4+4*x^2+4*x+1,x,method=_RETURNVERBOSE)
[Out] x+2*x^2+4/3*x^3+4/5*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int (1+4x+4x^2+4x^4)\ dx = \frac{4}{5}\,x^5+\frac{4}{3}\,x^3+2\,x^2+x$$

```
[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="fricas")
[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int (1+4x+4x^2+4x^4)\ dx = \frac{4x^5}{5} + \frac{4x^3}{3} + 2x^2 + x$$

```
[In] integrate(4*x**4+4*x**2+4*x+1,x)
[Out] 4*x**5/5 + 4*x**3/3 + 2*x**2 + x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int (1+4x+4x^2+4x^4)\ dx = \frac{4}{5}\,x^5 + \frac{4}{3}\,x^3 + 2\,x^2 + x$$

```
[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="maxima")
[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int (1+4x+4x^2+4x^4)\ dx = \frac{4}{5}\,x^5 + \frac{4}{3}\,x^3 + 2\,x^2 + x$$

```
[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="giac")
[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int (1+4x+4x^2+4x^4)\ dx = \frac{4\,x^5}{5} + \frac{4\,x^3}{3} + 2\,x^2 + x$$

```
[In] int(4*x + 4*x^2 + 4*x^4 + 1,x)
[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

3.55 $\int \frac{1}{1+4x+4x^2+4x^4} \, dx$

Optimal result 513
Rubi [A] (verified) 514
Mathematica [C] (verified) 517
Maple [C] (verified) 518
Fricas [C] (verification not implemented) 518
Sympy [B] (verification not implemented) 519
Maxima [F] 521
Giac [C] (verification not implemented) 521
Mupad [B] (verification not implemented) 523

Optimal result

Integrand size = 17, antiderivative size = 234

$$\begin{aligned}\int \frac{1}{1+4x+4x^2+4x^4} \, dx &= \frac{1}{2}\arctan\left(\frac{1}{2}\left(-1+\left(1+\frac{1}{x}\right)^2\right)\right)\\ &\quad-\frac{1}{2}\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\arctan\left(\frac{2-\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right)\\ &\quad-\frac{1}{2}\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\arctan\left(\frac{2+\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right)\\ &\quad-\frac{1}{4}\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\log\left(\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)\right.\\ &\quad\left.+\left(1+\frac{1}{x}\right)^2\right)+\frac{1}{4}\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\log\left(\sqrt{5}\right.\\ &\quad\left.+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right)\end{aligned}$$

```
[Out] 1/2*arctan(-1/2+1/2*(1+1/x)^2)-1/20*ln((1+1/x)^2+5^(1/2)-(1+1/x)*(2+2*5^(1/2))^(1/2))*(-10+5*5^(1/2))^(1/2)+1/20*ln((1+1/x)^2+5^(1/2)+(1+1/x)*(2+2*5^(1/2))^(1/2))*(-10+5*5^(1/2))^(1/2)-1/10*arctan((2+2/x-(2+2*5^(1/2))^(1/2))/(-2+2*5^(1/2))^(1/2))*(10+5*5^(1/2))^(1/2)-1/10*arctan((2+2/x+(2+2*5^(1/2))^(1/2))/(-2+2*5^(1/2))^(1/2))*(10+5*5^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {2094, 1687, 1183, 648, 632, 210, 642, 12, 1121}

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = \frac{1}{2}\arctan\left(\frac{1}{2}\left(\left(\frac{1}{x}+1\right)^2-1\right)\right) - \frac{1}{2}\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\arctan\left(\frac{\frac{2}{x}-\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right) - \frac{1}{2}\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\arctan\left(\frac{\frac{2}{x}+\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right) - \frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right) + \frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)$$

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-1),x]
```

```
[Out] ArcTan[(-1 + (1 + x^(-1))^2)/2]/2 - (Sqrt[(2 + Sqrt[5])/5]*ArcTan[(2 - Sqrt
[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/2 - (Sqrt[(2 + Sqrt[5])/5
]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/2 - (Sq
rt[(-2 + Sqrt[5])/5]*Log[Sqrt[5] - Sqrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1
+ x^(-1))^2])/4 + (Sqrt[(-2 + Sqrt[5])/5]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5]
)]*(1 + x^(-1)) + (1 + x^(-1))^2])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(16\text{Subst}\left(\int \frac{(4-4x)^2}{1280-512x^2+256x^4}\,dx, x, 1+\frac{1}{x}\right)\right)\\
&= -\left(16\text{Subst}\left(\int -\frac{32x}{1280-512x^2+256x^4}\,dx, x, 1+\frac{1}{x}\right)\right)\\
&\quad -16\text{Subst}\left(\int \frac{16+16x^2}{1280-512x^2+256x^4}\,dx, x, 1+\frac{1}{x}\right)\\
&= 512\text{Subst}\left(\int \frac{x}{1280-512x^2+256x^4}\,dx, x, 1+\frac{1}{x}\right)\\
&\quad -\frac{\text{Subst}\left(\int \frac{16\sqrt{2\left(1+\sqrt{5}\right)}-\left(16-16\sqrt{5}\right)x}{\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}x+x^2}\,dx, x, 1+\frac{1}{x}\right)}{32\sqrt{10\left(1+\sqrt{5}\right)}}\\
&\quad -\frac{\text{Subst}\left(\int \frac{16\sqrt{2\left(1+\sqrt{5}\right)}+\left(16-16\sqrt{5}\right)x}{\sqrt{5}+\sqrt{2\left(1+\sqrt{5}\right)}x+x^2}\,dx, x, 1+\frac{1}{x}\right)}{32\sqrt{10\left(1+\sqrt{5}\right)}}\\
&= 256\text{Subst}\left(\int \frac{1}{1280-512x+256x^2}\,dx, x, \left(1+\frac{1}{x}\right)^2\right)\\
&\quad +\frac{\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{-\sqrt{2\left(1+\sqrt{5}\right)}+2x}{\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}x+x^2}\,dx, x, 1+\frac{1}{x}\right)}{4\sqrt{10\left(1+\sqrt{5}\right)}}\\
&\quad -\frac{\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{5}\right)}+2x}{\sqrt{5}+\sqrt{2\left(1+\sqrt{5}\right)}x+x^2}\,dx, x, 1+\frac{1}{x}\right)}{4\sqrt{10\left(1+\sqrt{5}\right)}}\\
&\quad -\frac{1}{20}\left(5+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}x+x^2}\,dx, x, 1+\frac{1}{x}\right)\\
&\quad -\frac{1}{20}\left(5+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\sqrt{5}+\sqrt{2\left(1+\sqrt{5}\right)}x+x^2}\,dx, x, 1+\frac{1}{x}\right)
\end{aligned}$$

$$= -\frac{1}{4}\sqrt{-\frac{2}{5}+\frac{1}{\sqrt{5}}}\log\left(\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right)$$
$$+\frac{1}{4}\sqrt{-\frac{2}{5}+\frac{1}{\sqrt{5}}}\log\left(\sqrt{5}+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right)$$
$$-512\text{Subst}\left(\int\frac{1}{-1048576-x^2}\,dx,x,-512+512\left(1+\frac{1}{x}\right)^2\right)$$
$$+\frac{1}{10}\left(5+\sqrt{5}\right)\text{Subst}\left(\int\frac{1}{2\left(1-\sqrt{5}\right)-x^2}\,dx,x,-\sqrt{2\left(1+\sqrt{5}\right)}+2\left(1+\frac{1}{x}\right)\right)$$
$$+\frac{1}{10}\left(5+\sqrt{5}\right)\text{Subst}\left(\int\frac{1}{2\left(1-\sqrt{5}\right)-x^2}\,dx,x,\sqrt{2\left(1+\sqrt{5}\right)}+2\left(1+\frac{1}{x}\right)\right)$$

$$=\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\left(-1+\left(1+\frac{1}{x}\right)^2\right)\right)-\frac{\left(1+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\frac{2-\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right)}{4\sqrt{10}}$$
$$-\frac{\left(1+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\frac{2+\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right)}{4\sqrt{10}}$$
$$-\frac{1}{4}\sqrt{-\frac{2}{5}+\frac{1}{\sqrt{5}}}\log\left(\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right)$$
$$+\frac{1}{4}\sqrt{-\frac{2}{5}+\frac{1}{\sqrt{5}}}\log\left(\sqrt{5}+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.20

$$\int\frac{1}{1+4x+4x^2+4x^4}\,dx=\frac{1}{4}\text{RootSum}\left[1+4\#1+4\#1^2+4\#1^4\&,\frac{\log(x-\#1)}{1+2\#1+4\#1^3}\&\right]$$

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-1),x]
[Out] RootSum[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , Log[x - #1]/(1 + 2*#1 + 4*#1^3) & ]/
4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.18

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^{4}+4\_Z^{2}+4\_Z+1\right)}\frac{\ln\left(x-\_R\right)}{4\_R^{3}+2\_R+1}\right)}{4}$	41
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^{4}+4\_Z^{2}+4\_Z+1\right)}\frac{\ln\left(x-\_R\right)}{4\_R^{3}+2\_R+1}\right)}{4}$	41

```
[In] int(1/(4*x^4+4*x^2+4*x+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(1/(4*_R^3+2*_R+1)*ln(x-_R),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.13

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="fricas")
[Out] -1/20*(sqrt(10)*sqrt(-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(2*sqrt(1/10*
I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*I - 1/5) + I
)^2 - 9) - 5*sqrt(1/10*I - 1/5) - 5*sqrt(-1/10*I - 1/5))*log(5/2*(2*sqrt(1/
10*I - 1/5) - I)^2*(12*sqrt(-1/10*I - 1/5) + 6*I - 1) + 15*(2*sqrt(1/10*I -
 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)^2 - 5/2*(2*sqrt(-1/10*I - 1/5) + I)^
2 + ((6*sqrt(10)*(2*sqrt(-1/10*I - 1/5) + I) - sqrt(10))*(2*sqrt(1/10*I - 1
/5) - I) - sqrt(10)*(2*sqrt(-1/10*I - 1/5) + I))*sqrt(-15/8*(2*sqrt(1/10*I
- 1/5) - I)^2 - 5/4*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)
- 15/8*(2*sqrt(-1/10*I - 1/5) + I)^2 - 9) + 8*x + 3) + 1/20*(sqrt(10)*sqrt(
-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt
(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*I - 1/5) + I)^2 - 9) + 5*sqrt(1/1
0*I - 1/5) + 5*sqrt(-1/10*I - 1/5))*log(5/2*(2*sqrt(1/10*I - 1/5) - I)^2*(1
2*sqrt(-1/10*I - 1/5) + 6*I - 1) + 15*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1
/10*I - 1/5) + I)^2 - 5/2*(2*sqrt(-1/10*I - 1/5) + I)^2 - ((6*sqrt(10)*(2*s
qrt(-1/10*I - 1/5) + I) - sqrt(10))*(2*sqrt(1/10*I - 1/5) - I) - sqrt(10)*(
2*sqrt(-1/10*I - 1/5) + I))*sqrt(-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(
2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*
```

```
I - 1/5) + I)^2 - 9) + 8*x + 3) - 1/4*(2*sqrt(1/10*I - 1/5) - I)*log(-5*(2*
sqrt(1/10*I - 1/5) - I)^2*(12*sqrt(-1/10*I - 1/5) + 6*I - 1) - 30*(2*sqrt(1
/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)^2 - 30*(2*sqrt(-1/10*I - 1/5)
 + I)^3 + 8*x - 216*sqrt(-1/10*I - 1/5) - 108*I + 21) - 1/4*(2*sqrt(-1/10*I
 - 1/5) + I)*log(30*(2*sqrt(-1/10*I - 1/5) + I)^3 + 5*(2*sqrt(-1/10*I - 1/5
) + I)^2 + 8*x + 216*sqrt(-1/10*I - 1/5) + 108*I - 27)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3432 vs. $2(190) = 380$.

Time = 1.34 (sec) , antiderivative size = 3432, normalized size of antiderivative = 14.67

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(4*x**4+4*x**2+4*x+1),x)
```

```
[Out] sqrt(-1/40 + sqrt(5)/80)*log(x**2 + x*(-8 - 21*sqrt(5)*sqrt(-2 + sqrt(5))/1
0 - sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/2 - sqrt(5)/2 + 12*s
qrt(-2 + sqrt(5)) + 9*sqrt(5)*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2 +
sqrt(5)) + sqrt(5) + 19)/5) - 841*sqrt(5)*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)
) + sqrt(5) + 19)/20 - 14351/40 - 441*sqrt(-2 + sqrt(5))/4 - 75*sqrt(5)*sqr
t(-2 + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/8 - 3*sq
rt(-2 + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 301*s
qrt(5)*sqrt(-2 + sqrt(5))/10 + 7407*sqrt(5)/40 + 3913*sqrt(-2*sqrt(5)*sqrt(
-2 + sqrt(5)) + sqrt(5) + 19)/40) - sqrt(-1/40 + sqrt(5)/80)*log(x**2 + x*(
-8 - 12*sqrt(-2 + sqrt(5)) - sqrt(5)/2 + 21*sqrt(5)*sqrt(-2 + sqrt(5))/10 +
 sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/2 + 9*sqrt(5)*sqrt(-2 +
sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/5) - 3913*sqrt(2
*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/40 - 14351/40 - 75*sqrt(5)*sqrt
(-2 + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/8 - 301*sq
rt(5)*sqrt(-2 + sqrt(5))/10 - 3*sqrt(-2 + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 +
 sqrt(5)) + sqrt(5) + 19) + 441*sqrt(-2 + sqrt(5))/4 + 7407*sqrt(5)/40 + 84
1*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/20) - 2*sqrt(3/
80 + 3*sqrt(5)/80 + sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/40)*
atan(-20*x/(-27*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sq
rt(5)) + sqrt(5) + 19)) + 5*sqrt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(
-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(
5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*sqrt(5)*
sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) - 18*sqrt(5)*sqrt(-2 + sqrt(5))*sqrt(-2
*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/(-27*sqrt(5)*sqrt(3 + 3*sqrt(5)
 + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sqrt(-2 + sqrt
(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) +
19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5))
+ sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) - 120*
```

sqrt(-2 + sqrt(5))/(-27*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt
(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sqrt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) +
2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 +
3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*
sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sqrt(5)/(-27*sqrt(5)*sqrt(3
+ 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sq
rt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5))
+ sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2
+ sqrt(5)) + sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) +
19)) + 5*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/(-27*sqrt(5)*s
qrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) +
5*sqrt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt
(5)) + sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sq
rt(-2 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt
(5) + 19)) + 21*sqrt(5)*sqrt(-2 + sqrt(5))/(-27*sqrt(5)*sqrt(3 + 3*sqrt(5)
+ 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sqrt(-2 + sqrt(
5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 1
9)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) +
sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 80/(-
27*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt
(5) + 19)) + 5*sqrt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sq
rt(-2 + sqrt(5)) + sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-
2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqr
t(5)) + sqrt(5) + 19))) - 2*sqrt(-sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(
5) + 19)/40 + 3/80 + 3*sqrt(5)/80)*atan(20*x/(5*sqrt(-2 + sqrt(5))*sqrt(-2*
sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5)) + 27*sqr
t(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt
(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)*sqrt(-2*
sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5))) - 80/(5
*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 1
9) + 3 + 3*sqrt(5)) + 27*sqrt(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5))
+ sqrt(5) + 19) + 3 + 3*sqrt(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(
5)) + sqrt(5) + 19)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 1
9) + 3 + 3*sqrt(5))) - 120*sqrt(-2 + sqrt(5))/(5*sqrt(-2 + sqrt(5))*sqrt(-2
*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5)) + 27*sq
rt(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqr
t(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)*sqrt(-2
*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5))) - 5*sq
rt(5)/(5*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqr
t(5) + 19) + 3 + 3*sqrt(5)) + 27*sqrt(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + s
qrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2
+ sqrt(5)) + sqrt(5) + 19)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqr
t(5) + 19) + 3 + 3*sqrt(5))) + 21*sqrt(5)*sqrt(-2 + sqrt(5))/(5*sqrt(-2 + s
qrt(5))*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*s
qrt(5)) + 27*sqrt(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) +

```
19) + 3 + 3*sqrt(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5
) + 19)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*s
qrt(5))) + 5*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/(5*sqrt(-2 +
 sqrt(5))*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3
*sqrt(5)) + 27*sqrt(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5)
+ 19) + 3 + 3*sqrt(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt
(5) + 19)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3
*sqrt(5))) + 18*sqrt(5)*sqrt(-2 + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)
) + sqrt(5) + 19)/(5*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sq
rt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5)) + 27*sqrt(5)*sqrt(-2*sqrt(2*sqrt(5)
*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5)) + 6*sqrt(5)*sqrt(2*sqr
t(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sq
rt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5))))
```

Maxima [F]

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx=\int \frac{1}{4\,x^4+4\,x^2+4\,x+1}\,dx$$

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.13

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = -\frac{1}{20}\left((i+2)\sqrt{\sqrt{5}-2}\left(\frac{i}{\sqrt{5}-2}+1\right)+5i\right)\log\left((406i+174)\sqrt{5}x+(868i+372)\,x+29\sqrt{5}\sqrt{29\sqrt{5}+62}+(87i-203)\sqrt{5}+(19i+62)\sqrt{29\sqrt{5}+62}+186i-434\right)-\frac{1}{20}\left((i+2)\sqrt{\sqrt{5}-2}\left(-\frac{i}{\sqrt{5}-2}-1\right)+5i\right)\log\left((406i+174)\sqrt{5}x+(868i+372)\,x-29\sqrt{5}\sqrt{29\sqrt{5}+62}+(87i-203)\sqrt{5}-(19i+62)\sqrt{29\sqrt{5}+62}+186i-434\right)-\frac{1}{20}\left((2i+1)\sqrt{\sqrt{5}+2}\left(-\frac{i}{\sqrt{5}+2}-1\right)-5i\right)\log\left((26i+130)\sqrt{5}x-(44i+220)\,x+13\sqrt{5}\sqrt{13\sqrt{5}-22}-(65i-13)\sqrt{5}+(19i-22)\sqrt{13\sqrt{5}-22}+110i-22\right)-\frac{1}{20}\left((2i+1)\sqrt{\sqrt{5}+2}\left(\frac{i}{\sqrt{5}+2}+1\right)-5i\right)\log\left((26i+130)\sqrt{5}x-(44i+220)\,x-13\sqrt{5}\sqrt{13\sqrt{5}-22}-(65i-13)\sqrt{5}-(19i-22)\sqrt{13\sqrt{5}-22}+110i-22\right)$$

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="giac")
```

```
[Out] -1/20*((I + 2)*sqrt(sqrt(5) - 2)*(I/(sqrt(5) - 2) + 1) + 5*I)*log((406*I +
174)*sqrt(5)*x + (868*I + 372)*x + 29*sqrt(5)*sqrt(29*sqrt(5) + 62) + (87*I
 - 203)*sqrt(5) + (19*I + 62)*sqrt(29*sqrt(5) + 62) + 186*I - 434) - 1/20*(
(I + 2)*sqrt(sqrt(5) - 2)*(-I/(sqrt(5) - 2) - 1) + 5*I)*log((406*I + 174)*s
qrt(5)*x + (868*I + 372)*x - 29*sqrt(5)*sqrt(29*sqrt(5) + 62) + (87*I - 203
)*sqrt(5) - (19*I + 62)*sqrt(29*sqrt(5) + 62) + 186*I - 434) - 1/20*((2*I +
 1)*sqrt(sqrt(5) + 2)*(-I/(sqrt(5) + 2) - 1) - 5*I)*log((26*I + 130)*sqrt(5
)*x - (44*I + 220)*x + 13*sqrt(5)*sqrt(13*sqrt(5) - 22) - (65*I - 13)*sqrt(
5) + (19*I - 22)*sqrt(13*sqrt(5) - 22) + 110*I - 22) - 1/20*((2*I + 1)*sqrt
(sqrt(5) + 2)*(I/(sqrt(5) + 2) + 1) - 5*I)*log((26*I + 130)*sqrt(5)*x - (44
*I + 220)*x - 13*sqrt(5)*sqrt(13*sqrt(5) - 22) - (65*I - 13)*sqrt(5) - (19*
I - 22)*sqrt(13*sqrt(5) - 22) + 110*I - 22)
```

Mupad [B] (verification not implemented)

Time = 10.97 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.37

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = \sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)\left(\frac{x}{4}+\mathrm{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)\left(6\,x+\mathrm{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)(36\,x+16)\right)\right)\right)\mathrm{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)$$

```
[In] int(1/(4*x + 4*x^2 + 4*x^4 + 1),x)
```

```
[Out] symsum(log(-root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k)*(x/4 + root(z^4 +
(9*z^2)/40 + z/40 + 1/1280, z, k)*(6*x + root(z^4 + (9*z^2)/40 + z/40 + 1/1
280, z, k)*(36*x + 16))))*root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k), k,
1, 4)
```

3.56 $\int \frac{1}{(1+4x+4x^2+4x^4)^2} dx$

Optimal result 524
Rubi [A] (verified) 525
Mathematica [C] (verified) 530
Maple [C] (verified) 530
Fricas [C] (verification not implemented) 531
Sympy [B] (verification not implemented) 532
Maxima [F] 535
Giac [C] (verification not implemented) 535
Mupad [B] (verification not implemented) 538

Optimal result

Integrand size = 17, antiderivative size = 317

$$\begin{aligned}
&\int \frac{1}{(1+4x+4x^2+4x^4)^2} \, dx \\
&= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)}+\frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} \\
&\quad+\frac{7}{4}\arctan\left(\frac{1}{2}\left(-1+\left(1+\frac{1}{x}\right)^2\right)\right) \\
&\quad-\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\arctan\left(\frac{2-\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right) \\
&\quad-\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\arctan\left(\frac{2+\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right) \\
&\quad+\frac{1}{40}\sqrt{\frac{1}{10}\left(-5959+2665\sqrt{5}\right)}\log\left(\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right) \\
&\quad-\frac{1}{40}\sqrt{\frac{1}{10}\left(-5959+2665\sqrt{5}\right)}\log\left(\sqrt{5}+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right)
\end{aligned}$$

```
[Out] 1/2*(-17+(1+1/x)^2)/(5-2*(1+1/x)^2+(1+1/x)^4)+1/10*(59-17*(1+1/x)^2)*(1+1/x
)/(5-2*(1+1/x)^2+(1+1/x)^4)+7/4*arctan(-1/2+1/2*(1+1/x)^2)+1/400*ln((1+1/x)
^2+5^(1/2)-(1+1/x)*(2+2*5^(1/2))^(1/2))*(-59590+26650*5^(1/2))^(1/2)-1/400*
ln((1+1/x)^2+5^(1/2)+(1+1/x)*(2+2*5^(1/2))^(1/2))*(-59590+26650*5^(1/2))^(1
/2)-1/200*arctan((2+2/x-(2+2*5^(1/2))^(1/2))/(-2+2*5^(1/2))^(1/2))*(59590+2
```

```
6650*5^(1/2))^(1/2)-1/200*arctan((2+2/x+(2+2*5^(1/2))^(1/2))/(-2+2*5^(1/2))
^(1/2))*(59590+26650*5^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {2094, 1687, 1692, 1183, 648, 632, 210, 642, 1677, 1674, 12}

$$\begin{aligned}
&\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx \\
&= \frac{7}{4}\arctan\left(\frac{1}{2}\left(\left(\frac{1}{x}+1\right)^2-1\right)\right) \\
&\quad -\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\arctan\left(\frac{\frac{2}{x}-\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right) \\
&\quad -\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\arctan\left(\frac{\frac{2}{x}+\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right) \\
&\quad -\frac{17-\left(\frac{1}{x}+1\right)^2}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)} \\
&\quad +\frac{1}{40}\sqrt{\frac{1}{10}\left(2665\sqrt{5}-5959\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right) \\
&\quad -\frac{1}{40}\sqrt{\frac{1}{10}\left(2665\sqrt{5}-5959\right)}\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)
\end{aligned}$$

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-2),x]
```

```
[Out] -1/2*(17 - (1 + x^(-1))^2)/(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4) + ((59 -
 17*(1 + x^(-1))^2)*(1 + x^(-1)))/(10*(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^
4)) + (7*ArcTan[(-1 + (1 + x^(-1))^2)/2])/4 - (Sqrt[(5959 + 2665*Sqrt[5])/1
0]*ArcTan[(2 - Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/20 - (
Sqrt[(5959 + 2665*Sqrt[5])/10]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqr
t[2*(-1 + Sqrt[5])]])/20 + (Sqrt[(-5959 + 2665*Sqrt[5])/10]*Log[Sqrt[5] - S
qrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1 + x^(-1))^2])/40 - (Sqrt[(-5959 + 26
65*Sqrt[5])/10]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1 + x^(
-1))^2])/40
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(16\text{Subst}\left(\int \frac{(4-4x)^6}{(1280-512x^2+256x^4)^2}\,dx, x, 1+\frac{1}{x}\right)\right)\\ &= -\left(16\text{Subst}\left(\int \frac{x(-24576-81920x^2-24576x^4)}{(1280-512x^2+256x^4)^2}\,dx, x, 1+\frac{1}{x}\right)\right)\\ &\quad -16\text{Subst}\left(\int \frac{4096+61440x^2+61440x^4+4096x^6}{(1280-512x^2+256x^4)^2}\,dx, x, 1+\frac{1}{x}\right)\end{aligned}$$

$$= \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} - \frac{\text{Subst}\left(\int \frac{261993005056+115964116992x^2}{1280-512x^2+256x^4}\,dx, x, 1+\frac{1}{x}\right)}{167772160}$$

$$- 8\text{Subst}\left(\int \frac{-24576-81920x-24576x^2}{\left(1280-512x+256x^2\right)^2}\,dx, x, \left(1+\frac{1}{x}\right)^2\right)$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)}$$

$$- \frac{\text{Subst}\left(\int -\frac{117440512}{1280-512x+256x^2}\,dx, x, \left(1+\frac{1}{x}\right)^2\right)}{131072}$$

$$- \frac{\text{Subst}\left(\int \frac{261993005056\sqrt{2\left(1+\sqrt{5}\right)}-\left(261993005056-115964116992\sqrt{5}\right)x}{\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}x+x^2}\,dx, x, 1+\frac{1}{x}\right)}{85899345920\sqrt{10\left(1+\sqrt{5}\right)}}$$

$$- \frac{\text{Subst}\left(\int \frac{261993005056\sqrt{2\left(1+\sqrt{5}\right)}+\left(261993005056-115964116992\sqrt{5}\right)x}{\sqrt{5}+\sqrt{2\left(1+\sqrt{5}\right)}x+x^2}\,dx, x, 1+\frac{1}{x}\right)}{85899345920\sqrt{10\left(1+\sqrt{5}\right)}}$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)}$$

$$+ 896\text{Subst}\left(\int \frac{1}{1280-512x+256x^2}\,dx, x, \left(1+\frac{1}{x}\right)^2\right)$$

$$+ \frac{\left(61-27\sqrt{5}\right)\text{Subst}\left(\int \frac{-\sqrt{2\left(1+\sqrt{5}\right)}+2x}{\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}x+x^2}\,dx, x, 1+\frac{1}{x}\right)}{40\sqrt{10\left(1+\sqrt{5}\right)}}$$

$$- \frac{\left(61-27\sqrt{5}\right)\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{5}\right)}+2x}{\sqrt{5}+\sqrt{2\left(1+\sqrt{5}\right)}x+x^2}\,dx, x, 1+\frac{1}{x}\right)}{40\sqrt{10\left(1+\sqrt{5}\right)}}$$

$$- \frac{1}{20}\sqrt{\frac{1}{10}\left(3683+1647\sqrt{5}\right)}\text{Subst}\left(\int \frac{1}{\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}x+x^2}\,dx, x, 1+\frac{1}{x}\right)$$

$$- \frac{1}{20}\sqrt{\frac{1}{10}\left(3683+1647\sqrt{5}\right)}\text{Subst}\left(\int \frac{1}{\sqrt{5}+\sqrt{2\left(1+\sqrt{5}\right)}x+x^2}\,dx, x, 1+\frac{1}{x}\right)$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)}+\frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)}$$
$$+\frac{1}{40}\sqrt{-\frac{5959}{10}+\frac{533\sqrt{5}}{2}}\log\left(\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right)$$
$$-\frac{1}{40}\sqrt{-\frac{5959}{10}+\frac{533\sqrt{5}}{2}}\log\left(\sqrt{5}+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right)$$
$$-1792\text{Subst}\left(\int\frac{1}{-1048576-x^2}\,dx,x,-512+512\left(1+\frac{1}{x}\right)^2\right)$$
$$+\frac{1}{10}\sqrt{\frac{1}{10}\left(3683+1647\sqrt{5}\right)}\text{Subst}\left(\int\frac{1}{2\left(1-\sqrt{5}\right)-x^2}\,dx,x,-\sqrt{2\left(1+\sqrt{5}\right)}+2\left(1+\frac{1}{x}\right)\right)$$
$$+\frac{1}{10}\sqrt{\frac{1}{10}\left(3683+1647\sqrt{5}\right)}\text{Subst}\left(\int\frac{1}{2\left(1-\sqrt{5}\right)-x^2}\,dx,x,\sqrt{2\left(1+\sqrt{5}\right)}+2\left(1+\frac{1}{x}\right)\right)$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)}+\frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)}$$
$$+\frac{7}{4}\tan^{-1}\left(\frac{1}{2}\left(-1+\left(1+\frac{1}{x}\right)^2\right)\right)$$
$$-\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\tan^{-1}\left(\frac{2-\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right)$$
$$-\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\tan^{-1}\left(\frac{2+\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right)$$
$$+\frac{1}{40}\sqrt{-\frac{5959}{10}+\frac{533\sqrt{5}}{2}}\log\left(\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right)$$
$$-\frac{1}{40}\sqrt{-\frac{5959}{10}+\frac{533\sqrt{5}}{2}}\log\left(\sqrt{5}+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.34

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx = \frac{1}{40}\left(\frac{38+84x-32x^2+72x^3}{1+4x+4x^2+4x^4}+\text{RootSum}\left[1+4\#1+4\#1^2+4\#1^4\&,\frac{27\log(x-\#1)-16\log(x-\#1)\#1+18\log(x-\#1)\#1^2}{1+2\#1+4\#1^3}\&\right]\right)$$

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-2),x]
```

```
[Out] ((38 + 84*x - 32*x^2 + 72*x^3)/(1 + 4*x + 4*x^2 + 4*x^4) + RootSum[1 + 4*#1
 + 4*#1^2 + 4*#1^4 & , (27*Log[x - #1] - 16*Log[x - #1]*#1 + 18*Log[x - #1]
*#1^2)/(1 + 2*#1 + 4*#1^3) & ])/40
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.25

method	result	size
default	$\frac{\frac{9}{20}x^3-\frac{1}{5}x^2+\frac{21}{40}x+\frac{19}{80}}{x^4+x^2+x+\frac{1}{4}}+\frac{\left(\sum_{\_R=\text{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)}\frac{\left(18\_R^2-16\_R+27\right)\ln\left(x-\_R\right)}{4\_R^3+2\_R+1}\right)}{40}$	79
risch	$\frac{\frac{9}{20}x^3-\frac{1}{5}x^2+\frac{21}{40}x+\frac{19}{80}}{x^4+x^2+x+\frac{1}{4}}+\frac{\left(\sum_{\_R=\text{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)}\frac{\left(18\_R^2-16\_R+27\right)\ln\left(x-\_R\right)}{4\_R^3+2\_R+1}\right)}{40}$	79

```
[In] int(1/(4*x^4+4*x^2+4*x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (9/20*x^3-1/5*x^2+21/40*x+19/80)/(x^4+x^2+x+1/4)+1/40*sum((18*_R^2-16*_R+27
)/(4*_R^3+2*_R+1)*ln(x-_R),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 704, normalized size of antiderivative = 2.22

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/400*(720*x^3 - 50*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(19/1000*I - 5959/2000
) + 7*I)*log(33368250*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^3 - 11755375/4*
(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 541735337*x + 25784243612*sqrt(19
/1000*I - 5959/2000) + 45122426321*I - 71080995) - 50*(4*x^4 + 4*x^2 + 4*x
+ 1)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)*log(-33368250*(4*sqrt(19/1000*I
 - 5959/2000) + 7*I)^3 - 125/4*(4271136*sqrt(19/1000*I - 5959/2000) + 74744
88*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 25*(1334730*(4*sqr
t(19/1000*I - 5959/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000)
 - 7*I) + 541735337*x - 25806203712*sqrt(19/1000*I - 5959/2000) - 451608564
96*I - 355111539) - 320*x^2 - (4*sqrt(10)*(4*x^4 + 4*x^2 + 4*x + 1)*sqrt(-3
75/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*I -
5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sqrt(-
19/1000*I - 5959/2000) - 7*I)^2 - 3021) - 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*s
qrt(19/1000*I - 5959/2000) + 7*I) - 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(-1
9/1000*I - 5959/2000) - 7*I))*log(125/8*(4271136*sqrt(19/1000*I - 5959/2000
) + 7474488*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 + 11755375/
8*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 25/2*(1334730*(4*sqrt(19/1000*I
 - 5959/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) + 1
/2*sqrt(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19
/1000*I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32
*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021)*(5*(1067784*sqrt(10)*(4*s
qrt(19/1000*I - 5959/2000) + 7*I) + 94043*sqrt(10))*(4*sqrt(-19/1000*I - 59
59/2000) - 7*I) + 470215*sqrt(10)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) - 8
78404*sqrt(10)) + 541735337*x + 10980050*sqrt(19/1000*I - 5959/2000) + 3843
0175/2*I + 213096267) + (4*sqrt(10)*(4*x^4 + 4*x^2 + 4*x + 1)*sqrt(-375/32*
(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*I - 5959/2
000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sqrt(-19/100
0*I - 5959/2000) - 7*I)^2 - 3021) + 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(19
/1000*I - 5959/2000) + 7*I) + 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(-19/1000
*I - 5959/2000) - 7*I))*log(125/8*(4271136*sqrt(19/1000*I - 5959/2000) + 74
74488*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 + 11755375/8*(4*s
qrt(19/1000*I - 5959/2000) + 7*I)^2 + 25/2*(1334730*(4*sqrt(19/1000*I - 595
9/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 1/2*sqr
t(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*
I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sq
```

```
rt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021)*(5*(1067784*sqrt(10)*(4*sqrt(19
/1000*I - 5959/2000) + 7*I) + 94043*sqrt(10))*(4*sqrt(-19/1000*I - 5959/200
0) - 7*I) + 470215*sqrt(10)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) - 878404*
sqrt(10)) + 541735337*x + 10980050*sqrt(19/1000*I - 5959/2000) + 38430175/2
*I + 213096267) + 840*x + 380)/(4*x^4 + 4*x^2 + 4*x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3834 vs. 2(257) = 514.

Time = 1.94 (sec) , antiderivative size = 3834, normalized size of antiderivative = 12.09

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(4*x**4+4*x**2+4*x+1)**2,x)
```

```
[Out] (36*x**3 - 16*x**2 + 42*x + 19)/(80*x**4 + 80*x**2 + 80*x + 20) - sqrt(-595
9/16000 + 533*sqrt(5)/3200)*log(x**2 + x*(-1601676*sqrt(10)*sqrt(-5959 + 26
65*sqrt(5))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5)
+ 36004639)/13543383425 - 1067784*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/101638
9 + 3131659367*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/13543383425 + 291689395/
1083470674 + 470215*sqrt(5)/2032778 + 94043*sqrt(-665*sqrt(10)*sqrt(-5959 +
 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/541735337) - 40634464149111451*
sqrt(5)*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36
004639)/27530691871904650 - 288583554422522791728299 7/14673858767725178450 0
 - 83803227754187*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/100111606806926 - 5020
8805356*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))*sqrt(-665*sqrt(10)*sqrt(-5959 +
2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/550613837438093 - 53848575489193
3*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*
sqrt(5)) + 221195*sqrt(5) + 36004639)/14673858767725178450 - 92532195509690
1411*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/29347717535450356900 + 48430461193
8766076267*sqrt(5)/55061383743809300 + 22013036087014785403*sqrt(-665*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/666993580351144
4750) + sqrt(-5959/16000 + 533*sqrt(5)/3200)*log(x**2 + x*(-94043*sqrt(665*
sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/541735337
- 1601676*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(665*sqrt(10)*sqrt(-5959
+ 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/13543383425 - 3131659367*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5))/13543383425 + 291689395/1083470674 + 1067784
*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/1016389 + 470215*sqrt(5)/2032778) - 220
13036087014785403*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqr
t(5) + 36004639)/6669935803511444750 - 2885835544225227917282997/1467385876
77251784500 - 50208805356*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))*sqrt(665*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/550613837438093
 - 538485754891933*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(665*sqrt(10)*sq
```

rt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/14673858767725178450
+ 925321955096901411*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/293477175354503569
00 + 83803227754187*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/100111606806926 + 48
4304611938766076267*sqrt(5)/55061383743809300 + 40634464149111451*sqrt(5)*s
qrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/27
530691871904650) + 2*sqrt(6291/16000 + 1599*sqrt(5)/3200 + sqrt(-665*sqrt(1
0)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/4000)*atan(54173
533700*x/(-6440570878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt
(-5959 + 2665*sqrt(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt
(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqr
t(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 2
21195*sqrt(5) + 36004639))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) +
221195*sqrt(5) + 36004639)) - 3203352*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*s
qrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/(
-6440570878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-
5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt(-5959 + 2
665*sqrt(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2
665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqrt(6291 + 7
995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt
(5) + 36004639))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqr
t(5) + 36004639)) - 28456443600*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/(-644057
0878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 +
2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt(-5959 + 2665*sqr
t(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqr
t(5)) + 221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqrt(6291 + 7995*sqr
t(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 3
6004639))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) +
36004639)) + 6263318734*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/(-6440570878*sq
rt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sq
rt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt(-5959 + 2665*sqrt(5))*s
qrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) +
 221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqrt(6291 + 7995*sqrt(5) +
4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639
))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 3600463
9)) + 7292234875/(-6440570878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-6
65*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351
075*sqrt(-5959 + 2665*sqrt(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 1067784*sqrt
(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt
(5)) + 221195*sqrt(5) + 36004639))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqr
t(5)) + 221195*sqrt(5) + 36004639)) + 6265614875*sqrt(5)/(-6440570878*sqrt(
10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(
5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt(-5959 + 2665*sqrt(5))*sqrt
(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 22

1195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*s
qrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639))*
sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639))
 + 4702150*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) +
 36004639)/(-6440570878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqr
t(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sq
rt(-5959 + 2665*sqrt(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sq
rt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*s
qrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) +
 221195*sqrt(5) + 36004639))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))
+ 221195*sqrt(5) + 36004639))) - 2*sqrt(-sqrt(665*sqrt(10)*sqrt(-5959 + 266
5*sqrt(5)) + 221195*sqrt(5) + 36004639)/4000 + 6291/16000 + 1599*sqrt(5)/32
00)*atan(54173533700*x/(2351075*sqrt(-5959 + 2665*sqrt(5))*sqrt(-4*sqrt(665
*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 +
7995*sqrt(5)) + 6440570878*sqrt(10)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 +
2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5)) + 1067784
*sqrt(10)*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 3
6004639)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt
(5) + 36004639) + 6291 + 7995*sqrt(5))) - 4702150*sqrt(665*sqrt(10)*sqrt(-5
959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/(2351075*sqrt(-5959 + 2665
*sqrt(5))*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqr
t(5) + 36004639) + 6291 + 7995*sqrt(5)) + 6440570878*sqrt(10)*sqrt(-4*sqrt(
665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291
 + 7995*sqrt(5)) + 1067784*sqrt(10)*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqr
t(5)) + 221195*sqrt(5) + 36004639)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2
665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5))) - 3203352
*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sq
rt(5)) + 221195*sqrt(5) + 36004639)/(2351075*sqrt(-5959 + 2665*sqrt(5))*sqr
t(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 360046
39) + 6291 + 7995*sqrt(5)) + 6440570878*sqrt(10)*sqrt(-4*sqrt(665*sqrt(10)*
sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(
5)) + 1067784*sqrt(10)*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 22119
5*sqrt(5) + 36004639)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))
+ 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5))) - 6263318734*sqrt(10)*
sqrt(-5959 + 2665*sqrt(5))/(2351075*sqrt(-5959 + 2665*sqrt(5))*sqrt(-4*sqrt
(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 629
1 + 7995*sqrt(5)) + 6440570878*sqrt(10)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-595
9 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5)) + 106
7784*sqrt(10)*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5)
 + 36004639)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*
sqrt(5) + 36004639) + 6291 + 7995*sqrt(5))) + 7292234875/(2351075*sqrt(-595
9 + 2665*sqrt(5))*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 22
1195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5)) + 6440570878*sqrt(10)*sqrt(
-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639
) + 6291 + 7995*sqrt(5)) + 1067784*sqrt(10)*sqrt(665*sqrt(10)*sqrt(-5959 +

```
2665*sqrt(5)) + 221195*sqrt(5) + 36004639)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-
5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5))) +
 28456443600*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/(2351075*sqrt(-5959 + 2665*
sqrt(5))*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt
(5) + 36004639) + 6291 + 7995*sqrt(5)) + 6440570878*sqrt(10)*sqrt(-4*sqrt(6
65*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291
+ 7995*sqrt(5)) + 1067784*sqrt(10)*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt
(5)) + 221195*sqrt(5) + 36004639)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 26
65*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5))) + 62656148
75*sqrt(5)/(2351075*sqrt(-5959 + 2665*sqrt(5))*sqrt(-4*sqrt(665*sqrt(10)*sq
rt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5)
) + 6440570878*sqrt(10)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)
) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5)) + 1067784*sqrt(10)*sq
rt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)*sqr
t(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 360046
39) + 6291 + 7995*sqrt(5)))
```

Maxima [F]

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx = \int \frac{1}{(4\,x^4+4\,x^2+4\,x+1)^2}\,dx$$

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="maxima")
```

```
[Out] 1/20*(36*x^3 - 16*x^2 + 42*x + 19)/(4*x^4 + 4*x^2 + 4*x + 1) + 1/10*integra
te((18*x^2 - 16*x + 27)/(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.99

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx$$
$$= -\frac{1}{400}\left(-(i+3)\sqrt{2665\sqrt{5}-4790}\left(\frac{709i}{533\sqrt{5}-958}+1\right)-350i\right)\log\left((2534636224790i + 16853816172010)\sqrt{5}x - (3913528401620i + 26022625108780)x + 5049076145\sqrt{5}\sqrt{1424281\sqrt{5}-2199118} - (8426908086005i - 1267318112395)\sqrt{5} + (8166407345i - 7795873310)\sqrt{1424281\sqrt{5}-2199118} + 13011312554390i - 1956764200810\right)$$
$$- \frac{1}{400}\left((i+3)\sqrt{2665\sqrt{5}-4790}\left(\frac{709i}{533\sqrt{5}-958}+1\right)-350i\right)\log\left((2534636224790i + 16853816172010)\sqrt{5}x - (3913528401620i + 26022625108780)x - 5049076145\sqrt{5}\sqrt{1424281\sqrt{5}-2199118} - (8426908086005i - 1267318112395)\sqrt{5} - (8166407345i - 7795873310)\sqrt{1424281\sqrt{5}-2199118} + 13011312554390i - 1956764200810\right)$$
$$- \frac{1}{400}\left((3i+1)\sqrt{2665\sqrt{5}+4790}\left(\frac{709i}{533\sqrt{5}+958}+1\right)+350i\right)\log\left((16722951192450i + 2480822188910)\sqrt{5}x + (25712356272300i + 3814385585140)x + 5021907265\sqrt{5}\sqrt{1416617\sqrt{5}+2178118} + (1240411094455i - 8361475596225)\sqrt{5} + (8153361745i + 7721428310)\sqrt{1416617\sqrt{5}+2178118} + 1907192792570i - 12856178136150\right)$$
$$- \frac{1}{400}\left(-(3i+1)\sqrt{2665\sqrt{5}+4790}\left(\frac{709i}{533\sqrt{5}+958}+1\right)+350i\right)\log\left((16722951192450i + 2480822188910)\sqrt{5}x + (25712356272300i + 3814385585140)x - 5021907265\sqrt{5}\sqrt{1416617\sqrt{5}+2178118} + (1240411094455i - 8361475596225)\sqrt{5} - (8153361745i + 7721428310)\sqrt{1416617\sqrt{5}+2178118} + 1907192792570i - 12856178136150\right) + \frac{36\,x^3 - 16\,x^2 + 42\,x + 19}{20\,(4\,x^4 + 4\,x^2 + 4\,x + 1)}$$

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="giac")
[Out] -1/400*(-(I + 3)*sqrt(2665*sqrt(5) - 4790)*(709*I/(533*sqrt(5) - 958) + 1)
- 350*I)*log((2534636224790*I + 16853816172010)*sqrt(5)*x - (3913528401620*
```

```
I + 26022625108780)*x + 5049076145*sqrt(5)*sqrt(1424281*sqrt(5) - 2199118)
- (8426908086005*I - 1267318112395)*sqrt(5) + (8166407345*I - 7795873310)*s
qrt(1424281*sqrt(5) - 2199118) + 13011312554390*I - 1956764200810) - 1/400*
((I + 3)*sqrt(2665*sqrt(5) - 4790)*(709*I/(533*sqrt(5) - 958) + 1) - 350*I)
*log((2534636224790*I + 16853816172010)*sqrt(5)*x - (3913528401620*I + 2602
2625108780)*x - 5049076145*sqrt(5)*sqrt(1424281*sqrt(5) - 2199118) - (84269
08086005*I - 1267318112395)*sqrt(5) - (8166407345*I - 7795873310)*sqrt(1424
281*sqrt(5) - 2199118) + 13011312554390*I - 1956764200810) - 1/400*((3*I +
1)*sqrt(2665*sqrt(5) + 4790)*(709*I/(533*sqrt(5) + 958) + 1) + 350*I)*log((
16722951192450*I + 2480822188910)*sqrt(5)*x + (25712356272300*I + 381438558
5140)*x + 5021907265*sqrt(5)*sqrt(1416617*sqrt(5) + 2178118) + (12404110944
55*I - 8361475596225)*sqrt(5) + (8153361745*I + 7721428310)*sqrt(1416617*sq
rt(5) + 2178118) + 1907192792570*I - 12856178136150) - 1/400*(-(3*I + 1)*sq
rt(2665*sqrt(5) + 4790)*(709*I/(533*sqrt(5) + 958) + 1) + 350*I)*log((16722
951192450*I + 2480822188910)*sqrt(5)*x + (25712356272300*I + 3814385585140)
*x - 5021907265*sqrt(5)*sqrt(1416617*sqrt(5) + 2178118) + (1240411094455*I
- 8361475596225)*sqrt(5) - (8153361745*I + 7721428310)*sqrt(1416617*sqrt(5)
 + 2178118) + 1907192792570*I - 12856178136150) + 1/20*(36*x^3 - 16*x^2 + 4
2*x + 19)/(4*x^4 + 4*x^2 + 4*x + 1)
```

Mupad [B] (verification not implemented)

Time = 10.66 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.55

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx = \left(\sum_{k=1}^{4}\ln\left(-\frac{169\,\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)}{100}+\frac{11\,x}{1600}+\frac{\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)\,x\,131}{100}-\frac{\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^2\,x\,72}{5}-\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^3\,x\,36+\frac{59\,\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^2}{20}-16\,\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^3+\frac{27}{1600}\right)\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)\right)+\frac{\frac{9\,x^3}{20}-\frac{x^2}{5}+\frac{21\,x}{40}+\frac{19}{80}}{x^4+x^2+x+\frac{1}{4}}$$

```
[In] int(1/(4*x + 4*x^2 + 4*x^4 + 1)^2,x)
```

```
[Out] symsum(log((11*x)/1600 - (169*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 2
9/64000, z, k))/100 + (131*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/6
4000, z, k)*x)/100 - (72*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/640
00, z, k)^2*x)/5 - 36*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000,
 z, k)^3*x + (59*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k
)^2)/20 - 16*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k)^3
+ 27/1600)*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k), k,
1, 4) + ((21*x)/40 - x^2/5 + (9*x^3)/20 + 19/80)/(x + x^2 + x^4 + 1/4)
```

3.57 $\int \left(8+24x+8x^2-15x^3+8x^4\right)^4 \, dx$

Optimal result 539
Rubi [A] (verified) 539
Mathematica [A] (verified) 540
Maple [A] (verified) 541
Fricas [A] (verification not implemented) 541
Sympy [A] (verification not implemented) 542
Maxima [A] (verification not implemented) 542
Giac [A] (verification not implemented) 543
Mupad [B] (verification not implemented) 543

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^4 \, dx = 4096x+24576x^2+\frac{237568x^3}{3}+139776x^4+\frac{538624x^5}{5} -30720x^6-\frac{566912x^7}{7}+36384x^8+\frac{641152x^9}{9} -\frac{169584x^{10}}{5}-\frac{331040x^{11}}{11}+31128x^{12}-\frac{12095x^{13}}{13} -\frac{75504x^{14}}{7}+\frac{102784x^{15}}{15}-1920x^{16}+\frac{4096x^{17}}{17}$$

```
[Out] 4096*x+24576*x^2+237568/3*x^3+139776*x^4+538624/5*x^5-30720*x^6-566912/7*x^
7+36384*x^8+641152/9*x^9-169584/5*x^10-331040/11*x^11+31128*x^12-12095/13*x
^13-75504/7*x^14+102784/15*x^15-1920*x^16+4096/17*x^17
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2086}

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^4 \, dx = \frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13} +31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^9}{9} +36384x^8-\frac{566912x^7}{7}-30720x^6+\frac{538624x^5}{5} +139776x^4+\frac{237568x^3}{3}+24576x^2+4096x$$

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x
^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (33104
0*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/
15 - 1920*x^16 + (4096*x^17)/17
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(4096+49152x+237568x^2+559104x^3+538624x^4-184320x^5-566912x^6+291072x^7+641152x^8-339168x^9-331040x^{10}+373536x^{11}-12095x^{12}-151008x^{13}+102784x^{14}-30720x^{15}+4096x^{16}\right)\,dx\\ &= 4096x+24576x^2+\frac{237568x^3}{3}+139776x^4+\frac{538624x^5}{5}-30720x^6-\frac{566912x^7}{7}+36384x^8+\frac{641152x^9}{9}-\frac{169584x^{10}}{5}-\frac{331040x^{11}}{11}+31128x^{12}-\frac{12095x^{13}}{13}-\frac{75504x^{14}}{7}+\frac{102784x^{15}}{15}-1920x^{16}+\frac{4096x^{17}}{17}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^4\,dx = 4096x+24576x^2+\frac{237568x^3}{3}+139776x^4+\frac{538624x^5}{5}-30720x^6-\frac{566912x^7}{7}+36384x^8+\frac{641152x^9}{9}-\frac{169584x^{10}}{5}-\frac{331040x^{11}}{11}+31128x^{12}-\frac{12095x^{13}}{13}-\frac{75504x^{14}}{7}+\frac{102784x^{15}}{15}-1920x^{16}+\frac{4096x^{17}}{17}$$

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x
^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (33104
0*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/
15 - 1920*x^16 + (4096*x^17)/17
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.82

method	result
gosper	$4096x+24576x^2+\frac{237568}{3}x^3+139776x^4+\frac{538624}{5}x^5-30720x^6-\frac{566912}{7}x^7+36384x^8+\frac{64115}{9}$
default	$4096x+24576x^2+\frac{237568}{3}x^3+139776x^4+\frac{538624}{5}x^5-30720x^6-\frac{566912}{7}x^7+36384x^8+\frac{64115}{9}$
norman	$4096x+24576x^2+\frac{237568}{3}x^3+139776x^4+\frac{538624}{5}x^5-30720x^6-\frac{566912}{7}x^7+36384x^8+\frac{64115}{9}$
risch	$4096x+24576x^2+\frac{237568}{3}x^3+139776x^4+\frac{538624}{5}x^5-30720x^6-\frac{566912}{7}x^7+36384x^8+\frac{64115}{9}$
parallelrisch	$4096x+24576x^2+\frac{237568}{3}x^3+139776x^4+\frac{538624}{5}x^5-30720x^6-\frac{566912}{7}x^7+36384x^8+\frac{64115}{9}$

```
[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 4096*x+24576*x^2+237568/3*x^3+139776*x^4+538624/5*x^5-30720*x^6-566912/7*x^
7+36384*x^8+641152/9*x^9-169584/5*x^10-331040/11*x^11+31128*x^12-12095/13*x
^13-75504/7*x^14+102784/15*x^15-1920*x^16+4096/17*x^17
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^4\,dx=\frac{4096}{17}\,x^{17}-1920\,x^{16}+\frac{102784}{15}\,x^{15}-\frac{75504}{7}\,x^{14}-\frac{12095}{13}\,x^{13}+31128\,x^{12}-\frac{331040}{11}\,x^{11}-\frac{169584}{5}\,x^{10}+\frac{641152}{9}\,x^{9}+36384\,x^{8}-\frac{566912}{7}\,x^{7}-30720\,x^{6}+\frac{538624}{5}\,x^{5}+139776\,x^{4}+\frac{237568}{3}\,x^{3}+24576\,x^{2}+4096\,x$$

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="fricas")
```

```
[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 56
6912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x
^2 + 4096*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^4 \,dx = \frac{4096x^{17}}{17} - 1920x^{16} + \frac{102784x^{15}}{15} - \frac{75504x^{14}}{7} - \frac{12095x^{13}}{13} + 31128x^{12} - \frac{331040x^{11}}{11} - \frac{169584x^{10}}{5} + \frac{641152x^9}{9} + 36384x^8 - \frac{566912x^7}{7} - 30720x^6 + \frac{538624x^5}{5} + 139776x^4 + \frac{237568x^3}{3} + 24576x^2 + 4096x$$

```
[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**4,x)
```

```
[Out] 4096*x**17/17 - 1920*x**16 + 102784*x**15/15 - 75504*x**14/7 - 12095*x**13/
13 + 31128*x**12 - 331040*x**11/11 - 169584*x**10/5 + 641152*x**9/9 + 36384
*x**8 - 566912*x**7/7 - 30720*x**6 + 538624*x**5/5 + 139776*x**4 + 237568*x
**3/3 + 24576*x**2 + 4096*x
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^4 \,dx = \frac{4096}{17}\,x^{17} - 1920\,x^{16} + \frac{102784}{15}\,x^{15} - \frac{75504}{7}\,x^{14} - \frac{12095}{13}\,x^{13} + 31128\,x^{12} - \frac{331040}{11}\,x^{11} - \frac{169584}{5}\,x^{10} + \frac{641152}{9}\,x^9 + 36384\,x^8 - \frac{566912}{7}\,x^7 - 30720\,x^6 + \frac{538624}{5}\,x^5 + 139776\,x^4 + \frac{237568}{3}\,x^3 + 24576\,x^2 + 4096\,x$$

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="maxima")
```

```
[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 56
6912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x
^2 + 4096*x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^4 \, dx = \frac{4096}{17}\,x^{17} - 1920\,x^{16} + \frac{102784}{15}\,x^{15} - \frac{75504}{7}\,x^{14} - \frac{12095}{13}\,x^{13} + 31128\,x^{12} - \frac{331040}{11}\,x^{11} - \frac{169584}{5}\,x^{10} + \frac{641152}{9}\,x^{9} + 36384\,x^{8} - \frac{566912}{7}\,x^{7} - 30720\,x^{6} + \frac{538624}{5}\,x^{5} + 139776\,x^{4} + \frac{237568}{3}\,x^{3} + 24576\,x^{2} + 4096\,x$$

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="giac")
```

```
[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 56
6912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x
^2 + 4096*x
```

Mupad [B] (verification not implemented)

Time = 10.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^4 \, dx = \frac{4096\,x^{17}}{17} - 1920\,x^{16} + \frac{102784\,x^{15}}{15} - \frac{75504\,x^{14}}{7} - \frac{12095\,x^{13}}{13} + 31128\,x^{12} - \frac{331040\,x^{11}}{11} - \frac{169584\,x^{10}}{5} + \frac{641152\,x^{9}}{9} + 36384\,x^{8} - \frac{566912\,x^{7}}{7} - 30720\,x^{6} + \frac{538624\,x^{5}}{5} + 139776\,x^{4} + \frac{237568\,x^{3}}{3} + 24576\,x^{2} + 4096\,x$$

```
[In] int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^4,x)
```

```
[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x
^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (33104
0*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/
15 - 1920*x^16 + (4096*x^17)/17
```

3.58 $\int (8+24x+8x^2-15x^3+8x^4)^3\,dx$

Optimal result . . . 544
Rubi [A] (verified) . . . 544
Mathematica [A] (verified) . . . 545
Maple [A] (verified) . . . 545
Fricas [A] (verification not implemented) . . . 546
Sympy [A] (verification not implemented) . . . 546
Maxima [A] (verification not implemented) . . . 547
Giac [A] (verification not implemented) . . . 547
Mupad [B] (verification not implemented) . . . 547

Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3\,dx = 512x+2304x^2+5120x^3+5040x^4-\frac{384x^5}{5} - 2976x^6+\frac{5528x^7}{7}+2097x^8-\frac{2936x^9}{3} - \frac{4527x^{10}}{10}+\frac{6936x^{11}}{11}-240x^{12}+\frac{512x^{13}}{13}$$

```
[Out] 512*x+2304*x^2+5120*x^3+5040*x^4-384/5*x^5-2976*x^6+5528/7*x^7+2097*x^8-293
6/3*x^9-4527/10*x^10+6936/11*x^11-240*x^12+512/13*x^13
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2086}

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3\,dx = \frac{512x^{13}}{13}-240x^{12}+\frac{6936x^{11}}{11}-\frac{4527x^{10}}{10} - \frac{2936x^9}{3}+2097x^8+\frac{5528x^7}{7}-2976x^6 - \frac{384x^5}{5}+5040x^4+5120x^3+2304x^2+512x$$

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7
)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12
+ (512*x^13)/13
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(512+4608x+15360x^2+20160x^3-384x^4-17856x^5+5528x^6+16776x^7 \right.\\ &\left. -8808x^8-4527x^9+6936x^{10}-2880x^{11}+512x^{12}\right)\,dx\\ &= 512x+2304x^2+5120x^3+5040x^4-\frac{384x^5}{5}-2976x^6+\frac{5528x^7}{7}\\ &+2097x^8-\frac{2936x^9}{3}-\frac{4527x^{10}}{10}+\frac{6936x^{11}}{11}-240x^{12}+\frac{512x^{13}}{13}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3\,dx = 512x+2304x^2+5120x^3+5040x^4-\frac{384x^5}{5}-2976x^6+\frac{5528x^7}{7}+2097x^8-\frac{2936x^9}{3}-\frac{4527x^{10}}{10}+\frac{6936x^{11}}{11}-240x^{12}+\frac{512x^{13}}{13}$$

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7
)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12
+ (512*x^13)/13
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

method	result
gosper	$512x+2304x^2+5120x^3+5040x^4-\frac{384}{5}x^5-2976x^6+\frac{5528}{7}x^7+2097x^8-\frac{2936}{3}x^9-\frac{4527}{10}x^{10}$
default	$512x+2304x^2+5120x^3+5040x^4-\frac{384}{5}x^5-2976x^6+\frac{5528}{7}x^7+2097x^8-\frac{2936}{3}x^9-\frac{4527}{10}x^{10}$
norman	$512x+2304x^2+5120x^3+5040x^4-\frac{384}{5}x^5-2976x^6+\frac{5528}{7}x^7+2097x^8-\frac{2936}{3}x^9-\frac{4527}{10}x^{10}$
risch	$512x+2304x^2+5120x^3+5040x^4-\frac{384}{5}x^5-2976x^6+\frac{5528}{7}x^7+2097x^8-\frac{2936}{3}x^9-\frac{4527}{10}x^{10}$
parallelrisch	$512x+2304x^2+5120x^3+5040x^4-\frac{384}{5}x^5-2976x^6+\frac{5528}{7}x^7+2097x^8-\frac{2936}{3}x^9-\frac{4527}{10}x^{10}$

```
[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 512*x+2304*x^2+5120*x^3+5040*x^4-384/5*x^5-2976*x^6+5528/7*x^7+2097*x^8-293
6/3*x^9-4527/10*x^10+6936/11*x^11-240*x^12+512/13*x^13
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3\,dx = \frac{512}{13}\,x^{13} - 240\,x^{12} + \frac{6936}{11}\,x^{11} - \frac{4527}{10}\,x^{10} - \frac{2936}{3}\,x^{9} + 2097\,x^{8} + \frac{5528}{7}\,x^{7} - 2976\,x^{6} - \frac{384}{5}\,x^{5} + 5040\,x^{4} + 5120\,x^{3} + 2304\,x^{2} + 512\,x$$

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="fricas")
```

```
[Out] 512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x^
8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 + 51
2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3\,dx = \frac{512x^{13}}{13} - 240x^{12} + \frac{6936x^{11}}{11} - \frac{4527x^{10}}{10} - \frac{2936x^{9}}{3} + 2097x^{8} + \frac{5528x^{7}}{7} - 2976x^{6} - \frac{384x^{5}}{5} + 5040x^{4} + 5120x^{3} + 2304x^{2} + 512x$$

```
[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**3,x)
```

```
[Out] 512*x**13/13 - 240*x**12 + 6936*x**11/11 - 4527*x**10/10 - 2936*x**9/3 + 20
97*x**8 + 5528*x**7/7 - 2976*x**6 - 384*x**5/5 + 5040*x**4 + 5120*x**3 + 23
04*x**2 + 512*x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3 \, dx = \frac{512}{13}\,x^{13} - 240\,x^{12} + \frac{6936}{11}\,x^{11} - \frac{4527}{10}\,x^{10} - \frac{2936}{3}\,x^9 + 2097\,x^8 + \frac{5528}{7}\,x^7 - 2976\,x^6 - \frac{384}{5}\,x^5 + 5040\,x^4 + 5120\,x^3 + 2304\,x^2 + 512\,x$$

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="maxima")
[Out] 512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x^
8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 + 51
2*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3 \, dx = \frac{512}{13}\,x^{13} - 240\,x^{12} + \frac{6936}{11}\,x^{11} - \frac{4527}{10}\,x^{10} - \frac{2936}{3}\,x^9 + 2097\,x^8 + \frac{5528}{7}\,x^7 - 2976\,x^6 - \frac{384}{5}\,x^5 + 5040\,x^4 + 5120\,x^3 + 2304\,x^2 + 512\,x$$

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="giac")
[Out] 512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x^
8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 + 51
2*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3 \, dx = \frac{512\,x^{13}}{13} - 240\,x^{12} + \frac{6936\,x^{11}}{11} - \frac{4527\,x^{10}}{10} - \frac{2936\,x^9}{3} + 2097\,x^8 + \frac{5528\,x^7}{7} - 2976\,x^6 - \frac{384\,x^5}{5} + 5040\,x^4 + 5120\,x^3 + 2304\,x^2 + 512\,x$$

```
[In] int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^3,x)
```

```
[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7
)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12
+ (512*x^13)/13
```

3.59 $\int (8+24x+8x^2-15x^3+8x^4)^2 \, dx$

Optimal result 549
Rubi [A] (verified) 549
Mathematica [A] (verified) 550
Maple [A] (verified) 550
Fricas [A] (verification not implemented) 551
Sympy [A] (verification not implemented) 551
Maxima [A] (verification not implemented) 551
Giac [A] (verification not implemented) 552
Mupad [B] (verification not implemented) 552

Optimal result

Integrand size = 22, antiderivative size = 52

$$\int (8+24x+8x^2-15x^3+8x^4)^2 \, dx = 64x+192x^2+\frac{704x^3}{3}+36x^4-\frac{528x^5}{5}+24x^6+\frac{353x^7}{7}-30x^8+\frac{64x^9}{9}$$

```
[Out] 64*x+192*x^2+704/3*x^3+36*x^4-528/5*x^5+24*x^6+353/7*x^7-30*x^8+64/9*x^9
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2086}

$$\int (8+24x+8x^2-15x^3+8x^4)^2 \, dx = \frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
- 30*x^8 + (64*x^9)/9
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(64 + 384x + 704x^2 + 144x^3 - 528x^4 + 144x^5 + 353x^6 - 240x^7 + 64x^8\right) dx$$
$$= 64x + 192x^2 + \frac{704x^3}{3} + 36x^4 - \frac{528x^5}{5} + 24x^6 + \frac{353x^7}{7} - 30x^8 + \frac{64x^9}{9}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \left(8 + 24x + 8x^2 - 15x^3 + 8x^4\right)^2 dx = 64x + 192x^2 + \frac{704x^3}{3} + 36x^4 - \frac{528x^5}{5} + 24x^6 + \frac{353x^7}{7} - 30x^8 + \frac{64x^9}{9}$$

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2,x]
[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
- 30*x^8 + (64*x^9)/9
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

method	result	size
gosper	$64x + 192x^2 + \frac{704}{3}x^3 + 36x^4 - \frac{528}{5}x^5 + 24x^6 + \frac{353}{7}x^7 - 30x^8 + \frac{64}{9}x^9$	45
default	$64x + 192x^2 + \frac{704}{3}x^3 + 36x^4 - \frac{528}{5}x^5 + 24x^6 + \frac{353}{7}x^7 - 30x^8 + \frac{64}{9}x^9$	45
norman	$64x + 192x^2 + \frac{704}{3}x^3 + 36x^4 - \frac{528}{5}x^5 + 24x^6 + \frac{353}{7}x^7 - 30x^8 + \frac{64}{9}x^9$	45
risch	$64x + 192x^2 + \frac{704}{3}x^3 + 36x^4 - \frac{528}{5}x^5 + 24x^6 + \frac{353}{7}x^7 - 30x^8 + \frac{64}{9}x^9$	45
parallelrisch	$64x + 192x^2 + \frac{704}{3}x^3 + 36x^4 - \frac{528}{5}x^5 + 24x^6 + \frac{353}{7}x^7 - 30x^8 + \frac{64}{9}x^9$	45

```
[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^2,x,method=_RETURNVERBOSE)
[Out] 64*x+192*x^2+704/3*x^3+36*x^4-528/5*x^5+24*x^6+353/7*x^7-30*x^8+64/9*x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2\,dx = \frac{64}{9}\,x^9-30\,x^8+\frac{353}{7}\,x^7+24\,x^6-\frac{528}{5}\,x^5+36\,x^4+\frac{704}{3}\,x^3+192\,x^2+64\,x$$

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")
[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 1
92*x^2 + 64*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2\,dx = \frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

```
[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
[Out] 64*x**9/9 - 30*x**8 + 353*x**7/7 + 24*x**6 - 528*x**5/5 + 36*x**4 + 704*x**
3/3 + 192*x**2 + 64*x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2\,dx = \frac{64}{9}\,x^9-30\,x^8+\frac{353}{7}\,x^7+24\,x^6-\frac{528}{5}\,x^5+36\,x^4+\frac{704}{3}\,x^3+192\,x^2+64\,x$$

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="maxima")
[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 1
92*x^2 + 64*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2\,dx = \frac{64}{9}\,x^9 - 30\,x^8 + \frac{353}{7}\,x^7 + 24\,x^6 - \frac{528}{5}\,x^5 + 36\,x^4 + \frac{704}{3}\,x^3 + 192\,x^2 + 64\,x$$

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="giac")
[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 1
92*x^2 + 64*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2\,dx = \frac{64\,x^9}{9} - 30\,x^8 + \frac{353\,x^7}{7} + 24\,x^6 - \frac{528\,x^5}{5} + 36\,x^4 + \frac{704\,x^3}{3} + 192\,x^2 + 64\,x$$

```
[In] int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^2,x)
[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
- 30*x^8 + (64*x^9)/9
```

3.60 $\int (8+24x+8x^2-15x^3+8x^4)\ dx$

Optimal result 553
Rubi [A] (verified) 553
Mathematica [A] (verified) 554
Maple [A] (verified) 554
Fricas [A] (verification not implemented) 554
Sympy [A] (verification not implemented) 555
Maxima [A] (verification not implemented) 555
Giac [A] (verification not implemented) 555
Mupad [B] (verification not implemented) 555

Optimal result

Integrand size = 20, antiderivative size = 30

$$\int (8+24x+8x^2-15x^3+8x^4)\ dx = 8x+12x^2+\frac{8x^3}{3}-\frac{15x^4}{4}+\frac{8x^5}{5}$$

```
[Out] 8*x+12*x^2+8/3*x^3-15/4*x^4+8/5*x^5
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (8+24x+8x^2-15x^3+8x^4)\ dx = \frac{8x^5}{5}-\frac{15x^4}{4}+\frac{8x^3}{3}+12x^2+8x$$

```
[In] Int[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]
[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

Rubi steps

$$\text{integral} = 8x+12x^2+\frac{8x^3}{3}-\frac{15x^4}{4}+\frac{8x^5}{5}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)\,dx=8x+12x^2+\frac{8x^3}{3}-\frac{15x^4}{4}+\frac{8x^5}{5}$$

```
[In] Integrate[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]
[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25
default	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25
norman	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25
risch	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25
parallelrisch	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25
parts	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25

```
[In] int(8*x^4-15*x^3+8*x^2+24*x+8,x,method=_RETURNVERBOSE)
[Out] 8*x+12*x^2+8/3*x^3-15/4*x^4+8/5*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)\,dx=\frac{8}{5}\,x^5-\frac{15}{4}\,x^4+\frac{8}{3}\,x^3+12\,x^2+8\,x$$

```
[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="fricas")
[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)\,dx=\frac{8x^5}{5}-\frac{15x^4}{4}+\frac{8x^3}{3}+12x^2+8x$$

```
[In] integrate(8*x**4-15*x**3+8*x**2+24*x+8,x)
[Out] 8*x**5/5 - 15*x**4/4 + 8*x**3/3 + 12*x**2 + 8*x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)\,dx=\frac{8}{5}\,x^5-\frac{15}{4}\,x^4+\frac{8}{3}\,x^3+12\,x^2+8\,x$$

```
[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="maxima")
[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)\,dx=\frac{8}{5}\,x^5-\frac{15}{4}\,x^4+\frac{8}{3}\,x^3+12\,x^2+8\,x$$

```
[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="giac")
[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)\,dx=\frac{8\,x^5}{5}-\frac{15\,x^4}{4}+\frac{8\,x^3}{3}+12\,x^2+8\,x$$

```
[In] int(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8,x)
[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

3.61 $\int \frac{1}{8+24x+8x^2-15x^3+8x^4} dx$

Optimal result . . . 556
Rubi [A] (verified) . . . 557
Mathematica [C] (verified) . . . 560
Maple [C] (verified) . . . 561
Fricas [C] (verification not implemented) . . . 561
Sympy [A] (verification not implemented) . . . 563
Maxima [F] . . . 563
Giac [F] . . . 563
Mupad [B] (verification not implemented) . . . 564

Optimal result

Integrand size = 22, antiderivative size = 263

$$\begin{aligned}
\int \frac{1}{8+24x+8x^2-15x^3+8x^4} dx
&= -\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\arctan\left(\frac{6-\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}}{\sqrt{2\left(-19+\sqrt{517}\right)}}\right) \\
&\quad -\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\arctan\left(\frac{6+\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}}{\sqrt{2\left(-19+\sqrt{517}\right)}}\right) \\
&\quad +\frac{1}{4}\sqrt{\frac{3}{13}}\arctan\left(\frac{8+12x-5x^2}{\sqrt{39}x^2}\right) \\
&\quad -\frac{1}{8}\sqrt{\frac{-5167+235\sqrt{517}}{40326}}\log\left(\sqrt{517}-\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right) \\
&\quad +\frac{1}{8}\sqrt{\frac{-5167+235\sqrt{517}}{40326}}\log\left(\sqrt{517}+\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right)
\end{aligned}$$

```
[Out] 1/52*arctan(1/39*(-5*x^2+12*x+8)/x^2*39^(1/2))*39^(1/2)-1/322608*ln((3+4/x)
^2+517^(1/2)-(3+4/x)*(38+2*517^(1/2))^(1/2))*(-208364442+9476610*517^(1/2))
^(1/2)+1/322608*ln((3+4/x)^2+517^(1/2)+(3+4/x)*(38+2*517^(1/2))^(1/2))*(-20
8364442+9476610*517^(1/2))^(1/2)-1/161304*arctan((6+8/x-(38+2*517^(1/2))^(1
/2))/(-38+2*517^(1/2))^(1/2))*(208364442+9476610*517^(1/2))^(1/2)-1/161304*
arctan((6+8/x+(38+2*517^(1/2))^(1/2))/(-38+2*517^(1/2))^(1/2))*(208364442+9
476610*517^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {2094, 12, 1687, 1183, 648, 632, 210, 642, 1121}

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$$
$$= \frac{1}{4}\sqrt{\frac{3}{13}}\arctan\left(\frac{-5x^2+12x+8}{\sqrt{39}x^2}\right)$$
$$-\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\arctan\left(\frac{\frac{8}{x}-\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)$$
$$-\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\arctan\left(\frac{\frac{8}{x}+\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)$$
$$-\frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2-\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)$$
$$+\frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2+\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)$$

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1),x]
```

```
[Out] -1/4*(Sqrt[(5167 + 235*Sqrt[517])/40326]*ArcTan[(6 - Sqrt[2*(19 + Sqrt[517]
)] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]]) - (Sqrt[(5167 + 235*Sqrt[517])/40326]
*ArcTan[(6 + Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/4
+ (Sqrt[3/13]*ArcTan[(8 + 12*x - 5*x^2)/(Sqrt[39]*x^2)])/4 - (Sqrt[(-5167 +
 235*Sqrt[517])/40326]*Log[Sqrt[517] - Sqrt[2*(19 + Sqrt[517])]*(3 + 4/x) +
 (3 + 4/x)^2])/8 + (Sqrt[(-5167 + 235*Sqrt[517])/40326]*Log[Sqrt[517] + Sqr
t[2*(19 + Sqrt[517])]*(3 + 4/x) + (3 + 4/x)^2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\left(1024\text{Subst}\left(\int \frac{(24-32x)^2}{8\,(2117632-2490368x^2+1048576x^4)}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$
$$= -\left(128\text{Subst}\left(\int \frac{(24-32x)^2}{2117632-2490368x^2+1048576x^4}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$
$$= -\left(128\text{Subst}\left(\int -\frac{1536x}{2117632-2490368x^2+1048576x^4}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right) - 128\text{Subst}\left(\int \frac{576+1024x^2}{2117632-2490368x^2+1048576x^4}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)$$
$$= 196608\text{Subst}\left(\int \frac{x}{2117632-2490368x^2+1048576x^4}\,dx, x, \frac{3}{4}+\frac{1}{x}\right) - \frac{\text{Subst}\left(\int \frac{144\sqrt{2\left(19+\sqrt{517}\right)}-\left(576-64\sqrt{517}\right)x}{\frac{\sqrt{517}}{16}-\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}x+x^2}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)}{256\sqrt{1034\left(19+\sqrt{517}\right)}} - \frac{\text{Subst}\left(\int \frac{144\sqrt{2\left(19+\sqrt{517}\right)}+\left(576-64\sqrt{517}\right)x}{\frac{\sqrt{517}}{16}+\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}x+x^2}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)}{256\sqrt{1034\left(19+\sqrt{517}\right)}}$$
$$= 98304\text{Subst}\left(\int \frac{1}{2117632-2490368x+1048576x^2}\,dx, x, \left(\frac{3}{4}+\frac{1}{x}\right)^2\right) - \frac{\left(517+9\sqrt{517}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{517}}{16}-\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}x+x^2}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)}{16544} - \frac{\left(517+9\sqrt{517}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{517}}{16}+\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}x+x^2}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)}{16544} - \frac{1}{8}\sqrt{\frac{-5167+235\sqrt{517}}{40326}}\text{Subst}\left(\int \frac{-\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}+2x}{\frac{\sqrt{517}}{16}-\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}x+x^2}\,dx, x, \frac{3}{4}+\frac{1}{x}\right) + \frac{1}{8}\sqrt{\frac{-5167+235\sqrt{517}}{40326}}\text{Subst}\left(\int \frac{\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}+2x}{\frac{\sqrt{517}}{16}+\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}x+x^2}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)$$

$$= -\frac{1}{8}\sqrt{\frac{-5167+235\sqrt{517}}{40326}}\log\left(\sqrt{517}-\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right)$$
$$+\frac{1}{8}\sqrt{\frac{-5167+235\sqrt{517}}{40326}}\log\left(\sqrt{517}+\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right)$$
$$-196608\text{Subst}\left(\int\frac{1}{-2680059592704-x^2}\,dx,x,-2490368+2097152\left(\frac{3}{4}+\frac{1}{x}\right)^2\right)$$
$$+\frac{\left(517+9\sqrt{517}\right)\text{Subst}\left(\int\frac{1}{\frac{1}{8}\left(19-\sqrt{517}\right)-x^2}\,dx,x,-\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}+2\left(\frac{3}{4}+\frac{1}{x}\right)\right)}{8272}$$
$$+\frac{\left(517+9\sqrt{517}\right)\text{Subst}\left(\int\frac{1}{\frac{1}{8}\left(19-\sqrt{517}\right)-x^2}\,dx,x,\frac{1}{4}\left(6+\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}\right)\right)}{8272}$$
$$= -\frac{1}{4}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{19-\left(3+\frac{4}{x}\right)^2}{2\sqrt{39}}\right)$$
$$-\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\tan^{-1}\left(\frac{6+\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}}{\sqrt{2\left(-19+\sqrt{517}\right)}}\right)$$
$$-\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\tan^{-1}\left(\frac{8+\left(6-\sqrt{2\left(19+\sqrt{517}\right)}\right)x}{\sqrt{2\left(-19+\sqrt{517}\right)}x}\right)$$
$$-\frac{1}{8}\sqrt{\frac{-5167+235\sqrt{517}}{40326}}\log\left(\sqrt{517}-\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right)$$
$$+\frac{1}{8}\sqrt{\frac{-5167+235\sqrt{517}}{40326}}\log\left(\sqrt{517}+\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.21

$$\int\frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx = \text{RootSum}\left[8+24\#1+8\#1^2-15\#1^3+8\#1^4\&,\frac{\log(x-\#1)}{24+16\#1-45\#1^2+32\#1^3}\&\right]$$

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1),x]
[Out] RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , Log[x - #1]/(24 + 16*#1 -
 45*#1^2 + 32*#1^3) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.19

method	result	size
default	$\sum_{\_R=\mathrm{RootOf}(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8)} \frac{\ln(x-\_R)}{32\_R^3-45\_R^2+16\_R+24}$	49
risch	$\sum_{\_R=\mathrm{RootOf}(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8)} \frac{\ln(x-\_R)}{32\_R^3-45\_R^2+16\_R+24}$	49

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8),x,method=_RETURNVERBOSE)
```

```
[Out] sum(1/(32*_R^3-45*_R^2+16*_R+24)*ln(x-_R),_R=RootOf(8*_Z^4-15*_Z^3+8*_Z^2+2
4*_Z+8))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 1297, normalized size of antiderivative = 4.93

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="fricas")
```

```
[Out] -1/104*(-I*sqrt(13)*sqrt(3) + 52*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/
322608))*log(37895495846208*(1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(109/161304
*I*sqrt(13)*sqrt(3) - 5167/322608))^3 - 537872704512*(1/104*I*sqrt(13)*sqrt
(3) - 1/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2 + 5614027117
*I*sqrt(13)*sqrt(3) + 1789133960*x - 291929410084*sqrt(109/161304*I*sqrt(13
)*sqrt(3) - 5167/322608) + 2270349121) - 1/104*(I*sqrt(13)*sqrt(3) + 52*sqr
t(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))*log(-4736936980776*(1/104*
I*sqrt(13)*sqrt(3) - 1/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))
^3 + 20163*(-1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(-109/161304*I*sqrt(13)*sqr
t(3) - 5167/322608))^2*(-2258963*I*sqrt(13)*sqrt(3) + 117466076*sqrt(109/16
1304*I*sqrt(13)*sqrt(3) - 5167/322608) - 3334528) + 517*(88099557*(1/104*I*
sqrt(13)*sqrt(3) - 1/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2
 - 16507)*(I*sqrt(13)*sqrt(3) + 52*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 51
67/322608)) - 5545754165/8*I*sqrt(13)*sqrt(3) + 223641745*x + 72094804145/2
*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608) - 916562824) + 1/80652*(
sqrt(40326)*sqrt(-120978*(1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(109/161304*I*
sqrt(13)*sqrt(3) - 5167/322608))^2 - 120978*(-1/104*I*sqrt(13)*sqrt(3) - 1/
```

2*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2 - 1551/208*(I*sqrt(
13)*sqrt(3) + 52*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))*(-I*sq
rt(13)*sqrt(3) + 52*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608)) - 24
55) + 20163*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608) + 20163*sqrt(
-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))*log(-60489/2*(-1/104*I*sqrt(
13)*sqrt(3) - 1/2*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2*(-2
258963*I*sqrt(13)*sqrt(3) + 117466076*sqrt(109/161304*I*sqrt(13)*sqrt(3) -
5167/322608) - 3334528) - 1551/2*(88099557*(1/104*I*sqrt(13)*sqrt(3) - 1/2*
sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2 - 16507)*(I*sqrt(13)*s
qrt(3) + 52*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608)) + 100851132
096*(1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 51
67/322608))^2 + 1/416*(3*(2258963*sqrt(40326)*(-I*sqrt(13)*sqrt(3) + 52*sqr
t(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608)) - 3334528*sqrt(40326))*(I*s
qrt(13)*sqrt(3) + 52*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608)) -
10003584*sqrt(40326)*(-I*sqrt(13)*sqrt(3) + 52*sqrt(109/161304*I*sqrt(13)*s
qrt(3) - 5167/322608)) + 13733824*sqrt(40326))*sqrt(-120978*(1/104*I*sqrt(1
3)*sqrt(3) - 1/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2 - 120
978*(-1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(-109/161304*I*sqrt(13)*sqrt(3) -
5167/322608))^2 - 1551/208*(I*sqrt(13)*sqrt(3) + 52*sqrt(-109/161304*I*sqrt
(13)*sqrt(3) - 5167/322608))*(-I*sqrt(13)*sqrt(3) + 52*sqrt(109/161304*I*sq
rt(13)*sqrt(3) - 5167/322608)) - 2455) - 25602357/2*I*sqrt(13)*sqrt(3) + 67
0925235*x + 665661282*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608) + 3
20161368) - 1/80652*(sqrt(40326)*sqrt(-120978*(1/104*I*sqrt(13)*sqrt(3) - 1
/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2 - 120978*(-1/104*I*
sqrt(13)*sqrt(3) - 1/2*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^
2 - 1551/208*(I*sqrt(13)*sqrt(3) + 52*sqrt(-109/161304*I*sqrt(13)*sqrt(3) -
5167/322608))*(-I*sqrt(13)*sqrt(3) + 52*sqrt(109/161304*I*sqrt(13)*sqrt(3)
- 5167/322608)) - 2455) - 20163*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/
322608) - 20163*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))*log(-60
489/2*(-1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(-109/161304*I*sqrt(13)*sqrt(3)
- 5167/322608))^2*(-2258963*I*sqrt(13)*sqrt(3) + 117466076*sqrt(109/161304*
I*sqrt(13)*sqrt(3) - 5167/322608) - 3334528) - 1551/2*(88099557*(1/104*I*sq
rt(13)*sqrt(3) - 1/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2 -
16507)*(I*sqrt(13)*sqrt(3) + 52*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5167
/322608)) + 100851132096*(1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(109/161304*I*
sqrt(13)*sqrt(3) - 5167/322608))^2 - 1/416*(3*(2258963*sqrt(40326)*(-I*sqrt
(13)*sqrt(3) + 52*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608)) - 3334
528*sqrt(40326))*(I*sqrt(13)*sqrt(3) + 52*sqrt(-109/161304*I*sqrt(13)*sqrt(
3) - 5167/322608)) - 10003584*sqrt(40326)*(-I*sqrt(13)*sqrt(3) + 52*sqrt(10
9/161304*I*sqrt(13)*sqrt(3) - 5167/322608)) + 13733824*sqrt(40326))*sqrt(-1
20978*(1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) -
5167/322608))^2 - 120978*(-1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(-109/161304*
I*sqrt(13)*sqrt(3) - 5167/322608))^2 - 1551/208*(I*sqrt(13)*sqrt(3) + 52*sq
rt(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))*(-I*sqrt(13)*sqrt(3) + 52
*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608)) - 2455) - 25602357/2*I*

```
sqrt(13)*sqrt(3) + 670925235*x + 665661282*sqrt(109/161304*I*sqrt(13)*sqrt(
3) - 5167/322608) + 320161368)
```

Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.16

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$$
$$=\text{RootSum}\left(50326848t^4+765960t^2+12753t+64,\left(t\mapsto t\log\left(\frac{100785893208t^3}{4758335}-\frac{1430512512t^2}{4758335}+\frac{7298\ldots}{223\ldots}\right.\right.\right.$$

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8),x)
[Out] RootSum(50326848*_t**4 + 765960*_t**2 + 12753*_t + 64, Lambda(_t, _t*log(10
0785893208*_t**3/4758335 - 1430512512*_t**2/4758335 + 72982352521*_t/223641
745 + x + 2270349121/1789133960)))
```

Maxima [F]

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx=\int \frac{1}{8\,x^4-15\,x^3+8\,x^2+24\,x+8}\,dx$$

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="maxima")
[Out] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Giac [F]

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx=\int \frac{1}{8\,x^4-15\,x^3+8\,x^2+24\,x+8}\,dx$$

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="giac")
[Out] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.47

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$$

$$=\sum_{k=1}^{4}\ln\left(-\frac{\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)\left(2184\,\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)+256\,x\right.}{}\right.$$

$$+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\Big)$$

```
[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8),x)
```

```
[Out] symsum(log(-(root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k
)*(2184*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k) + 2
56*x + 38259*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k
)*x + 1531920*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z,
k)^2*x + 805896*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z
, k)^2 - 120))/4096)*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/7863
57, z, k), k, 1, 4)
```

3.62 $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx$

Optimal result 565
Rubi [A] (verified) 566
Mathematica [C] (verified) 571
Maple [C] (verified) 572
Fricas [C] (verification not implemented) 572
Sympy [B] (verification not implemented) 575
Maxima [F] 581
Giac [F] 582
Mupad [B] (verification not implemented) 582

Optimal result

Integrand size = 22, antiderivative size = 366

$$\begin{aligned}
&\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx \\
&= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)}+\frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} \\
&\quad-\frac{\sqrt{\frac{19+\sqrt{517}}{40326}}\left(1678181+74897\sqrt{517}\right)\arctan\left(\frac{6-\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}}{\sqrt{2\left(-19+\sqrt{517}\right)}}\right)}{645216} \\
&\quad-\frac{\sqrt{\frac{19+\sqrt{517}}{40326}}\left(1678181+74897\sqrt{517}\right)\arctan\left(\frac{6+\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}}{\sqrt{2\left(-19+\sqrt{517}\right)}}\right)}{645216} \\
&\quad+\frac{73}{208}\sqrt{\frac{3}{13}}\arctan\left(\frac{8+12x-5x^2}{\sqrt{39}x^2}\right) \\
&\quad-\frac{\sqrt{\frac{-59644114671451+2623170438295\sqrt{517}}{40326}}\log\left(\sqrt{517}-\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right)}{645216} \\
&\quad+\frac{\sqrt{\frac{-59644114671451+2623170438295\sqrt{517}}{40326}}\log\left(\sqrt{517}+\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right)}{645216}
\end{aligned}$$

```
[Out] -3/208*(3359-107*(3+4/x)^2)/(517-38*(3+4/x)^2+(3+4/x)^4)+1/322608*(3327931-
129631*(3+4/x)^2)*(3+4/x)/(517-38*(3+4/x)^2+(3+4/x)^4)+73/2704*arctan(1/39*
(-5*x^2+12*x+8)/x^2*39^(1/2))*39^(1/2)-1/26018980416*arctan((6+8/x-(38+2*51
7^(1/2))^(1/2))/(-38+2*517^(1/2))^(1/2))*(1678181+74897*517^(1/2))*(766194+
```

```
40326*517^(1/2))^(1/2)-1/26018980416*arctan((6+8/x+(38+2*517^(1/2))^(1/2))/
(-38+2*517^(1/2))^(1/2))*(1678181+74897*517^(1/2))*(766194+40326*517^(1/2))
^(1/2)-1/26018980416*ln((3+4/x)^2+517^(1/2)-(3+4/x)*(38+2*517^(1/2))^(1/2))
*(-240520856824093302 6+10578197109468417 0*517^(1/2))^(1/2)+1/26018980416*ln
((3+4/x)^2+517^(1/2)+(3+4/x)*(38+2*517^(1/2))^(1/2))*(-240520856824093302 6+
10578197109468417 0*517^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2094, 12, 1687, 1692, 1183, 648, 632, 210, 642, 1677, 1674}

$$\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^2}\,dx$$
$$= \frac{73}{208}\sqrt{\frac{3}{13}}\arctan\left(\frac{-5x^2+12x+8}{\sqrt{39}x^2}\right)$$
$$-\frac{\sqrt{\frac{19+\sqrt{517}}{40326}}\left(1678181+74897\sqrt{517}\right)\arctan\left(\frac{\frac{8}{x}-\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)}{645216}$$
$$-\frac{\sqrt{\frac{19+\sqrt{517}}{40326}}\left(1678181+74897\sqrt{517}\right)\arctan\left(\frac{\frac{8}{x}+\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)}{645216}$$
$$-\frac{3\left(3359-107\left(\frac{4}{x}+3\right)^2\right)}{208\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}+\frac{\left(3327931-129631\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)}{322608\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}$$
$$-\frac{\sqrt{\frac{2623170438295\sqrt{517}-59644114671451}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2-\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)}{645216}$$
$$+\frac{\sqrt{\frac{2623170438295\sqrt{517}-59644114671451}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2+\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)}{645216}$$

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2),x]
```

```
[Out] (-3*(3359 - 107*(3 + 4/x)^2))/(208*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)) +
((3327931 - 129631*(3 + 4/x)^2)*(3 + 4/x))/(322608*(517 - 38*(3 + 4/x)^2 +
(3 + 4/x)^4)) - (Sqrt[(19 + Sqrt[517])/40326]*(1678181 + 74897*Sqrt[517])*A
rcTan[(6 - Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/6452
16 - (Sqrt[(19 + Sqrt[517])/40326]*(1678181 + 74897*Sqrt[517])*ArcTan[(6 +
Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/645216 + (73*Sq
rt[3/13]*ArcTan[(8 + 12*x - 5*x^2)/(Sqrt[39]*x^2)])/208 - (Sqrt[(-596441146
```

```
71451 + 2623170438295*Sqrt[517])/40326]*Log[Sqrt[517] - Sqrt[2*(19 + Sqrt[5
17])]*(3 + 4/x) + (3 + 4/x)^2])/645216 + (Sqrt[(-59644114671451 + 262317043
8295*Sqrt[517])/40326]*Log[Sqrt[517] + Sqrt[2*(19 + Sqrt[517])]*(3 + 4/x) +
 (3 + 4/x)^2])/645216
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\left(1024\text{Subst}\left(\int \frac{(24-32x)^6}{64\left(2117632-2490368x^2+1048576x^4\right)^2}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(16\text{Subst}\left(\int \frac{(24-32x)^6}{\left(2117632-2490368x^2+1048576x^4\right)^2}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(16\text{Subst}\left(\int \frac{x(-1528823808-9059696640x^2-4831838208x^4)}{\left(2117632-2490368x^2+1048576x^4\right)^2}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right) - 16\text{Subst}\left(\int \frac{191102976+5096079360x^2+9059696640x^4+1073741824x^6}{\left(2117632-2490368x^2+1048576x^4\right)^2}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)$$

$$= \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} - \frac{\text{Subst}\left(\int \frac{1209256852201639415644 16+863503619305390179614 72x^2}{2117632-2490368x^2+1048576x^4}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)}{709422494427119616} - 8\text{Subst}\left(\int \frac{-1528823808-9059696640x-4831838208x^2}{\left(2117632-2490368x+1048576x^2\right)^2}\,dx, x, \left(\frac{3}{4}+\frac{1}{x}\right)^2\right)$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} + \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} - \frac{\text{Subst}\left(\int -\frac{46232264924725248}{2117632-2490368x+1048576x^2}\,dx, x, \left(\frac{3}{4}+\frac{1}{x}\right)^2\right)}{335007449088}$$
$$- \frac{\text{Subst}\left(\int \frac{302314213050409853911 04\sqrt{2\left(19+\sqrt{517}\right)}-\left(120925685220163941564416-53968976206586886225 92\sqrt{517}\right)x}{\frac{\sqrt{517}}{16}-\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}x+x^2}\,dx,}{232463562973878555770 88\sqrt{1034\left(19+\sqrt{517}\right)}}$$
$$- \frac{\text{Subst}\left(\int \frac{302314213050409853911 04\sqrt{2\left(19+\sqrt{517}\right)}+\left(120925685220163941564416-53968976206586886225 92\sqrt{517}\right)x}{\frac{\sqrt{517}}{16}+\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}x+x^2}\,dx,}{232463562973878555770 88\sqrt{1034\left(19+\sqrt{517}\right)}}$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)}+\frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)}$$

$$+\frac{1794048}{13}\text{Subst}\left(\int\frac{1}{2117632-2490368x+1048576x^2}\,dx,x,\left(\frac{3}{4}+\frac{1}{x}\right)^2\right)$$

$$+\frac{\left(1678181-74897\sqrt{517}\right)\text{Subst}\left(\int\frac{-\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}+2x}{\frac{\sqrt{517}}{16}-\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}x+x^2}\,dx,x,\frac{3}{4}+\frac{1}{x}\right)}{645216\sqrt{1034\left(19+\sqrt{517}\right)}}$$

$$-\frac{\left(1678181-74897\sqrt{517}\right)\text{Subst}\left(\int\frac{\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}+2x}{\frac{\sqrt{517}}{16}+\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}x+x^2}\,dx,x,\frac{3}{4}+\frac{1}{x}\right)}{645216\sqrt{1034\left(19+\sqrt{517}\right)}}$$

$$-\frac{\left(38721749+1678181\sqrt{517}\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{517}}{16}-\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}x+x^2}\,dx,x,\frac{3}{4}+\frac{1}{x}\right)}{1334306688}$$

$$-\frac{\left(38721749+1678181\sqrt{517}\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{517}}{16}+\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}x+x^2}\,dx,x,\frac{3}{4}+\frac{1}{x}\right)}{1334306688}$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)}+\frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)}$$

$$-\frac{\sqrt{-\frac{59644114671451}{40326}+\frac{5073830635\sqrt{517}}{78}}\log\left(\sqrt{517}-\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right)}{645216}$$

$$+\frac{\sqrt{-\frac{59644114671451}{40326}+\frac{5073830635\sqrt{517}}{78}}\log\left(\sqrt{517}+\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right)}{645216}$$

$$-\frac{3588096}{13}\text{Subst}\left(\int\frac{1}{-2680059592704-x^2}\,dx,x,-2490368+2097152\left(\frac{3}{4}+\frac{1}{x}\right)^2\right)$$

$$+\frac{\left(38721749+1678181\sqrt{517}\right)\text{Subst}\left(\int\frac{1}{\frac{1}{8}\left(19-\sqrt{517}\right)-x^2}\,dx,x,-\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}+2\left(\frac{3}{4}+\frac{1}{x}\right)\right)}{667153344}$$

$$+\frac{\left(38721749+1678181\sqrt{517}\right)\text{Subst}\left(\int\frac{1}{\frac{1}{8}\left(19-\sqrt{517}\right)-x^2}\,dx,x,\frac{1}{4}\left(6+\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}\right)\right)}{667153344}$$

$$
\begin{aligned}
&= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} \\
&+ \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)}-\frac{73}{208}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{19-\left(3+\frac{4}{x}\right)^2}{2\sqrt{39}}\right) \\
&- \frac{\left(1678181+74897\sqrt{517}\right)\tan^{-1}\left(\frac{6+\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}}{\sqrt{2\left(-19+\sqrt{517}\right)}}\right)}{322608\sqrt{1034\left(-19+\sqrt{517}\right)}} \\
&- \frac{\left(1678181+74897\sqrt{517}\right)\tan^{-1}\left(\frac{8+\left(6-\sqrt{2\left(19+\sqrt{517}\right)}\right)x}{\sqrt{2\left(-19+\sqrt{517}\right)}x}\right)}{322608\sqrt{1034\left(-19+\sqrt{517}\right)}} \\
&- \frac{\sqrt{-\frac{59644114671451}{40326}+\frac{5073830635\sqrt{517}}{78}}\log\left(\sqrt{517}-\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right)}{645216} \\
&+ \frac{\sqrt{-\frac{59644114671451}{40326}+\frac{5073830635\sqrt{517}}{78}}\log\left(\sqrt{517}+\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right)}{645216}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.35

$$
\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx = \frac{72888+89033x-94314x^2+39280x^3}{161304\left(8+24x+8x^2-15x^3+8x^4\right)} + \frac{\text{RootSum}\left[8+24\#1+8\#1^2-15\#1^3+8\#1^4\&,\frac{74897\log(x-\#1)-57489\log(x-\#1)\#1+19640\log(x-\#1)\#1^2}{24+16\#1-45\#1^2+32\#1^3}\&\right]}{80652}
$$

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2),x]
```

```
[Out] (72888 + 89033*x - 94314*x^2 + 39280*x^3)/(161304*(8 + 24*x + 8*x^2 - 15*x^
3 + 8*x^4)) + RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , (74897*Log[
x - #1] - 57489*Log[x - #1]*#1 + 19640*Log[x - #1]*#1^2)/(24 + 16*#1 - 45*#
1^2 + 32*#1^3) & ]/80652
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.26

method	result
default	$\frac{\frac{2455}{80652}x^3-\frac{1429}{19552}x^2+\frac{89033}{1290432}x+\frac{3037}{53768}}{x^4-\frac{15}{8}x^3+x^2+3x+1}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)}\frac{\left(19640\_R^2-57489\_R+74897\right)\ln\left(x-\_R\right)}{32\_R^3-45\_R^2+16\_R+24}\right)}{80652}$
risch	$\frac{\frac{2455}{80652}x^3-\frac{1429}{19552}x^2+\frac{89033}{1290432}x+\frac{3037}{53768}}{x^4-\frac{15}{8}x^3+x^2+3x+1}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)}\frac{\left(19640\_R^2-57489\_R+74897\right)\ln\left(x-\_R\right)}{32\_R^3-45\_R^2+16\_R+24}\right)}{80652}$

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (2455/80652*x^3-1429/19552*x^2+89033/1290432*x+3037/53768)/(x^4-15/8*x^3+x^
2+3*x+1)+1/80652*sum((19640*_R^2-57489*_R+74897)/(32*_R^3-45*_R^2+16*_R+24)
*ln(x-_R),_R=RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 1540, normalized size of antiderivative = 4.21

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")
```

```
[Out] 1/26018980416*(6336021120*x^3 - 4811202*(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)
*(-73*I*sqrt(13)*sqrt(3) + 2704*sqrt(537508757/26903625750144*I*sqrt(13)*sq
rt(3) - 59644114671451/2098482808511232))*log(-131155175531952*(21770028828
7626155772963*I*sqrt(13)*sqrt(3) + 806385725383207020835742 4*sqrt(-53750875
7/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232) - 29
045321766899257717 12)*(73/5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt(537508757/2690
3625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232))^2 + 21152
3322718189916535976363749069682 3296*(-73/5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt
(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/209848280851
1232))^3 - 801867*(48777466142704809483333222 0336*(-73/5408*I*sqrt(13)*sqrt
(3) - 1/2*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 5964411467145
1/2098482808511232))^2 + 32817075305772686518 99)*(-73*I*sqrt(13)*sqrt(3) +
2704*sqrt(537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098
482808511232)) - 324626519616115661477605155278 4488493/4*I*sqrt(13)*sqrt(3)
```

+ 15093053140299407988153390321526 5*x - 300613051041772859121727513655111 5
3716*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/209
8482808511232) + 1090507114917617311013907313810175 2) - 4811202*(8*x^4 - 15
*x^3 + 8*x^2 + 24*x + 8)*(73*I*sqrt(13)*sqrt(3) + 2704*sqrt(-537508757/2690
3625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232))*log(-1692
18658174551933228781090999255745863 68*(-73/5408*I*sqrt(13)*sqrt(3) - 1/2*sq
rt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808
511232))^3 + 30475554197757586862436544297099345 92*(-73/5408*I*sqrt(13)*sqr
t(3) - 1/2*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 596441146714
51/2098482808511232))^2 + 649252885553041733545293676366844411 4*I*sqrt(13)*
sqrt(3) + 120744425122395263905227122572212 0*x + 240490383908962307877599 19
1903554423072*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 596441146
71451/2098482808511232) - 907404798791025007870824774437492 95) - 1521322545
6*x^2 - (8*sqrt(20163)*(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)*sqrt(-3934655265
95856*(73/5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt(537508757/26903625750144*I*sqr
t(13)*sqrt(3) - 59644114671451/2098482808511232))^2 - 393465526595856*(-73/
5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt(-537508757/26903625750144*I*sqrt(13)*sqr
t(3) - 59644114671451/2098482808511232))^2 - 3731087151/416*(73*I*sqrt(13)*
sqrt(3) + 2704*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114
671451/2098482808511232))*(-73*I*sqrt(13)*sqrt(3) + 2704*sqrt(537508757/269
03625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232)) - 149116
25619311) - 2405601*(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)*(73*I*sqrt(13)*sqrt
(3) + 2704*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 596441146714
51/2098482808511232)) - 2405601*(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)*(-73*I*
sqrt(13)*sqrt(3) + 2704*sqrt(537508757/26903625750144*I*sqrt(13)*sqrt(3) -
59644114671451/2098482808511232)))*log(196732763297928*(2177002882876261557
72963*I*sqrt(13)*sqrt(3) + 806385725383207020835742 4*sqrt(-537508757/269036
25750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232) - 2904532176
689925771712)*(73/5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt(537508757/269036257501
44*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232))^2 - 5714166412079
5475367068520557061273 6*(-73/5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt(-537508757/
26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232))^2 + 2
405601/2*(487774661427048094833332220336*(-73/5408*I*sqrt(13)*sqrt(3) - 1/2
*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482
808511232))^2 + 32817075305772686518 99)*(-73*I*sqrt(13)*sqrt(3) + 2704*sqrt
(537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511
232)) + 1/208*sqrt(-393465526595856*(73/5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt(
537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/20984828085112
32))^2 - 393465526595856*(-73/5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt(-537508757
/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232))^2 -
3731087151/416*(73*I*sqrt(13)*sqrt(3) + 2704*sqrt(-537508757/26903625750144
*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232))*(-73*I*sqrt(13)*sqr
t(3) + 2704*sqrt(537508757/26903625750144*I*sqrt(13)*sqrt(3) - 596441146714
51/2098482808511232)) - 14911625619311)*(4653*(29821957299674815859 31*sqrt(
20163)*(73*I*sqrt(13)*sqrt(3) + 2704*sqrt(-537508757/26903625750144*I*sqrt(

13)*sqrt(3) - 59644114671451/2098482808511232)) - 2904532176689925771712*sq
rt(20163))*(-73*I*sqrt(13)*sqrt(3) + 2704*sqrt(537508757/26903625750144*I*s
qrt(13)*sqrt(3) - 59644114671451/2098482808511232)) - 135147882181382246157
75936*sqrt(20163)*(73*I*sqrt(13)*sqrt(3) + 2704*sqrt(-537508757/26903625750
144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232)) - 27303806654402
87518379968*sqrt(20163)) + 5762969609602871873782126998 27/2*I*sqrt(13)*sqrt
(3) + 4527915942089822396446017096457 95*x + 106733354961412092785663502762 4
8*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/209848
2808511232) + 231741133996538382702540710757432) + (8*sqrt(20163)*(8*x^4 -
15*x^3 + 8*x^2 + 24*x + 8)*sqrt(-393465526595856*(73/5408*I*sqrt(13)*sqrt(3
) - 1/2*sqrt(537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2
098482808511232))^2 - 393465526595856*(-73/5408*I*sqrt(13)*sqrt(3) - 1/2*sq
rt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808
511232))^2 - 3731087151/416*(73*I*sqrt(13)*sqrt(3) + 2704*sqrt(-537508757/2
6903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232))*(-73*I
*sqrt(13)*sqrt(3) + 2704*sqrt(537508757/26903625750144*I*sqrt(13)*sqrt(3) -
 59644114671451/2098482808511232)) - 14911625619311) + 2405601*(8*x^4 - 15*
x^3 + 8*x^2 + 24*x + 8)*(73*I*sqrt(13)*sqrt(3) + 2704*sqrt(-537508757/26903
625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232)) + 2405601*
(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)*(-73*I*sqrt(13)*sqrt(3) + 2704*sqrt(537
508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232)
))*log(196732763297928*(21770028828762615577 2963*I*sqrt(13)*sqrt(3) + 80638
57253832070208357424*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59
644114671451/2098482808511232) - 2904532176689925771712)*(73/5408*I*sqrt(13
)*sqrt(3) - 1/2*sqrt(537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114
671451/2098482808511232))^2 - 571416641207954753670685205570612736*(-73/540
8*I*sqrt(13)*sqrt(3) - 1/2*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3
) - 59644114671451/2098482808511232))^2 + 2405601/2*(48777466142704809483 33
32220336*(-73/5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt(-537508757/26903625750144*
I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232))^2 + 3281707530577268
651899)*(-73*I*sqrt(13)*sqrt(3) + 2704*sqrt(537508757/26903625750144*I*sqrt
(13)*sqrt(3) - 59644114671451/2098482808511232)) - 1/208*sqrt(-393465526595
856*(73/5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt(537508757/26903625750144*I*sqrt(
13)*sqrt(3) - 59644114671451/2098482808511232))^2 - 393465526595856*(-73/54
08*I*sqrt(13)*sqrt(3) - 1/2*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(
3) - 59644114671451/2098482808511232))^2 - 3731087151/416*(73*I*sqrt(13)*sq
rt(3) + 2704*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 5964411467
1451/2098482808511232))*(-73*I*sqrt(13)*sqrt(3) + 2704*sqrt(537508757/26903
625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232)) - 14911625
619311)*(4653*(29821957299674815859 31*sqrt(20163)*(73*I*sqrt(13)*sqrt(3) +
2704*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/209
8482808511232)) - 2904532176689925771712*sqrt(20163))*(-73*I*sqrt(13)*sqrt(
3) + 2704*sqrt(537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451
/2098482808511232)) - 13514788218138224615775936*sqrt(20163)*(73*I*sqrt(13)
*sqrt(3) + 2704*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 5964411

```
4671451/2098482808511232)) - 27303806654402875183799 68*sqrt(20163)) + 57629
696096028718737821269982 7/2*I*sqrt(13)*sqrt(3) + 4527915942089822396446 0170
9645795*x + 10673335496141209278566350276248*sqrt(-537508757/26903625750144
*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232) + 231741133996538382
702540710757432) + 14361379032*x + 11757125952)/(8*x^4 - 15*x^3 + 8*x^2 + 2
4*x + 8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3839 vs. 2(292) = 584.

Time = 2.23 (sec) , antiderivative size = 3839, normalized size of antiderivative = 10.49

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
```

```
[Out] (39280*x**3 - 94314*x**2 + 89033*x + 72888)/(1290432*x**4 - 2419560*x**3 +
1290432*x**2 + 3871296*x + 1290432) + sqrt(-59644114671451/1678786246808985
6 + 5073830635*sqrt(517)/32471687559168)*log(x**2 + x*(-1123969950204685033
06932567484755463/603722125611976319526135612861060 - 296438698298128332309
07750777733957*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))/
19364193987923944616378551419122383960 80 - 18153326104312036 0732*sqrt(-7120
427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) +
626362156858715004293 5*sqrt(517) + 35575799716919912947693826 75)/1509305314
029940798815339032152 65 - 46926347979646613249222*sqrt(517)/297468603626329
12338339 + 994065243322493861977*sqrt(78)*sqrt(-59644114671451 + 2623170438
295*sqrt(517))/1427849297406379792240272 + 994065243322493861977*sqrt(40326
)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-7120427417275887*sq
rt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715
0042935*sqrt(517) + 35575799716919912947693826 75)/12909462658615963077585 70
094608158930720) - 4597149706773066968921854791223560238809189313591735 1760
29*sqrt(517)*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623
170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919912
94769382675)/18432767186998698626450604048374763890148748053806275728419558
40 - 10221327637202671758827804250636131310886019359583038780811587109497 15
459967411486447/302201812380681690634631534385892067350866441656553353409 62
4708723614680800 - 106380947173342801261761115266827766437283828355653383 69
93*sqrt(78)*sqrt(-59644114671451 + 2623170438295*sqrt(517))/689619370306997
243674472351962660786270618994938298086 6560 - 89036038929850064673184559559
30346703260445958708241693 13*sqrt(40326)*sqrt(-59644114671451 + 26231704382
95*sqrt(517))*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 262
3170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991
294769382675)/93524738841216770796017496134898898986728492582298910146112 09
```

554309158400 - 451139763274888093250945016338260146717 91*sqrt(78)*sqrt(-596
44114671451 + 2623170438295*sqrt(517))*sqrt(-7120427417275887*sqrt(40326)*s
qrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715004 2935*sqr
t(517) + 355757997169199129476938 2675)/10775302661046831932413630499416574 7
8547842179591090760 40 + 4269800963651546871890094273427400521225528229955 28
023712838213081212 07*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(
517))/935247388412167707960174961348988989867284925822989101461120955430915
8400 + 685487767096696740818928514074132093732180070609343531371525732094 05
073*sqrt(517)/460819179674967465661265101209369097253718701345156893210488 9
60 + 274196431933554153007434570764680602132735098624644758583157528631167 0
33*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*
sqrt(517)) + 626362156858715004 2935*sqrt(517) + 355757997169199129476938267
5)/4835228998090907050154104550174273077613863066504853654553995339577834 89
2800) - sqrt(-59644114671451/16787862468089856 + 5073830635*sqrt(517)/32471
687559168)*log(x**2 + x*(-112396995020468503306932567484755463/603722125611
976319526135612861060 - 9940652433224938 61977*sqrt(78)*sqrt(-59644114671451
 + 2623170438295*sqrt(517))/142784929740637979 2240272 - 469263479796466132 4
9222*sqrt(517)/29746860362632912338339 + 181533261043120360732*sqrt(7120427
417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626
362156858715004 2935*sqrt(517) + 355757997169199129476938 2675)/1509305314029
940798815339032152 65 + 296438698298128332309077507777339 57*sqrt(40326)*sqrt
(-59644114671451 + 2623170438295*sqrt(517))/193641939879239446163785514191 2
238396080 + 9940652433224938 61977*sqrt(40326)*sqrt(-59644114671451 + 262317
0438295*sqrt(517))*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 +
 2623170438295*sqrt(517)) + 626362156858715004 2935*sqrt(517) + 355757997169
199129476938 2675)/1290946265861596307758570094608158930720) - 2741964319335
541530074345707646806021327350986246447585831575286311670 33*sqrt(7120427417
275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362
156858715004 2935*sqrt(517) + 355757997169199129476938 2675)/4835228998090907
050154104550174273077613863066504853654553995339577834892800 - 102213276372
026717588278042506361313108860193595830387808115871094971545996741148 6447/3
022018123806816906346315343858920673508664416565533534096247087236146808 00
- 890360389298500646731845595593034670326044595870824169313*sqrt(40326)*sqr
t(-59644114671451 + 2623170438295*sqrt(517))*sqrt(7120427417275887*sqrt(403
26)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215685871500429 3
5*sqrt(517) + 355757997169199129476938 2675)/935247388412167707960174961348 9
88989867284925822989101461120955430915 8400 - 4269800963651546871890094273 42
740052122552822995528023712838213081212 07*sqrt(40326)*sqrt(-59644114671451
+ 2623170438295*sqrt(517))/935247388412167707960174961348988989867284925822
98910146112095543091 58400 - 451139763274888093250945016338260146717 91*sqrt(
78)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(7120427417275887*s
qrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215685871
50042935*sqrt(517) + 355757997169199129476938 2675)/10775302661046831932413 6
304994165747854784217959109076040 + 10638094717334280126176111526682776643 7
283828355653383699 3*sqrt(78)*sqrt(-59644114671451 + 2623170438295*sqrt(517)

)/6896193703069972436744723519626607862706189949382980866560 + 685487767096
69674081892851407413209373218007060934353137152573209405073*sqrt(517)/46081
9179674967465661265101209369097253718701345156893210488960 + 45971497067730
66968921854791223560238809189313591735176029*sqrt(517)*sqrt(712042741727588
7*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215685
87150042935*sqrt(517) + 3557579971691991294769382675)/184327671869986986264
5060404837476389014874805380627572841955840) - 2*sqrt(59653665894623/167878
62468089856 + 5073830635*sqrt(517)/10823895853056 + sqrt(-7120427417275887*
sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587
150042935*sqrt(517) + 3557579971691991294769382675)/4196965617022464)*atan(
-7745677595169577846551420567648953584320*x/(-59292486929118917272637172801
533436*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-
7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)
) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 232
9048502925820708785386304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*s
qrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqr
t(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150
042935*sqrt(517) + 3557579971691991294769382675)) + 994065243322493861977*s
qrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-71204274
17275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263
621568587150042935*sqrt(517) + 3557579971691991294769382675))*sqrt(-7120427
417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626
3621568587150042935*sqrt(517) + 3557579971691991294769382675)) - 2982195729
967481585931*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sq
rt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(
517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/(-
59292486929118917272637172801533436*sqrt(40326)*sqrt(59653665894623 + 78695
11314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671
451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579
971691991294769382675)) + 2329048502925820708785386304*sqrt(-59644114671451
 + 2623170438295*sqrt(517))*sqrt(59653665894623 + 7869511314885*sqrt(517) +
 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*
sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997169199129476938267
5)) + 994065243322493861977*sqrt(40326)*sqrt(59653665894623 + 7869511314885
*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 26
23170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997169199
1294769382675))*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2
623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919
91294769382675)) - 2696261047060775175517112572266328310*sqrt(78)*sqrt(-596
44114671451 + 2623170438295*sqrt(517))/(-5929248692911891727263717280153343
6*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-71204
27417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6
263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 23290485
02925820708785386304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(5
9653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(403

26)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919912947693826 75)) + 994065243322493861977*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675))*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 6109491182230238698537149348154570111680*sqrt(517)/(-59292486929118917272637172801533436*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 232904850292582070878538 6304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 994065243322493861977*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675))*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 4658097005851641417570772608*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/(-59292486929118917272637172801533436*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 2329048502925820708785386304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 994065243322493861977*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675))*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 5928773965962566646181550155546791 4*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))/(-59292486929118917272637172801533436*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 2329048502925820708785386304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 994065243322493861977*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517

)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675))*sqrt
(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(51
7)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 7
21019529648624138729760776730387270795368/(-5929248692911891727263717280153
3436*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-71
20427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))
+ 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 23290
48502925820708785386304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqr
t(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(
40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715004
2935*sqrt(517) + 3557579971691991294769382675)) + 994065243322493861977*sqr
t(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417
275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362
1568587150042935*sqrt(517) + 3557579971691991294769382675))*sqrt(-712042741
7275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636
21568587150042935*sqrt(517) + 3557579971691991294769382675))) - 2*sqrt(-sqr
t(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(51
7)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/4196
965617022464 + 59653665894623/16787862468089856 + 5073830635*sqrt(517)/1082
3895853056)*atan(7745677595169577846551420567648953584320*x/(23290485029258
20708785386304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-4*sqrt
(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517
)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675) + 596
53665894623 + 7869511314885*sqrt(517)) + 5929248692911891727263717280153343
6*sqrt(40326)*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-5964411467145
1 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997
1691991294769382675) + 59653665894623 + 7869511314885*sqrt(517)) + 99406524
3322493861977*sqrt(40326)*sqrt(7120427417275887*sqrt(40326)*sqrt(-596441146
71451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575
79971691991294769382675)*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59
644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517)
+ 3557579971691991294769382675) + 59653665894623 + 7869511314885*sqrt(517))
) - 721019529648624138729760776730387270795368/(232904850292582070878538630
4*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-4*sqrt(712042741727
5887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215
68587150042935*sqrt(517) + 3557579971691991294769382675) + 59653665894623 +
 7869511314885*sqrt(517)) + 59292486929118917272637172801533436*sqrt(40326)
*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 262317043
8295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769
382675) + 59653665894623 + 7869511314885*sqrt(517)) + 994065243322493861977
*sqrt(40326)*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 26231
70438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997169199129
4769382675)*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451
+ 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716
91991294769382675) + 59653665894623 + 7869511314885*sqrt(517))) - 269626104

7060775175517112572266328310*sqrt(78)*sqrt(-59644114671451 + 2623170438295*
sqrt(517))/(2329048502925820708785386304*sqrt(-59644114671451 + 26231704382
95*sqrt(517))*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-5964411467145
1 + 2623170438295*sqrt(517)) + 626362156858715004293 5*sqrt(517) + 355757997
1691991294769382675) + 59653665894623 + 7869511314885*sqrt(517)) + 59292486
9291189172726371728015334 36*sqrt(40326)*sqrt(-4*sqrt(7120427417275887*sqrt(
40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715004
2935*sqrt(517) + 3557579971691991294769382675) + 59653665894623 + 786951131
4885*sqrt(517)) + 994065243322493861977*sqrt(40326)*sqrt(7120427417275887*s
qrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215685871
50042935*sqrt(517) + 3557579971691991294769382675)*sqrt(-4*sqrt(71204274172
75887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621
568587150042935*sqrt(517) + 3557579971691991294769382675) + 59653665894623
+ 7869511314885*sqrt(517))) - 61094911822302386985371493481545701116 80*sqrt
(517)/(2329048502925820708785386304*sqrt(-59644114671451 + 2623170438295*sq
rt(517))*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2
623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919
91294769382675) + 59653665894623 + 7869511314885*sqrt(517)) + 5929248692911
891727263717280153343 6*sqrt(40326)*sqrt(-4*sqrt(7120427417275887*sqrt(40326
)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*
sqrt(517) + 3557579971691991294769382675) + 59653665894623 + 7869511314885*
sqrt(517)) + 994065243322493861977*sqrt(40326)*sqrt(7120427417275887*sqrt(4
0326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715004 2
935*sqrt(517) + 3557579971691991294769382675)*sqrt(-4*sqrt(7120427417275887
*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858
7150042935*sqrt(517) + 3557579971691991294769382675) + 59653665894623 + 786
9511314885*sqrt(517))) + 4658097005851641417570772608*sqrt(7120427417275887
*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858
7150042935*sqrt(517) + 3557579971691991294769382675)/(232904850292582070878
5386304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-4*sqrt(712042
7417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62
63621568587150042935*sqrt(517) + 3557579971691991294769382675) + 5965366589
4623 + 7869511314885*sqrt(517)) + 592924869291189172726371728015334 36*sqrt(
40326)*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 262
3170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991
294769382675) + 59653665894623 + 7869511314885*sqrt(517)) + 994065243322493
861977*sqrt(40326)*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 +
 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997169
1991294769382675)*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-596441146
71451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575
79971691991294769382675) + 59653665894623 + 7869511314885*sqrt(517))) + 592
87739659625666461815501555467914*sqrt(40326)*sqrt(-59644114671451 + 2623170
438295*sqrt(517))/(2329048502925820708785386304*sqrt(-59644114671451 + 2623
170438295*sqrt(517))*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-596441
14671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35

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57579971691991294769382675) + 59653665894623 + 7869511314885*sqrt(517)) + 5
929248692911891727263717280153343 6*sqrt(40326)*sqrt(-4*sqrt(712042741727588
7*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215685
87150042935*sqrt(517) + 35575799716919912947693826 75) + 59653665894623 + 78
69511314885*sqrt(517)) + 994065243322493861977*sqrt(40326)*sqrt(71204274172
75887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621
568587150042935*sqrt(517) + 35575799716919912947693826 75)*sqrt(-4*sqrt(7120
427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) +
6263621568587150042935*sqrt(517) + 35575799716919912947693826 75) + 59653665
894623 + 7869511314885*sqrt(517))) + 298219572996748158593 1*sqrt(40326)*sqr
t(-59644114671451 + 2623170438295*sqrt(517))*sqrt(7120427417275887*sqrt(403
26)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715004293
5*sqrt(517) + 35575799716919912947693826 75)/(2329048502925820708785386304*s
qrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-4*sqrt(712042741727588
7*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215685
87150042935*sqrt(517) + 35575799716919912947693826 75) + 59653665894623 + 78
69511314885*sqrt(517)) + 5929248692911891727263717280153343 6*sqrt(40326)*sq
rt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 262317043829
5*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919912947693 82
675) + 59653665894623 + 7869511314885*sqrt(517)) + 994065243322493861977*sq
rt(40326)*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 26231704
38295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294 76
9382675)*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2
623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919
91294769382675) + 59653665894623 + 7869511314885*sqrt(517))))
```

Maxima [F]

$$\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^2}\,dx=\int \frac{1}{\left(8\,x^4-15\,x^3+8\,x^2+24\,x+8\right)^2}\,dx$$

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="maxima")
```

```
[Out] 1/161304*(39280*x^3 - 94314*x^2 + 89033*x + 72888)/(8*x^4 - 15*x^3 + 8*x^2
+ 24*x + 8) + 1/80652*integrate((19640*x^2 - 57489*x + 74897)/(8*x^4 - 15*x
^3 + 8*x^2 + 24*x + 8), x)
```

Giac [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^2}\,dx$$

[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="giac")

[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-2), x)

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.49

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx = \frac{\frac{2455\,x^3}{80652}-\frac{1429\,x^2}{19552}+\frac{89033\,x}{1290432}+\frac{3037}{53768}}{x^4-\frac{15\,x^3}{8}+x^2+3\,x+1} + \left(\sum_{k=1}^{4}\ln\left(\frac{2146659825\,\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)}{2960381771776} + \frac{2222183\,x}{338246745408} + \frac{\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)\,x\,924124364159}{26643435945984} - \frac{\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)^2\,x\,72451101}{8470528} - \frac{\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)^3\,x\,95745}{256} + \frac{389551\,\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)^2}{264704} - \frac{100737\,\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)^3}{512} + \frac{271033}{624455529984}\right)\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)\right)$$

[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^2,x)

```
[Out] ((89033*x)/1290432 - (1429*x^2)/19552 + (2455*x^3)/80652 + 3037/53768)/(3*x
 + x^2 - (15*x^3)/8 + x^4 + 1) + symsum(log((2146659825*root(z^4 + (1491162
5619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/
44204510553294663, z, k))/2960381771776 + (2222183*x)/338246745408 + (92412
4364159*root(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/3
730636104019968 + 43023440/44204510553294663, z, k)*x)/26643435945984 - (72
451101*root(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/37
30636104019968 + 43023440/44204510553294663, z, k)^2*x)/8470528 - (95745*ro
ot(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/37306361040
19968 + 43023440/44204510553294663, z, k)^3*x)/256 + (389551*root(z^4 + (14
911625619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 4302
3440/44204510553294663, z, k)^2)/264704 - (100737*root(z^4 + (1491162561931
1*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/442045
10553294663, z, k)^3)/512 + 271033/624455529984)*root(z^4 + (14911625619311
*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/4420451
0553294663, z, k), k, 1, 4)
```

3.63 $\int (a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)$

Optimal result 584
Rubi [A] (verified) 584
Mathematica [A] (verified) 585
Maple [B] (verified) 585
Fricas [B] (verification not implemented) 586
Sympy [B] (verification not implemented) 586
Maxima [B] (verification not implemented) 587
Giac [B] (verification not implemented) 588
Mupad [B] (verification not implemented) 588

Optimal result

Integrand size = 51, antiderivative size = 14

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3\,dx=\frac{(a+bx)^{16}}{16b}$$

```
[Out] 1/16*(b*x+a)^16/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2084, 32}

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3\,dx=\frac{(a+bx)^{16}}{16b}$$

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^3,x]
```

```
[Out] (a + b*x)^16/(16*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2084

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int (a+bx)^{15}\,dx$$
$$= \frac{(a+bx)^{16}}{16b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3\,dx = \frac{(a+bx)^{16}}{16b}$$

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^3,x]
```

```
[Out] (a + b*x)^16/(16*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(12) = 24.

Time = 0.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 11.71

method	result
default	$a^{15}x+\frac{15}{2}a^{14}b\,x^2+35a^{13}b^2x^3+\frac{455}{4}a^{12}b^3x^4+273a^{11}b^4x^5+\frac{1001}{2}a^{10}b^5x^6+715a^9b^6x^7+\frac{6435}{8}a^8$
norman	$a^{15}x+\frac{15}{2}a^{14}b\,x^2+35a^{13}b^2x^3+\frac{455}{4}a^{12}b^3x^4+273a^{11}b^4x^5+\frac{1001}{2}a^{10}b^5x^6+715a^9b^6x^7+\frac{6435}{8}a^8$
risch	$a^{15}x+\frac{15}{2}a^{14}b\,x^2+35a^{13}b^2x^3+\frac{455}{4}a^{12}b^3x^4+273a^{11}b^4x^5+\frac{1001}{2}a^{10}b^5x^6+715a^9b^6x^7+\frac{6435}{8}a^8$
parallelrisch	$a^{15}x+\frac{15}{2}a^{14}b\,x^2+35a^{13}b^2x^3+\frac{455}{4}a^{12}b^3x^4+273a^{11}b^4x^5+\frac{1001}{2}a^{10}b^5x^6+715a^9b^6x^7+\frac{6435}{8}a^8$
gosper	$\frac{x(b^{15}x^{15}+16a\,b^{14}x^{14}+120a^2b^{13}x^{13}+560a^3b^{12}x^{12}+1820a^4b^{11}x^{11}+4368a^5b^{10}x^{10}+8008a^6b^9x^9+11440a^7b^8x^8+12870a^8b^7x^7-}{16}$

```
[In] int((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] a^15*x+15/2*a^14*b*x^2+35*a^13*b^2*x^3+455/4*a^12*b^3*x^4+273*a^11*b^4*x^5+
1001/2*a^10*b^5*x^6+715*a^9*b^6*x^7+6435/8*a^8*b^7*x^8+715*a^7*b^8*x^9+1001
/2*a^6*b^9*x^10+273*a^5*b^10*x^11+455/4*a^4*b^11*x^12+35*a^3*b^12*x^13+15/2
*a^2*b^13*x^14+a*b^14*x^15+1/16*b^15*x^16
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 11.64

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3\,dx = \frac{1}{16}\,b^{15}x^{16}+ab^{14}x^{15}+\frac{15}{2}\,a^2b^{13}x^{14}+35\,a^3b^{12}x^{13}+\frac{455}{4}\,a^4b^{11}x^{12}+273\,a^5b^{10}x^{11}+\frac{1001}{2}\,a^6b^9x^{10}+715\,a^7b^8x^9+\frac{6435}{8}\,a^8b^7x^8+715\,a^9b^6x^7+\frac{1001}{2}\,a^{10}b^5x^6+273\,a^{11}b^4x^5+\frac{455}{4}\,a^{12}b^3x^4+35\,a^{13}b^2x^3+\frac{15}{2}\,a^{14}bx^2+a^{15}x$$

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^3,x, algorithm="fricas")
```

```
[Out] 1/16*b^15*x^16 + a*b^14*x^15 + 15/2*a^2*b^13*x^14 + 35*a^3*b^12*x^13 + 455/
4*a^4*b^11*x^12 + 273*a^5*b^10*x^11 + 1001/2*a^6*b^9*x^10 + 715*a^7*b^8*x^9
 + 6435/8*a^8*b^7*x^8 + 715*a^9*b^6*x^7 + 1001/2*a^10*b^5*x^6 + 273*a^11*b^
4*x^5 + 455/4*a^12*b^3*x^4 + 35*a^13*b^2*x^3 + 15/2*a^14*b*x^2 + a^15*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(8) = 16$.

Time = 0.04 (sec) , antiderivative size = 185, normalized size of antiderivative = 13.21

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3\,dx = a^{15}x+\frac{15a^{14}bx^2}{2}+35a^{13}b^2x^3+\frac{455a^{12}b^3x^4}{4}+273a^{11}b^4x^5+\frac{1001a^{10}b^5x^6}{2}+715a^9b^6x^7+\frac{6435a^8b^7x^8}{8}+715a^7b^8x^9+\frac{1001a^6b^9x^{10}}{2}+273a^5b^{10}x^{11}+\frac{455a^4b^{11}x^{12}}{4}+35a^3b^{12}x^{13}+\frac{15a^2b^{13}x^{14}}{2}+ab^{14}x^{15}+\frac{b^{15}x^{16}}{16}$$

```
[In] integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a*
*4*b*x+a**5)**3,x)
```

```
[Out] a**15*x + 15*a**14*b*x**2/2 + 35*a**13*b**2*x**3 + 455*a**12*b**3*x**4/4 +
273*a**11*b**4*x**5 + 1001*a**10*b**5*x**6/2 + 715*a**9*b**6*x**7 + 6435*a*
*8*b**7*x**8/8 + 715*a**7*b**8*x**9 + 1001*a**6*b**9*x**10/2 + 273*a**5*b**
10*x**11 + 455*a**4*b**11*x**12/4 + 35*a**3*b**12*x**13 + 15*a**2*b**13*x**
14/2 + a*b**14*x**15 + b**15*x**16/16
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(12) = 24$.

Time = 0.21 (sec) , antiderivative size = 592, normalized size of antiderivative = 42.29

$$\int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5\right)^3 \, dx$$
$$= \frac{1}{16}\,b^{15}x^{16} + ab^{14}x^{15} + \frac{75}{14}\,a^2b^{13}x^{14} + \frac{125}{13}\,a^3b^{12}x^{13} + 100\,a^6b^9x^{10} + \frac{1000}{7}\,a^9b^6x^7$$
$$+ \frac{125}{4}\,a^{12}b^3x^4 + a^{15}x + \frac{1}{2}\left(b^5x^6 + 6\,ab^4x^5 + 15\,a^2b^3x^4 + 20\,a^3b^2x^3 + 15\,a^4bx^2\right)a^{10}$$
$$+ \frac{25}{56}\left(21\,b^5x^8 + 120\,ab^4x^7 + 280\,a^2b^3x^6 + 336\,a^3b^2x^5\right)a^8b^2$$
$$+ \frac{5}{3}\left(18\,b^5x^{10} + 100\,ab^4x^9 + 225\,a^2b^3x^8\right)a^6b^4 + \frac{25}{11}\left(11\,b^5x^{12} + 60\,ab^4x^{11}\right)a^4b^6$$
$$+ \frac{1}{462}\left(126\,b^{10}x^{11} + 1386\,ab^9x^{10} + 3850\,a^2b^8x^9 + 19800\,a^4b^6x^7 + 27720\,a^6b^4x^5 + 11550\,a^8b^2x^3 + 330\,(6\,b^5\right.$$
$$+ \frac{5}{308}\left(77\,b^{10}x^{12} + 840\,ab^9x^{11} + 4158\,a^2b^8x^{10} + 12320\,a^3b^7x^9 + 23100\,a^4b^6x^8 + 26400\,a^5b^5x^7 + 15400\,a^6b^4\right.$$
$$+ \frac{5}{429}\left(198\,b^{10}x^{13} + 2145\,ab^9x^{12} + 10530\,a^2b^8x^{11} + 25740\,a^3b^7x^{10} + 28600\,a^4b^6x^9\right)a^3b^2$$
$$+ \frac{5}{182}\left(78\,b^{10}x^{14} + 840\,ab^9x^{13} + 2275\,a^2b^8x^{12}\right)a^2b^3$$

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^15*x^16 + a*b^14*x^15 + 75/14*a^2*b^13*x^14 + 125/13*a^3*b^12*x^13 +
 100*a^6*b^9*x^10 + 1000/7*a^9*b^6*x^7 + 125/4*a^12*b^3*x^4 + a^15*x + 1/2*
(b^5*x^6 + 6*a*b^4*x^5 + 15*a^2*b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^2)*a^
10 + 25/56*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6 + 336*a^3*b^2*x^5)
*a^8*b^2 + 5/3*(18*b^5*x^10 + 100*a*b^4*x^9 + 225*a^2*b^3*x^8)*a^6*b^4 + 25
/11*(11*b^5*x^12 + 60*a*b^4*x^11)*a^4*b^6 + 1/462*(126*b^10*x^11 + 1386*a*b
^9*x^10 + 3850*a^2*b^8*x^9 + 19800*a^4*b^6*x^7 + 27720*a^6*b^4*x^5 + 11550*
a^8*b^2*x^3 + 330*(6*b^5*x^7 + 35*a*b^4*x^6 + 84*a^2*b^3*x^5 + 105*a^3*b^2*
x^4)*a^4*b + 165*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6)*a^3*b^2 + 3
85*(8*b^5*x^9 + 45*a*b^4*x^8)*a^2*b^3)*a^5 + 5/308*(77*b^10*x^12 + 840*a*b^
9*x^11 + 4158*a^2*b^8*x^10 + 12320*a^3*b^7*x^9 + 23100*a^4*b^6*x^8 + 26400*
a^5*b^5*x^7 + 15400*a^6*b^4*x^6)*a^4*b + 5/429*(198*b^10*x^13 + 2145*a*b^9*
x^12 + 10530*a^2*b^8*x^11 + 25740*a^3*b^7*x^10 + 28600*a^4*b^6*x^9)*a^3*b^2
 + 5/182*(78*b^10*x^14 + 840*a*b^9*x^13 + 2275*a^2*b^8*x^12)*a^2*b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 11.64

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3\,dx = \frac{1}{16}\,b^{15}x^{16}+ab^{14}x^{15}+\frac{15}{2}\,a^2b^{13}x^{14}+35\,a^3b^{12}x^{13}+\frac{455}{4}\,a^4b^{11}x^{12} + 273\,a^5b^{10}x^{11}+\frac{1001}{2}\,a^6b^9x^{10}+715\,a^7b^8x^9+\frac{6435}{8}\,a^8b^7x^8+715\,a^9b^6x^7 + \frac{1001}{2}\,a^{10}b^5x^6+273\,a^{11}b^4x^5+\frac{455}{4}\,a^{12}b^3x^4+35\,a^{13}b^2x^3+\frac{15}{2}\,a^{14}bx^2+a^{15}x$$

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^3,x, algorithm="giac")
```

```
[Out] 1/16*b^15*x^16 + a*b^14*x^15 + 15/2*a^2*b^13*x^14 + 35*a^3*b^12*x^13 + 455/
4*a^4*b^11*x^12 + 273*a^5*b^10*x^11 + 1001/2*a^6*b^9*x^10 + 715*a^7*b^8*x^9
 + 6435/8*a^8*b^7*x^8 + 715*a^9*b^6*x^7 + 1001/2*a^10*b^5*x^6 + 273*a^11*b^
4*x^5 + 455/4*a^12*b^3*x^4 + 35*a^13*b^2*x^3 + 15/2*a^14*b*x^2 + a^15*x
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 11.64

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3\,dx = a^{15}\,x+\frac{15\,a^{14}\,b\,x^2}{2}+35\,a^{13}\,b^2\,x^3+\frac{455\,a^{12}\,b^3\,x^4}{4}+273\,a^{11}\,b^4\,x^5+\frac{1001\,a^{10}\,b^5\,x^6}{2} + 715\,a^9\,b^6\,x^7+\frac{6435\,a^8\,b^7\,x^8}{8}+715\,a^7\,b^8\,x^9+\frac{1001\,a^6\,b^9\,x^{10}}{2}+273\,a^5\,b^{10}\,x^{11} + \frac{455\,a^4\,b^{11}\,x^{12}}{4}+35\,a^3\,b^{12}\,x^{13}+\frac{15\,a^2\,b^{13}\,x^{14}}{2}+a\,b^{14}\,x^{15}+\frac{b^{15}\,x^{16}}{16}$$

```
[In] int((a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*
b*x)^3,x)
```

```
[Out] a^15*x + (b^15*x^16)/16 + (15*a^14*b*x^2)/2 + a*b^14*x^15 + 35*a^13*b^2*x^3
 + (455*a^12*b^3*x^4)/4 + 273*a^11*b^4*x^5 + (1001*a^10*b^5*x^6)/2 + 715*a^
9*b^6*x^7 + (6435*a^8*b^7*x^8)/8 + 715*a^7*b^8*x^9 + (1001*a^6*b^9*x^10)/2
+ 273*a^5*b^10*x^11 + (455*a^4*b^11*x^12)/4 + 35*a^3*b^12*x^13 + (15*a^2*b^
13*x^14)/2
```

3.64 $\int (a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5$

Optimal result 589
Rubi [A] (verified) 589
Mathematica [A] (verified) 590
Maple [B] (verified) 590
Fricas [B] (verification not implemented) 591
Sympy [B] (verification not implemented) 591
Maxima [B] (verification not implemented) 592
Giac [B] (verification not implemented) 592
Mupad [B] (verification not implemented) 593

Optimal result

Integrand size = 51, antiderivative size = 14

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2\,dx=\frac{(a+bx)^{11}}{11b}$$

```
[Out] 1/11*(b*x+a)^11/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2084, 32}

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2\,dx=\frac{(a+bx)^{11}}{11b}$$

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^2,x]
```

```
[Out] (a + b*x)^11/(11*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2084

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int (a+bx)^{10}\,dx$$
$$= \frac{(a+bx)^{11}}{11b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2\,dx = \frac{(a+bx)^{11}}{11b}$$

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^2,x]
```

```
[Out] (a + b*x)^11/(11*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(12) = 24.

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 7.79

method	result
default	$\frac{1}{11}b^{10}x^{11}+a\,b^9x^{10}+5a^2b^8x^9+15a^3b^7x^8+30a^4b^6x^7+42a^5b^5x^6+42a^6b^4x^5+30a^7b^3x^4+15a$
norman	$\frac{1}{11}b^{10}x^{11}+a\,b^9x^{10}+5a^2b^8x^9+15a^3b^7x^8+30a^4b^6x^7+42a^5b^5x^6+42a^6b^4x^5+30a^7b^3x^4+15a$
risch	$\frac{1}{11}b^{10}x^{11}+a\,b^9x^{10}+5a^2b^8x^9+15a^3b^7x^8+30a^4b^6x^7+42a^5b^5x^6+42a^6b^4x^5+30a^7b^3x^4+15a$
parallelrisch	$\frac{1}{11}b^{10}x^{11}+a\,b^9x^{10}+5a^2b^8x^9+15a^3b^7x^8+30a^4b^6x^7+42a^5b^5x^6+42a^6b^4x^5+30a^7b^3x^4+15a$
gosper	$\frac{x\left(b^{10}x^{10}+11a\,b^9x^9+55a^2b^8x^8+165a^3b^7x^7+330a^4b^6x^6+462a^5b^5x^5+462a^6b^4x^4+330a^7b^3x^3+165a^8b^2x^2+55a^9bx+11a^{10}\right)}{11}$

```
[In] int((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/11*b^10*x^11+a*b^9*x^10+5*a^2*b^8*x^9+15*a^3*b^7*x^8+30*a^4*b^6*x^7+42*a^
5*b^5*x^6+42*a^6*b^4*x^5+30*a^7*b^3*x^4+15*a^8*b^2*x^3+5*a^9*b*x^2+a^10*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(12) = 24$.

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 7.71

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2 dx$$
$$= \frac{1}{11}\,b^{10}x^{11}+ab^9x^{10}+5\,a^2b^8x^9+15\,a^3b^7x^8+30\,a^4b^6x^7$$
$$+42\,a^5b^5x^6+42\,a^6b^4x^5+30\,a^7b^3x^4+15\,a^8b^2x^3+5\,a^9bx^2+a^{10}x$$

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^2,x, algorithm="fricas")
```

```
[Out] 1/11*b^10*x^11 + a*b^9*x^10 + 5*a^2*b^8*x^9 + 15*a^3*b^7*x^8 + 30*a^4*b^6*x
^7 + 42*a^5*b^5*x^6 + 42*a^6*b^4*x^5 + 30*a^7*b^3*x^4 + 15*a^8*b^2*x^3 + 5*
a^9*b*x^2 + a^10*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(8) = 16$.

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 8.14

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2 dx$$
$$= a^{10}x+5a^9bx^2+15a^8b^2x^3+30a^7b^3x^4+42a^6b^4x^5+42a^5b^5x^6$$
$$+30a^4b^6x^7+15a^3b^7x^8+5a^2b^8x^9+ab^9x^{10}+\frac{b^{10}x^{11}}{11}$$

```
[In] integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a*
*4*b*x+a**5)**2,x)
```

```
[Out] a**10*x + 5*a**9*b*x**2 + 15*a**8*b**2*x**3 + 30*a**7*b**3*x**4 + 42*a**6*b
**4*x**5 + 42*a**5*b**5*x**6 + 30*a**4*b**6*x**7 + 15*a**3*b**7*x**8 + 5*a*
*2*b**8*x**9 + a*b**9*x**10 + b**10*x**11/11
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(12) = 24$.

Time = 0.21 (sec) , antiderivative size = 228, normalized size of antiderivative = 16.29

$$\int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5\right)^2 dx$$
$$= \frac{1}{11}b^{10}x^{11} + ab^9x^{10} + \frac{25}{9}a^2b^8x^9 + \frac{100}{7}a^4b^6x^7 + 20\,a^6b^4x^5 + \frac{25}{3}a^8b^2x^3 + a^{10}x + \frac{1}{3}\left(b^5x^6 + 6\,ab^4x^5 + 15\,a^2b^3x^4 + 20\,a^3b^2x^3 + 15\,a^4bx^2\right)a^5 + \frac{5}{21}\left(6\,b^5x^7 + 35\,ab^4x^6 + 84\,a^2b^3x^5 + 105\,a^3b^2x^4\right)a^4b + \frac{5}{42}\left(21\,b^5x^8 + 120\,ab^4x^7 + 280\,a^2b^3x^6\right)a^3b^2 + \frac{5}{18}\left(8\,b^5x^9 + 45\,ab^4x^8\right)a^2b^3$$

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^10*x^11 + a*b^9*x^10 + 25/9*a^2*b^8*x^9 + 100/7*a^4*b^6*x^7 + 20*a^6
*b^4*x^5 + 25/3*a^8*b^2*x^3 + a^10*x + 1/3*(b^5*x^6 + 6*a*b^4*x^5 + 15*a^2*
b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^2)*a^5 + 5/21*(6*b^5*x^7 + 35*a*b^4*x
^6 + 84*a^2*b^3*x^5 + 105*a^3*b^2*x^4)*a^4*b + 5/42*(21*b^5*x^8 + 120*a*b^4
*x^7 + 280*a^2*b^3*x^6)*a^3*b^2 + 5/18*(8*b^5*x^9 + 45*a*b^4*x^8)*a^2*b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 7.71

$$\int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5\right)^2 dx$$
$$= \frac{1}{11}b^{10}x^{11} + ab^9x^{10} + 5\,a^2b^8x^9 + 15\,a^3b^7x^8 + 30\,a^4b^6x^7 + 42\,a^5b^5x^6 + 42\,a^6b^4x^5 + 30\,a^7b^3x^4 + 15\,a^8b^2x^3 + 5\,a^9bx^2 + a^{10}x$$

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^2,x, algorithm="giac")
```

```
[Out] 1/11*b^10*x^11 + a*b^9*x^10 + 5*a^2*b^8*x^9 + 15*a^3*b^7*x^8 + 30*a^4*b^6*x
^7 + 42*a^5*b^5*x^6 + 42*a^6*b^4*x^5 + 30*a^7*b^3*x^4 + 15*a^8*b^2*x^3 + 5*
a^9*b*x^2 + a^10*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 7.71

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2\,dx$$
$$= a^{10}\,x+5\,a^9\,b\,x^2+15\,a^8\,b^2\,x^3+30\,a^7\,b^3\,x^4+42\,a^6\,b^4\,x^5+42\,a^5\,b^5\,x^6$$
$$+30\,a^4\,b^6\,x^7+15\,a^3\,b^7\,x^8+5\,a^2\,b^8\,x^9+a\,b^9\,x^{10}+\frac{b^{10}\,x^{11}}{11}$$

```
[In] int((a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*
b*x)^2,x)
```

```
[Out] a^10*x + (b^10*x^11)/11 + 5*a^9*b*x^2 + a*b^9*x^10 + 15*a^8*b^2*x^3 + 30*a^
7*b^3*x^4 + 42*a^6*b^4*x^5 + 42*a^5*b^5*x^6 + 30*a^4*b^6*x^7 + 15*a^3*b^7*x
^8 + 5*a^2*b^8*x^9
```

3.65 $\int (a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)$

Optimal result 594
Rubi [B] (verified) 594
Mathematica [B] (verified) 595
Maple [A] (verified) 595
Fricas [B] (verification not implemented) 596
Sympy [B] (verification not implemented) 596
Maxima [B] (verification not implemented) 596
Giac [B] (verification not implemented) 597
Mupad [B] (verification not implemented) 597

Optimal result

Integrand size = 49, antiderivative size = 14

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)\,dx=\frac{(a+bx)^6}{6b}$$

```
[Out] 1/6*(b*x+a)^6/b
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 61 vs. 2(14) = 28.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 4.36, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)\,dx$$
$$=a^5x+\frac{5}{2}a^4bx^2+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^2b^3x^4+ab^4x^5+\frac{b^5x^6}{6}$$

```
[In] Int[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x
^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^2)/2 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2 + a*b^4*x^
5 + (b^5*x^6)/6
```

Rubi steps

$$\text{integral}=a^5x+\frac{5}{2}a^4bx^2+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^2b^3x^4+ab^4x^5+\frac{b^5x^6}{6}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 61 vs. $2(14) = 28$.

Time = 0.00 (sec) , antiderivative size = 61, normalized size of antiderivative = 4.36

$$\int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5\right)\,dx$$
$$= a^5x + \frac{5}{2}a^4bx^2 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^2b^3x^4 + ab^4x^5 + \frac{b^5x^6}{6}$$

```
[In] Integrate[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 +
 b^5*x^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^2)/2 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2 + a*b^4*x^
5 + (b^5*x^6)/6
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{(bx+a)^6}{6b}$	13
norman	$\frac{1}{6}b^5x^6 + a\,b^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3x^3b^2 + \frac{5}{2}x^2a^4b + a^5x$	54
risch	$\frac{1}{6}b^5x^6 + a\,b^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3x^3b^2 + \frac{5}{2}x^2a^4b + a^5x$	54
parallelrisch	$\frac{1}{6}b^5x^6 + a\,b^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3x^3b^2 + \frac{5}{2}x^2a^4b + a^5x$	54
parts	$\frac{1}{6}b^5x^6 + a\,b^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3x^3b^2 + \frac{5}{2}x^2a^4b + a^5x$	54
gosper	$\frac{x\left(b^5x^5+6a\,b^4x^4+15a^2b^3x^3+20a^3b^2x^2+15x\,a^4b+6a^5\right)}{6}$	55

```
[In] int(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/6*(b*x+a)^6/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int (a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5) \, dx = \frac{1}{6} b^5x^6 + ab^4x^5 + \frac{5}{2} a^2b^3x^4 + \frac{10}{3} a^3b^2x^3 + \frac{5}{2} a^4bx^2 + a^5x$$

```
[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x
, algorithm="fricas")
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(8) = 16$.

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 4.29

$$\int (a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5) \, dx = a^5x + \frac{5a^4bx^2}{2} + \frac{10a^3b^2x^3}{3} + \frac{5a^2b^3x^4}{2} + ab^4x^5 + \frac{b^5x^6}{6}$$

```
[In] integrate(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**
4*b*x+a**5,x)
[Out] a**5*x + 5*a**4*b*x**2/2 + 10*a**3*b**2*x**3/3 + 5*a**2*b**3*x**4/2 + a*b**
4*x**5 + b**5*x**6/6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int (a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5) \, dx = \frac{1}{6} b^5x^6 + ab^4x^5 + \frac{5}{2} a^2b^3x^4 + \frac{10}{3} a^3b^2x^3 + \frac{5}{2} a^4bx^2 + a^5x$$

```
[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x
, algorithm="maxima")
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)\,dx$$
$$=\frac{1}{6}\,b^5x^6+ab^4x^5+\frac{5}{2}\,a^2b^3x^4+\frac{10}{3}\,a^3b^2x^3+\frac{5}{2}\,a^4bx^2+a^5x$$

```
[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x
, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)\,dx$$
$$=a^5\,x+\frac{5\,a^4\,b\,x^2}{2}+\frac{10\,a^3\,b^2\,x^3}{3}+\frac{5\,a^2\,b^3\,x^4}{2}+a\,b^4\,x^5+\frac{b^5\,x^6}{6}$$

```
[In] int(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b
*x,x)
```

```
[Out] a^5*x + (b^5*x^6)/6 + (5*a^4*b*x^2)/2 + a*b^4*x^5 + (10*a^3*b^2*x^3)/3 + (5
*a^2*b^3*x^4)/2
```

3.66 $\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5} dx$

Optimal result . . . 598
Rubi [A] (verified) . . . 598
Mathematica [A] (verified) . . . 599
Maple [A] (verified) . . . 599
Fricas [B] (verification not implemented) . . . 600
Sympy [B] (verification not implemented) . . . 600
Maxima [B] (verification not implemented) . . . 600
Giac [A] (verification not implemented) . . . 601
Mupad [B] (verification not implemented) . . . 601

Optimal result

Integrand size = 51, antiderivative size = 14

$$\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5} dx = -\frac{1}{4b(a+bx)^4}$$

```
[Out] -1/4/b/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2083, 32}

$$\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5} dx = -\frac{1}{4b(a+bx)^4}$$

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^(-1),x]
```

```
[Out] -1/4*1/(b*(a + b*x)^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(a+bx)^5}\,dx$$
$$= -\frac{1}{4b(a+bx)^4}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx = -\frac{1}{4b(a+bx)^4}$$

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^(-1),x]
```

```
[Out] -1/4*1/(b*(a + b*x)^4)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{4b(bx+a)^4}$	13
norman	$-\frac{1}{4b(bx+a)^4}$	13
gosper	$-\frac{1}{4b(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)}$	46
risch	$-\frac{1}{4b(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)}$	46
parallelrisch	$-\frac{1}{4b(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)}$	46

```
[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/4/b/(b*x+a)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(12) = 24$.

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.29

$$\int \frac{1}{a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5}\,dx = -\frac{1}{4\,(b^5x^4 + 4\,ab^4x^3 + 6\,a^2b^3x^2 + 4\,a^3b^2x + a^4b)}$$

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5),x, algorithm="fricas")
```

```
[Out] -1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.50

$$\int \frac{1}{a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5}\,dx = -\frac{1}{4a^4b + 16a^3b^2x + 24a^2b^3x^2 + 16ab^4x^3 + 4b^5x^4}$$

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*
a**4*b*x+a**5),x)
```

```
[Out] -1/(4*a**4*b + 16*a**3*b**2*x + 24*a**2*b**3*x**2 + 16*a*b**4*x**3 + 4*b**5
*x**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(12) = 24$.

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.29

$$\int \frac{1}{a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5}\,dx = -\frac{1}{4\,(b^5x^4 + 4\,ab^4x^3 + 6\,a^2b^3x^2 + 4\,a^3b^2x + a^4b)}$$

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5),x, algorithm="maxima")
```

```
[Out] -1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx=-\frac{1}{4\,(bx+a)^4b}$$

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5),x, algorithm="giac")
```

```
[Out] -1/4/((b*x + a)^4*b)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 3.43

$$\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx =-\frac{1}{4\,a^4\,b+16\,a^3\,b^2\,x+24\,a^2\,b^3\,x^2+16\,a\,b^4\,x^3+4\,b^5\,x^4}$$

```
[In] int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^
4*b*x),x)
```

```
[Out] -1/(4*a^4*b + 4*b^5*x^4 + 16*a^3*b^2*x + 16*a*b^4*x^3 + 24*a^2*b^3*x^2)
```

3.67 $\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^2} dx$

Optimal result 602
Rubi [A] (verified) 602
Mathematica [A] (verified) 603
Maple [A] (verified) 603
Fricas [B] (verification not implemented) 604
Sympy [B] (verification not implemented) 604
Maxima [B] (verification not implemented) 604
Giac [A] (verification not implemented) 605
Mupad [B] (verification not implemented) 605

Optimal result

Integrand size = 51, antiderivative size = 14

$$\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^2} dx = -\frac{1}{9b(a+bx)^9}$$

```
[Out] -1/9/b/(b*x+a)^9
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2083, 32}

$$\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^2} dx = -\frac{1}{9b(a+bx)^9}$$

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^(-2),x]
```

```
[Out] -1/9*1/(b*(a + b*x)^9)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(a+bx)^{10}}\,dx$$
$$= -\frac{1}{9b(a+bx)^9}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2}\,dx = -\frac{1}{9b(a+bx)^9}$$

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^(-2),x]
```

```
[Out] -1/9*1/(b*(a + b*x)^9)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{9b(bx+a)^9}$	13
norman	$-\frac{1}{9b(bx+a)^9}$	13
risch	$-\frac{1}{9b(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)^2(bx+a)}$	53
gosper	$-\frac{1}{9(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5x\,a^4b+a^5)b}$	97
parallelrisch	$-\frac{1}{9(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5x\,a^4b+a^5)b}$	97

```
[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x
,method=_RETURNVERBOSE)
```

```
[Out] -1/9/b/(b*x+a)^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 7.21

$$\int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^2}\,dx = -\frac{1}{9\,(b^{10}x^9 + 9\,ab^9x^8 + 36\,a^2b^8x^7 + 84\,a^3b^7x^6 + 126\,a^4b^6x^5 + 126\,a^5b^5x^4 + 84\,a^6b^4x^3 + 36\,a^7b^3x^2 + 9\,a^8b^2x}$$

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^2,x, algorithm="fricas")
```

```
[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^
6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a
^9*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 7.79

$$\int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^2}\,dx = -\frac{1}{9a^9b + 81a^8b^2x + 324a^7b^3x^2 + 756a^6b^4x^3 + 1134a^5b^5x^4 + 1134a^4b^6x^5 + 756a^3b^7x^6 + 324a^2b^8x^7 + 81ab^9}$$

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*
a**4*b*x+a**5)**2,x)
```

```
[Out] -1/(9*a**9*b + 81*a**8*b**2*x + 324*a**7*b**3*x**2 + 756*a**6*b**4*x**3 + 1
134*a**5*b**5*x**4 + 1134*a**4*b**6*x**5 + 756*a**3*b**7*x**6 + 324*a**2*b*
*8*x**7 + 81*a*b**9*x**8 + 9*b**10*x**9)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(12) = 24$.

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 7.21

$$\int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^2}\,dx = -\frac{1}{9\,(b^{10}x^9 + 9\,ab^9x^8 + 36\,a^2b^8x^7 + 84\,a^3b^7x^6 + 126\,a^4b^6x^5 + 126\,a^5b^5x^4 + 84\,a^6b^4x^3 + 36\,a^7b^3x^2 + 9\,a^8b^2x}$$

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^2,x, algorithm="maxima")
```

```
[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^
6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a
^9*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^2}\,dx = -\frac{1}{9\,(bx+a)^9b}$$

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^2,x, algorithm="giac")
```

```
[Out] -1/9/((b*x + a)^9*b)
```

Mupad [B] (verification not implemented)

Time = 10.73 (sec) , antiderivative size = 103, normalized size of antiderivative = 7.36

$$\int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^2}\,dx = -\frac{1}{9\,a^9\,b + 81\,a^8\,b^2\,x + 324\,a^7\,b^3\,x^2 + 756\,a^6\,b^4\,x^3 + 1134\,a^5\,b^5\,x^4 + 1134\,a^4\,b^6\,x^5 + 756\,a^3\,b^7\,x^6 + 324\,a^2\,b^8\,x}$$

```
[In] int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^
4*b*x)^2,x)
```

```
[Out] -1/(9*a^9*b + 9*b^10*x^9 + 81*a^8*b^2*x + 81*a*b^9*x^8 + 324*a^7*b^3*x^2 +
756*a^6*b^4*x^3 + 1134*a^5*b^5*x^4 + 1134*a^4*b^6*x^5 + 756*a^3*b^7*x^6 + 3
24*a^2*b^8*x^7)
```

3.68 $\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^3}\,dx$

Optimal result 606
Rubi [A] (verified) 606
Mathematica [A] (verified) 607
Maple [A] (verified) 607
Fricas [B] (verification not implemented) 608
Sympy [B] (verification not implemented) 608
Maxima [B] (verification not implemented) 609
Giac [A] (verification not implemented) 609
Mupad [B] (verification not implemented) 609

Optimal result

Integrand size = 51, antiderivative size = 14

$$\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^3}\,dx=-\frac{1}{14b(a+bx)^{14}}$$

```
[Out] -1/14/b/(b*x+a)^14
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2083, 32}

$$\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^3}\,dx=-\frac{1}{14b(a+bx)^{14}}$$

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^(-3),x]
```

```
[Out] -1/14*1/(b*(a + b*x)^14)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(a+bx)^{15}}\,dx$$
$$= -\frac{1}{14b(a+bx)^{14}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^3}\,dx = -\frac{1}{14b(a+bx)^{14}}$$

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^(-3),x]
[Out] -1/14*1/(b*(a + b*x)^14)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{14b(bx+a)^{14}}$	13
norman	$-\frac{1}{14b(bx+a)^{14}}$	13
risch	$-\frac{1}{14b(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)^3(bx+a)^2}$	53
gosper	$-\frac{1}{14(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5x\,a^4b+a^5)^2b}$	97
parallelrisch	$-\frac{1}{14(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5x\,a^4b+a^5)^2b}$	97

```
[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x
,method=_RETURNVERBOSE)
[Out] -1/14/b/(b*x+a)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 11.14

$$\int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^3}\,dx = -\frac{1}{14\,(b^{15}x^{14} + 14\,ab^{14}x^{13} + 91\,a^2b^{13}x^{12} + 364\,a^3b^{12}x^{11} + 1001\,a^4b^{11}x^{10} + 2002\,a^5b^{10}x^9 + 3003\,a^6b^9x^8 + 343}$$

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^3,x, algorithm="fricas")
```

```
[Out] -1/14/(b^15*x^14 + 14*a*b^14*x^13 + 91*a^2*b^13*x^12 + 364*a^3*b^12*x^11 +
1001*a^4*b^11*x^10 + 2002*a^5*b^10*x^9 + 3003*a^6*b^9*x^8 + 3432*a^7*b^8*x^
7 + 3003*a^8*b^7*x^6 + 2002*a^9*b^6*x^5 + 1001*a^10*b^5*x^4 + 364*a^11*b^4*
x^3 + 91*a^12*b^3*x^2 + 14*a^13*b^2*x + a^14*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(12) = 24$.

Time = 0.43 (sec) , antiderivative size = 168, normalized size of antiderivative = 12.00

$$\int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^3}\,dx = -\frac{}{14a^{14}b + 196a^{13}b^2x + 1274a^{12}b^3x^2 + 5096a^{11}b^4x^3 + 14014a^{10}b^5x^4 + 28028a^9b^6x^5 + 42042a^8b^7x^6 + 48048}$$

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*
a**4*b*x+a**5)**3,x)
```

```
[Out] -1/(14*a**14*b + 196*a**13*b**2*x + 1274*a**12*b**3*x**2 + 5096*a**11*b**4*
x**3 + 14014*a**10*b**5*x**4 + 28028*a**9*b**6*x**5 + 42042*a**8*b**7*x**6
+ 48048*a**7*b**8*x**7 + 42042*a**6*b**9*x**8 + 28028*a**5*b**10*x**9 + 140
14*a**4*b**11*x**10 + 5096*a**3*b**12*x**11 + 1274*a**2*b**13*x**12 + 196*a
*b**14*x**13 + 14*b**15*x**14)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(12) = 24$.

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 11.14

$$\int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^3}\,dx =$$

$$-\frac{1}{14\,(b^{15}x^{14} + 14\,ab^{14}x^{13} + 91\,a^2b^{13}x^{12} + 364\,a^3b^{12}x^{11} + 1001\,a^4b^{11}x^{10} + 2002\,a^5b^{10}x^9 + 3003\,a^6b^9x^8 + 34}$$

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^3,x, algorithm="maxima")
```

```
[Out] -1/14/(b^15*x^14 + 14*a*b^14*x^13 + 91*a^2*b^13*x^12 + 364*a^3*b^12*x^11 +
1001*a^4*b^11*x^10 + 2002*a^5*b^10*x^9 + 3003*a^6*b^9*x^8 + 3432*a^7*b^8*x^
7 + 3003*a^8*b^7*x^6 + 2002*a^9*b^6*x^5 + 1001*a^10*b^5*x^4 + 364*a^11*b^4*
x^3 + 91*a^12*b^3*x^2 + 14*a^13*b^2*x + a^14*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^3}\,dx = -\frac{1}{14\,(bx+a)^{14}b}$$

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^3,x, algorithm="giac")
```

```
[Out] -1/14/((b*x + a)^14*b)
```

Mupad [B] (verification not implemented)

Time = 11.83 (sec) , antiderivative size = 158, normalized size of antiderivative = 11.29

$$\int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^3}\,dx =$$

$$-\frac{}{14\,a^{14}\,b + 196\,a^{13}\,b^2\,x + 1274\,a^{12}\,b^3\,x^2 + 5096\,a^{11}\,b^4\,x^3 + 14014\,a^{10}\,b^5\,x^4 + 28028\,a^9\,b^6\,x^5 + 42042\,a^8\,b^7\,x}$$

```
[In] int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^
4*b*x)^3,x)
```

```
[Out] -1/(14*a^14*b + 14*b^15*x^14 + 196*a^13*b^2*x + 196*a*b^14*x^13 + 1274*a^12
*b^3*x^2 + 5096*a^11*b^4*x^3 + 14014*a^10*b^5*x^4 + 28028*a^9*b^6*x^5 + 420
42*a^8*b^7*x^6 + 48048*a^7*b^8*x^7 + 42042*a^6*b^9*x^8 + 28028*a^5*b^10*x^9
 + 14014*a^4*b^11*x^10 + 5096*a^3*b^12*x^11 + 1274*a^2*b^13*x^12)
```

3.69 $\int \frac{1}{1+x^2+x^3+x^5} dx$

Optimal result 610
Rubi [A] (verified) 610
Mathematica [A] (verified) 611
Maple [A] (verified) 612
Fricas [A] (verification not implemented) 612
Sympy [A] (verification not implemented) 612
Maxima [A] (verification not implemented) 613
Giac [A] (verification not implemented) 613
Mupad [B] (verification not implemented) 613

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{1}{1+x^2+x^3+x^5} dx = \frac{\arctan(x)}{2} + \frac{1}{6}\log(1+x) + \frac{1}{4}\log\left(1+x^2\right) - \frac{1}{3}\log\left(1-x+x^2\right)$$

```
[Out] 1/2*arctan(x)+1/6*ln(1+x)+1/4*ln(x^2+1)-1/3*ln(x^2-x+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2083, 649, 209, 266, 642}

$$\int \frac{1}{1+x^2+x^3+x^5} dx = \frac{\arctan(x)}{2} + \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{3}\log\left(x^2-x+1\right) + \frac{1}{6}\log(x+1)$$

```
[In] Int[(1 + x^2 + x^3 + x^5)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{6(1+x)}+\frac{1+x}{2\left(1+x^2\right)}+\frac{1-2x}{3\left(1-x+x^2\right)}\right) dx\\
&= \frac{1}{6}\log(1+x)+\frac{1}{3}\int \frac{1-2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1+x}{1+x^2}\,dx\\
&= \frac{1}{6}\log(1+x)-\frac{1}{3}\log\left(1-x+x^2\right)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{1}{2}\int \frac{x}{1+x^2}\,dx\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{6}\log(1+x)+\frac{1}{4}\log\left(1+x^2\right)-\frac{1}{3}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$
\int \frac{1}{1+x^2+x^3+x^5}\,dx=\frac{\arctan(x)}{2}+\frac{1}{6}\log(1+x)+\frac{1}{4}\log\left(1+x^2\right)-\frac{1}{3}\log\left(1-x+x^2\right)
$$

```
[In] Integrate[(1 + x^2 + x^3 + x^5)^(-1),x]
[Out] ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{6}+\frac{\ln(x^2+1)}{4}-\frac{\ln(x^2-x+1)}{3}$	31
risch	$\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{6}+\frac{\ln(x^2+1)}{4}-\frac{\ln(x^2-x+1)}{3}$	31
parallelrisch	$\frac{\ln(x+1)}{6}+\frac{\ln(x-i)}{4}-\frac{i\ln(x-i)}{4}+\frac{\ln(x+i)}{4}+\frac{i\ln(x+i)}{4}-\frac{\ln(x^2-x+1)}{3}$	49

```
[In] int(1/(x^5+x^3+x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x)+1/6*ln(x+1)+1/4*ln(x^2+1)-1/3*ln(x^2-x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^3+x^5}\,dx=\frac{1}{2}\arctan(x)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{6}\log(x+1)$$

```
[In] integrate(1/(x^5+x^3+x^2+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+x^2+x^3+x^5}\,dx=\frac{\log(x+1)}{6}+\frac{\log(x^2+1)}{4}-\frac{\log(x^2-x+1)}{3}+\frac{\operatorname{atan}(x)}{2}$$

```
[In] integrate(1/(x**5+x**3+x**2+1),x)
[Out] log(x + 1)/6 + log(x**2 + 1)/4 - log(x**2 - x + 1)/3 + atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^3+x^5}\,dx = \frac{1}{2}\arctan\left(x\right) - \frac{1}{3}\log\left(x^2-x+1\right) + \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{6}\log\left(x+1\right)$$

```
[In] integrate(1/(x^5+x^3+x^2+1),x, algorithm="maxima")
[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+x^2+x^3+x^5}\,dx = \frac{1}{2}\arctan\left(x\right) - \frac{1}{3}\log\left(x^2-x+1\right) + \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{6}\log\left(|x+1|\right)$$

```
[In] integrate(1/(x^5+x^3+x^2+1),x, algorithm="giac")
[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(abs(x + 1
))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{1}{1+x^2+x^3+x^5}\,dx = \frac{\ln\left(x+1\right)}{6} - \frac{\ln\left(x^2-x+1\right)}{3} + \ln\left(x-\mathrm{i}\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

```
[In] int(1/(x^2 + x^3 + x^5 + 1),x)
[Out] log(x + 1)/6 + log(x - 1i)*(1/4 - 1i/4) + log(x + 1i)*(1/4 + 1i/4) - log(x^
2 - x + 1)/3
```

3.70 $\int (3-19x^2+32x^4-16x^6)^4\,dx$

Optimal result 614
Rubi [A] (verified) 614
Mathematica [A] (verified) 616
Maple [A] (verified) 616
Fricas [A] (verification not implemented) 617
Sympy [A] (verification not implemented) 617
Maxima [A] (verification not implemented) 618
Giac [A] (verification not implemented) 618
Mupad [B] (verification not implemented) 619

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int (3-19x^2+32x^4-16x^6)^4\,dx = 81x-684x^3+4590x^5-\frac{149700x^7}{7}+\frac{634321x^9}{9}-\frac{1841600x^{11}}{11}+\frac{3764416x^{13}}{13}-\frac{1094656x^{15}}{3}+\frac{5633536x^{17}}{17}-\frac{4014080x^{19}}{19}+\frac{1884160x^{21}}{21}-\frac{524288x^{23}}{23}+\frac{65536x^{25}}{25}$$

```
[Out] 81*x-684*x^3+4590*x^5-149700/7*x^7+634321/9*x^9-1841600/11*x^11+3764416/13*
x^13-1094656/3*x^15+5633536/17*x^17-4014080/19*x^19+1884160/21*x^21-524288/
23*x^23+65536/25*x^25
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2084, 531, 535}

$$\int (3-19x^2+32x^4-16x^6)^4\,dx = \frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{634321x^9}{9}-\frac{149700x^7}{7}+4590x^5-684x^3+81x$$

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4,x]
```

```
[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11
)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*
x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2084

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (-1+x)^4(1+x)^4(-1+2x)^4(1+2x)^4\left(-3+4x^2\right)^4\,dx\\
&= \int (-1+2x)^4(1+2x)^4\left(-1+x^2\right)^4\left(-3+4x^2\right)^4\,dx\\
&= \int \left(-1+x^2\right)^4\left(-3+4x^2\right)^4\left(-1+4x^2\right)^4\,dx\\
&= \int \left(81-2052x^2+22950x^4-149700x^6+634321x^8-1841600x^{10}+3764416x^{12}\right.\\
&\quad \left.-5473280x^{14}+5633536x^{16}-4014080x^{18}+1884160x^{20}-524288x^{22}+65536x^{24}\right)\,dx\\
&= 81x-684x^3+4590x^5-\frac{149700x^7}{7}+\frac{634321x^9}{9}-\frac{1841600x^{11}}{11}+\frac{3764416x^{13}}{13}\\
&\quad -\frac{1094656x^{15}}{3}+\frac{5633536x^{17}}{17}-\frac{4014080x^{19}}{19}+\frac{1884160x^{21}}{21}-\frac{524288x^{23}}{23}+\frac{65536x^{25}}{25}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \left(3-19x^2+32x^4-16x^6\right)^4\,dx = 81x-684x^3+4590x^5-\frac{149700x^7}{7}+\frac{634321x^9}{9}-\frac{1841600x^{11}}{11}+\frac{3764416x^{13}}{13}-\frac{1094656x^{15}}{3}+\frac{5633536x^{17}}{17}-\frac{4014080x^{19}}{19}+\frac{1884160x^{21}}{21}-\frac{524288x^{23}}{23}+\frac{65536x^{25}}{25}$$

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4,x]
```

```
[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11
)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*
x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

method	result
default	$81x-684x^3+4590x^5-\frac{149700}{7}x^7+\frac{634321}{9}x^9-\frac{1841600}{11}x^{11}+\frac{3764416}{13}x^{13}-\frac{1094656}{3}x^{15}+\frac{5633536}{17}x$
norman	$81x-684x^3+4590x^5-\frac{149700}{7}x^7+\frac{634321}{9}x^9-\frac{1841600}{11}x^{11}+\frac{3764416}{13}x^{13}-\frac{1094656}{3}x^{15}+\frac{5633536}{17}x$
risch	$81x-684x^3+4590x^5-\frac{149700}{7}x^7+\frac{634321}{9}x^9-\frac{1841600}{11}x^{11}+\frac{3764416}{13}x^{13}-\frac{1094656}{3}x^{15}+\frac{5633536}{17}x$
parallelrisch	$81x-684x^3+4590x^5-\frac{149700}{7}x^7+\frac{634321}{9}x^9-\frac{1841600}{11}x^{11}+\frac{3764416}{13}x^{13}-\frac{1094656}{3}x^{15}+\frac{5633536}{17}x$
gosper	$x(4386184298496x^{24}-38140733030400x^{22}+150122379264000x^{20}-353491826688000x^{18}+554471344627200x^{16}-61052487175$

```
[In] int((-16*x^6+32*x^4-19*x^2+3)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 81*x-684*x^3+4590*x^5-149700/7*x^7+634321/9*x^9-1841600/11*x^11+3764416/13*
x^13-1094656/3*x^15+5633536/17*x^17-4014080/19*x^19+1884160/21*x^21-524288/
23*x^23+65536/25*x^25
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int \left(3-19x^2+32x^4-16x^6\right)^4 \, dx = \frac{65536}{25}\,x^{25}-\frac{524288}{23}\,x^{23}+\frac{1884160}{21}\,x^{21}-\frac{4014080}{19}\,x^{19}+\frac{5633536}{17}\,x^{17}-\frac{1094656}{3}\,x^{15}+\frac{3764416}{13}\,x^{13}-\frac{1841600}{11}\,x^{11}+\frac{634321}{9}\,x^{9}-\frac{149700}{7}\,x^{7}+4590\,x^{5}-684\,x^{3}+81\,x$$

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="fricas")
```

```
[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 563353
6/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x
^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \left(3-19x^2+32x^4-16x^6\right)^4 \, dx = \frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{634321x^{9}}{9}-\frac{149700x^{7}}{7}+4590x^{5}-684x^{3}+81x$$

```
[In] integrate((-16*x**6+32*x**4-19*x**2+3)**4,x)
```

```
[Out] 65536*x**25/25 - 524288*x**23/23 + 1884160*x**21/21 - 4014080*x**19/19 + 56
33536*x**17/17 - 1094656*x**15/3 + 3764416*x**13/13 - 1841600*x**11/11 + 63
4321*x**9/9 - 149700*x**7/7 + 4590*x**5 - 684*x**3 + 81*x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int \left(3-19x^2+32x^4-16x^6\right)^4\,dx = \frac{65536}{25}\,x^{25} - \frac{524288}{23}\,x^{23} + \frac{1884160}{21}\,x^{21} - \frac{4014080}{19}\,x^{19} + \frac{5633536}{17}\,x^{17} - \frac{1094656}{3}\,x^{15} + \frac{3764416}{13}\,x^{13} - \frac{1841600}{11}\,x^{11} + \frac{634321}{9}\,x^{9} - \frac{149700}{7}\,x^{7} + 4590\,x^{5} - 684\,x^{3} + 81\,x$$

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="maxima")
```

```
[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 563353
6/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x
^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int \left(3-19x^2+32x^4-16x^6\right)^4\,dx = \frac{65536}{25}\,x^{25} - \frac{524288}{23}\,x^{23} + \frac{1884160}{21}\,x^{21} - \frac{4014080}{19}\,x^{19} + \frac{5633536}{17}\,x^{17} - \frac{1094656}{3}\,x^{15} + \frac{3764416}{13}\,x^{13} - \frac{1841600}{11}\,x^{11} + \frac{634321}{9}\,x^{9} - \frac{149700}{7}\,x^{7} + 4590\,x^{5} - 684\,x^{3} + 81\,x$$

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="giac")
```

```
[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 563353
6/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x
^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int \left(3-19x^2+32x^4-16x^6\right)^4\,dx=\frac{65536\,x^{25}}{25}-\frac{524288\,x^{23}}{23}+\frac{1884160\,x^{21}}{21}-\frac{4014080\,x^{19}}{19}+\frac{5633536\,x^{17}}{17}-\frac{1094656\,x^{15}}{3}+\frac{3764416\,x^{13}}{13}-\frac{1841600\,x^{11}}{11}+\frac{634321\,x^{9}}{9}-\frac{149700\,x^{7}}{7}+4590\,x^5-684\,x^3+81\,x$$

```
[In] int((19*x^2 - 32*x^4 + 16*x^6 - 3)^4,x)
```

```
[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11
)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*
x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25
```

3.71 $\int (3-19x^2+32x^4-16x^6)^3\,dx$

Optimal result . . . 620
Rubi [A] (verified) . . . 620
Mathematica [A] (verified) . . . 621
Maple [A] (verified) . . . 622
Fricas [A] (verification not implemented) . . . 622
Sympy [A] (verification not implemented) . . . 622
Maxima [A] (verification not implemented) . . . 623
Giac [A] (verification not implemented) . . . 623
Mupad [B] (verification not implemented) . . . 623

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \left(3-19x^2+32x^4-16x^6\right)^3\,dx=27x-171x^3+\frac{4113x^5}{5}-2605x^7+\frac{16448x^9}{3}-\frac{84912x^{11}}{11}+\frac{93440x^{13}}{13}-\frac{21248x^{15}}{5}+\frac{24576x^{17}}{17}-\frac{4096x^{19}}{19}$$

```
[Out] 27*x-171*x^3+4113/5*x^5-2605*x^7+16448/3*x^9-84912/11*x^11+93440/13*x^13-21
248/5*x^15+24576/17*x^17-4096/19*x^19
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2084, 531, 535}

$$\int \left(3-19x^2+32x^4-16x^6\right)^3\,dx=-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^9}{3}-2605x^7+\frac{4113x^5}{5}-171x^3+27x$$

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3,x]
```

```
[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11
+ (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
```

```
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2084

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int (-1+x)^3(1+x)^3(-1+2x)^3(1+2x)^3\left(-3+4x^2\right)^3\,dx\\ &= -\int (-1+2x)^3(1+2x)^3\left(-1+x^2\right)^3\left(-3+4x^2\right)^3\,dx\\ &= -\int \left(-1+x^2\right)^3\left(-3+4x^2\right)^3\left(-1+4x^2\right)^3\,dx\\ &= -\int \left(-27+513x^2-4113x^4+18235x^6-49344x^8+84912x^{10}-93440x^{12}\right.\\ &\qquad \left.+63744x^{14}-24576x^{16}+4096x^{18}\right)\,dx\\ &= 27x-171x^3+\frac{4113x^5}{5}-2605x^7+\frac{16448x^9}{3}-\frac{84912x^{11}}{11}\\ &\quad+\frac{93440x^{13}}{13}-\frac{21248x^{15}}{5}+\frac{24576x^{17}}{17}-\frac{4096x^{19}}{19}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \left(3-19x^2+32x^4-16x^6\right)^3\,dx = 27x-171x^3+\frac{4113x^5}{5}-2605x^7+\frac{16448x^9}{3}-\frac{84912x^{11}}{11}+\frac{93440x^{13}}{13}-\frac{21248x^{15}}{5}+\frac{24576x^{17}}{17}-\frac{4096x^{19}}{19}$$

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3,x]
[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11
+ (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

method	result
default	$27x-171x^{3}+\frac{4113}{5}x^{5}-2605x^{7}+\frac{16448}{3}x^{9}-\frac{84912}{11}x^{11}+\frac{93440}{13}x^{13}-\frac{21248}{5}x^{15}+\frac{24576}{17}x^{17}-\frac{4096}{19}x$
norman	$27x-171x^{3}+\frac{4113}{5}x^{5}-2605x^{7}+\frac{16448}{3}x^{9}-\frac{84912}{11}x^{11}+\frac{93440}{13}x^{13}-\frac{21248}{5}x^{15}+\frac{24576}{17}x^{17}-\frac{4096}{19}x$
risch	$27x-171x^{3}+\frac{4113}{5}x^{5}-2605x^{7}+\frac{16448}{3}x^{9}-\frac{84912}{11}x^{11}+\frac{93440}{13}x^{13}-\frac{21248}{5}x^{15}+\frac{24576}{17}x^{17}-\frac{4096}{19}x$
parallelrisch	$27x-171x^{3}+\frac{4113}{5}x^{5}-2605x^{7}+\frac{16448}{3}x^{9}-\frac{84912}{11}x^{11}+\frac{93440}{13}x^{13}-\frac{21248}{5}x^{15}+\frac{24576}{17}x^{17}-\frac{4096}{19}x$
gosper	$-\frac{x\left(149360640x^{18}-1001594880x^{16}+2944271616x^{14}-4979884800x^{12}+5348182320x^{10}-3798583360x^{8}+1804835175x^{6}-56992\right.}{692835}$

[In] int((-16*x^6+32*x^4-19*x^2+3)^3,x,method=_RETURNVERBOSE)

[Out] 27*x-171*x^3+4113/5*x^5-2605*x^7+16448/3*x^9-84912/11*x^11+93440/13*x^13-21248/5*x^15+24576/17*x^17-4096/19*x^19

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \left(3-19x^2+32x^4-16x^6\right)^3\,dx=-\frac{4096}{19}\,x^{19}+\frac{24576}{17}\,x^{17}-\frac{21248}{5}\,x^{15}+\frac{93440}{13}\,x^{13}-\frac{84912}{11}\,x^{11}+\frac{16448}{3}\,x^{9}-2605\,x^{7}+\frac{4113}{5}\,x^{5}-171\,x^{3}+27\,x$$

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="fricas")

[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^11 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \left(3-19x^2+32x^4-16x^6\right)^3\,dx=-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^{9}}{3}-2605x^{7}+\frac{4113x^{5}}{5}-171x^{3}+27x$$

[In] integrate((-16*x**6+32*x**4-19*x**2+3)**3,x)

[Out] -4096*x**19/19 + 24576*x**17/17 - 21248*x**15/5 + 93440*x**13/13 - 84912*x**11/11 + 16448*x**9/3 - 2605*x**7 + 4113*x**5/5 - 171*x**3 + 27*x

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \left(3-19x^2+32x^4-16x^6\right)^3\,dx=-\frac{4096}{19}\,x^{19}+\frac{24576}{17}\,x^{17}-\frac{21248}{5}\,x^{15}+\frac{93440}{13}\,x^{13}-\frac{84912}{11}\,x^{11}+\frac{16448}{3}\,x^{9}-2605\,x^{7}+\frac{4113}{5}\,x^{5}-171\,x^{3}+27\,x$$

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="maxima")
[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^1
1 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \left(3-19x^2+32x^4-16x^6\right)^3\,dx=-\frac{4096}{19}\,x^{19}+\frac{24576}{17}\,x^{17}-\frac{21248}{5}\,x^{15}+\frac{93440}{13}\,x^{13}-\frac{84912}{11}\,x^{11}+\frac{16448}{3}\,x^{9}-2605\,x^{7}+\frac{4113}{5}\,x^{5}-171\,x^{3}+27\,x$$

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="giac")
[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^1
1 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \left(3-19x^2+32x^4-16x^6\right)^3\,dx=-\frac{4096\,x^{19}}{19}+\frac{24576\,x^{17}}{17}-\frac{21248\,x^{15}}{5}+\frac{93440\,x^{13}}{13}-\frac{84912\,x^{11}}{11}+\frac{16448\,x^{9}}{3}-2605\,x^{7}+\frac{4113\,x^{5}}{5}-171\,x^{3}+27\,x$$

```
[In] int(-(19*x^2 - 32*x^4 + 16*x^6 - 3)^3,x)
[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11
+ (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19
```

3.72 $\int (3-19x^2+32x^4-16x^6)^2 \, dx$

Optimal result 624
Rubi [A] (verified) 624
Mathematica [A] (verified) 625
Maple [A] (verified) 626
Fricas [A] (verification not implemented) 626
Sympy [A] (verification not implemented) 626
Maxima [A] (verification not implemented) 627
Giac [A] (verification not implemented) 627
Mupad [B] (verification not implemented) 627

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int (3-19x^2+32x^4-16x^6)^2 \, dx = 9x-38x^3+\frac{553x^5}{5}-\frac{1312x^7}{7}+\frac{544x^9}{3}-\frac{1024x^{11}}{11}+\frac{256x^{13}}{13}$$

```
[Out] 9*x-38*x^3+553/5*x^5-1312/7*x^7+544/3*x^9-1024/11*x^11+256/13*x^13
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2084, 531, 535}

$$\int (3-19x^2+32x^4-16x^6)^2 \, dx = \frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^9}{3}-\frac{1312x^7}{7}+\frac{553x^5}{5}-38x^3+9x$$

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]
```

```
[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 +
(256*x^13)/13
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2084

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int (-1+x)^2(1+x)^2(-1+2x)^2(1+2x)^2\left(-3+4x^2\right)^2\,dx\\ &= \int (-1+2x)^2(1+2x)^2\left(-1+x^2\right)^2\left(-3+4x^2\right)^2\,dx\\ &= \int \left(-1+x^2\right)^2\left(-3+4x^2\right)^2\left(-1+4x^2\right)^2\,dx\\ &= \int \left(9-114x^2+553x^4-1312x^6+1632x^8-1024x^{10}+256x^{12}\right)dx\\ &= 9x-38x^3+\frac{553x^5}{5}-\frac{1312x^7}{7}+\frac{544x^9}{3}-\frac{1024x^{11}}{11}+\frac{256x^{13}}{13}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \left(3-19x^2+32x^4-16x^6\right)^2\,dx=9x-38x^3+\frac{553x^5}{5}-\frac{1312x^7}{7}+\frac{544x^9}{3}-\frac{1024x^{11}}{11}+\frac{256x^{13}}{13}$$

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]
[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 +
(256*x^13)/13
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

method	result	size
default	$9x-38x^3+\frac{553}{5}x^5-\frac{1312}{7}x^7+\frac{544}{3}x^9-\frac{1024}{11}x^{11}+\frac{256}{13}x^{13}$	35
norman	$9x-38x^3+\frac{553}{5}x^5-\frac{1312}{7}x^7+\frac{544}{3}x^9-\frac{1024}{11}x^{11}+\frac{256}{13}x^{13}$	35
risch	$9x-38x^3+\frac{553}{5}x^5-\frac{1312}{7}x^7+\frac{544}{3}x^9-\frac{1024}{11}x^{11}+\frac{256}{13}x^{13}$	35
parallelrisch	$9x-38x^3+\frac{553}{5}x^5-\frac{1312}{7}x^7+\frac{544}{3}x^9-\frac{1024}{11}x^{11}+\frac{256}{13}x^{13}$	35
gosper	$\frac{x(295680x^{12}-1397760x^{10}+2722720x^8-2814240x^6+1660659x^4-570570x^2+135135)}{15015}$	36

```
[In] int((-16*x^6+32*x^4-19*x^2+3)^2,x,method=_RETURNVERBOSE)
[Out] 9*x-38*x^3+553/5*x^5-1312/7*x^7+544/3*x^9-1024/11*x^11+256/13*x^13
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(3-19x^2+32x^4-16x^6\right)^2\,dx=\frac{256}{13}\,x^{13}-\frac{1024}{11}\,x^{11}+\frac{544}{3}\,x^9-\frac{1312}{7}\,x^7+\frac{553}{5}\,x^5-38\,x^3+9\,x$$

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="fricas")
[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 +
9*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \left(3-19x^2+32x^4-16x^6\right)^2\,dx=\frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^9}{3}-\frac{1312x^7}{7}+\frac{553x^5}{5}-38x^3+9x$$

```
[In] integrate((-16*x**6+32*x**4-19*x**2+3)**2,x)
[Out] 256*x**13/13 - 1024*x**11/11 + 544*x**9/3 - 1312*x**7/7 + 553*x**5/5 - 38*x
**3 + 9*x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(3-19x^2+32x^4-16x^6\right)^2 \,dx = \frac{256}{13}\,x^{13}-\frac{1024}{11}\,x^{11}+\frac{544}{3}\,x^9-\frac{1312}{7}\,x^7+\frac{553}{5}\,x^5-38\,x^3+9\,x$$

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 +
9*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(3-19x^2+32x^4-16x^6\right)^2 \,dx = \frac{256}{13}\,x^{13}-\frac{1024}{11}\,x^{11}+\frac{544}{3}\,x^9-\frac{1312}{7}\,x^7+\frac{553}{5}\,x^5-38\,x^3+9\,x$$

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 +
9*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(3-19x^2+32x^4-16x^6\right)^2 \,dx = \frac{256\,x^{13}}{13}-\frac{1024\,x^{11}}{11}+\frac{544\,x^9}{3}-\frac{1312\,x^7}{7}+\frac{553\,x^5}{5}-38\,x^3+9\,x$$

```
[In] int((19*x^2 - 32*x^4 + 16*x^6 - 3)^2,x)
```

```
[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 +
(256*x^13)/13
```

3.73 $\int (3-19x^2+32x^4-16x^6)\ dx$

Optimal result 628
Rubi [A] (verified) 628
Mathematica [A] (verified) 629
Maple [A] (verified) 629
Fricas [A] (verification not implemented) 629
Sympy [A] (verification not implemented) 630
Maxima [A] (verification not implemented) 630
Giac [A] (verification not implemented) 630
Mupad [B] (verification not implemented) 630

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int (3-19x^2+32x^4-16x^6)\ dx = 3x-\frac{19x^3}{3}+\frac{32x^5}{5}-\frac{16x^7}{7}$$

```
[Out] 3*x-19/3*x^3+32/5*x^5-16/7*x^7
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (3-19x^2+32x^4-16x^6)\ dx = -\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

```
[In] Int[3 - 19*x^2 + 32*x^4 - 16*x^6,x]
[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

Rubi steps

$$\text{integral} = 3x-\frac{19x^3}{3}+\frac{32x^5}{5}-\frac{16x^7}{7}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left(3-19x^2+32x^4-16x^6\right)\,dx=3x-\frac{19x^3}{3}+\frac{32x^5}{5}-\frac{16x^7}{7}$$

```
[In] Integrate[3 - 19*x^2 + 32*x^4 - 16*x^6,x]
[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$3x-\frac{19}{3}x^3+\frac{32}{5}x^5-\frac{16}{7}x^7$	20
norman	$3x-\frac{19}{3}x^3+\frac{32}{5}x^5-\frac{16}{7}x^7$	20
risch	$3x-\frac{19}{3}x^3+\frac{32}{5}x^5-\frac{16}{7}x^7$	20
parallelrisch	$3x-\frac{19}{3}x^3+\frac{32}{5}x^5-\frac{16}{7}x^7$	20
parts	$3x-\frac{19}{3}x^3+\frac{32}{5}x^5-\frac{16}{7}x^7$	20
gosper	$-\frac{x\left(240x^6-672x^4+665x^2-315\right)}{105}$	21

```
[In] int(-16*x^6+32*x^4-19*x^2+3,x,method=_RETURNVERBOSE)
[Out] 3*x-19/3*x^3+32/5*x^5-16/7*x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(3-19x^2+32x^4-16x^6\right)\,dx=-\frac{16}{7}\,x^7+\frac{32}{5}\,x^5-\frac{19}{3}\,x^3+3\,x$$

```
[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="fricas")
[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \left(3-19x^2+32x^4-16x^6\right) \, dx = -\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

```
[In] integrate(-16*x**6+32*x**4-19*x**2+3,x)
[Out] -16*x**7/7 + 32*x**5/5 - 19*x**3/3 + 3*x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(3-19x^2+32x^4-16x^6\right) \, dx = -\frac{16}{7}\,x^7+\frac{32}{5}\,x^5-\frac{19}{3}\,x^3+3\,x$$

```
[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="maxima")
[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(3-19x^2+32x^4-16x^6\right) \, dx = -\frac{16}{7}\,x^7+\frac{32}{5}\,x^5-\frac{19}{3}\,x^3+3\,x$$

```
[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="giac")
[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(3-19x^2+32x^4-16x^6\right) \, dx = -\frac{16\,x^7}{7}+\frac{32\,x^5}{5}-\frac{19\,x^3}{3}+3\,x$$

```
[In] int(32*x^4 - 19*x^2 - 16*x^6 + 3,x)
[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

3.74 $\int \frac{1}{3-19x^2+32x^4-16x^6} \, dx$

Optimal result 631
Rubi [A] (verified) 631
Mathematica [A] (verified) 632
Maple [A] (verified) 632
Fricas [B] (verification not implemented) 633
Sympy [B] (verification not implemented) 633
Maxima [B] (verification not implemented) 633
Giac [B] (verification not implemented) 634
Mupad [B] (verification not implemented) 634

Optimal result

Integrand size = 19, antiderivative size = 31

$$\int \frac{1}{3-19x^2+32x^4-16x^6} \, dx = \frac{\operatorname{arctanh}(x)}{3} + \frac{1}{3}\operatorname{arctanh}(2x) - \frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*arctanh(x)+1/3*arctanh(2*x)-1/3*arctanh(2/3*x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2082, 213}

$$\int \frac{1}{3-19x^2+32x^4-16x^6} \, dx = \frac{\operatorname{arctanh}(x)}{3} + \frac{1}{3}\operatorname{arctanh}(2x) - \frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1),x]
```

```
[Out] ArcTanh[x]/3 + ArcTanh[2*x]/3 - ArcTanh[(2*x)/Sqrt[3]]/Sqrt[3]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2082

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
```

```
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{1}{3(-1+x^2)}+\frac{2}{-3+4x^2}-\frac{2}{3(-1+4x^2)}\right)dx\\ &= -\left(\frac{1}{3}\int \frac{1}{-1+x^2}\,dx\right)-\frac{2}{3}\int \frac{1}{-1+4x^2}\,dx+2\int \frac{1}{-3+4x^2}\,dx\\ &= \frac{1}{3}\tanh^{-1}(x)+\frac{1}{3}\tanh^{-1}(2x)-\frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.00

$$\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx = \frac{1}{6}\Big(\sqrt{3}\log\left(\sqrt{3}-2x\right)-\sqrt{3}\log\left(\sqrt{3}+2x\right) - \log\left(1-3x+2x^2\right)+\log\left(1+3x+2x^2\right)\Big)$$

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1),x]
[Out] (Sqrt[3]*Log[Sqrt[3] - 2*x] - Sqrt[3]*Log[Sqrt[3] + 2*x] - Log[1 - 3*x + 2*
x^2] + Log[1 + 3*x + 2*x^2])/6
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{\ln(1+2x)}{6}-\frac{\ln(2x-1)}{6}+\frac{\ln(x+1)}{6}-\frac{\ln(x-1)}{6}-\frac{\operatorname{arctanh}\left(\frac{2x\sqrt{3}}{3}\right)\sqrt{3}}{3}$	42
risch	$\frac{\sqrt{3}\,\ln\left(2x-\sqrt{3}\right)}{6}-\frac{\sqrt{3}\,\ln\left(2x+\sqrt{3}\right)}{6}+\frac{\ln(2x^2+3x+1)}{6}-\frac{\ln(2x^2-3x+1)}{6}$	56

```
[In] int(1/(-16*x^6+32*x^4-19*x^2+3),x,method=_RETURNVERBOSE)
[Out] 1/6*ln(1+2*x)-1/6*ln(2*x-1)+1/6*ln(x+1)-1/6*ln(x-1)-1/3*arctanh(2/3*x*3^(1/
2))*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(23) = 46$.

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{4x^2-4\sqrt{3}x+3}{4x^2-3}\right) + \frac{1}{6}\log\left(2x^2+3x+1\right) - \frac{1}{6}\log\left(2x^2-3x+1\right)$$

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log((4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) + 1/6*log(2*x^2 + 3*
x + 1) - 1/6*log(2*x^2 - 3*x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(29) = 58$.

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.03

$$\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx = \frac{\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}\right)}{6} - \frac{\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}\right)}{6} - \frac{\log\left(x^2-\frac{3x}{2}+\frac{1}{2}\right)}{6} + \frac{\log\left(x^2+\frac{3x}{2}+\frac{1}{2}\right)}{6}$$

```
[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3),x)
[Out] sqrt(3)*log(x - sqrt(3)/2)/6 - sqrt(3)*log(x + sqrt(3)/2)/6 - log(x**2 - 3*
x/2 + 1/2)/6 + log(x**2 + 3*x/2 + 1/2)/6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(23) = 46$.

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{2x-\sqrt{3}}{2x+\sqrt{3}}\right) + \frac{1}{6}\log\left(2x+1\right) - \frac{1}{6}\log\left(2x-1\right) + \frac{1}{6}\log\left(x+1\right) - \frac{1}{6}\log\left(x-1\right)$$

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="maxima")
[Out] 1/6*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) + 1/6*log(2*x + 1) - 1/6*l
og(2*x - 1) + 1/6*log(x + 1) - 1/6*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(23) = 46$.

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.00

$$\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{|8x-4\sqrt{3}|}{|8x+4\sqrt{3}|}\right)+\frac{1}{6}\log\left(|2x+1|\right)-\frac{1}{6}\log\left(|2x-1|\right)+\frac{1}{6}\log\left(|x+1|\right)-\frac{1}{6}\log\left(|x-1|\right)$$

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) + 1/6*log(abs(2*
x + 1)) - 1/6*log(abs(2*x - 1)) + 1/6*log(abs(x + 1)) - 1/6*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 10.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx = \frac{\operatorname{atanh}\left(\frac{x}{4608\left(\frac{x^2}{6912}+\frac{1}{13824}\right)}\right)}{3}-\frac{\sqrt{3}\operatorname{atanh}\left(\frac{2\sqrt{3}x}{3}\right)}{3}$$

```
[In] int(-1/(19*x^2 - 32*x^4 + 16*x^6 - 3),x)
```

```
[Out] atanh(x/(4608*(x^2/6912 + 1/13824)))/3 - (3^(1/2)*atanh((2*3^(1/2)*x)/3))/3
```

3.75 $\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx$

Optimal result 635
Rubi [A] (verified) 635
Mathematica [A] (verified) 637
Maple [A] (verified) 637
Fricas [B] (verification not implemented) 637
Sympy [A] (verification not implemented) 638
Maxima [A] (verification not implemented) 638
Giac [A] (verification not implemented) 639
Mupad [B] (verification not implemented) 639

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx = \frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(1+x)} - \frac{1}{18(1+2x)}+\frac{2x}{3(3-4x^2)}+\frac{67\text{arctanh}(x)}{54} - \frac{7}{27}\text{arctanh}(2x)-\frac{5\text{arctanh}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 1/18/(1-2*x)+1/36/(1-x)-1/36/(1+x)-1/18/(1+2*x)+2/3*x/(-4*x^2+3)+67/54*arct
anh(x)-7/27*arctanh(2*x)-5/9*arctanh(2/3*x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2082, 213, 205}

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx = \frac{67\text{arctanh}(x)}{54}-\frac{7}{27}\text{arctanh}(2x) - \frac{5\text{arctanh}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x}{3(3-4x^2)}+\frac{1}{18(1-2x)} + \frac{1}{36(1-x)}-\frac{1}{36(x+1)}-\frac{1}{18(2x+1)}$$

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2),x]
```

```
[Out] 1/(18*(1 - 2*x)) + 1/(36*(1 - x)) - 1/(36*(1 + x)) - 1/(18*(1 + 2*x)) + (2*
x)/(3*(3 - 4*x^2)) + (67*ArcTanh[x])/54 - (7*ArcTanh[2*x])/27 - (5*ArcTanh[
(2*x)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2082

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{36(-1+x)^2}+\frac{1}{36(1+x)^2}+\frac{1}{9(-1+2x)^2}+\frac{1}{9(1+2x)^2}-\frac{67}{54\,(-1+x^2)}\right.\\
&\qquad\left.+\frac{4}{(-3+4x^2)^2}+\frac{4}{-3+4x^2}+\frac{14}{27\,(-1+4x^2)}\right)dx\\
&= \frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(1+x)}-\frac{1}{18(1+2x)}+\frac{14}{27}\int \frac{1}{-1+4x^2}\,dx\\
&\quad-\frac{67}{54}\int \frac{1}{-1+x^2}\,dx+4\int \frac{1}{(-3+4x^2)^2}\,dx+4\int \frac{1}{-3+4x^2}\,dx\\
&= \frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(1+x)}-\frac{1}{18(1+2x)}+\frac{2x}{3\,(3-4x^2)}\\
&\quad+\frac{67}{54}\tanh^{-1}(x)-\frac{7}{27}\tanh^{-1}(2x)-\frac{2\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\int \frac{1}{-3+4x^2}\,dx\\
&= \frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(1+x)}-\frac{1}{18(1+2x)}+\frac{2x}{3\,(3-4x^2)}\\
&\quad+\frac{67}{54}\tanh^{-1}(x)-\frac{7}{27}\tanh^{-1}(2x)-\frac{5\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx = \frac{1}{108}\left(-\frac{6x(27-104x^2+80x^4)}{-3+19x^2-32x^4+16x^6}+14\log(1-2x)\right.$$
$$+30\sqrt{3}\log\left(\sqrt{3}-2x\right)-67\log(1-x)+67\log(1+x)$$
$$\left.-14\log(1+2x)-30\sqrt{3}\log\left(\sqrt{3}+2x\right)\right)$$

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2),x]
```

```
[Out] ((-6*x*(27 - 104*x^2 + 80*x^4))/(-3 + 19*x^2 - 32*x^4 + 16*x^6) + 14*Log[1
- 2*x] + 30*Sqrt[3]*Log[Sqrt[3] - 2*x] - 67*Log[1 - x] + 67*Log[1 + x] - 14
*Log[1 + 2*x] - 30*Sqrt[3]*Log[Sqrt[3] + 2*x])/108
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

method	result
default	$-\frac{1}{18(1+2x)}-\frac{7\ln(1+2x)}{54}-\frac{1}{18(2x-1)}+\frac{7\ln(2x-1)}{54}-\frac{1}{36(x+1)}+\frac{67\ln(x+1)}{108}-\frac{1}{36(x-1)}-\frac{67\ln(x-1)}{108}-\frac{x}{6(x^2-\frac{3}{4})}-$
risch	$\frac{-\frac{5}{18}x^5+\frac{13}{36}x^3-\frac{3}{32}x}{x^6-2x^4+\frac{19}{16}x^2-\frac{3}{16}}+\frac{67\ln(x+1)}{108}-\frac{7\ln(1+2x)}{54}+\frac{7\ln(2x-1)}{54}+\frac{5\sqrt{3}\,\ln\left(2x-\sqrt{3}\right)}{18}-\frac{5\sqrt{3}\,\ln\left(2x+\sqrt{3}\right)}{18}-\frac{67\ln(x-1)}{108}$

```
[In] int(1/(-16*x^6+32*x^4-19*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18/(1+2*x)-7/54*ln(1+2*x)-1/18/(2*x-1)+7/54*ln(2*x-1)-1/36/(x+1)+67/108*
ln(x+1)-1/36/(x-1)-67/108*ln(x-1)-1/6*x/(x^2-3/4)-5/9*arctanh(2/3*x*3^(1/2)
)*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(67) = 134$.

Time = 0.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.99

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx =$$
$$-\frac{480\,x^5-624\,x^3-30\,\sqrt{3}(16\,x^6-32\,x^4+19\,x^2-3)\log\left(\frac{4\,x^2-4\,\sqrt{3}x+3}{4\,x^2-3}\right)+14\,(16\,x^6-32\,x^4+19\,x^2-3)}{}$$

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/108*(480*x^5 - 624*x^3 - 30*sqrt(3)*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log((
4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) + 14*(16*x^6 - 32*x^4 + 19*x^2 - 3)*l
og(2*x + 1) - 14*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(2*x - 1) - 67*(16*x^6 -
 32*x^4 + 19*x^2 - 3)*log(x + 1) + 67*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(x
- 1) + 162*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3)
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx = \frac{-80x^5+104x^3-27x}{288x^6-576x^4+342x^2-54} - \frac{67\log(x-1)}{108} + \frac{7\log\left(x-\frac{1}{2}\right)}{54} - \frac{7\log\left(x+\frac{1}{2}\right)}{54} + \frac{67\log(x+1)}{108} + \frac{5\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}\right)}{18} - \frac{5\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}\right)}{18}$$

```
[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3)**2,x)
```

```
[Out] (-80*x**5 + 104*x**3 - 27*x)/(288*x**6 - 576*x**4 + 342*x**2 - 54) - 67*log
(x - 1)/108 + 7*log(x - 1/2)/54 - 7*log(x + 1/2)/54 + 67*log(x + 1)/108 + 5
*sqrt(3)*log(x - sqrt(3)/2)/18 - 5*sqrt(3)*log(x + sqrt(3)/2)/18
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx = \frac{5}{18}\sqrt{3}\log\left(\frac{2\,x-\sqrt{3}}{2\,x+\sqrt{3}}\right) - \frac{80\,x^5-104\,x^3+27\,x}{18\,(16\,x^6-32\,x^4+19\,x^2-3)} - \frac{7}{54}\log(2\,x+1) + \frac{7}{54}\log(2\,x-1) + \frac{67}{108}\log(x+1) - \frac{67}{108}\log(x-1)$$

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/18*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) - 1/18*(80*x^5 - 104*x^3
+ 27*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3) - 7/54*log(2*x + 1) + 7/54*log(2*x -
 1) + 67/108*log(x + 1) - 67/108*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx = \frac{5}{18}\,\sqrt{3}\log\left(\frac{|8\,x-4\,\sqrt{3}|}{|8\,x+4\,\sqrt{3}|}\right) - \frac{80\,x^5-104\,x^3+27\,x}{18\,(16\,x^6-32\,x^4+19\,x^2-3)} - \frac{7}{54}\,\log\left(|2\,x+1|\right) + \frac{7}{54}\,\log\left(|2\,x-1|\right) + \frac{67}{108}\,\log\left(|x+1|\right) - \frac{67}{108}\,\log\left(|x-1|\right)$$

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/18*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) - 1/18*(80*x^5
- 104*x^3 + 27*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3) - 7/54*log(abs(2*x + 1)) +
 7/54*log(abs(2*x - 1)) + 67/108*log(abs(x + 1)) - 67/108*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx = -\frac{\mathrm{atan}(x\,1\mathrm{i})\,67\mathrm{i}}{54} + \frac{\mathrm{atan}(x\,2\mathrm{i})\,7\mathrm{i}}{27} - \frac{\frac{5\,x^5}{18}-\frac{13\,x^3}{36}+\frac{3\,x}{32}}{x^6-2\,x^4+\frac{19\,x^2}{16}-\frac{3}{16}} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x\,2\mathrm{i}}{3}\right)\,5\mathrm{i}}{9}$$

```
[In] int(1/(19*x^2 - 32*x^4 + 16*x^6 - 3)^2,x)
```

```
[Out] (atan(x*2i)*7i)/27 - (atan(x*1i)*67i)/54 - ((3*x)/32 - (13*x^3)/36 + (5*x^5
)/18)/((19*x^2)/16 - 2*x^4 + x^6 - 3/16) + (3^(1/2)*atan((3^(1/2)*x*2i)/3)*
5i)/9
```

3.76 $\int \frac{1}{(3-19x^2+32x^4-16x^6)^3}\,dx$

Optimal result 640
Rubi [A] (verified) 640
Mathematica [A] (verified) 643
Maple [A] (verified) 643
Fricas [B] (verification not implemented) 643
Sympy [A] (verification not implemented) 644
Maxima [A] (verification not implemented) 644
Giac [A] (verification not implemented) 645
Mupad [B] (verification not implemented) 645

Optimal result

Integrand size = 19, antiderivative size = 161

$$\begin{aligned}\int \frac{1}{(3-19x^2+32x^4-16x^6)^3}\,dx &= \frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}\\&\quad+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}-\frac{67}{432(1+x)}\\&\quad-\frac{1}{108(1+2x)^2}+\frac{7}{108(1+2x)}-\frac{2x}{3(3-4x^2)^2}\\&\quad+\frac{5x}{3(3-4x^2)}+\frac{3913\text{arctanh}(x)}{648}+\frac{67}{162}\text{arctanh}(2x)\\&\quad+\frac{5\text{arctanh}\left(\frac{2x}{\sqrt{3}}\right)}{6\sqrt{3}}-4\sqrt{3}\text{arctanh}\left(\frac{2x}{\sqrt{3}}\right)\end{aligned}$$

```
[Out] 1/108/(1-2*x)^2-7/108/(1-2*x)+1/432/(1-x)^2+67/432/(1-x)-1/432/(1+x)^2-67/4
32/(1+x)-1/108/(1+2*x)^2+7/108/(1+2*x)-2/3*x/(-4*x^2+3)^2+5/3*x/(-4*x^2+3)+
3913/648*arctanh(x)+67/162*arctanh(2*x)-67/18*arctanh(2/3*x*3^(1/2))*3^(1/2
)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used

= {2082, 213, 205}

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^3}\,dx = \frac{3913\text{arctanh}(x)}{648}+\frac{67}{162}\text{arctanh}(2x) - 4\sqrt{3}\text{arctanh}\left(\frac{2x}{\sqrt{3}}\right)+\frac{5\text{arctanh}\left(\frac{2x}{\sqrt{3}}\right)}{6\sqrt{3}}+\frac{5x}{3(3-4x^2)} - \frac{2x}{3(3-4x^2)^2}-\frac{7}{108(1-2x)}+\frac{67}{432(1-x)} - \frac{67}{432(x+1)}+\frac{7}{108(2x+1)}+\frac{1}{108(1-2x)^2} + \frac{1}{432(1-x)^2}-\frac{1}{432(x+1)^2}-\frac{1}{108(2x+1)^2}$$

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3),x]
```

```
[Out] 1/(108*(1 - 2*x)^2) - 7/(108*(1 - 2*x)) + 1/(432*(1 - x)^2) + 67/(432*(1 -
x)) - 1/(432*(1 + x)^2) - 67/(432*(1 + x)) - 1/(108*(1 + 2*x)^2) + 7/(108*(
1 + 2*x)) - (2*x)/(3*(3 - 4*x^2)^2) + (5*x)/(3*(3 - 4*x^2)) + (3913*ArcTanh
[x])/648 + (67*ArcTanh[2*x])/162 + (5*ArcTanh[(2*x)/Sqrt[3]])/(6*Sqrt[3]) -
 4*Sqrt[3]*ArcTanh[(2*x)/Sqrt[3]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2082

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{216(-1+x)^3}+\frac{67}{432(-1+x)^2}+\frac{1}{216(1+x)^3}+\frac{67}{432(1+x)^2}-\frac{1}{27(-1+2x)^3}\right.\\
&\quad -\frac{7}{54(-1+2x)^2}+\frac{1}{27(1+2x)^3}-\frac{7}{54(1+2x)^2}-\frac{3913}{648\,(-1+x^2)}+\frac{8}{(-3+4x^2)^3}\\
&\quad \left.+\frac{12}{(-3+4x^2)^2}+\frac{24}{-3+4x^2}-\frac{67}{81\,(-1+4x^2)}\right) dx\\
&= \frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}-\frac{67}{432(1+x)}\\
&\quad -\frac{1}{108(1+2x)^2}+\frac{7}{108(1+2x)}-\frac{67}{81}\int\frac{1}{-1+4x^2}\,dx-\frac{3913}{648}\int\frac{1}{-1+x^2}\,dx\\
&\quad +8\int\frac{1}{(-3+4x^2)^3}\,dx+12\int\frac{1}{(-3+4x^2)^2}\,dx+24\int\frac{1}{-3+4x^2}\,dx\\
&= \frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}-\frac{67}{432(1+x)}\\
&\quad -\frac{1}{108(1+2x)^2}+\frac{7}{108(1+2x)}-\frac{2x}{3\,(3-4x^2)^2}+\frac{2x}{3-4x^2}+\frac{3913}{648}\tanh^{-1}(x)\\
&\quad +\frac{67}{162}\tanh^{-1}(2x)-4\sqrt{3}\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)-2\int\frac{1}{(-3+4x^2)^2}\,dx-2\int\frac{1}{-3+4x^2}\,dx\\
&= \frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}-\frac{67}{432(1+x)}\\
&\quad -\frac{1}{108(1+2x)^2}+\frac{7}{108(1+2x)}-\frac{2x}{3\,(3-4x^2)^2}+\frac{5x}{3\,(3-4x^2)}+\frac{3913}{648}\tanh^{-1}(x)\\
&\quad +\frac{67}{162}\tanh^{-1}(2x)+\frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}-4\sqrt{3}\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)+\frac{1}{3}\int\frac{1}{-3+4x^2}\,dx\\
&= \frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}\\
&\quad -\frac{67}{432(1+x)}-\frac{1}{108(1+2x)^2}+\frac{7}{108(1+2x)}-\frac{2x}{3\,(3-4x^2)^2}+\frac{5x}{3\,(3-4x^2)}\\
&\quad +\frac{3913}{648}\tanh^{-1}(x)+\frac{67}{162}\tanh^{-1}(2x)+\frac{5\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{6\sqrt{3}}-4\sqrt{3}\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.85

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^3}\,dx = \frac{\frac{36x(27-104x^2+80x^4)}{(3-19x^2+32x^4-16x^6)^2}-\frac{6x(345-2384x^2+2288x^4)}{-3+19x^2-32x^4+16x^6}-268\log(1-2x)+2412\sqrt{3}\log\left(\sqrt{3}-2x\right)-3913\log(1-x)+}{1296}$$

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3),x]
```

```
[Out] ((36*x*(27 - 104*x^2 + 80*x^4))/(3 - 19*x^2 + 32*x^4 - 16*x^6)^2 - (6*x*(34
5 - 2384*x^2 + 2288*x^4))/(-3 + 19*x^2 - 32*x^4 + 16*x^6) - 268*Log[1 - 2*x
] + 2412*Sqrt[3]*Log[Sqrt[3] - 2*x] - 3913*Log[1 - x] + 3913*Log[1 + x] + 2
68*Log[1 + 2*x] - 2412*Sqrt[3]*Log[Sqrt[3] + 2*x])/1296
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.67

method	result
risch	$\frac{-\frac{4576}{27}x^{11}+\frac{4640}{9}x^9-580x^7+\frac{7960}{27}x^5-\frac{4777}{72}x^3+\frac{133}{24}x}{(16x^6-32x^4+19x^2-3)^2}+\frac{67\ln(1+2x)}{324}+\frac{3913\ln(x+1)}{1296}+\frac{67\sqrt{3}\,\ln\left(2x-\sqrt{3}\right)}{36}-\frac{67\sqrt{3}\,\ln\left(2x+\sqrt{3}\right.}{36}$
default	$-\frac{1}{108(1+2x)^2}+\frac{7}{108(1+2x)}+\frac{67\ln(1+2x)}{324}+\frac{1}{108(2x-1)^2}+\frac{7}{108(2x-1)}-\frac{67\ln(2x-1)}{324}-\frac{1}{432(x+1)^2}-\frac{67}{432(x+1)}+\frac{3}{}$

```
[In] int(1/(-16*x^6+32*x^4-19*x^2+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 256*(-143/216*x^11+145/72*x^9-145/64*x^7+995/864*x^5-4777/18432*x^3+133/614
4*x)/(16*x^6-32*x^4+19*x^2-3)^2+67/324*ln(1+2*x)+3913/1296*ln(x+1)+67/36*3^
(1/2)*ln(2*x-3^(1/2))-67/36*3^(1/2)*ln(2*x+3^(1/2))-67/324*ln(2*x-1)-3913/1
296*ln(x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(111) = 222$.

Time = 0.30 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.75

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^3}\,dx = -\frac{219648\,x^{11}-668160\,x^9+751680\,x^7-382080\,x^5+85986\,x^3-2412\,\sqrt{3}(256\,x^{12}-1024\,x^{10}+1632\,x^8-}{}$$

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/1296*(219648*x^11 - 668160*x^9 + 751680*x^7 - 382080*x^5 + 85986*x^3 - 2
412*sqrt(3)*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2
 + 9)*log((4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) - 268*(256*x^12 - 1024*x^1
0 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(2*x + 1) + 268*(256*x^
12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(2*x - 1)
- 3913*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)
*log(x + 1) + 3913*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 -
114*x^2 + 9)*log(x - 1) - 7182*x)/(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x
^6 + 553*x^4 - 114*x^2 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.83

$$
\begin{aligned}
&\int \frac{1}{(3-19x^2+32x^4-16x^6)^3}\,dx \\
&= -\frac{36608x^{11}-111360x^9+125280x^7-63680x^5+14331x^3-1197x}{55296x^{12}-221184x^{10}+352512x^8-283392x^6+119448x^4-24624x^2+1944} \\
&\quad -\frac{3913\log(x-1)}{1296}-\frac{67\log\left(x-\frac{1}{2}\right)}{324}+\frac{67\log\left(x+\frac{1}{2}\right)}{324} \\
&\quad +\frac{3913\log(x+1)}{1296}+\frac{67\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}\right)}{36}-\frac{67\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}\right)}{36}
\end{aligned}
$$

```
[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3)**3,x)
```

```
[Out] -(36608*x**11 - 111360*x**9 + 125280*x**7 - 63680*x**5 + 14331*x**3 - 1197*
x)/(55296*x**12 - 221184*x**10 + 352512*x**8 - 283392*x**6 + 119448*x**4 -
24624*x**2 + 1944) - 3913*log(x - 1)/1296 - 67*log(x - 1/2)/324 + 67*log(x
+ 1/2)/324 + 3913*log(x + 1)/1296 + 67*sqrt(3)*log(x - sqrt(3)/2)/36 - 67*s
qrt(3)*log(x + sqrt(3)/2)/36
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.74

$$
\begin{aligned}
&\int \frac{1}{(3-19x^2+32x^4-16x^6)^3}\,dx \\
&= \frac{67}{36}\sqrt{3}\log\left(\frac{2\,x-\sqrt{3}}{2\,x+\sqrt{3}}\right) \\
&\quad -\frac{36608\,x^{11}-111360\,x^9+125280\,x^7-63680\,x^5+14331\,x^3-1197\,x}{216\,(256\,x^{12}-1024\,x^{10}+1632\,x^8-1312\,x^6+553\,x^4-114\,x^2+9)} \\
&\quad +\frac{67}{324}\log(2\,x+1)-\frac{67}{324}\log(2\,x-1)+\frac{3913}{1296}\log(x+1)-\frac{3913}{1296}\log(x-1)
\end{aligned}
$$

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 67/36*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) - 1/216*(36608*x^11 - 11
1360*x^9 + 125280*x^7 - 63680*x^5 + 14331*x^3 - 1197*x)/(256*x^12 - 1024*x^
10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9) + 67/324*log(2*x + 1) - 6
7/324*log(2*x - 1) + 3913/1296*log(x + 1) - 3913/1296*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^3}\,dx = \frac{67}{36}\sqrt{3}\log\left(\frac{|8\,x-4\,\sqrt{3}|}{|8\,x+4\,\sqrt{3}|}\right) - \frac{36608\,x^{11}-111360\,x^9+125280\,x^7-63680\,x^5+14331\,x^3-1197\,x}{216\,(16\,x^6-32\,x^4+19\,x^2-3)^2} + \frac{67}{324}\log\left(|2\,x+1|\right) - \frac{67}{324}\log\left(|2\,x-1|\right) + \frac{3913}{1296}\log\left(|x+1|\right) - \frac{3913}{1296}\log\left(|x-1|\right)$$

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="giac")
```

```
[Out] 67/36*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) - 1/216*(36608
*x^11 - 111360*x^9 + 125280*x^7 - 63680*x^5 + 14331*x^3 - 1197*x)/(16*x^6 -
 32*x^4 + 19*x^2 - 3)^2 + 67/324*log(abs(2*x + 1)) - 67/324*log(abs(2*x - 1
)) + 3913/1296*log(abs(x + 1)) - 3913/1296*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.58

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^3}\,dx = \frac{-\frac{143\,x^{11}}{216}+\frac{145\,x^9}{72}-\frac{145\,x^7}{64}+\frac{995\,x^5}{864}-\frac{4777\,x^3}{18432}+\frac{133\,x}{6144}}{x^{12}-4\,x^{10}+\frac{51\,x^8}{8}-\frac{41\,x^6}{8}+\frac{553\,x^4}{256}-\frac{57\,x^2}{128}+\frac{9}{256}} - \frac{\mathrm{atan}(x\,2\mathrm{i})\,67\mathrm{i}}{162} - \frac{\mathrm{atan}(x\,1\mathrm{i})\,3913\mathrm{i}}{648} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x\,2\mathrm{i}}{3}\right)\,67\mathrm{i}}{18}$$

```
[In] int(-1/(19*x^2 - 32*x^4 + 16*x^6 - 3)^3,x)
```

```
[Out] ((133*x)/6144 - (4777*x^3)/18432 + (995*x^5)/864 - (145*x^7)/64 + (145*x^9)
/72 - (143*x^11)/216)/((553*x^4)/256 - (57*x^2)/128 - (41*x^6)/8 + (51*x^8)
/8 - 4*x^10 + x^12 + 9/256) - (atan(x*2i)*67i)/162 - (atan(x*1i)*3913i)/648
 + (3^(1/2)*atan((3^(1/2)*x*2i)/3)*67i)/18
```

3.77 $\int \frac{1}{(-1+7x^2-7x^4+x^6)^2} dx$

Optimal result 646
Rubi [B] (verified) 647
Mathematica [A] (verified) 649
Maple [A] (verified) 650
Fricas [B] (verification not implemented) 650
Sympy [B] (verification not implemented) 651
Maxima [A] (verification not implemented) 652
Giac [A] (verification not implemented) 652
Mupad [B] (verification not implemented) 653

Optimal result

Integrand size = 17, antiderivative size = 91

$$\int \frac{1}{(-1+7x^2-7x^4+x^6)^2} dx = \frac{x}{32(1-x^2)} + \frac{x(99-17x^2)}{128(1-6x^2+x^4)} + \frac{5\text{arctanh}(x)}{32} + \frac{1}{512}\left(-4+3\sqrt{2}\right)\text{arctanh}\left(\left(-1+\sqrt{2}\right)x\right) + \frac{1}{512}\left(4+3\sqrt{2}\right)\text{arctanh}\left(\left(1+\sqrt{2}\right)x\right)$$

```
[Out] 1/32*x/(-x^2+1)+1/128*x*(-17*x^2+99)/(x^4-6*x^2+1)+5/32*arctanh(x)+1/512*ar
ctanh(x*(2^(1/2)-1))*(-4+3*2^(1/2))+1/512*arctanh(x*(1+2^(1/2)))*(4+3*2^(1/
2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 205 vs. 2(91) = 182.

Time = 0.10 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.25, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2082, 213, 652, 632, 212, 646, 31}

$$\int \frac{1}{(-1+7x^2-7x^4+x^6)^2} dx = -\frac{17\text{arctanh}\left(\frac{1-x}{\sqrt{2}}\right)}{256\sqrt{2}} + \frac{5\text{arctanh}(x)}{32} + \frac{17\text{arctanh}\left(\frac{x+1}{\sqrt{2}}\right)}{256\sqrt{2}} - \frac{41-17x}{256(-x^2+2x+1)} + \frac{17x+41}{256(-x^2-2x+1)} + \frac{1}{64(1-x)} - \frac{1}{64(x+1)} + \frac{1}{512}\left(2-7\sqrt{2}\right)\log\left(-x-\sqrt{2}+1\right) + \frac{1}{512}\left(2+7\sqrt{2}\right)\log\left(-x+\sqrt{2}+1\right) - \frac{1}{512}\left(2-7\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right) - \frac{1}{512}\left(2+7\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

```
[In] Int[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2),x]
```

```
[Out] 1/(64*(1 - x)) - 1/(64*(1 + x)) + (41 + 17*x)/(256*(1 - 2*x - x^2)) - (41 -
 17*x)/(256*(1 + 2*x - x^2)) - (17*ArcTanh[(1 - x)/Sqrt[2]])/(256*Sqrt[2])
+ (5*ArcTanh[x])/32 + (17*ArcTanh[(1 + x)/Sqrt[2]])/(256*Sqrt[2]) + ((2 - 7
*Sqrt[2])*Log[1 - Sqrt[2] - x])/512 + ((2 + 7*Sqrt[2])*Log[1 + Sqrt[2] - x]
)/512 - ((2 - 7*Sqrt[2])*Log[1 - Sqrt[2] + x])/512 - ((2 + 7*Sqrt[2])*Log[1
 + Sqrt[2] + x])/512
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2082

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{64(-1+x)^2}+\frac{1}{64(1+x)^2}-\frac{5}{32\left(-1+x^2\right)}+\frac{29-12x}{64\left(-1-2x+x^2\right)^2}\right.\\
&\qquad \left.+\frac{6+x}{128\left(-1-2x+x^2\right)}+\frac{29+12x}{64\left(-1+2x+x^2\right)^2}+\frac{6-x}{128\left(-1+2x+x^2\right)}\right)dx\\
&= \frac{1}{64(1-x)}-\frac{1}{64(1+x)}+\frac{1}{128}\int\frac{6+x}{-1-2x+x^2}\,dx+\frac{1}{128}\int\frac{6-x}{-1+2x+x^2}\,dx\\
&\quad +\frac{1}{64}\int\frac{29-12x}{\left(-1-2x+x^2\right)^2}\,dx+\frac{1}{64}\int\frac{29+12x}{\left(-1+2x+x^2\right)^2}\,dx-\frac{5}{32}\int\frac{1}{-1+x^2}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{64(1-x)} - \frac{1}{64(1+x)} + \frac{41+17x}{256\,(1-2x-x^2)} - \frac{41-17x}{256\,(1+2x-x^2)} \\
&\quad + \frac{5}{32}\tanh^{-1}(x) - \frac{17}{256}\int \frac{1}{-1-2x+x^2}\,dx - \frac{17}{256}\int \frac{1}{-1+2x+x^2}\,dx \\
&\quad + \frac{1}{512}\left(2-7\sqrt{2}\right)\int \frac{1}{-1+\sqrt{2}+x}\,dx + \frac{1}{512}\left(-2+7\sqrt{2}\right)\int \frac{1}{1-\sqrt{2}+x}\,dx \\
&\quad + \frac{1}{512}\left(2+7\sqrt{2}\right)\int \frac{1}{-1-\sqrt{2}+x}\,dx - \frac{1}{512}\left(2+7\sqrt{2}\right)\int \frac{1}{1+\sqrt{2}+x}\,dx \\
&= \frac{1}{64(1-x)} - \frac{1}{64(1+x)} + \frac{41+17x}{256\,(1-2x-x^2)} - \frac{41-17x}{256\,(1+2x-x^2)} + \frac{5}{32}\tanh^{-1}(x) \\
&\quad + \frac{1}{512}\left(2-7\sqrt{2}\right)\log\left(1-\sqrt{2}-x\right) + \frac{1}{512}\left(2+7\sqrt{2}\right)\log\left(1+\sqrt{2}-x\right) \\
&\quad - \frac{1}{512}\left(2-7\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right) - \frac{1}{512}\left(2+7\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right) \\
&\quad + \frac{17}{128}\text{Subst}\left(\int \frac{1}{8-x^2}\,dx, x, -2+2x\right) + \frac{17}{128}\text{Subst}\left(\int \frac{1}{8-x^2}\,dx, x, 2+2x\right) \\
&= \frac{1}{64(1-x)} - \frac{1}{64(1+x)} + \frac{41+17x}{256\,(1-2x-x^2)} - \frac{41-17x}{256\,(1+2x-x^2)} \\
&\quad - \frac{17\tanh^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{256\sqrt{2}} + \frac{5}{32}\tanh^{-1}(x) + \frac{17\tanh^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{256\sqrt{2}} \\
&\quad + \frac{1}{512}\left(2-7\sqrt{2}\right)\log\left(1-\sqrt{2}-x\right) + \frac{1}{512}\left(2+7\sqrt{2}\right)\log\left(1+\sqrt{2}-x\right) \\
&\quad - \frac{1}{512}\left(2-7\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right) - \frac{1}{512}\left(2+7\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.45

$$
\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx = \frac{-\frac{8x(103-140x^2+21x^4)}{-1+7x^2-7x^4+x^6} - 80\log(1-x) - \left(4+3\sqrt{2}\right)\log\left(-1+\sqrt{2}-x\right) + \left(4-3\sqrt{2}\right)\log\left(1+\sqrt{2}-x\right) + 80}{1024}
$$

```
[In] Integrate[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2),x]
```

```
[Out] ((-8*x*(103 - 140*x^2 + 21*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 80*Log[1 - x]
 - (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (4 - 3*Sqrt[2])*Log[1 + Sqrt[2]
- x] + 80*Log[1 + x] + (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] + x] + (-4 + 3*Sqrt
[2])*Log[1 + Sqrt[2] + x])/1024
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

method	result
default	$-\frac{1}{64(x+1)}+\frac{5\ln(x+1)}{64}-\frac{\frac{17x}{2}+\frac{41}{2}}{128(x^2+2x-1)}-\frac{\ln(x^2+2x-1)}{256}+\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{512}-\frac{1}{64(x-1)}-\frac{5\ln(x-1)}{64}+\frac{-1}{128x^2-}$
risch	$\frac{-\frac{21}{128}x^5+\frac{35}{32}x^3-\frac{103}{128}x}{x^6-7x^4+7x^2-1}+\frac{\ln\left(x-1+\sqrt{2}\right)}{256}+\frac{3\ln\left(x-1+\sqrt{2}\right)\sqrt{2}}{1024}+\frac{\ln\left(x-1-\sqrt{2}\right)}{256}-\frac{3\ln\left(x-1-\sqrt{2}\right)\sqrt{2}}{1024}-\frac{\ln\left(1+x+\sqrt{2}\right)}{256}+\frac{3\ln}{}$

```
[In] int(1/(x^6-7*x^4+7*x^2-1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/(x+1)+5/64*ln(x+1)-1/128*(17/2*x+41/2)/(x^2+2*x-1)-1/256*ln(x^2+2*x-1
)+3/512*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))-1/64/(x-1)-5/64*ln(x-1)+1/128*
(-17/2*x+41/2)/(x^2-2*x-1)+1/256*ln(x^2-2*x-1)+3/512*2^(1/2)*arctanh(1/4*(2
*x-2)*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(71) = 142.

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.45

$$\int \frac{1}{\left(-1+7x^2-7x^4+x^6\right)^2}\,dx = -\frac{168\,x^5-1120\,x^3-3\,\sqrt{2}(x^6-7\,x^4+7\,x^2-1)\log\left(\frac{x^2+2\,\sqrt{2}(x+1)+2\,x+3}{x^2+2\,x-1}\right)-3\,\sqrt{2}(x^6-7\,x^4+7\,x^2-1)\log\left(\right.}{}$$

```
[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="fricas")
```

```
[Out] -1/1024*(168*x^5 - 1120*x^3 - 3*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2
+ 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) - 3*sqrt(2)*(x^6 - 7*x^4 +
7*x^2 - 1)*log((x^2 + 2*sqrt(2)*(x - 1) - 2*x + 3)/(x^2 - 2*x - 1)) + 4*(x^
6 - 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*x - 1) - 4*(x^6 - 7*x^4 + 7*x^2 - 1)*log
(x^2 - 2*x - 1) - 80*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) + 80*(x^6 - 7*x^4
 + 7*x^2 - 1)*log(x - 1) + 824*x)/(x^6 - 7*x^4 + 7*x^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(75) = 150.

Time = 0.81 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.25

$$\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx$$
$$= \frac{-21x^5+140x^3-103x}{128x^6-896x^4+896x^2-128} - \frac{5\log(x-1)}{64} + \frac{5\log(x+1)}{64} + \left(-\frac{1}{256} + \frac{3\sqrt{2}}{1024}\right)\log\left(x - \frac{8071264001}{202624020} - \frac{471550901878784\left(-\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^3}{2979765} + \frac{1299552375287054336\left(-\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^5}{50656005}\right.$$
$$+ \left(-\frac{3\sqrt{2}}{1024} - \frac{1}{256}\right)\log\left(x - \frac{8071264001\sqrt{2}}{270165360} - \frac{8071264001}{202624020} + \frac{1299552375287054336\left(-\frac{3\sqrt{2}}{1024}-\frac{1}{256}\right)^5}{50656005} - \frac{471550901\ldots}{}\right.$$
$$+ \left(\frac{1}{256} - \frac{3\sqrt{2}}{1024}\right)\log\left(x - \frac{8071264001\sqrt{2}}{270165360} + \frac{1299552375287054336\left(\frac{1}{256}-\frac{3\sqrt{2}}{1024}\right)^5}{50656005} - \frac{471550901878784\left(\frac{1}{256}-\frac{3\sqrt{2}}{10\ldots}\right.}{2979765}\right.$$
$$+ \left(\frac{1}{256} + \frac{3\sqrt{2}}{1024}\right)\log\left(x - \frac{471550901878784\left(\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^3}{2979765} + \frac{1299552375287054336\left(\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^5}{50656005} + \frac{807126400\ldots}{202624020\ldots}\right.$$

```
[In] integrate(1/(x**6-7*x**4+7*x**2-1)**2,x)
```

```
[Out] (-21*x**5 + 140*x**3 - 103*x)/(128*x**6 - 896*x**4 + 896*x**2 - 128) - 5*lo
g(x - 1)/64 + 5*log(x + 1)/64 + (-1/256 + 3*sqrt(2)/1024)*log(x - 807126400
1/202624020 - 471550901878784*(-1/256 + 3*sqrt(2)/1024)**3/2979765 + 129955
2375287054336*(-1/256 + 3*sqrt(2)/1024)**5/50656005 + 8071264001*sqrt(2)/27
0165360) + (-3*sqrt(2)/1024 - 1/256)*log(x - 8071264001*sqrt(2)/270165360 -
 8071264001/202624020 + 1299552375287054336*(-3*sqrt(2)/1024 - 1/256)**5/50
656005 - 471550901878784*(-3*sqrt(2)/1024 - 1/256)**3/2979765) + (1/256 - 3
*sqrt(2)/1024)*log(x - 8071264001*sqrt(2)/270165360 + 1299552375287054336*(
1/256 - 3*sqrt(2)/1024)**5/50656005 - 471550901878784*(1/256 - 3*sqrt(2)/10
```

```
24)**3/2979765 + 8071264001/202624020) + (1/256 + 3*sqrt(2)/1024)*log(x - 4
71550901878784*(1/256 + 3*sqrt(2)/1024)**3/2979765 + 1299552375287054336*(1
/256 + 3*sqrt(2)/1024)**5/50656005 + 8071264001/202624020 + 8071264001*sqrt
(2)/270165360)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.25

$$\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx = -\frac{3}{1024}\,\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)-\frac{3}{1024}\,\sqrt{2}\log\left(\frac{x-\sqrt{2}-1}{x+\sqrt{2}-1}\right) - \frac{21\,x^5-140\,x^3+103\,x}{128\,(x^6-7\,x^4+7\,x^2-1)} - \frac{1}{256}\,\log\left(x^2+2\,x-1\right) + \frac{1}{256}\,\log\left(x^2-2\,x-1\right) + \frac{5}{64}\,\log\left(x+1\right) - \frac{5}{64}\,\log\left(x-1\right)$$

```
[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="maxima")
```

```
[Out] -3/1024*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) - 3/1024*sqrt(2)*l
og((x - sqrt(2) - 1)/(x + sqrt(2) - 1)) - 1/128*(21*x^5 - 140*x^3 + 103*x)/
(x^6 - 7*x^4 + 7*x^2 - 1) - 1/256*log(x^2 + 2*x - 1) + 1/256*log(x^2 - 2*x
- 1) + 5/64*log(x + 1) - 5/64*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.47

$$\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx = -\frac{3}{1024}\,\sqrt{2}\log\left(\frac{\left|2\,x-2\,\sqrt{2}+2\right|}{\left|2\,x+2\,\sqrt{2}+2\right|}\right) - \frac{3}{1024}\,\sqrt{2}\log\left(\frac{\left|2\,x-2\,\sqrt{2}-2\right|}{\left|2\,x+2\,\sqrt{2}-2\right|}\right) - \frac{21\,x^5-140\,x^3+103\,x}{128\,(x^6-7\,x^4+7\,x^2-1)} - \frac{1}{256}\,\log\left(\left|x^2+2\,x-1\right|\right) + \frac{1}{256}\,\log\left(\left|x^2-2\,x-1\right|\right) + \frac{5}{64}\,\log\left(\left|x+1\right|\right) - \frac{5}{64}\,\log\left(\left|x-1\right|\right)$$

```
[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="giac")
```

```
[Out] -3/1024*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) - 3/
1024*sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2)/abs(2*x + 2*sqrt(2) - 2)) - 1/128
*(21*x^5 - 140*x^3 + 103*x)/(x^6 - 7*x^4 + 7*x^2 - 1) - 1/256*log(abs(x^2 +
 2*x - 1)) + 1/256*log(abs(x^2 - 2*x - 1)) + 5/64*log(abs(x + 1)) - 5/64*lo
g(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 10.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.38

$$\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx = -\frac{\operatorname{atan}(x\,1\mathrm{i})\,5\mathrm{i}}{32} - \frac{\frac{21\,x^5}{128}-\frac{35\,x^3}{32}+\frac{103\,x}{128}}{x^6-7\,x^4+7\,x^2-1} - \operatorname{atan}\left(\frac{x\,940311\mathrm{i}}{134217728\,\left(\frac{275445\,\sqrt{2}}{134217728}-\frac{389421}{134217728}\right)} - \frac{\sqrt{2}\,x\,332433\mathrm{i}}{67108864\,\left(\frac{275445\,\sqrt{2}}{134217728}-\frac{389421}{134217728}\right)}\right)\left(\frac{\sqrt{2}\,3\mathrm{i}}{512}-\frac{1}{128}\mathrm{i}\right) - \operatorname{atan}\left(\frac{x\,940311\mathrm{i}}{134217728\,\left(\frac{275445\,\sqrt{2}}{134217728}+\frac{389421}{134217728}\right)} + \frac{\sqrt{2}\,x\,332433\mathrm{i}}{67108864\,\left(\frac{275445\,\sqrt{2}}{134217728}+\frac{389421}{134217728}\right)}\right)\left(\frac{\sqrt{2}\,3\mathrm{i}}{512}+\frac{1}{128}\mathrm{i}\right)$$

```
[In] int(1/(7*x^2 - 7*x^4 + x^6 - 1)^2,x)
```

```
[Out] - (atan(x*1i)*5i)/32 - ((103*x)/128 - (35*x^3)/32 + (21*x^5)/128)/(7*x^2 -
7*x^4 + x^6 - 1) - atan((x*940311i)/(134217728*((275445*2^(1/2))/134217728
- 389421/134217728)) - (2^(1/2)*x*332433i)/(67108864*((275445*2^(1/2))/1342
17728 - 389421/134217728)))*((2^(1/2)*3i)/512 - 1i/128) - atan((x*940311i)/
(134217728*((275445*2^(1/2))/134217728 + 389421/134217728)) + (2^(1/2)*x*33
2433i)/(67108864*((275445*2^(1/2))/134217728 + 389421/134217728)))*((2^(1/2
)*3i)/512 + 1i/128)
```

3.78 $\int \frac{x^3}{c+(a+bx)^2} dx$

Optimal result 654
Rubi [A] (verified) 654
Mathematica [A] (verified) 656
Maple [A] (verified) 656
Fricas [A] (verification not implemented) 656
Sympy [B] (verification not implemented) 657
Maxima [A] (verification not implemented) 657
Giac [A] (verification not implemented) 658
Mupad [B] (verification not implemented) 658

Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{x^3}{c+(a+bx)^2} dx = -\frac{3ax}{b^3} + \frac{(a+bx)^2}{2b^4} - \frac{a(a^2-3c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}} + \frac{(3a^2-c)\log\left(c+(a+bx)^2\right)}{2b^4}$$

```
[Out] -3*a*x/b^3+1/2*(b*x+a)^2/b^4+1/2*(3*a^2-c)*ln(c+(b*x+a)^2)/b^4-a*(a^2-3*c)*
arctan((b*x+a)/c^(1/2))/b^4/c^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {378, 716, 649, 209, 266}

$$\int \frac{x^3}{c+(a+bx)^2} dx = -\frac{a(a^2-3c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}} + \frac{(3a^2-c)\log\left((a+bx)^2+c\right)}{2b^4} + \frac{(a+bx)^2}{2b^4} - \frac{3ax}{b^3}$$

```
[In] Int[x^3/(c + (a + b*x)^2),x]
```

```
[Out] (-3*a*x)/b^3 + (a + b*x)^2/(2*b^4) - (a*(a^2 - 3*c)*ArcTan[(a + b*x)/Sqrt[c
]])/(b^4*Sqrt[c]) + ((3*a^2 - c)*Log[c + (a + b*x)^2])/(2*b^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-a+x)^3}{c+x^2}\,dx, x, a+bx\right)}{b^4}\\
&= \frac{\text{Subst}\left(\int \left(-3a+x-\frac{a^3-3ac-(3a^2-c)x}{c+x^2}\right)\,dx, x, a+bx\right)}{b^4}\\
&= -\frac{3ax}{b^3}+\frac{(a+bx)^2}{2b^4}-\frac{\text{Subst}\left(\int \frac{a^3-3ac-(3a^2-c)x}{c+x^2}\,dx, x, a+bx\right)}{b^4}\\
&= -\frac{3ax}{b^3}+\frac{(a+bx)^2}{2b^4}-\frac{(a(a^2-3c))\,\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{b^4}\\
&\quad+\frac{(3a^2-c)\,\text{Subst}\left(\int \frac{x}{c+x^2}\,dx, x, a+bx\right)}{b^4}\\
&= -\frac{3ax}{b^3}+\frac{(a+bx)^2}{2b^4}-\frac{a(a^2-3c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{(3a^2-c)\log\left(c+(a+bx)^2\right)}{2b^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{c+(a+bx)^2}\,dx = \frac{bx(-4a+bx)-\frac{2(a^3-3ac)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+(3a^2-c)\log\left(a^2+c+2abx+b^2x^2\right)}{2b^4}$$

[In] Integrate[x^3/(c + (a + b*x)^2),x]

[Out] (b*x*(-4*a + b*x) - (2*(a^3 - 3*a*c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + (3*a^2 - c)*Log[a^2 + c + 2*a*b*x + b^2*x^2])/(2*b^4)

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.42

method	result
default	$-\frac{-\frac{1}{2}b\,x^2+2ax}{b^3}+\frac{\frac{\left(3a^2b-bc\right)\ln\left(b^2x^2+2abx+a^2+c\right)}{2b^2}+\frac{\left(2a^3+2ac-\frac{\left(3a^2b-bc\right)a}{b}\right)\arctan\left(\frac{2b^2x+2ab}{2\sqrt{c}\,b}\right)}{\sqrt{c}\,b}}{b^3}$
risch	$\frac{x^2}{2b^2}-\frac{2ax}{b^3}+\frac{3\ln\left(-a^3c-\sqrt{-c\,a^2(a^2-3c)^2}\,bx+3a\,c^2-\sqrt{-c\,a^2(a^2-3c)^2}\,a\right)a^2}{2b^4}-\frac{c\ln\left(-a^3c-\sqrt{-c\,a^2(a^2-3c)^2}\,bx+3a\,c^2-\sqrt{-c\,a^2 \right.}{2b^4}$

[In] int(x^3/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)

[Out] -1/b^3*(-1/2*b*x^2+2*a*x)+1/b^3*(1/2*(3*a^2*b-b*c)/b^2*ln(b^2*x^2+2*a*b*x+a^2+c)+(2*a^3+2*a*c-(3*a^2*b-b*c)*a/b)/c^(1/2)/b*arctan(1/2*(2*b^2*x+2*a*b)/c^(1/2)/b))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.54

$$\int \frac{x^3}{c+(a+bx)^2}\,dx = \left[\frac{b^2cx^2-4\,abcx+(a^3-3\,ac)\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)+(3\,a^2c-c^2)\log\left(b^2x^2+2\,abx+a^2+\right.}{2\,b^4c}\right.$$

[In] integrate(x^3/(c+(b*x+a)^2),x, algorithm="fricas")

```
[Out] [1/2*(b^2*c*x^2 - 4*a*b*c*x + (a^3 - 3*a*c)*sqrt(-c)*log((b^2*x^2 + 2*a*b*x
 + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + (3*a^2*
c - c^2)*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^4*c), 1/2*(b^2*c*x^2 - 4*a*b*
c*x - 2*(a^3 - 3*a*c)*sqrt(c)*arctan((b*x + a)/sqrt(c)) + (3*a^2*c - c^2)*l
og(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^4*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(71) = 142$.

Time = 0.40 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.68

$$\int \frac{x^3}{c+(a+bx)^2}\,dx = -\frac{2ax}{b^3} + \left(-\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4}\right)\log\left(x + \frac{a^4 - 2b^4c\left(-\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4}\right) - c^2}{a^3b - 3abc}\right) + \left(\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4}\right)\log\left(x + \frac{a^4 - 2b^4c\left(\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4}\right) - c^2}{a^3b - 3abc}\right) + \frac{x^2}{2b^2}$$

```
[In] integrate(x**3/(c+(b*x+a)**2),x)
```

```
[Out] -2*a*x/b**3 + (-a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4))
*log(x + (a**4 - 2*b**4*c*(-a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2 -
c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c)) + (a*sqrt(-c)*(a**2 - 3*c)/(2*b**4
*c) + (3*a**2 - c)/(2*b**4))*log(x + (a**4 - 2*b**4*c*(a*sqrt(-c)*(a**2 - 3
*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c)) + x**2/
(2*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04

$$\int \frac{x^3}{c+(a+bx)^2}\,dx = \frac{bx^2 - 4\,ax}{2\,b^3} + \frac{(3\,a^2 - c)\log\left(b^2x^2 + 2\,abx + a^2 + c\right)}{2\,b^4} - \frac{(a^3 - 3\,ac)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^4\sqrt{c}}$$

```
[In] integrate(x^3/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 - 4*a*x)/b^3 + 1/2*(3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/
b^4 - (a^3 - 3*a*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/(b^4*sqrt(c))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{c+(a+bx)^2}\,dx = \frac{(3\,a^2-c)\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^4} - \frac{(a^3-3\,ac)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^4\sqrt{c}} + \frac{b^2x^2-4\,abx}{2\,b^4}$$

```
[In] integrate(x^3/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^4 - (a^3 - 3*a*c)*arctan
((b*x + a)/sqrt(c))/(b^4*sqrt(c)) + 1/2*(b^2*x^2 - 4*a*b*x)/b^4
```

Mupad [B] (verification not implemented)

Time = 9.96 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

$$\int \frac{x^3}{c+(a+bx)^2}\,dx = \frac{x^2}{2\,b^2} - \frac{2\,a\,x}{b^3} - \frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+c\right)\,\left(4\,b^4\,c^2-12\,a^2\,b^4\,c\right)}{8\,b^8\,c} + \frac{a\,\mathrm{atan}\left(\frac{a+b\,x}{\sqrt{c}}\right)\,\left(3\,c-a^2\right)}{b^4\,\sqrt{c}}$$

```
[In] int(x^3/(c + (a + b*x)^2),x)
```

```
[Out] x^2/(2*b^2) - (2*a*x)/b^3 - (log(c + a^2 + b^2*x^2 + 2*a*b*x)*(4*b^4*c^2 -
12*a^2*b^4*c))/(8*b^8*c) + (a*atan((a + b*x)/c^(1/2))*(3*c - a^2))/(b^4*c^(
1/2))
```

3.79 $\int \frac{x^2}{c+(a+bx)^2}\,dx$

Optimal result . . . 659
Rubi [A] (verified) . . . 659
Mathematica [A] (verified) . . . 661
Maple [A] (verified) . . . 661
Fricas [A] (verification not implemented) . . . 661
Sympy [B] (verification not implemented) . . . 662
Maxima [A] (verification not implemented) . . . 662
Giac [A] (verification not implemented) . . . 662
Mupad [B] (verification not implemented) . . . 663

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{x^2}{c+(a+bx)^2}\,dx=\frac{x}{b^2}+\frac{(a^2-c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}}-\frac{a\log\left(c+(a+bx)^2\right)}{b^3}$$

```
[Out] x/b^2-a*ln(c+(b*x+a)^2)/b^3+(a^2-c)*arctan((b*x+a)/c^(1/2))/b^3/c^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {378, 716, 649, 209, 266}

$$\int \frac{x^2}{c+(a+bx)^2}\,dx=\frac{(a^2-c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}}-\frac{a\log\left((a+bx)^2+c\right)}{b^3}+\frac{x}{b^2}$$

```
[In] Int[x^2/(c + (a + b*x)^2),x]
```

```
[Out] x/b^2 + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(b^3*Sqrt[c]) - (a*Log[c + (a
 + b*x)^2])/b^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{c+x^2}\,dx, x, a+bx\right)}{b^3}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{a^2-c-2ax}{c+x^2}\right)\,dx, x, a+bx\right)}{b^3}\\
&= \frac{x}{b^2}+\frac{\text{Subst}\left(\int \frac{a^2-c-2ax}{c+x^2}\,dx, x, a+bx\right)}{b^3}\\
&= \frac{x}{b^2}-\frac{(2a)\text{Subst}\left(\int \frac{x}{c+x^2}\,dx, x, a+bx\right)}{b^3}+\frac{(a^2-c)\,\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{b^3}\\
&= \frac{x}{b^2}+\frac{(a^2-c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}}-\frac{a\log\left(c+(a+bx)^2\right)}{b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{c+(a+bx)^2} dx = \frac{bx + \frac{(a^2-c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}} - a\log\left(a^2+c+2abx+b^2x^2\right)}{b^3}$$

[In] Integrate[x^2/(c + (a + b*x)^2),x]

[Out] (b*x + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - a*Log[a^2 + c + 2*a*b*x + b^2*x^2])/b^3

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

method	result
default	$\frac{x}{b^2}+\frac{-\frac{a\ln\left(b^2x^2+2abx+a^2+c\right)}{b}+\frac{\left(a^2-c\right)\arctan\left(\frac{2b^2x+2ab}{2\sqrt{c}\,b}\right)}{\sqrt{c}\,b}}{b^2}$
risch	$\frac{x}{b^2}-\frac{\ln\left(-\sqrt{-c(a^2-c)^2}\,bx+c\,a^2-\sqrt{-c(a^2-c)^2}\,a-c^2\right)a}{b^3}+\frac{\ln\left(-\sqrt{-c(a^2-c)^2}\,bx+c\,a^2-\sqrt{-c(a^2-c)^2}\,a-c^2\right)\sqrt{-c(a^2-c)^2}}{2b^3c}-$

[In] int(x^2/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)

[Out] x/b^2+1/b^2*(-a/b*ln(b^2*x^2+2*a*b*x+a^2+c)+(a^2-c)/c^(1/2)/b*arctan(1/2*(2*b^2*x+2*a*b)/c^(1/2)/b))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 3.14

$$\int \frac{x^2}{c+(a+bx)^2} dx$$
$$= \left[\frac{2\,bcx - 2\,ac\log\left(b^2x^2+2\,abx+a^2+c\right) + (a^2-c)\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)}{2\,b^3c}, \frac{bcx - ac\log\left(b^2\right.}{}\right.$$

[In] integrate(x^2/(c+(b*x+a)^2),x, algorithm="fricas")

[Out] [1/2*(2*b*c*x - 2*a*c*log(b^2*x^2 + 2*a*b*x + a^2 + c) + (a^2 - c)*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)))/(b^3*c), (b*c*x - a*c*log(b^2*x^2 + 2*a*b*x + a^2 + c) + (a^2 - c)*sqrt(c)*arctan((b*x + a)/sqrt(c)))/(b^3*c)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(44) = 88$.

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.06

$$\int \frac{x^2}{c+(a+bx)^2}\,dx = \left(-\frac{a}{b^3}-\frac{\sqrt{-c}(a^2-c)}{2b^3c}\right)\log\left(x+\frac{a^3+ac+2b^3c\left(-\frac{a}{b^3}-\frac{\sqrt{-c}(a^2-c)}{2b^3c}\right)}{a^2b-bc}\right) + \left(-\frac{a}{b^3} + \frac{\sqrt{-c}(a^2-c)}{2b^3c}\right)\log\left(x+\frac{a^3+ac+2b^3c\left(-\frac{a}{b^3}+\frac{\sqrt{-c}(a^2-c)}{2b^3c}\right)}{a^2b-bc}\right)+\frac{x}{b^2}$$

```
[In] integrate(x**2/(c+(b*x+a)**2),x)
```

```
[Out] (-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c + 2*b**3*c*(
-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*c)) + (-a/b**3 + sqr
t(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c + 2*b**3*c*(-a/b**3 + sqrt
(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*c)) + x/b**2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{c+(a+bx)^2}\,dx = \frac{x}{b^2} - \frac{a\log\left(b^2x^2+2\,abx+a^2+c\right)}{b^3} + \frac{(a^2-c)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^3\sqrt{c}}$$

```
[In] integrate(x^2/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] x/b^2 - a*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^3 + (a^2 - c)*arctan((b^2*x +
a*b)/(b*sqrt(c)))/(b^3*sqrt(c))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{c+(a+bx)^2}\,dx = \frac{x}{b^2} - \frac{a\log\left(b^2x^2+2\,abx+a^2+c\right)}{b^3} + \frac{(a^2-c)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^3\sqrt{c}}$$

```
[In] integrate(x^2/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] x/b^2 - a*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^3 + (a^2 - c)*arctan((b*x + a)
/sqrt(c))/(b^3*sqrt(c))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 206, normalized size of antiderivative = 4.12

$$\int \frac{x^2}{c+(a+bx)^2}\,dx = \frac{x}{b^2} - \frac{a\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+c\right)}{b^3} + \frac{\sqrt{c}\,\mathrm{atan}\left(\frac{a^3}{\sqrt{c}\,(c-a^2)} - \frac{\sqrt{c}\,x}{\frac{c}{b}-\frac{a^2}{b}} - \frac{a\,\sqrt{c}}{c-a^2} + \frac{a^2\,x}{\sqrt{c}\,\left(\frac{c}{b}-\frac{a^2}{b}\right)}\right)}{b^3} - \frac{a^2\,\mathrm{atan}\left(\frac{a^3}{\sqrt{c}\,(c-a^2)} - \frac{\sqrt{c}\,x}{\frac{c}{b}-\frac{a^2}{b}} - \frac{a\,\sqrt{c}}{c-a^2} + \frac{a^2\,x}{\sqrt{c}\,\left(\frac{c}{b}-\frac{a^2}{b}\right)}\right)}{b^3\,\sqrt{c}}$$

```
[In] int(x^2/(c + (a + b*x)^2),x)
```

```
[Out] x/b^2 - (a*log(c + a^2 + b^2*x^2 + 2*a*b*x))/b^3 + (c^(1/2)*atan(a^3/(c^(1/
2)*(c - a^2)) - (c^(1/2)*x)/(c/b - a^2/b) - (a*c^(1/2))/(c - a^2) + (a^2*x)
/(c^(1/2)*(c/b - a^2/b))))/b^3 - (a^2*atan(a^3/(c^(1/2)*(c - a^2)) - (c^(1/
2)*x)/(c/b - a^2/b) - (a*c^(1/2))/(c - a^2) + (a^2*x)/(c^(1/2)*(c/b - a^2/b
))))/(b^3*c^(1/2))
```

3.80 $\int \frac{x}{c+(a+bx)^2}\,dx$

Optimal result 664
Rubi [A] (verified) 664
Mathematica [A] (verified) 665
Maple [A] (verified) 666
Fricas [A] (verification not implemented) 666
Sympy [B] (verification not implemented) 667
Maxima [A] (verification not implemented) 667
Giac [A] (verification not implemented) 667
Mupad [B] (verification not implemented) 668

Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{x}{c+(a+bx)^2}\,dx=-\frac{a\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}+\frac{\log\left(c+(a+bx)^2\right)}{2b^2}$$

```
[Out] 1/2*ln(c+(b*x+a)^2)/b^2-a*arctan((b*x+a)/c^(1/2))/b^2/c^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {378, 649, 209, 266}

$$\int \frac{x}{c+(a+bx)^2}\,dx=\frac{\log\left((a+bx)^2+c\right)}{2b^2}-\frac{a\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}$$

```
[In] Int[x/(c + (a + b*x)^2),x]
```

```
[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]])/(b^2*Sqrt[c])) + Log[c + (a + b*x)^2]/(2*b^
2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-a+x}{c+x^2}\,dx, x, a+bx\right)}{b^2}\\
&= \frac{\text{Subst}\left(\int \frac{x}{c+x^2}\,dx, x, a+bx\right)}{b^2} - \frac{a\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{b^2}\\
&= -\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}} + \frac{\log\left(c+(a+bx)^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{x}{c+(a+bx)^2}\,dx = \frac{-\frac{2a\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}} + \log\left(c+(a+bx)^2\right)}{2b^2}$$

```
[In] Integrate[x/(c + (a + b*x)^2),x]
[Out] ((-2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + Log[c + (a + b*x)^2])/(2*b^2)
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{\ln\left(b^{2} x^{2}+2 a b x+a^{2}+c\right)}{2 b^{2}}-\frac{a \arctan\left(\frac{2 b^{2} x+2 a b}{2 \sqrt{c}\, b}\right)}{b^{2} \sqrt{c}}$	54
risch	$\frac{\ln\left(-\sqrt{-c}\, b x-a \sqrt{-c}-c\right) a \sqrt{-c}}{2 c\, b^{2}}+\frac{\ln\left(-\sqrt{-c}\, b x-a \sqrt{-c}-c\right)}{2 b^{2}}-\frac{\ln\left(\sqrt{-c}\, b x+a \sqrt{-c}-c\right) a \sqrt{-c}}{2 c\, b^{2}}+\frac{\ln\left(\sqrt{-c}\, b x+a \sqrt{-c}-c\right)}{2 b^{2}}$	124

```
[In] int(x/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^2*ln(b^2*x^2+2*a*b*x+a^2+c)-a/b^2/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/
c^(1/2)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.32

$$\int \frac{x}{c+(a+bx)^2}\,dx = \left[-\frac{a\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)-c\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2c}, -\frac{2\,a\sqrt{c}\arctan\left(\frac{bx+a}{\sqrt{c}}\right)-c\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2c}\right]$$

```
[In] integrate(x/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/
(b^2*x^2 + 2*a*b*x + a^2 + c)) - c*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^2*c
), -1/2*(2*a*sqrt(c)*arctan((b*x + a)/sqrt(c)) - c*log(b^2*x^2 + 2*a*b*x +
a^2 + c))/(b^2*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(36) = 72$.

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.02

$$\int \frac{x}{c+(a+bx)^2}\,dx = \left(-\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)\log\left(x+\frac{a^2-2b^2c\left(-\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)+c}{ab}\right) + \left(\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)\log\left(x+\frac{a^2-2b^2c\left(\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)+c}{ab}\right)$$

```
[In] integrate(x/(c+(b*x+a)**2),x)
```

```
[Out] (-a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2))*log(x + (a**2 - 2*b**2*c*(-a*sqrt(-c)
/(2*b**2*c) + 1/(2*b**2)) + c)/(a*b)) + (a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2)
)*log(x + (a**2 - 2*b**2*c*(a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2)) + c)/(a*b))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int \frac{x}{c+(a+bx)^2}\,dx = -\frac{a\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^2\sqrt{c}}+\frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2}$$

```
[In] integrate(x/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -a*arctan((b^2*x + a*b)/(b*sqrt(c)))/(b^2*sqrt(c)) + 1/2*log(b^2*x^2 + 2*a*
b*x + a^2 + c)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \frac{x}{c+(a+bx)^2}\,dx = -\frac{a\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^2\sqrt{c}}+\frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2}$$

```
[In] integrate(x/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -a*arctan((b*x + a)/sqrt(c))/(b^2*sqrt(c)) + 1/2*log(b^2*x^2 + 2*a*b*x + a^
2 + c)/b^2
```

Mupad [B] (verification not implemented)

Time = 9.99 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{x}{c+(a+bx)^2}\,dx=\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+c\right)}{2\,b^2}-\frac{a\,\mathrm{atan}\left(\frac{a}{\sqrt{c}}+\frac{b\,x}{\sqrt{c}}\right)}{b^2\,\sqrt{c}}$$

```
[In] int(x/(c + (a + b*x)^2),x)
```

```
[Out] log(c + a^2 + b^2*x^2 + 2*a*b*x)/(2*b^2) - (a*atan(a/c^(1/2) + (b*x)/c^(1/2
)))/(b^2*c^(1/2))
```

3.81 $\int \frac{1}{c+(a+bx)^2} \, dx$

Optimal result 669
Rubi [A] (verified) 669
Mathematica [A] (verified) 670
Maple [A] (verified) 670
Fricas [A] (verification not implemented) 671
Sympy [B] (verification not implemented) 671
Maxima [A] (verification not implemented) 671
Giac [A] (verification not implemented) 672
Mupad [B] (verification not implemented) 672

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \frac{1}{c+(a+bx)^2} \, dx = \frac{\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

```
[Out] arctan((b*x+a)/c^(1/2))/b/c^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {253, 209}

$$\int \frac{1}{c+(a+bx)^2} \, dx = \frac{\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

```
[In] Int[(c + (a + b*x)^2)^(-1),x]
```

```
[Out] ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
```

```
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{c+(a+bx)^2}\,dx = \frac{\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

```
[In] Integrate[(c + (a + b*x)^2)^(-1),x]
[Out] ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

method	result	size
default	$\frac{\arctan\left(\frac{2b^2x+2ab}{2\sqrt{c}\,b}\right)}{\sqrt{c}\,b}$	28
risch	$-\frac{\ln\left(bx+\sqrt{-c}+a\right)}{2\sqrt{-c}\,b}+\frac{\ln\left(-bx+\sqrt{-c}-a\right)}{2\sqrt{-c}\,b}$	47

```
[In] int(1/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
[Out] 1/c^(1/2)/b*arctan(1/2*(2*b^2*x+2*a*b)/c^(1/2)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.95

$$\int \frac{1}{c+(a+bx)^2}\,dx = \left[-\frac{\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)}{2\,bc}, \frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}}\right]$$

```
[In] integrate(1/(c+(b*x+a)^2),x, algorithm="fricas")
[Out] [-1/2*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^
2*x^2 + 2*a*b*x + a^2 + c))/(b*c), arctan((b*x + a)/sqrt(c))/(b*sqrt(c))]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(17) = 34.

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.57

$$\int \frac{1}{c+(a+bx)^2}\,dx = \frac{-\frac{\sqrt{-\frac{1}{c}}\log\left(x+\frac{a-c\sqrt{-\frac{1}{c}}}{b}\right)}{2}+\frac{\sqrt{-\frac{1}{c}}\log\left(x+\frac{a+c\sqrt{-\frac{1}{c}}}{b}\right)}{2}}{b}$$

```
[In] integrate(1/(c+(b*x+a)**2),x)
[Out] (-sqrt(-1/c)*log(x + (a - c*sqrt(-1/c))/b)/2 + sqrt(-1/c)*log(x + (a + c*sq
rt(-1/c))/b)/2)/b
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{1}{c+(a+bx)^2}\,dx = \frac{\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b\sqrt{c}}$$

```
[In] integrate(1/(c+(b*x+a)^2),x, algorithm="maxima")
[Out] arctan((b^2*x + a*b)/(b*sqrt(c)))/(b*sqrt(c))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{c+(a+bx)^2} \, dx = \frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}}$$

```
[In] integrate(1/(c+(b*x+a)^2),x, algorithm="giac")
[Out] arctan((b*x + a)/sqrt(c))/(b*sqrt(c))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{c+(a+bx)^2} \, dx = \frac{\operatorname{atan}\left(\frac{a+b\,x}{\sqrt{c}}\right)}{b\,\sqrt{c}}$$

```
[In] int(1/(c + (a + b*x)^2),x)
[Out] atan((a + b*x)/c^(1/2))/(b*c^(1/2))
```

3.82 $\int \frac{1}{x(c+(a+bx)^2)}\,dx$

Optimal result 673
Rubi [A] (verified) 673
Mathematica [A] (verified) 675
Maple [A] (verified) 675
Fricas [A] (verification not implemented) 675
Sympy [B] (verification not implemented) 676
Maxima [A] (verification not implemented) 677
Giac [A] (verification not implemented) 677
Mupad [B] (verification not implemented) 677

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx=-\frac{a\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2+c)}+\frac{\log(x)}{a^2+c}-\frac{\log\left(c+(a+bx)^2\right)}{2\,(a^2+c)}$$

```
[Out] ln(x)/(a^2+c)-1/2*ln(c+(b*x+a)^2)/(a^2+c)-a*arctan((b*x+a)/c^(1/2))/(a^2+c)
/c^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {378, 720, 31, 649, 209, 266}

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx=-\frac{a\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2+c)}-\frac{\log\left((a+bx)^2+c\right)}{2\,(a^2+c)}+\frac{\log(x)}{a^2+c}$$

```
[In] Int[1/(x*(c + (a + b*x)^2)),x]
```

```
[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c))) + Log[x]/(a^2 + c) - L
og[c + (a + b*x)^2]/(2*(a^2 + c))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(-a+x)\left(c+x^2\right)}\,dx, x, a+bx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{-a+x}\,dx, x, a+bx\right)}{a^2+c} + \frac{\text{Subst}\left(\int \frac{-a-x}{c+x^2}\,dx, x, a+bx\right)}{a^2+c}\\
&= \frac{\log(x)}{a^2+c} - \frac{\text{Subst}\left(\int \frac{x}{c+x^2}\,dx, x, a+bx\right)}{a^2+c} - \frac{a\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{a^2+c}\\
&= -\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)} + \frac{\log(x)}{a^2+c} - \frac{\log\left(c+(a+bx)^2\right)}{2\left(a^2+c\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx=-\frac{\frac{2a\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}-2\log(bx)+\log\left(c+(a+bx)^2\right)}{2\left(a^2+c\right)}$$

```
[In] Integrate[1/(x*(c + (a + b*x)^2)),x]
```

```
[Out] -1/2*((2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - 2*Log[b*x] + Log[c + (a + b
*x)^2])/(a^2 + c)
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.25

method	result	size
default	$\frac{\ln(x)}{a^2+c}-\frac{b\left(\frac{\ln\left(b^2x^2+2abx+a^2+c\right)}{2b}+\frac{a\arctan\left(\frac{2b^2x+2ab}{2\sqrt{c}\,b}\right)}{\sqrt{c}\,b}\right)}{a^2+c}$	74
risch	$\frac{\ln(x)}{a^2+c}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(c\,a^2+c^2\right)\_Z^2+2c\_Z\right)}\_R\ln\left(\left(\left(-a^2b+3bc\right)\_R+3b\right)x+\left(-a^3-ac\right)\_R+2a\right)\right)}{2}$	75

```
[In] int(1/x/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/(a^2+c)-b/(a^2+c)*(1/2/b*ln(b^2*x^2+2*a*b*x+a^2+c)+a/c^(1/2)/b*arctan
(1/2*(2*b^2*x+2*a*b)/c^(1/2)/b))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.61

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx = \left[-\frac{a\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)+c\log\left(b^2x^2+2\,abx+a^2+c\right)-2\,c\log(x)}{2\left(a^2c+c^2\right)},\right.$$
$$\left.-\frac{2\,a\sqrt{c}\arctan\left(\frac{bx+a}{\sqrt{c}}\right)+c\log\left(b^2x^2+2\,abx+a^2+c\right)-2\,c\log(x)}{2\left(a^2c+c^2\right)}\right]$$

```
[In] integrate(1/x/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/
(b^2*x^2 + 2*a*b*x + a^2 + c)) + c*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*c*l
og(x))/(a^2*c + c^2), -1/2*(2*a*sqrt(c)*arctan((b*x + a)/sqrt(c)) + c*log(b
^2*x^2 + 2*a*b*x + a^2 + c) - 2*c*log(x))/(a^2*c + c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(49) = 98$.

Time = 1.85 (sec) , antiderivative size = 738, normalized size of antiderivative = 12.51

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx = \left(-\frac{a\sqrt{-c}}{2c\left(a^2+c\right)} - \frac{1}{2\left(a^2+c\right)}\right)\log\left(x+\frac{-4a^6c\left(-\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2+4a^4c^2\left(-\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2-6a^4c\left(-\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(}\right.}{}\right) + \left(\frac{a\sqrt{-c}}{2c\left(a^2+c\right)} - \frac{1}{2\left(a^2+c\right)}\right)\log\left(x+\frac{-4a^6c\left(\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2+4a^4c^2\left(\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2-6a^4c\left(\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)}{}\right) + \frac{\log\left(x+\frac{-\frac{4a^6c}{(a^2+c)^2}+\frac{4a^4c^2}{(a^2+c)^2}-\frac{6a^4c}{a^2+c}+\frac{20a^2c^3}{(a^2+c)^2}-\frac{12a^2c^2}{a^2+c}+10a^2c+\frac{12c^4}{(a^2+c)^2}-\frac{6c^3}{a^2+c}-6c^2}{a^3b+9abc}\right)}{a^2+c}$$

```
[In] integrate(1/x/(c+(b*x+a)**2),x)
```

```
[Out] (-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*log(x + (-4*a**6*c*(-a*sq
rt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 + 4*a**4*c**2*(-a*sqrt(-c)/(
2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 6*a**4*c*(-a*sqrt(-c)/(2*c*(a**2 +
 c)) - 1/(2*(a**2 + c))) + 20*a**2*c**3*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(
2*(a**2 + c)))**2 - 12*a**2*c**2*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2
 + c))) + 10*a**2*c + 12*c**4*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 +
c)))**2 - 6*c**3*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) - 6*c**2
)/(a**3*b + 9*a*b*c)) + (a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*lo
g(x + (-4*a**6*c*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 + 4*a*
*4*c**2*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 6*a**4*c*(a*s
qrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) + 20*a**2*c**3*(a*sqrt(-c)/(2*
c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 12*a**2*c**2*(a*sqrt(-c)/(2*c*(a**2
+ c)) - 1/(2*(a**2 + c))) + 10*a**2*c + 12*c**4*(a*sqrt(-c)/(2*c*(a**2 + c)
) - 1/(2*(a**2 + c)))**2 - 6*c**3*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2
```

```
 + c))) - 6*c**2)/(a**3*b + 9*a*b*c)) + log(x + (-4*a**6*c/(a**2 + c)**2 +
4*a**4*c**2/(a**2 + c)**2 - 6*a**4*c/(a**2 + c) + 20*a**2*c**3/(a**2 + c)**
2 - 12*a**2*c**2/(a**2 + c) + 10*a**2*c + 12*c**4/(a**2 + c)**2 - 6*c**3/(a
**2 + c) - 6*c**2)/(a**3*b + 9*a*b*c))/(a**2 + c)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx=-\frac{a\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{(a^2+c)\sqrt{c}}-\frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,(a^2+c)}+\frac{\log\left(x\right)}{a^2+c}$$

```
[In] integrate(1/x/(c+(b*x+a)^2),x, algorithm="maxima")
[Out] -a*arctan((b^2*x + a*b)/(b*sqrt(c)))/((a^2 + c)*sqrt(c)) - 1/2*log(b^2*x^2
+ 2*a*b*x + a^2 + c)/(a^2 + c) + log(x)/(a^2 + c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx=-\frac{a\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{(a^2+c)\sqrt{c}}-\frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,(a^2+c)}+\frac{\log\left(|x|\right)}{a^2+c}$$

```
[In] integrate(1/x/(c+(b*x+a)^2),x, algorithm="giac")
[Out] -a*arctan((b*x + a)/sqrt(c))/((a^2 + c)*sqrt(c)) - 1/2*log(b^2*x^2 + 2*a*b*
x + a^2 + c)/(a^2 + c) + log(abs(x))/(a^2 + c)
```

Mupad [B] (verification not implemented)

Time = 10.58 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.93

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx=\frac{\ln\left(x\right)}{a^2+c}-\frac{\ln\left(2\,a\,b^3+3\,b^4\,x+\frac{b^3\,\left(c+a\,\sqrt{-c}\right)\,\left(a^3+b\,x\,a^2+c\,a-3\,b\,c\,x\right)}{c\,\left(a^2+c\right)}\right)\,\left(c+a\,\sqrt{-c}\right)}{2\,\left(a^2\,c+c^2\right)}-\frac{\ln\left(2\,a\,b^3+3\,b^4\,x+\frac{b^3\,\left(c-a\,\sqrt{-c}\right)\,\left(a^3+b\,x\,a^2+c\,a-3\,b\,c\,x\right)}{c\,\left(a^2+c\right)}\right)\,\left(c-a\,\sqrt{-c}\right)}{2\,\left(a^2\,c+c^2\right)}$$

```
[In] int(1/(x*(c + (a + b*x)^2)),x)
```

```
[Out] log(x)/(c + a^2) - (log(2*a*b^3 + 3*b^4*x + (b^3*(c + a*(-c)^(1/2))*(a*c +
a^3 - 3*b*c*x + a^2*b*x))/(c*(c + a^2)))*(c + a*(-c)^(1/2)))/(2*(a^2*c + c^
2)) - (log(2*a*b^3 + 3*b^4*x + (b^3*(c - a*(-c)^(1/2))*(a*c + a^3 - 3*b*c*x
 + a^2*b*x))/(c*(c + a^2)))*(c - a*(-c)^(1/2)))/(2*(a^2*c + c^2))
```

3.83 $\int \frac{1}{x^2(c+(a+bx)^2)}\,dx$

Optimal result . . . 679
Rubi [A] (verified) . . . 679
Mathematica [A] (verified) . . . 681
Maple [A] (verified) . . . 681
Fricas [A] (verification not implemented) . . . 682
Sympy [B] (verification not implemented) . . . 682
Maxima [A] (verification not implemented) . . . 683
Giac [A] (verification not implemented) . . . 684
Mupad [B] (verification not implemented) . . . 684

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \frac{1}{x^2\left(c+(a+bx)^2\right)}\,dx = -\frac{1}{(a^2+c)\,x}+\frac{b(a^2-c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2+c)^2} - \frac{2ab\log(x)}{(a^2+c)^2}+\frac{ab\log\left(c+(a+bx)^2\right)}{(a^2+c)^2}$$

```
[Out] -1/(a^2+c)/x-2*a*b*ln(x)/(a^2+c)^2+a*b*ln(c+(b*x+a)^2)/(a^2+c)^2+b*(a^2-c)*
arctan((b*x+a)/c^(1/2))/(a^2+c)^2/c^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {378, 724, 815, 649, 209, 266}

$$\int \frac{1}{x^2\left(c+(a+bx)^2\right)}\,dx = \frac{b(a^2-c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2+c)^2} - \frac{2ab\log(x)}{(a^2+c)^2} + \frac{ab\log\left((a+bx)^2+c\right)}{(a^2+c)^2} - \frac{1}{x\,(a^2+c)}$$

```
[In] Int[1/(x^2*(c + (a + b*x)^2)),x]
```

```
[Out] -(1/((a^2 + c)*x)) + (b*(a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2
+ c)^2) - (2*a*b*Log[x])/(a^2 + c)^2 + (a*b*Log[c + (a + b*x)^2])/(a^2 + c)
^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b\text{Subst}\left(\int \frac{1}{(-a+x)^2\left(c+x^2\right)}\,dx, x, a+bx\right)\\
&= -\frac{1}{\left(a^2+c\right)x}+\frac{b\text{Subst}\left(\int \frac{-a-x}{(-a+x)(c+x^2)}\,dx, x, a+bx\right)}{a^2+c}\\
&= -\frac{1}{\left(a^2+c\right)x}+\frac{b\text{Subst}\left(\int \left(\frac{2a}{(a^2+c)(a-x)}+\frac{a^2-c+2ax}{(a^2+c)(c+x^2)}\right)\,dx, x, a+bx\right)}{a^2+c}
\end{aligned}
$$

$$= -\frac{1}{(a^2+c)\,x} - \frac{2ab\log(x)}{(a^2+c)^2} + \frac{b\text{Subst}\left(\int \frac{a^2-c+2ax}{c+x^2}\,dx, x, a+bx\right)}{(a^2+c)^2}$$

$$= -\frac{1}{(a^2+c)\,x} - \frac{2ab\log(x)}{(a^2+c)^2} + \frac{(2ab)\text{Subst}\left(\int \frac{x}{c+x^2}\,dx, x, a+bx\right)}{(a^2+c)^2} + \frac{(b(a^2-c))\,\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{(a^2+c)^2}$$

$$= -\frac{1}{(a^2+c)\,x} + \frac{b(a^2-c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2+c)^2} - \frac{2ab\log(x)}{(a^2+c)^2} + \frac{ab\log\left(c+(a+bx)^2\right)}{(a^2+c)^2}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2\,(c+(a+bx)^2)}\,dx = \frac{b(a^2-c)\,x\arctan\left(\frac{a+bx}{\sqrt{c}}\right) - \sqrt{c}(a^2+c+2abx\log(x) - abx\log\left(a^2+c+2abx+b^2x^2\right))}{\sqrt{c}\,(a^2+c)^2\,x}$$

```
[In] Integrate[1/(x^2*(c + (a + b*x)^2)),x]
[Out] (b*(a^2 - c)*x*ArcTan[(a + b*x)/Sqrt[c]] - Sqrt[c]*(a^2 + c + 2*a*b*x*Log[x
] - a*b*x*Log[a^2 + c + 2*a*b*x + b^2*x^2]))/(Sqrt[c]*(a^2 + c)^2*x)
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.22

method	result
default	$-\frac{1}{(a^2+c)x} - \frac{2ab\ln(x)}{(a^2+c)^2} + \frac{b^2\left(\frac{a\ln\left(b^2x^2+2abx+a^2+c\right)}{b} + \frac{\left(a^2-c\right)\arctan\left(\frac{2b^2x+2ab}{2\sqrt{c}\,b}\right)}{\sqrt{c}\,b}\right)}{(a^2+c)^2}$
risch	$-\frac{1}{(a^2+c)x} - \frac{2ab\ln(x)}{a^4+2c\,a^2+c^2} + \frac{\left(\sum_{\_R=\text{RootOf}\left(\left(c\,a^4+2a^2c^2+c^3\right)\_Z^2-4abc\_Z+b^2\right)} \_R\ln\left(\left(\left(-a^6b+a^4bc+5a^2b\,c^2+3b\,c^3\right)\_R^2+(\right.\right.\right.}{}$

```
[In] int(1/x^2/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
[Out] -1/(a^2+c)/x-2*a*b*ln(x)/(a^2+c)^2+b^2/(a^2+c)^2*(a/b*ln(b^2*x^2+2*a*b*x+a^
2+c)+(a^2-c)/c^(1/2)/b*arctan(1/2*(2*b^2*x+2*a*b)/c^(1/2)/b))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.90

$$\int \frac{1}{x^2 (c+(a+bx)^2)} dx$$
$$= \left[\frac{2 abcx \log (b^2x^2 + 2 abx + a^2 + c) - 4 abcx \log (x) + (a^2b - bc)\sqrt{-c}x \log \left(\frac{b^2x^2+2 abx+a^2+2 (bx+a)\sqrt{-c}-c}{b^2x^2+2 abx+a^2+c}\right) - 2}{2 (a^4c + 2 a^2c^2 + c^3)x} \right.$$

```
[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*c*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 4*a*b*c*x*log(x) + (a^2*
b - b*c)*sqrt(-c)*x*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c
)/(b^2*x^2 + 2*a*b*x + a^2 + c)) - 2*a^2*c - 2*c^2)/((a^4*c + 2*a^2*c^2 + c
^3)*x), (a*b*c*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*a*b*c*x*log(x) + (a^2
*b - b*c)*sqrt(c)*x*arctan((b*x + a)/sqrt(c)) - a^2*c - c^2)/((a^4*c + 2*a^
2*c^2 + c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1620 vs. 2(73) = 146.

Time = 6.88 (sec) , antiderivative size = 1620, normalized size of antiderivative = 20.51

$$\int \frac{1}{x^2 (c+(a+bx)^2)} dx = \text{Too large to display}$$

```
[In] integrate(1/x**2/(c+(b*x+a)**2),x)
```

```
[Out] -2*a*b*log(x + (-16*a**13*b**2*c/(a**2 + c)**4 + 48*a**11*b**2*c**2/(a**2 +
 c)**4 + 352*a**9*b**2*c**3/(a**2 + c)**4 - 20*a**9*b**2*c/(a**2 + c)**2 +
608*a**7*b**2*c**4/(a**2 + c)**4 - 64*a**7*b**2*c**2/(a**2 + c)**2 + 432*a*
*5*b**2*c**5/(a**2 + c)**4 - 72*a**5*b**2*c**3/(a**2 + c)**2 + 36*a**5*b**2
*c + 112*a**3*b**2*c**6/(a**2 + c)**4 - 32*a**3*b**2*c**4/(a**2 + c)**2 - 8
8*a**3*b**2*c**2 - 4*a*b**2*c**5/(a**2 + c)**2 + 4*a*b**2*c**3)/(a**6*b**3
+ 33*a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3))/(a**2 + c)**2 + (a*b/(a*
*2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*log(x +
(-4*a**11*c*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*
c + c**2)))**2 + 12*a**9*c**2*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2
*c*(a**4 + 2*a**2*c + c**2)))**2 + 10*a**8*b*c*(a*b/(a**2 + c)**2 - b*sqrt(
-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 88*a**7*c**3*(a*b/(a**2 +
c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 32*a**6*
b*c**2*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c
```

```
**2))) + 36*a**5*b**2*c + 152*a**5*c**4*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a*
*2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 36*a**4*b*c**3*(a*b/(a**2 + c)
**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) - 88*a**3*b**2*
c**2 + 108*a**3*c**5*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4
+ 2*a**2*c + c**2)))**2 + 16*a**2*b*c**4*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a
**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 4*a*b**2*c**3 + 28*a*c**6*(a*b/(
a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 2
*b*c**5*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c +
c**2))))/(a**6*b**3 + 33*a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3)) + (a
*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*lo
g(x + (-4*a**11*c*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2
*a**2*c + c**2)))**2 + 12*a**9*c**2*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 -
 c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 10*a**8*b*c*(a*b/(a**2 + c)**2 + b
*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 88*a**7*c**3*(a*b/(a
**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 32
*a**6*b*c**2*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2
*c + c**2))) + 36*a**5*b**2*c + 152*a**5*c**4*(a*b/(a**2 + c)**2 + b*sqrt(-
c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 36*a**4*b*c**3*(a*b/(a**
2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) - 88*a**3
*b**2*c**2 + 108*a**3*c**5*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*
(a**4 + 2*a**2*c + c**2)))**2 + 16*a**2*b*c**4*(a*b/(a**2 + c)**2 + b*sqrt(
-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 4*a*b**2*c**3 + 28*a*c**6*
(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*
*2 + 2*b*c**5*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**
2*c + c**2))))/(a**6*b**3 + 33*a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3)
) - 1/(x*(a**2 + c))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.56

$$\int \frac{1}{x^2\left(c+(a+bx)^2\right)}\,dx = \frac{ab\log\left(b^2x^2+2\,abx+a^2+c\right)}{a^4+2\,a^2c+c^2} - \frac{2\,ab\log\left(x\right)}{a^4+2\,a^2c+c^2} + \frac{\left(a^2b^2-b^2c\right)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{\left(a^4+2\,a^2c+c^2\right)b\sqrt{c}} - \frac{1}{\left(a^2+c\right)x}$$

```
[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] a*b*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^4 + 2*a^2*c + c^2) - 2*a*b*log(x)/(
a^4 + 2*a^2*c + c^2) + (a^2*b^2 - b^2*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/
((a^4 + 2*a^2*c + c^2)*b*sqrt(c)) - 1/((a^2 + c)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^2 (c + (a + bx)^2)} dx = \frac{ab \log (b^2x^2 + 2\,abx + a^2 + c)}{a^4 + 2\,a^2c + c^2} - \frac{2\,ab \log (|x|)}{a^4 + 2\,a^2c + c^2} + \frac{(a^2b^2 - b^2c) \arctan \left(\frac{bx+a}{\sqrt{c}}\right)}{(a^4 + 2\,a^2c + c^2)b\sqrt{c}} - \frac{1}{(a^2 + c)x}$$

```
[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] a*b*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^4 + 2*a^2*c + c^2) - 2*a*b*log(abs(
x))/(a^4 + 2*a^2*c + c^2) + (a^2*b^2 - b^2*c)*arctan((b*x + a)/sqrt(c))/((a
^4 + 2*a^2*c + c^2)*b*sqrt(c)) - 1/((a^2 + c)*x)
```

Mupad [B] (verification not implemented)

Time = 10.10 (sec) , antiderivative size = 425, normalized size of antiderivative = 5.38

$$\int \frac{1}{x^2 (c + (a + bx)^2)} dx = \frac{\ln \left((-c)^{13/2} - 35\,a^2\,(-c)^{11/2} + 34\,a^4\,(-c)^{9/2} + 34\,a^6\,(-c)^{7/2} - 35\,a^8\,(-c)^{5/2} + a^{10}\,(-c)^{3/2} + a\,c^6 - a^{11}\,c + \cdots \right.}{} - \frac{1}{x\,(a^2 + c)} - \frac{\ln \left((-c)^{13/2} - 35\,a^2\,(-c)^{11/2} + 34\,a^4\,(-c)^{9/2} + 34\,a^6\,(-c)^{7/2} - 35\,a^8\,(-c)^{5/2} + a^{10}\,(-c)^{3/2} - a\,c^6 + a^{11}\,c \cdots \right.}{} - \frac{2\,a\,b\,\ln (x)}{(a^2 + c)^2}$$

```
[In] int(1/(x^2*(c + (a + b*x)^2)),x)
```

```
[Out] (log((-c)^(13/2) - 35*a^2*(-c)^(11/2) + 34*a^4*(-c)^(9/2) + 34*a^6*(-c)^(7/
2) - 35*a^8*(-c)^(5/2) + a^10*(-c)^(3/2) + a*c^6 - a^11*c + 35*a^3*c^5 + 34
*a^5*c^4 - 34*a^7*c^3 - 35*a^9*c^2 + b*c^6*x - a^10*b*c*x + 35*a^2*b*c^5*x
+ 34*a^4*b*c^4*x - 34*a^6*b*c^3*x - 35*a^8*b*c^2*x)*(b*(-c)^(3/2) + 2*a*b*c
 + a^2*b*(-c)^(1/2)))/(2*(a^4*c + c^3 + 2*a^2*c^2)) - 1/(x*(c + a^2)) - (lo
g((-c)^(13/2) - 35*a^2*(-c)^(11/2) + 34*a^4*(-c)^(9/2) + 34*a^6*(-c)^(7/2)
- 35*a^8*(-c)^(5/2) + a^10*(-c)^(3/2) - a*c^6 + a^11*c - 35*a^3*c^5 - 34*a^
5*c^4 + 34*a^7*c^3 + 35*a^9*c^2 - b*c^6*x + a^10*b*c*x - 35*a^2*b*c^5*x - 3
4*a^4*b*c^4*x + 34*a^6*b*c^3*x + 35*a^8*b*c^2*x)*(b*(-c)^(3/2) - 2*a*b*c +
a^2*b*(-c)^(1/2)))/(2*(a^4*c + c^3 + 2*a^2*c^2)) - (2*a*b*log(x))/(c + a^2)
^2
```

3.84 $\int \frac{1}{x^3(c+(a+bx)^2)}\,dx$

Optimal result 685
Rubi [A] (verified) 685
Mathematica [A] (verified) 687
Maple [A] (verified) 687
Fricas [A] (verification not implemented) 688
Sympy [B] (verification not implemented) 689
Maxima [A] (verification not implemented) 691
Giac [A] (verification not implemented) 691
Mupad [B] (verification not implemented) 692

Optimal result

Integrand size = 15, antiderivative size = 121

$$\int \frac{1}{x^3\left(c+(a+bx)^2\right)}\,dx = -\frac{1}{2\left(a^2+c\right)x^2}+\frac{2ab}{\left(a^2+c\right)^2x}-\frac{ab^2(a^2-3c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^3}+\frac{b^2(3a^2-c)\log(x)}{\left(a^2+c\right)^3}-\frac{b^2(3a^2-c)\log\left(c+(a+bx)^2\right)}{2\left(a^2+c\right)^3}$$

```
[Out] -1/2/(a^2+c)/x^2+2*a*b/(a^2+c)^2/x+b^2*(3*a^2-c)*ln(x)/(a^2+c)^3-1/2*b^2*(3
*a^2-c)*ln(c+(b*x+a)^2)/(a^2+c)^3-a*b^2*(a^2-3*c)*arctan((b*x+a)/c^(1/2))/(
a^2+c)^3/c^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {378, 724, 815, 649, 209, 266}

$$\int \frac{1}{x^3\left(c+(a+bx)^2\right)}\,dx = -\frac{ab^2(a^2-3c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^3}+\frac{b^2(3a^2-c)\log(x)}{\left(a^2+c\right)^3}-\frac{b^2(3a^2-c)\log\left((a+bx)^2+c\right)}{2\left(a^2+c\right)^3}+\frac{2ab}{x\left(a^2+c\right)^2}-\frac{1}{2x^2\left(a^2+c\right)}$$

```
[In] Int[1/(x^3*(c + (a + b*x)^2)),x]
```

```
[Out] -1/2*1/((a^2 + c)*x^2) + (2*a*b)/((a^2 + c)^2*x) - (a*b^2*(a^2 - 3*c)*ArcTa
n[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c)^3) + (b^2*(3*a^2 - c)*Log[x])/(a^2
 + c)^3 - (b^2*(3*a^2 - c)*Log[c + (a + b*x)^2])/(2*(a^2 + c)^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = b^2\text{Subst}\left(\int \frac{1}{(-a+x)^3\,(c+x^2)}\,dx, x, a+bx\right)$$

$$= -\frac{1}{2\,(a^2+c)\,x^2} + \frac{b^2\text{Subst}\left(\int \frac{-a-x}{(-a+x)^2(c+x^2)}\,dx, x, a+bx\right)}{a^2+c}$$

$$= -\frac{1}{2\,(a^2+c)\,x^2} + \frac{b^2\text{Subst}\left(\int \left(-\frac{2a}{(a^2+c)(a-x)^2} + \frac{-3a^2+c}{(a^2+c)^2(a-x)} + \frac{-a(a^2-3c)-(3a^2-c)x}{(a^2+c)^2(c+x^2)}\right)\,dx, x, a+bx\right)}{a^2+c}$$

$$= -\frac{1}{2(a^2+c)x^2} + \frac{2ab}{(a^2+c)^2 x} + \frac{b^2(3a^2-c)\log(x)}{(a^2+c)^3} + \frac{b^2\text{Subst}\left(\int \frac{-a(a^2-3c)-(3a^2-c)x}{c+x^2}\,dx, x, a+bx\right)}{(a^2+c)^3}$$

$$= -\frac{1}{2(a^2+c)x^2} + \frac{2ab}{(a^2+c)^2 x} + \frac{b^2(3a^2-c)\log(x)}{(a^2+c)^3} - \frac{(ab^2(a^2-3c))\,\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{(a^2+c)^3} - \frac{(b^2(3a^2-c))\,\text{Subst}\left(\int \frac{x}{c+x^2}\,dx, x, a+bx\right)}{(a^2+c)^3}$$

$$= -\frac{1}{2(a^2+c)x^2} + \frac{2ab}{(a^2+c)^2 x} - \frac{ab^2(a^2-3c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2+c)^3} + \frac{b^2(3a^2-c)\log(x)}{(a^2+c)^3} - \frac{b^2(3a^2-c)\log\left(c+(a+bx)^2\right)}{2(a^2+c)^3}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3\left(c+(a+bx)^2\right)}\,dx = -\frac{\frac{(a^2+c)(a^2+c-4abx)}{x^2} + \frac{2ab^2(a^2-3c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}} + 2b^2(-3a^2+c)\log(x) + b^2(3a^2-c)\log\left(a^2+c+2abx+b^2x\right.}{2(a^2+c)^3}$$

```
[In] Integrate[1/(x^3*(c + (a + b*x)^2)),x]
```

```
[Out] -1/2*(((a^2 + c)*(a^2 + c - 4*a*b*x))/x^2 + (2*a*b^2*(a^2 - 3*c)*ArcTan[(a
+ b*x)/Sqrt[c]])/Sqrt[c] + 2*b^2*(-3*a^2 + c)*Log[x] + b^2*(3*a^2 - c)*Log[
a^2 + c + 2*a*b*x + b^2*x^2])/(a^2 + c)^3
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.25

method	result
default	$-\frac{1}{2(a^2+c)x^2}+\frac{b^2(3a^2-c)\ln(x)}{(a^2+c)^3}+\frac{2ab}{(a^2+c)^2x}-\frac{b^3\left(\frac{(3a^2b-bc)\ln(b^2x^2+2abx+a^2+c)}{2b^2}+\frac{\left(4a^3-4ac-\frac{(3a^2b-bc)a}{b}\right)\arctan\left(\frac{2b^2x+2ab}{2\sqrt{c}\,b}\right)}{\sqrt{c}\,b}\right)}{(a^2+c)^3}$
risch	$\frac{\frac{2abx}{a^4+2ca^2+c^2}-\frac{1}{2(a^2+c)}}{x^2}+\frac{3b^2\ln(x)a^2}{a^6+3ca^4+3a^2c^2+c^3}-\frac{b^2\ln(x)c}{a^6+3ca^4+3a^2c^2+c^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(ca^6+3a^4c^2+3a^2c^3+c^4\right)\_Z^2+\left(6a^2b^2c-2b\right.\right.}\right.}{}$

[In] int(1/x^3/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)

[Out] -1/2/(a^2+c)/x^2+b^2*(3*a^2-c)*ln(x)/(a^2+c)^3+2*a*b/(a^2+c)^2/x-b^3/(a^2+c
)^3*(1/2*(3*a^2*b-b*c)/b^2*ln(b^2*x^2+2*a*b*x+a^2+c)+(4*a^3-4*a*c-(3*a^2*b-
b*c)*a/b)/c^(1/2)/b*arctan(1/2*(2*b^2*x+2*a*b)/c^(1/2)/b))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 371, normalized size of antiderivative = 3.07

$$\int \frac{1}{x^3\,(c+(a+bx)^2)}\,dx$$
$$=\left[-\frac{a^4c-(a^3b^2-3\,ab^2c)\sqrt{-c}x^2\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)+2\,a^2c^2+(3\,a^2b^2c-b^2c^2)x^2\log\left(b^2x^2+2\right.}{2\,(a^6c+3\,a^4c^2+3\,a^2c^3+c^4)x^2}\right.$$
$$-\frac{a^4c+2\,(a^3b^2-3\,ab^2c)\sqrt{c}x^2\arctan\left(\frac{bx+a}{\sqrt{c}}\right)+2\,a^2c^2+(3\,a^2b^2c-b^2c^2)x^2\log\left(b^2x^2+2\,abx+a^2+c\right)-2\,(}{2\,(a^6c+3\,a^4c^2+3\,a^2c^3+c^4)x^2}$$

[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="fricas")

[Out] [-1/2*(a^4*c - (a^3*b^2 - 3*a*b^2*c)*sqrt(-c)*x^2*log((b^2*x^2 + 2*a*b*x +
a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + 2*a^2*c^2
+ (3*a^2*b^2*c - b^2*c^2)*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*(3*a^2*b
^2*c - b^2*c^2)*x^2*log(x) + c^3 - 4*(a^3*b*c + a*b*c^2)*x)/((a^6*c + 3*a^4
*c^2 + 3*a^2*c^3 + c^4)*x^2), -1/2*(a^4*c + 2*(a^3*b^2 - 3*a*b^2*c)*sqrt(c)
*x^2*arctan((b*x + a)/sqrt(c)) + 2*a^2*c^2 + (3*a^2*b^2*c - b^2*c^2)*x^2*lo
g(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*(3*a^2*b^2*c - b^2*c^2)*x^2*log(x) + c^3
 - 4*(a^3*b*c + a*b*c^2)*x)/((a^6*c + 3*a^4*c^2 + 3*a^2*c^3 + c^4)*x^2)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3284 vs. 2(109) = 218.

Time = 42.25 (sec) , antiderivative size = 3284, normalized size of antiderivative = 27.14

$$\int \frac{1}{x^3 (c + (a + bx)^2)} \, dx = \text{Too large to display}$$

```
[In] integrate(1/x**3/(c+(b*x+a)**2),x)
```

```
[Out] b**2*(3*a**2 - c)*log(x + (-4*a**16*b**4*c*(3*a**2 - c)**2/(a**2 + c)**6 +
24*a**14*b**4*c**2*(3*a**2 - c)**2/(a**2 + c)**6 + 216*a**12*b**4*c**3*(3*a
**2 - c)**2/(a**2 + c)**6 - 14*a**12*b**4*c*(3*a**2 - c)/(a**2 + c)**3 + 56
8*a**10*b**4*c**4*(3*a**2 - c)**2/(a**2 + c)**6 - 44*a**10*b**4*c**2*(3*a**
2 - c)/(a**2 + c)**3 + 720*a**8*b**4*c**5*(3*a**2 - c)**2/(a**2 + c)**6 - 4
2*a**8*b**4*c**3*(3*a**2 - c)/(a**2 + c)**3 + 78*a**8*b**4*c + 456*a**6*b**
4*c**6*(3*a**2 - c)**2/(a**2 + c)**6 - 8*a**6*b**4*c**4*(3*a**2 - c)/(a**2
+ c)**3 - 464*a**6*b**4*c**2 + 104*a**4*b**4*c**7*(3*a**2 - c)**2/(a**2 + c
)**6 - 2*a**4*b**4*c**5*(3*a**2 - c)/(a**2 + c)**3 + 380*a**4*b**4*c**3 - 2
4*a**2*b**4*c**8*(3*a**2 - c)**2/(a**2 + c)**6 - 12*a**2*b**4*c**6*(3*a**2
- c)/(a**2 + c)**3 - 96*a**2*b**4*c**4 - 12*b**4*c**9*(3*a**2 - c)**2/(a**2
 + c)**6 - 6*b**4*c**7*(3*a**2 - c)/(a**2 + c)**3 + 6*b**4*c**5)/(a**9*b**5
 + 72*a**7*b**5*c - 270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 - 27*a*b**5*c**
4))/(a**2 + c)**3 + (-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))*log(x + (-4*a**
16*c*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c
**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 24*a**14*c**2*(-a*b**2*sq
rt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*
a**2 - c)/(2*(a**2 + c)**3))**2 - 14*a**12*b**2*c*(-a*b**2*sqrt(-c)*(a**2 -
 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(
a**2 + c)**3)) + 216*a**12*c**3*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 +
 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2
- 44*a**10*b**2*c**2*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 568*a**10*c*
*4*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**
3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 78*a**8*b**4*c - 42*a**8*b*
*2*c**3*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2
+ c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 720*a**8*c**5*(-a*b**2*sq
rt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*
a**2 - c)/(2*(a**2 + c)**3))**2 - 464*a**6*b**4*c**2 - 8*a**6*b**2*c**4*(-a
*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) -
b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 456*a**6*c**6*(-a*b**2*sqrt(-c)*(a**
2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(
2*(a**2 + c)**3))**2 + 380*a**4*b**4*c**3 - 2*a**4*b**2*c**5*(-a*b**2*sqrt(
-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**
```

2 - c)/(2*(a**2 + c)**3)) + 104*a**4*c**7*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2
c(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c
)**3))**2 - 96*a**2*b**4*c**4 - 12*a**2*b**2*c**6*(-a*b**2*sqrt(-c)*(a**2 -
 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(
a**2 + c)**3)) - 24*a**2*c**8*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3
*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 +
6*b**4*c**5 - 6*b**2*c**7*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**
4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) - 12*c**9
*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)
) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2)/(a**9*b**5 + 72*a**7*b**5*c -
270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 - 27*a*b**5*c**4)) + (a*b**2*sqrt(-
c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2
 - c)/(2*(a**2 + c)**3))*log(x + (-4*a**16*c*(a*b**2*sqrt(-c)*(a**2 - 3*c)/
(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 +
 c)**3))**2 + 24*a**14*c**2*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a*
*4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 14*
a**12*b**2*c*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c
2 + c3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 216*a**12*c**3*(a*b**
2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2
*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 44*a**10*b**2*c**2*(a*b**2*sqrt(-c)*(
a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c
)/(2*(a**2 + c)**3)) + 568*a**10*c**4*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a
6 + 3*a4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)
)**2 + 78*a**8*b**4*c - 42*a**8*b**2*c**3*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*
c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)
3)) + 720*a8*c**5*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 464*a**6*
b**4*c**2 - 8*a**6*b**2*c**4*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a
4*c + 3*a2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 456*a
6*c6*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2
+ c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 380*a**4*b**4*c**3 - 2
*a**4*b**2*c**5*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**
2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 104*a**4*c**7*(a*b
2*sqrt(-c)*(a2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b*
2(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 96*a**2*b**4*c**4 - 12*a**2*b**2*c*
6(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3
)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) - 24*a**2*c**8*(a*b**2*sqrt(-c)*(
a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c
)/(2*(a**2 + c)**3))**2 + 6*b**4*c**5 - 6*b**2*c**7*(a*b**2*sqrt(-c)*(a**2
- 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*
(a**2 + c)**3)) - 12*c**9*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4
*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2)/(a**9*
b**5 + 72*a**7*b**5*c - 270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 - 27*a*b**5
*c**4)) + (-a**2 + 4*a*b*x - c)/(x**2*(2*a**4 + 4*a**2*c + 2*c**2))

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.63

$$\int \frac{1}{x^3 (c + (a + bx)^2)} dx = -\frac{(3 a^2 b^2 - b^2 c) \log (b^2 x^2 + 2 abx + a^2 + c)}{2 (a^6 + 3 a^4 c + 3 a^2 c^2 + c^3)} + \frac{(3 a^2 b^2 - b^2 c) \log (x)}{a^6 + 3 a^4 c + 3 a^2 c^2 + c^3} - \frac{(a^3 b^3 - 3 ab^3 c) \arctan \left(\frac{b^2 x + ab}{b\sqrt{c}}\right)}{(a^6 + 3 a^4 c + 3 a^2 c^2 + c^3) b\sqrt{c}} + \frac{4 abx - a^2 - c}{2 (a^4 + 2 a^2 c + c^2) x^2}$$

```
[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(3*a^2*b^2 - b^2*c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^6 + 3*a^4*c +
3*a^2*c^2 + c^3) + (3*a^2*b^2 - b^2*c)*log(x)/(a^6 + 3*a^4*c + 3*a^2*c^2 +
c^3) - (a^3*b^3 - 3*a*b^3*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/((a^6 + 3*a^
4*c + 3*a^2*c^2 + c^3)*b*sqrt(c)) + 1/2*(4*a*b*x - a^2 - c)/((a^4 + 2*a^2*c
 + c^2)*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.61

$$\int \frac{1}{x^3 (c + (a + bx)^2)} dx = -\frac{(3 a^2 b^2 - b^2 c) \log (b^2 x^2 + 2 abx + a^2 + c)}{2 (a^6 + 3 a^4 c + 3 a^2 c^2 + c^3)} + \frac{(3 a^2 b^2 - b^2 c) \log (|x|)}{a^6 + 3 a^4 c + 3 a^2 c^2 + c^3} - \frac{(a^3 b^3 - 3 ab^3 c) \arctan \left(\frac{bx + a}{\sqrt{c}}\right)}{(a^6 + 3 a^4 c + 3 a^2 c^2 + c^3) b\sqrt{c}} - \frac{a^4 + 2 a^2 c + c^2 - 4 (a^3 b + abc) x}{2 (a^2 + c)^3 x^2}$$

```
[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/2*(3*a^2*b^2 - b^2*c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^6 + 3*a^4*c +
3*a^2*c^2 + c^3) + (3*a^2*b^2 - b^2*c)*log(abs(x))/(a^6 + 3*a^4*c + 3*a^2*c
^2 + c^3) - (a^3*b^3 - 3*a*b^3*c)*arctan((b*x + a)/sqrt(c))/((a^6 + 3*a^4*c
 + 3*a^2*c^2 + c^3)*b*sqrt(c)) - 1/2*(a^4 + 2*a^2*c + c^2 - 4*(a^3*b + a*b*
c)*x)/((a^2 + c)^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 10.43 (sec) , antiderivative size = 573, normalized size of antiderivative = 4.74

$$\int \frac{1}{x^3\,(c+(a+bx)^2)}\,dx=\ln(x)\,\left(\frac{3\,b^2}{(a^2+c)^2}-\frac{4\,b^2\,c}{(a^2+c)^3}\right)-\frac{\frac{1}{2\,(a^2+c)}-\frac{2\,a\,b\,x}{(a^2+c)^2}}{x^2}$$
$$-\frac{\ln\Big(27\,(-c)^{15/2}+90\,a^2\,(-c)^{13/2}+9\,a^4\,(-c)^{11/2}-324\,a^6\,(-c)^{9/2}+125\,a^8\,(-c)^{7/2}+74\,a^{10}\,(-c)^{5/2}-a^{12}\,(\cdots}{\cdots}$$
$$+\frac{\ln\Big(27\,(-c)^{15/2}+90\,a^2\,(-c)^{13/2}+9\,a^4\,(-c)^{11/2}-324\,a^6\,(-c)^{9/2}+125\,a^8\,(-c)^{7/2}+74\,a^{10}\,(-c)^{5/2}-a^{12}\,(\cdots}{\cdots}$$

```
[In] int(1/(x^3*(c + (a + b*x)^2)),x)
```

```
[Out] log(x)*((3*b^2)/(c + a^2)^2 - (4*b^2*c)/(c + a^2)^3) - (1/(2*(c + a^2)) - (
2*a*b*x)/(c + a^2)^2)/x^2 - (log(27*(-c)^(15/2) + 90*a^2*(-c)^(13/2) + 9*a^
4*(-c)^(11/2) - 324*a^6*(-c)^(9/2) + 125*a^8*(-c)^(7/2) + 74*a^10*(-c)^(5/2
) - a^12*(-c)^(3/2) - 27*a*c^7 + a^13*c + 90*a^3*c^6 - 9*a^5*c^5 - 324*a^7*
c^4 - 125*a^9*c^3 + 74*a^11*c^2 - 27*b*c^7*x + a^12*b*c*x + 90*a^2*b*c^6*x
- 9*a^4*b*c^5*x - 324*a^6*b*c^4*x - 125*a^8*b*c^3*x + 74*a^10*b*c^2*x)*(a^3
*b^2*(-c)^(1/2) - b^2*c^2 + 3*a^2*b^2*c + 3*a*b^2*(-c)^(3/2)))/(2*(a^6*c +
c^4 + 3*a^2*c^3 + 3*a^4*c^2)) + (log(27*(-c)^(15/2) + 90*a^2*(-c)^(13/2) +
9*a^4*(-c)^(11/2) - 324*a^6*(-c)^(9/2) + 125*a^8*(-c)^(7/2) + 74*a^10*(-c)^
(5/2) - a^12*(-c)^(3/2) + 27*a*c^7 - a^13*c - 90*a^3*c^6 + 9*a^5*c^5 + 324*
a^7*c^4 + 125*a^9*c^3 - 74*a^11*c^2 + 27*b*c^7*x - a^12*b*c*x - 90*a^2*b*c^
6*x + 9*a^4*b*c^5*x + 324*a^6*b*c^4*x + 125*a^8*b*c^3*x - 74*a^10*b*c^2*x)*
(b^2*c^2 + a^3*b^2*(-c)^(1/2) - 3*a^2*b^2*c + 3*a*b^2*(-c)^(3/2)))/(2*(a^6*
c + c^4 + 3*a^2*c^3 + 3*a^4*c^2))
```

3.85 $\int \frac{1}{a+b(c+dx)^2} \, dx$

Optimal result 693
Rubi [A] (verified) 693
Mathematica [A] (verified) 694
Maple [A] (verified) 694
Fricas [A] (verification not implemented) 695
Sympy [B] (verification not implemented) 695
Maxima [A] (verification not implemented) 695
Giac [A] (verification not implemented) 696
Mupad [B] (verification not implemented) 696

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{1}{a+b(c+dx)^2} \, dx = \frac{\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

```
[Out] arctan((d*x+c)*b^(1/2)/a^(1/2))/d/a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {253, 211}

$$\int \frac{1}{a+b(c+dx)^2} \, dx = \frac{\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

```
[In] Int[(a + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
```

```
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b(c+dx)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

```
[In] Integrate[(a + b*(c + d*x)^2)^(-1),x]
[Out] ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\arctan\left(\frac{2b\,d^2x+2bcd}{2d\sqrt{ab}}\right)}{d\sqrt{ab}}$	34
risch	$-\frac{\ln\left(bdx+bc+\sqrt{-ab}\right)}{2\sqrt{-ab}\,d}+\frac{\ln\left(-bdx-bc+\sqrt{-ab}\right)}{2\sqrt{-ab}\,d}$	56

```
[In] int(1/(a+b*(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d/(a*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.52

$$\int \frac{1}{a+b(c+dx)^2}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{bd^2x^2+2\,bcdx+bc^2-2\,\sqrt{-ab}(dx+c)-a}{bd^2x^2+2\,bcdx+bc^2+a}\right)}{2\,abd}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}(dx+c)}{a}\right)}{abd}\right]$$

```
[In] integrate(1/(a+b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c
) - a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(a*b*d), sqrt(a*b)*arctan(sqrt(
a*b)*(d*x + c)/a)/(a*b*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(27) = 54.

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int \frac{1}{a+b(c+dx)^2}\,dx = \frac{-\frac{\sqrt{-\frac{1}{ab}}\log\left(x+\frac{-a\sqrt{-\frac{1}{ab}}+c}{d}\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(x+\frac{a\sqrt{-\frac{1}{ab}}+c}{d}\right)}{2}}{d}$$

```
[In] integrate(1/(a+b*(d*x+c)**2),x)
```

```
[Out] (-sqrt(-1/(a*b))*log(x + (-a*sqrt(-1/(a*b)) + c)/d)/2 + sqrt(-1/(a*b))*log(
x + (a*sqrt(-1/(a*b)) + c)/d)/2)/d
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{a+b(c+dx)^2}\,dx = \frac{\arctan\left(\frac{bd^2x+bcd}{\sqrt{ab}d}\right)}{\sqrt{ab}d}$$

```
[In] integrate(1/(a+b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] arctan((b*d^2*x + b*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{a+b(c+dx)^2}\,dx = \frac{\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{\sqrt{ab}d}$$

```
[In] integrate(1/(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*d)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{a+b(c+dx)^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,c+\sqrt{b}\,d\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

```
[In] int(1/(a + b*(c + d*x)^2),x)
```

```
[Out] atan((b^(1/2)*c + b^(1/2)*d*x)/a^(1/2))/(a^(1/2)*b^(1/2)*d)
```

3.86 $\int \frac{1}{\left(a+b(c+dx)^2\right)^2} dx$

Optimal result 697
Rubi [A] (verified) 697
Mathematica [A] (verified) 698
Maple [A] (verified) 699
Fricas [A] (verification not implemented) 699
Sympy [B] (verification not implemented) 699
Maxima [A] (verification not implemented) 700
Giac [A] (verification not implemented) 700
Mupad [B] (verification not implemented) 700

Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{1}{\left(a+b(c+dx)^2\right)^2} dx = \frac{c+dx}{2ad\left(a+b(c+dx)^2\right)} + \frac{\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d}$$

```
[Out] 1/2*(d*x+c)/a/d/(a+b*(d*x+c)^2)+1/2*arctan((d*x+c)*b^(1/2)/a^(1/2))/a^(3/2)
/d/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {253, 205, 211}

$$\int \frac{1}{\left(a+b(c+dx)^2\right)^2} dx = \frac{\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d} + \frac{c+dx}{2ad\left(a+b(c+dx)^2\right)}$$

```
[In] Int[(a + b*(c + d*x)^2)^(-2),x]
```

```
[Out] (c + d*x)/(2*a*d*(a + b*(c + d*x)^2)) + ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]
/(2*a^(3/2)*Sqrt[b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
```

```
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, c+dx\right)}{d}\\ &= \frac{c+dx}{2ad\,(a+b(c+dx)^2)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, c+dx\right)}{2ad}\\ &= \frac{c+dx}{2ad\,(a+b(c+dx)^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+b(c+dx)^2)^2}\,dx = \frac{\frac{\sqrt{a}(c+dx)}{a+b(c+dx)^2} + \frac{\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{2a^{3/2}d}$$

```
[In] Integrate[(a + b*(c + d*x)^2)^(-2),x]
[Out] ((Sqrt[a]*(c + d*x))/(a + b*(c + d*x)^2) + ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[
a]]/Sqrt[b])/(2*a^(3/2)*d)
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

method	result	size
default	$\frac{2b\,d^2x+2bcd}{4ab\,d^2(b\,d^2x^2+2bcdx+b\,c^2+a)}+\frac{\arctan\left(\frac{2b\,d^2x+2bcd}{2d\sqrt{ab}}\right)}{2da\sqrt{ab}}$	86
risch	$\frac{\frac{x}{2a}+\frac{c}{2da}}{b\,d^2x^2+2bcdx+b\,c^2+a}-\frac{\ln(bdx+bc+\sqrt{-ab})}{4\sqrt{-ab}\,da}+\frac{\ln(-bdx-bc+\sqrt{-ab})}{4\sqrt{-ab}\,da}$	102

```
[In] int(1/(a+b*(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*(2*b*d^2*x+2*b*c*d)/a/b/d^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)+1/2/d/a/(a*b)
^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 4.02

$$\int \frac{1}{(a+b(c+dx)^2)^2}\,dx = \left[\frac{2\,abdx+2\,abc-(bd^2x^2+2\,bcdx+bc^2+a)\sqrt{-ab}\log\left(\frac{bd^2x^2+2\,bcdx+bc^2-2\,\sqrt{-ab}(dx+c)-a}{bd^2x^2+2\,bcdx+bc^2+a}\right)}{4\,(a^2b^2d^3x^2+2\,a^2b^2cd^2x+(a^2b^2c^2+a^3b)d)}, \frac{abdx+abc+(bd}{2\,(a^2b^2}\right.$$

```
[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="fricas")
[Out] [1/4*(2*a*b*d*x + 2*a*b*c - (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*sqrt(-a*b)*
log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c) - a)/(b*d^2*x^2
 + 2*b*c*d*x + b*c^2 + a)))/(a^2*b^2*d^3*x^2 + 2*a^2*b^2*c*d^2*x + (a^2*b^2
*c^2 + a^3*b)*d), 1/2*(a*b*d*x + a*b*c + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a
)*sqrt(a*b)*arctan(sqrt(a*b)*(d*x + c)/a))/(a^2*b^2*d^3*x^2 + 2*a^2*b^2*c*d
^2*x + (a^2*b^2*c^2 + a^3*b)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(51) = 102$.

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

$$\int \frac{1}{(a+b(c+dx)^2)^2}\,dx = \frac{c+dx}{2a^2d+2abc^2d+4abcd^2x+2abd^3x^2} + \frac{-\frac{\sqrt{-\frac{1}{a^3b}}\log\left(x+\frac{-a^2\sqrt{-\frac{1}{a^3b}}+c}{d}\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b}}\log\left(x+\frac{a^2\sqrt{-\frac{1}{a^3b}}+c}{d}\right)}{4}}{d}$$

```
[In] integrate(1/(a+b*(d*x+c)**2)**2,x)
```

```
[Out] (c + d*x)/(2*a**2*d + 2*a*b*c**2*d + 4*a*b*c*d**2*x + 2*a*b*d**3*x**2) + (-
sqrt(-1/(a**3*b))*log(x + (-a**2*sqrt(-1/(a**3*b)) + c)/d)/4 + sqrt(-1/(a**
3*b))*log(x + (a**2*sqrt(-1/(a**3*b)) + c)/d)/4)/d
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int \frac{1}{(a+b(c+dx)^2)^2}\,dx = \frac{dx+c}{2\,(abd^3x^2+2\,abcd^2x+(abc^2+a^2)d)} + \frac{\arctan\left(\frac{bd^2x+bcd}{\sqrt{ab}d}\right)}{2\,\sqrt{ab}ad}$$

```
[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*x + c)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (a*b*c^2 + a^2)*d) + 1/2*arcta
n((b*d^2*x + b*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a+b(c+dx)^2)^2}\,dx = \frac{\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ad} + \frac{dx+c}{2\,(bd^2x^2+2\,bcdx+bc^2+a)ad}$$

```
[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*a*d) + 1/2*(d*x + c)/((b*d^2
*x^2 + 2*b*c*d*x + b*c^2 + a)*a*d)
```

Mupad [B] (verification not implemented)

Time = 9.91 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+b(c+dx)^2)^2}\,dx = \frac{\frac{x}{2\,a}+\frac{c}{2\,a\,d}}{b\,c^2+2\,b\,c\,d\,x+b\,d^2\,x^2+a} + \frac{\mathrm{atan}\left(2\,a\,\left(\frac{\sqrt{b}\,c}{2\,a^{3/2}}+\frac{\sqrt{b}\,d\,x}{2\,a^{3/2}}\right)\right)}{2\,a^{3/2}\,\sqrt{b}\,d}$$

```
[In] int(1/(a + b*(c + d*x)^2)^2,x)
```

```
[Out] (x/(2*a) + c/(2*a*d))/(a + b*c^2 + b*d^2*x^2 + 2*b*c*d*x) + atan(2*a*((b^(1
/2)*c)/(2*a^(3/2)) + (b^(1/2)*d*x)/(2*a^(3/2))))/(2*a^(3/2)*b^(1/2)*d)
```

3.87 $\int \frac{1}{(a+b(c+dx)^2)^3} dx$

Optimal result 701
Rubi [A] (verified) 701
Mathematica [A] (verified) 702
Maple [A] (verified) 703
Fricas [B] (verification not implemented) 703
Sympy [B] (verification not implemented) 704
Maxima [B] (verification not implemented) 704
Giac [A] (verification not implemented) 705
Mupad [B] (verification not implemented) 705

Optimal result

Integrand size = 13, antiderivative size = 91

$$\int \frac{1}{(a+b(c+dx)^2)^3} dx = \frac{c+dx}{4ad(a+b(c+dx)^2)^2} + \frac{3(c+dx)}{8a^2d(a+b(c+dx)^2)} + \frac{3\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d}$$

```
[Out] 1/4*(d*x+c)/a/d/(a+b*(d*x+c)^2)^2+3/8*(d*x+c)/a^2/d/(a+b*(d*x+c)^2)+3/8*arc
tan((d*x+c)*b^(1/2)/a^(1/2))/a^(5/2)/d/b^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {253, 205, 211}

$$\int \frac{1}{(a+b(c+dx)^2)^3} dx = \frac{3\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d} + \frac{3(c+dx)}{8a^2d(a+b(c+dx)^2)} + \frac{c+dx}{4ad(a+b(c+dx)^2)^2}$$

```
[In] Int[(a + b*(c + d*x)^2)^(-3),x]
```

```
[Out] (c + d*x)/(4*a*d*(a + b*(c + d*x)^2)^2) + (3*(c + d*x))/(8*a^2*d*(a + b*(c
+ d*x)^2)) + (3*ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
```

```
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^3}\,dx, x, c+dx\right)}{d} \\
&= \frac{c+dx}{4ad\,(a+b(c+dx)^2)^2} + \frac{3\text{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, c+dx\right)}{4ad} \\
&= \frac{c+dx}{4ad\,(a+b(c+dx)^2)^2} + \frac{3(c+dx)}{8a^2d\,(a+b(c+dx)^2)} + \frac{3\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, c+dx\right)}{8a^2d} \\
&= \frac{c+dx}{4ad\,(a+b(c+dx)^2)^2} + \frac{3(c+dx)}{8a^2d\,(a+b(c+dx)^2)} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a+b(c+dx)^2)^3}\,dx = \frac{\frac{\sqrt{a}(c+dx)(5a+3b(c+dx)^2)}{(a+b(c+dx)^2)^2} + \frac{3\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{8a^{5/2}d}$$

```
[In] Integrate[(a + b*(c + d*x)^2)^(-3),x]
[Out] ((Sqrt[a]*(c + d*x)*(5*a + 3*b*(c + d*x)^2))/(a + b*(c + d*x)^2)^2 + (3*Arc
Tan[(Sqrt[b]*(c + d*x))/Sqrt[a]])/Sqrt[b])/(8*a^(5/2)*d)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.53

method	result	size
default	$\frac{2b d^2 x+2bcd}{8ab d^2 (b d^2 x^2+2bcdx+b c^2+a)^2}+\frac{\frac{3\left(2b d^2 x+2bcd\right)}{16ab d^2 \left(b d^2 x^2+2bcdx+b c^2+a\right)}+\frac{3\arctan\left(\frac{2b d^2 x+2bcd}{2d\sqrt{ab}}\right)}{8da\sqrt{ab}}}{a}$	139
risch	$\frac{\frac{3b d^2 x^3}{8a^2}+\frac{9c x^2 bd}{8a^2}+\frac{\left(9b c^2+5a\right)x}{8a^2}+\frac{c\left(3b c^2+5a\right)}{8d a^2}}{\left(b d^2 x^2+2bcdx+b c^2+a\right)^2}-\frac{3\ln\left(bdx+bc+\sqrt{-ab}\right)}{16\sqrt{-ab}\, d a^2}+\frac{3\ln\left(-bdx-bc+\sqrt{-ab}\right)}{16\sqrt{-ab}\, d a^2}$	145

```
[In] int(1/(a+b*(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(2*b*d^2*x+2*b*c*d)/a/b/d^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)^2+3/4/a*(1/4*
(2*b*d^2*x+2*b*c*d)/a/b/d^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)+1/2/d/a/(a*b)^(1/
2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(a*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(77) = 154$.

Time = 0.28 (sec) , antiderivative size = 595, normalized size of antiderivative = 6.54

$$\int \frac{1}{(a+b(c+dx)^2)^3}\,dx$$
$$= \left[\frac{6\,ab^2d^3x^3+18\,ab^2cd^2x^2+6\,ab^2c^3+10\,a^2bc+2\,(9\,ab^2c^2+5\,a^2b)dx-3\,(b^2d^4x^4+4\,b^2cd^3x^3+b^2c^4+2\,(3}{16\,(a^3b^3d^5x^4+4\,a^3b^3cd^4x^3+2\,(3\,a^3b^3c^2+a^4b^2)d^3x^2+4\,(a^3}\right.$$

```
[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*b^2*d^3*x^3 + 18*a*b^2*c*d^2*x^2 + 6*a*b^2*c^3 + 10*a^2*b*c + 2*
(9*a*b^2*c^2 + 5*a^2*b)*d*x - 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + b^2*c^4 +
2*(3*b^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c^2 + 4*(b^2*c^3 + a*b*c)*d*x + a^2)*sq
rt(-a*b)*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c) - a)/(
b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)))/(a^3*b^3*d^5*x^4 + 4*a^3*b^3*c*d^4*x^3
 + 2*(3*a^3*b^3*c^2 + a^4*b^2)*d^3*x^2 + 4*(a^3*b^3*c^3 + a^4*b^2*c)*d^2*x
+ (a^3*b^3*c^4 + 2*a^4*b^2*c^2 + a^5*b)*d), 1/8*(3*a*b^2*d^3*x^3 + 9*a*b^2*
c*d^2*x^2 + 3*a*b^2*c^3 + 5*a^2*b*c + (9*a*b^2*c^2 + 5*a^2*b)*d*x + 3*(b^2*
d^4*x^4 + 4*b^2*c*d^3*x^3 + b^2*c^4 + 2*(3*b^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c
^2 + 4*(b^2*c^3 + a*b*c)*d*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)*(d*x + c)/a)
)/(a^3*b^3*d^5*x^4 + 4*a^3*b^3*c*d^4*x^3 + 2*(3*a^3*b^3*c^2 + a^4*b^2)*d^3*
x^2 + 4*(a^3*b^3*c^3 + a^4*b^2*c)*d^2*x + (a^3*b^3*c^4 + 2*a^4*b^2*c^2 + a^
5*b)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(82) = 164$.

Time = 0.62 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.82

$$\int \frac{1}{(a+b(c+dx)^2)^3}\,dx$$
$$= \frac{5ac+3bc^3+9bcd^2x^2+3bd^3x^3+x(5ad+9bc^2d)}{8a^4d+16a^3bc^2d+8a^2b^2c^4d+32a^2b^2cd^4x^3+8a^2b^2d^5x^4+x^2\cdot(16a^3bd^3+48a^2b^2c^2d^3)+x\,(32a^3bcd^2+32a}$$
$$+ \frac{-\frac{3\sqrt{-\frac{1}{a^5b}}\log\left(x+\frac{-3a^3\sqrt{-\frac{1}{a^5b}}+3c}{3d}\right)}{16}+\frac{3\sqrt{-\frac{1}{a^5b}}\log\left(x+\frac{3a^3\sqrt{-\frac{1}{a^5b}}+3c}{3d}\right)}{16}}{d}$$

```
[In] integrate(1/(a+b*(d*x+c)**2)**3,x)
```

```
[Out] (5*a*c + 3*b*c**3 + 9*b*c*d**2*x**2 + 3*b*d**3*x**3 + x*(5*a*d + 9*b*c**2*d
))/(8*a**4*d + 16*a**3*b*c**2*d + 8*a**2*b**2*c**4*d + 32*a**2*b**2*c*d**4*
x**3 + 8*a**2*b**2*d**5*x**4 + x**2*(16*a**3*b*d**3 + 48*a**2*b**2*c**2*d**
3) + x*(32*a**3*b*c*d**2 + 32*a**2*b**2*c**3*d**2)) + (-3*sqrt(-1/(a**5*b))
*log(x + (-3*a**3*sqrt(-1/(a**5*b)) + 3*c)/(3*d))/16 + 3*sqrt(-1/(a**5*b))*
log(x + (3*a**3*sqrt(-1/(a**5*b)) + 3*c)/(3*d))/16)/d
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(77) = 154$.

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.02

$$\int \frac{1}{(a+b(c+dx)^2)^3}\,dx$$
$$= \frac{3\,bd^3x^3+9\,bcd^2x^2+3\,bc^3+(9\,bc^2+5\,a)dx+5\,ac}{8\,(a^2b^2d^5x^4+4\,a^2b^2cd^4x^3+2\,(3\,a^2b^2c^2+a^3b)d^3x^2+4\,(a^2b^2c^3+a^3bc)d^2x+(a^2b^2c^4+2\,a^3bc^2+a^4)d)}$$
$$+ \frac{3\,\arctan\left(\frac{bd^2x+bcd}{\sqrt{ab}d}\right)}{8\,\sqrt{ab}a^2d}$$

```
[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*b*d^3*x^3 + 9*b*c*d^2*x^2 + 3*b*c^3 + (9*b*c^2 + 5*a)*d*x + 5*a*c)/(
a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + 2*(3*a^2*b^2*c^2 + a^3*b)*d^3*x^2 +
 4*(a^2*b^2*c^3 + a^3*b*c)*d^2*x + (a^2*b^2*c^4 + 2*a^3*b*c^2 + a^4)*d) + 3
/8*arctan((b*d^2*x + b*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*a^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13

$$\int \frac{1}{(a+b(c+dx)^2)^3} \, dx = \frac{3 \arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{8\sqrt{ab}a^2d} + \frac{3\,bd^3x^3 + 9\,bcd^2x^2 + 9\,bc^2dx + 3\,bc^3 + 5\,adx + 5\,ac}{8\,(bd^2x^2 + 2\,bcdx + bc^2 + a)^2a^2d}$$

```
[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*a^2*d) + 1/8*(3*b*d^3*x^3 +
9*b*c*d^2*x^2 + 9*b*c^2*d*x + 3*b*c^3 + 5*a*d*x + 5*a*c)/((b*d^2*x^2 + 2*b*
c*d*x + b*c^2 + a)^2*a^2*d)
```

Mupad [B] (verification not implemented)

Time = 10.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.99

$$\int \frac{1}{(a+b(c+dx)^2)^3} \, dx = \frac{\frac{x\,(9\,b\,c^2+5\,a)}{8\,a^2} + \frac{3\,b\,c^3+5\,a\,c}{8\,a^2\,d} + \frac{3\,b\,d^2\,x^3}{8\,a^2} + \frac{9\,b\,c\,d\,x^2}{8\,a^2}}{x^2\,\left(6\,b^2\,c^2\,d^2+2\,a\,b\,d^2\right) + x\,\left(4\,d\,b^2\,c^3+4\,a\,d\,b\,c\right) + a^2 + b^2\,c^4 + b^2\,d^4\,x^4 + 2\,a\,b\,c^2 + 4\,b^2\,c\,d^3\,x^3} + \frac{3\,\mathrm{atan}\left(\frac{8\,a^2\,\left(\frac{3\,\sqrt{b}\,c}{8\,a^{5/2}}+\frac{3\,\sqrt{b}\,d\,x}{8\,a^{5/2}}\right)}{3}\right)}{8\,a^{5/2}\,\sqrt{b}\,d}$$

```
[In] int(1/(a + b*(c + d*x)^2)^3,x)
```

```
[Out] ((x*(5*a + 9*b*c^2))/(8*a^2) + (5*a*c + 3*b*c^3)/(8*a^2*d) + (3*b*d^2*x^3)/
(8*a^2) + (9*b*c*d*x^2)/(8*a^2))/(x^2*(6*b^2*c^2*d^2 + 2*a*b*d^2) + x*(4*b^
2*c^3*d + 4*a*b*c*d) + a^2 + b^2*c^4 + b^2*d^4*x^4 + 2*a*b*c^2 + 4*b^2*c*d^
3*x^3) + (3*atan((8*a^2*((3*b^(1/2)*c)/(8*a^(5/2)) + (3*b^(1/2)*d*x)/(8*a^(
5/2))))/3))/(8*a^(5/2)*b^(1/2)*d)
```

3.88 $\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx$

Optimal result 706
Rubi [A] (verified) 706
Mathematica [B] (verified) 707
Maple [A] (verified) 707
Fricas [A] (verification not implemented) 708
Sympy [B] (verification not implemented) 708
Maxima [B] (verification not implemented) 709
Giac [A] (verification not implemented) 709
Mupad [B] (verification not implemented) 709

Optimal result

Integrand size = 19, antiderivative size = 35

$$\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\sqrt{b}d}$$

```
[Out] arctan((d*x+c)*b^(1/2)/(-a)^(1/4))/(-a)^(1/4)/d/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {253, 211}

$$\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\sqrt{b}d}$$

```
[In] Int[(Sqrt[-a] + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/(-a)^(1/4)]/((-a)^(1/4)*Sqrt[b]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
```

```
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a}+bx^2}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\sqrt{b}d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 198 vs. $2(35) = 70$.

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 5.66

$$\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx = \frac{2(\sqrt{-a}-\sqrt{a})\arctan\left(1-\frac{\sqrt{2}\sqrt{b}(c+dx)}{\sqrt[4]{a}}\right)-2(\sqrt{-a}-\sqrt{a})\arctan\left(1+\frac{\sqrt{2}\sqrt{b}(c+dx)}{\sqrt[4]{a}}\right)+(\sqrt{-a}+\sqrt{a})\left(\log\left(\right.\right.}{4\sqrt{2}a^{3/4}\sqrt{b}d}$$

```
[In] Integrate[(Sqrt[-a] + b*(c + d*x)^2)^(-1),x]
```

```
[Out] (2*(Sqrt[-a] - Sqrt[a])*ArcTan[1 - (Sqrt[2]*Sqrt[b]*(c + d*x))/a^(1/4)] - 2
*(Sqrt[-a] - Sqrt[a])*ArcTan[1 + (Sqrt[2]*Sqrt[b]*(c + d*x))/a^(1/4)] + (Sq
rt[-a] + Sqrt[a])*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*Sqrt[b]*(c + d*x) + b*(c +
 d*x)^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*Sqrt[b]*(c + d*x) + b*(c + d*x)^2]
))/(4*Sqrt[2]*a^(3/4)*Sqrt[b]*d)
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

method	result	size
default	$\frac{\arctan\left(\frac{2b\,d^2x+2bcd}{2d\sqrt{b\sqrt{-a}}}\right)}{d\sqrt{b\sqrt{-a}}}$	42

```
[In] int(1/(b*(d*x+c)^2+(-a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(b*(-a)^(1/2))^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(b*(-a)^(1/2))^(1
/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 279, normalized size of antiderivative = 7.97

$$\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx = \left[\frac{\sqrt{\frac{\sqrt{-a}}{ab}}\log\left(\frac{b^2d^4x^4+4\,b^2cd^3x^3+6\,b^2c^2d^2x^2+4\,b^2c^3dx+b^2c^4-2\,(bd^2x^2+2\,bcdx+bc^2)\sqrt{-a}+2\,(abdx+abc+(b^2d^3x^3+3\,b^2cd^2x^2+3\,b^2c^2dx+b^2c\ldots}{b^2d^4x^4+4\,b^2cd^3x^3+6\,b^2c^2d^2x^2+4\,b^2c^3dx+b^2c^4+a}\right.}{2\,d}\right.$$

```
[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(sqrt(-a)/(a*b))*log((b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^
2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(-a
) + 2*(a*b*d*x + a*b*c + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*sqrt(-a))*sqrt(sqrt(-a)/(a*b)) - a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3
+ 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + a))/d, sqrt(-sqrt(-a)/(a*b)
)*arctan((b*d*x + b*c)*sqrt(-sqrt(-a)/(a*b)))/d]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(31) = 62$.

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.63

$$\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx = \frac{-\frac{\sqrt{-\frac{1}{b\sqrt{-a}}}\log\left(x+\frac{c-\sqrt{-a}\sqrt{-\frac{1}{b\sqrt{-a}}}}{d}\right)}{2}+\frac{\sqrt{-\frac{1}{b\sqrt{-a}}}\log\left(x+\frac{c+\sqrt{-a}\sqrt{-\frac{1}{b\sqrt{-a}}}}{d}\right)}{2}}{d}$$

```
[In] integrate(1/(b*(d*x+c)**2+(-a)**(1/2)),x)
```

```
[Out] (-sqrt(-1/(b*sqrt(-a)))*log(x + (c - sqrt(-a)*sqrt(-1/(b*sqrt(-a))))/d)/2 +
 sqrt(-1/(b*sqrt(-a)))*log(x + (c + sqrt(-a)*sqrt(-1/(b*sqrt(-a))))/d)/2)/d
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(27) = 54.

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.89

$$\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx = \frac{\log\left(\frac{bd^2x+bcd-\sqrt{-\sqrt{-a}b}d}{bd^2x+bcd+\sqrt{-\sqrt{-a}b}d}\right)}{2\sqrt{-\sqrt{-a}b}d}$$

```
[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*log((b*d^2*x + b*c*d - sqrt(-sqrt(-a)*b)*d)/(b*d^2*x + b*c*d + sqrt(-sq
rt(-a)*b)*d))/(sqrt(-sqrt(-a)*b)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx = \frac{\arctan\left(\frac{bdx+bc}{(-a)^{\frac{1}{4}}\sqrt{b}}\right)}{(-a)^{\frac{1}{4}}\sqrt{b}d}$$

```
[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] arctan((b*d*x + b*c)/((-a)^(1/4)*sqrt(b)))/((-a)^(1/4)*sqrt(b)*d)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,c+\sqrt{b}\,d\,x}{(-a)^{1/4}}\right)}{(-a)^{1/4}\sqrt{b}\,d}$$

```
[In] int(1/(b*(c + d*x)^2 + (-a)^(1/2)),x)
```

```
[Out] atan((b^(1/2)*c + b^(1/2)*d*x)/(-a)^(1/4))/((-a)^(1/4)*b^(1/2)*d)
```

3.89 $\int \frac{1}{1+(c+dx)^2}\,dx$

Optimal result . . . 710
Rubi [A] (verified) . . . 710
Mathematica [A] (verified) . . . 711
Maple [A] (verified) . . . 711
Fricas [A] (verification not implemented) . . . 711
Sympy [C] (verification not implemented) . . . 712
Maxima [A] (verification not implemented) . . . 712
Giac [A] (verification not implemented) . . . 712
Mupad [B] (verification not implemented) . . . 713

Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{1}{1+(c+dx)^2}\,dx=\frac{\arctan(c+dx)}{d}$$

```
[Out] arctan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {253, 209}

$$\int \frac{1}{1+(c+dx)^2}\,dx=\frac{\arctan(c+dx)}{d}$$

```
[In] Int[(1 + (c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[c + d*x]/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\tan^{-1}(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+(c+dx)^2}\,dx = \frac{\arctan(c+dx)}{d}$$

```
[In] Integrate[(1 + (c + d*x)^2)^(-1),x]
[Out] ArcTan[c + d*x]/d
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\arctan(dx+c)}{d}$	11
risch	$\frac{\arctan(dx+c)}{d}$	11
parallelrisch	$-\frac{i\ln(dx+c-i)-i\ln(dx+c+i)}{2d}$	29

```
[In] int(1/(1+(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] arctan(d*x+c)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+(c+dx)^2}\,dx = \frac{\arctan\left(dx+c\right)}{d}$$

```
[In] integrate(1/(1+(d*x+c)^2),x, algorithm="fricas")
[Out] arctan(d*x + c)/d
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.40

$$\int \frac{1}{1+(c+dx)^2}\,dx = \frac{-\frac{i\log\left(x+\frac{c-i}{d}\right)}{2}+\frac{i\log\left(x+\frac{c+i}{d}\right)}{2}}{d}$$

```
[In] integrate(1/(1+(d*x+c)**2),x)
[Out] (-I*log(x + (c - I)/d)/2 + I*log(x + (c + I)/d)/2)/d
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{1}{1+(c+dx)^2}\,dx = \frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d}$$

```
[In] integrate(1/(1+(d*x+c)^2),x, algorithm="maxima")
[Out] arctan((d^2*x + c*d)/d)/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+(c+dx)^2}\,dx = \frac{\arctan(dx+c)}{d}$$

```
[In] integrate(1/(1+(d*x+c)^2),x, algorithm="giac")
[Out] arctan(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+(c+dx)^2}\,dx = \frac{\operatorname{atan}(c+d\,x)}{d}$$

```
[In] int(1/((c + d*x)^2 + 1),x)
[Out] atan(c + d*x)/d
```

3.90 $\int \frac{1}{(1+(c+dx)^2)^2} \, dx$

Optimal result 714
Rubi [A] (verified) 714
Mathematica [A] (verified) 715
Maple [A] (verified) 715
Fricas [A] (verification not implemented) 716
Sympy [C] (verification not implemented) 716
Maxima [A] (verification not implemented) 716
Giac [A] (verification not implemented) 717
Mupad [B] (verification not implemented) 717

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{1}{(1+(c+dx)^2)^2} \, dx = \frac{c+dx}{2d(1+(c+dx)^2)} + \frac{\arctan(c+dx)}{2d}$$

```
[Out] 1/2*(d*x+c)/d/(1+(d*x+c)^2)+1/2*arctan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {253, 205, 209}

$$\int \frac{1}{(1+(c+dx)^2)^2} \, dx = \frac{\arctan(c+dx)}{2d} + \frac{c+dx}{2d((c+dx)^2+1)}$$

```
[In] Int[(1 + (c + d*x)^2)^(-2),x]
```

```
[Out] (c + d*x)/(2*d*(1 + (c + d*x)^2)) + ArcTan[c + d*x]/(2*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x, c+dx\right)}{d}\\ &= \frac{c+dx}{2d\left(1+(c+dx)^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{2d}\\ &= \frac{c+dx}{2d\left(1+(c+dx)^2\right)} + \frac{\tan^{-1}(c+dx)}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(1+(c+dx)^2\right)^2}\,dx = \frac{\frac{c+dx}{1+(c+dx)^2} + \arctan(c+dx)}{2d}$$

```
[In] Integrate[(1 + (c + d*x)^2)^(-2),x]
[Out] ((c + d*x)/(1 + (c + d*x)^2) + ArcTan[c + d*x])/(2*d)
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

method	result
risch	$\frac{\frac{x}{2}+\frac{c}{2d}}{d^2x^2+2cdx+c^2+1} + \frac{\arctan(dx+c)}{2d}$
default	$\frac{2d^2x+2cd}{4d^2(d^2x^2+2cdx+c^2+1)} + \frac{\arctan\left(\frac{2d^2x+2cd}{2d}\right)}{2d}$
parallelrisch	$-\frac{i\ln(dx+c-i)x^2d^3-i\ln(dx+c+i)x^2d^3+2i\ln(dx+c-i)xc\,d^2-2i\ln(dx+c+i)xc\,d^2+i\ln(dx+c-i)c^2d-i\ln(dx+c+i)c^2d+i\ln(}{4d^2(d^2x^2+2cdx+c^2+1)}$

```
[In] int(1/(1+(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] (1/2*x+1/2*c/d)/(d^2*x^2+2*c*d*x+c^2+1)+1/2*arctan(d*x+c)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.49

$$\int \frac{1}{(1+(c+dx)^2)^2}\,dx = \frac{dx + (d^2x^2 + 2\,cdx + c^2 + 1)\arctan(dx + c) + c}{2\,(d^3x^2 + 2\,cd^2x + (c^2 + 1)d)}$$

```
[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(d*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c) + c)/(d^3*x^2 + 2*
c*d^2*x + (c^2 + 1)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.51

$$\int \frac{1}{(1+(c+dx)^2)^2}\,dx = \frac{c+dx}{2c^2d + 4cd^2x + 2d^3x^2 + 2d} + \frac{-\frac{i\log\left(x+\frac{c-i}{d}\right)}{4} + \frac{i\log\left(x+\frac{c+i}{d}\right)}{4}}{d}$$

```
[In] integrate(1/(1+(d*x+c)**2)**2,x)
```

```
[Out] (c + d*x)/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2 + 2*d) + (-I*log(x + (c - I)
/d)/4 + I*log(x + (c + I)/d)/4)/d
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

$$\int \frac{1}{(1+(c+dx)^2)^2}\,dx = \frac{dx + c}{2\,(d^3x^2 + 2\,cd^2x + (c^2 + 1)d)} + \frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{2\,d}$$

```
[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*x + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 + 1)*d) + 1/2*arctan((d^2*x + c*d
)/d)/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(1+(c+dx)^2\right)^2}\,dx = \frac{\arctan\left(dx+c\right)}{2\,d} + \frac{dx+c}{2\left(d^2x^2+2\,cdx+c^2+1\right)d}$$

```
[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(d*x + c)/d + 1/2*(d*x + c)/((d^2*x^2 + 2*c*d*x + c^2 + 1)*d)
```

Mupad [B] (verification not implemented)

Time = 10.89 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(1+(c+dx)^2\right)^2}\,dx = \frac{\frac{x}{2}+\frac{c}{2\,d}}{c^2+2\,c\,d\,x+d^2\,x^2+1} + \frac{\mathrm{atan}(c+d\,x)}{2\,d}$$

```
[In] int(1/((c + d*x)^2 + 1)^2,x)
```

```
[Out] (x/2 + c/(2*d))/(c^2 + d^2*x^2 + 2*c*d*x + 1) + atan(c + d*x)/(2*d)
```

3.91 $\int \frac{1}{\left(1+(c+dx)^2\right)^3} dx$

Optimal result 718
Rubi [A] (verified) 718
Mathematica [A] (verified) 719
Maple [A] (verified) 720
Fricas [B] (verification not implemented) 720
Sympy [C] (verification not implemented) 720
Maxima [B] (verification not implemented) 721
Giac [A] (verification not implemented) 721
Mupad [B] (verification not implemented) 722

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{1}{\left(1+(c+dx)^2\right)^3} dx = \frac{c+dx}{4d\left(1+(c+dx)^2\right)^2}+\frac{3(c+dx)}{8d\left(1+(c+dx)^2\right)}+\frac{3\arctan(c+dx)}{8d}$$

```
[Out] 1/4*(d*x+c)/d/(1+(d*x+c)^2)^2+3/8*(d*x+c)/d/(1+(d*x+c)^2)+3/8*arctan(d*x+c)
/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {253, 205, 209}

$$\int \frac{1}{\left(1+(c+dx)^2\right)^3} dx = \frac{3\arctan(c+dx)}{8d}+\frac{3(c+dx)}{8d\left((c+dx)^2+1\right)}+\frac{c+dx}{4d\left((c+dx)^2+1\right)^2}$$

```
[In] Int[(1 + (c + d*x)^2)^(-3),x]
```

```
[Out] (c + d*x)/(4*d*(1 + (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 + (c + d*x)^2))
 + (3*ArcTan[c + d*x])/(8*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3}\,dx, x, c+dx\right)}{d}\\
&= \frac{c+dx}{4d\,(1+(c+dx)^2)^2} + \frac{3\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x, c+dx\right)}{4d}\\
&= \frac{c+dx}{4d\,(1+(c+dx)^2)^2} + \frac{3(c+dx)}{8d\,(1+(c+dx)^2)} + \frac{3\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{8d}\\
&= \frac{c+dx}{4d\,(1+(c+dx)^2)^2} + \frac{3(c+dx)}{8d\,(1+(c+dx)^2)} + \frac{3\tan^{-1}(c+dx)}{8d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{1}{(1+(c+dx)^2)^3}\,dx = \frac{\frac{2(c+dx)}{(1+(c+dx)^2)^2} + \frac{3(c+dx)}{1+(c+dx)^2} + 3\arctan(c+dx)}{8d}$$

```
[In] Integrate[(1 + (c + d*x)^2)^(-3),x]
```

```
[Out] ((2*(c + d*x))/(1 + (c + d*x)^2)^2 + (3*(c + d*x))/(1 + (c + d*x)^2) + 3*Ar
cTan[c + d*x])/(8*d)
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

method	result
risch	$\frac{\frac{3d^2x^3}{8}+\frac{9x^2cd}{8}+\left(\frac{9c^2}{8}+\frac{5}{8}\right)x+\frac{c\left(3c^2+5\right)}{8d}}{\left(d^2x^2+2cdx+c^2+1\right)^2}+\frac{3\arctan(dx+c)}{8d}$
default	$\frac{2d^2x+2cd}{8d^2\left(d^2x^2+2cdx+c^2+1\right)^2}+\frac{\frac{3}{8}d^2x+\frac{3}{8}cd}{d^2\left(d^2x^2+2cdx+c^2+1\right)}+\frac{3\arctan\left(\frac{2d^2x+2cd}{2d}\right)}{8d}$
parallelrisch	$-\frac{-3i\ln(dx+c+i)c^4d^3+6i\ln(dx+c-i)c^2d^3-6i\ln(dx+c+i)c^2d^3+3i\ln(dx+c-i)x^4d^7-3i\ln(dx+c+i)x^4d^7+6i\ln(dx+c-i)x^2d^5-}{}$

```
[In] int(1/(1+(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (3/8*d^2*x^3+9/8*x^2*c*d+(9/8*c^2+5/8)*x+1/8*c/d*(3*c^2+5))/(d^2*x^2+2*c*d*
x+c^2+1)^2+3/8*arctan(d*x+c)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(54) = 108$.

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.55

$$\int \frac{1}{\left(1+(c+dx)^2\right)^3}\,dx = \frac{3\,d^3x^3+9\,cd^2x^2+3\,c^3+(9\,c^2+5)dx+3\,(d^4x^4+4\,cd^3x^3+2\,(3\,c^2+1)d^2x^2+c^4+4\,(c^3+c)dx+2\,c^2+1)}{8\,(d^5x^4+4\,cd^4x^3+2\,(3\,c^2+1)d^3x^2+4\,(c^3+c)d^2x+(c^4+2\,c^2+1)d)}$$

```
[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 + 5)*d*x + 3*(d^4*x^4 + 4*c*d
^3*x^3 + 2*(3*c^2 + 1)*d^2*x^2 + c^4 + 4*(c^3 + c)*d*x + 2*c^2 + 1)*arctan(
d*x + c) + 5*c)/(d^5*x^4 + 4*c*d^4*x^3 + 2*(3*c^2 + 1)*d^3*x^2 + 4*(c^3 + c
)*d^2*x + (c^4 + 2*c^2 + 1)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.43

$$\int \frac{1}{\left(1+(c+dx)^2\right)^3}\,dx = \frac{3c^3+9cd^2x^2+5c+3d^3x^3+x(9c^2d+5d)}{8c^4d+16c^2d+32cd^4x^3+8d^5x^4+8d+x^2\cdot(48c^2d^3+16d^3)+x\,(32c^3d^2+32cd^2)} + \frac{-\frac{3i\log\left(x+\frac{3c-3i}{3d}\right)}{16}+\frac{3i\log\left(x+\frac{3c+3i}{3d}\right)}{16}}{d}$$

```
[In] integrate(1/(1+(d*x+c)**2)**3,x)
```

```
[Out] (3*c**3 + 9*c*d**2*x**2 + 5*c + 3*d**3*x**3 + x*(9*c**2*d + 5*d))/(8*c**4*d
 + 16*c**2*d + 32*c*d**4*x**3 + 8*d**5*x**4 + 8*d + x**2*(48*c**2*d**3 + 16
*d**3) + x*(32*c**3*d**2 + 32*c*d**2)) + (-3*I*log(x + (3*c - 3*I)/(3*d))/1
6 + 3*I*log(x + (3*c + 3*I)/(3*d))/16)/d
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(54) = 108$.

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.92

$$\int \frac{1}{\left(1+(c+dx)^2\right)^3}\,dx = \frac{3\,d^3x^3+9\,cd^2x^2+3\,c^3+(9\,c^2+5)dx+5\,c}{8\,(d^5x^4+4\,cd^4x^3+2\,(3\,c^2+1)d^3x^2+4\,(c^3+c)d^2x+(c^4+2\,c^2+1)d)} + \frac{3\,\arctan\left(\frac{d^2x+cd}{d}\right)}{8\,d}$$

```
[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 + 5)*d*x + 5*c)/(d^5*x^4 + 4*
c*d^4*x^3 + 2*(3*c^2 + 1)*d^3*x^2 + 4*(c^3 + c)*d^2*x + (c^4 + 2*c^2 + 1)*d
) + 3/8*arctan((d^2*x + c*d)/d)/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(1+(c+dx)^2\right)^3}\,dx = \frac{3\,\arctan\left(dx+c\right)}{8\,d} + \frac{3\,d^3x^3+9\,cd^2x^2+9\,c^2dx+3\,c^3+5\,dx+5\,c}{8\,\left(d^2x^2+2\,cdx+c^2+1\right)^2d}$$

```
[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(d*x + c)/d + 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 9*c^2*d*x + 3*c^3 +
5*d*x + 5*c)/((d^2*x^2 + 2*c*d*x + c^2 + 1)^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.85

$$\int \frac{1}{(1+(c+dx)^2)^3}\,dx$$
$$=\frac{3\,\mathrm{atan}(c+d\,x)}{8\,d}+\frac{x\,\left(\frac{9\,c^2}{8}+\frac{5}{8}\right)+\frac{3\,c^3+5\,c}{8\,d}+\frac{3\,d^2\,x^3}{8}+\frac{9\,c\,d\,x^2}{8}}{x^2\,\left(6\,c^2\,d^2+2\,d^2\right)+2\,c^2+c^4+x\,\left(4\,d\,c^3+4\,d\,c\right)+d^4\,x^4+4\,c\,d^3\,x^3+1}$$

```
[In] int(1/((c + d*x)^2 + 1)^3,x)
```

```
[Out] (3*atan(c + d*x))/(8*d) + (x*((9*c^2)/8 + 5/8) + (5*c + 3*c^3)/(8*d) + (3*d
^2*x^3)/8 + (9*c*d*x^2)/8)/(x^2*(2*d^2 + 6*c^2*d^2) + 2*c^2 + c^4 + x*(4*c*
d + 4*c^3*d) + d^4*x^4 + 4*c*d^3*x^3 + 1)
```

3.92 $\int \frac{1}{1-(c+dx)^2} \, dx$

Optimal result . . . 723
Rubi [A] (verified) . . . 723
Mathematica [B] (verified) . . . 724
Maple [B] (verified) . . . 724
Fricas [B] (verification not implemented) . . . 725
Sympy [B] (verification not implemented) . . . 725
Maxima [B] (verification not implemented) . . . 725
Giac [B] (verification not implemented) . . . 726
Mupad [B] (verification not implemented) . . . 726

Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \frac{1}{1-(c+dx)^2} \, dx = \frac{\operatorname{arctanh}(c+dx)}{d}$$

```
[Out] arctanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {253, 212}

$$\int \frac{1}{1-(c+dx)^2} \, dx = \frac{\operatorname{arctanh}(c+dx)}{d}$$

```
[In] Int[(1 - (c + d*x)^2)^(-1),x]
```

```
[Out] ArcTanh[c + d*x]/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\tanh^{-1}(c+dx)}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 32 vs. 2(10) = 20.

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 3.20

$$\int \frac{1}{1-(c+dx)^2}\,dx = -\frac{\log(1-c-dx)}{2d} + \frac{\log(1+c+dx)}{2d}$$

```
[In] Integrate[(1 - (c + d*x)^2)^(-1),x]
[Out] -1/2*Log[1 - c - d*x]/d + Log[1 + c + d*x]/(2*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(10) = 20.

Time = 0.88 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

method	result	size
parallelrisch	$-\frac{\ln(dx+c-1)-\ln(dx+c+1)}{2d}$	23
default	$-\frac{\ln(dx+c-1)}{2d}+\frac{\ln(dx+c+1)}{2d}$	26
norman	$-\frac{\ln(dx+c-1)}{2d}+\frac{\ln(dx+c+1)}{2d}$	26
risch	$-\frac{\ln(dx+c-1)}{2d}+\frac{\ln(-dx-c-1)}{2d}$	29

```
[In] int(1/(1-(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] -1/2*(ln(d*x+c-1)-ln(d*x+c+1))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(10) = 20.

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.20

$$\int \frac{1}{1-(c+dx)^2}\,dx = \frac{\log(dx+c+1)-\log(dx+c-1)}{2\,d}$$

```
[In] integrate(1/(1-(d*x+c)^2),x, algorithm="fricas")
[Out] 1/2*(log(d*x + c + 1) - log(d*x + c - 1))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(7) = 14.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.20

$$\int \frac{1}{1-(c+dx)^2}\,dx = -\frac{\frac{\log\left(x+\frac{c-1}{d}\right)}{2}-\frac{\log\left(x+\frac{c+1}{d}\right)}{2}}{d}$$

```
[In] integrate(1/(1-(d*x+c)**2),x)
[Out] -(log(x + (c - 1)/d)/2 - log(x + (c + 1)/d)/2)/d
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.50

$$\int \frac{1}{1-(c+dx)^2}\,dx = \frac{\log(dx+c+1)}{2\,d}-\frac{\log(dx+c-1)}{2\,d}$$

```
[In] integrate(1/(1-(d*x+c)^2),x, algorithm="maxima")
[Out] 1/2*log(d*x + c + 1)/d - 1/2*log(d*x + c - 1)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.70

$$\int \frac{1}{1-(c+dx)^2}\,dx = \frac{\log(|dx+c+1|)}{2\,d} - \frac{\log(|dx+c-1|)}{2\,d}$$

```
[In] integrate(1/(1-(d*x+c)^2),x, algorithm="giac")
[Out] 1/2*log(abs(d*x + c + 1))/d - 1/2*log(abs(d*x + c - 1))/d
```

Mupad [B] (verification not implemented)

Time = 10.47 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-(c+dx)^2}\,dx = \frac{\operatorname{atanh}(c+d\,x)}{d}$$

```
[In] int(-1/((c + d*x)^2 - 1),x)
[Out] atanh(c + d*x)/d
```

3.93 $\int \frac{1}{\left(1-(c+dx)^2\right)^2} \, dx$

Optimal result 727
Rubi [A] (verified) 727
Mathematica [A] (verified) 728
Maple [A] (verified) 728
Fricas [B] (verification not implemented) 729
Sympy [A] (verification not implemented) 729
Maxima [A] (verification not implemented) 729
Giac [A] (verification not implemented) 730
Mupad [B] (verification not implemented) 730

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{1}{\left(1-(c+dx)^2\right)^2} \, dx=\frac{c+dx}{2d\left(1-(c+dx)^2\right)}+\frac{\text{arctanh}(c+dx)}{2d}$$

```
[Out] 1/2*(d*x+c)/d/(1-(d*x+c)^2)+1/2*arctanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {253, 205, 212}

$$\int \frac{1}{\left(1-(c+dx)^2\right)^2} \, dx=\frac{\text{arctanh}(c+dx)}{2d}+\frac{c+dx}{2d\left(1-(c+dx)^2\right)}$$

```
[In] Int[(1 - (c + d*x)^2)^(-2),x]
```

```
[Out] (c + d*x)/(2*d*(1 - (c + d*x)^2)) + ArcTanh[c + d*x]/(2*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2}\,dx, x, c+dx\right)}{d}\\ &= \frac{c+dx}{2d\left(1-(c+dx)^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, c+dx\right)}{2d}\\ &= \frac{c+dx}{2d\left(1-(c+dx)^2\right)} + \frac{\tanh^{-1}(c+dx)}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(1-(c+dx)^2\right)^2}\,dx = \frac{-\frac{2(c+dx)}{-1+(c+dx)^2} - \log(1-c-dx) + \log(1+c+dx)}{4d}$$

```
[In] Integrate[(1 - (c + d*x)^2)^(-2),x]
[Out] ((-2*(c + d*x))/(-1 + (c + d*x)^2) - Log[1 - c - d*x] + Log[1 + c + d*x])/(
4*d)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

method	result
default	$-\frac{1}{4d(dx+c-1)} - \frac{\ln(dx+c-1)}{4d} - \frac{1}{4d(dx+c+1)} + \frac{\ln(dx+c+1)}{4d}$
norman	$\frac{-\frac{c}{2d}-\frac{x}{2}}{d^2x^2+2cdx+c^2-1} - \frac{\ln(dx+c-1)}{4d} + \frac{\ln(dx+c+1)}{4d}$
risch	$\frac{-\frac{c}{2d}-\frac{x}{2}}{d^2x^2+2cdx+c^2-1} - \frac{\ln(dx+c-1)}{4d} + \frac{\ln(-dx-c-1)}{4d}$
parallelrisch	$-\frac{\ln(dx+c-1)x^2d^3-\ln(dx+c+1)x^2d^3+2\ln(dx+c-1)xc\,d^2-2\ln(dx+c+1)xc\,d^2+\ln(dx+c-1)c^2d-\ln(dx+c+1)c^2d+2d^2x-\ln(d}{4d^2(d^2x^2+2cdx+c^2-1)}$

```
[In] int(1/(1-(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d/(d*x+c-1)-1/4/d*ln(d*x+c-1)-1/4/d/(d*x+c+1)+1/4/d*ln(d*x+c+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(33) = 66$.

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.18

$$\int \frac{1}{(1-(c+dx)^2)^2}\,dx = -\frac{2\,dx-(d^2x^2+2\,cdx+c^2-1)\log(dx+c+1)+(d^2x^2+2\,cdx+c^2-1)\log(dx+c-1)+2\,c}{4\,(d^3x^2+2\,cd^2x+(c^2-1)d)}$$

```
[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d*x - (d^2*x^2 + 2*c*d*x + c^2 - 1)*log(d*x + c + 1) + (d^2*x^2 + 2
*c*d*x + c^2 - 1)*log(d*x + c - 1) + 2*c)/(d^3*x^2 + 2*c*d^2*x + (c^2 - 1)*
d)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\int \frac{1}{(1-(c+dx)^2)^2}\,dx = \frac{-c-dx}{2c^2d+4cd^2x+2d^3x^2-2d}+\frac{-\frac{\log\left(x+\frac{c-1}{d}\right)}{4}+\frac{\log\left(x+\frac{c+1}{d}\right)}{4}}{d}$$

```
[In] integrate(1/(1-(d*x+c)**2)**2,x)
```

```
[Out] (-c - d*x)/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2 - 2*d) + (-log(x + (c - 1)/
d)/4 + log(x + (c + 1)/d)/4)/d
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int \frac{1}{(1-(c+dx)^2)^2}\,dx = -\frac{dx+c}{2\,(d^3x^2+2\,cd^2x+(c^2-1)d)}+\frac{\log(dx+c+1)}{4\,d}-\frac{\log(dx+c-1)}{4\,d}$$

```
[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d*x + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 - 1)*d) + 1/4*log(d*x + c + 1)/d
 - 1/4*log(d*x + c - 1)/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int \frac{1}{(1-(c+dx)^2)^2} \, dx = \frac{\log(|dx+c+1|)}{4\,d} - \frac{\log(|dx+c-1|)}{4\,d} - \frac{dx+c}{2\,(d^2x^2+2\,cdx+c^2-1)d}$$

```
[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="giac")
[Out] 1/4*log(abs(d*x + c + 1))/d - 1/4*log(abs(d*x + c - 1))/d - 1/2*(d*x + c)/(
(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)
```

Mupad [B] (verification not implemented)

Time = 10.36 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{1}{(1-(c+dx)^2)^2} \, dx = \frac{\operatorname{atanh}(c+d\,x)}{2\,d} - \frac{\frac{x}{2}+\frac{c}{2\,d}}{c^2+2\,c\,d\,x+d^2\,x^2-1}$$

```
[In] int(1/((c + d*x)^2 - 1)^2,x)
[Out] atanh(c + d*x)/(2*d) - (x/2 + c/(2*d))/(c^2 + d^2*x^2 + 2*c*d*x - 1)
```

3.94 $\int \frac{1}{(1-(c+dx)^2)^3} dx$

Optimal result 731
Rubi [A] (verified) 731
Mathematica [A] (verified) 732
Maple [A] (verified) 733
Fricas [B] (verification not implemented) 733
Sympy [B] (verification not implemented) 734
Maxima [B] (verification not implemented) 734
Giac [A] (verification not implemented) 735
Mupad [B] (verification not implemented) 735

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{1}{(1-(c+dx)^2)^3} dx = \frac{c+dx}{4d(1-(c+dx)^2)^2} + \frac{3(c+dx)}{8d(1-(c+dx)^2)} + \frac{3\text{arctanh}(c+dx)}{8d}$$

```
[Out] 1/4*(d*x+c)/d/(1-(d*x+c)^2)^2+3/8*(d*x+c)/d/(1-(d*x+c)^2)+3/8*arctanh(d*x+c
)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {253, 205, 212}

$$\int \frac{1}{(1-(c+dx)^2)^3} dx = \frac{3\text{arctanh}(c+dx)}{8d} + \frac{3(c+dx)}{8d(1-(c+dx)^2)} + \frac{c+dx}{4d(1-(c+dx)^2)^2}$$

```
[In] Int[(1 - (c + d*x)^2)^(-3),x]
```

```
[Out] (c + d*x)/(4*d*(1 - (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 - (c + d*x)^2))
 + (3*ArcTanh[c + d*x])/(8*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3}\,dx, x, c+dx\right)}{d}\\
&= \frac{c+dx}{4d\left(1-(c+dx)^2\right)^2}+\frac{3\text{Subst}\left(\int \frac{1}{(1-x^2)^2}\,dx, x, c+dx\right)}{4d}\\
&= \frac{c+dx}{4d\left(1-(c+dx)^2\right)^2}+\frac{3(c+dx)}{8d\left(1-(c+dx)^2\right)}+\frac{3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, c+dx\right)}{8d}\\
&= \frac{c+dx}{4d\left(1-(c+dx)^2\right)^2}+\frac{3(c+dx)}{8d\left(1-(c+dx)^2\right)}+\frac{3\tanh^{-1}(c+dx)}{8d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(1-(c+dx)^2\right)^3}\,dx = \frac{\frac{4(c+dx)}{(-1+(c+dx)^2)^2}-\frac{6(c+dx)}{-1+(c+dx)^2}-3\log(1-c-dx)+3\log(1+c+dx)}{16d}$$

```
[In] Integrate[(1 - (c + d*x)^2)^(-3),x]
[Out] ((4*(c + d*x))/(-1 + (c + d*x)^2)^2 - (6*(c + d*x))/(-1 + (c + d*x)^2) - 3*
Log[1 - c - d*x] + 3*Log[1 + c + d*x])/(16*d)
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

method	result
default	$\frac{1}{16d(dx+c-1)^2}-\frac{3}{16d(dx+c-1)}-\frac{3\ln(dx+c-1)}{16d}-\frac{1}{16d(dx+c+1)^2}-\frac{3}{16d(dx+c+1)}+\frac{3\ln(dx+c+1)}{16d}$
risch	$\frac{-\frac{3d^2x^3}{8}-\frac{9x^2cd}{8}+\left(-\frac{9c^2}{8}+\frac{5}{8}\right)x-\frac{c(3c^2-5)}{8d}}{(d^2x^2+2cdx+c^2-1)^2}-\frac{3\ln(dx+c-1)}{16d}+\frac{3\ln(-dx-c-1)}{16d}$
norman	$\frac{\frac{-3c^3d^3+5d^3c}{8d^4}+\frac{(-9c^2d^3+5d^3)x}{8d^3}-\frac{3d^2x^3}{8}-\frac{9x^2cd}{8}}{(d^2x^2+2cdx+c^2-1)^2}-\frac{3\ln(dx+c-1)}{16d}+\frac{3\ln(dx+c+1)}{16d}$
parallelrisch	$-\frac{3\ln(dx+c-1)c^4d^3+6d^6x^3+18x^2c\,d^5+18x\,c^2d^4-6\ln(dx+c-1)c^2d^3+6c^3d^3-10d^3c+12\ln(dx+c-1)x^3c\,d^6-12\ln(dx+c+1)x}{}$

[In] int(1/(1-(d*x+c)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/16/d/(d*x+c-1)^2-3/16/d/(d*x+c-1)-3/16/d*ln(d*x+c-1)-1/16/d/(d*x+c+1)^2-3/16/d/(d*x+c+1)+3/16/d*ln(d*x+c+1)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(54) = 108.

Time = 0.26 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.44

$$\int \frac{1}{(1-(c+dx)^2)^3}\,dx = -\frac{6\,d^3x^3+18\,cd^2x^2+6\,c^3+2\,(9\,c^2-5)dx-3\,(d^4x^4+4\,cd^3x^3+2\,(3\,c^2-1)d^2x^2+c^4+4\,(c^3-c)dx-2}{16\,(d^5x^4+4\,cd^4x^3+2\,(3\,c^2-}$$

[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/16*(6*d^3*x^3 + 18*c*d^2*x^2 + 6*c^3 + 2*(9*c^2 - 5)*d*x - 3*(d^4*x^4 + 4*c*d^3*x^3 + 2*(3*c^2 - 1)*d^2*x^2 + c^4 + 4*(c^3 - c)*d*x - 2*c^2 + 1)*log(d*x + c + 1) + 3*(d^4*x^4 + 4*c*d^3*x^3 + 2*(3*c^2 - 1)*d^2*x^2 + c^4 + 4*(c^3 - c)*d*x - 2*c^2 + 1)*log(d*x + c - 1) - 10*c)/(d^5*x^4 + 4*c*d^4*x^3 + 2*(3*c^2 - 1)*d^3*x^2 + 4*(c^3 - c)*d^2*x + (c^4 - 2*c^2 + 1)*d)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(53) = 106.

Time = 0.51 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.20

$$\int \frac{1}{(1-(c+dx)^2)^3}\,dx$$
$$= -\frac{3c^3+9cd^2x^2-5c+3d^3x^3+x(9c^2d-5d)}{8c^4d-16c^2d+32cd^4x^3+8d^5x^4+8d+x^2\cdot(48c^2d^3-16d^3)+x\left(32c^3d^2-32cd^2\right)} - \frac{\frac{3\log\left(x+\frac{3c-3}{3d}\right)}{16}-\frac{3\log\left(x+\frac{3c+3}{3d}\right)}{16}}{d}$$

```
[In] integrate(1/(1-(d*x+c)**2)**3,x)
```

```
[Out] -(3*c**3 + 9*c*d**2*x**2 - 5*c + 3*d**3*x**3 + x*(9*c**2*d - 5*d))/(8*c**4*
d - 16*c**2*d + 32*c*d**4*x**3 + 8*d**5*x**4 + 8*d + x**2*(48*c**2*d**3 - 1
6*d**3) + x*(32*c**3*d**2 - 32*c*d**2)) - (3*log(x + (3*c - 3)/(3*d))/16 -
3*log(x + (3*c + 3)/(3*d))/16)/d
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(54) = 108.

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.91

$$\int \frac{1}{(1-(c+dx)^2)^3}\,dx$$
$$= -\frac{3\,d^3x^3+9\,cd^2x^2+3\,c^3+(9\,c^2-5)dx-5\,c}{8\left(d^5x^4+4\,cd^4x^3+2\,(3\,c^2-1)d^3x^2+4\,(c^3-c)d^2x+(c^4-2\,c^2+1)d\right)} + \frac{3\,\log\left(dx+c+1\right)}{16\,d} - \frac{3\,\log\left(dx+c-1\right)}{16\,d}$$

```
[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 - 5)*d*x - 5*c)/(d^5*x^4 + 4
*c*d^4*x^3 + 2*(3*c^2 - 1)*d^3*x^2 + 4*(c^3 - c)*d^2*x + (c^4 - 2*c^2 + 1)*
d) + 3/16*log(d*x + c + 1)/d - 3/16*log(d*x + c - 1)/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(1-(c+dx)^2\right)^3}\,dx = \frac{3\,\log\left(|dx+c+1|\right)}{16\,d} - \frac{3\,\log\left(|dx+c-1|\right)}{16\,d} - \frac{3\,d^3x^3+9\,cd^2x^2+9\,c^2dx+3\,c^3-5\,dx-5\,c}{8\left(d^2x^2+2\,cdx+c^2-1\right)^2d}$$

```
[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/16*log(abs(d*x + c + 1))/d - 3/16*log(abs(d*x + c - 1))/d - 1/8*(3*d^3*x^
3 + 9*c*d^2*x^2 + 9*c^2*d*x + 3*c^3 - 5*d*x - 5*c)/((d^2*x^2 + 2*c*d*x + c^
2 - 1)^2*d)
```

Mupad [B] (verification not implemented)

Time = 10.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.78

$$\int \frac{1}{\left(1-(c+dx)^2\right)^3}\,dx = \frac{3\,\mathrm{atanh}(c+d\,x)}{8\,d} - \frac{x\left(\frac{9\,c^2}{8}-\frac{5}{8}\right)-\frac{5\,c-3\,c^3}{8\,d}+\frac{3\,d^2\,x^3}{8}+\frac{9\,c\,d\,x^2}{8}}{c^4-2\,c^2-x^2\,\left(2\,d^2-6\,c^2\,d^2\right)-x\,\left(4\,c\,d-4\,c^3\,d\right)+d^4\,x^4+4\,c\,d^3\,x^3+1}$$

```
[In] int(-1/((c + d*x)^2 - 1)^3,x)
```

```
[Out] (3*atanh(c + d*x))/(8*d) - (x*((9*c^2)/8 - 5/8) - (5*c - 3*c^3)/(8*d) + (3*
d^2*x^3)/8 + (9*c*d*x^2)/8)/(c^4 - 2*c^2 - x^2*(2*d^2 - 6*c^2*d^2) - x*(4*c
*d - 4*c^3*d) + d^4*x^4 + 4*c*d^3*x^3 + 1)
```

3.95 $\int \frac{1}{1-(1+x)^2} dx$

Optimal result 736
Rubi [A] (verified) 736
Mathematica [B] (verified) 737
Maple [B] (verified) 737
Fricas [B] (verification not implemented) 738
Sympy [B] (verification not implemented) 738
Maxima [B] (verification not implemented) 738
Giac [B] (verification not implemented) 739
Mupad [B] (verification not implemented) 739

Optimal result

Integrand size = 11, antiderivative size = 4

$$\int \frac{1}{1-(1+x)^2} dx = \operatorname{arctanh}(1+x)$$

```
[Out] arctanh(1+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {253, 212}

$$\int \frac{1}{1-(1+x)^2} dx = \operatorname{arctanh}(x+1)$$

```
[In] Int[(1 - (1 + x)^2)^(-1),x]
```

```
[Out] ArcTanh[1 + x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, 1+x\right)\\ &= \tanh^{-1}(1+x) \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 15 vs. $2(4) = 8$.

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \frac{1}{1-(1+x)^2}\,dx = -\frac{\log(x)}{2} + \frac{1}{2}\log(2+x)$$

```
[In] Integrate[(1 - (1 + x)^2)^(-1),x]
[Out] -1/2*Log[x] + Log[2 + x]/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(4) = 8$.

Time = 0.71 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

method	result	size
default	$-\frac{\ln(x)}{2}+\frac{\ln(x+2)}{2}$	12
norman	$-\frac{\ln(x)}{2}+\frac{\ln(x+2)}{2}$	12
risch	$-\frac{\ln(x)}{2}+\frac{\ln(x+2)}{2}$	12
parallelrisch	$-\frac{\ln(x)}{2}+\frac{\ln(x+2)}{2}$	12
meijerg	$\frac{\ln\left(1+\frac{x}{2}\right)}{2}-\frac{\ln(x)}{2}+\frac{\ln(2)}{2}$	18

```
[In] int(1/(1-(x+1)^2),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(x)+1/2*ln(x+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(4) = 8$.

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.75

$$\int \frac{1}{1-(1+x)^2}\,dx = \frac{1}{2}\,\log\left(x+2\right) - \frac{1}{2}\,\log\left(x\right)$$

```
[In] integrate(1/(1-(1+x)^2),x, algorithm="fricas")
[Out] 1/2*log(x + 2) - 1/2*log(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(3) = 6$.

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.50

$$\int \frac{1}{1-(1+x)^2}\,dx = -\frac{\log\left(x\right)}{2} + \frac{\log\left(x+2\right)}{2}$$

```
[In] integrate(1/(1-(1+x)**2),x)
[Out] -log(x)/2 + log(x + 2)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(4) = 8$.

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.75

$$\int \frac{1}{1-(1+x)^2}\,dx = \frac{1}{2}\,\log\left(x+2\right) - \frac{1}{2}\,\log\left(x\right)$$

```
[In] integrate(1/(1-(1+x)^2),x, algorithm="maxima")
[Out] 1/2*log(x + 2) - 1/2*log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(4) = 8$.

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 3.25

$$\int \frac{1}{1-(1+x)^2}\,dx = \frac{1}{2}\log\left(|x+2|\right) - \frac{1}{2}\log\left(|x|\right)$$

```
[In] integrate(1/(1-(1+x)^2),x, algorithm="giac")
[Out] 1/2*log(abs(x + 2)) - 1/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-(1+x)^2}\,dx = \operatorname{atanh}(x+1)$$

```
[In] int(-1/((x + 1)^2 - 1),x)
[Out] atanh(x + 1)
```

3.96 $\int \frac{1}{\left(1-(1+x)^2\right)^2} dx$

Optimal result 740
Rubi [A] (verified) 740
Mathematica [A] (verified) 741
Maple [A] (verified) 741
Fricas [A] (verification not implemented) 742
Sympy [A] (verification not implemented) 742
Maxima [A] (verification not implemented) 742
Giac [A] (verification not implemented) 743
Mupad [B] (verification not implemented) 743

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \frac{1}{(1-(1+x)^2)^2} dx = \frac{1+x}{2(1-(1+x)^2)} + \frac{1}{2}\text{arctanh}(1+x)$$

```
[Out] 1/2*(1+x)/(1-(1+x)^2)+1/2*arctanh(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {253, 205, 212}

$$\int \frac{1}{(1-(1+x)^2)^2} dx = \frac{1}{2}\text{arctanh}(x+1) + \frac{x+1}{2(1-(x+1)^2)}$$

```
[In] Int[(1 - (1 + x)^2)^(-2),x]
```

```
[Out] (1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{(1-x^2)^2}\,dx, x, 1+x\right)\\ &= \frac{1+x}{2\,(1-(1+x)^2)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, 1+x\right)\\ &= \frac{1+x}{2\,(1-(1+x)^2)} + \frac{1}{2}\tanh^{-1}(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{(1-(1+x)^2)^2}\,dx = \frac{1}{4}\left(-\frac{2(1+x)}{x(2+x)} - \log(x) + \log(2+x)\right)$$

```
[In] Integrate[(1 - (1 + x)^2)^(-2),x]
[Out] ((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{1}{4x} - \frac{\ln(x)}{4} - \frac{1}{4(x+2)} + \frac{\ln(x+2)}{4}$	24
norman	$\frac{-\frac{1}{2}-\frac{x}{2}}{x(x+2)} - \frac{\ln(x)}{4} + \frac{\ln(x+2)}{4}$	26
risch	$\frac{-\frac{1}{2}-\frac{x}{2}}{x(x+2)} - \frac{\ln(x)}{4} + \frac{\ln(x+2)}{4}$	26
meijerg	$\frac{3x}{16\left(\frac{3x}{2}+3\right)} + \frac{\ln\left(1+\frac{x}{2}\right)}{4} - \frac{1}{8} - \frac{\ln(x)}{4} + \frac{\ln(2)}{4} - \frac{1}{4x}$	34
parallelrisch	$-\frac{\ln(x)x^2-\ln(x+2)x^2+2+2\ln(x)x-2\ln(x+2)x+2x}{4x(x+2)}$	43

```
[In] int(1/(1-(x+1)^2)^2,x,method=_RETURNVERBOSE)
[Out] -1/4/x-1/4*ln(x)-1/4/(x+2)+1/4*ln(x+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{1}{(1-(1+x)^2)^2} \, dx = \frac{(x^2+2\,x)\log(x+2)-(x^2+2\,x)\log(x)-2\,x-2}{4\,(x^2+2\,x)}$$

```
[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="fricas")
[Out] 1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{(1-(1+x)^2)^2} \, dx = \frac{-x-1}{2x^2+4x} - \frac{\log(x)}{4} + \frac{\log(x+2)}{4}$$

```
[In] integrate(1/(1-(1+x)**2)**2,x)
[Out] (-x - 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{(1-(1+x)^2)^2} \, dx = -\frac{x+1}{2\,(x^2+2\,x)} + \frac{1}{4}\,\log(x+2) - \frac{1}{4}\,\log(x)$$

```
[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="maxima")
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(1-(1+x)^2\right)^2}\,dx = -\frac{x+1}{2\left(x^2+2\,x\right)} + \frac{1}{4}\,\log\left(|x+2|\right) - \frac{1}{4}\,\log\left(|x|\right)$$

```
[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="giac")
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(abs(x + 2)) - 1/4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(1-(1+x)^2\right)^2}\,dx = \frac{\operatorname{atanh}(x+1)}{2} - \frac{x+1}{2\,\left((x+1)^2-1\right)}$$

```
[In] int(1/((x + 1)^2 - 1)^2,x)
[Out] atanh(x + 1)/2 - (x + 1)/(2*((x + 1)^2 - 1))
```

3.97 $\int \frac{1}{\left(1-(1+x)^2\right)^3} dx$

Optimal result 744
Rubi [A] (verified) 744
Mathematica [A] (verified) 745
Maple [A] (verified) 745
Fricas [B] (verification not implemented) 746
Sympy [A] (verification not implemented) 746
Maxima [A] (verification not implemented) 747
Giac [A] (verification not implemented) 747
Mupad [B] (verification not implemented) 747

Optimal result

Integrand size = 11, antiderivative size = 45

$$\int \frac{1}{\left(1-(1+x)^2\right)^3} dx = \frac{1+x}{4\left(1-(1+x)^2\right)^2} + \frac{3(1+x)}{8\left(1-(1+x)^2\right)} + \frac{3}{8}\text{arctanh}(1+x)$$

```
[Out] 1/4*(1+x)/(1-(1+x)^2)^2+3/8*(1+x)/(1-(1+x)^2)+3/8*arctanh(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {253, 205, 212}

$$\int \frac{1}{\left(1-(1+x)^2\right)^3} dx = \frac{3}{8}\text{arctanh}(x+1) + \frac{3(x+1)}{8\left(1-(x+1)^2\right)} + \frac{x+1}{4\left(1-(x+1)^2\right)^2}$$

```
[In] Int[(1 - (1 + x)^2)^(-3),x]
```

```
[Out] (1 + x)/(4*(1 - (1 + x)^2)^2) + (3*(1 + x))/(8*(1 - (1 + x)^2)) + (3*ArcTan
h[1 + x])/8
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(1-x^2)^3}\,dx, x, 1+x\right)\\
&= \frac{1+x}{4\left(1-(1+x)^2\right)^2}+\frac{3}{4}\text{Subst}\left(\int \frac{1}{(1-x^2)^2}\,dx, x, 1+x\right)\\
&= \frac{1+x}{4\left(1-(1+x)^2\right)^2}+\frac{3(1+x)}{8\left(1-(1+x)^2\right)}+\frac{3}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, 1+x\right)\\
&= \frac{1+x}{4\left(1-(1+x)^2\right)^2}+\frac{3(1+x)}{8\left(1-(1+x)^2\right)}+\frac{3}{8}\tanh^{-1}(1+x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(1-(1+x)^2\right)^3}\,dx = \frac{1}{16}\left(\frac{1}{x^2}-\frac{3}{x}-\frac{1}{(2+x)^2}-\frac{3}{2+x}-3\log(x)+3\log(2+x)\right)$$

```
[In] Integrate[(1 - (1 + x)^2)^(-3),x]
[Out] (x^(-2) - 3/x - (2 + x)^(-2) - 3/(2 + x) - 3*Log[x] + 3*Log[2 + x])/16
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{1}{16x^{2}}-\frac{3}{16x}-\frac{3\ln(x)}{16}-\frac{1}{16(x+2)^{2}}-\frac{3}{16(x+2)}+\frac{3\ln(x+2)}{16}$	36
norman	$\frac{\frac{1}{4}-\frac{9}{8}x^{2}-\frac{3}{8}x^{3}-\frac{1}{2}x}{x^{2}(x+2)^{2}}-\frac{3\ln(x)}{16}+\frac{3\ln(x+2)}{16}$	36
risch	$\frac{\frac{1}{4}-\frac{9}{8}x^{2}-\frac{3}{8}x^{3}-\frac{1}{2}x}{x^{2}(x+2)^{2}}-\frac{3\ln(x)}{16}+\frac{3\ln(x+2)}{16}$	36
meijerg	$\frac{x\left(\frac{7x}{2}+8\right)}{128\left(1+\frac{x}{2}\right)^{2}}+\frac{3\ln\left(1+\frac{x}{2}\right)}{16}-\frac{7}{64}-\frac{3\ln(x)}{16}+\frac{3\ln(2)}{16}+\frac{1}{16x^{2}}-\frac{3}{16x}$	44
parallelrisch	$-\frac{3\ln(x)x^{4}-3\ln(x+2)x^{4}-4+12\ln(x)x^{3}-12\ln(x+2)x^{3}+12\ln(x)x^{2}-12\ln(x+2)x^{2}+6x^{3}+18x^{2}+8x}{16x^{2}(x+2)^{2}}$	74

```
[In] int(1/(1-(x+1)^2)^3,x,method=_RETURNVERBOSE)
[Out] 1/16/x^2-3/16/x-3/16*ln(x)-1/16/(x+2)^2-3/16/(x+2)+3/16*ln(x+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(35) = 70$.

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58

$$\int \frac{1}{\left(1-(1+x)^2\right)^3}\,dx = -\frac{6\,x^3+18\,x^2-3\,(x^4+4\,x^3+4\,x^2)\log(x+2)+3\,(x^4+4\,x^3+4\,x^2)\log(x)+8\,x-4}{16\,(x^4+4\,x^3+4\,x^2)}$$

```
[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="fricas")
[Out] -1/16*(6*x^3 + 18*x^2 - 3*(x^4 + 4*x^3 + 4*x^2)*log(x + 2) + 3*(x^4 + 4*x^3
 + 4*x^2)*log(x) + 8*x - 4)/(x^4 + 4*x^3 + 4*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(1-(1+x)^2\right)^3}\,dx = -\frac{3\log(x)}{16}+\frac{3\log(x+2)}{16}-\frac{3x^3+9x^2+4x-2}{8x^4+32x^3+32x^2}$$

```
[In] integrate(1/(1-(1+x)**2)**3,x)
[Out] -3*log(x)/16 + 3*log(x + 2)/16 - (3*x**3 + 9*x**2 + 4*x - 2)/(8*x**4 + 32*x
**3 + 32*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(1-(1+x)^2\right)^3}\,dx = -\frac{3\,x^3+9\,x^2+4\,x-2}{8\,(x^4+4\,x^3+4\,x^2)}+\frac{3}{16}\,\log\left(x+2\right)-\frac{3}{16}\,\log\left(x\right)$$

```
[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*x^3 + 9*x^2 + 4*x - 2)/(x^4 + 4*x^3 + 4*x^2) + 3/16*log(x + 2) - 3/
16*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(1-(1+x)^2\right)^3}\,dx = -\frac{3\,x^3+9\,x^2+4\,x-2}{8\,\left(x^2+2\,x\right)^2}+\frac{3}{16}\,\log\left(|x+2|\right)-\frac{3}{16}\,\log\left(|x|\right)$$

```
[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*x^3 + 9*x^2 + 4*x - 2)/(x^2 + 2*x)^2 + 3/16*log(abs(x + 2)) - 3/16*
log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 10.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(1-(1+x)^2\right)^3}\,dx = \frac{3\,\mathrm{atanh}(x+1)}{8}+\frac{\frac{5\,x}{8}-\frac{3\,(x+1)^3}{8}+\frac{5}{8}}{(x+1)^4-2\,(x+1)^2+1}$$

```
[In] int(-1/((x + 1)^2 - 1)^3,x)
```

```
[Out] (3*atanh(x + 1))/8 + ((5*x)/8 - (3*(x + 1)^3)/8 + 5/8)/((x + 1)^4 - 2*(x +
1)^2 + 1)
```

3.98 $\int \frac{(1+(a+bx)^2)^2}{x} \, dx$

Optimal result 748
Rubi [A] (verified) 748
Mathematica [A] (verified) 749
Maple [A] (verified) 749
Fricas [A] (verification not implemented) 750
Sympy [A] (verification not implemented) 750
Maxima [A] (verification not implemented) 750
Giac [A] (verification not implemented) 751
Mupad [B] (verification not implemented) 751

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{(1+(a+bx)^2)^2}{x} \, dx = a(2+a^2) \, bx + \frac{1}{2}(2+a^2) \, (a+bx)^2 + \frac{1}{3}a(a+bx)^3 + \frac{1}{4}(a+bx)^4 + (1+a^2)^2 \log(x)$$

```
[Out] a*(a^2+2)*b*x+1/2*(a^2+2)*(b*x+a)^2+1/3*a*(b*x+a)^3+1/4*(b*x+a)^4+(a^2+1)^2
*ln(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {378, 711}

$$\int \frac{(1+(a+bx)^2)^2}{x} \, dx = \frac{1}{2}(a^2+2) \, (a+bx)^2 + a(a^2+2) \, bx + (a^2+1)^2 \log(x) + \frac{1}{4}(a+bx)^4 + \frac{1}{3}a(a+bx)^3$$

```
[In] Int[(1 + (a + b*x)^2)^2/x,x]
```

```
[Out] a*(2 + a^2)*b*x + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a + b*x)
^4/4 + (1 + a^2)^2*Log[x]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
```

```
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{(1+x^2)^2}{-a+x}\,dx, x, a+bx\right)\\ &= \text{Subst}\left(\int \left(a(2+a^2) - \frac{(1+a^2)^2}{a-x} + (2+a^2)\,x + ax^2 + x^3\right)dx, x, a+bx\right)\\ &= a(2+a^2)\,bx + \frac{1}{2}(2+a^2)\,(a+bx)^2 + \frac{1}{3}a(a+bx)^3 + \frac{1}{4}(a+bx)^4 + (1+a^2)^2\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int \frac{(1+(a+bx)^2)^2}{x}\,dx = a(2+a^2)\,(a+bx) + \frac{1}{2}(2+a^2)\,(a+bx)^2 + \frac{1}{3}a(a+bx)^3 + \frac{1}{4}(a+bx)^4 + (1+a^2)^2\log(bx)$$

```
[In] Integrate[(1 + (a + b*x)^2)^2/x,x]
[Out] a*(2 + a^2)*(a + b*x) + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a
+ b*x)^4/4 + (1 + a^2)^2*Log[b*x]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

method	result	size
norman	$(3a^2b^2+b^2)\,x^2+(4a^3b+4ab)\,x+\frac{b^4x^4}{4}+\frac{4a\,b^3x^3}{3}+(a^4+2a^2+1)\ln(x)$	61
default	$\frac{b^4x^4}{4}+\frac{4a\,b^3x^3}{3}+3a^2b^2x^2+4a^3bx+b^2x^2+4abx+(a^4+2a^2+1)\ln(x)$	62
risch	$\frac{b^4x^4}{4}+\frac{4a\,b^3x^3}{3}+3a^2b^2x^2+b^2x^2+4a^3bx+4abx+a^4\ln(x)+2a^2\ln(x)+\ln(x)$	64
parallelrisch	$\frac{b^4x^4}{4}+\frac{4a\,b^3x^3}{3}+3a^2b^2x^2+b^2x^2+4a^3bx+4abx+a^4\ln(x)+2a^2\ln(x)+\ln(x)$	64

```
[In] int((1+(b*x+a)^2)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] (3*a^2*b^2+b^2)*x^2+(4*a^3*b+4*a*b)*x+1/4*b^4*x^4+4/3*a*b^3*x^3+(a^4+2*a^2+
1)*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{(1+(a+bx)^2)^2}{x}\,dx = \frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + \left(3\,a^2+1\right)b^2x^2 + 4\,\left(a^3+a\right)bx + \left(a^4+2\,a^2+1\right)\log\left(x\right)$$

```
[In] integrate((1+(b*x+a)^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b*x + (a^4
+ 2*a^2 + 1)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \frac{(1+(a+bx)^2)^2}{x}\,dx = \frac{4ab^3x^3}{3} + \frac{b^4x^4}{4} + x^2\cdot\left(3a^2b^2+b^2\right) + x\left(4a^3b+4ab\right) + \left(a^2+1\right)^2\log\left(x\right)$$

```
[In] integrate((1+(b*x+a)**2)**2/x,x)
```

```
[Out] 4*a*b**3*x**3/3 + b**4*x**4/4 + x**2*(3*a**2*b**2 + b**2) + x*(4*a**3*b + 4
*a*b) + (a**2 + 1)**2*log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{(1+(a+bx)^2)^2}{x}\,dx = \frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + \left(3\,a^2+1\right)b^2x^2 + 4\,\left(a^3+a\right)bx + \left(a^4+2\,a^2+1\right)\log\left(x\right)$$

```
[In] integrate((1+(b*x+a)^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b*x + (a^4
+ 2*a^2 + 1)*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

$$\int \frac{(1+(a+bx)^2)^2}{x}\,dx = \frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + 3\,a^2b^2x^2 + 4\,a^3bx + b^2x^2 + 4\,abx + \left(a^4 + 2\,a^2 + 1\right)\log\left(|x|\right)$$

```
[In] integrate((1+(b*x+a)^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + b^2*x^2 + 4*a*b*x
 + (a^4 + 2*a^2 + 1)*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{(1+(a+bx)^2)^2}{x}\,dx = \ln(x)\,\left(a^4 + 2\,a^2 + 1\right) + \frac{b^4\,x^4}{4} + \frac{4\,a\,b^3\,x^3}{3} + b^2\,x^2\,\left(3\,a^2 + 1\right) + 4\,a\,b\,x\,\left(a^2 + 1\right)$$

```
[In] int(((a + b*x)^2 + 1)^2/x,x)
```

```
[Out] log(x)*(2*a^2 + a^4 + 1) + (b^4*x^4)/4 + (4*a*b^3*x^3)/3 + b^2*x^2*(3*a^2 +
 1) + 4*a*b*x*(a^2 + 1)
```

3.99 $\int \frac{x^2}{1+(-1+x)^2} dx$

Optimal result 752
Rubi [A] (verified) 752
Mathematica [A] (verified) 753
Maple [A] (verified) 753
Fricas [A] (verification not implemented) 754
Sympy [A] (verification not implemented) 754
Maxima [A] (verification not implemented) 754
Giac [A] (verification not implemented) 754
Mupad [B] (verification not implemented) 755

Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \frac{x^2}{1+(-1+x)^2} dx = x + \log\left(1+(-1+x)^2\right)$$

```
[Out] x+ln(1+(-1+x)^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {378, 716, 266}

$$\int \frac{x^2}{1+(-1+x)^2} dx = x + \log\left((x-1)^2+1\right)$$

```
[In] Int[x^2/(1 + (-1 + x)^2),x]
```

```
[Out] x + Log[1 + (-1 + x)^2]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(1+x)^2}{1+x^2}\,dx, x, -1+x\right)\\
&= \text{Subst}\left(\int \left(1+\frac{2x}{1+x^2}\right)\,dx, x, -1+x\right)\\
&= x + 2\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, -1+x\right)\\
&= x + \log\left(1+(-1+x)^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{1+(-1+x)^2}\,dx = x + \log\left(2-2x+x^2\right)$$

```
[In] Integrate[x^2/(1 + (-1 + x)^2),x]
[Out] x + Log[2 - 2*x + x^2]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

method	result	size
default	$x+\ln\left(x^2-2x+2\right)$	12
norman	$x+\ln\left(x^2-2x+2\right)$	12
risch	$x+\ln\left(x^2-2x+2\right)$	12
parallelrisch	$x+\ln\left(x^2-2x+2\right)$	12

```
[In] int(x^2/(1+(x-1)^2),x,method=_RETURNVERBOSE)
[Out] x+ln(x^2-2*x+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{1+(-1+x)^2}\,dx = x + \log\left(x^2 - 2\,x + 2\right)$$

```
[In] integrate(x^2/(1+(-1+x)^2),x, algorithm="fricas")
[Out] x + log(x^2 - 2*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{1+(-1+x)^2}\,dx = x + \log\left(x^2 - 2x + 2\right)$$

```
[In] integrate(x**2/(1+(-1+x)**2),x)
[Out] x + log(x**2 - 2*x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{1+(-1+x)^2}\,dx = x + \log\left(x^2 - 2\,x + 2\right)$$

```
[In] integrate(x^2/(1+(-1+x)^2),x, algorithm="maxima")
[Out] x + log(x^2 - 2*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{1+(-1+x)^2}\,dx = x + \log\left(x^2 - 2\,x + 2\right)$$

```
[In] integrate(x^2/(1+(-1+x)^2),x, algorithm="giac")
[Out] x + log(x^2 - 2*x + 2)
```

Mupad [B] (verification not implemented)

Time = 10.37 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{1+(-1+x)^2} \, dx = x + \ln\left(x^2 - 2\,x + 2\right)$$

```
[In] int(x^2/((x - 1)^2 + 1),x)
[Out] x + log(x^2 - 2*x + 2)
```

3.100 $\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx$

Optimal result . . . 756
Rubi [A] (verified) . . . 756
Mathematica [A] (verified) . . . 757
Maple [A] (verified) . . . 758
Fricas [A] (verification not implemented) . . . 758
Sympy [A] (verification not implemented) . . . 758
Maxima [A] (verification not implemented) . . . 759
Giac [A] (verification not implemented) . . . 759
Mupad [F(-1)] . . . 759

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx=\frac{3}{2}\sqrt{1-(1+x)^2}-\frac{1}{2}x\sqrt{1-(1+x)^2}+\frac{3}{2}\arcsin(1+x)$$

```
[Out] 3/2*arcsin(1+x)+3/2*(1-(1+x)^2)^(1/2)-1/2*x*(1-(1+x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {378, 685, 655, 222}

$$\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx=\frac{3}{2}\arcsin(x+1)-\frac{1}{2}\sqrt{1-(x+1)^2}x+\frac{3}{2}\sqrt{1-(x+1)^2}$$

```
[In] Int[x^2/Sqrt[1 - (1 + x)^2],x]
[Out] (3*Sqrt[1 - (1 + x)^2])/2 - (x*Sqrt[1 - (1 + x)^2])/2 + (3*ArcSin[1 + x])/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
```

```
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(-1+x)^2}{\sqrt{1-x^2}}\,dx, x, 1+x\right)\\
&= -\frac{1}{2}x\sqrt{1-(1+x)^2} - \frac{3}{2}\text{Subst}\left(\int \frac{-1+x}{\sqrt{1-x^2}}\,dx, x, 1+x\right)\\
&= \frac{3}{2}\sqrt{1-(1+x)^2} - \frac{1}{2}x\sqrt{1-(1+x)^2} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1+x\right)\\
&= \frac{3}{2}\sqrt{1-(1+x)^2} - \frac{1}{2}x\sqrt{1-(1+x)^2} + \frac{3}{2}\sin^{-1}(1+x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx = \frac{x(-6-x+x^2) - 6\sqrt{x}\sqrt{2+x}\log\left(-\sqrt{x}+\sqrt{2+x}\right)}{2\sqrt{-x(2+x)}}$$

```
[In] Integrate[x^2/Sqrt[1 - (1 + x)^2],x]
[Out] (x*(-6 - x + x^2) - 6*Sqrt[x]*Sqrt[2 + x]*Log[-Sqrt[x] + Sqrt[2 + x]])/(2*S
qrt[-(x*(2 + x))])
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{(-3+x)x(x+2)}{2\sqrt{-x(x+2)}}+\frac{3\arcsin(x+1)}{2}$	25
pseudoelliptic	$-3\arctan\left(\frac{\sqrt{-x(x+2)}}{x}\right)+\frac{(3-x)\sqrt{-x(x+2)}}{2}$	32
default	$-\frac{x\sqrt{-x^2-2x}}{2}+\frac{3\sqrt{-x^2-2x}}{2}+\frac{3\arcsin(x+1)}{2}$	35
meijerg	$-\frac{4i\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-5x+15)\sqrt{1+\frac{x}{2}}}{40}+\frac{3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)}{4}\right)}{\sqrt{\pi}}$	45
trager	$\left(\frac{3}{2}-\frac{x}{2}\right)\sqrt{-x^2-2x}-\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2-2x}+\mathrm{RootOf}\left(\_Z^2+1\right)\right)}{2}$	54

```
[In] int(x^2/(1-(x+1)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(-3+x)*x*(x+2)/(-x*(x+2))^(1/2)+3/2*arcsin(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx = -\frac{1}{2}\,\sqrt{-x^2-2\,x}(x-3)-3\,\arctan\left(\frac{\sqrt{-x^2-2\,x}}{x}\right)$$

```
[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-x^2 - 2*x)*(x - 3) - 3*arctan(sqrt(-x^2 - 2*x)/x)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx = \left(\frac{3}{2}-\frac{x}{2}\right)\sqrt{-x^2-2x}+\frac{3\,\mathrm{asin}\,(x+1)}{2}$$

```
[In] integrate(x**2/(1-(1+x)**2)**(1/2),x)
[Out] (3/2 - x/2)*sqrt(-x**2 - 2*x) + 3*asin(x + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx = -\frac{1}{2}\sqrt{-x^2-2\,x}x + \frac{3}{2}\sqrt{-x^2-2\,x} - \frac{3}{2}\arcsin\left(-x-1\right)$$

```
[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(-x^2 - 2*x)*x + 3/2*sqrt(-x^2 - 2*x) - 3/2*arcsin(-x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.52

$$\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx = -\frac{1}{2}\sqrt{-x^2-2\,x}(x-3) + \frac{3}{2}\arcsin\left(x+1\right)$$

```
[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-x^2 - 2*x)*(x - 3) + 3/2*arcsin(x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx = \int \frac{x^2}{\sqrt{1-(x+1)^2}}\,dx$$

```
[In] int(x^2/(1 - (x + 1)^2)^(1/2),x)
[Out] int(x^2/(1 - (x + 1)^2)^(1/2), x)
```

3.101 $\int \frac{x^2}{\sqrt{1-(a+bx)^2}} \, dx$

Optimal result 760
Rubi [A] (verified) 760
Mathematica [B] (verified) 761
Maple [A] (verified) 762
Fricas [A] (verification not implemented) 762
Sympy [B] (verification not implemented) 763
Maxima [B] (verification not implemented) 763
Giac [A] (verification not implemented) 764
Mupad [F(-1)] 764

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}} \, dx = \frac{3a\sqrt{1-(a+bx)^2}}{2b^3} - \frac{x\sqrt{1-(a+bx)^2}}{2b^2} + \frac{(1+2a^2)\arcsin(a+bx)}{2b^3}$$

```
[Out] 1/2*(2*a^2+1)*arcsin(b*x+a)/b^3+3/2*a*(1-(b*x+a)^2)^(1/2)/b^3-1/2*x*(1-(b*x
+a)^2)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {378, 757, 655, 222}

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}} \, dx = \frac{(2a^2+1)\arcsin(a+bx)}{2b^3} + \frac{3a\sqrt{1-(a+bx)^2}}{2b^3} - \frac{x\sqrt{1-(a+bx)^2}}{2b^2}$$

```
[In] Int[x^2/Sqrt[1 - (a + b*x)^2],x]
```

```
[Out] (3*a*Sqrt[1 - (a + b*x)^2])/(2*b^3) - (x*Sqrt[1 - (a + b*x)^2])/(2*b^2) + (
(1 + 2*a^2)*ArcSin[a + b*x])/(2*b^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{b^3} \\
&= -\frac{x\sqrt{1-(a+bx)^2}}{2b^2} - \frac{\text{Subst}\left(\int \frac{-1-2a^2+3ax}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{2b^3} \\
&= \frac{3a\sqrt{1-(a+bx)^2}}{2b^3} - \frac{x\sqrt{1-(a+bx)^2}}{2b^2} + \frac{(1+2a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{2b^3} \\
&= \frac{3a\sqrt{1-(a+bx)^2}}{2b^3} - \frac{x\sqrt{1-(a+bx)^2}}{2b^2} + \frac{(1+2a^2)\sin^{-1}(a+bx)}{2b^3}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 163 vs. 2(67) = 134.

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.43

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx$$
$$= \frac{-2b(-3a+bx)\sqrt{1-a^2-2abx-b^2x^2} - 2(1+2a^2)\,b\arctan\left(\frac{-\sqrt{-b^2}x+\sqrt{1-a^2-2abx-b^2x^2}}{a}\right) + (1+2a^2)\sqrt{-b^2}}{4b^4}$$

```
[In] Integrate[x^2/Sqrt[1 - (a + b*x)^2],x]
```

```
[Out] (-2*b*(-3*a + b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] - 2*(1 + 2*a^2)*b*ArcT
an[(-(Sqrt[-b^2]*x) + Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2])/a] + (1 + 2*a^2)*S
qrt[-b^2]*Log[-1 + 2*a*b*x + 2*b^2*x^2 + 2*Sqrt[-b^2]*x*Sqrt[1 - a^2 - 2*a*
b*x - b^2*x^2]])/(4*b^4)
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{(-bx+3a)(b^2x^2+2abx+a^2-1)}{2b^3\sqrt{-b^2x^2-2abx-a^2+1}}+\frac{(2a^2+1)\arctan\left(\frac{\sqrt{b^2}\left(\frac{a}{b}+x\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{2b^2\sqrt{b^2}}$
default	$-\frac{x\sqrt{-b^2x^2-2abx-a^2+1}}{2b^2}-\frac{3a\left(-\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{b^2}-\frac{a\arctan\left(\frac{\sqrt{b^2}\left(\frac{a}{b}+x\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{b\sqrt{b^2}}\right)}{2b}+\frac{(-a^2+1)\arctan\left(\frac{\sqrt{b^2}\left(\frac{a}{b}+x\right)}{\sqrt{-b^2x^2-2abx-a^2}}\right.}{2b^2\sqrt{b^2}}$

```
[In] int(x^2/(1-(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-b*x+3*a)*(b^2*x^2+2*a*b*x+a^2-1)/b^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+
1/2/b^2*(2*a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(a/b+x)/(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx = -\frac{(2\,a^2+1)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)+\sqrt{-b^2x^2-2\,abx-a^2+1}(bx-3\,a)}{2\,b^3}$$

```
[In] integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((2*a^2 + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*
x^2 + 2*a*b*x + a^2 - 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x - 3*a))
/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(54) = 108$.

Time = 0.75 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.45

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx = \begin{cases} \left(\frac{3a}{2b^3}-\frac{x}{2b^2}\right)\sqrt{-a^2-2abx-b^2x^2+1}+\frac{\left(\frac{3a^2}{2b^2}+\frac{1-a^2}{2b^2}\right)\log\left(-2ab-2b^2x+2\sqrt{-b^2}\sqrt{-a^2-2abx-b^2x^2+1}\right)}{\sqrt{-b^2}} & \text{for } b^2\neq 0 \\ -\frac{a^4\sqrt{-a^2-2abx+1}-2a^2\sqrt{-a^2-2abx+1}+\frac{(2a^2-2)(-a^2-2abx+1)^{\frac{3}{2}}}{3}+\frac{(-a^2-2abx+1)^{\frac{5}{2}}}{5}+\sqrt{-a^2-2abx+1}}{4a^3b^3} & \text{for } ab\neq 0 \\ \frac{x^3}{3\sqrt{1-a^2}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2/(1-(b*x+a)**2)**(1/2),x)
```

```
[Out] Piecewise(((3*a/(2*b**3) - x/(2*b**2))*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1
) + (3*a**2/(2*b**2) + (1 - a**2)/(2*b**2))*log(-2*a*b - 2*b**2*x + 2*sqrt(
-b**2)*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1))/sqrt(-b**2), Ne(b**2, 0)), (-
(a**4*sqrt(-a**2 - 2*a*b*x + 1) - 2*a**2*sqrt(-a**2 - 2*a*b*x + 1) + (2*a**
2 - 2)*(-a**2 - 2*a*b*x + 1)**(3/2)/3 + (-a**2 - 2*a*b*x + 1)**(5/2)/5 + sq
rt(-a**2 - 2*a*b*x + 1))/(4*a**3*b**3), Ne(a*b, 0)), (x**3/(3*sqrt(1 - a**2
)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(57) = 114$.

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx = -\frac{3\,a^2\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{2\,b^3}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}x}{2\,b^2}+\frac{(a^2-1)\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{2\,b^3}+\frac{3\,\sqrt{-b^2x^2-2\,abx-a^2+1}a}{2\,b^3}$$

```
[In] integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*a^2*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 - 1/2*sqr
t(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x/b^2 + 1/2*(a^2 - 1)*arcsin(-(b^2*x + a*b)
/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 + 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1
)*a/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx = -\frac{1}{2}\sqrt{-b^2x^2-2\,abx-a^2+1}\left(\frac{x}{b^2}-\frac{3\,a}{b^3}\right) - \frac{(2\,a^2+1)\arcsin\left(-bx-a\right)\operatorname{sgn}(b)}{2\,b^2|b|}$$

```
[In] integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 + 1)
*arcsin(-b*x - a)*sgn(b)/(b^2*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx = \int \frac{x^2}{\sqrt{1-(a+b\,x)^2}}\,dx$$

```
[In] int(x^2/(1 - (a + b*x)^2)^(1/2),x)
[Out] int(x^2/(1 - (a + b*x)^2)^(1/2), x)
```

3.102 $\int \frac{x^2}{\sqrt{1+(a+bx)^2}} \, dx$

Optimal result 765
Rubi [A] (verified) 765
Mathematica [A] (verified) 766
Maple [A] (verified) 767
Fricas [A] (verification not implemented) 767
Sympy [B] (verification not implemented) 767
Maxima [B] (verification not implemented) 768
Giac [A] (verification not implemented) 768
Mupad [F(-1)] 769

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}} \, dx = -\frac{3a\sqrt{1+(a+bx)^2}}{2b^3}+\frac{x\sqrt{1+(a+bx)^2}}{2b^2}-\frac{(1-2a^2)\,\text{arcsinh}(a+bx)}{2b^3}$$

```
[Out] -1/2*(-2*a^2+1)*arcsinh(b*x+a)/b^3-3/2*a*(1+(b*x+a)^2)^(1/2)/b^3+1/2*x*(1+(
b*x+a)^2)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {378, 757, 655, 221}

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}} \, dx = -\frac{(1-2a^2)\,\text{arcsinh}(a+bx)}{2b^3}-\frac{3a\sqrt{(a+bx)^2+1}}{2b^3}+\frac{x\sqrt{(a+bx)^2+1}}{2b^2}$$

```
[In] Int[x^2/Sqrt[1 + (a + b*x)^2],x]
```

```
[Out] (-3*a*Sqrt[1 + (a + b*x)^2])/(2*b^3) + (x*Sqrt[1 + (a + b*x)^2])/(2*b^2) -
((1 - 2*a^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b^3} \\
&= \frac{x\sqrt{1+(a+bx)^2}}{2b^2} + \frac{\text{Subst}\left(\int \frac{-1+2a^2-3ax}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b^3} \\
&= -\frac{3a\sqrt{1+(a+bx)^2}}{2b^3} + \frac{x\sqrt{1+(a+bx)^2}}{2b^2} - \frac{(1-2a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b^3} \\
&= -\frac{3a\sqrt{1+(a+bx)^2}}{2b^3} + \frac{x\sqrt{1+(a+bx)^2}}{2b^2} - \frac{(1-2a^2)\sinh^{-1}(a+bx)}{2b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = \frac{(-3a+bx)\sqrt{1+a^2+2abx+b^2x^2}}{2b^3} + \frac{(1-2a^2)\,\text{arctanh}\left(\frac{bx}{\sqrt{1+a^2}-\sqrt{1+a^2+2abx+b^2x^2}}\right)}{b^3}$$

```
[In] Integrate[x^2/Sqrt[1 + (a + b*x)^2],x]
[Out] ((-3*a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/(2*b^3) + ((1 - 2*a^2)*Arc
Tanh[(b*x)/(Sqrt[1 + a^2] - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])])/b^3
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

method	result
risch	$-\frac{(-bx+3a)\sqrt{b^2x^2+2abx+a^2+1}}{2b^3}+\frac{(2a^2-1)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2b^2\sqrt{b^2}}$
default	$\frac{x\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{3a\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^2}-\frac{a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{b\sqrt{b^2}}\right)}{2b}-\frac{(a^2+1)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2-\right.}}{2b^2\sqrt{b^2}}$

```
[In] int(x^2/(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-b*x+3*a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^3+1/2/b^2*(2*a^2-1)*ln((b^2
*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = -\frac{(2\,a^2-1)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\sqrt{b^2x^2+2\,abx+a^2+1}(bx-3\,a)}{2\,b^3}$$

```
[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((2*a^2 - 1)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(54) = 108$.

Time = 0.64 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.62

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = \begin{cases} \left(-\frac{3a}{2b^3}+\frac{x}{2b^2}\right)\sqrt{a^2+2abx+b^2x^2+1}+\frac{\left(\frac{3a^2}{2b^2}-\frac{a^2+1}{2b^2}\right)\log\left(2ab+2b^2x+2\sqrt{a^2+2abx+b^2x^2+1}\sqrt{b^2}\right)}{\sqrt{b^2}} & \text{for } b^2 \neq 0 \\ \frac{a^4\sqrt{a^2+2abx+1}+2a^2\sqrt{a^2+2abx+1}+\frac{(-2a^2-2)(a^2+2abx+1)^{\frac{3}{2}}}{3}+\frac{(a^2+2abx+1)^{\frac{5}{2}}}{5}+\sqrt{a^2+2abx+1}}{4a^3b^3} & \text{for } ab \neq 0 \\ \frac{x^3}{3\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2/(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] Piecewise(((-3*a/(2*b**3) + x/(2*b**2))*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1
) + (3*a**2/(2*b**2) - (a**2 + 1)/(2*b**2))*log(2*a*b + 2*b**2*x + 2*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1)*sqrt(b**2))/sqrt(b**2), Ne(b**2, 0)), ((a**4
*sqrt(a**2 + 2*a*b*x + 1) + 2*a**2*sqrt(a**2 + 2*a*b*x + 1) + (-2*a**2 - 2)
*(a**2 + 2*a*b*x + 1)**(3/2)/3 + (a**2 + 2*a*b*x + 1)**(5/2)/5 + sqrt(a**2
+ 2*a*b*x + 1))/(4*a**3*b**3), Ne(a*b, 0)), (x**3/(3*sqrt(a**2 + 1)), True)
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(53) = 106$.

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.14

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = \frac{3\,a^2\,\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^3} + \frac{\sqrt{b^2x^2+2\,abx+a^2+1}x}{2\,b^2} - \frac{(a^2+1)\,\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^3} - \frac{3\,\sqrt{b^2x^2+2\,abx+a^2+1}a}{2\,b^3}$$

```
[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 + 1
/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^2 - 1/2*(a^2 + 1)*arcsinh(2*(b^2*x
 + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 3/2*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)*a/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = \frac{1}{2}\,\sqrt{b^2x^2+2\,abx+a^2+1}\left(\frac{x}{b^2}-\frac{3\,a}{b^3}\right) - \frac{(2\,a^2-1)\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{2\,b^2|b|}$$

```
[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 - 1)*l
og(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^2*abs(b
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = \int \frac{x^2}{\sqrt{(a+b\,x)^2+1}}\,dx$$

```
[In] int(x^2/((a + b*x)^2 + 1)^(1/2),x)
[Out] int(x^2/((a + b*x)^2 + 1)^(1/2), x)
```

3.103 $\int \frac{x^3}{a+b(c+dx)^3}\,dx$

Optimal result 770
Rubi [A] (verified) 770
Mathematica [C] (verified) 774
Maple [C] (verified) 774
Fricas [C] (verification not implemented) 775
Sympy [A] (verification not implemented) 775
Maxima [F] 776
Giac [F] 776
Mupad [B] (verification not implemented) 776

Optimal result

Integrand size = 17, antiderivative size = 234

$$\begin{aligned}\int \frac{x^3}{a+b(c+dx)^3}\,dx &= \frac{x}{bd^3}+\frac{\left(a-3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}d^4}\\&\quad-\frac{\left(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{4/3}d^4}\\&\quad+\frac{\left(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{4/3}d^4}\\&\quad-\frac{c\log\left(a+b(c+dx)^3\right)}{bd^4}\end{aligned}$$

```
[Out] x/b/d^3-1/3*(a+3*a^(1/3)*b^(2/3)*c^2+b*c^3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(
2/3)/b^(4/3)/d^4+1/6*(a+3*a^(1/3)*b^(2/3)*c^2+b*c^3)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/b^(4/3)/d^4-c*ln(a+b*(d*x+c)^3)/b/d
^4+1/3*(a-3*a^(1/3)*b^(2/3)*c^2+b*c^3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c
))/a^(1/3)*3^(1/2))/a^(2/3)/b^(4/3)/d^4*3^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules

used = {378, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx = \frac{\left(-3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}d^4} - \frac{\left(3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{4/3}d^4} + \frac{\left(3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{4/3}d^4} - \frac{c\log\left(a+b(c+dx)^3\right)}{bd^4} + \frac{x}{bd^3}$$

```
[In] Int[x^3/(a + b*(c + d*x)^3),x]
```

```
[Out] x/(b*d^3) + ((a - 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*ArcTan[(a^(1/3) - 2*b^(1/3
)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(4/3)*d^4) - ((a + 3*a^
(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(
4/3)*d^4) + ((a + 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(4/3)*d^4) - (c*Log[a +
b*(c + d*x)^3])/(b*d^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+bx^3}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b}-\frac{a+bc^3-3bc^2x+3bcx^2}{b(a+bx^3)}\right)\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{x}{bd^3} - \frac{\text{Subst}\left(\int \frac{a+bc^3-3bc^2x+3bcx^2}{a+bx^3}\,dx, x, c+dx\right)}{bd^4}$$

$$= \frac{x}{bd^3} - \frac{\text{Subst}\left(\int \frac{a+bc^3-3bc^2x}{a+bx^3}\,dx, x, c+dx\right)}{bd^4} - \frac{(3c)\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{x}{bd^3} - \frac{c\log\left(a+b(c+dx)^3\right)}{bd^4} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(-3\sqrt[3]{a}bc^2+2\sqrt[3]{b}(a+bc^3)\right)+\sqrt[3]{b}\left(-3\sqrt[3]{a}bc^2-\sqrt[3]{b}(a+bc^3)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}b^{4/3}d^4} - \frac{\left(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, c+dx\right)}{3a^{2/3}bd^4}$$

$$= \frac{x}{bd^3} - \frac{\left(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{4/3}d^4} - \frac{c\log\left(a+b(c+dx)^3\right)}{bd^4} - \frac{\left(a-3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{2\sqrt[3]{a}bd^4} + \frac{\left(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{6a^{2/3}b^{4/3}d^4}$$

$$= \frac{x}{bd^3} - \frac{\left(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{4/3}d^4} + \frac{\left(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{4/3}d^4} - \frac{c\log\left(a+b(c+dx)^3\right)}{bd^4} - \frac{\left(a-3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{a^{2/3}b^{4/3}d^4}$$

$$= \frac{x}{bd^3} + \frac{(a - 3\sqrt[3]{a}b^{2/3}c^2 + bc^3)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}b^{4/3}d^4} - \frac{(a + 3\sqrt[3]{a}b^{2/3}c^2 + bc^3)\log\left(\sqrt[3]{a} + \sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{4/3}d^4} + \frac{(a + 3\sqrt[3]{a}b^{2/3}c^2 + bc^3)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{4/3}d^4} - \frac{c\log(a + b(c+dx)^3)}{bd^4}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.56

$$\int \frac{x^3}{a + b(c+dx)^3}\,dx = -\frac{-3bdx + \text{RootSum}\left[a + bc^3 + 3bc^2d\#1 + 3bcd^2\#1^2 + bd^3\#1^3\&, \frac{a\log(x-\#1) + bc^3\log(x-\#1) + 3bc^2d\log(x-\#1)\#}{c^2+2cd\#1+d^2\#1^2}\right.}{3b^2d^4}$$

```
[In] Integrate[x^3/(a + b*(c + d*x)^3),x]
[Out] -1/3*(-3*b*d*x + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*
#1^3 & , (a*Log[x - #1] + b*c^3*Log[x - #1] + 3*b*c^2*d*Log[x - #1]*#1 + 3*
b*c*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(b^2*d^4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.46

method	result	size
default	$\frac{x}{b\,d^3} + \frac{\sum_{\_R=\text{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)} \frac{\left(-3\_R^2bc\,d^2-3\_Rb\,c^2d-b\,c^3-a\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3b^2d^4}$	108
risch	$\frac{x}{b\,d^3} + \frac{\sum_{\_R=\text{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)} \frac{\left(-3\_R^2bc\,d^2-3\_Rb\,c^2d-b\,c^3-a\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3b^2d^4}$	108

```
[In] int(x^3/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] x/b/d^3+1/3/b^2/d^4*sum((-3*_R^2*b*c*d^2-3*_R*b*c^2*d-b*c^3-a)/(_R^2*d^2+2*
_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3
+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.97 (sec) , antiderivative size = 6315, normalized size of antiderivative = 26.99

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx$$
$$= \operatorname{RootSum}\left(27t^3a^2b^4d^{12} + 81t^2a^2b^3cd^8 + t\left(54a^2b^2c^2d^4 - 27ab^3c^5d^4\right) + a^3 + 3a^2bc^3 + 3ab^2c^6 + b^3c^9, \left(t \mapsto t\right.\right.$$
$$+ \frac{x}{bd^3}$$

```
[In] integrate(x**3/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**4*d**12 + 81*_t**2*a**2*b**3*c*d**8 + _t*(54*a**2*
b**2*c**2*d**4 - 27*a*b**3*c**5*d**4) + a**3 + 3*a**2*b*c**3 + 3*a*b**2*c**
6 + b**3*c**9, Lambda(_t, _t*log(x + (-27*_t**2*a**2*b**3*c**2*d**8 - 3*_t*
a**3*b*d**4 - 60*_t*a**2*b**2*c**3*d**4 - 3*_t*a*b**3*c**6*d**4 - 2*a**3*c
- 12*a**2*b*c**4 - 9*a*b**2*c**7 + b**3*c**10)/(a**3*d + 3*a**2*b*c**3*d -
24*a*b**2*c**6*d + b**3*c**9*d)))) + x/(b*d**3)
```

Maxima [F]

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx=\int \frac{x^3}{(dx+c)^3b+a}\,dx$$

```
[In] integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] x/(b*d^3) - integrate((3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b*d^3*x^3
+ 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(b*d^3)
```

Giac [F]

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx=\int \frac{x^3}{(dx+c)^3b+a}\,dx$$

```
[In] integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(x^3/((d*x + c)^3*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.60

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx=\left(\sum_{k=1}^{3}\ln\left(\frac{3\left(b\,c^5+a\,c^2\right)}{d^2}-\mathrm{root}\left(27\,a^2\,b^4\,d^{12}\,z^3+81\,a^2\,b^3\,c\,d^8\,z^2+54\,a^2\,b^2\,c^2\,d^4\,z-27\,a\,b^3\,c^5\,d^4\,z+3\,a\,b^2\,c^6+3\,a^2\,b\,c^3+b^3\,c^9+a^3,z,k\right)\left(\right.\right.\right.$$
$$\left.-\frac{3\,x\left(a\,c-2\,b\,c^4\right)}{d}\right)\mathrm{root}\left(27\,a^2\,b^4\,d^{12}\,z^3+81\,a^2\,b^3\,c\,d^8\,z^2+54\,a^2\,b^2\,c^2\,d^4\,z-27\,a\,b^3\,c^5\,d^4\,z\right.$$
$$\left.\left.+3\,a\,b^2\,c^6+3\,a^2\,b\,c^3+b^3\,c^9+a^3,z,k\right)\right)+\frac{x}{b\,d^3}$$

```
[In] int(x^3/(a + b*(c + d*x)^3),x)
```

```
[Out] symsum(log((3*(a*c^2 + b*c^5))/d^2 - root(27*a^2*b^4*d^12*z^3 + 81*a^2*b^3*
c*d^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4*z + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + b^3*c^9 + a^3, z, k)*((3*(b^2*c^4*d^4 - 5*a*b*c*d^4))/d^2 + (3*x*(
b^2*c^3*d^4 + a*b*d^4))/d - 9*root(27*a^2*b^4*d^12*z^3 + 81*a^2*b^3*c*d^8*z
^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4*z + 3*a*b^2*c^6 + 3*a^2*b*c^3
+ b^3*c^9 + a^3, z, k)*a*b^2*d^6) - (3*x*(a*c - 2*b*c^4))/d)*root(27*a^2*b^
4*d^12*z^3 + 81*a^2*b^3*c*d^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4
*z + 3*a*b^2*c^6 + 3*a^2*b*c^3 + b^3*c^9 + a^3, z, k), k, 1, 3) + x/(b*d^3)
```

3.104 $\int \frac{x^2}{a+b(c+dx)^3} \, dx$

Optimal result . . . 777
Rubi [A] (verified) . . . 777
Mathematica [C] (verified) . . . 781
Maple [C] (verified) . . . 781
Fricas [C] (verification not implemented) . . . 782
Sympy [A] (verification not implemented) . . . 784
Maxima [F] . . . 785
Giac [F] . . . 785
Mupad [B] (verification not implemented) . . . 785

Optimal result

Integrand size = 17, antiderivative size = 210

$$\int \frac{x^2}{a+b(c+dx)^3} \, dx = \frac{c\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^3} + \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^3} - \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^3} + \frac{\log\left(a+b(c+dx)^3\right)}{3bd^3}$$

```
[Out] 1/3*c*(2*a^(1/3)+b^(1/3)*c)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/b^(2/3)/d^3
-1/6*c*(2*a^(1/3)+b^(1/3)*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*
x+c)^2)/a^(2/3)/b^(2/3)/d^3+1/3*ln(a+b*(d*x+c)^3)/b/d^3+1/3*c*(2*a^(1/3)-b^
(1/3)*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/b^
(2/3)/d^3*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used

$= \{378, 1885, 1874, 31, 648, 631, 210, 642, 266\}$

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \frac{c\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^3} + \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^3} - \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^3} + \frac{\log\left(a+b(c+dx)^3\right)}{3bd^3}$$

```
[In] Int[x^2/(a + b*(c + d*x)^3),x]
```

```
[Out] (c*(2*a^(1/3) - b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)*d^3) + (c*(2*a^(1/3) + b^(1/3)*c)*Log[a
^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(2/3)*d^3) - (c*(2*a^(1/3) + b^(1
/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a
^(2/3)*b^(2/3)*d^3) + Log[a + b*(c + d*x)^3]/(3*b*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+bx^3}\,dx, x, c+dx\right)}{d^3}\\ &= \frac{\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{d^3}+\frac{\text{Subst}\left(\int \frac{c^2-2cx}{a+bx^3}\,dx, x, c+dx\right)}{d^3}\end{aligned}$$

$$= \frac{\log\left(a+b(c+dx)^3\right)}{3bd^3} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(-2\sqrt[3]{a}c+2\sqrt[3]{b}c^2\right)+\sqrt[3]{b}\left(-2\sqrt[3]{a}c-\sqrt[3]{b}c^2\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}\sqrt[3]{b}d^3} + -\frac{\left(-2\sqrt[3]{a}c-\sqrt[3]{b}c^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, c+dx\right)}{3a^{2/3}\sqrt[3]{b}d^3}$$

$$= \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^3} + \frac{\log\left(a+b(c+dx)^3\right)}{3bd^3} - \frac{\left(c\left(\frac{2}{\sqrt[3]{b}}-\frac{c}{\sqrt[3]{a}}\right)\right)\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{2d^3} - \frac{\left(c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\right)\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{6a^{2/3}b^{2/3}d^3}$$

$$= \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^3} - \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^3} + \frac{\log\left(a+b(c+dx)^3\right)}{3bd^3} - \frac{\left(c\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{a^{2/3}b^{2/3}d^3}$$

$$= \frac{c\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^3} + \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^3} - \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^3} + \frac{\log\left(a+b(c+dx)^3\right)}{3bd^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \frac{\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&,\frac{\log(x-\#1)\#1^2}{c^2+2cd\#1+d^2\#1^2}\&\right]}{3bd}$$

```
[In] Integrate[x^2/(a + b*(c + d*x)^3),x]
[Out] RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (Log[x -
 #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ]/(3*b*d)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{\sum_{\_R=\text{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	74
risch	$\frac{\sum_{\_R=\text{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	74

```
[In] int(x^2/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
[Out] 1/3/b/d*sum(_R^2/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z
^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 4759, normalized size of antiderivative = 22.66

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
[Out] -1/12*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^
2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/
(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*
(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d
^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b
*d^3))*b*d^3*log(-1/2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2
*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*
b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) +
 (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3
- a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))
^(1/3) - 2/(b*d^3))^2*a^2*b^2*d^6 + b^2*c^6 - a*b*c^3 - 1/2*(a*b^2*c^3 + 4*
a^2*b)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*
d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b
^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9
) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d
^3))*d^3 + (b^2*c^5 - 8*a*b*c^2)*d*x - 2*a^2) - ((2*(1/2)^(2/3)*(-I*sqrt(3)
 + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2
*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 +
a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/
(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*
b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*b*d^3 - 3*sqrt(1/3)*b*d^3*sq
rt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^
6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3
*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*s
qrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9)
 + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3
))^2*a*b^2*d^6 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^
6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3
*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1
/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a
)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1
/3) - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6)) + 6)*log(1/2*(2*(1/
2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3
 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^
```

2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)
*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^
9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))^2*a^2*b^
2*d^6 + 2*b^2*c^6 - 23*a*b*c^3 + 1/2*(a*b^2*c^3 + 4*a^2*b)*(2*(1/2)^(2/3)*(
-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^
3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*
a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 -
 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*
c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*d^3 + 2*(b^2*c^5 -
 8*a*b*c^2)*d*x + 2*a^2 + 3/2*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((
2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) +
3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^
2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2
d^9) + 3(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 +
a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*a^2*b^2*d^6 - (a*b^2*c^3 - 2*a^2*b)*
d^3)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/
(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) +
 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^
3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2
/(b*d^3))^2*a*b^2*d^6 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a
*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)
/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/
3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*
c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d
^9))^(1/3) - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6))) - ((2*(1/2)
^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 -
 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*
c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(
(b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9)
 + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*b*d^3 + 3*
sqrt(1/3)*b*d^3*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^
2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a
*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3)
+ (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3
 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9)
)^(1/3) - 2/(b*d^3))^2*a*b^2*d^6 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*
c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2
*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^
3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9
) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)
/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6)) +
 6)*log(1/2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/
(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) +
 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/

```
3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^
3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2
/(b*d^3))^2*a^2*b^2*d^6 + 2*b^2*c^6 - 23*a*b*c^3 + 1/2*(a*b^2*c^3 + 4*a^2*b
)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))
/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^
9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt
(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2
/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*
d^3 + 2*(b^2*c^5 - 8*a*b*c^2)*d*x + 2*a^2 - 3/2*sqrt(1/3)*((2*(1/2)^(2/3)*(
-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^
3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*
a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 -
 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*
c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*a^2*b^2*d^6 - (a*b
^2*c^3 - 2*a^2*b)*d^3)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a
)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3
- a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(
2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b
^3*d^9))^(1/3) - 2/(b*d^3))^2*a*b^2*d^6 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)
*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9)
 + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/
(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*
b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3
 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*
d^6))))/(b*d^3)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx$$
$$= \operatorname{RootSum}\left(27t^3a^2b^3d^9 - 27t^2a^2b^2d^6 + t\left(9a^2bd^3 - 18ab^2c^3d^3\right) - a^2 - 2abc^3 - b^2c^6, \left(t \mapsto t\log\left(x + \frac{18t^2a^2b\cdots}{}\right.\right.\right.$$

```
[In] integrate(x**2/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**3*d**9 - 27*_t**2*a**2*b**2*d**6 + _t*(9*a**2*b*d*
*3 - 18*a*b**2*c**3*d**3) - a**2 - 2*a*b*c**3 - b**2*c**6, Lambda(_t, _t*lo
g(x + (18*_t**2*a**2*b**2*d**6 - 12*_t*a**2*b*d**3 - 3*_t*a*b**2*c**3*d**3
+ 2*a**2 + a*b*c**3 - b**2*c**6)/(8*a*b*c**2*d - b**2*c**5*d))))
```

Maxima [F]

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \int \frac{x^2}{(dx+c)^3 b+a}\,dx$$

```
[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
[Out] integrate(x^2/((d*x + c)^3*b + a), x)
```

Giac [F]

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \int \frac{x^2}{(dx+c)^3 b+a}\,dx$$

```
[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="giac")
[Out] integrate(x^2/((d*x + c)^3*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 9.86 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.08

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \sum_{k=1}^{3} \ln\Big(a + b\,c^3 - \mathrm{root}\big(27\,a^2\,b^3\,d^9\,z^3 - 27\,a^2\,b^2\,d^6\,z^2 - 18\,a\,b^2\,c^3\,d^3\,z + 9\,a^2\,b\,d^3\,z - 2\,a\,b\,c^3 - b^2\,c^6 - a^2, z, k\big)\;a\,b\,d^3\,6 + 3\,b\,c^2\,d\,x + \mathrm{root}\big(27\,a^2\,b^3\,d^9\,z^3 - 27\,a^2\,b^2\,d^6\,z^2 - 18\,a\,b^2\,c^3\,d^3\,z + 9\,a^2\,b\,d^3\,z - 2\,a\,b\,c^3 - b^2\,c^6 - a^2, z, k\big)^2\;a\,b^2\,d^6\,9 + \mathrm{root}\big(27\,a^2\,b^3\,d^9\,z^3 - 27\,a^2\,b^2\,d^6\,z^2 - 18\,a\,b^2\,c^3\,d^3\,z + 9\,a^2\,b\,d^3\,z - 2\,a\,b\,c^3 - b^2\,c^6 - a^2, z, k\big)\;b^2\,c^3\,d^3\,3 + \mathrm{root}\big(27\,a^2\,b^3\,d^9\,z^3 - 27\,a^2\,b^2\,d^6\,z^2 - 18\,a\,b^2\,c^3\,d^3\,z + 9\,a^2\,b\,d^3\,z - 2\,a\,b\,c^3 - b^2\,c^6 - a^2, z, k\big)\;b^2\,c^2\,d^4\,x\,3\Big)\;\mathrm{root}\big(27\,a^2\,b^3\,d^9\,z^3 - 27\,a^2\,b^2\,d^6\,z^2 - 18\,a\,b^2\,c^3\,d^3\,z + 9\,a^2\,b\,d^3\,z - 2\,a\,b\,c^3 - b^2\,c^6 - a^2, z, k\big)$$

```
[In] int(x^2/(a + b*(c + d*x)^3),x)
[Out] symsum(log(a + b*c^3 - 6*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*
a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)*a*b*d^3
+ 3*b*c^2*d*x + 9*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c
^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)^2*a*b^2*d^6 + 3
*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*
b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)*b^2*c^3*d^3 + 3*root(27*a^2*b^3*
d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c
^3 - b^2*c^6 - a^2, z, k)*b^2*c^2*d^4*x)*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b
^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2
, z, k), k, 1, 3)
```

3.105 $\int \frac{x}{a+b(c+dx)^3}\,dx$

Optimal result 786
Rubi [A] (verified) 786
Mathematica [C] (verified) 789
Maple [C] (verified) 789
Fricas [C] (verification not implemented) 790
Sympy [A] (verification not implemented) 791
Maxima [F] 791
Giac [F] 792
Mupad [B] (verification not implemented) 792

Optimal result

Integrand size = 15, antiderivative size = 180

$$\int \frac{x}{a+b(c+dx)^3}\,dx = -\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^2} - \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2} + \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^2}$$

```
[Out] -1/3*(a^(1/3)+b^(1/3)*c)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/b^(2/3)/d^2+1/
6*(a^(1/3)+b^(1/3)*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)
/a^(2/3)/b^(2/3)/d^2-1/3*(a^(1/3)-b^(1/3)*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*
(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/b^(2/3)/d^2*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {378, 1874, 31, 648, 631, 210, 642}

$$\int \frac{x}{a+b(c+dx)^3}\,dx = -\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^2} - \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2} + \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^2}$$

```
[In] Int[x/(a + b*(c + d*x)^3),x]
```

```
[Out] -(((a^(1/3) - b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)*d^2)) - ((a^(1/3) + b^(1/3)*c)*Log[a^(1/3
) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(2/3)*d^2) + ((a^(1/3) + b^(1/3)*c)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^
(2/3)*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-c+x}{a+bx^3}\,dx, x, c+dx\right)}{d^2}\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(\sqrt[3]{a}-2\sqrt[3]{b}c\right)+\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}\sqrt[3]{b}d^2}\\
&\quad -\frac{\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, c+dx\right)}{3a^{2/3}d^2}\\
&= -\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2}\\
&\quad +\frac{\left(\frac{1}{\sqrt[3]{b}}-\frac{c}{\sqrt[3]{a}}\right)\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{2d^2}\\
&\quad +\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{6a^{2/3}b^{2/3}d^2}\\
&= -\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2}\\
&\quad +\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^2}\\
&\quad +\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{a^{2/3}b^{2/3}d^2}\\
&= -\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^2}-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2}\\
&\quad +\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.44

$$\int \frac{x}{a+b(c+dx)^3}\,dx = \frac{\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&,\frac{\log(x-\#1)\#1}{c^2+2cd\#1+d^2\#1^2}\&\right]}{3bd}$$

```
[In] Integrate[x/(a + b*(c + d*x)^3),x]
```

```
[Out] RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (Log[x -
 #1]*#1)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ]/(3*b*d)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.40

method	result	size
default	$\frac{\sum_{\_R=\text{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\_R\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	72
risch	$\frac{\sum_{\_R=\text{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\_R\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	72

```
[In] int(x/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b/d*sum(_R/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2
*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 1950, normalized size of antiderivative = 10.83

$$\int \frac{x}{a+b(c+dx)^3} \, dx = \text{Too large to display}$$

[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="fricas")

[Out] 1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) - 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4)) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*log(1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 - 1/6*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + 2*b*c^4 + 2*(b*c^3 - a)*d*x - 4*a*c + 1/12*sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a^2*b*d^4 + 6*a*b*c^2*d^2)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4))) + 1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4)) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*log(1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 - 1/6*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + 2*b*c^4 + 2*(b*c^3 - a)*d*x - 4*a*c - 1/12*sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a^2*b*d^4 + 6*a*b*c^2*d^2)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1

```
/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(
b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a*b*d^
4 - 144*c)/(a*b*d^4))) - 1/18*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^
2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^
4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3))
)*log(-1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c
^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 +
a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 + 1/
6*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a
^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^
2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + b*c^4 + (b*c
^3 - a)*d*x + a*c)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.46

$$\int \frac{x}{a+b(c+dx)^3}\,dx$$
$$=\text{RootSum}\left(27t^3a^2b^2d^6-9tabcd^2+a+bc^3,\left(t\mapsto t\log\left(x+\frac{9t^2a^2bd^4+3tabc^2d^2-ac-bc^4}{ad-bc^3d}\right)\right)\right)$$

```
[In] integrate(x/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2*d**6 - 9*_t*a*b*c*d**2 + a + b*c**3, Lambda(_t,
_t*log(x + (9*_t**2*a**2*b*d**4 + 3*_t*a*b*c**2*d**2 - a*c - b*c**4)/(a*d -
 b*c**3*d))))
```

Maxima [F]

$$\int \frac{x}{a+b(c+dx)^3}\,dx=\int \frac{x}{(dx+c)^3b+a}\,dx$$

```
[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(x/((d*x + c)^3*b + a), x)
```

Giac [F]

$$\int \frac{x}{a+b(c+dx)^3}\,dx=\int \frac{x}{(dx+c)^3b+a}\,dx$$

[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="giac")

[Out] integrate(x/((d*x + c)^3*b + a), x)

Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.81

$$\int \frac{x}{a+b(c+dx)^3}\,dx=\sum_{k=1}^{3}\ln\left(-\mathrm{root}\left(27\,a^2\,b^2\,d^6\,z^3-9\,a\,b\,c\,d^2\,z+b\,c^3+a,z,k\right)\,\left(3\,b^2\,c^2\,d^4-\mathrm{root}\left(27\,a^2\,b^2\,d^6\,z^3-9\,a\,b\,c\,d^2\,z+b\,c^3+a,z,k\right)\,a\,b^2\,d^6\,9+3\,b^2\,c\,d^5\,x\right)+b\,d^3\,x\right)\,\mathrm{root}\left(27\,a^2\,b^2\,d^6\,z^3-9\,a\,b\,c\,d^2\,z+b\,c^3+a,z,k\right)$$

[In] int(x/(a + b*(c + d*x)^3),x)

[Out] symsum(log(b*d^3*x - root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 + a, z, k)*(3*b^2*c^2*d^4 - 9*root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 + a, z, k)*a*b^2*d^6 + 3*b^2*c*d^5*x))*root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 + a, z, k), k, 1, 3)

3.106 $\int \frac{1}{a+b(c+dx)^3}\,dx$

Optimal result . . . 793
Rubi [A] (verified) . . . 793
Mathematica [A] (verified) . . . 795
Maple [C] (verified) . . . 796
Fricas [A] (verification not implemented) . . . 796
Sympy [A] (verification not implemented) . . . 797
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Mupad [B] (verification not implemented) . . . 798

Optimal result

Integrand size = 13, antiderivative size = 140

$$\int \frac{1}{a+b(c+dx)^3}\,dx=-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d}$$

```
[Out] 1/3*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/b^(1/3)/d-1/6*ln(a^(2/3)-a^(1/3)*b^
(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/b^(1/3)/d-1/3*arctan(1/3*(a^(1/3)-
2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/b^(1/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {253, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{a+b(c+dx)^3}\,dx=-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d}$$

```
[In] Int[(a + b*(c + d*x)^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*b^(1/3)*d)) + Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3)*d) - Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(6*a^(2/3)*b^(
1/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, c+dx\right)}{3a^{2/3}d} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}d}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{2\sqrt[3]{a}d} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{6a^{2/3}\sqrt[3]{b}d}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}d}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d}$$

```
[In] Integrate[(a + b*(c + d*x)^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] + 2*L
og[a^(1/3) + b^(1/3)*(c + d*x)] - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) +
 b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(1/3)*d)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	71
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	71

```
[In] int(1/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*
b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 442, normalized size of antiderivative = 3.16

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abd^3x^3+6\,abcd^2x^2+6\,abc^2dx+2\,abc^3-a^2+3\sqrt{\frac{1}{3}}\left(2\,abd^2x^2+4\,abcdx+2\,abc^2+(a^2b)^{\frac{2}{3}}(dx+c)-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\left(\cdots\right.}}{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}\right.}{\cdots}\right.$$

```
[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*d^3*x^3 + 6*a*b*c*d
^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a^2 + 3*sqrt(1/3)*(2*a*b*d^2*x^2 + 4*a
*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt(-(a^
2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*x + a*c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 +
```

```
3*b*c^2*d*x + b*c^3 + a)) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x +
a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(
a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2
*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)
*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x +
 a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log
(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d)]
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.19

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \frac{\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(x+\frac{3ta+c}{d}\right)\right)\right)}{d}$$

```
[In] integrate(1/(a+b*(d*x+c)**3),x)
[Out] RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(x + (3*_t*a + c)/d)))/d
```

Maxima [F]

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \int \frac{1}{(dx+c)^3b+a}\,dx$$

```
[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="maxima")
[Out] integrate(1/((d*x + c)^3*b + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{1}{a+b(c+dx)^3}\,dx &= \frac{1}{3}\sqrt{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+(ab^2)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}\,(ab^2)^{\frac{1}{3}}}\right)\\&\quad-\frac{1}{6}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}(ab^2)^{\frac{1}{3}}\right)^2\right.\\&\quad\left.+4\left(bdx+bc+(ab^2)^{\frac{1}{3}}\right)^2\right)+\frac{1}{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\log\left(\left|bdx+bc+(ab^2)^{\frac{1}{3}}\right|\right)\end{aligned}$$

```
[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3))/(sq
rt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - 1/6*(1/(a^2*b*d^3))^(
1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + 4*(b*d
*x + b*c + (a*b^2)^(1/3))^2) + 1/3*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x + b*
c + (a*b^2)^(1/3)))
```

Mupad [B] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.03

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \frac{\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{3\,a^{2/3}\,b^{1/3}\,d} + \frac{\ln\left(3\,b^2\,c\,d^5+3\,b^2\,d^6\,x+\frac{3\,a^{1/3}\,b^{5/3}\,d^5\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d} - \frac{\ln\left(3\,b^2\,c\,d^5+3\,b^2\,d^6\,x-\frac{3\,a^{1/3}\,b^{5/3}\,d^5\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d}$$

```
[In] int(1/(a + b*(c + d*x)^3),x)
```

```
[Out] log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x)/(3*a^(2/3)*b^(1/3)*d) + (log(3*b^2*c
*d^5 + 3*b^2*d^6*x + (3*a^(1/3)*b^(5/3)*d^5*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1
i - 1))/(6*a^(2/3)*b^(1/3)*d) - (log(3*b^2*c*d^5 + 3*b^2*d^6*x - (3*a^(1/3)
*b^(5/3)*d^5*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)*b^(1/3)*d)
```

3.107 $\int \frac{1}{x(a+b(c+dx)^3)}\,dx$

Optimal result . . . 799
Rubi [A] (verified) . . . 799
Mathematica [C] (verified) . . . 803
Maple [C] (verified) . . . 803
Fricas [C] (verification not implemented) . . . 804
Sympy [F(-1)] . . . 806
Maxima [F] . . . 807
Giac [F] . . . 807
Mupad [B] (verification not implemented) . . . 807

Optimal result

Integrand size = 17, antiderivative size = 224

$$\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx=\frac{\sqrt[3]{b}c\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}c+b^{2/3}c^2\right)}+\frac{\log(x)}{a+bc^3}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)}-\frac{\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}c+b^{2/3}c^2\right)}$$

```
[Out] ln(x)/(b*c^3+a)-1/3*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/(a^(1/3)+b^(1/3)*c)
-1/6*(2*a^(1/3)-b^(1/3)*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+
c)^2)/a^(2/3)/(a^(2/3)-a^(1/3)*b^(1/3)*c+b^(2/3)*c^2)+1/3*b^(1/3)*c*arctan(
1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(a^(2/3)-a^(1/3)*b
^(1/3)*c+b^(2/3)*c^2)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules

used = {378, 6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx = \frac{\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)} - \frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)} + \frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)} - \frac{\log\left(a+b(c+dx)^3\right)}{3\left(a+bc^3\right)} + \frac{\log(x)}{a+bc^3}$$

```
[In] Int[1/(x*(a + b*(c + d*x)^3)),x]
```

```
[Out] (b^(1/3)*c*(a^(1/3) + b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sq
rt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a + b*c^3)) + Log[x]/(a + b*c^3) + (b^(1
/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*(a
 + b*c^3)) - (b^(1/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*(a + b*c^3)) - Log[a + b*(c +
 d*x)^3]/(3*(a + b*c^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{(-c+x)\,(a+bx^3)}\,dx, x, c+dx\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{(a+bc^3)\,(c-x)}-\frac{b(c^2+cx+x^2)}{(a+bc^3)\,(a+bx^3)}\right)\,dx, x, c+dx\right)\end{aligned}$$

$$= \frac{\log(x)}{a+bc^3} - \frac{b\text{Subst}\left(\int \frac{c^2+cx+x^2}{a+bx^3}\,dx, x, c+dx\right)}{a+bc^3}$$

$$= \frac{\log(x)}{a+bc^3} - \frac{b\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{a+bc^3} - \frac{b\text{Subst}\left(\int \frac{c^2+cx}{a+bx^3}\,dx, x, c+dx\right)}{a+bc^3}$$

$$= \frac{\log(x)}{a+bc^3} - \frac{\log\left(a+b(c+dx)^3\right)}{3\left(a+bc^3\right)} - \frac{b^{2/3}\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(\sqrt[3]{a}c+2\sqrt[3]{b}c^2\right)+\sqrt[3]{b}\left(\sqrt[3]{a}c-\sqrt[3]{b}c^2\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}\left(a+bc^3\right)} + \frac{\left(b^{2/3}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, c+dx\right)}{3a^{2/3}\left(a+bc^3\right)}$$

$$= \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)} - \frac{\log\left(a+b(c+dx)^3\right)}{3\left(a+bc^3\right)} - \frac{\left(\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\right)\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{6a^{2/3}\left(a+bc^3\right)} - \frac{\left(b^{2/3}c\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\right)\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{2\sqrt[3]{a}\left(a+bc^3\right)}$$

$$= \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)} - \frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)} - \frac{\log\left(a+b(c+dx)^3\right)}{3\left(a+bc^3\right)} - \frac{\left(\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{a^{2/3}\left(a+bc^3\right)}$$

$$= \frac{\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)} + \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)} - \frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)} - \frac{\log\left(a+b(c+dx)^3\right)}{3\left(a+bc^3\right)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.53

$$\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx = -\frac{-3\log(x)+\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&,\frac{3c^2\log(x-\#1)+3cd\log(x-\#1)\#1+d^2\log(x}{c^2+2cd\#1+d^2\#1^2}\right.}{3\left(a+bc^3\right)}$$

```
[In] Integrate[1/(x*(a + b*(c + d*x)^3)),x]
```

```
[Out] -1/3*(-3*Log[x] + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3
*#1^3 & , (3*c^2*Log[x - #1] + 3*c*d*Log[x - #1]*#1 + d^2*Log[x - #1]*#1^2)
/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(a + b*c^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.47

method	result
default	$\frac{\ln(x)}{b\,c^3+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\left(d^2\_R^2+3cd\_R+3c^2\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3(b\,c^3+a)}$
risch	$\frac{\ln(-x)}{b\,c^3+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a^2b\,c^3+a^3\right)\_Z^3+3a^2\_Z^2+3a\_Z\right)}\_R\ln\left(\left(\left(2ab\,c^3d-4d\,a^2\right)\_R^2+\left(b\,c^3d-8da\right)\_R-4d\right)x+\left(ab\,c^4+c\,a^2\right.\right.\right.}{3}$

[In] int(1/x/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)

[Out] ln(x)/(b*c^3+a)-1/3*sum((_R^2*d^2+3*_R*c*d+3*c^2)/(_R^2*d^2+2*_R*c*d+c^2)*l
n(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/(b*c^3+a
)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 4370, normalized size of antiderivative = 19.51

$$\int \frac{1}{x\,(a+b(c+dx)^3)}\,dx = \text{Too large to display}$$

[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="fricas")

[Out] 1/12*(2*(b*c^3 + a)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/
(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c
^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) +
1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b
*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))*log(b*c^2*d*x + b*c^3
+ 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2)
- 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a
*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3
) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2
)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))^2 - 1/2*(a*b*c^3 -
 2*a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^
2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b
*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((
b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) -
 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + a) - ((b*c^3 + a)*(2*(1/2)^(2/3)
*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)
^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3
+ a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/
(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3
) - 2/(b*c^3 + a)) + 3*sqrt(1/3)*(b*c^3 + a)*sqrt(-(16*b*c^3 + (a*b^2*c^6 +

2*a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 4*a)/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) + 6)*log(2*b*c^2*d*x + 2*b*c^3 - 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))^2 + 1/2*(a*b*c^3 - 2*a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 3/4*sqrt(1/3)*(2*a*b*c^3 + (a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 2*a^2)*sqrt(-(16*b*c^3 + (a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 4*a)/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) - a) - ((b*c^3 + a)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) - 3*sqrt(1/3)*(b*c^3 + a)*sqrt(-(16*b*c^3 + (a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3

```
 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*(2*(
1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((
b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) -
 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2
*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 +
a)^3)^(1/3) - 2/(b*c^3 + a)) + 4*a)/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) + 6)*l
og(2*b*c^2*d*x + 2*b*c^3 - 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3)
 + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/
(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3
) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 +
 a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3
 + a))^2 + 1/2*(a*b*c^3 - 2*a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^
3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3
) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*
(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*
c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) - 3/4*sqr
t(1/3)*(2*a*b*c^3 + (a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a
*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3
+ a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(
1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/(
(a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 2*
a^2)*sqrt(-(16*b*c^3 + (a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*s
qrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2
) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3
)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b
*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/
(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*
c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a
^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3
)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*
b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 4*a)/
(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) - a) + 12*log(x))/(b*c^3 + a)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(a+b*(d*x+c)**3),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x\,(a+b(c+dx)^3)}\,dx = \int \frac{1}{\left((dx+c)^3b+a\right)x}\,dx$$

[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="maxima")

[Out] -b*d*integrate((d^2*x^2 + 3*c*d*x + 3*c^2)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(b*c^3 + a) + log(x)/(b*c^3 + a)

Giac [F]

$$\int \frac{1}{x\,(a+b(c+dx)^3)}\,dx = \int \frac{1}{\left((dx+c)^3b+a\right)x}\,dx$$

[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="giac")

[Out] integrate(1/(((d*x + c)^3*b + a)*x), x)

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 553, normalized size of antiderivative = 2.47

$$\begin{aligned}
&\int \frac{1}{x\,(a+b(c+dx)^3)}\,dx \\
&= \frac{\ln(x)}{b\,c^3+a} + \left(\sum_{k=1}^{3} \ln\left(\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^2 b^4\,c^4\,d^8\,3\right.\right. \\
&\quad - \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)\,b^3\,c\,d^8\,3 \\
&\quad - \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)\,b^3\,d^9\,x\,4 \\
&\quad - \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^2 a\,b^3\,c\,d^8\,6 \\
&\quad - \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^2 a\,b^3\,d^9\,x\,24 \\
&\quad + \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^3 a^2\,b^3\,c\,d^8\,9 \\
&\quad + \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^3 a\,b^4\,c^4\,d^8\,9 \\
&\quad - \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^3 a^2\,b^3\,d^9\,x\,36 \\
&\quad + \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^2 b^4\,c^3\,d^9\,x\,3 \\
&\quad \left. + \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^3 a\,b^4\,c^3\,d^9\,x\,18\right) \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3\right. \\
&\quad \left.\left. +27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)\right)
\end{aligned}$$

```
[In] int(1/(x*(a + b*(c + d*x)^3)),x)
```

```
[Out] log(x)/(a + b*c^3) + symsum(log(3*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a
^2*z^2 + 9*a*z + 1, z, k)^2*b^4*c^4*d^8 - 3*root(27*a^2*b*c^3*z^3 + 27*a^3*
z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)*b^3*c*d^8 - 4*root(27*a^2*b*c^3*z^3 + 2
7*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)*b^3*d^9*x - 6*root(27*a^2*b*c^3*z
^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*a*b^3*c*d^8 - 24*root(27*
a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*a*b^3*d^9*x +
9*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^3*a^2*
b^3*c*d^8 + 9*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1,
z, k)^3*a*b^4*c^4*d^8 - 36*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2
+ 9*a*z + 1, z, k)^3*a^2*b^3*d^9*x + 3*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 +
 27*a^2*z^2 + 9*a*z + 1, z, k)^2*b^4*c^3*d^9*x + 18*root(27*a^2*b*c^3*z^3 +
 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^3*a*b^4*c^3*d^9*x)*root(27*a^2*
b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k), k, 1, 3)
```

3.108 $\int \frac{1}{x^2(a+b(c+dx)^3)}\,dx$

Optimal result . . . 809
Rubi [A] (verified) . . . 810
Mathematica [C] (verified) . . . 814
Maple [C] (verified) . . . 814
Fricas [C] (verification not implemented) . . . 815
Sympy [F(-1)] . . . 815
Maxima [F] . . . 815
Giac [F] . . . 815
Mupad [B] (verification not implemented) . . . 816

Optimal result

Integrand size = 17, antiderivative size = 314

$$
\begin{aligned}
&\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx\\
&= -\frac{1}{(a+bc^3)\,x}+\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3 d\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)^2}\\
&\quad-\frac{3bc^2d\log(x)}{(a+bc^3)^2}+\frac{\sqrt[3]{b}\left(\sqrt[3]{a}(a-2bc^3)-\sqrt[3]{b}c(2a-bc^3)\right)d\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^2}\\
&\quad-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}(a-2bc^3)-\sqrt[3]{b}c(2a-bc^3)\right)d\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)^2}\\
&\quad+\frac{bc^2d\log\left(a+b(c+dx)^3\right)}{(a+bc^3)^2}
\end{aligned}
$$

```
[Out] -1/(b*c^3+a)/x-3*b*c^2*d*ln(x)/(b*c^3+a)^2+1/3*b^(1/3)*(a^(1/3)*(-2*b*c^3+a
)-b^(1/3)*c*(-b*c^3+2*a))*d*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/(b*c^3+a)^2
-1/6*b^(1/3)*(a^(1/3)*(-2*b*c^3+a)-b^(1/3)*c*(-b*c^3+2*a))*d*ln(a^(2/3)-a^(
1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/(b*c^3+a)^2+b*c^2*d*ln(a+b*
(d*x+c)^3)/(b*c^3+a)^2+1/3*b^(1/3)*(a^(1/3)-b^(1/3)*c)*(a^(1/3)+b^(1/3)*c)^
3*d*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(b*c^3+
a)^2*3^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {378, 6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{1}{x^2 (a+b(c+dx)^3)} \, dx = \frac{\sqrt[3]{b} d \left(\sqrt[3]{a}-\sqrt[3]{b} c\right) \left(\sqrt[3]{a}+\sqrt[3]{b} c\right)^3 \arctan\left(\frac{\sqrt[3]{a}-2 \sqrt[3]{b} (c+dx)}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3} \left(a+b c^3\right)^2} + \frac{b^{2/3} d \left(-\frac{\sqrt[3]{a} \left(a-2 b c^3\right)}{\sqrt[3]{b}}+2 a c-b c^4\right) \log \left(a^{2/3}-\sqrt[3]{a} \sqrt[3]{b} (c+dx)+b^{2/3} (c+dx)^2\right)}{6 a^{2/3} \left(a+b c^3\right)^2} + \frac{\sqrt[3]{b} d \left(\sqrt[3]{a} \left(a-2 b c^3\right)-\sqrt[3]{b} c \left(2 a-b c^3\right)\right) \log \left(\sqrt[3]{a}+\sqrt[3]{b} (c+dx)\right)}{3 a^{2/3} \left(a+b c^3\right)^2} - \frac{1}{x \left(a+b c^3\right)} - \frac{3 b c^2 d \log (x)}{\left(a+b c^3\right)^2} + \frac{b c^2 d \log \left(a+b (c+dx)^3\right)}{\left(a+b c^3\right)^2}$$

```
[In] Int[1/(x^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] -(1/((a + b*c^3)*x)) + (b^(1/3)*(a^(1/3) - b^(1/3)*c)*(a^(1/3) + b^(1/3)*c)
^3*d*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^
(2/3)*(a + b*c^3)^2) - (3*b*c^2*d*Log[x])/(a + b*c^3)^2 + (b^(1/3)*(a^(1/3)
*(a - 2*b*c^3) - b^(1/3)*c*(2*a - b*c^3))*d*Log[a^(1/3) + b^(1/3)*(c + d*x)
])/(3*a^(2/3)*(a + b*c^3)^2) + (b^(2/3)*(2*a*c - b*c^4 - (a^(1/3)*(a - 2*b*
c^3))/b^(1/3))*d*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x
)^2])/(6*a^(2/3)*(a + b*c^3)^2) + (b*c^2*d*Log[a + b*(c + d*x)^3])/(a + b*c
^3)^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

[n, 0]

Rubi steps

$$
\begin{aligned}
\text{integral} &= d\text{Subst}\left(\int \frac{1}{(-c+x)^2(a+bx^3)}\,dx, x, c+dx\right) \\
&= d\text{Subst}\left(\int \left(\frac{1}{(a+bc^3)(c-x)^2} + \frac{3bc^2}{(a+bc^3)^2(c-x)} + \frac{b(-c(2a-bc^3)-(a-2bc^3)x+3bc^2x^2)}{(a+bc^3)^2(a+bx^3)}\right) dx, x, c+dx\right) \\
&= -\frac{1}{(a+bc^3)x} - \frac{3bc^2 d\log(x)}{(a+bc^3)^2} + \frac{(bd)\text{Subst}\left(\int \frac{-c(2a-bc^3)-(a-2bc^3)x+3bc^2x^2}{a+bx^3}\,dx, x, c+dx\right)}{(a+bc^3)^2} \\
&= -\frac{1}{(a+bc^3)x} - \frac{3bc^2 d\log(x)}{(a+bc^3)^2} + \frac{(bd)\text{Subst}\left(\int \frac{-c(2a-bc^3)+(-a+2bc^3)x}{a+bx^3}\,dx, x, c+dx\right)}{(a+bc^3)^2} \\
&\quad + \frac{(3b^2c^2d)\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{(a+bc^3)^2} \\
&= -\frac{1}{(a+bc^3)x} - \frac{3bc^2 d\log(x)}{(a+bc^3)^2} + \frac{bc^2 d\log\left(a+b(c+dx)^3\right)}{(a+bc^3)^2} \\
&\quad + \frac{(b^{2/3}d)\,\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(-2\sqrt[3]{b}c(2a-bc^3)+\sqrt[3]{a}(-a+2bc^3)\right)+\sqrt[3]{b}\left(\sqrt[3]{b}c(2a-bc^3)+\sqrt[3]{a}(-a+2bc^3)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}(a+bc^3)^2} \\
&\quad - \frac{\left(b\left(2ac-bc^4-\frac{\sqrt[3]{a}(a-2bc^3)}{\sqrt[3]{b}}\right)d\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, c+dx\right)}{3a^{2/3}(a+bc^3)^2}
\end{aligned}
$$

$$= -\frac{1}{(a+bc^3)\,x} - \frac{3bc^2 d\log(x)}{(a+bc^3)^2} - \frac{b^{2/3}\left(2ac - bc^4 - \frac{\sqrt[3]{a}(a-2bc^3)}{\sqrt[3]{b}}\right) d\log\left(\sqrt[3]{a} + \sqrt[3]{b}(c+dx)\right)}{3a^{2/3}(a+bc^3)^2} + \frac{bc^2 d\log\left(a+b(c+dx)^3\right)}{(a+bc^3)^2} - \frac{\left(b^{2/3}\left(\sqrt[3]{a} - \sqrt[3]{b}c\right)\left(\sqrt[3]{a} + \sqrt[3]{b}c\right)^3 d\right)\text{Subst}\left(\int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx, x, c+dx\right)}{2\sqrt[3]{a}(a+bc^3)^2} + \frac{\left(b^{2/3}\left(2ac - bc^4 - \frac{\sqrt[3]{a}(a-2bc^3)}{\sqrt[3]{b}}\right) d\right)\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx, x, c+dx\right)}{6a^{2/3}(a+bc^3)^2}$$

$$= -\frac{1}{(a+bc^3)\,x} - \frac{3bc^2 d\log(x)}{(a+bc^3)^2} - \frac{b^{2/3}\left(2ac - bc^4 - \frac{\sqrt[3]{a}(a-2bc^3)}{\sqrt[3]{b}}\right) d\log\left(\sqrt[3]{a} + \sqrt[3]{b}(c+dx)\right)}{3a^{2/3}(a+bc^3)^2} + \frac{b^{2/3}\left(2ac - bc^4 - \frac{\sqrt[3]{a}(a-2bc^3)}{\sqrt[3]{b}}\right) d\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2\right)}{6a^{2/3}(a+bc^3)^2} + \frac{bc^2 d\log\left(a+b(c+dx)^3\right)}{(a+bc^3)^2} - \frac{\left(\sqrt[3]{b}\left(\sqrt[3]{a} - \sqrt[3]{b}c\right)\left(\sqrt[3]{a} + \sqrt[3]{b}c\right)^3 d\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{a^{2/3}(a+bc^3)^2}$$

$$= -\frac{1}{(a+bc^3)\,x} + \frac{\sqrt[3]{b}\left(\sqrt[3]{a} - \sqrt[3]{b}c\right)\left(\sqrt[3]{a} + \sqrt[3]{b}c\right)^3 d\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}(a+bc^3)^2} - \frac{3bc^2 d\log(x)}{(a+bc^3)^2} - \frac{b^{2/3}\left(2ac - bc^4 - \frac{\sqrt[3]{a}(a-2bc^3)}{\sqrt[3]{b}}\right) d\log\left(\sqrt[3]{a} + \sqrt[3]{b}(c+dx)\right)}{3a^{2/3}(a+bc^3)^2} + \frac{b^{2/3}\left(2ac - bc^4 - \frac{\sqrt[3]{a}(a-2bc^3)}{\sqrt[3]{b}}\right) d\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2\right)}{6a^{2/3}(a+bc^3)^2} + \frac{bc^2 d\log\left(a+b(c+dx)^3\right)}{(a+bc^3)^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^2 (a+b(c+dx)^3)} \, dx$$
$$= \frac{-3(a+bc^3+3bc^2dx\log(x)) + dx\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&, \frac{-3ac\log(x-\#1)+6bc^4}{}\right.}{3(a+bc^3)^2 x}$$

```
[In] Integrate[1/(x^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] (-3*(a + b*c^3 + 3*b*c^2*d*x*Log[x]) + d*x*RootSum[a + b*c^3 + 3*b*c^2*d*#1
 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (-3*a*c*Log[x - #1] + 6*b*c^4*Log[x - #1
] - a*d*Log[x - #1]*#1 + 8*b*c^3*d*Log[x - #1]*#1 + 3*b*c^2*d^2*Log[x - #1]
*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(3*(a + b*c^3)^2*x)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.46

method	result
default	$-\frac{1}{(b\,c^3+a)x}-\frac{3b\,c^2 d\ln(x)}{(b\,c^3+a)^2}-\frac{d\left(\sum_{\_R=\text{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2 d\_Z+b\,c^3+a\right)}\frac{\left(-3\_R^2 b\,c^2 d^2-8\_R b\,c^3 d-6b\,c^4+\_R ad+3a\right.}{d^2\_R^2+2cd\_R+c^2}\right.}{3(b\,c^3+a)^2}$
risch	$-\frac{1}{(b\,c^3+a)x}-\frac{3b\,c^2 d\ln(x)}{c^6 b^2+2ab\,c^3+a^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(a^2 b^2 c^6+2b\,c^3 a^3+a^4\right)\_Z^3-9a^2 b\,c^2 d\_Z^2+6abc\,d^2\_Z-b\,d^3\right)}\_R\ln\left(\left((2a\,b^3 c^9 d-6a\right.\right.\right.}{}$

```
[In] int(1/x^2/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(b*c^3+a)/x-3*b*c^2*d*ln(x)/(b*c^3+a)^2-1/3*d*sum((-3*_R^2*b*c^2*d^2-8*_
R*b*c^3*d-6*b*c^4+_R*a*d+3*a*c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(
_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/(b*c^3+a)^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 8919, normalized size of antiderivative = 28.40

$$\int \frac{1}{x^2 (a + b(c + dx)^3)} \, dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + b(c + dx)^3)} \, dx = \text{Timed out}$$

```
[In] integrate(1/x**2/(a+b*(d*x+c)**3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x^2 (a + b(c + dx)^3)} \, dx = \int \frac{1}{\left((dx + c)^3 b + a\right) x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
[Out] -3*b*c^2*d*log(x)/(b^2*c^6 + 2*a*b*c^3 + a^2) + b*d^2*integrate((3*b*c^2*d^
2*x^2 + 6*b*c^4 + (8*b*c^3 - a)*d*x - 3*a*c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3
*b*c^2*d*x + b*c^3 + a), x)/(b^2*c^6 + 2*a*b*c^3 + a^2) - 1/((b*c^3 + a)*x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + b(c + dx)^3)} \, dx = \int \frac{1}{\left((dx + c)^3 b + a\right) x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="giac")
[Out] integrate(1/(((d*x + c)^3*b + a)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.90 (sec) , antiderivative size = 1588, normalized size of antiderivative = 5.06

$$\int \frac{1}{x^2 (a+b(c+dx)^3)} dx = \text{Too large to display}$$

```
[In] int(1/(x^2*(a + b*(c + d*x)^3)),x)
```

```
[Out] symsum(log((b^4*d^12*x - 3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*
a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a^3*b^3*d^9
- 3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*
d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*b^6*c^9*d^9 - 9*root(27*a^2*b^2*c^6
*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z
- b*d^3, z, k)*b^5*c^5*d^10 + 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3
 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a^2*b^
4*c^3*d^9 + 27*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81
*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^3*b^4*c^4*d^8 + 27*roo
t(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 +
 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^2*b^5*c^7*d^8 - 9*root(27*a^2*b^2*c^6*z^
3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b
*d^3, z, k)*a*b^4*c^2*d^10 - 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 +
 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)*b^5*c^4*d^
11*x + 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b
*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^4*b^3*c*d^8 + 18*root(27*a^2
*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*
c*d^2*z - b*d^3, z, k)^2*a*b^5*c^6*d^9 + 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3
*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k
)^3*a*b^6*c^10*d^8 - 36*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4
*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^4*b^3*d^9*x -
 3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d
*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*b^6*c^8*d^10*x + 48*root(27*a^2*b^2*
c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2
*z - b*d^3, z, k)^2*a*b^5*c^5*d^10*x + 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*
b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)
^3*a*b^6*c^9*d^9*x - 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4
*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)*a*b^4*c*d^11*x +
51*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d
*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a^2*b^4*c^2*d^10*x - 54*root(27*a^2*
b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c
*d^2*z - b*d^3, z, k)^3*a^3*b^4*c^3*d^9*x)/(a^2 + b^2*c^6 + 2*a*b*c^3))*roo
t(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 +
 18*a*b*c*d^2*z - b*d^3, z, k), k, 1, 3) - 1/(a*x + b*c^3*x) - (3*b*c^2*d*l
og(x))/(a^2 + b^2*c^6 + 2*a*b*c^3)
```

3.109 $\int \frac{1}{x^3(a+b(c+dx)^3)}\,dx$

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Mupad [B] (verification not implemented) . . . 824

Optimal result

Integrand size = 17, antiderivative size = 393

$$\begin{aligned}\int \frac{1}{x^3\,(a+b(c+dx)^3)}\,dx &= -\frac{1}{2\,(a+bc^3)\,x^2}+\frac{3bc^2d}{(a+bc^3)^2\,x}\\
&+\frac{b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3\left(a-3a^{2/3}\sqrt[3]{b}c+bc^3\right)d^2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\,(a+bc^3)^3}\\
&-\frac{3bc(a-2bc^3)\,d^2\log(x)}{(a+bc^3)^3}\\
&-\frac{b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2-7abc^3-3\sqrt[3]{a}b^{5/3}c^5+b^2c^6\right)d^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\,(a+bc^3)^3}\\
&+\frac{b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2-7abc^3-3\sqrt[3]{a}b^{5/3}c^5+b^2c^6\right)d^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\,(a+bc^3)^3}\\
&+\frac{bc(a-2bc^3)\,d^2\log\left(a+b(c+dx)^3\right)}{(a+bc^3)^3}\end{aligned}$$

```
[Out] -1/2/(b*c^3+a)/x^2+3*b*c^2*d/(b*c^3+a)^2/x-3*b*c*(-2*b*c^3+a)*d^2*ln(x)/(b*
c^3+a)^3-1/3*b^(2/3)*(a^2+6*a^(4/3)*b^(2/3)*c^2-7*a*b*c^3-3*a^(1/3)*b^(5/3)
*c^5+b^2*c^6)*d^2*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/(b*c^3+a)^3+1/6*b^(2/
3)*(a^2+6*a^(4/3)*b^(2/3)*c^2-7*a*b*c^3-3*a^(1/3)*b^(5/3)*c^5+b^2*c^6)*d^2*
ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/(b*c^3+a)^3+b
*c*(-2*b*c^3+a)*d^2*ln(a+b*(d*x+c)^3)/(b*c^3+a)^3+1/3*b^(2/3)*(a^(1/3)+b^(1
/3)*c)^3*(a-3*a^(2/3)*b^(1/3)*c+b*c^3)*d^2*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d
*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(b*c^3+a)^3*3^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {378, 6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{1}{x^3 (a+b(c+dx)^3)} dx$$
$$= \frac{b^{2/3}d^2\left(-3a^{2/3}\sqrt[3]{b}c+a+bc^3\right)\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3 \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(a+bc^3)^3}$$
$$-\frac{b^{2/3}d^2\left(6a^{4/3}b^{2/3}c^2+a^2-3\sqrt[3]{a}b^{5/3}c^5-7abc^3+b^2c^6\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}(a+bc^3)^3}$$
$$+\frac{b^{2/3}d^2\left(6a^{4/3}b^{2/3}c^2+a^2-3\sqrt[3]{a}b^{5/3}c^5-7abc^3+b^2c^6\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}(a+bc^3)^3}$$
$$-\frac{3bcd^2\log(x)(a-2bc^3)}{(a+bc^3)^3}+\frac{bcd^2(a-2bc^3)\log(a+b(c+dx)^3)}{(a+bc^3)^3}$$
$$-\frac{1}{2x^2(a+bc^3)}+\frac{3bc^2d}{x(a+bc^3)^2}$$

```
[In] Int[1/(x^3*(a + b*(c + d*x)^3)),x]
```

```
[Out] -1/2*1/((a + b*c^3)*x^2) + (3*b*c^2*d)/((a + b*c^3)^2*x) + (b^(2/3)*(a^(1/3
) + b^(1/3)*c)^3*(a - 3*a^(2/3)*b^(1/3)*c + b*c^3)*d^2*ArcTan[(a^(1/3) - 2*
b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a + b*c^3)^3) - (3
*b*c*(a - 2*b*c^3)*d^2*Log[x])/(a + b*c^3)^3 - (b^(2/3)*(a^2 + 6*a^(4/3)*b^
(2/3)*c^2 - 7*a*b*c^3 - 3*a^(1/3)*b^(5/3)*c^5 + b^2*c^6)*d^2*Log[a^(1/3) +
b^(1/3)*(c + d*x)])/(3*a^(2/3)*(a + b*c^3)^3) + (b^(2/3)*(a^2 + 6*a^(4/3)*b
^(2/3)*c^2 - 7*a*b*c^3 - 3*a^(1/3)*b^(5/3)*c^5 + b^2*c^6)*d^2*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*(a + b*c^3)^3
) + (b*c*(a - 2*b*c^3)*d^2*Log[a + b*(c + d*x)^3])/(a + b*c^3)^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = d^2\text{Subst}\left(\int \frac{1}{(-c+x)^3\,(a+bx^3)}\,dx, x, c+dx\right)$$

$$= d^2\text{Subst}\left(\int \left(-\frac{1}{(a+bc^3)\,(c-x)^3} - \frac{3bc^2}{(a+bc^3)^2\,(c-x)^2} - \frac{3bc(-a+2bc^3)}{(a+bc^3)^3\,(c-x)} + \frac{b(-a^2+7abc^3-b^2c^6+3bc^2(2a-bc^3)\,x+3bc(a-2bc^3)\,x^2)}{(a+bc^3)^3\,(a+bx^3)}\right) dx, x, c+dx\right)$$

$$= -\frac{1}{2\,(a+bc^3)\,x^2} + \frac{3bc^2d}{(a+bc^3)^2\,x} - \frac{3bc(a-2bc^3)\,d^2\log(x)}{(a+bc^3)^3} + \frac{(bd^2)\,\text{Subst}\left(\int \frac{-a^2+7abc^3-b^2c^6+3bc^2(2a-bc^3)x+3bc(a-2bc^3)x^2}{a+bx^3}\,dx, x, c+dx\right)}{(a+bc^3)^3}$$

$$= -\frac{1}{2\,(a+bc^3)\,x^2} + \frac{3bc^2d}{(a+bc^3)^2\,x} - \frac{3bc(a-2bc^3)\,d^2\log(x)}{(a+bc^3)^3} + \frac{(bd^2)\,\text{Subst}\left(\int \frac{-a^2+7abc^3-b^2c^6+3bc^2(2a-bc^3)x}{a+bx^3}\,dx, x, c+dx\right)}{(a+bc^3)^3} + \frac{(3b^2c(a-2bc^3)\,d^2)\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{(a+bc^3)^3}$$

$$= -\frac{1}{2\,(a+bc^3)\,x^2} + \frac{3bc^2d}{(a+bc^3)^2\,x} - \frac{3bc(a-2bc^3)\,d^2\log(x)}{(a+bc^3)^3} + \frac{bc(a-2bc^3)\,d^2\log\left(a+b(c+dx)^3\right)}{(a+bc^3)^3} + \frac{(b^{2/3}d^2)\,\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(3\sqrt[3]{a}bc^2(2a-bc^3)+2\sqrt[3]{b}(-a^2+7abc^3-b^2c^6)\right)+\sqrt[3]{b}\left(3\sqrt[3]{a}bc^2(2a-bc^3)-\sqrt[3]{b}(-a^2+7abc^3-b^2c^6)\right)}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\right)}{3a^{2/3}\,(a+bc^3)^3} - \frac{\left(b\left(a^2+6a^{4/3}b^{2/3}c^2-7abc^3-3\sqrt[3]{a}b^{5/3}c^5+b^2c^6\right)d^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, c+dx\right)}{3a^{2/3}\,(a+bc^3)^3}$$

$$= -\frac{1}{2(a+bc^3)x^2} + \frac{3bc^2d}{(a+bc^3)^2 x} - \frac{3bc(a-2bc^3)d^2\log(x)}{(a+bc^3)^3} - \frac{b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2-7abc^3-3\sqrt[3]{a}b^{5/3}c^5+b^2c^6\right)d^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}(a+bc^3)^3} + \frac{bc(a-2bc^3)d^2\log\left(a+b(c+dx)^3\right)}{(a+bc^3)^3} - \frac{\left(b\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3\left(a-3a^{2/3}\sqrt[3]{b}c+bc^3\right)d^2\right)\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{2\sqrt[3]{a}(a+bc^3)^3} + \frac{\left(b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2-7abc^3-3\sqrt[3]{a}b^{5/3}c^5+b^2c^6\right)d^2\right)\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+a\right.}{6a^{2/3}(a+bc^3)^3}$$

$$= -\frac{1}{2(a+bc^3)x^2} + \frac{3bc^2d}{(a+bc^3)^2 x} - \frac{3bc(a-2bc^3)d^2\log(x)}{(a+bc^3)^3} - \frac{b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2-7abc^3-3\sqrt[3]{a}b^{5/3}c^5+b^2c^6\right)d^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}(a+bc^3)^3} + \frac{b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2-7abc^3-3\sqrt[3]{a}b^{5/3}c^5+b^2c^6\right)d^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right.}{6a^{2/3}(a+bc^3)^3} + \frac{bc(a-2bc^3)d^2\log\left(a+b(c+dx)^3\right)}{(a+bc^3)^3} - \frac{\left(b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3\left(a-3a^{2/3}\sqrt[3]{b}c+bc^3\right)d^2\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{a^{2/3}(a+bc^3)^3}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2 x}$$
$$+ \frac{b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3\left(a-3a^{2/3}\sqrt[3]{b}c+bc^3\right)d^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)^3}$$
$$- \frac{3bc(a-2bc^3)\,d^2\log(x)}{\left(a+bc^3\right)^3}$$
$$- \frac{b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2-7abc^3-3\sqrt[3]{a}b^{5/3}c^5+b^2c^6\right)d^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^3}$$
$$+ \frac{b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2-7abc^3-3\sqrt[3]{a}b^{5/3}c^5+b^2c^6\right)d^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)^3}$$
$$+ \frac{bc(a-2bc^3)\,d^2\log\left(a+b(c+dx)^3\right)}{\left(a+bc^3\right)^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx =$$
$$-\frac{3(a+bc^3)\left(a+bc^2(c-6dx)\right)+18bc(a-2bc^3)\,d^2x^2\log(x)+2d^2x^2\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\right.}{}$$

```
[In] Integrate[1/(x^3*(a + b*(c + d*x)^3)),x]
```

```
[Out] -1/6*(3*(a + b*c^3)*(a + b*c^2*(c - 6*d*x)) + 18*b*c*(a - 2*b*c^3)*d^2*x^2*
Log[x] + 2*d^2*x^2*RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^
3*#1^3 & , (a^2*Log[x - #1] - 16*a*b*c^3*Log[x - #1] + 10*b^2*c^6*Log[x - #
1] - 12*a*b*c^2*d*Log[x - #1]*#1 + 15*b^2*c^5*d*Log[x - #1]*#1 - 3*a*b*c*d^
2*Log[x - #1]*#1^2 + 6*b^2*c^4*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*
#1^2) & ])/((a + b*c^3)^3*x^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.51

method	result
default	$-\frac{1}{2(b\,c^3+a)x^2}+\frac{3b\,c^2d}{(b\,c^3+a)^2x}-\frac{3bc(-2b\,c^3+a)d^2\ln(x)}{(b\,c^3+a)^3}-\frac{d^2\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\left(6\_R^2b^2c^4\right.}{}\right.}{}$
risch	$\frac{\frac{3b\,c^2dx}{c^6b^2+2ab\,c^3+a^2}-\frac{1}{2\left(b\,c^3+a\right)}}{x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^2b^3c^9+3a^3b^2c^6+3a^4b\,c^3+a^5\right)\_Z^3+\left(18d^2c^4b^2a^2-9a^3bc\,d^2\right)\_Z^2+9a\,b^2c^2d^4\_Z+b^2d^6\right)}\right.}{}$

[In] int(1/x^3/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)

[Out] -1/2/(b*c^3+a)/x^2+3*b*c^2*d/(b*c^3+a)^2/x-3*b*c*(-2*b*c^3+a)*d^2*ln(x)/(b*c^3+a)^3-1/3*d^2*sum((6*_R^2*b^2*c^4*d^2+15*_R*b^2*c^5*d+10*b^2*c^6-3*_R^2*a*b*c*d^2-12*_R*a*b*c^2*d-16*a*b*c^3+a^2)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/(b*c^3+a)^3

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.88 (sec) , antiderivative size = 14765, normalized size of antiderivative = 37.57

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx = \text{Too large to display}$$

[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx = \text{Timed out}$$

[In] integrate(1/x**3/(a+b*(d*x+c)**3),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx=\int \frac{1}{\left((dx+c)^3b+a\right)x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -b*d^3*integrate((10*b^2*c^6 - 16*a*b*c^3 + 3*(2*b^2*c^4 - a*b*c)*d^2*x^2 +
 3*(5*b^2*c^5 - 4*a*b*c^2)*d*x + a^2)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*
d*x + b*c^3 + a), x)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) + 3*(2*b^2
*c^4 - a*b*c)*d^2*log(x)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) + 1/2*
(6*b*c^2*d*x - b*c^3 - a)/((b^2*c^6 + 2*a*b*c^3 + a^2)*x^2)
```

Giac [F]

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx=\int \frac{1}{\left((dx+c)^3b+a\right)x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^3*b + a)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 1328, normalized size of antiderivative = 3.38

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx=\left(\sum_{k=1}^{3}\ln\left(\frac{6\,b^6\,c^4\,d^{14}-3\,a\,b^5\,c\,d^{14}}{a^4+4\,a^3\,b\,c^3+6\,a^2\,b^2\,c^6+4\,a\,b^3\,c^9+b^4\,c^{12}}\right.\right.$$
$$-\mathrm{root}\left(81\,a^3\,b^2\,c^6\,z^3+27\,a^2\,b^3\,c^9\,z^3+81\,a^4\,b\,c^3\,z^3+27\,a^5\,z^3-81\,a^3\,b\,c\,d^2\,z^2+162\,a^2\,b^2\,c^4\,d^2\,z^2+27\,a\,b^2\,c^2\,d^4\,z-\right.$$
$$\left.-\frac{x\,\left(b^6\,c^3\,d^{15}+a\,b^5\,d^{15}\right)}{a^4+4\,a^3\,b\,c^3+6\,a^2\,b^2\,c^6+4\,a\,b^3\,c^9+b^4\,c^{12}}\right)\mathrm{root}\left(81\,a^3\,b^2\,c^6\,z^3+27\,a^2\,b^3\,c^9\,z^3\right.$$
$$\left.\left.+81\,a^4\,b\,c^3\,z^3+27\,a^5\,z^3-81\,a^3\,b\,c\,d^2\,z^2+162\,a^2\,b^2\,c^4\,d^2\,z^2+27\,a\,b^2\,c^2\,d^4\,z+b^2\,d^6,z,k\right)\right)$$
$$-\frac{1}{2\,\left(b\,c^3\,x^2+a\,x^2\right)}+\frac{3\,b\,c^2\,d}{x\,a^2+2\,x\,a\,b\,c^3+x\,b^2\,c^6}$$
$$+\frac{6\,b^2\,c^4\,d^2\,\ln\left(x\right)}{a^3+3\,a^2\,b\,c^3+3\,a\,b^2\,c^6+b^3\,c^9}-\frac{3\,a\,b\,c\,d^2\,\ln\left(x\right)}{a^3+3\,a^2\,b\,c^3+3\,a\,b^2\,c^6+b^3\,c^9}$$

```
[In] int(1/(x^3*(a + b*(c + d*x)^3)),x)
```

```
[Out] symsum(log((6*b^6*c^4*d^14 - 3*a*b^5*c*d^14)/(a^4 + b^4*c^12 + 4*a^3*b*c^3
+ 4*a*b^3*c^9 + 6*a^2*b^2*c^6) - root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9*z
^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^2*c^4*d
^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k)*((a^3*b^4*d^12 + 19*b^7*c^9*d^
12 + 12*a*b^6*c^6*d^12 - 6*a^2*b^5*c^3*d^12)/(a^4 + b^4*c^12 + 4*a^3*b*c^3
+ 4*a*b^3*c^9 + 6*a^2*b^2*c^6) - root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9*z
^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^2*c^4*d
^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k)*(root(81*a^3*b^2*c^6*z^3 + 27*
a^2*b^3*c^9*z^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*
a^2*b^2*c^4*d^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k)*((9*a^6*b^3*c*d^8
 + 9*a*b^8*c^16*d^8 + 45*a^5*b^4*c^4*d^8 + 90*a^4*b^5*c^7*d^8 + 90*a^3*b^6*
c^10*d^8 + 45*a^2*b^7*c^13*d^8)/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9
 + 6*a^2*b^2*c^6) - (x*(36*a^6*b^3*d^9 - 18*a*b^8*c^15*d^9 + 126*a^5*b^4*c^
3*d^9 + 144*a^4*b^5*c^6*d^9 + 36*a^3*b^6*c^9*d^9 - 36*a^2*b^7*c^12*d^9))/(a
^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6)) + (3*b^8*c^14*d
^10 - 42*a*b^7*c^11*d^10 + 30*a^4*b^4*c^2*d^10 + 12*a^3*b^5*c^5*d^10 - 63*a
^2*b^6*c^8*d^10)/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^
6) + (x*(3*b^8*c^13*d^11 + 66*a^4*b^4*c*d^11 - 87*a*b^7*c^10*d^11 + 39*a^3*
b^5*c^4*d^11 - 117*a^2*b^6*c^7*d^11))/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b
^3*c^9 + 6*a^2*b^2*c^6)) + (x*(18*b^7*c^8*d^13 + 90*a*b^6*c^5*d^13 - 9*a^2*
b^5*c^2*d^13))/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6)
) - (x*(a*b^5*d^15 + b^6*c^3*d^15))/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3
*c^9 + 6*a^2*b^2*c^6))*root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9*z^3 + 81*a^
4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^2*c^4*d^2*z^2 + 2
7*a*b^2*c^2*d^4*z + b^2*d^6, z, k), k, 1, 3) - 1/(2*(a*x^2 + b*c^3*x^2)) +
(3*b*c^2*d)/(a^2*x + b^2*c^6*x + 2*a*b*c^3*x) + (6*b^2*c^4*d^2*log(x))/(a^3
 + b^3*c^9 + 3*a^2*b*c^3 + 3*a*b^2*c^6) - (3*a*b*c*d^2*log(x))/(a^3 + b^3*c
^9 + 3*a^2*b*c^3 + 3*a*b^2*c^6)
```

3.110 $\int \frac{x^3}{a+b(c+dx)^4} dx$

Optimal result . . . 826
Rubi [A] (verified) . . . 827
Mathematica [C] (verified) . . . 831
Maple [C] (verified) . . . 831
Fricas [F(-1)] . . . 832
Sympy [A] (verification not implemented) . . . 832
Maxima [F] . . . 833
Giac [F] . . . 833
Mupad [B] (verification not implemented) . . . 833

Optimal result

Integrand size = 17, antiderivative size = 356

$$\begin{aligned}\int \frac{x^3}{a+b(c+dx)^4} dx = {} & \frac{3c^2 \arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^4} + \frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4} \\ & - \frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4} \\ & - \frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4} \\ & + \frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4} \\ & + \frac{\log\left(a+b(c+dx)^4\right)}{4bd^4}\end{aligned}$$

```
[Out] 1/4*ln(a+b*(d*x+c)^4)/b/d^4+3/2*c^2*arctan((d*x+c)^2*b^(1/2)/a^(1/2))/d^4/a
^(1/2)/b^(1/2)-1/8*c*ln(-a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*
b^(1/2))*(3*a^(1/2)-b^(1/2)*c^2)/a^(3/4)/b^(3/4)/d^4*2^(1/2)+1/8*c*ln(a^(1/
4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))*(3*a^(1/2)-b^(1/2)*c^
2)/a^(3/4)/b^(3/4)/d^4*2^(1/2)-1/4*c*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1
/4))*(3*a^(1/2)+b^(1/2)*c^2)/a^(3/4)/b^(3/4)/d^4*2^(1/2)-1/4*c*arctan(1+b^(
1/4)*(d*x+c)*2^(1/2)/a^(1/4))*(3*a^(1/2)+b^(1/2)*c^2)/a^(3/4)/b^(3/4)/d^4*2
^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {378, 1890, 1262, 649, 211, 266, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx = \frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4} - \frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4} - \frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4} + \frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4} + \frac{3c^2\arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^4} + \frac{\log\left(a+b(c+dx)^4\right)}{4bd^4}$$

```
[In] Int[x^3/(a + b*(c + d*x)^4),x]
```

```
[Out] (3*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]*d^4) + (c*
(3*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) - (c*(3*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) - (c*(
3*Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) +
Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) + (c*(3*Sqrt[a] - Sqr
t[b]*c^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*
x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) + Log[a + b*(c + d*x)^4]/(4*b*d^4)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+bx^4}\,dx, x, c+dx\right)}{d^4} \\
&= \frac{\text{Subst}\left(\int \left(\frac{x(3c^2+x^2)}{a+bx^4}+\frac{-c^3-3cx^2}{a+bx^4}\right)\,dx, x, c+dx\right)}{d^4} \\
&= \frac{\text{Subst}\left(\int \frac{x(3c^2+x^2)}{a+bx^4}\,dx, x, c+dx\right)}{d^4}+\frac{\text{Subst}\left(\int \frac{-c^3-3cx^2}{a+bx^4}\,dx, x, c+dx\right)}{d^4} \\
&= \frac{\text{Subst}\left(\int \frac{3c^2+x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^4}+\frac{\left(c\left(3-\frac{\sqrt{b}c^2}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx, x, c+dx\right)}{2bd^4} \\
&\quad -\frac{\left(c\left(3+\frac{\sqrt{b}c^2}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx, x, c+dx\right)}{2bd^4}
\end{aligned}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^4} + \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^4}$$

$$- \frac{\left(c\left(3\sqrt{a}-\sqrt{b}c^2\right)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, c+dx\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4}$$

$$- \frac{\left(c\left(3\sqrt{a}-\sqrt{b}c^2\right)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, c+dx\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4}$$

$$- \frac{\left(c\left(3+\frac{\sqrt{b}c^2}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4bd^4}$$

$$- \frac{\left(c\left(3+\frac{\sqrt{b}c^2}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4bd^4}$$

$$= \frac{3c^2\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^4}$$

$$- \frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4}$$

$$+ \frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4}$$

$$+ \frac{\log\left(a+b(c+dx)^4\right)}{4bd^4}$$

$$- \frac{\left(c\left(3\sqrt{a}+\sqrt{b}c^2\right)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4}$$

$$+ \frac{\left(c\left(3\sqrt{a}+\sqrt{b}c^2\right)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4}$$

$$= \frac{3c^2\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^4} + \frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4}$$
$$- \frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4}$$
$$- \frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4}$$
$$+ \frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4}$$
$$+ \frac{\log\left(a+b(c+dx)^4\right)}{4bd^4}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.30

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx$$
$$= \frac{\text{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&, \frac{\log(x-\#1)\#1^3}{c^3+3c^2d\#1+3cd^2\#1^2+d^3\#1^3}\&\right]}{4bd}$$

```
[In] Integrate[x^3/(a + b*(c + d*x)^4),x]
```

```
[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^
4*#1^4 & , (Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3)
& ]/(4*b*d)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.27

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\_R^3\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4bd}$	97
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\_R^3\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4bd}$	97

```
[In] int(x^3/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b/d*sum(_R^3/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(
_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx = \text{Timed out}$$

```
[In] integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx$$
$$= \text{RootSum}\left(256t^4a^3b^4d^{16} - 256t^3a^3b^3d^{12} + t^2\cdot\left(96a^3b^2d^8 + 480a^2b^3c^4d^8\right) + t\left(-16a^3bd^4 + 192a^2b^2c^4d^4 - 48\right.\right.$$

```
[In] integrate(x**3/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**4*d**16 - 256*_t**3*a**3*b**3*d**12 + _t**2*(96*a
**3*b**2*d**8 + 480*a**2*b**3*c**4*d**8) + _t*(-16*a**3*b*d**4 + 192*a**2*b
**2*c**4*d**4 - 48*a*b**3*c**8*d**4) + a**3 + 3*a**2*b*c**4 + 3*a*b**2*c**8
 + b**3*c**12, Lambda(_t, _t*log(x + (-1728*_t**3*a**4*b**3*d**12 - 960*_t*
*3*a**3*b**4*c**4*d**12 + 1296*_t**2*a**4*b**2*d**8 + 2016*_t**2*a**3*b**3*
c**4*d**8 - 48*_t**2*a**2*b**4*c**8*d**8 - 324*_t*a**4*b*d**4 - 4716*_t*a**
3*b**2*c**4*d**4 - 1452*_t*a**2*b**3*c**8*d**4 - 4*_t*a*b**4*c**12*d**4 + 2
7*a**4 - 390*a**3*b*c**4 - 444*a**2*b**2*c**8 - 26*a*b**3*c**12 + b**4*c**1
6)/(729*a**3*b*c**3*d - 1053*a**2*b**2*c**7*d - 117*a*b**3*c**11*d + b**4*c
**15*d))))
```

Maxima [F]

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx = \int \frac{x^3}{(dx+c)^4b+a}\,dx$$

```
[In] integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="maxima")
[Out] integrate(x^3/((d*x + c)^4*b + a), x)
```

Giac [F]

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx = \int \frac{x^3}{(dx+c)^4b+a}\,dx$$

```
[In] integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="giac")
[Out] integrate(x^3/((d*x + c)^4*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 1003, normalized size of antiderivative = 2.82

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx = \sum_{k=1}^{4} \ln\Big(b\,c^2\,d\,\Big(2\,a\,c+2\,b\,c^5-3\,a\,d\,x+5\,b\,c^4\,d\,x$$
$$-\mathrm{root}\big(256\,a^3\,b^4\,d^{16}\,z^4-256\,a^3\,b^3\,d^{12}\,z^3+480\,a^2\,b^3\,c^4\,d^8\,z^2+96\,a^3\,b^2\,d^8\,z^2+192\,a^2\,b^2\,c^4\,d^4\,z-48\,a\,b^3\,c^8\,d^4\,z-$$
$$+\mathrm{root}\big(256\,a^3\,b^4\,d^{16}\,z^4-256\,a^3\,b^3\,d^{12}\,z^3+480\,a^2\,b^3\,c^4\,d^8\,z^2+96\,a^3\,b^2\,d^8\,z^2+192\,a^2\,b^2\,c^4\,d^4\,z-48\,a\,b^3\,c^8\,d$$
$$+\mathrm{root}\big(256\,a^3\,b^4\,d^{16}\,z^4-256\,a^3\,b^3\,d^{12}\,z^3+480\,a^2\,b^3\,c^4\,d^8\,z^2+96\,a^3\,b^2\,d^8\,z^2+192\,a^2\,b^2\,c^4\,d^4\,z-48\,a\,b^3\,c^8\,d$$
$$-\mathrm{root}\big(256\,a^3\,b^4\,d^{16}\,z^4-256\,a^3\,b^3\,d^{12}\,z^3+480\,a^2\,b^3\,c^4\,d^8\,z^2+96\,a^3\,b^2\,d^8\,z^2+192\,a^2\,b^2\,c^4\,d^4\,z-48\,a\,b^3\,c^8\,d^4\,z-$$
$$+\mathrm{root}\big(256\,a^3\,b^4\,d^{16}\,z^4-256\,a^3\,b^3\,d^{12}\,z^3+480\,a^2\,b^3\,c^4\,d^8\,z^2+96\,a^3\,b^2\,d^8\,z^2+192\,a^2\,b^2\,c^4\,d^4\,z-48\,a\,b^3\,c^8\,d^4\,z-$$
$$+\mathrm{root}\big(256\,a^3\,b^4\,d^{16}\,z^4-256\,a^3\,b^3\,d^{12}\,z^3+480\,a^2\,b^3\,c^4\,d^8\,z^2+96\,a^3\,b^2\,d^8\,z^2+192\,a^2\,b^2\,c^4\,d^4\,z-48\,a\,b^3\,c^8\,d^4\,z-$$
$$-256\,a^3\,b^3\,d^{12}\,z^3+480\,a^2\,b^3\,c^4\,d^8\,z^2+96\,a^3\,b^2\,d^8\,z^2+192\,a^2\,b^2\,c^4\,d^4\,z-48\,a\,b^3\,c^8\,d^4\,z$$
$$-16\,a^3\,b\,d^4\,z+3\,a\,b^2\,c^8+3\,a^2\,b\,c^4+b^3\,c^{12}+a^3,z,k\big)$$

```
[In] int(x^3/(a + b*(c + d*x)^4),x)
[Out] symsum(log(2*b*c^2*d*(2*a*c + 2*b*c^5 - 3*a*d*x + 5*b*c^4*d*x - 2*root(256*
a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^3*
b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z +
 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*b^2*c^5*d^4 + 32*root(25
6*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^
```

```
3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z
 + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)^2*a*b^2*c*d^8 + 24*roo
t(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 9
6*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d
^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)^2*a*b^2*d^9*x - 2*
root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2
+ 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*
b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*b^2*c^4*d^5*x +
 38*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*
z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*
a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*a*b*c*d^4 +
 6*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z
^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a
^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*a*b*d^5*x))*
root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2
+ 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*
b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k), k, 1, 4)
```

3.111 $\int \frac{x^2}{a+b(c+dx)^4} dx$

Optimal result 835
Rubi [A] (verified) 836
Mathematica [C] (verified) 839
Maple [C] (verified) 840
Fricas [C] (verification not implemented) 840
Sympy [A] (verification not implemented) 840
Maxima [F] 841
Giac [F] 841
Mupad [B] (verification not implemented) 841

Optimal result

Integrand size = 17, antiderivative size = 318

$$\begin{aligned}\int \frac{x^2}{a+b(c+dx)^4} dx = & -\frac{c \arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3} - \frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3} \\ & + \frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3} \\ & + \frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3} \\ & - \frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3}\end{aligned}$$

```
[Out] -c*arctan((d*x+c)^2*b^(1/2)/a^(1/2))/d^3/a^(1/2)/b^(1/2)+1/8*ln(-a^(1/4)*b^
(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))*(a^(1/2)-b^(1/2)*c^2)/a^(3
/4)/b^(3/4)/d^3*2^(1/2)-1/8*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x
+c)^2*b^(1/2))*(a^(1/2)-b^(1/2)*c^2)/a^(3/4)/b^(3/4)/d^3*2^(1/2)+1/4*arctan
(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*(a^(1/2)+b^(1/2)*c^2)/a^(3/4)/b^(3/4)/
d^3*2^(1/2)+1/4*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*(a^(1/2)+b^(1/2)*
c^2)/a^(3/4)/b^(3/4)/d^3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {378, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx = -\frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3} + \frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3} + \frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3} - \frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3} - \frac{c\arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3}$$

```
[In] Int[x^2/(a + b*(c + d*x)^4),x]
```

```
[Out] -((c*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*d^3)) - ((Sqrt
[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*b^(3/4)*d^3) + ((Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^3) + ((Sqrt[a] - Sqr
t[b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*
x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^3) - ((Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a
^(3/4)*b^(3/4)*d^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+bx^4}\,dx,x,c+dx\right)}{d^3}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{2cx}{a+bx^4}+\frac{c^2+x^2}{a+bx^4}\right)\,dx,x,c+dx\right)}{d^3}\\
&= \frac{\text{Subst}\left(\int \frac{c^2+x^2}{a+bx^4}\,dx,x,c+dx\right)}{d^3}-\frac{(2c)\text{Subst}\left(\int \frac{x}{a+bx^4}\,dx,x,c+dx\right)}{d^3}\\
&= -\frac{c\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,(c+dx)^2\right)}{d^3}-\frac{\left(1-\frac{\sqrt{b}c^2}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx,x,c+dx\right)}{2bd^3}\\
&\quad+\frac{\left(1+\frac{\sqrt{b}c^2}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx,x,c+dx\right)}{2bd^3}\\
&= -\frac{c\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3}+\frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx,x,c+dx\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3}\\
&\quad+\frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx,x,c+dx\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3}\\
&\quad+\frac{\left(1+\frac{\sqrt{b}c^2}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,c+dx\right)}{4bd^3}\\
&\quad+\frac{\left(1+\frac{\sqrt{b}c^2}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,c+dx\right)}{4bd^3}
\end{aligned}$$

$$= -\frac{c\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3} + \frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3} - \frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3} + \frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3} - \frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3}$$

$$= -\frac{c\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3} - \frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3} + \frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3} + \frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3} - \frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.33

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx = \frac{\text{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&,\frac{\log(x-\#1)\#1^2}{c^3+3c^2d\#1+3cd^2\#1^2+d^3\#1^3}\&\right]}{4bd}$$

```
[In] Integrate[x^2/(a + b*(c + d*x)^4),x]
```

```
[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^
4*#1^4 & , (Log[x - #1]*#1^2)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3)
& ]/(4*b*d)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.31

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^{4}\_Z^{4}+4bc\,d^{3}\_Z^{3}+6b\,c^{2}d^{2}\_Z^{2}+4b\,c^{3}d\_Z+b\,c^{4}+a\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{d^{3}\_R^{3}+3c\,d^{2}\_R^{2}+3c^{2}d\_R+c^{3}}}{4bd}$	97
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^{4}\_Z^{4}+4bc\,d^{3}\_Z^{3}+6b\,c^{2}d^{2}\_Z^{2}+4b\,c^{3}d\_Z+b\,c^{4}+a\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{d^{3}\_R^{3}+3c\,d^{2}\_R^{2}+3c^{2}d\_R+c^{3}}}{4bd}$	97

```
[In] int(x^2/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
[Out] 1/4/b/d*sum(_R^2/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(
_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 58.70 (sec) , antiderivative size = 61993, normalized size of antiderivative = 194.95

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx = \mathrm{RootSum}\left(256t^4a^3b^3d^{12}+192t^2a^2b^2c^2d^6+t\left(-32a^2bcd^3+32ab^2c^5d^3\right)+a^2+2abc^4+b^2c^8,\left(t\mapsto t\log\left(x\right.\right.\right.$$

```
[In] integrate(x**2/(a+b*(d*x+c)**4),x)
[Out] RootSum(256*_t**4*a**3*b**3*d**12 + 192*_t**2*a**2*b**2*c**2*d**6 + _t*(-32
*a**2*b*c*d**3 + 32*a*b**2*c**5*d**3) + a**2 + 2*a*b*c**4 + b**2*c**8, Lamb
da(_t, _t*log(x + (64*_t**3*a**4*b**2*d**9 + 448*_t**3*a**3*b**3*c**4*d**9
```

```
+ 160*_t**2*a**3*b**2*c**3*d**6 - 32*_t**2*a**2*b**3*c**7*d**6 + 60*_t*a**3
*b*c**2*d**3 + 256*_t*a**2*b**2*c**6*d**3 + 4*_t*a*b**3*c**10*d**3 - 5*a**3
*c - 9*a**2*b*c**5 - 3*a*b**2*c**9 + b**3*c**13)/(a**3*d - 33*a**2*b*c**4*d
 - 33*a*b**2*c**8*d + b**3*c**12*d)))
```

Maxima [F]

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx=\int \frac{x^2}{(dx+c)^4b+a}\,dx$$

```
[In] integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="maxima")
[Out] integrate(x^2/((d*x + c)^4*b + a), x)
```

Giac [F]

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx=\int \frac{x^2}{(dx+c)^4b+a}\,dx$$

```
[In] integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="giac")
[Out] integrate(x^2/((d*x + c)^4*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.97

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx=\sum_{k=1}^{4}\ln\Big(-b\,d^4\,\Big(a+b\,c^4+4\,b\,c^3\,d\,x$$
$$+\mathrm{root}\left(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\right)\,b^2\,c^5\,d^3\,4$$
$$+\mathrm{root}\left(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\right)\,b^2\,c^4\,d^4\,x$$
$$-\mathrm{root}\left(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\right)\,a\,b\,c\,d^3\,20$$
$$-\mathrm{root}\left(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\right)\,a\,b\,d^4\,x\,4$$
$$+\mathrm{root}\left(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\right)^2\,a\,b^2$$
$$+\mathrm{root}\left(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\right)^2\,a\,b^2$$
$$+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\right)$$

```
[In] int(x^2/(a + b*(c + d*x)^4),x)
```

```
[Out] symsum(log(-b*d^4*(a + b*c^4 + 4*b*c^3*d*x + 4*root(256*a^3*b^3*d^12*z^4 +
192*a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4
 + b^2*c^8 + a^2, z, k)*b^2*c^5*d^3 + 4*root(256*a^3*b^3*d^12*z^4 + 192*a^2
*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*
c^8 + a^2, z, k)*b^2*c^4*d^4*x - 20*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2
*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8
+ a^2, z, k)*a*b*c*d^3 - 4*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*
z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z,
 k)*a*b*d^4*x + 48*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32
*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k)^2*a*
b^2*c^2*d^6 + 32*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32*a
*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k)^2*a*b^
2*c*d^7*x))*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*
c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k), k, 1, 4)
```

3.112 $\int \frac{x}{a+b(c+dx)^4} dx$

Optimal result 843
Rubi [A] (verified) 844
Mathematica [C] (verified) 847
Maple [C] (verified) 848
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Sympy [A] (verification not implemented) 848
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Mupad [B] (verification not implemented) 849

Optimal result

Integrand size = 15, antiderivative size = 261

$$\int \frac{x}{a+b(c+dx)^4} dx = \frac{\arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} - \frac{c\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} + \frac{c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} - \frac{c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}$$

```
[Out] -1/4*c*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/d^2*2^(1/
2)-1/4*c*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/d^2*2^(1
/2)+1/8*c*ln(-a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))/a^
(3/4)/b^(1/4)/d^2*2^(1/2)-1/8*c*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+
(d*x+c)^2*b^(1/2))/a^(3/4)/b^(1/4)/d^2*2^(1/2)+1/2*arctan((d*x+c)^2*b^(1/2)
/a^(1/2))/d^2/a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {378, 1890, 217, 1179, 642, 1176, 631, 210, 281, 211}

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \frac{c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} - \frac{c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} + \frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} - \frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} + \frac{\arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2}$$

```
[In] Int[x/(a + b*(c + d*x)^4),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]*d^2) + (c*ArcTan[1
 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) -
(c*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1
/4)*d^2) + (c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c
+ d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) - (c*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d^2
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{-c+x}{a+bx^4}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{c}{a+bx^4}+\frac{x}{a+bx^4}\right)\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{a+bx^4}\,dx, x, c+dx\right)}{d^2} - \frac{c\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^2} - \frac{c\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, c+dx\right)}{2\sqrt{a}d^2} - \frac{c\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, c+dx\right)}{2\sqrt{a}d^2}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2} - \frac{c\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4\sqrt{a}\sqrt{b}d^2} - \frac{c\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4\sqrt{a}\sqrt{b}d^2} + \frac{c\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, c+dx\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} + \frac{c\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, c+dx\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} - \frac{c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} - \frac{c\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} + \frac{c\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} - \frac{c\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} + \frac{c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} - \frac{c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.40

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \frac{\text{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&, \frac{\log(x-\#1)\#1}{c^3+3c^2d\#1+3cd^2\#1^2+d^3\#1^3}\&\right]}{4bd}$$

```
[In] Integrate[x/(a + b*(c + d*x)^4),x]
```

```
[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^
4*#1^4 & , (Log[x - #1]*#1)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) &
]/(4*b*d)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.36

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\_R\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4bd}$	95
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\_R\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4bd}$	95

```
[In] int(x/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
[Out] 1/4/b/d*sum(_R/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(_Z
^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.05 (sec) , antiderivative size = 40785, normalized size of antiderivative = 156.26

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.50

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \text{RootSum}\left(256t^4a^3b^2d^8+32t^2a^2bd^4-16tabc^2d^2+a+bc^4,\left(t\mapsto t\log\left(x+\frac{128t^3a^3bd^6+16t^2a^2bc^2d^4+8t\right.\right.\right.$$
$$4acd-$$

```
[In] integrate(x/(a+b*(d*x+c)**4),x)
[Out] RootSum(256*_t**4*a**3*b**2*d**8 + 32*_t**2*a**2*b*d**4 - 16*_t*a*b*c**2*d*
*2 + a + b*c**4, Lambda(_t, _t*log(x + (128*_t**3*a**3*b*d**6 + 16*_t**2*a*
*2*b*c**2*d**4 + 8*_t*a**2*d**2 + 4*_t*a*b*c**4*d**2 - a*c**2 - b*c**6)/(4*
a*c*d - b*c**5*d))))
```

Maxima [F]

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \int \frac{x}{(dx+c)^4b+a}\,dx$$

```
[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] integrate(x/((d*x + c)^4*b + a), x)
```

Giac [F]

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \int \frac{x}{(dx+c)^4b+a}\,dx$$

```
[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x + c)^4*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.79

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(256\,a^3\,b^2\,d^8\,z^4+32\,a^2\,b\,d^4\,z^2-16\,a\,b\,c^2\,d^2\,z+b\,c^4+a,z,k\right)\left(-\mathrm{root}\left(256\,a^3\,b^2\,d^8\,z^4+32\,a^2\,b\,d^4\,z^2-16\,a\,b\,c^2\,d^2\,z+b\,c^4+a,z,k\right)\left(16\,a\,x\,b^3\,d^{12}+32\,a\,c\,b^3\,d^{11}\right)+4\,b^3\,c^3\,d^9+4\,b^3\,c^2\,d^{10}\,x\right)+b^2\,d^8\,x\right)\mathrm{root}\left(256\,a^3\,b^2\,d^8\,z^4+32\,a^2\,b\,d^4\,z^2-16\,a\,b\,c^2\,d^2\,z+b\,c^4+a,z,k\right)$$

```
[In] int(x/(a + b*(c + d*x)^4),x)
```

```
[Out] symsum(log(b^2*d^8*x - root(256*a^3*b^2*d^8*z^4 + 32*a^2*b*d^4*z^2 - 16*a*b
*c^2*d^2*z + b*c^4 + a, z, k)*(4*b^3*c^3*d^9 - root(256*a^3*b^2*d^8*z^4 + 3
2*a^2*b*d^4*z^2 - 16*a*b*c^2*d^2*z + b*c^4 + a, z, k)*(32*a*b^3*c*d^11 + 16
*a*b^3*d^12*x) + 4*b^3*c^2*d^10*x))*root(256*a^3*b^2*d^8*z^4 + 32*a^2*b*d^4
*z^2 - 16*a*b*c^2*d^2*z + b*c^4 + a, z, k), k, 1, 4)
```

3.113 $\int \frac{1}{a+b(c+dx)^4}\,dx$

Optimal result 850
Rubi [A] (verified) 850
Mathematica [A] (verified) 853
Maple [C] (verified) 854
Fricas [C] (verification not implemented) 854
Sympy [A] (verification not implemented) 855
Maxima [F] 855
Giac [A] (verification not implemented) 855
Mupad [B] (verification not implemented) 856

Optimal result

Integrand size = 13, antiderivative size = 221

$$\int \frac{1}{a+b(c+dx)^4}\,dx=-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d}-\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

```
[Out] 1/4*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/d*2^(1/2)+1/
4*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/d*2^(1/2)-1/8*l
n(-a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))/a^(3/4)/b^(1/
4)/d*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/
2))/a^(3/4)/b^(1/4)/d*2^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used

= {253, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{a+b(c+dx)^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

```
[In] Int[(a + b*(c + d*x)^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(Sqrt[2]*a^(3/4)*b^(1/
4)*d) + ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*
b^(1/4)*d) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c +
 d*x)^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, c+dx\right)}{2\sqrt{a}d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, c+dx\right)}{2\sqrt{a}d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4\sqrt{a}\sqrt{b}d} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4\sqrt{a}\sqrt{b}d}\\
&\quad - \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, c+dx\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d} - \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, c+dx\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}
\end{aligned}$$

$$=-\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d}-\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

$$=-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d}-\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.73

$$\int\frac{1}{a+b(c+dx)^4}\,dx=\frac{-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

```
[In] Integrate[(a + b*(c + d*x)^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]
*b^(1/4)*(c + d*x))/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d
*x) + Sqrt[b]*(c + d*x)^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x
) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.43

method	result	size
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\,d^{4}\_Z^{4}+4bc\,d^{3}\_Z^{3}+6b\,c^{2}d^{2}\_Z^{2}+4b\,c^{3}d\_Z+b\,c^{4}+a\right)}\frac{\ln\left(x-\_R\right)}{d^{3}\_R^{3}+3c\,d^{2}\_R^{2}+3c^{2}d\_R+c^{3}}}{4bd}$	94
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\,d^{4}\_Z^{4}+4bc\,d^{3}\_Z^{3}+6b\,c^{2}d^{2}\_Z^{2}+4b\,c^{3}d\_Z+b\,c^{4}+a\right)}\frac{\ln\left(x-\_R\right)}{d^{3}\_R^{3}+3c\,d^{2}\_R^{2}+3c^{2}d\_R+c^{3}}}{4bd}$	94

```
[In] int(1/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b/d*sum(1/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(_Z^
4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.69

$$\begin{aligned}\int \frac{1}{a+b(c+dx)^4}\,dx &= \frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+dx+c\right)\\ &\quad+\frac{1}{4}i\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(i\,ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+dx+c\right)\\ &\quad-\frac{1}{4}i\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(-i\,ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+dx+c\right)\\ &\quad-\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(-ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+dx+c\right)\end{aligned}$$

```
[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*(-1/(a^3*b*d^4))^(1/4)*log(a*d*(-1/(a^3*b*d^4))^(1/4) + d*x + c) + 1/4*
I*(-1/(a^3*b*d^4))^(1/4)*log(I*a*d*(-1/(a^3*b*d^4))^(1/4) + d*x + c) - 1/4*
I*(-1/(a^3*b*d^4))^(1/4)*log(-I*a*d*(-1/(a^3*b*d^4))^(1/4) + d*x + c) - 1/4
*(-1/(a^3*b*d^4))^(1/4)*log(-a*d*(-1/(a^3*b*d^4))^(1/4) + d*x + c)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.12

$$\int \frac{1}{a+b(c+dx)^4}\,dx = \frac{\operatorname{RootSum}\left(256t^4a^3b+1,\left(t\mapsto t\log\left(x+\frac{4ta+c}{d}\right)\right)\right)}{d}$$

```
[In] integrate(1/(a+b*(d*x+c)**4),x)
[Out] RootSum(256*_t**4*a**3*b + 1, Lambda(_t, _t*log(x + (4*_t*a + c)/d)))/d
```

Maxima [F]

$$\int \frac{1}{a+b(c+dx)^4}\,dx = \int \frac{1}{(dx+c)^4b+a}\,dx$$

```
[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="maxima")
[Out] integrate(1/((d*x + c)^4*b + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.47

$$\begin{aligned}\int \frac{1}{a+b(c+dx)^4}\,dx = &-\frac{1}{2}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{bdx+bc}{(-ab^3)^{\frac{1}{4}}}\right) \\ &+\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\left|bdx+bc+\left(-ab^3\right)^{\frac{1}{4}}\right|\right) \\ &-\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\left|-bdx-bc+\left(-ab^3\right)^{\frac{1}{4}}\right|\right)\end{aligned}$$

```
[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="giac")
[Out] -1/2*(-1/(a^3*b*d^4))^(1/4)*arctan(-(b*d*x + b*c)/(-a*b^3)^(1/4)) + 1/4*(-1
/(a^3*b*d^4))^(1/4)*log(abs(b*d*x + b*c + (-a*b^3)^(1/4))) - 1/4*(-1/(a^3*b
*d^4))^(1/4)*log(abs(-b*d*x - b*c + (-a*b^3)^(1/4)))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.27

$$\int \frac{1}{a+b(c+dx)^4}\,dx = -\frac{\operatorname{atan}\left(\frac{b^{1/4}\,c}{(-a)^{1/4}}+\frac{b^{1/4}\,d\,x}{(-a)^{1/4}}\right)+\operatorname{atanh}\left(\frac{b^{1/4}\,c}{(-a)^{1/4}}+\frac{b^{1/4}\,d\,x}{(-a)^{1/4}}\right)}{2\,(-a)^{3/4}\,b^{1/4}\,d}$$

```
[In] int(1/(a + b*(c + d*x)^4),x)
```

```
[Out] -(atan((b^(1/4)*c)/(-a)^(1/4) + (b^(1/4)*d*x)/(-a)^(1/4)) + atanh((b^(1/4)*
c)/(-a)^(1/4) + (b^(1/4)*d*x)/(-a)^(1/4)))/(2*(-a)^(3/4)*b^(1/4)*d)
```

3.114 $\int \frac{1}{x(a+b(c+dx)^4)}\,dx$

Optimal result 857
Rubi [A] (verified) 858
Mathematica [C] (verified) 862
Maple [C] (verified) 863
Fricas [C] (verification not implemented) 863
Sympy [F(-1)] 864
Maxima [F] 864
Giac [F] 864
Mupad [B] (verification not implemented) 865

Optimal result

Integrand size = 17, antiderivative size = 393

$$\begin{aligned}\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx &= -\frac{\sqrt{b}c^2\arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(a+bc^4\right)}+\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)}\\
&\quad-\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)}+\frac{\log(x)}{a+bc^4}\\
&\quad-\frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)}\\
&\quad+\frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)}\\
&\quad-\frac{\log\left(a+b(c+dx)^4\right)}{4\left(a+bc^4\right)}\end{aligned}$$

```
[Out] ln(x)/(b*c^4+a)-1/4*ln(a+b*(d*x+c)^4)/(b*c^4+a)-1/2*c^2*arctan((d*x+c)^2*b^
(1/2)/a^(1/2))*b^(1/2)/(b*c^4+a)/a^(1/2)-1/8*b^(1/4)*c*ln(-a^(1/4)*b^(1/4)*
(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))*(a^(1/2)-b^(1/2)*c^2)/a^(3/4)/(b
*c^4+a)*2^(1/2)+1/8*b^(1/4)*c*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d
*x+c)^2*b^(1/2))*(a^(1/2)-b^(1/2)*c^2)/a^(3/4)/(b*c^4+a)*2^(1/2)-1/4*b^(1/4
)*c*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*(a^(1/2)+b^(1/2)*c^2)/a^(3/4
)/(b*c^4+a)*2^(1/2)-1/4*b^(1/4)*c*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))
*(a^(1/2)+b^(1/2)*c^2)/a^(3/4)/(b*c^4+a)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.765, Rules used = {378, 6857, 1890, 1262, 649, 211, 266, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{x\,(a+b(c+dx)^4)}\,dx = \frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(a+bc^4)} - \frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\,(a+bc^4)} - \frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\,(a+bc^4)} + \frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\,(a+bc^4)} - \frac{\sqrt{b}c^2\arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(a+bc^4)} - \frac{\log\left(a+b(c+dx)^4\right)}{4\,(a+bc^4)} + \frac{\log(x)}{a+bc^4}$$

```
[In] Int[1/(x*(a + b*(c + d*x)^4)),x]
```

```
[Out] -1/2*(Sqrt[b]*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(Sqrt[a]*(a + b*c^
4)) + (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d
*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)) - (b^(1/4)*c*(Sqrt[a] + Sqrt
[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4
)*(a + b*c^4)) + Log[x]/(a + b*c^4) - (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sq
rt[2]*a^(3/4)*(a + b*c^4)) + (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(
3/4)*(a + b*c^4)) - Log[a + b*(c + d*x)^4]/(4*(a + b*c^4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(-c+x)\left(a+bx^4\right)}\,dx, x, c+dx\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{\left(a+bc^4\right)(c-x)}-\frac{b\left(c^3+c^2x+cx^2+x^3\right)}{\left(a+bc^4\right)\left(a+bx^4\right)}\right)\,dx, x, c+dx\right)\\
&= \frac{\log(x)}{a+bc^4}-\frac{b\text{Subst}\left(\int \frac{c^3+c^2x+cx^2+x^3}{a+bx^4}\,dx, x, c+dx\right)}{a+bc^4}\\
&= \frac{\log(x)}{a+bc^4}-\frac{b\text{Subst}\left(\int \left(\frac{x\left(c^2+x^2\right)}{a+bx^4}+\frac{c^3+cx^2}{a+bx^4}\right)\,dx, x, c+dx\right)}{a+bc^4}\\
&= \frac{\log(x)}{a+bc^4}-\frac{b\text{Subst}\left(\int \frac{x\left(c^2+x^2\right)}{a+bx^4}\,dx, x, c+dx\right)}{a+bc^4}-\frac{b\text{Subst}\left(\int \frac{c^3+cx^2}{a+bx^4}\,dx, x, c+dx\right)}{a+bc^4}
\end{aligned}$$

$$= \frac{\log(x)}{a+bc^4} - \frac{b\text{Subst}\left(\int \frac{c^2+x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2(a+bc^4)} + \frac{\left(c\left(1-\frac{\sqrt{b}c^2}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx, x, c+dx\right)}{2(a+bc^4)} - \frac{\left(c\left(1+\frac{\sqrt{b}c^2}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx, x, c+dx\right)}{2(a+bc^4)}$$

$$= \frac{\log(x)}{a+bc^4} - \frac{b\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2(a+bc^4)} - \frac{(bc^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2(a+bc^4)} - \frac{\left(\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, c+dx\right)}{4\sqrt{2}a^{3/4}(a+bc^4)} - \frac{\left(\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, c+dx\right)}{4\sqrt{2}a^{3/4}(a+bc^4)} - \frac{\left(c\left(1+\frac{\sqrt{b}c^2}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4(a+bc^4)} - \frac{\left(c\left(1+\frac{\sqrt{b}c^2}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4(a+bc^4)}$$

$$= -\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(a+bc^4\right)} + \frac{\log(x)}{a+bc^4} - \frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)} + \frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)} - \frac{\log\left(a+b(c+dx)^4\right)}{4\left(a+bc^4\right)} - \frac{\left(\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)} + \frac{\left(\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)}$$

$$= -\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(a+bc^4\right)} + \frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)} - \frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)} + \frac{\log(x)}{a+bc^4} - \frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)} + \frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)} - \frac{\log\left(a+b(c+dx)^4\right)}{4\left(a+bc^4\right)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.41

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx = -\frac{-4\log(x)+\text{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&,\frac{4c^3\log(x-\#1)+6c^2d\log(x-\#\ldots}{c^3+3\ldots}\right.}{4\left(a+bc^4\right)}$$

```
[In] Integrate[1/(x*(a + b*(c + d*x)^4)),x]
```

```
[Out] -1/4*(-4*Log[x] + RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b
*c*d^3*#1^3 + b*d^4*#1^4 & , (4*c^3*Log[x - #1] + 6*c^2*d*Log[x - #1]*#1 +
4*c*d^2*Log[x - #1]*#1^2 + d^3*Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^
2*#1^2 + d^3*#1^3) & ])/(a + b*c^4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.35

method	result
default	$\frac{\ln(x)}{b\,c^4+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\left(d^3\_R^3+4c\,d^2\_R^2+6c^2d\_R+4c^3\right)\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4(b\,c^4+a)}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a^3b\,c^4+a^4\right)\_Z^4+4\_Z^3a^3+6a^2\_Z^2+4a\_Z\right)}\_R\ln\left(\left(\left(-3a^2b\,c^4d+5a^3d\right)\_R^3+\left(-3ab\,c^4d+15d\,a^2\right)\_R^2+\left(-b\,c^4d\right.\right.\right.\right.}{4}$

```
[In] int(1/x/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/(b*c^4+a)-1/4/(b*c^4+a)*sum((_R^3*d^3+4*_R^2*c*d^2+6*_R*c^2*d+4*c^3)/
(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(_Z^4*b*d^4+4*_Z^3
*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.19 (sec) , antiderivative size = 307773, normalized size of antiderivative = 783.14

$$\int \frac{1}{x\,(a+b(c+dx)^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(a+b*(d*x+c)**4),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx = \int \frac{1}{\left((dx+c)^4 b+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="maxima")
[Out] -b*d*integrate((d^3*x^3 + 4*c*d^2*x^2 + 6*c^2*d*x + 4*c^3)/(b*d^4*x^4 + 4*b
*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a), x)/(b*c^4 + a) + l
og(x)/(b*c^4 + a)
```

Giac [F]

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx = \int \frac{1}{\left((dx+c)^4 b+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="giac")
[Out] integrate(1/(((d*x + c)^4*b + a)*x), x)
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 882, normalized size of antiderivative = 2.24

$$\int \frac{1}{x\,(a+b(c+dx)^4)}\,dx = \frac{\ln(x)}{b\,c^4+a} + \left(\sum_{k=1}^{4} \ln\Big(-\mathrm{root}\big(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\big)^2\,b^5\,c^5\,d^{15}\,4 + \mathrm{root}\big(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\big)\,b^4\,c\,d^{15}\,4 + \mathrm{root}\big(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\big)\,b^4\,d^{16}\,x\,5 - \mathrm{root}\big(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\big)^4\,a^2\,b^5\,c^5\,d^{15}\,64 + \mathrm{root}\big(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\big)^2\,a\,b^4\,c\,d^{15}\,28 + \mathrm{root}\big(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\big)^2\,a\,b^4\,d^{16}\,x\,60 + \mathrm{root}\big(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\big)^3\,a^2\,b^4\,c\,d^{15}\,32 - \mathrm{root}\big(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\big)^4\,a^3\,b^4\,c\,d^{15}\,64 - \mathrm{root}\big(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\big)^3\,a\,b^5\,c^5\,d^{15}\,32 + \mathrm{root}\big(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\big)^3\,a^2\,b^4\,d^{16}\,x\,240 + \mathrm{root}\big(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\big)^4\,a^3\,b^4\,d^{16}\,x\,320 - \mathrm{root}\big(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\big)^2\,b^5\,c^4\,d^{16}\,x\,4 - \mathrm{root}\big(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\big)^3\,a\,b^5\,c^4\,d^{16}\,x\,48 - \mathrm{root}\big(256\,a^3\,b\,c^4 + 256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\big)\Big)$$

```
[In] int(1/(x*(a + b*(c + d*x)^4)),x)
```

```
[Out] log(x)/(a + b*c^4) + symsum(log(4*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 25
6*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)*b^4*c*d^15 - 4*root(256*a^3*b*c^
4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^2*b^5*c^
5*d^15 + 5*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2
+ 16*a*z + 1, z, k)*b^4*d^16*x - 64*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 +
256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^2*b^5*c^5*d^15 + 28*root(2
56*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z,
k)^2*a*b^4*c*d^15 + 60*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 +
 96*a^2*z^2 + 16*a*z + 1, z, k)^2*a*b^4*d^16*x + 32*root(256*a^3*b*c^4*z^4
+ 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a^2*b^4*c*d^
15 - 64*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 1
6*a*z + 1, z, k)^4*a^3*b^4*c*d^15 - 32*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4
 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a*b^5*c^5*d^15 + 240*root
(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z
, k)^3*a^2*b^4*d^16*x + 320*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*
```

```
z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^3*b^4*d^16*x - 4*root(256*a^3*b*c^
4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^2*b^5*c^
4*d^16*x - 48*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z
^2 + 16*a*z + 1, z, k)^3*a*b^5*c^4*d^16*x - 192*root(256*a^3*b*c^4*z^4 + 25
6*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^2*b^5*c^4*d^16
*x)*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*
z + 1, z, k), k, 1, 4)
```

3.115 $\int \frac{1}{x^2(a+b(c+dx)^4)}\,dx$

Optimal result . . . 867
Rubi [A] (verified) . . . 868
Mathematica [C] (verified) . . . 873
Maple [C] (verified) . . . 874
Fricas [C] (verification not implemented) . . . 874
Sympy [F(-1)] . . . 874
Maxima [F] . . . 875
Giac [F] . . . 875
Mupad [B] (verification not implemented) . . . 875

Optimal result

Integrand size = 17, antiderivative size = 496

$$\begin{aligned}
\int \frac{1}{x^2\left(a+b(c+dx)^4\right)}\,dx &= -\frac{1}{(a+bc^4)\,x}-\frac{\sqrt{b}c(a-bc^4)\,d\arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,(a+bc^4)^2} \\
&+\frac{\sqrt[4]{b}\left(\sqrt{a}(a-3bc^4)+\sqrt{b}c^2(3a-bc^4)\right)d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(a+bc^4)^2} \\
&-\frac{\sqrt[4]{b}\left(\sqrt{a}(a-3bc^4)+\sqrt{b}c^2(3a-bc^4)\right)d\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(a+bc^4)^2}-\frac{4bc^3d\log(x)}{(a+bc^4)^2} \\
&-\frac{\sqrt[4]{b}\left(\sqrt{a}(a-3bc^4)-\sqrt{b}c^2(3a-bc^4)\right)d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\,(a+bc^4)^2} \\
&+\frac{\sqrt[4]{b}\left(\sqrt{a}(a-3bc^4)-\sqrt{b}c^2(3a-bc^4)\right)d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\,(a+bc^4)^2} \\
&+\frac{bc^3d\log\left(a+b(c+dx)^4\right)}{(a+bc^4)^2}
\end{aligned}$$

```
[Out] -1/(b*c^4+a)/x-4*b*c^3*d*ln(x)/(b*c^4+a)^2+b*c^3*d*ln(a+b*(d*x+c)^4)/(b*c^4
+a)^2-c*(-b*c^4+a)*d*arctan((d*x+c)^2*b^(1/2)/a^(1/2))*b^(1/2)/(b*c^4+a)^2/
a^(1/2)-1/8*b^(1/4)*d*ln(-a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2
*b^(1/2))*((-3*b*c^4+a)*a^(1/2)-c^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^4+a)
^2*2^(1/2)+1/8*b^(1/4)*d*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)
^2*b^(1/2))*((-3*b*c^4+a)*a^(1/2)-c^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^4+
```

```
a)^2*2^(1/2)-1/4*b^(1/4)*d*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*((-3*
b*c^4+a)*a^(1/2)+c^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^4+a)^2*2^(1/2)-1/4*
b^(1/4)*d*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*((-3*b*c^4+a)*a^(1/2)+c
^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^4+a)^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {378, 6857, 1890, 1262, 649, 211, 266, 1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}
&\int \frac{1}{x^2 (a+b(c+dx)^4)} \, dx \\
&= \frac{\sqrt[4]{b} d \left(\sqrt{a}(a-3bc^4)+\sqrt{b}c^2(3a-bc^4)\right) \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(a+bc^4)^2} \\
&- \frac{\sqrt[4]{b} d \left(\sqrt{a}(a-3bc^4)+\sqrt{b}c^2(3a-bc^4)\right) \arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}(a+bc^4)^2} \\
&- \frac{\sqrt[4]{b} d \left(\sqrt{a}(a-3bc^4)-\sqrt{b}c^2(3a-bc^4)\right) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}(a+bc^4)^2} \\
&+ \frac{\sqrt[4]{b} d \left(\sqrt{a}(a-3bc^4)-\sqrt{b}c^2(3a-bc^4)\right) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}(a+bc^4)^2} \\
&- \frac{\sqrt{b}cd(a-bc^4) \arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}(a+bc^4)^2} - \frac{1}{x(a+bc^4)} \\
&- \frac{4bc^3 d \log(x)}{(a+bc^4)^2} + \frac{bc^3 d \log\left(a+b(c+dx)^4\right)}{(a+bc^4)^2}
\end{aligned}$$

```
[In] Int[1/(x^2*(a + b*(c + d*x)^4)),x]
```

```
[Out] -(1/((a + b*c^4)*x)) - (Sqrt[b]*c*(a - b*c^4)*d*ArcTan[(Sqrt[b]*(c + d*x)^2
)/Sqrt[a]])/(Sqrt[a]*(a + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) + Sqr
t[b]*c^2*(3*a - b*c^4))*d*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) - (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) + Sqrt[
b]*c^2*(3*a - b*c^4))*d*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2
*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) - (4*b*c^3*d*Log[x])/(a + b*c^4)^2 - (b^(1/
4)*(Sqrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a - b*c^4))*d*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*(a
 + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a - b*c^4))
*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/
(4*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) + (b*c^3*d*Log[a + b*(c + d*x)^4])/(a + b
*c^4)^2
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

<u>Rule 211</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

<u>Rule 266</u>

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

<u>Rule 378</u>

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

<u>Rule 631</u>

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

<u>Rule 642</u>

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

<u>Rule 649</u>

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

<u>Rule 1176</u>

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\text{Subst}\left(\int \frac{1}{(-c+x)^2\,(a+bx^4)}\,dx, x, c+dx\right)\\
&= d\text{Subst}\left(\int \left(\frac{1}{(a+bc^4)\,(c-x)^2}+\frac{4bc^3}{(a+bc^4)^2\,(c-x)}\right.\right.\\
&\qquad \left.\left.+\frac{b(-c^2(3a-bc^4)-2c(a-bc^4)\,x-(a-3bc^4)\,x^2+4bc^3x^3)}{(a+bc^4)^2\,(a+bx^4)}\right)\,dx, x, c+dx\right)
\end{aligned}$$

$$= -\frac{1}{(a+bc^4)\,x} - \frac{4bc^3d\log(x)}{(a+bc^4)^2} + \frac{(bd)\text{Subst}\left(\int \frac{-c^2(3a-bc^4)-2c(a-bc^4)x-(a-3bc^4)x^2+4bc^3x^3}{a+bx^4}\,dx, x, c+dx\right)}{(a+bc^4)^2}$$

$$= -\frac{1}{(a+bc^4)\,x} - \frac{4bc^3d\log(x)}{(a+bc^4)^2} + \frac{(bd)\text{Subst}\left(\int \left(\frac{x(-2c(a-bc^4)+4bc^3x^2)}{a+bx^4} + \frac{-c^2(3a-bc^4)+(-a+3bc^4)x^2}{a+bx^4}\right)\,dx, x, c+dx\right)}{(a+bc^4)^2}$$

$$= -\frac{1}{(a+bc^4)\,x} - \frac{4bc^3d\log(x)}{(a+bc^4)^2} + \frac{(bd)\text{Subst}\left(\int \frac{x(-2c(a-bc^4)+4bc^3x^2)}{a+bx^4}\,dx, x, c+dx\right)}{(a+bc^4)^2} + \frac{(bd)\text{Subst}\left(\int \frac{-c^2(3a-bc^4)+(-a+3bc^4)x^2}{a+bx^4}\,dx, x, c+dx\right)}{(a+bc^4)^2}$$

$$= -\frac{1}{(a+bc^4)\,x} - \frac{4bc^3d\log(x)}{(a+bc^4)^2} + \frac{(bd)\text{Subst}\left(\int \frac{-2c(a-bc^4)+4bc^3x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2\,(a+bc^4)^2} + \frac{\left(\left(a-3bc^4-\frac{\sqrt{b}c^2(3a-bc^4)}{\sqrt{a}}\right)d\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx, x, c+dx\right)}{2\,(a+bc^4)^2} - \frac{\left(\left(a-3bc^4+\frac{\sqrt{b}c^2(3a-bc^4)}{\sqrt{a}}\right)d\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx, x, c+dx\right)}{2\,(a+bc^4)^2}$$

$$= -\frac{1}{(a+bc^4)\,x} - \frac{4bc^3 d\log(x)}{(a+bc^4)^2} + \frac{(2b^2c^3d)\,\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{(a+bc^4)^2}$$

$$- \frac{(bc(a-bc^4)\,d)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right)}{(a+bc^4)^2}$$

$$- \frac{\left(\sqrt[4]{b}\left(a-3bc^4-\frac{\sqrt{b}c^2(3a-bc^4)}{\sqrt{a}}\right)d\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, c+dx\right)}{4\sqrt{2}\sqrt[4]{a}\,(a+bc^4)^2}$$

$$- \frac{\left(\sqrt[4]{b}\left(a-3bc^4-\frac{\sqrt{b}c^2(3a-bc^4)}{\sqrt{a}}\right)d\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx, x, c+dx\right)}{4\sqrt{2}\sqrt[4]{a}\,(a+bc^4)^2}$$

$$- \frac{\left(\left(a-3bc^4+\frac{\sqrt{b}c^2(3a-bc^4)}{\sqrt{a}}\right)d\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4\,(a+bc^4)^2}$$

$$- \frac{\left(\left(a-3bc^4+\frac{\sqrt{b}c^2(3a-bc^4)}{\sqrt{a}}\right)d\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4\,(a+bc^4)^2}$$

$$= -\frac{1}{(a+bc^4)\,x} - \frac{\sqrt{b}c(a-bc^4)\,d\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,(a+bc^4)^2} - \frac{4bc^3 d\log(x)}{(a+bc^4)^2}$$

$$- \frac{\sqrt[4]{b}\left(a-3bc^4-\frac{\sqrt{b}c^2(3a-bc^4)}{\sqrt{a}}\right)d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(a+bc^4)^2}$$

$$+ \frac{\sqrt[4]{b}\left(a-3bc^4-\frac{\sqrt{b}c^2(3a-bc^4)}{\sqrt{a}}\right)d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(a+bc^4)^2}$$

$$+ \frac{bc^3 d\log\left(a+b(c+dx)^4\right)}{(a+bc^4)^2}$$

$$- \frac{\left(\sqrt[4]{b}\left(a-3bc^4+\frac{\sqrt{b}c^2(3a-bc^4)}{\sqrt{a}}\right)d\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(a+bc^4)^2}$$

$$+ \frac{\left(\sqrt[4]{b}\left(a-3bc^4+\frac{\sqrt{b}c^2(3a-bc^4)}{\sqrt{a}}\right)d\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(a+bc^4)^2}$$

$$= -\frac{1}{(a+bc^4)\,x} - \frac{\sqrt{b}c(a-bc^4)\,d\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,(a+bc^4)^2}$$
$$+ \frac{\sqrt[4]{b}\left(a-3bc^4+\frac{\sqrt{b}c^2(3a-bc^4)}{\sqrt{a}}\right)d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(a+bc^4)^2}$$
$$- \frac{\sqrt[4]{b}\left(a-3bc^4+\frac{\sqrt{b}c^2(3a-bc^4)}{\sqrt{a}}\right)d\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(a+bc^4)^2} - \frac{4bc^3d\log(x)}{(a+bc^4)^2}$$
$$- \frac{\sqrt[4]{b}\left(a-3bc^4-\frac{\sqrt{b}c^2(3a-bc^4)}{\sqrt{a}}\right)d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(a+bc^4)^2}$$
$$+ \frac{\sqrt[4]{b}\left(a-3bc^4-\frac{\sqrt{b}c^2(3a-bc^4)}{\sqrt{a}}\right)d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(a+bc^4)^2}$$
$$+ \frac{bc^3d\log\left(a+b(c+dx)^4\right)}{(a+bc^4)^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^2\,(a+b(c+dx)^4)}\,dx$$
$$= \frac{-4(a+bc^4+4bc^3dx\log(x))+dx\text{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&,\ \frac{-6c}{}\right.}{}$$

```
[In] Integrate[1/(x^2*(a + b*(c + d*x)^4)),x]
```

```
[Out] (-4*(a + b*c^4 + 4*b*c^3*d*x*Log[x]) + d*x*RootSum[a + b*c^4 + 4*b*c^3*d*#1
 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^4*#1^4 & , (-6*a*c^2*Log[x - #1]
 + 10*b*c^6*Log[x - #1] - 4*a*c*d*Log[x - #1]*#1 + 20*b*c^5*d*Log[x - #1]*#
1 - a*d^2*Log[x - #1]*#1^2 + 15*b*c^4*d^2*Log[x - #1]*#1^2 + 4*b*c^3*d^3*Lo
g[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) & ])/(4*(a + b
*c^4)^2*x)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.37

method	result
default	$-\frac{1}{(b\,c^4+a)x}-\frac{4b\,c^3 d\ln(x)}{(b\,c^4+a)^2}-\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\left(-4b\,d^3c^3\_R^3+d^2\left(-15b\,c^4+a\right)\_\right.}{d^3\_R^3}\right.}{4(b\,c^4+a)^2}$
risch	$-\frac{1}{(b\,c^4+a)x}-\frac{4b\,c^3d\ln(x)}{b^2c^8+2ab\,c^4+a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^3b^2c^8+2a^4b\,c^4+a^5\right)\_Z^4-16a^3b\,c^3d\_Z^3+20a^2b\,c^2d^2\_Z^2-8abc\,d^3\_Z+b\,d^4\right)}\_R\ln\right.}{}$

```
[In] int(1/x^2/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(b*c^4+a)/x-4*b*c^3*d*ln(x)/(b*c^4+a)^2-1/4*d/(b*c^4+a)^2*sum((-4*b*d^3*
c^3*_R^3+d^2*(-15*b*c^4+a)*_R^2+4*c*d*(-5*b*c^4+a)*_R-10*b*c^6+6*a*c^2)/(_R
^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(_Z^4*b*d^4+4*_Z^3*b*
c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 78.19 (sec) , antiderivative size = 1128605, normalized size of antiderivative = 2275.41

$$\int \frac{1}{x^2\left(a+b(c+dx)^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+b(c+dx)^4\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**2/(a+b*(d*x+c)**4),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x^2 (a + b(c + dx)^4)} \, dx = \int \frac{1}{((dx + c)^4 b + a)x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -4*b*c^3*d*log(x)/(b^2*c^8 + 2*a*b*c^4 + a^2) + b*d^2*integrate((4*b*c^3*d^
3*x^3 + 10*b*c^6 + (15*b*c^4 - a)*d^2*x^2 - 6*a*c^2 + 4*(5*b*c^5 - a*c)*d*x
)/(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a),
x)/(b^2*c^8 + 2*a*b*c^4 + a^2) - 1/((b*c^4 + a)*x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + b(c + dx)^4)} \, dx = \int \frac{1}{((dx + c)^4 b + a)x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^4*b + a)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 2440, normalized size of antiderivative = 4.92

$$\int \frac{1}{x^2 (a + b(c + dx)^4)} \, dx = \text{Too large to display}$$

```
[In] int(1/(x^2*(a + b*(c + d*x)^4)),x)
```

```
[Out] symsum(log(-(4*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 -
 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z,
k)^2*b^7*c^11*d^17 - 16*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*
a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b
*d^4, z, k)^3*a^4*b^4*d^16 - b^5*d^20*x + 16*root(256*a^3*b^2*c^8*z^4 + 512
*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2
 - 32*a*b*c*d^3*z + b*d^4, z, k)*b^6*c^6*d^18 - 60*root(256*a^3*b^2*c^8*z^4
 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d
^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a^2*b^5*c^3*d^17 + 176*root(256*a^
3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 32
0*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^3*b^5*c^4*d^16 + 19
2*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c
^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^4*b^5*
```

c^5*d^15 + 144*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^2*b^6*c^8*d^16 + 192*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^3*b^6*c^9*d^15 + 64*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^2*b^7*c^13*d^15 + 16*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)*b^6*c^5*d^19*x + 64*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^5*b^4*c*d^15 - 184*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a*b^6*c^7*d^17 - 48*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a*b^7*c^12*d^16 - 320*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^5*b^4*d^16*x + 4*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*b^7*c^10*d^18*x - 248*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a*b^6*c^6*d^18*x - 64*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a*b^7*c^11*d^17*x + 32*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)*a*b^5*c*d^19*x - 316*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a^2*b^5*c^2*d^18*x + 704*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^3*b^5*c^3*d^17*x - 448*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^4*b^5*c^4*d^16*x + 640*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^2*b^6*c^7*d^17*x + 64*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^3*b^6*c^8*d^16*x + 192*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^2*b^7*c^12*d^16*x)/(a^2 + b^2*c^8 + 2*a*b*c^4))*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k), k, 1, 4) - 1/(a*x + b*c^4*x) - (4*b*c^3*d*log(x))/(a^2 + b^2*c^8 + 2*a*b*c^4)

3.116 $\int (a+8x-8x^2+4x^3-x^4)^4\,dx$

Optimal result 877
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Optimal result

Integrand size = 22, antiderivative size = 123

$$\begin{aligned}\int (a+8x-8x^2+4x^3-x^4)^4\,dx &= -\frac{8}{3}(3+a)^3(-1+x)^3+\frac{4}{5}(3-a)(3+a)^2(-1+x)^5\\&\quad+\frac{8}{7}(3+a)(5+3a)(-1+x)^7-\frac{2}{9}\left(37+6a-3a^2\right)(-1+x)^9\\&\quad-\frac{8}{11}(5+3a)(-1+x)^{11}+\frac{4}{13}(3-a)(-1+x)^{13}\\&\quad+\frac{8}{15}(-1+x)^{15}+\frac{1}{17}(-1+x)^{17}+(3+a)^4x\end{aligned}$$

```
[Out] -8/3*(3+a)^3*(-1+x)^3+4/5*(3-a)*(3+a)^2*(-1+x)^5+8/7*(3+a)*(5+3*a)*(-1+x)^7
-2/9*(-3*a^2+6*a+37)*(-1+x)^9-8/11*(5+3*a)*(-1+x)^11+4/13*(3-a)*(-1+x)^13+8
/15*(-1+x)^15+1/17*(-1+x)^17+(3+a)^4*x
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1120, 1104}

$$\begin{aligned}\int (a+8x-8x^2+4x^3-x^4)^4\,dx &= -\frac{2}{9}\left(-3a^2+6a+37\right)(x-1)^9+\frac{4}{13}(3-a)(x-1)^{13}\\&\quad-\frac{8}{11}(3a+5)(x-1)^{11}+\frac{8}{7}(a+3)(3a+5)(x-1)^7\\&\quad+\frac{4}{5}(3-a)(a+3)^2(x-1)^5-\frac{8}{3}(a+3)^3(x-1)^3\\&\quad+(a+3)^4x+\frac{1}{17}(x-1)^{17}+\frac{8}{15}(x-1)^{15}\end{aligned}$$

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (-8*(3 + a)^3*(-1 + x)^3)/3 + (4*(3 - a)*(3 + a)^2*(-1 + x)^5)/5 + (8*(3 +
a)*(5 + 3*a)*(-1 + x)^7)/7 - (2*(37 + 6*a - 3*a^2)*(-1 + x)^9)/9 - (8*(5 +
3*a)*(-1 + x)^11)/11 + (4*(3 - a)*(-1 + x)^13)/13 + (8*(-1 + x)^15)/15 + (-
1 + x)^17/17 + (3 + a)^4*x
```

Rule 1104

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \left(3+a-2x^2-x^4\right)^4\,dx, x, -1+x\right)\\
&= \text{Subst}\Bigg(\int \Bigg(81\left(1+\frac{1}{81}a\left(108+54a+12a^2+a^3\right)\right)-216\left(1+a\left(1+\frac{1}{27}a(9+a)\right)\right)x^2\\
&\quad +108\left(1-\frac{1}{27}a\left(-9+3a+a^2\right)\right)x^4+120\left(1+\frac{1}{15}a(14+3a)\right)x^6\\
&\quad -74\left(1-\frac{3}{37}(-2+a)a\right)x^8-40\left(1+\frac{3a}{5}\right)x^{10}+12\left(1-\frac{a}{3}\right)x^{12}+8x^{14}+x^{16}\Bigg)\,dx, x,\\
&\quad -1+x\Bigg)\\
&= -\frac{8}{3}(3+a)^3(-1+x)^3+\frac{4}{5}(3-a)(3+a)^2(-1+x)^5+\frac{8}{7}(3+a)(5+3a)(-1+x)^7\\
&\quad -\frac{2}{9}\left(37+6a-3a^2\right)(-1+x)^9-\frac{8}{11}(5+3a)(-1+x)^{11}\\
&\quad +\frac{4}{13}(3-a)(-1+x)^{13}+\frac{8}{15}(-1+x)^{15}+\frac{1}{17}(-1+x)^{17}+(3+a)^4x
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.59

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^4\,dx = a^4x+16a^3x^2-\frac{32}{3}(-12+a)a^2x^3+4a\left(128-48a+a^2\right)x^4 -\frac{4}{5}\left(-1024+1536a-192a^2+a^3\right)x^5 -\frac{16}{3}\left(512-288a+15a^2\right)x^6+\frac{64}{7}\left(512-140a+3a^2\right)x^7 -6\left(896-128a+a^2\right)x^8+\frac{2}{9}\left(20480-1536a+3a^2\right)x^9 +\frac{16}{5}(-928+35a)x^{10}-\frac{32}{11}(-524+9a)x^{11} +\frac{4}{3}(-464+3a)x^{12}-\frac{4}{13}(-640+a)x^{13} -48x^{14}+\frac{128x^{15}}{15}-x^{16}+\frac{x^{17}}{17}$$

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] a^4*x + 16*a^3*x^2 - (32*(-12 + a)*a^2*x^3)/3 + 4*a*(128 - 48*a + a^2)*x^4
- (4*(-1024 + 1536*a - 192*a^2 + a^3)*x^5)/5 - (16*(512 - 288*a + 15*a^2)*x
^6)/3 + (64*(512 - 140*a + 3*a^2)*x^7)/7 - 6*(896 - 128*a + a^2)*x^8 + (2*(
20480 - 1536*a + 3*a^2)*x^9)/9 + (16*(-928 + 35*a)*x^10)/5 - (32*(-524 + 9*
a)*x^11)/11 + (4*(-464 + 3*a)*x^12)/3 - (4*(-640 + a)*x^13)/13 - 48*x^14 +
(128*x^15)/15 - x^16 + x^17/17
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.46

method	result
norman	$a^4x+16a^3x^2+\left(-\frac{32}{3}a^3+128a^2\right)x^3+\left(4a^3-192a^2+512a\right)x^4+\left(-\frac{4}{5}a^3+\frac{768}{5}a^2-\frac{6144}{5}a+\right.$
gosper	$-\frac{14848}{5}x^{10}+\frac{16768}{11}x^{11}-\frac{1856}{3}x^{12}+\frac{2560}{13}x^{13}+\frac{4096}{5}x^5+\frac{40960}{9}x^9+\frac{32768}{7}x^7+512a\,x^4-48x^{14}-$
risch	$-\frac{14848}{5}x^{10}+\frac{16768}{11}x^{11}-\frac{1856}{3}x^{12}+\frac{2560}{13}x^{13}+\frac{4096}{5}x^5+\frac{40960}{9}x^9+\frac{32768}{7}x^7+512a\,x^4-48x^{14}-$
parallelrisch	$-\frac{14848}{5}x^{10}+\frac{16768}{11}x^{11}-\frac{1856}{3}x^{12}+\frac{2560}{13}x^{13}+\frac{4096}{5}x^5+\frac{40960}{9}x^9+\frac{32768}{7}x^7+512a\,x^4-48x^{14}-$
default	$\frac{x^{17}}{17}-x^{16}+\frac{128x^{15}}{15}-48x^{14}+\frac{(-4a+2560)x^{13}}{13}+\frac{(48a-7424)x^{12}}{12}+\frac{(-288a+16768)x^{11}}{11}+\frac{(1120a-29696)x^{10}}{10}$

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*x+16*a^3*x^2+(-32/3*a^3+128*a^2)*x^3+(4*a^3-192*a^2+512*a)*x^4+(-4/5*a^
3+768/5*a^2-6144/5*a+4096/5)*x^5+(-80*a^2+1536*a-8192/3)*x^6+(192/7*a^2-128
0*a+32768/7)*x^7+(-6*a^2+768*a-5376)*x^8+(2/3*a^2-1024/3*a+40960/9)*x^9+(11
```

```
2*a-14848/5)*x^10+(-288/11*a+16768/11)*x^11+(4*a-1856/3)*x^12+(-4/13*a+2560
/13)*x^13-48*x^14+128/15*x^15-x^16+1/17*x^17
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.46

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^4\,dx = \frac{1}{17}\,x^{17}-x^{16}+\frac{128}{15}\,x^{15}-\frac{4}{13}\,(a-640)x^{13} - 48\,x^{14}+\frac{4}{3}\,(3\,a-464)x^{12}-\frac{32}{11}\,(9\,a-524)x^{11} + \frac{16}{5}\,(35\,a-928)x^{10}+\frac{2}{9}\,\left(3\,a^2-1536\,a+20480\right)x^9 - 6\,\left(a^2-128\,a+896\right)x^8+\frac{64}{7}\,\left(3\,a^2-140\,a+512\right)x^7 - \frac{16}{3}\,\left(15\,a^2-288\,a+512\right)x^6 - \frac{4}{5}\,\left(a^3-192\,a^2+1536\,a-1024\right)x^5+a^4x+16\,a^3x^2 + 4\,\left(a^3-48\,a^2+128\,a\right)x^4-\frac{32}{3}\,\left(a^3-12\,a^2\right)x^3$$

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

```
[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 4/13*(a - 640)*x^13 - 48*x^14 + 4/3*(3*a -
 464)*x^12 - 32/11*(9*a - 524)*x^11 + 16/5*(35*a - 928)*x^10 + 2/9*(3*a^2 -
 1536*a + 20480)*x^9 - 6*(a^2 - 128*a + 896)*x^8 + 64/7*(3*a^2 - 140*a + 51
2)*x^7 - 16/3*(15*a^2 - 288*a + 512)*x^6 - 4/5*(a^3 - 192*a^2 + 1536*a - 10
24)*x^5 + a^4*x + 16*a^3*x^2 + 4*(a^3 - 48*a^2 + 128*a)*x^4 - 32/3*(a^3 - 1
2*a^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.62

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^4\,dx = a^4x+16a^3x^2+\frac{x^{17}}{17}-x^{16}+\frac{128x^{15}}{15}-48x^{14}+x^{13}\cdot\left(\frac{2560}{13}-\frac{4a}{13}\right)+x^{12}\cdot\left(4a-\frac{1856}{3}\right)+x^{11}\cdot\left(\frac{16768}{11}-\frac{288a}{11}\right)+x^{10}\cdot\left(112a-\frac{14848}{5}\right)+x^9\cdot\left(\frac{2a^2}{3}-\frac{1024a}{3}+\frac{40960}{9}\right)+x^8\left(-6a^2+768a-5376\right)+x^7\cdot\left(\frac{192a^2}{7}-1280a+\frac{32768}{7}\right)+x^6\left(-80a^2+1536a-\frac{8192}{3}\right)+x^5\left(-\frac{4a^3}{5}+\frac{768a^2}{5}-\frac{6144a}{5}+\frac{4096}{5}\right)+x^4\cdot\left(4a^3-192a^2+512a\right)+x^3\left(-\frac{32a^3}{3}+128a^2\right)$$

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] a**4*x + 16*a**3*x**2 + x**17/17 - x**16 + 128*x**15/15 - 48*x**14 + x**13*
(2560/13 - 4*a/13) + x**12*(4*a - 1856/3) + x**11*(16768/11 - 288*a/11) + x
**10*(112*a - 14848/5) + x**9*(2*a**2/3 - 1024*a/3 + 40960/9) + x**8*(-6*a*
*2 + 768*a - 5376) + x**7*(192*a**2/7 - 1280*a + 32768/7) + x**6*(-80*a**2
+ 1536*a - 8192/3) + x**5*(-4*a**3/5 + 768*a**2/5 - 6144*a/5 + 4096/5) + x*
*4*(4*a**3 - 192*a**2 + 512*a) + x**3*(-32*a**3/3 + 128*a**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.56

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^4\,dx = \frac{1}{17}\,x^{17}-x^{16}+\frac{128}{15}\,x^{15}-48\,x^{14}+\frac{2560}{13}\,x^{13}-\frac{1856}{3}\,x^{12}+\frac{16768}{11}\,x^{11}-\frac{14848}{5}\,x^{10}+\frac{40960}{9}\,x^9-5376\,x^8+\frac{32768}{7}\,x^7-\frac{8192}{3}\,x^6+a^4x+\frac{4096}{5}\,x^5-\frac{4}{15}\left(3\,x^5-15\,x^4+40\,x^3-60\,x^2\right)a^3+\frac{2}{105}\left(35\,x^9-315\,x^8+1440\,x^7-4200\,x^6+8064\,x^5-10080\,x^4+6720\,x^3\right)a^2-\frac{4}{2145}\left(165\,x^{13}-2145\,x^{12}+14040\,x^{11}-60060\,x^{10}+183040\,x^9-411840\,x^8+686400\,x^7-823680\,x^6+\right.$$

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

```
[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 48*x^14 + 2560/13*x^13 - 1856/3*x^12 + 167
68/11*x^11 - 14848/5*x^10 + 40960/9*x^9 - 5376*x^8 + 32768/7*x^7 - 8192/3*x
^6 + a^4*x + 4096/5*x^5 - 4/15*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a^3 + 2/1
05*(35*x^9 - 315*x^8 + 1440*x^7 - 4200*x^6 + 8064*x^5 - 10080*x^4 + 6720*x^
3)*a^2 - 4/2145*(165*x^13 - 2145*x^12 + 14040*x^11 - 60060*x^10 + 183040*x^
9 - 411840*x^8 + 686400*x^7 - 823680*x^6 + 658944*x^5 - 274560*x^4)*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(103) = 206$.

Time = 0.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.78

$$\begin{aligned}\int \left(a+8x-8x^2+4x^3-x^4\right)^4\,dx &= \frac{1}{17}\,x^{17}-x^{16}+\frac{128}{15}\,x^{15}-\frac{4}{13}\,ax^{13}-48\,x^{14}+4\,ax^{12}\\&+\frac{2560}{13}\,x^{13}-\frac{288}{11}\,ax^{11}-\frac{1856}{3}\,x^{12}+\frac{2}{3}\,a^2x^9+112\,ax^{10}\\&+\frac{16768}{11}\,x^{11}-6\,a^2x^8-\frac{1024}{3}\,ax^9-\frac{14848}{5}\,x^{10}\\&+\frac{192}{7}\,a^2x^7+768\,ax^8+\frac{40960}{9}\,x^9-\frac{4}{5}\,a^3x^5-80\,a^2x^6\\&-1280\,ax^7-5376\,x^8+4\,a^3x^4+\frac{768}{5}\,a^2x^5+1536\,ax^6\\&+\frac{32768}{7}\,x^7-\frac{32}{3}\,a^3x^3-192\,a^2x^4-\frac{6144}{5}\,ax^5-\frac{8192}{3}\,x^6\\&+a^4x+16\,a^3x^2+128\,a^2x^3+512\,ax^4+\frac{4096}{5}\,x^5\end{aligned}$$

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

```
[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 4/13*a*x^13 - 48*x^14 + 4*a*x^12 + 2560/13
*x^13 - 288/11*a*x^11 - 1856/3*x^12 + 2/3*a^2*x^9 + 112*a*x^10 + 16768/11*x
^11 - 6*a^2*x^8 - 1024/3*a*x^9 - 14848/5*x^10 + 192/7*a^2*x^7 + 768*a*x^8 +
 40960/9*x^9 - 4/5*a^3*x^5 - 80*a^2*x^6 - 1280*a*x^7 - 5376*x^8 + 4*a^3*x^4
 + 768/5*a^2*x^5 + 1536*a*x^6 + 32768/7*x^7 - 32/3*a^3*x^3 - 192*a^2*x^4 -
6144/5*a*x^5 - 8192/3*x^6 + a^4*x + 16*a^3*x^2 + 128*a^2*x^3 + 512*a*x^4 +
4096/5*x^5
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.42

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^4\,dx = x^{12}\left(4\,a-\frac{1856}{3}\right)-x^{13}\left(\frac{4\,a}{13}-\frac{2560}{13}\right)+x^{10}\left(112\,a-\frac{14848}{5}\right)-x^{11}\left(\frac{288\,a}{11}-\frac{16768}{11}\right)-x^8\left(6\,a^2-768\,a+5376\right)-x^6\left(80\,a^2-1536\,a+\frac{8192}{3}\right)+x^7\left(\frac{192\,a^2}{7}-1280\,a+\frac{32768}{7}\right)+x^9\left(\frac{2\,a^2}{3}-\frac{1024\,a}{3}+\frac{40960}{9}\right)-x^5\left(\frac{4\,a^3}{5}-\frac{768\,a^2}{5}+\frac{6144\,a}{5}-\frac{4096}{5}\right)+a^4\,x-48\,x^{14}+\frac{128\,x^{15}}{15}-x^{16}+\frac{x^{17}}{17}+16\,a^3\,x^2+4\,a\,x^4\left(a^2-48\,a+128\right)-\frac{32\,a^2\,x^3\,(a-12)}{3}$$

```
[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)
```

```
[Out] x^12*(4*a - 1856/3) - x^13*((4*a)/13 - 2560/13) + x^10*(112*a - 14848/5) -
x^11*((288*a)/11 - 16768/11) - x^8*(6*a^2 - 768*a + 5376) - x^6*(80*a^2 - 1
536*a + 8192/3) + x^7*((192*a^2)/7 - 1280*a + 32768/7) + x^9*((2*a^2)/3 - (
1024*a)/3 + 40960/9) - x^5*((6144*a)/5 - (768*a^2)/5 + (4*a^3)/5 - 4096/5)
+ a^4*x - 48*x^14 + (128*x^15)/15 - x^16 + x^17/17 + 16*a^3*x^2 + 4*a*x^4*(
a^2 - 48*a + 128) - (32*a^2*x^3*(a - 12))/3
```

3.117 $\int (a+8x-8x^2+4x^3-x^4)^3\,dx$

Optimal result . . . 884
Rubi [A] (verified) . . . 884
Mathematica [A] (verified) . . . 885
Maple [A] (verified) . . . 885
Fricas [A] (verification not implemented) . . . 886
Sympy [A] (verification not implemented) . . . 886
Maxima [A] (verification not implemented) . . . 887
Giac [A] (verification not implemented) . . . 887
Mupad [B] (verification not implemented) . . . 888

Optimal result

Integrand size = 22, antiderivative size = 120

$$\int (a+8x-8x^2+4x^3-x^4)^3\,dx = a^3x+12a^2x^2+8(8-a)ax^3+(128-96a+3a^2)\,x^4 -\frac{3}{5}(512-128a+a^2)\,x^5+8(48-5a)x^6-\frac{32}{7}(70-3a)x^7 +3(64-a)x^8-\frac{1}{3}(256-a)x^9+28x^{10}-\frac{72x^{11}}{11}+x^{12}-\frac{x^{13}}{13}$$

```
[Out] a^3*x+12*a^2*x^2+8*(8-a)*a*x^3+(3*a^2-96*a+128)*x^4-3/5*(a^2-128*a+512)*x^5
+8*(48-5*a)*x^6-32/7*(70-3*a)*x^7+3*(64-a)*x^8-1/3*(256-a)*x^9+28*x^10-72/1
1*x^11+x^12-1/13*x^13
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2086}

$$\int (a+8x-8x^2+4x^3-x^4)^3\,dx = a^3x-\frac{3}{5}(a^2-128a+512)\,x^5+(3a^2-96a+128)\,x^4 +12a^2x^2-\frac{1}{3}(256-a)x^9+3(64-a)x^8-\frac{32}{7}(70-3a)x^7 +8(48-5a)x^6+8(8-a)ax^3-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}$$

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] a^3*x + 12*a^2*x^2 + 8*(8 - a)*a*x^3 + (128 - 96*a + 3*a^2)*x^4 - (3*(512 -
 128*a + a^2)*x^5)/5 + 8*(48 - 5*a)*x^6 - (32*(70 - 3*a)*x^7)/7 + 3*(64 - a
)*x^8 - ((256 - a)*x^9)/3 + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^3+24a^2x+24(8-a)ax^2+4\left(128-96a+3a^2\right)x^3-3\left(512-128a+a^2\right)x^4\right.\\&\quad+48(48-5a)x^5-32(70-3a)x^6+24(64-a)x^7-3(256-a)x^8+280x^9-72x^{10}\\&\quad\left.+12x^{11}-x^{12}\right)dx\\&=a^3x+12a^2x^2+8(8-a)ax^3+\left(128-96a+3a^2\right)x^4-\frac{3}{5}\left(512-128a+a^2\right)x^5+8(48\\&\quad-5a)x^6-\frac{32}{7}(70-3a)x^7+3(64-a)x^8-\frac{1}{3}(256-a)x^9+28x^{10}-\frac{72x^{11}}{11}+x^{12}-\frac{x^{13}}{13}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \left(a+8x-8x^2+4x^3-x^4\right)^3 dx &= a^3x+12a^2x^2-8(-8+a)ax^3+\left(128-96a+3a^2\right)x^4\\&\quad-\frac{3}{5}\left(512-128a+a^2\right)x^5-8(-48+5a)x^6\\&\quad+\frac{32}{7}(-70+3a)x^7-3(-64+a)x^8\\&\quad+\frac{1}{3}(-256+a)x^9+28x^{10}-\frac{72x^{11}}{11}+x^{12}-\frac{x^{13}}{13}\end{aligned}$$

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
[Out] a^3*x + 12*a^2*x^2 - 8*(-8 + a)*a*x^3 + (128 - 96*a + 3*a^2)*x^4 - (3*(512
- 128*a + a^2)*x^5)/5 - 8*(-48 + 5*a)*x^6 + (32*(-70 + 3*a)*x^7)/7 - 3*(-64
 + a)*x^8 + ((-256 + a)*x^9)/3 + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

method	result
norman	$-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}+\left(\frac{a}{3}-\frac{256}{3}\right)x^{9}+\left(-3a+192\right)x^{8}+\left(\frac{96a}{7}-320\right)x^{7}+\left(-40a+38\right.$
gosper	$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+28x^{10}+\frac{1}{3}x^{9}a-\frac{256}{3}x^{9}-3a\,x^{8}+192x^{8}+\frac{96}{7}a\,x^{7}-320x^{7}-40a\,x^{6}+$
risch	$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+28x^{10}+\frac{1}{3}x^{9}a-\frac{256}{3}x^{9}-3a\,x^{8}+192x^{8}+\frac{96}{7}a\,x^{7}-320x^{7}-40a\,x^{6}+$
parallelrisch	$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+28x^{10}+\frac{1}{3}x^{9}a-\frac{256}{3}x^{9}-3a\,x^{8}+192x^{8}+\frac{96}{7}a\,x^{7}-320x^{7}-40a\,x^{6}+$
default	$-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}+\frac{(3a-768)x^{9}}{9}+\frac{(-24a+1536)x^{8}}{8}+\frac{(96a-2240)x^{7}}{7}+\frac{(-240a+2304)x^{6}}{6}+\frac{(a(-2a}{}$

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
[Out] -1/13*x^13+x^12-72/11*x^11+28*x^10+(1/3*a-256/3)*x^9+(-3*a+192)*x^8+(96/7*a
-320)*x^7+(-40*a+384)*x^6+(-3/5*a^2+384/5*a-1536/5)*x^5+(3*a^2-96*a+128)*x^
4+(-8*a^2+64*a)*x^3+12*a^2*x^2+a^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \left(a+8x-8x^2+4x^3-x^4\right)^3\,dx = &-\frac{1}{13}\,x^{13}+x^{12}-\frac{72}{11}\,x^{11}+\frac{1}{3}\,(a-256)x^9+28\,x^{10}\\&-3\,(a-64)x^8+\frac{32}{7}\,(3\,a-70)x^7-8\,(5\,a-48)x^6\\&-\frac{3}{5}\,\left(a^2-128\,a+512\right)x^5+\left(3\,a^2-96\,a+128\right)x^4\\&+a^3x+12\,a^2x^2-8\,\left(a^2-8\,a\right)x^3\end{aligned}$$

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 1/3*(a - 256)*x^9 + 28*x^10 - 3*(a - 64)*x
^8 + 32/7*(3*a - 70)*x^7 - 8*(5*a - 48)*x^6 - 3/5*(a^2 - 128*a + 512)*x^5 +
 (3*a^2 - 96*a + 128)*x^4 + a^3*x + 12*a^2*x^2 - 8*(a^2 - 8*a)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \left(a+8x-8x^2+4x^3-x^4\right)^3\,dx = &\ a^3x+12a^2x^2-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}\\&+x^9\left(\frac{a}{3}-\frac{256}{3}\right)+x^8\cdot(192-3a)+x^7\cdot\left(\frac{96a}{7}-320\right)\\&+x^6\cdot(384-40a)+x^5\left(-\frac{3a^2}{5}+\frac{384a}{5}-\frac{1536}{5}\right)\\&+x^4\cdot\left(3a^2-96a+128\right)+x^3\left(-8a^2+64a\right)\end{aligned}$$

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x + 12*a**2*x**2 - x**13/13 + x**12 - 72*x**11/11 + 28*x**10 + x**9*(a
/3 - 256/3) + x**8*(192 - 3*a) + x**7*(96*a/7 - 320) + x**6*(384 - 40*a) +
x**5*(-3*a**2/5 + 384*a/5 - 1536/5) + x**4*(3*a**2 - 96*a + 128) + x**3*(-8
*a**2 + 64*a)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99

$$\begin{aligned}
&\int \left(a+8x-8x^2+4x^3-x^4\right)^3\,dx \\
&= -\frac{1}{13}\,x^{13}+x^{12}-\frac{72}{11}\,x^{11}+28\,x^{10}-\frac{256}{3}\,x^{9}+192\,x^{8}-320\,x^{7}+384\,x^{6} \\
&\quad -\frac{1536}{5}\,x^{5}+a^{3}x+128\,x^{4}-\frac{1}{5}\left(3\,x^{5}-15\,x^{4}+40\,x^{3}-60\,x^{2}\right)a^{2} \\
&\quad +\frac{1}{105}\left(35\,x^{9}-315\,x^{8}+1440\,x^{7}-4200\,x^{6}+8064\,x^{5}-10080\,x^{4}+6720\,x^{3}\right)a
\end{aligned}$$

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 28*x^10 - 256/3*x^9 + 192*x^8 - 320*x^7 +
384*x^6 - 1536/5*x^5 + a^3*x + 128*x^4 - 1/5*(3*x^5 - 15*x^4 + 40*x^3 - 60*
x^2)*a^2 + 1/105*(35*x^9 - 315*x^8 + 1440*x^7 - 4200*x^6 + 8064*x^5 - 10080
*x^4 + 6720*x^3)*a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.07

$$\begin{aligned}
\int \left(a+8x-8x^2+4x^3-x^4\right)^3\,dx = &-\frac{1}{13}\,x^{13}+x^{12}-\frac{72}{11}\,x^{11}+\frac{1}{3}\,ax^{9}+28\,x^{10}-3\,ax^{8} \\
&-\frac{256}{3}\,x^{9}+\frac{96}{7}\,ax^{7}+192\,x^{8}-\frac{3}{5}\,a^{2}x^{5}-40\,ax^{6} \\
&-320\,x^{7}+3\,a^{2}x^{4}+\frac{384}{5}\,ax^{5}+384\,x^{6}-8\,a^{2}x^{3} \\
&-96\,ax^{4}-\frac{1536}{5}\,x^{5}+a^{3}x+12\,a^{2}x^{2}+64\,ax^{3}+128\,x^{4}
\end{aligned}$$

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 1/3*a*x^9 + 28*x^10 - 3*a*x^8 - 256/3*x^9
+ 96/7*a*x^7 + 192*x^8 - 3/5*a^2*x^5 - 40*a*x^6 - 320*x^7 + 3*a^2*x^4 + 384
/5*a*x^5 + 384*x^6 - 8*a^2*x^3 - 96*a*x^4 - 1536/5*x^5 + a^3*x + 12*a^2*x^2
 + 64*a*x^3 + 128*x^4
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.90

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^3 \, dx = x^9 \left(\frac{a}{3}-\frac{256}{3}\right)-x^8 \left(3\,a-192\right)-x^6 \left(40\,a-384\right) + x^7 \left(\frac{96\,a}{7}-320\right)+x^4 \left(3\,a^2-96\,a+128\right) - x^5 \left(\frac{3\,a^2}{5}-\frac{384\,a}{5}+\frac{1536}{5}\right)+a^3\,x+28\,x^{10} - \frac{72\,x^{11}}{11}+x^{12}-\frac{x^{13}}{13}+12\,a^2\,x^2-8\,a\,x^3\,\left(a-8\right)$$

```
[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] x^9*(a/3 - 256/3) - x^8*(3*a - 192) - x^6*(40*a - 384) + x^7*((96*a)/7 - 32
0) + x^4*(3*a^2 - 96*a + 128) - x^5*((3*a^2)/5 - (384*a)/5 + 1536/5) + a^3*
x + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13 + 12*a^2*x^2 - 8*a*x^3*(a - 8)
```

3.118 $\int (a+8x-8x^2+4x^3-x^4)^2\,dx$

Optimal result 889
Rubi [A] (verified) 889
Mathematica [A] (verified) 890
Maple [A] (verified) 890
Fricas [A] (verification not implemented) 891
Sympy [A] (verification not implemented) 891
Maxima [A] (verification not implemented) 891
Giac [A] (verification not implemented) 892
Mupad [B] (verification not implemented) 892

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int (a+8x-8x^2+4x^3-x^4)^2\,dx = a^2x+8ax^2+\frac{16}{3}(4-a)x^3-2(16-a)x^4 +\frac{2}{5}(64-a)x^5-\frac{40x^6}{3}+\frac{32x^7}{7}-x^8+\frac{x^9}{9}$$

```
[Out] a^2*x+8*a*x^2+16/3*(4-a)*x^3-2*(16-a)*x^4+2/5*(64-a)*x^5-40/3*x^6+32/7*x^7-
x^8+1/9*x^9
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2086}

$$\int (a+8x-8x^2+4x^3-x^4)^2\,dx = a^2x+\frac{2}{5}(64-a)x^5-2(16-a)x^4+\frac{16}{3}(4-a)x^3 +8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}$$

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] a^2*x + 8*a*x^2 + (16*(4 - a)*x^3)/3 - 2*(16 - a)*x^4 + (2*(64 - a)*x^5)/5
- (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^2+16ax+16(4-a)x^2-8(16-a)x^3+2(64-a)x^4-80x^5+32x^6-8x^7+x^8\right)\,dx$$

$$= a^2x+8ax^2+\frac{16}{3}(4-a)x^3-2(16-a)x^4+\frac{2}{5}(64-a)x^5-\frac{40x^6}{3}+\frac{32x^7}{7}-x^8+\frac{x^9}{9}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^2\,dx = a^2x+8ax^2-\frac{16}{3}(-4+a)x^3+2(-16+a)x^4-\frac{2}{5}(-64+a)x^5-\frac{40x^6}{3}+\frac{32x^7}{7}-x^8+\frac{x^9}{9}$$

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] a^2*x + 8*a*x^2 - (16*(-4 + a)*x^3)/3 + 2*(-16 + a)*x^4 - (2*(-64 + a)*x^5)
/5 - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

method	result	size
norman	$\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}+\left(-\frac{2a}{5}+\frac{128}{5}\right)x^5+\left(2a-32\right)x^4+\left(-\frac{16a}{3}+\frac{64}{3}\right)x^3+8a\,x^2+a^2x$	60
default	$\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}+\frac{(-2a+128)x^5}{5}+\frac{(8a-128)x^4}{4}+\frac{(-16a+64)x^3}{3}+8a\,x^2+a^2x$	63
gosper	$\frac{1}{9}x^9-x^8+\frac{32}{7}x^7-\frac{40}{3}x^6-\frac{2}{5}a\,x^5+\frac{128}{5}x^5+2a\,x^4-32x^4-\frac{16}{3}a\,x^3+\frac{64}{3}x^3+8a\,x^2+a^2x$	66
risch	$\frac{1}{9}x^9-x^8+\frac{32}{7}x^7-\frac{40}{3}x^6-\frac{2}{5}a\,x^5+\frac{128}{5}x^5+2a\,x^4-32x^4-\frac{16}{3}a\,x^3+\frac{64}{3}x^3+8a\,x^2+a^2x$	66
parallelrisch	$\frac{1}{9}x^9-x^8+\frac{32}{7}x^7-\frac{40}{3}x^6-\frac{2}{5}a\,x^5+\frac{128}{5}x^5+2a\,x^4-32x^4-\frac{16}{3}a\,x^3+\frac{64}{3}x^3+8a\,x^2+a^2x$	66

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*x^9-x^8+32/7*x^7-40/3*x^6+(-2/5*a+128/5)*x^5+(2*a-32)*x^4+(-16/3*a+64/3
)*x^3+8*a*x^2+a^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{1}{9}\,x^9-x^8+\frac{32}{7}\,x^7-\frac{2}{5}\,(a-64)x^5-\frac{40}{3}\,x^6+2\,(a-16)x^4-\frac{16}{3}\,(a-4)x^3+a^2x+8\,ax^2$$

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 2/5*(a - 64)*x^5 - 40/3*x^6 + 2*(a - 16)*x^4 - 1
6/3*(a - 4)*x^3 + a^2*x + 8*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=a^2x+8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}+x^5\cdot\left(\frac{128}{5}-\frac{2a}{5}\right)+x^4\cdot(2a-32)+x^3\cdot\left(\frac{64}{3}-\frac{16a}{3}\right)$$

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**2,x)
[Out] a**2*x + 8*a*x**2 + x**9/9 - x**8 + 32*x**7/7 - 40*x**6/3 + x**5*(128/5 - 2
*a/5) + x**4*(2*a - 32) + x**3*(64/3 - 16*a/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{1}{9}\,x^9-x^8+\frac{32}{7}\,x^7-\frac{40}{3}\,x^6+\frac{128}{5}\,x^5-32\,x^4+a^2x+\frac{64}{3}\,x^3-\frac{2}{15}\,\left(3\,x^5-15\,x^4+40\,x^3-60\,x^2\right)a$$

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 40/3*x^6 + 128/5*x^5 - 32*x^4 + a^2*x + 64/3*x^3
 - 2/15*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^2 \, dx = \frac{1}{9}\,x^9 - x^8 + \frac{32}{7}\,x^7 - \frac{2}{5}\,ax^5 - \frac{40}{3}\,x^6 + 2\,ax^4 + \frac{128}{5}\,x^5 - \frac{16}{3}\,ax^3 - 32\,x^4 + a^2x + 8\,ax^2 + \frac{64}{3}\,x^3$$

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 2/5*a*x^5 - 40/3*x^6 + 2*a*x^4 + 128/5*x^5 - 16/
3*a*x^3 - 32*x^4 + a^2*x + 8*a*x^2 + 64/3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^2 \, dx = x^4\,(2\,a-32) - x^3\left(\frac{16\,a}{3}-\frac{64}{3}\right) - x^5\left(\frac{2\,a}{5}-\frac{128}{5}\right) + 8\,a\,x^2 + a^2\,x - \frac{40\,x^6}{3} + \frac{32\,x^7}{7} - x^8 + \frac{x^9}{9}$$

```
[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] x^4*(2*a - 32) - x^3*((16*a)/3 - 64/3) - x^5*((2*a)/5 - 128/5) + 8*a*x^2 +
a^2*x - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

3.119 $\int (a+8x-8x^2+4x^3-x^4)\,dx$

Optimal result 893
Rubi [A] (verified) 893
Mathematica [A] (verified) 894
Maple [A] (verified) 894
Fricas [A] (verification not implemented) 894
Sympy [A] (verification not implemented) 895
Maxima [A] (verification not implemented) 895
Giac [A] (verification not implemented) 895
Mupad [B] (verification not implemented) 895

Optimal result

Integrand size = 20, antiderivative size = 26

$$\int (a+8x-8x^2+4x^3-x^4)\,dx=ax+4x^2-\frac{8x^3}{3}+x^4-\frac{x^5}{5}$$

```
[Out] a*x+4*x^2-8/3*x^3+x^4-1/5*x^5
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int (a+8x-8x^2+4x^3-x^4)\,dx=ax-\frac{x^5}{5}+x^4-\frac{8x^3}{3}+4x^2$$

```
[In] Int[a + 8*x - 8*x^2 + 4*x^3 - x^4,x]
```

```
[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

Rubi steps

$$\text{integral}=ax+4x^2-\frac{8x^3}{3}+x^4-\frac{x^5}{5}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \left(a+8x-8x^2+4x^3-x^4\right)\,dx=ax+4x^2-\frac{8x^3}{3}+x^4-\frac{x^5}{5}$$

```
[In] Integrate[a + 8*x - 8*x^2 + 4*x^3 - x^4,x]
[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
gosper	$ax+4x^2-\frac{8}{3}x^3+x^4-\frac{1}{5}x^5$	23
default	$ax+4x^2-\frac{8}{3}x^3+x^4-\frac{1}{5}x^5$	23
norman	$ax+4x^2-\frac{8}{3}x^3+x^4-\frac{1}{5}x^5$	23
risch	$ax+4x^2-\frac{8}{3}x^3+x^4-\frac{1}{5}x^5$	23
parallelrisch	$ax+4x^2-\frac{8}{3}x^3+x^4-\frac{1}{5}x^5$	23
parts	$ax+4x^2-\frac{8}{3}x^3+x^4-\frac{1}{5}x^5$	23

```
[In] int(-x^4+4*x^3-8*x^2+a+8*x,x,method=_RETURNVERBOSE)
[Out] a*x+4*x^2-8/3*x^3+x^4-1/5*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \left(a+8x-8x^2+4x^3-x^4\right)\,dx=-\frac{1}{5}\,x^5+x^4-\frac{8}{3}\,x^3+ax+4\,x^2$$

```
[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="fricas")
[Out] -1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \left(a + 8x - 8x^2 + 4x^3 - x^4\right) \, dx = ax - \frac{x^5}{5} + x^4 - \frac{8x^3}{3} + 4x^2$$

```
[In] integrate(-x**4+4*x**3-8*x**2+a+8*x,x)
[Out] a*x - x**5/5 + x**4 - 8*x**3/3 + 4*x**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \left(a + 8x - 8x^2 + 4x^3 - x^4\right) \, dx = -\frac{1}{5}\,x^5 + x^4 - \frac{8}{3}\,x^3 + ax + 4\,x^2$$

```
[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="maxima")
[Out] -1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \left(a + 8x - 8x^2 + 4x^3 - x^4\right) \, dx = -\frac{1}{5}\,x^5 + x^4 - \frac{8}{3}\,x^3 + ax + 4\,x^2$$

```
[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="giac")
[Out] -1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \left(a + 8x - 8x^2 + 4x^3 - x^4\right) \, dx = -\frac{x^5}{5} + x^4 - \frac{8\,x^3}{3} + 4\,x^2 + a\,x$$

```
[In] int(a + 8*x - 8*x^2 + 4*x^3 - x^4,x)
[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

3.120 $\int \frac{1}{a+8x-8x^2+4x^3-x^4} \, dx$

Optimal result . . . 896
Rubi [A] (verified) . . . 896
Mathematica [C] (verified) . . . 897
Maple [C] (verified) . . . 898
Fricas [B] (verification not implemented) . . . 898
Sympy [A] (verification not implemented) . . . 900
Maxima [F] . . . 900
Giac [B] (verification not implemented) . . . 900
Mupad [B] (verification not implemented) . . . 902

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4} \, dx = -\frac{\arctan\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}}+\frac{\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}}$$

```
[Out] -1/2*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))/(4+a)^(1/2)/(1-(4+a)^(1/2))^(1/2)
+1/2*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2))/(4+a)^(1/2)/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1120, 1107, 210}

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4} \, dx = \frac{\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}-\frac{\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}$$

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(Sqrt[4 + a]*Sqrt[1 - Sqrt[4 +
a]]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqrt[
4 + a]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{3+a-2x^2-x^4}\,dx, x, -1+x\right) \\
&= -\frac{\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{2\sqrt{4+a}} + \frac{\text{Subst}\left(\int \frac{1}{-1+\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{2\sqrt{4+a}} \\
&= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.64

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx = -\frac{1}{4}\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3 - \#1^4\&, \frac{\log(x-\#1)}{-2+4\#1-3\#1^2+\#1^3}\&\right]$$

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1),x]
[Out] -1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , Log[x - #1]/(-2 + 4*#1 -
 3*#1^2 + #1^3) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}\right)}{4}$	51
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}\right)}{4}$	51

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(1/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-
a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(65) = 130$.

Time = 0.25 (sec) , antiderivative size = 457, normalized size of antiderivative = 5.13

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx$$
$$= \frac{1}{4}\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}\log\left(\left(a-\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}+x-1\right)$$
$$-\frac{1}{4}\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}\log\left(-\left(a-\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}+x-1\right)$$
$$+\frac{1}{4}\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}\log\left(\left(a+\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}+x-1\right)$$
$$-\frac{1}{4}\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}\log\left(-\left(a+\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}+x-1\right)$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a +
 12))*log((a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*sqrt(((
a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12)) + x -
 1) - 1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 +
 7*a + 12))*log(-(a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*
sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12)
) + x - 1) + 1/4*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1
)/(a^2 + 7*a + 12))*log((a + (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36
) + 4)*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)/(a^2 + 7
*a + 12)) + x - 1) - 1/4*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a +
 36) - 1)/(a^2 + 7*a + 12))*log(-(a + (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 +
33*a + 36) + 4)*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)
/(a^2 + 7*a + 12)) + x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.74

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx = -\operatorname{RootSum}\left(t^4\cdot\left(256a^3+2816a^2+10240a+12288\right)+t^2(-32a-128)-1,\left(t\mapsto t\log\left(64t^3a^2+448t^3a\right.\right.\right.$$

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-32*a - 12
8) - 1, Lambda(_t, _t*log(64*_t**3*a**2 + 448*_t**3*a + 768*_t**3 - 4*_t*a
- 20*_t + x - 1)))
```

Maxima [F]

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx = \int -\frac{1}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2669 vs. $2(65) = 130$.

Time = 2.52 (sec) , antiderivative size = 2669, normalized size of antiderivative = 29.99

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(((a + 4)^(3/2) + a + 4)/(a^3 + 11*a^2 + 40*a + 48))*log(abs(sqrt(
a + 4)*a^5 + sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^4*x + a^
5 + sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^3*x -
 sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^4 + 17*sqrt(a + 4)*a
^4 + 14*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^3*x - sqrt(a^
2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^3 + 17*a^4 + 10*
sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^2*x - 14*
sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^3 + 111*sqrt(a + 4)*a
^3 + 69*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^2*x - 10*sqrt
(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^2 + 111*a^3 +
 29*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a*x - 6
```

9*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^2 + 351*sqrt(a + 4)
*a^2 + 144*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a*x - 29*sqr
t(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a + 351*a^2 +
28*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*x - 144*
sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a + 544*sqrt(a + 4)*a +
112*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*x - 28*sqrt(a^2 +
(a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4) + 544*a - 112*sqrt(a^2
+ (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12) + 336*sqrt(a + 4) + 336)) + 1/4
*sqrt(((a + 4)^(3/2) + a + 4)/(a^3 + 11*a^2 + 40*a + 48))*log(abs(sqrt(a +
4)*a^5 - sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^4*x + a^5 -
sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^3*x + sqr
t(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^4 + 17*sqrt(a + 4)*a^4 -
14*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^3*x + sqrt(a^2 +
(a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^3 + 17*a^4 - 10*sqrt
(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^2*x + 14*sqrt
(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^3 + 111*sqrt(a + 4)*a^3 -
69*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^2*x + 10*sqrt(a^2
+ (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^2 + 111*a^3 - 29*
sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a*x + 69*sq
rt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^2 + 351*sqrt(a + 4)*a^2
- 144*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a*x + 29*sqrt(a^
2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a + 351*a^2 - 28*s
qrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*x + 144*sqrt
(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a + 544*sqrt(a + 4)*a - 112
*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*x + 28*sqrt(a^2 + (a^2
+ 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4) + 544*a + 112*sqrt(a^2 + (
a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12) + 336*sqrt(a + 4) + 336)) - 1/4*sqr
t(-((a + 4)^(3/2) - a - 4)/(a^3 + 11*a^2 + 40*a + 48))*log(abs(-sqrt(a + 4)
*a^5 + sqrt(a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^4*x + a^5 - sq
rt(a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^3*x - sqrt(
a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^4 - 17*sqrt(a + 4)*a^4 + 1
4*sqrt(a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^3*x + sqrt(a^2 - (a
^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^3 + 17*a^4 - 10*sqrt(a
^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^2*x - 14*sqrt(a
^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^3 - 111*sqrt(a + 4)*a^3 + 6
9*sqrt(a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^2*x + 10*sqrt(a^2 -
(a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^2 + 111*a^3 - 29*sq
rt(a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a*x - 69*sqrt
(a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^2 - 351*sqrt(a + 4)*a^2 +
144*sqrt(a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a*x + 29*sqrt(a^2
- (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a + 351*a^2 - 28*sqr
t(a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*x - 144*sqrt(a
^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a - 544*sqrt(a + 4)*a + 112*s
qrt(a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*x + 28*sqrt(a^2 - (a^2 +
7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4) + 544*a - 112*sqrt(a^2 - (a^

```
2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12) - 336*sqrt(a + 4) + 336)) + 1/4*sqrt(
-((a + 4)^(3/2) - a - 4)/(a^3 + 11*a^2 + 40*a + 48))*log(abs(-sqrt(a + 4)*a
^5 - sqrt(a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^4*x + a^5 + sqrt
(a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^3*x + sqrt(a^
2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^4 - 17*sqrt(a + 4)*a^4 - 14*
sqrt(a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^3*x - sqrt(a^2 - (a^2
 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^3 + 17*a^4 + 10*sqrt(a^2
 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^2*x + 14*sqrt(a^2
 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^3 - 111*sqrt(a + 4)*a^3 - 69*
sqrt(a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^2*x - 10*sqrt(a^2 - (
a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^2 + 111*a^3 + 29*sqrt
(a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a*x + 69*sqrt(a
^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^2 - 351*sqrt(a + 4)*a^2 - 1
44*sqrt(a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a*x - 29*sqrt(a^2 -
(a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a + 351*a^2 + 28*sqrt(
a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*x + 144*sqrt(a^2
 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a - 544*sqrt(a + 4)*a - 112*sqr
t(a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*x - 28*sqrt(a^2 - (a^2 + 7
*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4) + 544*a + 112*sqrt(a^2 - (a^2
+ 7*a + 12)*sqrt(a + 4) + 7*a + 12) - 336*sqrt(a + 4) + 336))
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 571, normalized size of antiderivative = 6.42

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx =$$
$$-\mathrm{atan}\left(-\frac{a\,8\mathrm{i}-x\,16\mathrm{i}+x\,\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a\,x\,8\mathrm{i}-\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a^2\,x}{44\,a^2\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+4\,a^3\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+160\,a\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+192}\right.$$
$$-\mathrm{atan}\left(-\frac{a\,8\mathrm{i}-x\,16\mathrm{i}-x\,\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a\,x\,8\mathrm{i}+\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a^2\,x}{160\,a\,\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+192\,\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+44\,a^2\,\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+4\,a^3}\right.$$

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

```
[Out] - atan(-(a*8i - x*16i + x*(48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a*x*8i - (4
8*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a^2*x*1i + a^2*1i + 16i)/(44*a^2*((a -
(48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/
2) + 4*a^3*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 1
6*a^3 + 768))^(1/2) + 160*a*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(64
0*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 192*((a - (48*a + 12*a^2 + a^3 + 64)
^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)))*((a - (48*a + 12*a^2
+ a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)*2i - atan(-(
```

```
a*8i - x*16i - x*(48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a*x*8i + (48*a + 12*
a^2 + a^3 + 64)^(1/2)*1i - a^2*x*1i + a^2*1i + 16i)/(160*a*((a + (48*a + 12
*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 192*(
(a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768)
)^(1/2) + 44*a^2*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a
^2 + 16*a^3 + 768))^(1/2) + 4*a^3*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) +
4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)))*((a + (48*a + 12*a^2 + a^3 + 6
4)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)*2i
```

3.121 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2} \, dx$

Optimal result . . . 904
Rubi [A] (verified) . . . 904
Mathematica [C] (verified) . . . 906
Maple [C] (verified) . . . 906
Fricas [B] (verification not implemented) . . . 907
Sympy [B] (verification not implemented) . . . 909
Maxima [F] . . . 909
Giac [B] (verification not implemented) . . . 910
Mupad [B] (verification not implemented) . . . 916

Optimal result

Integrand size = 22, antiderivative size = 169

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2} \, dx = \frac{(5+a+(-1+x)^2)\,(-1+x)}{4\,(12+7a+a^2)\,(3+a-2(-1+x)^2-(-1+x)^4)} - \frac{\left(10+3a+\sqrt{4+a}\right)\arctan\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1-\sqrt{4+a}}} + \frac{\left(10+3a-\sqrt{4+a}\right)\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1+\sqrt{4+a}}}$$

```
[Out] 1/4*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)-1/8*arctan
((-1+x)/(1-(4+a)^(1/2))^(1/2))*(10+3*a+(4+a)^(1/2))/(3+a)/(4+a)^(3/2)/(1-(4
+a)^(1/2))^(1/2)+1/8*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2))*(10+3*a-(4+a)^(1/
2))/(3+a)/(4+a)^(3/2)/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1120, 1106, 1180, 210}

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2} \, dx = \frac{(x-1)\,(a+(x-1)^2+5)}{4\,(a^2+7a+12)\,(a-(x-1)^4-2(x-1)^2+3)} - \frac{\left(3a+\sqrt{a+4}+10\right)\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}} + \frac{\left(3a-\sqrt{a+4}+10\right)\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{\sqrt{a+4}+1}}$$

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2),x]
```

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4)) - ((10 + 3*a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 +
 a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt[4 + a]]) + ((10 + 3*a - Sqrt[
4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sq
rt[1 + Sqrt[4 + a]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule steps

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{(3+a-2x^2-x^4)^2}\,dx, x, -1+x\right)\\ &= \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} - \frac{\text{Subst}\left(\int \frac{4+2(3+a)-2(4+4(3+a))-2x^2}{3+a-2x^2-x^4}\,dx, x, -1+x\right)}{8(12+7a+a^2)}\end{aligned}$$

$$= \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} - \frac{(10+3a-\sqrt{4+a})\,\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{8(3+a)(4+a)^{3/2}} + \frac{(10+3a+\sqrt{4+a})\,\text{Subst}\left(\int \frac{1}{-1+\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{8(3+a)(4+a)^{3/2}}$$

$$= \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{(10+3a+\sqrt{4+a})\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1-\sqrt{4+a}}} - \frac{(10+3a-\sqrt{4+a})\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1+\sqrt{4+a}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \frac{(-1+x)(6+a-2x+x^2)}{4(3+a)(4+a)(a-x(-8+8x-4x^2+x^3))} - \frac{\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&, \frac{12\log(x-\#1)+3a\log(x-\#1)-2\log(x-\#1)\#1+\log(x-\#1)\#1^2}{-2+4\#1-3\#1^2+\#1^3}\&\right]}{16(12+7a+a^2)}$$

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2),x]
```

```
[Out] ((-1 + x)*(6 + a - 2*x + x^2))/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2
+ x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (12*Log[x - #1] +
3*a*Log[x - #1] - 2*Log[x - #1]*#1 + Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2
+ #1^3) & ]/(16*(12 + 7*a + a^2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93

method	result
default	$\frac{\frac{x^3}{4a^2+28a+48}-\frac{3x^2}{4(4+a)(3+a)}+\frac{(a+8)x}{4a^2+28a+48}-\frac{6+a}{4\left(a^2+7a+12\right)}}{-x^4+4x^3-8x^2+a+8x}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(\_R^2-2\_R+3a+12\right)\ln\left(x-\right.}{-\_R^3+3\_R^2-4\_R+}}{16(4+a)(3+a)}$
risch	$\frac{\frac{x^3}{4a^2+28a+48}-\frac{3x^2}{4(4+a)(3+a)}+\frac{(a+8)x}{4a^2+28a+48}-\frac{6+a}{4\left(a^2+7a+12\right)}}{-x^4+4x^3-8x^2+a+8x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(\frac{\_R^2}{a^2+7a+12}-\frac{2\_R}{a^2+7a+12}+\frac{3}{3+}\right.}{-\_R^3+3\_R^2-4}\right.}{16}$

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4/(a^2+7*a+12)*x^3-3/4/(4+a)/(3+a)*x^2+1/4*(a+8)/(a^2+7*a+12)*x-1/4*(6+a
)/(a^2+7*a+12))/(-x^4+4*x^3-8*x^2+a+8*x)+1/16/(4+a)/(3+a)*sum((_R^2-2*_R+3*
a+12)/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1948 vs. $2(139) = 278$.

Time = 0.27 (sec) , antiderivative size = 1948, normalized size of antiderivative = 11.53

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*x^3 - ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^
2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 + (a^
6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 5
58*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 11
1105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 1
83*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a
 + 992)*x + (27*a^4 + 408*a^3 + 2309*a^2 - 2*(2*a^7 + 49*a^6 + 513*a^5 + 29
75*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 558*a + 96
1)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3
+ 156492*a^2 + 128304*a + 46656)) + 5800*a + 5456)*sqrt((15*a^2 + (a^6 + 21
*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a +
 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a
^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4
 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) + ((a^2 + 7*a + 12)*
x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^
2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 +
2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a
^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a
+ 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 302
```

4*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a + 992)*x - (27*a^4 + 408*a^3 + 2
309*a^2 - 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 +
24624*a + 12096)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088
*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656))
 + 5800*a + 5456)*sqrt((15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a
^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3
088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 4665
6)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a +
1728)) - 567*a - 992) - ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^
3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*
a^2 - (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((8
1*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598
*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 2
1*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^
2 + 567*a + 992)*x + (27*a^4 + 408*a^3 + 2309*a^2 + 2*(2*a^7 + 49*a^6 + 513
*a^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 5
58*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 11
1105*a^3 + 156492*a^2 + 128304*a + 46656)) + 5800*a + 5456)*sqrt((15*a^2 -
(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2
+ 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 +
 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5
+ 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) + ((a^2 + 7
*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^
2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 8
47*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^
8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 +
128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*
a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a + 992)*x - (27*a^4 + 40
8*a^3 + 2309*a^2 + 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 214
20*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a
^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a
+ 46656)) + 5800*a + 5456)*sqrt((15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 847*a^3
 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 39
9*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304
*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 +
3024*a + 1728)) - 567*a - 992) + 4*(a + 8)*x - 12*x^2 - 4*a - 24)/((a^2 + 7
*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^
2 - 8*(a^2 + 7*a + 12)*x - 12*a)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(144) = 288.

Time = 3.34 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.74

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$$
$$= \frac{a-x^3+3x^2+x(-a-8)+6}{-4a^3-28a^2-48a+x^4\cdot(4a^2+28a+48)+x^3\,(-16a^2-112a-192)+x^2\cdot(32a^2+224a+384)+x\,(-}$$
$$+\mathrm{RootSum}\left(t^4\cdot(65536a^9+2162688a^8+31653888a^7+269680640a^6+1473773568a^5+5357174784a^4\right.$$

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] (a - x**3 + 3*x**2 + x*(-a - 8) + 6)/(-4*a**3 - 28*a**2 - 48*a + x**4*(4*a*
*2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2 + 224*a + 3
84) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 + 2162688*a**
8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357174784*a**4 + 12
952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**2*(-7
680*a**5 - 145920*a**4 - 1107968*a**3 - 4202496*a**2 - 7962624*a - 6029312)
 - 81*a**2 - 576*a - 1024, Lambda(_t, _t*log(x + (-16384*_t**3*a**7 - 40140
8*_t**3*a**6 - 4202496*_t**3*a**5 - 24371200*_t**3*a**4 - 84549632*_t**3*a*
*3 - 175472640*_t**3*a**2 - 201719808*_t**3*a - 99090432*_t**3 + 432*_t*a**
4 + 7488*_t*a**3 + 47024*_t*a**2 + 128096*_t*a + 128512*_t - 81*a**2 - 567*
a - 992)/(81*a**2 + 567*a + 992))))
```

Maxima [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \int \frac{1}{(x^4-4\,x^3+8\,x^2-a-8\,x)^2}\,dx$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(x^3 + (a + 8)*x - 3*x^2 - a - 6)/((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a
 + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x -
12*a) - 1/4*integrate((x^2 + 3*a - 2*x + 12)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x
), x)/(a^2 + 7*a + 12)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8503 vs. $2(139) = 278$.

Time = 6.37 (sec) , antiderivative size = 8503, normalized size of antiderivative = 50.31

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] 1/16*(sqrt((15*a^3 + 165*a^2 + (9*a^3 + 103*a^2 + 392*a + 496)*sqrt(a + 4)
+ 604*a + 736)/(a^3 + 11*a^2 + 40*a + 48))*log(abs(243*sqrt(a + 4)*a^10 + 3
24*a^10 + 8640*sqrt(a + 4)*a^9 + 81*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a
^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^8*x
+ 11466*a^9 + 81*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*
a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^7*x - 81*sq
rt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488
)*sqrt(a + 4) + 2548*a + 2208)*a^8 + 138027*sqrt(a + 4)*a^8 + 2340*sqrt(15*
a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt
(a + 4) + 2548*a + 2208)*a^7*x - 81*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a
^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a
 + 4)*a^7 + 182314*a^8 + 2016*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 1
30*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*
a^6*x - 2340*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2
+ 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^7 + 1304648*sqrt(a + 4)*a^7
 + 29518*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 16
72*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^6*x - 2016*sqrt(15*a^4 + 210*a^
3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 25
48*a + 2208)*sqrt(a + 4)*a^6 + 1715172*a^7 + 21454*sqrt(15*a^4 + 210*a^3 +
1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a
 + 2208)*sqrt(a + 4)*a^5*x - 29518*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^
4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^6 + 8
079749*sqrt(a + 4)*a^6 + 212356*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 +
 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^5*x - 21
454*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a
+ 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^5 + 10572392*a^6 + 12654
0*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a +
1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^4*x - 212356*sqrt(15*a^4 +
 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a +
4) + 2548*a + 2208)*a^5 + 34255200*sqrt(a + 4)*a^5 + 952845*sqrt(15*a^4 + 2
10*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4)
 + 2548*a + 2208)*a^4*x - 126540*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4
+ 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a +
4)*a^4 + 44613658*a^5 + 446685*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 +
130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)
```

```
*a^3*x - 952845*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a
^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^4 + 100679657*sqrt(a + 4
)*a^4 + 2730184*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a
^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^3*x - 446685*sqrt(15*a^4
 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a
+ 4) + 2548*a + 2208)*sqrt(a + 4)*a^3 + 130513730*a^4 + 943444*sqrt(15*a^4
+ 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a +
 4) + 2548*a + 2208)*sqrt(a + 4)*a^2*x - 2730184*sqrt(15*a^4 + 210*a^3 + 10
99*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a +
 2208)*a^3 + 202540404*sqrt(a + 4)*a^3 + 4877364*sqrt(15*a^4 + 210*a^3 + 10
99*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a +
 2208)*a^2*x - 943444*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 +
 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^2 + 26
1341928*a^3 + 1103588*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 +
 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a*x - 48
77364*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*
a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^2 + 266882676*sqrt(a + 4)*a^2 + 49
65684*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*
a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a*x - 1103588*sqrt(15*a^4 + 210*a^3
+ 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548
*a + 2208)*sqrt(a + 4)*a + 342778384*a^2 + 551332*sqrt(15*a^4 + 210*a^3 + 1
099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a
+ 2208)*sqrt(a + 4)*x - 4965684*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 +
 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a + 207974
132*sqrt(a + 4)*a + 2205328*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130
*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*x - 551332*sqr
t(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)
*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4) + 265897256*a - 2205328*sqrt(15*a
^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(
a + 4) + 2548*a + 2208) + 72775824*sqrt(a + 4) + 92623776)) - sqrt((15*a^3
+ 165*a^2 + (9*a^3 + 103*a^2 + 392*a + 496)*sqrt(a + 4) + 604*a + 736)/(a^3
 + 11*a^2 + 40*a + 48))*log(abs(243*sqrt(a + 4)*a^10 + 324*a^10 + 8640*sqrt
(a + 4)*a^9 - 81*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*
a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^8*x + 11466*a^9 - 81*sq
rt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488
)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^7*x + 81*sqrt(15*a^4 + 210*a^3
 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 254
8*a + 2208)*a^8 + 138027*sqrt(a + 4)*a^8 - 2340*sqrt(15*a^4 + 210*a^3 + 109
9*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a +
2208)*a^7*x + 81*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*
a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^7 + 182314*
a^8 - 2016*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 +
1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^6*x + 2340*sqrt(1
5*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sq
rt(a + 4) + 2548*a + 2208)*a^7 + 1304648*sqrt(a + 4)*a^7 - 29518*sqrt(15*a^
```

4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^6*x + 2016*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^6 + 1715172*a^7 - 21454*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^5*x + 29518*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^6 + 8079749*sqrt(a + 4)*a^6 - 212356*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^5*x + 21454*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^5 + 10572392*a^6 - 126540*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^4*x + 212356*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^5 + 34255200*sqrt(a + 4)*a^5 - 952845*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^4*x + 126540*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^4 + 44613658*a^5 - 446685*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^3*x + 952845*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^4 + 100679657*sqrt(a + 4)*a^4 - 2730184*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^3*x + 446685*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^3 + 130513730*a^4 - 943444*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^2*x + 2730184*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^3 + 202540404*sqrt(a + 4)*a^3 - 4877364*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^2*x + 943444*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^2 + 261341928*a^3 - 1103588*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a*x + 4877364*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^2 + 266882676*sqrt(a + 4)*a^2 - 4965684*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a*x + 1103588*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a + 342778384*a^2 - 551332*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*x + 4965684*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a + 207974132*sqrt(a + 4)*a - 2205328*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 16

72*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*x + 551332*sqrt(15*a^4 + 210*a^3
+ 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548
*a + 2208)*sqrt(a + 4) + 265897256*a + 2205328*sqrt(15*a^4 + 210*a^3 + 1099
*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2
208) + 72775824*sqrt(a + 4) + 92623776)) + sqrt((15*a^3 + 165*a^2 - (9*a^3
+ 103*a^2 + 392*a + 496)*sqrt(a + 4) + 604*a + 736)/(a^3 + 11*a^2 + 40*a +
48))*log(abs(-243*sqrt(a + 4)*a^10 + 324*a^10 - 8640*sqrt(a + 4)*a^9 + 81*s
qrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 148
8)*sqrt(a + 4) + 2548*a + 2208)*a^8*x + 11466*a^9 - 81*sqrt(15*a^4 + 210*a^
3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 25
48*a + 2208)*sqrt(a + 4)*a^7*x - 81*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a
^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^8 -
138027*sqrt(a + 4)*a^8 + 2340*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 1
30*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^7*x + 81*s
qrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 148
8)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^7 + 182314*a^8 - 2016*sqrt(15
*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqr
t(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^6*x - 2340*sqrt(15*a^4 + 210*a^3 +
1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a
 + 2208)*a^7 - 1304648*sqrt(a + 4)*a^7 + 29518*sqrt(15*a^4 + 210*a^3 + 1099
*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2
208)*a^6*x + 2016*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701
*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^6 + 171517
2*a^7 - 21454*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2
 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^5*x - 29518*sq
rt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488
)*sqrt(a + 4) + 2548*a + 2208)*a^6 - 8079749*sqrt(a + 4)*a^6 + 212356*sqrt(
15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*s
qrt(a + 4) + 2548*a + 2208)*a^5*x + 21454*sqrt(15*a^4 + 210*a^3 + 1099*a^2
- (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*
sqrt(a + 4)*a^5 + 10572392*a^6 - 126540*sqrt(15*a^4 + 210*a^3 + 1099*a^2 -
(9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sq
rt(a + 4)*a^4*x - 212356*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^
3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^5 - 34255200*sq
rt(a + 4)*a^5 + 952845*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3
+ 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^4*x + 126540*sqrt
(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*
sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^4 + 44613658*a^5 - 446685*sqrt(1
5*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sq
rt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^3*x - 952845*sqrt(15*a^4 + 210*a^3
 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 254
8*a + 2208)*a^4 - 100679657*sqrt(a + 4)*a^4 + 2730184*sqrt(15*a^4 + 210*a^3
 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 254
8*a + 2208)*a^3*x + 446685*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*
a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^3

+ 130513730*a^4 - 943444*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a
^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^2*
x - 2730184*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 +
1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^3 - 202540404*sqrt(a + 4)*a^
3 + 4877364*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 +
1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^2*x + 943444*sqrt(15*a^4 + 2
10*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4)
+ 2548*a + 2208)*sqrt(a + 4)*a^2 + 261341928*a^3 - 1103588*sqrt(15*a^4 + 2
10*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4)
+ 2548*a + 2208)*sqrt(a + 4)*a*x - 4877364*sqrt(15*a^4 + 210*a^3 + 1099*a^
2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208
)*a^2 - 266882676*sqrt(a + 4)*a^2 + 4965684*sqrt(15*a^4 + 210*a^3 + 1099*a^
2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208
)*a*x + 1103588*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a
^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a + 342778384*
a^2 - 551332*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2
+ 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*x - 4965684*sqrt(
15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*s
qrt(a + 4) + 2548*a + 2208)*a - 207974132*sqrt(a + 4)*a + 2205328*sqrt(15*a
^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(
a + 4) + 2548*a + 2208)*x + 551332*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^
4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a
+ 4) + 265897256*a - 2205328*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 13
0*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208) - 72775824*sq
rt(a + 4) + 92623776)) - sqrt((15*a^3 + 165*a^2 - (9*a^3 + 103*a^2 + 392*a
+ 496)*sqrt(a + 4) + 604*a + 736)/(a^3 + 11*a^2 + 40*a + 48))*log(abs(-243*
sqrt(a + 4)*a^10 + 324*a^10 - 8640*sqrt(a + 4)*a^9 - 81*sqrt(15*a^4 + 210*a
^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2
548*a + 2208)*a^8*x + 11466*a^9 + 81*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*
a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(
a + 4)*a^7*x + 81*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701
*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^8 - 138027*sqrt(a + 4)
*a^8 - 2340*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 +
1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^7*x - 81*sqrt(15*a^4 + 210*a
^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2
548*a + 2208)*sqrt(a + 4)*a^7 + 182314*a^8 + 2016*sqrt(15*a^4 + 210*a^3 + 1
099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a
+ 2208)*sqrt(a + 4)*a^6*x + 2340*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4
+ 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^7 - 130
4648*sqrt(a + 4)*a^7 - 29518*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 13
0*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^6*x - 2016*
sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 14
88)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^6 + 1715172*a^7 + 21454*sqrt
(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*
sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^5*x + 29518*sqrt(15*a^4 + 210*a^

3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 25
48*a + 2208)*a^6 - 8079749*sqrt(a + 4)*a^6 - 212356*sqrt(15*a^4 + 210*a^3 +
1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*
a + 2208)*a^5*x - 21454*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3
+ 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^5 +
10572392*a^6 + 126540*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 +
701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^4*x +
212356*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672
*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^5 - 34255200*sqrt(a + 4)*a^5 - 95
2845*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a
+ 1488)*sqrt(a + 4) + 2548*a + 2208)*a^4*x - 126540*sqrt(15*a^4 + 210*a^3
+ 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548
*a + 2208)*sqrt(a + 4)*a^4 + 44613658*a^5 + 446685*sqrt(15*a^4 + 210*a^3 +
1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a
+ 2208)*sqrt(a + 4)*a^3*x + 952845*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a
^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^4 -
100679657*sqrt(a + 4)*a^4 - 2730184*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a
^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^3*x
- 446685*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 16
72*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^3 + 130513730*a^4 +
943444*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 167
2*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^2*x + 2730184*sqrt(1
5*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sq
rt(a + 4) + 2548*a + 2208)*a^3 - 202540404*sqrt(a + 4)*a^3 - 4877364*sqrt(1
5*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sq
rt(a + 4) + 2548*a + 2208)*a^2*x - 943444*sqrt(15*a^4 + 210*a^3 + 1099*a^2
- (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*
sqrt(a + 4)*a^2 + 261341928*a^3 + 1103588*sqrt(15*a^4 + 210*a^3 + 1099*a^2
- (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*
sqrt(a + 4)*a*x + 4877364*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a
^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^2 - 266882676*
sqrt(a + 4)*a^2 - 4965684*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a
^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a*x - 1103588*sq
rt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488
)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a + 342778384*a^2 + 551332*sqrt(
15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*s
qrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*x + 4965684*sqrt(15*a^4 + 210*a^3 +
1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*
a + 2208)*a - 207974132*sqrt(a + 4)*a - 2205328*sqrt(15*a^4 + 210*a^3 + 109
9*a^2 - (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a +
2208)*x - 551332*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*
a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4) + 265897256*a
+ 2205328*sqrt(15*a^4 + 210*a^3 + 1099*a^2 - (9*a^4 + 130*a^3 + 701*a^2 +
1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208) - 72775824*sqrt(a + 4) + 926237
76)))/(a^2 + 7*a + 12) - 1/4*(x^3 + a*x - 3*x^2 - a + 8*x - 6)/((x^4 - 4*x^

```
3 + 8*x^2 - a - 8*x)*(a^2 + 7*a + 12))
```

Mupad [B] (verification not implemented)

Time = 11.03 (sec) , antiderivative size = 4591, normalized size of antiderivative = 27.17

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
[Out] atan(-(((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2
+ 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632
*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592))
)^(1/2)*((((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^
5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576
)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/
(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((15552*a - 9*a*((a + 4)^9)^(1/2
) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/
(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6
 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) - (733184*a + 396288*a^2 + 1067
52*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*
a^4 + a^5 + 576)))*((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2)
 + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*
a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^
9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460
*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a
+ 73*a^2 + 14*a^3 + a^4 + 144)))*1i + ((15552*a - 9*a*((a + 4)^9)^(1/2) - 3
1*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*
(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 48
3*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640*a + 10878976*a^2 + 3997
696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2
 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 +
 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*(
(15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a
^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 8
1744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)
+ (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(
816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*((15552*a - 9*a*((a + 4)^
9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 +
11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4
115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 1
44*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(
61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*1i)/((9*a +
 32)/(32*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + ((15552*a - 9*
```

a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) - (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144))) - ((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))))*((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*2i + atan(-(((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 +

110592)))^(1/2) - (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + 768*a^
5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*((15552*
a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 28
5*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^
4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (5568
*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a
^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 1
44)))*1i + ((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*
a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 19
7632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 1105
92)))^(1/2)*((((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 9011
2*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 +
576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 14745
6))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((15552*a + 9*a*((a + 4)^9)^
(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 117
76)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115
*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (733184*a + 396288*a^2 +
106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 +
18*a^4 + a^5 + 576)))*((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(
1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306
432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8
+ a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a +
460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(16
8*a + 73*a^2 + 14*a^3 + a^4 + 144)))*1i)/((9*a + 32)/(32*(816*a + 460*a^2 +
129*a^3 + 18*a^4 + a^5 + 576)) + ((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((
a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276
480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^
7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640*a + 10878976*a^2 + 3997696*
a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 1
29*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 460
8*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((155
52*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 +
285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744
*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) - (7
33184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*
a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*((15552*a + 9*a*((a + 4)^9)^(
1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 1177
6)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*
a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a
^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a
+ 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144))) - ((15552*a + 9
a((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4
+ 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 2
2488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640
*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 94371

```
84)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a +
117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 +
 14*a^3 + a^4 + 144)))*((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(
1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306
432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8
+ a^9 + 110592)))^(1/2) + (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 +
 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*(
(15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a
^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 8
1744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)
+ (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*
a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 +
a^4 + 144)))))*((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8
208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2
+ 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 +
110592)))^(1/2)*2i + (x^3/(4*(7*a + a^2 + 12)) - (a + 6)/(4*(a + 3)*(a + 4)
) - (3*x^2)/(4*(a + 3)*(a + 4)) + (x*(a + 8))/(4*(a + 3)*(a + 4)))/(a + 8*x
 - 8*x^2 + 4*x^3 - x^4)
```

3.122 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx$

Optimal result 920
Rubi [A] (verified) 921
Mathematica [C] (verified) 923
Maple [C] (verified) 924
Fricas [B] (verification not implemented) 924
Sympy [B] (verification not implemented) 928
Maxima [F] 929
Giac [B] (verification not implemented) 929
Mupad [B] (verification not implemented) 942

Optimal result

Integrand size = 22, antiderivative size = 252

$$\begin{aligned}\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx &= \frac{(5+a+(-1+x)^2)\,(-1+x)}{8\,(12+7a+a^2)\,(3+a-2(-1+x)^2-(-1+x)^4)^2} \\ &\quad + \frac{((6+a)(25+7a)+6(7+2a)(-1+x)^2)\,(-1+x)}{32(3+a)^2(4+a)^2\,(3+a-2(-1+x)^2-(-1+x)^4)} \\ &\quad - \frac{3\big(80+7a^2+14\sqrt{4+a}+a\big(47+4\sqrt{4+a}\big)\big)\arctan\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{64(3+a)^2(4+a)^{5/2}\sqrt{1-\sqrt{4+a}}} \\ &\quad - \frac{3\left(14+4a-\frac{80+47a+7a^2}{\sqrt{4+a}}\right)\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{64(3+a)^2(4+a)^2\sqrt{1+\sqrt{4+a}}}\end{aligned}$$

```
[Out] 1/8*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^2+1/32*((6
+a)*(25+7*a)+6*(7+2*a)*(-1+x)^2)*(-1+x)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-1+
x)^4)-3/64*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(80+7*a^2+14*(4+a)^(1/2)+a*
(47+4*(4+a)^(1/2)))/(3+a)^2/(4+a)^(5/2)/(1-(4+a)^(1/2))^(1/2)-3/64*arctan((
-1+x)/(1+(4+a)^(1/2))^(1/2))*(14+4*a+(-7*a^2-47*a-80)/(4+a)^(1/2))/(3+a)^2/
(4+a)^2/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1120, 1106, 1192, 1180, 210}

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx$$
$$= -\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}} - \frac{3\left(-\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{64(a+3)^2(a+4)^2\sqrt{\sqrt{a+4}+1}} + \frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2} + \frac{(x-1)\left(6(2a+7)(x-1)^2+(a+6)(7a+25)\right)}{32(a+3)^2(a+4)^2\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3),x]
```

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4)^2) + (((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/
(32*(3 + a)^2*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - (3*(80 + 7*a
^2 + 14*Sqrt[4 + a] + a*(47 + 4*Sqrt[4 + a]))*ArcTan[(-1 + x)/Sqrt[1 - Sqrt
[4 + a]]])/(64*(3 + a)^2*(4 + a)^(5/2)*Sqrt[1 - Sqrt[4 + a]]) - (3*(14 + 4*
a - (80 + 47*a + 7*a^2)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]
)/(64*(3 + a)^2*(4 + a)^2*Sqrt[1 + Sqrt[4 + a]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
```

```
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(3+a-2x^2-x^4)^3}\,dx, x, -1+x\right)$$

$$= \frac{(5+a+(-1+x)^2)(-1+x)}{8(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^2} - \frac{\text{Subst}\left(\int \frac{4+2(3+a)-4(4+4(3+a))-10x^2}{(3+a-2x^2-x^4)^2}\,dx, x, -1+x\right)}{16(12+7a+a^2)}$$

$$= -\frac{((6+a)(25+7a)+6(7+2a)(1-x)^2)(1-x)}{32(12+7a+a^2)^2(3+a-2(1-x)^2-(1-x)^4)} + \frac{(5+a+(-1+x)^2)(-1+x)}{8(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^2} + \frac{\text{Subst}\left(\int \frac{12(94+51a+7a^2)+24(7+2a)x^2}{3+a-2x^2-x^4}\,dx, x, -1+x\right)}{128(12+7a+a^2)^2}$$

$$= -\frac{((6+a)(25+7a)+6(7+2a)(1-x)^2)(1-x)}{32(12+7a+a^2)^2(3+a-2(1-x)^2-(1-x)^4)} + \frac{(5+a+(-1+x)^2)(-1+x)}{8(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^2} + \frac{\left(3\left(14+4a-\frac{80+47a+7a^2}{\sqrt{4+a}}\right)\right)\mathrm{Subst}\left(\int\frac{1}{-1-\sqrt{4+a}-x^2}\,dx,x,-1+x\right)}{64(12+7a+a^2)^2} + \frac{\left(3\left(80+7a^2+14\sqrt{4+a}+a\left(47+4\sqrt{4+a}\right)\right)\right)\mathrm{Subst}\left(\int\frac{1}{-1+\sqrt{4+a}-x^2}\,dx,x,-1+x\right)}{64\sqrt{4+a}(12+7a+a^2)^2}$$

$$= -\frac{((6+a)(25+7a)+6(7+2a)(1-x)^2)(1-x)}{32(12+7a+a^2)^2(3+a-2(1-x)^2-(1-x)^4)} + \frac{(5+a+(-1+x)^2)(-1+x)}{8(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^2} + \frac{3\left(80+47a+7a^2+\sqrt{4+a}(14+4a)\right)\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{64(3+a)^2(4+a)^{5/2}\sqrt{1-\sqrt{4+a}}} + \frac{3\left(14+4a-\frac{80+47a+7a^2}{\sqrt{4+a}}\right)\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{64(12+7a+a^2)^2\sqrt{1+\sqrt{4+a}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \frac{1}{128}\left(\frac{16(-1+x)(6+a-2x+x^2)}{(3+a)(4+a)(a-x(-8+8x-4x^2+x^3))^2} + \frac{4(-1+x)(7a^2+6(32-14x+7x^2)+a(79-24x+12x^2))}{(3+a)^2(4+a)^2(a-x(-8+8x-4x^2+x^3))} - \frac{3\mathrm{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&,\frac{108\log(x-\#1)+55a\log(x-\#1)+7a^2\log(x-\#1)-28\log(x-\#1)\#1-}{-2+4\#1-3\#1^2}\right.}{(12+7a+a^2)^2}\right.$$

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3),x]
```

```
[Out] ((16*(-1 + x)*(6 + a - 2*x + x^2))/((3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^
2 + x^3))^2) + (4*(-1 + x)*(7*a^2 + 6*(32 - 14*x + 7*x^2) + a*(79 - 24*x +
12*x^2)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - (3*RootS
um[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (108*Log[x - #1] + 55*a*Log[x - #1
```

```
] + 7*a^2*Log[x - #1] - 28*Log[x - #1]*#1 - 8*a*Log[x - #1]*#1 + 14*Log[x -
 #1]*#1^2 + 4*a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ])/(12 + 7*
a + a^2)^2)/128
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.59

method	result
default	$-\frac{\frac{3(7+2a)x^7}{16\left(a^4+14a^3+73a^2+168a+144\right)}-\frac{21(7+2a)x^6}{16\left(a^2+8a+16\right)\left(a^2+6a+9\right)}+\frac{\left(7a^2+343a+1116\right)x^5}{32a^4+448a^3+2336a^2+5376a+4608}-\frac{5\left(7a^2+175a+528\right)x^4}{32\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{(3\cdots}{16a^4+22\cdots}}{(-x^4+4x^3-8x^2+a+\cdots}$
risch	$\frac{-\frac{3(7+2a)x^7}{16\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{21(7+2a)x^6}{16\left(a^2+8a+16\right)\left(a^2+6a+9\right)}-\frac{\left(7a^2+343a+1116\right)x^5}{32\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{5\left(7a^2+175a+528\right)x^4}{32\left(a^4+14a^3+73a^2+168a+144\right)}-\frac{(34a^2+\cdots}{16\left(a^4+14a^3\cdots\right.}}{(-x^4+4x^3-8x^2+a+8\cdots}$

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(3/16*(7+2*a)/(a^4+14*a^3+73*a^2+168*a+144)*x^7-21/16*(7+2*a)/(a^2+8*a+16)
/(a^2+6*a+9)*x^6+1/32*(7*a^2+343*a+1116)/(a^4+14*a^3+73*a^2+168*a+144)*x^5-
5/32*(7*a^2+175*a+528)/(a^4+14*a^3+73*a^2+168*a+144)*x^4+1/16*(34*a^2+679*a
+1968)/(a^4+14*a^3+73*a^2+168*a+144)*x^3-1/16*(32*a^2+623*a+1800)/(a^4+14*a
^3+73*a^2+168*a+144)*x^2-1/32*(11*a^3+107*a^2-84*a-1152)/(a^4+14*a^3+73*a^2
+168*a+144)*x+1/32*(11*a^3+131*a^2+408*a+288)/(3+a)/(a^3+11*a^2+40*a+48))/(
-x^4+4*x^3-8*x^2+a+8*x)^2-3/128/(a^3+10*a^2+33*a+36)/(4+a)*sum((-108+2*(-2*
a-7)*_R^2+4*(7+2*a)*_R-7*a^2-55*a)/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf
(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3971 vs. 2(220) = 440.

Time = 0.31 (sec) , antiderivative size = 3971, normalized size of antiderivative = 15.76

$$\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(24*(2*a + 7)*x^7 - 168*(2*a + 7)*x^6 + 4*(7*a^2 + 343*a + 1116)*x^5
 - 20*(7*a^2 + 175*a + 528)*x^4 + 8*(34*a^2 + 679*a + 1968)*x^3 + 44*a^3 -
8*(32*a^2 + 623*a + 1800)*x^2 - 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^
8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2
```

+ 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 +
14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^
4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16
*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 +
14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 +
(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4
+ 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 +
171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 +
167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 2
41870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^
2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 +
5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 +
725760*a + 248832))*log(-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^
4 + 33614*a^3 + 177061*a^2 + 415884*a + 367536)*x + 27*(343*a^7 + 8981*a^6
+ 100811*a^5 + 628887*a^4 + 2354874*a^3 + 5293208*a^2 - (11*a^12 + 462*a^11
+ 8881*a^10 + 103320*a^9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410
692*a^5 + 117844800*a^4 + 181238400*a^3 + 187875072*a^2 + 117863424*a + 338
41152)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 +
50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^
9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940
*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 6613472*a
+ 3543424)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*a^9 + 550*a^8
+ 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2
+ 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 3
46921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^1
0 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a
^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176
)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129
367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)) - 1122
8868*a - 9923472) + 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 +
14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 1
44)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(
a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 -
2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a
^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*
a^3 + 168*a^2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*
a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3
+ 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2
+ 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11
+ 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6
+ 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 27713664
0*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 3
1085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 2
48832))*log(-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^
3 + 177061*a^2 + 415884*a + 367536)*x - 27*(343*a^7 + 8981*a^6 + 100811*a^5

+ 628887*a^4 + 2354874*a^3 + 5293208*a^2 - (11*a^12 + 462*a^11 + 8881*a^10
+ 103320*a^9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 11
7844800*a^4 + 181238400*a^3 + 187875072*a^2 + 117863424*a + 33841152)*sqrt(
(2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1
165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320
*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 78207
1200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 6613472*a + 3543424)*
sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 +
31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a +
248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15
+ 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*
a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 7143179
40*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228*a
+ 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 37
3020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)) - 11228868*a - 992
3472) - 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73
*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^
6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4
- 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151
*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3
- 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^
2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^
8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^
2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a +
346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a
^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313
*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 604661
76)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 1
29367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832))*log(
-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^3 + 177061*a
^2 + 415884*a + 367536)*x + 27*(343*a^7 + 8981*a^6 + 100811*a^5 + 628887*a^
4 + 2354874*a^3 + 5293208*a^2 + (11*a^12 + 462*a^11 + 8881*a^10 + 103320*a^
9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 117844800*a^4
+ 181238400*a^3 + 187875072*a^2 + 117863424*a + 33841152)*sqrt((2401*a^4 +
33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 1
6780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320
045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 5
92064640*a^2 + 277136640*a + 60466176)) + 6613472*a + 3543424)*sqrt((105*a^
4 + 1470*a^3 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 +
129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqr
t((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 +
1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 285703
20*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782
071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a
^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 7

35840*a^3 + 950400*a^2 + 725760*a + 248832)) - 11228868*a - 9923472) + 3*((
a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a
 + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4
+ 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 -
 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*
a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 -
528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x
)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^8 + 5110*a^7
 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a
 + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^
15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 672213
0*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 71431
7940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228
*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 +
373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832))*log(-64827*a^4 -
 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^3 + 177061*a^2 + 415884*
a + 367536)*x - 27*(343*a^7 + 8981*a^6 + 100811*a^5 + 628887*a^4 + 2354874*
a^3 + 5293208*a^2 + (11*a^12 + 462*a^11 + 8881*a^10 + 103320*a^9 + 810205*a
^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 117844800*a^4 + 181238400*
a^3 + 187875072*a^2 + 117863424*a + 33841152)*sqrt((2401*a^4 + 33124*a^3 +
171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 +
167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 24
1870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2
 + 277136640*a + 60466176)) + 6613472*a + 3543424)*sqrt((105*a^4 + 1470*a^3
 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5
+ 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4
+ 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 +
 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 943
20045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 +
 592064640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9
 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 +
950400*a^2 + 725760*a + 248832)) - 11228868*a - 9923472) + 524*a^2 - 4*(11*
a^3 + 107*a^2 - 84*a - 1152)*x + 1632*a + 1152)/((a^4 + 14*a^3 + 73*a^2 + 1
68*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 1
4*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a
 + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 921
6)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3
+ 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^
2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(230) = 460$.

Time = 8.02 (sec) , antiderivative size = 697, normalized size of antiderivative = 2.77

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx =$$

$$-\frac{}{32a^6+448a^5+2336a^4+5376a^3+4608a^2+x^8\cdot(32a^4+448a^3+2336a^2+5376a+4608)+x^7\,(-256a}$$

$$-\mathrm{RootSum}\left(t^4\cdot\left(268435456a^{15}+14763950080a^{14}+378493992960a^{13}+5999532441600a^{12}+657572914\right.\right.$$

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] -(11*a**3 + 131*a**2 + 408*a + x**7*(12*a + 42) + x**6*(-84*a - 294) + x**5
*(7*a**2 + 343*a + 1116) + x**4*(-35*a**2 - 875*a - 2640) + x**3*(68*a**2 +
 1358*a + 3936) + x**2*(-64*a**2 - 1246*a - 3600) + x*(-11*a**3 - 107*a**2
+ 84*a + 1152) + 288)/(32*a**6 + 448*a**5 + 2336*a**4 + 5376*a**3 + 4608*a*
*2 + x**8*(32*a**4 + 448*a**3 + 2336*a**2 + 5376*a + 4608) + x**7*(-256*a**
4 - 3584*a**3 - 18688*a**2 - 43008*a - 36864) + x**6*(1024*a**4 + 14336*a**
3 + 74752*a**2 + 172032*a + 147456) + x**5*(-2560*a**4 - 35840*a**3 - 18688
0*a**2 - 430080*a - 368640) + x**4*(-64*a**5 + 3200*a**4 + 52672*a**3 + 288
256*a**2 + 678912*a + 589824) + x**3*(256*a**5 - 512*a**4 - 38656*a**3 - 25
6000*a**2 - 651264*a - 589824) + x**2*(-512*a**5 - 5120*a**4 - 8704*a**3 +
63488*a**2 + 270336*a + 294912) + x*(512*a**5 + 7168*a**4 + 37376*a**3 + 86
016*a**2 + 73728*a)) - RootSum(_t**4*(268435456*a**15 + 14763950080*a**14 +
 378493992960*a**13 + 5999532441600*a**12 + 65757291479040*a**11 + 52787590
8304896*a**10 + 3206246773555200*a**9 + 15003759578972160*a**8 + 5453715112
7224320*a**7 + 153980418717122560*a**6 + 334927734494986240*a**5 + 55115219
3655275520*a**4 + 664192984106926080*a**3 + 553362212027105280*a**2 + 28499
3413919539200*a + 68398419340689408) + _t**2*(-30965760*a**9 - 1052835840*a
**8 - 15910207488*a**7 - 140262506496*a**6 - 795007254528*a**5 - 3004516270
080*a**4 - 7571263979520*a**3 - 12268037210112*a**2 - 11598827618304*a - 48
75324751872) - 194481*a**4 - 2762424*a**3 - 14762736*a**2 - 35178624*a - 31
539456, Lambda(_t, _t*log(x + (23068672*_t**3*a**12 + 968884224*_t**3*a**11
 + 18624806912*_t**3*a**10 + 216677744640*_t**3*a**9 + 1699123036160*_t**3*
a**8 + 9461389328384*_t**3*a**7 + 38361186172928*_t**3*a**6 + 1141074915491
84*_t**3*a**5 + 247138458009600*_t**3*a**4 + 380084473036800*_t**3*a**3 + 3
94002582994944*_t**3*a**2 + 247177515368448*_t**3*a + 70970039599104*_t**3
- 395136*_t*a**7 - 11676672*_t*a**6 - 144076032*_t*a**5 - 969518592*_t*a**4
 - 3861475200*_t*a**3 - 9133300224*_t*a**2 - 11906574336*_t*a - 6611337216*
_t - 64827*a**4 - 907578*a**3 - 4780647*a**2 - 11228868*a - 9923472)/(64827
*a**4 + 907578*a**3 + 4780647*a**2 + 11228868*a + 9923472))))
```

Maxima [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \int -\frac{1}{(x^4-4\,x^3+8\,x^2-a-8\,x)^3}\,dx$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(6*(2*a + 7)*x^7 - 42*(2*a + 7)*x^6 + (7*a^2 + 343*a + 1116)*x^5 - 5*
(7*a^2 + 175*a + 528)*x^4 + 2*(34*a^2 + 679*a + 1968)*x^3 + 11*a^3 - 2*(32*
a^2 + 623*a + 1800)*x^2 + 131*a^2 - (11*a^3 + 107*a^2 - 84*a - 1152)*x + 40
8*a + 288)/((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 7
3*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a
^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^
4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 15
1*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3
 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a
^2 + 144*a)*x) - 3/32*integrate((2*(2*a + 7)*x^2 + 7*a^2 - 4*(2*a + 7)*x +
55*a + 108)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^4 + 14*a^3 + 73*a^2 + 16
8*a + 144)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16632 vs. $2(220) = 440$.

Time = 11.96 (sec) , antiderivative size = 16632, normalized size of antiderivative = 66.00

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] -3/128*(sqrt((105*a^5 + 1890*a^4 + 13629*a^3 + 49224*a^2 + (49*a^5 + 926*a^
4 + 6997*a^3 + 26428*a^2 + 49904*a + 37696)*sqrt(a + 4) + 89056*a + 64576)/
(a^3 + 11*a^2 + 40*a + 48))*log(abs(16807*sqrt(a + 4)*a^15 + 26411*a^15 + 9
08950*sqrt(a + 4)*a^14 + 1420804*a^14 + 22929088*sqrt(a + 4)*a^13 + 2401*sq
rt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073
*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) +
 331744*a + 193728)*a^12*x + 35650176*a^13 + 2401*sqrt(105*a^6 + 2205*a^5 +
 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419
*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqr
t(a + 4)*a^11*x - 2401*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 23
6728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*
a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^12 + 357887692*sqrt(a + 4)*a
^12 + 105154*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 +
 (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088
```

)*sqrt(a + 4) + 331744*a + 193728)*a^11*x - 2401*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^11 + 553458148*a^12 + 95550*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^10*x - 105154*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^11 + 3865394166*sqrt(a + 4)*a^11 + 2109279*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^10*x - 95550*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^10 + 5945365998*a^11 + 1727079*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^9*x - 2109279*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^10 + 30600511272*sqrt(a + 4)*a^10 + 25624212*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^9*x - 1727079*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^9 + 46810709868*a^10 + 18715896*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^8*x - 25624212*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^9 + 183431725500*sqrt(a + 4)*a^9 + 209972223*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^8*x - 18715896*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^8 + 279067335420*a^9 + 135108639*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^7*x - 209972223*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^8 + 847810669320*sqrt(a + 4)*a^8 + 1222644882*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^7*x - 135108639*sqrt(105*a^6 + 2205*a^5 +

19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*
a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt
(a + 4)*a^7 + 1282741275000*a^8 + 682210326*sqrt(105*a^6 + 2205*a^5 + 19299
*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 +
129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a +
4)*a^6*x - 1222644882*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236
728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a
+ 113088)*sqrt(a + 4) + 331744*a + 193728)*a^7 + 3046208716923*sqrt(a + 4)
*a^7 + 5187446733*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*
a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 1
13088)*sqrt(a + 4) + 331744*a + 193728)*a^6*x - 682210326*sqrt(105*a^6 + 22
05*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4
+ 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193
728)*sqrt(a + 4)*a^6 + 4583471076759*a^7 + 2458605429*sqrt(105*a^6 + 2205*a
^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 4
7419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)
*sqrt(a + 4)*a^5*x - 5187446733*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111
*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2
+ 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^6 + 8508527261290*s
qrt(a + 4)*a^6 + 16158435972*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^
3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 1
87408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^5*x - 2458605429*sqrt(
105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^
5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 33
1744*a + 193728)*sqrt(a + 4)*a^5 + 12731345334296*a^6 + 6324014256*sqrt(105
*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 +
9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 33174
4*a + 193728)*sqrt(a + 4)*a^4*x - 16158435972*sqrt(105*a^6 + 2205*a^5 + 192
99*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3
+ 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^5 + 1
8324012543204*sqrt(a + 4)*a^5 + 36673732452*sqrt(105*a^6 + 2205*a^5 + 19299
*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 +
129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^4*x - 6
324014256*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (4
9*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*s
qrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^4 + 27265747047380*a^5 + 1137
7675428*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*
a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqr
t(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^3*x - 36673732452*sqrt(105*a^6
+ 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775
*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a +
193728)*a^4 + 29880339194272*sqrt(a + 4)*a^4 + 59146408708*sqrt(105*a^6 +
2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a
^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 1
93728)*a^3*x - 11377675428*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3

+ 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187
408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^3 + 44213263
379848*a^4 + 13635706996*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 +
236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 18740
8*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^2*x - 59146408
708*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6
+ 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a
+ 4) + 331744*a + 193728)*a^3 + 35712575419864*sqrt(a + 4)*a^3 + 6434003664
0*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 +
1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a +
4) + 331744*a + 193728)*a^2*x - 13635706996*sqrt(105*a^6 + 2205*a^5 + 19299
*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 +
129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a +
4)*a^2 + 52547642661032*a^3 + 9797208656*sqrt(105*a^6 + 2205*a^5 + 19299*a^
4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 12
9188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*
a*x - 64340036640*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*
a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 1
13088)*sqrt(a + 4) + 331744*a + 193728)*a^2 + 29533943648028*sqrt(a + 4)*a^
2 + 42385864836*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^
2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113
088)*sqrt(a + 4) + 331744*a + 193728)*a*x - 9797208656*sqrt(105*a^6 + 2205*
a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 +
47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728
)*sqrt(a + 4)*a + 43213040637212*a^2 + 3197030212*sqrt(105*a^6 + 2205*a^5 +
19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419
*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqr
t(a + 4)*x - 42385864836*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 +
236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 18740
8*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a + 15111479733208*sqrt(a +
4)*a + 12788120848*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728
*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a +
113088)*sqrt(a + 4) + 331744*a + 193728)*x - 3197030212*sqrt(105*a^6 + 2205
*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 +
47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 19372
8)*sqrt(a + 4) + 21986673204304*a - 12788120848*sqrt(105*a^6 + 2205*a^5 + 1
9299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a
^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728) + 360
6250079136*sqrt(a + 4) + 5217553305984)) - sqrt((105*a^5 + 1890*a^4 + 13629
*a^3 + 49224*a^2 + (49*a^5 + 926*a^4 + 6997*a^3 + 26428*a^2 + 49904*a + 376
96)*sqrt(a + 4) + 89056*a + 64576)/(a^3 + 11*a^2 + 40*a + 48))*log(abs(1680
7*sqrt(a + 4)*a^15 + 26411*a^15 + 908950*sqrt(a + 4)*a^14 + 1420804*a^14 +
22929088*sqrt(a + 4)*a^13 - 2401*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 9011
1*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2
+ 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^12*x + 35650176*a^

13 - 2401*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (4
9*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*s
qrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^11*x + 2401*sqrt(105*a^6 + 22
05*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4
 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193
728)*a^12 + 357887692*sqrt(a + 4)*a^12 - 105154*sqrt(105*a^6 + 2205*a^5 + 1
9299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a
^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^11*
x + 2401*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49
*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sq
rt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^11 + 553458148*a^12 - 95550*sq
rt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073
*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) +
 331744*a + 193728)*sqrt(a + 4)*a^10*x + 105154*sqrt(105*a^6 + 2205*a^5 + 1
9299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a
^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^11
+ 3865394166*sqrt(a + 4)*a^11 - 2109279*sqrt(105*a^6 + 2205*a^5 + 19299*a^4
 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129
188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^10*x + 9555
0*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 +
1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a +
4) + 331744*a + 193728)*sqrt(a + 4)*a^10 + 5945365998*a^11 - 1727079*sqrt(1
05*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5
 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331
744*a + 193728)*sqrt(a + 4)*a^9*x + 2109279*sqrt(105*a^6 + 2205*a^5 + 19299
*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 +
 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^10 + 30
600511272*sqrt(a + 4)*a^10 - 25624212*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 +
 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 12918
8*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^9*x + 1727079
*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1
073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4
) + 331744*a + 193728)*sqrt(a + 4)*a^9 + 46810709868*a^10 - 18715896*sqrt(1
05*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5
 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331
744*a + 193728)*sqrt(a + 4)*a^8*x + 25624212*sqrt(105*a^6 + 2205*a^5 + 1929
9*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3
+ 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^9 + 18
3431725500*sqrt(a + 4)*a^9 - 209972223*sqrt(105*a^6 + 2205*a^5 + 19299*a^4
+ 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 1291
88*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^8*x + 187158
96*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 +
 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a +
 4) + 331744*a + 193728)*sqrt(a + 4)*a^8 + 279067335420*a^9 - 135108639*sqr
t(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*

a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) +
331744*a + 193728)*sqrt(a + 4)*a^7*x + 209972223*sqrt(105*a^6 + 2205*a^5 +
19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*
a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^8
+ 847810669320*sqrt(a + 4)*a^8 - 1222644882*sqrt(105*a^6 + 2205*a^5 + 19299
*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 +
129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^7*x + 1
35108639*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49
*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sq
rt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^7 + 1282741275000*a^8 - 682210
326*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6
+ 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a
+ 4) + 331744*a + 193728)*sqrt(a + 4)*a^6*x + 1222644882*sqrt(105*a^6 + 220
5*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4
+ 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 1937
28)*a^7 + 3046208716923*sqrt(a + 4)*a^7 - 5187446733*sqrt(105*a^6 + 2205*a^
5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47
419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*
a^6*x + 682210326*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*
a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 1
13088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^6 + 4583471076759*a^7
- 2458605429*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2
+ (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 11308
8)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^5*x + 5187446733*sqrt(105
*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 +
9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 33174
4*a + 193728)*a^6 + 8508527261290*sqrt(a + 4)*a^6 - 16158435972*sqrt(105*a^
6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 97
75*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a
+ 193728)*a^5*x + 2458605429*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a
^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 +
187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^5 + 12731
345334296*a^6 - 6324014256*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3
+ 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187
408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^4*x + 161584
35972*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^
6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(
a + 4) + 331744*a + 193728)*a^5 + 18324012543204*sqrt(a + 4)*a^5 - 36673732
452*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6
+ 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a
+ 4) + 331744*a + 193728)*a^4*x + 6324014256*sqrt(105*a^6 + 2205*a^5 + 1929
9*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3
+ 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a +
4)*a^4 + 27265747047380*a^5 - 11377675428*sqrt(105*a^6 + 2205*a^5 + 19299*
a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 +

129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4
)*a^3*x + 36673732452*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236
728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a
 + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^4 + 29880339194272*sqrt(a + 4
)*a^4 - 59146408708*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 23672
8*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a +
 113088)*sqrt(a + 4) + 331744*a + 193728)*a^3*x + 11377675428*sqrt(105*a^6
+ 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775
*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a +
 193728)*sqrt(a + 4)*a^3 + 44213263379848*a^4 - 13635706996*sqrt(105*a^6 +
2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a
^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 1
93728)*sqrt(a + 4)*a^2*x + 59146408708*sqrt(105*a^6 + 2205*a^5 + 19299*a^4
+ 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 1291
88*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^3 + 35712575
419864*sqrt(a + 4)*a^3 - 64340036640*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 +
90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188
*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^2*x + 13635706
996*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6
+ 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a
+ 4) + 331744*a + 193728)*sqrt(a + 4)*a^2 + 52547642661032*a^3 - 9797208656
*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1
073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4
) + 331744*a + 193728)*sqrt(a + 4)*a*x + 64340036640*sqrt(105*a^6 + 2205*a^
5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47
419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*
a^2 + 29533943648028*sqrt(a + 4)*a^2 - 42385864836*sqrt(105*a^6 + 2205*a^5
+ 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 4741
9*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a*
x + 9797208656*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2
 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 1130
88)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a + 43213040637212*a^2 - 3
197030212*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (4
9*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*s
qrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*x + 42385864836*sqrt(105*a^6 +
2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a
^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 1
93728)*a + 15111479733208*sqrt(a + 4)*a - 12788120848*sqrt(105*a^6 + 2205*a
^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 4
7419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)
*x + 3197030212*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^
2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113
088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4) + 21986673204304*a + 1278
8120848*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*
a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqr

t(a + 4) + 331744*a + 193728) + 3606250079136*sqrt(a + 4) + 5217553305984))
 + sqrt((105*a^5 + 1890*a^4 + 13629*a^3 + 49224*a^2 - (49*a^5 + 926*a^4 + 6
997*a^3 + 26428*a^2 + 49904*a + 37696)*sqrt(a + 4) + 89056*a + 64576)/(a^3
 + 11*a^2 + 40*a + 48))*log(abs(-16807*sqrt(a + 4)*a^15 + 26411*a^15 - 90895
0*sqrt(a + 4)*a^14 + 1420804*a^14 - 22929088*sqrt(a + 4)*a^13 + 2401*sqrt(1
05*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5
 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331
744*a + 193728)*a^12*x + 35650176*a^13 - 2401*sqrt(105*a^6 + 2205*a^5 + 192
99*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3
 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a
 + 4)*a^11*x - 2401*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728
*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a +
113088)*sqrt(a + 4) + 331744*a + 193728)*a^12 - 357887692*sqrt(a + 4)*a^12
 + 105154*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49
*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sq
rt(a + 4) + 331744*a + 193728)*a^11*x + 2401*sqrt(105*a^6 + 2205*a^5 + 1929
9*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3
 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a +
 4)*a^11 + 553458148*a^12 - 95550*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 901
11*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^
2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^10*x
 - 105154*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49
*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sq
rt(a + 4) + 331744*a + 193728)*a^11 - 3865394166*sqrt(a + 4)*a^11 + 2109279
*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1
073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4
) + 331744*a + 193728)*a^10*x + 95550*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 +
 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 12918
8*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^1
0 + 5945365998*a^11 - 1727079*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a
^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 +
187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^9*x - 210
9279*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6
 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a
 + 4) + 331744*a + 193728)*a^10 - 30600511272*sqrt(a + 4)*a^10 + 25624212*s
qrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 107
3*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4)
 + 331744*a + 193728)*a^9*x + 1727079*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 +
90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188
*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^9
 + 46810709868*a^10 - 18715896*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a
^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 +
187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^8*x - 256
24212*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^
6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(

a + 4) + 331744*a + 193728)*a^9 - 183431725500*sqrt(a + 4)*a^9 + 209972223*
sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 10
73*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4)
 + 331744*a + 193728)*a^8*x + 18715896*sqrt(105*a^6 + 2205*a^5 + 19299*a^4
 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 1291
88*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^
8 + 279067335420*a^9 - 135108639*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 9011
1*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2
 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^7*x -
209972223*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (4
9*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*s
qrt(a + 4) + 331744*a + 193728)*a^8 - 847810669320*sqrt(a + 4)*a^8 + 122264
4882*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6
 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a
 + 4) + 331744*a + 193728)*a^7*x + 135108639*sqrt(105*a^6 + 2205*a^5 + 1929
9*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3
 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a +
 4)*a^7 + 1282741275000*a^8 - 682210326*sqrt(105*a^6 + 2205*a^5 + 19299*a^4
 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129
188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a
^6*x - 1222644882*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*
a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 1
13088)*sqrt(a + 4) + 331744*a + 193728)*a^7 - 3046208716923*sqrt(a + 4)*a^7
 + 5187446733*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2
 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 11308
8)*sqrt(a + 4) + 331744*a + 193728)*a^6*x + 682210326*sqrt(105*a^6 + 2205*a
^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 4
7419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)
*sqrt(a + 4)*a^6 + 4583471076759*a^7 - 2458605429*sqrt(105*a^6 + 2205*a^5 +
 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419
*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqr
t(a + 4)*a^5*x - 5187446733*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3
 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 18
7408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^6 - 8508527261290*sqrt(
a + 4)*a^6 + 16158435972*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 +
236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 18740
8*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^5*x + 2458605429*sqrt(105*
a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 +
9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744
*a + 193728)*sqrt(a + 4)*a^5 + 12731345334296*a^6 - 6324014256*sqrt(105*a^6
 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 977
5*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a
 + 193728)*sqrt(a + 4)*a^4*x - 16158435972*sqrt(105*a^6 + 2205*a^5 + 19299*a
^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 1
29188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^5 - 18324

012543204*sqrt(a + 4)*a^5 + 36673732452*sqrt(105*a^6 + 2205*a^5 + 19299*a^4
 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129
188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^4*x + 63240
14256*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^
6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(
a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^4 + 27265747047380*a^5 - 11377675
428*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6
+ 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a
+ 4) + 331744*a + 193728)*sqrt(a + 4)*a^3*x - 36673732452*sqrt(105*a^6 + 22
05*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4
 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193
728)*a^4 - 29880339194272*sqrt(a + 4)*a^4 + 59146408708*sqrt(105*a^6 + 2205
*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 +
 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 19372
8)*a^3*x + 11377675428*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 23
6728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*
a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^3 + 442132633798
48*a^4 - 13635706996*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 2367
28*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a
+ 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^2*x - 59146408708*
sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 10
73*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4)
 + 331744*a + 193728)*a^3 - 35712575419864*sqrt(a + 4)*a^3 + 64340036640*sq
rt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073
*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) +
 331744*a + 193728)*a^2*x + 13635706996*sqrt(105*a^6 + 2205*a^5 + 19299*a^4
 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129
188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a
^2 + 52547642661032*a^3 - 9797208656*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 +
90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188
*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a*x
- 64340036640*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2
- (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 11308
8)*sqrt(a + 4) + 331744*a + 193728)*a^2 - 29533943648028*sqrt(a + 4)*a^2 +
42385864836*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 -
(49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)
*sqrt(a + 4) + 331744*a + 193728)*a*x + 9797208656*sqrt(105*a^6 + 2205*a^5
+ 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 4741
9*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sq
rt(a + 4)*a + 43213040637212*a^2 - 3197030212*sqrt(105*a^6 + 2205*a^5 + 192
99*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3
 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a
+ 4)*x - 42385864836*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 2367
28*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a
+ 113088)*sqrt(a + 4) + 331744*a + 193728)*a - 15111479733208*sqrt(a + 4)*a

+ 12788120848*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2
- (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 1130
88)*sqrt(a + 4) + 331744*a + 193728)*x + 3197030212*sqrt(105*a^6 + 2205*a^5
+ 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 474
19*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*s
qrt(a + 4) + 21986673204304*a - 12788120848*sqrt(105*a^6 + 2205*a^5 + 19299
*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 +
129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728) - 3606250
079136*sqrt(a + 4) + 5217553305984)) - sqrt((105*a^5 + 1890*a^4 + 13629*a^3
+ 49224*a^2 - (49*a^5 + 926*a^4 + 6997*a^3 + 26428*a^2 + 49904*a + 37696)*
sqrt(a + 4) + 89056*a + 64576)/(a^3 + 11*a^2 + 40*a + 48))*log(abs(-16807*s
qrt(a + 4)*a^15 + 26411*a^15 - 908950*sqrt(a + 4)*a^14 + 1420804*a^14 - 229
29088*sqrt(a + 4)*a^13 - 2401*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a
^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 +
187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^12*x + 35650176*a^13
+ 2401*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a
^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt
(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^11*x + 2401*sqrt(105*a^6 + 2205*
a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 +
47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728
)*a^12 - 357887692*sqrt(a + 4)*a^12 - 105154*sqrt(105*a^6 + 2205*a^5 + 1929
9*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3
+ 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^11*x -
2401*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^
6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(
a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^11 + 553458148*a^12 + 95550*sqrt(
105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^
5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 33
1744*a + 193728)*sqrt(a + 4)*a^10*x + 105154*sqrt(105*a^6 + 2205*a^5 + 1929
9*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3
+ 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^11 - 3
865394166*sqrt(a + 4)*a^11 - 2109279*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 +
90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188
*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^10*x - 95550*s
qrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 107
3*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4)
+ 331744*a + 193728)*sqrt(a + 4)*a^10 + 5945365998*a^11 + 1727079*sqrt(105*
a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 +
9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744
*a + 193728)*sqrt(a + 4)*a^9*x + 2109279*sqrt(105*a^6 + 2205*a^5 + 19299*a^
4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 12
9188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^10 - 30600
511272*sqrt(a + 4)*a^10 - 25624212*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90
111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a
^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^9*x - 1727079*sq

rt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073
*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) +
 331744*a + 193728)*sqrt(a + 4)*a^9 + 46810709868*a^10 + 18715896*sqrt(105*
a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 +
9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744
*a + 193728)*sqrt(a + 4)*a^8*x + 25624212*sqrt(105*a^6 + 2205*a^5 + 19299*a
^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 1
29188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^9 - 18343
1725500*sqrt(a + 4)*a^9 - 209972223*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 9
0111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*
a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^8*x - 18715896*
sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 10
73*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4)
 + 331744*a + 193728)*sqrt(a + 4)*a^8 + 279067335420*a^9 + 135108639*sqrt(1
05*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5
 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331
744*a + 193728)*sqrt(a + 4)*a^7*x + 209972223*sqrt(105*a^6 + 2205*a^5 + 192
99*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3
 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^8 - 8
47810669320*sqrt(a + 4)*a^8 - 1222644882*sqrt(105*a^6 + 2205*a^5 + 19299*a^
4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 12
9188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^7*x - 1351
08639*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^
6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(
a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^7 + 1282741275000*a^8 + 682210326
*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1
073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4
) + 331744*a + 193728)*sqrt(a + 4)*a^6*x + 1222644882*sqrt(105*a^6 + 2205*a
^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 4
7419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)
*a^7 - 3046208716923*sqrt(a + 4)*a^7 - 5187446733*sqrt(105*a^6 + 2205*a^5 +
 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419
*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^6
*x - 682210326*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2
 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 1130
88)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^6 + 4583471076759*a^7 +
2458605429*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (
49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*
sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^5*x + 5187446733*sqrt(105*a^
6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 97
75*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a
 + 193728)*a^6 - 8508527261290*sqrt(a + 4)*a^6 - 16158435972*sqrt(105*a^6 +
 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*
a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a +
193728)*a^5*x - 2458605429*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3

+ 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187
408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^5 + 12731345
334296*a^6 + 6324014256*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 2
36728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408
*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a^4*x + 161584359
72*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 +
1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a +
4) + 331744*a + 193728)*a^5 - 18324012543204*sqrt(a + 4)*a^5 - 36673732452
*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1
073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4
) + 331744*a + 193728)*a^4*x - 6324014256*sqrt(105*a^6 + 2205*a^5 + 19299*a
^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 1
29188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)
*a^4 + 27265747047380*a^5 + 11377675428*sqrt(105*a^6 + 2205*a^5 + 19299*a^4
+ 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129
188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a
^3*x + 36673732452*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728
*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a +
113088)*sqrt(a + 4) + 331744*a + 193728)*a^4 - 29880339194272*sqrt(a + 4)*a
^4 - 59146408708*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a
^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 11
3088)*sqrt(a + 4) + 331744*a + 193728)*a^3*x - 11377675428*sqrt(105*a^6 + 2
205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^
4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 19
3728)*sqrt(a + 4)*a^3 + 44213263379848*a^4 + 13635706996*sqrt(105*a^6 + 220
5*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4
+ 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 1937
28)*sqrt(a + 4)*a^2*x + 59146408708*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 9
0111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*
a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^3 - 35712575419
864*sqrt(a + 4)*a^3 - 64340036640*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 901
11*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^
2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^2*x - 13635706996
*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1
073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4
) + 331744*a + 193728)*sqrt(a + 4)*a^2 + 52547642661032*a^3 + 9797208656*sq
rt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073
*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) +
331744*a + 193728)*sqrt(a + 4)*a*x + 64340036640*sqrt(105*a^6 + 2205*a^5 +
19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419
*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^2
- 29533943648028*sqrt(a + 4)*a^2 - 42385864836*sqrt(105*a^6 + 2205*a^5 + 1
9299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a
^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a*x -
9797208656*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 -

```
(49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)
*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4)*a + 43213040637212*a^2 + 3197
030212*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a
^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt
(a + 4) + 331744*a + 193728)*sqrt(a + 4)*x + 42385864836*sqrt(105*a^6 + 220
5*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4
+ 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 1937
28)*a - 15111479733208*sqrt(a + 4)*a - 12788120848*sqrt(105*a^6 + 2205*a^5
+ 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6 + 1073*a^5 + 9775*a^4 + 4741
9*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*x
- 3197030212*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 -
 (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088
)*sqrt(a + 4) + 331744*a + 193728)*sqrt(a + 4) + 21986673204304*a + 1278812
0848*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 - (49*a^6
 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a
 + 4) + 331744*a + 193728) - 3606250079136*sqrt(a + 4) + 5217553305984)))/(
a^4 + 14*a^3 + 73*a^2 + 168*a + 144) - 1/32*(12*a*x^7 + 7*a^2*x^5 - 84*a*x^
6 + 42*x^7 - 35*a^2*x^4 + 343*a*x^5 - 294*x^6 + 68*a^2*x^3 - 875*a*x^4 + 11
16*x^5 - 11*a^3*x - 64*a^2*x^2 + 1358*a*x^3 - 2640*x^4 + 11*a^3 - 107*a^2*x
 - 1246*a*x^2 + 3936*x^3 + 131*a^2 + 84*a*x - 3600*x^2 + 408*a + 1152*x + 2
88)/((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^
2)
```

Mupad [B] (verification not implemented)

Time = 12.12 (sec) , antiderivative size = 8242, normalized size of antiderivative = 32.71

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Too large to display}$$

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] atan(((((52357496832*a + 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^
4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^
9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 +
 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) + (
(4290672328704*a + 6001143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^
4 + 1090200272896*a^5 + 302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8
 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 1391569403904)/(16384*(94
0032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5
564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3510632448*a + 402024038
4*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*a^5 + 61407232*a^6 + 74
38336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(256*(48384*a + 49248*a^2
 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9
*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*(
```

(a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^
5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(106168
3200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5
+ 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^1
0 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1
/2))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) -
49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 26
95744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384
*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703
040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 196
6491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 2548039
68)))^(1/2) + (108343296*a + 74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 61
4016*a^5 + 28224*a^6 + 66207744)/(16384*(940032*a + 1195776*a^2 + 899328*a^
3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^1
0 + 331776)) - (x*(73476*a + 31545*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*
(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7
+ a^8 + 20736)))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^
15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187
840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531
456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a
^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 119442
00*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^
15 + 254803968)))^(1/2)*1i - (((52357496832*a + 57139003392*a^2 + 363221483
52*a^3 + 14822473728*a^4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 +
5726208*a^8 + 172032*a^9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 89
9328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^
9 + a^10 + 331776)) - ((4290672328704*a + 6001143054336*a^2 + 5025917042688
*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 302556119040*a^6 + 598621552
64*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 13
91569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149
208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3
510632448*a + 4020240384*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*
a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(2
56*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a
^7 + a^8 + 20736)))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a +
4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10
187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16
531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 205320192
0*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 119
44200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 +
a^15 + 254803968)))^(1/2))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 58
9*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*
a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*
a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2
053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a

^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 5
5*a^14 + a^15 + 254803968)))^(1/2) - (108343296*a + 74059776*a^2 + 27065088
*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 + 66207744)/(16384*(940032*a +
1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 +
582*a^8 + 36*a^9 + a^10 + 331776)) + (x*(73476*a + 31545*a^2 + 6066*a^3 +
441*a^4 + 64656))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*
a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a - 338*a*((a + 4)^15
)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2
+ 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 357
0*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 247431
1680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7
+ 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1
410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2)*1i)/((((52357496832*a + 5713
9003392*a^2 + 36322148352*a^3 + 14822473728*a^4 + 4027170816*a^5 + 72850636
8*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^9 + 21290287104)/(16384*(9400
32*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 556
4*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) + ((4290672328704*a + 6001143054
336*a^2 + 5025917042688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 30255
6119040*a^6 + 59862155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a
^10 + 1048576*a^11 + 1391569403904)/(16384*(940032*a + 1195776*a^2 + 899328
*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 +
a^10 + 331776)) - (x*(3510632448*a + 4020240384*a^2 + 2678587392*a^3 + 1144
324096*a^4 + 325074944*a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8 + 1638
4*a^9 + 1358954496))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 23
80*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a - 338*a*((a + 4)
^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*
a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 +
3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 247
4311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a
^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12
+ 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2))*((9*(39329792*a - 338*a*
((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 4
1598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 5394
9*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a
^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203
166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 223
50*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) + (108343296*a +
74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 + 662077
44)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 +
34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(73476*a +
31545*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*(48384*a + 49248*a^2 + 28560*
a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(3932979
2*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^1
5)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 47560
8*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a +

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2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 5736217
60*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 24496
5*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) + (((
52357496832*a + 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^4 + 40271
70816*a^5 + 728506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^9 + 21290
287104)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a
^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - ((42906723
28704*a + 6001143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^4 + 10902
00272896*a^5 + 302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8 + 761266
176*a^9 + 41943040*a^10 + 1048576*a^11 + 1391569403904)/(16384*(940032*a +
1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 +
 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3510632448*a + 4020240384*a^2 + 2
678587392*a^3 + 1144324096*a^4 + 325074944*a^5 + 61407232*a^6 + 7438336*a^7
 + 524288*a^8 + 16384*a^9 + 1358954496))/(256*(48384*a + 49248*a^2 + 28560*
a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(3932979
2*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^1
5)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 47560
8*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a +
2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 5736217
60*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 24496
5*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2))*((9*
(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((
a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5
 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683
200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 +
 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10
 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/
2) - (108343296*a + 74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 614016*a^5
+ 28224*a^6 + 66207744)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 44286
4*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 33177
6)) + (x*(73476*a + 31545*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*(48384*a
+ 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 2
0736)))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2)
 - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 +
 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16
384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247
703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 +
1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 2548
03968)))^(1/2) - (99468*a + 28053*a^2 + 2646*a^3 + 117936)/(8192*(940032*a
+ 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7
 + 582*a^8 + 36*a^9 + a^10 + 331776))))*((9*(39329792*a - 338*a*((a + 4)^15
)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2
 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 357
0*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 247431
1680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7
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+ 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1
410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2)*2i + atan(((((52357496832*a
+ 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^4 + 4027170816*a^5 + 72
8506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^9 + 21290287104)/(16384
*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6
+ 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) + ((4290672328704*a + 6001
143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 +
302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 4194
3040*a^10 + 1048576*a^11 + 1391569403904)/(16384*(940032*a + 1195776*a^2 +
899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*
a^9 + a^10 + 331776)) - (x*(3510632448*a + 4020240384*a^2 + 2678587392*a^3
+ 1144324096*a^4 + 325074944*a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8
+ 16384*a^9 + 1358954496))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^
4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a + 338*a*((
a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 415
98976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*
a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2
+ 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 20316
6720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350
*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2))*((9*(39329792*a +
338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/
2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6
+ 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 206143
4880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6
+ 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11
+ 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) + (10834329
6*a + 74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 +
66207744)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208
*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(7347
6*a + 31545*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*(48384*a + 49248*a^2 +
28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(3
9329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a
+ 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 +
475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(106168320
0*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 5
73621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 +
244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2)
*1i - (((52357496832*a + 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^
4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^
9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 +
149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (
(4290672328704*a + 6001143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^
4 + 1090200272896*a^5 + 302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8
+ 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 1391569403904)/(16384*(94
0032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5

564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3510632448*a + 402024038
4*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*a^5 + 61407232*a^6 + 74
38336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(256*(48384*a + 49248*a^2
 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9
*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*(
(a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^
5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(106168
3200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5
+ 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^1
0 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1
/2))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) +
49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 26
95744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384
*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703
040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 196
6491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 2548039
68)))^(1/2) - (108343296*a + 74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 61
4016*a^5 + 28224*a^6 + 66207744)/(16384*(940032*a + 1195776*a^2 + 899328*a^
3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^1
0 + 331776)) + (x*(73476*a + 31545*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*
(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7
+ a^8 + 20736)))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^
15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187
840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531
456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a
^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 119442
00*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^
15 + 254803968)))^(1/2)*1i)/((((52357496832*a + 57139003392*a^2 + 363221483
52*a^3 + 14822473728*a^4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 +
5726208*a^8 + 172032*a^9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 89
9328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^
9 + a^10 + 331776)) + ((4290672328704*a + 6001143054336*a^2 + 5025917042688
*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 302556119040*a^6 + 598621552
64*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 13
91569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149
208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3
510632448*a + 4020240384*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*
a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(2
56*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a
^7 + a^8 + 20736)))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a +
4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10
187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16
531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 205320192
0*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 119
44200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 +

a^15 + 254803968)))^(1/2))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 58
9*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*
a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*
a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2
053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a
^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 5
5*a^14 + a^15 + 254803968)))^(1/2) + (108343296*a + 74059776*a^2 + 27065088
*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 + 66207744)/(16384*(940032*a +
1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 +
582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(73476*a + 31545*a^2 + 6066*a^3 +
441*a^4 + 64656))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*
a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a + 338*a*((a + 4)^15
)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2
+ 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 357
0*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 247431
1680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7
+ 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1
410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) + (((52357496832*a + 5713900
3392*a^2 + 36322148352*a^3 + 14822473728*a^4 + 4027170816*a^5 + 728506368*a
^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^9 + 21290287104)/(16384*(940032*
a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a
^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - ((4290672328704*a + 6001143054336
*a^2 + 5025917042688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 30255611
9040*a^6 + 59862155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^10
+ 1048576*a^11 + 1391569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^
3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^1
0 + 331776)) - (x*(3510632448*a + 4020240384*a^2 + 2678587392*a^3 + 1144324
096*a^4 + 325074944*a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8 + 16384*a
^9 + 1358954496))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*
a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a + 338*a*((a + 4)^15
)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2
+ 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 357
0*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 247431
1680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7
+ 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1
410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2))*((9*(39329792*a + 338*a*((a
+ 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 4159
8976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a
^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2
+ 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166
720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*
a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) - (108343296*a + 740
59776*a^2 + 27065088*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 + 66207744)
/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34
833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) + (x*(73476*a + 315

45*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) - (99468*a + 28053*a^2 + 2646*a^3 + 117936)/(8192*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776))))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2)*2i - ((408*a + 131*a^2 + 11*a^3 + 288)/(32*(a + 4)*(33*a + 10*a^2 + a^3 + 36)) - (21*x^6*(2*a + 7))/(16*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)) + (3*x^7*(2*a + 7))/(16*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)) + (x*(84*a - 107*a^2 - 11*a^3 + 1152))/(32*(a + 4)*(33*a + 10*a^2 + a^3 + 36)) - (5*x^4*(175*a + 7*a^2 + 528))/(32*(a + 4)*(33*a + 10*a^2 + a^3 + 36)) + (x^5*(343*a + 7*a^2 + 1116))/(32*(a + 4)*(33*a + 10*a^2 + a^3 + 36)) - (x^2*(623*a + 32*a^2 + 1800))/(16*(a + 4)*(33*a + 10*a^2 + a^3 + 36)) + (x^3*(679*a + 34*a^2 + 1968))/(16*(a + 4)*(33*a + 10*a^2 + a^3 + 36)))/(16*a*x - x^2*(16*a - 64) - x^4*(2*a - 128) + x^3*(8*a - 128) + a^2 - 80*x^5 + 32*x^6 - 8*x^7 + x^8)

3.123 $\int x(a+8x-8x^2+4x^3-x^4)^4\,dx$

Optimal result . . . 950
Rubi [A] (verified) . . . 951
Mathematica [A] (verified) . . . 952
Maple [A] (verified) . . . 953
Fricas [A] (verification not implemented) . . . 953
Sympy [A] (verification not implemented) . . . 954
Maxima [A] (verification not implemented) . . . 954
Giac [A] (verification not implemented) . . . 955
Mupad [B] (verification not implemented) . . . 956

Optimal result

Integrand size = 24, antiderivative size = 210

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx=\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+8(12-a)a^2x^4+\frac{16}{5}a\left(128-48a+a^2\right)x^5+\frac{2}{3}\left(1024-1536a+192a^2-a^3\right)x^6-\frac{32}{7}\left(512-288a+15a^2\right)x^7+8(128-3a)(4-a)x^8-\frac{16}{3}\left(896-128a+a^2\right)x^9+\frac{1}{5}\left(20480-1536a+3a^2\right)x^{10}-\frac{32}{11}(928-35a)x^{11}+\frac{8}{3}(524-9a)x^{12}-\frac{16}{13}(464-3a)x^{13}+\frac{2}{7}(640-a)x^{14}-\frac{224x^{15}}{5}+8x^{16}-\frac{16x^{17}}{17}+\frac{x^{18}}{18}$$

```
[Out] 1/2*a^4*x^2+32/3*a^3*x^3+8*(12-a)*a^2*x^4+16/5*a*(a^2-48*a+128)*x^5+2/3*(-a
^3+192*a^2-1536*a+1024)*x^6-32/7*(15*a^2-288*a+512)*x^7+8*(128-3*a)*(4-a)*x
^8-16/3*(a^2-128*a+896)*x^9+1/5*(3*a^2-1536*a+20480)*x^10-32/11*(928-35*a)*
x^11+8/3*(524-9*a)*x^12-16/13*(464-3*a)*x^13+2/7*(640-a)*x^14-224/5*x^15+8*
x^16-16/17*x^17+1/18*x^18
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6874}

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx=\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10}-\frac{16}{3}\left(a^2-128a+896\right)x^9-\frac{32}{7}\left(15a^2-288a+512\right)x^7+\frac{16}{5}a\left(a^2-48a+128\right)x^5+8(12-a)a^2x^4+\frac{2}{3}\left(-a^3+192a^2-1536a+1024\right)x^6+\frac{2}{7}(640-a)x^{14}-\frac{16}{13}(464-3a)x^{13}+\frac{8}{3}(524-9a)x^{12}-\frac{32}{11}(928-35a)x^{11}+8(128-3a)(4-a)x^8+\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}$$

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^2)/2 + (32*a^3*x^3)/3 + 8*(12 - a)*a^2*x^4 + (16*a*(128 - 48*a + a^2
)*x^5)/5 + (2*(1024 - 1536*a + 192*a^2 - a^3)*x^6)/3 - (32*(512 - 288*a + 1
5*a^2)*x^7)/7 + 8*(128 - 3*a)*(4 - a)*x^8 - (16*(896 - 128*a + a^2)*x^9)/3
+ ((20480 - 1536*a + 3*a^2)*x^10)/5 - (32*(928 - 35*a)*x^11)/11 + (8*(524 -
 9*a)*x^12)/3 - (16*(464 - 3*a)*x^13)/13 + (2*(640 - a)*x^14)/7 - (224*x^15
)/5 + 8*x^16 - (16*x^17)/17 + x^18/18
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral}=\int\left(a^4x+32a^3x^2-32(-12+a)a^2x^3+16a\left(128-48a+a^2\right)x^4-4\left(-1024+1536a-192a^2+a^3\right)x^5-32\left(512-288a+15a^2\right)x^6+64(128-3a)(4-a)x^7-48\left(896-128a+a^2\right)x^8+2\left(20480-1536a+3a^2\right)x^9+32(-928+35a)x^{10}-32(-524+9a)x^{11}+16(-464+3a)x^{12}-4(-640+a)x^{13}-672x^{14}+128x^{15}-16x^{16}+x^{17}\right)dx$$

$$
\begin{aligned}
&= \frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+8(12-a)a^2x^4+\frac{16}{5}a\left(128-48a+a^2\right)x^5 \\
&+\frac{2}{3}\left(1024-1536a+192a^2-a^3\right)x^6-\frac{32}{7}\left(512-288a+15a^2\right)x^7+8(128-3a)(4-a)x^8 \\
&-\frac{16}{3}\left(896-128a+a^2\right)x^9+\frac{1}{5}\left(20480-1536a+3a^2\right)x^{10}-\frac{32}{11}(928-35a)x^{11} \\
&+\frac{8}{3}(524-9a)x^{12}-\frac{16}{13}(464-3a)x^{13}+\frac{2}{7}(640-a)x^{14}-\frac{224x^{15}}{5}+8x^{16}-\frac{16x^{17}}{17}+\frac{x^{18}}{18}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.97

$$
\begin{aligned}
\int x\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx &= \frac{a^4x^2}{2}+\frac{32a^3x^3}{3}-8(-12+a)a^2x^4+\frac{16}{5}a\left(128-48a+a^2\right)x^5 \\
&-\frac{2}{3}\left(-1024+1536a-192a^2+a^3\right)x^6 \\
&-\frac{32}{7}\left(512-288a+15a^2\right)x^7 \\
&+8\left(512-140a+3a^2\right)x^8-\frac{16}{3}\left(896-128a+a^2\right)x^9 \\
&+\frac{1}{5}\left(20480-1536a+3a^2\right)x^{10}+\frac{32}{11}(-928+35a)x^{11} \\
&-\frac{8}{3}(-524+9a)x^{12}+\frac{16}{13}(-464+3a)x^{13} \\
&-\frac{2}{7}(-640+a)x^{14}-\frac{224x^{15}}{5}+8x^{16}-\frac{16x^{17}}{17}+\frac{x^{18}}{18}
\end{aligned}
$$

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^2)/2 + (32*a^3*x^3)/3 - 8*(-12 + a)*a^2*x^4 + (16*a*(128 - 48*a + a^
2)*x^5)/5 - (2*(-1024 + 1536*a - 192*a^2 + a^3)*x^6)/3 - (32*(512 - 288*a +
 15*a^2)*x^7)/7 + 8*(512 - 140*a + 3*a^2)*x^8 - (16*(896 - 128*a + a^2)*x^9
)/3 + ((20480 - 1536*a + 3*a^2)*x^10)/5 + (32*(-928 + 35*a)*x^11)/11 - (8*(
-524 + 9*a)*x^12)/3 + (16*(-464 + 3*a)*x^13)/13 - (2*(-640 + a)*x^14)/7 - (
224*x^15)/5 + 8*x^16 - (16*x^17)/17 + x^18/18
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

method	result
norman	$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\left(-8a^3+96a^2\right)x^4+\left(\frac{16}{5}a^3-\frac{768}{5}a^2+\frac{2048}{5}a\right)x^5+\left(-\frac{2}{3}a^3+128a^2-1024a+\right.$
gosper	$4096x^{10}-\frac{29696}{11}x^{11}+\frac{4192}{3}x^{12}-\frac{7424}{13}x^{13}-\frac{14336}{3}x^9-\frac{16384}{7}x^7-\frac{2}{7}x^{14}a+\frac{3}{5}x^{10}a^2-\frac{2}{3}x^6a^3+\frac{128}{7}$
risch	$4096x^{10}-\frac{29696}{11}x^{11}+\frac{4192}{3}x^{12}-\frac{7424}{13}x^{13}-\frac{14336}{3}x^9-\frac{16384}{7}x^7-\frac{2}{7}x^{14}a+\frac{3}{5}x^{10}a^2-\frac{2}{3}x^6a^3+\frac{128}{7}$
parallelrisch	$4096x^{10}-\frac{29696}{11}x^{11}+\frac{4192}{3}x^{12}-\frac{7424}{13}x^{13}-\frac{14336}{3}x^9-\frac{16384}{7}x^7-\frac{2}{7}x^{14}a+\frac{3}{5}x^{10}a^2-\frac{2}{3}x^6a^3+\frac{128}{7}$
default	$\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}+\frac{(-4a+2560)x^{14}}{14}+\frac{(48a-7424)x^{13}}{13}+\frac{(-288a+16768)x^{12}}{12}+\frac{(1120a-29696)x^{11}}{11}$

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^4*x^2+32/3*a^3*x^3+(-8*a^3+96*a^2)*x^4+(16/5*a^3-768/5*a^2+2048/5*a)*
x^5+(-2/3*a^3+128*a^2-1024*a+2048/3)*x^6+(-480/7*a^2+9216/7*a-16384/7)*x^7+
(24*a^2-1120*a+4096)*x^8+(-16/3*a^2+2048/3*a-14336/3)*x^9+(3/5*a^2-1536/5*a
+4096)*x^10+(1120/11*a-29696/11)*x^11+(-24*a+4192/3)*x^12+(48/13*a-7424/13)
*x^13+(-2/7*a+1280/7)*x^14-224/5*x^15+8*x^16-16/17*x^17+1/18*x^18
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

$$\begin{aligned}\int x\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx &= \frac{1}{18}x^{18}-\frac{16}{17}x^{17}+8x^{16}-\frac{2}{7}(a-640)x^{14}-\frac{224}{5}x^{15}+\frac{16}{13}(3a-464)x^{13}\\ &\quad-\frac{8}{3}(9a-524)x^{12}+\frac{32}{11}(35a-928)x^{11}+\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10}\\ &\quad-\frac{16}{3}\left(a^2-128a+896\right)x^9+8\left(3a^2-140a+512\right)x^8\\ &\quad-\frac{32}{7}\left(15a^2-288a+512\right)x^7-\frac{2}{3}\left(a^3-192a^2+1536a-1024\right)x^6\\ &\quad+\frac{1}{2}a^4x^2+\frac{32}{3}a^3x^3+\frac{16}{5}\left(a^3-48a^2+128a\right)x^5-8\left(a^3-12a^2\right)x^4\end{aligned}$$

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

```
[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*(a - 640)*x^14 - 224/5*x^15 + 16/13*(
3*a - 464)*x^13 - 8/3*(9*a - 524)*x^12 + 32/11*(35*a - 928)*x^11 + 1/5*(3*a
^2 - 1536*a + 20480)*x^10 - 16/3*(a^2 - 128*a + 896)*x^9 + 8*(3*a^2 - 140*a
 + 512)*x^8 - 32/7*(15*a^2 - 288*a + 512)*x^7 - 2/3*(a^3 - 192*a^2 + 1536*a
 - 1024)*x^6 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 16/5*(a^3 - 48*a^2 + 128*a)*x^5
 - 8*(a^3 - 12*a^2)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$\int x(a+8x-8x^2+4x^3-x^4)^4\,dx = \frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}+x^{14}\cdot\left(\frac{1280}{7}-\frac{2a}{7}\right)+x^{13}\cdot\left(\frac{48a}{13}-\frac{7424}{13}\right)+x^{12}\cdot\left(\frac{4192}{3}-24a\right)+x^{11}\cdot\left(\frac{1120a}{11}-\frac{29696}{11}\right)+x^{10}\cdot\left(\frac{3a^2}{5}-\frac{1536a}{5}+4096\right)+x^9\left(-\frac{16a^2}{3}+\frac{2048a}{3}-\frac{14336}{3}\right)+x^8\cdot\left(24a^2-1120a+4096\right)+x^7\left(-\frac{480a^2}{7}+\frac{9216a}{7}-\frac{16384}{7}\right)+x^6\left(-\frac{2a^3}{3}+128a^2-1024a+\frac{2048}{3}\right)+x^5\cdot\left(\frac{16a^3}{5}-\frac{768a^2}{5}+\frac{2048a}{5}\right)+x^4\left(-8a^3+96a^2\right)$$

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] a**4*x**2/2 + 32*a**3*x**3/3 + x**18/18 - 16*x**17/17 + 8*x**16 - 224*x**15
/5 + x**14*(1280/7 - 2*a/7) + x**13*(48*a/13 - 7424/13) + x**12*(4192/3 - 2
4*a) + x**11*(1120*a/11 - 29696/11) + x**10*(3*a**2/5 - 1536*a/5 + 4096) +
x**9*(-16*a**2/3 + 2048*a/3 - 14336/3) + x**8*(24*a**2 - 1120*a + 4096) + x
**7*(-480*a**2/7 + 9216*a/7 - 16384/7) + x**6*(-2*a**3/3 + 128*a**2 - 1024*
a + 2048/3) + x**5*(16*a**3/5 - 768*a**2/5 + 2048*a/5) + x**4*(-8*a**3 + 96
*a**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

$$\int x(a+8x-8x^2+4x^3-x^4)^4\,dx$$
$$= \frac{1}{18}x^{18}-\frac{16}{17}x^{17}+8x^{16}-\frac{2}{7}(a-640)x^{14}-\frac{224}{5}x^{15}+\frac{16}{13}(3a-464)x^{13} - \frac{8}{3}(9a-524)x^{12}+\frac{32}{11}(35a-928)x^{11}+\frac{1}{5}(3a^2-1536a+20480)x^{10} - \frac{16}{3}(a^2-128a+896)x^9+8(3a^2-140a+512)x^8 - \frac{32}{7}(15a^2-288a+512)x^7-\frac{2}{3}(a^3-192a^2+1536a-1024)x^6 + \frac{1}{2}a^4x^2+\frac{32}{3}a^3x^3+\frac{16}{5}(a^3-48a^2+128a)x^5-8(a^3-12a^2)x^4$$

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

```
[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*(a - 640)*x^14 - 224/5*x^15 + 16/13*(
3*a - 464)*x^13 - 8/3*(9*a - 524)*x^12 + 32/11*(35*a - 928)*x^11 + 1/5*(3*a
^2 - 1536*a + 20480)*x^10 - 16/3*(a^2 - 128*a + 896)*x^9 + 8*(3*a^2 - 140*a
 + 512)*x^8 - 32/7*(15*a^2 - 288*a + 512)*x^7 - 2/3*(a^3 - 192*a^2 + 1536*a
 - 1024)*x^6 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 16/5*(a^3 - 48*a^2 + 128*a)*x^5
 - 8*(a^3 - 12*a^2)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.06

$$\int x(a+8x-8x^2+4x^3-x^4)^4\,dx = \frac{1}{18}x^{18}-\frac{16}{17}x^{17}+8x^{16}-\frac{2}{7}ax^{14}-\frac{224}{5}x^{15}+\frac{48}{13}ax^{13} + \frac{1280}{7}x^{14}-24ax^{12}-\frac{7424}{13}x^{13}+\frac{3}{5}a^2x^{10}+\frac{1120}{11}ax^{11} + \frac{4192}{3}x^{12}-\frac{16}{3}a^2x^9-\frac{1536}{5}ax^{10}-\frac{29696}{11}x^{11} + 24a^2x^8+\frac{2048}{3}ax^9+4096x^{10}-\frac{2}{3}a^3x^6-\frac{480}{7}a^2x^7 - 1120ax^8-\frac{14336}{3}x^9+\frac{16}{5}a^3x^5+128a^2x^6+\frac{9216}{7}ax^7 + 4096x^8-8a^3x^4-\frac{768}{5}a^2x^5-1024ax^6-\frac{16384}{7}x^7 + \frac{1}{2}a^4x^2+\frac{32}{3}a^3x^3+96a^2x^4+\frac{2048}{5}ax^5+\frac{2048}{3}x^6$$

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

```
[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*a*x^14 - 224/5*x^15 + 48/13*a*x^13 +
1280/7*x^14 - 24*a*x^12 - 7424/13*x^13 + 3/5*a^2*x^10 + 1120/11*a*x^11 + 41
92/3*x^12 - 16/3*a^2*x^9 - 1536/5*a*x^10 - 29696/11*x^11 + 24*a^2*x^8 + 204
8/3*a*x^9 + 4096*x^10 - 2/3*a^3*x^6 - 480/7*a^2*x^7 - 1120*a*x^8 - 14336/3*
x^9 + 16/5*a^3*x^5 + 128*a^2*x^6 + 9216/7*a*x^7 + 4096*x^8 - 8*a^3*x^4 - 76
8/5*a^2*x^5 - 1024*a*x^6 - 16384/7*x^7 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 96*a^
2*x^4 + 2048/5*a*x^5 + 2048/3*x^6
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.85

$$\int x(a+8x-8x^2+4x^3-x^4)^4\,dx = x^{13}\left(\frac{48\,a}{13}-\frac{7424}{13}\right)-x^{12}\left(24\,a-\frac{4192}{3}\right)-x^{14}\left(\frac{2\,a}{7}-\frac{1280}{7}\right)+x^{11}\left(\frac{1120\,a}{11}-\frac{29696}{11}\right)+x^8\left(24\,a^2-1120\,a+4096\right)+x^{10}\left(\frac{3\,a^2}{5}-\frac{1536\,a}{5}+4096\right)-x^9\left(\frac{16\,a^2}{3}-\frac{2048\,a}{3}+\frac{14336}{3}\right)-x^7\left(\frac{480\,a^2}{7}-\frac{9216\,a}{7}+\frac{16384}{7}\right)-x^6\left(\frac{2\,a^3}{3}-128\,a^2+1024\,a-\frac{2048}{3}\right)-\frac{224\,x^{15}}{5}+8\,x^{16}-\frac{16\,x^{17}}{17}+\frac{x^{18}}{18}+\frac{32\,a^3\,x^3}{3}+\frac{a^4\,x^2}{2}+\frac{16\,a\,x^5\,\left(a^2-48\,a+128\right)}{5}-8\,a^2\,x^4\,\left(a-12\right)$$

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)
```

```
[Out] x^13*((48*a)/13 - 7424/13) - x^12*(24*a - 4192/3) - x^14*((2*a)/7 - 1280/7)
 + x^11*((1120*a)/11 - 29696/11) + x^8*(24*a^2 - 1120*a + 4096) + x^10*((3*
a^2)/5 - (1536*a)/5 + 4096) - x^9*((16*a^2)/3 - (2048*a)/3 + 14336/3) - x^7
*((480*a^2)/7 - (9216*a)/7 + 16384/7) - x^6*(1024*a - 128*a^2 + (2*a^3)/3 -
 2048/3) - (224*x^15)/5 + 8*x^16 - (16*x^17)/17 + x^18/18 + (32*a^3*x^3)/3
+ (a^4*x^2)/2 + (16*a*x^5*(a^2 - 48*a + 128))/5 - 8*a^2*x^4*(a - 12)
```

3.124 $\int x(a+8x-8x^2+4x^3-x^4)^3\,dx$

Optimal result . . . 957
Rubi [A] (verified) . . . 957
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Giac [A] (verification not implemented) . . . 961
Mupad [B] (verification not implemented) . . . 961

Optimal result

Integrand size = 24, antiderivative size = 134

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=\frac{a^3x^2}{2}+8a^2x^3+6(8-a)ax^4+\frac{4}{5}\left(128-96a+3a^2\right)x^5-\frac{1}{2}\left(512-128a+a^2\right)x^6+\frac{48}{7}(48-5a)x^7-4(70-3a)x^8+\frac{8}{3}(64-a)x^9-\frac{3}{10}(256-a)x^{10}+\frac{280x^{11}}{11}-6x^{12}+\frac{12x^{13}}{13}-\frac{x^{14}}{14}$$

```
[Out] 1/2*a^3*x^2+8*a^2*x^3+6*(8-a)*a*x^4+4/5*(3*a^2-96*a+128)*x^5-1/2*(a^2-128*a
+512)*x^6+48/7*(48-5*a)*x^7-4*(70-3*a)*x^8+8/3*(64-a)*x^9-3/10*(256-a)*x^10
+280/11*x^11-6*x^12+12/13*x^13-1/14*x^14
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6874}

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=\frac{a^3x^2}{2}-\frac{1}{2}\left(a^2-128a+512\right)x^6+\frac{4}{5}\left(3a^2-96a+128\right)x^5+8a^2x^3-\frac{3}{10}(256-a)x^{10}+\frac{8}{3}(64-a)x^9-4(70-3a)x^8+\frac{48}{7}(48-5a)x^7+6(8-a)ax^4-\frac{x^{14}}{14}+\frac{12x^{13}}{13}-6x^{12}+\frac{280x^{11}}{11}$$

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^2)/2 + 8*a^2*x^3 + 6*(8 - a)*a*x^4 + (4*(128 - 96*a + 3*a^2)*x^5)/5
- ((512 - 128*a + a^2)*x^6)/2 + (48*(48 - 5*a)*x^7)/7 - 4*(70 - 3*a)*x^8 +
(8*(64 - a)*x^9)/3 - (3*(256 - a)*x^10)/10 + (280*x^11)/11 - 6*x^12 + (12*x
^13)/13 - x^14/14
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^3x+24a^2x^2-24(-8+a)ax^3+4\left(128-96a+3a^2\right)x^4-3\left(512-128a+a^2\right)x^5 \right.\\ &\quad -48(-48+5a)x^6+32(-70+3a)x^7-24(-64+a)x^8+3(-256+a)x^9+280x^{10}\\ &\quad \left. -72x^{11}+12x^{12}-x^{13}\right)\,dx\\ &= \frac{a^3x^2}{2}+8a^2x^3+6(8-a)ax^4+\frac{4}{5}\left(128-96a+3a^2\right)x^5-\frac{1}{2}\left(512-128a+a^2\right)x^6+\frac{48}{7}(48\\ &\quad -5a)x^7-4(70-3a)x^8+\frac{8}{3}(64-a)x^9-\frac{3}{10}(256-a)x^{10}+\frac{280x^{11}}{11}-6x^{12}+\frac{12x^{13}}{13}-\frac{x^{14}}{14}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.97

$$\begin{aligned}\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx &= \frac{a^3x^2}{2}+8a^2x^3-6(-8+a)ax^4+\frac{4}{5}\left(128-96a+3a^2\right)x^5\\ &\quad +\frac{1}{2}\left(-512+128a-a^2\right)x^6-\frac{48}{7}(-48+5a)x^7\\ &\quad +4(-70+3a)x^8-\frac{8}{3}(-64+a)x^9\\ &\quad +\frac{3}{10}(-256+a)x^{10}+\frac{280x^{11}}{11}-6x^{12}+\frac{12x^{13}}{13}-\frac{x^{14}}{14}\end{aligned}$$

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^2)/2 + 8*a^2*x^3 - 6*(-8 + a)*a*x^4 + (4*(128 - 96*a + 3*a^2)*x^5)/5
 + ((-512 + 128*a - a^2)*x^6)/2 - (48*(-48 + 5*a)*x^7)/7 + 4*(-70 + 3*a)*x^
8 - (8*(-64 + a)*x^9)/3 + (3*(-256 + a)*x^10)/10 + (280*x^11)/11 - 6*x^12 +
 (12*x^13)/13 - x^14/14
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

method	result
norman	$\frac{a^3x^2}{2}+8a^2x^3+\left(-6a^2+48a\right)x^4+\left(\frac{12}{5}a^2-\frac{384}{5}a+\frac{512}{5}\right)x^5+\left(-\frac{1}{2}a^2+64a-256\right)x^6+\left(-\frac{24}{7}\right.$
gosper	$\frac{1}{2}a^3x^2+8a^2x^3-6a^2x^4+48a\,x^4+\frac{12}{5}a^2x^5-\frac{384}{5}a\,x^5+\frac{512}{5}x^5-\frac{1}{2}a^2x^6+64a\,x^6-256x^6-\frac{24}{7}$
risch	$\frac{1}{2}a^3x^2+8a^2x^3-6a^2x^4+48a\,x^4+\frac{12}{5}a^2x^5-\frac{384}{5}a\,x^5+\frac{512}{5}x^5-\frac{1}{2}a^2x^6+64a\,x^6-256x^6-\frac{24}{7}$
parallelrisch	$\frac{1}{2}a^3x^2+8a^2x^3-6a^2x^4+48a\,x^4+\frac{12}{5}a^2x^5-\frac{384}{5}a\,x^5+\frac{512}{5}x^5-\frac{1}{2}a^2x^6+64a\,x^6-256x^6-\frac{24}{7}$
default	$-\frac{x^{14}}{14}+\frac{12x^{13}}{13}-6x^{12}+\frac{280x^{11}}{11}+\frac{(3a-768)x^{10}}{10}+\frac{(-24a+1536)x^9}{9}+\frac{(96a-2240)x^8}{8}+\frac{(-240a+2304)x^7}{7}+\frac{(a}{}$

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^3*x^2+8*a^2*x^3+(-6*a^2+48*a)*x^4+(12/5*a^2-384/5*a+512/5)*x^5+(-1/2*
a^2+64*a-256)*x^6+(-240/7*a+2304/7)*x^7+(12*a-280)*x^8+(-8/3*a+512/3)*x^9+(
3/10*a-384/5)*x^10+280/11*x^11-6*x^12+12/13*x^13-1/14*x^14
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\begin{aligned}\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=&-\frac{1}{14}x^{14}+\frac{12}{13}x^{13}-6\,x^{12}+\frac{3}{10}\,(a-256)x^{10}+\frac{280}{11}x^{11}\\&-\frac{8}{3}\,(a-64)x^9+4\,(3\,a-70)x^8-\frac{48}{7}\,(5\,a-48)x^7\\&-\frac{1}{2}\,\left(a^2-128\,a+512\right)x^6+\frac{4}{5}\,\left(3\,a^2-96\,a+128\right)x^5\\&+\frac{1}{2}\,a^3x^2+8\,a^2x^3-6\,\left(a^2-8\,a\right)x^4\end{aligned}$$

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*(a - 256)*x^10 + 280/11*x^11 - 8/3*
(a - 64)*x^9 + 4*(3*a - 70)*x^8 - 48/7*(5*a - 48)*x^7 - 1/2*(a^2 - 128*a +
512)*x^6 + 4/5*(3*a^2 - 96*a + 128)*x^5 + 1/2*a^3*x^2 + 8*a^2*x^3 - 6*(a^2
- 8*a)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=\frac{a^3x^2}{2}+8a^2x^3-\frac{x^{14}}{14}+\frac{12x^{13}}{13}-6x^{12}+\frac{280x^{11}}{11}+x^{10}\cdot\left(\frac{3a}{10}-\frac{384}{5}\right)+x^9\cdot\left(\frac{512}{3}-\frac{8a}{3}\right)+x^8\cdot(12a-280)+x^7\cdot\left(\frac{2304}{7}-\frac{240a}{7}\right)+x^6\left(-\frac{a^2}{2}+64a-256\right)+x^5\cdot\left(\frac{12a^2}{5}-\frac{384a}{5}+\frac{512}{5}\right)+x^4\left(-6a^2+48a\right)$$

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x**2/2 + 8*a**2*x**3 - x**14/14 + 12*x**13/13 - 6*x**12 + 280*x**11/11
 + x**10*(3*a/10 - 384/5) + x**9*(512/3 - 8*a/3) + x**8*(12*a - 280) + x**7
*(2304/7 - 240*a/7) + x**6*(-a**2/2 + 64*a - 256) + x**5*(12*a**2/5 - 384*a
/5 + 512/5) + x**4*(-6*a**2 + 48*a)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=-\frac{1}{14}\,x^{14}+\frac{12}{13}\,x^{13}-6\,x^{12}+\frac{3}{10}\,(a-256)x^{10}+\frac{280}{11}\,x^{11}-\frac{8}{3}\,(a-64)x^9+4\,(3\,a-70)x^8-\frac{48}{7}\,(5\,a-48)x^7-\frac{1}{2}\,\left(a^2-128\,a+512\right)x^6+\frac{4}{5}\,\left(3\,a^2-96\,a+128\right)x^5+\frac{1}{2}\,a^3x^2+8\,a^2x^3-6\,\left(a^2-8\,a\right)x^4$$

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*(a - 256)*x^10 + 280/11*x^11 - 8/3*
(a - 64)*x^9 + 4*(3*a - 70)*x^8 - 48/7*(5*a - 48)*x^7 - 1/2*(a^2 - 128*a +
512)*x^6 + 4/5*(3*a^2 - 96*a + 128)*x^5 + 1/2*a^3*x^2 + 8*a^2*x^3 - 6*(a^2
- 8*a)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=-\frac{1}{14}\,x^{14}+\frac{12}{13}\,x^{13}-6\,x^{12}+\frac{3}{10}\,ax^{10}+\frac{280}{11}\,x^{11}-\frac{8}{3}\,ax^9-\frac{384}{5}\,x^{10}+12\,ax^8+\frac{512}{3}\,x^9-\frac{1}{2}\,a^2x^6-\frac{240}{7}\,ax^7-280\,x^8+\frac{12}{5}\,a^2x^5+64\,ax^6+\frac{2304}{7}\,x^7-6\,a^2x^4-\frac{384}{5}\,ax^5-256\,x^6+\frac{1}{2}\,a^3x^2+8\,a^2x^3+48\,ax^4+\frac{512}{5}\,x^5$$

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] -1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*a*x^10 + 280/11*x^11 - 8/3*a*x^9 -
384/5*x^10 + 12*a*x^8 + 512/3*x^9 - 1/2*a^2*x^6 - 240/7*a*x^7 - 280*x^8 + 1
2/5*a^2*x^5 + 64*a*x^6 + 2304/7*x^7 - 6*a^2*x^4 - 384/5*a*x^5 - 256*x^6 + 1
/2*a^3*x^2 + 8*a^2*x^3 + 48*a*x^4 + 512/5*x^5
```

Mupad [B] (verification not implemented)

Time = 9.64 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=x^8\,(12\,a-280)+x^{10}\,\left(\frac{3\,a}{10}-\frac{384}{5}\right)-x^9\,\left(\frac{8\,a}{3}-\frac{512}{3}\right)-x^7\,\left(\frac{240\,a}{7}-\frac{2304}{7}\right)-x^6\,\left(\frac{a^2}{2}-64\,a+256\right)+x^5\,\left(\frac{12\,a^2}{5}-\frac{384\,a}{5}+\frac{512}{5}\right)+\frac{280\,x^{11}}{11}-6\,x^{12}+\frac{12\,x^{13}}{13}-\frac{x^{14}}{14}+8\,a^2\,x^3+\frac{a^3\,x^2}{2}-6\,a\,x^4\,(a-8)$$

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] x^8*(12*a - 280) + x^10*((3*a)/10 - 384/5) - x^9*((8*a)/3 - 512/3) - x^7*((
240*a)/7 - 2304/7) - x^6*(a^2/2 - 64*a + 256) + x^5*((12*a^2)/5 - (384*a)/5
 + 512/5) + (280*x^11)/11 - 6*x^12 + (12*x^13)/13 - x^14/14 + 8*a^2*x^3 + (
a^3*x^2)/2 - 6*a*x^4*(a - 8)
```

3.125 $\int x(a+8x-8x^2+4x^3-x^4)^2\,dx$

Optimal result . . . 962
Rubi [A] (verified) . . . 962
Mathematica [A] (verified) . . . 963
Maple [A] (verified) . . . 963
Fricas [A] (verification not implemented) . . . 964
Sympy [A] (verification not implemented) . . . 964
Maxima [A] (verification not implemented) . . . 964
Giac [A] (verification not implemented) . . . 965
Mupad [B] (verification not implemented) . . . 965

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{a^2x^2}{2}+\frac{16ax^3}{3}+4(4-a)x^4-\frac{8}{5}(16-a)x^5+\frac{1}{3}(64-a)x^6-\frac{80x^7}{7}+4x^8-\frac{8x^9}{9}+\frac{x^{10}}{10}$$

```
[Out] 1/2*a^2*x^2+16/3*a*x^3+4*(4-a)*x^4-8/5*(16-a)*x^5+1/3*(64-a)*x^6-80/7*x^7+4
*x^8-8/9*x^9+1/10*x^10
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6874}

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{a^2x^2}{2}+\frac{1}{3}(64-a)x^6-\frac{8}{5}(16-a)x^5+4(4-a)x^4+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}$$

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (16*a*x^3)/3 + 4*(4 - a)*x^4 - (8*(16 - a)*x^5)/5 + ((64 - a)
*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2x + 16ax^2 - 16(-4+a)x^3 + 8(-16+a)x^4 - 2(-64+a)x^5 - 80x^6 + 32x^7 - 8x^8 + x^9\right) dx \\ &= \frac{a^2x^2}{2} + \frac{16ax^3}{3} + 4(4-a)x^4 - \frac{8}{5}(16-a)x^5 + \frac{1}{3}(64-a)x^6 - \frac{80x^7}{7} + 4x^8 - \frac{8x^9}{9} + \frac{x^{10}}{10}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^2 dx = \frac{a^2x^2}{2} + \frac{16ax^3}{3} - 4(-4+a)x^4 + \frac{8}{5}(-16+a)x^5 + \frac{1}{3}(64-a)x^6 - \frac{80x^7}{7} + 4x^8 - \frac{8x^9}{9} + \frac{x^{10}}{10}$$

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (16*a*x^3)/3 - 4*(-4 + a)*x^4 + (8*(-16 + a)*x^5)/5 + ((64 -
a)*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

method	result
norman	$\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}+\left(\frac{64}{3}-\frac{a}{3}\right)x^6+\left(\frac{8a}{5}-\frac{128}{5}\right)x^5+\left(-4a+16\right)x^4+\frac{16a\,x^3}{3}+\frac{a^2x^2}{2}$
default	$\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}+\frac{(-2a+128)x^6}{6}+\frac{(8a-128)x^5}{5}+\frac{(-16a+64)x^4}{4}+\frac{16a\,x^3}{3}+\frac{a^2x^2}{2}$
gosper	$\frac{1}{10}x^{10}-\frac{8}{9}x^9+4x^8-\frac{80}{7}x^7+\frac{64}{3}x^6-\frac{1}{3}a\,x^6+\frac{8}{5}a\,x^5-\frac{128}{5}x^5-4a\,x^4+16x^4+\frac{16}{3}a\,x^3+\frac{1}{2}a^2x^2$
risch	$\frac{1}{10}x^{10}-\frac{8}{9}x^9+4x^8-\frac{80}{7}x^7+\frac{64}{3}x^6-\frac{1}{3}a\,x^6+\frac{8}{5}a\,x^5-\frac{128}{5}x^5-4a\,x^4+16x^4+\frac{16}{3}a\,x^3+\frac{1}{2}a^2x^2$
parallelrisch	$\frac{1}{10}x^{10}-\frac{8}{9}x^9+4x^8-\frac{80}{7}x^7+\frac{64}{3}x^6-\frac{1}{3}a\,x^6+\frac{8}{5}a\,x^5-\frac{128}{5}x^5-4a\,x^4+16x^4+\frac{16}{3}a\,x^3+\frac{1}{2}a^2x^2$

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*x^10-8/9*x^9+4*x^8-80/7*x^7+(64/3-1/3*a)*x^6+(8/5*a-128/5)*x^5+(-4*a+1
6)*x^4+16/3*a*x^3+1/2*a^2*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{1}{10}\,x^{10}-\frac{8}{9}\,x^9+4\,x^8-\frac{1}{3}\,(a-64)x^6-\frac{80}{7}\,x^7+\frac{8}{5}\,(a-16)x^5-4\,(a-4)x^4+\frac{1}{2}\,a^2x^2+\frac{16}{3}\,ax^3$$

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*(a - 64)*x^6 - 80/7*x^7 + 8/5*(a - 16)*x^
5 - 4*(a - 4)*x^4 + 1/2*a^2*x^2 + 16/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{a^2x^2}{2}+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}+x^6\cdot\left(\frac{64}{3}-\frac{a}{3}\right)+x^5\cdot\left(\frac{8a}{5}-\frac{128}{5}\right)+x^4\cdot(16-4a)$$

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] a**2*x**2/2 + 16*a*x**3/3 + x**10/10 - 8*x**9/9 + 4*x**8 - 80*x**7/7 + x**6
*(64/3 - a/3) + x**5*(8*a/5 - 128/5) + x**4*(16 - 4*a)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{1}{10}\,x^{10}-\frac{8}{9}\,x^9+4\,x^8-\frac{1}{3}\,(a-64)x^6-\frac{80}{7}\,x^7+\frac{8}{5}\,(a-16)x^5-4\,(a-4)x^4+\frac{1}{2}\,a^2x^2+\frac{16}{3}\,ax^3$$

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*(a - 64)*x^6 - 80/7*x^7 + 8/5*(a - 16)*x^
5 - 4*(a - 4)*x^4 + 1/2*a^2*x^2 + 16/3*a*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int x(a+8x-8x^2+4x^3-x^4)^2 \, dx = \frac{1}{10}\,x^{10} - \frac{8}{9}\,x^9 + 4\,x^8 - \frac{1}{3}\,ax^6 - \frac{80}{7}\,x^7 + \frac{8}{5}\,ax^5 + \frac{64}{3}\,x^6 - 4\,ax^4 - \frac{128}{5}\,x^5 + \frac{1}{2}\,a^2x^2 + \frac{16}{3}\,ax^3 + 16\,x^4$$

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*a*x^6 - 80/7*x^7 + 8/5*a*x^5 + 64/3*x^6 -
 4*a*x^4 - 128/5*x^5 + 1/2*a^2*x^2 + 16/3*a*x^3 + 16*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

$$\int x(a+8x-8x^2+4x^3-x^4)^2 \, dx = x^5\left(\frac{8\,a}{5}-\frac{128}{5}\right) - x^6\left(\frac{a}{3}-\frac{64}{3}\right) - x^4\,(4\,a-16) + \frac{16\,a\,x^3}{3} - \frac{80\,x^7}{7} + 4\,x^8 - \frac{8\,x^9}{9} + \frac{x^{10}}{10} + \frac{a^2\,x^2}{2}$$

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] x^5*((8*a)/5 - 128/5) - x^6*(a/3 - 64/3) - x^4*(4*a - 16) + (16*a*x^3)/3 -
(80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10 + (a^2*x^2)/2
```

3.126 $\int x(a+8x-8x^2+4x^3-x^4)\,dx$

Optimal result 966
Rubi [A] (verified) 966
Mathematica [A] (verified) 967
Maple [A] (verified) 967
Fricas [A] (verification not implemented) 967
Sympy [A] (verification not implemented) 968
Maxima [A] (verification not implemented) 968
Giac [A] (verification not implemented) 968
Mupad [B] (verification not implemented) 968

Optimal result

Integrand size = 22, antiderivative size = 35

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)\,dx=\frac{ax^2}{2}+\frac{8x^3}{3}-2x^4+\frac{4x^5}{5}-\frac{x^6}{6}$$

```
[Out] 1/2*a*x^2+8/3*x^3-2*x^4+4/5*x^5-1/6*x^6
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)\,dx=\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(ax+8x^2-8x^3+4x^4-x^5\right)\,dx\\&=\frac{ax^2}{2}+\frac{8x^3}{3}-2x^4+\frac{4x^5}{5}-\frac{x^6}{6}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)\,dx=\frac{ax^2}{2}+\frac{8x^3}{3}-2x^4+\frac{4x^5}{5}-\frac{x^6}{6}$$

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{1}{2}a\,x^2+\frac{8}{3}x^3-2x^4+\frac{4}{5}x^5-\frac{1}{6}x^6$	28
default	$\frac{1}{2}a\,x^2+\frac{8}{3}x^3-2x^4+\frac{4}{5}x^5-\frac{1}{6}x^6$	28
norman	$\frac{1}{2}a\,x^2+\frac{8}{3}x^3-2x^4+\frac{4}{5}x^5-\frac{1}{6}x^6$	28
risch	$\frac{1}{2}a\,x^2+\frac{8}{3}x^3-2x^4+\frac{4}{5}x^5-\frac{1}{6}x^6$	28
parallelrisch	$\frac{1}{2}a\,x^2+\frac{8}{3}x^3-2x^4+\frac{4}{5}x^5-\frac{1}{6}x^6$	28

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x^2+8/3*x^3-2*x^4+4/5*x^5-1/6*x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)\,dx=-\frac{1}{6}\,x^6+\frac{4}{5}\,x^5-2\,x^4+\frac{1}{2}\,ax^2+\frac{8}{3}\,x^3$$

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x(a+8x-8x^2+4x^3-x^4)\,dx=\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x),x)
[Out] a*x**2/2 - x**6/6 + 4*x**5/5 - 2*x**4 + 8*x**3/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x(a+8x-8x^2+4x^3-x^4)\,dx=-\frac{1}{6}\,x^6+\frac{4}{5}\,x^5-2\,x^4+\frac{1}{2}\,ax^2+\frac{8}{3}\,x^3$$

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x(a+8x-8x^2+4x^3-x^4)\,dx=-\frac{1}{6}\,x^6+\frac{4}{5}\,x^5-2\,x^4+\frac{1}{2}\,ax^2+\frac{8}{3}\,x^3$$

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x(a+8x-8x^2+4x^3-x^4)\,dx=-\frac{x^6}{6}+\frac{4\,x^5}{5}-2\,x^4+\frac{8\,x^3}{3}+\frac{a\,x^2}{2}$$

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

3.127 $\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal result 969
Rubi [A] (verified) 969
Mathematica [C] (verified) 971
Maple [C] (verified) 972
Fricas [C] (verification not implemented) 972
Sympy [A] (verification not implemented) 972
Maxima [F] 973
Giac [F] 973
Mupad [B] (verification not implemented) 973

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = -\frac{\arctan\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} + \frac{\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}} + \frac{\operatorname{arctanh}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{2\sqrt{4+a}}$$

```
[Out] 1/2*arctanh((1+(-1+x)^2)/(4+a)^(1/2))/(4+a)^(1/2)-1/2*arctan((-1+x)/(1-(4+a
)^(1/2))^(1/2))/(4+a)^(1/2)/(1-(4+a)^(1/2))^(1/2)+1/2*arctan((-1+x)/(1+(4+a
)^(1/2))^(1/2))/(4+a)^(1/2)/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1694, 1687, 1107, 210, 1121, 632, 212}

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = -\frac{\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}} + \frac{\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}} + \frac{\operatorname{arctanh}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2\sqrt{a+4}}$$

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -1/2*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(Sqrt[4 + a]*Sqrt[1 - Sqrt[4 +
a]]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqrt[
4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(2*Sqrt[4 + a])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1+x}{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$
$$= \text{Subst}\left(\int \frac{1}{3+a-2x^2-x^4}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{3+a-2x-x^2}\,dx, x, (-1+x)^2\right) - \frac{\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{2\sqrt{4+a}} + \frac{\text{Subst}\left(\int \frac{1}{-1+\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{2\sqrt{4+a}}$$
$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}} - \text{Subst}\left(\int \frac{1}{4(4+a)-x^2}\,dx, x, -2\left(1+(-1+x)^2\right)\right)$$
$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}} + \frac{\tanh^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{2\sqrt{4+a}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.51

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = -\frac{1}{4}\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&, \frac{\log(x-\#1)\#1}{-2+4\#1-3\#1^2+\#1^3}\&\right]$$

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
[Out] -1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (Log[x - #1]*#1)/(-2 + 4
*#1 - 3*#1^2 + #1^3) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.45

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}-4\_Z^{3}+8\_Z^{2}-8\_Z-a\right)}\frac{\_R\ln\left(x-\_R\right)}{-\_R^{3}+3\_R^{2}-4\_R+2}\right)}{4}$	52
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}-4\_Z^{3}+8\_Z^{2}-8\_Z-a\right)}\frac{\_R\ln\left(x-\_R\right)}{-\_R^{3}+3\_R^{2}-4\_R+2}\right)}{4}$	52

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z
-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.45 (sec) , antiderivative size = 140500, normalized size of antiderivative = 1211.21

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.34

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx =$$
$$-\operatorname{RootSum}\left(t^4\cdot\left(256a^3+2816a^2+10240a+12288\right)+t^2\left(-32a^2-256a-512\right)+t(-16a-64)+a,\left(t\right.\right.$$

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x),x)
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-32*a**2 -
 256*a - 512) + _t*(-16*a - 64) + a, Lambda(_t, _t*log(x + (-128*_t**3*a**4
 - 1728*_t**3*a**3 - 8640*_t**3*a**2 - 18944*_t**3*a - 15360*_t**3 + 48*_t*
*2*a**3 + 464*_t**2*a**2 + 1472*_t**2*a + 1536*_t**2 + 8*_t*a**3 + 88*_t*a*
*2 + 312*_t*a + 352*_t - a**2 - 2*a)/(4*a**2 + 21*a + 28))))
```

Maxima [F]

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx=\int -\frac{x}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx$$

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
[Out] -integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Giac [F]

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx=\int -\frac{x}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx$$

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
[Out] integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Mupad [B] (verification not implemented)

Time = 9.58 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.37

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx=\sum_{k=1}^{4}\ln\Big(-x-\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-32\,a^2\,z^2-256\,a\,z^2-512\,z^2+16\,a\,z+64\,z+a,z,k\big)\,\big(\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-32\,a^2\,z^2-256\,a\,z^2-512\,z^2+16\,a\,z+64\,z+a,\ldots-8\big)\Big)\,\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-32\,a^2\,z^2-256\,a\,z^2-512\,z^2+16\,a\,z+64\,z+a,z,k\big)$$

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
[Out] symsum(log(- x - root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4
- 32*a^2*z^2 - 256*a*z^2 - 512*z^2 + 16*a*z + 64*z + a, z, k)*(root(2816*a^
2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 - 51
2*z^2 + 16*a*z + 64*z + a, z, k)*(32*a - root(2816*a^2*z^4 + 256*a^3*z^4 +
10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 - 512*z^2 + 16*a*z + 64*z
+ a, z, k)*(64*a - x*(64*a + 256) + 256) - x*(16*a + 64) + 128) - 8))*root(
2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z
^2 - 512*z^2 + 16*a*z + 64*z + a, z, k), k, 1, 4)
```

3.128 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2} \, dx$

Optimal result 974
Rubi [A] (verified) 975
Mathematica [C] (verified) 978
Maple [C] (verified) 978
Fricas [F(-1)] 979
Sympy [B] (verification not implemented) 979
Maxima [F] 980
Giac [F] 980
Mupad [B] (verification not implemented) 980

Optimal result

Integrand size = 24, antiderivative size = 231

$$\begin{aligned}\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2} \, dx = & \frac{1+(-1+x)^2}{4(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} \\ & + \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} \\ & - \frac{\left(10+3a+\sqrt{4+a}\right)\arctan\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1-\sqrt{4+a}}} \\ & + \frac{\left(10+3a-\sqrt{4+a}\right)\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1+\sqrt{4+a}}} \\ & + \frac{\operatorname{arctanh}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{4(4+a)^{3/2}}\end{aligned}$$

```
[Out] 1/4*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/4*(5+a+(-1+x)^2)*(-1+x)/
(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/4*arctanh((1+(-1+x)^2)/(4+a)^(1/2)
)/(4+a)^(3/2)-1/8*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(10+3*a+(4+a)^(1/2))
/(3+a)/(4+a)^(3/2)/(1-(4+a)^(1/2))^(1/2)+1/8*arctan((-1+x)/(1+(4+a)^(1/2))^
(1/2))*(10+3*a-(4+a)^(1/2))/(3+a)/(4+a)^(3/2)/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1694, 1687, 1106, 1180, 210, 1121, 628, 632, 212}

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)} - \frac{\left(3a+\sqrt{a+4}+10\right)\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}} + \frac{\left(3a-\sqrt{a+4}+10\right)\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{\sqrt{a+4}+1}} + \frac{\operatorname{arctanh}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{4(a+4)^{3/2}} + \frac{(x-1)^2+1}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (1 + (-1 + x)^2)/(4*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((5 + a
+ (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x
)^4)) - ((10 + 3*a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/(
8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt[4 + a]]) + ((10 + 3*a - Sqrt[4 + a])*
ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 + S
qrt[4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(4*(4 + a)^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
```

```
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1+x}{(3+a-2x^2-x^4)^2}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{(3+a-2x^2-x^4)^2}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{(3+a-2x^2-x^4)^2}\,dx, x, -1+x\right)$$

$$= \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(3+a-2x-x^2)^2}\,dx, x, (-1+x)^2\right) - \frac{\text{Subst}\left(\int \frac{4+2(3+a)-2(4+4(3+a))-2x^2}{3+a-2x^2-x^4}\,dx, x, -1+x\right)}{8(12+7a+a^2)}$$

$$= \frac{1+(-1+x)^2}{4(4+a)(3+a-2(1-x)^2-(1-x)^4)} + \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{\text{Subst}\left(\int \frac{1}{3+a-2x-x^2}\,dx, x, (-1+x)^2\right)}{4(4+a)} - \frac{(10+3a-\sqrt{4+a})\,\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{8(3+a)(4+a)^{3/2}} + \frac{(10+3a+\sqrt{4+a})\,\text{Subst}\left(\int \frac{1}{-1+\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{8(3+a)(4+a)^{3/2}}$$

$$= \frac{1+(-1+x)^2}{4(4+a)(3+a-2(1-x)^2-(1-x)^4)} + \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{(10+3a+\sqrt{4+a})\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1-\sqrt{4+a}}} - \frac{(10+3a-\sqrt{4+a})\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1+\sqrt{4+a}}} - \frac{\text{Subst}\left(\int \frac{1}{4(4+a)-x^2}\,dx, x, -2(1+(-1+x)^2)\right)}{2(4+a)}$$

$$= \frac{1+(-1+x)^2}{4(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{\left(10+3a+\sqrt{4+a}\right)\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1-\sqrt{4+a}}} - \frac{\left(10+3a-\sqrt{4+a}\right)\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1+\sqrt{4+a}}} + \frac{\tanh^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{4(4+a)^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.72

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \frac{a+2x-ax+ax^2+x^3}{4(3+a)(4+a)\left(a-x\left(-8+8x-4x^2+x^3\right)\right)} - \frac{\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&, \frac{6\log(x-\#1)+a\log(x-\#1)+4\log(x-\#1)\#1+2a\log(x-\#1)\#1+\log(x-\right.}{-2+4\#1-3\#1^2+\#1^3}}{16\left(12+7a+a^2\right)}$$

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a + 2*x - a*x + a*x^2 + x^3)/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2 +
 x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (6*Log[x - #1] + a*
Log[x - #1] + 4*Log[x - #1]*#1 + 2*a*Log[x - #1]*#1 + Log[x - #1]*#1^2)/(-2
 + 4*#1 - 3*#1^2 + #1^3) & ]/(16*(12 + 7*a + a^2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.68

method	result
default	$\frac{\frac{x^3}{4a^2+28a+48}+\frac{a\,x^2}{4(4+a)(3+a)}-\frac{(a-2)x}{4\left(a^2+7a+12\right)}+\frac{a}{4a^2+28a+48}}{-x^4+4x^3-8x^2+a+8x} + \frac{\sum_{\_R=\text{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)} \frac{\left(6+\_R^2+2(a+2)\_R+a\right)\ln\left(x\right.}{-\_R^3+3\_R^2-4\_R+}}{16a^2+112a+192}$
risch	$\frac{\frac{x^3}{4a^2+28a+48}+\frac{a\,x^2}{4(4+a)(3+a)}-\frac{(a-2)x}{4\left(a^2+7a+12\right)}+\frac{a}{4a^2+28a+48}}{-x^4+4x^3-8x^2+a+8x} + \frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)} \frac{\left(\frac{\_R^2}{a^2+7a+12}+\frac{2(a+2)\_R}{a^2+7a+12}+\frac{}{a^2}\right.}{-\_R^3+3\_R^2-}\right.}{16}$

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4/(a^2+7*a+12)*x^3+1/4*a/(4+a)/(3+a)*x^2-1/4*(a-2)/(a^2+7*a+12)*x+1/4/(a
^2+7*a+12)*a)/(-x^4+4*x^3-8*x^2+a+8*x)+1/16/(a^2+7*a+12)*sum((6+_R^2+2*(a+2
)*_R+a)/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a)
)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(197) = 394$.

Time = 16.40 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.33

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \frac{-ax^2-a-x^3+x(a-2)}{-4a^3-28a^2-48a+x^4\cdot(4a^2+28a+48)+x^3\,(-16a^2-112a-192)+x^2\cdot(32a^2+224a+384)+x\,(-} + \text{RootSum}\left(t^4\cdot(65536a^9+2162688a^8+31653888a^7+269680640a^6+1473773568a^5+5357174784a^4\right.$$

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] (-a*x**2 - a - x**3 + x*(a - 2))/(-4*a**3 - 28*a**2 - 48*a + x**4*(4*a**2 +
 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2 + 224*a + 384)
+ x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 + 2162688*a**8 +
31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357174784*a**4 + 129520
10752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**2*(-2048*
a**6 - 50688*a**5 - 520704*a**4 - 2842624*a**3 - 8699904*a**2 - 14155776*a
- 9568256) + _t*(1152*a**4 + 17792*a**3 + 102912*a**2 + 264192*a + 253952)
+ 16*a**3 - 57*a**2 - 984*a - 2064, Lambda(_t, _t*log(x + (98304*_t**3*a**1
2 + 3948544*_t**3*a**11 + 72196096*_t**3*a**10 + 793837568*_t**3*a**9 + 583
9372288*_t**3*a**8 + 30226464768*_t**3*a**7 + 112668450816*_t**3*a**6 + 303
864643584*_t**3*a**5 + 586157391872*_t**3*a**4 + 784017129472*_t**3*a**3 +
683648483328*_t**3*a**2 + 343136010240*_t**3*a + 72477573120*_t**3 + 30208*
_t**2*a**10 + 986624*_t**2*a**9 + 14420992*_t**2*a**8 + 124156928*_t**2*a**
```

```
7 + 696815104*_t**2*a**6 + 2661758464*_t**2*a**5 + 7001485312*_t**2*a**4 +
12506562560*_t**2*a**3 + 14494924800*_t**2*a**2 + 9820569600*_t**2*a + 2944
401408*_t**2 - 1536*_t*a**9 - 52048*_t*a**8 - 757040*_t*a**7 - 6200656*_t*a
**6 - 31380496*_t*a**5 - 100736416*_t*a**4 - 200813696*_t*a**3 - 228144640*
_t*a**2 - 114632704*_t*a - 2490368*_t + 248*a**7 + 6797*a**6 + 71132*a**5 +
 369745*a**4 + 987758*a**3 + 1128896*a**2 - 129568*a - 956416)/(576*a**7 +
10985*a**6 + 88746*a**5 + 396609*a**4 + 1076268*a**3 + 1826304*a**2 + 18677
76*a + 917504))))
```

Maxima [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \int \frac{x}{(x^4-4\,x^3+8\,x^2-a-8\,x)^2}\,dx$$

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a*x^2 + x^3 - (a - 2)*x + a)/((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 1
2)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a
) - 1/4*integrate((2*(a + 2)*x + x^2 + a + 6)/(x^4 - 4*x^3 + 8*x^2 - a - 8*
x), x)/(a^2 + 7*a + 12)
```

Giac [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \int \frac{x}{(x^4-4\,x^3+8\,x^2-a-8\,x)^2}\,dx$$

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 10.33 (sec) , antiderivative size = 1167, normalized size of antiderivative = 5.05

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Too large to display}$$

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] symsum(log((35*a + 4*a^2 + 68)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^
5 + 576)) - root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 +
 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z
```

^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*
z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 141
55776*a*z^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 26419
2*a*z + 253952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k)*((12800*a + 3600*a
^2 + 336*a^3 + 15104)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))
 + root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^
9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357
174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 284
2624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z
^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 2
53952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k)*(root(12952010752*a^3*z^4 +
 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 2
0082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^
6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2
 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z^2 - 9568256*z^2 + 102912*a^2
*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 253952*z - 984*a - 57*a^2 + 16
a^3 - 2064, z, k)((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 +
 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 +
a^5 + 576)) - (x*(3932160*a + 2719744*a^2 + 999424*a^3 + 205824*a^4 + 22528
*a^5 + 1024*a^6 + 2359296))/(16*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 +
 576))) - (1150976*a + 631808*a^2 + 172800*a^3 + 23552*a^4 + 1280*a^5 + 835
584)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + (x*(104448*a +
 58880*a^2 + 16512*a^3 + 2304*a^4 + 128*a^5 + 73728))/(16*(816*a + 460*a^2
+ 129*a^3 + 18*a^4 + a^5 + 576))) - (x*(864*a + 228*a^2 + 20*a^3 + 1088))/(
16*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))) + (x*(9*a + 2*a^2 + 8
))/(16*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*root(12952010752*
a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*
a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269
680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 2842624*a^3*z^2 - 52070
4*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z^2 - 9568256*z^2 + 1
02912*a^2*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 253952*z - 984*a - 57
*a^2 + 16*a^3 - 2064, z, k), k, 1, 4) + (x^3/(4*(7*a + a^2 + 12)) + a/(4*(a
 + 3)*(a + 4)) - (x*(a - 2))/(4*(a + 3)*(a + 4)) + (a*x^2)/(4*(a + 3)*(a +
4)))/(a + 8*x - 8*x^2 + 4*x^3 - x^4)

3.129 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx$

Optimal result 982
Rubi [A] (verified) 983
Mathematica [C] (verified) 987
Maple [C] (verified) 988
Fricas [F(-1)] 988
Sympy [B] (verification not implemented) 989
Maxima [F] 990
Giac [F] 991
Mupad [B] (verification not implemented) 991

Optimal result

Integrand size = 24, antiderivative size = 349

$$\begin{aligned}
&\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx \\
&= \frac{1+(-1+x)^2}{8(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^2} \\
&\quad+\frac{3(1+(-1+x)^2)}{16(4+a)^2\left(3+a-2(-1+x)^2-(-1+x)^4\right)} \\
&\quad+\frac{(5+a+(-1+x)^2)(-1+x)}{8\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^2} \\
&\quad+\frac{((6+a)(25+7a)+6(7+2a)(-1+x)^2)(-1+x)}{32(3+a)^2(4+a)^2\left(3+a-2(-1+x)^2-(-1+x)^4\right)} \\
&\quad-\frac{3\left(80+7a^2+14\sqrt{4+a}+a\left(47+4\sqrt{4+a}\right)\right)\arctan\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{64(3+a)^2(4+a)^{5/2}\sqrt{1-\sqrt{4+a}}} \\
&\quad-\frac{3\left(14+4a-\frac{80+47a+7a^2}{\sqrt{4+a}}\right)\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{64(3+a)^2(4+a)^2\sqrt{1+\sqrt{4+a}}}+\frac{3\operatorname{arctanh}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{16(4+a)^{5/2}}
\end{aligned}$$

```
[Out] 1/8*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^2+3/16*(1+(-1+x)^2)/(4+a)^
2/(3+a-2*(-1+x)^2-(-1+x)^4)+1/8*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(
-1+x)^2-(-1+x)^4)^2+1/32*((6+a)*(25+7*a)+6*(7+2*a)*(-1+x)^2)*(-1+x)/(a^2+7*
a+12)^2/(3+a-2*(-1+x)^2-(-1+x)^4)+3/16*arctanh((1+(-1+x)^2)/(4+a)^(1/2))/(4
+a)^(5/2)-3/64*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(80+7*a^2+14*(4+a)^(1/2
)+a*(47+4*(4+a)^(1/2)))/(3+a)^2/(4+a)^(5/2)/(1-(4+a)^(1/2))^(1/2)-3/64*arct
an((-1+x)/(1+(4+a)^(1/2))^(1/2))*(14+4*a+(-7*a^2-47*a-80)/(4+a)^(1/2))/(3+a
)^2/(4+a)^2/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1694, 1687, 1106, 1192, 1180, 210, 1121, 628, 632, 212}

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx$$
$$= -\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}$$
$$-\frac{3\left(-\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{64(a+3)^2(a+4)^2\sqrt{\sqrt{a+4}+1}}$$
$$+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}+\frac{3\text{arctanh}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{16(a+4)^{5/2}}$$
$$+\frac{3((x-1)^2+1)}{16(a+4)^2\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{(x-1)^2+1}{8(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}$$
$$+\frac{(x-1)\left(6(2a+7)(x-1)^2+(a+6)(7a+25)\right)}{32(a+3)^2(a+4)^2\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (1 + (-1 + x)^2)/(8*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^2) + (3*(1
+ (-1 + x)^2))/(16*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((5 + a
 + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 +
x)^4)^2) + (((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/(32*(3
+ a)^2*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - (3*(80 + 7*a^2 + 14
*Sqrt[4 + a] + a*(47 + 4*Sqrt[4 + a]))*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]
]])/(64*(3 + a)^2*(4 + a)^(5/2)*Sqrt[1 - Sqrt[4 + a]]) - (3*(14 + 4*a - (80
 + 47*a + 7*a^2)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(64*(
3 + a)^2*(4 + a)^2*Sqrt[1 + Sqrt[4 + a]]) + (3*ArcTanh[(1 + (-1 + x)^2)/Sqr
t[4 + a]])/(16*(4 + a)^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1+x}{(3+a-2x^2-x^4)^3}\,dx, x, -1+x\right)\\
&= \text{Subst}\left(\int \frac{1}{(3+a-2x^2-x^4)^3}\,dx, x, -1+x\right)\\
&\quad + \text{Subst}\left(\int \frac{x}{(3+a-2x^2-x^4)^3}\,dx, x, -1+x\right)\\
&= \frac{(5+a+(-1+x)^2)(-1+x)}{8(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^2}\\
&\quad + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(3+a-2x-x^2)^3}\,dx, x, (-1+x)^2\right)\\
&\quad - \frac{\text{Subst}\left(\int \frac{4+2(3+a)-4(4+4(3+a))-10x^2}{(3+a-2x^2-x^4)^2}\,dx, x, -1+x\right)}{16(12+7a+a^2)}\\
&= \frac{1+(-1+x)^2}{8(4+a)(3+a-2(1-x)^2-(1-x)^4)^2}\\
&\quad - \frac{((6+a)(25+7a)+6(7+2a)(1-x)^2)(1-x)}{32(12+7a+a^2)^2(3+a-2(1-x)^2-(1-x)^4)}\\
&\quad + \frac{(5+a+(-1+x)^2)(-1+x)}{8(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^2}\\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{(3+a-2x-x^2)^2}\,dx, x, (-1+x)^2\right)}{8(4+a)}\\
&\quad + \frac{\text{Subst}\left(\int \frac{12(94+51a+7a^2)+24(7+2a)x^2}{3+a-2x^2-x^4}\,dx, x, -1+x\right)}{128(12+7a+a^2)^2}
\end{aligned}$$

$$= \frac{1+(-1+x)^2}{8(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^2} + \frac{3(1+(-1+x)^2)}{16(4+a)^2\left(3+a-2(1-x)^2-(1-x)^4\right)} - \frac{\left((6+a)(25+7a)+6(7+2a)(1-x)^2\right)(1-x)}{32\left(12+7a+a^2\right)^2\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{8\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^2} + \frac{3\text{Subst}\left(\int \frac{1}{3+a-2x-x^2}\,dx, x, (-1+x)^2\right)}{16(4+a)^2} + \frac{\left(3\left(14+4a-\frac{80+47a+7a^2}{\sqrt{4+a}}\right)\right)\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{64\left(12+7a+a^2\right)^2} + \frac{\left(3\left(80+7a^2+14\sqrt{4+a}+a\left(47+4\sqrt{4+a}\right)\right)\right)\text{Subst}\left(\int \frac{1}{-1+\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{64\sqrt{4+a}\left(12+7a+a^2\right)^2}$$

$$= \frac{1+(-1+x)^2}{8(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^2} + \frac{3(1+(-1+x)^2)}{16(4+a)^2\left(3+a-2(1-x)^2-(1-x)^4\right)} - \frac{\left((6+a)(25+7a)+6(7+2a)(1-x)^2\right)(1-x)}{32\left(12+7a+a^2\right)^2\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{8\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^2} + \frac{3\left(80+47a+7a^2+\sqrt{4+a}(14+4a)\right)\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{64(3+a)^2(4+a)^{5/2}\sqrt{1-\sqrt{4+a}}} + \frac{3\left(14+4a-\frac{80+47a+7a^2}{\sqrt{4+a}}\right)\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{64\left(12+7a+a^2\right)^2\sqrt{1+\sqrt{4+a}}} - \frac{3\text{Subst}\left(\int \frac{1}{4(4+a)-x^2}\,dx, x, -2(1+(-1+x)^2)\right)}{8(4+a)^2}$$

$$= \frac{1+(-1+x)^2}{8(4+a)\,(3+a-2(1-x)^2-(1-x)^4)^2}$$
$$+ \frac{3(1+(-1+x)^2)}{16(4+a)^2\,(3+a-2(1-x)^2-(1-x)^4)}$$
$$- \frac{((6+a)(25+7a)+6(7+2a)(1-x)^2)\,(1-x)}{32\,(12+7a+a^2)^2\,(3+a-2(1-x)^2-(1-x)^4)}$$
$$+ \frac{(5+a+(-1+x)^2)\,(-1+x)}{8\,(12+7a+a^2)\,(3+a-2(-1+x)^2-(-1+x)^4)^2}$$
$$+ \frac{3\big(80+47a+7a^2+\sqrt{4+a}(14+4a)\big)\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{64(3+a)^2(4+a)^{5/2}\sqrt{1-\sqrt{4+a}}}$$
$$+ \frac{3\left(14+4a-\frac{80+47a+7a^2}{\sqrt{4+a}}\right)\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{64\,(12+7a+a^2)^2\sqrt{1+\sqrt{4+a}}} + \frac{3\tanh^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{16(4+a)^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.81

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \frac{1}{128}\left(\frac{16(a+2x-ax+ax^2+x^3)}{(3+a)(4+a)\,(a-x\,(-8+8x-4x^2+x^3))^2}\right.$$
$$+ \frac{4(a^2(5-5x+6x^2)+6(-14+28x-12x^2+7x^3)+a(-7+31x+12x^3))}{(3+a)^2(4+a)^2\,(a-x\,(-8+8x-4x^2+x^3))}$$
$$- \frac{3\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&,\frac{72\log(x-\#1)+31a\log(x-\#1)+3a^2\log(x-\#1)+8\log(x-\#1)\#1+16\ldots}{-2+4\#\ldots}\right.}{(12+7a+a^2)^2}$$

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] ((16*(a + 2*x - a*x + a*x^2 + x^3))/((3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x
^2 + x^3))^2) + (4*(a^2*(5 - 5*x + 6*x^2) + 6*(-14 + 28*x - 12*x^2 + 7*x^3)
 + a*(-7 + 31*x + 12*x^3)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8*x - 4*x^2 +
 x^3))) - (3*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (72*Log[x - #1]
+ 31*a*Log[x - #1] + 3*a^2*Log[x - #1] + 8*Log[x - #1]*#1 + 16*a*Log[x - #1
]*#1 + 4*a^2*Log[x - #1]*#1 + 14*Log[x - #1]*#1^2 + 4*a*Log[x - #1]*#1^2)/(
-2 + 4*#1 - 3*#1^2 + #1^3) & ])/(12 + 7*a + a^2)^2)/128
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.17

method	result
default	$-\frac{\frac{3(7+2a)x^7}{16(a^4+14a^3+73a^2+168a+144)}+\frac{3(a^2-8a-40)x^6}{16(a^4+14a^3+73a^2+168a+144)}-\frac{(29a^2-127a-792)x^5}{32(a^4+14a^3+73a^2+168a+144)}+\frac{(73a^2-227a-1668)x^4}{32a^4+448a^3+2336a^2+5376a+4608}-\frac{}{16(}}{(-x^4+4x^3-8x^2+a}$
risch	$\frac{-\frac{3(7+2a)x^7}{16(a^4+14a^3+73a^2+168a+144)}-\frac{3(a^2-8a-40)x^6}{16(a^4+14a^3+73a^2+168a+144)}+\frac{(29a^2-127a-792)x^5}{32a^4+448a^3+2336a^2+5376a+4608}-\frac{(73a^2-227a-1668)x^4}{32(a^4+14a^3+73a^2+168a+144)}+\frac{}{16a^4}}{(-x^4+4x^3-8x^2+}$

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(3/16*(7+2*a)/(a^4+14*a^3+73*a^2+168*a+144)*x^7+3/16*(a^2-8*a-40)/(a^4+14*
a^3+73*a^2+168*a+144)*x^6-1/32*(29*a^2-127*a-792)/(a^4+14*a^3+73*a^2+168*a+
144)*x^5+1/32*(73*a^2-227*a-1668)/(a^4+14*a^3+73*a^2+168*a+144)*x^4-1/16*(6
2*a^2-103*a-1104)/(a^4+14*a^3+73*a^2+168*a+144)*x^3-1/16*(5*a^3-26*a^2+140*
a+1008)/(a^4+14*a^3+73*a^2+168*a+144)*x^2+3/32*(3*a^3-17*a^2-40*a+192)/(a^4
+14*a^3+73*a^2+168*a+144)*x-3/32*a*(3*a^2+7*a-12)/(a^4+14*a^3+73*a^2+168*a+
144))/(-x^4+4*x^3-8*x^2+a+8*x)^2-3/128/(a^4+14*a^3+73*a^2+168*a+144)*sum((-
72+2*(-2*a-7)*_R^2+4*(-a^2-4*a-2)*_R-3*a^2-31*a)/(-_R^3+3*_R^2-4*_R+2)*ln(x
-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1102 vs. 2(318) = 636.

Time = 42.84 (sec) , antiderivative size = 1102, normalized size of antiderivative = 3.16

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] -(-9*a**3 - 21*a**2 + 36*a + x**7*(12*a + 42) + x**6*(6*a**2 - 48*a - 240)
+ x**5*(-29*a**2 + 127*a + 792) + x**4*(73*a**2 - 227*a - 1668) + x**3*(-12
4*a**2 + 206*a + 2208) + x**2*(-10*a**3 + 52*a**2 - 280*a - 2016) + x*(9*a*
*3 - 51*a**2 - 120*a + 576))/(32*a**6 + 448*a**5 + 2336*a**4 + 5376*a**3 +
4608*a**2 + x**8*(32*a**4 + 448*a**3 + 2336*a**2 + 5376*a + 4608) + x**7*(-
256*a**4 - 3584*a**3 - 18688*a**2 - 43008*a - 36864) + x**6*(1024*a**4 + 14
336*a**3 + 74752*a**2 + 172032*a + 147456) + x**5*(-2560*a**4 - 35840*a**3
- 186880*a**2 - 430080*a - 368640) + x**4*(-64*a**5 + 3200*a**4 + 52672*a**
3 + 288256*a**2 + 678912*a + 589824) + x**3*(256*a**5 - 512*a**4 - 38656*a*
*3 - 256000*a**2 - 651264*a - 589824) + x**2*(-512*a**5 - 5120*a**4 - 8704*
a**3 + 63488*a**2 + 270336*a + 294912) + x*(512*a**5 + 7168*a**4 + 37376*a*
*3 + 86016*a**2 + 73728*a)) - RootSum(_t**4*(268435456*a**15 + 14763950080*
a**14 + 378493992960*a**13 + 5999532441600*a**12 + 65757291479040*a**11 + 5
27875908304896*a**10 + 3206246773555200*a**9 + 15003759578972160*a**8 + 545
37151127224320*a**7 + 153980418717122560*a**6 + 334927734494986240*a**5 + 5
51152193655275520*a**4 + 664192984106926080*a**3 + 553362212027105280*a**2
+ 284993413919539200*a + 68398419340689408) + _t**2*(-4718592*a**10 - 19611
6480*a**9 - 3648061440*a**8 - 40022212608*a**7 - 286939938816*a**6 - 140543
7345792*a**5 - 4764645457920*a**4 - 11043392716800*a**3 - 16752587046912*a*
*2 - 15023392948224*a - 6049461436416) + _t*(-2709504*a**7 - 72880128*a**6
- 839890944*a**5 - 5375877120*a**4 - 20640890880*a**3 - 47542173696*a**2 -
60827369472*a - 33351008256) + 20736*a**5 - 155601*a**4 - 4706424*a**3 - 29
249424*a**2 - 74027520*a - 68345856, Lambda(_t, _t*log(x + (-469762048*_t**
3*a**20 - 31417434112*_t**3*a**19 - 992305217536*_t**3*a**18 - 196635766292
48*_t**3*a**17 - 273880031690752*_t**3*a**16 - 2846116194287616*_t**3*a**15
 - 22853982892326912*_t**3*a**14 - 144840417605582848*_t**3*a**13 - 7331931
54773123072*_t**3*a**12 - 2977941469704224768*_t**3*a**11 - 967719737311730
0736*_t**3*a**10 - 24850421452415959040*_t**3*a**9 - 48984708931769073664*_
t**3*a**8 - 69124682329943441408*_t**3*a**7 - 54921507243737219072*_t**3*a*
*6 + 18833423088924753920*_t**3*a**5 + 128767022044444360704*_t**3*a**4 + 1
97893824476545548288*_t**3*a**3 + 170576989286005997568*_t**3*a**2 + 837098
68624400351232*_t**3*a + 18392762450832261120*_t**3 + 136642560*_t**2*a**17
 + 7616593920*_t**2*a**16 + 198980665344*_t**2*a**15 + 3234300690432*_t**2*
a**14 + 36614363283456*_t**2*a**13 + 306155605721088*_t**2*a**12 + 19563396
56687616*_t**2*a**11 + 9747894775578624*_t**2*a**10 + 38291841445330944*_t*
```

```
*2*a**9 + 119050488573591552*_t**2*a**8 + 292236772188880896*_t**2*a**7 + 5
61261720373297152*_t**2*a**6 + 828898581078343680*_t**2*a**5 + 914439454498
750464*_t**2*a**4 + 718255692208668672*_t**2*a**3 + 369227414724673536*_t**
2*a**2 + 104815442748506112*_t**2*a + 10263520138493952*_t**2 + 4128768*_t*
a**15 + 235608192*_t*a**14 + 6050117376*_t*a**13 + 92875570560*_t*a**12 + 9
50838962688*_t*a**11 + 6825858397056*_t*a**10 + 34932826734336*_t*a**9 + 12
5262778564224*_t*a**8 + 287989861404672*_t*a**7 + 257684685023232*_t*a**6 -
 836263788945408*_t*a**5 - 4002432415137792*_t*a**4 - 8409454278082560*_t*a
**3 - 10371340262965248*_t*a**2 - 7285247072796672*_t*a - 2270140431335424*
_t + 1000512*a**12 + 42546357*a**11 + 777344580*a**10 + 7998006582*a**9 + 5
0045408388*a**8 + 182866499613*a**7 + 247394170512*a**6 - 1063305068832*a**
5 - 6960658344192*a**4 - 19132655580288*a**3 - 30001872614400*a**2 - 261928
92672000*a - 9953981595648)/(1354752*a**12 + 44550027*a**11 + 663517980*a**
10 + 5951170602*a**9 + 36270700668*a**8 + 162289912419*a**7 + 567868212432*
a**6 + 1626099007104*a**5 + 3825839091456*a**4 + 7035734732544*a**3 + 92167
60449024*a**2 + 7467334520832*a + 2773884911616)))
```

Maxima [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \int -\frac{x}{\left(x^4-4\,x^3+8\,x^2-a-8\,x\right)^3}\,dx$$

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(6*(2*a + 7)*x^7 + 6*(a^2 - 8*a - 40)*x^6 - (29*a^2 - 127*a - 792)*x^
5 + (73*a^2 - 227*a - 1668)*x^4 - 2*(62*a^2 - 103*a - 1104)*x^3 - 9*a^3 - 2
*(5*a^3 - 26*a^2 + 140*a + 1008)*x^2 - 21*a^2 + 3*(3*a^3 - 17*a^2 - 40*a +
192)*x + 36*a)/((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3
 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6
 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 5
0*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4
- 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17
*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 1
68*a^2 + 144*a)*x) - 3/32*integrate((2*(2*a + 7)*x^2 + 3*a^2 + 4*(a^2 + 4*a
 + 2)*x + 31*a + 72)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^4 + 14*a^3 + 73
*a^2 + 168*a + 144)
```

Giac [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \int -\frac{x}{(x^4-4\,x^3+8\,x^2-a-8\,x)^3}\,dx$$

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")

[Out] integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3, x)

Mupad [B] (verification not implemented)

Time = 9.93 (sec) , antiderivative size = 2200, normalized size of antiderivative = 6.30

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Too large to display}$$

[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)

[Out] symsum(log(root(15003759578972160*a^8*z^4 + 54537151127224320*a^7*z^4 + 153980418717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 551152193655275520*a^4*z^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z^4 + 5999532441600*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200*a*z^4 + 3206246773555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a^11*z^4 + 378493992960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^4 - 4718592*a^10*z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 150233929482224*a*z^2 - 16752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608*a^7*z^2 - 11043392716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z^2 - 6049461436416*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a^2*z + 72880128*a^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z + 33351008256*z - 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 20736*a^5 - 68345856, z, k)*((242823168*a + 170044416*a^2 + 63509760*a^3 + 13340736*a^4 + 1494144*a^5 + 69696*a^6 + 144506880)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) + root(15003759578972160*a^8*z^4 + 54537151127224320*a^7*z^4 + 153980418717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 551152193655275520*a^4*z^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z^4 + 5999532441600*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200*a*z^4 + 3206246773555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a^11*z^4 + 378493992960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^4 - 4718592*a^10*z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 150233929482224*a*z^2 - 16752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608*a^7*z^2 - 11043392716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z^2 - 6049461436416*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a^2*z + 72880128*a^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z + 3

3351008256*z - 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 20736
a^5 - 68345856, z, k)(root(15003759578972160*a^8*z^4 + 54537151127224320*
a^7*z^4 + 153980418717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 5511521
93655275520*a^4*z^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z
^4 + 5999532441600*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200
*a*z^4 + 3206246773555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a
^11*z^4 + 378493992960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^
4 - 4718592*a^10*z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 15023392
948224*a*z^2 - 16752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608
*a^7*z^2 - 11043392716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z
^2 - 6049461436416*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a
^2*z + 72880128*a^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z
 + 33351008256*z - 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 2
0736*a^5 - 68345856, z, k)*((4290672328704*a + 6001143054336*a^2 + 50259170
42688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 302556119040*a^6 + 5986
2155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11
 + 1391569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4
+ 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) -
(x*(536334041088*a + 750142881792*a^2 + 628239630336*a^3 + 350065000448*a^4
 + 136275034112*a^5 + 37819514880*a^6 + 7482769408*a^7 + 1034420224*a^8 + 9
5158272*a^9 + 5242880*a^10 + 131072*a^11 + 173946175488))/(2048*(940032*a +
 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7
+ 582*a^8 + 36*a^9 + a^10 + 331776))) - (73421291520*a + 81260445696*a^2 +
52393672704*a^3 + 21688418304*a^4 + 5977620480*a^5 + 1096949760*a^6 + 12924
5184*a^7 + 8871936*a^8 + 270336*a^9 + 29444014080)/(16384*(940032*a + 11957
76*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*
a^8 + 36*a^9 + a^10 + 331776)) + (x*(2632974336*a + 3015180288*a^2 + 200894
0544*a^3 + 858243072*a^4 + 243806208*a^5 + 46055424*a^6 + 5578752*a^7 + 393
216*a^8 + 12288*a^9 + 1019215872))/(2048*(940032*a + 1195776*a^2 + 899328*a
^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^
10 + 331776))) - (x*(10805760*a + 7173504*a^2 + 2539872*a^3 + 505800*a^4 +
53712*a^5 + 2376*a^6 + 6782976))/(2048*(940032*a + 1195776*a^2 + 899328*a^3
 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10
 + 331776))) - (133812*a + 56187*a^2 + 10098*a^3 + 648*a^4 + 115776)/(16384
*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6
 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(1971*a^2 - 1539*a +
918*a^3 + 108*a^4 - 6372))/(2048*(940032*a + 1195776*a^2 + 899328*a^3 + 442
864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331
776)))*root(15003759578972160*a^8*z^4 + 54537151127224320*a^7*z^4 + 1539804
18717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 551152193655275520*a^4*z
^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z^4 + 599953244160
0*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200*a*z^4 + 32062467
73555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a^11*z^4 + 3784939
92960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^4 - 4718592*a^10*
z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 15023392948224*a*z^2 - 16

752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608*a^7*z^2 - 110433
92716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z^2 - 604946143641
6*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a^2*z + 72880128*a
^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z + 33351008256*z
- 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 20736*a^5 - 683458
56, z, k), k, 1, 4) + ((3*(7*a^2 - 12*a + 3*a^3))/(32*(6*a + a^2 + 9)*(8*a
+ a^2 + 16)) - (3*x^7*(2*a + 7))/(16*(168*a + 73*a^2 + 14*a^3 + a^4 + 144))
 + (x^2*(140*a - 26*a^2 + 5*a^3 + 1008))/(16*(6*a + a^2 + 9)*(8*a + a^2 + 1
6)) + (3*x*(40*a + 17*a^2 - 3*a^3 - 192))/(32*(6*a + a^2 + 9)*(8*a + a^2 +
16)) + (3*x^6*(8*a - a^2 + 40))/(16*(6*a + a^2 + 9)*(8*a + a^2 + 16)) - (x^
5*(127*a - 29*a^2 + 792))/(32*(6*a + a^2 + 9)*(8*a + a^2 + 16)) - (x^3*(103
*a - 62*a^2 + 1104))/(16*(6*a + a^2 + 9)*(8*a + a^2 + 16)) + (x^4*(227*a -
73*a^2 + 1668))/(32*(6*a + a^2 + 9)*(8*a + a^2 + 16)))/(16*a*x - x^2*(16*a
- 64) - x^4*(2*a - 128) + x^3*(8*a - 128) + a^2 - 80*x^5 + 32*x^6 - 8*x^7 +
 x^8)

3.130 $\int x^2(a+8x-8x^2+4x^3-x^4)^4\,dx$

Optimal result 994
Rubi [A] (verified) 995
Mathematica [A] (verified) 996
Maple [A] (verified) 997
Fricas [A] (verification not implemented) 997
Sympy [A] (verification not implemented) 998
Maxima [A] (verification not implemented) 999
Giac [A] (verification not implemented) 999
Mupad [B] (verification not implemented) 1001

Optimal result

Integrand size = 26, antiderivative size = 210

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx=\frac{a^4x^3}{3}+8a^3x^4+\frac{32}{5}(12-a)a^2x^5+\frac{8}{3}a\left(128-48a+a^2\right)x^6+\frac{4}{7}\left(1024-1536a+192a^2-a^3\right)x^7-4\left(512-288a+15a^2\right)x^8+\frac{64}{9}(128-3a)(4-a)x^9-\frac{24}{5}\left(896-128a+a^2\right)x^{10}+\frac{2}{11}\left(20480-1536a+3a^2\right)x^{11}-\frac{8}{3}(928-35a)x^{12}+\frac{32}{13}(524-9a)x^{13}-\frac{8}{7}(464-3a)x^{14}+\frac{4}{15}(640-a)x^{15}-42x^{16}+\frac{128x^{17}}{17}-\frac{8x^{18}}{9}+\frac{x^{19}}{19}$$

```
[Out] 1/3*a^4*x^3+8*a^3*x^4+32/5*(12-a)*a^2*x^5+8/3*a*(a^2-48*a+128)*x^6+4/7*(-a^
3+192*a^2-1536*a+1024)*x^7-4*(15*a^2-288*a+512)*x^8+64/9*(128-3*a)*(4-a)*x^
9-24/5*(a^2-128*a+896)*x^10+2/11*(3*a^2-1536*a+20480)*x^11-8/3*(928-35*a)*x
^12+32/13*(524-9*a)*x^13-8/7*(464-3*a)*x^14+4/15*(640-a)*x^15-42*x^16+128/1
7*x^17-8/9*x^18+1/19*x^19
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {6874}

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx=\frac{a^4x^3}{3}+8a^3x^4+\frac{2}{11}\left(3a^2-1536a+20480\right)x^{11}-\frac{24}{5}\left(a^2-128a+896\right)x^{10}-4\left(15a^2-288a+512\right)x^8+\frac{8}{3}a\left(a^2-48a+128\right)x^6+\frac{32}{5}(12-a)a^2x^5+\frac{4}{7}\left(-a^3+192a^2-1536a+1024\right)x^7+\frac{4}{15}(640-a)x^{15}-\frac{8}{7}(464-3a)x^{14}+\frac{32}{13}(524-9a)x^{13}-\frac{8}{3}(928-35a)x^{12}+\frac{64}{9}(128-3a)(4-a)x^9+\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}$$

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^3)/3 + 8*a^3*x^4 + (32*(12 - a)*a^2*x^5)/5 + (8*a*(128 - 48*a + a^2)
*x^6)/3 + (4*(1024 - 1536*a + 192*a^2 - a^3)*x^7)/7 - 4*(512 - 288*a + 15*a
^2)*x^8 + (64*(128 - 3*a)*(4 - a)*x^9)/9 - (24*(896 - 128*a + a^2)*x^10)/5
+ (2*(20480 - 1536*a + 3*a^2)*x^11)/11 - (8*(928 - 35*a)*x^12)/3 + (32*(524
 - 9*a)*x^13)/13 - (8*(464 - 3*a)*x^14)/7 + (4*(640 - a)*x^15)/15 - 42*x^16
 + (128*x^17)/17 - (8*x^18)/9 + x^19/19
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral}=\int\left(a^4x^2+32a^3x^3-32(-12+a)a^2x^4+16a\left(128-48a+a^2\right)x^5-4\left(-1024+1536a-192a^2+a^3\right)x^6-32\left(512-288a+15a^2\right)x^7+64(128-3a)(4-a)x^8-48\left(896-128a+a^2\right)x^9+2\left(20480-1536a+3a^2\right)x^{10}+32(-928+35a)x^{11}-32(-524+9a)x^{12}+16(-464+3a)x^{13}-4(-640+a)x^{14}-672x^{15}+128x^{16}-16x^{17}+x^{18}\right)dx$$

$$= \frac{a^4x^3}{3} + 8a^3x^4 + \frac{32}{5}(12-a)a^2x^5 + \frac{8}{3}a\left(128-48a+a^2\right)x^6 + \frac{4}{7}\left(1024-1536a+192a^2-a^3\right)x^7 - 4\left(512-288a+15a^2\right)x^8 + \frac{64}{9}(128-3a)(4-a)x^9 - \frac{24}{5}\left(896-128a+a^2\right)x^{10} + \frac{2}{11}\left(20480-1536a+3a^2\right)x^{11} - \frac{8}{3}(928-35a)x^{12} + \frac{32}{13}(524-9a)x^{13} - \frac{8}{7}(464-3a)x^{14} + \frac{4}{15}(640-a)x^{15} - 42x^{16} + \frac{128x^{17}}{17} - \frac{8x^{18}}{9} + \frac{x^{19}}{19}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.97

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx = \frac{a^4x^3}{3} + 8a^3x^4 - \frac{32}{5}(-12+a)a^2x^5 + \frac{8}{3}a\left(128-48a+a^2\right)x^6 - \frac{4}{7}\left(-1024+1536a-192a^2+a^3\right)x^7 - 4\left(512-288a+15a^2\right)x^8 + \frac{64}{9}\left(512-140a+3a^2\right)x^9 - \frac{24}{5}\left(896-128a+a^2\right)x^{10} + \frac{2}{11}\left(20480-1536a+3a^2\right)x^{11} + \frac{8}{3}(-928+35a)x^{12} - \frac{32}{13}(-524+9a)x^{13} + \frac{8}{7}(-464+3a)x^{14} - \frac{4}{15}(-640+a)x^{15} - 42x^{16} + \frac{128x^{17}}{17} - \frac{8x^{18}}{9} + \frac{x^{19}}{19}$$

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^3)/3 + 8*a^3*x^4 - (32*(-12 + a)*a^2*x^5)/5 + (8*a*(128 - 48*a + a^2
)*x^6)/3 - (4*(-1024 + 1536*a - 192*a^2 + a^3)*x^7)/7 - 4*(512 - 288*a + 15
*a^2)*x^8 + (64*(512 - 140*a + 3*a^2)*x^9)/9 - (24*(896 - 128*a + a^2)*x^10
)/5 + (2*(20480 - 1536*a + 3*a^2)*x^11)/11 + (8*(-928 + 35*a)*x^12)/3 - (32
*(-524 + 9*a)*x^13)/13 + (8*(-464 + 3*a)*x^14)/7 - (4*(-640 + a)*x^15)/15 -
 42*x^16 + (128*x^17)/17 - (8*x^18)/9 + x^19/19
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

method	result
norman	$\frac{a^4x^3}{3}+8a^3x^4+\left(-\frac{32}{5}a^3+\frac{384}{5}a^2\right)x^5+\left(\frac{8}{3}a^3-128a^2+\frac{1024}{3}a\right)x^6+\left(-\frac{4}{7}a^3+\frac{768}{7}a^2-\frac{6144}{7}a+\right.$
gosper	$-\frac{21504}{5}x^{10}+\frac{40960}{11}x^{11}-\frac{7424}{3}x^{12}+\frac{16768}{13}x^{13}+\frac{32768}{9}x^9+\frac{4096}{7}x^7+\frac{24}{7}x^{14}a-\frac{24}{5}x^{10}a^2+\frac{8}{3}x^6a^3-$
risch	$-\frac{21504}{5}x^{10}+\frac{40960}{11}x^{11}-\frac{7424}{3}x^{12}+\frac{16768}{13}x^{13}+\frac{32768}{9}x^9+\frac{4096}{7}x^7+\frac{24}{7}x^{14}a-\frac{24}{5}x^{10}a^2+\frac{8}{3}x^6a^3-$
parallelrisch	$-\frac{21504}{5}x^{10}+\frac{40960}{11}x^{11}-\frac{7424}{3}x^{12}+\frac{16768}{13}x^{13}+\frac{32768}{9}x^9+\frac{4096}{7}x^7+\frac{24}{7}x^{14}a-\frac{24}{5}x^{10}a^2+\frac{8}{3}x^6a^3-$
default	$\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}+\frac{(-4a+2560)x^{15}}{15}+\frac{(48a-7424)x^{14}}{14}+\frac{(-288a+16768)x^{13}}{13}+\frac{(1120a-29696)x^{12}}{12}$

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^4*x^3+8*a^3*x^4+(-32/5*a^3+384/5*a^2)*x^5+(8/3*a^3-128*a^2+1024/3*a)*
x^6+(-4/7*a^3+768/7*a^2-6144/7*a+4096/7)*x^7+(-60*a^2+1152*a-2048)*x^8+(64/
3*a^2-8960/9*a+32768/9)*x^9+(-24/5*a^2+3072/5*a-21504/5)*x^10+(6/11*a^2-307
2/11*a+40960/11)*x^11+(280/3*a-7424/3)*x^12+(-288/13*a+16768/13)*x^13+(24/7
*a-3712/7)*x^14+(-4/15*a+512/3)*x^15-42*x^16+128/17*x^17-8/9*x^18+1/19*x^19
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

$$\begin{aligned}\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx=&\ \frac{1}{19}\,x^{19}-\frac{8}{9}\,x^{18}+\frac{128}{17}\,x^{17}-\frac{4}{15}\,(a-640)x^{15}\\&-42\,x^{16}+\frac{8}{7}\,(3\,a-464)x^{14}-\frac{32}{13}\,(9\,a-524)x^{13}\\&+\frac{8}{3}\,(35\,a-928)x^{12}+\frac{2}{11}\,\left(3\,a^2-1536\,a+20480\right)x^{11}\\&-\frac{24}{5}\,\left(a^2-128\,a+896\right)x^{10}\\&+\frac{64}{9}\,\left(3\,a^2-140\,a+512\right)x^9\\&-4\,\left(15\,a^2-288\,a+512\right)x^8\\&-\frac{4}{7}\,\left(a^3-192\,a^2+1536\,a-1024\right)x^7+\frac{1}{3}\,a^4x^3\\&+8\,a^3x^4+\frac{8}{3}\,\left(a^3-48\,a^2+128\,a\right)x^6\\&-\frac{32}{5}\,\left(a^3-12\,a^2\right)x^5\end{aligned}$$

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

```
[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*(a - 640)*x^15 - 42*x^16 + 8/7*(3
*a - 464)*x^14 - 32/13*(9*a - 524)*x^13 + 8/3*(35*a - 928)*x^12 + 2/11*(3*a
^2 - 1536*a + 20480)*x^11 - 24/5*(a^2 - 128*a + 896)*x^10 + 64/9*(3*a^2 - 1
40*a + 512)*x^9 - 4*(15*a^2 - 288*a + 512)*x^8 - 4/7*(a^3 - 192*a^2 + 1536*
a - 1024)*x^7 + 1/3*a^4*x^3 + 8*a^3*x^4 + 8/3*(a^3 - 48*a^2 + 128*a)*x^6 -
32/5*(a^3 - 12*a^2)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.04

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx=\frac{a^4x^3}{3}+8a^3x^4+\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}+x^{15}\cdot\left(\frac{512}{3}-\frac{4a}{15}\right)+x^{14}\cdot\left(\frac{24a}{7}-\frac{3712}{7}\right)+x^{13}\cdot\left(\frac{16768}{13}-\frac{288a}{13}\right)+x^{12}\cdot\left(\frac{280a}{3}-\frac{7424}{3}\right)+x^{11}\cdot\left(\frac{6a^2}{11}-\frac{3072a}{11}+\frac{40960}{11}\right)+x^{10}\left(-\frac{24a^2}{5}+\frac{3072a}{5}-\frac{21504}{5}\right)+x^9\cdot\left(\frac{64a^2}{3}-\frac{8960a}{9}+\frac{32768}{9}\right)+x^8\left(-60a^2+1152a-2048\right)+x^7\left(-\frac{4a^3}{7}+\frac{768a^2}{7}-\frac{6144a}{7}+\frac{4096}{7}\right)+x^6\cdot\left(\frac{8a^3}{3}-128a^2+\frac{1024a}{3}\right)+x^5\left(-\frac{32a^3}{5}+\frac{384a^2}{5}\right)$$

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] a**4*x**3/3 + 8*a**3*x**4 + x**19/19 - 8*x**18/9 + 128*x**17/17 - 42*x**16
+ x**15*(512/3 - 4*a/15) + x**14*(24*a/7 - 3712/7) + x**13*(16768/13 - 288*
a/13) + x**12*(280*a/3 - 7424/3) + x**11*(6*a**2/11 - 3072*a/11 + 40960/11)
 + x**10*(-24*a**2/5 + 3072*a/5 - 21504/5) + x**9*(64*a**2/3 - 8960*a/9 + 3
2768/9) + x**8*(-60*a**2 + 1152*a - 2048) + x**7*(-4*a**3/7 + 768*a**2/7 -
6144*a/7 + 4096/7) + x**6*(8*a**3/3 - 128*a**2 + 1024*a/3) + x**5*(-32*a**3
/5 + 384*a**2/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

$$\int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^4 \, dx = \frac{1}{19} x^{19} - \frac{8}{9} x^{18} + \frac{128}{17} x^{17} - \frac{4}{15} (a - 640) x^{15} - 42 x^{16} + \frac{8}{7} (3a - 464) x^{14} - \frac{32}{13} (9a - 524) x^{13} + \frac{8}{3} (35a - 928) x^{12} + \frac{2}{11} (3a^2 - 1536a + 20480) x^{11} - \frac{24}{5} (a^2 - 128a + 896) x^{10} + \frac{64}{9} (3a^2 - 140a + 512) x^9 - 4 (15a^2 - 288a + 512) x^8 - \frac{4}{7} (a^3 - 192a^2 + 1536a - 1024) x^7 + \frac{1}{3} a^4 x^3 + 8a^3 x^4 + \frac{8}{3} (a^3 - 48a^2 + 128a) x^6 - \frac{32}{5} (a^3 - 12a^2) x^5$$

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

```
[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*(a - 640)*x^15 - 42*x^16 + 8/7*(3
*a - 464)*x^14 - 32/13*(9*a - 524)*x^13 + 8/3*(35*a - 928)*x^12 + 2/11*(3*a
^2 - 1536*a + 20480)*x^11 - 24/5*(a^2 - 128*a + 896)*x^10 + 64/9*(3*a^2 - 1
40*a + 512)*x^9 - 4*(15*a^2 - 288*a + 512)*x^8 - 4/7*(a^3 - 192*a^2 + 1536*
a - 1024)*x^7 + 1/3*a^4*x^3 + 8*a^3*x^4 + 8/3*(a^3 - 48*a^2 + 128*a)*x^6 -
32/5*(a^3 - 12*a^2)*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.06

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx=\frac{1}{19}x^{19}-\frac{8}{9}x^{18}+\frac{128}{17}x^{17}-\frac{4}{15}ax^{15}-42x^{16}+\frac{24}{7}ax^{14}+\frac{512}{3}x^{15}-\frac{288}{13}ax^{13}-\frac{3712}{7}x^{14}+\frac{6}{11}a^2x^{11}+\frac{280}{3}ax^{12}+\frac{16768}{13}x^{13}-\frac{24}{5}a^2x^{10}-\frac{3072}{11}ax^{11}-\frac{7424}{3}x^{12}+\frac{64}{3}a^2x^9+\frac{3072}{5}ax^{10}+\frac{40960}{11}x^{11}-\frac{4}{7}a^3x^7-60a^2x^8-\frac{8960}{9}ax^9-\frac{21504}{5}x^{10}+\frac{8}{3}a^3x^6+\frac{768}{7}a^2x^7+1152ax^8+\frac{32768}{9}x^9-\frac{32}{5}a^3x^5-128a^2x^6-\frac{6144}{7}ax^7-2048x^8+\frac{1}{3}a^4x^3+8a^3x^4+\frac{384}{5}a^2x^5+\frac{1024}{3}ax^6+\frac{4096}{7}x^7$$

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

```
[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*a*x^15 - 42*x^16 + 24/7*a*x^14 +
512/3*x^15 - 288/13*a*x^13 - 3712/7*x^14 + 6/11*a^2*x^11 + 280/3*a*x^12 + 1
6768/13*x^13 - 24/5*a^2*x^10 - 3072/11*a*x^11 - 7424/3*x^12 + 64/3*a^2*x^9
+ 3072/5*a*x^10 + 40960/11*x^11 - 4/7*a^3*x^7 - 60*a^2*x^8 - 8960/9*a*x^9 -
 21504/5*x^10 + 8/3*a^3*x^6 + 768/7*a^2*x^7 + 1152*a*x^8 + 32768/9*x^9 - 32
/5*a^3*x^5 - 128*a^2*x^6 - 6144/7*a*x^7 - 2048*x^8 + 1/3*a^4*x^3 + 8*a^3*x^
4 + 384/5*a^2*x^5 + 1024/3*a*x^6 + 4096/7*x^7
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.85

$$\int x^2(a+8x-8x^2+4x^3-x^4)^4\,dx = x^{14}\left(\frac{24\,a}{7}-\frac{3712}{7}\right)-x^{15}\left(\frac{4\,a}{15}-\frac{512}{3}\right)+x^{12}\left(\frac{280\,a}{3}-\frac{7424}{3}\right)-x^{13}\left(\frac{288\,a}{13}-\frac{16768}{13}\right)-x^8\left(60\,a^2-1152\,a+2048\right)-x^{10}\left(\frac{24\,a^2}{5}-\frac{3072\,a}{5}+\frac{21504}{5}\right)+x^9\left(\frac{64\,a^2}{3}-\frac{8960\,a}{9}+\frac{32768}{9}\right)+x^{11}\left(\frac{6\,a^2}{11}-\frac{3072\,a}{11}+\frac{40960}{11}\right)-x^7\left(\frac{4\,a^3}{7}-\frac{768\,a^2}{7}+\frac{6144\,a}{7}-\frac{4096}{7}\right)-42\,x^{16}+\frac{128\,x^{17}}{17}-\frac{8\,x^{18}}{9}+\frac{x^{19}}{19}+8\,a^3\,x^4+\frac{a^4\,x^3}{3}+\frac{8\,a\,x^6\,(a^2-48\,a+128)}{3}-\frac{32\,a^2\,x^5\,(a-12)}{5}$$

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)
```

```
[Out] x^14*((24*a)/7 - 3712/7) - x^15*((4*a)/15 - 512/3) + x^12*((280*a)/3 - 7424
/3) - x^13*((288*a)/13 - 16768/13) - x^8*(60*a^2 - 1152*a + 2048) - x^10*((
24*a^2)/5 - (3072*a)/5 + 21504/5) + x^9*((64*a^2)/3 - (8960*a)/9 + 32768/9)
 + x^11*((6*a^2)/11 - (3072*a)/11 + 40960/11) - x^7*((6144*a)/7 - (768*a^2)
/7 + (4*a^3)/7 - 4096/7) - 42*x^16 + (128*x^17)/17 - (8*x^18)/9 + x^19/19 +
 8*a^3*x^4 + (a^4*x^3)/3 + (8*a*x^6*(a^2 - 48*a + 128))/3 - (32*a^2*x^5*(a
- 12))/5
```

3.131 $\int x^2(a+8x-8x^2+4x^3-x^4)^3\,dx$

Optimal result 1002
Rubi [A] (verified) 1002
Mathematica [A] (verified) 1003
Maple [A] (verified) 1004
Fricas [A] (verification not implemented) 1004
Sympy [A] (verification not implemented) 1005
Maxima [A] (verification not implemented) 1005
Giac [A] (verification not implemented) 1006
Mupad [B] (verification not implemented) 1006

Optimal result

Integrand size = 26, antiderivative size = 138

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=\frac{a^3x^3}{3}+6a^2x^4+\frac{24}{5}(8-a)ax^5+\frac{2}{3}\left(128-96a+3a^2\right)x^6-\frac{3}{7}\left(512-128a+a^2\right)x^7+6(48-5a)x^8-\frac{32}{9}(70-3a)x^9+\frac{12}{5}(64-a)x^{10}-\frac{3}{11}(256-a)x^{11}+\frac{70x^{12}}{3}-\frac{72x^{13}}{13}+\frac{6x^{14}}{7}-\frac{x^{15}}{15}$$

```
[Out] 1/3*a^3*x^3+6*a^2*x^4+24/5*(8-a)*a*x^5+2/3*(3*a^2-96*a+128)*x^6-3/7*(a^2-12
8*a+512)*x^7+6*(48-5*a)*x^8-32/9*(70-3*a)*x^9+12/5*(64-a)*x^10-3/11*(256-a)
*x^11+70/3*x^12-72/13*x^13+6/7*x^14-1/15*x^15
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {6874}

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=\frac{a^3x^3}{3}-\frac{3}{7}\left(a^2-128a+512\right)x^7+\frac{2}{3}\left(3a^2-96a+128\right)x^6+6a^2x^4-\frac{3}{11}(256-a)x^{11}+\frac{12}{5}(64-a)x^{10}-\frac{32}{9}(70-3a)x^9+6(48-5a)x^8+\frac{24}{5}(8-a)ax^5-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}$$

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + 6*a^2*x^4 + (24*(8 - a)*a*x^5)/5 + (2*(128 - 96*a + 3*a^2)*x^
6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 + 6*(48 - 5*a)*x^8 - (32*(70 - 3*a)*x^
9)/9 + (12*(64 - a)*x^10)/5 - (3*(256 - a)*x^11)/11 + (70*x^12)/3 - (72*x^1
3)/13 + (6*x^14)/7 - x^15/15
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^3x^2+24a^2x^3-24(-8+a)ax^4+4\left(128-96a+3a^2\right)x^5-3\left(512-128a+a^2\right)x^6 \right.\\ &\quad -48(-48+5a)x^7+32(-70+3a)x^8-24(-64+a)x^9+3(-256+a)x^{10}\\ &\quad \left. +280x^{11}-72x^{12}+12x^{13}-x^{14}\right)\,dx\\ &= \frac{a^3x^3}{3}+6a^2x^4+\frac{24}{5}(8-a)ax^5+\frac{2}{3}\left(128-96a+3a^2\right)x^6-\frac{3}{7}\left(512-128a+a^2\right)x^7+6(48\\ &\quad -5a)x^8\\ &\quad -\frac{32}{9}(70-3a)x^9+\frac{12}{5}(64-a)x^{10}-\frac{3}{11}(256-a)x^{11}+\frac{70x^{12}}{3}-\frac{72x^{13}}{13}+\frac{6x^{14}}{7}-\frac{x^{15}}{15}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

$$\begin{aligned}\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx &= \frac{a^3x^3}{3}+6a^2x^4-\frac{24}{5}(-8+a)ax^5+\frac{2}{3}\left(128-96a+3a^2\right)x^6\\ &\quad -\frac{3}{7}\left(512-128a+a^2\right)x^7-6(-48+5a)x^8\\ &\quad +\frac{32}{9}(-70+3a)x^9-\frac{12}{5}(-64+a)x^{10}\\ &\quad +\frac{3}{11}(-256+a)x^{11}+\frac{70x^{12}}{3}-\frac{72x^{13}}{13}+\frac{6x^{14}}{7}-\frac{x^{15}}{15}\end{aligned}$$

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + 6*a^2*x^4 - (24*(-8 + a)*a*x^5)/5 + (2*(128 - 96*a + 3*a^2)*x
^6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 - 6*(-48 + 5*a)*x^8 + (32*(-70 + 3*a)
*x^9)/9 - (12*(-64 + a)*x^10)/5 + (3*(-256 + a)*x^11)/11 + (70*x^12)/3 - (7
2*x^13)/13 + (6*x^14)/7 - x^15/15
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

method	result
norman	$\frac{a^3x^3}{3}+6a^2x^4+\left(-\frac{24}{5}a^2+\frac{192}{5}a\right)x^5+\left(2a^2-64a+\frac{256}{3}\right)x^6+\left(-\frac{3}{7}a^2+\frac{384}{7}a-\frac{1536}{7}\right)x^7+(-30$
gosper	$\frac{1}{3}a^3x^3+6a^2x^4-\frac{24}{5}a^2x^5+\frac{192}{5}a\,x^5+2a^2x^6-64a\,x^6+\frac{256}{3}x^6-\frac{3}{7}a^2x^7+\frac{384}{7}a\,x^7-\frac{1536}{7}x^7-30$
risch	$\frac{1}{3}a^3x^3+6a^2x^4-\frac{24}{5}a^2x^5+\frac{192}{5}a\,x^5+2a^2x^6-64a\,x^6+\frac{256}{3}x^6-\frac{3}{7}a^2x^7+\frac{384}{7}a\,x^7-\frac{1536}{7}x^7-30$
parallelrisch	$\frac{1}{3}a^3x^3+6a^2x^4-\frac{24}{5}a^2x^5+\frac{192}{5}a\,x^5+2a^2x^6-64a\,x^6+\frac{256}{3}x^6-\frac{3}{7}a^2x^7+\frac{384}{7}a\,x^7-\frac{1536}{7}x^7-30$
default	$-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}+\frac{(3a-768)x^{11}}{11}+\frac{(-24a+1536)x^{10}}{10}+\frac{(96a-2240)x^9}{9}+\frac{(-240a+2304)x^8}{8}+\frac{(a(-$

[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)

[Out] 1/3*a^3*x^3+6*a^2*x^4+(-24/5*a^2+192/5*a)*x^5+(2*a^2-64*a+256/3)*x^6+(-3/7*a^2+384/7*a-1536/7)*x^7+(-30*a+288)*x^8+(32/3*a-2240/9)*x^9+(-12/5*a+768/5)*x^10+(3/11*a-768/11)*x^11+70/3*x^12-72/13*x^13+6/7*x^14-1/15*x^15

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=-\frac{1}{15}x^{15}+\frac{6}{7}x^{14}-\frac{72}{13}x^{13}+\frac{3}{11}(a-256)x^{11}+\frac{70}{3}x^{12}-\frac{12}{5}(a-64)x^{10}+\frac{32}{9}(3\,a-70)x^9-6\,(5\,a-48)x^8-\frac{3}{7}\left(a^2-128\,a+512\right)x^7+\frac{2}{3}\left(3\,a^2-96\,a+128\right)x^6+\frac{1}{3}a^3x^3+6\,a^2x^4-\frac{24}{5}\left(a^2-8\,a\right)x^5$$

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")

[Out] -1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*(a - 256)*x^11 + 70/3*x^12 - 12/5*(a - 64)*x^10 + 32/9*(3*a - 70)*x^9 - 6*(5*a - 48)*x^8 - 3/7*(a^2 - 128*a + 512)*x^7 + 2/3*(3*a^2 - 96*a + 128)*x^6 + 1/3*a^3*x^3 + 6*a^2*x^4 - 24/5*(a^2 - 8*a)*x^5

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.97

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=\frac{a^3x^3}{3}+6a^2x^4-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}+x^{11}\cdot\left(\frac{3a}{11}-\frac{768}{11}\right)+x^{10}\cdot\left(\frac{768}{5}-\frac{12a}{5}\right)+x^9\cdot\left(\frac{32a}{3}-\frac{2240}{9}\right)+x^8\cdot(288-30a)+x^7\left(-\frac{3a^2}{7}+\frac{384a}{7}-\frac{1536}{7}\right)+x^6\cdot\left(2a^2-64a+\frac{256}{3}\right)+x^5\left(-\frac{24a^2}{5}+\frac{192a}{5}\right)$$

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x**3/3 + 6*a**2*x**4 - x**15/15 + 6*x**14/7 - 72*x**13/13 + 70*x**12/3
 + x**11*(3*a/11 - 768/11) + x**10*(768/5 - 12*a/5) + x**9*(32*a/3 - 2240/9
) + x**8*(288 - 30*a) + x**7*(-3*a**2/7 + 384*a/7 - 1536/7) + x**6*(2*a**2
- 64*a + 256/3) + x**5*(-24*a**2/5 + 192*a/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=-\frac{1}{15}\,x^{15}+\frac{6}{7}\,x^{14}-\frac{72}{13}\,x^{13}+\frac{3}{11}\,(a-256)x^{11}+\frac{70}{3}\,x^{12}-\frac{12}{5}\,(a-64)x^{10}+\frac{32}{9}\,(3\,a-70)x^9-6\,(5\,a-48)x^8-\frac{3}{7}\,\left(a^2-128\,a+512\right)x^7+\frac{2}{3}\,\left(3\,a^2-96\,a+128\right)x^6+\frac{1}{3}\,a^3x^3+6\,a^2x^4-\frac{24}{5}\,\left(a^2-8\,a\right)x^5$$

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*(a - 256)*x^11 + 70/3*x^12 - 12/5
*(a - 64)*x^10 + 32/9*(3*a - 70)*x^9 - 6*(5*a - 48)*x^8 - 3/7*(a^2 - 128*a
+ 512)*x^7 + 2/3*(3*a^2 - 96*a + 128)*x^6 + 1/3*a^3*x^3 + 6*a^2*x^4 - 24/5*
(a^2 - 8*a)*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int x^2(a+8x-8x^2+4x^3-x^4)^3\,dx = -\frac{1}{15}x^{15}+\frac{6}{7}x^{14}-\frac{72}{13}x^{13}+\frac{3}{11}ax^{11}+\frac{70}{3}x^{12} - \frac{12}{5}ax^{10}-\frac{768}{11}x^{11}+\frac{32}{3}ax^9+\frac{768}{5}x^{10} - \frac{3}{7}a^2x^7-30\,ax^8-\frac{2240}{9}x^9+2\,a^2x^6+\frac{384}{7}ax^7 + 288\,x^8-\frac{24}{5}a^2x^5-64\,ax^6-\frac{1536}{7}x^7 + \frac{1}{3}a^3x^3+6\,a^2x^4+\frac{192}{5}ax^5+\frac{256}{3}x^6$$

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] -1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*a*x^11 + 70/3*x^12 - 12/5*a*x^10
- 768/11*x^11 + 32/3*a*x^9 + 768/5*x^10 - 3/7*a^2*x^7 - 30*a*x^8 - 2240/9*x
^9 + 2*a^2*x^6 + 384/7*a*x^7 + 288*x^8 - 24/5*a^2*x^5 - 64*a*x^6 - 1536/7*x
^7 + 1/3*a^3*x^3 + 6*a^2*x^4 + 192/5*a*x^5 + 256/3*x^6
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int x^2(a+8x-8x^2+4x^3-x^4)^3\,dx = x^{11}\left(\frac{3\,a}{11}-\frac{768}{11}\right)-x^{10}\left(\frac{12\,a}{5}-\frac{768}{5}\right)-x^8\,(30\,a-288) + x^9\left(\frac{32\,a}{3}-\frac{2240}{9}\right)+x^6\left(2\,a^2-64\,a+\frac{256}{3}\right) - x^7\left(\frac{3\,a^2}{7}-\frac{384\,a}{7}+\frac{1536}{7}\right)+\frac{70\,x^{12}}{3}-\frac{72\,x^{13}}{13} + \frac{6\,x^{14}}{7}-\frac{x^{15}}{15}+6\,a^2\,x^4+\frac{a^3\,x^3}{3}-\frac{24\,a\,x^5\,(a-8)}{5}$$

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] x^11*((3*a)/11 - 768/11) - x^10*((12*a)/5 - 768/5) - x^8*(30*a - 288) + x^9
*((32*a)/3 - 2240/9) + x^6*(2*a^2 - 64*a + 256/3) - x^7*((3*a^2)/7 - (384*a
)/7 + 1536/7) + (70*x^12)/3 - (72*x^13)/13 + (6*x^14)/7 - x^15/15 + 6*a^2*x
^4 + (a^3*x^3)/3 - (24*a*x^5*(a - 8))/5
```

3.132 $\int x^2(a+8x-8x^2+4x^3-x^4)^2\,dx$

Optimal result 1007
Rubi [A] (verified) 1007
Mathematica [A] (verified) 1008
Maple [A] (verified) 1008
Fricas [A] (verification not implemented) 1009
Sympy [A] (verification not implemented) 1009
Maxima [A] (verification not implemented) 1009
Giac [A] (verification not implemented) 1010
Mupad [B] (verification not implemented) 1010

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{a^2x^3}{3}+4ax^4+\frac{16}{5}(4-a)x^5-\frac{4}{3}(16-a)x^6+\frac{2}{7}(64-a)x^7-10x^8+\frac{32x^9}{9}-\frac{4x^{10}}{5}+\frac{x^{11}}{11}$$

```
[Out] 1/3*a^2*x^3+4*a*x^4+16/5*(4-a)*x^5-4/3*(16-a)*x^6+2/7*(64-a)*x^7-10*x^8+32/
9*x^9-4/5*x^10+1/11*x^11
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {6874}

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{a^2x^3}{3}+\frac{2}{7}(64-a)x^7-\frac{4}{3}(16-a)x^6+\frac{16}{5}(4-a)x^5+4ax^4+\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^9}{9}-10x^8$$

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + 4*a*x^4 + (16*(4 - a)*x^5)/5 - (4*(16 - a)*x^6)/3 + (2*(64 -
a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(a^2x^2 + 16ax^3 - 16(-4+a)x^4 + 8(-16+a)x^5 - 2(-64+a)x^6 - 80x^7 + 32x^8 - 8x^9 + x^{10}\right) dx$$
$$= \frac{a^2x^3}{3} + 4ax^4 + \frac{16}{5}(4-a)x^5 - \frac{4}{3}(16-a)x^6 + \frac{2}{7}(64-a)x^7 - 10x^8 + \frac{32x^9}{9} - \frac{4x^{10}}{5} + \frac{x^{11}}{11}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^2 dx = \frac{a^2x^3}{3} + 4ax^4 - \frac{16}{5}(-4+a)x^5 + \frac{4}{3}(-16+a)x^6 - \frac{2}{7}(-64+a)x^7 - 10x^8 + \frac{32x^9}{9} - \frac{4x^{10}}{5} + \frac{x^{11}}{11}$$

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + 4*a*x^4 - (16*(-4 + a)*x^5)/5 + (4*(-16 + a)*x^6)/3 - (2*(-64
 + a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

method	result
norman	$\frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8 + \left(-\frac{2a}{7} + \frac{128}{7}\right)x^7 + \left(\frac{4a}{3} - \frac{64}{3}\right)x^6 + \left(-\frac{16a}{5} + \frac{64}{5}\right)x^5 + 4a\,x^4 + \frac{a^2x^3}{3}$
default	$\frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8 + \frac{(-2a+128)x^7}{7} + \frac{(8a-128)x^6}{6} + \frac{(-16a+64)x^5}{5} + 4a\,x^4 + \frac{a^2x^3}{3}$
gosper	$\frac{1}{11}x^{11} - \frac{4}{5}x^{10} + \frac{32}{9}x^9 - 10x^8 - \frac{2}{7}a\,x^7 + \frac{128}{7}x^7 + \frac{4}{3}a\,x^6 - \frac{64}{3}x^6 - \frac{16}{5}a\,x^5 + \frac{64}{5}x^5 + 4a\,x^4 + \frac{1}{3}a^2x^3$
risch	$\frac{1}{11}x^{11} - \frac{4}{5}x^{10} + \frac{32}{9}x^9 - 10x^8 - \frac{2}{7}a\,x^7 + \frac{128}{7}x^7 + \frac{4}{3}a\,x^6 - \frac{64}{3}x^6 - \frac{16}{5}a\,x^5 + \frac{64}{5}x^5 + 4a\,x^4 + \frac{1}{3}a^2x^3$
parallelrisch	$\frac{1}{11}x^{11} - \frac{4}{5}x^{10} + \frac{32}{9}x^9 - 10x^8 - \frac{2}{7}a\,x^7 + \frac{128}{7}x^7 + \frac{4}{3}a\,x^6 - \frac{64}{3}x^6 - \frac{16}{5}a\,x^5 + \frac{64}{5}x^5 + 4a\,x^4 + \frac{1}{3}a^2x^3$

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*x^11-4/5*x^10+32/9*x^9-10*x^8+(-2/7*a+128/7)*x^7+(4/3*a-64/3)*x^6+(-16
/5*a+64/5)*x^5+4*a*x^4+1/3*a^2*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int x^2(a+8x-8x^2+4x^3-x^4)^2\,dx = \frac{1}{11}\,x^{11} - \frac{4}{5}\,x^{10} + \frac{32}{9}\,x^9 - \frac{2}{7}\,(a-64)x^7 - 10\,x^8 + \frac{4}{3}\,(a-16)x^6 - \frac{16}{5}\,(a-4)x^5 + \frac{1}{3}\,a^2x^3 + 4\,ax^4$$

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*(a - 64)*x^7 - 10*x^8 + 4/3*(a - 16)*
x^6 - 16/5*(a - 4)*x^5 + 1/3*a^2*x^3 + 4*a*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int x^2(a+8x-8x^2+4x^3-x^4)^2\,dx = \frac{a^2x^3}{3} + 4ax^4 + \frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8 + x^7 \cdot \left(\frac{128}{7} - \frac{2a}{7}\right) + x^6 \cdot \left(\frac{4a}{3} - \frac{64}{3}\right) + x^5 \cdot \left(\frac{64}{5} - \frac{16a}{5}\right)$$

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
[Out] a**2*x**3/3 + 4*a*x**4 + x**11/11 - 4*x**10/5 + 32*x**9/9 - 10*x**8 + x**7*
(128/7 - 2*a/7) + x**6*(4*a/3 - 64/3) + x**5*(64/5 - 16*a/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int x^2(a+8x-8x^2+4x^3-x^4)^2\,dx = \frac{1}{11}\,x^{11} - \frac{4}{5}\,x^{10} + \frac{32}{9}\,x^9 - \frac{2}{7}\,(a-64)x^7 - 10\,x^8 + \frac{4}{3}\,(a-16)x^6 - \frac{16}{5}\,(a-4)x^5 + \frac{1}{3}\,a^2x^3 + 4\,ax^4$$

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*(a - 64)*x^7 - 10*x^8 + 4/3*(a - 16)*
x^6 - 16/5*(a - 4)*x^5 + 1/3*a^2*x^3 + 4*a*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{1}{11}\,x^{11}-\frac{4}{5}\,x^{10}+\frac{32}{9}\,x^9-\frac{2}{7}\,ax^7-10\,x^8+\frac{4}{3}\,ax^6+\frac{128}{7}\,x^7-\frac{16}{5}\,ax^5-\frac{64}{3}\,x^6+\frac{1}{3}\,a^2x^3+4\,ax^4+\frac{64}{5}\,x^5$$

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*a*x^7 - 10*x^8 + 4/3*a*x^6 + 128/7*x^
7 - 16/5*a*x^5 - 64/3*x^6 + 1/3*a^2*x^3 + 4*a*x^4 + 64/5*x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=x^6\left(\frac{4\,a}{3}-\frac{64}{3}\right)-x^5\left(\frac{16\,a}{5}-\frac{64}{5}\right)-x^7\left(\frac{2\,a}{7}-\frac{128}{7}\right)+4\,a\,x^4-10\,x^8+\frac{32\,x^9}{9}-\frac{4\,x^{10}}{5}+\frac{x^{11}}{11}+\frac{a^2\,x^3}{3}$$

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] x^6*((4*a)/3 - 64/3) - x^5*((16*a)/5 - 64/5) - x^7*((2*a)/7 - 128/7) + 4*a*
x^4 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11 + (a^2*x^3)/3
```

3.133 $\int x^2(a+8x-8x^2+4x^3-x^4)\,dx$

Optimal result 1011
Rubi [A] (verified) 1011
Mathematica [A] (verified) 1012
Maple [A] (verified) 1012
Fricas [A] (verification not implemented) 1012
Sympy [A] (verification not implemented) 1013
Maxima [A] (verification not implemented) 1013
Giac [A] (verification not implemented) 1013
Mupad [B] (verification not implemented) 1013

Optimal result

Integrand size = 24, antiderivative size = 35

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)\,dx=\frac{ax^3}{3}+2x^4-\frac{8x^5}{5}+\frac{2x^6}{3}-\frac{x^7}{7}$$

```
[Out] 1/3*a*x^3+2*x^4-8/5*x^5+2/3*x^6-1/7*x^7
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {14}

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)\,dx=\frac{ax^3}{3}-\frac{x^7}{7}+\frac{2x^6}{3}-\frac{8x^5}{5}+2x^4$$

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(ax^2+8x^3-8x^4+4x^5-x^6\right)\,dx\\&=\frac{ax^3}{3}+2x^4-\frac{8x^5}{5}+\frac{2x^6}{3}-\frac{x^7}{7}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)\,dx=\frac{ax^3}{3}+2x^4-\frac{8x^5}{5}+\frac{2x^6}{3}-\frac{x^7}{7}$$

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28
default	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28
norman	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28
risch	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28
parallelrisch	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
[Out] 1/3*a*x^3+2*x^4-8/5*x^5+2/3*x^6-1/7*x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)\,dx=-\frac{1}{7}\,x^7+\frac{2}{3}\,x^6-\frac{8}{5}\,x^5+\frac{1}{3}\,ax^3+2\,x^4$$

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x^2(a+8x-8x^2+4x^3-x^4)\,dx=\frac{ax^3}{3}-\frac{x^7}{7}+\frac{2x^6}{3}-\frac{8x^5}{5}+2x^4$$

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x),x)
[Out] a*x**3/3 - x**7/7 + 2*x**6/3 - 8*x**5/5 + 2*x**4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x^2(a+8x-8x^2+4x^3-x^4)\,dx=-\frac{1}{7}\,x^7+\frac{2}{3}\,x^6-\frac{8}{5}\,x^5+\frac{1}{3}\,ax^3+2\,x^4$$

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x^2(a+8x-8x^2+4x^3-x^4)\,dx=-\frac{1}{7}\,x^7+\frac{2}{3}\,x^6-\frac{8}{5}\,x^5+\frac{1}{3}\,ax^3+2\,x^4$$

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x^2(a+8x-8x^2+4x^3-x^4)\,dx=-\frac{x^7}{7}+\frac{2\,x^6}{3}-\frac{8\,x^5}{5}+2\,x^4+\frac{a\,x^3}{3}$$

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

3.134 $\int \frac{x^2}{a+8x-8x^2+4x^3-x^4} \, dx$

Optimal result 1014
Rubi [A] (verified) 1014
Mathematica [C] (verified) 1016
Maple [C] (verified) 1017
Fricas [C] (verification not implemented) 1017
Sympy [B] (verification not implemented) 1018
Maxima [F] 1018
Giac [F] 1018
Mupad [B] (verification not implemented) 1019

Optimal result

Integrand size = 26, antiderivative size = 99

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4} \, dx = -\frac{\arctan\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}} - \frac{\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}} + \frac{\operatorname{arctanh}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{\sqrt{4+a}}$$

```
[Out] arctanh((1+(-1+x)^2)/(4+a)^(1/2))/(4+a)^(1/2)-1/2*arctan((-1+x)/(1-(4+a)^(1
/2))^(1/2))/(1-(4+a)^(1/2))^(1/2)-1/2*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2))/
(1+(4+a)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1694, 1687, 1180, 210, 12, 1121, 632, 212}

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4} \, dx = -\frac{\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}} - \frac{\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}} + \frac{\operatorname{arctanh}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

```
[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -1/2*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/Sqrt[1 - Sqrt[4 + a]] - ArcTan[
(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh[(1 + (-
1 + x)^2)/Sqrt[4 + a]]/Sqrt[4 + a]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
```

```
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] && !IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(1+x)^2}{3+a-2x^2-x^4}\,dx, x, -1+x\right)\\
&= \text{Subst}\left(\int \frac{2x}{3+a-2x^2-x^4}\,dx, x, -1+x\right)+\text{Subst}\left(\int \frac{1+x^2}{3+a-2x^2-x^4}\,dx, x, -1+x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\,dx, x, -1+x\right)\\
&\quad+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+\sqrt{4+a}-x^2}\,dx, x, -1+x\right)\\
&\quad+2\text{Subst}\left(\int \frac{x}{3+a-2x^2-x^4}\,dx, x, -1+x\right)\\
&= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}}+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}}+\text{Subst}\left(\int \frac{1}{3+a-2x-x^2}\,dx, x, (-1+x)^2\right)\\
&= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}}+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}}\\
&\quad-2\text{Subst}\left(\int \frac{1}{4(4+a)-x^2}\,dx, x, -2\left(1+(-1+x)^2\right)\right)\\
&= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}}+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}}+\frac{\tanh^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{\sqrt{4+a}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.62

$$
\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx = -\frac{1}{4}\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&, \frac{\log(x-\#1)\#1^2}{-2+4\#1-3\#1^2+\#1^3}\&\right]
$$

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (Log[x - #1]*#1^2)/(-2 +
 4*#1 - 3*#1^2 + #1^3) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}\right)}{4}$	54
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}\right)}{4}$	54

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R^2/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*
_Z-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.66 (sec) , antiderivative size = 1515766, normalized size of antiderivative = 15310.77

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(82) = 164.

Time = 4.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.74

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx = -\operatorname{RootSum}\left(t^4\cdot\left(256a^3+2816a^2+10240a+12288\right)+t^2\left(-160a^2-1152a-2048\right)+t\left(-32a^2-256a-\right.\right.$$

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x),x)
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-160*a**2
- 1152*a - 2048) + _t*(-32*a**2 - 256*a - 512) - a**2, Lambda(_t, _t*log(x
+ (-64*_t**3*a**4 - 448*_t**3*a**3 - 256*_t**3*a**2 + 3584*_t**3*a + 6144*_
t**3 - 224*_t**2*a**3 - 2208*_t**2*a**2 - 7168*_t**2*a - 7680*_t**2 + 56*_t
*a**3 + 400*_t*a**2 + 864*_t*a + 512*_t + 5*a**3 + 34*a**2 + 56*a)/(a**3 +
60*a**2 + 320*a + 448)))
```

Maxima [F]

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx = \int -\frac{x^2}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx$$

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
[Out] -integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Giac [F]

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx = \int -\frac{x^2}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx$$

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
[Out] integrate(-x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 878, normalized size of antiderivative = 8.87

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx = \sum_{k=1}^{4} \ln\Big(64\,\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4 + 12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256\,a\,z+512\,z-a^2,z,k\big)-a - 8\,x+\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2 - 2048\,z^2+32\,a^2\,z+256\,a\,z+512\,z-a^2,z,k\big)\ a\,20$$
$$-\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256\,a$$
$$+\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256\,a$$
$$+\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256\,a$$
$$-\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256\,a$$
$$-192\,\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+2$$
$$+256\,\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+2$$
$$-\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2 + 32\,a^2\,z+256\,a\,z+512\,z-a^2,z,k\big)\ a\,x\,4$$
$$+\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256\,a$$
$$-\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256\,a$$
$$+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256\,a\,z + 512\,z-a^2,z,k\big)$$

```
[In] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

```
[Out] symsum(log(64*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 1
60*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k
) - a - 8*x + 20*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4
- 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z
, k)*a - 48*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160
*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^
2*a + 64*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^
2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3*a
 + 128*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*
z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2*x -
 256*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^
2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3*x - 1
92*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2
- 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2 + 256*r
oot(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 11
52*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3 - 4*root(28
```

```
16*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z
^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)*a*x + 32*root(2816*
a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2
- 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2*a*x - 64*root(2816*a
^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 -
 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3*a*x)*root(2816*a^2*z^
4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048
*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k), k, 1, 4)
```

3.135 $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

Optimal result 1021
Rubi [A] (verified) 1022
Mathematica [C] (verified) 1025
Maple [C] (verified) 1026
Fricas [F(-1)] 1026
Sympy [B] (verification not implemented) 1026
Maxima [F] 1027
Giac [F] 1027
Mupad [B] (verification not implemented) 1028

Optimal result

Integrand size = 26, antiderivative size = 225

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \frac{1+(-1+x)^2}{2(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} - \frac{\left(4+a+\sqrt{4+a}\right)\arctan\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{8(3+a)(4+a)\sqrt{1-\sqrt{4+a}}} - \frac{\left(4+a-\sqrt{4+a}\right)\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{8(3+a)(4+a)\sqrt{1+\sqrt{4+a}}} + \frac{\operatorname{arctanh}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{2(4+a)^{3/2}}$$

```
[Out] 1/2*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/4*(4+a)*(2+(-1+x)^2)*(-1
+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/2*arctanh((1+(-1+x)^2)/(4+a)^(
1/2))/(4+a)^(3/2)-1/8*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(4+a+(4+a)^(1/2)
)/(3+a)/(4+a)/(1-(4+a)^(1/2))^(1/2)-1/8*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2)
)*(4+a-(4+a)^(1/2))/(3+a)/(4+a)/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1694, 1687, 1192, 1180, 210, 12, 1121, 628, 632, 212}

$$\begin{aligned}\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx &= \frac{(a+4)\left((x-1)^2+2\right)(x-1)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)} \\ &\quad - \frac{\left(a+\sqrt{a+4}+4\right)\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)\sqrt{1-\sqrt{a+4}}} \\ &\quad - \frac{\left(a-\sqrt{a+4}+4\right)\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)\sqrt{\sqrt{a+4}+1}} \\ &\quad + \frac{\operatorname{arctanh}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2(a+4)^{3/2}} \\ &\quad + \frac{(x-1)^2+1}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}\end{aligned}$$

```
[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (1 + (-1 + x)^2)/(2*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((4 + a)
*(2 + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1
 + x)^4)) - ((4 + a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/
(8*(3 + a)*(4 + a)*Sqrt[1 - Sqrt[4 + a]]) - ((4 + a - Sqrt[4 + a])*ArcTan[(
-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)*Sqrt[1 + Sqrt[4 + a]]) +
 ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(2*(4 + a)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
```

```
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(1+x)^2}{(3+a-2x^2-x^4)^2}\,dx, x, -1+x\right)\\
&= \text{Subst}\left(\int \frac{2x}{(3+a-2x^2-x^4)^2}\,dx, x, -1+x\right)\\
&\quad + \text{Subst}\left(\int \frac{1+x^2}{(3+a-2x^2-x^4)^2}\,dx, x, -1+x\right)\\
&= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)}\\
&\quad + 2\text{Subst}\left(\int \frac{x}{(3+a-2x^2-x^4)^2}\,dx, x, -1+x\right)\\
&\quad - \frac{\text{Subst}\left(\int \frac{-4(4+a)-2(4+a)x^2}{3+a-2x^2-x^4}\,dx, x, -1+x\right)}{8\left(12+7a+a^2\right)}\\
&= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)}\\
&\quad + \frac{\left(4+a-\sqrt{4+a}\right)\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{8\left(12+7a+a^2\right)}\\
&\quad + \frac{\left(4+a+\sqrt{4+a}\right)\text{Subst}\left(\int \frac{1}{-1+\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{8\left(12+7a+a^2\right)}\\
&\quad + \text{Subst}\left(\int \frac{1}{(3+a-2x-x^2)^2}\,dx, x, (-1+x)^2\right)\\
&= \frac{1+(-1+x)^2}{2(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)}\\
&\quad + \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)}\\
&\quad + \frac{\left(4+a+\sqrt{4+a}\right)\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{8\left(12+7a+a^2\right)\sqrt{1-\sqrt{4+a}}} + \frac{\left(4+a-\sqrt{4+a}\right)\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{8\left(12+7a+a^2\right)\sqrt{1+\sqrt{4+a}}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{3+a-2x-x^2}\,dx, x, (-1+x)^2\right)}{2(4+a)}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1+(-1+x)^2}{2(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} \\
&\quad + \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} \\
&\quad + \frac{\left(4+a+\sqrt{4+a}\right)\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{8\left(12+7a+a^2\right)\sqrt{1-\sqrt{4+a}}} + \frac{\left(4+a-\sqrt{4+a}\right)\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{8\left(12+7a+a^2\right)\sqrt{1+\sqrt{4+a}}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{4(4+a)-x^2}\,dx, x, -2\left(1+(-1+x)^2\right)\right)}{4+a} \\
&= \frac{1+(-1+x)^2}{2(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} \\
&\quad + \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} \\
&\quad + \frac{\left(4+a+\sqrt{4+a}\right)\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{8\left(12+7a+a^2\right)\sqrt{1-\sqrt{4+a}}} \\
&\quad + \frac{\left(4+a-\sqrt{4+a}\right)\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{8\left(12+7a+a^2\right)\sqrt{1+\sqrt{4+a}}} + \frac{\tanh^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{2(4+a)^{3/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.81

$$
\begin{aligned}
\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx &= \frac{2x(4-3x+2x^2)+a(1+x-x^2+x^3)}{4(3+a)(4+a)\left(a-x\left(-8+8x-4x^2+x^3\right)\right)} \\
&\quad - \frac{\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&, \frac{-a\log(x-\#1)+4\log(x-\#1)\#1+2a\log(x-\#1)\#1+4\log(x-\#1)\#1^2}{-2+4\#1-3\#1^2+\#1^3}\right.}{16\left(12+7a+a^2\right)}
\end{aligned}
$$

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (2*x*(4 - 3*x + 2*x^2) + a*(1 + x - x^2 + x^3))/(4*(3 + a)*(4 + a)*(a - x*(
-8 + 8*x - 4*x^2 + x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (
-(a*Log[x - #1]) + 4*Log[x - #1]*#1 + 2*a*Log[x - #1]*#1 + 4*Log[x - #1]*#1
^2 + a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ]/(16*(12 + 7*a + a^
2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.71

method	result
default	$\frac{\frac{x^3}{4a+12}-\frac{(6+a)x^2}{4(3+a)(4+a)}+\frac{(a+8)x}{4(3+a)(4+a)}+\frac{a}{4(4+a)(3+a)}}{-x^4+4x^3-8x^2+a+8x}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a)}\frac{(\_R^2(4+a)+2(a+2)\_R-a)\ln(x-\_}{-\_R^3+3\_R^2-4\_R+2}}{16(4+a)(3+a)}$
risch	$\frac{\frac{x^3}{4a+12}-\frac{(6+a)x^2}{4(3+a)(4+a)}+\frac{(a+8)x}{4(3+a)(4+a)}+\frac{a}{4(4+a)(3+a)}}{-x^4+4x^3-8x^2+a+8x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a)}\frac{\left(\frac{\_R^2}{3+a}+\frac{2(a+2)\_R}{(3+a)(4+a)}-\frac{a}{(4+a)(3+a)}\right)}{-\_R^3+3\_R^2-4\_R+2}\right.}{16}$

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4/(3+a)*x^3-1/4*(6+a)/(3+a)/(4+a)*x^2+1/4*(a+8)/(3+a)/(4+a)*x+1/4*a/(4+a
)/(3+a))/(-x^4+4*x^3-8*x^2+a+8*x)+1/16/(4+a)/(3+a)*sum((_R^2*(4+a)+2*(a+2)*
_R-a)/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx=\text{Timed out}$$

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(185) = 370.

Time = 18.36 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.49

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$$
$$=\frac{-a+x^3(-a-4)+x^2(a+6)+x(-a-8)}{-4a^3-28a^2-48a+x^4\cdot(4a^2+28a+48)+x^3(-16a^2-112a-192)+x^2\cdot(32a^2+224a+384)+x(-}$$
$$+\mathrm{RootSum}\Big(t^4\cdot(65536a^9+2162688a^8+31653888a^7+269680640a^6+1473773568a^5+5357174784a^4+$$

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] (-a + x**3*(-a - 4) + x**2*(a + 6) + x*(-a - 8))/(-4*a**3 - 28*a**2 - 48*a
+ x**4*(4*a**2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2
 + 224*a + 384) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 +
 2162688*a**8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 53571747
84*a**4 + 12952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312)
 + _t**2*(-9728*a**6 - 209408*a**5 - 1878016*a**4 - 8986624*a**3 - 24215552
*a**2 - 34865152*a - 20971520) + _t*(256*a**5 + 5888*a**4 + 53248*a**3 + 23
7568*a**2 + 524288*a + 458752) - a**4 + 144*a**3 + 1024*a**2 + 1792*a, Lamb
da(_t, _t*log(x + (4096*_t**3*a**12 - 61440*_t**3*a**11 - 5480448*_t**3*a**
10 - 111403008*_t**3*a**9 - 1227173888*_t**3*a**8 - 8682876928*_t**3*a**7 -
 42187440128*_t**3*a**6 - 144630284288*_t**3*a**5 - 350972280832*_t**3*a**4
 - 591750234112*_t**3*a**3 - 660716126208*_t**3*a**2 - 439848271872*_t**3*a
 - 132271570944*_t**3 - 28672*_t**2*a**10 - 993280*_t**2*a**9 - 15400960*_t
**2*a**8 - 140742656*_t**2*a**7 - 839462912*_t**2*a**6 - 3414427648*_t**2*a
**5 - 9590087680*_t**2*a**4 - 18363547648*_t**2*a**3 - 22938255360*_t**2*a*
*2 - 16873684992*_t**2*a - 5549064192*_t**2 - 848*_t*a**9 - 6096*_t*a**8 +
174608*_t*a**7 + 3323792*_t*a**6 + 26276224*_t*a**5 + 119009280*_t*a**4 + 3
32017664*_t*a**3 + 566497280*_t*a**2 + 544112640*_t*a + 225837056*_t + 11*a
**8 + 958*a**7 + 17419*a**6 + 142964*a**5 + 632632*a**4 + 1567552*a**3 + 20
49792*a**2 + 1100800*a)/(a**8 + 870*a**7 + 18289*a**6 + 165176*a**5 + 82456
0*a**4 + 2452288*a**3 + 4340224*a**2 + 4229120*a + 1748992))))
```

Maxima [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \int \frac{x^2}{(x^4-4\,x^3+8\,x^2-a-8\,x)^2}\,dx$$

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((a + 4)*x^3 - (a + 6)*x^2 + (a + 8)*x + a)/((a^2 + 7*a + 12)*x^4 - 4*
(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a
+ 12)*x - 12*a) - 1/4*integrate(((a + 4)*x^2 + 2*(a + 2)*x - a)/(x^4 - 4*x^
3 + 8*x^2 - a - 8*x), x)/(a^2 + 7*a + 12)
```

Giac [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \int \frac{x^2}{(x^4-4\,x^3+8\,x^2-a-8\,x)^2}\,dx$$

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 1218, normalized size of antiderivative = 5.41

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Too large to display}$$

```
[In] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] symsum(log((x*(40*a + 7*a^2 + 56))/(8*(816*a + 460*a^2 + 129*a^3 + 18*a^4 +
 a^5 + 576)) - (48*a + 12*a^2 - a^3)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^
4 + a^5 + 576)) - root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8
*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568
*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 242155
52*a^2*z^2 - 8986624*a^3*z^2 - 1878016*a^4*z^2 - 209408*a^5*z^2 - 9728*a^6*
z^2 - 34865152*a*z^2 - 20971520*z^2 + 237568*a^2*z + 53248*a^3*z + 5888*a^4
*z + 256*a^5*z + 524288*a*z + 458752*z + 1792*a + 1024*a^2 + 144*a^3 - a^4,
 z, k)*((28160*a + 11328*a^2 + 2064*a^3 + 144*a^4 + 26624)/(64*(816*a + 460
*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + root(12952010752*a^3*z^4 + 31653888
*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 2008232755
2*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7
247757312*z^4 - 24215552*a^2*z^2 - 8986624*a^3*z^2 - 1878016*a^4*z^2 - 2094
08*a^5*z^2 - 9728*a^6*z^2 - 34865152*a*z^2 - 20971520*z^2 + 237568*a^2*z +
53248*a^3*z + 5888*a^4*z + 256*a^5*z + 524288*a*z + 458752*z + 1792*a + 102
4*a^2 + 144*a^3 - a^4, z, k)*(root(12952010752*a^3*z^4 + 31653888*a^7*z^4 +
 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4
+ 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*
z^4 - 24215552*a^2*z^2 - 8986624*a^3*z^2 - 1878016*a^4*z^2 - 209408*a^5*z^2
 - 9728*a^6*z^2 - 34865152*a*z^2 - 20971520*z^2 + 237568*a^2*z + 53248*a^3*
z + 5888*a^4*z + 256*a^5*z + 524288*a*z + 458752*z + 1792*a + 1024*a^2 + 14
4*a^3 - a^4, z, k)*((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 +
 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 +
a^5 + 576)) - (x*(1966080*a + 1359872*a^2 + 499712*a^3 + 102912*a^4 + 11264
*a^5 + 512*a^6 + 1179648))/(8*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 5
76))) - (1359872*a + 749568*a^2 + 205824*a^3 + 28160*a^4 + 1536*a^5 + 98304
0)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + (x*(104448*a + 5
8880*a^2 + 16512*a^3 + 2304*a^4 + 128*a^5 + 73728))/(8*(816*a + 460*a^2 + 1
29*a^3 + 18*a^4 + a^5 + 576))) + (x*(448*a + 104*a^2 - 2*a^3 - 2*a^4 + 512)
)/(8*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))))*root(12952010752*a
^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a
*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 2696
80640*a^6*z^4 + 7247757312*z^4 - 24215552*a^2*z^2 - 8986624*a^3*z^2 - 18780
16*a^4*z^2 - 209408*a^5*z^2 - 9728*a^6*z^2 - 34865152*a*z^2 - 20971520*z^2
+ 237568*a^2*z + 53248*a^3*z + 5888*a^4*z + 256*a^5*z + 524288*a*z + 458752
*z + 1792*a + 1024*a^2 + 144*a^3 - a^4, z, k), k, 1, 4) + (x^3/(4*(a + 3))
```

```
+ a/(4*(a + 3)*(a + 4)) - (x^2*(a + 6))/(4*(a + 3)*(a + 4)) + (x*(a + 8))/(
4*(a + 3)*(a + 4)))/(a + 8*x - 8*x^2 + 4*x^3 - x^4)
```

3.136 $\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal result 1030
Rubi [A] (verified) 1031
Mathematica [C] (verified) 1034
Maple [C] (verified) 1034
Fricas [F(-2)] 1035
Sympy [F(-1)] 1035
Maxima [F] 1035
Giac [F] 1036
Mupad [B] (verification not implemented) 1036

Optimal result

Integrand size = 46, antiderivative size = 545

$$\begin{aligned}
&\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx \\
&= -\frac{\sqrt[3]{-1}\left(2\sqrt[3]{-1}b+3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{5/6}b^2\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}c^{2/3}} \\
&\quad-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}a^{5/6}b^2\sqrt{4b-3\sqrt[3]{a}c^{2/3}}c^{2/3}} \\
&\quad-\frac{(-1)^{2/3}\left(2b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{5/6}b^2\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}c^{2/3}} \\
&\quad-\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}+\frac{\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{6\left(1+\sqrt[3]{-1}\right)^2a^{2/3}b^2\sqrt[3]{c}} \\
&\quad+\frac{\sqrt[3]{-1}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}
\end{aligned}$$

```
[Out] -1/18*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(2/3)/b^2/c^(1/3)+1/6*ln(3*a-3*(-
1)^(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(2/3)/b^2/c^(1/3)+1/18
*(-1)^(1/3)*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(2/3)/b^2/c^(1/3
)-1/27*(2*b-3*a^(1/3)*c^(2/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)
/a^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2))/a^(5/6)/b^2/c^(2/3)*3^(1/2)/(4*b-3*
a^(1/3)*c^(2/3))^(1/2)-1/9*(-1)^(2/3)*(2*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))*ar
ctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3*(-1)^(
1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(5/6)/b^2/c^
```

```
(2/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)-1/9*(-1)^(1/3)*(2*(-
1)^(1/3)*b+3*a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*
x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))
^2/a^(5/6)/b^2/c^(2/3)*3^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {2122, 648, 632, 210, 642}

$$\begin{aligned}
&\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx \\
&= -\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}c^{2/3}+2\sqrt[3]{-1}b\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{5/6}b^2c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}} \\
&\quad -\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}a^{5/6}b^2c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}} \\
&\quad -\frac{(-1)^{2/3}\left(3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+2b\right)\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{3\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{5/6}b^2c^{2/3}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}} \\
&\quad -\frac{\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}+\frac{\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{6\left(1+\sqrt[3]{-1}\right)^2a^{2/3}b^2\sqrt[3]{c}} \\
&\quad +\frac{\sqrt[3]{-1}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}
\end{aligned}$$

```
[In] Int[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] -1/3*((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*
a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c
^(2/3)])])/(Sqrt[3]*(1 + (-1)^(1/3))^2*a^(5/6)*b^2*Sqrt[4*b - 3*(-1)^(2/3)*
a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^
(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]
*a^(5/6)*b^2*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) - ((-1)^(2/3)*(2*b + 3*
(-1)^(1/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(
Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(3*Sqrt[3]*(1 -
 (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(5/6)*b^2*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)
*c^(2/3)]*c^(2/3)) - Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(18*a^(2/3)*b^2
*c^(1/3)) + Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(6*(1 + (-1)^
(1/3))^2*a^(2/3)*b^2*c^(1/3)) + ((-1)^(1/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*
c^(1/3)*x + b*x^2])/(18*a^(2/3)*b^2*c^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\text{integral} = \left(19683a^6\right)\int \left(\frac{-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{c}x}{59049\left(1+\sqrt[3]{-1}\right)^2 a^{20/3}bc^{2/3}\left(-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-bx^2\right)} + \frac{-\sqrt[3]{a}-\sqrt[3]{c}x}{177147a^{20/3}bc^{2/3}\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)} + \frac{(-1)^{2/3}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{c}x\right)}{59049\left(-1+\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2 a^{20/3}bc^{2/3}\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}\right)dx$$

$$=\frac{\int \frac{-\sqrt[3]{a}-\sqrt[3]{c}x}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{9a^{2/3}bc^{2/3}}-\frac{(-1)^{2/3}\int \frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{c}x}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{9a^{2/3}bc^{2/3}}+\frac{\int \frac{-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{c}x}{-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-bx^2}\,dx}{3\left(1+\sqrt[3]{-1}\right)^2 a^{2/3}bc^{2/3}}$$

$$=\frac{\left(3-\frac{2b}{\sqrt[3]{a}c^{2/3}}\right)\int \frac{1}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{18b^2}+\frac{\left(3-\frac{2(-1)^{2/3}b}{\sqrt[3]{a}c^{2/3}}\right)\int \frac{1}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{18b^2}$$
$$-\frac{\left(b+i\sqrt{3}b+3\sqrt[3]{a}c^{2/3}\right)\int \frac{1}{-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-bx^2}\,dx}{18\sqrt[3]{a}b^2c^{2/3}}-\frac{\int \frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{18a^{2/3}b^2\sqrt[3]{c}}$$
$$+\frac{\sqrt[3]{-1}\int \frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{18a^{2/3}b^2\sqrt[3]{c}}+\frac{\int \frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-bx^2}\,dx}{6\left(1+\sqrt[3]{-1}\right)^2 a^{2/3}b^2\sqrt[3]{c}}$$

$$=-\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}+\frac{\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{6\left(1+\sqrt[3]{-1}\right)^2 a^{2/3}b^2\sqrt[3]{c}}$$
$$+\frac{\sqrt[3]{-1}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}$$
$$-\frac{\left(3-\frac{2b}{\sqrt[3]{a}c^{2/3}}\right)\text{Subst}\left(\int \frac{1}{-3a\left(4b-3\sqrt[3]{a}c^{2/3}\right)-x^2}\,dx,x,3a^{2/3}\sqrt[3]{c}+2bx\right)}{9b^2}$$
$$-\frac{\left(3-\frac{2(-1)^{2/3}b}{\sqrt[3]{a}c^{2/3}}\right)\text{Subst}\left(\int \frac{1}{-3a\left(4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)-x^2}\,dx,x,3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx\right)}{9b^2}$$
$$+\frac{\left(b+i\sqrt{3}b+3\sqrt[3]{a}c^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3a\left(4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)-x^2}\,dx,x,3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx\right)}{9\sqrt[3]{a}b^2c^{2/3}}$$

$$=-\frac{\left(3ib+\sqrt{3}\left(b+3\sqrt[3]{a}c^{2/3}\right)\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{27a^{5/6}b^2\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}c^{2/3}}$$
$$+\frac{\left(3-\frac{2b}{\sqrt[3]{a}c^{2/3}}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\sqrt{a}b^2\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}$$
$$+\frac{\left(3-\frac{2(-1)^{2/3}b}{\sqrt[3]{a}c^{2/3}}\right)\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\sqrt{a}b^2\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}-\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}$$
$$+\frac{\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{6\left(1+\sqrt[3]{-1}\right)^2 a^{2/3}b^2\sqrt[3]{c}}+\frac{\sqrt[3]{-1}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.18

$$\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\&,\frac{\log(x-\#1)\#1^3}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\right]$$

```
[In] Integrate[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6
),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, (Log[x - #1]*#1^3)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4)
& ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.17

method	result	size
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27ca^2\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3ab^2+27\_R^2a^2c+18a^2b\_R}}{3}$	93
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27ca^2\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3ab^2+27\_R^2a^2c+18a^2b\_R}}{3}$	93

```
[In] int(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3*sum(_R^4/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),
_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \text{Timed out}$$

```
[In] integrate(x**4/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \int \frac{x^4}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

```
[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

Giac [F]

$$\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \int \frac{x^4}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx$$

```
[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 1563, normalized size of antiderivative = 2.87

$$\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \text{Too large to display}$$

```
[In] int(x^4/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(-19683*a^8*b^3*(c*x - b + 6561*root(918330048*a^5*b^9*c^4*z^6 -
387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z
^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*
a^3*c^4 + 2*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 15
94323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 3280
5*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)*b^4*x - 198*root(918330048*a^5*b
^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*
a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z
+ 1, z, k)*a*b^2*c - 8991*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^
6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*
c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*a^2*b^3*c^2 - 19
683*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^
4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^
2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^3*a^3*b^4*c^3 + 104976*root(918330048*a^
5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 10235
16*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c
*z + 1, z, k)^4*a^3*b^8*c^2 - 8503056*root(918330048*a^5*b^9*c^4*z^6 - 3874
20489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 -
 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^5*a^4*
b^9*c^3 + 4782969*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^
6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3
+ 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^5*a^5*b^6*c^5 + 108*root(9
```

```
18330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*
z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2
+ 324*a*b*c*z + 1, z, k)^2*a*b^5*c*x + 108*root(918330048*a^5*b^9*c^4*z^6 -
 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*
z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)*a
*b*c^2*x + 1458*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6
+ 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 +
32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*a^2*b^2*c^3*x - 2916*root(
918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4
*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2
 + 324*a*b*c*z + 1, z, k)^3*a^2*b^6*c^2*x + 78732*root(918330048*a^5*b^9*c^
4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b
^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1,
z, k)^4*a^3*b^7*c^3*x + 1062882*root(918330048*a^5*b^9*c^4*z^6 - 387420489*
a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 53144
1*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^5*a^4*b^8*c^
4*x))*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*
a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*
b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k), k, 1, 6)
```

3.137 $\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal result 1038
Rubi [A] (verified) 1039
Mathematica [C] (verified) 1042
Maple [C] (verified) 1042
Fricas [F(-2)] 1043
Sympy [F(-1)] 1043
Maxima [F] 1043
Giac [F] 1044
Mupad [B] (verification not implemented) 1044

Optimal result

Integrand size = 46, antiderivative size = 487

$$\begin{aligned}
&\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx \\
&= -\frac{\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{7/6}b\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}}-\frac{\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}a^{7/6}b\sqrt{4b-3\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} \\
&\quad+\frac{\sqrt[3]{-1}\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{7/6}b\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} \\
&\quad+\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54a^{4/3}bc^{2/3}}-\frac{(-1)^{2/3}\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{18\left(1+\sqrt[3]{-1}\right)^2a^{4/3}bc^{2/3}} \\
&\quad+\frac{(-1)^{2/3}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54a^{4/3}bc^{2/3}}
\end{aligned}$$

```
[Out] 1/54*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(4/3)/b/c^(2/3)-1/18*(-1)^(2/3)*ln
(3*a-3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(4/3)/b/c^(2/
3)+1/54*(-1)^(2/3)*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(4/3)/b/c
^(2/3)-1/27*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(
1/3)*c^(2/3))^(1/2))/a^(7/6)/b/c^(1/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2
)+1/9*(-1)^(1/3)*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^
(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3
))^2/a^(7/6)/b/c^(1/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)-1/9
*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1
)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(7/6)/b/c^(1/3)*3^(1/2)/
(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {2122, 648, 632, 210, 642}

$$\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$
$$= -\frac{\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{7/6}b\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}} - \frac{\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}a^{7/6}b\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}$$
$$+\frac{\sqrt[3]{-1}\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{3\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{7/6}b\sqrt[3]{c}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}$$
$$+\frac{\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}} - \frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{18\left(1+\sqrt[3]{-1}\right)^2a^{4/3}bc^{2/3}}$$
$$+\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}}$$

```
[In] Int[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] -1/3*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*
b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(Sqrt[3]*(1 + (-1)^(1/3))^2*a^(7/6)*b*S
qrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTan[(3*a^(2/3)*c^(1/3
) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*a^(7
/6)*b*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(
2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1
/3)*c^(2/3)])])/(3*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(7/6)*b*Sq
rt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[3*a + 3*a^(2/3)*c^(1/
3)*x + b*x^2]/(54*a^(4/3)*b*c^(2/3)) - ((-1)^(2/3)*Log[3*a - 3*(-1)^(1/3)*a
^(2/3)*c^(1/3)*x + b*x^2])/(18*(1 + (-1)^(1/3))^2*a^(4/3)*b*c^(2/3)) + ((-1
)^(2/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(54*a^(4/3)*b*c^
(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\text{integral} = \left(19683a^6\right)\int \left(\frac{(-1)^{2/3}x}{177147\left(1+\sqrt[3]{-1}\right)^2 a^{22/3}c^{2/3}\left(-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-bx^2\right)}+\frac{x}{531441a^{22/3}c^{2/3}\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}-\frac{(-1)^{2/3}x}{177147\left(-1+\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2 a^{22/3}c^{2/3}\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}\right)dx$$

$$=\frac{\int \frac{x}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{27a^{4/3}c^{2/3}}+\frac{(-1)^{2/3}\int \frac{x}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{27a^{4/3}c^{2/3}}+\frac{(-1)^{2/3}\int \frac{x}{-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-bx^2}\,dx}{9\left(1+\sqrt[3]{-1}\right)^2 a^{4/3}c^{2/3}}$$

$$= \frac{\int \frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{54a^{4/3}bc^{2/3}} + \frac{(-1)^{2/3}\int \frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{54a^{4/3}bc^{2/3}}$$
$$- \frac{(-1)^{2/3}\int \frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-bx^2}\,dx}{18\left(1+\sqrt[3]{-1}\right)^2 a^{4/3}bc^{2/3}} - \frac{\int \frac{1}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{18a^{2/3}b\sqrt[3]{c}}$$
$$+ \frac{\sqrt[3]{-1}\int \frac{1}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{18a^{2/3}b\sqrt[3]{c}} - \frac{\int \frac{1}{-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-bx^2}\,dx}{6\left(1+\sqrt[3]{-1}\right)^2 a^{2/3}b\sqrt[3]{c}}$$

$$= \frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54a^{4/3}bc^{2/3}} - \frac{(-1)^{2/3}\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{18\left(1+\sqrt[3]{-1}\right)^2 a^{4/3}bc^{2/3}}$$
$$+ \frac{(-1)^{2/3}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54a^{4/3}bc^{2/3}}$$
$$+ \frac{\text{Subst}\left(\int \frac{1}{-3a\left(4b-3\sqrt[3]{a}c^{2/3}\right)-x^2}\,dx, x, 3a^{2/3}\sqrt[3]{c}+2bx\right)}{9a^{2/3}b\sqrt[3]{c}}$$
$$- \frac{\sqrt[3]{-1}\text{Subst}\left(\int \frac{1}{-3a\left(4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)-x^2}\,dx, x, 3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx\right)}{9a^{2/3}b\sqrt[3]{c}}$$
$$+ \frac{\text{Subst}\left(\int \frac{1}{-3a\left(4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)-x^2}\,dx, x, 3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx\right)}{3\left(1+\sqrt[3]{-1}\right)^2 a^{2/3}b\sqrt[3]{c}}$$

$$= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{7/6}b\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}}$$
$$- \frac{\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}a^{7/6}b\sqrt{4b-3\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} + \frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}a^{7/6}b\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}}$$
$$+ \frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54a^{4/3}bc^{2/3}} - \frac{(-1)^{2/3}\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{18\left(1+\sqrt[3]{-1}\right)^2 a^{4/3}bc^{2/3}}$$
$$+ \frac{(-1)^{2/3}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54a^{4/3}bc^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.20

$$\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$
$$=\frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4 +b^3\#1^6\&,\frac{\log(x-\#1)\#1^2}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\right]$$

```
[In] Integrate[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6
),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, (Log[x - #1]*#1^2)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4)
& ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.19

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27c\,a^2\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	93
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27c\,a^2\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	93

```
[In] int(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),
_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} \, dx = \text{Timed out}$$

```
[In] integrate(x**3/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} \, dx = \int \frac{x^3}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3} \, dx$$

```
[In] integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

Giac [F]

$$\int \frac{x^3}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx$$
$$= \int \frac{x^3}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

```
[In] integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 1354, normalized size of antiderivative = 2.78

$$\int \frac{x^3}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \text{Too large to display}$$

```
[In] int(x^3/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(4782969*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c
^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4
*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^2*a^9*b^6*c^3 - 729*a^5*b^7*x + 12914
0163*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348
907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a
^3*b*c^2*z^2 - 1, z, k)^3*a^10*b^8*c^3 + 1549681956*root(10460353203*a^9*b^
3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928
*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^4*a^
11*b^10*c^3 + 167365651248*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a
^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441
*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^5*a^12*b^12*c^3 - 94143178827
*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*
a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b
*c^2*z^2 - 1, z, k)^5*a^13*b^9*c^5 + 98415*root(10460353203*a^9*b^3*c^6*z^6
 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*
c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)*a^7*b^7*c + 4
374*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 143489
07*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^
3*b*c^2*z^2 - 1, z, k)*a^6*b^9*x - 2125764*root(10460353203*a^9*b^3*c^6*z^6
 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*
c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^2*a^8*b^9*c -
 59049*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 143
```

```
48907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683
*a^3*b*c^2*z^2 - 1, z, k)*a^7*b^6*c^2*x - 531441*root(10460353203*a^9*b^3*c
^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^
4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^2*a^8*b
^8*c^2*x - 688747536*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6
*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c
^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^4*a^10*b^12*c^2*x + 1162261467*root
(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b
^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*
z^2 - 1, z, k)^4*a^11*b^9*c^4*x - 20920706406*root(10460353203*a^9*b^3*c^6*
z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b
^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^5*a^12*b^1
1*c^4*x)*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 1
4348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 196
83*a^3*b*c^2*z^2 - 1, z, k), k, 1, 6)
```

3.138 $\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal result 1046
Rubi [A] (verified) 1047
Mathematica [C] (verified) 1049
Maple [C] (verified) 1049
Fricas [C] (verification not implemented) 1050
Sympy [F(-1)] 1050
Maxima [F] 1050
Giac [F] 1051
Mupad [B] (verification not implemented) 1051

Optimal result

Integrand size = 46, antiderivative size = 334

$$\begin{aligned}
&\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx \\
&= \frac{2(-1)^{2/3}\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{11/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}c^{2/3}} + \frac{2\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{11/6}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}c^{2/3}} \\
&\quad + \frac{2(-1)^{2/3}\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{11/6}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}c^{2/3}}
\end{aligned}$$

```
[Out] 2/81*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3)*c^
(2/3))^(1/2))/a^(11/6)/c^(2/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)+2/27*(
-1)^(2/3)*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(
4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^
(11/6)/c^(2/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)+2/27*(-1)^(
2/3)*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3
*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(11/6)/c^(2/3)*3^(1/
2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2122, 632, 210}

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$
$$=\frac{2(-1)^{2/3}\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{11/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}+\frac{2\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{11/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}$$
$$+\frac{2(-1)^{2/3}\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{11/6}c^{2/3}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}$$

```
[In] Int[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] (2*(-1)^(2/3)*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a
]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 + (-1)^(1/3))^2
*a^(11/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) + (2*ArcTan[(3*
a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/
(27*Sqrt[3]*a^(11/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + (2*(-1)^(2/3)
*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b +
3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3
))^2*a^(11/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
```

```
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\text{integral} = \left(19683a^6\right)\int \left(\frac{(-1)^{2/3}}{177147\left(1+\sqrt[3]{-1}\right)^2 a^{22/3}c^{2/3}\left(-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-bx^2\right)}+\frac{1}{531441a^{22/3}c^{2/3}\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}-\frac{(-1)^{2/3}}{177147\left(-1+\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2 a^{22/3}c^{2/3}\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}\right)dx$$

$$=\frac{\int \frac{1}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{27a^{4/3}c^{2/3}}+\frac{(-1)^{2/3}\int \frac{1}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{27a^{4/3}c^{2/3}}+\frac{(-1)^{2/3}\int \frac{1}{-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-bx^2}\,dx}{9\left(1+\sqrt[3]{-1}\right)^2 a^{4/3}c^{2/3}}$$

$$=-\frac{2\text{Subst}\left(\int \frac{1}{-3a\left(4b-3\sqrt[3]{a}c^{2/3}\right)-x^2}\,dx,x,3a^{2/3}\sqrt[3]{c}+2bx\right)}{27a^{4/3}c^{2/3}}-\frac{\left(2(-1)^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3a\left(4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)-x^2}\,dx,x,3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx\right)}{27a^{4/3}c^{2/3}}-\frac{\left(2(-1)^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3a\left(4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)-x^2}\,dx,x,3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx\right)}{9\left(1+\sqrt[3]{-1}\right)^2 a^{4/3}c^{2/3}}$$

$$=\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{11/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}c^{2/3}}+\frac{2\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{11/6}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}c^{2/3}}+\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{11/6}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}c^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.29

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \frac{1}{3}\text{RootSum}\Big[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4 + b^3\#1^6\&,\frac{\log(x-\#1)\#1}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\Big]$$

```
[In] Integrate[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6
),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, (Log[x - #1]*#1)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) &
]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.28

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27c\,a^2\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	93
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27c\,a^2\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	93

```
[In] int(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3*sum(_R^2/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),
_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.43 (sec) , antiderivative size = 27094, normalized size of antiderivative = 81.12

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \int \frac{x^2}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx$$

```
[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

Giac [F]

$$\int \frac{x^2}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \int \frac{x^2}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

```
[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 825, normalized size of antiderivative = 2.47

$$\int \frac{x^2}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx$$

$$= \sum_{k=1}^{6} \ln \Big(-a^3\, b^9 \Big(-\mathrm{root}\big(669462604992\, a^{11}\, b^3\, c^4\, z^6 - 282429536481\, a^{12}\, c^6\, z^6 + 129140163\, a^8\, c^4\, z^4 - 19683\, a$$

$$-\mathrm{root}\big(669462604992\, a^{11}\, b^3\, c^4\, z^6 - 282429536481\, a^{12}\, c^6\, z^6 + 129140163\, a^8\, c^4\, z^4 - 19683\, a^4\, c^2\, z^2 + 1, z, k\big)$$

$$+\mathrm{root}\big(669462604992\, a^{11}\, b^3\, c^4\, z^6 - 282429536481\, a^{12}\, c^6\, z^6 + 129140163\, a^8\, c^4\, z^4 - 19683\, a^4\, c^2\, z^2 + 1, z, k\big)$$

$$+\mathrm{root}\big(669462604992\, a^{11}\, b^3\, c^4\, z^6 - 282429536481\, a^{12}\, c^6\, z^6 + 129140163\, a^8\, c^4\, z^4 - 19683\, a^4\, c^2\, z^2 + 1, z, k\big)$$

$$+\mathrm{root}\big(669462604992\, a^{11}\, b^3\, c^4\, z^6 - 282429536481\, a^{12}\, c^6\, z^6 + 129140163\, a^8\, c^4\, z^4 - 19683\, a^4\, c^2\, z^2 + 1, z, k\big)\; a^2$$

$$+\mathrm{root}\big(669462604992\, a^{11}\, b^3\, c^4\, z^6 - 282429536481\, a^{12}\, c^6\, z^6 + 129140163\, a^8\, c^4\, z^4 - 19683\, a^4\, c^2\, z^2 + 1, z, k\big)\; a\, b$$

$$-\mathrm{root}\big(669462604992\, a^{11}\, b^3\, c^4\, z^6 - 282429536481\, a^{12}\, c^6\, z^6 + 129140163\, a^8\, c^4\, z^4 - 19683\, a^4\, c^2\, z^2 + 1, z, k\big)$$

$$-\mathrm{root}\big(669462604992\, a^{11}\, b^3\, c^4\, z^6 - 282429536481\, a^{12}\, c^6\, z^6 + 129140163\, a^8\, c^4\, z^4 - 19683\, a^4\, c^2\, z^2 + 1, z, k\big)$$

$$+\mathrm{root}\big(669462604992\, a^{11}\, b^3\, c^4\, z^6 - 282429536481\, a^{12}\, c^6\, z^6 + 129140163\, a^8\, c^4\, z^4 - 19683\, a^4\, c^2\, z^2 + 1, z, k\big)$$

$$+\mathrm{root}\big(669462604992\, a^{11}\, b^3\, c^4\, z^6 - 282429536481\, a^{12}\, c^6\, z^6 + 129140163\, a^8\, c^4\, z^4 - 19683\, a^4\, c^2\, z^2 + 1, z, k\big)$$

$$+\mathrm{root}\big(669462604992\, a^{11}\, b^3\, c^4\, z^6 - 282429536481\, a^{12}\, c^6\, z^6 + 129140163\, a^8\, c^4\, z^4 - 19683\, a^4\, c^2\, z^2 + 1, z, k\big)$$

$$+\mathrm{root}\big(669462604992\, a^{11}\, b^3\, c^4\, z^6 - 282429536481\, a^{12}\, c^6\, z^6 + 129140163\, a^8\, c^4\, z^4 - 19683\, a^4\, c^2\, z^2 + 1, z, k\big)$$

$$+ 1\Big)\; 27\Big)\; \mathrm{root}\big(669462604992\, a^{11}\, b^3\, c^4\, z^6 - 282429536481\, a^{12}\, c^6\, z^6 + 129140163\, a^8\, c^4\, z^4 - 19683\, a^4\, c^2\, z^2 + 1, z, k\big)$$

```
[In] int(x^2/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(-27*a^3*b^9*(43046721*root(669462604992*a^11*b^3*c^4*z^6 - 28242
9536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)
```

```
^4*a^8*c^4 - 1062882*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12
*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^3*a^6*c^3 -
 13122*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129
140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^2*a^4*c^2 + 3486784401*ro
ot(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^
8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^10*c^5 + 81*root(669462604992*
a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 1968
3*a^4*c^2*z^2 + 1, z, k)*a^2*c + 18*root(669462604992*a^11*b^3*c^4*z^6 - 28
2429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z,
 k)*a*b^2*x - 25509168*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^
12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^4*a^7*b^3
*c^2 - 6198727824*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^
6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^9*b^3*c^3
+ 5832*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129
140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^2*a^3*b^2*c*x + 708588*ro
ot(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^
8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^3*a^5*b^2*c^2*x + 38263752*root(66
9462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4
*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^4*a^7*b^2*c^3*x + 774840978*root(669462
604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4
 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^9*b^2*c^4*x + 1))*root(669462604992*a^1
1*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a
^4*c^2*z^2 + 1, z, k), k, 1, 6)
```

3.139 $\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal result . . . 1053
Rubi [A] (verified) . . . 1054
Mathematica [C] (verified) . . . 1056
Maple [C] (verified) . . . 1057
Fricas [F(-2)] . . . 1057
Sympy [F(-1)] . . . 1057
Maxima [F] . . . 1058
Giac [F] . . . 1058
Mupad [B] (verification not implemented) . . . 1059

Optimal result

Integrand size = 44, antiderivative size = 469

$$\begin{aligned}
&\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx \\
&= -\frac{\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{13/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} - \frac{\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{13/6}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} \\
&\quad + \frac{\sqrt[3]{-1}\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{13/6}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} \\
&\quad - \frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{7/3}c^{2/3}} + \frac{(-1)^{2/3}\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{7/3}c^{2/3}} \\
&\quad - \frac{(-1)^{2/3}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{7/3}c^{2/3}}
\end{aligned}$$

```
[Out] -1/162*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(7/3)/c^(2/3)+1/54*(-1)^(2/3)*ln
(3*a-3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(7/3)/c^(2/3)
-1/162*(-1)^(2/3)*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(7/3)/c^(2
/3)-1/81*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3
)*c^(2/3))^(1/2))/a^(13/6)/c^(1/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)+1/
27*(-1)^(1/3)*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/
2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^
2/a^(13/6)/c^(1/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)-1/27*ar
ctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(
2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(13/6)/c^(1/3)*3^(1/2)/(4*b
-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2122, 648, 632, 210, 642}

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$
$$= -\frac{\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{13/6}\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{13/6}\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}$$
$$+\frac{\sqrt[3]{-1}\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{13/6}\sqrt[3]{c}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}$$
$$-\frac{\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162a^{7/3}c^{2/3}}+\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{7/3}c^{2/3}}$$
$$-\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162a^{7/3}c^{2/3}}$$

```
[In] Int[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] -1/9*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*
b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(Sqrt[3]*(1 + (-1)^(1/3))^2*a^(13/6)*Sq
rt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTan[(3*a^(2/3)*c^(1/3)
 + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(27*Sqrt[3]*a^(1
3/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(2
/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/
3)*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(13/6)*Sqrt
[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - Log[3*a + 3*a^(2/3)*c^(1/3)
*x + b*x^2]/(162*a^(7/3)*c^(2/3)) + ((-1)^(2/3)*Log[3*a - 3*(-1)^(1/3)*a^(2
/3)*c^(1/3)*x + b*x^2])/(54*(1 + (-1)^(1/3))^2*a^(7/3)*c^(2/3)) - ((-1)^(2/
3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162*a^(7/3)*c^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\text{integral} = \left(19683a^6\right)\int\left(\frac{-3a^{2/3}\sqrt[3]{c}-(-1)^{2/3}bx}{531441\left(1+\sqrt[3]{-1}\right)^2a^{25/3}c^{2/3}\left(-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-bx^2\right)}+\frac{-3a^{2/3}\sqrt[3]{c}-bx}{1594323a^{25/3}c^{2/3}\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}+\frac{(-1)^{2/3}\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+bx\right)}{531441\left(-1+\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{25/3}c^{2/3}\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}\right)dx$$

$$=\frac{\int\frac{-3a^{2/3}\sqrt[3]{c}-bx}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{81a^{7/3}c^{2/3}}-\frac{(-1)^{2/3}\int\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{81a^{7/3}c^{2/3}}+\frac{\int\frac{-3a^{2/3}\sqrt[3]{c}-(-1)^{2/3}bx}{-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-bx^2}\,dx}{27\left(1+\sqrt[3]{-1}\right)^2a^{7/3}c^{2/3}}$$

$$=-\frac{\int\frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{162a^{7/3}c^{2/3}}-\frac{(-1)^{2/3}\int\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{162a^{7/3}c^{2/3}}+\frac{(-1)^{2/3}\int\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-bx^2}\,dx}{54\left(1+\sqrt[3]{-1}\right)^2a^{7/3}c^{2/3}}-\frac{\int\frac{1}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{54a^{5/3}\sqrt[3]{c}}+\frac{\sqrt[3]{-1}\int\frac{1}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{54a^{5/3}\sqrt[3]{c}}-\frac{\int\frac{1}{-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-bx^2}\,dx}{18\left(1+\sqrt[3]{-1}\right)^2a^{5/3}\sqrt[3]{c}}$$

$$= -\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{7/3}c^{2/3}} + \frac{(-1)^{2/3}\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2 a^{7/3}c^{2/3}} - \frac{(-1)^{2/3}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{7/3}c^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3a\left(4b-3\sqrt[3]{a}c^{2/3}\right)-x^2}dx, x, 3a^{2/3}\sqrt[3]{c}+2bx\right)}{27a^{5/3}\sqrt[3]{c}} - \frac{\sqrt[3]{-1}\text{Subst}\left(\int \frac{1}{-3a\left(4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)-x^2}dx, x, 3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx\right)}{27a^{5/3}\sqrt[3]{c}} + \frac{\text{Subst}\left(\int \frac{1}{-3a\left(4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)-x^2}dx, x, 3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx\right)}{9\left(1+\sqrt[3]{-1}\right)^2 a^{5/3}\sqrt[3]{c}}$$

$$= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{13/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} - \frac{\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{13/6}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} + \frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{13/6}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} - \frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{7/3}c^{2/3}} + \frac{(-1)^{2/3}\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2 a^{7/3}c^{2/3}} - \frac{(-1)^{2/3}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{7/3}c^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.20

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4 + b^3\#1^6\&, \frac{\log(x-\#1)}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\right]$$

```
[In] Integrate[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),
x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, Log[x - #1]/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.19

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^{3}\_Z^{6}+9b^{2}a\_Z^{4}+27c\,a^{2}\_Z^{3}+27a^{2}b\_Z^{2}+27a^{3}\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^{5}b^{3}+12\_R^{3}a\,b^{2}+27\_R^{2}a^{2}c+18a^{2}b\_R}\right)}{3}$	91
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^{3}\_Z^{6}+9b^{2}a\_Z^{4}+27c\,a^{2}\_Z^{3}+27a^{2}b\_Z^{2}+27a^{3}\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^{5}b^{3}+12\_R^{3}a\,b^{2}+27\_R^{2}a^{2}c+18a^{2}b\_R}\right)}{3}$	91

```
[In] int(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_RETU
RNVERBOSE)
[Out] 1/3*sum(_R/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R
=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="fricas")
[Out] Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \text{Timed out}$$

```
[In] integrate(x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3)
,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \int \frac{x}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

```
[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="maxima")
```

```
[Out] integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

Giac [F]

$$\int \frac{x}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \int \frac{x}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

```
[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="giac")
```

```
[Out] integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 1057, normalized size of antiderivative = 2.25

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx=\sum_{k=1}^{6}\ln\Big(b^{12}\,x$$
$$+\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2$$
$$+\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2$$
$$-\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2$$
$$+\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4$$
$$+8503056\,a^7\,b^3\,c^2\,z^3-14348907\,a^8\,c^4\,z^3+177147\,a^5\,b^2\,c^2\,z^2+b^3,z,k\big)\;a^2\,b^{13}\,x\,54$$
$$+\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2$$
$$+\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2$$
$$-\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2$$
$$+\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2$$
$$+\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2$$
$$-\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2$$
$$-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2\,z^3$$
$$-14348907\,a^8\,c^4\,z^3+177147\,a^5\,b^2\,c^2\,z^2+b^3,z,k\big)$$

```
[In] int(x/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(b^12*x + 1033121304*root(18075490334784*a^14*b^3*c^4*z^6 - 76255
97484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3
 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^4*a^10*b^11*c
^3 + 167365651248*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15
*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a
^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^5*a^12*b^12*c^3 - 94143178
827*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 116
2261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 1
77147*a^5*b^2*c^2*z^2 + b^3, z, k)^5*a^13*b^9*c^5 + 54*root(18075490334784*
a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 +
 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 +
b^3, z, k)*a^2*b^13*x + 177147*root(18075490334784*a^14*b^3*c^4*z^6 - 76255
97484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3
 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^2*a^5*b^11*c^
2*x + 17006112*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^
6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*
c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^3*a^7*b^12*c^2*x - 14348907*r
oot(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 11622614
67*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147
```

```
*a^5*b^2*c^2*z^2 + b^3, z, k)^3*a^8*b^9*c^4*x + 229582512*root(180754903347
84*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^
4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2
 + b^3, z, k)^4*a^9*b^13*c^2*x + 387420489*root(18075490334784*a^14*b^3*c^4
*z^6 - 7625597484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7
*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^4
*a^10*b^10*c^4*x - 20920706406*root(18075490334784*a^14*b^3*c^4*z^6 - 76255
97484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3
 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^5*a^12*b^11*c
^4*x)*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 1
162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 +
 177147*a^5*b^2*c^2*z^2 + b^3, z, k), k, 1, 6)
```

3.140 $\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal result 1061
Rubi [A] (verified) 1062
Mathematica [C] (verified) 1065
Maple [C] (verified) 1066
Fricas [F(-2)] 1066
Sympy [F(-1)] 1066
Maxima [F] 1067
Giac [F] 1067
Mupad [B] (verification not implemented) 1067

Optimal result

Integrand size = 42, antiderivative size = 522

$$\begin{aligned}\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx &= -\frac{\sqrt[3]{-1}\left(2\sqrt[3]{-1}b+3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{17/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}c^{2/3}} \\ &\quad -\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}a^{17/6}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}c^{2/3}} \\ &\quad -\frac{\left(2(-1)^{2/3}b-3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{17/6}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}c^{2/3}} \\ &\quad +\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}-\frac{\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{8/3}\sqrt[3]{c}} \\ &\quad -\frac{\sqrt[3]{-1}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}\end{aligned}$$

```
[Out] 1/162*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(8/3)/c^(1/3)-1/54*ln(3*a-3*(-1)^
(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(8/3)/c^(1/3)-1/162*(-1)^
(1/3)*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(8/3)/c^(1/3)-1/243*(2
*b-3*a^(1/3)*c^(2/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/
(4*b-3*a^(1/3)*c^(2/3))^(1/2))/a^(17/6)/c^(2/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2
/3))^(1/2)-1/81*(2*(-1)^(2/3)*b-3*a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(2/3)
*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^
(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(17/6)/c^(2/3)*3^(1/2)/(4*b+3*(-1)
^(1/3)*a^(1/3)*c^(2/3))^(1/2)-1/81*(-1)^(1/3)*(2*(-1)^(1/3)*b+3*a^(1/3)*c^(
```

```
2/3))*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-
3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(17/6)/c^(2/3)*3^(1
/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2095, 648, 632, 210, 642}

$$\begin{aligned}
&\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx \\
&= -\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}c^{2/3}+2\sqrt[3]{-1}b\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{17/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}} \\
&\quad -\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}a^{17/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}} \\
&\quad -\frac{\left(2(-1)^{2/3}b-3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{27\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{17/6}c^{2/3}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}} \\
&\quad +\frac{\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}-\frac{\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{8/3}\sqrt[3]{c}} \\
&\quad -\frac{\sqrt[3]{-1}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}
\end{aligned}$$

```
[In] Int[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-1),x]
```

```
[Out] -1/27*((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)
*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*
c^(2/3)]])])/(Sqrt[3]*(1 + (-1)^(1/3))^2*a^(17/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^
(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1
/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(81*Sqrt[3]*
a^(17/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*(-1)^(2/3)*b - 3*a^(1
/3)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]
*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(27*Sqrt[3]*(1 - (-1)^(1/3))*(
1 + (-1)^(1/3))^2*a^(17/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3)
) + Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(162*a^(8/3)*c^(1/3)) - Log[3*a
- 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(54*(1 + (-1)^(1/3))^2*a^(8/3)*c^
(1/3)) - ((-1)^(1/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(16
2*a^(8/3)*c^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2095

```
Int[(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2]
, c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3
^(3*p)*a^(2*p)), Int[ExpandIntegrand[(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2
)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3
)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0] && Eq
Q[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x,
1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\text{integral} = \left(19683a^6\right)\int\left(\frac{-(-1)^{2/3}\sqrt[3]{a}b-3\sqrt[3]{-1}a^{2/3}c^{2/3}+b\sqrt[3]{c}x}{531441\left(1+\sqrt[3]{-1}\right)^2a^{26/3}c^{2/3}\left(-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-bx^2\right)}+\frac{-\sqrt[3]{a}b+3a^{2/3}c^{2/3}+b\sqrt[3]{c}x}{1594323a^{26/3}c^{2/3}\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}+\frac{(-1)^{2/3}\sqrt[3]{a}b-3a^{2/3}c^{2/3}+\sqrt[3]{-1}b\sqrt[3]{c}x}{531441\left(-1+\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{26/3}c^{2/3}\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}\right)dx$$

$$=\frac{\int \frac{-\sqrt[3]{a}b+3a^{2/3}c^{2/3}+b\sqrt[3]{c}x}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{81a^{8/3}c^{2/3}}-\frac{\int \frac{(-1)^{2/3}\sqrt[3]{a}b-3a^{2/3}c^{2/3}+\sqrt[3]{-1}b\sqrt[3]{c}x}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{81a^{8/3}c^{2/3}}$$

$$+\frac{\int \frac{-(-1)^{2/3}\sqrt[3]{a}b-3\sqrt[3]{-1}a^{2/3}c^{2/3}+b\sqrt[3]{c}x}{-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-bx^2}\,dx}{27\left(1+\sqrt[3]{-1}\right)^2a^{8/3}c^{2/3}}$$

$$=-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\int \frac{1}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{162a^{7/3}c^{2/3}}$$

$$-\frac{\left(2(-1)^{2/3}b-3\sqrt[3]{a}c^{2/3}\right)\int \frac{1}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{162a^{7/3}c^{2/3}}$$

$$-\frac{\left(-2b\left(-(-1)^{2/3}\sqrt[3]{a}b-3\sqrt[3]{-1}a^{2/3}c^{2/3}\right)-3\sqrt[3]{-1}a^{2/3}bc^{2/3}\right)\int \frac{1}{-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-bx^2}\,dx}{54\left(1+\sqrt[3]{-1}\right)^2a^{8/3}bc^{2/3}}$$

$$+\frac{\int \frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{162a^{8/3}\sqrt[3]{c}}-\frac{\sqrt[3]{-1}\int \frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{162a^{8/3}\sqrt[3]{c}}$$

$$-\frac{\int \frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{-3a+3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x-bx^2}\,dx}{54\left(1+\sqrt[3]{-1}\right)^2a^{8/3}\sqrt[3]{c}}$$

$$=\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}-\frac{\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{8/3}\sqrt[3]{c}}$$

$$-\frac{\sqrt[3]{-1}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}$$

$$+\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3a\left(4b-3\sqrt[3]{a}c^{2/3}\right)-x^2}\,dx,x,3a^{2/3}\sqrt[3]{c}+2bx\right)}{81a^{7/3}c^{2/3}}$$

$$+\frac{\left(2(-1)^{2/3}b-3\sqrt[3]{a}c^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3a\left(4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)-x^2}\,dx,x,3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx\right)}{81a^{7/3}c^{2/3}}$$

$$+\frac{\left(-2b\left(-(-1)^{2/3}\sqrt[3]{a}b-3\sqrt[3]{-1}a^{2/3}c^{2/3}\right)-3\sqrt[3]{-1}a^{2/3}bc^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3a\left(4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)-x^2}\,dx,\right.}{27\left(1+\sqrt[3]{-1}\right)^2a^{8/3}bc^{2/3}}$$

$$= -\frac{\left(2(-1)^{2/3}b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{17/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}c^{2/3}} - \frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}a^{17/6}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}c^{2/3}} - \frac{\left(2(-1)^{2/3}b-3\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}a^{17/6}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}c^{2/3}} + \frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{8/3}\sqrt[3]{c}} - \frac{\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{8/3}\sqrt[3]{c}} - \frac{\sqrt[3]{-1}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.19

$$\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\&,\frac{\log(x-\#1)}{18a^2b\#1+27a^2c\#1^2+12ab^2\#1^3+2b^3\#1^5}\&\right]$$

```
[In] Integrate[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-
1),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, Log[x - #1]/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12*a*b^2*#1^3 + 2*b^3*#1^5) &
]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.17

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^{3}\_Z^{6}+9b^{2}a\_Z^{4}+27c\,a^{2}\_Z^{3}+27a^{2}b\_Z^{2}+27a^{3}\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}b^{3}+12\_R^{3}a\,b^{2}+27\_R^{2}a^{2}c+18a^{2}b\_R}\right)}{3}$	90
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^{3}\_Z^{6}+9b^{2}a\_Z^{4}+27c\,a^{2}\_Z^{3}+27a^{2}b\_Z^{2}+27a^{3}\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}b^{3}+12\_R^{3}a\,b^{2}+27\_R^{2}a^{2}c+18a^{2}b\_R}\right)}{3}$	90

```
[In] int(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3*sum(1/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R=
RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3)
,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \int \frac{1}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

```
[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="maxima")
[Out] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

Giac [F]

$$\int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \int \frac{1}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

```
[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="giac")
[Out] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

Mupad [B] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 1394, normalized size of antiderivative = 2.67

$$\int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \text{Too large to display}$$

```
[In] int(1/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
[Out] symsum(log(6561*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^1
8*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 3874
20489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k
)^2*a^4*b^12*c^2 - 6*root(488038239039168*a^17*b^3*c^4*z^6 - 20589113209464
9*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 +
 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6,
 z, k)*b^15*x - 4782969*root(488038239039168*a^17*b^3*c^4*z^6 - 20589113209
4649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^
```

3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^3*a^7*b^11*c^3 - 229582512*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^9*b^13*c^2 - 387420489*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^10*b^10*c^4 + 167365651248*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a^12*b^12*c^3 - 94143178827*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a^13*b^9*c^5 + 14580*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^2*a^3*b^14*c*x - 10628820*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^3*a^6*b^13*c^2*x + 2238429492*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^9*b^12*c^3*x - 1162261467*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^10*b^9*c^5*x - 20920706406*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a^12*b^11*c^4*x)*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k), k, 1, 6)

3.141 $\int \frac{1}{x(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx$

Optimal result 1069
Rubi [A] (verified) 1070
Mathematica [C] (verified) 1074
Maple [C] (verified) 1074
Fricas [F(-2)] 1075
Sympy [F(-1)] 1075
Maxima [F] 1075
Giac [F] 1076
Mupad [B] (verification not implemented) 1076

Optimal result

Integrand size = 46, antiderivative size = 563

$$\begin{aligned}
&\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx \\
&= \frac{\left(b-(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{19/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} \\
&+\frac{\left(b-\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{19/6}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} \\
&+\frac{(-1)^{2/3}\left((-1)^{2/3}b-\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{19/6}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} \\
&+\frac{\log(x)}{27a^3}-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{486a^{10/3}} \\
&-\frac{\left(b+i\sqrt{3}b+6\sqrt[3]{a}c^{2/3}\right)\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{972a^{10/3}c^{2/3}} \\
&-\frac{\left(3\sqrt[3]{a}-\frac{(-1)^{2/3}b}{c^{2/3}}\right)\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{486a^{10/3}}
\end{aligned}$$

```
[Out] 1/27*ln(x)/a^3-1/486*(3*a^(1/3)-b/c^(2/3))*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2
)/a^(10/3)-1/486*(3*a^(1/3)-(-1)^(2/3)*b/c^(2/3))*ln(3*a+3*(-1)^(2/3)*a^(2/
3)*c^(1/3)*x+b*x^2)/a^(10/3)-1/972*ln(3*a-3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x+b*
x^2)*(b+6*a^(1/3)*c^(2/3)+I*b*3^(1/2))/a^(10/3)/c^(2/3)+1/81*(b-a^(1/3)*c^(
2/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3)*c
^(2/3))^(1/2))/a^(19/6)/c^(1/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)+1/27*
```

```
(-1)^(2/3)*((-1)^(2/3)*b-a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*
c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1
-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(19/6)/c^(1/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^
(1/3)*c^(2/3))^(1/2)+1/27*(b-(-1)^(2/3)*a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)
^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(
2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(19/6)/c^(1/3)*3^(1/2)/(4*b-3*(-1)^(2/3)*a^
(1/3)*c^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {2122, 648, 632, 210, 642}

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}dx$$
$$= \frac{\left(b-(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{19/6}\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}$$
$$+\frac{\left(b-\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{19/6}\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}$$
$$+\frac{(-1)^{2/3}\left((-1)^{2/3}b-\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{19/6}\sqrt[3]{c}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}$$
$$-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{486a^{10/3}}$$
$$-\frac{\left(6\sqrt[3]{a}c^{2/3}+i\sqrt{3}b+b\right)\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{972a^{10/3}c^{2/3}}$$
$$-\frac{\left(3\sqrt[3]{a}-\frac{(-1)^{2/3}b}{c^{2/3}}\right)\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{486a^{10/3}}+\frac{\log(x)}{27a^3}$$

```
[In] Int[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),x]
```

```
[Out] ((b - (-1)^(2/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*
b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3
]*(1 + (-1)^(1/3))^2*a^(19/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1
/3)) + ((b - a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*S
qrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(27*Sqrt[3]*a^(19/6)*Sqrt[4*b - 3*a
^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(2/3)*((-1)^(2/3)*b - a^(1/3)*c^(2/3))*Arc
Tan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-
1)^(1/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2
```

```
*a^(19/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[x]/(27*a^
3) - ((3*a^(1/3) - b/c^(2/3))*Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2])/(486*
a^(10/3)) - ((b + I*Sqrt[3]*b + 6*a^(1/3)*c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a
^(2/3)*c^(1/3)*x + b*x^2])/(972*a^(10/3)*c^(2/3)) - ((3*a^(1/3) - ((-1)^(2/
3)*b)/c^(2/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(1
0/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\text{integral} = \left(19683a^6\right)\int\left(\frac{1}{531441a^9x}+\frac{3a^{2/3}\left(2b-3\sqrt[3]{a}c^{2/3}\right)\sqrt[3]{c}+b\left(b-3\sqrt[3]{a}c^{2/3}\right)x}{4782969a^{28/3}c^{2/3}\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}+\frac{-3a^{2/3}\left(2b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\sqrt[3]{c}-\sqrt[3]{-1}b\left(\sqrt[3]{-1}b+3\sqrt[3]{a}c^{2/3}\right)x}{1594323\left(1+\sqrt[3]{-1}\right)^2a^{28/3}c^{2/3}\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}+\frac{(-1)^{2/3}\left(3a^{2/3}\left(2(-1)^{2/3}b-3\sqrt[3]{a}c^{2/3}\right)\sqrt[3]{c}+b\left(b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)x\right)}{1594323\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{28/3}c^{2/3}\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}\right)dx$$

$$=\frac{\log(x)}{27a^3}+\frac{\int\frac{3a^{2/3}\left(2b-3\sqrt[3]{a}c^{2/3}\right)\sqrt[3]{c}+b\left(b-3\sqrt[3]{a}c^{2/3}\right)x}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}dx}{243a^{10/3}c^{2/3}}+\frac{(-1)^{2/3}\int\frac{3a^{2/3}\left(2(-1)^{2/3}b-3\sqrt[3]{a}c^{2/3}\right)\sqrt[3]{c}+b\left(b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)x}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}dx}{243a^{10/3}c^{2/3}}+\frac{\int\frac{-3a^{2/3}\left(2b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\sqrt[3]{c}-\sqrt[3]{-1}b\left(\sqrt[3]{-1}b+3\sqrt[3]{a}c^{2/3}\right)x}{3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2}dx}{81\left(1+\sqrt[3]{-1}\right)^2a^{10/3}c^{2/3}}$$

$$=\frac{\log(x)}{27a^3}-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\int\frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}dx}{486a^{10/3}}-\frac{\left(3\sqrt[3]{a}-\frac{(-1)^{2/3}b}{c^{2/3}}\right)\int\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}dx}{486a^{10/3}}-\frac{\left(b+i\sqrt{3}b+6\sqrt[3]{a}c^{2/3}\right)\int\frac{-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}+2bx}{3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2}dx}{972a^{10/3}c^{2/3}}+\frac{\left(b-\sqrt[3]{a}c^{2/3}\right)\int\frac{1}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}dx}{54a^{8/3}\sqrt[3]{c}}+\frac{\left((-1)^{2/3}\left((-1)^{2/3}b-\sqrt[3]{a}c^{2/3}\right)\right)\int\frac{1}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}dx}{54a^{8/3}\sqrt[3]{c}}-\frac{\left(b-(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\int\frac{1}{3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2}dx}{18\left(1+\sqrt[3]{-1}\right)^2a^{8/3}\sqrt[3]{c}}$$

$$= \frac{\log(x)}{27a^3} - \frac{\left(3\sqrt[3]{a} - \frac{b}{c^{2/3}}\right)\log\left(3a + 3a^{2/3}\sqrt[3]{c}x + bx^2\right)}{486a^{10/3}} - \frac{\left(b + i\sqrt{3}b + 6\sqrt[3]{a}c^{2/3}\right)\log\left(3a - 3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + bx^2\right)}{972a^{10/3}c^{2/3}} - \frac{\left(3\sqrt[3]{a} - \frac{(-1)^{2/3}b}{c^{2/3}}\right)\log\left(3a + 3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x + bx^2\right)}{486a^{10/3}} - \frac{\left(b - \sqrt[3]{a}c^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3a\left(4b - 3\sqrt[3]{a}c^{2/3}\right) - x^2}dx, x, 3a^{2/3}\sqrt[3]{c} + 2bx\right)}{27a^{8/3}\sqrt[3]{c}} - \frac{\left((-1)^{2/3}\left((-1)^{2/3}b - \sqrt[3]{a}c^{2/3}\right)\right)\text{Subst}\left(\int \frac{1}{-3a\left(4b + 3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right) - x^2}dx, x, 3(-1)^{2/3}a^{2/3}\sqrt[3]{c} + 2bx\right)}{27a^{8/3}\sqrt[3]{c}} + \frac{\left(b - (-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3a\left(4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right) - x^2}dx, x, -3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c} + 2bx\right)}{9\left(1 + \sqrt[3]{-1}\right)^2 a^{8/3}\sqrt[3]{c}}$$

$$= \frac{\left(b - (-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c} - 2bx}{\sqrt{3}\sqrt{a}\sqrt{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1 + \sqrt[3]{-1}\right)^2 a^{19/6}\sqrt{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} + \frac{\left(b - \sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c} + 2bx}{\sqrt{3}\sqrt{a}\sqrt{4b - 3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{19/6}\sqrt{4b - 3\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} + \frac{(-1)^{2/3}\left((-1)^{2/3}b - \sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c} + 2bx}{\sqrt{3}\sqrt{a}\sqrt{4b + 3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{19/6}\sqrt{4b + 3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} + \frac{\log(x)}{27a^3} - \frac{\left(3\sqrt[3]{a} - \frac{b}{c^{2/3}}\right)\log\left(3a + 3a^{2/3}\sqrt[3]{c}x + bx^2\right)}{486a^{10/3}} - \frac{\left(b + i\sqrt{3}b + 6\sqrt[3]{a}c^{2/3}\right)\log\left(3a - 3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + bx^2\right)}{972a^{10/3}c^{2/3}} - \frac{\left(3\sqrt[3]{a} - \frac{(-1)^{2/3}b}{c^{2/3}}\right)\log\left(3a + 3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x + bx^2\right)}{486a^{10/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.28

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = -\frac{-3\log(x)+\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\&,\frac{27a^2b\log(x-\#1)+27a^2c\log(x-\#1}{18a^2b+27a^2c}\right.}{81a^3}$$

```
[In] Integrate[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^
6)),x]
```

```
[Out] -1/81*(-3*Log[x] + RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2
*#1^4 + b^3*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*#1 + 9*a*
b^2*Log[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b + 27*a^2*c*#1 + 12*a
*b^2*#1^2 + 2*b^3*#1^4) & ])/a^3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.24

method	result
default	$\frac{\ln(x)}{27a^3}-\frac{\sum_{\_R=\text{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27c\,a^2\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\left(\_R^5b^3+9\_R^3ab^2+27\_R^2a^2c+27a^2b\_R\right)\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3ab^2+27\_R^2a^2c+18a^2b\_R}}{81a^3}$
risch	$\frac{\ln(-x)}{27a^3}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(27a^{21}c^6-64a^{20}b^3c^4\right)\_Z^6+\left(243a^{18}c^6-576a^{17}b^3c^4\right)\_Z^5+\left(729a^{15}c^6-1755a^{14}c^4b^3\right)\_Z^4+\left(729a^{12}c^6-1917a^{11}c^4b^3\right.\right.}\right.}{}$

```
[In] int(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/27*ln(x)/a^3-1/81/a^3*sum((_R^5*b^3+9*_R^3*a*b^2+27*_R^2*a^2*c+27*_R*a^2*
b)/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R=RootOf(
_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\,(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\,(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**
3),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x\,(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx = \int \frac{1}{(b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3)x}\,dx$$

```
[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] -1/27*integrate((b^3*x^5 + 9*a*b^2*x^3 + 27*a^2*c*x^2 + 27*a^2*b*x)/(b^3*x^
6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)/a^3 + 1/27*log(
x)/a^3
```

Giac [F]

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = \int \frac{1}{(b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3)x}\,dx$$

```
[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
*x), x)
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 4002, normalized size of antiderivative = 7.11

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = \text{Too large to display}$$

```
[In] int(1/(x*(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3)),x)
```

```
[Out] log(x)/(27*a^3) + symsum(log(7*root(13177032454057536*a^20*b^3*c^4*z^6 - 55
59060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 2058911320
94649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^
6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 1046035
3203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6
561*a^4*b^6*c^2*z + b^9, z, k)*b^18*x - 162*root(13177032454057536*a^20*b^3
*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5
 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 25418658
28329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*
z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^
6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^2*a^3*b^18*x + 86093442*root(13
177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 48803823
9039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^1
4*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 -
 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c
^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^3*a^8*b
^13*c^3 + 34867844010*root(13177032454057536*a^20*b^3*c^4*z^6 - 55590605665
55523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^1
8*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 2
7506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12
*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b
^6*c^2*z + b^9, z, k)^4*a^11*b^13*c^3 - 10460353203*root(13177032454057536*
```

a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3
*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 -
2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*
b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 1009737
9*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^4*a^12*b^10*c^5 + 15062
90861232*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^
6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6
119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^
11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14
348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^
9, z, k)^5*a^14*b^13*c^3 - 564859072962*root(13177032454057536*a^20*b^3*c^4
*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 2
05891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 254186582832
9*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3
- 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^
2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^5*a^15*b^10*c^5 - 67783088755440*ro
ot(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488
038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 611930662375
5*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*
z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*
b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^6*
a^17*b^13*c^3 + 22876792454961*root(13177032454057536*a^20*b^3*c^4*z^6 - 55
59060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 2058911320
94649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^
6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 1046035
3203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6
561*a^4*b^6*c^2*z + b^9, z, k)^6*a^18*b^10*c^5 + 17496*root(131770324540575
36*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*
b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4
 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^
10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 1009
7379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^2*a^4*b^16*c - 47239
2*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 +
 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 61193066
23755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*
c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*
a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k
)^3*a^7*b^16*c - 39366*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566
555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^
18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 +
27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^1
2*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*
b^6*c^2*z + b^9, z, k)^2*a^4*b^15*c^2*x + 51372630*root(13177032454057536*a
^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*
c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2

541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b
^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379
*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^3*a^7*b^15*c^2*x + 71744
535*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6
 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 611930
6623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^
3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 1434890
7*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z,
 k)^3*a^8*b^12*c^4*x - 2008846980*root(13177032454057536*a^20*b^3*c^4*z^6 -
 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 2058911
32094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15
*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 1046
0353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2
 - 6561*a^4*b^6*c^2*z + b^9, z, k)^4*a^10*b^15*c^2*x + 108477736920*root(131
77032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239
039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14
*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 -
229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^
4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^4*a^11*b
^12*c^4*x - 41841412812*root(13177032454057536*a^20*b^3*c^4*z^6 - 555906056
6555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a
^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 +
 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^
12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4
*b^6*c^2*z + b^9, z, k)^4*a^12*b^9*c^6*x + 18596183472*root(131770324540575
36*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*
b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4
 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^
10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 1009
7379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^5*a^13*b^15*c^2*x +
16129864639026*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a
^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z
^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854
719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^
3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*
z + b^9, z, k)^5*a^14*b^12*c^4*x - 6778308875544*root(13177032454057536*a^2
0*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^
4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 254
1865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6
*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a
^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^5*a^15*b^9*c^6*x + 6456339
20395566*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^
6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6
119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^
11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14

348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^6*a^17*b^12*c^4*x - 274521509459532*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^6*a^18*b^9*c^6*x)*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k), k, 1, 6)

3.142 $\int \frac{1}{x^2(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx$

Optimal result 1080
Rubi [A] (verified) 1081
Mathematica [C] (verified) 1085
Maple [C] (verified) 1085
Fricas [F(-1)] 1086
Sympy [F(-1)] 1086
Maxima [F] 1086
Giac [F] 1087
Mupad [B] (verification not implemented) 1087

Optimal result

Integrand size = 46, antiderivative size = 645

$$\begin{aligned}
&\int \frac{1}{x^2\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx \\
&= -\frac{1}{27a^3x}+\frac{\left(2(-1)^{2/3}b^2+12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3}+9a^{2/3}c^{4/3}\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{23/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\,c^{2/3}} \\
&+\frac{\left(2b^2-12\sqrt[3]{a}bc^{2/3}+9a^{2/3}c^{4/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{243\sqrt{3}a^{23/6}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}\,c^{2/3}} \\
&+\frac{(-1)^{2/3}\left(2b^2+12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3}+9(-1)^{2/3}a^{2/3}c^{4/3}\right)\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{23/6}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}\,c^{2/3}} \\
&-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{486a^{11/3}\sqrt[3]{c}} \\
&+\frac{\left(2b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162\left(1+\sqrt[3]{-1}\right)^2a^{11/3}\sqrt[3]{c}} \\
&+\frac{\sqrt[3]{-1}\left(2b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{486a^{11/3}\sqrt[3]{c}}
\end{aligned}$$

```
[Out] -1/27/a^3/x-1/486*(2*b-3*a^(1/3)*c^(2/3))*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)
/a^(11/3)/c^(1/3)+1/162*(2*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))*ln(3*a-3*(-1)^(1
/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(11/3)/c^(1/3)+1/486*(-1)^(
1/3)*(2*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)
*x+b*x^2)/a^(11/3)/c^(1/3)+1/729*(2*b^2-12*a^(1/3)*b*c^(2/3)+9*a^(2/3)*c^(4
/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3)*c^
```

```
(2/3))^(1/2))/a^(23/6)/c^(2/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)+1/243*
(-1)^(2/3)*(2*b^2+12*(-1)^(1/3)*a^(1/3)*b*c^(2/3)+9*(-1)^(2/3)*a^(2/3)*c^(4
/3))*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3
*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(23/6
)/c^(2/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)+1/243*(2*(-1)^(2
/3)*b^2+12*(-1)^(1/3)*a^(1/3)*b*c^(2/3)+9*a^(2/3)*c^(4/3))*arctan(1/3*(3*(-
1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c
^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(23/6)/c^(2/3)*3^(1/2)/(4*b-3*(-1)^(2/3)*
a^(1/3)*c^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 640, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {2122, 648, 632, 210, 642}

$$\begin{aligned}
&\int \frac{1}{x^2 \left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)} \, dx \\
&= \frac{\left(9a^{2/3}c^{4/3}+12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3}+2(-1)^{2/3}b^2\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{23/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}} \\
&+\frac{\left(9a^{2/3}c^{4/3}-12\sqrt[3]{a}bc^{2/3}+2b^2\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{243\sqrt{3}a^{23/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}} \\
&+\frac{\left(-9\sqrt[3]{-1}a^{2/3}c^{4/3}-12\sqrt[3]{a}bc^{2/3}+2(-1)^{2/3}b^2\right)\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{81\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2 a^{23/6}c^{2/3}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}} \\
&-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{486a^{11/3}\sqrt[3]{c}} \\
&+\frac{\left(2b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162\left(1+\sqrt[3]{-1}\right)^2 a^{11/3}\sqrt[3]{c}} \\
&+\frac{\sqrt[3]{-1}\left(3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+2b\right)\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{486a^{11/3}\sqrt[3]{c}}-\frac{1}{27a^3x}
\end{aligned}$$

```
[In] Int[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),
x]
```

```
[Out] -1/27*1/(a^3*x) + ((2*(-1)^(2/3)*b^2 + 12*(-1)^(1/3)*a^(1/3)*b*c^(2/3) + 9*
a^(2/3)*c^(4/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqr
t[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]])/(81*Sqrt[3]*(1 + (-1)^(1/3
))^2*a^(23/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) + ((2*b^2 -
 12*a^(1/3)*b*c^(2/3) + 9*a^(2/3)*c^(4/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*
```

```
x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(243*Sqrt[3]*a^(23/6)*
Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + ((2*(-1)^(2/3)*b^2 - 12*a^(1/3)*b*
c^(2/3) - 9*(-1)^(1/3)*a^(2/3)*c^(4/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3
) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(81
*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(23/6)*Sqrt[4*b + 3*(-1)^(1/
3)*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*Log[3*a + 3*a^(2/
3)*c^(1/3)*x + b*x^2])/(486*a^(11/3)*c^(1/3)) + ((2*b - 3*(-1)^(2/3)*a^(1/3
)*c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162*(1 + (-1
)^(1/3))^2*a^(11/3)*c^(1/3)) + ((-1)^(1/3)*(2*b + 3*(-1)^(1/3)*a^(1/3)*c^(2
/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(11/3)*c^(1/
3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

integral

$$= \left(19683a^6\right) \int \left(\frac{1}{531441a^9x^2} + \frac{\sqrt[3]{a}\left(b^2-9\sqrt[3]{a}bc^{2/3}+9a^{2/3}c^{4/3}\right)-b\left(2b-3\sqrt[3]{a}c^{2/3}\right)\sqrt[3]{c}x}{1594323\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2 a^{29/3}c^{2/3}\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)} + \frac{-\sqrt[3]{a}\left((-1)^{2/3}b^2+9\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3}+9a^{2/3}c^{4/3}\right)+b\left(2b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\sqrt[3]{c}x}{1594323\left(1+\sqrt[3]{-1}\right)^2 a^{29/3}c^{2/3}\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)} + \frac{\sqrt[3]{a}\left((-1)^{2/3}b^2-9\sqrt[3]{a}bc^{2/3}-9\sqrt[3]{-1}a^{2/3}c^{4/3}\right)+\sqrt[3]{-1}b\left(2b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)\sqrt[3]{c}x}{1594323\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2 a^{29/3}c^{2/3}\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}\right)dx$$

$$= -\frac{1}{27a^3x} + \frac{\int \frac{\sqrt[3]{a}\left(b^2-9\sqrt[3]{a}bc^{2/3}+9a^{2/3}c^{4/3}\right)-b\left(2b-3\sqrt[3]{a}c^{2/3}\right)\sqrt[3]{c}x}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}dx}{243a^{11/3}c^{2/3}} + \frac{\int \frac{\sqrt[3]{a}\left((-1)^{2/3}b^2-9\sqrt[3]{a}bc^{2/3}-9\sqrt[3]{-1}a^{2/3}c^{4/3}\right)+\sqrt[3]{-1}b\left(2b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)\sqrt[3]{c}x}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}dx}{243a^{11/3}c^{2/3}} + \frac{\int \frac{-\sqrt[3]{a}\left((-1)^{2/3}b^2+9\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3}+9a^{2/3}c^{4/3}\right)+b\left(2b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\sqrt[3]{c}x}{3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2}dx}{81\left(1+\sqrt[3]{-1}\right)^2 a^{11/3}c^{2/3}}$$

$$= -\frac{1}{27a^3x} - \frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\int \frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}dx}{486a^{11/3}\sqrt[3]{c}} + \frac{\left(\sqrt[3]{-1}\left(2b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)\right)\int \frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}dx}{486a^{11/3}\sqrt[3]{c}} + \frac{\left(2b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\int \frac{-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}+2bx}{3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2}dx}{162\left(1+\sqrt[3]{-1}\right)^2 a^{11/3}\sqrt[3]{c}} + \frac{\left(2b^2-12\sqrt[3]{a}bc^{2/3}+9a^{2/3}c^{4/3}\right)\int \frac{1}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}dx}{486a^{10/3}c^{2/3}} - \frac{\left(2(-1)^{2/3}b^2+12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3}+9a^{2/3}c^{4/3}\right)\int \frac{1}{3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2}dx}{162\left(1+\sqrt[3]{-1}\right)^2 a^{10/3}c^{2/3}} + \frac{\left(2(-1)^{2/3}b^2-12\sqrt[3]{a}bc^{2/3}-9\sqrt[3]{-1}a^{2/3}c^{4/3}\right)\int \frac{1}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}dx}{486a^{10/3}c^{2/3}}$$

$$= -\frac{1}{27a^3x} - \frac{\left(2b - 3\sqrt[3]{a}c^{2/3}\right)\log\left(3a + 3a^{2/3}\sqrt[3]{c}x + bx^2\right)}{486a^{11/3}\sqrt[3]{c}}$$

$$+ \frac{\left(2b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\log\left(3a - 3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + bx^2\right)}{162\left(1 + \sqrt[3]{-1}\right)^2 a^{11/3}\sqrt[3]{c}}$$

$$+ \frac{\sqrt[3]{-1}\left(2b + 3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)\log\left(3a + 3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x + bx^2\right)}{486a^{11/3}\sqrt[3]{c}}$$

$$- \frac{\left(2b^2 - 12\sqrt[3]{a}bc^{2/3} + 9a^{2/3}c^{4/3}\right)\text{Subst}\left(\int \frac{1}{-3a\left(4b - 3\sqrt[3]{a}c^{2/3}\right) - x^2}\,dx, x, 3a^{2/3}\sqrt[3]{c} + 2bx\right)}{243a^{10/3}c^{2/3}}$$

$$+ \frac{\left(2(-1)^{2/3}b^2 + 12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3} + 9a^{2/3}c^{4/3}\right)\text{Subst}\left(\int \frac{1}{-3a\left(4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right) - x^2}\,dx, x, -3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c} + 2\right.}{81\left(1 + \sqrt[3]{-1}\right)^2 a^{10/3}c^{2/3}}$$

$$- \frac{\left(2(-1)^{2/3}b^2 - 12\sqrt[3]{a}bc^{2/3} - 9\sqrt[3]{-1}a^{2/3}c^{4/3}\right)\text{Subst}\left(\int \frac{1}{-3a\left(4b + 3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right) - x^2}\,dx, x, 3(-1)^{2/3}a^{2/3}\sqrt[3]{c} + 2\right.}{243a^{10/3}c^{2/3}}$$

$$= -\frac{1}{27a^3x} + \frac{\left(2(-1)^{2/3}b^2 + 12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3} + 9a^{2/3}c^{4/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c} - 2bx}{\sqrt{3}\sqrt{a}\sqrt{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}\left(1 + \sqrt[3]{-1}\right)^2 a^{23/6}\sqrt{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}c^{2/3}}$$

$$+ \frac{\left(2b^2 - 12\sqrt[3]{a}bc^{2/3} + 9a^{2/3}c^{4/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c} + 2bx}{\sqrt{3}\sqrt{a}\sqrt{4b - 3\sqrt[3]{a}c^{2/3}}}\right)}{243\sqrt{3}a^{23/6}\sqrt{4b - 3\sqrt[3]{a}c^{2/3}}c^{2/3}}$$

$$+ \frac{\left(2(-1)^{2/3}b^2 - 12\sqrt[3]{a}bc^{2/3} - 9\sqrt[3]{-1}a^{2/3}c^{4/3}\right)\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c} + 2bx}{\sqrt{3}\sqrt{a}\sqrt{4b + 3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{243\sqrt{3}a^{23/6}\sqrt{4b + 3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}c^{2/3}}$$

$$- \frac{\left(2b - 3\sqrt[3]{a}c^{2/3}\right)\log\left(3a + 3a^{2/3}\sqrt[3]{c}x + bx^2\right)}{486a^{11/3}\sqrt[3]{c}}$$

$$+ \frac{\left(2b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\log\left(3a - 3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + bx^2\right)}{162\left(1 + \sqrt[3]{-1}\right)^2 a^{11/3}\sqrt[3]{c}}$$

$$+ \frac{\sqrt[3]{-1}\left(2b + 3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)\log\left(3a + 3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x + bx^2\right)}{486a^{11/3}\sqrt[3]{c}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^2\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = -\frac{3+x\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\&,\frac{27a^2b\log(x-\#1)+27a^2c\log(x-\#1)\#1+9}{18a^2b\#1+27a^2c\#1^2}\right.}{81a^3x}$$

```
[In] Integrate[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*
x^6)),x]
```

```
[Out] -1/81*(3 + x*RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4
+ b^3*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*#1 + 9*a*b^2*Lo
g[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12*a*
b^2*#1^3 + 2*b^3*#1^5) & ])/(a^3*x)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.21

method	result
default	$\frac{\sum_{\_R=\text{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27c\,a^2\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\left(-\_R^4b^3-9\_R^2ab^2-27\_Ra^2c-27a^2b\right)\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3ab^2+27\_R^2a^2c+18a^2b\_R}}{81a^3}-\frac{1}{27a^3x}$
risch	$-\frac{1}{27a^3x}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(729a^{24}c^6-1728a^{23}b^3c^4\right)\_Z^6+\left(13122a^{17}bc^6-31347a^{16}b^4c^4\right)\_Z^4+\left(-19683c^7a^{14}+52488c^5b^3a^{13}-14472c^3b^6a^{1}\right.\right.}\right.}{}$

```
[In] int(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_
RETURNVERBOSE)
```

```
[Out] 1/81/a^3*sum((-_R^4*b^3-9*_R^2*a*b^2-27*_R*a^2*c-27*a^2*b)/(2*_R^5*b^3+12*_
R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R=RootOf(_Z^6*b^3+9*_Z^4*a*b
^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))-1/27/a^3/x
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} dx = \text{Timed out}$$

```
[In] integrate(1/x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*
a**3),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x^2 (27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} dx = \int \frac{1}{(b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3)x^2} dx$$

```
[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, a
lgorithm="maxima")
```

```
[Out] -1/27*integrate((b^3*x^4 + 9*a*b^2*x^2 + 27*a^2*c*x + 27*a^2*b)/(b^3*x^6 +
9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)/a^3 - 1/27/(a^3*x)
```

Giac [F]

$$\int \frac{1}{x^2 (27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} dx = \int \frac{1}{(b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3)x^2} dx$$

```
[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, a
lgorithm="giac")
```

```
[Out] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.68 (sec) , antiderivative size = 2663, normalized size of antiderivative = 4.13

$$\int \frac{1}{x^2 (27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} dx = \text{Too large to display}$$

```
[In] int(1/(x^2*(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3)),
x)
```

```
[Out] symsum(log(-282429536481*root(355779876259553472*a^23*b^3*c^4*z^6 - 1500946
35296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*
a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^
3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a
^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^23*b^9*(2*b^10*x + 2541
865828329*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^2
4*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^
4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 2824295
36481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2
+ 17496*a^4*b^10*c*z + b^12, z, k)^4*a^17*c^5 - 45*a*b^8*c + 387420489*root
(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45
753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 7531454306
16*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7
*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^
10*c*z + b^12, z, k)^2*a^10*c^6*x - 401769396*root(355779876259553472*a^23*
b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z
^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207
657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*
c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^2*a^
9*b^4*c^3 - 2066242608*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635
296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^
```

16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^3*a^12*b^5*c^2 + 6973568802*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^3*a^13*b^2*c^4 - 4518872583696*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^4*a^16*b^3*c^3 - 328050*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^5*b^6*c^2 - 177147*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^6*b^3*c^4 + 387420489*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^2*a^10*b*c^5 + 23328*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^4*b^8*c*x + 196830*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^5*b^5*c^3*x - 20920706406*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^3*a^13*b*c^5*x + 74401740*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^2*a^8*b^6*c^2*x - 746143164* root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^2*a^9*b^3*c^4*x + 55788550416*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^3*a^12*b^4*c^3*x + 564859072962*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^4*a^16*b^2*c^4*x))*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k), k, 1, 6) - 1/(27*a^3*x)

3.143 $\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal result 1090
Rubi [A] (verified) 1091
Mathematica [C] (verified) 1094
Maple [C] (verified) 1094
Fricas [F(-1)] 1095
Sympy [A] (verification not implemented) 1095
Maxima [F] 1096
Giac [F] 1096
Mupad [B] (verification not implemented) 1096

Optimal result

Integrand size = 26, antiderivative size = 395

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx = -\frac{\sqrt[3]{-2}\left(1+\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} + \frac{\sqrt[6]{\frac{3}{2}}\left(1-(-3)^{2/3}\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} - \frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}} + \frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)+\frac{1}{108}\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)$$

```
[Out] 1/108*(18-(-2)^(2/3)*3^(1/3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/108*(18-2^
(2/3)*3^(1/3))*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/216*ln(6-3*(-3)^(1/3)*2^(2/3
)*x+x^2)*(36+2^(2/3)*3^(1/3)*(1+I*3^(1/2)))+1/2*3^(1/6)*2^(5/6)*(1-(-3)^(2/
3)*2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3
))^(1/2))/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/6*(1-2^(1/3)*3^
(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))
*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)-1/3*(-2)^(1/3)*(1+(-2)^(1/3)*
```

```
3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2)
)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2122, 648, 632, 210, 642, 212}

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= -\frac{\sqrt[3]{-2}\left(1+\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} + \frac{\sqrt[6]{\frac{3}{2}}\left(1-(-3)^{2/3}\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} - \frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{\sqrt[6]{2}3^{5/6}\sqrt{3\sqrt[3]{2}3^{2/3}-4}} + \frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)+\frac{1}{108}\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+\right.$$

```
[In] Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] -(((-2)^(1/3)*(1 + (-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/
Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6
) + 3*2^(1/3)*3^(2/3)])) + ((3/2)^(1/6)*(1 - (-3)^(2/3)*2^(1/3))*ArcTan[(2^
(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/((1
+ (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 - 2^(1/3)*3^(2/3))*Ar
cTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/
(2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((36 + 2^(2/3)*3^(1/3)*(1
+ I*Sqrt[3]))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/216 + ((18 - (-2)^(2/3
)*3^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/108 + ((18 - 2^(2/3)*3^(1
/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/108
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\text{integral} = 1259712\int\left(\frac{(-1)^{2/3}\left(3\sqrt[3]{-3}2^{2/3}+\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)x\right)}{3779136\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}+\frac{(-1)^{2/3}\left(3(-2)^{2/3}\sqrt[3]{3}-\left(1+3\sqrt[3]{-2}3^{2/3}\right)x\right)}{3779136\sqrt[3]{2}3^{2/3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}+\frac{6\sqrt[3]{2}3^{2/3}+\left(18-2^{2/3}\sqrt[3]{3}\right)x}{68024448\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}\right)dx$$

$$=\frac{1}{54}\int\frac{6\sqrt[3]{2}3^{2/3}+\left(18-2^{2/3}\sqrt[3]{3}\right)x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx+\frac{(-1)^{2/3}\int\frac{3(-2)^{2/3}\sqrt[3]{3}-\left(1+3\sqrt[3]{-2}3^{2/3}\right)x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{9\sqrt[3]{2}3^{2/3}}+\frac{(-1)^{2/3}\int\frac{3\sqrt[3]{-3}2^{2/3}+\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)x}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}\,dx}{3\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$=\frac{\left((-1)^{2/3}\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)\right)\int\frac{-3\sqrt[3]{-3}2^{2/3}+2x}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}\,dx}{6\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\left((-1)^{2/3}\sqrt[3]{\frac{3}{2}}\left(6+\sqrt[3]{-3}2^{2/3}\right)\right)\int\frac{1}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}\,dx}{2\left(1+\sqrt[3]{-1}\right)^2}-\frac{\left((-1)^{2/3}\left(6-(-2)^{2/3}\sqrt[3]{3}\right)\right)\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{2\sqrt[3]{2}3^{2/3}}$$

$$+\frac{1}{108}\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\int\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx+\frac{1}{108}\left(18-2^{2/3}\sqrt[3]{3}\right)\int\frac{3\ 2^{2/3}\sqrt[3]{3}+2x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}$$

$$=\frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}\right.$$

$$\left.+i2^{2/3}3^{5/6}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)+\frac{1}{108}\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)+\frac{1}{108}\Big($$

$$= \frac{(-1)^{2/3}\left((-2)^{2/3}-2\ 3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{6^{5/6}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}} - \frac{(-1)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} - \frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}} + \frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}+i2^{2/3}3^{5/6}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)+\frac{1}{108}\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(6+3(-2)^{2/3}\right.$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.15

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)\#1^4}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

```
[In] Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^4)/(
36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.14

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^5\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	56
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^5\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	56

```
[In] int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sum(_R^5/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+
324*_Z^3+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx = \text{Timed out}$$

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.18

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \operatorname{RootSum}\left(72662865048t^6 - 72662865048t^5 + 24163559388t^4 - 2646786132t^3 - 6626610t^2 - 4374t - 1,\right.$$

```
[In] integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(72662865048*_t**6 - 72662865048*_t**5 + 24163559388*_t**4 - 2646786
132*_t**3 - 6626610*_t**2 - 4374*_t - 1, Lambda(_t, _t*log(-892364171310473
76*_t**5/833243797 + 89301949532998128*_t**4/833243797 - 29740560281805852*
_t**3/833243797 + 192466080408420*_t**2/49014341 + 5867255361684*_t/8332437
97 + x + 5365044886/2499731391))
```

Maxima [F]

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx=\int \frac{x^5}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx=\int \frac{x^5}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$=\sum_{k=1}^{6}\ln\left(\frac{362797056\left(19236852\,x\,\mathrm{root}(z^6+4374\,z^5+6626610\,z^4+2646786132\,z^3-24163559388\,z^2+7266\right.}{}\right.$$
$$\left.-z^5+\frac{421\,z^4}{1266}-\frac{100853\,z^3}{2768742}-\frac{505\,z^2}{5537484}-\frac{z}{16612452}-\frac{1}{72662865048},z,k\right)$$

```
[In] int(x^5/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
[Out] symsum(log((362797056*(19236852*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 26467
86132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k) - 19131876
*x - 6482268*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 2416355
9388*z^2 + 72662865048*z - 72662865048, z, k)^2 + 742851*x*root(z^6 + 4374*
z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 7266
2865048, z, k)^3 - 4130*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^
3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^4 + x*root(z^6 + 4
374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z -
72662865048, z, k)^5 - 154944576*root(z^6 + 4374*z^5 + 6626610*z^4 + 264678
6132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^2 + 1704742
2*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72
```

662865048*z - 72662865048, z, k)^3 + 27054*root(z^6 + 4374*z^5 + 6626610*z^
4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^4
 + 9*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 +
 72662865048*z - 72662865048, z, k)^5 + 465542316*root(z^6 + 4374*z^5 + 662
6610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048,
z, k) - 465542316))/root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24
163559388*z^2 + 72662865048*z - 72662865048, z, k)^5)*root(z^6 - z^5 + (421
*z^4)/1266 - (100853*z^3)/2768742 - (505*z^2)/5537484 - z/16612452 - 1/7266
2865048, z, k), k, 1, 6)

3.144 $\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6} \, dx$

Optimal result 1098
Rubi [A] (verified) 1099
Mathematica [C] (verified) 1102
Maple [C] (verified) 1103
Fricas [F(-1)] 1103
Sympy [A] (verification not implemented) 1103
Maxima [F] 1104
Giac [F] 1104
Mupad [B] (verification not implemented) 1104

Optimal result

Integrand size = 26, antiderivative size = 377

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6} \, dx = \frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{9\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} + \frac{\left(9-(-2)^{2/3}\sqrt[3]{3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{27\sqrt{3\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)}} - \frac{\left(9-2^{2/3}\sqrt[3]{3}\right)\text{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{27\sqrt{6\left(-4+3\sqrt[3]{2}3^{2/3}\right)}} + \frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{6\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2} + \frac{\sqrt[3]{-\frac{1}{3}}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{18\ 2^{2/3}} - \frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{18\ 2^{2/3}\sqrt[3]{3}}$$

```
[Out] 1/36*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(1/3)*3^(2/3)/(1+(-1)^(1/3))^2+1/10
8*(-1)^(1/3)*3^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(1/3)-1/108*ln(6+3*
2^(2/3)*3^(1/3)*x+x^2)*2^(1/3)*3^(2/3)+1/27*(-1)^(2/3)*(3*(-3)^(2/3)-2^(2/3
```

```
))*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*3^(5
/6)/(1+(-1)^(1/3))^2/(8-6*(-3)^(2/3)*2^(1/3))^(1/2)-1/27*(9-2^(2/3)*3^(1/3)
)*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))/(-24
+18*2^(1/3)*3^(2/3))^(1/2)+1/27*(9-(-2)^(2/3)*3^(1/3))*arctan((3*(-2)^(2/3)
*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(24+27*I*2^(1/3)*3^(1/6)+9*
2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2122, 648, 632, 210, 642, 212}

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx$$

$$= \frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{9\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}$$
$$+\frac{\left(9-(-2)^{2/3}\sqrt[3]{3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{27\sqrt{3\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)}}$$
$$-\frac{\left(9-2^{2/3}\sqrt[3]{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{27\sqrt{6\left(3\sqrt[3]{2}3^{2/3}-4\right)}}+\frac{\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{6\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}$$
$$+\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{18\ 2^{2/3}}-\frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{18\ 2^{2/3}\sqrt[3]{3}}$$

```
[In] Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sq
rt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(9*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[2*(4
- 3*(-3)^(2/3)*2^(1/3))]) + ((9 - (-2)^(2/3)*3^(1/3))*ArcTan[(3*(-2)^(2/3)*
3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(27*Sqrt[3*(8 + (9*I)*2
^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))]) - ((9 - 2^(2/3)*3^(1/3))*ArcTanh[(2^(
1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(27*Sqrt[6
```

```
*(-4 + 3*2^(1/3)*3^(2/3))]) + Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(6*2^(2
/3)*3^(1/3)*(1 + (-1)^(1/3))^2) + ((-1/3)^(1/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3
)*x + x^2])/(18*2^(2/3)) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(18*2^(2/3)*3
^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\text{integral} = 1259712 \int \left(\frac{(-1)^{2/3}\left(-2+\sqrt[3]{-3}2^{2/3}x\right)}{7558272\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}2^{2/3}x-x^2\right)} + \frac{(-1)^{2/3}\left(2+(-2)^{2/3}\sqrt[3]{3}x\right)}{7558272\sqrt[3]{2}3^{2/3}\left(-1+\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)} - \frac{\sqrt[3]{2}+\sqrt[3]{3}x}{11337408\ 6^{2/3}\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)} \right) dx$$

$$= -\frac{(-1)^{2/3}\int \frac{2+(-2)^{2/3}\sqrt[3]{3}x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{18\sqrt[3]{2}3^{2/3}} - \frac{\int \frac{\sqrt[3]{2}+\sqrt[3]{3}x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{9\ 6^{2/3}} + \frac{(-1)^{2/3}\int \frac{-2+\sqrt[3]{-3}2^{2/3}x}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{6\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\int \frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{18\ 2^{2/3}} - \frac{\int \frac{3\ 2^{2/3}\sqrt[3]{3}+2x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{18\ 2^{2/3}\sqrt[3]{3}} + \frac{\int \frac{3\sqrt[3]{-3}2^{2/3}-2x}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{6\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2} - \frac{\left(\sqrt[3]{-1}\left(9+\sqrt[3]{-3}2^{2/3}\right)\right)\int \frac{1}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{18\left(1+\sqrt[3]{-1}\right)^2} - \frac{1}{54}\left(-9+(-2)^{2/3}\sqrt[3]{3}\right)\int \frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx - \frac{1}{54}\left(-9+2^{2/3}\sqrt[3]{3}\right)\int \frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}$$

$$= \frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{6\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2} + \frac{\sqrt[3]{-\frac{1}{3}}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{18\ 2^{2/3}} - \frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{18\ 2^{2/3}\sqrt[3]{3}} + \frac{\left(\sqrt[3]{-1}\left(9+\sqrt[3]{-3}2^{2/3}\right)\right)\text{Subst}\left(\int \frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}\,dx, x, 3\sqrt[3]{-3}2^{2/3}-2x\right)}{9\left(1+\sqrt[3]{-1}\right)^2} - \frac{1}{27}\left(9-(-2)^{2/3}\sqrt[3]{3}\right)\text{Subst}\left(\int \frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}\,dx, x, 3(-2)^{2/3}\sqrt[3]{3}+2x\right) - \frac{1}{27}\left(9-2^{2/3}\right.$$

$$= -\frac{\sqrt[3]{-1}\left(9+\sqrt[3]{-3}2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{9\left(1+\sqrt[3]{-1}\right)^2\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}$$

$$-\frac{\left((-2)^{2/3}-3\ 3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{27\sqrt[6]{3}\sqrt{2\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}$$

$$+\frac{\left(2^{2/3}-3\ 3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{27\sqrt[6]{3}\sqrt{2\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}+\frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{6\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$+\frac{\sqrt[3]{-\frac{1}{3}}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{18\ 2^{2/3}}-\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{18\ 2^{2/3}\sqrt[3]{3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.16

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)\#1^3}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

```
[In] Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^3)/(
36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.15

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	56
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	56

```
[In] int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
[Out] 1/6*sum(_R^4/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+
324*_Z^3+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx=\text{Timed out}$$

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.17

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$=\mathrm{RootSum}\left(15695178850368t^6-2066242608t^4+1845163152t^3-1180980t^2-1944t-1,\left(t\mapsto t\log\left(\frac{61}{}\right.\right.\right.$$

```
[In] integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216),x)
[Out] RootSum(15695178850368*_t**6 - 2066242608*_t**4 + 1845163152*_t**3 - 118098
0*_t**2 - 1944*_t - 1, Lambda(_t, _t*log(6147145261785517462O8*_t**5/571212
95165 - 1270857362386176*_t**4/57121295165 - 80483053187684376*_t**3/571212
95165 + 72431318325103884*_t**2/57121295165 - 45358602689088*_t/57121295165
 + x - 44532180783/57121295165)))
```

Maxima [F]

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x^4}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x^4}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.03

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \sum_{k=1}^{6} \ln\left(-\frac{5038848\left(1377495072\,x+17006112\,x\,\mathrm{root}(z^6+1944\,z^5+1180980\,z^4-1845163152\,z^3+20662\right.}{}\right.$$
$$\left. -\frac{z^4}{7596}+\frac{217\,z^3}{1845828}-\frac{5\,z^2}{66449808}-\frac{z}{8073651672}-\frac{1}{15695178850368},z,k\right)$$

```
[In] int(x^4/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
[Out] symsum(log(-(5038848*(1377495072*x + 17006112*x*root(z^6 + 1944*z^5 + 11809
80*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k) - 104976*x
*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 1569
5178850368, z, k)^2 + 158112*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 18451631
52*z^3 + 2066242608*z^2 - 15695178850368, z, k)^3 + 1946*x*root(z^6 + 1944*
z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)
^4 + 3*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^
2 - 15695178850368, z, k)^5 - 4251528*root(z^6 + 1944*z^5 + 1180980*z^4 - 1
845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^2 + 3927852*root(z^6
 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 1569517885036
8, z, k)^3 - 1188*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066
```

```
242608*z^2 - 15695178850368, z, k)^4 - root(z^6 + 1944*z^5 + 1180980*z^4 -
1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^5 + 7558272*root(z^
6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 156951788503
68, z, k) + 33519046752))/root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^
3 + 2066242608*z^2 - 15695178850368, z, k)^5)*root(z^6 - z^4/7596 + (217*z^
3)/1845828 - (5*z^2)/66449808 - z/8073651672 - 1/15695178850368, z, k), k,
1, 6)
```

3.145 $\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal result 1106
Rubi [A] (verified) 1107
Mathematica [C] (verified) 1110
Maple [C] (verified) 1110
Fricas [F(-1)] 1111
Sympy [A] (verification not implemented) 1111
Maxima [F] 1111
Giac [F] 1112
Mupad [B] (verification not implemented) 1112

Optimal result

Integrand size = 26, antiderivative size = 361

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx = -\frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{6\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{\sqrt[3]{-1}\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{9\,2^{2/3}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{18\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}} - \frac{(-1)^{2/3}\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{36\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} + \frac{(-1)^{2/3}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{108\sqrt[3]{2}3^{2/3}} + \frac{\log\left(6+3\,2^{2/3}\sqrt[3]{3}x+x^2\right)}{108\sqrt[3]{2}3^{2/3}}$$

```
[Out] -1/216*(-1)^(2/3)*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^
(1/3))^2+1/648*(-1)^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)+
1/648*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/36*arctan((3*(-3)^(1/
```

```
3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(1/6)/(1+(-1)^(
1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)+1/108*arctanh(2^(1/6)*(3*3^(1/3)+2^(
1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3
))^(1/2)+1/54*(-1)^(1/3)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3
)*3^(2/3))^(1/2))*2^(1/3)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))
^(1/2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2122, 648, 632, 210, 642, 212}

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx = -\frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{6\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{\sqrt[3]{-1}\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{9\ 2^{2/3}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{18\sqrt[6]{2}3^{5/6}\sqrt{3\sqrt[3]{2}3^{2/3}-4}} - \frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{36\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} + \frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{108\sqrt[3]{2}3^{2/3}} + \frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{108\sqrt[3]{2}3^{2/3}}$$

```
[In] Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] -1/6*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]
]/(2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((-
1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/
3))]])/(9*2^(2/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3
)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2
```

```
/3))]]/(18*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*Log[
6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(36*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2)
+ ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(108*2^(1/3)*3^(2/3))
+ Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(108*2^(1/3)*3^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\text{integral} = 1259712 \int \left(\frac{(-1)^{2/3}x}{22674816\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}2^{2/3}x-x^2\right)} - \frac{(-1)^{2/3}x}{22674816\sqrt[3]{2}3^{2/3}\left(-1+\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)} + \frac{x}{68024448\sqrt[3]{2}3^{2/3}\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)} \right) dx$$

$$= \frac{\int \frac{x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{54\sqrt[3]{2}3^{2/3}} + \frac{(-1)^{2/3}\int \frac{x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{54\sqrt[3]{2}3^{2/3}} + \frac{(-1)^{2/3}\int \frac{x}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{18\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\int \frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{18\ 2^{2/3}} + \frac{\int \frac{3\ 2^{2/3}\sqrt[3]{3}+2x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{108\sqrt[3]{2}3^{2/3}} + \frac{(-1)^{2/3}\int \frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{108\sqrt[3]{2}3^{2/3}} - \frac{\int \frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{18\ 2^{2/3}\sqrt[3]{3}} - \frac{(-1)^{2/3}\int \frac{3\sqrt[3]{-3}2^{2/3}-2x}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{36\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} - \frac{\int \frac{1}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{6\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$= -\frac{(-1)^{2/3}\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{36\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} + \frac{(-1)^{2/3}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{108\sqrt[3]{2}3^{2/3}} + \frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{108\sqrt[3]{2}3^{2/3}} - \frac{\sqrt[3]{-\frac{1}{3}}\text{Subst}\left(\int \frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}\,dx, x, 3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{9\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-6\left(4-3\sqrt[3]{2}3^{2/3}\right)-x^2}\,dx, x, 3\ 2^{2/3}\sqrt[3]{3}+2x\right)}{9\ 2^{2/3}\sqrt[3]{3}} + \frac{\text{Subst}\left(\int \frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}\,dx, x, 3\sqrt[3]{-3}2^{2/3}-2x\right)}{3\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{6\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{18\sqrt[6]{2}3^{5/6}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$
$$+ \frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{18\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}} - \frac{(-1)^{2/3}\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{36\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$
$$+ \frac{(-1)^{2/3}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{108\sqrt[3]{2}3^{2/3}} + \frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{108\sqrt[3]{2}3^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.17

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4 + \#1^6\&,\frac{\log(x-\#1)\#1^2}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

```
[In] Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^2)/(
36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.16

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^3\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	56
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^3\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	56

```
[In] int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sum(_R^3/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+
324*_Z^3+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{216 + 108x^2 + 324x^3 + 18x^4 + x^6}\, dx = \text{Timed out}$$

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.17

$$\int \frac{x^3}{216 + 108x^2 + 324x^3 + 18x^4 + x^6}\, dx = \text{RootSum}\left(3390158631679488t^6 - 74384733888t^4 - 1332145440t^3 - 1417176t^2 - 1, \left(t \mapsto t\log\left(-\frac{84823\ldots}{}\right.\right.\right.$$

```
[In] integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216),x)
[Out] RootSum(3390158631679488*_t**6 - 74384733888*_t**4 - 1332145440*_t**3 - 141
7176*_t**2 - 1, Lambda(_t, _t*log(-8482372214243328*_t**5/415817 + 22160559
10930560*_t**4/415817 - 2062546612992*_t**3/415817 - 57027208896*_t**2/4158
17 - 416583756*_t/415817 + x - 89938/415817)))
```

Maxima [F]

$$\int \frac{x^3}{216 + 108x^2 + 324x^3 + 18x^4 + x^6}\, dx = \int \frac{x^3}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\, dx$$

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx=\int \frac{x^3}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$=\sum_{k=1}^{6}\ln\left(-\frac{23328\left(297538935552\,x-7992872640\,x\,\mathrm{root}(z^6+1417176\,z^4+1332145440\,z^3+74384733888\,z\right.}{}\right.$$
$$\left.-\frac{z^4}{45576}-\frac{235\,z^3}{598048272}-\frac{z^2}{2392193088}-\frac{1}{3390158631679488},z,k\right)$$

```
[In] int(x^3/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
[Out] symsum(log(-(23328*(297538935552*x - 7992872640*x*root(z^6 + 1417176*z^4 +
1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k) + 52488*x*root(z
^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z,
k)^3 + 2904*x*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3
390158631679488, z, k)^4 + x*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 7438
4733888*z^2 - 3390158631679488, z, k)^5 - 153055008*root(z^6 + 1417176*z^4
+ 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k)^2 - 2764368*ro
ot(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488,
 z, k)^3 - 1620*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 -
 3390158631679488, z, k)^4 - 3673320192*root(z^6 + 1417176*z^4 + 1332145440
*z^3 + 74384733888*z^2 - 3390158631679488, z, k) + 7240114098432))/root(z^6
 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k)
^5)*root(z^6 - z^4/45576 - (235*z^3)/598048272 - z^2/2392193088 - 1/3390158
631679488, z, k), k, 1, 6)
```

3.146 $\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal result 1113
Rubi [A] (verified) 1114
Mathematica [C] (verified) 1116
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Sympy [A] (verification not implemented) 1118
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Giac [F] 1119
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Optimal result

Integrand size = 26, antiderivative size = 248

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{(-1)^{2/3}\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{(-1)^{2/3}\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{81\sqrt[3]{2}\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{81\ 2^{5/6}\sqrt[6]{3}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$

```
[Out] 1/162*(-1)^(2/3)*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3
))^(1/2))*2^(1/6)*3^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1
/486*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2
^(1/6)*3^(5/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)+1/486*(-1)^(2/3)*arctan((3*(-2)
^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(2/3)*3^(5/6)/(8+9*
I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2122, 632, 210, 212}

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{(-1)^{2/3}\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\,2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{(-1)^{2/3}\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{81\sqrt[3]{2}\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{81\,2^{5/6}\sqrt[6]{3}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}$$

```
[In] Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] ((-1)^(2/3)*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(27*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((-1)^(2/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(81*2^(1/3)*3^(1/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(81*2^(5/6)*3^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\text{integral} = 1259712\int\left(\frac{(-1)^{2/3}}{22674816\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}2^{2/3}x-x^2\right)}-\frac{(-1)^{2/3}}{22674816\sqrt[3]{2}3^{2/3}\left(-1+\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}+\frac{1}{68024448\sqrt[3]{2}3^{2/3}\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}\right)dx$$

$$=\frac{\int\frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{54\sqrt[3]{2}3^{2/3}}+\frac{(-1)^{2/3}\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{54\sqrt[3]{2}3^{2/3}}+\frac{(-1)^{2/3}\int\frac{1}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{18\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$=-\frac{\text{Subst}\left(\int\frac{1}{-6\left(4-3\sqrt[3]{2}3^{2/3}\right)-x^2}\,dx,x,3\ 2^{2/3}\sqrt[3]{3}+2x\right)}{27\sqrt[3]{2}3^{2/3}}-\frac{(-1)^{2/3}\text{Subst}\left(\int\frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}\,dx,x,3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{27\sqrt[3]{2}3^{2/3}}-\frac{(-1)^{2/3}\text{Subst}\left(\int\frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}\,dx,x,3\sqrt[3]{-3}2^{2/3}-2x\right)}{9\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$= \frac{(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{(-1)^{2/3}\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{81\ 2^{5/6}\sqrt[6]{3}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}} - \frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{81\ 2^{5/6}\sqrt[6]{3}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4 + \#1^6\&, \frac{\log(x-\#1)\#1}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

```
[In] Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1)/(36
 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.23

method	result	size
default	$\frac{\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}}{6}$	56
risch	$\frac{\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}}{6}$	56

```
[In] int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
[Out] 1/6*sum(_R^2/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+
324*_Z^3+108*_Z^2+216))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1277 vs. 2(162) = 324.

Time = 0.93 (sec) , antiderivative size = 1277, normalized size of antiderivative = 5.15

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
[Out] 1/324*sqrt(1/633)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81)*log(1/211*sqrt(1/633)*
(3*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 3741*18^(2/3) - 4988*18^(1/3) - 24867
)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81) - 1/422*(6*18^(2/3) + 8*18^(1/3) + 81)
^2 + 2*x + 729/211*18^(2/3) + 972/211*18^(1/3) + 8289/422) - 1/324*sqrt(1/6
33)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81)*log(-1/211*sqrt(1/633)*(3*(6*18^(2/3
) + 8*18^(1/3) + 81)^2 - 3741*18^(2/3) - 4988*18^(1/3) - 24867)*sqrt(6*18^(
2/3) + 8*18^(1/3) + 81) - 1/422*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 2*x + 72
9/211*18^(2/3) + 972/211*18^(1/3) + 8289/422) - 1/136728*sqrt(1266)*sqrt(-2
/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 4
8*18^(1/3) + 371) - 8/9*18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)
^2 + 18*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(
1/3) + 371)*(6*18^(2/3) + 8*18^(1/3) + 81) + 1/211*(6*sqrt(1266)*(6*18^(2/3
) + 8*18^(1/3) + 81)^2 + 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36
*18^(2/3) + 48*18^(1/3) + 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)
 - 211*sqrt(1266)) - 1247*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273
*sqrt(1266))*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)
^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*
18^(2/3) - 3888*18^(1/3) - 16578) + 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3)
+ sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) +
 371) - 8/9*18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 18*sqrt
(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*
(6*18^(2/3) + 8*18^(1/3) + 81) - 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/
3) + 81)^2 + 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) +
48*18^(1/3) + 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(
1266)) - 1247*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))
*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(
2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3
888*18^(1/3) - 16578) - 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3) - sqrt(-1/27
*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*
18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 18*sqrt(-1/27*(6*18
^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*18^(2/3)
+ 8*18^(1/3) + 81) + 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)^2 -
 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3)
+ 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(1266)) - 124
7*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))*sqrt(-2/3*1
```

```
8^(2/3) - sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18
^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3888*18^(1/3)
 - 16578) + 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3) - sqrt(-1/27*(6*18^(2/3)
 + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 1
8)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 18*sqrt(-1/27*(6*18^(2/3) + 8*1
8^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*18^(2/3) + 8*18^(1/3)
 + 81) - 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 9*sqrt(-1/2
7*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*sq
rt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(1266)) - 1247*sqrt(1266)
*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))*sqrt(-2/3*18^(2/3) - sq
rt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371
) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3888*18^(1/3) - 16578)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.19

$$\int \frac{x^2}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} \, dx$$
$$= \text{RootSum}\left(732274264442769408t^6 - 2677850419968t^4 + 2834352t^2 - 1, \left(t \mapsto t \log\left(10170475895038464t\right.\right.\right.$$

```
[In] integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
[Out] RootSum(732274264442769408*_t**6 - 2677850419968*_t**4 + 2834352*_t**2 - 1,
 Lambda(_t, _t*log(10170475895038464*_t**5 - 5231726283456*_t**4 - 31809932
496*_t**3 + 19131876*_t**2 + 19683*_t + x - 27/2)))
```

Maxima [F]

$$\int \frac{x^2}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} \, dx = \int \frac{x^2}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216} \, dx$$

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x^2}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")

[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \sum_{k=1}^{6} \ln\left(-\frac{216\left(32134205039616\,x-1836660096\,\mathrm{root}(z^6-2834352\,z^4+2677850419968\,z^2-7322742644\right.}{}\right.$$
$$\left. -\frac{z^4}{273456}+\frac{z^2}{258356853504}-\frac{1}{732274264442769408},z,k\right)$$

[In] int(x^2/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)

[Out] symsum(log(-(216*(32134205039616*x - 1836660096*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^2 - 1889568*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^3 + 972*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^4 + root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^5 + 132239526912*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k) + 204073344*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^2 + 139968*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^3 + 36*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^4 + 863230245120*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k) + 781932322630656))/root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^5)*root(z^6 - z^4/273456 + z^2/258356853504 - 1/732274264442769408, z, k), k, 1, 6)

3.147 $\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal result 1120
Rubi [A] (verified) 1121
Mathematica [C] (verified) 1124
Maple [C] (verified) 1124
Fricas [F(-1)] 1125
Sympy [A] (verification not implemented) 1125
Maxima [F] 1125
Giac [F] 1126
Mupad [B] (verification not implemented) 1126

Optimal result

Integrand size = 24, antiderivative size = 361

$$\begin{aligned}\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx = &-\frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{36\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} \\ &+\frac{\sqrt[3]{-1}\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{54\ 2^{2/3}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} \\ &+\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{108\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}} \\ &+\frac{(-1)^{2/3}\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{216\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} \\ &-\frac{(-1)^{2/3}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{648\sqrt[3]{2}3^{2/3}} \\ &-\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{648\sqrt[3]{2}3^{2/3}}\end{aligned}$$

```
[Out] 1/1296*(-1)^(2/3)*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^
(1/3))^2-1/3888*(-1)^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)
-1/3888*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/216*arctan((3*(-3)^
```

```
(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(1/6)/(1+(-1
)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)+1/648*arctanh(2^(1/6)*(3*3^(1/3)+
2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(
2/3))^(1/2)+1/324*(-1)^(1/3)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^
(1/3)*3^(2/3))^(1/2))*2^(1/3)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2
/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2122, 648, 632, 210, 642, 212}

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx = -\frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{36\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\sqrt[3]{-1}\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{54\,2^{2/3}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{108\sqrt[6]{2}3^{5/6}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}+\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{216\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{648\sqrt[3]{2}3^{2/3}}-\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}{648\sqrt[3]{2}3^{2/3}}$$

```
[In] Int[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] -1/36*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))
]]/(2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((
-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2
/3))]])/(54*2^(2/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2
/3)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^
```

```
(2/3))]]/(108*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((-1)^(2/3)*L
og[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(216*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))
^2) - ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(648*2^(1/3)*3^(2/
3)) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(648*2^(1/3)*3^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\text{integral} = 1259712\int\left(\frac{(-1)^{2/3}\left(3\sqrt[3]{-3}2^{2/3}-x\right)}{136048896\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}2^{2/3}x-x^2\right)}+\frac{(-1)^{2/3}\left(3(-2)^{2/3}\sqrt[3]{3}+x\right)}{136048896\sqrt[3]{2}3^{2/3}\left(-1+\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}-\frac{6\sqrt[3]{3}+\sqrt[3]{2}x}{408146688\ 6^{2/3}\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}\right)dx$$

$$=-\frac{(-1)^{2/3}\int\frac{3(-2)^{2/3}\sqrt[3]{3}+x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{324\sqrt[3]{2}3^{2/3}}-\frac{\int\frac{6\sqrt[3]{3}+\sqrt[3]{2}x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{324\ 6^{2/3}}+\frac{(-1)^{2/3}\int\frac{3\sqrt[3]{-3}2^{2/3}-x}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{108\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$=\frac{\sqrt[3]{-\frac{1}{3}}\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{108\ 2^{2/3}}-\frac{\int\frac{3\ 2^{2/3}\sqrt[3]{3}+2x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{648\sqrt[3]{2}3^{2/3}}-\frac{(-1)^{2/3}\int\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{648\sqrt[3]{2}3^{2/3}}-\frac{\int\frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{108\ 2^{2/3}\sqrt[3]{3}}+\frac{(-1)^{2/3}\int\frac{3\sqrt[3]{-3}2^{2/3}-2x}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{216\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\int\frac{1}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{36\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$=\frac{(-1)^{2/3}\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{216\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{(-1)^{2/3}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{648\sqrt[3]{2}3^{2/3}}-\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{648\sqrt[3]{2}3^{2/3}}-\frac{\sqrt[3]{-\frac{1}{3}}\text{Subst}\left(\int\frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}\,dx,x,3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{54\ 2^{2/3}}+\frac{\text{Subst}\left(\int\frac{1}{-6\left(4-3\sqrt[3]{2}3^{2/3}\right)-x^2}\,dx,x,3\ 2^{2/3}\sqrt[3]{3}+2x\right)}{54\ 2^{2/3}\sqrt[3]{3}}+\frac{\text{Subst}\left(\int\frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}\,dx,x,3\sqrt[3]{-3}2^{2/3}-2x\right)}{18\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{36\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{108\sqrt[6]{2}3^{5/6}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{108\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}} + \frac{(-1)^{2/3}\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{216\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} - \frac{(-1)^{2/3}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{648\sqrt[3]{2}3^{2/3}} - \frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{648\sqrt[3]{2}3^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.16

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

```
[In] Integrate[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36 + 16
2*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.15

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	54
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	54

```
[In] int(x/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sum(_R/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+32
4*_Z^3+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx = \text{Timed out}$$

```
[In] integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.17

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \text{RootSum}\left(1581712411196381921 28t^6 - 96402615118848t^4 + 287743415040t^3 - 51018336t^2 - 1, \left(t \mapsto t\right.\right.$$

```
[In] integrate(x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(158171241119638192128*_t**6 - 96402615118848*_t**4 + 287743415040*_
t**3 - 51018336*_t**2 - 1, Lambda(_t, _t*log(65418399445721140961280*_t**5/
415817 + 2480926457425102848*_t**4/415817 - 39451802929737984*_t**3/415817
+ 118071997444800*_t**2/415817 - 16745884920*_t/415817 + x - 268790/415817)
))
```

Maxima [F]

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

```
[In] integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

```
[In] integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
[Out] integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.49

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx = \sum_{k=1}^{6} \ln\left(x+\mathrm{root}\left(z^6-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{15817124111963819212 8},z,k\right)\left(216\,x+\mathrm{root}\left(z^6-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{158171241119638192128},z,k\right)\left(51018336\,x-\right.\right.\right.$$
$$-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{158171241119638192128},z,k\Big)$$

```
[In] int(x/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
[Out] symsum(log(x + root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/310028
2242048 - 1/158171241119638192128, z, k)*(216*x + root(z^6 - z^4/1640736 +
(235*z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k)
*(51018336*x - root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/310028
2242048 - 1/158171241119638192128, z, k)*(277947894528*x - root(z^6 - z^4/1
640736 + (235*z^3)/129178426752 - z^2/3100282242048 - 1/15817124111963819212
28, z, k)*(33192121254912*x - root(z^6 - z^4/1640736 + (235*z^3)/1291784267
52 - z^2/3100282242048 - 1/158171241119638192128, z, k)*(6940988288557056*x
 + 1688973816882216 96) + 28563737812992)))))*root(z^6 - z^4/1640736 + (235*
z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k), k,
1, 6)
```

3.148 $\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal result 1127
Rubi [A] (verified) 1128
Mathematica [C] (verified) 1131
Maple [C] (verified) 1132
Fricas [F(-1)] 1132
Sympy [A] (verification not implemented) 1132
Maxima [F] 1133
Giac [F] 1133
Mupad [B] (verification not implemented) 1134

Optimal result

Integrand size = 22, antiderivative size = 377

$$
\begin{aligned}
\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx
&= \frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{324\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} \\
&\quad + \frac{\left(9-(-2)^{2/3}\sqrt[3]{3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{972\sqrt{3\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)}} \\
&\quad - \frac{\left(9-2^{2/3}\sqrt[3]{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{972\sqrt{6\left(-4+3\sqrt[3]{2}3^{2/3}\right)}} - \frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{216\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2} \\
&\quad - \frac{\sqrt[3]{-\frac{1}{3}}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{648\ 2^{2/3}} + \frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{648\ 2^{2/3}\sqrt[3]{3}}
\end{aligned}
$$

```
[Out] -1/1296*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(1/3)*3^(2/3)/(1+(-1)^(1/3))^2-1
/3888*(-1)^(1/3)*3^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(1/3)+1/3888*ln
(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(1/3)*3^(2/3)+1/972*(-1)^(2/3)*(3*(-3)^(2/3)-
```

```
2^(2/3))*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2)
)*3^(5/6)/(1+(-1)^(1/3))^2/(8-6*(-3)^(2/3)*2^(1/3))^(1/2)-1/972*(9-2^(2/3)*
3^(1/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2
))/(-24+18*2^(1/3)*3^(2/3))^(1/2)+1/972*(9-(-2)^(2/3)*3^(1/3))*arctan((3*(-
2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(24+27*I*2^(1/3)*3^
(1/6)+9*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2095, 648, 632, 210, 642, 212}

$$
\begin{aligned}
&\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx \\
&= \frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{324\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} \\
&\quad+\frac{\left(9-(-2)^{2/3}\sqrt[3]{3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{972\sqrt{3\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)}} \\
&\quad-\frac{\left(9-2^{2/3}\sqrt[3]{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{972\sqrt{6\left(3\sqrt[3]{2}3^{2/3}-4\right)}}-\frac{\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{216\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2} \\
&\quad-\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{648\ 2^{2/3}}+\frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{648\ 2^{2/3}\sqrt[3]{3}}
\end{aligned}
$$

```
[In] Int[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
```

```
[Out] ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sq
rt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(324*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[2*(
4 - 3*(-3)^(2/3)*2^(1/3))]) + ((9 - (-2)^(2/3)*3^(1/3))*ArcTan[(3*(-2)^(2/3
)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(972*Sqrt[3*(8 + (9*I
)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))]) - ((9 - 2^(2/3)*3^(1/3))*ArcTanh[(
2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(972*Sq
```

```
rt[6*(-4 + 3*2^(1/3)*3^(2/3))]) - Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(21
6*2^(2/3)*3^(1/3)*(1 + (-1)^(1/3))^2) - ((-1/3)^(1/3)*Log[6 + 3*(-2)^(2/3)*
3^(1/3)*x + x^2])/(648*2^(2/3)) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(648*2
^(2/3)*3^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2095

```
Int[(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2]
, c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3
^(3*p)*a^(2*p)), Int[ExpandIntegrand[(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2
)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3
)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0] && Eq
Q[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x,
1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\text{integral} = 1259712\int\left(\frac{(-1)^{2/3}\left(-2+6(-3)^{2/3}\sqrt[3]{2}-\sqrt[3]{-3}2^{2/3}x\right)}{272097792\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}2^{2/3}x-x^2\right)}+\frac{2(-1)^{2/3}-6\sqrt[3]{2}3^{2/3}+\sqrt[3]{-3}2^{2/3}x}{272097792\sqrt[3]{2}3^{2/3}\left(-1+\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}+\frac{18-2^{2/3}\sqrt[3]{3}+\sqrt[3]{2}3^{2/3}x}{2448880128\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}\right)dx$$

$$=\frac{\int\frac{18-2^{2/3}\sqrt[3]{3}+\sqrt[3]{2}3^{2/3}x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}dx}{1944}-\frac{\int\frac{2(-1)^{2/3}-6\sqrt[3]{2}3^{2/3}+\sqrt[3]{-3}2^{2/3}x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{648\sqrt[3]{2}3^{2/3}}+\frac{(-1)^{2/3}\int\frac{-2+6(-3)^{2/3}\sqrt[3]{2}-\sqrt[3]{-3}2^{2/3}x}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}dx}{216\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$=-\frac{\sqrt[3]{-\frac{1}{3}}\int\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{648\ 2^{2/3}}+\frac{\int\frac{3\ 2^{2/3}\sqrt[3]{3}+2x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}dx}{648\ 2^{2/3}\sqrt[3]{3}}-\frac{\int\frac{3\sqrt[3]{-3}2^{2/3}-2x}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}dx}{216\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\left(\sqrt[3]{-1}\left(9+\sqrt[3]{-3}2^{2/3}\right)\right)\int\frac{1}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}dx}{648\left(1+\sqrt[3]{-1}\right)^2}-\frac{\left(-9+(-2)^{2/3}\sqrt[3]{3}\right)\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{1944}+\frac{\left(9-2^{2/3}\sqrt[3]{3}\right)\int\frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}dx}{1944}$$

$$=-\frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{216\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\sqrt[3]{-\frac{1}{3}}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{648\ 2^{2/3}}+\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{648\ 2^{2/3}\sqrt[3]{3}}+\frac{\left(\sqrt[3]{-1}\left(9+\sqrt[3]{-3}2^{2/3}\right)\right)\text{Subst}\left(\int\frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}dx,x,3\sqrt[3]{-3}2^{2/3}-2x\right)}{324\left(1+\sqrt[3]{-1}\right)^2}-\frac{1}{972}\left(9-(-2)^{2/3}\sqrt[3]{3}\right)\text{Subst}\left(\int\frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}dx,x,3(-2)^{2/3}\sqrt[3]{3}+2x\right)+\frac{1}{972}\left(-9+2\right.$$

$$= -\frac{\sqrt[3]{-1}\left(9+\sqrt[3]{-3}2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{324\left(1+\sqrt[3]{-1}\right)^2\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}$$

$$- \frac{\left((-2)^{2/3}-3\ 3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{972\sqrt[6]{3}\sqrt{2\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}$$

$$+ \frac{\left(2^{2/3}-3\ 3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{972\sqrt[6]{3}\sqrt{2\left(-4+3\sqrt[3]{2}3^{2/3}\right)}} - \frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{216\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$- \frac{\sqrt[3]{-\frac{1}{3}}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{648\ 2^{2/3}} + \frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{648\ 2^{2/3}\sqrt[3]{3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.16

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)}{36\#1+162\#1^2+12\#1^3+\#1^5}\&\right]$$

```
[In] Integrate[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36*#1 +
 162*#1^2 + 12*#1^3 + #1^5) & ]/6
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.14

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	53
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	53

```
[In] int(1/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
[Out] 1/6*sum(1/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324
*_Z^3+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx = \text{Timed out}$$

```
[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.17

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \text{RootSum}\left(341649880818418494996448t^6 - 3470494144278528t^4 - 86087932019712t^3 - 1530550080t^2 + 69\right.$$

```
[In] integrate(1/(x**6+18*x**4+324*x**3+108*x**2+216),x)
[Out] RootSum(341649880818418494996448*_t**6 - 3470494144278528*_t**4 - 8608793201
9712*_t**3 - 1530550080*_t**2 + 69984*_t - 1, Lambda(_t, _t*log(18590444669
910961141057378 7136*_t**5/57121295165 + 637730125326791738276 6592*_t**4/571
21295165 - 189046360023885643115 52*_t**3/57121295165 - 469080552915181723 96
8*_t**2/57121295165 - 24358640509989936*_t/57121295165 + x + 152427895956/5
7121295165)))
```

Maxima [F]

$$\int \frac{1}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} \, dx = \int \frac{1}{x^6 + 18 \, x^4 + 324 \, x^3 + 108 \, x^2 + 216} \, dx$$

```
[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
[Out] integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{1}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} \, dx = \int \frac{1}{x^6 + 18 \, x^4 + 324 \, x^3 + 108 \, x^2 + 216} \, dx$$

```
[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
[Out] integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.81

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx = \sum_{k=1}^{6} \ln\Bigg(-\mathrm{root}\bigg(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{34164988081841849499648},z,k\bigg)\,x\,6$$

$$+\mathrm{root}\bigg(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{3416498808184184}$$

$$-\mathrm{root}\bigg(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{3416498808184184}$$

$$-\mathrm{root}\bigg(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{3416498808184184}$$

$$-\mathrm{root}\bigg(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{3416498808184184}$$

$$+944784\,\mathrm{root}\bigg(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{}{3416498808}$$

$$-16529940864\,\mathrm{root}\bigg(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{}{34164}$$

$$-33192121254912\,\mathrm{root}\bigg(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{}{34}$$

$$-168897381688221696\,\mathrm{root}\bigg(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}$$

$$-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{34164988081841849499648},z,k\bigg)$$

```
[In] int(1/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

```
[Out] symsum(log(349920*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/
111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^2
*x - 6*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161016071
3728 + z/488182842961846272 - 1/34164988081841849499648, z, k)*x - 61222003
20*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728
 + z/488182842961846272 - 1/34164988081841849499648, z, k)^3*x - 2582637960
59136*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713
728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^4*x - 6940988
288557056*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161016
0713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^5*x + 944
```

```
784*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161016071372
8 + z/488182842961846272 - 1/34164988081841849499648, z, k)^2 - 16529940864
*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 +
 z/488182842961846272 - 1/34164988081841849499648, z, k)^3 - 33192121254912
*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 +
 z/488182842961846272 - 1/34164988081841849499648, z, k)^4 - 16889738168822
1696*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/1116101607137
28 + z/488182842961846272 - 1/34164988081841849499648, z, k)^5)*root(z^6 -
z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 + z/488182842
961846272 - 1/34164988081841849499648, z, k), k, 1, 6)
```

3.149 $\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx$

Optimal result 1136
Rubi [A] (verified) 1137
Mathematica [C] (verified) 1141
Maple [C] (verified) 1142
Fricas [F(-1)] 1142
Sympy [A] (verification not implemented) 1142
Maxima [F] 1143
Giac [F] 1143
Mupad [B] (verification not implemented) 1144

Optimal result

Integrand size = 26, antiderivative size = 415

$$\int \frac{1}{x\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx = \frac{(-1)^{2/3}\left((-2)^{2/3}-2\,3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{216\sqrt[3]{2}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} - \frac{(-1)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{216\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} - \frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{216\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}} + \frac{\log(x)}{216} - \frac{\left(36+2^{2/3}\sqrt[3]{3}(1+i\sqrt{3})\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{46656} - \frac{\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{23328} - \frac{\left(18-2^{2/3}\sqrt[3]{3}\right)\log\left(6+3\,2^{2/3}\sqrt[3]{3}x+x^2\right)}{23328}$$

```
[Out] 1/216*ln(x)-1/23328*(18-(-2)^(2/3)*3^(1/3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2
)-1/23328*(18-2^(2/3)*3^(1/3))*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)-1/46656*ln(6-3
```

```
*(-3)^(1/3)*2^(2/3)*x+x^2)*(36+2^(2/3)*3^(1/3)*(1+I*3^(1/2)))-1/1296*(-1)^(
2/3)*((-3)^(1/3)+3*2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(
-3)^(2/3)*2^(1/3))^(1/2))*6^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))
^(1/2)-1/1296*(1-2^(1/3)*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-1
2+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)+1/
1296*(-1)^(2/3)*((-2)^(2/3)-2*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(2
4+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(2/3)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^
(1/3)*3^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2122, 648, 632, 210, 642, 212}

$$\int \frac{1}{x\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx$$
$$=\frac{(-1)^{2/3}\left((-2)^{2/3}-2\ 3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{216\sqrt[3]{2}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}-\frac{(-1)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{216\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\text{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{216\sqrt[6]{2}3^{5/6}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}-\frac{\left(36+2^{2/3}\sqrt[3]{3}(1+i\sqrt{3})\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{46656}-\frac{\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{23328}-\frac{\left(18-2^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{23328}+\frac{\log(x)}{216}$$

```
[In] Int[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] ((-1)^(2/3)*((-2)^(2/3) - 2*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sq
rt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(216*2^(1/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1
```

```
/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*((-3)^(1/3) + 3*2^(1/3))*Ar
cTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))
]])/(216*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 -
 2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*
2^(1/3)*3^(2/3))]])/(216*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + Lo
g[x]/216 - ((36 + 2^(2/3)*3^(1/3)*(1 + I*Sqrt[3]))*Log[6 - 3*(-3)^(1/3)*2^(
2/3)*x + x^2])/46656 - ((18 - (-2)^(2/3)*3^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1
/3)*x + x^2])/23328 - ((18 - 2^(2/3)*3^(1/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x +
 x^2])/23328
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
```

```
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\text{integral} = 1259712\int\left(\frac{1}{272097792x}+\frac{(-1)^{2/3}\left(6\left(9+\sqrt[3]{-3}2^{2/3}\right)-\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)x\right)}{816293376\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}+\frac{(-1)^{2/3}\left(-6\left(9-(-2)^{2/3}\sqrt[3]{3}\right)+\left(1+3\sqrt[3]{-2}3^{2/3}\right)x\right)}{816293376\sqrt[3]{2}3^{2/3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}+\frac{-6\sqrt[3]{6}\left(9\sqrt[3]{2}-2\sqrt[3]{3}\right)-\left(18-2^{2/3}\sqrt[3]{3}\right)x}{14693280768\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}\right)dx$$

$$=\frac{\log(x)}{216}+\frac{\int\frac{-6\sqrt[3]{6}\left(9\sqrt[3]{2}-2\sqrt[3]{3}\right)-\left(18-2^{2/3}\sqrt[3]{3}\right)x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}dx}{11664}+\frac{(-1)^{2/3}\int\frac{-6\left(9-(-2)^{2/3}\sqrt[3]{3}\right)+\left(1+3\sqrt[3]{-2}3^{2/3}\right)x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{1944\sqrt[3]{2}3^{2/3}}+\frac{(-1)^{2/3}\int\frac{6\left(9+\sqrt[3]{-3}2^{2/3}\right)-\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)x}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}dx}{648\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$= \frac{\log(x)}{216} + \frac{\left(\left(-\frac{1}{6}\right)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\right)\int \frac{1}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}\,dx}{72\left(1+\sqrt[3]{-1}\right)^2}$$

$$+ \frac{\left((-1)^{2/3}\left(-1+3(-3)^{2/3}\sqrt[3]{2}\right)\right)\int \frac{-3\sqrt[3]{-3}2^{2/3}+2x}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}\,dx}{1296\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$+ \frac{\left(-18+(-2)^{2/3}\sqrt[3]{3}\right)\int \frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{23328}$$

$$+ \frac{\left(-18+2^{2/3}\sqrt[3]{3}\right)\int \frac{3\ 2^{2/3}\sqrt[3]{3}+2x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{23328}$$

$$+ \frac{\left((-1)^{2/3}\left((-2)^{2/3}-2\ 3^{2/3}\right)\right)\int \frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{432\sqrt[3]{6}}$$

$$+ \frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\int \frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{216\ 2^{2/3}\sqrt[3]{3}}$$

$$= \frac{\log(x)}{216} - \frac{(-1)^{2/3}\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{1296\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$- \frac{\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{23328}$$

$$- \frac{\left(18-2^{2/3}\sqrt[3]{3}\right)\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{23328}$$

$$- \frac{\left(\left(-\frac{1}{6}\right)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\right)\mathrm{Subst}\left(\int \frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}\,dx, x, -3\sqrt[3]{-3}2^{2/3}+2x\right)}{36\left(1+\sqrt[3]{-1}\right)^2}$$

$$- \frac{\left((-1)^{2/3}\left((-2)^{2/3}-2\ 3^{2/3}\right)\right)\mathrm{Subst}\left(\int \frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}\,dx, x, 3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{216\sqrt[3]{6}}$$

$$- \frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{-6\left(4-3\sqrt[3]{2}3^{2/3}\right)-x^2}\,dx, x, 3\ 2^{2/3}\sqrt[3]{3}+2x\right)}{108\ 2^{2/3}\sqrt[3]{3}}$$

$$= \frac{(-1)^{2/3}\left((-2)^{2/3}-2\ 3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{216\ 6^{5/6}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$

$$- \frac{(-1)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{216\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$

$$- \frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{216\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}} + \frac{\log(x)}{216}$$

$$- \frac{(-1)^{2/3}\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{1296\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$- \frac{\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{23328}$$

$$- \frac{\left(18-2^{2/3}\sqrt[3]{3}\right)\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{23328}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.25

$$\int \frac{1}{x\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx = \frac{\log(x)}{216}$$

$$- \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{108\log(x-\#1)+324\log(x-\#1)\#1+18\log(x-\#1)\#1^2+\log}{36+162\#1+12\#1^2+\#1^4}\right.}{1296}$$

```
[In] Integrate[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] Log[x]/216 - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (108*Lo
g[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/(3
6 + 162*#1 + 12*#1^2 + #1^4) & ]/1296
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.18

method	result
risch	$\frac{\ln(x)}{216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(136728\_Z^6+1230552\_Z^5+3682908\_Z^4+3630708\_Z^3-81810\_Z^2+486\_Z-1\right)}\_R\ln\left(-23672342955240\_R^5-21\right.\right.}{}$
default	$\frac{\ln(x)}{216}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(\_R^5+18\_R^3+324\_R^2+108\_R\right)\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{1296}$

```
[In] int(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

```
[Out] 1/216*ln(x)+1/1944*sum(_R*ln(-23672342955240*_R^5-213056277916248*_R^4-6376
89647288592*_R^3-628763677061560*_R^2+14004611129596*_R+2499731391*x-551330
83786),_R=RootOf(136728*_Z^6+1230552*_Z^5+3682908*_Z^4+3630708*_Z^3-81810*_
Z^2+486*_Z-1))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx=\text{Timed out}$$

```
[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.20

$$\int \frac{1}{x\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx=\frac{\log(x)}{216}$$
$$+\mathrm{RootSum}\left(737963742567783949192396 8t^6+341649880818418494996 48t^5+52598809250685370368t^4\right.$$

```
[In] integrate(1/x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] log(x)/216 + RootSum(7379637425677839491923968*_t**6 + 34164988081841849499
648*_t**5 + 52598809250685370368*_t**4 + 26673506015311872*_t**3 - 30917111
6160*_t**2 + 944784*_t - 1, Lambda(_t, _t*log(814557009966881793678336211 51
```

```
19297360560128*_t**6/1434257993090524400063 + 977068766770806381087358257564
745728*_t**5/1434257993090524400063 - 11652952660885126428840097153906153881
6*_t**4/1434257993090524400063 - 239359794985242202542501440710766592*_t**3/
1434257993090524400063 - 136678312638137094439887341418240*_t**2/14342579930
9052440063 + 1563115569067663795735413696*_t/1434257993090524400063 + x - 31
64446315075236190044/1434257993090524400063)))
```

Maxima [F]

$$\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx = \int \frac{1}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)x}\,dx$$

```
[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
[Out] -1/216*integrate((x^5 + 18*x^3 + 324*x^2 + 108*x)/(x^6 + 18*x^4 + 324*x^3 +
 108*x^2 + 216), x) + 1/216*log(x)
```

Giac [F]

$$\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx = \int \frac{1}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)x}\,dx$$

```
[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
[Out] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x), x)
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx = \frac{\ln(x)}{216}$$

$$+\left(\sum_{k=1}^{6}\ln\left(\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{7810925487389540352}-\right.\right.\right.$$

$$-\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{7810925487389540352}-\frac{}{737}\right.$$

$$+\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{7810925487389540352}-\frac{}{737}\right.$$

$$-\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{7810925487389540352}-\frac{}{737}\right.$$

$$-\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{7810925487389540352}-\frac{}{737}\right.$$

$$-\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{7810925487389540352}-\frac{}{737}\right.$$

$$+839808\,\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{7810925487389540352}\right.$$

$$+594896472576\,\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{781092548738}\right.$$

$$-8483430130458624\,\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{}{7810925487}\right.$$

$$-3831425535283494912\,\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{}{781092}\right.$$

$$+1217393817906599165952\,\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{}{781}\right.$$

$$\left.\left.\left.+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{7810925487389540352}-\frac{1}{7379637425677839491923968},z,k\right)\right)\right)$$

```
[In] int(1/(x*(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)),x)
```

```
[Out] log(x)/216 + symsum(log(7*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853
*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352
- 1/7379637425677839491923968, z, k)*x - 5670000*root(z^6 + z^5/216 + (421*
z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 +
 z/7810925487389540352 - 1/7379637425677839491923968, z, k)^2*x + 154687594
```

```
7520*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432
- (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/73796374256778394
91923968, z, k)^3*x - 106961147905609728*root(z^6 + z^5/216 + (421*z^4)/590
66496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/78109
25487389540352 - 1/7379637425677839491923968, z, k)^4*x - 14051199585413401
8048*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432
- (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/73796374256778394
91923968, z, k)^5*x - 4560729056738761900032O*root(z^6 + z^5/216 + (421*z^4
)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/
7810925487389540352 - 1/7379637425677839491923968, z, k)^6*x + 839808*root(
z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2
)/12053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968,
z, k)^2 + 594896472576*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^
3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1
/7379637425677839491923968, z, k)^3 - 8483430130458624*root(z^6 + z^5/216 +
 (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/1205389735708
2624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^4 - 38314
25535283494912*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902
540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637
425677839491923968, z, k)^5 + 1217393817906599165952*root(z^6 + z^5/216 + (
421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/120538973570826
24 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^6)*root(z^6
 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/1
2053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z,
k), k, 1, 6)
```

3.150 $\int \frac{1}{x^2(216+108x^2+324x^3+18x^4+x^6)}\,dx$

Optimal result . . . 1146
Rubi [A] (verified) . . . 1147
Mathematica [C] (verified) . . . 1151
Maple [C] (verified) . . . 1152
Fricas [F(-1)] . . . 1152
Sympy [A] (verification not implemented) . . . 1152
Maxima [F] . . . 1153
Giac [F] . . . 1153
Mupad [B] (verification not implemented) . . . 1154

Optimal result

Integrand size = 26, antiderivative size = 448

$$\begin{aligned}
&\int \frac{1}{x^2\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx \\
&= -\frac{1}{216x} - \frac{\left(27\sqrt[3]{-6}-(-2)^{2/3}+12\,3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{5832\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} \\
&- \frac{(-1)^{2/3}\left(6(-6)^{2/3}+27\sqrt[3]{-3}-\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{1944\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} \\
&- \frac{\left(\sqrt[3]{2}+27\sqrt[3]{3}-6\,6^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{5832\sqrt[6]{6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}} \\
&- \frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}2^{2/3}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{1296\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} \\
&+ \frac{\left(3(-6)^{2/3}+2\sqrt[3]{-2}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{7776\sqrt[3]{3}} \\
&- \frac{\left(2^{2/3}-3\,3^{2/3}\right)\log\left(6+3\,2^{2/3}\sqrt[3]{3}x+x^2\right)}{3888\sqrt[3]{6}}
\end{aligned}$$

```
[Out] -1/216/x-1/7776*(-1)^(2/3)*(9+(-3)^(1/3)*2^(2/3))*ln(6-3*(-3)^(1/3)*2^(2/3)
*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^(1/3))^2+1/23328*(3*(-6)^(2/3)+2*(-2)^(1/3)
)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*3^(2/3)-1/23328*(2^(2/3)-3*3^(2/3))*ln(6
+3*2^(2/3)*3^(1/3)*x+x^2)*6^(2/3)-1/11664*(-1)^(2/3)*(6*(-6)^(2/3)+27*(-3)^
(1/3)-2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(
1/3))^(1/2))*6^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/3499
2*(2^(1/3)+27*3^(1/3)-6*6^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12
+9*2^(1/3)*3^(2/3))^(1/2))*6^(5/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)-1/17496*(27
*(-6)^(1/3)-(-2)^(2/3)+12*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18
*(-2)^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3
))^(1/2)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2122, 648, 632, 210, 642, 212}

$$\int \frac{1}{x^2(216+108x^2+324x^3+18x^4+x^6)}\,dx$$
$$= -\frac{\left(27\sqrt[3]{-6}-(-2)^{2/3}+12\ 3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{5832\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} - \frac{(-1)^{2/3}\left(6(-6)^{2/3}+27\sqrt[3]{-3}-\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{1944\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} - \frac{\left(\sqrt[3]{2}+27\sqrt[3]{3}-6\ 6^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{5832\sqrt[6]{6}\sqrt{3\sqrt[3]{2}3^{2/3}-4}} - \frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}2^{2/3}\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{1296\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} + \frac{\left(3(-6)^{2/3}+2\sqrt[3]{-2}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{7776\sqrt[3]{3}} - \frac{\left(2^{2/3}-3\ 3^{2/3}\right)\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{3888\sqrt[3]{6}} - \frac{1}{216x}$$

```
[In] Int[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] -1/216*1/x - ((27*(-6)^(1/3) - (-2)^(2/3) + 12*3^(2/3))*ArcTan[(3*(-2)^(2/3
)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(5832*3^(1/6)*Sqrt[8
+ (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*(6*(-6)^(2/3) +
 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[
3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(1944*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3
*(-3)^(2/3)*2^(1/3)]) - ((2^(1/3) + 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/6
)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(5832*6^(1/6)
*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*(9 + (-3)^(1/3)*2^(2/3))*Log[6
 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(1296*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2)
 + ((3*(-6)^(2/3) + 2*(-2)^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(7
776*3^(1/3)) - ((2^(2/3) - 3*3^(2/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/(
3888*6^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\text{integral} = 1259712 \int \left(\frac{1}{272097792x^2} + \frac{(-1)^{2/3} \left(-1 + 9(-3)^{2/3}\sqrt[3]{2} + 27\sqrt[3]{-3}2^{2/3} - \left(9 + \sqrt[3]{-3}2^{2/3}\right)x\right)}{816293376\sqrt[3]{2}3^{2/3}\left(1 + \sqrt[3]{-1}\right)^2\left(6 - 3\sqrt[3]{-3}2^{2/3}x + x^2\right)} + \frac{(-1)^{2/3}\left(1 + 27(-2)^{2/3}\sqrt[3]{3} + 9\sqrt[3]{-2}3^{2/3} + \left(9 - (-2)^{2/3}\sqrt[3]{3}\right)x\right)}{816293376\sqrt[3]{2}3^{2/3}\left(1 - \sqrt[3]{-1}\right)\left(1 + \sqrt[3]{-1}\right)^2\left(6 + 3(-2)^{2/3}\sqrt[3]{3}x + x^2\right)} + \frac{-54 + 2^{2/3}\sqrt[3]{3} + 54\sqrt[3]{2}3^{2/3} - 6^{2/3}\left(2^{2/3} - 3\ 3^{2/3}\right)x}{14693280768\left(6 + 3\ 2^{2/3}\sqrt[3]{3}x + x^2\right)} \right) dx$$

$$= -\frac{1}{216x} + \frac{\int \frac{-54+2^{2/3}\sqrt[3]{3}+54\sqrt[3]{2}3^{2/3}-6^{2/3}\left(2^{2/3}-3\ 3^{2/3}\right)x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2} dx}{11664} + \frac{(-1)^{2/3}\int \frac{1+27(-2)^{2/3}\sqrt[3]{3}+9\sqrt[3]{-2}3^{2/3}+\left(9-(-2)^{2/3}\sqrt[3]{3}\right)x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2} dx}{1944\sqrt[3]{2}3^{2/3}} + \frac{(-1)^{2/3}\int \frac{-1+9(-3)^{2/3}\sqrt[3]{2}+27\sqrt[3]{-3}2^{2/3}-\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{6-3\sqrt[3]{-3}2^{2/3}x+x^2} dx}{648\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$= -\frac{1}{216x} - \frac{\left((-1)^{2/3}\left(9+\sqrt[3]{-3}2^{2/3}\right)\right)\int \frac{-3\sqrt[3]{-3}2^{2/3}+2x}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}\,dx}{1296\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$
$$-\frac{\left((-1)^{2/3}\left((-2)^{2/3}-3\ 3^{2/3}\right)\right)\int \frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{3888\sqrt[3]{6}}$$
$$-\frac{\left(2^{2/3}-3\ 3^{2/3}\right)\int \frac{3\ 2^{2/3}\sqrt[3]{3}+2x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{3888\sqrt[3]{6}}$$
$$+\frac{\left((-1)^{2/3}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)\right)\int \frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{3888\sqrt[3]{2}3^{2/3}}$$
$$+\frac{\left((-1)^{2/3}\left(3\sqrt[3]{-3}2^{2/3}\left(-9-\sqrt[3]{-3}2^{2/3}\right)+2\left(-1+9(-3)^{2/3}\sqrt[3]{2}+27\sqrt[3]{-3}2^{2/3}\right)\right)\right)\int \frac{1}{6-3\sqrt[3]{-3}2^{2/3}x+}}{1296\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$
$$+\frac{\left(18\sqrt[3]{2}\left(2^{2/3}-3\ 3^{2/3}\right)+2\left(-54+2^{2/3}\sqrt[3]{3}+54\sqrt[3]{2}3^{2/3}\right)\right)\int \frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{23328}$$

$$= -\frac{1}{216x} - \frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}2^{2/3}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{1296\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$
$$-\frac{(-1)^{2/3}\left((-2)^{2/3}-3\ 3^{2/3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{3888\sqrt[3]{6}}$$
$$-\frac{\left(2^{2/3}-3\ 3^{2/3}\right)\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{3888\sqrt[3]{6}}$$
$$-\frac{\left((-1)^{2/3}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)\right)\text{Subst}\left(\int \frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}\,dx,x,3(-2)^{2/3}\sqrt[3]{3}+\right.}{1944\sqrt[3]{2}3^{2/3}}$$
$$-\frac{\left((-1)^{2/3}\left(3\sqrt[3]{-3}2^{2/3}\left(-9-\sqrt[3]{-3}2^{2/3}\right)+2\left(-1+9(-3)^{2/3}\sqrt[3]{2}+27\sqrt[3]{-3}2^{2/3}\right)\right)\right)\text{Subst}\left(\int \frac{}{-6\left(4-\right.}\right.}{648\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$
$$-\frac{\left(18\sqrt[3]{2}\left(2^{2/3}-3\ 3^{2/3}\right)+2\left(-54+2^{2/3}\sqrt[3]{3}+54\sqrt[3]{2}3^{2/3}\right)\right)\text{Subst}\left(\int \frac{1}{-6\left(4-3\sqrt[3]{2}3^{2/3}\right)-x^2}\,dx,x,3\ 2^{2/\right.}{11664}$$

$$= -\frac{1}{216x} + \frac{(-1)^{2/3}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{5832\ 2^{5/6}\sqrt[6]{3}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$
$$+ \frac{(-1)^{2/3}\left(2-12(-3)^{2/3}\sqrt[3]{2}-27\sqrt[3]{-3}2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{1944\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$
$$+ \frac{\left(18\ 2^{2/3}-27\ 3^{2/3}-\sqrt[3]{6}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{5832\sqrt[6]{2}\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}$$
$$- \frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}2^{2/3}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{1296\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$
$$- \frac{(-1)^{2/3}\left((-2)^{2/3}-3\ 3^{2/3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{3888\sqrt[3]{6}}$$
$$- \frac{\left(2^{2/3}-3\ 3^{2/3}\right)\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{3888\sqrt[3]{6}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^2\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx = -\frac{1}{216x} - \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{108\log(x-\#1)+324\log(x-\#1)\#1+18\log(x-\#1)\#1^2+\log(x-\#1)\#1^4}{36\#1+162\#1^2+12\#1^3+\#1^5}\&\right]}{1296}$$

```
[In] Integrate[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
[Out] -1/216*1/x - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (108*Lo
g[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/(3
6*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/1296
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.15

method	result
risch	$-\frac{1}{216x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(633\_Z^6+204849\_Z^4-5446980\_Z^3-80433\_Z^2-72\_Z-1\right)}\_R\ln\left(-462040439801351484393\_R^5+13642318\right.\right.}{}$
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-\_R^4-18\_R^2-324\_R-108\right)\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{1296}-\frac{1}{216x}$

```
[In] int(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

```
[Out] -1/216/x+1/11664*sum(_R*ln(-462040439801351484393*_R^5+13642318659339253O8*
_R^4-149523740969574483417612*_R^3+397631047190316263673604 2*_R^2+469674545
43463546461111*_R+247008995694079835 90*x-25597852658707816584),_R=RootOf(63
3*_Z^6+204849*_Z^4-5446980*_Z^3-80433*_Z^2-72*_Z-1))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx=\text{Timed out}$$

```
[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^2\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx$$
$$=\mathrm{RootSum}\left(1594001683946413330255577088t^6+37916120264603316387 84t^4-864367269958950912 0t^3-\right.$$
$$-\frac{1}{216x}$$

```
[In] integrate(1/x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(1594001683946413330255577088*_t**6 + 3791612026460331638784*_t**4 -
 864367269958950912 0*_t**3 - 10942820851968*_t**2 - 839808*_t - 1, Lambda(_
```

```
t, _t*log(-4987553276190249600329356123691446802841 6*_t**5/1235044978470399
1795 + 1262548987243162038800597520049766 4*_t**4/12350449784703991795 - 118
637692607573771238550798852644864*_t**3/12350449784703991795 + 270486324927
832147818193778754816*_t**2/12350449784703991795 + 273914194897479402961199
352*_t/12350449784703991795 + x - 12798926329353908292/12350449784703991795
))) - 1/(216*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (216+108x^2+324x^3+18x^4+x^6)} \, dx = \int \frac{1}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)x^2} \, dx$$

```
[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
[Out] -1/216/x - 1/216*integrate((x^4 + 18*x^2 + 324*x + 108)/(x^6 + 18*x^4 + 324
*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{1}{x^2 (216+108x^2+324x^3+18x^4+x^6)} \, dx = \int \frac{1}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)x^2} \, dx$$

```
[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
[Out] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx$$

$$= \left(\sum_{k=1}^{6}\ln\left(\frac{5\,\mathrm{root}\left(z^6+\frac{281\,z^4}{118132992}-\frac{50435\,z^3}{9300846726144}-\frac{331\,z^2}{48215589428330496}-\frac{z}{189805489343565830553 6}-\frac{1}{15940016839464133302 5\ldots}\right.}{8}\right.\right.$$

$$-\frac{\mathrm{root}\left(z^6+\frac{281\,z^4}{118132992}-\frac{50435\,z^3}{9300846726144}-\frac{331\,z^2}{48215589428330496}-\frac{z}{1898054893435658305536}-\frac{1}{1594001683946413330255577088},z,\ldots\right.}{216}$$

$$-\mathrm{root}\left(z^6+\frac{281\,z^4}{118132992}-\frac{50435\,z^3}{9300846726144}-\frac{331\,z^2}{48215589428330496}-\frac{z}{1898054893435658305536}-\frac{\ldots}{159400\ldots}\right.$$

$$-\mathrm{root}\left(z^6+\frac{281\,z^4}{118132992}-\frac{50435\,z^3}{9300846726144}-\frac{331\,z^2}{48215589428330496}-\frac{z}{1898054893435658305536}-\frac{\ldots}{159400\ldots}\right.$$

$$+\mathrm{root}\left(z^6+\frac{281\,z^4}{118132992}-\frac{50435\,z^3}{9300846726144}-\frac{331\,z^2}{48215589428330496}-\frac{z}{1898054893435658305536}-\frac{\ldots}{159400\ldots}\right.$$

$$-\mathrm{root}\left(z^6+\frac{281\,z^4}{118132992}-\frac{50435\,z^3}{9300846726144}-\frac{331\,z^2}{48215589428330496}-\frac{z}{1898054893435658305536}-\frac{\ldots}{159400\ldots}\right.$$

$$+2344464\,\mathrm{root}\left(z^6+\frac{281\,z^4}{118132992}-\frac{50435\,z^3}{9300846726144}-\frac{331\,z^2}{48215589428330496}-\frac{z}{1898054893435658305536}-\ldots\right.$$

$$-210297580992\,\mathrm{root}\left(z^6+\frac{281\,z^4}{118132992}-\frac{50435\,z^3}{9300846726144}-\frac{331\,z^2}{48215589428330496}-\frac{z}{18980548934356583\ldots}\right.$$

$$-10535082310656\,\mathrm{root}\left(z^6+\frac{281\,z^4}{118132992}-\frac{50435\,z^3}{9300846726144}-\frac{331\,z^2}{48215589428330496}-\frac{z}{1898054893435658\ldots}\right.$$

$$-168897381688221696\,\mathrm{root}\left(z^6+\frac{281\,z^4}{118132992}-\frac{50435\,z^3}{9300846726144}-\frac{331\,z^2}{48215589428330496}-\frac{z}{18980548934\ldots}\right.$$

$$+\frac{281\,z^4}{118132992}-\frac{50435\,z^3}{9300846726144}-\frac{331\,z^2}{48215589428330496}-\frac{z}{1898054893435658305536}$$

$$\left.\left.\left.-\frac{1}{1594001683946413330255577088},z,k\right)\right)\right)-\frac{1}{216\,x}$$

```
[In] int(1/(x^2*(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)),x)
```

```
[Out] symsum(log((5*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 -
(331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/1594001683946413
330255577088, z, k))/8 - (root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300
846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/1594
001683946413330255577088, z, k)*x)/216 - 396252*root(z^6 + (281*z^4)/118132
992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/189805489
```

```
3435658305536 - 1/15940016839464133302555770888, z, k)^2*x - 598229670528*ro
ot(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/482155
89428330496 - z/18980548934356583055536 - 1/15940016839464133302555770888, z,
 k)^3*x + 82120746212352*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/93008
46726144 - (331*z^2)/48215589428330496 - z/18980548934356583055536 - 1/15940
0168394641333025557708 8, z, k)^4*x - 6940988288557056*root(z^6 + (281*z^4)/
118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/189
8054893435658305536 - 1/15940016839464133302555770888, z, k)^5*x + 2344464*r
oot(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215
589428330496 - z/18980548934356583055536 - 1/15940016839464133302555770888, z
, k)^2 - 210297580992*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/93008467
26144 - (331*z^2)/48215589428330496 - z/18980548934356583055536 - 1/15940016
839464133302555770888, z, k)^3 - 10535082310656*root(z^6 + (281*z^4)/1181329
92 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893
435658305536 - 1/15940016839464133302555770888, z, k)^4 - 168897381688221696
*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/482
15589428330496 - z/18980548934356583055536 - 1/15940016839464133302555770888,
 z, k)^5)*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331
*z^2)/48215589428330496 - z/18980548934356583055536 - 1/159400168394641333302
55577088, z, k), k, 1, 6) - 1/(216*x)
```

3.151 $\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2} \, dx$

Optimal result 1157
Rubi [A] (verified) 1158
Mathematica [C] (verified) 1166
Maple [C] (verified) 1166
Fricas [F(-1)] 1167
Sympy [A] (verification not implemented) 1167
Maxima [F] 1167
Giac [F] 1168
Mupad [B] (verification not implemented) 1168

Optimal result

Integrand size = 26, antiderivative size = 1064

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6+\sqrt[3]{-3}2^{2/3}\right)+\left(2-2^{2/3}\left(6(-6)^{2/3}+27\sqrt[3]{-3}\right)\right)x\right)}{162\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$

$$-\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6-(-2)^{2/3}\sqrt[3]{3}\right)+\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)x\right)}{729\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{9\left(6-2^{2/3}\sqrt[3]{3}\right)+\left(2+2^{2/3}\left(27\sqrt[3]{3}-6\ 6^{2/3}\right)\right)x}{1458\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$-\frac{i\left((-2)^{2/3}+6\ 3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{162\ 2^{5/6}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$

$$-\frac{\sqrt[3]{-1}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{162\sqrt[6]{2}3^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$

$$-\frac{\sqrt[3]{-1}\left(6(-6)^{2/3}+27\sqrt[3]{-3}-\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{81\sqrt{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}}$$

$$+\frac{\left(i2^{2/3}-9\sqrt[6]{3}-3i3^{2/3}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{162\ 2^{5/6}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$

$$-\frac{\left(1+3\sqrt[3]{2}3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{1458\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$

$$+\frac{\left(\sqrt[3]{2}+27\sqrt[3]{3}-6\ 6^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{81\sqrt{2}3^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}-\frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{972\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^4}$$

$$i\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)\qquad\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)$$

```
[Out] -1/972*(-1)^(1/3)*3^(2/3)*(54+9*(-3)^(1/3)*2^(2/3)+(2-2^(2/3)*(6*(-6)^(2/3)
+27*(-3)^(1/3)))*x)*2^(1/3)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*
(-3)^(1/3)*2^(2/3)*x+x^2)-1/4374*(-1)^(1/3)*3^(2/3)*(54-9*(-2)^(2/3)*3^(1/3
)+(2+27*(-2)^(2/3)*3^(1/3)+12*(-2)^(1/3)*3^(2/3))*x)*2^(1/3)/(8+9*I*2^(1/3)
*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/8748*(54-9*2^(
2/3)*3^(1/3)+(2+2^(2/3)*(27*3^(1/3)-6*6^(2/3)))*x)*2^(1/3)*3^(2/3)/(4-3*2^(
1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)-1/972*(-1)^(1/3)*(2+27*(-2)^(2/3)
*3^(1/3)+12*(-2)^(1/3)*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-
2)^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/
(4+3*(-2)^(1/3)*3^(2/3))^(3/2)-1/5832*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2
/3)*3^(1/3)/(1+(-1)^(1/3))^4+1/5832*I*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2
/3)*3^(5/6)/(1+(-1)^(1/3))^5-1/52488*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*
3^(1/3)-1/486*(-1)^(1/3)*(6*(-6)^(2/3)+27*(-3)^(1/3)-2^(1/3))*arctan(2^(1/6
)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*3^(1/6)/(1+(-1)
^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(3/2)*2^(1/2)+1/486*(2^(1/3)+27*3^(1/3)-
6*6^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1
/2))*3^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-4+3*2^(1/3)*3^(2/3))^(3/2)
*2^(1/2)+1/972*(I*2^(2/3)-9*3^(1/6)-3*I*3^(2/3))*arctan(2^(1/6)*(3*(-3)^(1/
3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*2^(1/6)*3^(2/3)/(1+(-1)^(1/3
))^5/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/972*I*((-2)^(2/3)+6*3^(2/3))*arctan((
3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(1/6)*3^(2/3)
/(1+(-1)^(1/3))^5/(4+3*(-2)^(1/3)*3^(2/3))^(1/2)-1/8748*(1+3*2^(1/3)*3^(2/3
))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(
5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 1064, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used

$= \{2122, 652, 632, 210, 648, 642, 212\}$

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\left(\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)x+9\left(6-(-2)^{2/3}\sqrt[3]{3}\right)\right)}{729\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}$$

$$-\frac{\sqrt[3]{-1}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{162\sqrt[6]{2}3^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$

$$-\frac{i\left((-2)^{2/3}+6\ 3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{162\ 2^{5/6}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$

$$+\frac{\left(i2^{2/3}-9\sqrt[6]{3}-3i3^{2/3}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{162\ 2^{5/6}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$

$$-\frac{\sqrt[3]{-1}\left(6(-6)^{2/3}+27\sqrt[3]{-3}-\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{81\sqrt{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}}$$

$$+\frac{\left(\sqrt[3]{2}+27\sqrt[3]{3}-6\ 6^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{81\sqrt{2}3^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$

$$-\frac{\left(1+3\sqrt[3]{2}3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{1458\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}-\frac{\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{972\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^4}$$

$$+\frac{i\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{972\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}-\frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{8748\sqrt[3]{2}3^{2/3}}$$

$$-\frac{\sqrt[3]{-\frac{1}{3}}\left(\left(2-3\ 2^{2/3}\left(2(-6)^{2/3}+9\sqrt[3]{-3}\right)\right)x+9\left(6+\sqrt[3]{-3}2^{2/3}\right)\right)}{162\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}$$

$$+\frac{\left(2+2^{2/3}\left(27\sqrt[3]{3}-6\ 6^{2/3}\right)\right)x+9\left(6-2^{2/3}\sqrt[3]{3}\right)}{1458\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}$$

```
[In] Int[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] -1/162*((-1/3)^(1/3)*(9*(6 + (-3)^(1/3)*2^(2/3)) + (2 - 3*2^(2/3)*(2*(-6)^(
2/3) + 9*(-3)^(1/3)))*x))/(2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(
1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-1/3)^(1/3)*(9*(6 - (-2)^(2/3
)*3^(1/3)) + (2 + 27*(-2)^(2/3)*3^(1/3) + 12*(-2)^(1/3)*3^(2/3))*x))/(729*2
^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^
(1/3)*x + x^2)) + (9*(6 - 2^(2/3)*3^(1/3)) + (2 + 2^(2/3)*(27*3^(1/3) - 6*6
^(2/3)))*x)/(1458*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^
(1/3)*x + x^2)) - ((I/162)*((-2)^(2/3) + 6*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^
(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2^(5/6)*3^(1/3)*(1 + (-1
)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) - ((-1)^(1/3)*(2 + 27*(-2)^(2/3)
*3^(1/3) + 12*(-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[
6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(162*2^(1/6)*3^(5/6)*(1 - (-1)^(1/3))^2*(1
+ (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((-1)^(1/3)*(6*(-6)^(2/
3) + 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/S
qrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(81*Sqrt[2]*3^(5/6)*(1 + (-1)^(1/3))^4*
(4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) + ((I*2^(2/3) - 9*3^(1/6) - (3*I)*3^(2/3)
)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1
/3))]])/(162*2^(5/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/
3)]) - ((1 + 3*2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/S
qrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(1458*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)
*3^(2/3)]) + ((2^(1/3) + 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3
) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(81*Sqrt[2]*3^(5/6)*(1 -
 (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - Log[6 -
 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(972*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^4) + (
(I/972)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(
1/3))^5) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(8748*2^(1/3)*3^(2/3))
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 632</u>

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\text{integral} = 1586874322944 \int \left(\frac{\sqrt[3]{-\frac{1}{3}}\left(-1+3(-3)^{2/3}\sqrt[3]{2}+\left(9+\sqrt[3]{-3}2^{2/3}\right)x\right)}{42845606719488\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)^2} + \frac{27\left(2+(-1)^{2/3}\right)-\left(1+\sqrt[3]{-1}\right)x}{771220920950784\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^5\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} + \frac{\sqrt[3]{-\frac{1}{3}}\left(-1-3\sqrt[3]{-2}3^{2/3}+\left(9-(-2)^{2/3}\sqrt[3]{3}\right)x\right)}{42845606719488\ 2^{2/3}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2} + \frac{i(-27+x)}{771220920950784\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)} + \frac{1-3\sqrt[3]{2}3^{2/3}-\left(9-2^{2/3}\sqrt[3]{3}\right)x}{42845606719488\ 2^{2/3}\sqrt[3]{3}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)^2} - \frac{-27+x}{6940988288557056\sqrt[3]{2}3^{2/3}\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)} \right) dx$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\int \frac{-1-3\sqrt[3]{-2}3^{2/3}+\left(9-(-2)^{2/3}\sqrt[3]{3}\right)x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}dx}{243\ 2^{2/3}} - \frac{\int \frac{-27+x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}dx}{4374\sqrt[3]{2}3^{2/3}} + \frac{\int \frac{1-3\sqrt[3]{2}3^{2/3}-\left(9-2^{2/3}\sqrt[3]{3}\right)x}{\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)^2}dx}{243\ 2^{2/3}\sqrt[3]{3}} + \frac{\int \frac{27\left(2+(-1)^{2/3}\right)-\left(1+\sqrt[3]{-1}\right)x}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}dx}{486\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^5} + \frac{i\int \frac{-27+x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{486\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5} + \frac{\sqrt[3]{-\frac{1}{3}}\int \frac{-1+3(-3)^{2/3}\sqrt[3]{2}+\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)^2}dx}{27\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4}$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6+\sqrt[3]{-3}2^{2/3}\right)+\left(2-3\ 2^{2/3}\left(2(-6)^{2/3}+9\sqrt[3]{-3}\right)\right)x\right)}{162\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$

$$-\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6-(-2)^{2/3}\sqrt[3]{3}\right)+\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)x\right)}{1458\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{9\left(6-2^{2/3}\sqrt[3]{3}\right)+\left(2+2^{2/3}\left(27\sqrt[3]{3}-6\ 6^{2/3}\right)\right)x}{1458\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$-\frac{\int\frac{3\ 2^{2/3}\sqrt[3]{3}+2x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}dx}{8748\sqrt[3]{2}3^{2/3}}+\frac{i\int\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{972\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}-\frac{\int\frac{-3\sqrt[3]{-3}2^{2/3}+2x}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}dx}{972\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^4}$$

$$+\frac{\left(\sqrt[3]{-\frac{1}{3}}\left(6(-6)^{2/3}+27\sqrt[3]{-3}-\sqrt[3]{2}\right)\right)\int\frac{1}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}dx}{162\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}$$

$$-\frac{\left(i\left((-2)^{2/3}+6\ 3^{2/3}\right)\right)\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{108\sqrt[3]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}$$

$$-\frac{\left(\sqrt[3]{-\frac{1}{3}}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)\right)\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{1458\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)}$$

$$+\frac{\left(1+3\sqrt[3]{2}3^{2/3}\right)\int\frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}dx}{1458\ 2^{2/3}\sqrt[3]{3}}$$

$$+\frac{\left(3\sqrt[3]{-3}2^{2/3}\left(-1-\sqrt[3]{-1}\right)+54\left(2+(-1)^{2/3}\right)\right)\int\frac{1}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}dx}{972\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^5}$$

$$+\frac{\left(81+3^{2/3}\left(\sqrt[3]{2}-6\ 6^{2/3}\right)\right)\int\frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}dx}{4374\left(4-3\sqrt[3]{2}3^{2/3}\right)}$$

$$
\begin{aligned}
&= -\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6+\sqrt[3]{-3}2^{2/3}\right)+\left(2-3\ 2^{2/3}\left(2(-6)^{2/3}+9\sqrt[3]{-3}\right)\right)x\right)}{162\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} \\
&\quad -\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6-(-2)^{2/3}\sqrt[3]{3}\right)+\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)x\right)}{1458\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)} \\
&\quad +\frac{9\left(6-2^{2/3}\sqrt[3]{3}\right)+\left(2+2^{2/3}\left(27\sqrt[3]{3}-6\ 6^{2/3}\right)\right)x}{1458\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}-\frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{972\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^4} \\
&\quad +\frac{i\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{972\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}-\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{8748\sqrt[3]{2}3^{2/3}} \\
&\quad -\frac{\left(\sqrt[3]{-\frac{1}{3}}\left(6(-6)^{2/3}+27\sqrt[3]{-3}-\sqrt[3]{2}\right)\right)\text{Subst}\left(\int\frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}\,dx,x,-3\sqrt[3]{-3}2^{2/3}+2x\right)}{81\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)} \\
&\quad +\frac{\left(i\left((-2)^{2/3}+6\ 3^{2/3}\right)\right)\text{Subst}\left(\int\frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}\,dx,x,3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{54\sqrt[3]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5} \\
&\quad +\frac{\left(\sqrt[3]{-\frac{1}{3}}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)\right)\text{Subst}\left(\int\frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}\,dx,x,3(-2)^{2/3}\sqrt[3]{3}+2\right)}{729\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)} \\
&\quad -\frac{\left(1+3\sqrt[3]{2}3^{2/3}\right)\text{Subst}\left(\int\frac{1}{-6\left(4-3\sqrt[3]{2}3^{2/3}\right)-x^2}\,dx,x,3\ 2^{2/3}\sqrt[3]{3}+2x\right)}{729\ 2^{2/3}\sqrt[3]{3}} \\
&\quad -\frac{\left(3\sqrt[3]{-3}2^{2/3}\left(-1-\sqrt[3]{-1}\right)+54\left(2+(-1)^{2/3}\right)\right)\text{Subst}\left(\int\frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}\,dx,x,-3\sqrt[3]{-3}2^{2/3}+\right)}{486\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^5} \\
&\quad -\frac{\left(81+3^{2/3}\left(\sqrt[3]{2}-6\ 6^{2/3}\right)\right)\text{Subst}\left(\int\frac{1}{-6\left(4-3\sqrt[3]{2}3^{2/3}\right)-x^2}\,dx,x,3\ 2^{2/3}\sqrt[3]{3}+2x\right)}{2187\left(4-3\sqrt[3]{2}3^{2/3}\right)}
\end{aligned}
$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6+\sqrt[3]{-3}2^{2/3}\right)+\left(2-3\ 2^{2/3}\left(2(-6)^{2/3}+9\sqrt[3]{-3}\right)\right)x\right)}{162\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$

$$-\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6-(-2)^{2/3}\sqrt[3]{3}\right)+\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)x\right)}{1458\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{9\left(6-2^{2/3}\sqrt[3]{3}\right)+\left(2+2^{2/3}\left(27\sqrt[3]{3}-6\ 6^{2/3}\right)\right)x}{1458\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$-\frac{i\left((-2)^{2/3}+6\ 3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{162\ 2^{5/6}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$

$$-\frac{\sqrt[3]{-1}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1458\sqrt[6]{2}3^{5/6}\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$

$$-\frac{\sqrt[3]{-1}\left(6(-6)^{2/3}+27\sqrt[3]{-3}-\sqrt[3]{2}\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{81\sqrt{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}}$$

$$+\frac{\left(i3^{5/6}-9\sqrt[3]{2}\left(2+(-1)^{2/3}\right)\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{486\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$

$$-\frac{\left(1+3\sqrt[3]{2}3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{1458\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$

$$+\frac{\left(\sqrt[3]{2}+27\sqrt[3]{3}-6\ 6^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{729\sqrt{2}3^{5/6}\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$

$$-\frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{972\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^4}$$

$$+\frac{i\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{972\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}-\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{8748\sqrt[3]{2}3^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.16

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \frac{-7884+324x-3990x^2-11610x^3-203x^4-9x^5}{34182\,(216+108x^2+324x^3+18x^4+x^6)} - \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{324\log(x-\#1)-96\log(x-\#1)\#1+324\log(x-\#1)\#1^2+406\log \cdots}{36\#1+162\#1^2+12\#1^3+\# \cdots}\right.}{205092}$$

[In] Integrate[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] (-7884 + 324*x - 3990*x^2 - 11610*x^3 - 203*x^4 - 9*x^5)/(34182*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (324*Log[x - #1] - 96*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 + 406*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) &]/205092

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.11

method	result
default	$\frac{-\frac{1}{3798}x^5-\frac{203}{34182}x^4-\frac{215}{633}x^3-\frac{665}{5697}x^2+\frac{2}{211}x-\frac{146}{633}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\text{RootOf}(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216)}\frac{(-9\_R^4-406\_R^3-324\_ \cdots}{\_R^5+12\_R^3+ \cdots}\right.}{205092}$
risch	$\frac{-\frac{1}{3798}x^5-\frac{203}{34182}x^4-\frac{215}{633}x^3-\frac{665}{5697}x^2+\frac{2}{211}x-\frac{146}{633}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\text{RootOf}(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216)}\frac{(-9\_R^4-406\_R^3-324\_ \cdots}{\_R^5+12\_R^3+ \cdots}\right.}{205092}$

[In] int(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)

[Out] (-1/3798*x^5-203/34182*x^4-215/633*x^3-665/5697*x^2+2/211*x-146/633)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/205092*sum((-9*_R^4-406*_R^3-324*_R^2+96*_R-324)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\text{Timed out}$$

```
[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.11

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$=\text{RootSum}\Big(85256017052964187415123360664576t^6+501051915333854345687 04t^4+4888574805127748\ldots$$
$$+\frac{-9x^5-203x^4-11610x^3-3990x^2+324x-7884}{34182x^6+615276x^4+11074968x^3+3691656x^2+7383312}$$

```
[In] integrate(x**8/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
[Out] RootSum(85256017052964187415123360664576*_t**6 + 50105191533385434568704*_t
**4 + 4888574805127748 6016*_t**3 + 865447782603408*_t**2 + 3220532460*_t +
4513, Lambda(_t, _t*log(3549203620408417440411919313548348746659076 4032*_t*
*5/356900697070792948475845 - 19474160067218837086826809631017022308224*_t*
*4/71380139414158589695169 + 20779963076545132233894582764903396544*_t**3/3
56900697070792948475845 + 2026521915436700497216219801203 7344*_t**2/3569006
97070792948475845 + 275192468949210532049075145372*_t/356900697070792948475
845 + x + 12902851912921772896220 12/107070209121237884 5427535))) + (-9*x**5
 - 203*x**4 - 11610*x**3 - 3990*x**2 + 324*x - 7884)/(34182*x**6 + 615276*x
**4 + 11074968*x**3 + 3691656*x**2 + 7383312)
```

Maxima [F]

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\int \frac{x^8}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

```
[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
[Out] -1/34182*(9*x^5 + 203*x^4 + 11610*x^3 + 3990*x^2 - 324*x + 7884)/(x^6 + 18*
x^4 + 324*x^3 + 108*x^2 + 216) - 1/34182*integrate((9*x^4 + 406*x^3 + 324*x
^2 - 96*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^8}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

```
[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
[Out] integrate(x^8/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.36

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

```
[In] int(x^8/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
[Out] symsum(log((239491904*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/141
49992416343982992 + (5171*z^2)/5093997269883838712 + (505*z)/133686864350
83133627113728 + 4513/852560170529641874151233606645 76, z, k)*x)/876306843
- (275536*x)/638827688547 - (3848128*root(z^6 + (326*z^4)/554702231619 + (8
113597*z^3)/14149992416343982992 + (5171*z^2)/5093997269883838712 + (505*
z)/13368686435083133627113728 + 4513/8525601705296418741512336066457 6, z, k
))/3606201 - (152363520*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/1
4149992416343982992 + (5171*z^2)/5093997269883838712 + (505*z)/1336868643
5083133627113728 + 4513/85256017052964187415123360664576, z, k)^2*x)/44521
- (698075283456*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/141499924
16343982992 + (5171*z^2)/5093997269883838712 + (505*z)/133686864350831336
27113728 + 4513/85256017052964187415123360664576, z, k)^3*x)/44521 + (13078
9789876224*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343
982992 + (5171*z^2)/5093997269883838712 + (505*z)/1336868643508313362711 3
728 + 4513/85256017052964187415123360664576, z, k)^4*x)/211 - 6940988288557
056*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343982992
+ (5171*z^2)/5093997269883838712 + (505*z)/13368686435083133627113728 + 4
513/85256017052964187415123360664576, z, k)^5*x - (4264220928*root(z^6 + (3
26*z^4)/554702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/5093
997269883838712 + (505*z)/13368686435083133627113728 + 4513/8525601705296
41874151233606645 76, z, k)^2)/44521 - (5086414725120*root(z^6 + (326*z^4)/5
54702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988 3
83387712 + (505*z)/13368686435083133627113728 + 4513/852560170529641874151 2
3360664576, z, k)^3)/44521 + (243585208571904*root(z^6 + (326*z^4)/55470223
1619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387 71
2 + (505*z)/13368686435083133627113728 + 4513/852560170529641874151233606 64
```

```
576, z, k)^4)/211 - 168897381688221696*root(z^6 + (326*z^4)/554702231619 +
(8113597*z^3)/14149992416343982992 + (5171*z^2)/5093997269883833877l2 + (50
5*z)/1336868643508313362711372 8 + 4513/85256017052964187415123360664576, z,
 k)^5 - 48160/23660284761)*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3
)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/1336868
643508313362711372 8 + 4513/85256017052964187415123360664576, z, k), k, 1, 6
) - ((665*x^2)/5697 - (2*x)/211 + (215*x^3)/633 + (203*x^4)/34182 + x^5/379
8 + 146/633)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

3.152 $\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal result 1171
Rubi [A] (verified) 1172
Mathematica [C] (verified) 1180
Maple [C] (verified) 1180
Fricas [F(-1)] 1181
Sympy [A] (verification not implemented) 1181
Maxima [F] 1182
Giac [F] 1182
Mupad [B] (verification not implemented) 1182

Optimal result

Integrand size = 26, antiderivative size = 1005

$$
\begin{aligned}
&\int \frac{x^7}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx \\
&= -\frac{2\left(2\sqrt[3]{-1}3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)x}{972\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} \\
&\quad-\frac{\sqrt[3]{-6}\left(9\sqrt[3]{-2}+2\sqrt[3]{3}\right)-9\left(1+\sqrt[3]{-2}3^{2/3}\right)x}{4374\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)} \\
&\quad+\frac{2\left(2-3\sqrt[3]{2}3^{2/3}\right)-3\left(6-2^{2/3}\sqrt[3]{3}\right)x}{2916\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)} \\
&\quad+\frac{\left(9i+\sqrt[3]{3}\left(2i2^{2/3}-9\sqrt[6]{3}+2\ 2^{2/3}\sqrt{3}\right)\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{5832\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} \\
&\quad+\frac{\left(1+\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{54\sqrt{6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}} \\
&\quad+\frac{\left(9\sqrt[6]{3}+i\left(4\ 2^{2/3}-3\ 3^{2/3}\right)\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1944\ 3^{2/3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4+3\sqrt[3]{-2}3^{2/3}\right)}} \\
&\quad-\frac{\sqrt[3]{-1}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{54\sqrt{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}} \\
&\quad+\frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{54\sqrt{6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} \\
&\quad+\frac{\left(2\ 2^{2/3}+3\ 3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{26244\sqrt[6]{3}\sqrt{2\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}+\frac{i\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{648\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}
\end{aligned}
$$

```
[Out] 1/1944*(-4*(-1)^(1/3)*3^(2/3)-18*6^(1/3)+9*((-2)^(2/3)+2*(-1)^(1/3)*3^(2/3)
)*x)*2^(1/3)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2
/3)*x+x^2)+1/4374*(-(-6)^(1/3)*(9*(-2)^(1/3)+2*3^(1/3))+9*(1+(-2)^(1/3)*3^(
2/3))*x)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*
x+x^2)+1/17496*(4-6*2^(1/3)*3^(2/3)-3*(6-2^(2/3)*3^(1/3))*x)*2^(1/3)*3^(2/3
)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/3888*I*ln(6-3*(-3)^(1
/3)*2^(2/3)*x+x^2)*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^5-1/104976*ln(6+3*2^(2/3)
*3^(1/3)*x+x^2)*2^(1/3)*3^(2/3)-1/324*(-1)^(1/3)*((-3)^(1/3)+3*2^(1/3))*arc
tan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*3^(1/
6)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(3/2)*2^(1/2)-1/7776*ln(6+3*(-
2)^(2/3)*3^(1/3)*x+x^2)*(3^(1/2)+I)*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^5+1/324*
(1+(-2)^(1/3)*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*
3^(2/3))^(1/2))/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(1/3)*3^(2/3))^
(3/2)*6^(1/2)+1/324*(1-2^(1/3)*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*
x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-4+3*2
^(1/3)*3^(2/3))^(3/2)*6^(1/2)+1/5832*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-
18*(-3)^(2/3)*2^(1/3))^(1/2))*(9*I+3^(1/3)*(2*I*2^(2/3)-9*3^(1/6)+2*2^(2/3)
*3^(1/2)))/(1+(-1)^(1/3))^5/(8-6*(-3)^(2/3)*2^(1/3))^(1/2)+1/5832*(9*3^(1/6
)+I*(4*2^(2/3)-3*3^(2/3)))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1
/3)*3^(2/3))^(1/2))*3^(1/3)/(1+(-1)^(1/3))^5/(8+6*(-2)^(1/3)*3^(2/3))^(1/2)
+1/78732*(2*2^(2/3)+3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9
*2^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(-8+6*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 1005, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used

$$= \{2122, 648, 632, 210, 642, 652, 212\}$$

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$

$$= \frac{2\left(2-3\sqrt[3]{2}3^{2/3}\right)-3\left(6-2^{2/3}\sqrt[3]{3}\right)x}{2916\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}$$

$$+ \frac{\left(9i+\sqrt[3]{3}\left(2i2^{2/3}-9\sqrt[6]{3}+2\ 2^{2/3}\sqrt{3}\right)\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{5832\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}$$

$$+ \frac{\left(9\sqrt[6]{3}+i(4\ 2^{2/3}-3\ 3^{2/3})\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1944\ 3^{2/3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}$$

$$+ \frac{\left(1+\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{54\sqrt{6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$

$$- \frac{\sqrt[3]{-1}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{54\sqrt{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}}$$

$$+ \frac{\left(2\ 2^{2/3}+3\ 3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{26244\sqrt[6]{3}\sqrt{2\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}$$

$$+ \frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{54\sqrt{6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} + \frac{i\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{648\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}$$

$$- \frac{\left(i+\sqrt{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{1296\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5} - \frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{17496\ 2^{2/3}\sqrt[3]{3}}$$

$$- \frac{2\left(2\sqrt[3]{-1}3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)x}{972\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}$$

$$- \frac{\sqrt[3]{-6}\left(9\sqrt[3]{-2}+2\sqrt[3]{3}\right)-9\left(1+\sqrt[3]{-2}3^{2/3}\right)x}{\cdots}$$

```
[In] Int[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] -1/972*(2*(2*(-1)^(1/3)*3^(2/3) + 9*6^(1/3)) - 9*((-2)^(2/3) + 2*(-1)^(1/3)
*3^(2/3))*x)/(2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*
(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3)*(9*(-2)^(1/3) + 2*3^(1/3)) - 9*(
1 + (-2)^(1/3)*3^(2/3))*x)/(4374*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(
2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (2*(2 - 3*2^(1/3)*3^(2/3)) - 3*
(6 - 2^(2/3)*3^(1/3))*x)/(2916*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 +
 3*2^(2/3)*3^(1/3)*x + x^2)) + ((9*I + 3^(1/3)*((2*I)*2^(2/3) - 9*3^(1/6) +
 2*2^(2/3)*Sqrt[3]))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)
^(2/3)*2^(1/3))]])/(5832*(1 + (-1)^(1/3))^5*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3
))]) + ((1 + (-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6
*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(54*Sqrt[6]*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/
3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) + ((9*3^(1/6) + I*(4*2^(2/3) - 3*3^
(2/3)))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3
))]])/(1944*3^(2/3)*(1 + (-1)^(1/3))^5*Sqrt[2*(4 + 3*(-2)^(1/3)*3^(2/3))])
- ((-1)^(1/3)*((-3)^(1/3) + 3*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1
/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(54*Sqrt[2]*3^(5/6)*(1 + (-1)^
(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) + ((1 - 2^(1/3)*3^(2/3))*ArcTanh
[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(54*S
qrt[6]*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)
) + ((2*2^(2/3) + 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt
[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(26244*3^(1/6)*Sqrt[2*(-4 + 3*2^(1/3)*3^(2/3
))]) + ((I/648)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(2^(2/3)*3^(5/6)*(1
+ (-1)^(1/3))^5) - ((I + Sqrt[3])*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(1
296*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^5) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2
]/(17496*2^(2/3)*3^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\text{integral} = 1586874322944 \int \left(\frac{-27+3\ 2^{2/3}\sqrt[3]{3}+9i\sqrt{3}+i2^{2/3}3^{5/6}-3i\sqrt[3]{2}\sqrt[6]{3}x}{9254651051409408\left(1+\sqrt[3]{-1}\right)^5\left(-6+3\sqrt[3]{-3}2^{2/3}x-x^2\right)} + \frac{9(-2)^{2/3}+\sqrt[3]{3}\left(\sqrt[3]{-3}+9\sqrt[3]{2}\right)x}{771220920950784\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)^2} + \frac{9\ 2^{2/3}+\sqrt[3]{-1}3^{2/3}\left(1+3\sqrt[3]{-2}3^{2/3}\right)x}{771220920950784\ 2^{2/3}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2} + \frac{2\left(27-9i\sqrt{3}+2i2^{2/3}3^{5/6}\right)-3\sqrt[3]{2}\sqrt[6]{3}\left(i+\sqrt{3}\right)x}{18509302102818816\left(1+\sqrt[3]{-1}\right)^5\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)} + \frac{3\ 2^{2/3}\sqrt[3]{3}-\left(1-3\sqrt[3]{2}3^{2/3}\right)x}{257073640316928\ 2^{2/3}\sqrt[3]{3}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)^2} + \frac{-18-2\ 2^{2/3}\sqrt[3]{3}-\sqrt[3]{2}3^{2/3}x}{83291859462684672\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)} \right) dx$$

$$= \frac{\int \frac{-18-2\ 2^{2/3}\sqrt[3]{3}-\sqrt[3]{2}3^{2/3}x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{52488} + \frac{\int \frac{9\ 2^{2/3}+\sqrt[3]{-1}3^{2/3}\left(1+3\sqrt[3]{-2}3^{2/3}\right)x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}\,dx}{4374\ 2^{2/3}} + \frac{\int \frac{3\ 2^{2/3}\sqrt[3]{3}-\left(1-3\sqrt[3]{2}3^{2/3}\right)x}{\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)^2}\,dx}{1458\ 2^{2/3}\sqrt[3]{3}} + \frac{\int \frac{2\left(27-9i\sqrt{3}+2i2^{2/3}3^{5/6}\right)-3\sqrt[3]{2}\sqrt[6]{3}\left(i+\sqrt{3}\right)x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{11664\left(1+\sqrt[3]{-1}\right)^5} + \frac{\int \frac{-27+3\ 2^{2/3}\sqrt[3]{3}+9i\sqrt{3}+i2^{2/3}3^{5/6}-3i\sqrt[3]{2}\sqrt[6]{3}x}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{5832\left(1+\sqrt[3]{-1}\right)^5} + \frac{\int \frac{9(-2)^{2/3}+\sqrt[3]{3}\left(\sqrt[3]{-3}+9\sqrt[3]{2}\right)x}{\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)^2}\,dx}{486\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4}$$

$$= -\frac{2\left(2\sqrt[3]{-1}3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)x}{972\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$

$$-\frac{\sqrt[3]{-6}\left(9\sqrt[3]{-2}+2\sqrt[3]{3}\right)-9\left(1+\sqrt[3]{-2}3^{2/3}\right)x}{8748\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{2\left(2-3\sqrt[3]{2}3^{2/3}\right)-3\left(6-2^{2/3}\sqrt[3]{3}\right)x}{2916\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$-\frac{\int\frac{3\ 2^{2/3}\sqrt[3]{3}+2x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}dx}{17496\ 2^{2/3}\sqrt[3]{3}}+\frac{i\int\frac{3\sqrt[3]{-3}2^{2/3}-2x}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}dx}{648\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}$$

$$-\frac{\left(9+2\ 2^{2/3}\sqrt[3]{3}\right)\int\frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}dx}{52488}-\frac{\left(i+\sqrt{3}\right)\int\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{1296\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}$$

$$+\frac{\left(1+\sqrt[3]{-2}3^{2/3}\right)\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{972\left(4+3\sqrt[3]{-2}3^{2/3}\right)}+\frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\int\frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}dx}{972\left(4-3\sqrt[3]{2}3^{2/3}\right)}$$

$$+\frac{\left(18(-2)^{2/3}+3\sqrt[3]{-1}6^{2/3}\left(\sqrt[3]{-3}+9\sqrt[3]{2}\right)\right)\int\frac{1}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}dx}{486\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(24-18(-3)^{2/3}\sqrt[3]{2}\right)}$$

$$+\frac{\left(-18(-1)^{5/6}\sqrt{3}+2\left(-27+3\ 2^{2/3}\sqrt[3]{3}+9i\sqrt{3}+i2^{2/3}3^{5/6}\right)\right)\int\frac{1}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}dx}{11664\left(1+\sqrt[3]{-1}\right)^5}$$

$$+\frac{\left(18(-1)^{2/3}\sqrt{3}\left(i+\sqrt{3}\right)+4\left(27-9i\sqrt{3}+2i2^{2/3}3^{5/6}\right)\right)\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{23328\left(1+\sqrt[3]{-1}\right)^5}$$

$$= -\frac{2\left(2\sqrt[3]{-1}3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)x}{972\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$

$$-\frac{\sqrt[3]{-6}\left(9\sqrt[3]{-2}+2\sqrt[3]{3}\right)-9\left(1+\sqrt[3]{-2}3^{2/3}\right)x}{8748\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{2\left(2-3\sqrt[3]{2}3^{2/3}\right)-3\left(6-2^{2/3}\sqrt[3]{3}\right)x}{2916\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{i\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{648\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}$$

$$-\frac{\left(i+\sqrt{3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{1296\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}-\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{17496\ 2^{2/3}\sqrt[3]{3}}$$

$$+\frac{\left(9+2\ 2^{2/3}\sqrt[3]{3}\right)\text{Subst}\left(\int\frac{1}{-6\left(4-3\sqrt[3]{2}3^{2/3}\right)-x^2}\,dx,x,3\ 2^{2/3}\sqrt[3]{3}+2x\right)}{26244}$$

$$-\frac{\left(1+\sqrt[3]{-2}3^{2/3}\right)\text{Subst}\left(\int\frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}\,dx,x,3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{486\left(4+3\sqrt[3]{-2}3^{2/3}\right)}$$

$$-\frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\text{Subst}\left(\int\frac{1}{-6\left(4-3\sqrt[3]{2}3^{2/3}\right)-x^2}\,dx,x,3\ 2^{2/3}\sqrt[3]{3}+2x\right)}{486\left(4-3\sqrt[3]{2}3^{2/3}\right)}$$

$$-\frac{\left(18(-2)^{2/3}+3\sqrt[3]{-1}6^{2/3}\left(\sqrt[3]{-3}+9\sqrt[3]{2}\right)\right)\text{Subst}\left(\int\frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}\,dx,x,-3\sqrt[3]{-3}2^{2/3}+2x\right.}{243\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(24-18(-3)^{2/3}\sqrt[3]{2}\right)}$$

$$-\frac{\left(-18(-1)^{5/6}\sqrt{3}+2\left(-27+3\ 2^{2/3}\sqrt[3]{3}+9i\sqrt{3}+i2^{2/3}3^{5/6}\right)\right)\text{Subst}\left(\int\frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}\,dx,x\right.}{5832\left(1+\sqrt[3]{-1}\right)^5}$$

$$-\frac{\left(18(-1)^{2/3}\sqrt{3}\left(i+\sqrt{3}\right)+4\left(27-9i\sqrt{3}+2i2^{2/3}3^{5/6}\right)\right)\text{Subst}\left(\int\frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}\,dx,x,3(-2)\right.}{11664\left(1+\sqrt[3]{-1}\right)^5}$$

$$
\begin{aligned}
= & -\frac{2\left(2\sqrt[3]{-1}3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)x}{972\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} \\
& -\frac{\sqrt[3]{-6}\left(9\sqrt[3]{-2}+2\sqrt[3]{3}\right)-9\left(1+\sqrt[3]{-2}3^{2/3}\right)x}{8748\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)} \\
& +\frac{2\left(2-3\sqrt[3]{2}3^{2/3}\right)-3\left(6-2^{2/3}\sqrt[3]{3}\right)x}{2916\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)} \\
& -\frac{\left(27-6\ 2^{2/3}\sqrt[3]{3}-9i\sqrt{3}-2i2^{2/3}3^{5/6}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{5832\left(1+\sqrt[3]{-1}\right)^5\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} \\
& +\frac{\left(1+\sqrt[3]{-2}3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{486\sqrt{6}\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}} \\
& -\frac{\left(9i-4i2^{2/3}\sqrt[3]{3}-9\sqrt{3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{5832\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4+3\sqrt[3]{-2}3^{2/3}\right)}} \\
& -\frac{\sqrt[3]{-1}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{54\sqrt{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}} \\
& +\frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{486\sqrt{6}\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} \\
& +\frac{\left(2\ 2^{2/3}+3\ 3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{26244\sqrt[6]{3}\sqrt{2\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}+\frac{i\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{648\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5} \\
& -\frac{\left(i+\sqrt{3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{1296\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}-\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{17496\ 2^{2/3}\sqrt[3]{3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.17

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \frac{648-96x+432x^2+908x^3-18x^4+73x^5}{68364\,(216+108x^2+324x^3+18x^4+x^6)} + \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{96\log(x-\#1)-216\log(x-\#1)\#1+96\log(x-\#1)\#1^2-36\log}{36\#1+162\#1^2+12\#1^3+\#1}\right.}{410184}$$

```
[In] Integrate[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (648 - 96*x + 432*x^2 + 908*x^3 - 18*x^4 + 73*x^5)/(68364*(216 + 108*x^2 +
324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1
^6 & , (96*Log[x - #1] - 216*Log[x - #1]*#1 + 96*Log[x - #1]*#1^2 - 36*Log[
x - #1]*#1^3 + 73*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]
/410184
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.12

method	result
default	$\frac{\frac{73}{68364}x^5-\frac{1}{3798}x^4+\frac{227}{17091}x^3+\frac{4}{633}x^2-\frac{8}{5697}x+\frac{2}{211}}{x^6+18x^4+324x^3+108x^2+216} + \frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)} \frac{\left(73\_R^4-36\_R^3+96\_R^2\right.}{\_R^5+12\_R^3+1}\right.}{410184}$
risch	$\frac{\frac{73}{68364}x^5-\frac{1}{3798}x^4+\frac{227}{17091}x^3+\frac{4}{633}x^2-\frac{8}{5697}x+\frac{2}{211}}{x^6+18x^4+324x^3+108x^2+216} + \frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)} \frac{\left(73\_R^4-36\_R^3+96\_R^2\right.}{\_R^5+12\_R^3+1}\right.}{410184}$

```
[In] int(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (73/68364*x^5-1/3798*x^4+227/17091*x^3+4/633*x^2-8/5697*x+2/211)/(x^6+18*x^
4+324*x^3+108*x^2+216)+1/410184*sum((73*_R^4-36*_R^3+96*_R^2-216*_R+96)/(_R
^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^
2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.11

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \text{RootSum}\Big(58928958987008846341333266891354931 2t^6 - 539640290266075248405737472t^4 + 92182638$$
$$+ \frac{73x^5-18x^4+908x^3+432x^2-96x+648}{68364x^6+1230552x^4+22149936x^3+7383312x^2+14766624}$$

```
[In] integrate(x**7/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
[Out] RootSum(589289589870088463413332668913549312*_t**6 - 539640290266075248405737472*_t**4 + 92182638168509682392064*_t**3 - 553241442069170496*_t**2 - 3759837842016*_t - 7197829, Lambda(_t, _t*log(42996027639727447714003743305160746111018438501025999323136*_t**5/154206009791052044490694380303237521 - 42584766259508194684689715474422251405157209835847680*_t**4/154206009791052044490694380303237521 - 375124461288495881501083694493237540783173410821 12*_t**3/154206009791052044490694380303237521 + 715203759402167526763889071553 1672481920222144*_t**2/154206009791052044490694380303237521 - 44227546998835297723830291794974310524032*_t/154206009791052044490694380303237521 + x - 17457334903667604773413256958302485 5/154206009791052044490694380303237521))) + (73*x**5 - 18*x**4 + 908*x**3 + 432*x**2 - 96*x + 648)/(68364*x**6 + 1230552*x**4 + 22149936*x**3 + 7383312*x**2 + 14766624)
```

Maxima [F]

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^7}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

```
[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] 1/68364*(73*x^5 - 18*x^4 + 908*x^3 + 432*x^2 - 96*x + 648)/(x^6 + 18*x^4 +
324*x^3 + 108*x^2 + 216) + 1/68364*integrate((73*x^4 - 36*x^3 + 96*x^2 - 21
6*x + 96)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^7}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

```
[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^7/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.39

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

```
[In] int(x^7/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((8336932*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3
)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787
*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668913
549312, z, k))/97367427 - (480227*x)/851770251396 - (759164282*root(z^6 - (
292589*z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (247918
9*z^2)/2640728184707779481899008 - (1989787*z)/3118647171576193412533090467
84 - 7197829/589289589870088463413332668913549312, z, k)*x)/7886761587 - (2
07565888*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/754666262
20501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/3118647
17157619341253309046784 - 7197829/589289589870088463413332668913549312, z,
k)^2*x)/400689 - (108430970112*root(z^6 - (292589*z^4)/319508485412544 + (1
1805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008
 - (1989787*z)/311864717157619341253309046784 - 7197829/5892895898700884634
13332668913549312, z, k)^3*x)/44521 - (147138513610752*root(z^6 - (292589*z
```

```
^4)/319508485412544 + (11805253*z^3)/7546662622050124262 4 - (2479189*z^2)/2
640728184707779481899008 - (1989787*z)/3118647171576193412533090467 84 - 719
7829/58928958987008846341333266891354 9312, z, k)^4*x)/211 - 694098828855705
6*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/7546662622050124
2624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/31186471715761
9341253309046784 - 7197829/589289589870088463413332668913549312, z, k)^5*x
- (1156135728*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/7546
6626220501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/31
18647171576193412533090467 84 - 7197829/589289589870088463413332668913549312
, z, k)^2)/44521 + (6458021903232*root(z^6 - (292589*z^4)/319508485412544 +
 (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899
008 - (1989787*z)/311864717157619341253309046784 - 7197829/5892895898700884
634133326689135 49312, z, k)^3)/44521 - (102226052063232*root(z^6 - (292589*
z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/
2640728184707779481899008 - (1989787*z)/311864717157619341253309046784 - 71
97829/589289589870088463413332668913549312, z, k)^4)/211 - 1688973816882216
96*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/754666262205012
42624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/3118647171576
19341253309046784 - 7197829/589289589870088463413332668913549312, z, k)^5 +
 2207561/7665932262564)*root(z^6 - (292589*z^4)/319508485412544 + (11805253
*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (198
9787*z)/311864717157619341253309046784 - 7197829/589289589870088463413332266
8913549312, z, k), k, 1, 6) + ((4*x^2)/633 - (8*x)/5697 + (227*x^3)/17091 -
 x^4/3798 + (73*x^5)/68364 + 2/211)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216
)
```

3.153 $\int \frac{x^6}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal result 1185
Rubi [A] (verified) 1186
Mathematica [C] (verified) 1192
Maple [C] (verified) 1193
Fricas [F(-1)] 1193
Sympy [A] (verification not implemented) 1193
Maxima [F] 1194
Giac [F] 1194
Mupad [B] (verification not implemented) 1194

Optimal result

Integrand size = 26, antiderivative size = 677

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{2916\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$
$$+\frac{9\ 2^{2/3}+\sqrt[3]{-1}3^{2/3}\left(2+3\sqrt[3]{-2}3^{2/3}\right)x}{13122\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$+\frac{3\ 2^{2/3}\sqrt[3]{3}-\left(2-3\sqrt[3]{2}3^{2/3}\right)x}{8748\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$+\frac{\sqrt[3]{-1}\left(3(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{486\ 6^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}}$$
$$+\frac{\left(3(-3)^{2/3}+\sqrt[3]{-1}2^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{486\ 6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$
$$-\frac{\left(2^{2/3}-3\ 3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{486\ 6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$
$$+\frac{\sqrt[6]{-\frac{1}{3}}\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{5832\sqrt[3]{2}\left(1+\sqrt[3]{-1}\right)^5}$$
$$-\frac{i\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{5832\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}+\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{52488\sqrt[3]{2}3^{2/3}}$$

```
[Out] 1/5832*(9*(-2)^(2/3)+6^(1/3)*(9+(-3)^(1/3)*2^(2/3))*x)*2^(1/3)/(1+(-1)^(1/3
))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)+1/26244*(9*2^(
2/3)+(-1)^(1/3)*3^(2/3)*(2+3*(-2)^(1/3)*3^(2/3))*x)*2^(1/3)/(8+9*I*2^(1/3)*
3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/52488*(3*2^(2/3
)*3^(1/3)-(2-3*2^(1/3)*3^(2/3))*x)*2^(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6
```

```
+3*2^(2/3)*3^(1/3)*x+x^2)+1/2916*(-1)^(1/3)*(3*(-3)^(2/3)-2^(2/3))*arctan((
3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*6^(1/6)/(1+(-1)
^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(3/2)+1/2916*(3*(-3)^(2/3)+(-1)^(1/3)*2^
(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*
6^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(1/3)*3^(2/3))^(3/2)-1/
2916*(2^(2/3)-3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/
3)*3^(2/3))^(1/2))*6^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-4+3*2^(1/3)*
3^(2/3))^(3/2)+1/34992*(-1)^(1/6)*3^(5/6)*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*
2^(2/3)/(1+(-1)^(1/3))^5-1/34992*I*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)
*3^(5/6)/(1+(-1)^(1/3))^5+1/314928*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^
(1/3)
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 677, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used

$= \{2122, 652, 632, 210, 642, 212\}$

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$

$$= \frac{\sqrt[3]{-1}\left(3(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{486\ 6^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}}$$
$$+ \frac{\left(3(-3)^{2/3}+\sqrt[3]{-1}2^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{486\ 6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$
$$- \frac{\left(2^{2/3}-3\ 3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{486\ 6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(3\sqrt[3]{2}3^{2/3}-4\right)^{3/2}}$$
$$+ \frac{\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x+9(-2)^{2/3}}{2916\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}$$
$$+ \frac{\sqrt[3]{-1}3^{2/3}\left(2+3\sqrt[3]{-2}3^{2/3}\right)x+9\ 2^{2/3}}{13122\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}$$
$$+ \frac{3\ 2^{2/3}\sqrt[3]{3}-\left(2-3\sqrt[3]{2}3^{2/3}\right)x}{8748\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}$$
$$+ \frac{\sqrt[6]{-\frac{1}{3}}\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{5832\sqrt[3]{2}\left(1+\sqrt[3]{-1}\right)^5}$$
$$- \frac{i\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{5832\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5} + \frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{52488\sqrt[3]{2}3^{2/3}}$$

```
[In] Int[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (9*(-2)^(2/3) + 6^(1/3)*(9 + (-3)^(1/3)*2^(2/3))*x)/(2916*2^(2/3)*(1 + (-1)
^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) +
(9*2^(2/3) + (-1)^(1/3)*3^(2/3)*(2 + 3*(-2)^(1/3)*3^(2/3))*x)/(13122*2^(2/3
)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)
*x + x^2)) + (3*2^(2/3)*3^(1/3) - (2 - 3*2^(1/3)*3^(2/3))*x)/(8748*2^(2/3)*
3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ((-1)^(1
```

```
/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4
- 3*(-3)^(2/3)*2^(1/3))]])/(486*6^(5/6)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3
)*2^(1/3))^(3/2)) + ((3*(-3)^(2/3) + (-1)^(1/3)*2^(2/3))*ArcTan[(3*(-2)^(2/
3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(486*6^(5/6)*(1 - (-
1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((2^(2/3
) - 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(
1/3)*3^(2/3))]])/(486*6^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3
*2^(1/3)*3^(2/3))^(3/2)) + ((-1/3)^(1/6)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x
^2])/(5832*2^(1/3)*(1 + (-1)^(1/3))^5) - ((I/5832)*Log[6 + 3*(-2)^(2/3)*3^(
1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(
1/3)*x + x^2]/(52488*2^(1/3)*3^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
```

```
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

integral

$$= 1586874322944 \int \left(-\frac{-2\sqrt[3]{-1}3^{2/3} + 3(-2)^{2/3}x}{1542441841901568\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(-6+3\sqrt[3]{-3}2^{2/3}x-x^2\right)^2} + \frac{-3i3^{5/6} + \left(\sqrt[3]{-2}+\sqrt[3]{2}\right)x}{4627325525704704\ 6^{2/3}\left(1+\sqrt[3]{-1}\right)^5\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} - \frac{-2\sqrt[3]{-1}3^{2/3} + 3\ 2^{2/3}x}{1542441841901568\ 2^{2/3}\left(-1+\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2} + \frac{3+3\sqrt[3]{-1}-i\sqrt[3]{2}\sqrt[6]{3}x}{4627325525704704\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)} - \frac{2+2^{2/3}\sqrt[3]{3}x}{514147280633856\ 2^{2/3}\sqrt[3]{3}\left(-1+\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)^2} + \frac{3\sqrt[3]{3}+\sqrt[3]{2}x}{41645929731342336\ 6^{2/3}\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)} \right) dx$$

$$= -\frac{\int \frac{-2\sqrt[3]{-1}3^{2/3}+3\ 2^{2/3}x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}dx}{8748\ 2^{2/3}} - \frac{\int \frac{2+2^{2/3}\sqrt[3]{3}x}{\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)^2}dx}{2916\ 2^{2/3}\sqrt[3]{3}} + \frac{\int \frac{3\sqrt[3]{3}+\sqrt[3]{2}x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}dx}{26244\ 6^{2/3}} + \frac{\int \frac{3+3\sqrt[3]{-1}-i\sqrt[3]{2}\sqrt[6]{3}x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{2916\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5} + \frac{\int \frac{-3i3^{5/6}+\left(\sqrt[3]{-2}+\sqrt[3]{2}\right)x}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}dx}{2916\ 6^{2/3}\left(1+\sqrt[3]{-1}\right)^5} - \frac{\int \frac{-2\sqrt[3]{-1}3^{2/3}+3(-2)^{2/3}x}{\left(-6+3\sqrt[3]{-3}2^{2/3}x-x^2\right)^2}dx}{972\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4}$$

$$= \frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{2916\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$
$$+\frac{9\ 2^{2/3}+\sqrt[3]{-1}3^{2/3}\left(2+3\sqrt[3]{-2}3^{2/3}\right)x}{26244\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$+\frac{3\ 2^{2/3}\sqrt[3]{3}-\left(2-3\sqrt[3]{2}3^{2/3}\right)x}{8748\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}+\frac{\sqrt[6]{-\frac{1}{3}}\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{5832\sqrt[3]{2}\left(1+\sqrt[3]{-1}\right)^5}$$
$$-\frac{i\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{5832\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}+\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{52488\sqrt[3]{2}3^{2/3}}$$
$$-\frac{\left(-18\sqrt[3]{-6}(-1)^{2/3}+4\sqrt[3]{-1}3^{2/3}\right)\int\frac{1}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{972\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(24-18(-3)^{2/3}\sqrt[3]{2}\right)}$$
$$-\frac{\left(2\ 3^{2/3}-9\sqrt[3]{6}\right)\int\frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{52488\left(2\ 2^{2/3}-3\ 3^{2/3}\right)}+$$
$$-\frac{\left(-4\sqrt[3]{-1}3^{2/3}-18(-1)^{2/3}\sqrt[3]{6}\right)\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{8748\ 2^{2/3}\left(24+18\sqrt[3]{-2}3^{2/3}\right)}$$

$$
\begin{aligned}
&= \frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{2916\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} \\
&\quad+\frac{9\ 2^{2/3}+\sqrt[3]{-1}3^{2/3}\left(2+3\sqrt[3]{-2}3^{2/3}\right)x}{26244\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)} \\
&\quad+\frac{3\ 2^{2/3}\sqrt[3]{3}-\left(2-3\sqrt[3]{2}3^{2/3}\right)x}{8748\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}+\frac{\sqrt[6]{-\frac{1}{3}}\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{5832\sqrt[3]{2}\left(1+\sqrt[3]{-1}\right)^5} \\
&\quad-\frac{i\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{5832\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}+\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{52488\sqrt[3]{2}3^{2/3}} \\
&\quad+\frac{\left(-18\sqrt[3]{-6}(-1)^{2/3}+4\sqrt[3]{-1}3^{2/3}\right)\text{Subst}\left(\int\frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}\,dx,x,3\sqrt[3]{-3}2^{2/3}-2x\right)}{486\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(24-18(-3)^{2/3}\sqrt[3]{2}\right)} \\
&\quad+\frac{\left(2\ 3^{2/3}-9\sqrt[3]{6}\right)\text{Subst}\left(\int\frac{1}{-6\left(4-3\sqrt[3]{2}3^{2/3}\right)-x^2}\,dx,x,3\ 2^{2/3}\sqrt[3]{3}+2x\right)}{26244\left(2\ 2^{2/3}-3\ 3^{2/3}\right)}- \\
&\quad-\frac{\left(-4\sqrt[3]{-1}3^{2/3}-18(-1)^{2/3}\sqrt[3]{6}\right)\text{Subst}\left(\int\frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}\,dx,x,3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{4374\ 2^{2/3}\left(24+18\sqrt[3]{-2}3^{2/3}\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{2916\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} \\
&\quad + \frac{9\ 2^{2/3}+\sqrt[3]{-1}3^{2/3}\left(2+3\sqrt[3]{-2}3^{2/3}\right)x}{26244\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)} \\
&\quad + \frac{3\ 2^{2/3}\sqrt[3]{3}-\left(2-3\sqrt[3]{2}3^{2/3}\right)x}{8748\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)} \\
&\quad + \frac{\sqrt[3]{-1}\left(3(-3)^{2/3}-2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{486\ 6^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}} \\
&\quad + \frac{\left(3(-3)^{2/3}\sqrt[6]{2}+\sqrt[3]{-1}2^{5/6}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{8748\ 3^{5/6}\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}} \\
&\quad - \frac{\left(2^{2/3}-3\ 3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{4374\ 6^{5/6}\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} + \frac{\sqrt[6]{-\frac{1}{3}}\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{5832\sqrt[3]{2}\left(1+\sqrt[3]{-1}\right)^5} \\
&\quad - \frac{i\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{5832\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5} + \frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{52488\sqrt[3]{2}3^{2/3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.25

$$
\begin{aligned}
&\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \frac{-96+108x-64x^2-72x^3+73x^4-3x^5}{68364\,(216+108x^2+324x^3+18x^4+x^6)} \\
&\quad - \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{108\log(x-\#1)-32\log(x-\#1)\#1+108\log(x-\#1)\#1^2-1461}{36\#1+162\#1^2+12\#1^3+\#}\right.}{410184}
\end{aligned}
$$

```
[In] Integrate[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-96 + 108*x - 64*x^2 - 72*x^3 + 73*x^4 - 3*x^5)/(68364*(216 + 108*x^2 + 32
4*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6
```

```
& , (108*Log[x - #1] - 32*Log[x - #1]*#1 + 108*Log[x - #1]*#1^2 - 146*Log[
x - #1]*#1^3 + 3*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/
410184
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.18

method	result
default	$\frac{-\frac{1}{22788}x^5+\frac{73}{68364}x^4-\frac{2}{1899}x^3-\frac{16}{17091}x^2+\frac{1}{633}x-\frac{8}{5697}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-3\_R^4+146\_R^3-1\right.}{\_R^5+12\_}\right.}{410184}$
risch	$\frac{-\frac{1}{22788}x^5+\frac{73}{68364}x^4-\frac{2}{1899}x^3-\frac{16}{17091}x^2+\frac{1}{633}x-\frac{8}{5697}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-3\_R^4+146\_R^3-1\right.}{\_R^5+12\_}\right.}{410184}$

```
[In] int(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/22788*x^5+73/68364*x^4-2/1899*x^3-16/17091*x^2+1/633*x-8/5697)/(x^6+18*
x^4+324*x^3+108*x^2+216)+1/410184*sum((-3*_R^4+146*_R^3-108*_R^2+32*_R-108)
/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108
*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.17

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \mathrm{RootSum}\Big(397770473162309712803999551516645 7856t^6 - 1010314319415295961050951680t^4 - 201682$$
$$+ \frac{-3x^5+73x^4-72x^3-64x^2+108x-96}{68364x^6+1230552x^4+22149936x^3+7383312x^2+14766624}$$

```
[In] integrate(x**6/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(3977704731623097128039995515166457856*_t**6 - 10103143194152959610509516800*_t**4 - 20168224477093957151232*_t**3 - 112582856818899648*_t**2 - 50648453064*_t - 880007, Lambda(_t, _t*log(-27365556709001899157064994141439556098619968804064460 8*_t**5/4979785539613990026757339569 5 + 1183700847019604608530864623076435429280504457011 2*_t**4/49797855396139900267573395695 - 1057058190044671726637407748287331504778700 8*_t**3/49797855396139900267573395695 - 15525474115694698723875632187927893233 92*_t**2/49797855396139900267573395695 - 125429237911591408269090032502959 28*_t/49797855396139900267573395695 + x - 2306653387032032241083434829 6/49797855396139900267573395695)))
 + (-3*x**5 + 73*x**4 - 72*x**3 - 64*x**2 + 108*x - 96)/(68364*x**6 + 1230552*x**4 + 22149936*x**3 + 7383312*x**2 + 14766624)
```

Maxima [F]

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^6}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

```
[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] -1/68364*(3*x^5 - 73*x^4 + 72*x^3 + 64*x^2 - 108*x + 96)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) - 1/68364*integrate((3*x^4 - 146*x^3 + 108*x^2 - 32*x + 108)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^6}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

```
[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^6/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.57

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

```
[In] int(x^6/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((7028852*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)
/3056398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)
/311864717157619341253309046784 - 880007/3977704731623097128039995515166457
856, z, k))/2628920529 - (1980083*x)/310470256633842 - (235710556*root(z^6
- (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (16
8169*z^2)/5941638415592503834272768 - (3971*z)/3118647171576193412533090467
84 - 880007/3977704731623097128039995515166457856, z, k)*x)/70980854283 - (
6628544*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/30563983619
30300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/31186471715
7619341253309046784 - 880007/3977704731623097128039995515166457856, z, k)^2
*x)/44521 - (141776759808*root(z^6 - (60865*z^4)/239631364059408 - (1549690
9*z^3)/3056398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3
971*z)/311864717157619341253309046784 - 880007/397770473162309712803999515
166457856, z, k)^3*x)/44521 + (183701926508544*root(z^6 - (60865*z^4)/23963
1364059408 - (15496909*z^3)/3056398361930300326272 - (168169*z^2)/594163841
5592503834272768 - (3971*z)/311864717157619341253309046784 - 880007/3977704
731623097128039995515166457856, z, k)^4*x)/211 - 6940988288557056*root(z^6
- (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (16
8169*z^2)/5941638415592503834272768 - (3971*z)/3118647171576193412533090467
84 - 880007/3977704731623097128039995515166457856, z, k)^5*x + (100886752*r
oot(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/30563983619303003262
72 - (168169*z^2)/5941638415592503834272768 - (3971*z)/31186471715761934125
3309046784 - 880007/3977704731623097128039995515166457856, z, k)^2)/133563
+ (1715052538368*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/30
56398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/31
1864717157619341253309046784 - 880007/3977704731623097128039995515166457856
, z, k)^3)/44521 + (115004308571136*root(z^6 - (60865*z^4)/239631364059408
- (15496909*z^3)/3056398361930300326272 - (168169*z^2)/59416384155925038342
72768 - (3971*z)/311864717157619341253309046784 - 880007/397770473162309712
8039995515166457856, z, k)^4)/211 - 168897381688221696*root(z^6 - (60865*z^
4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (168169*z^2)/5
941638415592503834272768 - (3971*z)/311864717157619341253309046784 - 880007
/3977704731623097128039995515166457856, z, k)^5 - 265/5749449196923)*root(z
^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 -
(168169*z^2)/5941638415592503834272768 - (3971*z)/3118647171576193412533090
46784 - 880007/3977704731623097128039995515166457856, z, k), k, 1, 6) - ((1
6*x^2)/17091 - x/633 + (2*x^3)/1899 - (73*x^4)/68364 + x^5/22788 + 8/5697)/
(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

3.154 $\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2} \, dx$

Optimal result 1197
Rubi [A] (warning: unable to verify) 1198
Mathematica [C] (verified) 1203
Maple [C] (verified) 1204
Fricas [B] (verification not implemented) 1204
Sympy [A] (verification not implemented) 1206
Maxima [F] 1207
Giac [F] 1207
Mupad [B] (verification not implemented) 1208

Optimal result

Integrand size = 26, antiderivative size = 682

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}2^{2/3}x\right)}{1944\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$
$$+\frac{\sqrt[3]{-\frac{1}{3}}\left(4+(-2)^{2/3}\sqrt[3]{3}x\right)}{8748\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$-\frac{4+2^{2/3}\sqrt[3]{3}x}{17496\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$-\frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^4\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\sqrt{3}\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$
$$-\frac{i\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1458\ 2^{5/6}3^{2/3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{4374\sqrt{3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{8748\sqrt{6}\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{39366\ 2^{5/6}\sqrt[6]{3}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$

```
[Out] 1/11664*(-1)^(1/3)*3^(2/3)*(4-(-3)^(1/3)*2^(2/3)*x)*2^(1/3)/(1+(-1)^(1/3))^
4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)+1/52488*(-1)^(1/3
)*3^(2/3)*(4+(-2)^(2/3)*3^(1/3)*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)
*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/104976*(-4-2^(2/3)*3^(1/3)*x)*2^
(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/13122*arc
tan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))/(8-9*I*2^(
1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)*3^(1/2)-1/13122*arctan((3*(-2)^(2/3)*
3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(8+9*I*2^(1/3)*3^(1/6)+3*2^(
1/3)*3^(2/3))^(3/2)*3^(1/2)-1/52488*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(
-12+9*2^(1/3)*3^(2/3))^(1/2))/(-4+3*2^(1/3)*3^(2/3))^(3/2)*6^(1/2)-1/26244*
```

```
arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(1/6)
*3^(5/6)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/8748*I*arctan((3
*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(1/6)*3^(1/3)/
(1+(-1)^(1/3))^5/(4+3*(-2)^(1/3)*3^(2/3))^(1/2)-1/236196*arctanh(2^(1/6)*(3
*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(1/6)*3^(5/6)/(-4+3*2^
(1/3)*3^(2/3))^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 1.24 (sec) , antiderivative size = 682, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2122, 652, 632, 210, 212}

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\sqrt{3}\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} - \frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^4\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$
$$- \frac{\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{4374\sqrt{3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} - \frac{i\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1458\ 2^{5/6}3^{2/3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$
$$- \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{39366\ 2^{5/6}\sqrt[6]{3}\sqrt{3\sqrt[3]{2}3^{2/3}-4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{8748\sqrt{6}\left(3\sqrt[3]{2}3^{2/3}-4\right)^{3/2}}$$
$$+ \frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}2^{2/3}x\right)}{1944\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}$$
$$+ \frac{\sqrt[3]{-\frac{1}{3}}\left((-2)^{2/3}\sqrt[3]{3}x+4\right)}{8748\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}$$
$$- \frac{2^{2/3}\sqrt[3]{3}x+4}{17496\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}$$

```
[In] Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] ((-1/3)^(1/3)*(4 - (-3)^(1/3)*2^(2/3)*x))/(1944*2^(2/3)*(1 + (-1)^(1/3))^4*
(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) + ((-1/3)^(1
/3)*(4 + (-2)^(2/3)*3^(1/3)*x))/(8748*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) +
3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (4 + 2^(2/3)*3^(1/
3)*x)/(17496*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)
*x + x^2)) - ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2
^(1/3))]]/(4374*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^4*Sqrt[4 - 3*(-3)^(2/3)*2^
(1/3)]) + ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1
/3))]]/(4374*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2))
 - ((I/1458)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3
^(2/3))]])/(2^(5/6)*3^(2/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3
)]) - ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))
]]/(4374*Sqrt[3]*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - A
rcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/
(8748*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - ArcTanh[(2^(1/6)*(3*3^(1/3)
 + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(39366*2^(5/6)*3^(1/6)*Sqr
t[-4 + 3*2^(1/3)*3^(2/3)])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
```

```
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

integral

$$= 1586874322944 \int \left(-\frac{\sqrt[3]{-\frac{1}{3}}x}{1542441841901568\ 2^{2/3} \left(1+\sqrt[3]{-1}\right)^4 \left(-6+3\sqrt[3]{-3}2^{2/3}x-x^2\right)^2} - \frac{1}{4627325525704704\sqrt[3]{2}3^{2/3} \left(1+\sqrt[3]{-1}\right)^4 \left(-6+3\sqrt[3]{-3}2^{2/3}x-x^2\right)} - \frac{\sqrt[3]{-\frac{1}{3}}x}{1542441841901568\ 2^{2/3} \left(-1+\sqrt[3]{-1}\right)^2 \left(1+\sqrt[3]{-1}\right)^4 \left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2} - \frac{i}{4627325525704704\sqrt[3]{2}\sqrt[6]{3} \left(1+\sqrt[3]{-1}\right)^5 \left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)} + \frac{x}{1542441841901568\ 2^{2/3}\sqrt[3]{3} \left(-1+\sqrt[3]{-1}\right)^2 \left(1+\sqrt[3]{-1}\right)^4 \left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)^2} + \frac{1}{41645929731342336\sqrt[3]{2}3^{2/3} \left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)} \right) dx$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}} \int \frac{x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2} dx}{8748\ 2^{2/3}} + \frac{\int \frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2} dx}{26244\sqrt[3]{2}3^{2/3}} + \frac{\int \frac{x}{\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)^2} dx}{8748\ 2^{2/3}\sqrt[3]{3}} - \frac{i \int \frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2} dx}{2916\sqrt[3]{2}\sqrt[6]{3} \left(1+\sqrt[3]{-1}\right)^5} - \frac{\sqrt[3]{-\frac{1}{3}} \int \frac{x}{\left(-6+3\sqrt[3]{-3}2^{2/3}x-x^2\right)^2} dx}{972\ 2^{2/3} \left(1+\sqrt[3]{-1}\right)^4} - \frac{\int \frac{1}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2} dx}{2916\sqrt[3]{2}3^{2/3} \left(1+\sqrt[3]{-1}\right)^4}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}2^{2/3}x\right)}{1944\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$

$$+ \frac{\sqrt[3]{-\frac{1}{3}}\left(4+(-2)^{2/3}\sqrt[3]{3}x\right)}{17496\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$- \frac{4+2^{2/3}\sqrt[3]{3}x}{17496\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$- \frac{\text{Subst}\left(\int \frac{1}{-6\left(4-3\sqrt[3]{2}3^{2/3}\right)-x^2}\,dx, x, 3\ 2^{2/3}\sqrt[3]{3}+2x\right)}{13122\sqrt[3]{2}3^{2/3}}$$

$$+ \frac{i\text{Subst}\left(\int \frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}\,dx, x, 3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{1458\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}$$

$$+ \frac{\text{Subst}\left(\int \frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}\,dx, x, 3\sqrt[3]{-3}2^{2/3}-2x\right)}{1458\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^4} - \frac{\int \frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{17496\left(4+3\sqrt[3]{-2}3^{2/3}\right)}$$

$$- \frac{\int \frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{17496\left(4-3\sqrt[3]{2}3^{2/3}\right)} + \frac{\int \frac{1}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{8748\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}2^{2/3}x\right)}{1944\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$

$$+\frac{\sqrt[3]{-\frac{1}{3}}\left(4+(-2)^{2/3}\sqrt[3]{3}x\right)}{17496\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$-\frac{4+2^{2/3}\sqrt[3]{3}x}{17496\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^4\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$

$$-\frac{i\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1458\ 2^{5/6}3^{2/3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{39366\ 2^{5/6}\sqrt[6]{3}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$

$$+\frac{\text{Subst}\left(\int\frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}dx,x,3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{8748\left(4+3\sqrt[3]{-2}3^{2/3}\right)}$$

$$+\frac{\text{Subst}\left(\int\frac{1}{-6\left(4-3\sqrt[3]{2}3^{2/3}\right)-x^2}dx,x,3\ 2^{2/3}\sqrt[3]{3}+2x\right)}{8748\left(4-3\sqrt[3]{2}3^{2/3}\right)}$$

$$-\frac{\text{Subst}\left(\int\frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}dx,x,3\sqrt[3]{-3}2^{2/3}-2x\right)}{4374\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)}$$

$$
\begin{aligned}
&= \frac{\sqrt[3]{-\frac{1}{3}}\,(4-\sqrt[3]{-3}2^{2/3}x)}{1944\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} \\
&\quad+\frac{\sqrt[3]{-\frac{1}{3}}\left(4+(-2)^{2/3}\sqrt[3]{3}x\right)}{17496\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)} \\
&\quad-\frac{4+2^{2/3}\sqrt[3]{3}x}{17496\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)} \\
&\quad-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^4\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\sqrt{3}\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} \\
&\quad-\frac{\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{8748\sqrt{6}\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}-\frac{i\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1458\ 2^{5/6}3^{2/3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}} \\
&\quad-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{8748\sqrt{6}\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{39366\ 2^{5/6}\sqrt[6]{3}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.24

$$
\begin{aligned}
\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx &= \frac{972-144x+648x^2+729x^3-27x^4+4x^5}{615276\,(216+108x^2+324x^3+18x^4+x^6)} \\
&\quad+\frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{144\log(x-\#1)-324\log(x-\#1)\#1+2043\log(x-\#1)\#1^2-\cdots}{36\#1+162\#1^2+12\#1^3+\cdots}\right.}{3691656}
\end{aligned}
$$

```
[In] Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (972 - 144*x + 648*x^2 + 729*x^3 - 27*x^4 + 4*x^5)/(615276*(216 + 108*x^2 +
 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #
1^6 & , (144*Log[x - #1] - 324*Log[x - #1]*#1 + 2043*Log[x - #1]*#1^2 - 54*
Log[x - #1]*#1^3 + 4*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5)
& ]/3691656
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.18

method	result
default	$\frac{\frac{1}{153819}x^5-\frac{1}{22788}x^4+\frac{1}{844}x^3+\frac{2}{1899}x^2-\frac{4}{17091}x+\frac{1}{633}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(4\_R^4-54\_R^3+2043\_}{\_R^5+12\_R^3}\right.}{3691656}$
risch	$\frac{\frac{1}{153819}x^5-\frac{1}{22788}x^4+\frac{1}{844}x^3+\frac{2}{1899}x^2-\frac{4}{17091}x+\frac{1}{633}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(4\_R^4-54\_R^3+2043\_}{\_R^5+12\_R^3}\right.}{3691656}$

```
[In] int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/153819*x^5-1/22788*x^4+1/844*x^3+2/1899*x^2-4/17091*x+1/633)/(x^6+18*x^4
+324*x^3+108*x^2+216)+1/3691656*sum((4*_R^4-54*_R^3+2043*_R^2-324*_R+144)/(
_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_
Z^2+216))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1445 vs. 2(463) = 926.

Time = 0.96 (sec) , antiderivative size = 1445, normalized size of antiderivative = 2.12

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] 1/28041818976*(182304*x^5 - 1230552*x^4 + 33224904*x^3 + 422*sqrt(1/633)*(x
^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(5034474*18^(2/3) + 9367856*18^(
1/3) + 44687457)*log(2/1982119441*sqrt(1/633)*(7238020557*(5034474*18^(2/3)
 + 9367856*18^(1/3) + 44687457)^2 - 447902374840040617697673*18^(2/3) - 83
343065225076612586451l2*18^(1/3) - 268625598114228853471204 77)*sqrt(5034474
*18^(2/3) + 9367856*18^(1/3) + 44687457) - 7383041510/9393931*(5034474*18^(
2/3) + 9367856*18^(1/3) + 44687457)^2 + 247458158879850620*x + 513225545496
0803463351330/9393931*18^(2/3) + 954980203637704604075352 0/9393931*18^(1/3)
 + 272789282330339400324258 30/9393931) - 422*sqrt(1/633)*(x^6 + 18*x^4 + 32
4*x^3 + 108*x^2 + 216)*sqrt(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)
*log(-2/1982119441*sqrt(1/633)*(7238020557*(5034474*18^(2/3) + 9367856*18^(
1/3) + 44687457)^2 - 447902374840040617697673*18^(2/3) - 83343065225076612
58645112*18^(1/3) - 26862559811422885347120477)*sqrt(5034474*18^(2/3) + 936
7856*18^(1/3) + 44687457) - 7383041510/9393931*(5034474*18^(2/3) + 9367856*
```

18^(1/3) + 44687457)^2 + 247458158879850620*x + 51322554549608034633513330/9
393931*18^(2/3) + 95498020363770460407535520/9393931*18^(1/3) + 272789282330
33940032425830/9393931) - 9*sqrt(422)*(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 2
16)*sqrt(-20718*18^(2/3) + sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/
3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 27397
4962699) - 9367856/243*18^(1/3) + 367798)*log(14766083020/211*(5034474*18^(
2/3) + 9367856*18^(1/3) + 44687457)^2 + 3064230/211*sqrt(-1/19683*(5034474*
18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 34454787
01088/81*18^(1/3) + 273974962699)*(5895278433468*18^(2/3) + 10969590754592*
18^(1/3) + 57028339027521) + 9/9393931*(14476041114*sqrt(422)*(5034474*18^(
2/3) + 9367856*18^(1/3) + 44687457)^2 + 243*sqrt(-1/19683*(5034474*18^(2/3)
 + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81
18^(1/3) + 273974962699)(14476041114*sqrt(422)*(5034474*18^(2/3) + 936785
6*18^(1/3) + 44687457) - 161351097450615865*sqrt(422)) - 177934129698570542
9*sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) + 265058558805
69051992480475*sqrt(422))*sqrt(-20718*18^(2/3) + sqrt(-1/19683*(5034474*18^
(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 34454787010
88/81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 367798) + 440683387
65959317812080*x - 102645109099216069267026660/211*18^(2/3) - 19099604072754
092081507040/211*18^(1/3) - 54557856466067880064851660/211) + 9*sqrt(422)*(
x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(-20718*18^(2/3) + sqrt(-1/1968
3*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3)
 + 3445478701088/81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 36779
8)*log(14766083020/211*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 +
 3064230/211*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)
^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(5895
278433468*18^(2/3) + 10969590754592*18^(1/3) + 57028339027521) - 9/9393931*
(14476041114*sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 +
 243*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 228
60116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(14476041114*
sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) - 16135109745061
5865*sqrt(422)) - 1779341296985705429*sqrt(422)*(5034474*18^(2/3) + 9367856
*18^(1/3) + 44687457) + 26505855880569051992480475*sqrt(422))*sqrt(-20718*1
8^(2/3) + sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2
+ 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699) - 936785
6/243*18^(1/3) + 367798) + 44068338765959317812080*x - 10264510909921606926
702660/211*18^(2/3) - 19099604072754092081507040/211*18^(1/3) - 54557856466
067880064851660/211) - 9*sqrt(422)*(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)
*sqrt(-20718*18^(2/3) - sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3)
+ 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 27397496
2699) - 9367856/243*18^(1/3) + 367798)*log(14766083020/211*(5034474*18^(2/3
) + 9367856*18^(1/3) + 44687457)^2 - 3064230/211*sqrt(-1/19683*(5034474*18^
(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 34454787010
88/81*18^(1/3) + 273974962699)*(5895278433468*18^(2/3) + 10969590754592*18^
(1/3) + 57028339027521) + 9/9393931*(14476041114*sqrt(422)*(5034474*18^(2/3

```
) + 9367856*18^(1/3) + 44687457)^2 - 243*sqrt(-1/19683*(5034474*18^(2/3) +
9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18
^(1/3) + 273974962699)*(14476041114*sqrt(422)*(5034474*18^(2/3) + 9367856*1
8^(1/3) + 44687457) - 161351097450615865*sqrt(422)) - 1779341296985705429*s
qrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) + 265058558805690
51992480475*sqrt(422))*sqrt(-20718*18^(2/3) - sqrt(-1/19683*(5034474*18^(2/
3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/
81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 367798) + 440683387659
59317812080*x - 10264510909921606926702660/211*18^(2/3) - 19099604072754092
081507040/211*18^(1/3) - 54557856466067880064851660/211) + 9*sqrt(422)*(x^6
 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(-20718*18^(2/3) - sqrt(-1/19683*(
5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) +
3445478701088/81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 367798)*
log(14766083020/211*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 - 30
64230/211*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2
+ 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(5895278
433468*18^(2/3) + 10969590754592*18^(1/3) + 57028339027521) - 9/9393931*(14
476041114*sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 - 24
3*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 228601
16892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(14476041114*sqr
t(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) - 16135109745061586
5*sqrt(422)) - 1779341296985705429*sqrt(422)*(5034474*18^(2/3) + 9367856*18
^(1/3) + 44687457) + 26505855880569051992480475*sqrt(422))*sqrt(-20718*18^(
2/3) - sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 2
2860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699) - 9367856/2
43*18^(1/3) + 367798) + 44068338765959317812080*x - 10264510909921606926702
660/211*18^(2/3) - 19099604072754092081507040/211*18^(1/3) - 54557856466067
880064851660/211) + 29533248*x^2 - 6562944*x + 44299872)/(x^6 + 18*x^4 + 32
4*x^3 + 108*x^2 + 216)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.15

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \text{RootSum}\Big(2749389510497884734901244900083055670067 2t^6 - 131871818922695008886298319257 6t^4 + 1$$
$$+ \frac{4x^5-27x^4+729x^3+648x^2-144x+972}{615276x^6+11074968x^4+199349424x^3+66449808x^2+132899616}$$

```
[In] integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(27493895104978847349012449000830556700672*_t**6 - 13187181892269500
88862983192576*_t**4 + 12120917704776776448*_t**2 - 39753025, Lambda(_t, _t
```

```
*log(947842259001288723909832054550209950242045952*_t**5/61864539719962655
- 24345864681777560763965488948081 4592*_t**4/9811980923071 - 41682556475067
500431787310779667456*_t**3/61864539719962655 + 120268774426643286164622 72*
_t**2/9811980923071 + 21614261848885979366842 8*_t/61864539719962655 + x - 3
08574300024117/39247923692284))) + (4*x**5 - 27*x**4 + 729*x**3 + 648*x**2
- 144*x + 972)/(615276*x**6 + 11074968*x**4 + 199349424*x**3 + 66449808*x**
2 + 132899616)
```

Maxima [F]

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^5}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")

[Out] 1/615276*(4*x^5 - 27*x^4 + 729*x^3 + 648*x^2 - 144*x + 972)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) + 1/615276*integrate((4*x^4 - 54*x^3 + 2043*x^2 - 324*x + 144)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

Giac [F]

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^5}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")

[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.44

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$

$$= \left(\sum_{k=1}^{6} \ln\left(-\frac{4477969\,\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{140838836517748239034613 76}-\frac{39753025}{27493895104978847349012449000830556700}\right.}{189282278088}\right.\right.$$

$$+\frac{6305\,x}{4967524106141472}$$

$$-\frac{\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{39753025}{274938951049788473490124490008305567006 72},z,k\right)\,x\,16340}{5110621508376}$$

$$-\frac{\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{39753025}{27493895104978847349012449000830556700672},z,k\right)^2\,x\,43348}{10818603}$$

$$-\frac{\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{39753025}{27493895104978847349012449000830556700672},z,k\right)^3\,x\,65333}{44521}$$

$$-\frac{\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{39753025}{27493895104978847349012449000830556700672},z,k\right)^4\,x\,40024}{211}$$

$$-\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{39753025}{274938951049788473490124490008\right.$$

$$+\frac{5943884\,\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{39753025}{27493895104978847349012449000830556700672},z,k\right)}{400689}$$

$$+\frac{224442467136\,\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{39753025}{27493895104978847349012449000830556700672}\right.}{44521}$$

$$-\frac{137087493272064\,\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{39753025}{2749389510497884734901244900083055670}\right.}{211}$$

$$-168897381688221696\,\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{}{2749389510497}\right.$$

$$\left.-\frac{13082875}{178830867821092992}\right)\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}\right.$$

$$\left.\left.\left.-\frac{39753025}{27493895104978847349012449000830556700672},z,k\right)\right)\right)$$

$$+\frac{\frac{x^5}{153819}-\frac{x^4}{22788}+\frac{x^3}{844}+\frac{2\,x^2}{1899}-\frac{4\,x}{17091}+\frac{1}{633}}{x^6+18\,x^4+324\,x^3+108\,x^2+216}$$

```
[In] int(x^5/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)

[Out] symsum(log((6305*x)/4967524106141472 - (4477969*root(z^6 - (183899*z^4)/383
4101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/2749389510
4978847349012449000830556700672, z, k))/189282278088 - (16340881*root(z^6 -
 (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39
753025/27493895104978847349012449000830556700672, z, k)*x)/5110621508376 -
(43348696*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651
774823903461376 - 39753025/27493895104978847349012449000830556700672, z, k)
^2*x)/10818603 - (65333687616*root(z^6 - (183899*z^4)/3834101824950528 + (6
209*z^2)/14083883651774823903461376 - 39753025/2749389510497884734901244900
0830556700672, z, k)^3*x)/44521 - (40024496812032*root(z^6 - (183899*z^4)/3
834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/27493895
104978847349012449000830556700672, z, k)^4*x)/211 - 6940988288557056*root(z
^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376
- 39753025/27493895104978847349012449000830556700672, z, k)^5*x + (5943884*
root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/140838836517748239034
61376 - 39753025/27493895104978847349012449000830556700672, z, k)^2)/400689
 + (224442467136*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/1408
3883651774823903461376 - 39753025/27493895104978847349012449000830556700672
, z, k)^3)/44521 - (137087493272064*root(z^6 - (183899*z^4)/383410182495052
8 + (6209*z^2)/14083883651774823903461376 - 39753025/2749389510497884734901
2449000830556700672, z, k)^4)/211 - 168897381688221696*root(z^6 - (183899*z
^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/274
93895104978847349012449000830556700672, z, k)^5 - 13082875/1788308678210929
92)*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823
903461376 - 39753025/27493895104978847349012449000830556700672, z, k), k, 1
, 6) + ((2*x^2)/1899 - (4*x)/17091 + x^3/844 - x^4/22788 + x^5/153819 + 1/6
33)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

3.155 $\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal result 1211
Rubi [A] (warning: unable to verify) 1212
Mathematica [C] (verified) 1219
Maple [C] (verified) 1220
Fricas [F(-1)] 1220
Sympy [A] (verification not implemented) 1221
Maxima [F] 1221
Giac [F] 1222
Mupad [B] (verification not implemented) 1222

Optimal result

Integrand size = 26, antiderivative size = 850

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\,(3\sqrt[3]{-3}2^{2/3}-2x)}{5832\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$

$$-\frac{\sqrt[3]{-\frac{1}{3}}\left(3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{26244\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$-\frac{3\sqrt[3]{3}+\sqrt[3]{2}x}{52488\left(9\sqrt[3]{2}-4\sqrt[3]{3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{\sqrt[3]{-1}\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{729\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$

$$-\frac{\sqrt[3]{-1}\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{2916\sqrt[6]{2}3^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$

$$-\frac{(i+\sqrt{3})\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{11664\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$

$$-\frac{i\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{5832\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{26244\sqrt[6]{2}3^{5/6}\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$

$$+\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{52488\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}-\frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{34992\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^4}$$

$$+\frac{i\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{34992\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}-\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{314928\sqrt[3]{2}3^{2/3}}$$

```
[Out] 1/34992*(-1)^(1/3)*3^(2/3)*(3*(-3)^(1/3)*2^(2/3)-2*x)*2^(1/3)/(1+(-1)^(1/3)
)^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)-1/157464*(-1)^(
1/3)*3^(2/3)*(3*(-2)^(2/3)*3^(1/3)+2*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3*2^
(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/52488*(-3*3^(1/3)-2^(1/3)*x
)/(9*2^(1/3)-4*3^(1/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/4374*(-1)^(1/3)*arcta
n((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(1/3)*3^(1
/6)/(1+(-1)^(1/3))^4/(8-9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)-1/1749
6*(-1)^(1/3)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(
1/2))*2^(5/6)*3^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(1/3)*3^(
2/3))^(3/2)+1/157464*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3
^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(3/2)-1/209952*ln(6-3
*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^(1/3))^4+1/209952*I*ln(6
+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(5/6)/(1+(-1)^(1/3))^5-1/1889568*ln(
6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/34992*I*arctan(2^(1/6)*(3*(-3)
^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(2/3)/(1+(-1)^
(1/3))^5/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/69984*arctan((3*(-2)^(2/3)*3^(1/3
)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*(3^(1/2)+I)*2^(5/6)*3^(2/3)/(1+(-1
)^(1/3))^5/(4+3*(-2)^(1/3)*3^(2/3))^(1/2)+1/314928*arctanh(2^(1/6)*(3*3^(1/
3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*
3^(2/3))^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 1.45 (sec) , antiderivative size = 850, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used

$= \{2122, 628, 632, 210, 648, 642, 212\}$

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}2^{2/3}-2x\right)}{5832\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}$$

$$+ \frac{\sqrt[3]{-1}\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{729\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$

$$- \frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{11664\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$

$$- \frac{\sqrt[3]{-1}\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{2916\sqrt[6]{2}3^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$

$$- \frac{i\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{5832\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{52488\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$

$$+ \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{26244\sqrt[6]{2}3^{5/6}\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} - \frac{\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{34992\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^4}$$

$$+ \frac{i\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{34992\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5} - \frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{314928\sqrt[3]{2}3^{2/3}}$$

$$- \frac{\sqrt[3]{-\frac{1}{3}}\left(2x+3(-2)^{2/3}\sqrt[3]{3}\right)}{26244\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}$$

$$- \frac{\sqrt[3]{2}x+3\sqrt[3]{3}}{52488\left(9\sqrt[3]{2}-4\sqrt[3]{3}\right)\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}$$

```
[In] Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] ((-1/3)^(1/3)*(3*(-3)^(1/3)*2^(2/3) - 2*x))/(5832*2^(2/3)*(1 + (-1)^(1/3))^
4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-1/3)^
(1/3)*(3*(-2)^(2/3)*3^(1/3) + 2*x))/(26244*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/
6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (3*3^(1/3) +
2^(1/3)*x)/(52488*(9*2^(1/3) - 4*3^(1/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2))
+ ((-1)^(1/3)*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*
2^(1/3))]])/(729*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^4*(8 - (9*I)*2^(1/3)*3^(1
/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - ((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3)
+ 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2916*2^(1/6)*3^(5/6)*(1 - (-1)
^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((I + Sqrt
[3])*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]
])/(11664*2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]
) - ((I/5832)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3
)^(2/3)*2^(1/3))]])/(2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/
3)*2^(1/3)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(
1/3)*3^(2/3))]]/(26244*2^(1/6)*3^(5/6)*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + Ar
cTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(
52488*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - Log[6 - 3*(-3)^(1/3)*
2^(2/3)*x + x^2]/(34992*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^4) + ((I/34992)*Lo
g[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5) -
 Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(314928*2^(1/3)*3^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

integral

$$= 1586874322944 \int \left(-\frac{\sqrt[3]{-\frac{1}{3}}}{1542441841901568\ 2^{2/3} \left(1+\sqrt[3]{-1}\right)^4 \left(-6+3\sqrt[3]{-3}2^{2/3}x-x^2\right)^2} + \frac{6i3^{5/6}-\left(\sqrt[3]{-2}+\sqrt[3]{2}\right)x}{27763953154228224\ 6^{2/3} \left(1+\sqrt[3]{-1}\right)^5 \left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} - \frac{\sqrt[3]{-\frac{1}{3}}}{1542441841901568\ 2^{2/3} \left(-1+\sqrt[3]{-1}\right)^2 \left(1+\sqrt[3]{-1}\right)^4 \left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2} + \frac{i\left(9i-3\sqrt{3}+\sqrt[3]{2}\sqrt[6]{3}x\right)}{27763953154228224\ 2^{2/3}\sqrt[3]{3} \left(1+\sqrt[3]{-1}\right)^5 \left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)} + \frac{1}{1542441841901568\ 2^{2/3}\sqrt[3]{3} \left(-1+\sqrt[3]{-1}\right)^2 \left(1+\sqrt[3]{-1}\right)^4 \left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)^2} - \frac{3\ 2^{2/3}\sqrt[3]{3}+x}{249875578388054016\sqrt[3]{2}3^{2/3} \left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)} \right) dx$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}} \int \frac{1}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2} dx}{8748\ 2^{2/3}} - \frac{\int \frac{3\ 2^{2/3}\sqrt[3]{3}+x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2} dx}{157464\sqrt[3]{2}3^{2/3}} + \frac{\int \frac{1}{\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)^2} dx}{8748\ 2^{2/3}\sqrt[3]{3}} + \frac{\int \frac{6i3^{5/6}-\left(\sqrt[3]{-2}+\sqrt[3]{2}\right)x}{6-3\sqrt[3]{-3}2^{2/3}x+x^2} dx}{17496\ 6^{2/3} \left(1+\sqrt[3]{-1}\right)^5} - \frac{\sqrt[3]{-\frac{1}{3}} \int \frac{1}{\left(-6+3\sqrt[3]{-3}2^{2/3}x-x^2\right)^2} dx}{972\ 2^{2/3} \left(1+\sqrt[3]{-1}\right)^4} + \frac{\int \frac{9i-3\sqrt{3}+\sqrt[3]{2}\sqrt[6]{3}x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2} dx}{78732\ 2^{2/3}\sqrt[3]{3} \left(3i+\sqrt{3}\right)}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}(3\sqrt[3]{-3}2^{2/3}-2x)}{5832\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$

$$-\frac{\sqrt[3]{-\frac{1}{3}}\left(3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{52488\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{3\sqrt[3]{3}+\sqrt[3]{2}x}{52488\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}-\frac{\int\frac{3\ 2^{2/3}\sqrt[3]{3}+2x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{314928\sqrt[3]{2}3^{2/3}}$$

$$-\frac{\int\frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{52488\ 2^{2/3}\sqrt[3]{3}}+\frac{i\int\frac{1}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}\,dx}{1944\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}$$

$$-\frac{\int\frac{-3\sqrt[3]{-3}2^{2/3}+2x}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}\,dx}{34992\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^4}+\frac{\sqrt[3]{-\frac{1}{3}}\int\frac{1}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{2916\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}$$

$$+\frac{\int\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{157464\sqrt[3]{2}\sqrt[6]{3}\left(3i+\sqrt{3}\right)}-\frac{\left(1-i\sqrt{3}\right)\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{17496\ 2^{2/3}3^{5/6}\left(3i+\sqrt{3}\right)}$$

$$-\frac{\sqrt[3]{-\frac{1}{3}}\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{26244\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)}+\frac{\int\frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{26244\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)}$$

$$
= \frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}2^{2/3}-2x\right)}{5832\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}
$$

$$
- \frac{\sqrt[3]{-\frac{1}{3}}\left(3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{52488\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}
$$

$$
+ \frac{3\sqrt[3]{3}+\sqrt[3]{2}x}{52488\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)} - \frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{34992\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^4}
$$

$$
+ \frac{\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{157464\sqrt[3]{2}\sqrt[6]{3}\left(3i+\sqrt{3}\right)} - \frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{314928\sqrt[3]{2}3^{2/3}}
$$

$$
+ \frac{\text{Subst}\left(\int \frac{1}{-6\left(4-3\sqrt[3]{2}3^{2/3}\right)-x^2}\,dx, x, 3\ 2^{2/3}\sqrt[3]{3}+2x\right)}{26244\ 2^{2/3}\sqrt[3]{3}}
$$

$$
- \frac{i\text{Subst}\left(\int \frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}\,dx, x, -3\sqrt[3]{-3}2^{2/3}+2x\right)}{972\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}
$$

$$
- \frac{\sqrt[3]{-\frac{1}{3}}\text{Subst}\left(\int \frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}\,dx, x, 3\sqrt[3]{-3}2^{2/3}-2x\right)}{1458\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}
$$

$$
+ \frac{\left(1-i\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}\,dx, x, 3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{8748\ 2^{2/3}3^{5/6}\left(3i+\sqrt{3}\right)}
$$

$$
+ \frac{\sqrt[3]{-\frac{1}{3}}\text{Subst}\left(\int \frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}\,dx, x, 3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{13122\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)}
$$

$$
- \frac{\text{Subst}\left(\int \frac{1}{-6\left(4-3\sqrt[3]{2}3^{2/3}\right)-x^2}\,dx, x, 3\ 2^{2/3}\sqrt[3]{3}+2x\right)}{13122\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)}
$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}2^{2/3}-2x\right)}{5832\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$
$$- \frac{\sqrt[3]{-\frac{1}{3}}\left(3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{52488\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$+ \frac{3\sqrt[3]{3}+\sqrt[3]{2}x}{52488\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$+ \frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{2916\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}} - \frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{26244\sqrt[6]{2}3^{5/6}\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$
$$- \frac{\left(1-i\sqrt{3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{52488\sqrt[6]{2}\sqrt[3]{3}\left(3i+\sqrt{3}\right)\sqrt{4+3\sqrt[3]{-2}3^{2/3}}} - \frac{i\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{5832\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$
$$+ \frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{26244\sqrt[6]{2}3^{5/6}\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{52488\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$
$$- \frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{34992\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^4} + \frac{\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{157464\sqrt[3]{2}\sqrt[6]{3}\left(3i+\sqrt{3}\right)} - \frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{314928\sqrt[3]{2}3^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.20

$$\int \frac{x^4}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = \frac{-288+324x-1458x^2-216x^3+8x^4-9x^5}{1230552\left(216+108x^2+324x^3+18x^4+x^6\right)}$$
$$- \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{324\log(x-\#1)-2628\log(x-\#1)\#1+324\log(x-\#1)\#1^2-}{36\#1+162\#1^2+12\#1^3+}\right.}{7383312}$$

```
[In] Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-288 + 324*x - 1458*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(1230552*(216 + 108*x^2
 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 +
 #1^6 & , (324*Log[x - #1] - 2628*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 - 1
6*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5
) & ]/7383312
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.14

method	result
default	$\frac{-\frac{1}{136728}x^5+\frac{1}{153819}x^4-\frac{1}{5697}x^3-\frac{1}{844}x^2+\frac{1}{3798}x-\frac{4}{17091}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-9\_R^4+16\_R^3-32\right.}{\_R^5+12\_}\right.}{7383312}$
risch	$\frac{-\frac{1}{136728}x^5+\frac{1}{153819}x^4-\frac{1}{5697}x^3-\frac{1}{844}x^2+\frac{1}{3798}x-\frac{4}{17091}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-9\_R^4+16\_R^3-32\right.}{\_R^5+12\_}\right.}{7383312}$

```
[In] int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/136728*x^5+1/153819*x^4-1/5697*x^3-1/844*x^2+1/3798*x-4/17091)/(x^6+18*
x^4+324*x^3+108*x^2+216)+1/7383312*sum((-9*_R^4+16*_R^3-324*_R^2+2628*_R-32
4)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+1
08*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.13

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \mathrm{RootSum}\Big(185583791958607219605834030755606257729536t^6 - 1309367357962223565522033377280t^4$$
$$+ \frac{-9x^5+8x^4-216x^3-1458x^2+324x-288}{1230552x^6+22149936x^4+398698848x^3+132899616x^2+265799232}$$

```
[In] integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(185583791958607219605834030755606257729536*_t**6 - 1309367357962223
565522033377280*_t**4 + 4356336487052294744666112*_t**3 - 40529828454803873
28*_t**2 + 303890718384*_t - 880007, Lambda(_t, _t*log(3908346265795559347 6
84104470733356597641295275928063469158 4*_t**5/4979785539613990026757339569 5
 + 8836979346223785538912817601414711102396804462575616*_t**4/497978553961 3
9900267573395695 - 2649305813483085325888442495971346957068050677 76*_t**3/4
97978553961399002675733956 95 + 886135333547363185201515109826158376250624*_
t**2/4979785539613990026757339 5695 - 68232147957490990651139463585560193 6*_
t/4979785539613990026757339 5695 + x - 2137556077084648622429151956 8/4979785
5396139900267573395695))) + (-9*x**5 + 8*x**4 - 216*x**3 - 1458*x**2 + 324*
x - 288)/(1230552*x**6 + 22149936*x**4 + 398698848*x**3 + 132899616*x**2 +
265799232)
```

Maxima [F]

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^4}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] -1/1230552*(9*x^5 - 8*x^4 + 216*x^3 + 1458*x^2 - 324*x + 288)/(x^6 + 18*x^4
 + 324*x^3 + 108*x^2 + 216) - 1/1230552*integrate((9*x^4 - 16*x^3 + 324*x^2
 - 2628*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^4}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.46

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

```
[In] int(x^4/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
[Out] symsum(log((24389*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/
660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (397
1*z)/2425060040617647997585731147792384 - 880007/18558379195860721960583403
0755606257729536, z, k))/851770251396 + (288041*x)/804738905194918464 - (10
90723*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176
944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060
040617647997585731147792384 - 880007/18558379195860721960583403075560625772
9536, z, k)*x)/22997796787692 + (5850124*root(z^6 - (60865*z^4)/86267291061
38688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/770036338660
7884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185
583791958607219605834030755606257729536, z, k)^2*x)/3606201 - (64554687936*
root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870
474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617
647997585731147792384 - 880007/185583791958607219605834030755606257729536,
z, k)^3*x)/44521 + (31535589897216*root(z^6 - (60865*z^4)/8626729106138688
+ (15496909*z^3)/660182046176944870474752 - (168169*z^2)/770036338660788496
9217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791
958607219605834030755606257729536, z, k)^4*x)/211 - 6940988288557056*root(z
^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752
 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997
585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)^
5*x - (1697552*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660
182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z
)/2425060040617647997585731147792384 - 880007/18558379195860721960583403075
5606257729536, z, k)^2)/10818603 + (12229983936*root(z^6 - (60865*z^4)/8626
729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/77003
```

633866078849692175O7328 + (3971*z)/2425060040617647997585731147792384 - 880
007/185583791958607219605834030755606257729536, z, k)^3)/44521 + (253679492
45952*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176
944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060
040617647997585731147792384 - 880007/18558379195860721960583403075560625772
9536, z, k)^4)/211 - 168897381688221696*root(z^6 - (60865*z^4)/862672910613
8688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607
884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/1855
83791958607219605834030755606257729536, z, k)^5 - 971/22353858477636624)*ro
ot(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/66018204617694487047
4752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/242506004061764
7997585731147792384 - 880007/185583791958607219605834030755606257729536, z,
 k), k, 1, 6) - (x^2/844 - x/3798 + x^3/5697 - x^4/153819 + x^5/136728 + 4/
17091)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)

3.156 $\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal result 1225
Rubi [A] (warning: unable to verify) 1226
Mathematica [C] (verified) 1234
Maple [C] (verified) 1234
Fricas [F(-1)] 1235
Sympy [A] (verification not implemented) 1235
Maxima [F] 1236
Giac [F] 1236
Mupad [B] (verification not implemented) 1236

Optimal result

Integrand size = 26, antiderivative size = 873

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} - \frac{\sqrt[3]{-6}\left(9\sqrt[3]{-2}+2\sqrt[3]{3}\right)+3x}{157464\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)} - \frac{2\sqrt[3]{2}-3\ 6^{2/3}-\sqrt[3]{3}x}{104976\left(9\sqrt[3]{2}-4\sqrt[3]{3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)} + \frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{26244\sqrt{3}\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} - \frac{\left(9i-\sqrt[3]{3}\left(2i2^{2/3}+9\sqrt[6]{3}+2\ 2^{2/3}\sqrt{3}\right)\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{209952\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} - \frac{\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{26244\sqrt{3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} + \frac{\left(9i+\sqrt[3]{3}\left(4i2^{2/3}-9\sqrt[6]{3}\right)\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{209952\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4+3\sqrt[3]{-2}3^{2/3}\right)}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{52488\sqrt{6}\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} + \frac{\left(2\ 2^{2/3}-3\ 3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{944784\sqrt[6]{3}\sqrt{2\left(-4+3\sqrt[3]{2}3^{2/3}\right)}} - \frac{i\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{23328\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}$$

```
[Out] 1/157464*((-6)^(1/3)*(2*(-3)^(1/3)+9*2^(1/3))-3*x)/(8-9*I*2^(1/3)*3^(1/6)+3
*2^(1/3)*3^(2/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)+1/157464*(-(-6)^(1/3)*(9*(
-2)^(1/3)+2*3^(1/3))-3*x)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-
2)^(2/3)*3^(1/3)*x+x^2)+1/104976*(-2*2^(1/3)+3*6^(2/3)+3^(1/3)*x)/(9*2^(1/3
)-4*3^(1/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)-1/139968*I*ln(6-3*(-3)^(1/3)*2^(2/
3)*x+x^2)*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^5+1/3779136*ln(6+3*2^(2/3)*3^(1/3)
*x+x^2)*2^(1/3)*3^(2/3)+1/78732*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-
3)^(2/3)*2^(1/3))^(1/2))/(8-9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)*3^
(1/2)-1/78732*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^
(1/2))/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)*3^(1/2)+1/279936*ln(
6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*(3^(1/2)+I)*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^5-
1/314928*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2
))/(-4+3*2^(1/3)*3^(2/3))^(3/2)*6^(1/2)-1/209952*arctan((3*(-3)^(1/3)*2^(2/
3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*(9*I-3^(1/3)*(2*I*2^(2/3)+9*3^(1/
6)+2*2^(2/3)*3^(1/2)))/(1+(-1)^(1/3))^5/(8-6*(-3)^(2/3)*2^(1/3))^(1/2)+1/20
9952*(9*I+3^(1/3)*(4*I*2^(2/3)-9*3^(1/6)))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x
)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(1+(-1)^(1/3))^5/(8+6*(-2)^(1/3)*3^(2/3
))^(1/2)+1/2834352*(2*2^(2/3)-3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)
*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(-8+6*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 1.85 (sec) , antiderivative size = 873, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used

$$= \{2122, 652, 632, 210, 648, 642, 212\}$$

$$\int \frac{x^3}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$$

$$= \frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}$$

$$- \frac{\left(9i-\sqrt[3]{3}\left(2i2^{2/3}+9\sqrt[6]{3}+2\ 2^{2/3}\sqrt{3}\right)\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{209952\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}$$

$$+ \frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{26244\sqrt{3}\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$

$$+ \frac{\left(9i+\sqrt[3]{3}\left(4i2^{2/3}-9\sqrt[6]{3}\right)\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{209952\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}$$

$$- \frac{\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{26244\sqrt{3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$

$$+ \frac{\left(2\ 2^{2/3}-3\ 3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{944784\sqrt[6]{3}\sqrt{2\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}$$

$$- \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{52488\sqrt{6}\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} - \frac{i\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{23328\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}$$

$$+ \frac{\left(i+\sqrt{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{46656\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5} + \frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{629856\ 2^{2/3}\sqrt[3]{3}}$$

$$- \frac{3x+\sqrt[3]{-6}\left(9\sqrt[3]{-2}+2\sqrt[3]{3}\right)}{157464\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}$$

$$- \frac{-\sqrt[3]{3}x-3\ 6^{2/3}+2\sqrt[3]{2}}{\cdots}$$

```
[In] Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] ((-6)^(1/3)*(2*(-3)^(1/3) + 9*2^(1/3)) - 3*x)/(157464*(8 - (9*I)*2^(1/3)*3^
(1/6) + 3*2^(1/3)*3^(2/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3
)*(9*(-2)^(1/3) + 2*3^(1/3)) + 3*x)/(157464*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*
2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (2*2^(1/3) - 3*6^(2/
3) - 3^(1/3)*x)/(104976*(9*2^(1/3) - 4*3^(1/3))*(6 + 3*2^(2/3)*3^(1/3)*x +
x^2)) + ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3
))]]/(26244*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2))
- ((9*I - 3^(1/3)*((2*I)*2^(2/3) + 9*3^(1/6) + 2*2^(2/3)*Sqrt[3]))*ArcTan[(
3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(209952*(1
 + (-1)^(1/3))^5*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) - ArcTan[(3*(-2)^(2/3)
*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]]/(26244*Sqrt[3]*(8 + (9*
I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) + ((9*I + 3^(1/3)*((4*I)*2^(
2/3) - 9*3^(1/6)))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(
1/3)*3^(2/3))]])/(209952*(1 + (-1)^(1/3))^5*Sqrt[2*(4 + 3*(-2)^(1/3)*3^(2/3
))]) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(
2/3))]]/(52488*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + ((2*2^(2/3) - 3*3^
(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(
2/3))]])/(944784*3^(1/6)*Sqrt[2*(-4 + 3*2^(1/3)*3^(2/3))]) - ((I/23328)*Log
[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^5) +
((I + Sqrt[3])*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(46656*2^(2/3)*3^(5/6
)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(629856*2^(2/3)*
3^(1/3))
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 632</u>

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

<u>Rule 642</u>

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

integral

$$= 1586874322944 \int \left(-\frac{9(-2)^{2/3} - \sqrt[3]{-1}3^{2/3}x}{27763953154228224\ 2^{2/3} \left(1+\sqrt[3]{-1}\right)^4 \left(-6+3\sqrt[3]{-3}2^{2/3}x - x^2\right)^2} + \frac{27 + 3\ 2^{2/3}\sqrt[3]{3} - 9i\sqrt{3} + i2^{2/3}3^{5/6} + 3i\sqrt[3]{2}\sqrt[6]{3}x}{333167437850738688 \left(1+\sqrt[3]{-1}\right)^5 \left(-6+3\sqrt[3]{-3}2^{2/3}x - x^2\right)} - \frac{9\ 2^{2/3} - \sqrt[3]{-1}3^{2/3}x}{27763953154228224\ 2^{2/3} \left(-1+\sqrt[3]{-1}\right)^2 \left(1+\sqrt[3]{-1}\right)^4 \left(6+3(-2)^{2/3}\sqrt[3]{3}x + x^2\right)^2} + \frac{-2\left(27 - i\left(9\sqrt{3} + 2\ 2^{2/3}3^{5/6}\right)\right) + 3\sqrt[3]{2}\sqrt[6]{3}\left(i+\sqrt{3}\right)x}{666334875701477376 \left(1+\sqrt[3]{-1}\right)^5 \left(6+3(-2)^{2/3}\sqrt[3]{3}x + x^2\right)} - \frac{3\ 2^{2/3}\sqrt[3]{3} + x}{9254651051409408\ 2^{2/3}\sqrt[3]{3} \left(-1+\sqrt[3]{-1}\right)^2 \left(1+\sqrt[3]{-1}\right)^4 \left(6+3\ 2^{2/3}\sqrt[3]{3}x + x^2\right)^2} + \frac{18 - 2\ 2^{2/3}\sqrt[3]{3} + \sqrt[3]{2}3^{2/3}x}{2998506940656648192 \left(6+3\ 2^{2/3}\sqrt[3]{3}x + x^2\right)} \right) dx$$

$$= \frac{\int \frac{18-2\ 2^{2/3}\sqrt[3]{3}+\sqrt[3]{2}3^{2/3}x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{1889568} - \frac{\int \frac{9\ 2^{2/3}-\sqrt[3]{-1}3^{2/3}x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}\,dx}{157464\ 2^{2/3}} - \frac{\int \frac{3\ 2^{2/3}\sqrt[3]{3}+x}{\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)^2}\,dx}{52488\ 2^{2/3}\sqrt[3]{3}} + \frac{\int \frac{-2\left(27-i\left(9\sqrt{3}+2\ 2^{2/3}3^{5/6}\right)\right)+3\sqrt[3]{2}\sqrt[6]{3}\left(i+\sqrt{3}\right)x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{419904\left(1+\sqrt[3]{-1}\right)^5} + \frac{\int \frac{27+3\ 2^{2/3}\sqrt[3]{3}-9i\sqrt{3}+i2^{2/3}3^{5/6}+3i\sqrt[3]{2}\sqrt[6]{3}x}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{209952\left(1+\sqrt[3]{-1}\right)^5} - \frac{\int \frac{9(-2)^{2/3}-\sqrt[3]{-1}3^{2/3}x}{\left(-6+3\sqrt[3]{-3}2^{2/3}x-x^2\right)^2}\,dx}{17496\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4}$$

$$= \frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$

$$-\frac{\sqrt[3]{-6}\left(9\sqrt[3]{-2}+2\sqrt[3]{3}\right)+3x}{314928\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{2\sqrt[3]{2}-3\ 6^{2/3}-\sqrt[3]{3}x}{104976\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}+\frac{\int\frac{3\ 2^{2/3}\sqrt[3]{3}+2x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{629856\ 2^{2/3}\sqrt[3]{3}}$$

$$-\frac{i\int\frac{3\sqrt[3]{-3}2^{2/3}-2x}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{23328\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}+\frac{\left(9-2\ 2^{2/3}\sqrt[3]{3}\right)\int\frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{1889568}$$

$$+\frac{\left(i+\sqrt{3}\right)\int\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{46656\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}-\frac{\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{104976\left(4+3\sqrt[3]{-2}3^{2/3}\right)}$$

$$-\frac{\int\frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{104976\left(4-3\sqrt[3]{2}3^{2/3}\right)}+\frac{\int\frac{1}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{52488\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)}$$

$$+\frac{\left(18(-1)^{5/6}\sqrt{3}+2\left(27+3\ 2^{2/3}\sqrt[3]{3}-9i\sqrt{3}+i2^{2/3}3^{5/6}\right)\right)\int\frac{1}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}\,dx}{419904\left(1+\sqrt[3]{-1}\right)^5}$$

$$-\frac{\left(18(-1)^{2/3}\sqrt{3}\left(i+\sqrt{3}\right)+4\left(27-i\left(9\sqrt{3}+2\ 2^{2/3}3^{5/6}\right)\right)\right)\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{839808\left(1+\sqrt[3]{-1}\right)^5}$$

$$= \frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$

$$-\frac{\sqrt[3]{-6}\left(9\sqrt[3]{-2}+2\sqrt[3]{3}\right)+3x}{314928\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{2\sqrt[3]{2}-3\ 6^{2/3}-\sqrt[3]{3}x}{104976\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}-\frac{i\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{23328\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}$$

$$+\frac{\left(i+\sqrt{3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{46656\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}+\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{629856\ 2^{2/3}\sqrt[3]{3}}$$

$$+\frac{\left(-9+2\ 2^{2/3}\sqrt[3]{3}\right)\text{Subst}\left(\int\frac{1}{-6\left(4-3\sqrt[3]{2}3^{2/3}\right)-x^2}\,dx,x,3\ 2^{2/3}\sqrt[3]{3}+2x\right)}{944784}$$

$$+\frac{\text{Subst}\left(\int\frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}\,dx,x,3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{52488\left(4+3\sqrt[3]{-2}3^{2/3}\right)}$$

$$+\frac{\text{Subst}\left(\int\frac{1}{-6\left(4-3\sqrt[3]{2}3^{2/3}\right)-x^2}\,dx,x,3\ 2^{2/3}\sqrt[3]{3}+2x\right)}{52488\left(4-3\sqrt[3]{2}3^{2/3}\right)}$$

$$-\frac{\text{Subst}\left(\int\frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}\,dx,x,3\sqrt[3]{-3}2^{2/3}-2x\right)}{26244\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)}$$

$$-\frac{\left(18(-1)^{5/6}\sqrt{3}+2\left(27+3\ 2^{2/3}\sqrt[3]{3}-9i\sqrt{3}+i2^{2/3}3^{5/6}\right)\right)\text{Subst}\left(\int\frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}\,dx,x,3\sqrt[3]{-3}2^{2/3}\right.}{209952\left(1+\sqrt[3]{-1}\right)^5}$$

$$+\frac{\left(18(-1)^{2/3}\sqrt{3}\left(i+\sqrt{3}\right)+4\left(27-i\left(9\sqrt{3}+2\ 2^{2/3}3^{5/6}\right)\right)\right)\text{Subst}\left(\int\frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}\,dx,x,3(-2)^{2/3}\sqrt[3]{3}\right.}{419904\left(1+\sqrt[3]{-1}\right)^5}$$

$$
\begin{aligned}
&= \frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} \\
&\quad - \frac{\sqrt[3]{-6}\left(9\sqrt[3]{-2}+2\sqrt[3]{3}\right)+3x}{314928\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)} \\
&\quad + \frac{2\sqrt[3]{2}-3\ 6^{2/3}-\sqrt[3]{3}x}{104976\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)} \\
&\quad + \frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{26244\sqrt{3}\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} \\
&\quad + \frac{\left(27+6\ 2^{2/3}\sqrt[3]{3}-9i\sqrt{3}+2i2^{2/3}3^{5/6}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{209952\left(1+\sqrt[3]{-1}\right)^5\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} \\
&\quad - \frac{\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{52488\sqrt{6}\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}} - \frac{\left(27-9i\sqrt{3}-4i2^{2/3}3^{5/6}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{209952\left(1+\sqrt[3]{-1}\right)^5\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}} \\
&\quad - \frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{52488\sqrt{6}\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} \\
&\quad + \frac{\left(2\ 2^{2/3}-3\ 3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{944784\sqrt[6]{3}\sqrt{2\left(-4+3\sqrt[3]{2}3^{2/3}\right)}} - \frac{i\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{23328\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5} \\
&\quad + \frac{\left(i+\sqrt{3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{46656\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5} + \frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{629856\ 2^{2/3}\sqrt[3]{3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.19

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \frac{972-3942x+648x^2+96x^3-27x^4+4x^5}{3691656\,(216+108x^2+324x^3+18x^4+x^6)} + \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{1971\log(x-\#1)-162\log(x-\#1)\#1+72\log(x-\#1)\#1^2-27\text{l}}{36\#1+162\#1^2+12\#1^3+\#}\right.}{11074968}$$

```
[In] Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (972 - 3942*x + 648*x^2 + 96*x^3 - 27*x^4 + 4*x^5)/(3691656*(216 + 108*x^2
+ 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 +
#1^6 & , (1971*Log[x - #1] - 162*Log[x - #1]*#1 + 72*Log[x - #1]*#1^2 - 27*
Log[x - #1]*#1^3 + 2*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5)
& ]/11074968
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.14

method	result
default	$\frac{\frac{1}{922914}x^5-\frac{1}{136728}x^4+\frac{4}{153819}x^3+\frac{1}{5697}x^2-\frac{73}{68364}x+\frac{1}{3798}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(2\_R^4-27\_R^3+72\right.}{\_R^5+12\_}\right.}{11074968}$
risch	$\frac{\frac{1}{922914}x^5-\frac{1}{136728}x^4+\frac{4}{153819}x^3+\frac{1}{5697}x^2-\frac{73}{68364}x+\frac{1}{3798}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(2\_R^4-27\_R^3+72\right.}{\_R^5+12\_}\right.}{11074968}$

```
[In] int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/922914*x^5-1/136728*x^4+4/153819*x^3+1/5697*x^2-73/68364*x+1/3798)/(x^6+
18*x^4+324*x^3+108*x^2+216)+1/11074968*sum((2*_R^4-27*_R^3+72*_R^2-162*_R+1
971)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3
+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\text{Timed out}$$

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.13

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$=\text{RootSum}\Big(128275517001789310191552482058275045342655 2832t^6-90638846577554424442625114977$$
$$+\frac{4x^5-27x^4+96x^3+648x^2-3942x+972}{3691656x^6+66449808x^4+1196096544x^3+398698848x^2+797397696}$$

```
[In] integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
[Out] RootSum(1282755170017893101915524820582750453426552832*_t**6 - 906388465775
5442444262511497 70752*_t**4 - 430087316638998774168413 7984*_t**3 - 71700090
8921644962816*_t**2 + 135354162312576*_t - 7197829, Lambda(_t, _t*log(17257
93559281044990140955659789188299560400108333936804136148061 3888*_t**5/15420
600979105204449069438030323 7521 + 238960740062098552437635885357265220718 19
5632456058768405 2992*_t**4/154206009791052044490694380303237521 - 122860721
60883283930711715948878260078996992193488388096*_t**3/154206009791052044490
694380303237521 - 594905535739591731611254960135279097541565584 10752*_t**2/
154206009791052044490694380303237521 - 175201496798366911123670641977137530
04827200*_t/154206009791052044490694380303237521 + x + 766422988707229615 05
5855287040887332/154206009791052044490694380303237521))) + (4*x**5 - 27*x**
4 + 96*x**3 + 648*x**2 - 3942*x + 972)/(3691656*x**6 + 66449808*x**4 + 1196
096544*x**3 + 398698848*x**2 + 797397696)
```

Maxima [F]

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^3}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] 1/3691656*(4*x^5 - 27*x^4 + 96*x^3 + 648*x^2 - 3942*x + 972)/(x^6 + 18*x^4
+ 324*x^3 + 108*x^2 + 216) + 1/1845828*integrate((2*x^4 - 27*x^3 + 72*x^2 -
 162*x + 1971)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^3}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.44

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

```
[In] int(x^3/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((11*x)/603554178896188848 - (14059*root(z^6 - (292589*z^4)/41408
2997094657024 - (11805253*z^3)/352097091294370597586534 4 - (2479189*z^2)/44
354093106861417422692842 20928 + (1989787*z)/188572668758428308292266454052 3
3577984 - 7197829/128275517001789310191552482058275045342655283 2, z, k))/30
663729050256 - (5658601*root(z^6 - (292589*z^4)/414082997094657024 - (11805
253*z^3)/352097091294370597586534 4 - (2479189*z^2)/44354093106861417422692 8
4220928 + (1989787*z)/188572668758428308292266454052335779 84 - 7197829/1282
755170017893101915524820582750453426552832, z, k)*x)/6623365474855296 + (66
03523*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/352097091
2943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787
*z)/1885726687584283082922664540523357798 4 - 7197829/128275517001789310191 5
524820582750453426552832, z, k)^2*x)/584204562 - (1762321104*root(z^6 - (29
2589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (
2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/1885726687584283
```

0829226645405233577984 - 7197829/128275517001789310191552482058275045342655 2832, z, k)^3*x)/44521 - (59633904436992*root(z^6 - (292589*z^4)/4140829970 94657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409 310686141742269284220928 + (1989787*z)/1885726687584283082922664540523357798 4 - 7197829/1282755170017893101915524820582750453426552832, z, k)^4*x)/211 - 6940988288557056*root(z^6 - (292589*z^4)/414082997094657024 - (11805253* z^3)/3520970912943705975865344 - (2479189*z^2)/443540931068614174226928422 0928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/12827551 70017893101915524820582750453426552832, z, k)^5*x + (166697*root(z^6 - (292 589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2 479189*z^2)/443540931068614174226928422 0928 + (1989787*z)/18857266875842830 829226645405233577984 - 7197829/1282755170017893101915524820582750453426552 832, z, k)^2)/43274412 + (639193032*root(z^6 - (292589*z^4)/414082997094657 024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/443540931068 614174226928422 0928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)^3)/44521 - (9 815247601920*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/35 20970912943705975865344 - (2479189*z^2)/443540931068614174226928422 0928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/128275517001789 3101915524820582750453426552832, z, k)^4)/211 - 1688973816882216 96*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/352097091294370597586534 4 - (2479189*z^2)/443540931068614174226928422 0928 + (1989787*z)/1885726687 5842830829226645405233577984 - 7197829/128275517001789310191552482058275045 3426552832, z, k)^5 + 661/28970600587017064704)*root(z^6 - (292589*z^4)/414 082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/ 443540931068614174226928422 0928 + (1989787*z)/188572668758428308292266454 05 233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k), k, 1, 6) + (x^2/5697 - (73*x)/68364 + (4*x^3)/153819 - x^4/136728 + x^5/922 914 + 1/3798)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)

3.157 $\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal result 1239
Rubi [A] (warning: unable to verify) 1240
Mathematica [C] (verified) 1248
Maple [C] (verified) 1248
Fricas [F(-1)] 1249
Sympy [A] (verification not implemented) 1249
Maxima [F] 1249
Giac [F] 1250
Mupad [B] (verification not implemented) 1250

Optimal result

Integrand size = 26, antiderivative size = 986

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= -\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{104976\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$
$$-\frac{27\ 2^{2/3}\left(1+\sqrt[3]{-2}3^{2/3}\right)-\sqrt[3]{-1}3^{2/3}\left(2+3\sqrt[3]{-2}3^{2/3}\right)x}{472392\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$+\frac{9\left(6-2^{2/3}\sqrt[3]{3}\right)-\left(2-3\sqrt[3]{2}3^{2/3}\right)x}{314928\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$-\frac{\left(1+i\sqrt{3}+3\sqrt[3]{2}3^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{8748\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$
$$+\frac{\left(3(-3)^{2/3}+\sqrt[3]{-1}2^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{17496\ 6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$
$$+\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{34992\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$
$$+\frac{i\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{17496\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$
$$-\frac{\left(2^{2/3}-3\ 3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{17496\ 6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{157464\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}+\frac{\left(i+\sqrt{3}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{419904\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}$$
$$-\frac{i\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{209952\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}+\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{1889568\sqrt[3]{2}3^{2/3}}$$

```
[Out] 1/209952*(-27*(-2)^(2/3)-54*(-1)^(1/3)*3^(2/3)+6^(1/3)*(9+(-3)^(1/3)*2^(2/3
))*x)*2^(1/3)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(
2/3)*x+x^2)+1/944784*(-27*2^(2/3)-54*(-1)^(1/3)*3^(2/3)+(-1)^(1/3)*3^(2/3)*
(2+3*(-2)^(1/3)*3^(2/3))*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3
))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/1889568*(54-9*2^(2/3)*3^(1/3)-(2-3*2^(1
/3)*3^(2/3))*x)*2^(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*
x+x^2)+1/104976*(3*(-3)^(2/3)+(-1)^(1/3)*2^(2/3))*arctan((3*(-2)^(2/3)*3^(1
/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*6^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)
^(1/3))^4/(4+3*(-2)^(1/3)*3^(2/3))^(3/2)-1/104976*(2^(2/3)-3*3^(2/3))*arcta
nh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*6^(1/6)/(1-
(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-4+3*2^(1/3)*3^(2/3))^(3/2)-1/1259712*I*ln(
6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(5/6)/(1+(-1)^(1/3))^5+1/11337408*l
n(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/52488*arctan((3*(-3)^(1/3)*2
^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*(1+3*2^(1/3)*3^(2/3)+I*3^(1/2
))*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^4/(8-9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3
))^(3/2)+1/2519424*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*(3^(1/2)+I)*2^(2/3)*3^(
5/6)/(1+(-1)^(1/3))^5+1/104976*I*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(1
2-9*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(2/3)/(1+(-1)^(1/3))^5/(4-3*(-3)^(
2/3)*2^(1/3))^(1/2)+1/209952*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^
(1/3)*3^(2/3))^(1/2))*(3^(1/2)+I)*2^(5/6)*3^(2/3)/(1+(-1)^(1/3))^5/(4+3*(-2
)^(1/3)*3^(2/3))^(1/2)-1/944784*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+
9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 2.04 (sec) , antiderivative size = 986, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used

$= \{2122, 652, 632, 210, 648, 642, 212\}$

$$\int \frac{x^2}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$$

$$= -\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{104976\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}$$

$$-\frac{\left(1+i\sqrt{3}+3\sqrt[3]{2}3^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{8748\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$

$$+\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{34992\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$

$$+\frac{\left(3(-3)^{2/3}+\sqrt[3]{-1}2^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{17496\ 6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$

$$+\frac{i\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{17496\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{157464\sqrt[3]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$

$$-\frac{\left(2^{2/3}-3\ 3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{17496\ 6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$

$$+\frac{\left(i+\sqrt{3}\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{419904\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}$$

$$-\frac{i\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{209952\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}+\frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{1889568\sqrt[3]{2}3^{2/3}}$$

$$-\frac{27\ 2^{2/3}\left(1+\sqrt[3]{-2}3^{2/3}\right)-\sqrt[3]{-1}3^{2/3}\left(2+3\sqrt[3]{-2}3^{2/3}\right)x}{472392\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}$$

$$+\frac{9\left(6-2^{2/3}\sqrt[3]{3}\right)-\left(2-3\sqrt[3]{2}3^{2/3}\right)x}{314928\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}$$

```
[In] Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] -1/104976*(27*((-2)^(2/3) + 2*(-1)^(1/3)*3^(2/3)) - 6^(1/3)*(9 + (-3)^(1/3)
*2^(2/3))*x)/(2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*
(-3)^(1/3)*2^(2/3)*x + x^2)) - (27*2^(2/3)*(1 + (-2)^(1/3)*3^(2/3)) - (-1)^
(1/3)*3^(2/3)*(2 + 3*(-2)^(1/3)*3^(2/3))*x)/(472392*2^(2/3)*(8 + (9*I)*2^(1
/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (9*(
6 - 2^(2/3)*3^(1/3)) - (2 - 3*2^(1/3)*3^(2/3))*x)/(314928*2^(2/3)*3^(1/3)*(
4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - ((1 + I*Sqrt[3] +
 3*2^(1/3)*3^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^
(2/3)*2^(1/3))]])/(8748*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^4*(8 - (9*I)*2^(1/
3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) + ((3*(-3)^(2/3) + (-1)^(1/3)*2^(2/3
))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])
/(17496*6^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(
2/3))^(3/2)) + ((I + Sqrt[3])*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4
 + 3*(-2)^(1/3)*3^(2/3))]])/(34992*2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[
4 + 3*(-2)^(1/3)*3^(2/3)]) + ((I/17496)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(
1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(2^(1/6)*3^(1/3)*(1 + (-1)^(1
/3))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((2^(2/3) - 3*3^(2/3))*ArcTanh[(2^
(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(17496*6^
(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2))
 - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3)
)]]/(157464*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((I + Sqrt[3])*
Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(419904*2^(1/3)*3^(1/6)*(1 + (-1)^(1
/3))^5) - ((I/209952)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/
6)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(1889568*2^(1/3
)*3^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

integral

$$= 1586874322944 \int \left(\frac{2\sqrt[3]{-1}3^{2/3} + 18\sqrt[3]{6} + 3(-2)^{2/3}x}{55527906308456448\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(-6+3\sqrt[3]{-3}2^{2/3}x - x^2\right)^2} - \frac{i\left(18\ 3^{5/6} + \sqrt[3]{2}\left(3i-\sqrt{3}\right)x\right)}{333167437850738688\ 6^{2/3}\left(1+\sqrt[3]{-1}\right)^5\left(6-3\sqrt[3]{-3}2^{2/3}x + x^2\right)} + \frac{2\sqrt[3]{-1}3^{2/3} + 18(-1)^{2/3}\sqrt[3]{6} + 3\ 2^{2/3}x}{55527906308456448\ 2^{2/3}\left(-1+\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(6+3(-2)^{2/3}\sqrt[3]{3}x + x^2\right)^2} + \frac{9+9\sqrt[3]{-1} - i\sqrt[3]{2}\sqrt[6]{3}x}{166583718925369344\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\left(6+3(-2)^{2/3}\sqrt[3]{3}x + x^2\right)} + \frac{-2+6\sqrt[3]{2}3^{2/3} + 2^{2/3}\sqrt[3]{3}x}{18509302102818816\ 2^{2/3}\sqrt[3]{3}\left(-1+\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(6+3\ 2^{2/3}\sqrt[3]{3}x + x^2\right)^2} + \frac{9\sqrt[3]{3} + \sqrt[3]{2}x}{1499253470328324096\ 6^{2/3}\left(6+3\ 2^{2/3}\sqrt[3]{3}x + x^2\right)} \right) dx$$

$$= \frac{\int \frac{2\sqrt[3]{-1}3^{2/3} + 18(-1)^{2/3}\sqrt[3]{6} + 3\ 2^{2/3}x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x + x^2\right)^2} dx}{314928\ 2^{2/3}} + \frac{\int \frac{-2+6\sqrt[3]{2}3^{2/3} + 2^{2/3}\sqrt[3]{3}x}{\left(6+3\ 2^{2/3}\sqrt[3]{3}x + x^2\right)^2} dx}{104976\ 2^{2/3}\sqrt[3]{3}} + \frac{\int \frac{9\sqrt[3]{3} + \sqrt[3]{2}x}{6+3\ 2^{2/3}\sqrt[3]{3}x + x^2} dx}{944784\ 6^{2/3}} + \frac{\int \frac{9+9\sqrt[3]{-1} - i\sqrt[3]{2}\sqrt[6]{3}x}{6+3(-2)^{2/3}\sqrt[3]{3}x + x^2} dx}{104976\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5} - \frac{i\int \frac{18\ 3^{5/6} + \sqrt[3]{2}\left(3i-\sqrt{3}\right)x}{6-3\sqrt[3]{-3}2^{2/3}x + x^2} dx}{209952\ 6^{2/3}\left(1+\sqrt[3]{-1}\right)^5} + \frac{\int \frac{2\sqrt[3]{-1}3^{2/3} + 18\sqrt[3]{6} + 3(-2)^{2/3}x}{\left(-6+3\sqrt[3]{-3}2^{2/3}x - x^2\right)^2} dx}{34992\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4}$$

$$= -\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{104976\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$

$$-\frac{27\ 2^{2/3}\left(1+\sqrt[3]{-2}3^{2/3}\right)-\sqrt[3]{-1}3^{2/3}\left(2+3\sqrt[3]{-2}3^{2/3}\right)x}{944784\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{9\left(6-2^{2/3}\sqrt[3]{3}\right)-\left(2-3\sqrt[3]{2}3^{2/3}\right)x}{314928\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{\int\frac{3\ 2^{2/3}\sqrt[3]{3}+2x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}dx}{1889568\sqrt[3]{2}3^{2/3}}+\frac{\int\frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}dx}{157464\ 2^{2/3}\sqrt[3]{3}}-\frac{i\int\frac{1}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}dx}{5832\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}$$

$$-\frac{i\int\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{209952\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}+\frac{\left(i+\sqrt{3}\right)\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{11664\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}$$

$$+\frac{\left(i+\sqrt{3}\right)\int\frac{-3\sqrt[3]{-3}2^{2/3}+2x}{6-3\sqrt[3]{-3}2^{2/3}x+x^2}dx}{419904\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}-\frac{\left(2\ 3^{2/3}-9\sqrt[3]{6}\right)\int\frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}dx}{1889568\left(2\ 2^{2/3}-3\ 3^{2/3}\right)}$$

$$--\frac{\left(-18\sqrt[3]{-6}(-1)^{2/3}-2\left(2\sqrt[3]{-1}3^{2/3}+18\sqrt[3]{6}\right)\right)\int\frac{1}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2}dx}{34992\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(24-18(-3)^{2/3}\sqrt[3]{2}\right)}$$

$$+\frac{\left(-18(-1)^{2/3}\sqrt[3]{6}+2\left(2\sqrt[3]{-1}3^{2/3}+18(-1)^{2/3}\sqrt[3]{6}\right)\right)\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{314928\ 2^{2/3}\left(24+18\sqrt[3]{-2}3^{2/3}\right)}$$

$$= -\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{104976\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$

$$-\frac{27\ 2^{2/3}\left(1+\sqrt[3]{-2}3^{2/3}\right)-\sqrt[3]{-1}3^{2/3}\left(2+3\sqrt[3]{-2}3^{2/3}\right)x}{944784\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{9\left(6-2^{2/3}\sqrt[3]{3}\right)-\left(2-3\sqrt[3]{2}3^{2/3}\right)x}{314928\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{\left(i+\sqrt{3}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{419904\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}-\frac{i\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{209952\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}$$

$$+\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{1889568\sqrt[3]{2}3^{2/3}}-\frac{\text{Subst}\left(\int\frac{1}{-6\left(4-3\sqrt[3]{2}3^{2/3}\right)-x^2}\,dx,x,3\ 2^{2/3}\sqrt[3]{3}+2x\right)}{78732\ 2^{2/3}\sqrt[3]{3}}$$

$$+\frac{i\text{Subst}\left(\int\frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}\,dx,x,-3\sqrt[3]{-3}2^{2/3}+2x\right)}{2916\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}$$

$$-\frac{\left(i+\sqrt{3}\right)\text{Subst}\left(\int\frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}\,dx,x,3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{5832\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}$$

$$+\frac{\left(2\ 3^{2/3}-9\sqrt[3]{6}\right)\text{Subst}\left(\int\frac{1}{-6\left(4-3\sqrt[3]{2}3^{2/3}\right)-x^2}\,dx,x,3\ 2^{2/3}\sqrt[3]{3}+2x\right)}{944784\left(2\ 2^{2/3}-3\ 3^{2/3}\right)}+$$

$$-\frac{\left(-18\sqrt[3]{-6}(-1)^{2/3}-2\left(2\sqrt[3]{-1}3^{2/3}+18\sqrt[3]{6}\right)\right)\text{Subst}\left(\int\frac{1}{-6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)-x^2}\,dx,x,3\sqrt[3]{-3}2^{2/3}-2x\right)}{17496\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(24-18(-3)^{2/3}\sqrt[3]{2}\right)}$$

$$-\frac{\left(-18(-1)^{2/3}\sqrt[3]{6}+2\left(2\sqrt[3]{-1}3^{2/3}+18(-1)^{2/3}\sqrt[3]{6}\right)\right)\text{Subst}\left(\int\frac{1}{-6\left(4+3\sqrt[3]{-2}3^{2/3}\right)-x^2}\,dx,x,3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{157464\ 2^{2/3}\left(24+18\sqrt[3]{-2}3^{2/3}\right)}$$

$$= -\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{104976\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$

$$-\frac{27\ 2^{2/3}\left(1+\sqrt[3]{-2}3^{2/3}\right)-\sqrt[3]{-1}3^{2/3}\left(2+3\sqrt[3]{-2}3^{2/3}\right)x}{944784\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{9\left(6-2^{2/3}\sqrt[3]{3}\right)-\left(2-3\sqrt[3]{2}3^{2/3}\right)x}{314928\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$-\frac{\left(2\sqrt[3]{-1}+3\sqrt[3]{2}3^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{34992\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}}$$

$$+\frac{\left(3(-3)^{2/3}\sqrt[6]{2}+\sqrt[3]{-1}2^{5/6}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{314928\ 3^{5/6}\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$

$$+\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{34992\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$

$$+\frac{i\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{17496\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$

$$-\frac{\left(2^{2/3}-3\ 3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{157464\ 6^{5/6}\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$

$$-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{157464\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}+\frac{\left(i+\sqrt{3}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{419904\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}$$

$$-\frac{i\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{209952\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}+\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{1889568\sqrt[3]{2}3^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.17

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \frac{-7884+324x-2724x^2-216x^3+8x^4-9x^5}{7383312\,(216+108x^2+324x^3+18x^4+x^6)} - \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{324\log(x-\#1)+2436\log(x-\#1)\#1+324\log(x-\#1)\#1^2-16}{36\#1+162\#1^2+12\#1^3+\#}\right.}{44299872}$$

[In] Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] (-7884 + 324*x - 2724*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(7383312*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (324*Log[x - #1] + 2436*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 - 16*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) &]/44299872

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.12

method	result
default	$\frac{-\frac{1}{820368}x^5+\frac{1}{922914}x^4-\frac{1}{34182}x^3-\frac{227}{615276}x^2+\frac{1}{22788}x-\frac{73}{68364}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-9\_R^4+16\_R\right.}{\_R^5}\right.}{44299872}$
risch	$\frac{-\frac{1}{820368}x^5+\frac{1}{922914}x^4-\frac{1}{34182}x^3-\frac{227}{615276}x^2+\frac{1}{22788}x-\frac{73}{68364}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-9\_R^4+16\_R\right.}{\_R^5}\right.}{44299872}$

[In] int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)

[Out] (-1/820368*x^5+1/922914*x^4-1/34182*x^3-227/615276*x^2+1/22788*x-73/68364)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/44299872*sum((-9*_R^4+16*_R^3-324*_R^2-2436*_R-324)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.11

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \text{RootSum}\Big(865859739762077843792979253893356556062923161 6t^6 + 1090680958717701682488386456 1$$
$$+ \frac{-9x^5+8x^4-216x^3-2724x^2+324x-7884}{7383312x^6+132899616x^4+2392193088x^3+797397696x^2+1594795392}$$

```
[In] integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(8658597397620778437929792538933565560629231616*_t**6 + 109068095871
770168248838645612544*_t**4 - 492655707593366915713499136*_t**3 + 403783317
45144603648*_t**2 - 695635011360*_t + 4513, Lambda(_t, _t*log(1014425315618
04181113161287039859349851881619653631712165888*_t**5/356900697070792948475
845 - 149796550082359335112709434971975088967050210050048*_t**4/35690069707
0792948475845 + 122240975445827281850589877776867078361723699 2*_t**3/356900
697070792948475845 - 577505552425159572302290193855826145382 4*_t**2/3569006
97070792948475845 + 96165242200260265765603930470432*_t/7138013941415858969
5169 + x - 17059152341129698120545584/107070209121237884542 7535))) + (-9*x*
*5 + 8*x**4 - 216*x**3 - 2724*x**2 + 324*x - 7884)/(7383312*x**6 + 13289961
6*x**4 + 2392193088*x**3 + 797397696*x**2 + 1594795392)
```

Maxima [F]

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^2}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] -1/7383312*(9*x^5 - 8*x^4 + 216*x^3 + 2724*x^2 - 324*x + 7884)/(x^6 + 18*x^
4 + 324*x^3 + 108*x^2 + 216) - 1/7383312*integrate((9*x^4 - 16*x^3 + 324*x^
2 + 2436*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^2}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

```
[In] int(x^2/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
[Out] symsum(log((4897*x)/18772949180387057928192 - (8147*root(z^6 + (163*z^4)/12
940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2)/11
0885232767153543556732105232 - (505*z)/6285755625280943609742215135077859
328 + 4513/865859739762077843792979253893356556062923161 6, z, k))/1103894245
809216 - (1197643*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/14
2599321974220092022546432 + (5171*z^2)/110885232767153543556732105232 - (5
05*z)/6285755625280943609742215135077859328 + 4513/86585973976207784379297 9
25389335655606292316 16, z, k)*x)/29805144636848832 + (452809*root(z^6 + (16
3*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (517
1*z^2)/110885232767153543556732105232 - (505*z)/628575562528094360974221 51
35077859328 + 4513/8658597397620778437929792538933565560629231616, z, k)^2*
x)/194734854 - (1241776944*root(z^6 + (163*z^4)/12940093659208032 - (811359
7*z^3)/142599321974220092022546432 + (5171*z^2)/1108852327671535435567321 05
5232 - (505*z)/6285755625280943609742215135077859328 + 4513/865859739762077
8437929792538933565560629231616, z, k)^3*x)/44521 + (452407928832*root(z^6
+ (163*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 +
 (5171*z^2)/110885232767153543556732105232 - (505*z)/628575562528094360974
2215135077859328 + 4513/8658597397620778437929792538933565560629231616, z,
k)^4*x)/211 - 6940988288557056*root(z^6 + (163*z^4)/12940093659208032 - (81
13597*z^3)/142599321974220092022546432 + (5171*z^2)/110885232767153543556 73
21055232 - (505*z)/6285755625280943609742215135077859328 + 4513/86585973976
2077843792979253893356556062923161 6, z, k)^5*x + (114155*root(z^6 + (163*z^
4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^
2)/110885232767153543556732105232 - (505*z)/62857556252809436097422151350 7
7859328 + 4513/8658597397620778437929792538933565560629231616, z, k)^2)/292
102281 - (163984176*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/
```

```
142599321974220092022546432 + (5171*z^2)/1108852327671535435567321055232 -
(505*z)/6285755625280943609742215135077859328 + 4513/8658597397620778437929
792538933565560629231616, z, k)^3)/44521 + (94281884928*root(z^6 + (163*z^4
)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2
)/1108852327671535435567321055232 - (505*z)/6285755625280943609742215135077
859328 + 4513/8658597397620778437929792538933565560629231616, z, k)^4)/211
- 168897381688221696*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)
/142599321974220092022546432 + (5171*z^2)/1108852327671535435567321055232 -
 (505*z)/6285755625280943609742215135077859328 + 4513/865859739762077843792
9792538933565560629231616, z, k)^5 + 1/19313733724678043136)*root(z^6 + (16
3*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (517
1*z^2)/1108852327671535435567321055232 - (505*z)/62857556252809436097422151
35077859328 + 4513/8658597397620778437929792538933565560629231616, z, k), k
, 1, 6) - ((227*x^2)/615276 - x/22788 + x^3/34182 - x^4/922914 + x^5/820368
 + 73/68364)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

3.158 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx}\,dx$

Optimal result 1252
Rubi [A] (verified) 1252
Mathematica [A] (verified) 1253
Maple [A] (verified) 1253
Fricas [A] (verification not implemented) 1253
Sympy [A] (verification not implemented) 1254
Maxima [A] (verification not implemented) 1254
Giac [A] (verification not implemented) 1254
Mupad [B] (verification not implemented) 1255

Optimal result

Integrand size = 52, antiderivative size = 25

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx}\,dx = a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {1600}

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx}\,dx = a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
c + d*x),x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(a^2+2abx^2+b^2x^4\right)\,dx \\ &= a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx}\,dx = a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(c + d*x),x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$a^2x + \frac{2}{3}ab\,x^3 + \frac{1}{5}b^2x^5$	22
norman	$a^2x + \frac{2}{3}ab\,x^3 + \frac{1}{5}b^2x^5$	22
risch	$a^2x + \frac{2}{3}ab\,x^3 + \frac{1}{5}b^2x^5$	22
parallelrisch	$a^2x + \frac{2}{3}ab\,x^3 + \frac{1}{5}b^2x^5$	22
parts	$a^2x + \frac{2}{3}ab\,x^3 + \frac{1}{5}b^2x^5$	22
gosper	$\frac{x\left(3b^2x^4+10ab\,x^2+15a^2\right)}{15}$	25

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx}\,dx = \frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c),x, algorithm="fricas")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx}\,dx = a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(d*x+c),x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx}\,dx = \frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c),x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx}\,dx = \frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c),x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx}\,dx = a^2\,x + \frac{2\,a\,b\,x^3}{3} + \frac{b^2\,x^5}{5}$$

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
c + d*x),x)
```

```
[Out] a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

3.159 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx$

Optimal result 1256
Rubi [A] (verified) 1256
Mathematica [A] (verified) 1257
Maple [A] (verified) 1258
Fricas [A] (verification not implemented) 1258
Sympy [A] (verification not implemented) 1259
Maxima [A] (verification not implemented) 1259
Giac [B] (verification not implemented) 1259
Mupad [B] (verification not implemented) 1260

Optimal result

Integrand size = 52, antiderivative size = 94

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx = -\frac{bc(bc^2+2ad^2)\,x}{d^4}+\frac{b(bc^2+2ad^2)\,x^2}{2d^3}-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}+\frac{(bc^2+ad^2)^2\log(c+dx)}{d^5}$$

```
[Out] -b*c*(2*a*d^2+b*c^2)*x/d^4+1/2*b*(2*a*d^2+b*c^2)*x^2/d^3-1/3*b^2*c*x^3/d^2+
1/4*b^2*x^4/d+(a*d^2+b*c^2)^2*ln(d*x+c)/d^5
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.058, Rules used = {1600, 28, 711}

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx = \frac{(ad^2+bc^2)^2\log(c+dx)}{d^5}-\frac{bcx(2ad^2+bc^2)}{d^4}+\frac{bx^2(2ad^2+bc^2)}{2d^3}-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}$$

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
c + d*x)^2,x]
```

```
[Out] -((b*c*(b*c^2 + 2*a*d^2)*x)/d^4) + (b*(b*c^2 + 2*a*d^2)*x^2)/(2*d^3) - (b^2
*c*x^3)/(3*d^2) + (b^2*x^4)/(4*d) + ((b*c^2 + a*d^2)^2*Log[c + d*x])/d^5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{a^2+2abx^2+b^2x^4}{c+dx}\,dx \\
&= \frac{\int \frac{(ab+b^2x^2)^2}{c+dx}\,dx}{b^2} \\
&= \frac{\int \left(-\frac{b^3c(bc^2+2ad^2)}{d^4}+\frac{b^3(bc^2+2ad^2)x}{d^3}-\frac{b^4cx^2}{d^2}+\frac{b^4x^3}{d}+\frac{b^2(bc^2+ad^2)^2}{d^4(c+dx)}\right)dx}{b^2} \\
&= -\frac{bc(bc^2+2ad^2)\,x}{d^4}+\frac{b(bc^2+2ad^2)\,x^2}{2d^3}-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}+\frac{(bc^2+ad^2)^2\log(c+dx)}{d^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.84

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx$$
$$=\frac{bdx(12ad^2(-2c+dx)+b(-12c^3+6c^2dx-4cd^2x^2+3d^3x^3))+12(bc^2+ad^2)^2\log(c+dx)}{12d^5}$$

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(c + d*x)^2,x]
```

```
[Out] (b*d*x*(12*a*d^2*(-2*c + d*x) + b*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d^
3*x^3)) + 12*(b*c^2 + a*d^2)^2*Log[c + d*x])/(12*d^5)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

method	result
default	$-\frac{b\left(-\frac{b\,x^{4} d^{3}}{4}+\frac{b c\,x^{3} d^{2}}{3}-\frac{\left(2 a\,d^{2}+b\,c^{2}\right) x^{2} d}{2}+c\left(2 a\,d^{2}+b\,c^{2}\right) x\right)}{d^{4}}+\frac{\left(a^{2} d^{4}+2 b\,c^{2} d^{2} a+b^{2} c^{4}\right) \ln\left(d x+c\right)}{d^{5}}$
risch	$\frac{b^{2} x^{4}}{4 d}-\frac{b^{2} c\,x^{3}}{3 d^{2}}+\frac{b a\,x^{2}}{d}+\frac{b^{2} c^{2} x^{2}}{2 d^{3}}-\frac{2 b a c x}{d^{2}}-\frac{b^{2} c^{3} x}{d^{4}}+\frac{\ln\left(d x+c\right) a^{2}}{d}+\frac{2 \ln\left(d x+c\right) b\,c^{2} a}{d^{3}}+\frac{\ln\left(d x+c\right) b^{2} c^{4}}{d^{5}}$
parallelrisch	$\frac{3 x^{4} b^{2} d^{4}-4 b^{2} c\,x^{3} d^{3}+12 x^{2} a b\,d^{4}+6 x^{2} b^{2} c^{2} d^{2}+12 \ln\left(d x+c\right) a^{2} d^{4}+24 \ln\left(d x+c\right) a b\,c^{2} d^{2}+12 \ln\left(d x+c\right) b^{2} c^{4}-24 x a b c\,d^{3}-12 x\,b^{2} c^{3} d}{12 d^{5}}$
norman	$\frac{\frac{c\left(2 b\,c^{2} d^{2} a+b^{2} c^{4}\right)}{d^{5}}+\frac{b^{2} x^{5}}{4}+\frac{b\left(6 a\,d^{2}+b\,c^{2}\right) x^{3}}{6 d^{2}}-\frac{b^{2} c\,x^{4}}{12 d}-\frac{b c\left(2 a\,d^{2}+b\,c^{2}\right) x^{2}}{2 d^{3}}}{d x+c}+\frac{\left(a^{2} d^{4}+2 b\,c^{2} d^{2} a+b^{2} c^{4}\right) \ln\left(d x+c\right)}{d^{5}}$

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c)^2,x
,method=_RETURNVERBOSE)
```

```
[Out] -b/d^4*(-1/4*b*x^4*d^3+1/3*b*c*x^3*d^2-1/2*(2*a*d^2+b*c^2)*x^2*d+c*(2*a*d^2
+b*c^2)*x)+(a^2*d^4+2*a*b*c^2*d^2+b^2*c^4)/d^5*ln(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx = \frac{3\,b^2d^4x^4-4\,b^2cd^3x^3+6\,(b^2c^2d^2+2\,abd^4)x^2-12\,(b^2c^3d+2\,abcd^3)x+12\,(b^2c^4+2\,abc^2d^2+a^2d^4)\log{(dx+}}{12\,d^5}$$

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*d^4*x^4 - 4*b^2*c*d^3*x^3 + 6*(b^2*c^2*d^2 + 2*a*b*d^4)*x^2 - 1
2*(b^2*c^3*d + 2*a*b*c*d^3)*x + 12*(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*log(
d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx$$
$$=-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}+x^2\left(\frac{ab}{d}+\frac{b^2c^2}{2d^3}\right)+x\left(-\frac{2abc}{d^2}-\frac{b^2c^3}{d^4}\right)+\frac{\left(ad^2+bc^2\right)^2\log{(c+dx)}}{d^5}$$

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(d*x+c)**2,x)
```

```
[Out] -b**2*c*x**3/(3*d**2) + b**2*x**4/(4*d) + x**2*(a*b/d + b**2*c**2/(2*d**3))
 + x*(-2*a*b*c/d**2 - b**2*c**3/d**4) + (a*d**2 + b*c**2)**2*log(c + d*x)/d
**5
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx$$
$$=\frac{3\,b^2d^3x^4-4\,b^2cd^2x^3+6\,(b^2c^2d+2\,abd^3)x^2-12\,(b^2c^3+2\,abcd^2)x}{12\,d^4}$$
$$+\frac{(b^2c^4+2\,abc^2d^2+a^2d^4)\log{(dx+c)}}{d^5}$$

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^2*d^3*x^4 - 4*b^2*c*d^2*x^3 + 6*(b^2*c^2*d + 2*a*b*d^3)*x^2 - 12*
(b^2*c^3 + 2*a*b*c*d^2)*x)/d^4 + (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*log(d*
x + c)/d^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(88) = 176.

Time = 0.28 (sec) , antiderivative size = 365, normalized size of antiderivative = 3.88

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx$$
$$= -\frac{1}{12}b^2d\left(\frac{(dx+c)^4\left(\frac{20\,c}{dx+c}-\frac{60\,c^2}{(dx+c)^2}+\frac{120\,c^3}{(dx+c)^3}-3\right)}{d^6}+\frac{60\,c^4\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^6}-\frac{12\,c^5}{(dx+c)d^6}\right)$$
$$-\frac{1}{3}b^2c\left(\frac{(dx+c)^3\left(\frac{6\,c}{dx+c}-\frac{18\,c^2}{(dx+c)^2}-1\right)}{d^5}-\frac{12\,c^3\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^5}+\frac{3\,c^4}{(dx+c)d^5}\right)$$
$$-abd\left(\frac{(dx+c)^2\left(\frac{6\,c}{dx+c}-1\right)}{d^4}+\frac{6\,c^2\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^4}-\frac{2\,c^3}{(dx+c)d^4}\right)$$
$$+2\,abc\left(\frac{2\,c\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^3}+\frac{dx+c}{d^3}-\frac{c^2}{(dx+c)d^3}\right)$$
$$-a^2\left(\frac{\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d}-\frac{c}{(dx+c)d}\right)-\frac{a^2c}{(dx+c)d}$$

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c)^2,x, algorithm="giac")
```

```
[Out] -1/12*b^2*d*((d*x + c)^4*(20*c/(d*x + c) - 60*c^2/(d*x + c)^2 + 120*c^3/(d*
x + c)^3 - 3)/d^6 + 60*c^4*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^6 - 12*
c^5/((d*x + c)*d^6)) - 1/3*b^2*c*((d*x + c)^3*(6*c/(d*x + c) - 18*c^2/(d*x
+ c)^2 - 1)/d^5 - 12*c^3*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^5 + 3*c^4
/((d*x + c)*d^5)) - a*b*d*((d*x + c)^2*(6*c/(d*x + c) - 1)/d^4 + 6*c^2*log(
abs(d*x + c)/((d*x + c)^2*abs(d)))/d^4 - 2*c^3/((d*x + c)*d^4)) + 2*a*b*c*(
2*c*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^3 + (d*x + c)/d^3 - c^2/((d*x
+ c)*d^3)) - a^2*(log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d - c/((d*x + c)*d
)) - a^2*c/((d*x + c)*d)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx$$
$$= x^2\left(\frac{b^2\,c^2}{2\,d^3}+\frac{a\,b}{d}\right)+\frac{\ln\left(c+d\,x\right)\,\left(a^2\,d^4+2\,a\,b\,c^2\,d^2+b^2\,c^4\right)}{d^5}$$
$$+\frac{b^2\,x^4}{4\,d}-\frac{b^2\,c\,x^3}{3\,d^2}-\frac{c\,x\,\left(\frac{b^2\,c^2}{d^3}+\frac{2\,a\,b}{d}\right)}{d}$$

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
c + d*x)^2,x)
```

```
[Out] x^2*((b^2*c^2)/(2*d^3) + (a*b)/d) + (log(c + d*x)*(a^2*d^4 + b^2*c^4 + 2*a*
b*c^2*d^2))/d^5 + (b^2*x^4)/(4*d) - (b^2*c*x^3)/(3*d^2) - (c*x*((b^2*c^2)/d
^3 + (2*a*b)/d))/d
```

3.160 $\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx$

Optimal result 1262
Rubi [A] (verified) 1262
Mathematica [B] (verified) 1263
Maple [A] (verified) 1263
Fricas [B] (verification not implemented) 1264
Sympy [B] (verification not implemented) 1264
Maxima [A] (verification not implemented) 1265
Giac [A] (verification not implemented) 1265
Mupad [B] (verification not implemented) 1265

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(bx+cx^2\right)^{14}$$

```
[Out] 1/14*(c*x^2+b*x)^14
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {643}

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(bx+cx^2\right)^{14}$$

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b*x + c*x^2)^14/14
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{14}\left(bx+cx^2\right)^{14}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. $2(15) = 30$.

Time = 0.00 (sec) , antiderivative size = 172, normalized size of antiderivative = 11.47

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx = \frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17} +\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20} +\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24} +26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (1
43*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c
^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/
2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result
gosper	$\frac{(cx+b)^{14}x^{14}}{14}$
default	$\frac{(c\,x^2+bx)^{14}}{14}$
norman	$26b^3c^{11}x^{25}+\frac{13}{2}x^{26}b^2c^{12}+b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}x^{14}b^{14}+b^{13}c\,x^{15}+\frac{13}{2}x^{16}b^{12}c^2+26b^{11}c^3x^1$
risch	$26b^3c^{11}x^{25}+\frac{13}{2}x^{26}b^2c^{12}+b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}x^{14}b^{14}+b^{13}c\,x^{15}+\frac{13}{2}x^{16}b^{12}c^2+26b^{11}c^3x^1$
parallelrisch	$26b^3c^{11}x^{25}+\frac{13}{2}x^{26}b^2c^{12}+b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}x^{14}b^{14}+b^{13}c\,x^{15}+\frac{13}{2}x^{16}b^{12}c^2+26b^{11}c^3x^1$

```
[In] int((2*c*x+b)*(c*x^2+b*x)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*(c*x+b)^14*x^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 10.27

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx = \frac{1}{14}\,c^{14}x^{28} + bc^{13}x^{27} + \frac{13}{2}\,b^2c^{12}x^{26} + 26\,b^3c^{11}x^{25} + \frac{143}{2}\,b^4c^{10}x^{24} + 143\,b^5c^9x^{23} + \frac{429}{2}\,b^6c^8x^{22} + \frac{1716}{7}\,b^7c^7x^{21} + \frac{429}{2}\,b^8c^6x^{20} + 143\,b^9c^5x^{19} + \frac{143}{2}\,b^{10}c^4x^{18} + 26\,b^{11}c^3x^{17} + \frac{13}{2}\,b^{12}c^2x^{16} + b^{13}cx^{15} + \frac{1}{14}\,b^{14}x^{14}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="fricas")
```

```
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143/
2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*x^
21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^11*
c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 175, normalized size of antiderivative = 11.67

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx = \frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13b^{12}c^2x^{16}}{2} + 26b^{11}c^3x^{17} + \frac{143b^{10}c^4x^{18}}{2} + 143b^9c^5x^{19} + \frac{429b^8c^6x^{20}}{2} + \frac{1716b^7c^7x^{21}}{7} + \frac{429b^6c^8x^{22}}{2} + 143b^5c^9x^{23} + \frac{143b^4c^{10}x^{24}}{2} + 26b^3c^{11}x^{25} + \frac{13b^2c^{12}x^{26}}{2} + bc^{13}x^{27} + \frac{c^{14}x^{28}}{14}$$

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**13,x)
```

```
[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**1
7 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2 +
1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 143*
b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c**13*
x**27 + c**14*x**28/14
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(cx^2+bx\right)^{14}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="maxima")
[Out] 1/14*(c*x^2 + b*x)^14
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(cx^2+bx\right)^{14}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="giac")
[Out] 1/14*(c*x^2 + b*x)^14
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 154, normalized size of antiderivative = 10.27

$$\begin{aligned}\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=&\ \frac{b^{14}\,x^{14}}{14}+b^{13}\,c\,x^{15}+\frac{13\,b^{12}\,c^2\,x^{16}}{2}+26\,b^{11}\,c^3\,x^{17}\\&+\frac{143\,b^{10}\,c^4\,x^{18}}{2}+143\,b^9\,c^5\,x^{19}+\frac{429\,b^8\,c^6\,x^{20}}{2}\\&+\frac{1716\,b^7\,c^7\,x^{21}}{7}+\frac{429\,b^6\,c^8\,x^{22}}{2}+143\,b^5\,c^9\,x^{23}+\frac{143\,b^4\,c^{10}\,x^{24}}{2}\\&+26\,b^3\,c^{11}\,x^{25}+\frac{13\,b^2\,c^{12}\,x^{26}}{2}+b\,c^{13}\,x^{27}+\frac{c^{14}\,x^{28}}{14}\end{aligned}$$

```
[In] int((b*x + c*x^2)^13*(b + 2*c*x),x)
[Out] (b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2*
x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (42
9*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*
c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2
```

3.161 $\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx$

Optimal result . . . 1266
Rubi [A] (verified) . . . 1266
Mathematica [B] (verified) . . . 1267
Maple [A] (verified) . . . 1268
Fricas [B] (verification not implemented) . . . 1268
Sympy [B] (verification not implemented) . . . 1269
Maxima [B] (verification not implemented) . . . 1269
Giac [B] (verification not implemented) . . . 1270
Mupad [B] (verification not implemented) . . . 1270

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int x^{14}\left(b+2cx^2\right)\left(bx+cx^3\right)^{13}\,dx=\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

```
[Out] 1/28*x^28*(c*x^2+b)^14
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1598, 457, 75}

$$\int x^{14}\left(b+2cx^2\right)\left(bx+cx^3\right)^{13}\,dx=\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

```
[In] Int[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]
```

```
[Out] (x^28*(b + c*x^2)^14)/28
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^2\right)\\ &= \frac{1}{28}x^{28}\left(b+cx^2\right)^{14}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 182 vs. $2(16) = 32$.

Time = 0.00 (sec) , antiderivative size = 182, normalized size of antiderivative = 11.38

$$\begin{aligned}\int x^{14}\left(b+2cx^2\right)\left(bx+cx^3\right)^{13}\,dx = {} & \frac{b^{14}x^{28}}{28}+\frac{1}{2}b^{13}cx^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}\\ &+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}\\ &+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^6c^8x^{44}+\frac{143}{2}b^5c^9x^{46}+\frac{143}{4}b^4c^{10}x^{48}\\ &+13b^3c^{11}x^{50}+\frac{13}{4}b^2c^{12}x^{52}+\frac{1}{2}bc^{13}x^{54}+\frac{c^{14}x^{56}}{28}\end{aligned}$$

```
[In] Integrate[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]
```

```
[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
+ (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (85
8*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*
c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 +
(c^14*x^56)/28
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{x^{28}(c\,x^{2}+b)^{14}}{28}$
default	$\frac{1}{2}b\,c^{13}x^{54}+\frac{1}{28}c^{14}x^{56}+\frac{143}{2}x^{46}b^{5}c^{9}+\frac{143}{4}x^{48}b^{4}c^{10}+13x^{50}b^{3}c^{11}+\frac{13}{4}x^{52}b^{2}c^{12}+\frac{429}{4}x^{44}b^{6}c^{8}+\frac{858}{7}x$
risch	$\frac{1}{2}b\,c^{13}x^{54}+\frac{1}{28}c^{14}x^{56}+\frac{143}{2}x^{46}b^{5}c^{9}+\frac{143}{4}x^{48}b^{4}c^{10}+13x^{50}b^{3}c^{11}+\frac{13}{4}x^{52}b^{2}c^{12}+\frac{429}{4}x^{44}b^{6}c^{8}+\frac{858}{7}x$
parallelrisch	$\frac{1}{2}b\,c^{13}x^{54}+\frac{1}{28}c^{14}x^{56}+\frac{143}{2}x^{46}b^{5}c^{9}+\frac{143}{4}x^{48}b^{4}c^{10}+13x^{50}b^{3}c^{11}+\frac{13}{4}x^{52}b^{2}c^{12}+\frac{429}{4}x^{44}b^{6}c^{8}+\frac{858}{7}x$

```
[In] int(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*x^28*(c*x^2+b)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(14) = 28.

Time = 0.24 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx=\frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^2c^{12}x^{52}+13\,b^3c^{11}x^{50}+\frac{143}{4}\,b^4c^{10}x^{48}+\frac{143}{2}\,b^5c^9x^{46}+\frac{429}{4}\,b^6c^8x^{44}+\frac{858}{7}\,b^7c^7x^{42}+\frac{429}{4}\,b^8c^6x^{40}+\frac{143}{2}\,b^9c^5x^{38}+\frac{143}{4}\,b^{10}c^4x^{36}+13\,b^{11}c^3x^{34}+\frac{13}{4}\,b^{12}c^2x^{32}+\frac{1}{2}\,b^{13}cx^{30}+\frac{1}{28}\,b^{14}x^{28}$$

```
[In] integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="fricas")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(12) = 24$.

Time = 0.05 (sec) , antiderivative size = 182, normalized size of antiderivative = 11.38

$$\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx = \frac{b^{14}x^{28}}{28}+\frac{b^{13}cx^{30}}{2}+\frac{13b^{12}c^2x^{32}}{4}+13b^{11}c^3x^{34}+\frac{143b^{10}c^4x^{36}}{4}+\frac{143b^9c^5x^{38}}{2}+\frac{429b^8c^6x^{40}}{4}+\frac{858b^7c^7x^{42}}{7}+\frac{429b^6c^8x^{44}}{4}+\frac{143b^5c^9x^{46}}{2}+\frac{143b^4c^{10}x^{48}}{4}+13b^3c^{11}x^{50}+\frac{13b^2c^{12}x^{52}}{4}+\frac{bc^{13}x^{54}}{2}+\frac{c^{14}x^{56}}{28}$$

```
[In] integrate(x**14*(2*c*x**2+b)*(c*x**3+b*x)**13,x)
```

```
[Out] b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x*
*34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**40/
4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/2 +
 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 + b*c
**13*x**54/2 + c**14*x**56/28
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx = \frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^2c^{12}x^{52}+13\,b^3c^{11}x^{50}+\frac{143}{4}\,b^4c^{10}x^{48}+\frac{143}{2}\,b^5c^9x^{46}+\frac{429}{4}\,b^6c^8x^{44}+\frac{858}{7}\,b^7c^7x^{42}+\frac{429}{4}\,b^8c^6x^{40}+\frac{143}{2}\,b^9c^5x^{38}+\frac{143}{4}\,b^{10}c^4x^{36}+13\,b^{11}c^3x^{34}+\frac{13}{4}\,b^{12}c^2x^{32}+\frac{1}{2}\,b^{13}cx^{30}+\frac{1}{28}\,b^{14}x^{28}$$

```
[In] integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx = \frac{1}{28}\,c^{14}x^{56} + \frac{1}{2}\,bc^{13}x^{54} + \frac{13}{4}\,b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}\,b^4c^{10}x^{48} + \frac{143}{2}\,b^5c^9x^{46} + \frac{429}{4}\,b^6c^8x^{44} + \frac{858}{7}\,b^7c^7x^{42} + \frac{429}{4}\,b^8c^6x^{40} + \frac{143}{2}\,b^9c^5x^{38} + \frac{143}{4}\,b^{10}c^4x^{36} + 13\,b^{11}c^3x^{34} + \frac{13}{4}\,b^{12}c^2x^{32} + \frac{1}{2}\,b^{13}cx^{30} + \frac{1}{28}\,b^{14}x^{28}$$

```
[In] integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="giac")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx = \frac{b^{14}\,x^{28}}{28} + \frac{b^{13}\,c\,x^{30}}{2} + \frac{13\,b^{12}\,c^2\,x^{32}}{4} + 13\,b^{11}\,c^3\,x^{34} + \frac{143\,b^{10}\,c^4\,x^{36}}{4} + \frac{143\,b^9\,c^5\,x^{38}}{2} + \frac{429\,b^8\,c^6\,x^{40}}{4} + \frac{858\,b^7\,c^7\,x^{42}}{7} + \frac{429\,b^6\,c^8\,x^{44}}{4} + \frac{143\,b^5\,c^9\,x^{46}}{2} + \frac{143\,b^4\,c^{10}\,x^{48}}{4} + 13\,b^3\,c^{11}\,x^{50} + \frac{13\,b^2\,c^{12}\,x^{52}}{4} + \frac{b\,c^{13}\,x^{54}}{2} + \frac{c^{14}\,x^{56}}{28}$$

```
[In] int(x^14*(b*x + c*x^3)^13*(b + 2*c*x^2),x)
```

```
[Out] (b^14*x^28)/28 + (c^14*x^56)/28 + (b^13*c*x^30)/2 + (b*c^13*x^54)/2 + (13*b
^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x
^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4
 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^
2*c^12*x^52)/4
```

3.162 $\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13}\,dx$

Optimal result 1271
Rubi [A] (verified) 1271
Mathematica [B] (verified) 1272
Maple [A] (verified) 1273
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Giac [B] (verification not implemented) 1275
Mupad [B] (verification not implemented) 1275

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int x^{28}\left(b+2cx^3\right)\left(bx+cx^4\right)^{13}\,dx=\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

```
[Out] 1/42*x^42*(c*x^3+b)^14
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1598, 457, 75}

$$\int x^{28}\left(b+2cx^3\right)\left(bx+cx^4\right)^{13}\,dx=\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

```
[In] Int[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]
```

```
[Out] (x^42*(b + c*x^3)^14)/42
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx\\ &= \frac{1}{3}\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^3\right)\\ &= \frac{1}{42}x^{42}\left(b+cx^3\right)^{14}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 186 vs. $2(16) = 32$.

Time = 0.01 (sec) , antiderivative size = 186, normalized size of antiderivative = 11.62

$$\begin{aligned}\int x^{28}\left(b+2cx^3\right)\left(bx+cx^4\right)^{13}\,dx &= \frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}\\ &\quad+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}\\ &\quad+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{6}b^4c^{10}x^{72}\\ &\quad+\frac{26}{3}b^3c^{11}x^{75}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}bc^{13}x^{81}+\frac{c^{14}x^{84}}{42}\end{aligned}$$

```
[In] Integrate[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]
```

```
[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51
)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 +
 (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*
b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^
81)/3 + (c^14*x^84)/42
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{x^{42}(c x^3+b)^{14}}{42}$
default	$\frac{1}{42}c^{14}x^{84}+\frac{13}{6}x^{78}b^2c^{12}+\frac{1}{3}b\,c^{13}x^{81}+\frac{143}{3}x^{69}b^5c^9+\frac{143}{6}x^{72}b^4c^{10}+\frac{26}{3}x^{75}b^3c^{11}+\frac{572}{7}x^{63}b^7c^7+\frac{143}{2}$
risch	$\frac{1}{42}c^{14}x^{84}+\frac{13}{6}x^{78}b^2c^{12}+\frac{1}{3}b\,c^{13}x^{81}+\frac{143}{3}x^{69}b^5c^9+\frac{143}{6}x^{72}b^4c^{10}+\frac{26}{3}x^{75}b^3c^{11}+\frac{572}{7}x^{63}b^7c^7+\frac{143}{2}$
parallelrisch	$\frac{1}{42}c^{14}x^{84}+\frac{13}{6}x^{78}b^2c^{12}+\frac{1}{3}b\,c^{13}x^{81}+\frac{143}{3}x^{69}b^5c^9+\frac{143}{6}x^{72}b^4c^{10}+\frac{26}{3}x^{75}b^3c^{11}+\frac{572}{7}x^{63}b^7c^7+\frac{143}{2}$

```
[In] int(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/42*x^42*(c*x^3+b)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.23 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13}\,dx = \frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^2c^{12}x^{78}+\frac{26}{3}\,b^3c^{11}x^{75} + \frac{143}{6}\,b^4c^{10}x^{72}+\frac{143}{3}\,b^5c^9x^{69}+\frac{143}{2}\,b^6c^8x^{66} + \frac{572}{7}\,b^7c^7x^{63}+\frac{143}{2}\,b^8c^6x^{60}+\frac{143}{3}\,b^9c^5x^{57}+\frac{143}{6}\,b^{10}c^4x^{54} + \frac{26}{3}\,b^{11}c^3x^{51}+\frac{13}{6}\,b^{12}c^2x^{48}+\frac{1}{3}\,b^{13}cx^{45}+\frac{1}{42}\,b^{14}x^{42}$$

```
[In] integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="fricas")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(12) = 24$.

Time = 0.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 11.56

$$\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13}\,dx = \frac{b^{14}x^{42}}{42}+\frac{b^{13}cx^{45}}{3}+\frac{13b^{12}c^2x^{48}}{6}+\frac{26b^{11}c^3x^{51}}{3}+\frac{143b^{10}c^4x^{54}}{6}+\frac{143b^9c^5x^{57}}{3}+\frac{143b^8c^6x^{60}}{2}+\frac{572b^7c^7x^{63}}{7}+\frac{143b^6c^8x^{66}}{2}+\frac{143b^5c^9x^{69}}{3}+\frac{143b^4c^{10}x^{72}}{6}+\frac{26b^3c^{11}x^{75}}{3}+\frac{13b^2c^{12}x^{78}}{6}+\frac{bc^{13}x^{81}}{3}+\frac{c^{14}x^{84}}{42}$$

```
[In] integrate(x**28*(2*c*x**3+b)*(c*x**4+b*x)**13,x)
```

```
[Out] b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x**
*51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x**6
0/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**69/3
 + 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78/6 +
 b*c**13*x**81/3 + c**14*x**84/42
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13}\,dx = \frac{1}{42}c^{14}x^{84}+\frac{1}{3}bc^{13}x^{81}+\frac{13}{6}b^2c^{12}x^{78}+\frac{26}{3}b^3c^{11}x^{75}+\frac{143}{6}b^4c^{10}x^{72}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{2}b^6c^8x^{66}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^8c^6x^{60}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{6}b^{10}c^4x^{54}+\frac{26}{3}b^{11}c^3x^{51}+\frac{13}{6}b^{12}c^2x^{48}+\frac{1}{3}b^{13}cx^{45}+\frac{1}{42}b^{14}x^{42}$$

```
[In] integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13}\,dx = \frac{1}{42}c^{14}x^{84}+\frac{1}{3}bc^{13}x^{81}+\frac{13}{6}b^2c^{12}x^{78}+\frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^8c^6x^{60}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{6}b^{10}c^4x^{54} + \frac{26}{3}b^{11}c^3x^{51}+\frac{13}{6}b^{12}c^2x^{48}+\frac{1}{3}b^{13}cx^{45}+\frac{1}{42}b^{14}x^{42}$$

```
[In] integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="giac")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13}\,dx = \frac{b^{14}\,x^{42}}{42}+\frac{b^{13}\,c\,x^{45}}{3}+\frac{13\,b^{12}\,c^2\,x^{48}}{6} + \frac{26\,b^{11}\,c^3\,x^{51}}{3}+\frac{143\,b^{10}\,c^4\,x^{54}}{6}+\frac{143\,b^9\,c^5\,x^{57}}{3} + \frac{143\,b^8\,c^6\,x^{60}}{2}+\frac{572\,b^7\,c^7\,x^{63}}{7}+\frac{143\,b^6\,c^8\,x^{66}}{2} + \frac{143\,b^5\,c^9\,x^{69}}{3}+\frac{143\,b^4\,c^{10}\,x^{72}}{6}+\frac{26\,b^3\,c^{11}\,x^{75}}{3} + \frac{13\,b^2\,c^{12}\,x^{78}}{6}+\frac{b\,c^{13}\,x^{81}}{3}+\frac{c^{14}\,x^{84}}{42}$$

```
[In] int(x^28*(b*x + c*x^4)^13*(b + 2*c*x^3),x)
```

```
[Out] (b^14*x^42)/42 + (c^14*x^84)/42 + (b^13*c*x^45)/3 + (b*c^13*x^81)/3 + (13*b
^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c
^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^6
6)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3
+ (13*b^2*c^12*x^78)/6
```

3.163 $\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx$

Optimal result 1276
Rubi [A] (verified) 1276
Mathematica [A] (verified) 1277
Maple [B] (verified) 1277
Fricas [B] (verification not implemented) 1278
Sympy [F(-1)] 1278
Maxima [B] (verification not implemented) 1278
Giac [B] (verification not implemented) 1279
Mupad [B] (verification not implemented) 1279

Optimal result

Integrand size = 29, antiderivative size = 21

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

```
[Out] 1/14*x^(14*n)*(b+c*x^n)^14/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1598, 457, 75}

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

```
[In] Int[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{13+14(-1+n)}(b+cx^n)^{13}\,(b+2cx^n)\;dx\\ &= \frac{\operatorname{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^n\right)}{n}\\ &= \frac{x^{14n}(b+cx^n)^{14}}{14n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\;dx=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

```
[In] Integrate[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(19) = 38.

Time = 0.02 (sec) , antiderivative size = 230, normalized size of antiderivative = 10.95

$$\frac{c^{14}x^{28n}}{14n}+\frac{b\,c^{13}x^{27n}}{n}+\frac{13c^{12}x^{26n}b^{2}}{2n}+\frac{26c^{11}b^{3}x^{25n}}{n}+\frac{143c^{10}x^{24n}b^{4}}{2n}+\frac{143c^{9}b^{5}x^{23n}}{n}+\frac{429c^{8}x^{22n}b^{6}}{2n}+\frac{1716b^{7}c^{7}x^{21n}}{7n}+\frac{42}{}$$

```
[In] int(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x)
[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*c^12/n*(x^n)^26*b^2+26*c^11*b^3
/n*(x^n)^25+143/2*c^10/n*(x^n)^24*b^4+143*c^9*b^5/n*(x^n)^23+429/2*c^8/n*(x
^n)^22*b^6+1716/7*b^7*c^7/n*(x^n)^21+429/2*c^6/n*(x^n)^20*b^8+143*b^9*c^5/n
*(x^n)^19+143/2*c^4/n*(x^n)^18*b^10+26*b^11*c^3/n*(x^n)^17+13/2*c^2/n*(x^n)
^16*b^12+b^13*c/n*(x^n)^15+1/14/n*(x^n)^14*b^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(19) = 38.

Time = 0.25 (sec) , antiderivative size = 262, normalized size of antiderivative = 12.48

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx = \frac{b^{14}x^{14}x^{14\,n+14}+14\,b^{13}cx^{13}x^{15\,n+15}+91\,b^{12}c^2x^{12}x^{16\,n+16}+364\,b^{11}c^3x^{11}x^{17\,n+17}+1001\,b^{10}c^4x^{10}x^{18\,n+18}+200}{}$$

```
[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="fricas"
)
```

```
[Out] 1/14*(b^14*x^14*x^(14*n + 14) + 14*b^13*c*x^13*x^(15*n + 15) + 91*b^12*c^2*
x^12*x^(16*n + 16) + 364*b^11*c^3*x^11*x^(17*n + 17) + 1001*b^10*c^4*x^10*x
^(18*n + 18) + 2002*b^9*c^5*x^9*x^(19*n + 19) + 3003*b^8*c^6*x^8*x^(20*n +
20) + 3432*b^7*c^7*x^7*x^(21*n + 21) + 3003*b^6*c^8*x^6*x^(22*n + 22) + 200
2*b^5*c^9*x^5*x^(23*n + 23) + 1001*b^4*c^10*x^4*x^(24*n + 24) + 364*b^3*c^1
1*x^3*x^(25*n + 25) + 91*b^2*c^12*x^2*x^(26*n + 26) + 14*b*c^13*x*x^(27*n +
 27) + c^14*x^(28*n + 28))/(n*x^28)
```

Sympy [F(-1)]

Timed out.

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx = \text{Timed out}$$

```
[In] integrate(x**(-14+14*n)*(b+2*c*x**n)*(b*x+c*x**(1+n))**13,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(19) = 38.

Time = 0.20 (sec) , antiderivative size = 229, normalized size of antiderivative = 10.90

$$\begin{aligned}\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx &= \frac{c^{14}x^{28\,n}}{14\,n}+\frac{bc^{13}x^{27\,n}}{n}+\frac{13\,b^2c^{12}x^{26\,n}}{2\,n}\\&+\frac{26\,b^3c^{11}x^{25\,n}}{n}+\frac{143\,b^4c^{10}x^{24\,n}}{2\,n}+\frac{143\,b^5c^9x^{23\,n}}{n}\\&+\frac{429\,b^6c^8x^{22\,n}}{2\,n}+\frac{1716\,b^7c^7x^{21\,n}}{7\,n}+\frac{429\,b^8c^6x^{20\,n}}{2\,n}\\&+\frac{143\,b^9c^5x^{19\,n}}{n}+\frac{143\,b^{10}c^4x^{18\,n}}{2\,n}+\frac{26\,b^{11}c^3x^{17\,n}}{n}\\&+\frac{13\,b^{12}c^2x^{16\,n}}{2\,n}+\frac{b^{13}cx^{15\,n}}{n}+\frac{b^{14}x^{14\,n}}{14\,n}\end{aligned}$$

```
[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="maxima"
)
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b^
3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n + 42
9/2*b^6*c^8*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 429/2*b^8*c^6*x^(20*n)
/n + 143*b^9*c^5*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 26*b^11*c^3*x^(17
*n)/n + 13/2*b^12*c^2*x^(16*n)/n + b^13*c*x^(15*n)/n + 1/14*b^14*x^(14*n)/n
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(19) = 38$.

Time = 0.50 (sec) , antiderivative size = 189, normalized size of antiderivative = 9.00

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx = \frac{c^{14}x^{28\,n}+14\,bc^{13}x^{27\,n}+91\,b^2c^{12}x^{26\,n}+364\,b^3c^{11}x^{25\,n}+1001\,b^4c^{10}x^{24\,n}+2002\,b^5c^9x^{23\,n}+3003\,b^6c^8x^{22\,n}+}{}$$

```
[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

Mupad [B] (verification not implemented)

Time = 10.21 (sec) , antiderivative size = 229, normalized size of antiderivative = 10.90

$$\begin{aligned}\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx &= \frac{b^{14}\,x^{14\,n}}{14\,n}+\frac{c^{14}\,x^{28\,n}}{14\,n}+\frac{13\,b^{12}\,c^2\,x^{16\,n}}{2\,n}\\&+\frac{26\,b^{11}\,c^3\,x^{17\,n}}{n}+\frac{143\,b^{10}\,c^4\,x^{18\,n}}{2\,n}+\frac{143\,b^9\,c^5\,x^{19\,n}}{n}\\&+\frac{429\,b^8\,c^6\,x^{20\,n}}{2\,n}+\frac{1716\,b^7\,c^7\,x^{21\,n}}{7\,n}+\frac{429\,b^6\,c^8\,x^{22\,n}}{2\,n}\\&+\frac{143\,b^5\,c^9\,x^{23\,n}}{n}+\frac{143\,b^4\,c^{10}\,x^{24\,n}}{2\,n}+\frac{26\,b^3\,c^{11}\,x^{25\,n}}{n}\\&+\frac{13\,b^2\,c^{12}\,x^{26\,n}}{2\,n}+\frac{b^{13}\,c\,x^{15\,n}}{n}+\frac{b\,c^{13}\,x^{27\,n}}{n}\end{aligned}$$

```
[In] int(x^(14*n - 14)*(b*x + c*x^(n + 1))^13*(b + 2*c*x^n),x)
```

```
[Out] (b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(2
*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9*c
```

```
^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/(7*
n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4*c^1
0*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/(2*n)
 + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n
```

3.164 $\int \frac{b+2cx}{bx+cx^2} \, dx$

Optimal result 1281
Rubi [A] (verified) 1281
Mathematica [A] (verified) 1282
Maple [A] (verified) 1282
Fricas [A] (verification not implemented) 1282
Sympy [A] (verification not implemented) 1283
Maxima [A] (verification not implemented) 1283
Giac [A] (verification not implemented) 1283
Mupad [B] (verification not implemented) 1283

Optimal result

Integrand size = 18, antiderivative size = 10

$$\int \frac{b+2cx}{bx+cx^2} \, dx = \log \left(bx+cx^2\right)$$

```
[Out] ln(c*x^2+b*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {642}

$$\int \frac{b+2cx}{bx+cx^2} \, dx = \log \left(bx+cx^2\right)$$

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2),x]
```

```
[Out] Log[b*x + c*x^2]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\text{integral} = \log \left(bx+cx^2\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log(x)+\log(b+cx)$$

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2),x]
[Out] Log[x] + Log[b + c*x]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
default	$\ln\left(x(cx+b)\right)$	9
norman	$\ln\left(x\right)+\ln\left(cx+b\right)$	10
parallelrisch	$\ln\left(x\right)+\ln\left(cx+b\right)$	10
derivativedivides	$\ln\left(c\,x^{2}+bx\right)$	11
risch	$\ln\left(c\,x^{2}+bx\right)$	11

```
[In] int((2*c*x+b)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
[Out] ln(x*(c*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(cx^2+bx\right)$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="fricas")
[Out] log(c*x^2 + b*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(bx+cx^2\right)$$

```
[In] integrate((2*c*x+b)/(c*x**2+b*x),x)
[Out] log(b*x + c*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(cx^2+bx\right)$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="maxima")
[Out] log(c*x^2 + b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(\left|cx^2+bx\right|\right)$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="giac")
[Out] log(abs(c*x^2 + b*x))
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \ln\left(x\,\left(b+c\,x\right)\right)$$

```
[In] int((b + 2*c*x)/(b*x + c*x^2),x)
[Out] log(x*(b + c*x))
```

3.165 $\int \frac{b+2cx^2}{bx+cx^3}\,dx$

Optimal result . . . 1284
Rubi [A] (verified) . . . 1284
Mathematica [A] (verified) . . . 1285
Maple [A] (verified) . . . 1285
Fricas [A] (verification not implemented) . . . 1286
Sympy [A] (verification not implemented) . . . 1286
Maxima [A] (verification not implemented) . . . 1286
Giac [A] (verification not implemented) . . . 1287
Mupad [B] (verification not implemented) . . . 1287

Optimal result

Integrand size = 20, antiderivative size = 15

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx=\log(x)+\frac{1}{2}\log\left(b+cx^2\right)$$

```
[Out] ln(x)+1/2*ln(c*x^2+b)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1607, 457, 78}

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx=\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

```
[In] Int[(b + 2*c*x^2)/(b*x + c*x^3),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{b+2cx^2}{x\,(b+cx^2)}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx, x, x^2\right)\\ &= \log(x)+\frac{1}{2}\log\left(b+cx^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx = \log(x)+\frac{1}{2}\log\left(b+cx^2\right)$$

```
[In] Integrate[(b + 2*c*x^2)/(b*x + c*x^3),x]
[Out] Log[x] + Log[b + c*x^2]/2
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\ln(x)+\frac{\ln(c\,x^2+b)}{2}$	14
norman	$\ln(x)+\frac{\ln(c\,x^2+b)}{2}$	14
risch	$\ln(x)+\frac{\ln(c\,x^2+b)}{2}$	14
parallelrisch	$\ln(x)+\frac{\ln(c\,x^2+b)}{2}$	14

```
[In] int((2*c*x^2+b)/(c*x^3+b*x),x,method=_RETURNVERBOSE)
[Out] ln(x)+1/2*ln(c*x^2+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx=\frac{1}{2}\log\left(cx^2+b\right)+\log(x)$$

```
[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="fricas")
[Out] 1/2*log(c*x^2 + b) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx=\log(x)+\frac{\log\left(\frac{b}{c}+x^2\right)}{2}$$

```
[In] integrate((2*c*x**2+b)/(c*x**3+b*x),x)
[Out] log(x) + log(b/c + x**2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx=\frac{1}{2}\log\left(cx^2+b\right)+\log(x)$$

```
[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="maxima")
[Out] 1/2*log(c*x^2 + b) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx = \frac{1}{2}\,\log\left(x^2\right) + \frac{1}{2}\,\log\left(\left|cx^2+b\right|\right)$$

```
[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="giac")
[Out] 1/2*log(x^2) + 1/2*log(abs(c*x^2 + b))
```

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx = \frac{\ln\left(c\,x^2+b\right)}{2} + \ln\left(x\right)$$

```
[In] int((b + 2*c*x^2)/(b*x + c*x^3),x)
[Out] log(b + c*x^2)/2 + log(x)
```

3.166 $\int \frac{b+2cx^3}{bx+cx^4} \, dx$

Optimal result 1288
Rubi [A] (verified) 1288
Mathematica [A] (verified) 1289
Maple [A] (verified) 1289
Fricas [A] (verification not implemented) 1290
Sympy [A] (verification not implemented) 1290
Maxima [A] (verification not implemented) 1290
Giac [A] (verification not implemented) 1291
Mupad [B] (verification not implemented) 1291

Optimal result

Integrand size = 20, antiderivative size = 15

$$\int \frac{b+2cx^3}{bx+cx^4} \, dx = \log(x)+\frac{1}{3}\log\left(b+cx^3\right)$$

```
[Out] ln(x)+1/3*ln(c*x^3+b)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1607, 457, 78}

$$\int \frac{b+2cx^3}{bx+cx^4} \, dx = \frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

```
[In] Int[(b + 2*c*x^3)/(b*x + c*x^4),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx\\ &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx, x, x^3\right)\\ &= \log(x)+\frac{1}{3}\log\left(b+cx^3\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^3}{bx+cx^4}\,dx = \log(x)+\frac{1}{3}\log\left(b+cx^3\right)$$

```
[In] Integrate[(b + 2*c*x^3)/(b*x + c*x^4),x]
[Out] Log[x] + Log[b + c*x^3]/3
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\ln(x)+\frac{\ln(cx^3+b)}{3}$	14
norman	$\ln(x)+\frac{\ln(cx^3+b)}{3}$	14
risch	$\ln(x)+\frac{\ln(cx^3+b)}{3}$	14
parallelrisch	$\ln(x)+\frac{\ln(cx^3+b)}{3}$	14

```
[In] int((2*c*x^3+b)/(c*x^4+b*x),x,method=_RETURNVERBOSE)
[Out] ln(x)+1/3*ln(c*x^3+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^3}{bx+cx^4}\,dx=\frac{1}{3}\log\left(cx^3+b\right)+\log(x)$$

```
[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="fricas")
[Out] 1/3*log(c*x^3 + b) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx^3}{bx+cx^4}\,dx=\log(x)+\frac{\log\left(\frac{b}{c}+x^3\right)}{3}$$

```
[In] integrate((2*c*x**3+b)/(c*x**4+b*x),x)
[Out] log(x) + log(b/c + x**3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^3}{bx+cx^4}\,dx=\frac{1}{3}\log\left(cx^3+b\right)+\log(x)$$

```
[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="maxima")
[Out] 1/3*log(c*x^3 + b) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b + 2cx^3}{bx + cx^4} \, dx = \frac{1}{3} \log \left(\left| cx^3 + b \right| \right) + \log \left(|x| \right)$$

```
[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="giac")
[Out] 1/3*log(abs(c*x^3 + b)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b + 2cx^3}{bx + cx^4} \, dx = \frac{\ln \left(c\, x^3 + b \right)}{3} + \ln \left(x \right)$$

```
[In] int((b + 2*c*x^3)/(b*x + c*x^4),x)
[Out] log(b + c*x^3)/3 + log(x)
```

3.167 $\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx$

Optimal result 1292
Rubi [A] (verified) 1292
Mathematica [A] (verified) 1293
Maple [A] (verified) 1293
Fricas [A] (verification not implemented) 1294
Sympy [B] (verification not implemented) 1294
Maxima [B] (verification not implemented) 1294
Giac [F] 1295
Mupad [F(-1)] 1295

Optimal result

Integrand size = 22, antiderivative size = 15

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx=\log(x)+\frac{\log(b+cx^n)}{n}$$

```
[Out] ln(x)+ln(b+c*x^n)/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1607, 457, 78}

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx=\frac{\log(b+cx^n)}{n}+\log(x)$$

```
[In] Int[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]
```

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{b+2cx^n}{x\,(b+cx^n)}\,dx \\ &= \frac{\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx, x, x^n\right)}{n} \\ &= \log(x)+\frac{\log\left(b+cx^n\right)}{n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx = \frac{\log\left(x^n\right)+\log\left(n(b+cx^n)\right)}{n}$$

```
[In] Integrate[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]
[Out] (Log[x^n] + Log[n*(b + c*x^n)])/n
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

method	result	size
norman	$\ln\left(x\right)+\frac{\ln\left(c\,{\mathrm e}^{n \ln\left(x\right)}+b\right)}{n}$	18
risch	$\ln\left(x\right)+\frac{\ln\left(x^{n}+\frac{b}{c}\right)}{n}$	18

```
[In] int((b+2*c*x^n)/(b*x+c*x^(1+n)),x,method=_RETURNVERBOSE)
[Out] ln(x)+1/n*ln(c*exp(n*ln(x))+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx = \frac{(n-1)\log(x)+\log\left(bx+cx^{n+1}\right)}{n}$$

```
[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="fricas")
[Out] ((n - 1)*log(x) + log(b*x + c*x^(n + 1)))/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx = \begin{cases} \log(x) & \text{for } c=0 \wedge (c=0 \vee n=0) \\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n=0 \\ \log(x)+\frac{\log\left(\frac{b}{c}+x^n\right)}{n} & \text{otherwise} \end{cases}$$

```
[In] integrate((b+2*c*x**n)/(b*x+c*x**(1+n)),x)
[Out] Piecewise((log(x), Eq(c, 0) & (Eq(c, 0) | Eq(n, 0))), ((b + 2*c)*log(x)/(b
+ c), Eq(n, 0)), (log(x) + log(b/c + x**n)/n, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(15) = 30.

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.13

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx = b\left(\frac{\log(x)}{b}-\frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right)+\frac{2\log\left(\frac{cx^n+b}{c}\right)}{n}$$

```
[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="maxima")
[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

Giac [F]

$$\int \frac{b + 2cx^n}{bx + cx^{1+n}}\, dx = \int \frac{2\, cx^n + b}{bx + cx^{n+1}}\, dx$$

```
[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="giac")
[Out] integrate((2*c*x^n + b)/(b*x + c*x^(n + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{b + 2cx^n}{bx + cx^{1+n}}\, dx = \int \frac{b + 2\, c\, x^n}{b\, x + c\, x^{n+1}}\, dx$$

```
[In] int((b + 2*c*x^n)/(b*x + c*x^(n + 1)),x)
[Out] int((b + 2*c*x^n)/(b*x + c*x^(n + 1)), x)
```

3.168 $\int \frac{b+2cx}{(bx+cx^2)^8} dx$

Optimal result 1296
Rubi [A] (verified) 1296
Mathematica [A] (verified) 1297
Maple [A] (verified) 1297
Fricas [B] (verification not implemented) 1297
Sympy [B] (verification not implemented) 1298
Maxima [A] (verification not implemented) 1298
Giac [A] (verification not implemented) 1298
Mupad [B] (verification not implemented) 1299

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{b+2cx}{(bx+cx^2)^8} \, dx = -\frac{1}{7(bx+cx^2)^7}$$

```
[Out] -1/7/(c*x^2+b*x)^7
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {643}

$$\int \frac{b+2cx}{(bx+cx^2)^8} \, dx = -\frac{1}{7(bx+cx^2)^7}$$

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2)^8,x]
[Out] -1/7*1/(b*x + c*x^2)^7
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{7(bx+cx^2)^7}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7x^7(b+cx)^7}$$

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

```
[Out] -1/7*1/(x^7*(b + c*x)^7)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result
gosper	$-\frac{1}{7x^7(cx+b)^7}$
norman	$-\frac{1}{7x^7(cx+b)^7}$
risch	$-\frac{1}{7x^7(cx+b)^7}$
parallelrisch	$-\frac{1}{7x^7(cx+b)^7}$
derivativedivides	$-\frac{1}{7(c\,x^2+bx)^7}$
default	$-\frac{1}{7b^7x^7}-\frac{132c^6}{b^{13}x}+\frac{66c^5}{b^{12}x^2}-\frac{30c^4}{b^{11}x^3}+\frac{12c^3}{b^{10}x^4}-\frac{4c^2}{b^9x^5}+\frac{c}{b^8x^6}+\frac{132c^7}{b^{13}(cx+b)}+\frac{66c^7}{b^{12}(cx+b)^2}+\frac{30c^7}{b^{11}(cx+b)^3}+$

```
[In] int((2*c*x+b)/(c*x^2+b*x)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/x^7/(c*x+b)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.40

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx$$
$$= -\frac{1}{7\,(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7)}$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(14) = 28$.

Time = 0.46 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.80

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7b^7x^7+49b^6cx^8+147b^5c^2x^9+245b^4c^3x^{10}+245b^3c^4x^{11}+147b^2c^5x^{12}+49bc^6x^{13}+7c^7x^{14}}$$

```
[In] integrate((2*c*x+b)/(c*x**2+b*x)**8,x)
[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10
 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**
14)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7\,(cx^2+bx)^7}$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="maxima")
[Out] -1/7/(c*x^2 + b*x)^7
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7\,(cx^2+bx)^7}$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="giac")
[Out] -1/7/(c*x^2 + b*x)^7
```

Mupad [B] (verification not implemented)

Time = 10.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7\,x^7\,(b+c\,x)^7}$$

```
[In] int((b + 2*c*x)/(b*x + c*x^2)^8,x)
[Out] -1/(7*x^7*(b + c*x)^7)
```

3.169 $\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx$

Optimal result 1300
Rubi [A] (verified) 1300
Mathematica [A] (verified) 1301
Maple [A] (verified) 1301
Fricas [B] (verification not implemented) 1302
Sympy [B] (verification not implemented) 1302
Maxima [B] (verification not implemented) 1302
Giac [A] (verification not implemented) 1303
Mupad [B] (verification not implemented) 1303

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx = -\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

```
[Out] -1/14/x^14/(c*x^2+b)^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1598, 457, 75}

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx = -\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

```
[In] Int[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x]
```

```
[Out] -1/14*1/(x^14*(b + c*x^2)^7)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^2\right)\\ &= -\frac{1}{14x^{14}\left(b+cx^2\right)^7}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx = -\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

```
[In] Integrate[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x]
[Out] -1/14*1/(x^14*(b + c*x^2)^7)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{1}{14x^{14}\left(c\,x^2+b\right)^7}$
norman	$-\frac{1}{14x^{14}\left(c\,x^2+b\right)^7}$
risch	$-\frac{1}{14x^{14}\left(c\,x^2+b\right)^7}$
parallelrisch	$-\frac{1}{14x^{14}\left(c\,x^2+b\right)^7}$
default	$-\frac{1}{14b^7x^{14}}-\frac{66c^6}{b^{13}x^2}+\frac{33c^5}{b^{12}x^4}-\frac{15c^4}{b^{11}x^6}+\frac{6c^3}{b^{10}x^8}-\frac{2c^2}{b^9x^{10}}+\frac{c}{2b^8x^{12}}-\frac{c^8\left(-\frac{12b^3}{c\left(c\,x^2+b\right)^4}-\frac{b^5}{c\left(c\,x^2+b\right)^6}-\frac{66b}{c\left(c\,x^2+b\right)^2}-\cdots\right.}{2\cdots}$

```
[In] int((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x,method=_RETURNVERBOSE)
[Out] -1/14/x^14/(c*x^2+b)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx = -\frac{1}{14\left(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14}\right)}$$

```
[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="fricas")
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.

Time = 0.69 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx = -\frac{1}{14b^7x^{14}+98b^6cx^{16}+294b^5c^2x^{18}+490b^4c^3x^{20}+490b^3c^4x^{22}+294b^2c^5x^{24}+98bc^6x^{26}+14c^7x^{28}}$$

```
[In] integrate((2*c*x**2+b)/x**7/(c*x**3+b*x)**8,x)
[Out] -1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*x
**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c**
7*x**28)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx = -\frac{1}{14\left(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14}\right)}$$

```
[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="maxima")
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx=-\frac{1}{14\left(cx^4+bx^2\right)^7}$$

```
[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="giac")
[Out] -1/14/(c*x^4 + b*x^2)^7
```

Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx=-\frac{1}{14\,x^{14}\left(c\,x^2+b\right)^7}$$

```
[In] int((b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x)
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

3.170 $\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx$

Optimal result 1304
Rubi [A] (verified) 1304
Mathematica [A] (verified) 1305
Maple [A] (verified) 1305
Fricas [B] (verification not implemented) 1306
Sympy [B] (verification not implemented) 1306
Maxima [B] (verification not implemented) 1306
Giac [A] (verification not implemented) 1307
Mupad [B] (verification not implemented) 1307

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx=-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

```
[Out] -1/21/x^21/(c*x^3+b)^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1598, 457, 75}

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx=-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

```
[In] Int[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x]
```

```
[Out] -1/21*1/(x^21*(b + c*x^3)^7)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx\\ &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^3\right)\\ &= -\frac{1}{21x^{21}\left(b+cx^3\right)^7}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx = -\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

```
[In] Integrate[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x]
[Out] -1/21*1/(x^21*(b + c*x^3)^7)
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{1}{21x^{21}\left(c\,x^{3}+b\right)^{7}}$
risch	$-\frac{1}{21x^{21}\left(c\,x^{3}+b\right)^{7}}$
parallelrisch	$-\frac{1}{21x^{21}\left(c\,x^{3}+b\right)^{7}}$
default	$-\frac{1}{21b^{7}x^{21}}-\frac{44c^{6}}{b^{13}x^{3}}+\frac{22c^{5}}{b^{12}x^{6}}-\frac{10c^{4}}{b^{11}x^{9}}+\frac{4c^{3}}{b^{10}x^{12}}-\frac{4c^{2}}{3b^{9}x^{15}}+\frac{c}{3b^{8}x^{18}}-\frac{c^{8}\left(-\frac{66b}{c\left(c\,x^{3}+b\right)^{2}}-\frac{4b^{4}}{c\left(c\,x^{3}+b\right)^{5}}-\frac{132}{c\left(c\,x^{3}+b\right)}-\cdots\right.}{\cdots}$

```
[In] int((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/x^21/(c*x^3+b)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx = -\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

```
[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="fricas")
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.

Time = 0.93 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx = -\frac{1}{21b^7x^{21}+147b^6cx^{24}+441b^5c^2x^{27}+735b^4c^3x^{30}+735b^3c^4x^{33}+441b^2c^5x^{36}+147bc^6x^{39}+21c^7x^{42}}$$

```
[In] integrate((2*c*x**3+b)/x**14/(c*x**4+b*x)**8,x)
[Out] -1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3*
x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21*c
**7*x**42)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx = -\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

```
[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="maxima")
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx=-\frac{1}{21\left(cx^6+bx^3\right)^7}$$

```
[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="giac")
[Out] -1/21/(c*x^6 + b*x^3)^7
```

Mupad [B] (verification not implemented)

Time = 12.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx=-\frac{1}{21\,x^{21}\,\left(c\,x^3+b\right)^7}$$

```
[In] int((b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x)
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

3.171 $\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8} \, dx$

Optimal result 1308
Rubi [A] (verified) 1308
Mathematica [A] (verified) 1309
Maple [B] (verified) 1309
Fricas [B] (verification not implemented) 1310
Sympy [F(-1)] 1310
Maxima [B] (verification not implemented) 1310
Giac [F] 1311
Mupad [F(-1)] 1311

Optimal result

Integrand size = 29, antiderivative size = 21

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8} \, dx = -\frac{x^{-7n}}{7n(b+cx^n)^7}$$

```
[Out] -1/7/n/(x^(7*n))/(b+c*x^n)^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1598, 457, 75}

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8} \, dx = -\frac{x^{-7n}}{7n(b+cx^n)^7}$$

```
[In] Int[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8),x]
```

```
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^{-8-7(-1+n)}(b+2cx^n)}{(b+cx^n)^8}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^n\right)}{n}\\ &= -\frac{x^{-7n}}{7n\,(b+cx^n)^7}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx = -\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

```
[In] Integrate[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8),x]
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(21) = 42.

Time = 0.02 (sec) , antiderivative size = 203, normalized size of antiderivative = 9.67

$$-\frac{132c^6x^{-n}}{b^{13}n}+\frac{66c^5x^{-2n}}{b^{12}n}-\frac{30c^4x^{-3n}}{b^{11}n}+\frac{12c^3x^{-4n}}{b^{10}n}-\frac{4c^2x^{-5n}}{b^9n}+\frac{c\,x^{-6n}}{b^8n}-\frac{x^{-7n}}{7b^7n}+\frac{c^7(924x^{6n}c^6+6006b\,c^5x^{5n}+1638}{}$$

```
[In] int((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x)
[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*c
^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*c^
7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^3*(
x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(21) = 42$.

Time = 0.39 (sec) , antiderivative size = 143, normalized size of antiderivative = 6.81

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx = -\frac{x^{14}}{7\,(b^7nx^7x^{7\,n+7}+7\,b^6cnx^6x^{8\,n+8}+21\,b^5c^2nx^5x^{9\,n+9}+35\,b^4c^3nx^4x^{10\,n+10}+35\,b^3c^4nx^3x^{11\,n+11}+21\,b^2c^5nx$$

```
[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="fricas")
```

```
[Out] -1/7*x^14/(b^7*n*x^7*x^(7*n + 7) + 7*b^6*c*n*x^6*x^(8*n + 8) + 21*b^5*c^2*n
*x^5*x^(9*n + 9) + 35*b^4*c^3*n*x^4*x^(10*n + 10) + 35*b^3*c^4*n*x^3*x^(11*
n + 11) + 21*b^2*c^5*n*x^2*x^(12*n + 12) + 7*b*c^6*n*x*x^(13*n + 13) + c^7*
n*x^(14*n + 14))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx = \text{Timed out}$$

```
[In] integrate((b+2*c*x**n)/(x**(-7+7*n))/(b*x+c*x**(1+n))**8,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(21) = 42$.

Time = 0.24 (sec) , antiderivative size = 612, normalized size of antiderivative = 29.14

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx = -\frac{1}{105}\,b\left(\frac{360360\,c^{13}x^{13\,n}+2342340\,bc^{12}x^{12\,n}+6426420\,b^2c^{11}x^{11\,n}+9579570\,b^3c^{10}x^{10\,n}+8270262\,b^4c^9x^{9\,n}}{b^{14}c^7nx^{14\,n}+7\,b^{15}c^6nx^{13\,n}+21\,b^{16}c^5nx}\right. + \frac{1}{105}\,c\left(\frac{360360\,c^{12}x^{12\,n}+2342340\,bc^{11}x^{11\,n}+6426420\,b^2c^{10}x^{10\,n}+9579570\,b^3c^9x^{9\,n}+8270262\,b^4c^8x^{8\,n}+}{b^{13}c^7nx^{13\,n}+7\,b^{14}c^6nx^{12\,n}+21\,b^{15}c^5nx^{11\,n}+3}\right.$$

```
[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="maxima")
```

```
[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^1
1*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 4018014*
```

```
b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*b^8
*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^
(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*
n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*
n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 360360*
c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c*((36036
0*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10*n) + 957
9570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*x^(7*n) +
934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 100
1*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c
^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^
4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*b^19*c*n*x
^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c^6*log((c*x^n +
 b)/c)/(b^14*n))
```

Giac [F]

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx = \int \frac{2\,cx^n+b}{(bx+cx^{n+1})^8x^{7\,n-7}}\,dx$$

```
[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="giac")
[Out] integrate((2*c*x^n + b)/((b*x + c*x^(n + 1))^8*x^(7*n - 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx = \int \frac{x^{7-7\,n}\,(b+2\,c\,x^n)}{(b\,x+c\,x^{n+1})^8}\,dx$$

```
[In] int((x^(7 - 7*n)*(b + 2*c*x^n))/(b*x + c*x^(n + 1))^8,x)
[Out] int((x^(7 - 7*n)*(b + 2*c*x^n))/(b*x + c*x^(n + 1))^8, x)
```

3.172 $\int (b+2cx)\left(bx+cx^2\right)^p \,dx$

Optimal result 1312
Rubi [A] (verified) 1312
Mathematica [A] (verified) 1313
Maple [A] (verified) 1313
Fricas [A] (verification not implemented) 1313
Sympy [B] (verification not implemented) 1314
Maxima [A] (verification not implemented) 1314
Giac [A] (verification not implemented) 1314
Mupad [B] (verification not implemented) 1315

Optimal result

Integrand size = 18, antiderivative size = 19

$$\int (b+2cx)\left(bx+cx^2\right)^p \,dx=\frac{\left(bx+cx^2\right)^{1+p}}{1+p}$$

```
[Out] (c*x^2+b*x)^(p+1)/(p+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {643}

$$\int (b+2cx)\left(bx+cx^2\right)^p \,dx=\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

```
[Out] (b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral}=\frac{\left(bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{(x(b+cx))^{1+p}}{1+p}$$

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^p,x]
[Out] (x*(b + c*x))^(1 + p)/(1 + p)
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{\left(c\,x^2+bx\right)^{1+p}}{1+p}$	20
default	$\frac{\left(c\,x^2+bx\right)^{1+p}}{1+p}$	20
risch	$\frac{x(cx+b)(x(cx+b))^p}{1+p}$	22
gosper	$\frac{x(cx+b)\left(c\,x^2+bx\right)^p}{1+p}$	24
parallelrisch	$\frac{x^2(x(cx+b))^p bc+x(x(cx+b))^p b^2}{b(1+p)}$	40
norman	$\frac{bx\,\mathrm{e}^{p\ln\left(c\,x^2+bx\right)}}{1+p}+\frac{c\,x^2\mathrm{e}^{p\ln\left(c\,x^2+bx\right)}}{1+p}$	46

```
[In] int((2*c*x+b)*(c*x^2+b*x)^p,x,method=_RETURNVERBOSE)
[Out] (c*x^2+b*x)^(1+p)/(1+p)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{(cx^2+bx)(cx^2+bx)^p}{p+1}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="fricas")
[Out] (c*x^2 + b*x)*(c*x^2 + b*x)^p/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx = \begin{cases} \frac{bx(bx+cx^2)^p}{p+1} + \frac{cx^2(bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log(x) + \log\left(\frac{b}{c}+x\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**p,x)
[Out] Piecewise(((b*x*(b*x + c*x**2)**p/(p + 1) + c*x**2*(b*x + c*x**2)**p/(p + 1)
, Ne(p, -1)), (log(x) + log(b/c + x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx = \frac{(cx^2+bx)^{p+1}}{p+1}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="maxima")
[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx = \frac{(cx^2+bx)^{p+1}}{p+1}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="giac")
[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int (b+2cx) \left(bx+cx^2\right)^p \, dx = \frac{x\,(c\,x^2+b\,x)^p\,(b+c\,x)}{p+1}$$

```
[In] int((b*x + c*x^2)^p*(b + 2*c*x),x)
[Out] (x*(b*x + c*x^2)^p*(b + c*x))/(p + 1)
```

3.173 $\int x^{1+p}(b+2cx^2)\left(bx+cx^3\right)^p\,dx$

Optimal result 1316
Rubi [A] (verified) 1316
Mathematica [C] (verified) 1317
Maple [A] (verified) 1317
Fricas [A] (verification not implemented) 1317
Sympy [B] (verification not implemented) 1318
Maxima [A] (verification not implemented) 1318
Giac [B] (verification not implemented) 1318
Mupad [B] (verification not implemented) 1319

Optimal result

Integrand size = 25, antiderivative size = 27

$$\int x^{1+p}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx=\frac{x^{1+p}\left(bx+cx^3\right)^{1+p}}{2(1+p)}$$

```
[Out] 1/2*x^(p+1)*(c*x^3+b*x)^(p+1)/(p+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1604}

$$\int x^{1+p}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx=\frac{x^{p+1}\left(bx+cx^3\right)^{p+1}}{2(p+1)}$$

```
[In] Int[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]
```

```
[Out] (x^(1 + p)*(b*x + c*x^3)^(1 + p))/(2*(1 + p))
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\text{integral}=\frac{x^{1+p}\left(bx+cx^3\right)^{1+p}}{2(1+p)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

$$\int x^{1+p}(b+2cx^2)\left(bx+cx^3\right)^p\,dx$$
$$=\frac{x^{2+p}(x(b+cx^2))^p\left(1+\frac{cx^2}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^2}{b}\right)+2c(1+p)x^2\,\text{Hyp}\right.}{2(1+p)(2+p)}$$

```
[In] Integrate[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]
[Out] (x^(2 + p)*(x*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p,
 -((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x
^2)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

method	result	siz
gosper	$\frac{x^{2+p}(c\,x^2+b)(c\,x^3+bx)^p}{2+2p}$	31
parallelrisch	$\frac{x^3x^{1+p}(x(c\,x^2+b))^p bc+x\,x^{1+p}(x(c\,x^2+b))^p b^2}{2b(1+p)}$	55
risch	$\frac{(c\,x^2+b)x\,x^{1+p}(c\,x^2+b)^p x^p \mathrm{e}^{-\frac{i\,\mathrm{csgn}(ix(c\,x^2+b))\pi p(-\mathrm{csgn}(ix(c\,x^2+b))+\mathrm{csgn}(i(c\,x^2+b)))(-\mathrm{csgn}(ix(c\,x^2+b))+\mathrm{csgn}(ix))}{2}}}{2+2p}$	97

```
[In] int(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x,method=_RETURNVERBOSE)
[Out] 1/2*x^(2+p)/(1+p)*(c*x^2+b)*(c*x^3+b*x)^p
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x^{1+p}(b+2cx^2)\left(bx+cx^3\right)^p\,dx=\frac{(cx^3+bx)(cx^3+bx)^p x^{p+1}}{2\,(p+1)}$$

```
[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="fricas")
[Out] 1/2*(c*x^3 + b*x)*(c*x^3 + b*x)^p*x^(p + 1)/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(20) = 40$.

Time = 28.85 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.96

$$\int x^{1+p}(b+2cx^2)\left(bx+cx^3\right)^p\,dx = \begin{cases} \frac{bxx^{p+1}(bx+cx^3)^p}{2p+2} + \frac{cx^3x^{p+1}(bx+cx^3)^p}{2p+2} & \text{for}\ p \neq -1 \\ \log(x) + \frac{\log\left(x-\sqrt{-\frac{b}{c}}\right)}{2} + \frac{\log\left(x+\sqrt{-\frac{b}{c}}\right)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(1+p)*(2*c*x**2+b)*(c*x**3+b*x)**p,x)
```

```
[Out] Piecewise((b*x*x**(p + 1)*(b*x + c*x**3)**p/(2*p + 2) + c*x**3*x**(p + 1)*(
b*x + c*x**3)**p/(2*p + 2), Ne(p, -1)), (log(x) + log(x - sqrt(-b/c))/2 + l
og(x + sqrt(-b/c))/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int x^{1+p}(b+2cx^2)\left(bx+cx^3\right)^p\,dx = \frac{(cx^4+bx^2)e^{(p\log(cx^2+b)+2p\log(x))}}{2(p+1)}$$

```
[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int x^{1+p}(b+2cx^2)\left(bx+cx^3\right)^p\,dx = \frac{cx^3e^{(p\log(cx^2+b)+2p\log(x)+\log(x))} + bxe^{(p\log(cx^2+b)+2p\log(x)+\log(x))}}{2(p+1)}$$

```
[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="giac")
```

```
[Out] 1/2*(c*x^3*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)) + b*x*e^(p*log(c*x^2
 + b) + 2*p*log(x) + log(x)))/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int x^{1+p}(b+2cx^2)\left(bx+cx^3\right)^p\,dx=\left(c\,x^3+b\,x\right)^p\left(\frac{b\,x\,x^{p+1}}{2\,p+2}+\frac{c\,x^{p+1}\,x^3}{2\,p+2}\right)$$

```
[In] int(x^(p + 1)*(b*x + c*x^3)^p*(b + 2*c*x^2),x)
[Out] (b*x + c*x^3)^p*((b*x*x^(p + 1))/(2*p + 2) + (c*x^(p + 1)*x^3)/(2*p + 2))
```

3.174 $\int \left(bx^{1+p}(bx+cx^3)^p+2cx^{3+p}(bx+cx^3)^p\right)\,dx$

Optimal result 1320
Rubi [C] (verified) 1320
Mathematica [C] (verified) 1322
Maple [C] (warning: unable to verify) 1322
Fricas [A] (verification not implemented) 1322
Sympy [F] 1323
Maxima [A] (verification not implemented) 1323
Giac [B] (verification not implemented) 1323
Mupad [F(-1)] 1324

Optimal result

Integrand size = 38, antiderivative size = 27

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx=\frac{x^{1+p}\left(bx+cx^3\right)^{1+p}}{2(1+p)}$$

```
[Out] 1/2*x^(p+1)*(c*x^3+b*x)^(p+1)/(p+1)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.30, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2057, 372, 371}

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx$$
$$=\frac{bx^{p+2}(bx+cx^3)^p\left(\frac{cx^2}{b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-p,p+1,p+2,-\frac{cx^2}{b}\right)}{2(p+1)}$$
$$+\frac{cx^{p+4}(bx+cx^3)^p\left(\frac{cx^2}{b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-p,p+2,p+3,-\frac{cx^2}{b}\right)}{p+2}$$

```
[In] Int[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]
```

```
[Out] (b*x^(2 + p)*(b*x + c*x^3)^p*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x^2)/
b)])/(2*(1 + p)*(1 + (c*x^2)/b)^p) + (c*x^(4 + p)*(b*x + c*x^3)^p*Hypergeom
etric2F1[-p, 2 + p, 3 + p, -((c*x^2)/b)])/((2 + p)*(1 + (c*x^2)/b)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b\int x^{1+p}\left(bx+cx^3\right)^p\,dx+(2c)\int x^{3+p}\left(bx+cx^3\right)^p\,dx\\
&= \left(bx^{-p}\left(b+cx^2\right)^{-p}\left(bx+cx^3\right)^p\right)\int x^{1+2p}\left(b+cx^2\right)^p\,dx\\
&\quad+\left(2cx^{-p}\left(b+cx^2\right)^{-p}\left(bx+cx^3\right)^p\right)\int x^{3+2p}\left(b+cx^2\right)^p\,dx\\
&= \left(bx^{-p}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx+cx^3\right)^p\right)\int x^{1+2p}\left(1+\frac{cx^2}{b}\right)^p\,dx\\
&\quad+\left(2cx^{-p}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx+cx^3\right)^p\right)\int x^{3+2p}\left(1+\frac{cx^2}{b}\right)^p\,dx\\
&= \frac{bx^{2+p}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx+cx^3\right)^p\,{}_2F_1\left(-p,1+p;2+p;-\frac{cx^2}{b}\right)}{2(1+p)}\\
&\quad+\frac{cx^{4+p}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx+cx^3\right)^p\,{}_2F_1\left(-p,2+p;3+p;-\frac{cx^2}{b}\right)}{2+p}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx$$
$$=\frac{x^{2+p}(x(b+cx^2))^p\left(1+\frac{cx^2}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^2}{b}\right)+2c(1+p)x^2\,\text{Hyper}\right.}{2(1+p)(2+p)}$$

```
[In] Integrate[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]
```

```
[Out] (x^(2 + p)*(x*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p,
 -((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x
^2)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.74 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

method	result	size
risch	$\frac{(c\,x^2+b)x\,x^{1+p}(c\,x^2+b)^p x^p \mathrm{e}^{-\frac{i\,\mathrm{csgn}\left(ix\left(c\,x^2+b\right)\right)\pi p\left(-\,\mathrm{csgn}\left(ix\left(c\,x^2+b\right)\right)+\mathrm{csgn}\left(i\left(c\,x^2+b\right)\right)\right)\left(-\,\mathrm{csgn}\left(ix\left(c\,x^2+b\right)\right)+\mathrm{csgn}(ix)\right)}{2}}}{2+2p}$	97

```
[In] int(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x,method=_RETURNVERBO
SE)
```

```
[Out] 1/2*(c*x^2+b)*x*x^(1+p)/(1+p)*(c*x^2+b)^p*x^p*exp(-1/2*I*csgn(I*x*(c*x^2+b)
)*Pi*p*(-csgn(I*x*(c*x^2+b))+csgn(I*(c*x^2+b)))*(-csgn(I*x*(c*x^2+b))+csgn(
I*x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx=\frac{(cx^2+b)(cx^3+bx)^p x^{p+3}}{2\,(p+1)x}$$

```
[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="f
ricas")
```

```
[Out] 1/2*(c*x^2 + b)*(c*x^3 + b*x)^p*x^(p + 3)/((p + 1)*x)
```

Sympy [F]

$$\int \left(bx^{1+p}(bx+cx^3)^p+2cx^{3+p}(bx+cx^3)^p\right)\,dx=\int \left(x(b+cx^2)\right)^p\left(bx^{p+1}+2cx^{p+3}\right)\,dx$$

```
[In] integrate(b*x**(1+p)*(c*x**3+b*x)**p+2*c*x**(3+p)*(c*x**3+b*x)**p,x)
[Out] Integral((x*(b + c*x**2))**p*(b*x**(p + 1) + 2*c*x**(p + 3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \left(bx^{1+p}(bx+cx^3)^p+2cx^{3+p}(bx+cx^3)^p\right)\,dx=\frac{(cx^4+bx^2)e^{(p\log(cx^2+b)+2\,p\log(x))}}{2\,(p+1)}$$

```
[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="m
axima")
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \left(bx^{1+p}(bx+cx^3)^p+2cx^{3+p}(bx+cx^3)^p\right)\,dx$$
$$=\frac{cx^3e^{(p\log(cx^2+b)+2\,p\log(x)+\log(x))}+bxe^{(p\log(cx^2+b)+2\,p\log(x)+\log(x))}}{2\,(p+1)}$$

```
[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="g
iac")
[Out] 1/2*(c*x^3*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)) + b*x*e^(p*log(c*x^2
 + b) + 2*p*log(x) + log(x)))/(p + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \left(bx^{1+p}(bx+cx^3)^p+2cx^{3+p}(bx+cx^3)^p\right)\,dx$$
$$=\int b\,x^{p+1}\,\left(c\,x^3+b\,x\right)^p+2\,c\,x^{p+3}\,\left(c\,x^3+b\,x\right)^p\,dx$$

```
[In] int(b*x^(p + 1)*(b*x + c*x^3)^p + 2*c*x^(p + 3)*(b*x + c*x^3)^p,x)
[Out] int(b*x^(p + 1)*(b*x + c*x^3)^p + 2*c*x^(p + 3)*(b*x + c*x^3)^p, x)
```

3.175 $\int x^{2(1+p)}(b+2cx^3)\left(bx+cx^4\right)^p\,dx$

Optimal result 1325
Rubi [A] (verified) 1325
Mathematica [C] (verified) 1326
Maple [A] (verified) 1326
Fricas [A] (verification not implemented) 1326
Sympy [F(-1)] 1327
Maxima [A] (verification not implemented) 1327
Giac [B] (verification not implemented) 1327
Mupad [B] (verification not implemented) 1328

Optimal result

Integrand size = 27, antiderivative size = 29

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\frac{x^{2(1+p)}\left(bx+cx^4\right)^{1+p}}{3(1+p)}$$

```
[Out] 1/3*x^(2+2*p)*(c*x^4+b*x)^(p+1)/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1604}

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\frac{x^{2(p+1)}\left(bx+cx^4\right)^{p+1}}{3(p+1)}$$

```
[In] Int[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]
[Out] (x^(2*(1 + p))*(b*x + c*x^4)^(1 + p))/(3*(1 + p))
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\text{integral}=\frac{x^{2(1+p)}\left(bx+cx^4\right)^{1+p}}{3(1+p)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.41

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx$$
$$=\frac{x^{3+2p}(x(b+cx^3))^p\left(1+\frac{cx^3}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^3}{b}\right)+2c(1+p)x^3\,\text{Hype}\right.}{3(1+p)(2+p)}$$

```
[In] Integrate[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]
```

```
[Out] (x^(3 + 2*p)*(x*(b + c*x^3))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 +
p, -((c*x^3)/b)] + 2*c*(1 + p)*x^3*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c
*x^3)/b)]))/(3*(1 + p)*(2 + p)*(1 + (c*x^3)/b)^p)
```

Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

method	result	siz
gosper	$\frac{x^{3+2p}\left(c\,x^{3}+b\right)\left(c\,x^{4}+b x\right)^{p}}{3+3p}$	33
parallelrisch	$\frac{x^{4} x^{2+2p}\left(x\left(c\,x^{3}+b\right)\right)^{p} c^{2}+x\,x^{2+2p}\left(x\left(c\,x^{3}+b\right)\right)^{p} b c}{3c(1+p)}$	59
risch	$\frac{\left(c\,x^{3}+b\right) x\,x^{2+2p}\left(c\,x^{3}+b\right)^{p} x^{p} {\mathrm e}^{-\frac{i \pi\,\mathrm{csgn}\left(i x\left(c\,x^{3}+b\right)\right) p\left(-\mathrm{csgn}\left(i x\left(c\,x^{3}+b\right)\right)+\mathrm{csgn}\left(i\left(c\,x^{3}+b\right)\right)\right)\left(-\mathrm{csgn}\left(i x\left(c\,x^{3}+b\right)\right)+\mathrm{csgn}\left(i x\right)\right)}{2}}}{3+3p}$	99

```
[In] int(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(3+2*p)/(1+p)*(c*x^3+b)*(c*x^4+b*x)^p
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\frac{(cx^4+bx)(cx^4+bx)^p x^{2\,p+2}}{3\,(p+1)}$$

```
[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="fricas")
```

```
[Out] 1/3*(c*x^4 + b*x)*(c*x^4 + b*x)^p*x^(2*p + 2)/(p + 1)
```

Sympy [F(-1)]

Timed out.

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x**(2+2*p)*(2*c*x**3+b)*(c*x**4+b*x)**p,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\frac{(cx^6+bx^3)e^{\left(p\log\left(cx^3+b\right)+3\,p\log\left(x\right)\right)}}{3\,(p+1)}$$

```
[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="maxima")
[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(27) = 54.

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.00

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx =\frac{cx^4e^{\left(p\log\left(cx^3+b\right)+3\,p\log\left(x\right)+2\,\log\left(x\right)\right)}+bxe^{\left(p\log\left(cx^3+b\right)+3\,p\log\left(x\right)+2\,\log\left(x\right)\right)}}{3\,(p+1)}$$

```
[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="giac")
[Out] 1/3*(c*x^4*e^(p*log(c*x^3 + b) + 3*p*log(x) + 2*log(x)) + b*x*e^(p*log(c*x^
3 + b) + 3*p*log(x) + 2*log(x)))/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\left(c\,x^4+b\,x\right)^p\,\left(\frac{c\,x^{2\,p+2}\,x^4}{3\,p+3}+\frac{b\,x\,x^{2\,p+2}}{3\,p+3}\right)$$

```
[In] int(x^(2*p + 2)*(b*x + c*x^4)^p*(b + 2*c*x^3),x)
[Out] (b*x + c*x^4)^p*((c*x^(2*p + 2)*x^4)/(3*p + 3) + (b*x*x^(2*p + 2))/(3*p + 3
))
```

3.176 $\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx$

Optimal result 1329
Rubi [A] (verified) 1329
Mathematica [C] (verified) 1330
Maple [F] 1330
Fricas [A] (verification not implemented) 1330
Sympy [F(-1)] 1331
Maxima [A] (verification not implemented) 1331
Giac [F] 1331
Mupad [F(-1)] 1331

Optimal result

Integrand size = 31, antiderivative size = 36

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\frac{x^{-((1-n)(1+p))}\left(bx+cx^{1+n}\right)^{1+p}}{n(1+p)}$$

```
[Out] (b*x+c*x^(1+n))^(p+1)/n/(p+1)/(x^((1-n)*(p+1)))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2061}

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\frac{x^{-((1-n)(p+1))}\left(bx+cx^{n+1}\right)^{p+1}}{n(p+1)}$$

```
[In] Int[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]
[Out] (b*x + c*x^(1 + n))^(1 + p)/(n*(1 + p)*x^((1 - n)*(1 + p)))
```

Rule 2061

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] /; FreeQ[{a, b, c, d, e, j, m
, n, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && EqQ[
a*d*(m + j*p + 1) - b*c*(m + n + p*(j + n) + 1), 0] && (GtQ[e, 0] || Intege
rsQ[j]) && NeQ[m + j*p + 1, 0]
```

Rubi steps

$$\text{integral}=\frac{x^{-((1-n)(1+p))}\left(bx+cx^{1+n}\right)^{1+p}}{n(1+p)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.00

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx$$
$$=\frac{x^{-p}(x(b+cx^n))^p\left(1+\frac{cx^n}{b}\right)^{-p}\left(b(2+p)x^{n(1+p)}\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^n}{b}\right)+2c(1+p)x^{n(2}\right.}{n(1+p)(2+p)}$$

```
[In] Integrate[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]
```

```
[Out] ((x*(b + c*x^n))^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p, 2
+ p, -((c*x^n)/b)] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2 + p,
 3 + p, -((c*x^n)/b)]))/(n*(1 + p)*(2 + p)*x^p*(1 + (c*x^n)/b)^p)
```

Maple [F]

$$\int x^{(-1+n)(1+p)}(b+2c\,x^n)\left(bx+c\,x^{1+n}\right)^p dx$$

```
[In] int(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)
```

```
[Out] int(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\frac{(bx+cx^{n+1})(bx+cx^{n+1})^p x^{(n-1)p+n-1}}{np+n}$$

```
[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="fric
as")
```

```
[Out] (b*x + c*x^(n + 1))*(b*x + c*x^(n + 1))^p*x^((n - 1)*p + n - 1)/(n*p + n)
```

Sympy [F(-1)]

Timed out.

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x**((-1+n)*(1+p))*(b+2*c*x**n)*(b*x+c*x**(1+n))**p,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\frac{(cx^{2n}+bx^n)e^{(np\log(x)+p\log(cx^n+b))}}{n(p+1)}$$

```
[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="maxi
ma")
[Out] (c*x^(2*n) + b*x^n)*e^(n*p*log(x) + p*log(c*x^n + b))/(n*(p + 1))
```

Giac [F]

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\int(2\,cx^n+b)\left(bx+cx^{n+1}\right)^px^{(n-1)(p+1)}\,dx$$

```
[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="giac
")
[Out] integrate((2*c*x^n + b)*(b*x + c*x^(n + 1))^p*x^((n - 1)*(p + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\int x^{(n-1)\,(p+1)}\,\left(b\,x+c\,x^{n+1}\right)^p\,\left(b+2\,c\,x^n\right)\,dx$$

```
[In] int(x^((n - 1)*(p + 1))*(b*x + c*x^(n + 1))^p*(b + 2*c*x^n),x)
[Out] int(x^((n - 1)*(p + 1))*(b*x + c*x^(n + 1))^p*(b + 2*c*x^n), x)
```

3.177 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2} dx$

Optimal result 1332
Rubi [A] (verified) 1332
Mathematica [A] (verified) 1333
Maple [A] (verified) 1333
Fricas [A] (verification not implemented) 1333
Sympy [A] (verification not implemented) 1334
Maxima [A] (verification not implemented) 1334
Giac [A] (verification not implemented) 1334
Mupad [B] (verification not implemented) 1335

Optimal result

Integrand size = 54, antiderivative size = 32

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2} dx = acx+\frac{1}{2}adx^2+\frac{1}{3}bcx^3+\frac{1}{4}bdx^4$$

```
[Out] a*c*x+1/2*a*d*x^2+1/3*b*c*x^3+1/4*b*d*x^4
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {1600}

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2} dx = acx+\frac{1}{2}adx^2+\frac{1}{3}bcx^3+\frac{1}{4}bdx^4$$

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
a + b*x^2),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(ac+adx+bcx^2+bdx^3\right) dx \\ &= acx+\frac{1}{2}adx^2+\frac{1}{3}bcx^3+\frac{1}{4}bdx^4 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{a + bx^2}\,dx = acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(a + b*x^2),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
default	$acx+\frac{1}{2}ad\,x^2+\frac{1}{3}bc\,x^3+\frac{1}{4}bd\,x^4$	27
norman	$acx+\frac{1}{2}ad\,x^2+\frac{1}{3}bc\,x^3+\frac{1}{4}bd\,x^4$	27
risch	$acx+\frac{1}{2}ad\,x^2+\frac{1}{3}bc\,x^3+\frac{1}{4}bd\,x^4$	27
parallelrisch	$acx+\frac{1}{2}ad\,x^2+\frac{1}{3}bc\,x^3+\frac{1}{4}bd\,x^4$	27
parts	$acx+\frac{1}{2}ad\,x^2+\frac{1}{3}bc\,x^3+\frac{1}{4}bd\,x^4$	27
gosper	$\frac{x(3x^3bd+4bc\,x^2+6adx+12ac)}{12}$	28

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a),x
,method=_RETURNVERBOSE)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*b*c*x^3+1/4*b*d*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{a + bx^2}\,dx = \frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,adx^2 + acx$$

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a),x, algorithm="fricas")
```

```
[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{a + bx^2}\,dx = acx + \frac{adx^2}{2} + \frac{bcx^3}{3} + \frac{bdx^4}{4}$$

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(b*x**2+a),x)
```

```
[Out] a*c*x + a*d*x**2/2 + b*c*x**3/3 + b*d*x**4/4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{a + bx^2}\,dx = \frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,adx^2 + acx$$

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{a + bx^2}\,dx = \frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,adx^2 + acx$$

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a),x, algorithm="giac")
```

```
[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2}\,dx=\frac{b\,d\,x^4}{4}+\frac{b\,c\,x^3}{3}+\frac{a\,d\,x^2}{2}+a\,c\,x$$

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
a + b*x^2),x)
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

3.178 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}\,dx$

Optimal result 1336
Rubi [A] (verified) 1336
Mathematica [A] (verified) 1337
Maple [A] (verified) 1337
Fricas [A] (verification not implemented) 1338
Sympy [A] (verification not implemented) 1338
Maxima [A] (verification not implemented) 1338
Giac [A] (verification not implemented) 1339
Mupad [B] (verification not implemented) 1339

Optimal result

Integrand size = 54, antiderivative size = 12

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}\,dx = cx+\frac{dx^2}{2}$$

```
[Out] c*x+1/2*d*x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {1600}

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}\,dx = cx+\frac{dx^2}{2}$$

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
a + b*x^2)^2,x]
```

```
[Out] c*x + (d*x^2)/2
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{ac+adx+bcx^2+bdx^3}{a+bx^2}\,dx\\ &= \int (c+dx)\,dx\\ &= cx+\frac{dx^2}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}\,dx = cx+\frac{dx^2}{2}$$

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(a + b*x^2)^2,x]
```

```
[Out] c*x + (d*x^2)/2
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$\frac{x(dx+2c)}{2}$	11
default	$cx+\frac{1}{2}d\,x^2$	11
risch	$cx+\frac{1}{2}d\,x^2$	11
parallelrisch	$cx+\frac{1}{2}d\,x^2$	11
parts	$cx+\frac{1}{2}d\,x^2$	11
norman	$\frac{acx+bc\,x^3-\frac{a^2d}{2b}+\frac{bd\,x^4}{2}}{b\,x^2+a}$	38

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^2
,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(d*x+2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^2} dx = \frac{1}{2} dx^2 + cx$$

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*d*x^2 + c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^2} dx = cx + \frac{dx^2}{2}$$

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(b*x**2+a)**2,x)
```

```
[Out] c*x + d*x**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^2} dx = \frac{1}{2} dx^2 + cx$$

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2 + c*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^2}\, dx = \frac{1}{2}\, dx^2 + cx$$

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d*x^2 + c*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^2}\, dx = \frac{d\,x^2}{2} + c\,x$$

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
a + b*x^2)^2,x)
```

```
[Out] c*x + (d*x^2)/2
```

3.179 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^3} dx$

Optimal result 1340
Rubi [A] (verified) 1340
Mathematica [A] (verified) 1341
Maple [A] (verified) 1342
Fricas [A] (verification not implemented) 1342
Sympy [B] (verification not implemented) 1342
Maxima [A] (verification not implemented) 1343
Giac [A] (verification not implemented) 1343
Mupad [B] (verification not implemented) 1343

Optimal result

Integrand size = 54, antiderivative size = 42

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^3}\,dx=\frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{d\log(a+bx^2)}{2b}$$

```
[Out] 1/2*d*ln(b*x^2+a)/b+c*arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1600, 649, 211, 266}

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^3}\,dx=\frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{d\log(a+bx^2)}{2b}$$

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
a + b*x^2)^3,x]
```

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*b
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{ac+adx+bcx^2+bdx^3}{(a+bx^2)^2}\,dx \\ &= \int \frac{c+dx}{a+bx^2}\,dx \\ &= c\int \frac{1}{a+bx^2}\,dx + d\int \frac{x}{a+bx^2}\,dx \\ &= \frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{d\log(a+bx^2)}{2b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^3}\,dx = \frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{d\log(a+bx^2)}{2b}$$

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(a + b*x^2)^3,x]
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*b
)
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{d\ln\left(b\,x^{2}+a\right)}{2b}+\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	32
risch	$\frac{\ln\left(-\sqrt{-ab}\,x+a\right)c\sqrt{-ab}}{2ab}+\frac{\ln\left(-\sqrt{-ab}\,x+a\right)d}{2b}-\frac{\ln\left(\sqrt{-ab}\,x+a\right)c\sqrt{-ab}}{2ab}+\frac{\ln\left(\sqrt{-ab}\,x+a\right)d}{2b}$	90

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^3
,x,method=_RETURNVERBOSE)
[Out] 1/2*d*ln(b*x^2+a)/b+c/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.33

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^3}\,dx = \left[\frac{ad\log\left(bx^2+a\right)-\sqrt{-ab}c\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab},\frac{ad\log\left(bx^2+a\right)+2\sqrt{ab}c\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\,ab}\right]$$

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^3,x, algorithm="fricas")
[Out] [1/2*(a*d*log(b*x^2 + a) - sqrt(-a*b)*c*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)))/(a*b), 1/2*(a*d*log(b*x^2 + a) + 2*sqrt(a*b)*c*arctan(sqrt(a*b)
*x/a))/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(37) = 74.

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.95

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^3}\,dx = \left(\frac{d}{2b}-\frac{c\sqrt{-ab^3}}{2ab^2}\right)\log\left(x+\frac{2ab\left(\frac{d}{2b}-\frac{c\sqrt{-ab^3}}{2ab^2}\right)-ad}{bc}\right) + \left(\frac{d}{2b}+\frac{c\sqrt{-ab^3}}{2ab^2}\right)\log\left(x+\frac{2ab\left(\frac{d}{2b}+\frac{c\sqrt{-ab^3}}{2ab^2}\right)-ad}{bc}\right)$$

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(b*x**2+a)**3,x)
```

```
[Out] (d/(2*b) - c*sqrt(-a*b**3)/(2*a*b**2))*log(x + (2*a*b*(d/(2*b) - c*sqrt(-a*
b**3)/(2*a*b**2)) - a*d)/(b*c)) + (d/(2*b) + c*sqrt(-a*b**3)/(2*a*b**2))*lo
g(x + (2*a*b*(d/(2*b) + c*sqrt(-a*b**3)/(2*a*b**2)) - a*d)/(b*c))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^3}\, dx = \frac{c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{d \log\left(bx^2 + a\right)}{2\, b}$$

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^3,x, algorithm="maxima")
```

```
[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*d*log(b*x^2 + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^3}\, dx = \frac{c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{d \log\left(bx^2 + a\right)}{2\, b}$$

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^3,x, algorithm="giac")
```

```
[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*d*log(b*x^2 + a)/b
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^3}\, dx = \frac{d\, \ln\left(b\, x^2 + a\right)}{2\, b} + \frac{c\, \mathrm{atan}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{\sqrt{a}\, \sqrt{b}}$$

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
a + b*x^2)^3,x)
```

```
[Out] (d*log(a + b*x^2))/(2*b) + (c*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2))
```

3.180 $\int (b+2cx+3dx^2) (a+bx+cx^2+dx^3)^n \, dx$

Optimal result 1344
Rubi [A] (verified) 1344
Mathematica [A] (verified) 1345
Maple [A] (verified) 1345
Fricas [A] (verification not implemented) 1345
Sympy [F(-1)] 1346
Maxima [A] (verification not implemented) 1346
Giac [A] (verification not implemented) 1346
Mupad [B] (verification not implemented) 1346

Optimal result

Integrand size = 30, antiderivative size = 25

$$\int (b+2cx+3dx^2) (a+bx+cx^2+dx^3)^n \, dx = \frac{(a+bx+cx^2+dx^3)^{1+n}}{1+n}$$

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1602}

$$\int (b+2cx+3dx^2) (a+bx+cx^2+dx^3)^n \, dx = \frac{(a+bx+cx^2+dx^3)^{n+1}}{n+1}$$

```
[In] Int[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(a+bx+cx^2+dx^3)^{1+n}}{1+n}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^n\,dx=\frac{(a+x(b+x(c+dx)))^{1+n}}{1+n}$$

```
[In] Integrate[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]
[Out] (a + x*(b + x*(c + d*x)))^(1 + n)/(1 + n)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	si
gosper	$\frac{\left(x^3d+c\,x^2+bx+a\right)^{1+n}}{1+n}$	26
derivativedivides	$\frac{\left(x^3d+c\,x^2+bx+a\right)^{1+n}}{1+n}$	26
default	$\frac{\left(x^3d+c\,x^2+bx+a\right)^{1+n}}{1+n}$	26
risch	$\frac{\left(x^3d+c\,x^2+bx+a\right)\left(x^3d+c\,x^2+bx+a\right)^n}{1+n}$	39
parallelrisch	$\frac{x^3\left(x^3d+c\,x^2+bx+a\right)^n cd+x^2\left(x^3d+c\,x^2+bx+a\right)^n c^2+x\left(x^3d+c\,x^2+bx+a\right)^n bc+\left(x^3d+c\,x^2+bx+a\right)^n ac}{c(1+n)}$	99
norman	$\frac{a\,\mathrm{e}^{n\ln\left(x^3d+c\,x^2+bx+a\right)}}{1+n}+\frac{bx\,\mathrm{e}^{n\ln\left(x^3d+c\,x^2+bx+a\right)}}{1+n}+\frac{c\,x^2\mathrm{e}^{n\ln\left(x^3d+c\,x^2+bx+a\right)}}{1+n}+\frac{d\,x^3\mathrm{e}^{n\ln\left(x^3d+c\,x^2+bx+a\right)}}{1+n}$	11

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x,method=_RETURNVERBOSE)
[Out] (d*x^3+c*x^2+b*x+a)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^n\,dx=\frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^n}{n+1}$$

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="fricas")
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^n/(n + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^n\,dx=\text{Timed out}$$

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**n,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^n\,dx=\frac{\left(dx^3+cx^2+bx+a\right)^{n+1}}{n+1}$$

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="maxima")
[Out] (d*x^3 + c*x^2 + b*x + a)^(n + 1)/(n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^n\,dx=\frac{\left(dx^3+cx^2+bx+a\right)^{n+1}}{n+1}$$

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="giac")
[Out] (d*x^3 + c*x^2 + b*x + a)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^n\,dx =\left(d\,x^3+c\,x^2+b\,x+a\right)^n\left(\frac{a}{n+1}+\frac{b\,x}{n+1}+\frac{c\,x^2}{n+1}+\frac{d\,x^3}{n+1}\right)$$

```
[In] int((b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x)
[Out] (a + b*x + c*x^2 + d*x^3)^n*(a/(n + 1) + (b*x)/(n + 1) + (c*x^2)/(n + 1) +
(d*x^3)/(n + 1))
```

3.181 $\int (b+2cx+3dx^2)(bx+cx^2+dx^3)^n\,dx$

Optimal result 1347
Rubi [A] (verified) 1347
Mathematica [A] (verified) 1348
Maple [A] (verified) 1348
Fricas [A] (verification not implemented) 1348
Sympy [F(-1)] 1349
Maxima [A] (verification not implemented) 1349
Giac [A] (verification not implemented) 1349
Mupad [B] (verification not implemented) 1349

Optimal result

Integrand size = 29, antiderivative size = 24

$$\int (b+2cx+3dx^2)\left(bx+cx^2+dx^3\right)^n\,dx=\frac{(bx+cx^2+dx^3)^{1+n}}{1+n}$$

```
[Out] (d*x^3+c*x^2+b*x)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1602}

$$\int (b+2cx+3dx^2)\left(bx+cx^2+dx^3\right)^n\,dx=\frac{(bx+cx^2+dx^3)^{n+1}}{n+1}$$

```
[In] Int[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]
[Out] (b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{(bx+cx^2+dx^3)^{1+n}}{1+n}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int (b+2cx+3dx^2) (bx+cx^2+dx^3)^n \, dx = \frac{(x(b+x(c+dx)))^{1+n}}{1+n}$$

```
[In] Integrate[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]
[Out] (x*(b + x*(c + d*x)))^(1 + n)/(1 + n)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{(x^3 d+c\,x^2+bx)^{1+n}}{1+n}$	25
default	$\frac{(x^3 d+c\,x^2+bx)^{1+n}}{1+n}$	25
risch	$\frac{x(d\,x^2+cx+b)\,(x(d\,x^2+cx+b))^n}{1+n}$	32
gosper	$\frac{x(d\,x^2+cx+b)\,(x^3 d+c\,x^2+bx)^n}{1+n}$	34
parallelrisch	$\frac{x^3 (x(d\,x^2+cx+b))^n d^2+x^2 (x(d\,x^2+cx+b))^n cd+x(x(d\,x^2+cx+b))^n bd}{d(1+n)}$	70
norman	$\frac{bx\,\mathrm{e}^{n\ln(x^3 d+c\,x^2+bx)}}{1+n}+\frac{c\,x^2\mathrm{e}^{n\ln(x^3 d+c\,x^2+bx)}}{1+n}+\frac{d\,x^3\mathrm{e}^{n\ln(x^3 d+c\,x^2+bx)}}{1+n}$	84

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x,method=_RETURNVERBOSE)
[Out] (d*x^3+c*x^2+b*x)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int (b+2cx+3dx^2) (bx+cx^2+dx^3)^n \, dx = \frac{(dx^3+cx^2+bx)(dx^3+cx^2+bx)^n}{n+1}$$

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="fricas")
[Out] (d*x^3 + c*x^2 + b*x)*(d*x^3 + c*x^2 + b*x)^n/(n + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^n\,dx=\text{Timed out}$$

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**n,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^n\,dx=\frac{\left(dx^3+cx^2+bx\right)^{n+1}}{n+1}$$

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="maxima")
[Out] (d*x^3 + c*x^2 + b*x)^(n + 1)/(n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^n\,dx=\frac{\left(dx^3+cx^2+bx\right)^{n+1}}{n+1}$$

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="giac")
[Out] (d*x^3 + c*x^2 + b*x)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^n\,dx=\left(\frac{b\,x}{n+1}+\frac{c\,x^2}{n+1}+\frac{d\,x^3}{n+1}\right)\,{\left(d\,x^3+c\,x^2+b\,x\right)}^n$$

```
[In] int((b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x)
[Out] ((b*x)/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(b*x + c*x^2 + d*x^3)^n
```

3.182 $\int x^n (b+cx+dx^2)^n (b+2cx+3dx^2)\,dx$

Optimal result 1350
Rubi [A] (verified) 1350
Mathematica [A] (verified) 1351
Maple [A] (verified) 1351
Fricas [A] (verification not implemented) 1351
Sympy [F(-1)] 1352
Maxima [A] (verification not implemented) 1352
Giac [B] (verification not implemented) 1352
Mupad [B] (verification not implemented) 1353

Optimal result

Integrand size = 28, antiderivative size = 25

$$\int x^n (b+cx+dx^2)^n (b+2cx+3dx^2)\,dx = \frac{x^{1+n}(b+cx+dx^2)^{1+n}}{1+n}$$

```
[Out] x^(1+n)*(d*x^2+c*x+b)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1604}

$$\int x^n (b+cx+dx^2)^n (b+2cx+3dx^2)\,dx = \frac{x^{n+1}(b+cx+dx^2)^{n+1}}{n+1}$$

```
[In] Int[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]
[Out] (x^(1 + n)*(b + c*x + d*x^2)^(1 + n))/(1 + n)
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{x^{1+n}(b+cx+dx^2)^{1+n}}{1+n}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int x^n \left(b+cx+dx^2\right)^n \left(b+2cx+3dx^2\right)\,dx=\frac{x^{1+n}(b+x(c+dx))^{1+n}}{1+n}$$

```
[In] Integrate[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]
[Out] (x^(1 + n)*(b + x*(c + d*x))^(1 + n))/(1 + n)
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{x^{1+n}\left(d\,x^{2}+cx+b\right)^{1+n}}{1+n}$	26
risch	$\frac{x\left(d\,x^{2}+cx+b\right)x^{n}\left(d\,x^{2}+cx+b\right)^{n}}{1+n}$	33
parallelrisch	$\frac{x^{3}x^{n}\left(d\,x^{2}+cx+b\right)^{n}d^{2}+x^{2}x^{n}\left(d\,x^{2}+cx+b\right)^{n}cd+x\,x^{n}\left(d\,x^{2}+cx+b\right)^{n}bd}{d(1+n)}$	73

```
[In] int(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x,method=_RETURNVERBOSE)
[Out] x^(1+n)*(d*x^2+c*x+b)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int x^n \left(b+cx+dx^2\right)^n \left(b+2cx+3dx^2\right)\,dx=\frac{(dx^3+cx^2+bx)(dx^2+cx+b)^n x^n}{n+1}$$

```
[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="fricas")
[Out] (d*x^3 + c*x^2 + b*x)*(d*x^2 + c*x + b)^n*x^n/(n + 1)
```

Sympy [F(-1)]

Timed out.

$$\int x^n\left(b+cx+dx^2\right)^n\left(b+2cx+3dx^2\right)\,dx=\text{Timed out}$$

```
[In] integrate(x**n*(d*x**2+c*x+b)**n*(3*d*x**2+2*c*x+b),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int x^n\left(b+cx+dx^2\right)^n\left(b+2cx+3dx^2\right)\,dx=\frac{(dx^3+cx^2+bx)e^{\left(n\log\left(dx^2+cx+b\right)+n\log(x)\right)}}{n+1}$$

```
[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="maxima")
[Out] (d*x^3 + c*x^2 + b*x)*e^(n*log(d*x^2 + c*x + b) + n*log(x))/(n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(25) = 50.

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.60

$$\int x^n\left(b+cx+dx^2\right)^n\left(b+2cx+3dx^2\right)\,dx=\frac{(dx^2+cx+b)^n dx^3x^n+(dx^2+cx+b)^n cx^2x^n+(dx^2+cx+b)^n bxx^n}{n+1}$$

```
[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="giac")
[Out] ((d*x^2 + c*x + b)^n*d*x^3*x^n + (d*x^2 + c*x + b)^n*c*x^2*x^n + (d*x^2 + c
*x + b)^n*b*x*x^n)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int x^n \left(b + cx + dx^2\right)^n \left(b + 2cx + 3dx^2\right)\,dx = \left(\frac{c\,x^n\,x^2}{n+1} + \frac{d\,x^n\,x^3}{n+1} + \frac{b\,x\,x^n}{n+1}\right)\left(d\,x^2 + c\,x + b\right)^n$$

```
[In] int(x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x)
[Out] ((c*x^n*x^2)/(n + 1) + (d*x^n*x^3)/(n + 1) + (b*x*x^n)/(n + 1))*(b + c*x +
d*x^2)^n
```

3.183 $\int (b+3dx^2)(a+bx+dx^3)^n \, dx$

Optimal result 1354
Rubi [A] (verified) 1354
Mathematica [A] (verified) 1355
Maple [A] (verified) 1355
Fricas [A] (verification not implemented) 1355
Sympy [F(-1)] 1356
Maxima [A] (verification not implemented) 1356
Giac [A] (verification not implemented) 1356
Mupad [B] (verification not implemented) 1356

Optimal result

Integrand size = 21, antiderivative size = 20

$$\int (b+3dx^2)\left(a+bx+dx^3\right)^n \, dx = \frac{(a+bx+dx^3)^{1+n}}{1+n}$$

```
[Out] (d*x^3+b*x+a)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1602}

$$\int (b+3dx^2)\left(a+bx+dx^3\right)^n \, dx = \frac{(a+bx+dx^3)^{n+1}}{n+1}$$

```
[In] Int[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]
[Out] (a + b*x + d*x^3)^(1 + n)/(1 + n)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(a+bx+dx^3)^{1+n}}{1+n}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (b+3dx^2)(a+bx+dx^3)^n\,dx = \frac{(a+bx+dx^3)^{1+n}}{1+n}$$

```
[In] Integrate[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]
[Out] (a + b*x + d*x^3)^(1 + n)/(1 + n)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{(x^3d+bx+a)^{1+n}}{1+n}$	21
derivativedivides	$\frac{(x^3d+bx+a)^{1+n}}{1+n}$	21
default	$\frac{(x^3d+bx+a)^{1+n}}{1+n}$	21
risch	$\frac{(x^3d+bx+a)(x^3d+bx+a)^n}{1+n}$	29
parallelrisch	$\frac{x^3(x^3d+bx+a)^n d^2+x(x^3d+bx+a)^n bd+(x^3d+bx+a)^n ad}{d(1+n)}$	61
norman	$\frac{a\,e^{n\ln(x^3d+bx+a)}}{1+n}+\frac{bx\,e^{n\ln(x^3d+bx+a)}}{1+n}+\frac{d\,x^3e^{n\ln(x^3d+bx+a)}}{1+n}$	69

```
[In] int((3*d*x^2+b)*(d*x^3+b*x+a)^n,x,method=_RETURNVERBOSE)
[Out] (d*x^3+b*x+a)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int (b+3dx^2)(a+bx+dx^3)^n\,dx = \frac{(dx^3+bx+a)(dx^3+bx+a)^n}{n+1}$$

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="fricas")
[Out] (d*x^3 + b*x + a)*(d*x^3 + b*x + a)^n/(n + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^n\,dx=\text{Timed out}$$

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x+a)**n,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^n\,dx=\frac{\left(dx^3+bx+a\right)^{n+1}}{n+1}$$

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="maxima")
[Out] (d*x^3 + b*x + a)^(n + 1)/(n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^n\,dx=\frac{\left(dx^3+bx+a\right)^{n+1}}{n+1}$$

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="giac")
[Out] (d*x^3 + b*x + a)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^n\,dx=\left(\frac{a}{n+1}+\frac{b\,x}{n+1}+\frac{d\,x^3}{n+1}\right)\left(d\,x^3+b\,x+a\right)^n$$

```
[In] int((b + 3*d*x^2)*(a + b*x + d*x^3)^n,x)
[Out] (a/(n + 1) + (b*x)/(n + 1) + (d*x^3)/(n + 1))*(a + b*x + d*x^3)^n
```

3.184 $\int (b+3dx^2)(bx+dx^3)^n \, dx$

Optimal result 1357
Rubi [A] (verified) 1357
Mathematica [C] (verified) 1358
Maple [A] (verified) 1358
Fricas [A] (verification not implemented) 1359
Sympy [B] (verification not implemented) 1359
Maxima [A] (verification not implemented) 1359
Giac [A] (verification not implemented) 1360
Mupad [B] (verification not implemented) 1360

Optimal result

Integrand size = 20, antiderivative size = 19

$$\int (b+3dx^2)\left(bx+dx^3\right)^n \, dx = \frac{\left(bx+dx^3\right)^{1+n}}{1+n}$$

```
[Out] (d*x^3+b*x)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1602}

$$\int (b+3dx^2)\left(bx+dx^3\right)^n \, dx = \frac{\left(bx+dx^3\right)^{n+1}}{n+1}$$

```
[In] Int[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]
```

```
[Out] (b*x + d*x^3)^(1 + n)/(1 + n)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\left(bx+dx^3\right)^{1+n}}{1+n}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 5.58

$$\int (b+3dx^2)\,(bx+dx^3)^n\,dx$$
$$= \frac{x(x(b+dx^2))^n\left(1+\frac{dx^2}{b}\right)^{-n}\left(b(3+n)\,\text{Hypergeometric2F1}\left(-n,\frac{1+n}{2},\frac{3+n}{2},-\frac{dx^2}{b}\right)+3d(1+n)x^2\,\text{Hypergeo}\right.}{(1+n)(3+n)}$$

```
[In] Integrate[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]
```

```
[Out] (x*(x*(b + d*x^2))^n*(b*(3 + n)*Hypergeometric2F1[-n, (1 + n)/2, (3 + n)/2,
 -((d*x^2)/b)] + 3*d*(1 + n)*x^2*Hypergeometric2F1[-n, (3 + n)/2, (5 + n)/2
, -((d*x^2)/b)]))/((1 + n)*(3 + n)*(1 + (d*x^2)/b)^n)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{(x^3d+bx)^{1+n}}{1+n}$	20
default	$\frac{(x^3d+bx)^{1+n}}{1+n}$	20
gosper	$\frac{x(d\,x^2+b)(x^3d+bx)^n}{1+n}$	26
risch	$\frac{x(d\,x^2+b)(x(d\,x^2+b))^n}{1+n}$	26
parallelrisch	$\frac{x^3(x(d\,x^2+b))^n bd+x(x(d\,x^2+b))^n b^2}{b(1+n)}$	44
norman	$\frac{bx\,\mathrm{e}^{n\ln(x^3d+bx)}}{1+n}+\frac{d\,x^3\mathrm{e}^{n\ln(x^3d+bx)}}{1+n}$	46

```
[In] int((3*d*x^2+b)*(d*x^3+b*x)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (d*x^3+b*x)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \left(b+3dx^2\right) \left(bx+dx^3\right)^n \, dx = \frac{(dx^3+bx)(dx^3+bx)^n}{n+1}$$

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="fricas")
[Out] (d*x^3 + b*x)*(d*x^3 + b*x)^n/(n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(14) = 28.

Time = 5.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.32

$$\int \left(b+3dx^2\right) \left(bx+dx^3\right)^n \, dx = \begin{cases} \frac{bx(bx+dx^3)^n}{n+1} + \frac{dx^3(bx+dx^3)^n}{n+1} & \text{for } n \neq -1 \\ \log(x) + \log\left(x - \sqrt{-\frac{b}{d}}\right) + \log\left(x + \sqrt{-\frac{b}{d}}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x)**n,x)
[Out] Piecewise((b*x*(b*x + d*x**3)**n/(n + 1) + d*x**3*(b*x + d*x**3)**n/(n + 1)
, Ne(n, -1)), (log(x) + log(x - sqrt(-b/d)) + log(x + sqrt(-b/d)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \left(b+3dx^2\right) \left(bx+dx^3\right)^n \, dx = \frac{(dx^3+bx)^{n+1}}{n+1}$$

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="maxima")
[Out] (d*x^3 + b*x)^(n + 1)/(n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (b+3dx^2)(bx+dx^3)^n \, dx = \frac{(dx^3+bx)^{n+1}}{n+1}$$

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="giac")
[Out] (d*x^3 + b*x)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int (b+3dx^2)(bx+dx^3)^n \, dx = \frac{x\,(d\,x^3+b\,x)^n\,(d\,x^2+b)}{n+1}$$

```
[In] int((b*x + d*x^3)^n*(b + 3*d*x^2),x)
[Out] (x*(b*x + d*x^3)^n*(b + d*x^2))/(n + 1)
```

3.185 $\int x^n(b+dx^2)^n(b+3dx^2)\,dx$

Optimal result . . . 1361
Rubi [A] (verified) . . . 1361
Mathematica [C] (verified) . . . 1362
Maple [A] (verified) . . . 1362
Fricas [A] (verification not implemented) . . . 1362
Sympy [B] (verification not implemented) . . . 1363
Maxima [A] (verification not implemented) . . . 1363
Giac [A] (verification not implemented) . . . 1363
Mupad [B] (verification not implemented) . . . 1364

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int x^n\left(b+dx^2\right)^n\left(b+3dx^2\right)\,dx=\frac{x^{1+n}(b+dx^2)^{1+n}}{1+n}$$

```
[Out] x^(1+n)*(d*x^2+b)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {460}

$$\int x^n\left(b+dx^2\right)^n\left(b+3dx^2\right)\,dx=\frac{x^{n+1}(b+dx^2)^{n+1}}{n+1}$$

```
[In] Int[x^n*(b + d*x^2)^n*(b + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(b + d*x^2)^(1 + n))/(1 + n)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{x^{1+n}(b+dx^2)^{1+n}}{1+n}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.91

$$\int x^n (b+dx^2)^n (b+3dx^2)\, dx = \frac{x^{1+n}(b+dx^2)^n \left(1+\frac{dx^2}{b}\right)^{-n} \left(b(3+n)\,\text{Hypergeometric2F1}\left(-n,\frac{1+n}{2},\frac{3+n}{2},-\frac{dx^2}{b}\right)+3d(1+n)x^2\,\text{Hypergeo}\right.}{(1+n)(3+n)}$$

```
[In] Integrate[x^n*(b + d*x^2)^n*(b + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(b + d*x^2)^n*(b*(3 + n)*Hypergeometric2F1[-n, (1 + n)/2, (3 + n
)/2, -((d*x^2)/b)] + 3*d*(1 + n)*x^2*Hypergeometric2F1[-n, (3 + n)/2, (5 +
n)/2, -((d*x^2)/b)]))/((1 + n)*(3 + n)*(1 + (d*x^2)/b)^n)
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{x^{1+n} \left(d\,x^2+b\right)^{1+n}}{1+n}$	23
risch	$\frac{x \left(d\,x^2+b\right) x^n \left(d\,x^2+b\right)^n}{1+n}$	27
parallelrisch	$\frac{x^3 x^n \left(d\,x^2+b\right)^n b d+x\,x^n \left(d\,x^2+b\right)^n b^2}{b(1+n)}$	46

```
[In] int(x^n*(d*x^2+b)^n*(3*d*x^2+b),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1+n)*(d*x^2+b)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int x^n (b+dx^2)^n (b+3dx^2)\, dx = \frac{(dx^3+bx)(dx^2+b)^n x^n}{n+1}$$

```
[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="fricas")
```

```
[Out] (d*x^3 + b*x)*(d*x^2 + b)^n*x^n/(n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(17) = 34$.

Time = 23.67 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.00

$$\int x^n \left(b+dx^2\right)^n \left(b+3dx^2\right)\,dx = \begin{cases} \frac{bxx^n\left(b+dx^2\right)^n}{n+1} + \frac{dx^3x^n\left(b+dx^2\right)^n}{n+1} & \text{for } n \neq -1 \\ \log\left(x\right) + \log\left(x - \sqrt{-\frac{b}{d}}\right) + \log\left(x + \sqrt{-\frac{b}{d}}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(x**n*(d*x**2+b)**n*(3*d*x**2+b),x)
[Out] Piecewise((b*x*x**n*(b + d*x**2)**n/(n + 1) + d*x**3*x**n*(b + d*x**2)**n/(
n + 1), Ne(n, -1)), (log(x) + log(x - sqrt(-b/d)) + log(x + sqrt(-b/d)), Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int x^n \left(b+dx^2\right)^n \left(b+3dx^2\right)\,dx = \frac{(dx^3+bx)e^{\left(n\log\left(dx^2+b\right)+n\log\left(x\right)\right)}}{n+1}$$

```
[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="maxima")
[Out] (d*x^3 + b*x)*e^(n*log(d*x^2 + b) + n*log(x))/(n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int x^n \left(b+dx^2\right)^n \left(b+3dx^2\right)\,dx = \frac{\left(dx^2+b\right)^n dx^3x^n + \left(dx^2+b\right)^n bxx^n}{n+1}$$

```
[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="giac")
[Out] ((d*x^2 + b)^n*d*x^3*x^n + (d*x^2 + b)^n*b*x*x^n)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int x^n \left(b+dx^2\right)^n \left(b+3dx^2\right) \, dx = \frac{x \, x^n \, (d \, x^2+b)^n \, (d \, x^2+b)}{n+1}$$

```
[In] int(x^n*(b + d*x^2)^n*(b + 3*d*x^2),x)
[Out] (x*x^n*(b + d*x^2)^n*(b + d*x^2))/(n + 1)
```

3.186 $\int (2cx+3dx^2)(a+cx^2+dx^3)^n\,dx$

Optimal result . . . 1365
Rubi [A] (verified) . . . 1365
Mathematica [A] (verified) . . . 1366
Maple [A] (verified) . . . 1366
Fricas [A] (verification not implemented) . . . 1366
Sympy [F(-1)] . . . 1367
Maxima [A] (verification not implemented) . . . 1367
Giac [A] (verification not implemented) . . . 1367
Mupad [B] (verification not implemented) . . . 1367

Optimal result

Integrand size = 26, antiderivative size = 22

$$\int (2cx+3dx^2)(a+cx^2+dx^3)^n\,dx=\frac{(a+cx^2+dx^3)^{1+n}}{1+n}$$

```
[Out] (d*x^3+c*x^2+a)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1602}

$$\int (2cx+3dx^2)(a+cx^2+dx^3)^n\,dx=\frac{(a+cx^2+dx^3)^{n+1}}{n+1}$$

```
[In] Int[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{(a+cx^2+dx^3)^{1+n}}{1+n}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(a+x^2(c+dx)\right)^{1+n}}{1+n}$$

```
[In] Integrate[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]
[Out] (a + x^2*(c + d*x))^(1 + n)/(1 + n)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{\left(x^3d+c\,x^2+a\right)^{1+n}}{1+n}$	23
derivativedivides	$\frac{\left(x^3d+c\,x^2+a\right)^{1+n}}{1+n}$	23
default	$\frac{\left(x^3d+c\,x^2+a\right)^{1+n}}{1+n}$	23
risch	$\frac{\left(x^3d+c\,x^2+a\right)^{n}\left(x^3d+c\,x^2+a\right)}{1+n}$	33
parallelrisch	$\frac{x^3\left(x^3d+c\,x^2+a\right)^{n}d^2+x^2\left(x^3d+c\,x^2+a\right)^{n}cd+\left(x^3d+c\,x^2+a\right)^{n}ad}{d(1+n)}$	69
norman	$\frac{a\,\mathrm{e}^{n\ln\left(x^3d+c\,x^2+a\right)}}{1+n}+\frac{c\,x^2\mathrm{e}^{n\ln\left(x^3d+c\,x^2+a\right)}}{1+n}+\frac{d\,x^3\mathrm{e}^{n\ln\left(x^3d+c\,x^2+a\right)}}{1+n}$	77

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x,method=_RETURNVERBOSE)
[Out] (d*x^3+c*x^2+a)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx=\frac{(dx^3+cx^2+a)(dx^3+cx^2+a)^n}{n+1}$$

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="fricas")
[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx=\text{Timed out}$$

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**n,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(dx^3+cx^2+a\right)^{n+1}}{n+1}$$

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="maxima")
[Out] (d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(dx^3+cx^2+a\right)^{n+1}}{n+1}$$

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="giac")
[Out] (d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.95

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx=\left(\frac{a}{n+1}+\frac{c\,x^2}{n+1}+\frac{d\,x^3}{n+1}\right)\left(d\,x^3+c\,x^2+a\right)^n$$

```
[In] int((2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x)
[Out] (a/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(a + c*x^2 + d*x^3)^n
```

3.187 $\int (2cx+3dx^2) (cx^2+dx^3)^n \, dx$

Optimal result 1368
Rubi [A] (verified) 1368
Mathematica [A] (verified) 1369
Maple [A] (verified) 1369
Fricas [A] (verification not implemented) 1369
Sympy [B] (verification not implemented) 1370
Maxima [A] (verification not implemented) 1370
Giac [A] (verification not implemented) 1370
Mupad [B] (verification not implemented) 1371

Optimal result

Integrand size = 25, antiderivative size = 21

$$\int (2cx+3dx^2) (cx^2+dx^3)^n \, dx = \frac{(cx^2+dx^3)^{1+n}}{1+n}$$

```
[Out] (d*x^3+c*x^2)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{1602\}$

$$\int (2cx+3dx^2) (cx^2+dx^3)^n \, dx = \frac{(cx^2+dx^3)^{n+1}}{n+1}$$

```
[In] Int[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]
```

```
[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(cx^2+dx^3)^{1+n}}{1+n}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int (2cx+3dx^2)\left(cx^2+dx^3\right)^n\,dx=\frac{(x^2(c+dx))^{1+n}}{1+n}$$

```
[In] Integrate[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]
[Out] (x^2*(c + d*x))^(1 + n)/(1 + n)
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{\left(x^3d+c\,x^2\right)^{1+n}}{1+n}$	22
default	$\frac{\left(x^3d+c\,x^2\right)^{1+n}}{1+n}$	22
risch	$\frac{x^2(dx+c)\left(x^2(dx+c)\right)^n}{1+n}$	26
gosper	$\frac{\left(x^3d+c\,x^2\right)^n x^2(dx+c)}{1+n}$	28
parallelrisch	$\frac{x^3\left(x^2(dx+c)\right)^n cd+x^2\left(x^2(dx+c)\right)^n c^2}{c(1+n)}$	46
norman	$\frac{c\,x^2\mathrm{e}^{n\ln\left(x^3d+c\,x^2\right)}}{1+n}+\frac{d\,x^3\mathrm{e}^{n\ln\left(x^3d+c\,x^2\right)}}{1+n}$	52

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x,method=_RETURNVERBOSE)
[Out] (d*x^3+c*x^2)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int (2cx+3dx^2)\left(cx^2+dx^3\right)^n\,dx=\frac{(dx^3+cx^2)(dx^3+cx^2)^n}{n+1}$$

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="fricas")
[Out] (d*x^3 + c*x^2)*(d*x^3 + c*x^2)^n/(n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.

Time = 0.43 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.52

$$\int \left(2cx + 3dx^2\right) \left(cx^2 + dx^3\right)^n \, dx = \begin{cases} \frac{cx^2 (cx^2+dx^3)^n}{n+1} + \frac{dx^3 (cx^2+dx^3)^n}{n+1} & \text{for } n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**n,x)
[Out] Piecewise((c*x**2*(c*x**2 + d*x**3)**n/(n + 1) + d*x**3*(c*x**2 + d*x**3)**
n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(2cx + 3dx^2\right) \left(cx^2 + dx^3\right)^n \, dx = \frac{\left(dx^3 + cx^2\right)^{n+1}}{n+1}$$

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="maxima")
[Out] (d*x^3 + c*x^2)^(n + 1)/(n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(2cx + 3dx^2\right) \left(cx^2 + dx^3\right)^n \, dx = \frac{\left(dx^3 + cx^2\right)^{n+1}}{n+1}$$

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="giac")
[Out] (d*x^3 + c*x^2)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int (2cx + 3dx^2) (cx^2 + dx^3)^n \, dx = \frac{x^2 (d\,x^3 + c\,x^2)^n (c + d\,x)}{n + 1}$$

```
[In] int((2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x)
[Out] (x^2*(c*x^2 + d*x^3)^n*(c + d*x))/(n + 1)
```

3.188 $\int x^n(cx+dx^2)^n (2cx+3dx^2) \, dx$

Optimal result 1372
Rubi [A] (verified) 1372
Mathematica [A] (verified) 1373
Maple [A] (verified) 1373
Fricas [A] (verification not implemented) 1374
Sympy [B] (verification not implemented) 1374
Maxima [A] (verification not implemented) 1374
Giac [B] (verification not implemented) 1375
Mupad [B] (verification not implemented) 1375

Optimal result

Integrand size = 26, antiderivative size = 24

$$\int x^n (cx+dx^2)^n (2cx+3dx^2) \, dx = \frac{x^{1+n}(cx+dx^2)^{1+n}}{1+n}$$

```
[Out] x^(1+n)*(d*x^2+c*x)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1598, 777}

$$\int x^n (cx+dx^2)^n (2cx+3dx^2) \, dx = \frac{x^{n+1}(cx+dx^2)^{n+1}}{n+1}$$

```
[In] Int[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(c*x + d*x^2)^(1 + n))/(1 + n)
```

Rule 777

```
Int[((e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.
), x_Symbol] :> Simp[g*(e*x)^m*((b*x + c*x^2)^(p + 1)/(c*(m + 2*p + 2))), x
] /; FreeQ[{b, c, e, f, g, m, p}, x] && EqQ[b*g*(m + p + 1) - c*f*(m + 2*p
+ 2), 0] && NeQ[m + 2*p + 2, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int x^{1+n}(2c+3dx)\left(cx+dx^2\right)^n\,dx$$
$$= \frac{x^{1+n}\left(cx+dx^2\right)^{1+n}}{1+n}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^n\left(cx+dx^2\right)^n\left(2cx+3dx^2\right)\,dx = \frac{x^{1+n}(x(c+dx))^{1+n}}{1+n}$$

```
[In] Integrate[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x]
[Out] (x^(1 + n)*(x*(c + d*x))^(1 + n))/(1 + n)
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

method	result	size
gosper	$\frac{x^{2+n}(dx+c)\left(d\,x^2+cx\right)^n}{1+n}$	28
parallelrisch	$\frac{x^3x^n((dx+c)x)^n cd+x^2x^n((dx+c)x)^n c^2}{c(1+n)}$	48
risch	$\frac{(dx+c)x^2x^{2n}(dx+c)^n\mathrm{e}^{-\frac{i\,\mathrm{csgn}(ix(dx+c))\pi n(-\,\mathrm{csgn}(ix(dx+c))+\mathrm{csgn}(i(dx+c)))(-\,\mathrm{csgn}(ix(dx+c))+\mathrm{csgn}(ix))}{2}}}{1+n}$	83

```
[In] int(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x,method=_RETURNVERBOSE)
[Out] x^(2+n)/(1+n)*(d*x+c)*(d*x^2+c*x)^n
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int x^n (cx + dx^2)^n (2cx + 3dx^2) \, dx = \frac{(dx^3 + cx^2)(dx^2 + cx)^n x^n}{n + 1}$$

```
[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="fricas")
[Out] (d*x^3 + c*x^2)*(d*x^2 + c*x)^n*x^n/(n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(19) = 38.

Time = 1.55 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int x^n (cx + dx^2)^n (2cx + 3dx^2) \, dx = \begin{cases} \frac{cx^2 x^n (cx+dx^2)^n}{n+1} + \frac{dx^3 x^n (cx+dx^2)^n}{n+1} & \text{for } n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(x**n*(d*x**2+c*x)**n*(3*d*x**2+2*c*x),x)
[Out] Piecewise((c*x**2*x**n*(c*x + d*x**2)**n/(n + 1) + d*x**3*x**n*(c*x + d*x**
2)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int x^n (cx + dx^2)^n (2cx + 3dx^2) \, dx = \frac{(dx^3 + cx^2)e^{(n\log(dx+c) + 2n\log(x))}}{n + 1}$$

```
[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="maxima")
[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(24) = 48.

Time = 0.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int x^n \left(cx+dx^2\right)^n \left(2cx+3dx^2\right)\,dx=\frac{dx^3x^ne^{(n\log(dx+c)+n\log(x))}+cx^2x^ne^{(n\log(dx+c)+n\log(x))}}{n+1}$$

```
[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="giac")
```

```
[Out] (d*x^3*x^n*e^(n*log(d*x + c) + n*log(x)) + c*x^2*x^n*e^(n*log(d*x + c) + n*
log(x)))/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int x^n \left(cx+dx^2\right)^n \left(2cx+3dx^2\right)\,dx=\frac{x^n\,x^2\,{\left(d\,x^2+c\,x\right)}^n\,\left(c+d\,x\right)}{n+1}$$

```
[In] int(x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x)
```

```
[Out] (x^n*x^2*(c*x + d*x^2)^n*(c + d*x))/(n + 1)
```

3.189 $\int x^{2n}(c+dx)^n \left(2cx+3dx^2\right) \, dx$

Optimal result 1376
Rubi [A] (verified) 1376
Mathematica [A] (verified) 1377
Maple [A] (verified) 1377
Fricas [A] (verification not implemented) 1377
Sympy [B] (verification not implemented) 1378
Maxima [A] (verification not implemented) 1378
Giac [A] (verification not implemented) 1378
Mupad [B] (verification not implemented) 1379

Optimal result

Integrand size = 24, antiderivative size = 22

$$\int x^{2n}(c+dx)^n \left(2cx+3dx^2\right) \, dx = \frac{x^{2(1+n)}(c+dx)^{1+n}}{1+n}$$

```
[Out] x^(2+2*n)*(d*x+c)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {859}

$$\int x^{2n}(c+dx)^n \left(2cx+3dx^2\right) \, dx = \frac{x^{2(n+1)}(c+dx)^{n+1}}{n+1}$$

```
[In] Int[x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(2*(1 + n))*(c + d*x)^(1 + n))/(1 + n)
```

Rule 859

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((b_.)*(x_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[c*x^(m + 2)*((f + g*x)^(n + 1)/(g*(m + n + 3))), x] /; FreeQ[{
b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n
 + 3, 0]
```

Rubi steps

$$\text{integral} = \frac{x^{2(1+n)}(c+dx)^{1+n}}{1+n}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^{2n}(c+dx)^n\left(2cx+3dx^2\right)\,dx=\frac{x^{2+2n}(c+dx)^{1+n}}{1+n}$$

```
[In] Integrate[x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2),x]
[Out] (x^(2 + 2*n)*(c + d*x)^(1 + n))/(1 + n)
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{x^{2+2n}(dx+c)^{1+n}}{1+n}$	23
risch	$\frac{(dx+c)^n x^{2n} x^2 (dx+c)}{1+n}$	27
parallelrisch	$\frac{x^3 x^{2n} (dx+c)^n cd+x^2 x^{2n} (dx+c)^n c^2}{c(1+n)}$	48

```
[In] int(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x,method=_RETURNVERBOSE)
[Out] x^(2+2*n)*(d*x+c)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int x^{2n}(c+dx)^n\left(2cx+3dx^2\right)\,dx=\frac{(dx^3+cx^2)(dx+c)^n x^{2n}}{n+1}$$

```
[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="fricas")
[Out] (d*x^3 + c*x^2)*(d*x + c)^n*x^(2*n)/(n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(17) = 34$.

Time = 1.53 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int x^{2n}(c+dx)^n \left(2cx+3dx^2\right) dx = \begin{cases} \frac{cx^2x^{2n}(c+dx)^n}{n+1} + \frac{dx^3x^{2n}(c+dx)^n}{n+1} & \text{for } n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d}+x\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(2*n)*(d*x+c)**n*(3*d*x**2+2*c*x),x)
[Out] Piecewise((c*x**2*x**(2*n)*(c + d*x)**n/(n + 1) + d*x**3*x**(2*n)*(c + d*x)
**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int x^{2n}(c+dx)^n \left(2cx+3dx^2\right) dx = \frac{(dx^3+cx^2)e^{(n\log(dx+c)+2n\log(x))}}{n+1}$$

```
[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="maxima")
[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int x^{2n}(c+dx)^n \left(2cx+3dx^2\right) dx = \frac{(dx+c)^n dx^3x^{2n} + (dx+c)^n cx^2x^{2n}}{n+1}$$

```
[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="giac")
[Out] ((d*x + c)^n*d*x^3*x^(2*n) + (d*x + c)^n*c*x^2*x^(2*n))/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int x^{2n}(c+dx)^n\left(2cx+3dx^2\right)\,dx=\frac{x^{2\,n}\,x^2\,{\left(c+d\,x\right)}^n\,\left(c+d\,x\right)}{n+1}$$

```
[In] int(x^(2*n)*(2*c*x + 3*d*x^2)*(c + d*x)^n,x)
[Out] (x^(2*n)*x^2*(c + d*x)^n*(c + d*x))/(n + 1)
```

3.190 $\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx$

Optimal result 1380
Rubi [A] (verified) 1380
Mathematica [A] (verified) 1381
Maple [A] (verified) 1381
Fricas [A] (verification not implemented) 1381
Sympy [F(-1)] 1382
Maxima [A] (verification not implemented) 1382
Giac [A] (verification not implemented) 1382
Mupad [B] (verification not implemented) 1382

Optimal result

Integrand size = 24, antiderivative size = 22

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(a+cx^2+dx^3\right)^{1+n}}{1+n}$$

```
[Out] (d*x^3+c*x^2+a)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1602}

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

```
[In] Int[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{\left(a+cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(a+x^2(c+dx)\right)^{1+n}}{1+n}$$

```
[In] Integrate[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]
[Out] (a + x^2*(c + d*x))^(1 + n)/(1 + n)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{\left(x^3d+c\,x^2+a\right)^{1+n}}{1+n}$	23
risch	$\frac{\left(x^3d+c\,x^2+a\right)^n\left(x^3d+c\,x^2+a\right)}{1+n}$	33
parallelrisch	$\frac{x^3\left(x^3d+c\,x^2+a\right)^n d^2+x^2\left(x^3d+c\,x^2+a\right)^n cd+\left(x^3d+c\,x^2+a\right)^n ad}{d(1+n)}$	69
norman	$\frac{a\,\mathrm{e}^{n\ln\left(x^3d+c\,x^2+a\right)}}{1+n}+\frac{c\,x^2\mathrm{e}^{n\ln\left(x^3d+c\,x^2+a\right)}}{1+n}+\frac{d\,x^3\mathrm{e}^{n\ln\left(x^3d+c\,x^2+a\right)}}{1+n}$	77

```
[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x,method=_RETURNVERBOSE)
[Out] (d*x^3+c*x^2+a)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\frac{(dx^3+cx^2+a)(dx^3+cx^2+a)^n}{n+1}$$

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="fricas")
[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

Sympy [F(-1)]

Timed out.

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\text{Timed out}$$

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**n,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\frac{(dx^3+cx^2+a)(dx^3+cx^2+a)^n}{n+1}$$

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="maxima")
[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\frac{(dx^3+cx^2+a)^{n+1}}{n+1}$$

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="giac")
[Out] (d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.95

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\left(\frac{a}{n+1}+\frac{c\,x^2}{n+1}+\frac{d\,x^3}{n+1}\right)\left(d\,x^3+c\,x^2+a\right)^n$$

```
[In] int(x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x)
[Out] (a/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(a + c*x^2 + d*x^3)^n
```

3.191 $\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx$

Optimal result 1383
Rubi [A] (verified) 1383
Mathematica [A] (verified) 1384
Maple [A] (verified) 1384
Fricas [A] (verification not implemented) 1384
Sympy [B] (verification not implemented) 1385
Maxima [A] (verification not implemented) 1385
Giac [A] (verification not implemented) 1385
Mupad [B] (verification not implemented) 1386

Optimal result

Integrand size = 23, antiderivative size = 21

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\frac{\left(cx^2+dx^3\right)^{1+n}}{1+n}$$

```
[Out] (d*x^3+c*x^2)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1602}

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

```
[In] Int[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]
[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{\left(cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\frac{\left(x^2(c+dx)\right)^{1+n}}{1+n}$$

```
[In] Integrate[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]
[Out] (x^2*(c + d*x))^(1 + n)/(1 + n)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

method	result	size
risch	$\frac{x^2(dx+c)\left(x^2(dx+c)\right)^n}{1+n}$	26
gosper	$\frac{\left(x^3d+c\,x^2\right)^n x^2(dx+c)}{1+n}$	28
parallelrisch	$\frac{x^3\left(x^2(dx+c)\right)^n cd+x^2\left(x^2(dx+c)\right)^n c^2}{c(1+n)}$	46
norman	$\frac{c\,x^2\mathrm{e}^{n\ln\left(x^3d+c\,x^2\right)}}{1+n}+\frac{d\,x^3\mathrm{e}^{n\ln\left(x^3d+c\,x^2\right)}}{1+n}$	52

```
[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x,method=_RETURNVERBOSE)
[Out] x^2*(d*x+c)/(1+n)*(x^2*(d*x+c))^n
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\frac{(dx^3+cx^2)(dx^3+cx^2)^n}{n+1}$$

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="fricas")
[Out] (d*x^3 + c*x^2)*(d*x^3 + c*x^2)^n/(n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.

Time = 0.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.52

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\begin{cases}\frac{cx^2(cx^2+dx^3)^n}{n+1}+\frac{dx^3(cx^2+dx^3)^n}{n+1} & \text{for } n\neq -1\\ 2\log\left(x\right)+\log\left(\frac{c}{d}+x\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**n,x)
[Out] Piecewise((c*x**2*(c*x**2 + d*x**3)**n/(n + 1) + d*x**3*(c*x**2 + d*x**3)**
n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.52

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\frac{(dx^3+cx^2)e^{(n\log(dx+c)+2\,n\log(x))}}{n+1}$$

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="maxima")
[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\frac{\left(dx^3+cx^2\right)^{n+1}}{n+1}$$

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="giac")
[Out] (d*x^3 + c*x^2)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\frac{x^2\,{\left(d\,x^3+c\,x^2\right)}^n\,\left(c+d\,x\right)}{n+1}$$

```
[In] int(x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x)
[Out] (x^2*(c*x^2 + d*x^3)^n*(c + d*x))/(n + 1)
```

3.192 $\int (b+2cx+3dx^2)(a+bx+cx^2+dx^3)^7\,dx$

Optimal result 1387
Rubi [A] (verified) 1387
Mathematica [B] (verified) 1388
Maple [A] (verified) 1388
Fricas [B] (verification not implemented) 1389
Sympy [B] (verification not implemented) 1390
Maxima [A] (verification not implemented) 1391
Giac [B] (verification not implemented) 1391
Mupad [B] (verification not implemented) 1392

Optimal result

Integrand size = 30, antiderivative size = 21

$$\int (b+2cx+3dx^2)(a+bx+cx^2+dx^3)^7\,dx=\frac{1}{8}(a+bx+cx^2+dx^3)^8$$

```
[Out] 1/8*(d*x^3+c*x^2+b*x+a)^8
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1602}

$$\int (b+2cx+3dx^2)(a+bx+cx^2+dx^3)^7\,dx=\frac{1}{8}(a+bx+cx^2+dx^3)^8$$

```
[In] Int[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^8/8
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{8}(a+bx+cx^2+dx^3)^8$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 143 vs. 2(21) = 42.

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 6.81

$$\int (b+2cx+3dx^2)\left(a+bx+cx^2+dx^3\right)^7\,dx$$
$$=\frac{1}{8}x(b+x(c+dx))\left(8a^7+28a^6x(b+x(c+dx))+56a^5x^2(b+x(c+dx))^2+70a^4x^3(b+x(c+dx))^3+56a^3x^4(b+x(c+dx))^4+28a^2x^5(b+x(c+dx))^5+8ax^6(b+x(c+dx))^6+x^7(b+x(c+dx))^7\right)$$

```
[In] Integrate[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (x*(b + x*(c + d*x))*(8*a^7 + 28*a^6*x*(b + x*(c + d*x)) + 56*a^5*x^2*(b +
x*(c + d*x))^2 + 70*a^4*x^3*(b + x*(c + d*x))^3 + 56*a^3*x^4*(b + x*(c + d*
x))^4 + 28*a^2*x^5*(b + x*(c + d*x))^5 + 8*a*x^6*(b + x*(c + d*x))^6 + x^7*
(b + x*(c + d*x))^7))/8
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\left(x^3d+c\,x^2+bx+a\right)^8}{8}$	20
norman	Expression too large to display	1579
gosper	Expression too large to display	1957
parallelrisch	Expression too large to display	1957
risch	Expression too large to display	1962

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(d*x^3+c*x^2+b*x+a)^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1528 vs. $2(19) = 38$.

Time = 0.26 (sec) , antiderivative size = 1528, normalized size of antiderivative = 72.76

$$\int (b + 2cx + 3dx^2) (a + bx + cx^2 + dx^3)^7 \, dx = \text{Too large to display}$$

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="fricas")
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 1/2*(7*c^2*d^6 + 2*b*d^7)*x^22 + (7*c^3*d^5 + 7
*b*c*d^6 + a*d^7)*x^21 + 7/4*(5*c^4*d^4 + 12*b*c^2*d^5 + 2*(b^2 + 2*a*c)*d^
6)*x^20 + 7*(c^5*d^3 + 5*b*c^3*d^4 + a*b*d^6 + 3*(b^2*c + a*c^2)*d^5)*x^19
+ 7/2*(c^6*d^2 + 10*b*c^4*d^3 + a^2*d^6 + 2*(b^3 + 6*a*b*c)*d^5 + 5*(3*b^2*
c^2 + 2*a*c^3)*d^4)*x^18 + (c^7*d + 21*b*c^5*d^2 + 21*(a*b^2 + a^2*c)*d^5 +
 35*(b^3*c + 3*a*b*c^2)*d^4 + 35*(2*b^2*c^3 + a*c^4)*d^3)*x^17 + 1/8*(c^8 +
 56*b*c^6*d + 168*a^2*b*d^5 + 70*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4 + 560*(
b^3*c^2 + 2*a*b*c^3)*d^3 + 84*(5*b^2*c^4 + 2*a*c^5)*d^2)*x^16 + (b*c^7 + 7*
a^3*d^5 + 35*(a*b^3 + 3*a^2*b*c)*d^4 + 35*(b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)
*d^3 + 35*(2*b^3*c^3 + 3*a*b*c^4)*d^2 + 7*(3*b^2*c^5 + a*c^6)*d)*x^15 + 1/2
*(7*b^2*c^6 + 2*a*c^7 + 35*(3*a^2*b^2 + 2*a^3*c)*d^4 + 14*(b^5 + 20*a*b^3*c
 + 30*a^2*b*c^2)*d^3 + 105*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^2 + 14*(5*b^
3*c^4 + 6*a*b*c^5)*d)*x^14 + 7*(b^3*c^5 + a*b*c^6 + 5*a^3*b*d^4 + 5*(a*b^4
+ 6*a^2*b^2*c + 2*a^3*c^2)*d^3 + 3*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^
2 + (5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*d)*x^13 + 7/4*(5*b^4*c^4 + 12*a*
b^2*c^5 + 2*a^2*c^6 + 5*a^4*d^4 + 40*(a^2*b^3 + 2*a^3*b*c)*d^3 + 2*(b^6 + 3
0*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d^2 + 4*(3*b^5*c^2 + 20*a*b^3*c^3
+ 15*a^2*b*c^4)*d)*x^12 + 7*(b^5*c^3 + 5*a*b^3*c^4 + 3*a^2*b*c^5 + 5*(2*a^3
*b^2 + a^4*c)*d^3 + 3*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d^2 + (b^6*c +
15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3*c^4)*d)*x^11 + 1/2*(7*b^6*c^2 + 70*a*
b^4*c^3 + 105*a^2*b^2*c^4 + 14*a^3*c^5 + 70*a^4*b*d^3 + 105*(a^2*b^4 + 4*a^
3*b^2*c + a^4*c^2)*d^2 + 2*(b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b*
c^3)*d)*x^10 + (b^7*c + 21*a*b^5*c^2 + 70*a^2*b^3*c^3 + 35*a^3*b*c^4 + 7*a^
5*d^3 + 35*(2*a^3*b^3 + 3*a^4*b*c)*d^2 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b
^2*c^2 + 5*a^4*c^3)*d)*x^9 + a^7*b*x + 1/8*(b^8 + 56*a*b^6*c + 420*a^2*b^4*
c^2 + 560*a^3*b^2*c^3 + 70*a^4*c^4 + 84*(5*a^4*b^2 + 2*a^5*c)*d^2 + 56*(3*a
^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*d)*x^8 + (a*b^7 + 21*a^2*b^5*c + 70*a
^3*b^3*c^2 + 35*a^4*b*c^3 + 21*a^5*b*d^2 + 7*(5*a^3*b^4 + 15*a^4*b^2*c + 3*
a^5*c^2)*d)*x^7 + 7/2*(a^2*b^6 + 10*a^3*b^4*c + 15*a^4*b^2*c^2 + 2*a^5*c^3
+ a^6*d^2 + 2*(5*a^4*b^3 + 6*a^5*b*c)*d)*x^6 + 7*(a^3*b^5 + 5*a^4*b^3*c + 3
*a^5*b*c^2 + (3*a^5*b^2 + a^6*c)*d)*x^5 + 7/4*(5*a^4*b^4 + 12*a^5*b^2*c + 2
*a^6*c^2 + 4*a^6*b*d)*x^4 + (7*a^5*b^3 + 7*a^6*b*c + a^7*d)*x^3 + 1/2*(7*a^
6*b^2 + 2*a^7*c)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1771 vs. $2(17) = 34$.

Time = 0.18 (sec) , antiderivative size = 1771, normalized size of antiderivative = 84.33

$$\int (b + 2cx + 3dx^2) (a + bx + cx^2 + dx^3)^7 \, dx = \text{Too large to display}$$

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**7,x)
```

```
[Out] a**7*b*x + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7*c**2*d**6/2) + x
**21*(a*d**7 + 7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 7*b**2*d**6/
2 + 21*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(7*a*b*d**6 + 21*a*c**2*d**5 +
 21*b**2*c*d**5 + 35*b*c**3*d**4 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 42
*a*b*c*d**5 + 35*a*c**3*d**4 + 7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c*
*4*d**3 + 7*c**6*d**2/2) + x**17*(21*a**2*c*d**5 + 21*a*b**2*d**5 + 105*a*b
*c**2*d**4 + 35*a*c**4*d**3 + 35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**
5*d**2 + c**7*d) + x**16*(21*a**2*b*d**5 + 105*a**2*c**2*d**4/2 + 105*a*b**
2*c*d**4 + 140*a*b*c**3*d**3 + 21*a*c**5*d**2 + 35*b**4*d**4/4 + 70*b**3*c*
*2*d**3 + 105*b**2*c**4*d**2/2 + 7*b*c**6*d + c**8/8) + x**15*(7*a**3*d**5
+ 105*a**2*b*c*d**4 + 70*a**2*c**3*d**3 + 35*a*b**3*d**4 + 210*a*b**2*c**2*
d**3 + 105*a*b*c**4*d**2 + 7*a*c**6*d + 35*b**4*c*d**3 + 70*b**3*c**3*d**2
+ 21*b**2*c**5*d + b*c**7) + x**14*(35*a**3*c*d**4 + 105*a**2*b**2*d**4/2 +
 210*a**2*b*c**2*d**3 + 105*a**2*c**4*d**2/2 + 140*a*b**3*c*d**3 + 210*a*b*
*2*c**3*d**2 + 42*a*b*c**5*d + a*c**7 + 7*b**5*d**3 + 105*b**4*c**2*d**2/2
+ 35*b**3*c**4*d + 7*b**2*c**6/2) + x**13*(35*a**3*b*d**4 + 70*a**3*c**2*d*
*3 + 210*a**2*b**2*c*d**3 + 210*a**2*b*c**3*d**2 + 21*a**2*c**5*d + 35*a*b*
*4*d**3 + 210*a*b**3*c**2*d**2 + 105*a*b**2*c**4*d + 7*a*b*c**6 + 21*b**5*c
*d**2 + 35*b**4*c**3*d + 7*b**3*c**5) + x**12*(35*a**4*d**4/4 + 140*a**3*b*
c*d**3 + 70*a**3*c**3*d**2 + 70*a**2*b**3*d**3 + 315*a**2*b**2*c**2*d**2 +
105*a**2*b*c**4*d + 7*a**2*c**6/2 + 105*a*b**4*c*d**2 + 140*a*b**3*c**3*d +
 21*a*b**2*c**5 + 7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4) + x**11*
(35*a**4*c*d**3 + 70*a**3*b**2*d**3 + 210*a**3*b*c**2*d**2 + 35*a**3*c**4*d
 + 210*a**2*b**3*c*d**2 + 210*a**2*b**2*c**3*d + 21*a**2*b*c**5 + 21*a*b**5
*d**2 + 105*a*b**4*c**2*d + 35*a*b**3*c**4 + 7*b**6*c*d + 7*b**5*c**3) + x*
*10*(35*a**4*b*d**3 + 105*a**4*c**2*d**2/2 + 210*a**3*b**2*c*d**2 + 140*a**
3*b*c**3*d + 7*a**3*c**5 + 105*a**2*b**4*d**2/2 + 210*a**2*b**3*c**2*d + 10
5*a**2*b**2*c**4/2 + 42*a*b**5*c*d + 35*a*b**4*c**3 + b**7*d + 7*b**6*c**2/
2) + x**9*(7*a**5*d**3 + 105*a**4*b*c*d**2 + 35*a**4*c**3*d + 70*a**3*b**3*
d**2 + 210*a**3*b**2*c**2*d + 35*a**3*b*c**4 + 105*a**2*b**4*c*d + 70*a**2*
b**3*c**3 + 7*a*b**6*d + 21*a*b**5*c**2 + b**7*c) + x**8*(21*a**5*c*d**2 +
105*a**4*b**2*d**2/2 + 105*a**4*b*c**2*d + 35*a**4*c**4/4 + 140*a**3*b**3*c
*d + 70*a**3*b**2*c**3 + 21*a**2*b**5*d + 105*a**2*b**4*c**2/2 + 7*a*b**6*c
 + b**8/8) + x**7*(21*a**5*b*d**2 + 21*a**5*c**2*d + 105*a**4*b**2*c*d + 35
*a**4*b*c**3 + 35*a**3*b**4*d + 70*a**3*b**3*c**2 + 21*a**2*b**5*c + a*b**7
```

```
) + x**6*(7*a**6*d**2/2 + 42*a**5*b*c*d + 7*a**5*c**3 + 35*a**4*b**3*d + 10
5*a**4*b**2*c**2/2 + 35*a**3*b**4*c + 7*a**2*b**6/2) + x**5*(7*a**6*c*d + 2
1*a**5*b**2*d + 21*a**5*b*c**2 + 35*a**4*b**3*c + 7*a**3*b**5) + x**4*(7*a*
*6*b*d + 7*a**6*c**2/2 + 21*a**5*b**2*c + 35*a**4*b**4/4) + x**3*(a**7*d +
7*a**6*b*c + 7*a**5*b**3) + x**2*(a**7*c + 7*a**6*b**2/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int (b+2cx+3dx^2)\left(a+bx+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(dx^3+cx^2+bx+a\right)^8$$

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x + a)^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(19) = 38.

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 7.62

$$\begin{aligned}\int (b+2cx+3dx^2)\left(a+bx+cx^2+dx^3\right)^7\,dx\\ =\frac{1}{8}\left(dx^3+cx^2+bx\right)^8+\left(dx^3+cx^2+bx\right)^7a+\frac{7}{2}\left(dx^3+cx^2+bx\right)^6a^2\\ +7\left(dx^3+cx^2+bx\right)^5a^3+\frac{35}{4}\left(dx^3+cx^2+bx\right)^4a^4\\ +7\left(dx^3+cx^2+bx\right)^3a^5+\frac{7}{2}\left(dx^3+cx^2+bx\right)^2a^6+\left(dx^3+cx^2+bx\right)a^7\end{aligned}$$

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8 + (d*x^3 + c*x^2 + b*x)^7*a + 7/2*(d*x^3 + c*x^
2 + b*x)^6*a^2 + 7*(d*x^3 + c*x^2 + b*x)^5*a^3 + 35/4*(d*x^3 + c*x^2 + b*x)
^4*a^4 + 7*(d*x^3 + c*x^2 + b*x)^3*a^5 + 7/2*(d*x^3 + c*x^2 + b*x)^2*a^6 +
(d*x^3 + c*x^2 + b*x)*a^7
```

Mupad [B] (verification not implemented)

Time = 9.90 (sec) , antiderivative size = 1576, normalized size of antiderivative = 75.05

$$\int (b + 2cx + 3dx^2) (a + bx + cx^2 + dx^3)^7 \, dx = \text{Too large to display}$$

```
[In] int((b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x)
```

```
[Out] x^12*((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + (35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21*
a*b^2*c^5 + 21*b^5*c^2*d + 70*a^2*b^3*d^3 + 70*a^3*c^3*d^2 + 315*a^2*b^2*c^
2*d^2 + 140*a*b^3*c^3*d + 105*a*b^4*c*d^2 + 105*a^2*b*c^4*d + 140*a^3*b*c*d
^3) + x^11*(7*b^5*c^3 + 35*a*b^3*c^4 + 21*a^2*b*c^5 + 21*a*b^5*d^2 + 35*a^3
*c^4*d + 35*a^4*c*d^3 + 70*a^3*b^2*d^3 + 7*b^6*c*d + 210*a^2*b^2*c^3*d + 21
0*a^2*b^3*c*d^2 + 210*a^3*b*c^2*d^2 + 105*a*b^4*c^2*d) + x^13*(7*b^3*c^5 +
35*a*b^4*d^3 + 35*a^3*b*d^4 + 21*a^2*c^5*d + 35*b^4*c^3*d + 21*b^5*c*d^2 +
70*a^3*c^2*d^3 + 7*a*b*c^6 + 210*a*b^3*c^2*d^2 + 210*a^2*b*c^3*d^2 + 210*a^
2*b^2*c*d^3 + 105*a*b^2*c^4*d) + x^5*(7*a^3*b^5 + 35*a^4*b^3*c + 21*a^5*b*c
^2 + 21*a^5*b^2*d + 7*a^6*c*d) + x^19*(7*c^5*d^3 + 21*a*c^2*d^5 + 35*b*c^3*
d^4 + 21*b^2*c*d^5 + 7*a*b*d^6) + x^8*(b^8/8 + (35*a^4*c^4)/4 + 21*a^2*b^5*
d + 21*a^5*c*d^2 + (105*a^2*b^4*c^2)/2 + 70*a^3*b^2*c^3 + (105*a^4*b^2*d^2)
/2 + 7*a*b^6*c + 140*a^3*b^3*c*d + 105*a^4*b*c^2*d) + x^9*(b^7*c + 7*a^5*d^
3 + 21*a*b^5*c^2 + 35*a^3*b*c^4 + 35*a^4*c^3*d + 70*a^2*b^3*c^3 + 70*a^3*b^
3*d^2 + 7*a*b^6*d + 210*a^3*b^2*c^2*d + 105*a^2*b^4*c*d + 105*a^4*b*c*d^2)
+ x^16*(c^8/8 + (35*b^4*d^4)/4 + 21*a^2*b*d^5 + 21*a*c^5*d^2 + (105*a^2*c^2
*d^4)/2 + (105*b^2*c^4*d^2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d + 140*a*b*c^3*d^
3 + 105*a*b^2*c*d^4) + x^10*(b^7*d + 7*a^3*c^5 + (7*b^6*c^2)/2 + 35*a*b^4*c
^3 + 35*a^4*b*d^3 + (105*a^2*b^2*c^4)/2 + (105*a^2*b^4*d^2)/2 + (105*a^4*c^
2*d^2)/2 + 210*a^2*b^3*c^2*d + 210*a^3*b^2*c*d^2 + 42*a*b^5*c*d + 140*a^3*b
*c^3*d) + x^15*(b*c^7 + 7*a^3*d^5 + 35*a*b^3*d^4 + 21*b^2*c^5*d + 35*b^4*c*
d^3 + 70*a^2*c^3*d^3 + 70*b^3*c^3*d^2 + 7*a*c^6*d + 210*a*b^2*c^2*d^3 + 105
*a*b*c^4*d^2 + 105*a^2*b*c*d^4) + x^14*(a*c^7 + (7*b^2*c^6)/2 + 7*b^5*d^3 +
 35*a^3*c*d^4 + 35*b^3*c^4*d + (105*a^2*b^2*d^4)/2 + (105*a^2*c^4*d^2)/2 +
(105*b^4*c^2*d^2)/2 + 210*a*b^2*c^3*d^2 + 210*a^2*b*c^2*d^3 + 42*a*b*c^5*d
+ 140*a*b^3*c*d^3) + x^4*((35*a^4*b^4)/4 + (7*a^6*c^2)/2 + 21*a^5*b^2*c + 7
*a^6*b*d) + x^20*((7*b^2*d^6)/2 + (35*c^4*d^4)/4 + 21*b*c^2*d^5 + 7*a*c*d^6
) + x^6*((7*a^2*b^6)/2 + 7*a^5*c^3 + (7*a^6*d^2)/2 + 35*a^3*b^4*c + 35*a^4*
b^3*d + (105*a^4*b^2*c^2)/2 + 42*a^5*b*c*d) + x^7*(a*b^7 + 21*a^2*b^5*c + 3
5*a^4*b*c^3 + 35*a^3*b^4*d + 21*a^5*b*d^2 + 21*a^5*c^2*d + 70*a^3*b^3*c^2 +
 105*a^4*b^2*c*d) + x^18*((7*a^2*d^6)/2 + 7*b^3*d^5 + (7*c^6*d^2)/2 + 35*a*
c^3*d^4 + 35*b*c^4*d^3 + (105*b^2*c^2*d^4)/2 + 42*a*b*c*d^5) + x^17*(c^7*d
+ 21*a*b^2*d^5 + 35*a*c^4*d^3 + 21*a^2*c*d^5 + 21*b*c^5*d^2 + 35*b^3*c*d^4
+ 70*b^2*c^3*d^3 + 105*a*b*c^2*d^4) + x^3*(a^7*d + 7*a^5*b^3 + 7*a^6*b*c) +
 (d^8*x^24)/8 + x^2*(a^7*c + (7*a^6*b^2)/2) + c*d^7*x^23 + d^5*x^21*(a*d^2
+ 7*c^3 + 7*b*c*d) + (d^6*x^22*(2*b*d + 7*c^2))/2 + a^7*b*x
```

3.193 $\int (b+2cx+3dx^2)(bx+cx^2+dx^3)^7\,dx$

Optimal result 1393
Rubi [A] (verified) 1393
Mathematica [A] (verified) 1394
Maple [A] (verified) 1394
Fricas [B] (verification not implemented) 1395
Sympy [B] (verification not implemented) 1395
Maxima [A] (verification not implemented) 1397
Giac [A] (verification not implemented) 1397
Mupad [B] (verification not implemented) 1398

Optimal result

Integrand size = 29, antiderivative size = 20

$$\int (b+2cx+3dx^2)(bx+cx^2+dx^3)^7\,dx=\frac{1}{8}(bx+cx^2+dx^3)^8$$

```
[Out] 1/8*(d*x^3+c*x^2+b*x)^8
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1602}

$$\int (b+2cx+3dx^2)(bx+cx^2+dx^3)^7\,dx=\frac{1}{8}(bx+cx^2+dx^3)^8$$

```
[In] Int[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (b*x + c*x^2 + d*x^3)^8/8
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{8}(bx+cx^2+dx^3)^8$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (b+2cx+3dx^2)\left(bx+cx^2+dx^3\right)^7\,dx=\frac{1}{8}x^8(b+x(c+dx))^8$$

[In] Integrate[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]

[Out] (x^8*(b + x*(c + d*x))^8)/8

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result
gosper	$\frac{x^8(d x^2+cx+b)^8}{8}$
default	$\frac{(x^3 d+c x^2+bx)^8}{8}$
norman	$\frac{d^8x^{24}}{8}+c\,d^7x^{23}+\left(b\,d^7+\frac{7}{2}c^2d^6\right)x^{22}+\left(\frac{7}{2}b^2d^6+21b\,c^2d^5+\frac{35}{4}c^4d^4\right)x^{20}+\left(7bc\,d^6+7c^3d^5\right)x^{21}$
risch	$\frac{7}{2}x^{12}b^6d^2+\frac{35}{4}x^{12}b^4c^4+c\,d^7x^{23}+b\,c^7x^{15}+21b^2c\,d^5x^{19}+35b\,c^3d^4x^{19}+35b^3c\,d^4x^{17}+70b^2c^3d^3$
parallelrisch	$\frac{7}{2}x^{12}b^6d^2+\frac{35}{4}x^{12}b^4c^4+c\,d^7x^{23}+b\,c^7x^{15}+21b^2c\,d^5x^{19}+35b\,c^3d^4x^{19}+35b^3c\,d^4x^{17}+70b^2c^3d^3$

[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x,method=_RETURNVERBOSE)

[Out] 1/8*x^8*(d*x^2+c*x+b)^8

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 441, normalized size of antiderivative = 22.05

$$
\begin{aligned}
&\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^7\,dx\\
&=\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{1}{2}\left(7\,c^2d^6+2\,bd^7\right)x^{22}+7\left(c^3d^5+bcd^6\right)x^{21}\\
&+\frac{7}{4}\left(5\,c^4d^4+12\,bc^2d^5+2\,b^2d^6\right)x^{20}+7\left(c^5d^3+5\,bc^3d^4+3\,b^2cd^5\right)x^{19}\\
&+\frac{7}{2}\left(c^6d^2+10\,bc^4d^3+15\,b^2c^2d^4+2\,b^3d^5\right)x^{18}\\
&+\left(c^7d+21\,bc^5d^2+70\,b^2c^3d^3+35\,b^3cd^4\right)x^{17}+b^7cx^9\\
&+\frac{1}{8}\left(c^8+56\,bc^6d+420\,b^2c^4d^2+560\,b^3c^2d^3+70\,b^4d^4\right)x^{16}\\
&+\frac{1}{8}\,b^8x^8+\left(bc^7+21\,b^2c^5d+70\,b^3c^3d^2+35\,b^4cd^3\right)x^{15}\\
&+\frac{7}{2}\left(b^2c^6+10\,b^3c^4d+15\,b^4c^2d^2+2\,b^5d^3\right)x^{14}+7\left(b^3c^5+5\,b^4c^3d+3\,b^5cd^2\right)x^{13}\\
&+\frac{7}{4}\left(5\,b^4c^4+12\,b^5c^2d+2\,b^6d^2\right)x^{12}+7\left(b^5c^3+b^6cd\right)x^{11}+\frac{1}{2}\left(7\,b^6c^2+2\,b^7d\right)x^{10}
\end{aligned}
$$

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 1/2*(7*c^2*d^6 + 2*b*d^7)*x^22 + 7*(c^3*d^5 + b
*c*d^6)*x^21 + 7/4*(5*c^4*d^4 + 12*b*c^2*d^5 + 2*b^2*d^6)*x^20 + 7*(c^5*d^3
 + 5*b*c^3*d^4 + 3*b^2*c*d^5)*x^19 + 7/2*(c^6*d^2 + 10*b*c^4*d^3 + 15*b^2*c
^2*d^4 + 2*b^3*d^5)*x^18 + (c^7*d + 21*b*c^5*d^2 + 70*b^2*c^3*d^3 + 35*b^3*
c*d^4)*x^17 + b^7*c*x^9 + 1/8*(c^8 + 56*b*c^6*d + 420*b^2*c^4*d^2 + 560*b^3
*c^2*d^3 + 70*b^4*d^4)*x^16 + 1/8*b^8*x^8 + (b*c^7 + 21*b^2*c^5*d + 70*b^3*
c^3*d^2 + 35*b^4*c*d^3)*x^15 + 7/2*(b^2*c^6 + 10*b^3*c^4*d + 15*b^4*c^2*d^2
 + 2*b^5*d^3)*x^14 + 7*(b^3*c^5 + 5*b^4*c^3*d + 3*b^5*c*d^2)*x^13 + 7/4*(5*
b^4*c^4 + 12*b^5*c^2*d + 2*b^6*d^2)*x^12 + 7*(b^5*c^3 + b^6*c*d)*x^11 + 1/2
*(7*b^6*c^2 + 2*b^7*d)*x^10
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(15) = 30$.

Time = 0.08 (sec) , antiderivative size = 469, normalized size of antiderivative = 23.45

$$\int (b+2cx+3dx^2)\,(bx+cx^2+dx^3)^7\,dx = \frac{b^8x^8}{8}+b^7cx^9+cd^7x^{23}+\frac{d^8x^{24}}{8} + x^{22}\left(bd^7+\frac{7c^2d^6}{2}\right)+x^{21}\cdot\left(7bcd^6+7c^3d^5\right) + x^{20}\cdot\left(\frac{7b^2d^6}{2}+21bc^2d^5+\frac{35c^4d^4}{4}\right) + x^{19}\cdot\left(21b^2cd^5+35bc^3d^4+7c^5d^3\right)+x^{18} \cdot\left(7b^3d^5+\frac{105b^2c^2d^4}{2}+35bc^4d^3+\frac{7c^6d^2}{2}\right)+x^{17} \cdot\left(35b^3cd^4+70b^2c^3d^3+21bc^5d^2+c^7d\right)+x^{16} \cdot\left(\frac{35b^4d^4}{4}+70b^3c^2d^3+\frac{105b^2c^4d^2}{2}+7bc^6d+\frac{c^8}{8}\right) + x^{15}\cdot\left(35b^4cd^3+70b^3c^3d^2+21b^2c^5d+bc^7\right) + x^{14}\cdot\left(7b^5d^3+\frac{105b^4c^2d^2}{2}+35b^3c^4d+\frac{7b^2c^6}{2}\right) + x^{13}\cdot\left(21b^5cd^2+35b^4c^3d+7b^3c^5\right)+x^{12} \cdot\left(\frac{7b^6d^2}{2}+21b^5c^2d+\frac{35b^4c^4}{4}\right)+x^{11} \cdot\left(7b^6cd+7b^5c^3\right)+x^{10}\left(b^7d+\frac{7b^6c^2}{2}\right)$$

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**7,x)
```

```
[Out] b**8*x**8/8 + b**7*c*x**9 + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7
*c**2*d**6/2) + x**21*(7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*b**2*d**6/2 + 2
1*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(21*b**2*c*d**5 + 35*b*c**3*d**4 +
7*c**5*d**3) + x**18*(7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c**4*d**3 +
 7*c**6*d**2/2) + x**17*(35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**5*d**
2 + c**7*d) + x**16*(35*b**4*d**4/4 + 70*b**3*c**2*d**3 + 105*b**2*c**4*d**
2/2 + 7*b*c**6*d + c**8/8) + x**15*(35*b**4*c*d**3 + 70*b**3*c**3*d**2 + 21
*b**2*c**5*d + b*c**7) + x**14*(7*b**5*d**3 + 105*b**4*c**2*d**2/2 + 35*b**
3*c**4*d + 7*b**2*c**6/2) + x**13*(21*b**5*c*d**2 + 35*b**4*c**3*d + 7*b**3
*c**5) + x**12*(7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4) + x**11*(7
*b**6*c*d + 7*b**5*c**3) + x**10*(b**7*d + 7*b**6*c**2/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(dx^3+cx^2+bx\right)^8$$

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="maxima")
[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(dx^3+cx^2+bx\right)^8$$

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="giac")
[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8
```

Mupad [B] (verification not implemented)

Time = 9.40 (sec) , antiderivative size = 418, normalized size of antiderivative = 20.90

$$\int (b+2cx+3dx^2) (bx+cx^2+dx^3)^7 dx = x^{14} \left(7 b^5 d^3 + \frac{105 b^4 c^2 d^2}{2} + 35 b^3 c^4 d + \frac{7 b^2 c^6}{2}\right) + x^{18} \left(7 b^3 d^5 + \frac{105 b^2 c^2 d^4}{2} + 35 b c^4 d^3 + \frac{7 c^6 d^2}{2}\right) + x^{12} \left(\frac{7 b^6 d^2}{2} + 21 b^5 c^2 d + \frac{35 b^4 c^4}{4}\right) + x^{20} \left(\frac{7 b^2 d^6}{2} + 21 b c^2 d^5 + \frac{35 c^4 d^4}{4}\right) + x^{16} \left(\frac{35 b^4 d^4}{4} + 70 b^3 c^2 d^3 + \frac{105 b^2 c^4 d^2}{2} + 7 b c^6 d + \frac{c^8}{8}\right) + \frac{b^8 x^8}{8} + \frac{d^8 x^{24}}{8} + x^{10} \left(d b^7 + \frac{7 b^6 c^2}{2}\right) + b^7 c x^9 + c d^7 x^{23} + \frac{d^6 x^{22} (7 c^2 + 2 b d)}{2} + 7 b^3 c x^{13} (3 b^2 d^2 + 5 b c^2 d + c^4) + 7 c d^3 x^{19} (3 b^2 d^2 + 5 b c^2 d + c^4) + b c x^{15} (35 b^3 d^3 + 70 b^2 c^2 d^2 + 21 b c^4 d + c^6) + c d x^{17} (35 b^3 d^3 + 70 b^2 c^2 d^2 + 21 b c^4 d + c^6) + 7 b^5 c x^{11} (c^2 + b d) + 7 c d^5 x^{21} (c^2 + b d)$$

```
[In] int((b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x)
```

```
[Out] x^14*((7*b^2*c^6)/2 + 7*b^5*d^3 + 35*b^3*c^4*d + (105*b^4*c^2*d^2)/2) + x^1
8*(7*b^3*d^5 + (7*c^6*d^2)/2 + 35*b*c^4*d^3 + (105*b^2*c^2*d^4)/2) + x^12*(
(35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21*b^5*c^2*d) + x^20*((7*b^2*d^6)/2 + (35*
c^4*d^4)/4 + 21*b*c^2*d^5) + x^16*(c^8/8 + (35*b^4*d^4)/4 + (105*b^2*c^4*d^
2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d) + (b^8*x^8)/8 + (d^8*x^24)/8 + x^10*(b^7
*d + (7*b^6*c^2)/2) + b^7*c*x^9 + c*d^7*x^23 + (d^6*x^22*(2*b*d + 7*c^2))/2
 + 7*b^3*c*x^13*(c^4 + 3*b^2*d^2 + 5*b*c^2*d) + 7*c*d^3*x^19*(c^4 + 3*b^2*d
^2 + 5*b*c^2*d) + b*c*x^15*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d)
 + c*d*x^17*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d) + 7*b^5*c*x^11
*(b*d + c^2) + 7*c*d^5*x^21*(b*d + c^2)
```

3.194 $\int x^7(b+cx+dx^2)^7(b+2cx+3dx^2)\,dx$

Optimal result 1399
Rubi [A] (verified) 1399
Mathematica [A] (verified) 1400
Maple [B] (verified) 1400
Fricas [B] (verification not implemented) 1401
Sympy [B] (verification not implemented) 1401
Maxima [B] (verification not implemented) 1403
Giac [A] (verification not implemented) 1403
Mupad [B] (verification not implemented) 1404

Optimal result

Integrand size = 28, antiderivative size = 19

$$\int x^7(b+cx+dx^2)^7(b+2cx+3dx^2)\,dx=\frac{1}{8}x^8(b+cx+dx^2)^8$$

```
[Out] 1/8*x^8*(d*x^2+c*x+b)^8
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1602}

$$\int x^7(b+cx+dx^2)^7(b+2cx+3dx^2)\,dx=\frac{1}{8}x^8(b+cx+dx^2)^8$$

```
[In] Int[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x]
[Out] (x^8*(b + c*x + d*x^2)^8)/8
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{8}x^8(b+cx+dx^2)^8$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int x^7 \left(b+cx+dx^2\right)^7 \left(b+2cx+3dx^2\right) \, dx = \frac{1}{8}x^8(b+x(c+dx))^8$$

```
[In] Integrate[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x]
[Out] (x^8*(b + x*(c + d*x))^8)/8
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(17) = 34.

Time = 0.96 (sec) , antiderivative size = 439, normalized size of antiderivative = 23.11

method	result
norman	$\frac{d^8x^{24}}{8}+c\,d^7x^{23}+\left(b\,d^7+\frac{7}{2}c^2d^6\right)x^{22}+\left(\frac{7}{2}b^2d^6+21b\,c^2d^5+\frac{35}{4}c^4d^4\right)x^{20}+\left(7bc\,d^6+7c^3d^5\right)x^{21}$ -
gosper	$\frac{7}{2}x^{12}b^6d^2+\frac{35}{4}x^{12}b^4c^4+c\,d^7x^{23}+b\,c^7x^{15}+21b^2c\,d^5x^{19}+35b\,c^3d^4x^{19}+35b^3c\,d^4x^{17}+70b^2c^3d^3$
risch	$\frac{7}{2}x^{12}b^6d^2+\frac{35}{4}x^{12}b^4c^4+c\,d^7x^{23}+b\,c^7x^{15}+21b^2c\,d^5x^{19}+35b\,c^3d^4x^{19}+35b^3c\,d^4x^{17}+70b^2c^3d^3$
parallelrisch	$\frac{7}{2}x^{12}b^6d^2+\frac{35}{4}x^{12}b^4c^4+c\,d^7x^{23}+b\,c^7x^{15}+21b^2c\,d^5x^{19}+35b\,c^3d^4x^{19}+35b^3c\,d^4x^{17}+70b^2c^3d^3$
default	Expression too large to display

```
[In] int(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x,method=_RETURNVERBOSE)
[Out] 1/8*d^8*x^24+c*d^7*x^23+(b*d^7+7/2*c^2*d^6)*x^22+(7/2*b^2*d^6+21*b*c^2*d^5+
35/4*c^4*d^4)*x^20+(7*b*c*d^6+7*c^3*d^5)*x^21+(35*b^4*c*d^3+70*b^3*c^3*d^2+
21*b^2*c^5*d+b*c^7)*x^15+(35/4*b^4*d^4+70*b^3*c^2*d^3+105/2*b^2*c^4*d^2+7*b
*c^6*d+1/8*c^8)*x^16+(35*b^3*c*d^4+70*b^2*c^3*d^3+21*b*c^5*d^2+c^7*d)*x^17+
(7*b^3*d^5+105/2*b^2*c^2*d^4+35*b*c^4*d^3+7/2*c^6*d^2)*x^18+(21*b^2*c*d^5+3
5*b*c^3*d^4+7*c^5*d^3)*x^19+(7/2*b^6*d^2+21*b^5*c^2*d+35/4*b^4*c^4)*x^12+(2
1*b^5*c*d^2+35*b^4*c^3*d+7*b^3*c^5)*x^13+(7*b^5*d^3+105/2*b^4*c^2*d^2+35*b^
3*c^4*d+7/2*c^6*b^2)*x^14+b^7*c*x^9+(b^7*d+7/2*c^2*b^6)*x^10+(7*b^6*c*d+7*b
^5*c^3)*x^11+1/8*x^8*b^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 441, normalized size of antiderivative = 23.21

$$\begin{aligned}
&\int x^7(b+cx+dx^2)^7\,(b+2cx+3dx^2)\;dx \\
&= \frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{1}{2}\,(7\,c^2d^6+2\,bd^7)x^{22}+7\,(c^3d^5+bcd^6)x^{21} \\
&\quad+\frac{7}{4}\,(5\,c^4d^4+12\,bc^2d^5+2\,b^2d^6)x^{20}+7\,(c^5d^3+5\,bc^3d^4+3\,b^2cd^5)x^{19} \\
&\quad+\frac{7}{2}\,(c^6d^2+10\,bc^4d^3+15\,b^2c^2d^4+2\,b^3d^5)x^{18} \\
&\quad+(c^7d+21\,bc^5d^2+70\,b^2c^3d^3+35\,b^3cd^4)x^{17}+b^7cx^9 \\
&\quad+\frac{1}{8}\,(c^8+56\,bc^6d+420\,b^2c^4d^2+560\,b^3c^2d^3+70\,b^4d^4)x^{16} \\
&\quad+\frac{1}{8}\,b^8x^8+(bc^7+21\,b^2c^5d+70\,b^3c^3d^2+35\,b^4cd^3)x^{15} \\
&\quad+\frac{7}{2}\,(b^2c^6+10\,b^3c^4d+15\,b^4c^2d^2+2\,b^5d^3)x^{14}+7\,(b^3c^5+5\,b^4c^3d+3\,b^5cd^2)x^{13} \\
&\quad+\frac{7}{4}\,(5\,b^4c^4+12\,b^5c^2d+2\,b^6d^2)x^{12}+7\,(b^5c^3+b^6cd)x^{11}+\frac{1}{2}\,(7\,b^6c^2+2\,b^7d)x^{10}
\end{aligned}$$

```
[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 1/2*(7*c^2*d^6 + 2*b*d^7)*x^22 + 7*(c^3*d^5 + b
*c*d^6)*x^21 + 7/4*(5*c^4*d^4 + 12*b*c^2*d^5 + 2*b^2*d^6)*x^20 + 7*(c^5*d^3
 + 5*b*c^3*d^4 + 3*b^2*c*d^5)*x^19 + 7/2*(c^6*d^2 + 10*b*c^4*d^3 + 15*b^2*c
^2*d^4 + 2*b^3*d^5)*x^18 + (c^7*d + 21*b*c^5*d^2 + 70*b^2*c^3*d^3 + 35*b^3*
c*d^4)*x^17 + b^7*c*x^9 + 1/8*(c^8 + 56*b*c^6*d + 420*b^2*c^4*d^2 + 560*b^3
*c^2*d^3 + 70*b^4*d^4)*x^16 + 1/8*b^8*x^8 + (b*c^7 + 21*b^2*c^5*d + 70*b^3*
c^3*d^2 + 35*b^4*c*d^3)*x^15 + 7/2*(b^2*c^6 + 10*b^3*c^4*d + 15*b^4*c^2*d^2
 + 2*b^5*d^3)*x^14 + 7*(b^3*c^5 + 5*b^4*c^3*d + 3*b^5*c*d^2)*x^13 + 7/4*(5*
b^4*c^4 + 12*b^5*c^2*d + 2*b^6*d^2)*x^12 + 7*(b^5*c^3 + b^6*c*d)*x^11 + 1/2
*(7*b^6*c^2 + 2*b^7*d)*x^10
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(15) = 30$.

Time = 0.08 (sec) , antiderivative size = 469, normalized size of antiderivative = 24.68

$$\int x^7\left(b+cx+dx^2\right)^7\left(b+2cx+3dx^2\right)\,dx=\frac{b^8x^8}{8}+b^7cx^9+cd^7x^{23}+\frac{d^8x^{24}}{8}+x^{22}\left(bd^7+\frac{7c^2d^6}{2}\right)+x^{21}\cdot\left(7bcd^6+7c^3d^5\right)+x^{20}\cdot\left(\frac{7b^2d^6}{2}+21bc^2d^5+\frac{35c^4d^4}{4}\right)+x^{19}\cdot\left(21b^2cd^5+35bc^3d^4+7c^5d^3\right)+x^{18}\cdot\left(7b^3d^5+\frac{105b^2c^2d^4}{2}+35bc^4d^3+\frac{7c^6d^2}{2}\right)+x^{17}\cdot\left(35b^3cd^4+70b^2c^3d^3+21bc^5d^2+c^7d\right)+x^{16}\cdot\left(\frac{35b^4d^4}{4}+70b^3c^2d^3+\frac{105b^2c^4d^2}{2}+7bc^6d+\frac{c^8}{8}\right)+x^{15}\cdot\left(35b^4cd^3+70b^3c^3d^2+21b^2c^5d+bc^7\right)+x^{14}\cdot\left(7b^5d^3+\frac{105b^4c^2d^2}{2}+35b^3c^4d+\frac{7b^2c^6}{2}\right)+x^{13}\cdot\left(21b^5cd^2+35b^4c^3d+7b^3c^5\right)+x^{12}\cdot\left(\frac{7b^6d^2}{2}+21b^5c^2d+\frac{35b^4c^4}{4}\right)+x^{11}\cdot\left(7b^6cd+7b^5c^3\right)+x^{10}\left(b^7d+\frac{7b^6c^2}{2}\right)$$

```
[In] integrate(x**7*(d*x**2+c*x+b)**7*(3*d*x**2+2*c*x+b),x)
```

```
[Out] b**8*x**8/8 + b**7*c*x**9 + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7
*c**2*d**6/2) + x**21*(7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*b**2*d**6/2 + 2
1*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(21*b**2*c*d**5 + 35*b*c**3*d**4 +
7*c**5*d**3) + x**18*(7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c**4*d**3 +
 7*c**6*d**2/2) + x**17*(35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**5*d**
2 + c**7*d) + x**16*(35*b**4*d**4/4 + 70*b**3*c**2*d**3 + 105*b**2*c**4*d**
2/2 + 7*b*c**6*d + c**8/8) + x**15*(35*b**4*c*d**3 + 70*b**3*c**3*d**2 + 21
*b**2*c**5*d + b*c**7) + x**14*(7*b**5*d**3 + 105*b**4*c**2*d**2/2 + 35*b**
3*c**4*d + 7*b**2*c**6/2) + x**13*(21*b**5*c*d**2 + 35*b**4*c**3*d + 7*b**3
*c**5) + x**12*(7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4) + x**11*(7
*b**6*c*d + 7*b**5*c**3) + x**10*(b**7*d + 7*b**6*c**2/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 441, normalized size of antiderivative = 23.21

$$\begin{aligned}
&\int x^7(b+cx+dx^2)^7\,(b+2cx+3dx^2)\,\,dx\\
&=\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{1}{2}\,(7\,c^2d^6+2\,bd^7)x^{22}+7\,(c^3d^5+bcd^6)x^{21}\\
&+\frac{7}{4}\,(5\,c^4d^4+12\,bc^2d^5+2\,b^2d^6)x^{20}+7\,(c^5d^3+5\,bc^3d^4+3\,b^2cd^5)x^{19}\\
&+\frac{7}{2}\,(c^6d^2+10\,bc^4d^3+15\,b^2c^2d^4+2\,b^3d^5)x^{18}\\
&+(c^7d+21\,bc^5d^2+70\,b^2c^3d^3+35\,b^3cd^4)x^{17}+b^7cx^9\\
&+\frac{1}{8}\,(c^8+56\,bc^6d+420\,b^2c^4d^2+560\,b^3c^2d^3+70\,b^4d^4)x^{16}\\
&+\frac{1}{8}\,b^8x^8+(bc^7+21\,b^2c^5d+70\,b^3c^3d^2+35\,b^4cd^3)x^{15}\\
&+\frac{7}{2}\,(b^2c^6+10\,b^3c^4d+15\,b^4c^2d^2+2\,b^5d^3)x^{14}+7\,(b^3c^5+5\,b^4c^3d+3\,b^5cd^2)x^{13}\\
&+\frac{7}{4}\,(5\,b^4c^4+12\,b^5c^2d+2\,b^6d^2)x^{12}+7\,(b^5c^3+b^6cd)x^{11}+\frac{1}{2}\,(7\,b^6c^2+2\,b^7d)x^{10}
\end{aligned}$$

```
[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 1/2*(7*c^2*d^6 + 2*b*d^7)*x^22 + 7*(c^3*d^5 + b
*c*d^6)*x^21 + 7/4*(5*c^4*d^4 + 12*b*c^2*d^5 + 2*b^2*d^6)*x^20 + 7*(c^5*d^3
 + 5*b*c^3*d^4 + 3*b^2*c*d^5)*x^19 + 7/2*(c^6*d^2 + 10*b*c^4*d^3 + 15*b^2*c
^2*d^4 + 2*b^3*d^5)*x^18 + (c^7*d + 21*b*c^5*d^2 + 70*b^2*c^3*d^3 + 35*b^3*
c*d^4)*x^17 + b^7*c*x^9 + 1/8*(c^8 + 56*b*c^6*d + 420*b^2*c^4*d^2 + 560*b^3
*c^2*d^3 + 70*b^4*d^4)*x^16 + 1/8*b^8*x^8 + (b*c^7 + 21*b^2*c^5*d + 70*b^3*
c^3*d^2 + 35*b^4*c*d^3)*x^15 + 7/2*(b^2*c^6 + 10*b^3*c^4*d + 15*b^4*c^2*d^2
 + 2*b^5*d^3)*x^14 + 7*(b^3*c^5 + 5*b^4*c^3*d + 3*b^5*c*d^2)*x^13 + 7/4*(5*
b^4*c^4 + 12*b^5*c^2*d + 2*b^6*d^2)*x^12 + 7*(b^5*c^3 + b^6*c*d)*x^11 + 1/2
*(7*b^6*c^2 + 2*b^7*d)*x^10
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int x^7(b+cx+dx^2)^7\,(b+2cx+3dx^2)\,\,dx=\frac{1}{8}\,(dx^3+cx^2+bx)^8$$

```
[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 418, normalized size of antiderivative = 22.00

$$\int x^7 \left(b+cx+dx^2\right)^7 \left(b+2cx+3dx^2\right)\,dx = x^{14}\left(7\,b^5\,d^3+\frac{105\,b^4\,c^2\,d^2}{2}+35\,b^3\,c^4\,d+\frac{7\,b^2\,c^6}{2}\right) + x^{18}\left(7\,b^3\,d^5+\frac{105\,b^2\,c^2\,d^4}{2}+35\,b\,c^4\,d^3+\frac{7\,c^6\,d^2}{2}\right)+x^{12}\left(\frac{7\,b^6\,d^2}{2}+21\,b^5\,c^2\,d+\frac{35\,b^4\,c^4}{4}\right) + x^{20}\left(\frac{7\,b^2\,d^6}{2}+21\,b\,c^2\,d^5+\frac{35\,c^4\,d^4}{4}\right) + x^{16}\left(\frac{35\,b^4\,d^4}{4}+70\,b^3\,c^2\,d^3+\frac{105\,b^2\,c^4\,d^2}{2}+7\,b\,c^6\,d+\frac{c^8}{8}\right) + \frac{b^8\,x^8}{8}+\frac{d^8\,x^{24}}{8}+x^{10}\left(d\,b^7+\frac{7\,b^6\,c^2}{2}\right) + b^7\,c\,x^9+c\,d^7\,x^{23}+\frac{d^6\,x^{22}\,\left(7\,c^2+2\,b\,d\right)}{2} + 7\,b^3\,c\,x^{13}\left(3\,b^2\,d^2+5\,b\,c^2\,d+c^4\right) + 7\,c\,d^3\,x^{19}\left(3\,b^2\,d^2+5\,b\,c^2\,d+c^4\right) + b\,c\,x^{15}\left(35\,b^3\,d^3+70\,b^2\,c^2\,d^2+21\,b\,c^4\,d+c^6\right) + c\,d\,x^{17}\left(35\,b^3\,d^3+70\,b^2\,c^2\,d^2+21\,b\,c^4\,d+c^6\right) + 7\,b^5\,c\,x^{11}\left(c^2+b\,d\right)+7\,c\,d^5\,x^{21}\left(c^2+b\,d\right)$$

```
[In] int(x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x)
```

```
[Out] x^14*((7*b^2*c^6)/2 + 7*b^5*d^3 + 35*b^3*c^4*d + (105*b^4*c^2*d^2)/2) + x^1
8*(7*b^3*d^5 + (7*c^6*d^2)/2 + 35*b*c^4*d^3 + (105*b^2*c^2*d^4)/2) + x^12*(
(35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21*b^5*c^2*d) + x^20*((7*b^2*d^6)/2 + (35*
c^4*d^4)/4 + 21*b*c^2*d^5) + x^16*(c^8/8 + (35*b^4*d^4)/4 + (105*b^2*c^4*d^
2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d) + (b^8*x^8)/8 + (d^8*x^24)/8 + x^10*(b^7
*d + (7*b^6*c^2)/2) + b^7*c*x^9 + c*d^7*x^23 + (d^6*x^22*(2*b*d + 7*c^2))/2
 + 7*b^3*c*x^13*(c^4 + 3*b^2*d^2 + 5*b*c^2*d) + 7*c*d^3*x^19*(c^4 + 3*b^2*d
^2 + 5*b*c^2*d) + b*c*x^15*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d)
 + c*d*x^17*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d) + 7*b^5*c*x^11
*(b*d + c^2) + 7*c*d^5*x^21*(b*d + c^2)
```

3.195 $\int (b+3dx^2)(a+bx+dx^3)^7\,dx$

Optimal result 1405
Rubi [A] (verified) 1405
Mathematica [B] (verified) 1406
Maple [A] (verified) 1406
Fricas [B] (verification not implemented) 1406
Sympy [B] (verification not implemented) 1408
Maxima [A] (verification not implemented) 1409
Giac [B] (verification not implemented) 1409
Mupad [B] (verification not implemented) 1410

Optimal result

Integrand size = 21, antiderivative size = 16

$$\int (b+3dx^2)\left(a+bx+dx^3\right)^7\,dx=\frac{1}{8}\left(a+bx+dx^3\right)^8$$

```
[Out] 1/8*(d*x^3+b*x+a)^8
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1602}

$$\int (b+3dx^2)\left(a+bx+dx^3\right)^7\,dx=\frac{1}{8}\left(a+bx+dx^3\right)^8$$

```
[In] Int[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]
```

```
[Out] (a + b*x + d*x^3)^8/8
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{8}\left(a+bx+dx^3\right)^8$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 127 vs. $2(16) = 32$.

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 7.94

$$\int (b+3dx^2)\left(a+bx+dx^3\right)^7\,dx = \frac{1}{8}x(b+dx^2)\left(8a^7+28a^6x(b+dx^2)+56a^5x^2(b+dx^2)^2+70a^4x^3(b+dx^2)^3+56a^3x^4(b+dx^2)^4+28a^2x^5(b+dx^2)^5+8ax^6(b+dx^2)^6+x^7(b+dx^2)^7\right)$$

```
[In] Integrate[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]
[Out] (x*(b + d*x^2)*(8*a^7 + 28*a^6*x*(b + d*x^2) + 56*a^5*x^2*(b + d*x^2)^2 + 7
0*a^4*x^3*(b + d*x^2)^3 + 56*a^3*x^4*(b + d*x^2)^4 + 28*a^2*x^5*(b + d*x^2)
^5 + 8*a*x^6*(b + d*x^2)^6 + x^7*(b + d*x^2)^7))/8
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$\frac{(x^3d+bx+a)^8}{8}$
norman	$\frac{7x^{20}b^2d^6}{2}+a\,d^7x^{21}+x^{22}b\,d^7+\frac{d^8x^{24}}{8}+7ab\,d^6x^{19}+(7a^3d^5+35b^3a\,d^4)\,x^{15}+\left(21a^2b\,d^5+\frac{35}{4}b^4d^4\right.$
gosper	$\frac{7}{2}x^{12}b^6d^2+\frac{35}{4}x^4a^4b^4+\frac{7}{2}x^6a^6d^2+\frac{7}{2}x^6a^2b^6+\frac{7}{2}x^2b^2a^6+70a^3b^3d^2x^9+7a\,b^6d\,x^9+70a^3b^2d^3x^{11}$
parallelrisch	$\frac{7}{2}x^{12}b^6d^2+\frac{35}{4}x^4a^4b^4+\frac{7}{2}x^6a^6d^2+\frac{7}{2}x^6a^2b^6+\frac{7}{2}x^2b^2a^6+70a^3b^3d^2x^9+7a\,b^6d\,x^9+70a^3b^2d^3x^{11}$
risch	$\frac{7}{2}x^{12}b^6d^2+\frac{35}{4}x^4a^4b^4+\frac{7}{2}x^6a^6d^2+\frac{7}{2}x^6a^2b^6+\frac{7}{2}x^2b^2a^6+70a^3b^3d^2x^9+7a\,b^6d\,x^9+70a^3b^2d^3x^{11}$

```
[In] int((3*d*x^2+b)*(d*x^3+b*x+a)^7,x,method=_RETURNVERBOSE)
[Out] 1/8*(d*x^3+b*x+a)^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 456, normalized size of antiderivative = 28.50

$$\int (b+3dx^2)(a+bx+dx^3)^7\,dx = \frac{1}{8}d^8x^{24}+bd^7x^{22}+ad^7x^{21}+\frac{7}{2}b^2d^6x^{20}+7\,abd^6x^{19}+21\,ab^2d^5x^{17}+\frac{7}{2}\left(2\,b^3d^5+a^2d^6\right)x^{18}+\frac{7}{4}\left(5\,b^4d^4+12\,a^2bd^5\right)x^{16}+7\left(5\,ab^3d^4+a^3d^5\right)x^{15}+\frac{7}{2}\left(2\,b^5d^3+15\,a^2b^2d^4\right)x^{14}+35\left(ab^4d^3+a^3bd^4\right)x^{13}+\frac{7}{4}\left(2\,b^6d^2+40\,a^2b^3d^3+5\,a^4d^4\right)x^{12}+7\left(3\,ab^5d^2+10\,a^3b^2d^3\right)x^{11}+\frac{1}{2}\left(2\,b^7d+105\,a^2b^4d^2+70\,a^4bd^3\right)x^{10}+\frac{7}{2}a^6b^2x^2+7\left(ab^6d+10\,a^3b^3d^2+a^5d^3\right)x^9+a^7bx+\frac{1}{8}\left(b^8+168\,a^2b^5d+420\,a^4b^2d^2\right)x^8+\left(ab^7+35\,a^3b^4d+21\,a^5bd^2\right)x^7+\frac{7}{2}\left(a^2b^6+10\,a^4b^3d+a^6d^2\right)x^6+7\left(a^3b^5+3\,a^5b^2d\right)x^5+\frac{7}{4}\left(5\,a^4b^4+4\,a^6bd\right)x^4+\left(7\,a^5b^3+a^7d\right)x^3$$

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + b*d^7*x^22 + a*d^7*x^21 + 7/2*b^2*d^6*x^20 + 7*a*b*d^6*x^19
+ 21*a*b^2*d^5*x^17 + 7/2*(2*b^3*d^5 + a^2*d^6)*x^18 + 7/4*(5*b^4*d^4 + 12*
a^2*b*d^5)*x^16 + 7*(5*a*b^3*d^4 + a^3*d^5)*x^15 + 7/2*(2*b^5*d^3 + 15*a^2*
b^2*d^4)*x^14 + 35*(a*b^4*d^3 + a^3*b*d^4)*x^13 + 7/4*(2*b^6*d^2 + 40*a^2*b
^3*d^3 + 5*a^4*d^4)*x^12 + 7*(3*a*b^5*d^2 + 10*a^3*b^2*d^3)*x^11 + 1/2*(2*b
^7*d + 105*a^2*b^4*d^2 + 70*a^4*b*d^3)*x^10 + 7/2*a^6*b^2*x^2 + 7*(a*b^6*d
+ 10*a^3*b^3*d^2 + a^5*d^3)*x^9 + a^7*b*x + 1/8*(b^8 + 168*a^2*b^5*d + 420*
a^4*b^2*d^2)*x^8 + (a*b^7 + 35*a^3*b^4*d + 21*a^5*b*d^2)*x^7 + 7/2*(a^2*b^6
 + 10*a^4*b^3*d + a^6*d^2)*x^6 + 7*(a^3*b^5 + 3*a^5*b^2*d)*x^5 + 7/4*(5*a^4
*b^4 + 4*a^6*b*d)*x^4 + (7*a^5*b^3 + a^7*d)*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(12) = 24$.

Time = 0.08 (sec) , antiderivative size = 483, normalized size of antiderivative = 30.19

$$\int (b+3dx^2)(a+bx+dx^3)^7 \, dx = a^7bx + \frac{7a^6b^2x^2}{2} + 21ab^2d^5x^{17} + 7abd^6x^{19} + ad^7x^{21} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8} + x^{18} \cdot \left(\frac{7a^2d^6}{2} + 7b^3d^5\right) + x^{16} \cdot \left(21a^2bd^5 + \frac{35b^4d^4}{4}\right) + x^{15} \cdot (7a^3d^5 + 35ab^3d^4) + x^{14} \cdot \left(\frac{105a^2b^2d^4}{2} + 7b^5d^3\right) + x^{13} \cdot (35a^3bd^4 + 35ab^4d^3) + x^{12} \cdot \left(\frac{35a^4d^4}{4} + 70a^2b^3d^3 + \frac{7b^6d^2}{2}\right) + x^{11} \cdot (70a^3b^2d^3 + 21ab^5d^2) + x^{10} \cdot \left(35a^4bd^3 + \frac{105a^2b^4d^2}{2} + b^7d\right) + x^9 \cdot (7a^5d^3 + 70a^3b^3d^2 + 7ab^6d) + x^8 \cdot \left(\frac{105a^4b^2d^2}{2} + 21a^2b^5d + \frac{b^8}{8}\right) + x^7 \cdot (21a^5bd^2 + 35a^3b^4d + ab^7) + x^6 \cdot \left(\frac{7a^6d^2}{2} + 35a^4b^3d + \frac{7a^2b^6}{2}\right) + x^5 \cdot (21a^5b^2d + 7a^3b^5) + x^4 \cdot \left(7a^6bd + \frac{35a^4b^4}{4}\right) + x^3(a^7d + 7a^5b^3)$$

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x+a)**7,x)
```

```
[Out] a**7*b*x + 7*a**6*b**2*x**2/2 + 21*a*b**2*d**5*x**17 + 7*a*b*d**6*x**19 + a
*d**7*x**21 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 + d**8*x**24/8 + x**18*(7*
a**2*d**6/2 + 7*b**3*d**5) + x**16*(21*a**2*b*d**5 + 35*b**4*d**4/4) + x**1
5*(7*a**3*d**5 + 35*a*b**3*d**4) + x**14*(105*a**2*b**2*d**4/2 + 7*b**5*d**
3) + x**13*(35*a**3*b*d**4 + 35*a*b**4*d**3) + x**12*(35*a**4*d**4/4 + 70*a
**2*b**3*d**3 + 7*b**6*d**2/2) + x**11*(70*a**3*b**2*d**3 + 21*a*b**5*d**2)
 + x**10*(35*a**4*b*d**3 + 105*a**2*b**4*d**2/2 + b**7*d) + x**9*(7*a**5*d*
*3 + 70*a**3*b**3*d**2 + 7*a*b**6*d) + x**8*(105*a**4*b**2*d**2/2 + 21*a**2
*b**5*d + b**8/8) + x**7*(21*a**5*b*d**2 + 35*a**3*b**4*d + a*b**7) + x**6*
(7*a**6*d**2/2 + 35*a**4*b**3*d + 7*a**2*b**6/2) + x**5*(21*a**5*b**2*d + 7
*a**3*b**5) + x**4*(7*a**6*b*d + 35*a**4*b**4/4) + x**3*(a**7*d + 7*a**5*b*
*3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int (b+3dx^2)\left(a+bx+dx^3\right)^7\,dx=\frac{1}{8}\left(dx^3+bx+a\right)^8$$

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + b*x + a)^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(14) = 28.

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 7.50

$$\begin{aligned}\int (b+3dx^2)\left(a+bx+dx^3\right)^7\,dx=&\ \frac{1}{8}\left(dx^3+bx\right)^8+\left(dx^3+bx\right)^7a+\frac{7}{2}\left(dx^3+bx\right)^6a^2\\&+7\left(dx^3+bx\right)^5a^3+\frac{35}{4}\left(dx^3+bx\right)^4a^4\\&+7\left(dx^3+bx\right)^3a^5+\frac{7}{2}\left(dx^3+bx\right)^2a^6+\left(dx^3+bx\right)a^7\end{aligned}$$

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + b*x)^8 + (d*x^3 + b*x)^7*a + 7/2*(d*x^3 + b*x)^6*a^2 + 7*(d*x^
3 + b*x)^5*a^3 + 35/4*(d*x^3 + b*x)^4*a^4 + 7*(d*x^3 + b*x)^3*a^5 + 7/2*(d*
x^3 + b*x)^2*a^6 + (d*x^3 + b*x)*a^7
```

Mupad [B] (verification not implemented)

Time = 9.69 (sec) , antiderivative size = 438, normalized size of antiderivative = 27.38

$$\int (b+3dx^2)\left(a+bx+dx^3\right)^7\,dx = x^{12}\left(\frac{35\,a^4\,d^4}{4}+70\,a^2\,b^3\,d^3+\frac{7\,b^6\,d^2}{2}\right)+x^4\left(7\,d\,a^6\,b+\frac{35\,a^4\,b^4}{4}\right)+x^{18}\left(\frac{7\,a^2\,d^6}{2}+7\,b^3\,d^5\right)+x^6\left(\frac{7\,a^6\,d^2}{2}+35\,a^4\,b^3\,d+\frac{7\,a^2\,b^6}{2}\right)+x^8\left(\frac{105\,a^4\,b^2\,d^2}{2}+21\,a^2\,b^5\,d+\frac{b^8}{8}\right)+\frac{d^8\,x^{24}}{8}+x^3\left(d\,a^7+7\,a^5\,b^3\right)+a\,d^7\,x^{21}+b\,d^7\,x^{22}+\frac{7\,a^6\,b^2\,x^2}{2}+\frac{7\,b^2\,d^6\,x^{20}}{2}+a^7\,b\,x+21\,a\,b^2\,d^5\,x^{17}+a\,b\,x^7\left(21\,a^4\,d^2+35\,a^2\,b^3\,d+b^6\right)+7\,a\,d\,x^9\left(a^4\,d^2+10\,a^2\,b^3\,d+b^6\right)+7\,a^3\,b^2\,x^5\left(3\,d\,a^2+b^3\right)+7\,a\,d^4\,x^{15}\left(d\,a^2+5\,b^3\right)+\frac{7\,b\,d^4\,x^{16}\,(12\,d\,a^2+5\,b^3)}{4}+\frac{b\,d\,x^{10}\,(70\,a^4\,d^2+105\,a^2\,b^3\,d+2\,b^6)}{2}+7\,a\,b\,d^6\,x^{19}+\frac{7\,b^2\,d^3\,x^{14}\,(15\,d\,a^2+2\,b^3)}{2}+7\,a\,b^2\,d^2\,x^{11}\left(10\,d\,a^2+3\,b^3\right)+35\,a\,b\,d^3\,x^{13}\left(d\,a^2+b^3\right)$$

```
[In] int((b + 3*d*x^2)*(a + b*x + d*x^3)^7,x)
```

```
[Out] x^12*((35*a^4*d^4)/4 + (7*b^6*d^2)/2 + 70*a^2*b^3*d^3) + x^4*((35*a^4*b^4)/
4 + 7*a^6*b*d) + x^18*((7*a^2*d^6)/2 + 7*b^3*d^5) + x^6*((7*a^2*b^6)/2 + (7
*a^6*d^2)/2 + 35*a^4*b^3*d) + x^8*(b^8/8 + 21*a^2*b^5*d + (105*a^4*b^2*d^2)
/2) + (d^8*x^24)/8 + x^3*(a^7*d + 7*a^5*b^3) + a*d^7*x^21 + b*d^7*x^22 + (7
*a^6*b^2*x^2)/2 + (7*b^2*d^6*x^20)/2 + a^7*b*x + 21*a*b^2*d^5*x^17 + a*b*x^
7*(b^6 + 21*a^4*d^2 + 35*a^2*b^3*d) + 7*a*d*x^9*(b^6 + a^4*d^2 + 10*a^2*b^3
*d) + 7*a^3*b^2*x^5*(3*a^2*d + b^3) + 7*a*d^4*x^15*(a^2*d + 5*b^3) + (7*b*d
^4*x^16*(12*a^2*d + 5*b^3))/4 + (b*d*x^10*(2*b^6 + 70*a^4*d^2 + 105*a^2*b^3
*d))/2 + 7*a*b*d^6*x^19 + (7*b^2*d^3*x^14*(15*a^2*d + 2*b^3))/2 + 7*a*b^2*d
^2*x^11*(10*a^2*d + 3*b^3) + 35*a*b*d^3*x^13*(a^2*d + b^3)
```

3.196 $\int (b+3dx^2)(bx+dx^3)^7\,dx$

Optimal result . . . 1411
Rubi [A] (verified) . . . 1411
Mathematica [B] (verified) . . . 1412
Maple [A] (verified) . . . 1412
Fricas [B] (verification not implemented) . . . 1412
Sympy [B] (verification not implemented) . . . 1413
Maxima [A] (verification not implemented) . . . 1413
Giac [A] (verification not implemented) . . . 1413
Mupad [B] (verification not implemented) . . . 1414

Optimal result

Integrand size = 20, antiderivative size = 15

$$\int (b+3dx^2)\,(bx+dx^3)^7\,dx=\frac{1}{8}(bx+dx^3)^8$$

```
[Out] 1/8*(d*x^3+b*x)^8
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1602}

$$\int (b+3dx^2)\,(bx+dx^3)^7\,dx=\frac{1}{8}(bx+dx^3)^8$$

```
[In] Int[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]
[Out] (b*x + d*x^3)^8/8
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{8}(bx+dx^3)^8$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(15) = 30.

Time = 0.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 6.53

$$\int (b+3dx^2)(bx+dx^3)^7\,dx = \frac{b^8x^8}{8}+b^7dx^{10}+\frac{7}{2}b^6d^2x^{12}+7b^5d^3x^{14}+\frac{35}{4}b^4d^4x^{16}+7b^3d^5x^{18}+\frac{7}{2}b^2d^6x^{20}+bd^7x^{22}+\frac{d^8x^{24}}{8}$$

[In] Integrate[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]

[Out] (b^8*x^8)/8 + b^7*d*x^10 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 + (35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2 + b*d^7*x^22 + (d^8*x^24)/8

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result
default	$\frac{(x^3d+bx)^8}{8}$
gosper	$\frac{x^8(d\,x^2+b)^8}{8}$
norman	$\frac{1}{8}d^8x^{24}+x^{22}b\,d^7+7x^{18}b^3d^5+\frac{7}{2}x^{20}b^2d^6+7x^{14}b^5d^3+\frac{35}{4}x^{16}b^4d^4+\frac{1}{8}x^8b^8+x^{10}b^7d+\frac{7}{2}x^{12}b^6d^2$
risch	$\frac{1}{8}d^8x^{24}+x^{22}b\,d^7+7x^{18}b^3d^5+\frac{7}{2}x^{20}b^2d^6+7x^{14}b^5d^3+\frac{35}{4}x^{16}b^4d^4+\frac{1}{8}x^8b^8+x^{10}b^7d+\frac{7}{2}x^{12}b^6d^2$
parallelrisch	$\frac{1}{8}d^8x^{24}+x^{22}b\,d^7+7x^{18}b^3d^5+\frac{7}{2}x^{20}b^2d^6+7x^{14}b^5d^3+\frac{35}{4}x^{16}b^4d^4+\frac{1}{8}x^8b^8+x^{10}b^7d+\frac{7}{2}x^{12}b^6d^2$

[In] int((3*d*x^2+b)*(d*x^3+b*x)^7,x,method=_RETURNVERBOSE)

[Out] 1/8*(d*x^3+b*x)^8

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.87

$$\int (b+3dx^2)(bx+dx^3)^7\,dx = \frac{1}{8}d^8x^{24}+bd^7x^{22}+\frac{7}{2}b^2d^6x^{20}+7b^3d^5x^{18}+\frac{35}{4}b^4d^4x^{16}+7b^5d^3x^{14}+\frac{7}{2}b^6d^2x^{12}+b^7dx^{10}+\frac{1}{8}b^8x^8$$

[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="fricas")

[Out] 1/8*d^8*x^24 + b*d^7*x^22 + 7/2*b^2*d^6*x^20 + 7*b^3*d^5*x^18 + 35/4*b^4*d^4*x^16 + 7*b^5*d^3*x^14 + 7/2*b^6*d^2*x^12 + b^7*d*x^10 + 1/8*b^8*x^8

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.47

$$\int (b+3dx^2)\,(bx+dx^3)^7\,dx = \frac{b^8x^8}{8} + b^7dx^{10} + \frac{7b^6d^2x^{12}}{2} + 7b^5d^3x^{14} + \frac{35b^4d^4x^{16}}{4} + 7b^3d^5x^{18} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8}$$

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x)**7,x)
[Out] b**8*x**8/8 + b**7*d*x**10 + 7*b**6*d**2*x**12/2 + 7*b**5*d**3*x**14 + 35*b
**4*d**4*x**16/4 + 7*b**3*d**5*x**18 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 +
 d**8*x**24/8
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int (b+3dx^2)\,(bx+dx^3)^7\,dx = \frac{1}{8}\,(dx^3+bx)^8$$

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="maxima")
[Out] 1/8*(d*x^3 + b*x)^8
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int (b+3dx^2)\,(bx+dx^3)^7\,dx = \frac{1}{8}\,(dx^3+bx)^8$$

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="giac")
[Out] 1/8*(d*x^3 + b*x)^8
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.87

$$\int \left(b+3dx^2\right)\left(bx+dx^3\right)^7\,dx=\frac{b^8\,x^8}{8}+b^7\,d\,x^{10}+\frac{7\,b^6\,d^2\,x^{12}}{2}+7\,b^5\,d^3\,x^{14}+\frac{35\,b^4\,d^4\,x^{16}}{4}+7\,b^3\,d^5\,x^{18}+\frac{7\,b^2\,d^6\,x^{20}}{2}+b\,d^7\,x^{22}+\frac{d^8\,x^{24}}{8}$$

```
[In] int((b*x + d*x^3)^7*(b + 3*d*x^2),x)
[Out] (b^8*x^8)/8 + (d^8*x^24)/8 + b^7*d*x^10 + b*d^7*x^22 + (7*b^6*d^2*x^12)/2 +
 7*b^5*d^3*x^14 + (35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2
```

3.197 $\int x^7(b+dx^2)^7(b+3dx^2)\,dx$

Optimal result . . . 1415
Rubi [A] (verified) . . . 1415
Mathematica [B] (verified) . . . 1416
Maple [B] (verified) . . . 1416
Fricas [B] (verification not implemented) . . . 1417
Sympy [B] (verification not implemented) . . . 1417
Maxima [B] (verification not implemented) . . . 1417
Giac [A] (verification not implemented) . . . 1418
Mupad [B] (verification not implemented) . . . 1418

Optimal result

Integrand size = 21, antiderivative size = 16

$$\int x^7(b+dx^2)^7(b+3dx^2)\,dx=\frac{1}{8}x^8(b+dx^2)^8$$

```
[Out] 1/8*x^8*(d*x^2+b)^8
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {457, 75}

$$\int x^7(b+dx^2)^7(b+3dx^2)\,dx=\frac{1}{8}x^8(b+dx^2)^8$$

```
[In] Int[x^7*(b + d*x^2)^7*(b + 3*d*x^2),x]
```

```
[Out] (x^8*(b + d*x^2)^8)/8
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^3(b+dx)^7(b+3dx)\,dx, x, x^2\right)\\ &= \frac{1}{8}x^8\left(b+dx^2\right)^8\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. $2(16) = 32$.

Time = 0.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 6.12

$$\int x^7\left(b+dx^2\right)^7\left(b+3dx^2\right)\,dx = \frac{b^8x^8}{8} + b^7dx^{10} + \frac{7}{2}b^6d^2x^{12} + 7b^5d^3x^{14} + \frac{35}{4}b^4d^4x^{16} + 7b^3d^5x^{18} + \frac{7}{2}b^2d^6x^{20} + bd^7x^{22} + \frac{d^8x^{24}}{8}$$

```
[In] Integrate[x^7*(b + d*x^2)^7*(b + 3*d*x^2),x]
```

```
[Out] (b^8*x^8)/8 + b^7*d*x^10 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 + (35*b^4*d^
4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2 + b*d^7*x^22 + (d^8*x^24)/8
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(14) = 28$.

Time = 0.77 (sec) , antiderivative size = 89, normalized size of antiderivative = 5.56

method	result
gosper	$\frac{1}{8}d^8x^{24} + x^{22}b\,d^7 + 7x^{18}b^3d^5 + \frac{7}{2}x^{20}b^2d^6 + 7x^{14}b^5d^3 + \frac{35}{4}x^{16}b^4d^4 + \frac{1}{8}x^8b^8 + x^{10}b^7d + \frac{7}{2}x^{12}b^6d^2$
default	$\frac{1}{8}d^8x^{24} + x^{22}b\,d^7 + 7x^{18}b^3d^5 + \frac{7}{2}x^{20}b^2d^6 + 7x^{14}b^5d^3 + \frac{35}{4}x^{16}b^4d^4 + \frac{1}{8}x^8b^8 + x^{10}b^7d + \frac{7}{2}x^{12}b^6d^2$
norman	$\frac{1}{8}d^8x^{24} + x^{22}b\,d^7 + 7x^{18}b^3d^5 + \frac{7}{2}x^{20}b^2d^6 + 7x^{14}b^5d^3 + \frac{35}{4}x^{16}b^4d^4 + \frac{1}{8}x^8b^8 + x^{10}b^7d + \frac{7}{2}x^{12}b^6d^2$
risch	$\frac{1}{8}d^8x^{24} + x^{22}b\,d^7 + 7x^{18}b^3d^5 + \frac{7}{2}x^{20}b^2d^6 + 7x^{14}b^5d^3 + \frac{35}{4}x^{16}b^4d^4 + \frac{1}{8}x^8b^8 + x^{10}b^7d + \frac{7}{2}x^{12}b^6d^2$
parallelrisch	$\frac{1}{8}d^8x^{24} + x^{22}b\,d^7 + 7x^{18}b^3d^5 + \frac{7}{2}x^{20}b^2d^6 + 7x^{14}b^5d^3 + \frac{35}{4}x^{16}b^4d^4 + \frac{1}{8}x^8b^8 + x^{10}b^7d + \frac{7}{2}x^{12}b^6d^2$

```
[In] int(x^7*(d*x^2+b)^7*(3*d*x^2+b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*d^8*x^24+x^22*b*d^7+7*x^18*b^3*d^5+7/2*x^20*b^2*d^6+7*x^14*b^5*d^3+35/4
*x^16*b^4*d^4+1/8*x^8*b^8+x^10*b^7*d+7/2*x^12*b^6*d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.50

$$\int x^7 (b+dx^2)^7 (b+3dx^2) \, dx = \frac{1}{8} d^8x^{24} + bd^7x^{22} + \frac{7}{2} b^2d^6x^{20} + 7 b^3d^5x^{18} + \frac{35}{4} b^4d^4x^{16} + 7 b^5d^3x^{14} + \frac{7}{2} b^6d^2x^{12} + b^7dx^{10} + \frac{1}{8} b^8x^8$$

```
[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + b*d^7*x^22 + 7/2*b^2*d^6*x^20 + 7*b^3*d^5*x^18 + 35/4*b^4*d^
4*x^16 + 7*b^5*d^3*x^14 + 7/2*b^6*d^2*x^12 + b^7*d*x^10 + 1/8*b^8*x^8
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.06

$$\int x^7 (b+dx^2)^7 (b+3dx^2) \, dx = \frac{b^8x^8}{8} + b^7dx^{10} + \frac{7b^6d^2x^{12}}{2} + 7b^5d^3x^{14} + \frac{35b^4d^4x^{16}}{4} + 7b^3d^5x^{18} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8}$$

```
[In] integrate(x**7*(d*x**2+b)**7*(3*d*x**2+b),x)
```

```
[Out] b**8*x**8/8 + b**7*d*x**10 + 7*b**6*d**2*x**12/2 + 7*b**5*d**3*x**14 + 35*b
**4*d**4*x**16/4 + 7*b**3*d**5*x**18 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 +
 d**8*x**24/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.50

$$\int x^7 (b+dx^2)^7 (b+3dx^2) \, dx = \frac{1}{8} d^8x^{24} + bd^7x^{22} + \frac{7}{2} b^2d^6x^{20} + 7 b^3d^5x^{18} + \frac{35}{4} b^4d^4x^{16} + 7 b^5d^3x^{14} + \frac{7}{2} b^6d^2x^{12} + b^7dx^{10} + \frac{1}{8} b^8x^8$$

```
[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + b*d^7*x^22 + 7/2*b^2*d^6*x^20 + 7*b^3*d^5*x^18 + 35/4*b^4*d^
4*x^16 + 7*b^5*d^3*x^14 + 7/2*b^6*d^2*x^12 + b^7*d*x^10 + 1/8*b^8*x^8
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int x^7 \left(b + dx^2\right)^7 \left(b + 3dx^2\right) \, dx = \frac{1}{8} \left(dx^3 + bx\right)^8$$

```
[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="giac")
[Out] 1/8*(d*x^3 + b*x)^8
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.50

$$\int x^7 \left(b + dx^2\right)^7 \left(b + 3dx^2\right) \, dx = \frac{b^8 \, x^8}{8} + b^7 \, d \, x^{10} + \frac{7 \, b^6 \, d^2 \, x^{12}}{2} + 7 \, b^5 \, d^3 \, x^{14} + \frac{35 \, b^4 \, d^4 \, x^{16}}{4} + 7 \, b^3 \, d^5 \, x^{18} + \frac{7 \, b^2 \, d^6 \, x^{20}}{2} + b \, d^7 \, x^{22} + \frac{d^8 \, x^{24}}{8}$$

```
[In] int(x^7*(b + d*x^2)^7*(b + 3*d*x^2),x)
[Out] (b^8*x^8)/8 + (d^8*x^24)/8 + b^7*d*x^10 + b*d^7*x^22 + (7*b^6*d^2*x^12)/2 +
 7*b^5*d^3*x^14 + (35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2
```

3.198 $\int (2cx+3dx^2)(a+cx^2+dx^3)^7\,dx$

Optimal result 1419
Rubi [A] (verified) 1419
Mathematica [B] (verified) 1420
Maple [A] (verified) 1420
Fricas [B] (verification not implemented) 1420
Sympy [B] (verification not implemented) 1422
Maxima [A] (verification not implemented) 1423
Giac [B] (verification not implemented) 1423
Mupad [B] (verification not implemented) 1424

Optimal result

Integrand size = 26, antiderivative size = 18

$$\int (2cx+3dx^2)(a+cx^2+dx^3)^7\,dx=\frac{1}{8}(a+cx^2+dx^3)^8$$

```
[Out] 1/8*(d*x^3+c*x^2+a)^8
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1602}

$$\int (2cx+3dx^2)(a+cx^2+dx^3)^7\,dx=\frac{1}{8}(a+cx^2+dx^3)^8$$

```
[In] Int[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] (a + c*x^2 + d*x^3)^8/8
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{8}(a+cx^2+dx^3)^8$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 115 vs. 2(18) = 36.

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 6.39

$$\int (2cx+3dx^2)\left(a+cx^2+dx^3\right)^7\,dx=\frac{1}{8}x^2(c+dx)\left(8a^7+28a^6x^2(c+dx)+56a^5x^4(c+dx)^2+70a^4x^6(c+dx)^3+56a^3x^8(c+dx)^4+28a^2x^{10}(c+dx)^5+8ax^{12}(c+dx)^6+x^{14}(c+dx)^7\right)$$

```
[In] Integrate[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] (x^2*(c + d*x)*(8*a^7 + 28*a^6*x^2*(c + d*x) + 56*a^5*x^4*(c + d*x)^2 + 70*
a^4*x^6*(c + d*x)^3 + 56*a^3*x^8*(c + d*x)^4 + 28*a^2*x^10*(c + d*x)^5 + 8*
a*x^12*(c + d*x)^6 + x^14*(c + d*x)^7))/8
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(x^3d+c\,x^2+a\right)^8}{8}$
norman	$\frac{7x^{22}c^2d^6}{2}+c\,d^7x^{23}+\frac{d^8x^{24}}{8}+\left(7ac\,d^6+\frac{35}{4}c^4d^4\right)x^{20}+\left(d^7a+7c^3d^5\right)x^{21}+\left(21a\,c^2d^5+7c^5d^3\right)x^{19}$ …
parallelrisch	$\frac{7}{2}x^4a^6c^2+\frac{7}{2}x^6a^6d^2+7x^6a^5c^3+x^2c\,a^7+35a^4c^3d\,x^9+35a^4c\,d^3x^{11}+35a^3c^4d\,x^{11}+c\,d^7x^{23}+$ …
gosper	$x^2\left(d^8x^{22}+8c\,d^7x^{21}+28x^{20}c^2d^6+8a\,d^7x^{19}+56c^3d^5x^{19}+56x^{18}ac\,d^6+70x^{18}c^4d^4+168a\,c^2d^5x^{17}+56c^5d^3x^{17}+28x^{16}a^2d^6+280x^{16}\right.$ …
risch	$\frac{7}{2}x^4a^6c^2+\frac{7}{2}x^6a^6d^2+7x^6a^5c^3+x^2c\,a^7+35a^4c^3d\,x^9+35a^4c\,d^3x^{11}+35a^3c^4d\,x^{11}+c\,d^7x^{23}+$ …

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(d*x^3+c*x^2+a)^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(16) = 32.

Time = 0.25 (sec) , antiderivative size = 458, normalized size of antiderivative = 25.44

$$\int (2cx + 3dx^2) (a + cx^2 + dx^3)^7 \, dx = \frac{1}{8} d^8 x^{24} + cd^7 x^{23} + \frac{7}{2} c^2 d^6 x^{22} + (7 c^3 d^5 + a d^7) x^{21} + \frac{7}{4} (5 c^4 d^4 + 4 a c d^6) x^{20} + 7 (c^5 d^3 + 3 a c^2 d^5) x^{19} + \frac{7}{2} (c^6 d^2 + 10 a c^3 d^4 + a^2 d^6) x^{18} + (c^7 d + 35 a c^4 d^3 + 21 a^2 c d^5) x^{17} + \frac{1}{8} (c^8 + 168 a c^5 d^2 + 420 a^2 c^2 d^4) x^{16} + 7 (a c^6 d + 10 a^2 c^3 d^3 + a^3 d^5) x^{15} + 21 a^5 c^2 d x^7 + \frac{1}{2} (2 a c^7 + 105 a^2 c^4 d^2 + 70 a^3 c d^4) x^{14} + 7 (3 a^2 c^5 d + 10 a^3 c^2 d^3) x^{13} + 7 a^6 c d x^5 + \frac{7}{4} (2 a^2 c^6 + 40 a^3 c^3 d^2 + 5 a^4 d^4) x^{12} + \frac{7}{2} a^6 c^2 x^4 + 35 (a^3 c^4 d + a^4 c d^3) x^{11} + a^7 d x^3 + \frac{7}{2} (2 a^3 c^5 + 15 a^4 c^2 d^2) x^{10} + a^7 c x^2 + 7 (5 a^4 c^3 d + a^5 d^3) x^9 + \frac{7}{4} (5 a^4 c^4 + 12 a^5 c d^2) x^8 + \frac{7}{2} (2 a^5 c^3 + a^6 d^2) x^6$$

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + (7*c^3*d^5 + a*d^7)*x^21 + 7
/4*(5*c^4*d^4 + 4*a*c*d^6)*x^20 + 7*(c^5*d^3 + 3*a*c^2*d^5)*x^19 + 7/2*(c^6
*d^2 + 10*a*c^3*d^4 + a^2*d^6)*x^18 + (c^7*d + 35*a*c^4*d^3 + 21*a^2*c*d^5)
*x^17 + 1/8*(c^8 + 168*a*c^5*d^2 + 420*a^2*c^2*d^4)*x^16 + 7*(a*c^6*d + 10*
a^2*c^3*d^3 + a^3*d^5)*x^15 + 21*a^5*c^2*d*x^7 + 1/2*(2*a*c^7 + 105*a^2*c^4
*d^2 + 70*a^3*c*d^4)*x^14 + 7*(3*a^2*c^5*d + 10*a^3*c^2*d^3)*x^13 + 7*a^6*c
*d*x^5 + 7/4*(2*a^2*c^6 + 40*a^3*c^3*d^2 + 5*a^4*d^4)*x^12 + 7/2*a^6*c^2*x^
4 + 35*(a^3*c^4*d + a^4*c*d^3)*x^11 + a^7*d*x^3 + 7/2*(2*a^3*c^5 + 15*a^4*c
^2*d^2)*x^10 + a^7*c*x^2 + 7*(5*a^4*c^3*d + a^5*d^3)*x^9 + 7/4*(5*a^4*c^4 +
 12*a^5*c*d^2)*x^8 + 7/2*(2*a^5*c^3 + a^6*d^2)*x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(14) = 28$.

Time = 0.08 (sec) , antiderivative size = 484, normalized size of antiderivative = 26.89

$$\int (2cx + 3dx^2)(a + cx^2 + dx^3)^7 dx = a^7cx^2 + a^7dx^3 + \frac{7a^6c^2x^4}{2} + 7a^6cdx^5 + 21a^5c^2dx^7 + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8} + x^{21}(ad^7 + 7c^3d^5) + x^{20} \cdot \left(7acd^6 + \frac{35c^4d^4}{4}\right) + x^{19} \cdot (21ac^2d^5 + 7c^5d^3) + x^{18} \cdot \left(\frac{7a^2d^6}{2} + 35ac^3d^4 + \frac{7c^6d^2}{2}\right) + x^{17} \cdot (21a^2cd^5 + 35ac^4d^3 + c^7d) + x^{16} \cdot \left(\frac{105a^2c^2d^4}{2} + 21ac^5d^2 + \frac{c^8}{8}\right) + x^{15} \cdot (7a^3d^5 + 70a^2c^3d^3 + 7ac^6d) + x^{14} \cdot \left(35a^3cd^4 + \frac{105a^2c^4d^2}{2} + ac^7\right) + x^{13} \cdot (70a^3c^2d^3 + 21a^2c^5d) + x^{12} \cdot \left(\frac{35a^4d^4}{4} + 70a^3c^3d^2 + \frac{7a^2c^6}{2}\right) + x^{11} \cdot (35a^4cd^3 + 35a^3c^4d) + x^{10} \cdot \left(\frac{105a^4c^2d^2}{2} + 7a^3c^5\right) + x^9 \cdot (7a^5d^3 + 35a^4c^3d) + x^8 \cdot \left(21a^5cd^2 + \frac{35a^4c^4}{4}\right) + x^6 \cdot \left(\frac{7a^6d^2}{2} + 7a^5c^3\right)$$

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**7,x)
```

```
[Out] a**7*c*x**2 + a**7*d*x**3 + 7*a**6*c**2*x**4/2 + 7*a**6*c*d*x**5 + 21*a**5*
c**2*d*x**7 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8 + x**21*(a*
d**7 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 35*c**4*d**4/4) + x**19*(21*a*c**
2*d**5 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 35*a*c**3*d**4 + 7*c**6*d**2
/2) + x**17*(21*a**2*c*d**5 + 35*a*c**4*d**3 + c**7*d) + x**16*(105*a**2*c*
*2*d**4/2 + 21*a*c**5*d**2 + c**8/8) + x**15*(7*a**3*d**5 + 70*a**2*c**3*d*
*3 + 7*a*c**6*d) + x**14*(35*a**3*c*d**4 + 105*a**2*c**4*d**2/2 + a*c**7) +
 x**13*(70*a**3*c**2*d**3 + 21*a**2*c**5*d) + x**12*(35*a**4*d**4/4 + 70*a*
*3*c**3*d**2 + 7*a**2*c**6/2) + x**11*(35*a**4*c*d**3 + 35*a**3*c**4*d) + x
**10*(105*a**4*c**2*d**2/2 + 7*a**3*c**5) + x**9*(7*a**5*d**3 + 35*a**4*c**
3*d) + x**8*(21*a**5*c*d**2 + 35*a**4*c**4/4) + x**6*(7*a**6*d**2/2 + 7*a**
5*c**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int (2cx + 3dx^2) (a + cx^2 + dx^3)^7 \, dx = \frac{1}{8} (dx^3 + cx^2 + a)^8$$

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="maxima")
[Out] 1/8*(d*x^3 + c*x^2 + a)^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(16) = 32.

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 7.56

$$\begin{aligned}\int (2cx + 3dx^2) (a + cx^2 + dx^3)^7 \, dx = {} & \frac{1}{8} (dx^3 + cx^2)^8 + (dx^3 + cx^2)^7 a \\ & + \frac{7}{2} (dx^3 + cx^2)^6 a^2 + 7 (dx^3 + cx^2)^5 a^3 \\ & + \frac{35}{4} (dx^3 + cx^2)^4 a^4 + 7 (dx^3 + cx^2)^3 a^5 \\ & + \frac{7}{2} (dx^3 + cx^2)^2 a^6 + (dx^3 + cx^2) a^7\end{aligned}$$

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="giac")
[Out] 1/8*(d*x^3 + c*x^2)^8 + (d*x^3 + c*x^2)^7*a + 7/2*(d*x^3 + c*x^2)^6*a^2 + 7
*(d*x^3 + c*x^2)^5*a^3 + 35/4*(d*x^3 + c*x^2)^4*a^4 + 7*(d*x^3 + c*x^2)^3*a
^5 + 7/2*(d*x^3 + c*x^2)^2*a^6 + (d*x^3 + c*x^2)*a^7
```

Mupad [B] (verification not implemented)

Time = 9.58 (sec) , antiderivative size = 440, normalized size of antiderivative = 24.44

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^7\,dx = x^{12}\left(\frac{35\,a^4\,d^4}{4}+70\,a^3\,c^3\,d^2+\frac{7\,a^2\,c^6}{2}\right) + x^6\left(\frac{7\,a^6\,d^2}{2}+7\,a^5\,c^3\right)+x^{20}\left(\frac{35\,c^4\,d^4}{4}+7\,a\,c\,d^6\right) + x^{16}\left(\frac{105\,a^2\,c^2\,d^4}{2}+21\,a\,c^5\,d^2+\frac{c^8}{8}\right) + x^{18}\left(\frac{7\,a^2\,d^6}{2}+35\,a\,c^3\,d^4+\frac{7\,c^6\,d^2}{2}\right) + \frac{d^8\,x^{24}}{8}+x^{21}\left(7\,c^3\,d^5+a\,d^7\right)+a^7\,c\,x^2 + a^7\,d\,x^3+c\,d^7\,x^{23}+\frac{7\,a^6\,c^2\,x^4}{2}+\frac{7\,c^2\,d^6\,x^{22}}{2} + 21\,a^5\,c^2\,d\,x^7+7\,a\,d\,x^{15}\left(a^2\,d^4+10\,a\,c^3\,d^2+c^6\right) + c\,d\,x^{17}\left(21\,a^2\,d^4+35\,a\,c^3\,d^2+c^6\right) + \frac{7\,a^4\,c\,x^8\,(5\,c^3+12\,a\,d^2)}{4} + 7\,a^4\,d\,x^9\left(5\,c^3+a\,d^2\right)+7\,c^2\,d^3\,x^{19}\left(c^3+3\,a\,d^2\right) + \frac{a\,c\,x^{14}\,(70\,a^2\,d^4+105\,a\,c^3\,d^2+2\,c^6)}{2} + 7\,a^6\,c\,d\,x^5+\frac{7\,a^3\,c^2\,x^{10}\,(2\,c^3+15\,a\,d^2)}{2} + 7\,a^2\,c^2\,d\,x^{13}\left(3\,c^3+10\,a\,d^2\right)+35\,a^3\,c\,d\,x^{11}\left(c^3+a\,d^2\right)$$

```
[In] int((2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x)
```

```
[Out] x^12*((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + 70*a^3*c^3*d^2) + x^6*(7*a^5*c^3 + (
7*a^6*d^2)/2) + x^20*((35*c^4*d^4)/4 + 7*a*c*d^6) + x^16*(c^8/8 + 21*a*c^5*
d^2 + (105*a^2*c^2*d^4)/2) + x^18*((7*a^2*d^6)/2 + (7*c^6*d^2)/2 + 35*a*c^3
*d^4) + (d^8*x^24)/8 + x^21*(a*d^7 + 7*c^3*d^5) + a^7*c*x^2 + a^7*d*x^3 + c
*d^7*x^23 + (7*a^6*c^2*x^4)/2 + (7*c^2*d^6*x^22)/2 + 21*a^5*c^2*d*x^7 + 7*a
*d*x^15*(c^6 + a^2*d^4 + 10*a*c^3*d^2) + c*d*x^17*(c^6 + 21*a^2*d^4 + 35*a*
c^3*d^2) + (7*a^4*c*x^8*(12*a*d^2 + 5*c^3))/4 + 7*a^4*d*x^9*(a*d^2 + 5*c^3)
 + 7*c^2*d^3*x^19*(3*a*d^2 + c^3) + (a*c*x^14*(2*c^6 + 70*a^2*d^4 + 105*a*c
^3*d^2))/2 + 7*a^6*c*d*x^5 + (7*a^3*c^2*x^10*(15*a*d^2 + 2*c^3))/2 + 7*a^2*
c^2*d*x^13*(10*a*d^2 + 3*c^3) + 35*a^3*c*d*x^11*(a*d^2 + c^3)
```

3.199 $\int (2cx+3dx^2)(cx^2+dx^3)^7 \, dx$

Optimal result . . . 1425
Rubi [A] (verified) . . . 1425
Mathematica [B] (verified) . . . 1426
Maple [A] (verified) . . . 1426
Fricas [B] (verification not implemented) . . . 1426
Sympy [B] (verification not implemented) . . . 1427
Maxima [A] (verification not implemented) . . . 1427
Giac [A] (verification not implemented) . . . 1427
Mupad [B] (verification not implemented) . . . 1428

Optimal result

Integrand size = 25, antiderivative size = 17

$$\int (2cx+3dx^2)(cx^2+dx^3)^7 \, dx = \frac{1}{8}(cx^2+dx^3)^8$$

```
[Out] 1/8*(d*x^3+c*x^2)^8
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1602}

$$\int (2cx+3dx^2)(cx^2+dx^3)^7 \, dx = \frac{1}{8}(cx^2+dx^3)^8$$

```
[In] Int[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]
[Out] (c*x^2 + d*x^3)^8/8
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{8}(cx^2+dx^3)^8$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(17) = 34.

Time = 0.01 (sec) , antiderivative size = 98, normalized size of antiderivative = 5.76

$$\int (2cx+3dx^2)\left(cx^2+dx^3\right)^7\,dx=\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

```
[In] Integrate[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result
gosper	$\frac{x^{16}(dx+c)^8}{8}$
default	$\frac{(x^3d+c\,x^2)^8}{8}$
norman	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x^2$
risch	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x^2$
parallelrisch	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x^2$

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^16*(d*x+c)^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(15) = 30.

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.18

$$\int (2cx+3dx^2)\left(cx^2+dx^3\right)^7\,dx=\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(12) = 24$.

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 5.71

$$\int (2cx + 3dx^2) (cx^2 + dx^3)^7 \, dx = \frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**7,x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int (2cx + 3dx^2) (cx^2 + dx^3)^7 \, dx = \frac{1}{8} (dx^3 + cx^2)^8$$

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int (2cx + 3dx^2) (cx^2 + dx^3)^7 \, dx = \frac{1}{8} (dx^3 + cx^2)^8$$

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8
```

Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.18

$$\int (2cx+3dx^2) (cx^2+dx^3)^7 dx = \frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7 c^6 d^2 x^{18}}{2} + 7 c^5 d^3 x^{19} + \frac{35 c^4 d^4 x^{20}}{4} + 7 c^3 d^5 x^{21} + \frac{7 c^2 d^6 x^{22}}{2} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

```
[In] int((2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x)

[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

3.200 $\int x^7(cx+dx^2)^7\,(2cx+3dx^2)\;dx$

Optimal result 1429
Rubi [A] (verified) 1429
Mathematica [B] (verified) 1430
Maple [A] (verified) 1430
Fricas [B] (verification not implemented) 1431
Sympy [B] (verification not implemented) 1431
Maxima [B] (verification not implemented) 1431
Giac [A] (verification not implemented) 1432
Mupad [B] (verification not implemented) 1432

Optimal result

Integrand size = 26, antiderivative size = 14

$$\int x^7\left(cx+dx^2\right)^7\left(2cx+3dx^2\right)\,dx=\frac{1}{8}x^{16}(c+dx)^8$$

```
[Out] 1/8*x^16*(d*x+c)^8
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1598, 859}

$$\int x^7\left(cx+dx^2\right)^7\left(2cx+3dx^2\right)\,dx=\frac{1}{8}x^{16}(c+dx)^8$$

```
[In] Int[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rule 859

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((b_.)*(x_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[c*x^(m + 2)*((f + g*x)^(n + 1)/(g*(m + n + 3))), x] /; FreeQ[{
b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n
 + 3, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx\\ &= \frac{1}{8}x^{16}(c+dx)^8\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(14) = 28.

Time = 0.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 7.00

$$\int x^7\left(cx+dx^2\right)^7\left(2cx+3dx^2\right)\,dx=\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

```
[In] Integrate[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result
gosper	$\frac{x^{16}(dx+c)^8}{8}$
default	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x^2$
norman	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x^2$
risch	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x^2$
parallelrisch	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x^2$

```
[In] int(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^16*(d*x+c)^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^7 (cx+dx^2)^7 (2cx+3dx^2)\,dx = \frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

```
[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.93

$$\int x^7 (cx+dx^2)^7 (2cx+3dx^2)\,dx = \frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

```
[In] integrate(x**7*(d*x**2+c*x)**7*(3*d*x**2+2*c*x),x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^7 (cx+dx^2)^7 (2cx+3dx^2)\,dx = \frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

```
[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int x^7 (cx + dx^2)^7 (2cx + 3dx^2) \, dx = \frac{1}{8} (dx^3 + cx^2)^8$$

```
[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^7 (cx + dx^2)^7 (2cx + 3dx^2) \, dx = \frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7 c^6 d^2 x^{18}}{2} + 7 c^5 d^3 x^{19} + \frac{35 c^4 d^4 x^{20}}{4} + 7 c^3 d^5 x^{21} + \frac{7 c^2 d^6 x^{22}}{2} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

```
[In] int(x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

3.201 $\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx$

Optimal result 1433
Rubi [A] (verified) 1433
Mathematica [B] (verified) 1434
Maple [B] (verified) 1434
Fricas [B] (verification not implemented) 1435
Sympy [B] (verification not implemented) 1435
Maxima [B] (verification not implemented) 1435
Giac [A] (verification not implemented) 1436
Mupad [B] (verification not implemented) 1436

Optimal result

Integrand size = 22, antiderivative size = 14

$$\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx=\frac{1}{8}x^{16}(c+dx)^8$$

```
[Out] 1/8*x^16*(d*x+c)^8
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {859}

$$\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx=\frac{1}{8}x^{16}(c+dx)^8$$

```
[In] Int[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rule 859

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((b_.)*(x_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[c*x^(m + 2)*((f + g*x)^(n + 1)/(g*(m + n + 3))), x] /; FreeQ[{
b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n
 + 3, 0]
```

Rubi steps

$$\text{integral}=\frac{1}{8}x^{16}(c+dx)^8$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. $2(14) = 28$.

Time = 0.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 7.00

$$\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx=\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

[In] Integrate[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2),x]

[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/8

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(12) = 24$.

Time = 0.72 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

method	result
gosper	$\frac{x^{16}\left(d^8x^8+8c\,d^7x^7+28x^6c^2d^6+56c^3d^5x^5+70x^4c^4d^4+56c^5d^3x^3+28x^2c^6d^2+8c^7dx+c^8\right)}{8}$
default	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x^2$
norman	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x^2$
risch	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x^2$
parallelrisch	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x^2$

[In] int(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x,method=_RETURNVERBOSE)

[Out] 1/8*x^16*(d^8*x^8+8*c*d^7*x^7+28*c^2*d^6*x^6+56*c^3*d^5*x^5+70*c^4*d^4*x^4+56*c^5*d^3*x^3+28*c^6*d^2*x^2+8*c^7*d*x+c^8)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(12) = 24.

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx = \frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

```
[In] integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(10) = 20.

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.93

$$\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx = \frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

```
[In] integrate(x**14*(d*x+c)**7*(3*d*x**2+2*c*x),x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(12) = 24.

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx = \frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

```
[In] integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx=\frac{1}{8}\left(dx^3+cx^2\right)^8$$

```
[In] integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx=\frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7\,c^6\,d^2\,x^{18}}{2}+7\,c^5\,d^3\,x^{19}+\frac{35\,c^4\,d^4\,x^{20}}{4}+7\,c^3\,d^5\,x^{21}+\frac{7\,c^2\,d^6\,x^{22}}{2}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

```
[In] int(x^14*(2*c*x + 3*d*x^2)*(c + d*x)^7,x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

3.202 $\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx$

Optimal result 1437
Rubi [A] (verified) 1437
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Optimal result

Integrand size = 24, antiderivative size = 18

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

```
[Out] 1/8*(d*x^3+c*x^2+a)^8
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1602}

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

```
[In] Int[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] (a + c*x^2 + d*x^3)^8/8
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 115 vs. 2(18) = 36.

Time = 0.01 (sec) , antiderivative size = 115, normalized size of antiderivative = 6.39

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7 dx = \frac{1}{8}x^2(c+dx)\left(8a^7+28a^6x^2(c+dx)+56a^5x^4(c+dx)^2 + 70a^4x^6(c+dx)^3+56a^3x^8(c+dx)^4 + 28a^2x^{10}(c+dx)^5+8ax^{12}(c+dx)^6+x^{14}(c+dx)^7\right)$$

```
[In] Integrate[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] (x^2*(c + d*x)*(8*a^7 + 28*a^6*x^2*(c + d*x) + 56*a^5*x^4*(c + d*x)^2 + 70*
a^4*x^6*(c + d*x)^3 + 56*a^3*x^8*(c + d*x)^4 + 28*a^2*x^10*(c + d*x)^5 + 8*
a*x^12*(c + d*x)^6 + x^14*(c + d*x)^7))/8
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(x^3d+c\,x^2+a\right)^8}{8}$
norman	$\left(d^7a+7c^3d^5\right)x^{21}+\frac{7x^{22}c^2d^6}{2}+c\,d^7x^{23}+\frac{d^8x^{24}}{8}+\left(35a^3c\,d^4+\frac{105}{2}a^2c^4d^2+a\,c^7\right)x^{14}+\left(7a^3d^5+\right.$
gosper	$\frac{7}{2}x^4a^6c^2+\frac{7}{2}x^6a^6d^2+7x^6a^5c^3+x^2c\,a^7+35a^4c^3d\,x^9+35a^4c\,d^3x^{11}+35a^3c^4d\,x^{11}+c\,d^7x^{23}+$
parallelrisch	$\frac{7}{2}x^4a^6c^2+\frac{7}{2}x^6a^6d^2+7x^6a^5c^3+x^2c\,a^7+35a^4c^3d\,x^9+35a^4c\,d^3x^{11}+35a^3c^4d\,x^{11}+c\,d^7x^{23}+$
risch	$\frac{7}{2}x^4a^6c^2+\frac{7}{2}x^6a^6d^2+7x^6a^5c^3+x^2c\,a^7+35a^4c^3d\,x^9+35a^4c\,d^3x^{11}+35a^3c^4d\,x^{11}+c\,d^7x^{23}+$

```
[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(d*x^3+c*x^2+a)^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(16) = 32.

Time = 0.25 (sec) , antiderivative size = 458, normalized size of antiderivative = 25.44

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx = \frac{1}{8}d^8x^{24}+cd^7x^{23}+\frac{7}{2}c^2d^6x^{22}+\left(7c^3d^5+ad^7\right)x^{21} + \frac{7}{4}\left(5c^4d^4+4acd^6\right)x^{20}+7\left(c^5d^3+3ac^2d^5\right)x^{19} + \frac{7}{2}\left(c^6d^2+10ac^3d^4+a^2d^6\right)x^{18} + \left(c^7d+35ac^4d^3+21a^2cd^5\right)x^{17} + \frac{1}{8}\left(c^8+168ac^5d^2+420a^2c^2d^4\right)x^{16} + 7\left(ac^6d+10a^2c^3d^3+a^3d^5\right)x^{15}+21a^5c^2dx^7 + \frac{1}{2}\left(2ac^7+105a^2c^4d^2+70a^3cd^4\right)x^{14} + 7\left(3a^2c^5d+10a^3c^2d^3\right)x^{13}+7a^6cdx^5 + \frac{7}{4}\left(2a^2c^6+40a^3c^3d^2+5a^4d^4\right)x^{12} + \frac{7}{2}a^6c^2x^4+35\left(a^3c^4d+a^4cd^3\right)x^{11} + a^7dx^3+\frac{7}{2}\left(2a^3c^5+15a^4c^2d^2\right)x^{10} + a^7cx^2+7\left(5a^4c^3d+a^5d^3\right)x^9 + \frac{7}{4}\left(5a^4c^4+12a^5cd^2\right)x^8+\frac{7}{2}\left(2a^5c^3+a^6d^2\right)x^6$$

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + (7*c^3*d^5 + a*d^7)*x^21 + 7
/4*(5*c^4*d^4 + 4*a*c*d^6)*x^20 + 7*(c^5*d^3 + 3*a*c^2*d^5)*x^19 + 7/2*(c^6
*d^2 + 10*a*c^3*d^4 + a^2*d^6)*x^18 + (c^7*d + 35*a*c^4*d^3 + 21*a^2*c*d^5)
*x^17 + 1/8*(c^8 + 168*a*c^5*d^2 + 420*a^2*c^2*d^4)*x^16 + 7*(a*c^6*d + 10*
a^2*c^3*d^3 + a^3*d^5)*x^15 + 21*a^5*c^2*d*x^7 + 1/2*(2*a*c^7 + 105*a^2*c^4
*d^2 + 70*a^3*c*d^4)*x^14 + 7*(3*a^2*c^5*d + 10*a^3*c^2*d^3)*x^13 + 7*a^6*c
*d*x^5 + 7/4*(2*a^2*c^6 + 40*a^3*c^3*d^2 + 5*a^4*d^4)*x^12 + 7/2*a^6*c^2*x^
4 + 35*(a^3*c^4*d + a^4*c*d^3)*x^11 + a^7*d*x^3 + 7/2*(2*a^3*c^5 + 15*a^4*c
^2*d^2)*x^10 + a^7*c*x^2 + 7*(5*a^4*c^3*d + a^5*d^3)*x^9 + 7/4*(5*a^4*c^4 +
 12*a^5*c*d^2)*x^8 + 7/2*(2*a^5*c^3 + a^6*d^2)*x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 484, normalized size of antiderivative = 26.89

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx = a^7cx^2+a^7dx^3+\frac{7a^6c^2x^4}{2}+7a^6cdx^5+21a^5c^2dx^7+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}+x^{21}\left(ad^7+7c^3d^5\right)+x^{20}\cdot\left(7acd^6+\frac{35c^4d^4}{4}\right)+x^{19}\cdot\left(21ac^2d^5+7c^5d^3\right)+x^{18}\cdot\left(\frac{7a^2d^6}{2}+35ac^3d^4+\frac{7c^6d^2}{2}\right)+x^{17}\cdot\left(21a^2cd^5+35ac^4d^3+c^7d\right)+x^{16}\cdot\left(\frac{105a^2c^2d^4}{2}+21ac^5d^2+\frac{c^8}{8}\right)+x^{15}\cdot\left(7a^3d^5+70a^2c^3d^3+7ac^6d\right)+x^{14}\cdot\left(35a^3cd^4+\frac{105a^2c^4d^2}{2}+ac^7\right)+x^{13}\cdot\left(70a^3c^2d^3+21a^2c^5d\right)+x^{12}\cdot\left(\frac{35a^4d^4}{4}+70a^3c^3d^2+\frac{7a^2c^6}{2}\right)+x^{11}\cdot\left(35a^4cd^3+35a^3c^4d\right)+x^{10}\cdot\left(\frac{105a^4c^2d^2}{2}+7a^3c^5\right)+x^9\cdot\left(7a^5d^3+35a^4c^3d\right)+x^8\cdot\left(21a^5cd^2+\frac{35a^4c^4}{4}\right)+x^6\cdot\left(\frac{7a^6d^2}{2}+7a^5c^3\right)$$

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**7,x)
```

```
[Out] a**7*c*x**2 + a**7*d*x**3 + 7*a**6*c**2*x**4/2 + 7*a**6*c*d*x**5 + 21*a**5*
c**2*d*x**7 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8 + x**21*(a*
d**7 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 35*c**4*d**4/4) + x**19*(21*a*c**
2*d**5 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 35*a*c**3*d**4 + 7*c**6*d**2
/2) + x**17*(21*a**2*c*d**5 + 35*a*c**4*d**3 + c**7*d) + x**16*(105*a**2*c*
*2*d**4/2 + 21*a*c**5*d**2 + c**8/8) + x**15*(7*a**3*d**5 + 70*a**2*c**3*d*
*3 + 7*a*c**6*d) + x**14*(35*a**3*c*d**4 + 105*a**2*c**4*d**2/2 + a*c**7) +
 x**13*(70*a**3*c**2*d**3 + 21*a**2*c**5*d) + x**12*(35*a**4*d**4/4 + 70*a*
*3*c**3*d**2 + 7*a**2*c**6/2) + x**11*(35*a**4*c*d**3 + 35*a**3*c**4*d) + x
**10*(105*a**4*c**2*d**2/2 + 7*a**3*c**5) + x**9*(7*a**5*d**3 + 35*a**4*c**
3*d) + x**8*(21*a**5*c*d**2 + 35*a**4*c**4/4) + x**6*(7*a**6*d**2/2 + 7*a**
5*c**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(16) = 32$.

Time = 0.19 (sec) , antiderivative size = 458, normalized size of antiderivative = 25.44

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx = \frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+\left(7\,c^3d^5+ad^7\right)x^{21}+\frac{7}{4}\,\left(5\,c^4d^4+4\,acd^6\right)x^{20}+7\,\left(c^5d^3+3\,ac^2d^5\right)x^{19}+\frac{7}{2}\,\left(c^6d^2+10\,ac^3d^4+a^2d^6\right)x^{18}+\left(c^7d+35\,ac^4d^3+21\,a^2cd^5\right)x^{17}+\frac{1}{8}\,\left(c^8+168\,ac^5d^2+420\,a^2c^2d^4\right)x^{16}+7\,\left(ac^6d+10\,a^2c^3d^3+a^3d^5\right)x^{15}+21\,a^5c^2dx^7+\frac{1}{2}\,\left(2\,ac^7+105\,a^2c^4d^2+70\,a^3cd^4\right)x^{14}+7\,\left(3\,a^2c^5d+10\,a^3c^2d^3\right)x^{13}+7\,a^6cdx^5+\frac{7}{4}\,\left(2\,a^2c^6+40\,a^3c^3d^2+5\,a^4d^4\right)x^{12}+\frac{7}{2}\,a^6c^2x^4+35\,\left(a^3c^4d+a^4cd^3\right)x^{11}+a^7dx^3+\frac{7}{2}\,\left(2\,a^3c^5+15\,a^4c^2d^2\right)x^{10}+a^7cx^2+7\,\left(5\,a^4c^3d+a^5d^3\right)x^9+\frac{7}{4}\,\left(5\,a^4c^4+12\,a^5cd^2\right)x^8+\frac{7}{2}\,\left(2\,a^5c^3+a^6d^2\right)x^6$$

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + (7*c^3*d^5 + a*d^7)*x^21 + 7
/4*(5*c^4*d^4 + 4*a*c*d^6)*x^20 + 7*(c^5*d^3 + 3*a*c^2*d^5)*x^19 + 7/2*(c^6
*d^2 + 10*a*c^3*d^4 + a^2*d^6)*x^18 + (c^7*d + 35*a*c^4*d^3 + 21*a^2*c*d^5)
*x^17 + 1/8*(c^8 + 168*a*c^5*d^2 + 420*a^2*c^2*d^4)*x^16 + 7*(a*c^6*d + 10*
a^2*c^3*d^3 + a^3*d^5)*x^15 + 21*a^5*c^2*d*x^7 + 1/2*(2*a*c^7 + 105*a^2*c^4
*d^2 + 70*a^3*c*d^4)*x^14 + 7*(3*a^2*c^5*d + 10*a^3*c^2*d^3)*x^13 + 7*a^6*c
*d*x^5 + 7/4*(2*a^2*c^6 + 40*a^3*c^3*d^2 + 5*a^4*d^4)*x^12 + 7/2*a^6*c^2*x^
4 + 35*(a^3*c^4*d + a^4*c*d^3)*x^11 + a^7*d*x^3 + 7/2*(2*a^3*c^5 + 15*a^4*c
^2*d^2)*x^10 + a^7*c*x^2 + 7*(5*a^4*c^3*d + a^5*d^3)*x^9 + 7/4*(5*a^4*c^4 +
 12*a^5*c*d^2)*x^8 + 7/2*(2*a^5*c^3 + a^6*d^2)*x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(16) = 32.

Time = 0.29 (sec) , antiderivative size = 488, normalized size of antiderivative = 27.11

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx = \frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+ad^7x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,acd^6x^{20}+7\,c^5d^3x^{19}+21\,ac^2d^5x^{19}+\frac{7}{2}\,c^6d^2x^{18}+35\,ac^3d^4x^{18}+\frac{7}{2}\,a^2d^6x^{18}+c^7dx^{17}+35\,ac^4d^3x^{17}+21\,a^2cd^5x^{17}+\frac{1}{8}\,c^8x^{16}+21\,ac^5d^2x^{16}+\frac{105}{2}\,a^2c^2d^4x^{16}+7\,ac^6dx^{15}+70\,a^2c^3d^3x^{15}+7\,a^3d^5x^{15}+ac^7x^{14}+\frac{105}{2}\,a^2c^4d^2x^{14}+35\,a^3cd^4x^{14}+21\,a^2c^5dx^{13}+70\,a^3c^2d^3x^{13}+\frac{7}{2}\,a^2c^6x^{12}+70\,a^3c^3d^2x^{12}+\frac{35}{4}\,a^4d^4x^{12}+35\,a^3c^4dx^{11}+35\,a^4cd^3x^{11}+7\,a^3c^5x^{10}+\frac{105}{2}\,a^4c^2d^2x^{10}+35\,a^4c^3dx^9+7\,a^5d^3x^9+\frac{35}{4}\,a^4c^4x^8+21\,a^5cd^2x^8+21\,a^5c^2dx^7+7\,a^5c^3x^6+\frac{7}{2}\,a^6d^2x^6+7\,a^6cdx^5+\frac{7}{2}\,a^6c^2x^4+a^7dx^3+a^7cx^2$$

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + a*d^7*x^21
+ 35/4*c^4*d^4*x^20 + 7*a*c*d^6*x^20 + 7*c^5*d^3*x^19 + 21*a*c^2*d^5*x^19 +
 7/2*c^6*d^2*x^18 + 35*a*c^3*d^4*x^18 + 7/2*a^2*d^6*x^18 + c^7*d*x^17 + 35*
a*c^4*d^3*x^17 + 21*a^2*c*d^5*x^17 + 1/8*c^8*x^16 + 21*a*c^5*d^2*x^16 + 105
/2*a^2*c^2*d^4*x^16 + 7*a*c^6*d*x^15 + 70*a^2*c^3*d^3*x^15 + 7*a^3*d^5*x^15
 + a*c^7*x^14 + 105/2*a^2*c^4*d^2*x^14 + 35*a^3*c*d^4*x^14 + 21*a^2*c^5*d*x
^13 + 70*a^3*c^2*d^3*x^13 + 7/2*a^2*c^6*x^12 + 70*a^3*c^3*d^2*x^12 + 35/4*a
^4*d^4*x^12 + 35*a^3*c^4*d*x^11 + 35*a^4*c*d^3*x^11 + 7*a^3*c^5*x^10 + 105/
2*a^4*c^2*d^2*x^10 + 35*a^4*c^3*d*x^9 + 7*a^5*d^3*x^9 + 35/4*a^4*c^4*x^8 +
21*a^5*c*d^2*x^8 + 21*a^5*c^2*d*x^7 + 7*a^5*c^3*x^6 + 7/2*a^6*d^2*x^6 + 7*a
^6*c*d*x^5 + 7/2*a^6*c^2*x^4 + a^7*d*x^3 + a^7*c*x^2
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 440, normalized size of antiderivative = 24.44

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx = x^{12}\left(\frac{35\,a^4\,d^4}{4}+70\,a^3\,c^3\,d^2+\frac{7\,a^2\,c^6}{2}\right)+x^6\left(\frac{7\,a^6\,d^2}{2}+7\,a^5\,c^3\right)+x^{20}\left(\frac{35\,c^4\,d^4}{4}+7\,a\,c\,d^6\right)+x^{16}\left(\frac{105\,a^2\,c^2\,d^4}{2}+21\,a\,c^5\,d^2+\frac{c^8}{8}\right)+x^{18}\left(\frac{7\,a^2\,d^6}{2}+35\,a\,c^3\,d^4+\frac{7\,c^6\,d^2}{2}\right)+\frac{d^8\,x^{24}}{8}+x^{21}\left(7\,c^3\,d^5+a\,d^7\right)+a^7\,c\,x^2+a^7\,d\,x^3+c\,d^7\,x^{23}+\frac{7\,a^6\,c^2\,x^4}{2}+\frac{7\,c^2\,d^6\,x^{22}}{2}+21\,a^5\,c^2\,d\,x^7+7\,a\,d\,x^{15}\left(a^2\,d^4+10\,a\,c^3\,d^2+c^6\right)+c\,d\,x^{17}\left(21\,a^2\,d^4+35\,a\,c^3\,d^2+c^6\right)+\frac{7\,a^4\,c\,x^8\,(5\,c^3+12\,a\,d^2)}{4}+7\,a^4\,d\,x^9\left(5\,c^3+a\,d^2\right)+7\,c^2\,d^3\,x^{19}\left(c^3+3\,a\,d^2\right)+\frac{a\,c\,x^{14}\,(70\,a^2\,d^4+105\,a\,c^3\,d^2+2\,c^6)}{2}+7\,a^6\,c\,d\,x^5+\frac{7\,a^3\,c^2\,x^{10}\,(2\,c^3+15\,a\,d^2)}{2}+7\,a^2\,c^2\,d\,x^{13}\left(3\,c^3+10\,a\,d^2\right)+35\,a^3\,c\,d\,x^{11}\left(c^3+a\,d^2\right)$$

```
[In] int(x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x)
```

```
[Out] x^12*((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + 70*a^3*c^3*d^2) + x^6*(7*a^5*c^3 + (
7*a^6*d^2)/2) + x^20*((35*c^4*d^4)/4 + 7*a*c*d^6) + x^16*(c^8/8 + 21*a*c^5*
d^2 + (105*a^2*c^2*d^4)/2) + x^18*((7*a^2*d^6)/2 + (7*c^6*d^2)/2 + 35*a*c^3
*d^4) + (d^8*x^24)/8 + x^21*(a*d^7 + 7*c^3*d^5) + a^7*c*x^2 + a^7*d*x^3 + c
*d^7*x^23 + (7*a^6*c^2*x^4)/2 + (7*c^2*d^6*x^22)/2 + 21*a^5*c^2*d*x^7 + 7*a
*d*x^15*(c^6 + a^2*d^4 + 10*a*c^3*d^2) + c*d*x^17*(c^6 + 21*a^2*d^4 + 35*a*
c^3*d^2) + (7*a^4*c*x^8*(12*a*d^2 + 5*c^3))/4 + 7*a^4*d*x^9*(a*d^2 + 5*c^3)
 + 7*c^2*d^3*x^19*(3*a*d^2 + c^3) + (a*c*x^14*(2*c^6 + 70*a^2*d^4 + 105*a*c
^3*d^2))/2 + 7*a^6*c*d*x^5 + (7*a^3*c^2*x^10*(15*a*d^2 + 2*c^3))/2 + 7*a^2*
c^2*d*x^13*(10*a*d^2 + 3*c^3) + 35*a^3*c*d*x^11*(a*d^2 + c^3)
```

3.203 $\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx$

Optimal result 1444
Rubi [A] (verified) 1444
Mathematica [B] (verified) 1445
Maple [A] (verified) 1445
Fricas [B] (verification not implemented) 1446
Sympy [B] (verification not implemented) 1446
Maxima [B] (verification not implemented) 1446
Giac [B] (verification not implemented) 1447
Mupad [B] (verification not implemented) 1447

Optimal result

Integrand size = 23, antiderivative size = 14

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx=\frac{1}{8}x^{16}(c+dx)^8$$

```
[Out] 1/8*x^16*(d*x+c)^8
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1598, 75}

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx=\frac{1}{8}x^{16}(c+dx)^8$$

```
[In] Int[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int x^{15}(c+dx)^7(2c+3dx)\,dx$$
$$= \frac{1}{8}x^{16}(c+dx)^8$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. $2(14) = 28$.

Time = 0.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 7.00

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx = \frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7}{2}c^6d^2x^{18} + 7c^5d^3x^{19} + \frac{35}{4}c^4d^4x^{20} + 7c^3d^5x^{21} + \frac{7}{2}c^2d^6x^{22} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

```
[In] Integrate[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result
gosper	$\frac{x^{16}(dx+c)^8}{8}$
default	$\frac{(x^3d+c\,x^2)^8}{8}$
norman	$\frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24} + \frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x$
risch	$\frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24} + \frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x$
parallelrisch	$\frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24} + \frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x$

```
[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^16*(d*x+c)^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx = \frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.93

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx = \frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7c^6d^2x^{18}}{2}+7c^5d^3x^{19}+\frac{35c^4d^4x^{20}}{4}+7c^3d^5x^{21}+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**7,x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx = \frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx = \frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx = \frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7\,c^6\,d^2\,x^{18}}{2}+7\,c^5\,d^3\,x^{19}+\frac{35\,c^4\,d^4\,x^{20}}{4}+7\,c^3\,d^5\,x^{21}+\frac{7\,c^2\,d^6\,x^{22}}{2}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

```
[In] int(x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

3.204 $\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx$

Optimal result 1448
Rubi [A] (verified) 1448
Mathematica [B] (verified) 1449
Maple [A] (verified) 1449
Fricas [B] (verification not implemented) 1449
Sympy [B] (verification not implemented) 1450
Maxima [B] (verification not implemented) 1450
Giac [B] (verification not implemented) 1450
Mupad [B] (verification not implemented) 1451

Optimal result

Integrand size = 23, antiderivative size = 18

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx=\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

```
[Out] 1/8*x^8*(d*x^2+c*x)^8
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {777}

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx=\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

```
[In] Int[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]
```

```
[Out] (x^8*(c*x + d*x^2)^8)/8
```

Rule 777

```
Int[((e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.
), x_Symbol] :> Simp[g*(e*x)^m*((b*x + c*x^2)^(p + 1)/(c*(m + 2*p + 2))), x
] /; FreeQ[{b, c, e, f, g, m, p}, x] && EqQ[b*g*(m + p + 1) - c*f*(m + 2*p
+ 2), 0] && NeQ[m + 2*p + 2, 0]
```

Rubi steps

$$\text{integral}=\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. $2(18) = 36$.

Time = 0.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 5.44

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx=\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

```
[In] Integrate[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

method	result
gosper	$\frac{x^{16}(dx+c)^8}{8}$
default	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x$
norman	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x$
risch	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x$
parallelrisch	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x$

```
[In] int(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^16*(d*x+c)^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 4.89

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx=\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

```
[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(14) = 28$.

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 5.39

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx=\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7c^6d^2x^{18}}{2}+7c^5d^3x^{19}+\frac{35c^4d^4x^{20}}{4}+7c^3d^5x^{21}+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

```
[In] integrate(x**8*(3*d*x+2*c)*(d*x**2+c*x)**7,x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(16) = 32$.

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 4.89

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx=\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

```
[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 4.89

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx=\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

```
[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 4.89

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx=\frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7\,c^6\,d^2\,x^{18}}{2}+7\,c^5\,d^3\,x^{19}+\frac{35\,c^4\,d^4\,x^{20}}{4}+7\,c^3\,d^5\,x^{21}+\frac{7\,c^2\,d^6\,x^{22}}{2}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

```
[In] int(x^8*(c*x + d*x^2)^7*(2*c + 3*d*x),x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

3.205 $\int x^{15}(c+dx)^7(2c+3dx)\,dx$

Optimal result 1452
Rubi [A] (verified) 1452
Mathematica [B] (verified) 1453
Maple [B] (verified) 1453
Fricas [B] (verification not implemented) 1454
Sympy [B] (verification not implemented) 1454
Maxima [B] (verification not implemented) 1454
Giac [B] (verification not implemented) 1455
Mupad [B] (verification not implemented) 1455

Optimal result

Integrand size = 19, antiderivative size = 14

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx=\frac{1}{8}x^{16}(c+dx)^8$$

```
[Out] 1/8*x^16*(d*x+c)^8
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {75}

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx=\frac{1}{8}x^{16}(c+dx)^8$$

```
[In] Int[x^15*(c + d*x)^7*(2*c + 3*d*x),x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\text{integral}=\frac{1}{8}x^{16}(c+dx)^8$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. $2(14) = 28$.

Time = 0.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 7.00

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx = \frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7}{2}c^6d^2x^{18} + 7c^5d^3x^{19} + \frac{35}{4}c^4d^4x^{20} + 7c^3d^5x^{21} + \frac{7}{2}c^2d^6x^{22} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

```
[In] Integrate[x^15*(c + d*x)^7*(2*c + 3*d*x),x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.

Time = 0.68 (sec) , antiderivative size = 89, normalized size of antiderivative = 6.36

method	result
gosper	$\frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24} + \frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x$
default	$\frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24} + \frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x$
norman	$\frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24} + \frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x$
risch	$\frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24} + \frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x$
parallelrisch	$\frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24} + \frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x$

```
[In] int(x^15*(d*x+c)^7*(3*d*x+2*c),x,method=_RETURNVERBOSE)
```

```
[Out] 7/2*x^22*c^2*d^6+c*d^7*x^23+1/8*d^8*x^24+1/8*x^16*c^8+c^7*d*x^17+7/2*x^18*c
^6*d^2+7*c^5*d^3*x^19+35/4*x^20*c^4*d^4+7*c^3*d^5*x^21
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx = \frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

```
[In] integrate(x^15*(d*x+c)^7*(3*d*x+2*c),x, algorithm="fricas")
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(10) = 20$.

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.93

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx = \frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

```
[In] integrate(x**15*(d*x+c)**7*(3*d*x+2*c),x)
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx = \frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

```
[In] integrate(x^15*(d*x+c)^7*(3*d*x+2*c),x, algorithm="maxima")
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx = \frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

```
[In] integrate(x^15*(d*x+c)^7*(3*d*x+2*c),x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx = \frac{c^8\,x^{16}}{8} + c^7\,d\,x^{17} + \frac{7\,c^6\,d^2\,x^{18}}{2} + 7\,c^5\,d^3\,x^{19} + \frac{35\,c^4\,d^4\,x^{20}}{4} + 7\,c^3\,d^5\,x^{21} + \frac{7\,c^2\,d^6\,x^{22}}{2} + c\,d^7\,x^{23} + \frac{d^8\,x^{24}}{8}$$

```
[In] int(x^15*(2*c + 3*d*x)*(c + d*x)^7,x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

3.206 $\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx$

Optimal result 1456
Rubi [A] (verified) 1456
Mathematica [B] (verified) 1457
Maple [A] (verified) 1457
Fricas [B] (verification not implemented) 1458
Sympy [B] (verification not implemented) 1458
Maxima [B] (verification not implemented) 1458
Giac [A] (verification not implemented) 1459
Mupad [B] (verification not implemented) 1459

Optimal result

Integrand size = 22, antiderivative size = 28

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx=ax+\frac{bx^2}{2}+\frac{1}{160}x^5(2a+bx)^5$$

```
[Out] a*x+1/2*b*x^2+1/160*x^5*(b*x+2*a)^5
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1605}

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx=\frac{1}{160}x^5(2a+bx)^5+ax+\frac{bx^2}{2}$$

```
[In] Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4),x]
```

```
[Out] a*x + (b*x^2)/2 + (x^5*(2*a + b*x)^5)/160
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (1+x^4)\ dx, x, ax+\frac{bx^2}{2}\right)$$
$$= ax+\frac{bx^2}{2}+\frac{1}{160}x^5(2a+bx)^5$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 80 vs. 2(28) = 56.

Time = 0.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.86

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right) dx = ax+\frac{bx^2}{2}+\frac{a^5x^5}{5}+\frac{1}{2}a^4bx^6+\frac{1}{2}a^3b^2x^7+\frac{1}{4}a^2b^3x^8+\frac{1}{16}ab^4x^9+\frac{b^5x^{10}}{160}$$

```
[In] Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4),x]
```

```
[Out] a*x + (b*x^2)/2 + (a^5*x^5)/5 + (a^4*b*x^6)/2 + (a^3*b^2*x^7)/2 + (a^2*b^3*
x^8)/4 + (a*b^4*x^9)/16 + (b^5*x^10)/160
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\left(ax+\frac{1}{2}b\,x^2\right)^5}{5}+ax+\frac{b\,x^2}{2}$	25
gosper	$\frac{x\left(x^9b^5+10a\,x^8b^4+40a^2b^3x^7+80a^3b^2x^6+80a^4b\,x^5+32a^5x^4+80bx+160a\right)}{160}$	67
norman	$ax+\frac{1}{5}a^5x^5+\frac{1}{2}b\,x^2+\frac{1}{160}x^{10}b^5+\frac{1}{4}a^2b^3x^8+\frac{1}{2}a^3b^2x^7+\frac{1}{2}a^4b\,x^6+\frac{1}{16}b^4a\,x^9$	67
risch	$ax+\frac{1}{5}a^5x^5+\frac{1}{2}b\,x^2+\frac{1}{160}x^{10}b^5+\frac{1}{4}a^2b^3x^8+\frac{1}{2}a^3b^2x^7+\frac{1}{2}a^4b\,x^6+\frac{1}{16}b^4a\,x^9$	67
parallelrisch	$ax+\frac{1}{5}a^5x^5+\frac{1}{2}b\,x^2+\frac{1}{160}x^{10}b^5+\frac{1}{4}a^2b^3x^8+\frac{1}{2}a^3b^2x^7+\frac{1}{2}a^4b\,x^6+\frac{1}{16}b^4a\,x^9$	67

```
[In] int((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(a*x+1/2*b*x^2)^5+a*x+1/2*b*x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(24) = 48$.

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx = \frac{1}{160}\,b^5x^{10}+\frac{1}{16}\,ab^4x^9+\frac{1}{4}\,a^2b^3x^8+\frac{1}{2}\,a^3b^2x^7+\frac{1}{2}\,a^4bx^6+\frac{1}{5}\,a^5x^5+\frac{1}{2}\,bx^2+ax$$

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="fricas")
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/4*a^2*b^3*x^8 + 1/2*a^3*b^2*x^7 + 1/2*a
^4*b*x^6 + 1/5*a^5*x^5 + 1/2*b*x^2 + a*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(22) = 44$.

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.50

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx = \frac{a^5x^5}{5}+\frac{a^4bx^6}{2}+\frac{a^3b^2x^7}{2}+\frac{a^2b^3x^8}{4}+\frac{ab^4x^9}{16}+ax+\frac{b^5x^{10}}{160}+\frac{bx^2}{2}$$

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**4),x)
[Out] a**5*x**5/5 + a**4*b*x**6/2 + a**3*b**2*x**7/2 + a**2*b**3*x**8/4 + a*b**4*
x**9/16 + a*x + b**5*x**10/160 + b*x**2/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(24) = 48$.

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx = \frac{1}{160}\,b^5x^{10}+\frac{1}{16}\,ab^4x^9+\frac{1}{4}\,a^2b^3x^8+\frac{1}{2}\,a^3b^2x^7+\frac{1}{2}\,a^4bx^6+\frac{1}{5}\,a^5x^5+\frac{1}{2}\,bx^2+ax$$

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="maxima")
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/4*a^2*b^3*x^8 + 1/2*a^3*b^2*x^7 + 1/2*a
^4*b*x^6 + 1/5*a^5*x^5 + 1/2*b*x^2 + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx=\frac{1}{160}\left(bx^2+2\,ax\right)^5+\frac{1}{2}\,bx^2+ax$$

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="giac")
```

```
[Out] 1/160*(b*x^2 + 2*a*x)^5 + 1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx=\frac{a^5\,x^5}{5}+\frac{a^4\,b\,x^6}{2}+\frac{a^3\,b^2\,x^7}{2}+\frac{a^2\,b^3\,x^8}{4}+\frac{a\,b^4\,x^9}{16}+a\,x+\frac{b^5\,x^{10}}{160}+\frac{b\,x^2}{2}$$

```
[In] int(((a*x + (b*x^2)/2)^4 + 1)*(a + b*x),x)
```

```
[Out] a*x + (b*x^2)/2 + (a^5*x^5)/5 + (b^5*x^10)/160 + (a^4*b*x^6)/2 + (a*b^4*x^9
)/16 + (a^3*b^2*x^7)/2 + (a^2*b^3*x^8)/4
```

3.207 $\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx$

Optimal result 1460
Rubi [A] (verified) 1460
Mathematica [B] (verified) 1461
Maple [A] (verified) 1461
Fricas [B] (verification not implemented) 1462
Sympy [B] (verification not implemented) 1462
Maxima [B] (verification not implemented) 1463
Giac [B] (verification not implemented) 1463
Mupad [B] (verification not implemented) 1464

Optimal result

Integrand size = 23, antiderivative size = 31

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx = ax+\frac{bx^2}{2}+\frac{1}{5}\left(c+ax+\frac{bx^2}{2}\right)^5$$

```
[Out] a*x+1/2*b*x^2+1/5*(c+a*x+1/2*b*x^2)^5
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1605}

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx = \frac{1}{5}\left(ax+\frac{bx^2}{2}+c\right)^5+ax+\frac{bx^2}{2}$$

```
[In] Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4),x]
```

```
[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^5/5
```

<u>Rule 1605</u>

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (1+x^4)\, dx, x, c+ax+\frac{bx^2}{2}\right)$$
$$= ax + \frac{bx^2}{2} + \frac{1}{5}\left(c+ax+\frac{bx^2}{2}\right)^5$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(31) = 62.

Time = 0.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.48

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right) dx = \frac{1}{160}x(2a+bx)\left(80+80c^4+16a^4x^4+32a^3bx^5 + 24a^2b^2x^6+8ab^3x^7+b^4x^8+80c^3x(2a+bx) + 40c^2x^2(2a+bx)^2+10cx^3(2a+bx)^3\right)$$

```
[In] Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4),x]
[Out] (x*(2*a + b*x)*(80 + 80*c^4 + 16*a^4*x^4 + 32*a^3*b*x^5 + 24*a^2*b^2*x^6 +
8*a*b^3*x^7 + b^4*x^8 + 80*c^3*x*(2*a + b*x) + 40*c^2*x^2*(2*a + b*x)^2 + 1
0*c*x^3*(2*a + b*x)^3))/160
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result
default	$\frac{\left(c+ax+\frac{1}{2}b\,x^2\right)^5}{5}+c+ax+\frac{b\,x^2}{2}$
norman	$\left(\frac{1}{4}a^2b^3+\frac{1}{16}b^4c\right)x^8+\left(\frac{1}{2}a^3b^2+\frac{1}{2}a\,b^3c\right)x^7+\left(\frac{1}{5}a^5+2a^3bc+\frac{3}{2}b^2a\,c^2\right)x^5+\left(2a^2c^3+\frac{1}{2}b\,c^4+\frac{1}{2}\right.$
gosper	$\frac{x\left(x^9b^5+10a\,x^8b^4+40a^2b^3x^7+10b^4c\,x^7+80a^3b^2x^6+80a\,b^3c\,x^6+80a^4b\,x^5+240a^2b^2c\,x^5+40b^3c^2x^5+32a^5x^4+320x^4a^3bc+240x\right.}{160}$
risch	$\frac{1}{160}x^{10}b^5+\frac{1}{16}b^4a\,x^9+\frac{1}{4}a^2b^3x^8+\frac{1}{16}x^8b^4c+\frac{1}{2}a^3b^2x^7+\frac{1}{2}x^7a\,b^3c+\frac{1}{2}a^4b\,x^6+\frac{3}{2}x^6a^2b^2c+\frac{1}{4}x^6b$
parallelrisch	$\frac{1}{160}x^{10}b^5+\frac{1}{16}b^4a\,x^9+\frac{1}{4}a^2b^3x^8+\frac{1}{16}x^8b^4c+\frac{1}{2}a^3b^2x^7+\frac{1}{2}x^7a\,b^3c+\frac{1}{2}a^4b\,x^6+\frac{3}{2}x^6a^2b^2c+\frac{1}{4}x^6b$

```
[In] int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x,method=_RETURNVERBOSE)
[Out] 1/5*(c+a*x+1/2*b*x^2)^5+c+a*x+1/2*b*x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(27) = 54$.

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 6.03

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx = \frac{1}{160}\,b^5x^{10}+\frac{1}{16}\,ab^4x^9+\frac{1}{16}\,\left(4\,a^2b^3+b^4c\right)x^8+\frac{1}{2}\,\left(a^3b^2+ab^3c\right)x^7+\frac{1}{4}\,\left(2\,a^4b+6\,a^2b^2c+b^3c^2\right)x^6+\frac{1}{10}\,\left(2\,a^5+20\,a^3bc+15\,ab^2c^2\right)x^5+\frac{1}{2}\,\left(2\,a^4c+6\,a^2bc^2+b^2c^3\right)x^4+2\,\left(a^3c^2+abc^3\right)x^3+\frac{1}{2}\,\left(4\,a^2c^3+bc^4+b\right)x^2+\left(ac^4+a\right)x$$

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="fricas")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/16*(4*a^2*b^3 + b^4*c)*x^8 + 1/2*(a^3*b
^2 + a*b^3*c)*x^7 + 1/4*(2*a^4*b + 6*a^2*b^2*c + b^3*c^2)*x^6 + 1/10*(2*a^5
 + 20*a^3*b*c + 15*a*b^2*c^2)*x^5 + 1/2*(2*a^4*c + 6*a^2*b*c^2 + b^2*c^3)*x
^4 + 2*(a^3*c^2 + a*b*c^3)*x^3 + 1/2*(4*a^2*c^3 + b*c^4 + b)*x^2 + (a*c^4 +
 a)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(24) = 48$.

Time = 0.05 (sec) , antiderivative size = 194, normalized size of antiderivative = 6.26

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx = \frac{ab^4x^9}{16}+\frac{b^5x^{10}}{160}+x^8\left(\frac{a^2b^3}{4}+\frac{b^4c}{16}\right)+x^7\left(\frac{a^3b^2}{2}+\frac{ab^3c}{2}\right)+x^6\left(\frac{a^4b}{2}+\frac{3a^2b^2c}{2}+\frac{b^3c^2}{4}\right)+x^5\left(\frac{a^5}{5}+2a^3bc+\frac{3ab^2c^2}{2}\right)+x^4\left(a^4c+3a^2bc^2+\frac{b^2c^3}{2}\right)+x^3\cdot\left(2a^3c^2+2abc^3\right)+x^2\cdot\left(2a^2c^3+\frac{bc^4}{2}+\frac{b}{2}\right)+x\left(ac^4+a\right)$$

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**4),x)
```

```
[Out] a*b**4*x**9/16 + b**5*x**10/160 + x**8*(a**2*b**3/4 + b**4*c/16) + x**7*(a*
*3*b**2/2 + a*b**3*c/2) + x**6*(a**4*b/2 + 3*a**2*b**2*c/2 + b**3*c**2/4) +
 x**5*(a**5/5 + 2*a**3*b*c + 3*a*b**2*c**2/2) + x**4*(a**4*c + 3*a**2*b*c**
2 + b**2*c**3/2) + x**3*(2*a**3*c**2 + 2*a*b*c**3) + x**2*(2*a**2*c**3 + b*
c**4/2 + b/2) + x*(a*c**4 + a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(27) = 54$.

Time = 0.19 (sec) , antiderivative size = 187, normalized size of antiderivative = 6.03

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx = \frac{1}{160}\,b^5x^{10}+\frac{1}{16}\,ab^4x^9+\frac{1}{16}\,\left(4\,a^2b^3+b^4c\right)x^8+\frac{1}{2}\,\left(a^3b^2+ab^3c\right)x^7+\frac{1}{4}\,\left(2\,a^4b+6\,a^2b^2c+b^3c^2\right)x^6+\frac{1}{10}\,\left(2\,a^5+20\,a^3bc+15\,ab^2c^2\right)x^5+\frac{1}{2}\,\left(2\,a^4c+6\,a^2bc^2+b^2c^3\right)x^4+2\,\left(a^3c^2+abc^3\right)x^3+\frac{1}{2}\,\left(4\,a^2c^3+bc^4+b\right)x^2+\left(ac^4+a\right)x$$

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="maxima")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/16*(4*a^2*b^3 + b^4*c)*x^8 + 1/2*(a^3*b
^2 + a*b^3*c)*x^7 + 1/4*(2*a^4*b + 6*a^2*b^2*c + b^3*c^2)*x^6 + 1/10*(2*a^5
 + 20*a^3*b*c + 15*a*b^2*c^2)*x^5 + 1/2*(2*a^4*c + 6*a^2*b*c^2 + b^2*c^3)*x
^4 + 2*(a^3*c^2 + a*b*c^3)*x^3 + 1/2*(4*a^2*c^3 + b*c^4 + b)*x^2 + (a*c^4 +
 a)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(27) = 54$.

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.84

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx = \frac{1}{160}\,\left(bx^2+2\,ax\right)^5+\frac{1}{16}\,\left(bx^2+2\,ax\right)^4c+\frac{1}{4}\,\left(bx^2+2\,ax\right)^3c^2+\frac{1}{2}\,\left(bx^2+2\,ax\right)^2c^3+\frac{1}{2}\,\left(bx^2+2\,ax\right)c^4+\frac{1}{2}\,bx^2+ax$$

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="giac")
```

```
[Out] 1/160*(b*x^2 + 2*a*x)^5 + 1/16*(b*x^2 + 2*a*x)^4*c + 1/4*(b*x^2 + 2*a*x)^3*
c^2 + 1/2*(b*x^2 + 2*a*x)^2*c^3 + 1/2*(b*x^2 + 2*a*x)*c^4 + 1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 5.81

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx = x^6\left(\frac{a^4\,b}{2}+\frac{3\,a^2\,b^2\,c}{2}+\frac{b^3\,c^2}{4}\right) + x^4\left(a^4\,c+3\,a^2\,b\,c^2+\frac{b^2\,c^3}{2}\right) + x^2\left(2\,a^2\,c^3+\frac{b\,c^4}{2}+\frac{b}{2}\right) + x^5\left(\frac{a^5}{5}+2\,a^3\,b\,c+\frac{3\,a\,b^2\,c^2}{2}\right)+\frac{b^5\,x^{10}}{160} + x^8\left(\frac{a^2\,b^3}{4}+\frac{c\,b^4}{16}\right)+\frac{a\,b^4\,x^9}{16}+a\,x\left(c^4+1\right) + \frac{a\,b^2\,x^7\left(a^2+b\,c\right)}{2}+2\,a\,c^2\,x^3\left(a^2+b\,c\right)$$

```
[In] int(((c + a*x + (b*x^2)/2)^4 + 1)*(a + b*x),x)
```

```
[Out] x^6*((a^4*b)/2 + (b^3*c^2)/4 + (3*a^2*b^2*c)/2) + x^4*(a^4*c + (b^2*c^3)/2
+ 3*a^2*b*c^2) + x^2*(b/2 + (b*c^4)/2 + 2*a^2*c^3) + x^5*(a^5/5 + (3*a*b^2*
c^2)/2 + 2*a^3*b*c) + (b^5*x^10)/160 + x^8*((b^4*c)/16 + (a^2*b^3)/4) + (a*
b^4*x^9)/16 + a*x*(c^4 + 1) + (a*b^2*x^7*(b*c + a^2))/2 + 2*a*c^2*x^3*(b*c
+ a^2)
```

3.208 $\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx$

Optimal result 1465
Rubi [A] (verified) 1465
Mathematica [A] (verified) 1466
Maple [A] (verified) 1466
Fricas [A] (verification not implemented) 1467
Sympy [B] (verification not implemented) 1467
Maxima [A] (verification not implemented) 1468
Giac [A] (verification not implemented) 1468
Mupad [B] (verification not implemented) 1468

Optimal result

Integrand size = 22, antiderivative size = 34

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx = ax+\frac{bx^2}{2}+\frac{\left(ax+\frac{bx^2}{2}\right)^{1+n}}{1+n}$$

```
[Out] a*x+1/2*b*x^2+(a*x+1/2*b*x^2)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1605}

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx = \frac{\left(ax+\frac{bx^2}{2}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

```
[In] Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (1+x^n)\, dx, x, ax+\frac{bx^2}{2}\right)$$
$$= ax+\frac{bx^2}{2}+\frac{\left(ax+\frac{bx^2}{2}\right)^{1+n}}{1+n}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\, dx = \frac{x(2a+bx)\left(1+n+\left(ax+\frac{bx^2}{2}\right)^n\right)}{2(1+n)}$$

```
[In] Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n),x]
[Out] (x*(2*a + b*x)*(1 + n + (a*x + (b*x^2)/2)^n))/(2*(1 + n))
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$ax+\frac{b\,x^2}{2}+\frac{\left(ax+\frac{1}{2}b\,x^2\right)^{1+n}}{1+n}$	31
default	$ax+\frac{b\,x^2}{2}+\frac{\left(ax+\frac{1}{2}b\,x^2\right)^{1+n}}{1+n}$	31
risch	$ax+\frac{b\,x^2}{2}+\frac{x(bx+2a)(x(bx+2a))^n\left(\frac{1}{2}\right)^n}{2+2n}$	40
norman	$ax+\frac{ax\,\mathrm{e}^{n\ln\left(ax+\frac{1}{2}b\,x^2\right)}}{1+n}+\frac{b\,x^2}{2}+\frac{b\,x^2\mathrm{e}^{n\ln\left(ax+\frac{1}{2}b\,x^2\right)}}{2+2n}$	58
parallelrisch	$\frac{x^2\left(\frac{x(bx+2a)}{2}\right)^n b^2+x^2b^2n+b^2x^2+2x\left(\frac{x(bx+2a)}{2}\right)^n ab+2abnx+2abx-4a^2n-4a^2}{2b(1+n)}$	85

```
[In] int((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x,method=_RETURNVERBOSE)
[Out] a*x+1/2*b*x^2+(a*x+1/2*b*x^2)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx=\frac{(bn+b)x^2+(bx^2+2\,ax)\left(\frac{1}{2}\,bx^2+ax\right)^n+2\,(an+a)x}{2\,(n+1)}$$

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="fricas")
[Out] 1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x)*(1/2*b*x^2 + a*x)^n + 2*(a*n + a)*x)/(
n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(26) = 52$.

Time = 18.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 6.71

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx$$
$$=\begin{cases} a\left(x+\frac{\log(x)}{a}\right) & \text{for } b=0\wedge n=- \\ a\left(\frac{nx}{n+1}+\frac{x(ax)^n}{n+1}+\frac{x}{n+1}\right) & \text{for } b=0 \\ ax+\frac{bx^2}{2}+\log(x)+\log\left(\frac{2a}{b}+x\right) & \text{for } n=-1 \\ \frac{2\cdot 2^n abnx}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2\cdot 2^n abx}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2^n b^2nx^2}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2^n b^2x^2}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2abx(2ax+bx^2)^n}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{b^2x^2(2ax+bx^2)^n}{2\cdot 2^n bn+2\cdot 2^n b} & \text{otherwise} \end{cases}$$

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**n),x)
[Out] Piecewise((a*(x + log(x)/a), Eq(b, 0) & Eq(n, -1)), (a*(n*x/(n + 1) + x*(a*
x)**n/(n + 1) + x/(n + 1)), Eq(b, 0)), (a*x + b*x**2/2 + log(x) + log(2*a/b
 + x), Eq(n, -1)), (2*2**n*a*b*n*x/(2*2**n*b*n + 2*2**n*b) + 2*2**n*a*b*x/(
2*2**n*b*n + 2*2**n*b) + 2**n*b**2*n*x**2/(2*2**n*b*n + 2*2**n*b) + 2**n*b*
*2*x**2/(2*2**n*b*n + 2*2**n*b) + 2*a*b*x*(2*a*x + b*x**2)**n/(2*2**n*b*n +
 2*2**n*b) + b**2*x**2*(2*a*x + b*x**2)**n/(2*2**n*b*n + 2*2**n*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.53

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx=\frac{1}{2}\,bx^2+ax+\frac{(bx^2+2\,ax)e^{(n\log(bx+2\,a)+n\log(x))}}{2^{n+1}n+2^{n+1}}$$

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x + (b*x^2 + 2*a*x)*e^(n*log(b*x + 2*a) + n*log(x))/(2^(n + 1
)*n + 2^(n + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx=\frac{1}{2}\,bx^2+ax+\frac{\left(\frac{1}{2}\,bx^2+ax\right)^{n+1}}{n+1}$$

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x + (1/2*b*x^2 + a*x)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx=\frac{x\,(2\,a+b\,x)\,\left(n+\left(\frac{b\,x^2}{2}+a\,x\right)^n+1\right)}{2\,(n+1)}$$

```
[In] int(((a*x + (b*x^2)/2)^n + 1)*(a + b*x),x)
```

```
[Out] (x*(2*a + b*x)*(n + (a*x + (b*x^2)/2)^n + 1))/(2*(n + 1))
```

3.209 $\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx$

Optimal result 1469
Rubi [A] (verified) 1469
Mathematica [B] (verified) 1470
Maple [A] (verified) 1470
Fricas [A] (verification not implemented) 1471
Sympy [F(-1)] 1471
Maxima [A] (verification not implemented) 1471
Giac [A] (verification not implemented) 1472
Mupad [B] (verification not implemented) 1472

Optimal result

Integrand size = 23, antiderivative size = 35

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx=ax+\frac{bx^2}{2}+\frac{\left(c+ax+\frac{bx^2}{2}\right)^{1+n}}{1+n}$$

```
[Out] a*x+1/2*b*x^2+(c+a*x+1/2*b*x^2)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1605}

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx=\frac{\left(ax+\frac{bx^2}{2}+c\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

```
[In] Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (1+x^n)\ dx, x, c+ax+\frac{bx^2}{2}\right)$$
$$= ax+\frac{bx^2}{2}+\frac{\left(c+ax+\frac{bx^2}{2}\right)^{1+n}}{1+n}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)dx$$
$$= \frac{2c\left(c+ax+\frac{bx^2}{2}\right)^n+2ax\left(1+n+\left(c+ax+\frac{bx^2}{2}\right)^n\right)+bx^2\left(1+n+\left(c+ax+\frac{bx^2}{2}\right)^n\right)}{2(1+n)}$$

```
[In] Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n),x]
[Out] (2*c*(c + a*x + (b*x^2)/2)^n + 2*a*x*(1 + n + (c + a*x + (b*x^2)/2)^n) + b*
x^2*(1 + n + (c + a*x + (b*x^2)/2)^n))/(2*(1 + n))
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

method	result
derivativedivides	$c+ax+\frac{b\,x^2}{2}+\frac{\left(c+ax+\frac{1}{2}b\,x^2\right)^{1+n}}{1+n}$
default	$c+ax+\frac{b\,x^2}{2}+\frac{\left(c+ax+\frac{1}{2}b\,x^2\right)^{1+n}}{1+n}$
risch	$ax+\frac{b\,x^2}{2}+\frac{\left(b\,x^2+2ax+2c\right)\left(b\,x^2+2ax+2c\right)^n\left(\frac{1}{2}\right)^n}{2+2n}$
norman	$ax+\frac{c\,\mathrm{e}^{n\ln\left(c+ax+\frac{1}{2}b\,x^2\right)}}{1+n}+\frac{ax\,\mathrm{e}^{n\ln\left(c+ax+\frac{1}{2}b\,x^2\right)}}{1+n}+\frac{b\,x^2}{2}+\frac{b\,x^2\mathrm{e}^{n\ln\left(c+ax+\frac{1}{2}b\,x^2\right)}}{2+2n}$
parallelrisch	$\frac{\left(c+ax+\frac{1}{2}b\,x^2\right)^n b^2x^2+x^2b^2n+b^2x^2+2\left(c+ax+\frac{1}{2}b\,x^2\right)^n abx+2abnx+2abx+2\left(c+ax+\frac{1}{2}b\,x^2\right)^n bc-4a^2n-2bcn-4a^2-2bc}{2b(1+n)}$

```
[In] int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x,method=_RETURNVERBOSE)
[Out] c+a*x+1/2*b*x^2+(c+a*x+1/2*b*x^2)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.49

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx = \frac{(bn+b)x^2+(bx^2+2\,ax+2\,c)\left(\frac{1}{2}\,bx^2+ax+c\right)^n+2\,(an+a)x}{2\,(n+1)}$$

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="fricas")
[Out] 1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x + 2*c)*(1/2*b*x^2 + a*x + c)^n + 2*(a*n
 + a)*x)/(n + 1)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**n),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx = \frac{1}{2}\,bx^2+ax+\frac{(bx^2+2\,ax+2\,c)(bx^2+2\,ax+2\,c)^n}{2^{n+1}n+2^{n+1}}$$

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="maxima")
[Out] 1/2*b*x^2 + a*x + (b*x^2 + 2*a*x + 2*c)*(b*x^2 + 2*a*x + 2*c)^n/(2^(n + 1)*
n + 2^(n + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx=\frac{1}{2}\,bx^2+ax+c+\frac{\left(\frac{1}{2}\,bx^2+ax+c\right)^{n+1}}{n+1}$$

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="giac")
[Out] 1/2*b*x^2 + a*x + c + (1/2*b*x^2 + a*x + c)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.66

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx=a\,x+\left(\frac{b\,x^2}{2}+a\,x+c\right)^n\left(\frac{2\,c}{2\,n+2}+\frac{b\,x^2}{2\,n+2}+\frac{2\,a\,x}{2\,n+2}\right)+\frac{b\,x^2}{2}$$

```
[In] int(((c + a*x + (b*x^2)/2)^n + 1)*(a + b*x),x)
[Out] a*x + (c + a*x + (b*x^2)/2)^n*((2*c)/(2*n + 2) + (b*x^2)/(2*n + 2) + (2*a*x
)/(2*n + 2)) + (b*x^2)/2
```

3.210 $\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal result 1473
Rubi [A] (verified) 1473
Mathematica [B] (verified) 1474
Maple [A] (verified) 1474
Fricas [B] (verification not implemented) 1475
Sympy [B] (verification not implemented) 1475
Maxima [B] (verification not implemented) 1475
Giac [A] (verification not implemented) 1476
Mupad [B] (verification not implemented) 1476

Optimal result

Integrand size = 24, antiderivative size = 30

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx = ax+\frac{cx^3}{3}+\frac{1}{6}\left(ax+\frac{cx^3}{3}\right)^6$$

```
[Out] a*x+1/3*c*x^3+1/6*(a*x+1/3*c*x^3)^6
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1605}

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx = \frac{1}{6}\left(ax+\frac{cx^3}{3}\right)^6+ax+\frac{cx^3}{3}$$

```
[In] Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^6/6
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \left(1 + x^5\right)\, dx, x, ax + \frac{cx^3}{3}\right)$$
$$= ax + \frac{cx^3}{3} + \frac{1}{6}\left(ax + \frac{cx^3}{3}\right)^6$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 93 vs. 2(30) = 60.

Time = 0.01 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.10

$$\int \left(a + cx^2\right)\left(1 + \left(ax + \frac{cx^3}{3}\right)^5\right) dx = ax + \frac{cx^3}{3} + \frac{a^6x^6}{6} + \frac{1}{3}a^5cx^8 + \frac{5}{18}a^4c^2x^{10} + \frac{10}{81}a^3c^3x^{12} + \frac{5}{162}a^2c^4x^{14} + \frac{1}{243}ac^5x^{16} + \frac{c^6x^{18}}{4374}$$

```
[In] Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]
[Out] a*x + (c*x^3)/3 + (a^6*x^6)/6 + (a^5*c*x^8)/3 + (5*a^4*c^2*x^10)/18 + (10*a
^3*c^3*x^12)/81 + (5*a^2*c^4*x^14)/162 + (a*c^5*x^16)/243 + (c^6*x^18)/4374
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result
default	$ax+\frac{c\,x^3}{3}+\frac{\left(ax+\frac{1}{3}c\,x^3\right)^6}{6}$
norman	$ax+\frac{1}{6}a^6x^6+\frac{1}{3}c\,x^3+\frac{1}{4374}c^6x^{18}+\frac{1}{243}a\,c^5x^{16}+\frac{5}{162}a^2c^4x^{14}+\frac{5}{18}a^4c^2x^{10}+\frac{1}{3}c\,a^5x^8+\frac{10}{81}c^3a^3x^{12}$
risch	$ax+\frac{1}{6}a^6x^6+\frac{1}{3}c\,x^3+\frac{1}{4374}c^6x^{18}+\frac{1}{243}a\,c^5x^{16}+\frac{5}{162}a^2c^4x^{14}+\frac{5}{18}a^4c^2x^{10}+\frac{1}{3}c\,a^5x^8+\frac{10}{81}c^3a^3x^{12}$
parallelrisch	$ax+\frac{1}{6}a^6x^6+\frac{1}{3}c\,x^3+\frac{1}{4374}c^6x^{18}+\frac{1}{243}a\,c^5x^{16}+\frac{5}{162}a^2c^4x^{14}+\frac{5}{18}a^4c^2x^{10}+\frac{1}{3}c\,a^5x^8+\frac{10}{81}c^3a^3x^{12}$
gosper	$\frac{x\left(c^6x^{17}+18a\,c^5x^{15}+135a^2c^4x^{13}+540c^3a^3x^{11}+1215a^4c^2x^9+1458c\,a^5x^7+729a^6x^5+1458c\,x^2+4374a\right)}{4374}$

```
[In] int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
[Out] a*x+1/3*c*x^3+1/6*(a*x+1/3*c*x^3)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(24) = 48$.

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.57

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)dx = \frac{1}{4374}c^6x^{18}+\frac{1}{243}ac^5x^{16}+\frac{5}{162}a^2c^4x^{14}+\frac{10}{81}a^3c^3x^{12} + \frac{5}{18}a^4c^2x^{10}+\frac{1}{3}a^5cx^8+\frac{1}{6}a^6x^6+\frac{1}{3}cx^3+ax$$

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="fricas")
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 5/162*a^2*c^4*x^14 + 10/81*a^3*c^3*x^1
2 + 5/18*a^4*c^2*x^10 + 1/3*a^5*c*x^8 + 1/6*a^6*x^6 + 1/3*c*x^3 + a*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(22) = 44$.

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.90

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)dx = \frac{a^6x^6}{6}+\frac{a^5cx^8}{3}+\frac{5a^4c^2x^{10}}{18}+\frac{10a^3c^3x^{12}}{81} + \frac{5a^2c^4x^{14}}{162}+\frac{ac^5x^{16}}{243}+ax+\frac{c^6x^{18}}{4374}+\frac{cx^3}{3}$$

```
[In] integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**5),x)
[Out] a**6*x**6/6 + a**5*c*x**8/3 + 5*a**4*c**2*x**10/18 + 10*a**3*c**3*x**12/81
+ 5*a**2*c**4*x**14/162 + a*c**5*x**16/243 + a*x + c**6*x**18/4374 + c*x**3
/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(24) = 48$.

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.57

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)dx = \frac{1}{4374}c^6x^{18}+\frac{1}{243}ac^5x^{16}+\frac{5}{162}a^2c^4x^{14}+\frac{10}{81}a^3c^3x^{12} + \frac{5}{18}a^4c^2x^{10}+\frac{1}{3}a^5cx^8+\frac{1}{6}a^6x^6+\frac{1}{3}cx^3+ax$$

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="maxima")
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 5/162*a^2*c^4*x^14 + 10/81*a^3*c^3*x^1
2 + 5/18*a^4*c^2*x^10 + 1/3*a^5*c*x^8 + 1/6*a^6*x^6 + 1/3*c*x^3 + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{1}{4374}\left(cx^3+3\,ax\right)^6+\frac{1}{3}\,cx^3+ax$$

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="giac")
[Out] 1/4374*(c*x^3 + 3*a*x)^6 + 1/3*c*x^3 + a*x
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.57

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{a^6\,x^6}{6}+\frac{a^5\,c\,x^8}{3}+\frac{5\,a^4\,c^2\,x^{10}}{18}+\frac{10\,a^3\,c^3\,x^{12}}{81}+\frac{5\,a^2\,c^4\,x^{14}}{162}+\frac{a\,c^5\,x^{16}}{243}+a\,x+\frac{c^6\,x^{18}}{4374}+\frac{c\,x^3}{3}$$

```
[In] int((a + c*x^2)*((a*x + (c*x^3)/3)^5 + 1),x)
[Out] a*x + (c*x^3)/3 + (a^6*x^6)/6 + (c^6*x^18)/4374 + (a^5*c*x^8)/3 + (a*c^5*x^
16)/243 + (5*a^4*c^2*x^10)/18 + (10*a^3*c^3*x^12)/81 + (5*a^2*c^4*x^14)/162
```

3.211 $\int (a+cx^2) \left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right) dx$

Optimal result 1477
Rubi [A] (verified) 1477
Mathematica [B] (verified) 1478
Maple [A] (verified) 1478
Fricas [B] (verification not implemented) 1479
Sympy [B] (verification not implemented) 1479
Maxima [B] (verification not implemented) 1481
Giac [B] (verification not implemented) 1481
Mupad [B] (verification not implemented) 1482

Optimal result

Integrand size = 25, antiderivative size = 31

$$\int (a+cx^2) \left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right) dx = ax+\frac{cx^3}{3}+\frac{1}{6}\left(d+ax+\frac{cx^3}{3}\right)^6$$

```
[Out] a*x+1/3*c*x^3+1/6*(d+a*x+1/3*c*x^3)^6
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1605}

$$\int (a+cx^2) \left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right) dx = \frac{1}{6}\left(ax+\frac{cx^3}{3}+d\right)^6+ax+\frac{cx^3}{3}$$

```
[In] Int[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (c*x^3)/3 + (d + a*x + (c*x^3)/3)^6/6
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \left(1+x^5\right)\,dx, x, d+ax+\frac{cx^3}{3}\right)$$
$$= ax + \frac{cx^3}{3} + \frac{1}{6}\left(d+ax+\frac{cx^3}{3}\right)^6$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(31) = 62.

Time = 0.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 4.52

$$\int \left(a+cx^2\right)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)dx$$
$$= \frac{x(3a+cx^2)\left(1458+1458d^5+243a^5x^5+405a^4cx^7+270a^3c^2x^9+90a^2c^3x^{11}+15ac^4x^{13}+c^5x^{15}+1215d^4(\right.}{4374}$$

```
[In] Integrate[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]
```

```
[Out] (x*(3*a + c*x^2)*(1458 + 1458*d^5 + 243*a^5*x^5 + 405*a^4*c*x^7 + 270*a^3*c
^2*x^9 + 90*a^2*c^3*x^11 + 15*a*c^4*x^13 + c^5*x^15 + 1215*d^4*(3*a*x + c*x
^3) + 540*d^3*(3*a*x + c*x^3)^2 + 135*d^2*(3*a*x + c*x^3)^3 + 18*d*(3*a*x +
 c*x^3)^4))/4374
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result
default	$\frac{\left(d+ax+\frac{1}{3}c\,x^3\right)^6}{6}+d+ax+\frac{c\,x^3}{3}$
norman	$\left(\frac{5}{18}a^4c^2+\frac{10}{27}a\,c^3d^2\right)x^{10}+\left(\frac{5}{2}a^4d^2+\frac{5}{3}ac\,d^4\right)x^4+\left(\frac{1}{3}c\,a^5+\frac{5}{3}a^2c^2d^2\right)x^8+\left(\frac{10}{81}c^3a^3+\frac{5}{162}d^2c^4\right)x^1$
risch	$\frac{5}{2}a^2d^4x^2+\frac{10}{3}x^6a^3c\,d^2+\frac{10}{3}x^5a^2c\,d^3+\frac{5}{3}x^4ac\,d^4+\frac{5}{81}a\,c^4d\,x^{13}+\frac{1}{6}a^6x^6+\frac{1}{3}c\,x^3+ax+\frac{5}{162}x^{12}d^2$
parallelrisch	$\frac{5}{2}a^2d^4x^2+\frac{10}{3}x^6a^3c\,d^2+\frac{10}{3}x^5a^2c\,d^3+\frac{5}{3}x^4ac\,d^4+\frac{5}{81}a\,c^4d\,x^{13}+\frac{1}{6}a^6x^6+\frac{1}{3}c\,x^3+ax+\frac{5}{162}x^{12}d^2$
gosper	$\frac{x\left(c^6x^{17}+18a\,c^5x^{15}+18c^5d\,x^{14}+135a^2c^4x^{13}+270a\,c^4d\,x^{12}+540c^3a^3x^{11}+135x^{11}d^2c^4+1620a^2c^3d\,x^{10}+1215a^4c^2x^9+1620x^9a\,c^3\right.}{}$

```
[In] int((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(d+a*x+1/3*c*x^3)^6+d+a*x+1/3*c*x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(27) = 54$.

Time = 0.26 (sec) , antiderivative size = 280, normalized size of antiderivative = 9.03

$$\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)dx = \frac{1}{4374}c^6x^{18}+\frac{1}{243}ac^5x^{16}+\frac{1}{243}c^5dx^{15}+\frac{5}{162}a^2c^4x^{14}+\frac{5}{81}ac^4dx^{13}+\frac{10}{27}a^2c^3dx^{11}+\frac{5}{162}(4a^3c^3+c^4d^2)x^{12}+\frac{5}{54}(3a^4c^2+4ac^3d^2)x^{10}+\frac{10}{81}(9a^3c^2d+c^3d^3)x^9+\frac{1}{3}(a^5c+5a^2c^2d^2)x^8+\frac{5}{2}a^2d^4x^2+\frac{5}{9}(3a^4cd+2ac^2d^3)x^7+\frac{1}{18}(3a^6+60a^3cd^2+5c^2d^4)x^6+\frac{1}{3}(3a^5d+10a^2cd^3)x^5+\frac{5}{6}(3a^4d^2+2acd^4)x^4+\frac{1}{3}(10a^3d^3+cd^5+c)x^3+(ad^5+a)x$$

```
[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="fricas")
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 1/243*c^5*d*x^15 + 5/162*a^2*c^4*x^14
+ 5/81*a*c^4*d*x^13 + 10/27*a^2*c^3*d*x^11 + 5/162*(4*a^3*c^3 + c^4*d^2)*x^
12 + 5/54*(3*a^4*c^2 + 4*a*c^3*d^2)*x^10 + 10/81*(9*a^3*c^2*d + c^3*d^3)*x^
9 + 1/3*(a^5*c + 5*a^2*c^2*d^2)*x^8 + 5/2*a^2*d^4*x^2 + 5/9*(3*a^4*c*d + 2*
a*c^2*d^3)*x^7 + 1/18*(3*a^6 + 60*a^3*c*d^2 + 5*c^2*d^4)*x^6 + 1/3*(3*a^5*d
 + 10*a^2*c*d^3)*x^5 + 5/6*(3*a^4*d^2 + 2*a*c*d^4)*x^4 + 1/3*(10*a^3*d^3 +
c*d^5 + c)*x^3 + (a*d^5 + a)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(24) = 48$.

Time = 0.06 (sec) , antiderivative size = 314, normalized size of antiderivative = 10.13

$$\int (a + cx^2) \left(1 + \left(d + ax + \frac{cx^3}{3}\right)^5\right) dx = \frac{5a^2c^4x^{14}}{162} + \frac{10a^2c^3dx^{11}}{27} + \frac{5a^2d^4x^2}{2} + \frac{ac^5x^{16}}{243} + \frac{5ac^4dx^{13}}{81} + \frac{c^6x^{18}}{4374} + \frac{c^5dx^{15}}{243} + x^{12} \cdot \left(\frac{10a^3c^3}{81} + \frac{5c^4d^2}{162}\right) + x^{10} \cdot \left(\frac{5a^4c^2}{18} + \frac{10ac^3d^2}{27}\right) + x^9 \cdot \left(\frac{10a^3c^2d}{9} + \frac{10c^3d^3}{81}\right) + x^8\left(\frac{a^5c}{3} + \frac{5a^2c^2d^2}{3}\right) + x^7 \cdot \left(\frac{5a^4cd}{3} + \frac{10ac^2d^3}{9}\right) + x^6\left(\frac{a^6}{6} + \frac{10a^3cd^2}{3} + \frac{5c^2d^4}{18}\right) + x^5\left(a^5d + \frac{10a^2cd^3}{3}\right) + x^4 \cdot \left(\frac{5a^4d^2}{2} + \frac{5acd^4}{3}\right) + x^3 \cdot \left(\frac{10a^3d^3}{3} + \frac{cd^5}{3} + \frac{c}{3}\right) + x\left(ad^5 + a\right)$$

```
[In] integrate((c*x**2+a)*(1+(d+a*x+1/3*c*x**3)**5),x)
```

```
[Out] 5*a**2*c**4*x**14/162 + 10*a**2*c**3*d*x**11/27 + 5*a**2*d**4*x**2/2 + a*c*
*5*x**16/243 + 5*a*c**4*d*x**13/81 + c**6*x**18/4374 + c**5*d*x**15/243 + x
**12*(10*a**3*c**3/81 + 5*c**4*d**2/162) + x**10*(5*a**4*c**2/18 + 10*a*c**
3*d**2/27) + x**9*(10*a**3*c**2*d/9 + 10*c**3*d**3/81) + x**8*(a**5*c/3 + 5
*a**2*c**2*d**2/3) + x**7*(5*a**4*c*d/3 + 10*a*c**2*d**3/9) + x**6*(a**6/6
+ 10*a**3*c*d**2/3 + 5*c**2*d**4/18) + x**5*(a**5*d + 10*a**2*c*d**3/3) + x
**4*(5*a**4*d**2/2 + 5*a*c*d**4/3) + x**3*(10*a**3*d**3/3 + c*d**5/3 + c/3)
 + x*(a*d**5 + a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(27) = 54$.

Time = 0.19 (sec) , antiderivative size = 280, normalized size of antiderivative = 9.03

$$\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)dx = \frac{1}{4374}c^6x^{18}+\frac{1}{243}ac^5x^{16}+\frac{1}{243}c^5dx^{15}+\frac{5}{162}a^2c^4x^{14}+\frac{5}{81}ac^4dx^{13}+\frac{10}{27}a^2c^3dx^{11}+\frac{5}{162}\left(4a^3c^3+c^4d^2\right)x^{12}+\frac{5}{54}\left(3a^4c^2+4ac^3d^2\right)x^{10}+\frac{10}{81}\left(9a^3c^2d+c^3d^3\right)x^9+\frac{1}{3}\left(a^5c+5a^2c^2d^2\right)x^8+\frac{5}{2}a^2d^4x^2+\frac{5}{9}\left(3a^4cd+2ac^2d^3\right)x^7+\frac{1}{18}\left(3a^6+60a^3cd^2+5c^2d^4\right)x^6+\frac{1}{3}\left(3a^5d+10a^2cd^3\right)x^5+\frac{5}{6}\left(3a^4d^2+2acd^4\right)x^4+\frac{1}{3}\left(10a^3d^3+cd^5+c\right)x^3+\left(ad^5+a\right)x$$

```
[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 1/243*c^5*d*x^15 + 5/162*a^2*c^4*x^14
+ 5/81*a*c^4*d*x^13 + 10/27*a^2*c^3*d*x^11 + 5/162*(4*a^3*c^3 + c^4*d^2)*x^
12 + 5/54*(3*a^4*c^2 + 4*a*c^3*d^2)*x^10 + 10/81*(9*a^3*c^2*d + c^3*d^3)*x^
9 + 1/3*(a^5*c + 5*a^2*c^2*d^2)*x^8 + 5/2*a^2*d^4*x^2 + 5/9*(3*a^4*c*d + 2*
a*c^2*d^3)*x^7 + 1/18*(3*a^6 + 60*a^3*c*d^2 + 5*c^2*d^4)*x^6 + 1/3*(3*a^5*d
 + 10*a^2*c*d^3)*x^5 + 5/6*(3*a^4*d^2 + 2*a*c*d^4)*x^4 + 1/3*(10*a^3*d^3 +
c*d^5 + c)*x^3 + (a*d^5 + a)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(27) = 54$.

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.39

$$\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)\,dx = \frac{1}{4374}\left(cx^3+3\,ax\right)^6+\frac{1}{243}\left(cx^3+3\,ax\right)^5 d+\frac{5}{162}\left(cx^3+3\,ax\right)^4 d^2+\frac{10}{81}\left(cx^3+3\,ax\right)^3 d^3+\frac{5}{18}\left(cx^3+3\,ax\right)^2 d^4+\frac{1}{3}\left(cx^3+3\,ax\right)d^5+\frac{1}{3}\,cx^3+ax$$

[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="giac")

[Out] 1/4374*(c*x^3 + 3*a*x)^6 + 1/243*(c*x^3 + 3*a*x)^5*d + 5/162*(c*x^3 + 3*a*x)^4*d^2 + 10/81*(c*x^3 + 3*a*x)^3*d^3 + 5/18*(c*x^3 + 3*a*x)^2*d^4 + 1/3*(c*x^3 + 3*a*x)*d^5 + 1/3*c*x^3 + a*x

Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 266, normalized size of antiderivative = 8.58

$$\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)\,dx = x^5\left(a^5\,d+\frac{10\,c\,a^2\,d^3}{3}\right)+x^4\left(\frac{5\,a^4\,d^2}{2}+\frac{5\,c\,a\,d^4}{3}\right)+x^3\left(\frac{10\,a^3\,d^3}{3}+\frac{c\,d^5}{3}+\frac{c}{3}\right)+x^6\left(\frac{a^6}{6}+\frac{10\,a^3\,c\,d^2}{3}+\frac{5\,c^2\,d^4}{18}\right)+\frac{c^6\,x^{18}}{4374}+\frac{a\,c^5\,x^{16}}{243}+a\,x\left(d^5+1\right)+\frac{c^5\,d\,x^{15}}{243}+\frac{5\,a^2\,c^4\,x^{14}}{162}+\frac{5\,a^2\,d^4\,x^2}{2}+\frac{5\,c^3\,x^{12}\left(4\,a^3+c\,d^2\right)}{162}+\frac{a^2\,c\,x^8\left(a^3+5\,c\,d^2\right)}{3}+\frac{10\,a^2\,c^3\,d\,x^{11}}{27}+\frac{5\,a\,c^2\,x^{10}\left(3\,a^3+4\,c\,d^2\right)}{54}+\frac{10\,c^2\,d\,x^9\left(9\,a^3+c\,d^2\right)}{81}+\frac{5\,a\,c^4\,d\,x^{13}}{81}+\frac{5\,a\,c\,d\,x^7\left(3\,a^3+2\,c\,d^2\right)}{9}$$

[In] int(((d + a*x + (c*x^3)/3)^5 + 1)*(a + c*x^2),x)

[Out] x^5*(a^5*d + (10*a^2*c*d^3)/3) + x^4*((5*a^4*d^2)/2 + (5*a*c*d^4)/3) + x^3*(c/3 + (c*d^5)/3 + (10*a^3*d^3)/3) + x^6*(a^6/6 + (5*c^2*d^4)/18 + (10*a^3*c*d^2)/3) + (c^6*x^18)/4374 + (a*c^5*x^16)/243 + a*x*(d^5 + 1) + (c^5*d*x^15)/243 + (5*a^2*c^4*x^14)/162 + (5*a^2*d^4*x^2)/2 + (5*c^3*x^12*(c*d^2 + 4*a^3))/162 + (a^2*c*x^8*(5*c*d^2 + a^3))/3 + (10*a^2*c^3*d*x^11)/27 + (5*a*c^2*x^10*(4*c*d^2 + 3*a^3))/54 + (10*c^2*d*x^9*(c*d^2 + 9*a^3))/81 + (5*a*c^4*d*x^13)/81 + (5*a*c*d*x^7*(2*c*d^2 + 3*a^3))/9

3.212 $\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal result 1483
Rubi [A] (verified) 1483
Mathematica [B] (verified) 1484
Maple [A] (verified) 1484
Fricas [B] (verification not implemented) 1485
Sympy [B] (verification not implemented) 1485
Maxima [B] (verification not implemented) 1485
Giac [A] (verification not implemented) 1486
Mupad [B] (verification not implemented) 1486

Optimal result

Integrand size = 31, antiderivative size = 34

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{x^{12}(3b+2cx)^6}{279936}$$

```
[Out] 1/2*b*x^2+1/3*c*x^3+1/279936*x^12*(2*c*x+3*b)^6
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1605}

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{x^{12}(3b+2cx)^6}{279936}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[In] Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (x^12*(3*b + 2*c*x)^6)/279936
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \left(1+x^5\right)\,dx, x, \frac{bx^2}{2}+\frac{cx^3}{3}\right)$$
$$= \frac{bx^2}{2}+\frac{cx^3}{3}+\frac{x^{12}(3b+2cx)^6}{279936}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. $2(34) = 68$.

Time = 0.01 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.88

$$\int \left(bx+cx^2\right)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx = \frac{bx^2}{2}+\frac{cx^3}{3}+\frac{b^6x^{12}}{384}+\frac{1}{96}b^5cx^{13}+\frac{5}{288}b^4c^2x^{14}+\frac{5}{324}b^3c^3x^{15}+\frac{5}{648}b^2c^4x^{16}+\frac{1}{486}bc^5x^{17}+\frac{c^6x^{18}}{4374}$$

```
[In] Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5),x]
[Out] (b*x^2)/2 + (c*x^3)/3 + (b^6*x^12)/384 + (b^5*c*x^13)/96 + (5*b^4*c^2*x^14)
/288 + (5*b^3*c^3*x^15)/324 + (5*b^2*c^4*x^16)/648 + (b*c^5*x^17)/486 + (c^
6*x^18)/4374
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result
default	$\frac{\left(\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3\right)^6}{6}+\frac{b\,x^2}{2}+\frac{c\,x^3}{3}$
gosper	$\frac{x^2\left(64c^6x^{16}+576b\,c^5x^{15}+2160b^2c^4x^{14}+4320b^3c^3x^{13}+4860b^4c^2x^{12}+2916b^5c\,x^{11}+729b^6x^{10}+93312cx+139968b\right)}{279936}$
norman	$\frac{1}{2}b\,x^2+\frac{1}{384}b^6x^{12}+\frac{1}{3}c\,x^3+\frac{1}{4374}c^6x^{18}+\frac{1}{486}b\,c^5x^{17}+\frac{5}{648}b^2c^4x^{16}+\frac{5}{324}b^3c^3x^{15}+\frac{5}{288}b^4c^2x^{14}+\frac{1}{96}$
risch	$\frac{1}{2}b\,x^2+\frac{1}{384}b^6x^{12}+\frac{1}{3}c\,x^3+\frac{1}{4374}c^6x^{18}+\frac{1}{486}b\,c^5x^{17}+\frac{5}{648}b^2c^4x^{16}+\frac{5}{324}b^3c^3x^{15}+\frac{5}{288}b^4c^2x^{14}+\frac{1}{96}$
parallelrisch	$\frac{1}{2}b\,x^2+\frac{1}{384}b^6x^{12}+\frac{1}{3}c\,x^3+\frac{1}{4374}c^6x^{18}+\frac{1}{486}b\,c^5x^{17}+\frac{5}{648}b^2c^4x^{16}+\frac{5}{324}b^3c^3x^{15}+\frac{5}{288}b^4c^2x^{14}+\frac{1}{96}$

```
[In] int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
[Out] 1/6*(1/2*b*x^2+1/3*c*x^3)^6+1/2*b*x^2+1/3*c*x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(28) = 56$.

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35

$$\int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) dx = \frac{1}{4374} c^6 x^{18} + \frac{1}{486} bc^5 x^{17} + \frac{5}{648} b^2 c^4 x^{16} + \frac{5}{324} b^3 c^3 x^{15} + \frac{5}{288} b^4 c^2 x^{14} + \frac{1}{96} b^5 c x^{13} + \frac{1}{384} b^6 x^{12} + \frac{1}{3} c x^3 + \frac{1}{2} b x^2$$

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fricas")
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 5/324*b^3*c^3*x^1
5 + 5/288*b^4*c^2*x^14 + 1/96*b^5*c*x^13 + 1/384*b^6*x^12 + 1/3*c*x^3 + 1/2
*b*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(27) = 54$.

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

$$\int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) dx = \frac{b^6 x^{12}}{384} + \frac{b^5 c x^{13}}{96} + \frac{5b^4 c^2 x^{14}}{288} + \frac{5b^3 c^3 x^{15}}{324} + \frac{5b^2 c^4 x^{16}}{648} + \frac{bc^5 x^{17}}{486} + \frac{bx^2}{2} + \frac{c^6 x^{18}}{4374} + \frac{cx^3}{3}$$

```
[In] integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**5),x)
[Out] b**6*x**12/384 + b**5*c*x**13/96 + 5*b**4*c**2*x**14/288 + 5*b**3*c**3*x**1
5/324 + 5*b**2*c**4*x**16/648 + b*c**5*x**17/486 + b*x**2/2 + c**6*x**18/43
74 + c*x**3/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(28) = 56$.

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35

$$\int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) dx = \frac{1}{4374} c^6 x^{18} + \frac{1}{486} bc^5 x^{17} + \frac{5}{648} b^2 c^4 x^{16} + \frac{5}{324} b^3 c^3 x^{15} + \frac{5}{288} b^4 c^2 x^{14} + \frac{1}{96} b^5 c x^{13} + \frac{1}{384} b^6 x^{12} + \frac{1}{3} c x^3 + \frac{1}{2} b x^2$$

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 5/324*b^3*c^3*x^1
5 + 5/288*b^4*c^2*x^14 + 1/96*b^5*c*x^13 + 1/384*b^6*x^12 + 1/3*c*x^3 + 1/2
*b*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{1}{279936}\left(2\,cx^3+3\,bx^2\right)^6+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2$$

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac")
```

```
[Out] 1/279936*(2*c*x^3 + 3*b*x^2)^6 + 1/3*c*x^3 + 1/2*b*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{b^6\,x^{12}}{384}+\frac{b^5\,c\,x^{13}}{96}+\frac{5\,b^4\,c^2\,x^{14}}{288}+\frac{5\,b^3\,c^3\,x^{15}}{324}+\frac{5\,b^2\,c^4\,x^{16}}{648}+\frac{b\,c^5\,x^{17}}{486}+\frac{b\,x^2}{2}+\frac{c^6\,x^{18}}{4374}+\frac{c\,x^3}{3}$$

```
[In] int((b*x + c*x^2)*(((b*x^2)/2 + (c*x^3)/3)^5 + 1),x)
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (b^6*x^12)/384 + (c^6*x^18)/4374 + (b^5*c*x^13)/96
+ (b*c^5*x^17)/486 + (5*b^4*c^2*x^14)/288 + (5*b^3*c^3*x^15)/324 + (5*b^2*c
^4*x^16)/648
```

3.213 $\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal result 1487
Rubi [A] (verified) 1487
Mathematica [B] (verified) 1488
Maple [A] (verified) 1488
Fricas [B] (verification not implemented) 1489
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Optimal result

Integrand size = 32, antiderivative size = 41

$$\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6$$

```
[Out] 1/2*b*x^2+1/3*c*x^3+1/6*(d+1/2*b*x^2+1/3*c*x^3)^6
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {1605}

$$\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{1}{6}\left(\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[In] Int[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (d + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (1+x^5)\, dx, x, d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)$$
$$= \frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 146 vs. $2(41) = 82$.

Time = 0.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.56

$$\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right) dx$$
$$= \frac{x^2(3b+2cx)\left(46656+46656d^5+243b^5x^{10}+810b^4cx^{11}+1080b^3c^2x^{12}+720b^2c^3x^{13}+240bc^4x^{14}+32c^5x^{15}\right.}{279936}$$

```
[In] Integrate[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (x^2*(3*b + 2*c*x)*(46656 + 46656*d^5 + 243*b^5*x^10 + 810*b^4*c*x^11 + 108
0*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14 + 32*c^5*x^15 + 19440*d^
4*x^2*(3*b + 2*c*x) + 4320*d^3*x^4*(3*b + 2*c*x)^2 + 540*d^2*x^6*(3*b + 2*c
*x)^3 + 36*d*x^8*(3*b + 2*c*x)^4))/279936
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result
default	$\frac{\left(d+\frac{1}{2}bx^2+\frac{1}{3}cx^3\right)^6}{6}+d+\frac{bx^2}{2}+\frac{cx^3}{3}$
norman	$\left(\frac{1}{2}bd^5+\frac{1}{2}b\right)x^2+\left(\frac{5}{324}b^3c^3+\frac{1}{243}c^5d\right)x^{15}+\left(\frac{5}{12}b^3d^3+\frac{5}{18}c^2d^4\right)x^6+\left(\frac{5}{288}b^4c^2+\frac{5}{162}bc^4d\right)x^{14}+$
gosper	$\frac{x^2\left(64c^6x^{16}+576bc^5x^{15}+2160b^2c^4x^{14}+4320b^3c^3x^{13}+1152c^5dx^{13}+4860b^4c^2x^{12}+8640bc^4dx^{12}+2916b^5cx^{11}+25920b^2c^3dx^{11}\right.}{}$
risch	$\frac{5}{162}x^{14}bc^4d+\frac{5}{54}x^{13}b^2c^3d+\frac{5}{12}x^6b^3d^3+\frac{1}{2}bd^5x^2+\frac{1}{384}b^6x^{12}+\frac{1}{3}cx^3+\frac{1}{2}bx^2+\frac{5}{162}x^{12}d^2c^4+\frac{10}{81}x$
parallelrisch	$\frac{5}{162}x^{14}bc^4d+\frac{5}{54}x^{13}b^2c^3d+\frac{5}{12}x^6b^3d^3+\frac{1}{2}bd^5x^2+\frac{1}{384}b^6x^{12}+\frac{1}{3}cx^3+\frac{1}{2}bx^2+\frac{5}{162}x^{12}d^2c^4+\frac{10}{81}x$

```
[In] int((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(d+1/2*b*x^2+1/3*c*x^3)^6+d+1/2*b*x^2+1/3*c*x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(33) = 66$.

Time = 0.27 (sec) , antiderivative size = 289, normalized size of antiderivative = 7.05

$$\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$=\frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{5}{648}b^2c^4x^{16}+\frac{1}{972}\left(15\,b^3c^3+4\,c^5d\right)x^{15}$$
$$+\frac{5}{2592}\left(9\,b^4c^2+16\,bc^4d\right)x^{14}+\frac{1}{864}\left(9\,b^5c+80\,b^2c^3d\right)x^{13}+\frac{5}{6}b^2cd^3x^7$$
$$+\frac{1}{10368}\left(27\,b^6+1440\,b^3c^2d+320\,c^4d^2\right)x^{12}+\frac{5}{432}\left(9\,b^4cd+16\,bc^3d^2\right)x^{11}$$
$$+\frac{5}{6}bcd^4x^5+\frac{1}{96}\left(3\,b^5d+40\,b^2c^2d^2\right)x^{10}+\frac{5}{8}b^2d^4x^4+\frac{5}{324}\left(27\,b^3cd^2+8\,c^3d^3\right)x^9$$
$$+\frac{5}{288}\left(9\,b^4d^2+32\,bc^2d^3\right)x^8+\frac{5}{36}\left(3\,b^3d^3+2\,c^2d^4\right)x^6+\frac{1}{3}\left(cd^5+c\right)x^3+\frac{1}{2}\left(bd^5+b\right)x^2$$

```
[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fricas")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/972*(15*b^3*c^3
 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 16*b*c^4*d)*x^14 + 1/864*(9*b^5*c +
80*b^2*c^3*d)*x^13 + 5/6*b^2*c*d^3*x^7 + 1/10368*(27*b^6 + 1440*b^3*c^2*d +
 320*c^4*d^2)*x^12 + 5/432*(9*b^4*c*d + 16*b*c^3*d^2)*x^11 + 5/6*b*c*d^4*x^
5 + 1/96*(3*b^5*d + 40*b^2*c^2*d^2)*x^10 + 5/8*b^2*d^4*x^4 + 5/324*(27*b^3*
c*d^2 + 8*c^3*d^3)*x^9 + 5/288*(9*b^4*d^2 + 32*b*c^2*d^3)*x^8 + 5/36*(3*b^3
*d^3 + 2*c^2*d^4)*x^6 + 1/3*(c*d^5 + c)*x^3 + 1/2*(b*d^5 + b)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(31) = 62$.

Time = 0.06 (sec) , antiderivative size = 321, normalized size of antiderivative = 7.83

$$\int (bx + cx^2)\left(1 + \left(d + \frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) dx = \frac{5b^2c^4x^{16}}{648} + \frac{5b^2cd^3x^7}{6} + \frac{5b^2d^4x^4}{8} + \frac{bc^5x^{17}}{486} + \frac{5bcd^4x^5}{6} + \frac{c^6x^{18}}{4374} + x^{15}\cdot\left(\frac{5b^3c^3}{324} + \frac{c^5d}{243}\right) + x^{14}\cdot\left(\frac{5b^4c^2}{288} + \frac{5bc^4d}{162}\right) + x^{13}\left(\frac{b^5c}{96} + \frac{5b^2c^3d}{54}\right) + x^{12}\left(\frac{b^6}{384} + \frac{5b^3c^2d}{36} + \frac{5c^4d^2}{162}\right) + x^{11}\cdot\left(\frac{5b^4cd}{48} + \frac{5bc^3d^2}{27}\right) + x^{10}\left(\frac{b^5d}{32} + \frac{5b^2c^2d^2}{12}\right) + x^9\cdot\left(\frac{5b^3cd^2}{12} + \frac{10c^3d^3}{81}\right) + x^8\cdot\left(\frac{5b^4d^2}{32} + \frac{5bc^2d^3}{9}\right) + x^6\cdot\left(\frac{5b^3d^3}{12} + \frac{5c^2d^4}{18}\right) + x^3\left(\frac{cd^5}{3} + \frac{c}{3}\right) + x^2\left(\frac{bd^5}{2} + \frac{b}{2}\right)$$

```
[In] integrate((c*x**2+b*x)*(1+(d+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] 5*b**2*c**4*x**16/648 + 5*b**2*c*d**3*x**7/6 + 5*b**2*d**4*x**4/8 + b*c**5*
x**17/486 + 5*b*c*d**4*x**5/6 + c**6*x**18/4374 + x**15*(5*b**3*c**3/324 +
c**5*d/243) + x**14*(5*b**4*c**2/288 + 5*b*c**4*d/162) + x**13*(b**5*c/96 +
 5*b**2*c**3*d/54) + x**12*(b**6/384 + 5*b**3*c**2*d/36 + 5*c**4*d**2/162)
+ x**11*(5*b**4*c*d/48 + 5*b*c**3*d**2/27) + x**10*(b**5*d/32 + 5*b**2*c**2
*d**2/12) + x**9*(5*b**3*c*d**2/12 + 10*c**3*d**3/81) + x**8*(5*b**4*d**2/3
2 + 5*b*c**2*d**3/9) + x**6*(5*b**3*d**3/12 + 5*c**2*d**4/18) + x**3*(c*d**
5/3 + c/3) + x**2*(b*d**5/2 + b/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(33) = 66$.

Time = 0.20 (sec) , antiderivative size = 289, normalized size of antiderivative = 7.05

$$\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$= \frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{5}{648}b^2c^4x^{16} + \frac{1}{972}(15b^3c^3+4c^5d)x^{15} + \frac{5}{2592}(9b^4c^2+16bc^4d)x^{14} + \frac{1}{864}(9b^5c+80b^2c^3d)x^{13} + \frac{5}{6}b^2cd^3x^7 + \frac{1}{10368}(27b^6+1440b^3c^2d+320c^4d^2)x^{12} + \frac{5}{432}(9b^4cd+16bc^3d^2)x^{11} + \frac{5}{6}bcd^4x^5 + \frac{1}{96}(3b^5d+40b^2c^2d^2)x^{10} + \frac{5}{8}b^2d^4x^4 + \frac{5}{324}(27b^3cd^2+8c^3d^3)x^9 + \frac{5}{288}(9b^4d^2+32bc^2d^3)x^8 + \frac{5}{36}(3b^3d^3+2c^2d^4)x^6 + \frac{1}{3}(cd^5+c)x^3 + \frac{1}{2}(bd^5+b)x^2$$

```
[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/972*(15*b^3*c^3
 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 16*b*c^4*d)*x^14 + 1/864*(9*b^5*c +
80*b^2*c^3*d)*x^13 + 5/6*b^2*c*d^3*x^7 + 1/10368*(27*b^6 + 1440*b^3*c^2*d +
 320*c^4*d^2)*x^12 + 5/432*(9*b^4*c*d + 16*b*c^3*d^2)*x^11 + 5/6*b*c*d^4*x^
5 + 1/96*(3*b^5*d + 40*b^2*c^2*d^2)*x^10 + 5/8*b^2*d^4*x^4 + 5/324*(27*b^3*
c*d^2 + 8*c^3*d^3)*x^9 + 5/288*(9*b^4*d^2 + 32*b*c^2*d^3)*x^8 + 5/36*(3*b^3
*d^3 + 2*c^2*d^4)*x^6 + 1/3*(c*d^5 + c)*x^3 + 1/2*(b*d^5 + b)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(33) = 66.

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.07

$$\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$= \frac{1}{279936}(2cx^3+3bx^2)^6 + \frac{1}{7776}(2cx^3+3bx^2)^5d + \frac{5}{2592}(2cx^3+3bx^2)^4d^2 + \frac{5}{324}(2cx^3+3bx^2)^3d^3 + \frac{5}{72}(2cx^3+3bx^2)^2d^4 + \frac{1}{6}(2cx^3+3bx^2)d^5 + \frac{1}{3}cx^3 + \frac{1}{2}bx^2$$

```
[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac")
```

```
[Out] 1/279936*(2*c*x^3 + 3*b*x^2)^6 + 1/7776*(2*c*x^3 + 3*b*x^2)^5*d + 5/2592*(2
*c*x^3 + 3*b*x^2)^4*d^2 + 5/324*(2*c*x^3 + 3*b*x^2)^3*d^3 + 5/72*(2*c*x^3 +
 3*b*x^2)^2*d^4 + 1/6*(2*c*x^3 + 3*b*x^2)*d^5 + 1/3*c*x^3 + 1/2*b*x^2
```

Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 273, normalized size of antiderivative = 6.66

$$\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx = x^{13}\left(\frac{b^5\,c}{96}+\frac{5\,d\,b^2\,c^3}{54}\right)+x^{14}\left(\frac{5\,b^4\,c^2}{288}+\frac{5\,d\,b\,c^4}{162}\right) + x^{12}\left(\frac{b^6}{384}+\frac{5\,b^3\,c^2\,d}{36}+\frac{5\,c^4\,d^2}{162}\right) + \frac{c^6\,x^{18}}{4374}+x^{15}\left(\frac{5\,b^3\,c^3}{324}+\frac{d\,c^5}{243}\right) + \frac{5\,d^3\,x^6\,(3\,b^3+2\,d\,c^2)}{36}+\frac{b\,c^5\,x^{17}}{486}+\frac{5\,b^2\,c^4\,x^{16}}{648} + \frac{b\,x^2\,(d^5+1)}{2}+\frac{5\,b^2\,d^4\,x^4}{8}+\frac{c\,x^3\,(d^5+1)}{3} + \frac{5\,b^2\,c\,d^3\,x^7}{6}+\frac{5\,b\,d^2\,x^8\,(9\,b^3+32\,d\,c^2)}{288} + \frac{b^2\,d\,x^{10}\,(3\,b^3+40\,d\,c^2)}{96} + \frac{5\,c\,d^2\,x^9\,(27\,b^3+8\,d\,c^2)}{324}+\frac{5\,b\,c\,d^4\,x^5}{6} + \frac{5\,b\,c\,d\,x^{11}\,(9\,b^3+16\,d\,c^2)}{432}$$

```
[In] int((b*x + c*x^2)*((d + (b*x^2)/2 + (c*x^3)/3)^5 + 1),x)
```

```
[Out] x^13*((b^5*c)/96 + (5*b^2*c^3*d)/54) + x^14*((5*b^4*c^2)/288 + (5*b*c^4*d)/
162) + x^12*(b^6/384 + (5*c^4*d^2)/162 + (5*b^3*c^2*d)/36) + (c^6*x^18)/437
4 + x^15*((c^5*d)/243 + (5*b^3*c^3)/324) + (5*d^3*x^6*(2*c^2*d + 3*b^3))/36
 + (b*c^5*x^17)/486 + (5*b^2*c^4*x^16)/648 + (b*x^2*(d^5 + 1))/2 + (5*b^2*d
^4*x^4)/8 + (c*x^3*(d^5 + 1))/3 + (5*b^2*c*d^3*x^7)/6 + (5*b*d^2*x^8*(32*c^
2*d + 9*b^3))/288 + (b^2*d*x^10*(40*c^2*d + 3*b^3))/96 + (5*c*d^2*x^9*(8*c^
2*d + 27*b^3))/324 + (5*b*c*d^4*x^5)/6 + (5*b*c*d*x^11*(16*c^2*d + 9*b^3))/
432
```

3.214 $\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right) dx$

Optimal result 1493
Rubi [A] (verified) 1493
Mathematica [B] (verified) 1494
Maple [A] (verified) 1494
Fricas [B] (verification not implemented) 1495
Sympy [B] (verification not implemented) 1495
Maxima [B] (verification not implemented) 1496
Giac [A] (verification not implemented) 1497
Mupad [B] (verification not implemented) 1497

Optimal result

Integrand size = 35, antiderivative size = 46

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right) dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6$$

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+1/6*(a*x+1/2*b*x^2+1/3*c*x^3)^6
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1605}

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right) dx = \frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[In] Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (1+x^5)\ dx, x, ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)$$
$$= ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 244 vs. 2(46) = 92.

Time = 0.05 (sec) , antiderivative size = 244, normalized size of antiderivative = 5.30

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right) dx$$
$$= \frac{a^6x^6}{6}+\frac{1}{6}a^5x^7(3b+2cx)+\frac{5}{72}a^4x^8(3b+2cx)^2+\frac{5}{324}a^3x^9(3b+2cx)^3+\frac{5a^2x^{10}(3b+2cx)^4}{2592}$$
$$+a\left(x+\frac{b^5x^{11}}{32}+\frac{5}{48}b^4cx^{12}+\frac{5}{36}b^3c^2x^{13}+\frac{5}{54}b^2c^3x^{14}+\frac{5}{162}bc^4x^{15}+\frac{c^5x^{16}}{243}\right)$$
$$+\frac{x^2(729b^6x^{10}+2916b^5cx^{11}+4860b^4c^2x^{12}+4320b^3c^3x^{13}+2160b^2c^4x^{14}+576b(243+c^5x^{15})+64cx(1458$$
$$279936$$

```
[In] Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (a^6*x^6)/6 + (a^5*x^7*(3*b + 2*c*x))/6 + (5*a^4*x^8*(3*b + 2*c*x)^2)/72 +
(5*a^3*x^9*(3*b + 2*c*x)^3)/324 + (5*a^2*x^10*(3*b + 2*c*x)^4)/2592 + a*(x
+ (b^5*x^11)/32 + (5*b^4*c*x^12)/48 + (5*b^3*c^2*x^13)/36 + (5*b^2*c^3*x^14
)/54 + (5*b*c^4*x^15)/162 + (c^5*x^16)/243) + (x^2*(729*b^6*x^10 + 2916*b^5
*c*x^11 + 4860*b^4*c^2*x^12 + 4320*b^3*c^3*x^13 + 2160*b^2*c^4*x^14 + 576*b
*(243 + c^5*x^15) + 64*c*x*(1458 + c^5*x^15)))/279936
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result
default	$ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(ax+\frac{1}{2}bx^2+\frac{1}{3}cx^3\right)^6}{6}$
norman	$ax+\left(\frac{1}{243}ac^5+\frac{5}{648}b^2c^4\right)x^{16}+\left(\frac{1}{3}ca^5+\frac{5}{8}b^2a^4\right)x^8+\left(\frac{5}{162}abc^4+\frac{5}{324}b^3c^3\right)x^{15}+\left(\frac{5}{6}a^4bc+\frac{5}{12}a^3b\right.$
risch	$\frac{1}{2}a^5bx^7+\frac{5}{12}a^3b^3x^9+\frac{1}{32}ab^5x^{11}+\frac{1}{384}b^6x^{12}+\frac{1}{6}a^6x^6+\frac{1}{3}cx^3+ax+\frac{1}{2}bx^2+\frac{1}{96}b^5cx^{13}+\frac{5}{288}b^4c$
parallelrisch	$\frac{1}{2}a^5bx^7+\frac{5}{12}a^3b^3x^9+\frac{1}{32}ab^5x^{11}+\frac{1}{384}b^6x^{12}+\frac{1}{6}a^6x^6+\frac{1}{3}cx^3+ax+\frac{1}{2}bx^2+\frac{1}{96}b^5cx^{13}+\frac{5}{288}b^4c$
gosper	$\frac{x(64c^6x^{17}+576bc^5x^{16}+1152ac^5x^{15}+2160b^2c^4x^{15}+8640abc^4x^{14}+4320b^3c^3x^{14}+8640a^2c^4x^{13}+25920ab^2c^3x^{13}+4860b^4c^2x^{13}+}{}$

```
[In] int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
[Out] a*x+1/2*b*x^2+1/3*c*x^3+1/6*(a*x+1/2*b*x^2+1/3*c*x^3)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(37) = 74.

Time = 0.38 (sec) , antiderivative size = 289, normalized size of antiderivative = 6.28

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx = \frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{1}{1944}\left(15b^2c^4+8ac^5\right)x^{16}+\frac{5}{324}\left(b^3c^3+2abc^4\right)x^{15}+\frac{5}{2592}\left(9b^4c^2+48ab^2c^3+16a^2c^4\right)x^{14}+\frac{1}{864}\left(9b^5c+120ab^3c^2+160a^2bc^3\right)x^{13}+\frac{1}{2}a^5bx^7+\frac{1}{10368}\left(27b^6+1080ab^4c+4320a^2b^2c^2+1280a^3c^3\right)x^{12}+\frac{1}{6}a^6x^6+\frac{1}{288}\left(9ab^5+120a^2b^3c+160a^3bc^2\right)x^{11}+\frac{5}{288}\left(9a^2b^4+48a^3b^2c+16a^4c^2\right)x^{10}+\frac{5}{12}\left(a^3b^3+2a^4bc\right)x^9+\frac{1}{24}\left(15a^4b^2+8a^5c\right)x^8+\frac{1}{3}cx^3+\frac{1}{2}bx^2+ax$$

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="frica
s")
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 + 5
/324*(b^3*c^3 + 2*a*b*c^4)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^2*c^3 + 16*a^2
*c^4)*x^14 + 1/864*(9*b^5*c + 120*a*b^3*c^2 + 160*a^2*b*c^3)*x^13 + 1/2*a^5
*b*x^7 + 1/10368*(27*b^6 + 1080*a*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3)*
x^12 + 1/6*a^6*x^6 + 1/288*(9*a*b^5 + 120*a^2*b^3*c + 160*a^3*b*c^2)*x^11 +
 5/288*(9*a^2*b^4 + 48*a^3*b^2*c + 16*a^4*c^2)*x^10 + 5/12*(a^3*b^3 + 2*a^4
*b*c)*x^9 + 1/24*(15*a^4*b^2 + 8*a^5*c)*x^8 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(36) = 72.

Time = 0.07 (sec) , antiderivative size = 323, normalized size of antiderivative = 7.02

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$=\frac{a^6x^6}{6}+\frac{a^5bx^7}{2}+ax+\frac{bc^5x^{17}}{486}+\frac{bx^2}{2}+\frac{c^6x^{18}}{4374}+\frac{cx^3}{3}+x^{16}\left(\frac{ac^5}{243}+\frac{5b^2c^4}{648}\right)+x^{15}\cdot\left(\frac{5abc^4}{162}+\frac{5b^3c^3}{324}\right)+x^{14}\cdot\left(\frac{5a^2c^4}{162}+\frac{5ab^2c^3}{54}+\frac{5b^4c^2}{288}\right)+x^{13}\cdot\left(\frac{5a^2bc^3}{27}+\frac{5ab^3c^2}{36}+\frac{b^5c}{96}\right)+x^{12}\cdot\left(\frac{10a^3c^3}{81}+\frac{5a^2b^2c^2}{12}+\frac{5ab^4c}{48}+\frac{b^6}{384}\right)+x^{11}\cdot\left(\frac{5a^3bc^2}{9}+\frac{5a^2b^3c}{12}+\frac{ab^5}{32}\right)+x^{10}\cdot\left(\frac{5a^4c^2}{18}+\frac{5a^3b^2c}{6}+\frac{5a^2b^4}{32}\right)+x^9\cdot\left(\frac{5a^4bc}{6}+\frac{5a^3b^3}{12}\right)+x^8\left(\frac{a^5c}{3}+\frac{5a^4b^2}{8}\right)$$

```
[In] integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] a**6*x**6/6 + a**5*b*x**7/2 + a*x + b*c**5*x**17/486 + b*x**2/2 + c**6*x**1
8/4374 + c*x**3/3 + x**16*(a*c**5/243 + 5*b**2*c**4/648) + x**15*(5*a*b*c**
4/162 + 5*b**3*c**3/324) + x**14*(5*a**2*c**4/162 + 5*a*b**2*c**3/54 + 5*b*
*4*c**2/288) + x**13*(5*a**2*b*c**3/27 + 5*a*b**3*c**2/36 + b**5*c/96) + x*
*12*(10*a**3*c**3/81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + b**6/384) + x*
*11*(5*a**3*b*c**2/9 + 5*a**2*b**3*c/12 + a*b**5/32) + x**10*(5*a**4*c**2/1
8 + 5*a**3*b**2*c/6 + 5*a**2*b**4/32) + x**9*(5*a**4*b*c/6 + 5*a**3*b**3/12
) + x**8*(a**5*c/3 + 5*a**4*b**2/8)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(37) = 74.

Time = 0.19 (sec) , antiderivative size = 289, normalized size of antiderivative = 6.28

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$=\frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{1}{1944}(15b^2c^4+8ac^5)x^{16}+\frac{5}{324}(b^3c^3+2abc^4)x^{15}+\frac{5}{2592}(9b^4c^2+48ab^2c^3+16a^2c^4)x^{14}+\frac{1}{864}(9b^5c+120ab^3c^2+160a^2bc^3)x^{13}+\frac{1}{2}a^5bx^7+\frac{1}{10368}(27b^6+1080ab^4c+4320a^2b^2c^2+1280a^3c^3)x^{12}+\frac{1}{6}a^6x^6+\frac{1}{288}(9ab^5+120a^2b^3c+160a^3bc^2)x^{11}+\frac{5}{288}(9a^2b^4+48a^3b^2c+16a^4c^2)x^{10}+\frac{5}{12}(a^3b^3+2a^4bc)x^9+\frac{1}{24}(15a^4b^2+8a^5c)x^8+\frac{1}{3}cx^3+\frac{1}{2}bx^2+ax$$

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxim
a")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 + 5
/324*(b^3*c^3 + 2*a*b*c^4)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^2*c^3 + 16*a^2
*c^4)*x^14 + 1/864*(9*b^5*c + 120*a*b^3*c^2 + 160*a^2*b*c^3)*x^13 + 1/2*a^5
*b*x^7 + 1/10368*(27*b^6 + 1080*a*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3)*
x^12 + 1/6*a^6*x^6 + 1/288*(9*a*b^5 + 120*a^2*b^3*c + 160*a^3*b*c^2)*x^11 +
 5/288*(9*a^2*b^4 + 48*a^3*b^2*c + 16*a^4*c^2)*x^10 + 5/12*(a^3*b^3 + 2*a^4
*b*c)*x^9 + 1/24*(15*a^4*b^2 + 8*a^5*c)*x^8 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx = \frac{1}{279936}\left(2\,cx^3+3\,bx^2+6\,ax\right)^6+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac"
)
```

```
[Out] 1/279936*(2*c*x^3 + 3*b*x^2 + 6*a*x)^6 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 270, normalized size of antiderivative = 5.87

$$\begin{aligned}\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx &= x^{12}\left(\frac{10\,a^3\,c^3}{81}+\frac{5\,a^2\,b^2\,c^2}{12}+\frac{5\,a\,b^4\,c}{48}+\frac{b^6}{384}\right)+a\,x+\frac{b\,x^2}{2}\\ &+\frac{c\,x^3}{3}+\frac{a^6\,x^6}{6}+\frac{c^6\,x^{18}}{4374}+\frac{5\,a^2\,x^{10}\,(16\,a^2\,c^2+48\,a\,b^2\,c+9\,b^4)}{288}\\ &+\frac{5\,c^2\,x^{14}\,(16\,a^2\,c^2+48\,a\,b^2\,c+9\,b^4)}{2592}+\frac{a^5\,b\,x^7}{2}+\frac{b\,c^5\,x^{17}}{486}+\frac{a^4\,x^8\,(15\,b^2+8\,a\,c)}{24}\\ &+\frac{c^4\,x^{16}\,(15\,b^2+8\,a\,c)}{1944}+\frac{a\,b\,x^{11}\,(160\,a^2\,c^2+120\,a\,b^2\,c+9\,b^4)}{288}\\ &+\frac{b\,c\,x^{13}\,(160\,a^2\,c^2+120\,a\,b^2\,c+9\,b^4)}{864}+\frac{5\,a^3\,b\,x^9\,(b^2+2\,a\,c)}{12}+\frac{5\,b\,c^3\,x^{15}\,(b^2+2\,a\,c)}{324}\end{aligned}$$

```
[In] int(((a*x + (b*x^2)/2 + (c*x^3)/3)^5 + 1)*(a + b*x + c*x^2),x)
```

```
[Out] x^12*(b^6/384 + (10*a^3*c^3)/81 + (5*a^2*b^2*c^2)/12 + (5*a*b^4*c)/48) + a*
x + (b*x^2)/2 + (c*x^3)/3 + (a^6*x^6)/6 + (c^6*x^18)/4374 + (5*a^2*x^10*(9*
```

```
b^4 + 16*a^2*c^2 + 48*a*b^2*c))/288 + (5*c^2*x^14*(9*b^4 + 16*a^2*c^2 + 48*
a*b^2*c))/2592 + (a^5*b*x^7)/2 + (b*c^5*x^17)/486 + (a^4*x^8*(8*a*c + 15*b^
2))/24 + (c^4*x^16*(8*a*c + 15*b^2))/1944 + (a*b*x^11*(9*b^4 + 160*a^2*c^2
+ 120*a*b^2*c))/288 + (b*c*x^13*(9*b^4 + 160*a^2*c^2 + 120*a*b^2*c))/864 +
(5*a^3*b*x^9*(2*a*c + b^2))/12 + (5*b*c^3*x^15*(2*a*c + b^2))/324
```

3.215 $\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal result . . . 1499
Rubi [A] (verified) . . . 1499
Mathematica [B] (verified) . . . 1500
Maple [A] (verified) . . . 1500
Fricas [B] (verification not implemented) . . . 1501
Sympy [B] (verification not implemented) . . . 1502
Maxima [B] (verification not implemented) . . . 1504
Giac [B] (verification not implemented) . . . 1505
Mupad [B] (verification not implemented) . . . 1506

Optimal result

Integrand size = 36, antiderivative size = 47

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6$$

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+1/6*(d+a*x+1/2*b*x^2+1/3*c*x^3)^6
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1605}

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx = \frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[In] Int[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
```

```
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int (1+x^5)\,dx, x, d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 248 vs. 2(47) = 94.

Time = 0.08 (sec) , antiderivative size = 248, normalized size of antiderivative = 5.28

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$= \frac{x(6a+x(3b+2cx))\,(46656+46656d^5+7776a^5x^5+243b^5x^{10}+810b^4cx^{11}+1080b^3c^2x^{12}+720b^2c^3x^{13}+2}{}$$

```
[In] Integrate[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
[Out] (x*(6*a + x*(3*b + 2*c*x))*(46656 + 46656*d^5 + 7776*a^5*x^5 + 243*b^5*x^10
 + 810*b^4*c*x^11 + 1080*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14 +
 32*c^5*x^15 + 6480*a^4*x^6*(3*b + 2*c*x) + 2160*a^3*x^7*(3*b + 2*c*x)^2 +
360*a^2*x^8*(3*b + 2*c*x)^3 + 30*a*x^9*(3*b + 2*c*x)^4 + 19440*d^4*x*(6*a +
 x*(3*b + 2*c*x)) + 4320*d^3*x^2*(6*a + x*(3*b + 2*c*x))^2 + 540*d^2*x^3*(6
*a + x*(3*b + 2*c*x))^3 + 36*d*x^4*(6*a + x*(3*b + 2*c*x))^4))/279936
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

method	result
default	$\frac{\left(d+ax+\frac{1}{2}bx^2+\frac{1}{3}cx^3\right)^6}{6}+d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$
norman	$\left(\frac{1}{243}ac^5+\frac{5}{648}b^2c^4\right)x^{16}+\left(\frac{5}{2}a^2d^4+\frac{1}{2}bd^5+\frac{1}{2}b\right)x^2+\left(\frac{5}{162}abc^4+\frac{5}{324}b^3c^3+\frac{1}{243}c^5d\right)x^{15}+\left(\frac{5}{162}a\right.$
gosper	$\frac{x(64c^6x^{17}+576bc^5x^{16}+1152ac^5x^{15}+2160b^2c^4x^{15}+8640abc^4x^{14}+4320b^3c^3x^{14}+1152c^5dx^{14}+8640a^2c^4x^{13}+25920ab^2c^3x^{13}+}{}$
risch	$\frac{5}{162}x^{14}bc^4d+\frac{5}{54}x^{13}b^2c^3d+\frac{5}{2}a^2d^4x^2+\frac{5}{12}x^6b^3d^3+\frac{1}{2}bd^5x^2+\frac{10}{3}x^6a^3cd^2+\frac{10}{3}x^5a^2cd^3+\frac{5}{3}x^4acd$
parallelrisch	$\frac{5}{162}x^{14}bc^4d+\frac{5}{54}x^{13}b^2c^3d+\frac{5}{2}a^2d^4x^2+\frac{5}{12}x^6b^3d^3+\frac{1}{2}bd^5x^2+\frac{10}{3}x^6a^3cd^2+\frac{10}{3}x^5a^2cd^3+\frac{5}{3}x^4acd$

```
[In] int((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/6*(d+a*x+1/2*b*x^2+1/3*c*x^3)^6+d+a*x+1/2*b*x^2+1/3*c*x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(40) = 80$.

Time = 0.25 (sec) , antiderivative size = 773, normalized size of antiderivative = 16.45

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$\begin{aligned}
&= \frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{1}{1944}\left(15\,b^2c^4+8\,ac^5\right)x^{16} \\
&+ \frac{1}{972}\left(15\,b^3c^3+30\,abc^4+4\,c^5d\right)x^{15} + \frac{5}{2592}\left(9\,b^4c^2+48\,ab^2c^3+16\,a^2c^4+16\,bc^4d\right)x^{14} \\
&+ \frac{1}{2592}\left(27\,b^5c+360\,ab^3c^2+480\,a^2bc^3+80\left(3\,b^2c^3+2\,ac^4\right)d\right)x^{13} \\
&+ \frac{1}{10368}\left(27\,b^6+1080\,ab^4c+4320\,a^2b^2c^2+1280\,a^3c^3+320\,c^4d^2+480\left(3\,b^3c^2+8\,abc^3\right)d\right)x^{12} \\
&+ \frac{1}{864}\left(27\,ab^5+360\,a^2b^3c+480\,a^3bc^2+160\,bc^3d^2+10\left(9\,b^4c+72\,ab^2c^2+32\,a^2c^3\right)d\right)x^{11} \\
&+ \frac{1}{864}\left(135\,a^2b^4+720\,a^3b^2c+240\,a^4c^2+40\left(9\,b^2c^2+8\,ac^3\right)d^2+9\left(3\,b^5+80\,ab^3c+160\,a^2bc^2\right)d\right)x^{10} \\
&+ \frac{5}{1296}\left(108\,a^3b^3+216\,a^4bc+32\,c^3d^3+108\left(b^3c+4\,abc^2\right)d^2+9\left(9\,ab^4+72\,a^2b^2c+32\,a^3c^2\right)d\right)x^9 \\
&+ \frac{1}{288}\left(180\,a^4b^2+96\,a^5c+160\,bc^2d^3+15\left(3\,b^4+48\,ab^2c+32\,a^2c^2\right)d^2+120\left(3\,a^2b^3+8\,a^3bc\right)d\right)x^8 \\
&+ \frac{1}{36}\left(18\,a^5b+10\left(3\,b^2c+4\,ac^2\right)d^3+45\left(ab^3+4\,a^2bc\right)d^2+30\left(3\,a^3b^2+2\,a^4c\right)d\right)x^7 \\
&+ \frac{1}{36}\left(6\,a^6+90\,a^4bd+10\,c^2d^4+15\left(b^3+8\,abc\right)d^3+15\left(9\,a^2b^2+8\,a^3c\right)d^2\right)x^6 \\
&+ \frac{1}{6}\left(6\,a^5d+30\,a^3bd^2+5\,bcd^4+5\left(3\,ab^2+4\,a^2c\right)d^3\right)x^5 \\
&+ \frac{5}{24}\left(12\,a^4d^2+24\,a^2bd^3+\left(3\,b^2+8\,ac\right)d^4\right)x^4 \\
&+ \frac{1}{6}\left(20\,a^3d^3+15\,abd^4+2\,cd^5+2\,c\right)x^3 + \frac{1}{2}\left(5\,a^2d^4+bd^5+b\right)x^2 + \left(ad^5+a\right)x
\end{aligned}$$

```
[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fri
cas")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 + 1
/972*(15*b^3*c^3 + 30*a*b*c^4 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^
2*c^3 + 16*a^2*c^4 + 16*b*c^4*d)*x^14 + 1/2592*(27*b^5*c + 360*a*b^3*c^2 +
480*a^2*b*c^3 + 80*(3*b^2*c^3 + 2*a*c^4)*d)*x^13 + 1/10368*(27*b^6 + 1080*a
```

```
*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3 + 320*c^4*d^2 + 480*(3*b^3*c^2 + 8
*a*b*c^3)*d)*x^12 + 1/864*(27*a*b^5 + 360*a^2*b^3*c + 480*a^3*b*c^2 + 160*b
*c^3*d^2 + 10*(9*b^4*c + 72*a*b^2*c^2 + 32*a^2*c^3)*d)*x^11 + 1/864*(135*a^
2*b^4 + 720*a^3*b^2*c + 240*a^4*c^2 + 40*(9*b^2*c^2 + 8*a*c^3)*d^2 + 9*(3*b
^5 + 80*a*b^3*c + 160*a^2*b*c^2)*d)*x^10 + 5/1296*(108*a^3*b^3 + 216*a^4*b*
c + 32*c^3*d^3 + 108*(b^3*c + 4*a*b*c^2)*d^2 + 9*(9*a*b^4 + 72*a^2*b^2*c +
32*a^3*c^2)*d)*x^9 + 1/288*(180*a^4*b^2 + 96*a^5*c + 160*b*c^2*d^3 + 15*(3*
b^4 + 48*a*b^2*c + 32*a^2*c^2)*d^2 + 120*(3*a^2*b^3 + 8*a^3*b*c)*d)*x^8 + 1
/36*(18*a^5*b + 10*(3*b^2*c + 4*a*c^2)*d^3 + 45*(a*b^3 + 4*a^2*b*c)*d^2 + 3
0*(3*a^3*b^2 + 2*a^4*c)*d)*x^7 + 1/36*(6*a^6 + 90*a^4*b*d + 10*c^2*d^4 + 15
*(b^3 + 8*a*b*c)*d^3 + 15*(9*a^2*b^2 + 8*a^3*c)*d^2)*x^6 + 1/6*(6*a^5*d + 3
0*a^3*b*d^2 + 5*b*c*d^4 + 5*(3*a*b^2 + 4*a^2*c)*d^3)*x^5 + 5/24*(12*a^4*d^2
 + 24*a^2*b*d^3 + (3*b^2 + 8*a*c)*d^4)*x^4 + 1/6*(20*a^3*d^3 + 15*a*b*d^4 +
 2*c*d^5 + 2*c)*x^3 + 1/2*(5*a^2*d^4 + b*d^5 + b)*x^2 + (a*d^5 + a)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 930 vs. $2(37) = 74$.

Time = 0.12 (sec) , antiderivative size = 930, normalized size of antiderivative = 19.79

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$= \frac{bc^5x^{17}}{486}+\frac{c^6x^{18}}{4374}+x^{16}\left(\frac{ac^5}{243}+\frac{5b^2c^4}{648}\right)+x^{15}\cdot\left(\frac{5abc^4}{162}+\frac{5b^3c^3}{324}+\frac{c^5d}{243}\right)+x^{14}\cdot\left(\frac{5a^2c^4}{162}+\frac{5ab^2c^3}{54}+\frac{5b^4c^2}{288}+\frac{5bc^4d}{162}\right)+x^{13}\cdot\left(\frac{5a^2bc^3}{27}+\frac{5ab^3c^2}{36}+\frac{5ac^4d}{81}+\frac{b^5c}{96}+\frac{5b^2c^3d}{54}\right)+x^{12}\cdot\left(\frac{10a^3c^3}{81}+\frac{5a^2b^2c^2}{12}+\frac{5ab^4c}{48}+\frac{10abc^3d}{27}+\frac{b^6}{384}+\frac{5b^3c^2d}{36}+\frac{5c^4d^2}{162}\right)+x^{11}\cdot\left(\frac{5a^3bc^2}{9}+\frac{5a^2b^3c}{12}+\frac{10a^2c^3d}{27}+\frac{ab^5}{32}+\frac{5ab^2c^2d}{6}+\frac{5b^4cd}{48}+\frac{5bc^3d^2}{27}\right)+x^{10}\cdot\left(\frac{5a^4c^2}{18}+\frac{5a^3b^2c}{6}+\frac{5a^2b^4}{32}+\frac{5a^2bc^2d}{3}+\frac{5ab^3cd}{6}+\frac{10ac^3d^2}{27}+\frac{b^5d}{32}+\frac{5b^2c^2d^2}{12}\right)+x^9\cdot\left(\frac{5a^4bc}{6}+\frac{5a^3b^3}{12}+\frac{10a^3c^2d}{9}+\frac{5a^2b^2cd}{2}+\frac{5ab^4d}{16}+\frac{5abc^2d^2}{3}+\frac{5b^3cd^2}{12}+\frac{10c^3d^3}{81}\right)+x^8\left(\frac{a^5c}{3}+\frac{5a^4b^2}{8}+\frac{10a^3bcd}{3}+\frac{5a^2b^3d}{4}+\frac{5a^2c^2d^2}{3}+\frac{5ab^2cd^2}{2}+\frac{5b^4d^2}{32}+\frac{5bc^2d^3}{9}\right)+x^7\left(\frac{a^5b}{2}+\frac{5a^4cd}{3}+\frac{5a^3b^2d}{2}+5a^2bcd^2+\frac{5ab^3d^2}{4}+\frac{10ac^2d^3}{9}+\frac{5b^2cd^3}{6}\right)+x^6\left(\frac{a^6}{6}+\frac{5a^4bd}{2}+\frac{10a^3cd^2}{3}+\frac{15a^2b^2d^2}{4}+\frac{10abcd^3}{3}+\frac{5b^3d^3}{12}+\frac{5c^2d^4}{18}\right)+x^5\left(a^5d+5a^3bd^2+\frac{10a^2cd^3}{3}+\frac{5ab^2d^3}{2}+\frac{5bcd^4}{6}\right)+x^4\cdot\left(\frac{5a^4d^2}{2}+5a^2bd^3+\frac{5acd^4}{3}+\frac{5b^2d^4}{8}\right)+x^3\cdot\left(\frac{10a^3d^3}{3}+\frac{5abd^4}{2}+\frac{cd^5}{3}+\frac{c}{3}\right)+x^2\cdot\left(\frac{5a^2d^4}{2}+\frac{bd^5}{2}+\frac{b}{2}\right)+x(ad^5+a)$$

```
[In] integrate((c*x**2+b*x+a)*(1+(d+a*x+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] b*c**5*x**17/486 + c**6*x**18/4374 + x**16*(a*c**5/243 + 5*b**2*c**4/648) +
 x**15*(5*a*b*c**4/162 + 5*b**3*c**3/324 + c**5*d/243) + x**14*(5*a**2*c**4
/162 + 5*a*b**2*c**3/54 + 5*b**4*c**2/288 + 5*b*c**4*d/162) + x**13*(5*a**2
*b*c**3/27 + 5*a*b**3*c**2/36 + 5*a*c**4*d/81 + b**5*c/96 + 5*b**2*c**3*d/5
4) + x**12*(10*a**3*c**3/81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + 10*a*b*
c**3*d/27 + b**6/384 + 5*b**3*c**2*d/36 + 5*c**4*d**2/162) + x**11*(5*a**3*
b*c**2/9 + 5*a**2*b**3*c/12 + 10*a**2*c**3*d/27 + a*b**5/32 + 5*a*b**2*c**2
*d/6 + 5*b**4*c*d/48 + 5*b*c**3*d**2/27) + x**10*(5*a**4*c**2/18 + 5*a**3*b
**2*c/6 + 5*a**2*b**4/32 + 5*a**2*b*c**2*d/3 + 5*a*b**3*c*d/6 + 10*a*c**3*d
**2/27 + b**5*d/32 + 5*b**2*c**2*d**2/12) + x**9*(5*a**4*b*c/6 + 5*a**3*b**
3/12 + 10*a**3*c**2*d/9 + 5*a**2*b**2*c*d/2 + 5*a*b**4*d/16 + 5*a*b*c**2*d*
*2/3 + 5*b**3*c*d**2/12 + 10*c**3*d**3/81) + x**8*(a**5*c/3 + 5*a**4*b**2/8
 + 10*a**3*b*c*d/3 + 5*a**2*b**3*d/4 + 5*a**2*c**2*d**2/3 + 5*a*b**2*c*d**2
```

```
/2 + 5*b**4*d**2/32 + 5*b*c**2*d**3/9) + x**7*(a**5*b/2 + 5*a**4*c*d/3 + 5*
a**3*b**2*d/2 + 5*a**2*b*c*d**2 + 5*a*b**3*d**2/4 + 10*a*c**2*d**3/9 + 5*b*
*2*c*d**3/6) + x**6*(a**6/6 + 5*a**4*b*d/2 + 10*a**3*c*d**2/3 + 15*a**2*b**
2*d**2/4 + 10*a*b*c*d**3/3 + 5*b**3*d**3/12 + 5*c**2*d**4/18) + x**5*(a**5*
d + 5*a**3*b*d**2 + 10*a**2*c*d**3/3 + 5*a*b**2*d**3/2 + 5*b*c*d**4/6) + x*
*4*(5*a**4*d**2/2 + 5*a**2*b*d**3 + 5*a*c*d**4/3 + 5*b**2*d**4/8) + x**3*(1
0*a**3*d**3/3 + 5*a*b*d**4/2 + c*d**5/3 + c/3) + x**2*(5*a**2*d**4/2 + b*d*
*5/2 + b/2) + x*(a*d**5 + a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(40) = 80$.

Time = 0.20 (sec) , antiderivative size = 773, normalized size of antiderivative = 16.45

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$\begin{aligned}
&= \frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{1}{1944}\left(15\,b^2c^4 + 8\,ac^5\right)x^{16} \\
&+ \frac{1}{972}\left(15\,b^3c^3 + 30\,abc^4 + 4\,c^5d\right)x^{15} + \frac{5}{2592}\left(9\,b^4c^2 + 48\,ab^2c^3 + 16\,a^2c^4 + 16\,bc^4d\right)x^{14} \\
&+ \frac{1}{2592}\left(27\,b^5c + 360\,ab^3c^2 + 480\,a^2bc^3 + 80\left(3\,b^2c^3 + 2\,ac^4\right)d\right)x^{13} \\
&+ \frac{1}{10368}\left(27\,b^6 + 1080\,ab^4c + 4320\,a^2b^2c^2 + 1280\,a^3c^3 + 320\,c^4d^2 + 480\left(3\,b^3c^2 + 8\,abc^3\right)d\right)x^{12} \\
&+ \frac{1}{864}\left(27\,ab^5 + 360\,a^2b^3c + 480\,a^3bc^2 + 160\,bc^3d^2 + 10\left(9\,b^4c + 72\,ab^2c^2 + 32\,a^2c^3\right)d\right)x^{11} \\
&+ \frac{1}{864}\left(135\,a^2b^4 + 720\,a^3b^2c + 240\,a^4c^2 + 40\left(9\,b^2c^2 + 8\,ac^3\right)d^2 + 9\left(3\,b^5 + 80\,ab^3c + 160\,a^2bc^2\right)d\right)x^{10} \\
&+ \frac{5}{1296}\left(108\,a^3b^3 + 216\,a^4bc + 32\,c^3d^3 + 108\left(b^3c + 4\,abc^2\right)d^2 + 9\left(9\,ab^4 + 72\,a^2b^2c + 32\,a^3c^2\right)d\right)x^9 \\
&+ \frac{1}{288}\left(180\,a^4b^2 + 96\,a^5c + 160\,bc^2d^3 + 15\left(3\,b^4 + 48\,ab^2c + 32\,a^2c^2\right)d^2 + 120\left(3\,a^2b^3 + 8\,a^3bc\right)d\right)x^8 \\
&+ \frac{1}{36}\left(18\,a^5b + 10\left(3\,b^2c + 4\,ac^2\right)d^3 + 45\left(ab^3 + 4\,a^2bc\right)d^2 + 30\left(3\,a^3b^2 + 2\,a^4c\right)d\right)x^7 \\
&+ \frac{1}{36}\left(6\,a^6 + 90\,a^4bd + 10\,c^2d^4 + 15\left(b^3 + 8\,abc\right)d^3 + 15\left(9\,a^2b^2 + 8\,a^3c\right)d^2\right)x^6 \\
&+ \frac{1}{6}\left(6\,a^5d + 30\,a^3bd^2 + 5\,bcd^4 + 5\left(3\,ab^2 + 4\,a^2c\right)d^3\right)x^5 \\
&+ \frac{5}{24}\left(12\,a^4d^2 + 24\,a^2bd^3 + \left(3\,b^2 + 8\,ac\right)d^4\right)x^4 \\
&+ \frac{1}{6}\left(20\,a^3d^3 + 15\,abd^4 + 2\,cd^5 + 2\,c\right)x^3 + \frac{1}{2}\left(5\,a^2d^4 + bd^5 + b\right)x^2 + \left(ad^5 + a\right)x
\end{aligned}$$

```
[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="max
ima")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 + 1
/972*(15*b^3*c^3 + 30*a*b*c^4 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^
2*c^3 + 16*a^2*c^4 + 16*b*c^4*d)*x^14 + 1/2592*(27*b^5*c + 360*a*b^3*c^2 +
480*a^2*b*c^3 + 80*(3*b^2*c^3 + 2*a*c^4)*d)*x^13 + 1/10368*(27*b^6 + 1080*a
*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3 + 320*c^4*d^2 + 480*(3*b^3*c^2 + 8
*a*b*c^3)*d)*x^12 + 1/864*(27*a*b^5 + 360*a^2*b^3*c + 480*a^3*b*c^2 + 160*b
*c^3*d^2 + 10*(9*b^4*c + 72*a*b^2*c^2 + 32*a^2*c^3)*d)*x^11 + 1/864*(135*a^
2*b^4 + 720*a^3*b^2*c + 240*a^4*c^2 + 40*(9*b^2*c^2 + 8*a*c^3)*d^2 + 9*(3*b
^5 + 80*a*b^3*c + 160*a^2*b*c^2)*d)*x^10 + 5/1296*(108*a^3*b^3 + 216*a^4*b*
c + 32*c^3*d^3 + 108*(b^3*c + 4*a*b*c^2)*d^2 + 9*(9*a*b^4 + 72*a^2*b^2*c +
32*a^3*c^2)*d)*x^9 + 1/288*(180*a^4*b^2 + 96*a^5*c + 160*b*c^2*d^3 + 15*(3*
b^4 + 48*a*b^2*c + 32*a^2*c^2)*d^2 + 120*(3*a^2*b^3 + 8*a^3*b*c)*d)*x^8 + 1
/36*(18*a^5*b + 10*(3*b^2*c + 4*a*c^2)*d^3 + 45*(a*b^3 + 4*a^2*b*c)*d^2 + 3
0*(3*a^3*b^2 + 2*a^4*c)*d)*x^7 + 1/36*(6*a^6 + 90*a^4*b*d + 10*c^2*d^4 + 15
*(b^3 + 8*a*b*c)*d^3 + 15*(9*a^2*b^2 + 8*a^3*c)*d^2)*x^6 + 1/6*(6*a^5*d + 3
0*a^3*b*d^2 + 5*b*c*d^4 + 5*(3*a*b^2 + 4*a^2*c)*d^3)*x^5 + 5/24*(12*a^4*d^2
 + 24*a^2*b*d^3 + (3*b^2 + 8*a*c)*d^4)*x^4 + 1/6*(20*a^3*d^3 + 15*a*b*d^4 +
 2*c*d^5 + 2*c)*x^3 + 1/2*(5*a^2*d^4 + b*d^5 + b)*x^2 + (a*d^5 + a)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(40) = 80$.

Time = 0.44 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.26

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$=\frac{1}{279936}\left(2\,cx^3+3\,bx^2+6\,ax\right)^6+\frac{1}{7776}\left(2\,cx^3+3\,bx^2+6\,ax\right)^5d+\frac{5}{2592}\left(2\,cx^3+3\,bx^2+6\,ax\right)^4d^2+\frac{5}{324}\left(2\,cx^3+3\,bx^2+6\,ax\right)^3d^3+\frac{5}{72}\left(2\,cx^3+3\,bx^2+6\,ax\right)^2d^4+\frac{1}{6}\left(2\,cx^3+3\,bx^2+6\,ax\right)d^5+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

```
[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="gia
c")
```

```
[Out] 1/279936*(2*c*x^3 + 3*b*x^2 + 6*a*x)^6 + 1/7776*(2*c*x^3 + 3*b*x^2 + 6*a*x)
^5*d + 5/2592*(2*c*x^3 + 3*b*x^2 + 6*a*x)^4*d^2 + 5/324*(2*c*x^3 + 3*b*x^2
+ 6*a*x)^3*d^3 + 5/72*(2*c*x^3 + 3*b*x^2 + 6*a*x)^2*d^4 + 1/6*(2*c*x^3 + 3*
b*x^2 + 6*a*x)*d^5 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 753, normalized size of antiderivative = 16.02

$$\int \left(a+bx+cx^2\right)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$$
$$= x^{10}\left(\frac{5\,a^4\,c^2}{18}+\frac{5\,a^3\,b^2\,c}{6}+\frac{5\,a^2\,b^4}{32}+\frac{5\,a^2\,b\,c^2\,d}{3}+\frac{5\,a\,b^3\,c\,d}{6}+\frac{10\,a\,c^3\,d^2}{27}+\frac{b^5\,d}{32}+\frac{5\,b^2\,c^2\,d^2}{12}\right)$$
$$+x^8\left(\frac{a^5\,c}{3}+\frac{5\,a^4\,b^2}{8}+\frac{10\,a^3\,b\,c\,d}{3}+\frac{5\,a^2\,b^3\,d}{4}+\frac{5\,a^2\,c^2\,d^2}{3}+\frac{5\,a\,b^2\,c\,d^2}{2}+\frac{5\,b^4\,d^2}{32}+\frac{5\,b\,c^2\,d^3}{9}\right)$$
$$+x^9\left(\frac{5\,a^4\,b\,c}{6}+\frac{5\,a^3\,b^3}{12}+\frac{10\,a^3\,c^2\,d}{9}+\frac{5\,a^2\,b^2\,c\,d}{2}+\frac{5\,a\,b^4\,d}{16}+\frac{5\,a\,b\,c^2\,d^2}{3}+\frac{5\,b^3\,c\,d^2}{12}+\frac{10\,c^3\,d^3}{81}\right)$$
$$+x^{14}\left(\frac{5\,a^2\,c^4}{162}+\frac{5\,a\,b^2\,c^3}{54}+\frac{5\,b^4\,c^2}{288}+\frac{5\,d\,b\,c^4}{162}\right)$$
$$+x^{12}\left(\frac{10\,a^3\,c^3}{81}+\frac{5\,a^2\,b^2\,c^2}{12}+\frac{5\,a\,b^4\,c}{48}+\frac{10\,a\,b\,c^3\,d}{27}+\frac{b^6}{384}+\frac{5\,b^3\,c^2\,d}{36}+\frac{5\,c^4\,d^2}{162}\right)$$
$$+x^6\left(\frac{a^6}{6}+\frac{5\,a^4\,b\,d}{2}+\frac{10\,a^3\,c\,d^2}{3}+\frac{15\,a^2\,b^2\,d^2}{4}+\frac{10\,a\,b\,c\,d^3}{3}+\frac{5\,b^3\,d^3}{12}+\frac{5\,c^2\,d^4}{18}\right)$$
$$+x^3\left(\frac{10\,a^3\,d^3}{3}+\frac{5\,b\,a\,d^4}{2}+\frac{c\,d^5}{3}+\frac{c}{3}\right)$$
$$+x^{11}\left(\frac{5\,a^3\,b\,c^2}{9}+\frac{5\,a^2\,b^3\,c}{12}+\frac{10\,a^2\,c^3\,d}{27}+\frac{a\,b^5}{32}+\frac{5\,a\,b^2\,c^2\,d}{6}+\frac{5\,b^4\,c\,d}{48}+\frac{5\,b\,c^3\,d^2}{27}\right)$$
$$+x^7\left(\frac{a^5\,b}{2}+\frac{5\,a^4\,c\,d}{3}+\frac{5\,a^3\,b^2\,d}{2}+5\,a^2\,b\,c\,d^2+\frac{5\,a\,b^3\,d^2}{4}+\frac{10\,a\,c^2\,d^3}{9}+\frac{5\,b^2\,c\,d^3}{6}\right)$$
$$+x^2\left(\frac{5\,a^2\,d^4}{2}+\frac{b\,d^5}{2}+\frac{b}{2}\right)+x^{13}\left(\frac{5\,a^2\,b\,c^3}{27}+\frac{5\,a\,b^3\,c^2}{36}+\frac{5\,d\,a\,c^4}{81}+\frac{b^5\,c}{96}+\frac{5\,d\,b^2\,c^3}{54}\right)$$
$$+x^5\left(a^5\,d+5\,a^3\,b\,d^2+\frac{10\,c\,a^2\,d^3}{3}+\frac{5\,a\,b^2\,d^3}{2}+\frac{5\,c\,b\,d^4}{6}\right)+\frac{c^6\,x^{18}}{4374}$$
$$+\frac{5\,d^2\,x^4\,\left(12\,a^4+24\,a^2\,b\,d+8\,c\,a\,d^2+3\,b^2\,d^2\right)}{24}+a\,x\,\left(d^5+1\right)$$
$$+\frac{b\,c^5\,x^{17}}{486}+\frac{c^3\,x^{15}\,\left(15\,b^3+30\,a\,b\,c+4\,d\,c^2\right)}{972}+\frac{c^4\,x^{16}\,\left(15\,b^2+8\,a\,c\right)}{1944}$$

```
[In] int(((d + a*x + (b*x^2)/2 + (c*x^3)/3)^5 + 1)*(a + b*x + c*x^2),x)
```

```
[Out] x^10*((b^5*d)/32 + (5*a^2*b^4)/32 + (5*a^4*c^2)/18 + (5*a^3*b^2*c)/6 + (10*
a*c^3*d^2)/27 + (5*b^2*c^2*d^2)/12 + (5*a*b^3*c*d)/6 + (5*a^2*b*c^2*d)/3) +
 x^8*((a^5*c)/3 + (5*a^4*b^2)/8 + (5*b^4*d^2)/32 + (5*a^2*b^3*d)/4 + (5*b*c
^2*d^3)/9 + (5*a^2*c^2*d^2)/3 + (10*a^3*b*c*d)/3 + (5*a*b^2*c*d^2)/2) + x^9
*((5*a^3*b^3)/12 + (10*c^3*d^3)/81 + (10*a^3*c^2*d)/9 + (5*b^3*c*d^2)/12 +
(5*a^4*b*c)/6 + (5*a*b^4*d)/16 + (5*a*b*c^2*d^2)/3 + (5*a^2*b^2*c*d)/2) + x
^14*((5*a^2*c^4)/162 + (5*b^4*c^2)/288 + (5*a*b^2*c^3)/54 + (5*b*c^4*d)/162
```

) + x^12*(b^6/384 + (10*a^3*c^3)/81 + (5*c^4*d^2)/162 + (5*b^3*c^2*d)/36 + (5*a^2*b^2*c^2)/12 + (5*a*b^4*c)/48 + (10*a*b*c^3*d)/27) + x^6*(a^6/6 + (5*b^3*d^3)/12 + (5*c^2*d^4)/18 + (10*a^3*c*d^2)/3 + (15*a^2*b^2*d^2)/4 + (5*a^4*b*d)/2 + (10*a*b*c*d^3)/3) + x^3*(c/3 + (c*d^5)/3 + (10*a^3*d^3)/3 + (5*a*b*d^4)/2) + x^11*((a*b^5)/32 + (5*a^2*b^3*c)/12 + (5*a^3*b*c^2)/9 + (10*a^2*c^3*d)/27 + (5*b*c^3*d^2)/27 + (5*b^4*c*d)/48 + (5*a*b^2*c^2*d)/6) + x^7*((a^5*b)/2 + (5*a*b^3*d^2)/4 + (5*a^3*b^2*d)/2 + (10*a*c^2*d^3)/9 + (5*b^2*c*d^3)/6 + (5*a^4*c*d)/3 + 5*a^2*b*c*d^2) + x^2*(b/2 + (b*d^5)/2 + (5*a^2*d^4)/2) + x^13*((b^5*c)/96 + (5*a*b^3*c^2)/36 + (5*a^2*b*c^3)/27 + (5*b^2*c^3*d)/54 + (5*a*c^4*d)/81) + x^5*(a^5*d + (5*a*b^2*d^3)/2 + 5*a^3*b*d^2 + (10*a^2*c*d^3)/3 + (5*b*c*d^4)/6) + (c^6*x^18)/4374 + (5*d^2*x^4*(12*a^4 + 3*b^2*d^2 + 24*a^2*b*d + 8*a*c*d^2))/24 + a*x*(d^5 + 1) + (b*c^5*x^17)/486 + (c^3*x^15*(4*c^2*d + 15*b^3 + 30*a*b*c))/972 + (c^4*x^16*(8*a*c + 15*b^2))/1944

3.216 $\int (a+cx^2) \left(1+\left(ax+\frac{cx^3}{3}\right)^n\right) dx$

Optimal result 1508
Rubi [A] (verified) 1508
Mathematica [A] (verified) 1509
Maple [A] (verified) 1509
Fricas [A] (verification not implemented) 1510
Sympy [B] (verification not implemented) 1510
Maxima [A] (verification not implemented) 1510
Giac [A] (verification not implemented) 1511
Mupad [B] (verification not implemented) 1511

Optimal result

Integrand size = 24, antiderivative size = 34

$$\int (a+cx^2) \left(1+\left(ax+\frac{cx^3}{3}\right)^n\right) dx = ax+\frac{cx^3}{3}+\frac{\left(ax+\frac{cx^3}{3}\right)^{1+n}}{1+n}$$

```
[Out] a*x+1/3*c*x^3+(a*x+1/3*c*x^3)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1605}

$$\int (a+cx^2) \left(1+\left(ax+\frac{cx^3}{3}\right)^n\right) dx = \frac{\left(ax+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{cx^3}{3}$$

```
[In] Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]
```

```
[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (1+x^n)\, dx, x, ax + \frac{cx^3}{3}\right)$$
$$= ax + \frac{cx^3}{3} + \frac{\left(ax+\frac{cx^3}{3}\right)^{1+n}}{1+n}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right) dx = \frac{x(3a+cx^2)\left(1+n+\left(ax+\frac{cx^3}{3}\right)^n\right)}{3(1+n)}$$

```
[In] Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]
[Out] (x*(3*a + c*x^2)*(1 + n + (a*x + (c*x^3)/3)^n))/(3*(1 + n))
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$ax+\frac{c\,x^3}{3}+\frac{\left(ax+\frac{1}{3}c\,x^3\right)^{1+n}}{1+n}$	31
default	$ax+\frac{c\,x^3}{3}+\frac{\left(ax+\frac{1}{3}c\,x^3\right)^{1+n}}{1+n}$	31
risch	$ax+\frac{c\,x^3}{3}+\frac{x\left(c\,x^2+3a\right)\left(\frac{1}{3}\right)^n\left(x\left(c\,x^2+3a\right)\right)^n}{3+3n}$	44
norman	$ax+\frac{ax\,\mathrm{e}^{n\ln\left(ax+\frac{1}{3}c\,x^3\right)}}{1+n}+\frac{c\,x^3}{3}+\frac{c\,x^3\mathrm{e}^{n\ln\left(ax+\frac{1}{3}c\,x^3\right)}}{3+3n}$	58
parallelrisch	$\frac{x^3\left(\frac{x\left(c\,x^2+3a\right)}{3}\right)^n c^2+x^3c^2n+c^2x^3+3x\left(\frac{x\left(c\,x^2+3a\right)}{3}\right)^n ac+3xacn+3acx}{3c(1+n)}$	78

```
[In] int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x,method=_RETURNVERBOSE)
[Out] a*x+1/3*c*x^3+(a*x+1/3*c*x^3)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx = \frac{(cn+c)x^3+(cx^3+3\,ax)\left(\frac{1}{3}\,cx^3+ax\right)^n+3\,(an+a)x}{3\,(n+1)}$$

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="fricas")
```

```
[Out] 1/3*((c*n + c)*x^3 + (c*x^3 + 3*a*x)*(1/3*c*x^3 + a*x)^n + 3*(a*n + a)*x)/(
n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(26) = 52.

Time = 40.14 (sec) , antiderivative size = 190, normalized size of antiderivative = 5.59

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx$$

$$= \begin{cases} \frac{3\cdot 3^n anx}{3\cdot 3^n n+3\cdot 3^n} + \frac{3\cdot 3^n ax}{3\cdot 3^n n+3\cdot 3^n} + \frac{3^n cnx^3}{3\cdot 3^n n+3\cdot 3^n} + \frac{3^n cx^3}{3\cdot 3^n n+3\cdot 3^n} + \frac{3ax(3ax+cx^3)^n}{3\cdot 3^n n+3\cdot 3^n} + \frac{cx^3(3ax+cx^3)^n}{3\cdot 3^n n+3\cdot 3^n} & \text{for } n\neq -1 \\ ax+\frac{cx^3}{3}+\log(x)+\log\left(x-\sqrt{3}\sqrt{-\frac{a}{c}}\right)+\log\left(x+\sqrt{3}\sqrt{-\frac{a}{c}}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**n),x)
```

```
[Out] Piecewise((3*3**n*a*n*x/(3*3**n*n + 3*3**n) + 3*3**n*a*x/(3*3**n*n + 3*3**n
) + 3**n*c*n*x**3/(3*3**n*n + 3*3**n) + 3**n*c*x**3/(3*3**n*n + 3*3**n) + 3
*a*x*(3*a*x + c*x**3)**n/(3*3**n*n + 3*3**n) + c*x**3*(3*a*x + c*x**3)**n/(
3*3**n*n + 3*3**n), Ne(n, -1)), (a*x + c*x**3/3 + log(x) + log(x - sqrt(3)*
sqrt(-a/c)) + log(x + sqrt(3)*sqrt(-a/c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx = \frac{1}{3}\,cx^3+ax+\frac{(cx^3+3\,ax)e^{(n\log(cx^2+3\,a)+n\log(x))}}{3^{n+1}n+3^{n+1}}$$

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + a*x + (c*x^3 + 3*a*x)*e^(n*log(c*x^2 + 3*a) + n*log(x))/(3^(n +
 1)*n + 3^(n + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx=\frac{1}{3}\,cx^3+ax+\frac{\left(\frac{1}{3}\,cx^3+ax\right)^{n+1}}{n+1}$$

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="giac")
[Out] 1/3*c*x^3 + a*x + (1/3*c*x^3 + a*x)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx=\frac{x\,(c\,x^2+3\,a)\,\left(n+\left(\frac{c\,x^3}{3}+a\,x\right)^n+1\right)}{3\,(n+1)}$$

```
[In] int((a + c*x^2)*((a*x + (c*x^3)/3)^n + 1),x)
[Out] (x*(3*a + c*x^2)*(n + (a*x + (c*x^3)/3)^n + 1))/(3*(n + 1))
```

3.217 $\int (bx+cx^2) \left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right) \, dx$

Optimal result . . . 1512
Rubi [A] (verified) . . . 1512
Mathematica [A] (verified) . . . 1513
Maple [A] (verified) . . . 1513
Fricas [A] (verification not implemented) . . . 1514
Sympy [B] (verification not implemented) . . . 1514
Maxima [A] (verification not implemented) . . . 1514
Giac [A] (verification not implemented) . . . 1515
Mupad [B] (verification not implemented) . . . 1515

Optimal result

Integrand size = 31, antiderivative size = 44

$$\int (bx+cx^2) \left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right) \, dx=\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}}{1+n}$$

```
[Out] 1/2*b*x^2+1/3*c*x^3+(1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1605}

$$\int (bx+cx^2) \left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right) \, dx=\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[In] Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + ((b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (1+x^n)\,dx, x, \frac{bx^2}{2}+\frac{cx^3}{3}\right)$$
$$= \frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}}{1+n}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)dx = \frac{x^2(3b+2cx)\left(1+n+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)}{6(1+n)}$$

```
[In] Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n),x]
[Out] (x^2*(3*b + 2*c*x)*(1 + n + ((b*x^2)/2 + (c*x^3)/3)^n))/(6*(1 + n))
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{b\,x^2}{2}+\frac{c\,x^3}{3}+\frac{\left(\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3\right)^{1+n}}{1+n}$	37
default	$\frac{b\,x^2}{2}+\frac{c\,x^3}{3}+\frac{\left(\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3\right)^{1+n}}{1+n}$	37
risch	$\frac{b\,x^2}{2}+\frac{c\,x^3}{3}+\frac{x^2(2cx+3b)\left(\frac{1}{3}\right)^n\left(\frac{1}{2}\right)^n\left(x^2(2cx+3b)\right)^n}{6n+6}$	52
norman	$\frac{b\,x^2}{2}+\frac{c\,x^3}{3}+\frac{b\,x^2\mathrm{e}^{n\ln\left(\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3\right)}}{2+2n}+\frac{c\,x^3\mathrm{e}^{n\ln\left(\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3\right)}}{3+3n}$	70
parallelrisch	$\frac{2x^3\left(\frac{x^2(2cx+3b)}{6}\right)^n c^2+2x^3c^2n+2c^2x^3+3x^2\left(\frac{x^2(2cx+3b)}{6}\right)^n bc+3bcn\,x^2+3bc\,x^2}{6c(1+n)}$	89

```
[In] int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x,method=_RETURNVERBOSE)
[Out] 1/2*b*x^2+1/3*c*x^3+(1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)dx = \frac{2\,(cn+c)x^3+3\,(bn+b)x^2+(2\,cx^3+3\,bx^2)\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2\right)^n}{6\,(n+1)}$$

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="fricas")
[Out] 1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2)*(1/3*c*x^3 + 1
/2*b*x^2)^n)/(n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(32) = 64.

Time = 73.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.30

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)dx = \begin{cases} \frac{3\cdot6^n bnx^2}{6\cdot6^n n+6\cdot6^n}+\frac{3\cdot6^n bx^2}{6\cdot6^n n+6\cdot6^n}+\frac{2\cdot6^n cnx^3}{6\cdot6^n n+6\cdot6^n}+\frac{2\cdot6^n cx^3}{6\cdot6^n n+6\cdot6^n}+\frac{3bx^2(3bx^2+2cx^3)^n}{6\cdot6^n n+6\cdot6^n}+\frac{2cx^3(3bx^2+2cx^3)^n}{6\cdot6^n n+6\cdot6^n} & \text{for } n\neq -1 \\ \frac{bx^2}{2}+\frac{cx^3}{3}+2\log(x)+\log\left(\frac{3b}{2c}+x\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**n),x)
[Out] Piecewise((3*6**n*b*n*x**2/(6*6**n*n + 6*6**n) + 3*6**n*b*x**2/(6*6**n*n +
6*6**n) + 2*6**n*c*n*x**3/(6*6**n*n + 6*6**n) + 2*6**n*c*x**3/(6*6**n*n + 6
*6**n) + 3*b*x**2*(3*b*x**2 + 2*c*x**3)**n/(6*6**n*n + 6*6**n) + 2*c*x**3*(
3*b*x**2 + 2*c*x**3)**n/(6*6**n*n + 6*6**n), Ne(n, -1)), (b*x**2/2 + c*x**3
/3 + 2*log(x) + log(3*b/(2*c) + x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)dx = \frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+\frac{(2\,cx^3+3\,bx^2)e^{(n\log(2\,cx+3\,b)+2\,n\log(x))}}{3^{n+1}2^{n+1}n+3^{n+1}2^{n+1}}$$

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="maxima")
[Out] 1/3*c*x^3 + 1/2*b*x^2 + (2*c*x^3 + 3*b*x^2)*e^(n*log(2*c*x + 3*b) + 2*n*log
(x))/(3^(n + 1)*2^(n + 1)*n + 3^(n + 1)*2^(n + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx=\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+\frac{\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2\right)^{n+1}}{n+1}$$

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="giac")
[Out] 1/3*c*x^3 + 1/2*b*x^2 + (1/3*c*x^3 + 1/2*b*x^2)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx=\frac{x^2\,(3\,b+2\,c\,x)\,\left(n+\left(\frac{c\,x^3}{3}+\frac{b\,x^2}{2}\right)^n+1\right)}{6\,(n+1)}$$

```
[In] int((b*x + c*x^2)*(((b*x^2)/2 + (c*x^3)/3)^n + 1),x)
[Out] (x^2*(3*b + 2*c*x)*(n + ((b*x^2)/2 + (c*x^3)/3)^n + 1))/(6*(n + 1))
```

3.218 $\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx$

Optimal result . . . 1516
Rubi [A] (verified) . . . 1516
Mathematica [A] (verified) . . . 1517
Maple [A] (verified) . . . 1517
Fricas [A] (verification not implemented) . . . 1518
Sympy [F(-1)] . . . 1518
Maxima [A] (verification not implemented) . . . 1518
Giac [A] (verification not implemented) . . . 1519
Mupad [B] (verification not implemented) . . . 1519

Optimal result

Integrand size = 35, antiderivative size = 50

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx=ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}}{1+n}$$

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+(a*x+1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1605}

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx=\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[In] Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (1+x^n)\,dx, x, ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)$$
$$= ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}}{1+n}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)dx$$
$$= \frac{x(6a+x(3b+2cx))\left(1+n+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)}{6(1+n)}$$

```
[In] Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n),x]
[Out] (x*(6*a + x*(3*b + 2*c*x))*(1 + n + (a*x + (b*x^2)/2 + (c*x^3)/3)^n))/(6*(1
 + n))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

method	result
derivativedivides	$ax+\frac{b\,x^2}{2}+\frac{c\,x^3}{3}+\frac{\left(ax+\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3\right)^{1+n}}{1+n}$
default	$ax+\frac{b\,x^2}{2}+\frac{c\,x^3}{3}+\frac{\left(ax+\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3\right)^{1+n}}{1+n}$
risch	$ax+\frac{b\,x^2}{2}+\frac{c\,x^3}{3}+\frac{x\left(2c\,x^2+3bx+6a\right)\left(\frac{1}{3}\right)^n\left(\frac{1}{2}\right)^n\left(x\left(2c\,x^2+3bx+6a\right)\right)^n}{6n+6}$
norman	$ax+\frac{ax\,e^{n\ln\left(ax+\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3\right)}}{1+n}+\frac{b\,x^2}{2}+\frac{c\,x^3}{3}+\frac{b\,x^2 e^{n\ln\left(ax+\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3\right)}}{2+2n}+\frac{c\,x^3 e^{n\ln\left(ax+\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3\right)}}{3+3n}$
parallelrisch	$\frac{2x^3\left(\frac{x\left(2c\,x^2+3bx+6a\right)}{6}\right)^n c^2+2x^3c^2n+2c^2x^3+3x^2\left(\frac{x\left(2c\,x^2+3bx+6a\right)}{6}\right)^n bc+3bcn\,x^2+3bc\,x^2+6x\left(\frac{x\left(2c\,x^2+3bx+6a\right)}{6}\right)^n c}{6c(1+n)}$

```
[In] int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x,method=_RETURNVERBOSE)
[Out] a*x+1/2*b*x^2+1/3*c*x^3+(a*x+1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.44

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx = \frac{2\,(cn+c)x^3+3\,(bn+b)x^2+(2\,cx^3+3\,bx^2+6\,ax)\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax\right)^n+6\,(an+a)x}{6\,(n+1)}$$

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="frica
s")
```

```
[Out] 1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2 + 6*a*x)*(1/3*c
*x^3 + 1/2*b*x^2 + a*x)^n + 6*(a*n + a)*x)/(n + 1)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**n),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.66

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx = \frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax+\frac{(2\,cx^3+3\,bx^2+6\,ax)e^{\left(n\log\left(2\,cx^2+3\,bx+6\,a\right)+n\log\left(x\right)\right)}}{3^{n+1}2^{n+1}n+3^{n+1}2^{n+1}}$$

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="maxim
a")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x + (2*c*x^3 + 3*b*x^2 + 6*a*x)*e^(n*log(2*c*x^2
+ 3*b*x + 6*a) + n*log(x))/(3^(n + 1)*2^(n + 1)*n + 3^(n + 1)*2^(n + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx = \frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax+\frac{\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax\right)^{n+1}}{n+1}$$

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="giac"
)
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x + (1/3*c*x^3 + 1/2*b*x^2 + a*x)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.46

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx = a\,x+\left(\frac{3\,b\,x^2}{6\,n+6}+\frac{2\,c\,x^3}{6\,n+6}+\frac{6\,a\,x}{6\,n+6}\right)\left(\frac{c\,x^3}{3}+\frac{b\,x^2}{2}+a\,x\right)^n+\frac{b\,x^2}{2}+\frac{c\,x^3}{3}$$

```
[In] int(((a*x + (b*x^2)/2 + (c*x^3)/3)^n + 1)*(a + b*x + c*x^2),x)
```

```
[Out] a*x + ((3*b*x^2)/(6*n + 6) + (2*c*x^3)/(6*n + 6) + (6*a*x)/(6*n + 6))*(a*x
+ (b*x^2)/2 + (c*x^3)/3)^n + (b*x^2)/2 + (c*x^3)/3
```

3.219 $\int \left(-4+4x+x^2\right) \left(5-12x+6x^2+x^3\right) \, dx$

Optimal result 1520
Rubi [A] (verified) 1520
Mathematica [A] (verified) 1521
Maple [A] (verified) 1521
Fricas [A] (verification not implemented) 1521
Sympy [A] (verification not implemented) 1522
Maxima [A] (verification not implemented) 1522
Giac [A] (verification not implemented) 1522
Mupad [B] (verification not implemented) 1522

Optimal result

Integrand size = 22, antiderivative size = 19

$$\int \left(-4+4x+x^2\right) \left(5-12x+6x^2+x^3\right) \, dx=\frac{1}{6}\left(5-12x+6x^2+x^3\right)^2$$

```
[Out] 1/6*(x^3+6*x^2-12*x+5)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1602}

$$\int \left(-4+4x+x^2\right) \left(5-12x+6x^2+x^3\right) \, dx=\frac{1}{6}\left(x^3+6x^2-12x+5\right)^2$$

```
[In] Int[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]
[Out] (5 - 12*x + 6*x^2 + x^3)^2/6
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{6}\left(5-12x+6x^2+x^3\right)^2$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \left(-4+4x+x^2\right)\left(5-12x+6x^2+x^3\right)\,dx=-20x+34x^2-\frac{67x^3}{3}+2x^4+2x^5+\frac{x^6}{6}$$

```
[In] Integrate[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]
[Out] -20*x + 34*x^2 - (67*x^3)/3 + 2*x^4 + 2*x^5 + x^6/6
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\left(x^3+6x^2-12x+5\right)^2}{6}$	18
gosper	$\frac{x\left(x^5+12x^4+12x^3-134x^2+204x-120\right)}{6}$	27
norman	$\frac{1}{6}x^6+2x^5+2x^4-\frac{67}{3}x^3+34x^2-20x$	30
parallelrisch	$\frac{1}{6}x^6+2x^5+2x^4-\frac{67}{3}x^3+34x^2-20x$	30
risch	$\frac{1}{6}x^6+2x^5+2x^4-\frac{67}{3}x^3+34x^2-20x+\frac{25}{6}$	31

```
[In] int((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x,method=_RETURNVERBOSE)
[Out] 1/6*(x^3+6*x^2-12*x+5)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \left(-4+4x+x^2\right)\left(5-12x+6x^2+x^3\right)\,dx=\frac{1}{6}\,x^6+2\,x^5+2\,x^4-\frac{67}{3}\,x^3+34\,x^2-20\,x$$

```
[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="fricas")
[Out] 1/6*x^6 + 2*x^5 + 2*x^4 - 67/3*x^3 + 34*x^2 - 20*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int (-4+4x+x^2)\left(5-12x+6x^2+x^3\right)\,dx=\frac{x^6}{6}+2x^5+2x^4-\frac{67x^3}{3}+34x^2-20x$$

```
[In] integrate((x**2+4*x-4)*(x**3+6*x**2-12*x+5),x)
[Out] x**6/6 + 2*x**5 + 2*x**4 - 67*x**3/3 + 34*x**2 - 20*x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int (-4+4x+x^2)\left(5-12x+6x^2+x^3\right)\,dx=\frac{1}{6}\left(x^3+6\,x^2-12\,x+5\right)^2$$

```
[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="maxima")
[Out] 1/6*(x^3 + 6*x^2 - 12*x + 5)^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int (-4+4x+x^2)\left(5-12x+6x^2+x^3\right)\,dx=\frac{5}{3}\,x^3+\frac{1}{6}\left(x^3+6\,x^2-12\,x\right)^2+10\,x^2-20\,x$$

```
[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="giac")
[Out] 5/3*x^3 + 1/6*(x^3 + 6*x^2 - 12*x)^2 + 10*x^2 - 20*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int (-4+4x+x^2)\left(5-12x+6x^2+x^3\right)\,dx=\frac{x^6}{6}+2\,x^5+2\,x^4-\frac{67\,x^3}{3}+34\,x^2-20\,x$$

```
[In] int((4*x + x^2 - 4)*(6*x^2 - 12*x + x^3 + 5),x)
[Out] 34*x^2 - 20*x - (67*x^3)/3 + 2*x^4 + 2*x^5 + x^6/6
```

3.220 $\int (2x+x^3)(1+4x^2+x^4)\,dx$

Optimal result . . . 1523
Rubi [A] (verified) . . . 1523
Mathematica [A] (verified) . . . 1524
Maple [A] (verified) . . . 1524
Fricas [A] (verification not implemented) . . . 1524
Sympy [A] (verification not implemented) . . . 1525
Maxima [A] (verification not implemented) . . . 1525
Giac [A] (verification not implemented) . . . 1525
Mupad [B] (verification not implemented) . . . 1525

Optimal result

Integrand size = 18, antiderivative size = 16

$$\int (2x+x^3)\left(1+4x^2+x^4\right)\,dx=\frac{1}{8}\left(1+4x^2+x^4\right)^2$$

```
[Out] 1/8*(x^4+4*x^2+1)^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1602}

$$\int (2x+x^3)\left(1+4x^2+x^4\right)\,dx=\frac{1}{8}\left(x^4+4x^2+1\right)^2$$

```
[In] Int[(2*x + x^3)*(1 + 4*x^2 + x^4),x]
```

```
[Out] (1 + 4*x^2 + x^4)^2/8
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{8}\left(1+4x^2+x^4\right)^2$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int (2x + x^3) (1 + 4x^2 + x^4) \, dx = x^2 + \frac{9x^4}{4} + x^6 + \frac{x^8}{8}$$

```
[In] Integrate[(2*x + x^3)*(1 + 4*x^2 + x^4),x]
[Out] x^2 + (9*x^4)/4 + x^6 + x^8/8
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(x^4+4x^2+1)^2}{8}$	15
norman	$\frac{1}{8}x^8 + x^6 + \frac{9}{4}x^4 + x^2$	18
parallelrisch	$\frac{1}{8}x^8 + x^6 + \frac{9}{4}x^4 + x^2$	18
risch	$\frac{1}{8}x^8 + x^6 + \frac{9}{4}x^4 + x^2 + \frac{1}{8}$	19
gosper	$\frac{x^2(x^6+8x^4+18x^2+8)}{8}$	21

```
[In] int((x^3+2*x)*(x^4+4*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/8*(x^4+4*x^2+1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (2x + x^3) (1 + 4x^2 + x^4) \, dx = \frac{1}{8} x^8 + x^6 + \frac{9}{4} x^4 + x^2$$

```
[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="fricas")
[Out] 1/8*x^8 + x^6 + 9/4*x^4 + x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (2x + x^3)(1 + 4x^2 + x^4)\, dx = \frac{x^8}{8} + x^6 + \frac{9x^4}{4} + x^2$$

```
[In] integrate((x**3+2*x)*(x**4+4*x**2+1),x)
[Out] x**8/8 + x**6 + 9*x**4/4 + x**2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int (2x + x^3)(1 + 4x^2 + x^4)\, dx = \frac{1}{8}\left(x^4 + 4x^2 + 1\right)^2$$

```
[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="maxima")
[Out] 1/8*(x^4 + 4*x^2 + 1)^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (2x + x^3)(1 + 4x^2 + x^4)\, dx = \frac{1}{4}x^4 + \frac{1}{8}\left(x^4 + 4x^2\right)^2 + x^2$$

```
[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="giac")
[Out] 1/4*x^4 + 1/8*(x^4 + 4*x^2)^2 + x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (2x + x^3)(1 + 4x^2 + x^4)\, dx = \frac{x^8}{8} + x^6 + \frac{9\,x^4}{4} + x^2$$

```
[In] int((2*x + x^3)*(4*x^2 + x^4 + 1),x)
[Out] x^2 + (9*x^4)/4 + x^6 + x^8/8
```

3.221 $\int (1+2x) (x+x^2)^3 \left(-18+7(x+x^2)^3\right)^2 dx$

Optimal result 1526
Rubi [B] (verified) 1526
Mathematica [B] (verified) 1527
Maple [A] (verified) 1528
Fricas [B] (verification not implemented) 1528
Sympy [B] (verification not implemented) 1528
Maxima [B] (verification not implemented) 1529
Giac [A] (verification not implemented) 1529
Mupad [B] (verification not implemented) 1529

Optimal result

Integrand size = 26, antiderivative size = 33

$$\int (1+2x) (x+x^2)^3 \left(-18+7(x+x^2)^3\right)^2 dx = 81x^4(1+x)^4-36x^7(1+x)^7+\frac{49}{10}x^{10}(1+x)^{10}$$

```
[Out] 81*x^4*(1+x)^4-36*x^7*(1+x)^7+49/10*x^10*(1+x)^10
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 96 vs. $2(33) = 66$.

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1607, 1626}

$$\begin{aligned}&\int (1+2x) (x+x^2)^3 \left(-18+7(x+x^2)^3\right)^2 dx\\ &= \frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}\\ &\quad -1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4\end{aligned}$$

```
[In] Int[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^3(1+x)^3(1+2x)\left(-18+7\left(x+x^2\right)^3\right)^2\,dx\\ &= \int \left(324x^3+1620x^4+2916x^5+2016x^6-1368x^7-6804x^8-12551x^9-13321x^{10}-6426x^{11}\right.\\ &\quad +4368x^{12}+13902x^{13}+18522x^{14}+16464x^{15}+9996x^{16}+3969x^{17}+931x^{18}\\ &\quad \left.+98x^{19}\right)\,dx\\ &= 81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x^{10}}{10}-1211x^{11}-\frac{1071x^{12}}{2}\\ &\quad +336x^{13}+993x^{14}+\frac{6174x^{15}}{5}+1029x^{16}+588x^{17}+\frac{441x^{18}}{2}+49x^{19}+\frac{49x^{20}}{10}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 96 vs. $2(33) = 66$.

Time = 0.01 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.91

$$\begin{aligned}&\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx\\ &= 81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x^{10}}{10}-1211x^{11}-\frac{1071x^{12}}{2}\\ &\quad +336x^{13}+993x^{14}+\frac{6174x^{15}}{5}+1029x^{16}+588x^{17}+\frac{441x^{18}}{2}+49x^{19}+\frac{49x^{20}}{10}\end{aligned}$$

```
[In] Integrate[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result
default	$\frac{49(x^2+x)^{10}}{10}-36(x^2+x)^7+81(x^2+x)^4$
gosper	$\frac{(x+1)^3(49x^{13}+343x^{12}+1029x^{11}+1715x^{10}+1715x^9+1029x^8-17x^7-1391x^6-2160x^5-1440x^4-360x^3+810x+810)x^4}{10}$
norman	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}+$
risch	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}+$
parallelrisch	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}+$

```
[In] int((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x,method=_RETURNVERBOSE)
[Out] 49/10*(x^2+x)^10-36*(x^2+x)^7+81*(x^2+x)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(31) = 62.

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int(1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx = \frac{49}{10}\,x^{20}+49\,x^{19}+\frac{441}{2}\,x^{18}+588\,x^{17}+1029\,x^{16}+\frac{6174}{5}\,x^{15}+993\,x^{14}+336\,x^{13} - \frac{1071}{2}\,x^{12}-1211\,x^{11}-\frac{12551}{10}\,x^{10}-756\,x^9-171\,x^8+288\,x^7+486\,x^6+324\,x^5+81\,x^4$$

```
[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="fricas")
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(31) = 62.

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.85

$$\int(1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx = \frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2} - 1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

```
[In] integrate((1+2*x)*(x**2+x)**3*(-18+7*(x**2+x)**3)**2,x)
```

```
[Out] 49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6174*x**15/
5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 - 12551*x**10/10 - 75
6*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324*x**5 + 81*x**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(31) = 62.

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx = \frac{49}{10}\,x^{20}+49\,x^{19}+\frac{441}{2}\,x^{18}+588\,x^{17}+1029\,x^{16}+\frac{6174}{5}\,x^{15}+993\,x^{14}+336\,x^{13} - \frac{1071}{2}\,x^{12}-1211\,x^{11}-\frac{12551}{10}\,x^{10}-756\,x^{9}-171\,x^{8}+288\,x^{7}+486\,x^{6}+324\,x^{5}+81\,x^{4}$$

```
[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="maxima")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx = \frac{49}{10}\left(x^2+x\right)^{10}-36\left(x^2+x\right)^{7}+81\left(x^2+x\right)^{4}$$

```
[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="giac")
```

```
[Out] 49/10*(x^2 + x)^10 - 36*(x^2 + x)^7 + 81*(x^2 + x)^4
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx = \frac{49\,x^{20}}{10}+49\,x^{19}+\frac{441\,x^{18}}{2}+588\,x^{17}+1029\,x^{16}+\frac{6174\,x^{15}}{5}+993\,x^{14}+336\,x^{13} - \frac{1071\,x^{12}}{2}-1211\,x^{11}-\frac{12551\,x^{10}}{10}-756\,x^{9}-171\,x^{8}+288\,x^{7}+486\,x^{6}+324\,x^{5}+81\,x^{4}$$

```
[In] int((2*x + 1)*(x + x^2)^3*(7*(x + x^2)^3 - 18)^2,x)
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

3.222 $\int x^3(1+x)^3(1+2x) \left(-18+7x^3(1+x)^3\right)^2 \, dx$

Optimal result 1531
Rubi [B] (verified) 1531
Mathematica [B] (verified) 1532
Maple [B] (verified) 1532
Fricas [B] (verification not implemented) 1533
Sympy [B] (verification not implemented) 1533
Maxima [B] (verification not implemented) 1534
Giac [A] (verification not implemented) 1534
Mupad [B] (verification not implemented) 1534

Optimal result

Integrand size = 28, antiderivative size = 33

$$\int x^3(1+x)^3(1+2x) \left(-18+7x^3(1+x)^3\right)^2 \, dx = 81x^4(1+x)^4-36x^7(1+x)^7+\frac{49}{10}x^{10}(1+x)^{10}$$

```
[Out] 81*x^4*(1+x)^4-36*x^7*(1+x)^7+49/10*x^10*(1+x)^10
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 96 vs. 2(33) = 66.

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1626}

$$\int x^3(1+x)^3(1+2x) \left(-18+7x^3(1+x)^3\right)^2 \, dx = \frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13} -\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

```
[In] Int[x^3*(1 + x)^3*(1 + 2*x)*(-18 + 7*x^3*(1 + x)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(324x^3 + 1620x^4 + 2916x^5 + 2016x^6 - 1368x^7 - 6804x^8 - 12551x^9 - 13321x^{10} - 6426x^{11} + 4368x^{12} + 13902x^{13} + 18522x^{14} + 16464x^{15} + 9996x^{16} + 3969x^{17} + 931x^{18} + 98x^{19}\right) dx \\ &= 81x^4 + 324x^5 + 486x^6 + 288x^7 - 171x^8 - 756x^9 - \frac{12551x^{10}}{10} - 1211x^{11} - \frac{1071x^{12}}{2} + 336x^{13} + 993x^{14} + \frac{6174x^{15}}{5} + 1029x^{16} + 588x^{17} + \frac{441x^{18}}{2} + 49x^{19} + \frac{49x^{20}}{10}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 96 vs. $2(33) = 66$.

Time = 0.00 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.91

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2 dx = 81x^4 + 324x^5 + 486x^6 + 288x^7 - 171x^8 - 756x^9 - \frac{12551x^{10}}{10} - 1211x^{11} - \frac{1071x^{12}}{2} + 336x^{13} + 993x^{14} + \frac{6174x^{15}}{5} + 1029x^{16} + 588x^{17} + \frac{441x^{18}}{2} + 49x^{19} + \frac{49x^{20}}{10}$$

```
[In] Integrate[x^3*(1 + x)^3*(1 + 2*x)*(-18 + 7*x^3*(1 + x)^3)^2,x]
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(31) = 62$.

Time = 0.79 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

method	result
gosper	$\frac{x^4(49x^{16}+490x^{15}+2205x^{14}+5880x^{13}+10290x^{12}+12348x^{11}+9930x^{10}+3360x^9-5355x^8-12110x^7-12551x^6-7560x^5-1710x}{10}$
default	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}+$
norman	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}+$
risch	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}+$
parallelrisch	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}+$

```
[In] int(x^3*(x+1)^3*(1+2*x)*(-18+7*x^3*(x+1)^3)^2,x,method=_RETURNVERBOSE)
[Out] 1/10*x^4*(49*x^16+490*x^15+2205*x^14+5880*x^13+10290*x^12+12348*x^11+9930*x
^10+3360*x^9-5355*x^8-12110*x^7-12551*x^6-7560*x^5-1710*x^4+2880*x^3+4860*x
^2+3240*x+810)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(31) = 62$.

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx$$
$$=\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="fricas")
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(31) = 62$.

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.85

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx$$
$$=\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

```
[In] integrate(x**3*(1+x)**3*(1+2*x)*(-18+7*x**3*(1+x)**3)**2,x)
```

```
[Out] 49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6174*x**15/
5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 - 12551*x**10/10 - 75
6*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324*x**5 + 81*x**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(31) = 62$.

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx = \frac{49}{10}\,x^{20} + 49\,x^{19} + \frac{441}{2}\,x^{18} + 588\,x^{17} + 1029\,x^{16} + \frac{6174}{5}\,x^{15} + 993\,x^{14} + 336\,x^{13} - \frac{1071}{2}\,x^{12} - 1211\,x^{11} - \frac{12551}{10}\,x^{10} - 756\,x^9 - 171\,x^8 + 288\,x^7 + 486\,x^6 + 324\,x^5 + 81\,x^4$$

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="maxima")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx = \frac{49}{10}\left(x^2+x\right)^{10} - 36\left(x^2+x\right)^7 + 81\left(x^2+x\right)^4$$

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="giac")
```

```
[Out] 49/10*(x^2 + x)^10 - 36*(x^2 + x)^7 + 81*(x^2 + x)^4
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx = \frac{49\,x^{20}}{10} + 49\,x^{19} + \frac{441\,x^{18}}{2} + 588\,x^{17} + 1029\,x^{16} + \frac{6174\,x^{15}}{5} + 993\,x^{14} + 336\,x^{13} - \frac{1071\,x^{12}}{2} - 1211\,x^{11} - \frac{12551\,x^{10}}{10} - 756\,x^9 - 171\,x^8 + 288\,x^7 + 486\,x^6 + 324\,x^5 + 81\,x^4$$

```
[In] int(x^3*(2*x + 1)*(7*x^3*(x + 1)^3 - 18)^2*(x + 1)^3,x)
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

3.223 $\int \frac{2-x^2}{\left(1-6x+x^3\right)^5} dx$

Optimal result 1536
Rubi [A] (verified) 1536
Mathematica [A] (verified) 1537
Maple [A] (verified) 1537
Fricas [B] (verification not implemented) 1537
Sympy [B] (verification not implemented) 1538
Maxima [A] (verification not implemented) 1538
Giac [A] (verification not implemented) 1538
Mupad [B] (verification not implemented) 1539

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \frac{2-x^2}{\left(1-6x+x^3\right)^5} dx = \frac{1}{12\left(1-6x+x^3\right)^4}$$

```
[Out] 1/12/(x^3-6*x+1)^4
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1602}

$$\int \frac{2-x^2}{\left(1-6x+x^3\right)^5} dx = \frac{1}{12\left(x^3-6x+1\right)^4}$$

```
[In] Int[(2 - x^2)/(1 - 6*x + x^3)^5,x]
```

```
[Out] 1/(12*(1 - 6*x + x^3)^4)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{12\left(1-6x+x^3\right)^4}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{2-x^2}{(1-6x+x^3)^5}\,dx=\frac{1}{12(1-6x+x^3)^4}$$

```
[In] Integrate[(2 - x^2)/(1 - 6*x + x^3)^5,x]
[Out] 1/(12*(1 - 6*x + x^3)^4)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{1}{12(x^3-6x+1)^4}$	13
default	$\frac{1}{12(x^3-6x+1)^4}$	13
norman	$\frac{1}{12(x^3-6x+1)^4}$	13
risch	$\frac{1}{12(x^3-6x+1)^4}$	13
parallelrisch	$\frac{1}{12(x^3-6x+1)^4}$	13

```
[In] int((-x^2+2)/(x^3-6*x+1)^5,x,method=_RETURNVERBOSE)
[Out] 1/12/(x^3-6*x+1)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(12) = 24.

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.07

$$\int \frac{2-x^2}{(1-6x+x^3)^5}\,dx=\frac{1}{12(x^{12}-24x^{10}+4x^9+216x^8-72x^7-858x^6+432x^5+1224x^4-860x^3+216x^2-24x+1)}$$

```
[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="fricas")
[Out] 1/12/(x^12 - 24*x^10 + 4*x^9 + 216*x^8 - 72*x^7 - 858*x^6 + 432*x^5 + 1224*
x^4 - 860*x^3 + 216*x^2 - 24*x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(12) = 24$.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 4.00

$$\int \frac{2-x^2}{(1-6x+x^3)^5}\,dx = \frac{1}{12x^{12}-288x^{10}+48x^9+2592x^8-864x^7-10296x^6+5184x^5+14688x^4-10320x^3+2592x^2-288x+1}$$

```
[In] integrate((-x**2+2)/(x**3-6*x+1)**5,x)
[Out] 1/(12*x**12 - 288*x**10 + 48*x**9 + 2592*x**8 - 864*x**7 - 10296*x**6 + 518
4*x**5 + 14688*x**4 - 10320*x**3 + 2592*x**2 - 288*x + 12)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2-x^2}{(1-6x+x^3)^5}\,dx = \frac{1}{12\,(x^3-6\,x+1)^4}$$

```
[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="maxima")
[Out] 1/12/(x^3 - 6*x + 1)^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2-x^2}{(1-6x+x^3)^5}\,dx = \frac{1}{12\,(x^3-6\,x+1)^4}$$

```
[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="giac")
[Out] 1/12/(x^3 - 6*x + 1)^4
```

Mupad [B] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2-x^2}{(1-6x+x^3)^5}\,dx=\frac{1}{12\,{\left(x^3-6\,x+1\right)}^4}$$

```
[In] int(-(x^2 - 2)/(x^3 - 6*x + 1)^5,x)
```

```
[Out] 1/(12*(x^3 - 6*x + 1)^4)
```

3.224 $\int \frac{2x+x^2}{4+3x^2+x^3} \, dx$

Optimal result 1540
Rubi [A] (verified) 1540
Mathematica [A] (verified) 1541
Maple [A] (verified) 1541
Fricas [A] (verification not implemented) 1541
Sympy [A] (verification not implemented) 1542
Maxima [A] (verification not implemented) 1542
Giac [A] (verification not implemented) 1542
Mupad [B] (verification not implemented) 1542

Optimal result

Integrand size = 20, antiderivative size = 15

$$\int \frac{2x+x^2}{4+3x^2+x^3} \, dx = \frac{1}{3} \log \left(4+3x^2+x^3\right)$$

```
[Out] 1/3*ln(x^3+3*x^2+4)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1601}

$$\int \frac{2x+x^2}{4+3x^2+x^3} \, dx = \frac{1}{3} \log \left(x^3+3x^2+4\right)$$

```
[In] Int[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

```
[Out] Log[4 + 3*x^2 + x^3]/3
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\text{integral} = \frac{1}{3} \log \left(4+3x^2+x^3\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx=\frac{1}{3}\log\left(4+3x^2+x^3\right)$$

```
[In] Integrate[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
[Out] Log[4 + 3*x^2 + x^3]/3
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln\left(x^3+3x^2+4\right)}{3}$	14
norman	$\frac{\ln\left(x^3+3x^2+4\right)}{3}$	14
risch	$\frac{\ln\left(x^3+3x^2+4\right)}{3}$	14
parallelrisch	$\frac{\ln\left(x^3+3x^2+4\right)}{3}$	14

```
[In] int((x^2+2*x)/(x^3+3*x^2+4),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(x^3+3*x^2+4)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx=\frac{1}{3}\log\left(x^3+3\,x^2+4\right)$$

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="fricas")
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{2x + x^2}{4 + 3x^2 + x^3} \, dx = \frac{\log(x^3 + 3x^2 + 4)}{3}$$

```
[In] integrate((x**2+2*x)/(x**3+3*x**2+4),x)
[Out] log(x**3 + 3*x**2 + 4)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{2x + x^2}{4 + 3x^2 + x^3} \, dx = \frac{1}{3} \log(x^3 + 3x^2 + 4)$$

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="maxima")
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{2x + x^2}{4 + 3x^2 + x^3} \, dx = \frac{1}{3} \log(|x^3 + 3x^2 + 4|)$$

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="giac")
[Out] 1/3*log(abs(x^3 + 3*x^2 + 4))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{2x + x^2}{4 + 3x^2 + x^3} \, dx = \frac{\ln(x^3 + 3x^2 + 4)}{3}$$

```
[In] int((2*x + x^2)/(3*x^2 + x^3 + 4),x)
[Out] log(3*x^2 + x^3 + 4)/3
```

3.225 $\int \frac{1+x+x^3}{4x+2x^2+x^4} dx$

Optimal result 1543
Rubi [A] (verified) 1543
Mathematica [A] (verified) 1544
Maple [A] (verified) 1544
Fricas [A] (verification not implemented) 1544
Sympy [A] (verification not implemented) 1545
Maxima [A] (verification not implemented) 1545
Giac [A] (verification not implemented) 1545
Mupad [B] (verification not implemented) 1545

Optimal result

Integrand size = 21, antiderivative size = 17

$$\int \frac{1+x+x^3}{4x+2x^2+x^4} \, dx = \frac{1}{4} \log \left(4x+2x^2+x^4\right)$$

```
[Out] 1/4*ln(x^4+2*x^2+4*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1601}

$$\int \frac{1+x+x^3}{4x+2x^2+x^4} \, dx = \frac{1}{4} \log \left(x^4+2x^2+4x\right)$$

```
[In] Int[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

```
[Out] Log[4*x + 2*x^2 + x^4]/4
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\text{integral} = \frac{1}{4} \log \left(4x+2x^2+x^4\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx=\frac{\log(x)}{4}+\frac{1}{4}\log\left(4+2x+x^3\right)$$

```
[In] Integrate[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
[Out] Log[x]/4 + Log[4 + 2*x + x^3]/4
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\ln\left(x\left(x^3+2x+4\right)\right)}{4}$	14
risch	$\frac{\ln\left(x^4+2x^2+4x\right)}{4}$	16
norman	$\frac{\ln(x)}{4}+\frac{\ln\left(x^3+2x+4\right)}{4}$	17
parallelrisch	$\frac{\ln(x)}{4}+\frac{\ln\left(x^3+2x+4\right)}{4}$	17

```
[In] int((x^3+x+1)/(x^4+2*x^2+4*x),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(x*(x^3+2*x+4))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx=\frac{1}{4}\log\left(x^4+2\,x^2+4\,x\right)$$

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="fricas")
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx = \frac{\log\left(x^4+2x^2+4x\right)}{4}$$

```
[In] integrate((x**3+x+1)/(x**4+2*x**2+4*x),x)
[Out] log(x**4 + 2*x**2 + 4*x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx = \frac{1}{4}\,\log\left(x^4+2\,x^2+4\,x\right)$$

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="maxima")
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx = \frac{1}{4}\,\log\left(4\left|\frac{1}{4}\,x^4+\frac{1}{2}\,x^2+x\right|\right)$$

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="giac")
[Out] 1/4*log(4*abs(1/4*x^4 + 1/2*x^2 + x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx = \frac{\ln\left(x\,\left(x^3+2\,x+4\right)\right)}{4}$$

```
[In] int((x + x^3 + 1)/(4*x + 2*x^2 + x^4),x)
[Out] log(x*(2*x + x^3 + 4))/4
```

3.226 $\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx$

Optimal result 1546
Rubi [A] (verified) 1546
Mathematica [A] (verified) 1547
Maple [A] (verified) 1547
Fricas [A] (verification not implemented) 1548
Sympy [A] (verification not implemented) 1548
Maxima [A] (verification not implemented) 1549
Giac [A] (verification not implemented) 1549
Mupad [B] (verification not implemented) 1549

Optimal result

Integrand size = 52, antiderivative size = 40

$$\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx=\frac{a}{c+dx+ex^2+fx^3}+\frac{bx}{c+dx+ex^2+fx^3}$$

```
[Out] a/(f*x^3+e*x^2+d*x+c)+b*x/(f*x^3+e*x^2+d*x+c)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.058, Rules used = {6, 2127, 1602}

$$\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx=\frac{a}{c+dx+ex^2+fx^3}+\frac{bx}{c+dx+ex^2+fx^3}$$

```
[In] Int[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x + e*x^
2 + f*x^3)^2,x]
```

```
[Out] a/(c + d*x + e*x^2 + f*x^3) + (b*x)/(c + d*x + e*x^2 + f*x^3)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
```

```
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2127

```
Int[(Pm_)*(Qn_)^(p_.), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x
]}, Simp[Coeff[Pm, x, m]*x^(m - n + 1)*(Qn^(p + 1)/((m + n*p + 1)*Coeff[Qn,
 x, n])), x] + Dist[1/((m + n*p + 1)*Coeff[Qn, x, n]), Int[ExpandToSum[(m +
 n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn +
(p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 < 0
] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{bc-ad-2aex+(-be-3af)x^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx \\
&= \frac{bx}{c+dx+ex^2+fx^3} - \frac{\int \frac{2adf+4aefx+6af^2x^2}{(c+dx+ex^2+fx^3)^2}\,dx}{2f} \\
&= \frac{a}{c+dx+ex^2+fx^3} + \frac{bx}{c+dx+ex^2+fx^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$
\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx = \frac{a+bx}{c+dx+ex^2+fx^3}
$$

```
[In] Integrate[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x
+ e*x^2 + f*x^3)^2,x]
[Out] (a + b*x)/(c + d*x + e*x^2 + f*x^3)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{bx+a}{f\,x^{3}+e\,x^{2}+dx+c}$	24
norman	$\frac{bx+a}{f\,x^{3}+e\,x^{2}+dx+c}$	24
risch	$\frac{bx+a}{f\,x^{3}+e\,x^{2}+dx+c}$	24
default	$-\frac{-bx-a}{f\,x^{3}+e\,x^{2}+dx+c}$	28
parallelrisch	$\frac{bex+ae}{e(f\,x^{3}+e\,x^{2}+dx+c)}$	30

```
[In] int((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c)^2,x,
method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)/(f*x^3+e*x^2+d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx = \frac{bx+a}{fx^3+ex^2+dx+c}$$

```
[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c
)^2,x, algorithm="fricas")
```

```
[Out] (b*x + a)/(f*x^3 + e*x^2 + d*x + c)
```

Sympy [A] (verification not implemented)

Time = 12.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.55

$$\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx = -\frac{-a-bx}{c+dx+ex^2+fx^3}$$

```
[In] integrate((-2*b*f*x**3-3*a*f*x**2-b*e*x**2-2*a*e*x-a*d+b*c)/(f*x**3+e*x**2+
d*x+c)**2,x)
```

```
[Out] -(-a - b*x)/(c + d*x + e*x**2 + f*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx=\frac{bx+a}{fx^3+ex^2+dx+c}$$

```
[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c
)^2,x, algorithm="maxima")
```

```
[Out] (b*x + a)/(f*x^3 + e*x^2 + d*x + c)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx=\frac{bx+a}{fx^3+ex^2+dx+c}$$

```
[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c
)^2,x, algorithm="giac")
```

```
[Out] (b*x + a)/(f*x^3 + e*x^2 + d*x + c)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx=\frac{a+b\,x}{f\,x^3+e\,x^2+d\,x+c}$$

```
[In] int(-(a*d - b*c + 2*a*e*x + 3*a*f*x^2 + b*e*x^2 + 2*b*f*x^3)/(c + d*x + e*x
^2 + f*x^3)^2,x)
```

```
[Out] (a + b*x)/(c + d*x + e*x^2 + f*x^3)
```

3.227 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx$

Optimal result 1550
Rubi [A] (verified) 1551
Mathematica [C] (verified) 1554
Maple [A] (verified) 1554
Fricas [F(-1)] 1555
Sympy [F(-1)] 1555
Maxima [F] 1555
Giac [F(-2)] 1555
Mupad [F(-1)] 1556

Optimal result

Integrand size = 38, antiderivative size = 605

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx$$

$$= \frac{\left(4a^2B+b\left(b-\sqrt{8a^2+b^2-4ac}\right)D-a\left(A\left(b-\sqrt{8a^2+b^2-4ac}\right)+bC-\sqrt{8a^2+b^2-4ac}C+2cD\right)\right)\operatorname{arct}}{\sqrt{2}a\sqrt{8a^2+b^2-4ac}\sqrt{4a^2+2ac-b\left(b-\sqrt{8a^2+b^2-4ac}\right)}}$$

$$-\frac{\left(4a^2B+b\left(b+\sqrt{8a^2+b^2-4ac}\right)D-a\left(A\left(b+\sqrt{8a^2+b^2-4ac}\right)+bC+\sqrt{8a^2+b^2-4ac}C+2cD\right)\right)\operatorname{ar}}{\sqrt{2}a\sqrt{8a^2+b^2-4ac}\sqrt{4a^2+2ac-b\left(b+\sqrt{8a^2+b^2-4ac}\right)}}$$

$$-\frac{\left(2a(A-C)+\left(b-\sqrt{8a^2+b^2-4ac}\right)D\right)\log\left(2a+\left(b-\sqrt{8a^2+b^2-4ac}\right)x+2ax^2\right)}{4a\sqrt{8a^2+b^2-4ac}}$$

$$+\frac{\left(2a(A-C)+\left(b+\sqrt{8a^2+b^2-4ac}\right)D\right)\log\left(2a+\left(b+\sqrt{8a^2+b^2-4ac}\right)x+2ax^2\right)}{4a\sqrt{8a^2+b^2-4ac}}$$

```
[Out] -1/4*ln(2*a+2*a*x^2+x*(b-(8*a^2-4*a*c+b^2)^(1/2)))*(2*a*(A-C)+D*(b-(8*a^2-4
*a*c+b^2)^(1/2)))/a/(8*a^2-4*a*c+b^2)^(1/2)+1/4*ln(2*a+2*a*x^2+x*(b+(8*a^2-
4*a*c+b^2)^(1/2)))*(2*a*(A-C)+D*(b+(8*a^2-4*a*c+b^2)^(1/2)))/a/(8*a^2-4*a*c
+b^2)^(1/2)+1/2*arctan(1/2*(b+4*a*x-(8*a^2-4*a*c+b^2)^(1/2))*2^(1/2)/(4*a^2
+2*a*c-b*(b-(8*a^2-4*a*c+b^2)^(1/2)))^(1/2))*(4*a^2*B+b*D*(b-(8*a^2-4*a*c+b
^2)^(1/2))-a*(b*C+2*c*D+A*(b-(8*a^2-4*a*c+b^2)^(1/2))-C*(8*a^2-4*a*c+b^2)^(
1/2)))/a*2^(1/2)/(8*a^2-4*a*c+b^2)^(1/2)/(4*a^2+2*a*c-b*(b-(8*a^2-4*a*c+b^2
)^(1/2)))^(1/2)-1/2*arctan(1/2*(b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))*2^(1/2)/(4
*a^2+2*a*c-b*(b+(8*a^2-4*a*c+b^2)^(1/2)))^(1/2))*(4*a^2*B+b*D*(b+(8*a^2-4*a
*c+b^2)^(1/2))-a*(b*C+2*c*D+C*(8*a^2-4*a*c+b^2)^(1/2)+A*(b+(8*a^2-4*a*c+b^2
```

```
)^(1/2))))/a*2^(1/2)/(8*a^2-4*a*c+b^2)^(1/2)/(4*a^2+2*a*c-b*(b+(8*a^2-4*a*c
+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 3.07 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2111, 648, 632, 210, 642}

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx$$
$$= \frac{\arctan\left(\frac{-\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}\right)\left(-a\left(A\left(b-\sqrt{8a^2-4ac+b^2}\right)-C\sqrt{8a^2-4ac+b^2}+bC+2cD\right.\right.}{\sqrt{2}a\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}$$
$$-\frac{\arctan\left(\frac{\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(\sqrt{8a^2-4ac+b^2}+b\right)+4a^2+2ac}}\right)\left(-a\left(A\left(\sqrt{8a^2-4ac+b^2}+b\right)+C\sqrt{8a^2-4ac+b^2}+bC+2c\right.\right.}{\sqrt{2}a\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(\sqrt{8a^2-4ac+b^2}+b\right)+4a^2+2ac}}$$
$$-\frac{\log\left(x\left(b-\sqrt{8a^2-4ac+b^2}\right)+2ax^2+2a\right)\left(D\left(b-\sqrt{8a^2-4ac+b^2}\right)+2a(A-C)\right)}{4a\sqrt{8a^2-4ac+b^2}}$$
$$+\frac{\log\left(x\left(\sqrt{8a^2-4ac+b^2}+b\right)+2ax^2+2a\right)\left(D\left(\sqrt{8a^2-4ac+b^2}+b\right)+2a(A-C)\right)}{4a\sqrt{8a^2-4ac+b^2}}$$

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4),x]
```

```
[Out] ((4*a^2*B + b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])*D - a*(A*(b - Sqrt[8*a^2 + b^
2 - 4*a*c]) + b*C - Sqrt[8*a^2 + b^2 - 4*a*c]*C + 2*c*D))*ArcTan[(b - Sqrt[
8*a^2 + b^2 - 4*a*c] + 4*a*x)/(Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b - Sqrt[8*a
^2 + b^2 - 4*a*c])])])/(Sqrt[2]*a*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*
a*c - b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])]) - ((4*a^2*B + b*(b + Sqrt[8*a^2 +
 b^2 - 4*a*c])*D - a*(A*(b + Sqrt[8*a^2 + b^2 - 4*a*c]) + b*C + Sqrt[8*a^2
+ b^2 - 4*a*c]*C + 2*c*D))*ArcTan[(b + Sqrt[8*a^2 + b^2 - 4*a*c] + 4*a*x)/(
Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 - 4*a*c])])])/(Sqrt[2]
*a*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 -
 4*a*c])]) - ((2*a*(A - C) + (b - Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (
b - Sqrt[8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*a*Sqrt[8*a^2 + b^2 - 4*a*c]
) + ((2*a*(A - C) + (b + Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (b + Sqrt[
8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*a*Sqrt[8*a^2 + b^2 - 4*a*c])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2111

```
Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4),
 x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B =
 Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[
(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*
x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A -
2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{Ab-2aB-A\sqrt{8a^2+b^2-4ac}+2aD+\left(2aA-2aC+bD-\sqrt{8a^2+b^2-4ac}D\right)x}{2a+\left(b-\sqrt{8a^2+b^2-4ac}\right)x+2ax^2}\,dx}{\sqrt{8a^2+b^2-4ac}} + \frac{\int \frac{Ab-2aB+A\sqrt{8a^2+b^2-4ac}+2aD+\left(2aA-2aC+bD+\sqrt{8a^2+b^2-4ac}D\right)x}{2a+\left(b+\sqrt{8a^2+b^2-4ac}\right)x+2ax^2}\,dx}{\sqrt{8a^2+b^2-4ac}}$$

$$= -\frac{\left(2a(A-C)+\left(b-\sqrt{8a^2+b^2-4ac}\right)D\right)\int \frac{b-\sqrt{8a^2+b^2-4ac}+4ax}{2a+\left(b-\sqrt{8a^2+b^2-4ac}\right)x+2ax^2}\,dx}{4a\sqrt{8a^2+b^2-4ac}}$$
$$+\frac{\left(2a(A-C)+\left(b+\sqrt{8a^2+b^2-4ac}\right)D\right)\int \frac{b+\sqrt{8a^2+b^2-4ac}+4ax}{2a+\left(b+\sqrt{8a^2+b^2-4ac}\right)x+2ax^2}\,dx}{4a\sqrt{8a^2+b^2-4ac}}$$
$$+\frac{\left(4a^2B+b\left(b-\sqrt{8a^2+b^2-4ac}\right)D-a\left(A\left(b-\sqrt{8a^2+b^2-4ac}\right)+bC-\sqrt{8a^2+b^2-4ac}C+\right.\right.}{2a\sqrt{8a^2+b^2-4ac}}$$
$$-\frac{\left(4a^2B+b\left(b+\sqrt{8a^2+b^2-4ac}\right)D-a\left(A\left(b+\sqrt{8a^2+b^2-4ac}\right)+bC+\sqrt{8a^2+b^2-4ac}C+\right.\right.}{2a\sqrt{8a^2+b^2-4ac}}$$

$$= -\frac{\left(2a(A-C)+\left(b-\sqrt{8a^2+b^2-4ac}\right)D\right)\log\left(2a+\left(b-\sqrt{8a^2+b^2-4ac}\right)x+2ax^2\right)}{4a\sqrt{8a^2+b^2-4ac}}$$
$$+\frac{\left(2a(A-C)+\left(b+\sqrt{8a^2+b^2-4ac}\right)D\right)\log\left(2a+\left(b+\sqrt{8a^2+b^2-4ac}\right)x+2ax^2\right)}{4a\sqrt{8a^2+b^2-4ac}}$$
$$-\frac{\left(4a^2B+b\left(b-\sqrt{8a^2+b^2-4ac}\right)D-a\left(A\left(b-\sqrt{8a^2+b^2-4ac}\right)+bC-\sqrt{8a^2+b^2-4ac}C+\right.\right.}{a\sqrt{8a^2+b^2-4ac}}$$
$$+\frac{\left(4a^2B+b\left(b+\sqrt{8a^2+b^2-4ac}\right)D-a\left(A\left(b+\sqrt{8a^2+b^2-4ac}\right)+bC+\sqrt{8a^2+b^2-4ac}C+\right.\right.}{a\sqrt{8a^2+b^2-4ac}}$$

$$= \frac{\left(4a^2B+b\left(b-\sqrt{8a^2+b^2-4ac}\right)D-a\left(A\left(b-\sqrt{8a^2+b^2-4ac}\right)+bC-\sqrt{8a^2+b^2-4ac}C+2a\right.\right.}{\sqrt{2}a\sqrt{8a^2+b^2-4ac}\sqrt{4a^2+2ac-b\left(b-\sqrt{8a^2+b^2-4\right.}}}$$
$$-\frac{\left(4a^2B+b\left(b+\sqrt{8a^2+b^2-4ac}\right)D-a\left(A\left(b+\sqrt{8a^2+b^2-4ac}\right)+bC+\sqrt{8a^2+b^2-4ac}C+\right.\right.}{\sqrt{2}a\sqrt{8a^2+b^2-4ac}\sqrt{4a^2+2ac-b\left(b+\sqrt{8a^2+b^2-\right.}}}$$
$$-\frac{\left(2a(A-C)+\left(b-\sqrt{8a^2+b^2-4ac}\right)D\right)\log\left(2a+\left(b-\sqrt{8a^2+b^2-4ac}\right)x+2ax^2\right)}{4a\sqrt{8a^2+b^2-4ac}}$$
$$+\frac{\left(2a(A-C)+\left(b+\sqrt{8a^2+b^2-4ac}\right)D\right)\log\left(2a+\left(b+\sqrt{8a^2+b^2-4ac}\right)x+2ax^2\right)}{4a\sqrt{8a^2+b^2-4ac}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.16

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx = \text{RootSum}\Big[a+b\#1+c\#1^2+b\#1^3 \\ +a\#1^4\&, \frac{A\log(x-\#1)+B\log(x-\#1)\#1+C\log(x-\#1)\#1^2+D\log(x-\#1)\#1^3}{b+2c\#1+3b\#1^2+4a\#1^3}\&\Big]$$

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4),x]
```

```
[Out] RootSum[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , (A*Log[x - #1] + B*Log[x -
#1]*#1 + C*Log[x - #1]*#1^2 + D*Log[x - #1]*#1^3)/(b + 2*c*#1 + 3*b*#1^2 +
4*a*#1^3) & ]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 535, normalized size of antiderivative = 0.88

method	result
default	$4a\left(\frac{-\frac{\left(2Aa-2Ca-\sqrt{8a^2-4ac+b^2}\,D+Db\right)\ln\left(-2a\,x^2+\sqrt{8a^2-4ac+b^2}\,x-bx-2a\right)}{4a}+\frac{2\left(\frac{\left(2Aa-2Ca-\sqrt{8a^2-4ac+b^2}\,D+Db\right)\left(\sqrt{8a^2-4ac+b^2}-b\right)}{4a}-\cdots\right)}{\sqrt{8a^2+\cdots}}}{4a\sqrt{8a^2-4ac+b^2}}\cdots\right.$

```
[In] int((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4*a*(1/4/a/(8*a^2-4*a*c+b^2)^(1/2)*(-1/4*(2*A*a-2*C*a-(8*a^2-4*a*c+b^2)^(1/
2)*D+D*b)/a*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*a)+2*(1/4*(2*A*a-2*
C*a-(8*a^2-4*a*c+b^2)^(1/2)*D+D*b)/a*((8*a^2-4*a*c+b^2)^(1/2)-b)-(8*a^2-4*a
*c+b^2)^(1/2)*A+A*b-2*B*a+2*D*a)/(8*a^2+4*a*c-2*b^2+2*(8*a^2-4*a*c+b^2)^(1/
2)*b)^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*
(8*a^2-4*a*c+b^2)^(1/2)*b)^(1/2)))+1/4/a/(8*a^2-4*a*c+b^2)^(1/2)*(1/4*(2*A*
a-2*C*a+(8*a^2-4*a*c+b^2)^(1/2)*D+D*b)/a*ln(2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)
*x+b*x+2*a)+2*(-1/4*(2*A*a-2*C*a+(8*a^2-4*a*c+b^2)^(1/2)*D+D*b)/a*(b+(8*a^2
-4*a*c+b^2)^(1/2))+(8*a^2-4*a*c+b^2)^(1/2)*A+A*b-2*B*a+2*D*a)/(8*a^2+4*a*c-
2*b^2-2*(8*a^2-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^
(1/2))/(8*a^2+4*a*c-2*b^2-2*(8*a^2-4*a*c+b^2)^(1/2)*b)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx = \text{Timed out}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx = \text{Timed out}$$

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(a*x**4+b*x**3+c*x**2+b*x+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{ax^4+bx^3+cx^2+bx+a}\,dx$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="maxim
a")
[Out] integrate((D*x^3 + C*x^2 + B*x + A)/(a*x^4 + b*x^3 + c*x^2 + b*x + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="giac"
)
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument Va
lue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{a\,x^4+b\,x^3+c\,x^2+b\,x+a}\,dx$$

```
[In] int((A + B*x + C*x^2 + x^3*D)/(a + b*x + a*x^4 + b*x^3 + c*x^2),x)
[Out] int((A + B*x + C*x^2 + x^3*D)/(a + b*x + a*x^4 + b*x^3 + c*x^2), x)
```

3.228 $\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx$

Optimal result 1557
Rubi [A] (verified) 1557
Mathematica [A] (verified) 1558
Maple [A] (verified) 1558
Fricas [A] (verification not implemented) 1559
Sympy [A] (verification not implemented) 1559
Maxima [F] 1559
Giac [A] (verification not implemented) 1560
Mupad [B] (verification not implemented) 1560

Optimal result

Integrand size = 32, antiderivative size = 63

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx = -\frac{2\log\left(2-\left(1-\sqrt{5}\right)x+2x^2\right)}{1-\sqrt{5}}-\frac{2\log\left(2-\left(1+\sqrt{5}\right)x+2x^2\right)}{1+\sqrt{5}}$$

```
[Out] -2*ln(2+2*x^2-x*(-5^(1/2)+1))/(-5^(1/2)+1)-2*ln(2+2*x^2-x*(5^(1/2)+1))/(5^(
1/2)+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2111, 642}

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx = -\frac{2\log\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}{1-\sqrt{5}}-\frac{2\log\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}{1+\sqrt{5}}$$

```
[In] Int[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4),x]
```

```
[Out] (-2*Log[2 - (1 - Sqrt[5])*x + 2*x^2])/(1 - Sqrt[5]) - (2*Log[2 - (1 + Sqrt[
5])*x + 2*x^2])/(1 + Sqrt[5])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2111

```
Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4),
 x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B =
 Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[
(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*
x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A -
2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \frac{-2\sqrt{5}+\left(10-2\sqrt{5}\right)x}{2+\left(-1-\sqrt{5}\right)x+2x^2}\,dx}{\sqrt{5}}+\frac{\int \frac{2\sqrt{5}+\left(10+2\sqrt{5}\right)x}{2+\left(-1+\sqrt{5}\right)x+2x^2}\,dx}{\sqrt{5}}\\ &= -\frac{2\log\left(2-\left(1-\sqrt{5}\right)x+2x^2\right)}{1-\sqrt{5}}-\frac{2\log\left(2-\left(1+\sqrt{5}\right)x+2x^2\right)}{1+\sqrt{5}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx = \frac{1}{2}\left(-\left(\left(-1+\sqrt{5}\right)\log\left(-2+x+\sqrt{5}x-2x^2\right)\right)+\left(1+\sqrt{5}\right)\log\left(2+\left(-1+\sqrt{5}\right)x+2x^2\right)\right)$$

```
[In] Integrate[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4),x]
[Out] (-((-1 + Sqrt[5])*Log[-2 + x + Sqrt[5]*x - 2*x^2]) + (1 + Sqrt[5])*Log[2 +
(-1 + Sqrt[5])*x + 2*x^2])/2
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

method	result	size
default	$2\left(\frac{\sqrt{5}}{4}+\frac{1}{4}\right)\ln\left(x\sqrt{5}+2x^2-x+2\right)-2\left(\frac{\sqrt{5}}{4}-\frac{1}{4}\right)\ln\left(-x\sqrt{5}+2x^2-x+2\right)$	53
risch	$\frac{\ln\left(2+2x^2+\left(\sqrt{5}-1\right)x\right)}{2}+\frac{\ln\left(2+2x^2+\left(\sqrt{5}-1\right)x\right)\sqrt{5}}{2}+\frac{\ln\left(2+2x^2+\left(-\sqrt{5}-1\right)x\right)}{2}-\frac{\ln\left(2+2x^2+\left(-\sqrt{5}-1\right)x\right)\sqrt{5}}{2}$	80

```
[In] int((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x,method=_RETURNVERBOSE)
[Out] 2*(1/4*5^(1/2)+1/4)*ln(x*5^(1/2)+2*x^2-x+2)-2*(1/4*5^(1/2)-1/4)*ln(-x*5^(1/
2)+2*x^2-x+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4} dx = \frac{1}{2}\sqrt{5}\log\left(\frac{2x^4-2x^3+7x^2+\sqrt{5}(2x^3-x^2+2x)-2x+2}{x^4-x^3+x^2-x+1}\right) + \frac{1}{2}\log\left(x^4-x^3+x^2-x+1\right)$$

```
[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="fricas")
[Out] 1/2*sqrt(5)*log((2*x^4 - 2*x^3 + 7*x^2 + sqrt(5)*(2*x^3 - x^2 + 2*x) - 2*x
+ 2)/(x^4 - x^3 + x^2 - x + 1)) + 1/2*log(x^4 - x^3 + x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4} dx = \left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\log\left(x^2+x\left(-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+1\right) + \left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\log\left(x^2+x\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)+1\right)$$

```
[In] integrate((2*x**3-4*x**2+x+2)/(x**4-x**3+x**2-x+1),x)
[Out] (1/2 + sqrt(5)/2)*log(x**2 + x*(-1/2 + sqrt(5)/2) + 1) + (1/2 - sqrt(5)/2)*
log(x**2 + x*(-sqrt(5)/2 - 1/2) + 1)
```

Maxima [F]

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4} dx = \int \frac{2x^3-4x^2+x+2}{x^4-x^3+x^2-x+1} dx$$

```
[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="maxima")
[Out] integrate((2*x^3 - 4*x^2 + x + 2)/(x^4 - x^3 + x^2 - x + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx = -\frac{1}{2}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right) + \frac{1}{2}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right) + \frac{1}{2}\log\left(x^4-x^3+x^2-x+1\right)$$

```
[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="giac")
[Out] -1/2*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/2*sqrt(5)*log(x^2 + 1/2
*x*(sqrt(5) - 1) + 1) + 1/2*log(x^4 - x^3 + x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx = \frac{\ln\left(x^2-\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+1\right)}{2} + \frac{\ln\left(\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+x^2+1\right)}{2} - \frac{\sqrt{5}\,\ln\left(x^2-\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+1\right)}{2} + \frac{\sqrt{5}\,\ln\left(\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+x^2+1\right)}{2}$$

```
[In] int((x - 4*x^2 + 2*x^3 + 2)/(x^2 - x - x^3 + x^4 + 1),x)
[Out] log(x^2 - (5^(1/2)*x)/2 - x/2 + 1)/2 + log((5^(1/2)*x)/2 - x/2 + x^2 + 1)/2
 - (5^(1/2)*log(x^2 - (5^(1/2)*x)/2 - x/2 + 1))/2 + (5^(1/2)*log((5^(1/2)*x
)/2 - x/2 + x^2 + 1))/2
```

3.229 $\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4} \, dx$

Optimal result 1561
Rubi [A] (verified) 1561
Mathematica [A] (verified) 1562
Maple [A] (verified) 1562
Fricas [B] (verification not implemented) 1563
Sympy [A] (verification not implemented) 1563
Maxima [A] (verification not implemented) 1563
Giac [A] (verification not implemented) 1564
Mupad [B] (verification not implemented) 1564

Optimal result

Integrand size = 33, antiderivative size = 14

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4} \, dx = \frac{1}{3(1+x)^3} + \log(1+x)$$

```
[Out] 1/3/(1+x)^3+ln(1+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1608, 1694, 14}

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4} \, dx = \frac{1}{3(x+1)^3} + \log(x+1)$$

```
[In] Int[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

```
[Out] 1/(3*(1 + x)^3) + Log[1 + x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(3+3x+x^2)}{1+4x+6x^2+4x^3+x^4}\,dx \\
&= \text{Subst}\left(\int \frac{-1+x^3}{x^4}\,dx, x, 1+x\right) \\
&= \text{Subst}\left(\int \left(-\frac{1}{x^4}+\frac{1}{x}\right)\,dx, x, 1+x\right) \\
&= \frac{1}{3(1+x)^3}+\log(1+x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{1}{3(1+x)^3}+\log(1+x)$$

```
[In] Integrate[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
[Out] 1/(3*(1 + x)^3) + Log[1 + x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{1}{3(x+1)^3}+\ln(x+1)$	13
norman	$\frac{1}{3(x+1)^3}+\ln(x+1)$	13
risch	$\frac{1}{3x^3+9x^2+9x+3}+\ln(x+1)$	23
parallelrisch	$\frac{3\ln(x+1)x^3+1+9\ln(x+1)x^2+9\ln(x+1)x+3\ln(x+1)}{3x^3+9x^2+9x+3}$	51

```
[In] int((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x,method=_RETURNVERBOSE)
[Out] 1/3/(x+1)^3+ln(x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(12) = 24.

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.71

$$\int \frac{3x + 3x^2 + x^3}{1 + 4x + 6x^2 + 4x^3 + x^4}\,dx = \frac{3\,(x^3 + 3\,x^2 + 3\,x + 1)\log\left(x + 1\right) + 1}{3\,(x^3 + 3\,x^2 + 3\,x + 1)}$$

```
[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="fricas")
[Out] 1/3*(3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 1)/(x^3 + 3*x^2 + 3*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{3x + 3x^2 + x^3}{1 + 4x + 6x^2 + 4x^3 + x^4}\,dx = \log\left(x + 1\right) + \frac{1}{3x^3 + 9x^2 + 9x + 3}$$

```
[In] integrate((x**3+3*x**2+3*x)/(x**4+4*x**3+6*x**2+4*x+1),x)
[Out] log(x + 1) + 1/(3*x**3 + 9*x**2 + 9*x + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{3x + 3x^2 + x^3}{1 + 4x + 6x^2 + 4x^3 + x^4}\,dx = \frac{1}{3\,(x^3 + 3\,x^2 + 3\,x + 1)} + \log\left(x + 1\right)$$

```
[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")
[Out] 1/3/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{1}{3\,(x+1)^3} + \log\left(|x+1|\right)$$

```
[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")
[Out] 1/3/(x + 1)^3 + log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \ln\left(x+1\right) + \frac{1}{3\,(x+1)^3}$$

```
[In] int((3*x + 3*x^2 + x^3)/(4*x + 6*x^2 + 4*x^3 + x^4 + 1),x)
[Out] log(x + 1) + 1/(3*(x + 1)^3)
```

3.230 $\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx$

Optimal result 1565
Rubi [A] (verified) 1565
Mathematica [A] (verified) 1566
Maple [A] (verified) 1566
Fricas [A] (verification not implemented) 1567
Sympy [A] (verification not implemented) 1567
Maxima [A] (verification not implemented) 1567
Giac [A] (verification not implemented) 1567
Mupad [B] (verification not implemented) 1568

Optimal result

Integrand size = 34, antiderivative size = 28

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{8}{3(1+x)^3} - \frac{6}{(1+x)^2} + \frac{6}{1+x} + \log(1+x)$$

```
[Out] 8/3/(1+x)^3-6/(1+x)^2+6/(1+x)+ln(1+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1694, 45}

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{6}{x+1} - \frac{6}{(x+1)^2} + \frac{8}{3(x+1)^3} + \log(x+1)$$

```
[In] Int[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

```
[Out] 8/(3*(1 + x)^3) - 6/(1 + x)^2 + 6/(1 + x) + Log[1 + x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
```

```
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{(-2+x)^3}{x^4}\,dx, x, 1+x\right)\\ &= \text{Subst}\left(\int \left(-\frac{8}{x^4}+\frac{12}{x^3}-\frac{6}{x^2}+\frac{1}{x}\right)dx, x, 1+x\right)\\ &= \frac{8}{3(1+x)^3}-\frac{6}{(1+x)^2}+\frac{6}{1+x}+\log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{2(4+9x+9x^2)}{3(1+x)^3}+\log(1+x)$$

```
[In] Integrate[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
[Out] (2*(4 + 9*x + 9*x^2))/(3*(1 + x)^3) + Log[1 + x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

method	result	size
norman	$\frac{6x+6x^2+\frac{8}{3}}{(x+1)^3}+\ln(x+1)$	22
default	$\frac{8}{3(x+1)^3}-\frac{6}{(x+1)^2}+\frac{6}{x+1}+\ln(x+1)$	27
risch	$\frac{6x+6x^2+\frac{8}{3}}{x^3+3x^2+3x+1}+\ln(x+1)$	32
parallelrisch	$\frac{3\ln(x+1)x^3+8+9\ln(x+1)x^2+9\ln(x+1)x+18x^2+3\ln(x+1)+18x}{3x^3+9x^2+9x+3}$	59

```
[In] int((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x,method=_RETURNVERBOSE)
[Out] (6*x+6*x^2+8/3)/(x+1)^3+ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{18\,x^2+3\,(x^3+3\,x^2+3\,x+1)\log{(x+1)}+18\,x+8}{3\,(x^3+3\,x^2+3\,x+1)}$$

```
[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="fricas")
[Out] 1/3*(18*x^2 + 3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 8)/(x^3 + 3*x^2
 + 3*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{18x^2+18x+8}{3x^3+9x^2+9x+3}+\log{(x+1)}$$

```
[In] integrate((x**3-3*x**2+3*x-1)/(x**4+4*x**3+6*x**2+4*x+1),x)
[Out] (18*x**2 + 18*x + 8)/(3*x**3 + 9*x**2 + 9*x + 3) + log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{2\,(9\,x^2+9\,x+4)}{3\,(x^3+3\,x^2+3\,x+1)}+\log{(x+1)}$$

```
[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")
[Out] 2/3*(9*x^2 + 9*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{2\,(9\,x^2+9\,x+4)}{3\,(x+1)^3}+\log{(|x+1|)}$$

```
[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")
[Out] 2/3*(9*x^2 + 9*x + 4)/(x + 1)^3 + log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \ln(x+1)+\frac{6\,x^2+6\,x+\frac{8}{3}}{(x+1)^3}$$

```
[In] int((3*x - 3*x^2 + x^3 - 1)/(4*x + 6*x^2 + 4*x^3 + x^4 + 1),x)
[Out] log(x + 1) + (6*x + 6*x^2 + 8/3)/(x + 1)^3
```

3.231 $\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3} dx$

Optimal result 1569
Rubi [A] (verified) 1569
Mathematica [A] (verified) 1571
Maple [A] (verified) 1571
Fricas [A] (verification not implemented) 1572
Sympy [A] (verification not implemented) 1572
Maxima [A] (verification not implemented) 1572
Giac [A] (verification not implemented) 1573
Mupad [B] (verification not implemented) 1573

Optimal result

Integrand size = 43, antiderivative size = 59

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3} dx = \frac{2(1-2x^2)}{(3+2x^2+x^4)^2} - \frac{2x(18+13x^2)}{(3+2x^2+x^4)^2} + \frac{13x}{3+2x^2+x^4}$$

```
[Out] 2*(-2*x^2+1)/(x^4+2*x^2+3)^2-2*x*(13*x^2+18)/(x^4+2*x^2+3)^2+13*x/(x^4+2*x^
2+3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {1687, 1692, 1602, 1677, 1674, 8}

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3} dx = \frac{13x}{x^4+2x^2+3} - \frac{2(13x^2+18)x}{(x^4+2x^2+3)^2} + \frac{2(1-2x^2)}{(x^4+2x^2+3)^2}$$

```
[In] Int[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x^2 + x
^4)^3,x]
```

```
[Out] (2*(1 - 2*x^2))/(3 + 2*x^2 + x^4)^2 - (2*x*(18 + 13*x^2))/(3 + 2*x^2 + x^4)
^2 + (13*x)/(3 + 2*x^2 + x^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
```

```
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(-40+24x^4)}{(3+2x^2+x^4)^3}\,dx + \int \frac{9-18x^2+174x^4+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx \\
&= -\frac{2x(18+13x^2)}{(3+2x^2+x^4)^2} + \frac{1}{96}\int \frac{3744-2496x^2-3744x^4}{(3+2x^2+x^4)^2}\,dx \\
&\quad + \frac{1}{2}\text{Subst}\left(\int \frac{-40+24x^2}{(3+2x+x^2)^3}\,dx, x, x^2\right) \\
&= \frac{2(1-2x^2)}{(3+2x^2+x^4)^2} - \frac{2x(18+13x^2)}{(3+2x^2+x^4)^2} + \frac{13x}{3+2x^2+x^4} + \frac{1}{32}\text{Subst}\left(\int 0\,dx, x, x^2\right) \\
&= \frac{2(1-2x^2)}{(3+2x^2+x^4)^2} - \frac{2x(18+13x^2)}{(3+2x^2+x^4)^2} + \frac{13x}{3+2x^2+x^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = \frac{2+3x-4x^2+13x^5}{(3+2x^2+x^4)^2}$$

```
[In] Integrate[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x
^2 + x^4)^3,x]
[Out] (2 + 3*x - 4*x^2 + 13*x^5)/(3 + 2*x^2 + x^4)^2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{13x^5-4x^2+3x+2}{(x^4+2x^2+3)^2}$	29
norman	$\frac{13x^5-4x^2+3x+2}{(x^4+2x^2+3)^2}$	29
risch	$\frac{13x^5-4x^2+3x+2}{(x^4+2x^2+3)^2}$	29
parallelrisch	$\frac{13x^5-4x^2+3x+2}{(x^4+2x^2+3)^2}$	29
default	$-\frac{-13x^5+4x^2-3x-2}{(x^4+2x^2+3)^2}$	30

```
[In] int((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,method=
_RETURNVERBOSE)
```

```
[Out] (13*x^5-4*x^2+3*x+2)/(x^4+2*x^2+3)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = \frac{13\,x^5-4\,x^2+3\,x+2}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$

```
[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
algorithm="fricas")
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = -\frac{-13x^5+4x^2-3x-2}{x^8+4x^6+10x^4+12x^2+9}$$

```
[In] integrate((-39*x**8+26*x**6+24*x**5+174*x**4-18*x**2-40*x+9)/(x**4+2*x**2+3
)**3,x)
```

```
[Out] -(-13*x**5 + 4*x**2 - 3*x - 2)/(x**8 + 4*x**6 + 10*x**4 + 12*x**2 + 9)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = \frac{13\,x^5-4\,x^2+3\,x+2}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$

```
[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
algorithm="maxima")
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Giac [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = \frac{13\,x^5-4\,x^2+3\,x+2}{\left(x^4+2\,x^2+3\right)^2}$$

```
[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
algorithm="giac")
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^4 + 2*x^2 + 3)^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = \frac{13\,x^5-4\,x^2+3\,x+2}{\left(x^4+2\,x^2+3\right)^2}$$

```
[In] int((174*x^4 - 18*x^2 - 40*x + 24*x^5 + 26*x^6 - 39*x^8 + 9)/(2*x^2 + x^4 +
 3)^3,x)
```

```
[Out] (3*x - 4*x^2 + 13*x^5 + 2)/(2*x^2 + x^4 + 3)^2
```

3.232 $\int \frac{-1+4x^5}{(1+x+x^5)^2} \, dx$

Optimal result 1574
Rubi [A] (verified) 1574
Mathematica [A] (verified) 1575
Maple [A] (verified) 1575
Fricas [A] (verification not implemented) 1575
Sympy [A] (verification not implemented) 1576
Maxima [A] (verification not implemented) 1576
Giac [A] (verification not implemented) 1576
Mupad [B] (verification not implemented) 1576

Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \frac{-1+4x^5}{(1+x+x^5)^2} \, dx = -\frac{x}{1+x+x^5}$$

```
[Out] -x/(x^5+x+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1602}

$$\int \frac{-1+4x^5}{(1+x+x^5)^2} \, dx = -\frac{x}{x^5+x+1}$$

```
[In] Int[(-1 + 4*x^5)/(1 + x + x^5)^2,x]
```

```
[Out] -(x/(1 + x + x^5))
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{x}{1+x+x^5}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{1+x+x^5}$$

```
[In] Integrate[(-1 + 4*x^5)/(1 + x + x^5)^2,x]
[Out] -(x/(1 + x + x^5))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
gosper	$-\frac{x}{x^5+x+1}$	12
norman	$-\frac{x}{x^5+x+1}$	12
risch	$-\frac{x}{x^5+x+1}$	12
parallelrisch	$-\frac{x}{x^5+x+1}$	12
default	$-\frac{-3x^2+5x-1}{7(x^3-x^2+1)}+\frac{-3x-1}{7x^2+7x+7}$	41

```
[In] int((4*x^5-1)/(x^5+x+1)^2,x,method=_RETURNVERBOSE)
[Out] -x/(x^5+x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{x^5+x+1}$$

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="fricas")
[Out] -x/(x^5 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{x^5+x+1}$$

```
[In] integrate((4*x**5-1)/(x**5+x+1)**2,x)
[Out] -x/(x**5 + x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{x^5+x+1}$$

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="maxima")
[Out] -x/(x^5 + x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{x^5+x+1}$$

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="giac")
[Out] -x/(x^5 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{x^5+x+1}$$

```
[In] int((4*x^5 - 1)/(x + x^5 + 1)^2,x)
[Out] -x/(x + x^5 + 1)
```

3.233 $\int \frac{1+x^2}{\left(1-7x^2+7x^4-x^6\right)^2}\,dx$

Optimal result 1577
Rubi [B] (verified) 1578
Mathematica [A] (verified) 1580
Maple [A] (verified) 1581
Fricas [B] (verification not implemented) 1581
Sympy [B] (verification not implemented) 1581
Maxima [A] (verification not implemented) 1583
Giac [A] (verification not implemented) 1583
Mupad [B] (verification not implemented) 1584

Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx = \frac{x}{16(1-x^2)} + \frac{x(29-5x^2)}{32(1-6x^2+x^4)} + \frac{\operatorname{arctanh}(x)}{4} + \frac{1}{64}\left(\left(3-2\sqrt{2}\right)\operatorname{arctanh}\left(\left(-1+\sqrt{2}\right)x\right) - \left(3+2\sqrt{2}\right)\operatorname{arctanh}\left(\left(1+\sqrt{2}\right)x\right)\right)$$

```
[Out] 1/16*x/(-x^2+1)+1/32*x*(-5*x^2+29)/(x^4-6*x^2+1)+1/4*arctanh(x)+1/64*arctan
h(x*(2^(1/2)-1))*(3-2*2^(1/2))-1/64*arctanh(x*(1+2^(1/2)))*(3+2*2^(1/2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 205 vs. 2(91) = 182.

Time = 0.11 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.25, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2098, 213, 652, 632, 212, 646, 31}

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx = -\frac{5\text{arctanh}\left(\frac{1-x}{\sqrt{2}}\right)}{64\sqrt{2}}+\frac{\text{arctanh}(x)}{4}+\frac{5\text{arctanh}\left(\frac{x+1}{\sqrt{2}}\right)}{64\sqrt{2}} - \frac{12-5x}{64(-x^2+2x+1)}+\frac{5x+12}{64(-x^2-2x+1)}+\frac{1}{32(1-x)} - \frac{1}{32(x+1)}-\frac{3}{256}\left(2+3\sqrt{2}\right)\log\left(-x-\sqrt{2}+1\right) - \frac{3}{256}\left(2-3\sqrt{2}\right)\log\left(-x+\sqrt{2}+1\right) + \frac{3}{256}\left(2+3\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right) + \frac{3}{256}\left(2-3\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

```
[In] Int[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2,x]
```

```
[Out] 1/(32*(1 - x)) - 1/(32*(1 + x)) + (12 + 5*x)/(64*(1 - 2*x - x^2)) - (12 - 5
*x)/(64*(1 + 2*x - x^2)) - (5*ArcTanh[(1 - x)/Sqrt[2]])/(64*Sqrt[2]) + ArcT
anh[x]/4 + (5*ArcTanh[(1 + x)/Sqrt[2]])/(64*Sqrt[2]) - (3*(2 + 3*Sqrt[2])*L
og[1 - Sqrt[2] - x])/256 - (3*(2 - 3*Sqrt[2])*Log[1 + Sqrt[2] - x])/256 + (
3*(2 + 3*Sqrt[2])*Log[1 - Sqrt[2] + x])/256 + (3*(2 - 3*Sqrt[2])*Log[1 + Sq
rt[2] + x])/256
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2098

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{32(-1+x)^2}+\frac{1}{32(1+x)^2}-\frac{1}{4\left(-1+x^2\right)}+\frac{17-7x}{32\left(-1-2x+x^2\right)^2}\right. \\
&\qquad \left. -\frac{3(-4+x)}{64\left(-1-2x+x^2\right)}+\frac{17+7x}{32\left(-1+2x+x^2\right)^2}+\frac{3(4+x)}{64\left(-1+2x+x^2\right)}\right)dx \\
&= \frac{1}{32(1-x)}-\frac{1}{32(1+x)}+\frac{1}{32}\int\frac{17-7x}{\left(-1-2x+x^2\right)^2}\,dx+\frac{1}{32}\int\frac{17+7x}{\left(-1+2x+x^2\right)^2}\,dx \\
&\quad -\frac{3}{64}\int\frac{-4+x}{-1-2x+x^2}\,dx+\frac{3}{64}\int\frac{4+x}{-1+2x+x^2}\,dx-\frac{1}{4}\int\frac{1}{-1+x^2}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{32(1-x)} - \frac{1}{32(1+x)} + \frac{12+5x}{64\,(1-2x-x^2)} - \frac{12-5x}{64\,(1+2x-x^2)} \\
&\quad + \frac{1}{4}\tanh^{-1}(x) - \frac{5}{64}\int \frac{1}{-1-2x+x^2}\,dx - \frac{5}{64}\int \frac{1}{-1+2x+x^2}\,dx \\
&\quad - \frac{1}{256}\left(3\left(2-3\sqrt{2}\right)\right)\int \frac{1}{-1-\sqrt{2}+x}\,dx + \frac{1}{256}\left(3\left(2-3\sqrt{2}\right)\right)\int \frac{1}{1+\sqrt{2}+x}\,dx \\
&\quad + \frac{1}{256}\left(3\left(2+3\sqrt{2}\right)\right)\int \frac{1}{1-\sqrt{2}+x}\,dx - \frac{1}{256}\left(3\left(2+3\sqrt{2}\right)\right)\int \frac{1}{-1+\sqrt{2}+x}\,dx \\
&= \frac{1}{32(1-x)} - \frac{1}{32(1+x)} + \frac{12+5x}{64\,(1-2x-x^2)} - \frac{12-5x}{64\,(1+2x-x^2)} + \frac{1}{4}\tanh^{-1}(x) \\
&\quad - \frac{3}{256}\left(2+3\sqrt{2}\right)\log\left(1-\sqrt{2}-x\right) - \frac{3}{256}\left(2-3\sqrt{2}\right)\log\left(1+\sqrt{2}-x\right) \\
&\quad + \frac{3}{256}\left(2+3\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right) + \frac{3}{256}\left(2-3\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right) \\
&\quad + \frac{5}{32}\text{Subst}\left(\int \frac{1}{8-x^2}\,dx, x, -2+2x\right) + \frac{5}{32}\text{Subst}\left(\int \frac{1}{8-x^2}\,dx, x, 2+2x\right) \\
&= \frac{1}{32(1-x)} - \frac{1}{32(1+x)} + \frac{12+5x}{64\,(1-2x-x^2)} - \frac{12-5x}{64\,(1+2x-x^2)} \\
&\quad - \frac{5\tanh^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{64\sqrt{2}} + \frac{1}{4}\tanh^{-1}(x) + \frac{5\tanh^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{64\sqrt{2}} \\
&\quad - \frac{3}{256}\left(2+3\sqrt{2}\right)\log\left(1-\sqrt{2}-x\right) - \frac{3}{256}\left(2-3\sqrt{2}\right)\log\left(1+\sqrt{2}-x\right) \\
&\quad + \frac{3}{256}\left(2+3\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right) + \frac{3}{256}\left(2-3\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.45

$$
\begin{aligned}
\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx = \frac{1}{128}\Bigg(&-\frac{4x(31-46x^2+7x^4)}{-1+7x^2-7x^4+x^6} - 16\log(1-x) \\
&+ \left(3+2\sqrt{2}\right)\log\left(-1+\sqrt{2}-x\right) \\
&+ \left(-3+2\sqrt{2}\right)\log\left(1+\sqrt{2}-x\right) + 16\log(1+x) \\
&- \left(3+2\sqrt{2}\right)\log\left(-1+\sqrt{2}+x\right) \\
&+ \left(3-2\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)\Bigg)
\end{aligned}
$$

```
[In] Integrate[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2,x]
```

```
[Out] ((-4*x*(31 - 46*x^2 + 7*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 16*Log[1 - x] +
(3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (-3 + 2*Sqrt[2])*Log[1 + Sqrt[2] -
x] + 16*Log[1 + x] - (3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] + x] + (3 - 2*Sqrt[2]
)*Log[1 + Sqrt[2] + x])/128
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

method	result
default	$-\frac{1}{32(x+1)}+\frac{\ln(x+1)}{8}+\frac{-5x-12}{64x^2+128x-64}+\frac{3\ln(x^2+2x-1)}{128}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{32}-\frac{1}{32(x-1)}-\frac{\ln(x-1)}{8}-\frac{5x-}{64(x^2-}$
risch	$\frac{-\frac{7}{32}x^5+\frac{23}{16}x^3-\frac{31}{32}x}{x^6-7x^4+7x^2-1}-\frac{3\ln\left(x-1-\sqrt{2}\right)}{128}+\frac{\ln\left(x-1-\sqrt{2}\right)\sqrt{2}}{64}-\frac{3\ln\left(x-1+\sqrt{2}\right)}{128}-\frac{\ln\left(x-1+\sqrt{2}\right)\sqrt{2}}{64}+\frac{\ln(x+1)}{8}-\frac{\ln(x-1)}{8}$

```
[In] int((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/(x+1)+1/8*ln(x+1)+1/64*(-5*x-12)/(x^2+2*x-1)+3/128*ln(x^2+2*x-1)-1/32
*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))-1/32/(x-1)-1/8*ln(x-1)-1/64*(5*x-12)/
(x^2-2*x-1)-3/128*ln(x^2-2*x-1)-1/32*2^(1/2)*arctanh(1/4*(2*x-2)*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(71) = 142$.

Time = 0.34 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.45

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx = -\frac{28\,x^5-184\,x^3-2\,\sqrt{2}(x^6-7\,x^4+7\,x^2-1)\log\left(\frac{x^2-2\,\sqrt{2}(x+1)+2\,x+3}{x^2+2\,x-1}\right)-2\,\sqrt{2}(x^6-7\,x^4+7\,x^2-1)\log\left(\right.}{}$$

```
[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/128*(28*x^5 - 184*x^3 - 2*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2 - 2
*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) - 2*sqrt(2)*(x^6 - 7*x^4 + 7*x
^2 - 1)*log((x^2 - 2*sqrt(2)*(x - 1) - 2*x + 3)/(x^2 - 2*x - 1)) - 3*(x^6 -
 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*x - 1) + 3*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x^
2 - 2*x - 1) - 16*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) + 16*(x^6 - 7*x^4 +
7*x^2 - 1)*log(x - 1) + 124*x)/(x^6 - 7*x^4 + 7*x^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(73) = 146$.

Time = 0.80 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.99

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx$$
$$= \frac{-7x^5+46x^3-31x}{32x^6-224x^4+224x^2-32} - \frac{\log(x-1)}{8} + \frac{\log(x+1)}{8} + \left(-\frac{3}{128} - \frac{\sqrt{2}}{64}\right)\log\left(x - \frac{38423555}{909328} - \frac{38423555\sqrt{2}}{1363992} + \frac{9549859782656\left(-\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^5}{170499} - \frac{56267374592\left(-\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^3}{56833}\right)$$
$$+ \left(-\frac{3}{128} + \frac{\sqrt{2}}{64}\right)\log\left(x - \frac{38423555}{909328} + \frac{9549859782656\left(-\frac{3}{128}+\frac{\sqrt{2}}{64}\right)^5}{170499} - \frac{56267374592\left(-\frac{3}{128}+\frac{\sqrt{2}}{64}\right)^3}{56833} + \frac{38423555\sqrt{2}}{1363992}\right)$$
$$+ \left(\frac{3}{128} - \frac{\sqrt{2}}{64}\right)\log\left(x - \frac{38423555\sqrt{2}}{1363992} - \frac{56267374592\left(\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^3}{56833} + \frac{9549859782656\left(\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^5}{170499} + \frac{38423555}{909328}\right)$$
$$+ \left(\frac{\sqrt{2}}{64} + \frac{3}{128}\right)\log\left(x - \frac{56267374592\left(\frac{\sqrt{2}}{64}+\frac{3}{128}\right)^3}{56833} + \frac{9549859782656\left(\frac{\sqrt{2}}{64}+\frac{3}{128}\right)^5}{170499} + \frac{38423555\sqrt{2}}{1363992} + \frac{38423555}{909328}\right)$$

```
[In] integrate((x**2+1)/(-x**6+7*x**4-7*x**2+1)**2,x)
```

```
[Out] (-7*x**5 + 46*x**3 - 31*x)/(32*x**6 - 224*x**4 + 224*x**2 - 32) - log(x - 1
)/8 + log(x + 1)/8 + (-3/128 - sqrt(2)/64)*log(x - 38423555/909328 - 384235
55*sqrt(2)/1363992 + 9549859782656*(-3/128 - sqrt(2)/64)**5/170499 - 562673
74592*(-3/128 - sqrt(2)/64)**3/56833) + (-3/128 + sqrt(2)/64)*log(x - 38423
555/909328 + 9549859782656*(-3/128 + sqrt(2)/64)**5/170499 - 56267374592*(-
3/128 + sqrt(2)/64)**3/56833 + 38423555*sqrt(2)/1363992) + (3/128 - sqrt(2)
/64)*log(x - 38423555*sqrt(2)/1363992 - 56267374592*(3/128 - sqrt(2)/64)**3
/56833 + 9549859782656*(3/128 - sqrt(2)/64)**5/170499 + 38423555/909328) +
(sqrt(2)/64 + 3/128)*log(x - 56267374592*(sqrt(2)/64 + 3/128)**3/56833 + 95
49859782656*(sqrt(2)/64 + 3/128)**5/170499 + 38423555*sqrt(2)/1363992 + 384
23555/909328)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.25

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx = \frac{1}{64}\,\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{64}\,\sqrt{2}\log\left(\frac{x-\sqrt{2}-1}{x+\sqrt{2}-1}\right) - \frac{7\,x^5-46\,x^3+31\,x}{32\,(x^6-7\,x^4+7\,x^2-1)}+\frac{3}{128}\,\log\left(x^2+2\,x-1\right) - \frac{3}{128}\,\log\left(x^2-2\,x-1\right)+\frac{1}{8}\,\log\left(x+1\right)-\frac{1}{8}\,\log\left(x-1\right)$$

```
[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/64*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/64*sqrt(2)*log((x
 - sqrt(2) - 1)/(x + sqrt(2) - 1)) - 1/32*(7*x^5 - 46*x^3 + 31*x)/(x^6 - 7*
x^4 + 7*x^2 - 1) + 3/128*log(x^2 + 2*x - 1) - 3/128*log(x^2 - 2*x - 1) + 1/
8*log(x + 1) - 1/8*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.47

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx = \frac{1}{64}\,\sqrt{2}\log\left(\frac{|2\,x-2\,\sqrt{2}+2|}{|2\,x+2\,\sqrt{2}+2|}\right) + \frac{1}{64}\,\sqrt{2}\log\left(\frac{|2\,x-2\,\sqrt{2}-2|}{|2\,x+2\,\sqrt{2}-2|}\right)-\frac{7\,x^5-46\,x^3+31\,x}{32\,(x^6-7\,x^4+7\,x^2-1)} + \frac{3}{128}\,\log\left(|x^2+2\,x-1|\right)-\frac{3}{128}\,\log\left(|x^2-2\,x-1|\right) + \frac{1}{8}\,\log\left(|x+1|\right)-\frac{1}{8}\,\log\left(|x-1|\right)$$

```
[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/64*
sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2)/abs(2*x + 2*sqrt(2) - 2)) - 1/32*(7*x^
5 - 46*x^3 + 31*x)/(x^6 - 7*x^4 + 7*x^2 - 1) + 3/128*log(abs(x^2 + 2*x - 1)
) - 3/128*log(abs(x^2 - 2*x - 1)) + 1/8*log(abs(x + 1)) - 1/8*log(abs(x - 1
))
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.36

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx = -\frac{\mathrm{atan}(x\,1\mathrm{i})\,1\mathrm{i}}{4} - \frac{\frac{7\,x^5}{32}-\frac{23\,x^3}{16}+\frac{31\,x}{32}}{x^6-7\,x^4+7\,x^2-1} + \mathrm{atan}\left(\frac{x\,23313\mathrm{i}}{8192\,\left(\frac{27309\,\sqrt{2}}{32768}-\frac{19317}{16384}\right)} - \frac{\sqrt{2}\,x\,65943\mathrm{i}}{32768\,\left(\frac{27309\,\sqrt{2}}{32768}-\frac{19317}{16384}\right)}\right)\left(\frac{\sqrt{2}\,1\mathrm{i}}{32}-\frac{3}{64}\mathrm{i}\right) + \mathrm{atan}\left(\frac{x\,23313\mathrm{i}}{8192\,\left(\frac{27309\,\sqrt{2}}{32768}+\frac{19317}{16384}\right)} + \frac{\sqrt{2}\,x\,65943\mathrm{i}}{32768\,\left(\frac{27309\,\sqrt{2}}{32768}+\frac{19317}{16384}\right)}\right)\left(\frac{\sqrt{2}\,1\mathrm{i}}{32}+\frac{3}{64}\mathrm{i}\right)$$

```
[In] int((x^2 + 1)/(7*x^2 - 7*x^4 + x^6 - 1)^2,x)
```

```
[Out] atan((x*23313i)/(8192*((27309*2^(1/2))/32768 - 19317/16384)) - (2^(1/2)*x*6
5943i)/(32768*((27309*2^(1/2))/32768 - 19317/16384)))*((2^(1/2)*1i)/32 - 3i
/64) - ((31*x)/32 - (23*x^3)/16 + (7*x^5)/32)/(7*x^2 - 7*x^4 + x^6 - 1) - (
atan(x*1i)*1i)/4 + atan((x*23313i)/(8192*((27309*2^(1/2))/32768 + 19317/163
84)) + (2^(1/2)*x*65943i)/(32768*((27309*2^(1/2))/32768 + 19317/16384)))*((
2^(1/2)*1i)/32 + 3i/64)
```

3.234 $\int x^m(a+bx+cx^2+dx^3)^p(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx$

Optimal result 1585
Rubi [A] (verified) 1585
Mathematica [A] (verified) 1586
Maple [A] (verified) 1586
Fricas [A] (verification not implemented) 1587
Sympy [F(-1)] 1587
Maxima [A] (verification not implemented) 1587
Giac [B] (verification not implemented) 1588
Mupad [B] (verification not implemented) 1588

Optimal result

Integrand size = 56, antiderivative size = 25

$$\int x^m(a+bx+cx^2+dx^3)^p(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx=x^{1+m}(a+bx+cx^2+dx^3)^{1+p}$$

```
[Out] x^(1+m)*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {1604}

$$\int x^m(a+bx+cx^2+dx^3)^p(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx=x^{m+1}(a+bx+cx^2+dx^3)^{p+1}$$

```
[In] Int[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x*(c*(3
 + m + 2*p) + d*(4 + m + 3*p)*x))),x]
```

```
[Out] x^(1 + m)*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
```

```
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = x^{1+m}\left(a+bx+cx^2+dx^3\right)^{1+p}$$

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int x^m\left(a+bx+cx^2+dx^3\right)^p(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx=x^{1+m}(a+x(b+x(c+dx)))^{1+p}$$

```
[In] Integrate[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x
*(c*(3 + m + 2*p) + d*(4 + m + 3*p)*x))),x]
[Out] x^(1 + m)*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
gosper	$x^{1+m}(x^3d+c\,x^2+bx+a)^{1+p}$	26
risch	$(x^3d+c\,x^2+bx+a)^p\,x^m x(x^3d+c\,x^2+bx+a)$	38
parallelrisch	$\frac{x^4x^m\left(x^3d+c\,x^2+bx+a\right)^p ad+x^3x^m\left(x^3d+c\,x^2+bx+a\right)^p ac+x^2x^m\left(x^3d+c\,x^2+bx+a\right)^p ab+x\,x^m\left(x^3d+c\,x^2+bx+a\right)^p a^2}{a}$	109

```
[In] int(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+3
*p)*x))),x,method=_RETURNVERBOSE)
[Out] x^(1+m)*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Fricas [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int x^m \left(a + bx + cx^2 + dx^3\right)^p (a(1+m) + x(b(2+m+p) + x(c(3+m+2p)+d(4+m+3p)x)))\,dx = \left(dx^4 + cx^3 + bx^2 + ax\right)\left(dx^3 + cx^2 + bx + a\right)^p x^m$$

```
[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*
(4+m+3*p)*x))),x, algorithm="fricas")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p*x^m
```

Sympy [F(-1)]

Timed out.

$$\int x^m \left(a + bx + cx^2 + dx^3\right)^p (a(1+m) + x(b(2+m+p) + x(c(3+m+2p)+d(4+m+3p)x)))\,dx = \text{Timed out}$$

```
[In] integrate(x**m*(d*x**3+c*x**2+b*x+a)**p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p
)+d*(4+m+3*p)*x))),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int x^m \left(a + bx + cx^2 + dx^3\right)^p (a(1+m) + x(b(2+m+p) + x(c(3+m+2p)+d(4+m+3p)x)))\,dx = \left(dx^4 + cx^3 + bx^2 + ax\right)e^{\left(p\log\left(dx^3+cx^2+bx+a\right)+m\log\left(x\right)\right)}$$

```
[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*
(4+m+3*p)*x))),x, algorithm="maxima")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*e^(p*log(d*x^3 + c*x^2 + b*x + a) + m*log(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(25) = 50$.

Time = 0.54 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.96

$$\int x^m(a+bx+cx^2+dx^3)^p(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx = (dx^3+cx^2+bx+a)^p dx^4x^m + (dx^3+cx^2+bx+a)^p cx^3x^m + (dx^3+cx^2+bx+a)^p bx^2x^m + (dx^3+cx^2+bx+a)^p axx^m$$

```
[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*
(4+m+3*p)*x))),x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^4*x^m + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3*x
^m + (d*x^3 + c*x^2 + b*x + a)^p*b*x^2*x^m + (d*x^3 + c*x^2 + b*x + a)^p*a*
x*x^m
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int x^m(a+bx+cx^2+dx^3)^p(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx = (d\,x^3+c\,x^2+b\,x+a)^p\,(a\,x\,x^m+b\,x^m\,x^2+c\,x^m\,x^3+d\,x^m\,x^4)$$

```
[In] int(x^m*(a*(m + 1) + x*(x*(c*(m + 2*p + 3) + d*x*(m + 3*p + 4)) + b*(m + p
+ 2)))*(a + b*x + c*x^2 + d*x^3)^p,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x*x^m + b*x^m*x^2 + c*x^m*x^3 + d*x^m*x^4)
```

3.235 $\int x^2(a+bx+cx^2+dx^3)^p\,(3a+b(4+p)x+c(5+2$

Optimal result 1589
Rubi [A] (verified) 1589
Mathematica [A] (verified) 1590
Maple [A] (verified) 1590
Fricas [A] (verification not implemented) 1590
Sympy [F(-1)] 1591
Maxima [A] (verification not implemented) 1591
Giac [B] (verification not implemented) 1591
Mupad [B] (verification not implemented) 1592

Optimal result

Integrand size = 51, antiderivative size = 23

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx$$
$$=x^3\left(a+bx+cx^2+dx^3\right)^{1+p}$$

```
[Out] x^3*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {1602}

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx$$
$$=x^3\left(a+bx+cx^2+dx^3\right)^{p+1}$$

```
[In] Int[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*x^2 + d*(6 + 3*p)*x^3),x]
```

```
[Out] x^3*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = x^3\left(a+bx+cx^2+dx^3\right)^{1+p}$$

Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx$$
$$= x^3(a+x(b+x(c+dx)))^{1+p}$$

```
[In] Integrate[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*
x^2 + d*(6 + 3*p)*x^3),x]
[Out] x^3*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result
gosper	$x^3(x^3d+c\,x^2+bx+a)^{1+p}$
risch	$(x^3d+c\,x^2+bx+a)^p\,x^3(x^3d+c\,x^2+bx+a)$
norman	$a\,x^3\mathrm{e}^{p\ln(x^3d+c\,x^2+bx+a)}+b\,x^4\mathrm{e}^{p\ln(x^3d+c\,x^2+bx+a)}+c\,x^5\mathrm{e}^{p\ln(x^3d+c\,x^2+bx+a)}+d\,x^6\mathrm{e}^{p\ln(x^3d+c\,x^2+bx+a}$
parallelrisch	$\frac{x^6(x^3d+c\,x^2+bx+a)^p cd+x^5(x^3d+c\,x^2+bx+a)^p c^2+x^4(x^3d+c\,x^2+bx+a)^p bc+x^3(x^3d+c\,x^2+bx+a)^p ac}{c}$

```
[In] int(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3),x
,method=_RETURNVERBOSE)
[Out] x^3*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx$$
$$= \left(dx^6+cx^5+bx^4+ax^3\right)\left(dx^3+cx^2+bx+a\right)^p$$

```
[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*
x^3),x, algorithm="fricas")
[Out] (d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(d*x**3+c*x**2+b*x+a)**p*(3*a+b*(4+p)*x+c*(5+2*p)*x**2+d*(6+3*p)*x**3),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx = \left(dx^6+cx^5+bx^4+ax^3\right)\left(dx^3+cx^2+bx+a\right)^p$$

```
[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3),x, algorithm="maxima")
[Out] (d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(23) = 46.

Time = 0.53 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.87

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx = \left(dx^3+cx^2+bx+a\right)^p dx^6+\left(dx^3+cx^2+bx+a\right)^p cx^5 + \left(dx^3+cx^2+bx+a\right)^p bx^4+\left(dx^3+cx^2+bx+a\right)^p ax^3$$

```
[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3),x, algorithm="giac")
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^6 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^5 + (d*x^3 + c*x^2 + b*x + a)^p*b*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^3
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx$$
$$=\left(d\,x^3+c\,x^2+b\,x+a\right)^p\left(d\,x^6+c\,x^5+b\,x^4+a\,x^3\right)$$

```
[In] int(x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*x*(p + 4) + c*x^2*(2*p + 5) +
d*x^3*(3*p + 6)),x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x^3 + b*x^4 + c*x^5 + d*x^6)
```

3.236 $\int x(a+bx+cx^2+dx^3)^p (2a+b(3+p)x+c(4+2p$

Optimal result 1593
Rubi [A] (verified) 1593
Mathematica [A] (verified) 1594
Maple [A] (verified) 1594
Fricas [A] (verification not implemented) 1594
Sympy [F(-1)] 1595
Maxima [A] (verification not implemented) 1595
Giac [B] (verification not implemented) 1595
Mupad [B] (verification not implemented) 1596

Optimal result

Integrand size = 49, antiderivative size = 23

$$\int x\left(a+bx+cx^2+dx^3\right)^p \left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx = x^2\left(a+bx+cx^2+dx^3\right)^{1+p}$$

```
[Out] x^2*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {1602}

$$\int x\left(a+bx+cx^2+dx^3\right)^p \left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx = x^2\left(a+bx+cx^2+dx^3\right)^{p+1}$$

```
[In] Int[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^2 + d*
(5 + 3*p)*x^3),x]
```

```
[Out] x^2*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = x^2 \left(a + bx + cx^2 + dx^3\right)^{1+p}$$

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x \left(a + bx + cx^2 + dx^3\right)^p \left(2a + b(3+p)x + c(4+2p)x^2 + d(5+3p)x^3\right) \, dx$$
$$= x^2(a + x(b + x(c + dx)))^{1+p}$$

```
[In] Integrate[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^
2 + d*(5 + 3*p)*x^3),x]
[Out] x^2*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result
gosper	$x^{2} \left(x^{3} d +c \,x^{2}+b x +a \right)^{1+p}$
risch	$\left(x^{3} d +c \,x^{2}+b x +a \right)^{p} x^{2} \left(x^{3} d +c \,x^{2}+b x +a \right)$
norman	$a \,x^{2} {\mathrm e}^{p \ln \left(x^{3} d +c \,x^{2}+b x +a \right)}+b \,x^{3} {\mathrm e}^{p \ln \left(x^{3} d +c \,x^{2}+b x +a \right)}+c \,x^{4} {\mathrm e}^{p \ln \left(x^{3} d +c \,x^{2}+b x +a \right)}+x^{5} d \,{\mathrm e}^{p \ln \left(x^{3} d +c \,x^{2}+b x +a \right.}$
parallelrisch	$\frac{x^{5} \left(x^{3} d +c \,x^{2}+b x +a \right)^{p} a d +x^{4} \left(x^{3} d +c \,x^{2}+b x +a \right)^{p} a c +a b \left(x^{3} d +c \,x^{2}+b x +a \right)^{p} x^{3}+a^{2} \left(x^{3} d +c \,x^{2}+b x +a \right)^{p} x^{2}}{a}$

```
[In] int(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3),x,m
ethod=_RETURNVERBOSE)
[Out] x^2*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x \left(a + bx + cx^2 + dx^3\right)^p \left(2a + b(3+p)x + c(4+2p)x^2 + d(5+3p)x^3\right) \, dx$$
$$= \left(dx^5 + cx^4 + bx^3 + ax^2\right) \left(dx^3 + cx^2 + bx + a\right)^p$$

```
[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^
3),x, algorithm="fricas")
[Out] (d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)]

Timed out.

$$\int x(a+bx+cx^2+dx^3)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx=\text{Timed out}$$

```
[In] integrate(x*(d*x**3+c*x**2+b*x+a)**p*(2*a+b*(3+p)*x+c*(4+2*p)*x**2+d*(5+3*p
)*x**3),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x(a+bx+cx^2+dx^3)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx$$
$$=\left(dx^5+cx^4+bx^3+ax^2\right)\left(dx^3+cx^2+bx+a\right)^p$$

```
[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^
3),x, algorithm="maxima")
```

```
[Out] (d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(23) = 46.

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.87

$$\int x(a+bx+cx^2+dx^3)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx$$
$$=\left(dx^3+cx^2+bx+a\right)^p dx^5+\left(dx^3+cx^2+bx+a\right)^p cx^4$$
$$+\left(dx^3+cx^2+bx+a\right)^p bx^3+\left(dx^3+cx^2+bx+a\right)^p ax^2$$

```
[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^
3),x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^5 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^4 + (d*
x^3 + c*x^2 + b*x + a)^p*b*x^3 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^2
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x(a+bx+cx^2+dx^3)^p\,(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3)\ dx$$
$$=\left(d\,x^3+c\,x^2+b\,x+a\right)^p\,\left(d\,x^5+c\,x^4+b\,x^3+a\,x^2\right)$$

```
[In] int(x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*x*(p + 3) + c*x^2*(2*p + 4) + d*
x^3*(3*p + 5)),x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x^2 + b*x^3 + c*x^4 + d*x^5)
```

3.237 $\int (a+bx+cx^2+dx^3)^p (a+b(2+p)x+c(3+2p)x$

Optimal result 1597
Rubi [A] (verified) 1597
Mathematica [A] (verified) 1598
Maple [A] (verified) 1598
Fricas [A] (verification not implemented) 1598
Sympy [F(-1)] 1599
Maxima [A] (verification not implemented) 1599
Giac [B] (verification not implemented) 1599
Mupad [B] (verification not implemented) 1600

Optimal result

Integrand size = 46, antiderivative size = 21

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right)\,dx = x\left(a+bx+cx^2+dx^3\right)^{1+p}$$

```
[Out] x*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {1602}

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right)\,dx = x\left(a+bx+cx^2+dx^3\right)^{p+1}$$

```
[In] Int[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 + d*(4 +
 3*p)*x^3),x]
```

```
[Out] x*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = x\left(a + bx + cx^2 + dx^3\right)^{1+p}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a + bx + cx^2 + dx^3\right)^p \left(a + b(2+p)x + c(3+2p)x^2 + d(4+3p)x^3\right) dx \\ = x(a + x(b + x(c + dx)))^{1+p}$$

```
[In] Integrate[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 +
d*(4 + 3*p)*x^3),x]
[Out] x*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result
gosper	$x\left(x^3 d + c\,x^2 + bx + a\right)^{1+p}$
risch	$\left(x^3 d + c\,x^2 + bx + a\right)^p x\left(x^3 d + c\,x^2 + bx + a\right)$
norman	$ax\,e^{p\ln\left(x^3 d + c\,x^2 + bx + a\right)} + b\,x^2 e^{p\ln\left(x^3 d + c\,x^2 + bx + a\right)} + c\,x^3 e^{p\ln\left(x^3 d + c\,x^2 + bx + a\right)} + d\,x^4 e^{p\ln\left(x^3 d + c\,x^2 + bx + a\right)}$
parallelrisch	$\frac{x^4\left(x^3 d + c\,x^2 + bx + a\right)^p d^2 + x^3\left(x^3 d + c\,x^2 + bx + a\right)^p cd + x^2\left(x^3 d + c\,x^2 + bx + a\right)^p bd + x\left(x^3 d + c\,x^2 + bx + a\right)^p ad}{d}$

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x,metho
d=_RETURNVERBOSE)
[Out] x*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \left(a + bx + cx^2 + dx^3\right)^p \left(a + b(2+p)x + c(3+2p)x^2 + d(4+3p)x^3\right) dx \\ = \left(dx^4 + cx^3 + bx^2 + ax\right)\left(dx^3 + cx^2 + bx + a\right)^p$$

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x
, algorithm="fricas")
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right) \, dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(a+b*(2+p)*x+c*(3+2*p)*x**2+d*(4+3*p)*x**3),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right) \, dx = \left(dx^4+cx^3+bx^2+ax\right)\left(dx^3+cx^2+bx+a\right)^p$$

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x, algorithm="maxima")
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(21) = 42$.

Time = 0.40 (sec) , antiderivative size = 87, normalized size of antiderivative = 4.14

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right) \, dx = \left(dx^3+cx^2+bx+a\right)^p dx^4 + \left(dx^3+cx^2+bx+a\right)^p cx^3 + \left(dx^3+cx^2+bx+a\right)^p bx^2 + \left(dx^3+cx^2+bx+a\right)^p ax$$

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x, algorithm="giac")
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3 + (d*x^3 + c*x^2 + b*x + a)^p*b*x^2 + (d*x^3 + c*x^2 + b*x + a)^p*a*x
```

Mupad [B] (verification not implemented)

Time = 9.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right)\,dx$$
$$= \left(d\,x^3+c\,x^2+b\,x+a\right)^p\,\left(d\,x^4+c\,x^3+b\,x^2+a\,x\right)$$

```
[In] int((a + b*x + c*x^2 + d*x^3)^p*(a + b*x*(p + 2) + c*x^2*(2*p + 3) + d*x^3*
(3*p + 4)),x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x + b*x^2 + c*x^3 + d*x^4)
```

3.238 $\int \frac{(a+bx+cx^2+dx^3)^p(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx$

Optimal result 1601
Rubi [A] (verified) 1601
Mathematica [A] (verified) 1602
Maple [A] (verified) 1602
Fricas [A] (verification not implemented) 1603
Sympy [F(-1)] 1603
Maxima [A] (verification not implemented) 1603
Giac [B] (verification not implemented) 1604
Mupad [B] (verification not implemented) 1604

Optimal result

Integrand size = 48, antiderivative size = 19

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = \left(a+bx+cx^2+dx^3\right)^{1+p}$$

```
[Out] (d*x^3+c*x^2+b*x+a)^(p+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1599, 1602}

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = \left(a+bx+cx^2+dx^3\right)^{p+1}$$

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(3 + 3*
p)*x^3))/x,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(b(1+p)+c(2+2p)x+d(3+3p)x^2\right)\left(a+bx+cx^2+dx^3\right)^p\,dx\\ &= \left(a+bx+cx^2+dx^3\right)^{1+p}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = (a+x(b+x(c+dx)))^{1+p}$$

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(
3 + 3*p)*x^3))/x,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result
gosper	$(x^3d+c\,x^2+bx+a)^{1+p}$
risch	$(x^3d+c\,x^2+bx+a)^p\,(x^3d+c\,x^2+bx+a)$
norman	$a\,e^{p\ln(x^3d+c\,x^2+bx+a)}+bx\,e^{p\ln(x^3d+c\,x^2+bx+a)}+c\,x^2e^{p\ln(x^3d+c\,x^2+bx+a)}+x^3d\,e^{p\ln(x^3d+c\,x^2+bx+a)}$
parallelrisch	$\frac{x^3(x^3d+c\,x^2+bx+a)^pd^2+x^2(x^3d+c\,x^2+bx+a)^pcd+x(x^3d+c\,x^2+bx+a)^pbd+(x^3d+c\,x^2+bx+a)^pad}{d}$

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x,metho
d=_RETURNVERBOSE)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = (dx^3+cx^2+bx+a)\,(dx^3+cx^2+bx+a)^p$$

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x
, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(b*(1+p)*x+c*(2+2*p)*x**2+d*(3+3*p)*x**3
)/x,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = (dx^3+cx^2+bx+a)\,(dx^3+cx^2+bx+a)^p$$

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x
, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(19) = 38$.

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.74

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = \frac{(dx^3+cx^2+bx+a)^{p+1}p}{p+1} + \frac{(dx^3+cx^2+bx+a)^{p+1}}{p+1}$$

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x
, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^(p + 1)*p/(p + 1) + (d*x^3 + c*x^2 + b*x + a)^(p
+ 1)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = \left(d\,x^3+c\,x^2+b\,x+a\right)^{p+1}$$

```
[In] int(((b*x*(p + 1) + c*x^2*(2*p + 2) + d*x^3*(3*p + 3))*(a + b*x + c*x^2 + d
*x^3)^p)/x,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)
```

3.239 $\int \frac{(a+bx+cx^2+dx^3)^p(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx$

Optimal result 1605
Rubi [A] (verified) 1605
Mathematica [A] (verified) 1606
Maple [A] (verified) 1606
Fricas [A] (verification not implemented) 1607
Sympy [F(-1)] 1607
Maxima [A] (verification not implemented) 1607
Giac [F] 1608
Mupad [B] (verification not implemented) 1608

Optimal result

Integrand size = 49, antiderivative size = 23

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x}$$

```
[Out] (d*x^3+c*x^2+b*x+a)^(p+1)/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {1604}

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{(a+bx+cx^2+dx^3)^{p+1}}{x}$$

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2 + 3*p
)*x^3))/x^2,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
```

```
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{(a+x(b+x(c+dx)))^{1+p}}{x}$$

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2
 + 3*p)*x^3))/x^2,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{\left(x^{3} d+c\,x^{2}+b x+a\right)^{1+p}}{x}$	24
risch	$\frac{\left(x^{3} d+c\,x^{2}+b x+a\right)\left(x^{3} d+c\,x^{2}+b x+a\right)^{p}}{x}$	37
norman	$\frac{a\,{\mathrm e}^{p \ln\left(x^{3} d+c\,x^{2}+b x+a\right)}+b x\,{\mathrm e}^{p \ln\left(x^{3} d+c\,x^{2}+b x+a\right)}+c\,x^{2} {\mathrm e}^{p \ln\left(x^{3} d+c\,x^{2}+b x+a\right)}+x^{3} d\,{\mathrm e}^{p \ln\left(x^{3} d+c\,x^{2}+b x+a\right)}}{x}$	97
parallelrisch	$\frac{x^{3} \left(x^{3} d+c\,x^{2}+b x+a\right)^{p} d^{2}+x^{2} \left(x^{3} d+c\,x^{2}+b x+a\right)^{p} c d+x \left(x^{3} d+c\,x^{2}+b x+a\right)^{p} b d+\left(x^{3} d+c\,x^{2}+b x+a\right)^{p} a d}{d x}$	97

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,x,meth
od=_RETURNVERBOSE)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x}$$

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,
x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-a+b*p*x+c*(1+2*p)*x**2+d*(2+3*p)*x**3)
/x**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x}$$

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,
x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x
```

Giac [F]

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \int \frac{(d(3\,p+2)x^3+c(2\,p+1)x^2+bpx-a)(dx^3+cx^2+bx+a)^p}{x^2}\,dx$$

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,
x, algorithm="giac")
```

```
[Out] integrate((d*(3*p + 2)*x^3 + c*(2*p + 1)*x^2 + b*p*x - a)*(d*x^3 + c*x^2 +
b*x + a)^p/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{(d\,x^3+c\,x^2+b\,x+a)^{p+1}}{x}$$

```
[In] int(((a + b*x + c*x^2 + d*x^3)^p*(b*p*x - a + c*x^2*(2*p + 1) + d*x^3*(3*p
+ 2)))/x^2,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)/x
```

3.240 $\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx$

Optimal result 1609
Rubi [A] (verified) 1609
Mathematica [A] (verified) 1610
Maple [A] (verified) 1610
Fricas [A] (verification not implemented) 1611
Sympy [F(-1)] 1611
Maxima [A] (verification not implemented) 1611
Giac [F] 1612
Mupad [B] (verification not implemented) 1612

Optimal result

Integrand size = 48, antiderivative size = 23

$$\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^2}$$

```
[Out] (d*x^3+c*x^2+b*x+a)^(p+1)/x^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {1604}

$$\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(a+bx+cx^2+dx^3)^{p+1}}{x^2}$$

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d*(1 +
3*p)*x^3))/x^3,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^2
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
```

```
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^2}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(a+x(b+x(c+dx)))^{1+p}}{x^2}$$

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d
*(1 + 3*p)*x^3))/x^3,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x^2
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{(x^3d+c\,x^2+bx+a)^{1+p}}{x^2}$	24
risch	$\frac{(x^3d+c\,x^2+bx+a)\,(x^3d+c\,x^2+bx+a)^p}{x^2}$	37
norman	$\frac{a\,\mathrm{e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}+bx\,\mathrm{e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}+c\,x^2\mathrm{e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}+x^3d\,\mathrm{e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}}{x^2}$	97
parallelrisch	$\frac{x^3\,(x^3d+c\,x^2+bx+a)^p\,cd+x^2\,(x^3d+c\,x^2+bx+a)^p\,c^2+x\,(x^3d+c\,x^2+bx+a)^p\,bc+(x^3d+c\,x^2+bx+a)^p\,ac}{x^2c}$	97

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x^3,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^2}$$

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x
^3,x, algorithm="fricas")
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-2*a+b*(-1+p)*x+2*c*p*x**2+d*(1+3*p)*x*
*3)/x**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^2}$$

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x
^3,x, algorithm="maxima")
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2
```

Giac [F]

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx$$
$$=\int \frac{(d(3\,p+1)x^3+2\,cpx^2+b(p-1)x-2\,a)(dx^3+cx^2+bx+a)^p}{x^3}\,dx$$

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x
^3,x, algorithm="giac")
```

```
[Out] integrate((d*(3*p + 1)*x^3 + 2*c*p*x^2 + b*(p - 1)*x - 2*a)*(d*x^3 + c*x^2
+ b*x + a)^p/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx$$
$$=\frac{(d\,x^3+c\,x^2+b\,x+a)^{p+1}}{x^2}$$

```
[In] int(((a + b*x + c*x^2 + d*x^3)^p*(b*x*(p - 1) - 2*a + 2*c*p*x^2 + d*x^3*(3*
p + 1)))/x^3,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)/x^2
```

3.241 $\int \frac{(a+bx+cx^2+dx^3)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4} dx$

Optimal result . . . 1613
Rubi [A] (verified) . . . 1613
Mathematica [A] (verified) . . . 1614
Maple [A] (verified) . . . 1614
Fricas [A] (verification not implemented) . . . 1615
Sympy [F(-1)] . . . 1615
Maxima [A] (verification not implemented) . . . 1615
Giac [F] . . . 1616
Mupad [B] (verification not implemented) . . . 1616

Optimal result

Integrand size = 48, antiderivative size = 23

$$\int \frac{(a+bx+cx^2+dx^3)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4} dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^3}$$

```
[Out] (d*x^3+c*x^2+b*x+a)^(p+1)/x^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {1604}

$$\int \frac{(a+bx+cx^2+dx^3)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4} dx = \frac{(a+bx+cx^2+dx^3)^{p+1}}{x^3}$$

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*x^2 +
3*d*p*x^3))/x^4,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^3
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
```

```
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^3}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{(a+x(b+x(c+dx)))^{1+p}}{x^3}$$

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*
x^2 + 3*d*p*x^3))/x^4,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x^3
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{(x^3d+c\,x^2+bx+a)^{1+p}}{x^3}$	24
risch	$\frac{(x^3d+c\,x^2+bx+a)\,(x^3d+c\,x^2+bx+a)^p}{x^3}$	37
norman	$\frac{a\,\mathrm{e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}+bx\,\mathrm{e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}+c\,x^2\mathrm{e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}+x^3d\,\mathrm{e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}}{x^3}$	97
parallelrisch	$\frac{(x^3d+c\,x^2+bx+a)^p\,ad\,x^3+(x^3d+c\,x^2+bx+a)^p\,ac\,x^2+(x^3d+c\,x^2+bx+a)^p\,abx+(x^3d+c\,x^2+bx+a)^p\,a^2}{x^3a}$	97

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x,
method=_RETURNVERBOSE)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^3}$$

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/
x^4,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x**2+3*d*p*x
**3)/x**4,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^3}$$

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/
x^4,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3
```

Giac [F]

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx$$
$$=\int \frac{(3\,dpx^3+c(2\,p-1)x^2+b(p-2)x-3\,a)(dx^3+cx^2+bx+a)^p}{x^4}\,dx$$

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x, algorithm="giac")

[Out] integrate((3*d*p*x^3 + c*(2*p - 1)*x^2 + b*(p - 2)*x - 3*a)*(d*x^3 + c*x^2 + b*x + a)^p/x^4, x)

Mupad [B] (verification not implemented)

Time = 9.86 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx$$
$$=\frac{{\left(d\,x^3+c\,x^2+b\,x+a\right)}^{p+1}}{x^3}$$

[In] int(((a + b*x + c*x^2 + d*x^3)^p*(b*x*(p - 2) - 3*a + 3*d*p*x^3 + c*x^2*(2*p - 1)))/x^4,x)

[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)/x^3

3.242 $\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal result 1617
Rubi [A] (verified) 1617
Mathematica [A] (verified) 1619
Maple [A] (verified) 1619
Fricas [A] (verification not implemented) 1620
Sympy [A] (verification not implemented) 1620
Maxima [A] (verification not implemented) 1620
Giac [A] (verification not implemented) 1621
Mupad [B] (verification not implemented) 1621

Optimal result

Integrand size = 35, antiderivative size = 97

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{1}{24}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right) -\frac{10\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(1+x+x^2\right)-\frac{13}{48}\log\left(2-x+2x^2\right)$$

```
[Out] 5/4*x-3/4*x^2+1/3*x^3+1/4*x^4+1/3*ln(x^2+x+1)-13/48*ln(2*x^2-x+2)+1/72*arct
an(1/15*(1-4*x)*15^(1/2))*15^(1/2)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2100, 648, 632, 210, 642}

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{24}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{x^4}{4}+\frac{x^3}{3} -\frac{3x^2}{4}+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{5x}{4}$$

```
[In] Int[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (5*x)/4 - (3*x^2)/4 + x^3/3 + x^4/4 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]]
)/24 - (10*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 - (1
3*Log[2 - x + 2*x^2])/48
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2100

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{5}{4}-\frac{3x}{2}+x^2+x^3+\frac{2(-2+x)}{3\left(1+x+x^2\right)}+\frac{2-13x}{12\left(2-x+2x^2\right)}\right)dx \\
&= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{1}{12}\int \frac{2-13x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{-2+x}{1+x+x^2}\,dx \\
&= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}-\frac{5}{48}\int \frac{1}{2-x+2x^2}\,dx \\
&\quad -\frac{13}{48}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{5}{3}\int \frac{1}{1+x+x^2}\,dx \\
&= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{1}{3}\log\left(1+x+x^2\right)-\frac{13}{48}\log\left(2-x+2x^2\right) \\
&\quad +\frac{5}{24}\text{Subst}\left(\int \frac{1}{-15-x^2}\,dx,x,-1+4x\right)+\frac{10}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)
\end{aligned}$$

$$= \frac{5x}{4} - \frac{3x^2}{4} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)$$
$$- \frac{10\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{3}\log\left(1+x+x^2\right) - \frac{13}{48}\log\left(2-x+2x^2\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{144}\left(180x - 108x^2 + 48x^3 + 36x^4 - 160\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 2\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right) + 48\log\left(1+x+x^2\right) - 39\log\left(2-x+2x^2\right)\right)$$

```
[In] Integrate[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (180*x - 108*x^2 + 48*x^3 + 36*x^4 - 160*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]
- 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x + x^2] - 39*Log[2 -
 x + 2*x^2])/144
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

method	result
default	$\frac{x^4}{4} + \frac{x^3}{3} - \frac{3x^2}{4} + \frac{5x}{4} + \frac{\ln(x^2+x+1)}{3} - \frac{10\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9} - \frac{13\ln(2x^2-x+2)}{48} - \frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{72}$
risch	$\frac{x^4}{4} + \frac{x^3}{3} - \frac{3x^2}{4} + \frac{5x}{4} + \frac{\ln(4x^2+4x+4)}{3} - \frac{10\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9} - \frac{13\ln(16x^2-8x+16)}{48} - \frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{72}$

```
[In] int(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4+1/3*x^3-3/4*x^2+5/4*x+1/3*ln(x^2+x+1)-10/9*arctan(1/3*(1+2*x)*3^(1/
2))*3^(1/2)-13/48*ln(2*x^2-x+2)-1/72*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{4}\,x^4 + \frac{1}{3}\,x^3 - \frac{3}{4}\,x^2 - \frac{1}{72}\,\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\,\sqrt{5}\sqrt{3}(4\,x-1)\right) - \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{5}{4}\,x - \frac{13}{48}\,\log\left(2\,x^2-x+2\right) + \frac{1}{3}\,\log\left(x^2+x+1\right)$$

```
[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt
(3)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48
*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{x^4}{4} + \frac{x^3}{3} - \frac{3x^2}{4} + \frac{5x}{4} - \frac{13\log\left(x^2-\frac{x}{2}+1\right)}{48} + \frac{\log\left(x^2+x+1\right)}{3} - \frac{\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{72} - \frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate(x**4*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x**4/4 + x**3/3 - 3*x**2/4 + 5*x/4 - 13*log(x**2 - x/2 + 1)/48 + log(x**2 +
 x + 1)/3 - sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/72 - 10*sqrt(3)*at
an(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{4}\,x^4 + \frac{1}{3}\,x^3 - \frac{3}{4}\,x^2 - \frac{1}{72}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) - \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{5}{4}\,x - \frac{13}{48}\,\log\left(2\,x^2-x+2\right) + \frac{1}{3}\,\log\left(x^2+x+1\right)$$

```
[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1))
 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2 - x
 + 2) + 1/3*log(x^2 + x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{4}\,x^4+\frac{1}{3}\,x^3-\frac{3}{4}\,x^2-\frac{1}{72}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) - \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{5}{4}\,x - \frac{13}{48}\,\log\left(2\,x^2-x+2\right)+\frac{1}{3}\,\log\left(x^2+x+1\right)$$

```
[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1))
 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2 - x
 + 2) + 1/3*log(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{5\,x}{4}+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) + \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\,\left(-\frac{13}{48}+\frac{\sqrt{15}\,1\mathrm{i}}{144}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\,\left(\frac{13}{48}+\frac{\sqrt{15}\,1\mathrm{i}}{144}\right)-\frac{3\,x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}$$

```
[In] int((x^4*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] (5*x)/4 + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - log(x + (3
^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) + log(x - (15^(1/2)*1i)/4 - 1/4)
*((15^(1/2)*1i)/144 - 13/48) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i
)/144 + 13/48) - (3*x^2)/4 + x^3/3 + x^4/4
```

3.243 $\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal result 1622
Rubi [A] (verified) 1622
Mathematica [A] (verified) 1624
Maple [A] (verified) 1624
Fricas [A] (verification not implemented) 1625
Sympy [A] (verification not implemented) 1625
Maxima [A] (verification not implemented) 1625
Giac [A] (verification not implemented) 1626
Mupad [B] (verification not implemented) 1626

Optimal result

Integrand size = 35, antiderivative size = 90

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{5}{12}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{24}\log\left(2-x+2x^2\right)$$

```
[Out] -3/2*x+1/2*x^2+1/3*x^3+2/3*ln(x^2+x+1)-1/24*ln(2*x^2-x+2)+5/36*arctan(1/15*
(1-4*x)*15^(1/2))*15^(1/2)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2100, 648, 632, 210, 642}

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{5}{12}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{x^3}{3}+\frac{x^2}{2}+\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{3x}{2}$$

```
[In] Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (-3*x)/2 + x^2/2 + x^3/3 + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 + (8
*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 -
x + 2*x^2]/24
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2100

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{3}{2}+x+x^2+\frac{2(3+2x)}{3\left(1+x+x^2\right)}+\frac{-6-x}{6\left(2-x+2x^2\right)}\right) dx \\
&= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{1}{6}\int \frac{-6-x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{3+2x}{1+x+x^2}\,dx \\
&= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}-\frac{1}{24}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{1+2x}{1+x+x^2}\,dx \\
&\quad -\frac{25}{24}\int \frac{1}{2-x+2x^2}\,dx+\frac{4}{3}\int \frac{1}{1+x+x^2}\,dx \\
&= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{24}\log\left(2-x+2x^2\right) \\
&\quad +\frac{25}{12}\text{Subst}\left(\int \frac{1}{-15-x^2}\,dx,x,-1+4x\right)-\frac{8}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)
\end{aligned}$$

$$= -\frac{3x}{2} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{8\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{2}{3}\log\left(1+x+x^2\right) - \frac{1}{24}\log\left(2-x+2x^2\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{72}\left(-108x+36x^2+24x^3+64\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 10\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right) + 48\log\left(1+x+x^2\right) - 3\log\left(2-x+2x^2\right)\right)$$

```
[In] Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
[Out] (-108*x + 36*x^2 + 24*x^3 + 64*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 10*Sqrt[
15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x + x^2] - 3*Log[2 - x + 2*x^2
])/72
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{x^3}{3}+\frac{x^2}{2}-\frac{3x}{2}+\frac{2\ln(x^2+x+1)}{3}+\frac{8\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{\ln(2x^2-x+2)}{24}-\frac{5\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{36}$	69
risch	$\frac{x^3}{3}+\frac{x^2}{2}-\frac{3x}{2}-\frac{\ln(16x^2-8x+16)}{24}-\frac{5\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{36}+\frac{2\ln(4x^2+4x+4)}{3}+\frac{8\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}$	73

```
[In] int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3+1/2*x^2-3/2*x+2/3*ln(x^2+x+1)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/
2)-1/24*ln(2*x^2-x+2)-5/36*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.82

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{3}\,x^3+\frac{1}{2}\,x^2-\frac{5}{36}\,\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\,\sqrt{5}\sqrt{3}(4\,x-1)\right)+\frac{8}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{3}{2}\,x-\frac{1}{24}\,\log\left(2\,x^2-x+2\right)+\frac{2}{3}\,\log\left(x^2+x+1\right)$$

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x -
 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 -
 x + 2) + 2/3*log(x^2 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{x^3}{3}+\frac{x^2}{2}-\frac{3x}{2}-\frac{\log\left(x^2-\frac{x}{2}+1\right)}{24}+\frac{2\log\left(x^2+x+1\right)}{3}-\frac{5\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{36}+\frac{8\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x**3/3 + x**2/2 - 3*x/2 - log(x**2 - x/2 + 1)/24 + 2*log(x**2 + x + 1)/3 -
5*sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/36 + 8*sqrt(3)*atan(2*sqrt(3
)*x/3 + sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{3}\,x^3+\frac{1}{2}\,x^2-\frac{5}{36}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right)+\frac{8}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{3}{2}\,x-\frac{1}{24}\,\log\left(2\,x^2-x+2\right)+\frac{2}{3}\,\log\left(x^2+x+1\right)$$

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqr
t(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2/3*
log(x^2 + x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{3}\,x^3+\frac{1}{2}\,x^2-\frac{5}{36}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right)+\frac{8}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{3}{2}\,x-\frac{1}{24}\,\log\left(2\,x^2-x+2\right)+\frac{2}{3}\,\log\left(x^2+x+1\right)$$

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqr
t(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2/3*
log(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{x^2}{2}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{24}+\frac{\sqrt{15}\,5\mathrm{i}}{72}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{24}+\frac{\sqrt{15}\,5\mathrm{i}}{72}\right)-\frac{3\,x}{2}+\frac{x^3}{3}$$

```
[In] int((x^3*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) - log(x - (3^(1/2)*1i)
/2 + 1/2)*((3^(1/2)*4i)/9 - 2/3) - (3*x)/2 + log(x - (15^(1/2)*1i)/4 - 1/4)
*((15^(1/2)*5i)/72 - 1/24) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*5i)/
72 + 1/24) + x^2/2 + x^3/3
```

3.244 $\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} \, dx$

Optimal result 1627
Rubi [A] (verified) 1627
Mathematica [A] (verified) 1629
Maple [A] (verified) 1629
Fricas [A] (verification not implemented) 1629
Sympy [A] (verification not implemented) 1630
Maxima [A] (verification not implemented) 1630
Giac [A] (verification not implemented) 1631
Mupad [B] (verification not implemented) 1631

Optimal result

Integrand size = 35, antiderivative size = 77

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} \, dx = x+\frac{x^2}{2}+\frac{1}{6}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} -\log\left(1+x+x^2\right)+\frac{1}{4}\log\left(2-x+2x^2\right)$$

```
[Out] x+1/2*x^2-ln(x^2+x+1)+1/4*ln(2*x^2-x+2)+1/18*arctan(1/15*(1-4*x)*15^(1/2))*
15^(1/2)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2100, 648, 632, 210, 642}

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} \, dx = \frac{1}{6}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{x^2}{2} -\log\left(x^2+x+1\right)+\frac{1}{4}\log\left(2x^2-x+2\right)+x$$

```
[In] Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x + x^2/2 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/
Sqrt[3]])/(3*Sqrt[3]) - Log[1 + x + x^2] + Log[2 - x + 2*x^2]/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2100

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(1+x-\frac{2(1+3x)}{3\left(1+x+x^2\right)}+\frac{-2+3x}{3\left(2-x+2x^2\right)}\right) dx \\
&= x+\frac{x^2}{2}+\frac{1}{3}\int \frac{-2+3x}{2-x+2x^2}\, dx-\frac{2}{3}\int \frac{1+3x}{1+x+x^2}\, dx \\
&= x+\frac{x^2}{2}+\frac{1}{4}\int \frac{-1+4x}{2-x+2x^2}\, dx+\frac{1}{3}\int \frac{1}{1+x+x^2}\, dx \\
&\quad -\frac{5}{12}\int \frac{1}{2-x+2x^2}\, dx-\int \frac{1+2x}{1+x+x^2}\, dx \\
&= x+\frac{x^2}{2}-\log\left(1+x+x^2\right)+\frac{1}{4}\log\left(2-x+2x^2\right) \\
&\quad -\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\, dx,x,1+2x\right)+\frac{5}{6}\text{Subst}\left(\int \frac{1}{-15-x^2}\, dx,x,-1+4x\right) \\
&= x+\frac{x^2}{2}+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\log\left(1+x+x^2\right)+\frac{1}{4}\log\left(2-x\right. \\
&\quad \left. +2x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{36}\left(8\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-2\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right)+9\left(2x(2+x)-4\log\left(1+x+x^2\right)+\log\left(2-x+2x^2\right)\right)\right)$$

```
[In] Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
[Out] (8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15
]] + 9*(2*x*(2 + x) - 4*Log[1 + x + x^2] + Log[2 - x + 2*x^2]))/36
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x^2}{2}+x-\ln\left(x^2+x+1\right)+\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{\ln(2x^2-x+2)}{4}-\frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{18}$	62
risch	$\frac{x^2}{2}+x+\frac{\ln(16x^2-8x+16)}{4}-\frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{18}-\ln\left(4x^2+4x+4\right)+\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}$	66

```
[In] int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+x-ln(x^2+x+1)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/4*ln(2*x^2-
x+2)-1/18*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{2}\,x^2-\frac{1}{18}\,\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\,\sqrt{5}\sqrt{3}(4\,x-1)\right)+\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+x+\frac{1}{4}\,\log\left(2\,x^2-x+2\right)-\log\left(x^2+x+1\right)$$

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/2*x^2 - 1/18*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 2/9
*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x
^2 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{x^2}{2}+x+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{4}-\log\left(x^2+x+1\right) - \frac{\sqrt{15}\,\text{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{18}+\frac{2\sqrt{3}\,\text{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x**2/2 + x + log(x**2 - x/2 + 1)/4 - log(x**2 + x + 1) - sqrt(15)*atan(4*sq
rt(15)*x/15 - sqrt(15)/15)/18 + 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{2}\,x^2-\frac{1}{18}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) + \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+x + \frac{1}{4}\,\log\left(2\,x^2-x+2\right)-\log\left(x^2+x+1\right)$$

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{2}\,x^2 - \frac{1}{18}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) + \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + x + \frac{1}{4}\,\log\left(2\,x^2-x+2\right) - \log\left(x^2+x+1\right)$$

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = x - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(1+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-1+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right) + \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{36}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{36}\right) + \frac{x^2}{2}$$

```
[In] int((x^2*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] x - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 + 1) + log(x + (3^(1/2)*1
i)/2 + 1/2)*((3^(1/2)*1i)/9 - 1) + log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2
)*1i)/36 + 1/4) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/36 - 1/4) +
 x^2/2
```

3.245 $\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} \, dx$

Optimal result 1632
Rubi [A] (verified) 1632
Mathematica [A] (verified) 1634
Maple [A] (verified) 1634
Fricas [A] (verification not implemented) 1634
Sympy [A] (verification not implemented) 1635
Maxima [A] (verification not implemented) 1635
Giac [A] (verification not implemented) 1635
Mupad [B] (verification not implemented) 1636

Optimal result

Integrand size = 33, antiderivative size = 72

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} \, dx = x - \frac{1}{3}\sqrt{\frac{5}{3}} \arctan\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{3}\log\left(1+x+x^2\right) + \frac{1}{6}\log\left(2-x+2x^2\right)$$

```
[Out] x+1/3*ln(x^2+x+1)+1/6*ln(2*x^2-x+2)-1/9*arctan(1/15*(1-4*x)*15^(1/2))*15^(1
/2)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2100, 648, 632, 210, 642}

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} \, dx = -\frac{1}{3}\sqrt{\frac{5}{3}} \arctan\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{3}\log\left(x^2+x+1\right) + \frac{1}{6}\log\left(2x^2-x+2\right) + x$$

```
[In] Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x - (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 - (10*ArcTan[(1 + 2*x)/Sqrt[3]
])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 + Log[2 - x + 2*x^2]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2100

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(1+\frac{2(-2+x)}{3\left(1+x+x^2\right)}+\frac{2(1+x)}{3\left(2-x+2x^2\right)}\right) dx \\
&= x+\frac{2}{3}\int \frac{-2+x}{1+x+x^2}\,dx+\frac{2}{3}\int \frac{1+x}{2-x+2x^2}\,dx \\
&= x+\frac{1}{6}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{5}{6}\int \frac{1}{2-x+2x^2}\,dx-\frac{5}{3}\int \frac{1}{1+x+x^2}\,dx \\
&= x+\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{6}\log\left(2-x+2x^2\right) \\
&\quad -\frac{5}{3}\text{Subst}\left(\int \frac{1}{-15-x^2}\,dx,x,-1+4x\right)+\frac{10}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right) \\
&= x-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{6}\log\left(2-x+2x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{18}\left(-20\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+2\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right)+3\left(6x+2\log\left(1+x+x^2\right)+\log\left(2-x+2x^2\right)\right)\right)$$

```
[In] Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
[Out] (-20*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[
15]] + 3*(6*x + 2*Log[1 + x + x^2] + Log[2 - x + 2*x^2]))/18
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

method	result	size
default	$x+\frac{\ln(x^2+x+1)}{3}-\frac{10\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{\ln(2x^2-x+2)}{6}+\frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{9}$	57
risch	$x+\frac{\ln(4x^2+4x+4)}{3}-\frac{10\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{\ln(16x^2-8x+16)}{6}+\frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{9}$	61

```
[In] int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
[Out] x+1/3*ln(x^2+x+1)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/6*ln(2*x^2-x+2
)+1/9*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{9}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
[Out] 1/9*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 10/9*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x
 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = x+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{6}+\frac{\log\left(x^2+x+1\right)}{3}+\frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{9}-\frac{10\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
[Out] x + log(x**2 - x/2 + 1)/6 + log(x**2 + x + 1)/3 + sqrt(15)*atan(4*sqrt(15)*
x/15 - sqrt(15)/15)/9 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = x+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) - \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{6}+\frac{\sqrt{15}\,1\mathrm{i}}{18}\right) + \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{6}+\frac{\sqrt{15}\,1\mathrm{i}}{18}\right)$$

```
[In] int((x*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] x + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - log(x + (3^(1/2)
*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) - log(x - (15^(1/2)*1i)/4 - 1/4)*((15^
(1/2)*1i)/18 - 1/6) + log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/18 + 1/
6)
```

3.246 $\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal result 1637
Rubi [A] (verified) 1637
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Mupad [B] (verification not implemented) 1641

Optimal result

Integrand size = 32, antiderivative size = 71

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = -\frac{1}{3}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} +\frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{6}\log\left(2-x+2x^2\right)$$

```
[Out] 2/3*ln(x^2+x+1)-1/6*ln(2*x^2-x+2)-1/9*arctan(1/15*(1-4*x)*15^(1/2))*15^(1/2
)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2099, 648, 632, 210, 642}

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = -\frac{1}{3}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}} +\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{6}\log\left(2x^2-x+2\right)$$

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] -1/3*(Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]]) + (8*ArcTan[(1 + 2*x)/Sqrt[3]])
/(3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 - x + 2*x^2]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{2(3+2x)}{3(1+x+x^2)}+\frac{3-2x}{3(2-x+2x^2)}\right) dx\\
&= \frac{1}{3}\int \frac{3-2x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{3+2x}{1+x+x^2}\,dx\\
&= -\left(\frac{1}{6}\int \frac{-1+4x}{2-x+2x^2}\,dx\right)+\frac{2}{3}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{5}{6}\int \frac{1}{2-x+2x^2}\,dx+\frac{4}{3}\int \frac{1}{1+x+x^2}\,dx\\
&= \frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{6}\log\left(2-x+2x^2\right)\\
&\quad -\frac{5}{3}\text{Subst}\left(\int \frac{1}{-15-x^2}\,dx,x,-1+4x\right)-\frac{8}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= -\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{6}\log\left(2-x+2x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{18}\left(16\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+2\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right)+12\log\left(1+x+x^2\right)-3\log\left(2-x+2x^2\right)\right)$$

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
[Out] (16*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[1
5]] + 12*Log[1 + x + x^2] - 3*Log[2 - x + 2*x^2])/18
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{2\ln(x^2+x+1)}{3}+\frac{8\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{\ln(2x^2-x+2)}{6}+\frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{9}$	56
risch	$\frac{2\ln(4x^2+4x+4)}{3}+\frac{8\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{\ln(16x^2-8x+16)}{6}+\frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{9}$	60

```
[In] int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
[Out] 2/3*ln(x^2+x+1)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/6*ln(2*x^2-x+2)+1
/9*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{9}\,\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\,\sqrt{5}\sqrt{3}(4\,x-1)\right)+\frac{8}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{1}{6}\,\log\left(2\,x^2-x+2\right)+\frac{2}{3}\,\log\left(x^2+x+1\right)$$

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
[Out] 1/9*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 8/9*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = -\frac{\log\left(x^2-\frac{x}{2}+1\right)}{6}+\frac{2\log\left(x^2+x+1\right)}{3} + \frac{\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{9}+\frac{8\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
[Out] -log(x**2 - x/2 + 1)/6 + 2*log(x**2 + x + 1)/3 + sqrt(15)*atan(4*sqrt(15)*x
/15 - sqrt(15)/15)/9 + 8*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.11

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = -\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,4i}{9}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,4i}{9}\right) - \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1i}{4}\right)\left(\frac{1}{6}+\frac{\sqrt{15}\,1i}{18}\right) + \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1i}{4}\right)\left(-\frac{1}{6}+\frac{\sqrt{15}\,1i}{18}\right)$$

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) - log(x - (3^(1/2)*1i)
/2 + 1/2)*((3^(1/2)*4i)/9 - 2/3) - log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2
)*1i)/18 + 1/6) + log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/18 - 1/6)
```

3.247 $\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)} \, dx$

Optimal result 1642
Rubi [A] (verified) 1642
Mathematica [A] (verified) 1644
Maple [A] (verified) 1644
Fricas [A] (verification not implemented) 1645
Sympy [A] (verification not implemented) 1645
Maxima [A] (verification not implemented) 1645
Giac [A] (verification not implemented) 1646
Mupad [B] (verification not implemented) 1646

Optimal result

Integrand size = 35, antiderivative size = 75

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)} \, dx = \frac{1}{6}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} +\frac{5\log(x)}{2}-\log\left(1+x+x^2\right)-\frac{1}{4}\log\left(2-x+2x^2\right)$$

```
[Out] 5/2*ln(x)-ln(x^2+x+1)-1/4*ln(2*x^2-x+2)+1/18*arctan(1/15*(1-4*x)*15^(1/2))*
15^(1/2)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2112, 814, 648, 632, 210, 642}

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)} \, dx = \frac{1}{6}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}} -\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(2x^2-x+2\right)+\frac{5\log(x)}{2}$$

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(3
*Sqrt[3]) + (5*Log[x])/2 - Log[1 + x + x^2] - Log[2 - x + 2*x^2]/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2112

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[x^m*((b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x)/(2*a + (b + q)*x + 2*a*x^2)), x], x] - Dist[1/q, Int[x^m*((b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2)),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{3}\int \frac{-6+4x}{x\,(4-2x+4x^2)}\,dx\right)+\frac{1}{3}\int \frac{24+16x}{x\,(4+4x+4x^2)}\,dx \\
&= \frac{1}{3}\int \left(\frac{6}{x}-\frac{2(1+3x)}{1+x+x^2}\right)\,dx-\frac{1}{3}\int \left(-\frac{3}{2x}+\frac{1+6x}{2\,(2-x+2x^2)}\right)\,dx
\end{aligned}$$

$$= \frac{5\log(x)}{2} - \frac{1}{6}\int \frac{1+6x}{2-x+2x^2}\,dx - \frac{2}{3}\int \frac{1+3x}{1+x+x^2}\,dx$$

$$= \frac{5\log(x)}{2} - \frac{1}{4}\int \frac{-1+4x}{2-x+2x^2}\,dx + \frac{1}{3}\int \frac{1}{1+x+x^2}\,dx - \frac{5}{12}\int \frac{1}{2-x+2x^2}\,dx - \int \frac{1+2x}{1+x+x^2}\,dx$$

$$= \frac{5\log(x)}{2} - \log\left(1+x+x^2\right) - \frac{1}{4}\log\left(2-x+2x^2\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right) + \frac{5}{6}\text{Subst}\left(\int \frac{1}{-15-x^2}\,dx, x, -1+4x\right)$$

$$= \frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{5\log(x)}{2} - \log\left(1+x+x^2\right) - \frac{1}{4}\log\left(2-x+2x^2\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)}\,dx = \frac{1}{36}\left(8\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 2\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right) + 90\log(x) - 36\log\left(1+x+x^2\right) - 9\log\left(2-x+2x^2\right)\right)$$

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
[Out] (8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15
]] + 90*Log[x] - 36*Log[1 + x + x^2] - 9*Log[2 - x + 2*x^2])/36
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{5\ln(x)}{2} - \ln\left(x^2+x+1\right) + \frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9} - \frac{\ln(2x^2-x+2)}{4} - \frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{18}$	60
risch	$-\ln\left(4x^2+4x+4\right) + \frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9} - \frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{18} - \frac{\ln(16x^2-8x+16)}{4} + \frac{5\ln(x)}{2}$	64

```
[In] int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
[Out] 5/2*ln(x)-ln(x^2+x+1)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/4*ln(2*x^2-
x+2)-1/18*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int \frac{5+x+3x^2+2x^3}{x\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\frac{1}{18}\,\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\,\sqrt{5}\sqrt{3}(4\,x-1)\right) + \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{4}\,\log\left(2\,x^2-x+2\right) - \log\left(x^2+x+1\right) + \frac{5}{2}\,\log\left(x\right)$$

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 2/9*sqrt(3)*
arctan(1/3*sqrt(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) +
 5/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

$$\int \frac{5+x+3x^2+2x^3}{x\left(2+x+3x^2+x^3+2x^4\right)}\,dx = \frac{5\log\left(x\right)}{2} - \frac{\log\left(x^2-\frac{x}{2}+1\right)}{4} - \log\left(x^2+x+1\right) - \frac{\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{18} + \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] 5*log(x)/2 - log(x**2 - x/2 + 1)/4 - log(x**2 + x + 1) - sqrt(15)*atan(4*sq
rt(15)*x/15 - sqrt(15)/15)/18 + 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int \frac{5+x+3x^2+2x^3}{x\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\frac{1}{18}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) + \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{4}\,\log\left(2\,x^2-x+2\right) - \log\left(x^2+x+1\right) + \frac{5}{2}\,\log\left(x\right)$$

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) + 5/2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{4}\log\left(2x^2-x+2\right) - \log\left(x^2+x+1\right) + \frac{5}{2}\log\left(|x|\right)$$

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) + 5/2*log(abs(x
))
```

Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)}\,dx = \frac{5\,\ln(x)}{2} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(1+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-1+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right) + \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{36}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{36}\right)$$

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] (5*log(x))/2 - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 + 1) + log(x +
 (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 - 1) + log(x - (15^(1/2)*1i)/4 - 1/4
)*((15^(1/2)*1i)/36 - 1/4) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/
36 + 1/4)
```

3.248 $\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx$

Optimal result 1647
Rubi [A] (verified) 1647
Mathematica [A] (verified) 1649
Maple [A] (verified) 1649
Fricas [A] (verification not implemented) 1650
Sympy [A] (verification not implemented) 1650
Maxima [A] (verification not implemented) 1651
Giac [A] (verification not implemented) 1651
Mupad [B] (verification not implemented) 1652

Optimal result

Integrand size = 35, antiderivative size = 84

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{5}{2x}+\frac{5}{12}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} -\frac{3\log(x)}{4}+\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{24}\log\left(2-x+2x^2\right)$$

```
[Out] -5/2/x-3/4*ln(x)+1/3*ln(x^2+x+1)+1/24*ln(2*x^2-x+2)+5/36*arctan(1/15*(1-4*x
)*15^(1/2))*15^(1/2)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2112, 814, 648, 632, 210, 642}

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx = \frac{5}{12}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right) -\frac{10\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(x^2+x+1\right) +\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{5}{2x}-\frac{3\log(x)}{4}$$

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(2*x) + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 - (10*ArcTan[(1 + 2*
x)/Sqrt[3]])/(3*Sqrt[3]) - (3*Log[x])/4 + Log[1 + x + x^2]/3 + Log[2 - x +
2*x^2]/24
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2112

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[x^m*((b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x)/(2*a + (b + q)*x + 2*a*x^2)), x], x] - Dist[1/q, Int[x^m*((b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2)),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{3}\int \frac{-6+4x}{x^2\,(4-2x+4x^2)}\,dx\right)+\frac{1}{3}\int \frac{24+16x}{x^2\,(4+4x+4x^2)}\,dx$$

$$= \frac{1}{3}\int \left(\frac{6}{x^2} - \frac{2}{x} + \frac{2(-2+x)}{1+x+x^2}\right) dx - \frac{1}{3}\int \left(-\frac{3}{2x^2} + \frac{1}{4x} + \frac{13-2x}{4(2-x+2x^2)}\right) dx$$
$$= -\frac{5}{2x} - \frac{3\log(x)}{4} - \frac{1}{12}\int \frac{13-2x}{2-x+2x^2} dx + \frac{2}{3}\int \frac{-2+x}{1+x+x^2} dx$$
$$= -\frac{5}{2x} - \frac{3\log(x)}{4} + \frac{1}{24}\int \frac{-1+4x}{2-x+2x^2} dx + \frac{1}{3}\int \frac{1+2x}{1+x+x^2} dx - \frac{25}{24}\int \frac{1}{2-x+2x^2} dx - \frac{5}{3}\int \frac{1}{1+x+x^2} dx$$
$$= -\frac{5}{2x} - \frac{3\log(x)}{4} + \frac{1}{3}\log\left(1+x+x^2\right) + \frac{1}{24}\log\left(2-x+2x^2\right) + \frac{25}{12}\text{Subst}\left(\int \frac{1}{-15-x^2} dx, x, -1+4x\right) + \frac{10}{3}\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2x\right)$$
$$= -\frac{5}{2x} + \frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{3\log(x)}{4} + \frac{1}{3}\log\left(1+x+x^2\right) + \frac{1}{24}\log\left(2-x+2x^2\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+3x^2+x^3+2x^4\right)} dx = -\frac{180+80\sqrt{3}x\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+10\sqrt{15}x\arctan\left(\frac{-1+4x}{\sqrt{15}}\right)+54x\log(x)-24x\log\left(1+x+x^2\right)-3x\log\left(2\right.}{72x}$$

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -1/72*(180 + 80*Sqrt[3]*x*ArcTan[(1 + 2*x)/Sqrt[3]] + 10*Sqrt[15]*x*ArcTan[
(-1 + 4*x)/Sqrt[15]] + 54*x*Log[x] - 24*x*Log[1 + x + x^2] - 3*x*Log[2 - x
+ 2*x^2])/x
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

method	result	si
default	$-\frac{5}{2x}-\frac{3\ln(x)}{4}+\frac{\ln(x^2+x+1)}{3}-\frac{10\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{\ln(2x^2-x+2)}{24}-\frac{5\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{36}$	65
risch	$-\frac{5}{2x}-\frac{5\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{36}+\frac{\ln(16x^2-8x+16)}{24}-\frac{3\ln(x)}{4}+\frac{\ln(25x^2+25x+25)}{3}-\frac{10\sqrt{3}\,\arctan\left(\frac{2\left(5x+\frac{5}{2}\right)\sqrt{3}}{15}\right)}{9}$	69

```
[In] int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
[Out] -5/2/x-3/4*ln(x)+1/3*ln(x^2+x+1)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1
/24*ln(2*x^2-x+2)-5/36*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{10\sqrt{5}\sqrt{3}x\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)+80\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-3x\log\left(2x^2-x+2\right)-24x\,\mathrm{lo}}{72x}$$

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
[Out] -1/72*(10*sqrt(5)*sqrt(3)*x*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 80*sqr
t(3)*x*arctan(1/3*sqrt(3)*(2*x + 1)) - 3*x*log(2*x^2 - x + 2) - 24*x*log(x^
2 + x + 1) + 54*x*log(x) + 180)/x
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{3\log(x)}{4}+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{24}+\frac{\log(x^2+x+1)}{3}-\frac{5\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{36}-\frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}-\frac{5}{2x}$$

```
[In] integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+3*x**2+x+2),x)
[Out] -3*log(x)/4 + log(x**2 - x/2 + 1)/24 + log(x**2 + x + 1)/3 - 5*sqrt(15)*ata
n(4*sqrt(15)*x/15 - sqrt(15)/15)/36 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(
3)/3)/9 - 5/(2*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{5}{2x} + \frac{1}{24}\log\left(2x^2-x+2\right) + \frac{1}{3}\log\left(x^2+x+1\right) - \frac{3}{4}\log\left(x\right)$$

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] -5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) - 5/2/x + 1/24*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1) -
 3/4*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{5}{2x} + \frac{1}{24}\log\left(2x^2-x+2\right) + \frac{1}{3}\log\left(x^2+x+1\right) - \frac{3}{4}\log\left(|x|\right)$$

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] -5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) - 5/2/x + 1/24*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1) -
 3/4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+3x^2+x^3+2x^4\right)}\,dx=-\frac{3\,\ln\left(x\right)}{4}+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\,\left(\frac{1}{24}+\frac{\sqrt{15}\,5\mathrm{i}}{72}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\,\left(-\frac{1}{24}+\frac{\sqrt{15}\,5\mathrm{i}}{72}\right)-\frac{5}{2\,x}$$

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^2*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - (3*log(x))/4 - log(x
 + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) + log(x - (15^(1/2)*1i)/4 -
 1/4)*((15^(1/2)*5i)/72 + 1/24) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)
*5i)/72 - 1/24) - 5/(2*x)
```

3.249 $\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)} dx$

Optimal result 1653
Rubi [A] (verified) 1653
Mathematica [A] (verified) 1655
Maple [A] (verified) 1656
Fricas [A] (verification not implemented) 1656
Sympy [A] (verification not implemented) 1656
Maxima [A] (verification not implemented) 1657
Giac [A] (verification not implemented) 1657
Mupad [B] (verification not implemented) 1658

Optimal result

Integrand size = 35, antiderivative size = 91

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)} dx = -\frac{5}{4x^2}+\frac{3}{4x}+\frac{1}{24}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{15\log(x)}{8}+\frac{2}{3}\log\left(1+x+x^2\right)+\frac{13}{48}\log\left(2-x+2x^2\right)$$

```
[Out] -5/4/x^2+3/4/x-15/8*ln(x)+2/3*ln(x^2+x+1)+13/48*ln(2*x^2-x+2)+1/72*arctan(1
/15*(1-4*x)*15^(1/2))*15^(1/2)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2112, 814, 648, 632, 210, 642}

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)} dx = \frac{1}{24}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{5}{4x^2}+\frac{2}{3}\log\left(x^2+x+1\right) + \frac{13}{48}\log\left(2x^2-x+2\right)+\frac{3}{4x}-\frac{15\log(x)}{8}$$

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(4*x^2) + 3/(4*x) + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/24 + (8*ArcTa
n[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (15*Log[x])/8 + (2*Log[1 + x + x^2])/3
+ (13*Log[2 - x + 2*x^2])/48
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2112

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[x^m*((b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x)/(2*a + (b + q)*x + 2*a*x^2)), x], x] - Dist[1/q, Int[x^m*((b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2)),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{3}\int \frac{-6+4x}{x^3\,(4-2x+4x^2)}\,dx\right)+\frac{1}{3}\int \frac{24+16x}{x^3\,(4+4x+4x^2)}\,dx$$

$$= \frac{1}{3}\int\left(\frac{6}{x^3}-\frac{2}{x^2}-\frac{4}{x}+\frac{2(3+2x)}{1+x+x^2}\right)dx-\frac{1}{3}\int\left(-\frac{3}{2x^3}+\frac{1}{4x^2}+\frac{13}{8x}+\frac{9-26x}{8(2-x+2x^2)}\right)dx$$

$$= -\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\log(x)}{8}-\frac{1}{24}\int\frac{9-26x}{2-x+2x^2}\,dx+\frac{2}{3}\int\frac{3+2x}{1+x+x^2}\,dx$$

$$= -\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\log(x)}{8}-\frac{5}{48}\int\frac{1}{2-x+2x^2}\,dx + \frac{13}{48}\int\frac{-1+4x}{2-x+2x^2}\,dx+\frac{2}{3}\int\frac{1+2x}{1+x+x^2}\,dx+\frac{4}{3}\int\frac{1}{1+x+x^2}\,dx$$

$$= -\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\log(x)}{8}+\frac{2}{3}\log\left(1+x+x^2\right)+\frac{13}{48}\log\left(2-x+2x^2\right) + \frac{5}{24}\text{Subst}\left(\int\frac{1}{-15-x^2}\,dx,x,-1+4x\right)-\frac{8}{3}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+2x\right)$$

$$= -\frac{5}{4x^2}+\frac{3}{4x}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{15\log(x)}{8}+\frac{2}{3}\log\left(1+x+x^2\right)+\frac{13}{48}\log\left(2-x+2x^2\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int\frac{5+x+3x^2+2x^3}{x^3\left(2+x+3x^2+x^3+2x^4\right)}\,dx=\frac{1}{144}\left(128\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 2\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right)+3\left(-\frac{60}{x^2}+\frac{36}{x}-90\log(x) + 32\log\left(1+x+x^2\right)+13\log\left(2-x+2x^2\right)\right)\right)$$

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
[Out] (128*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[
15]] + 3*(-60/x^2 + 36/x - 90*Log[x] + 32*Log[1 + x + x^2] + 13*Log[2 - x +
 2*x^2]))/144
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

method	result	siz
default	$-\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\ln(x)}{8}+\frac{2\ln(x^2+x+1)}{3}+\frac{8\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{13\ln(2x^2-x+2)}{48}-\frac{\sqrt{15}\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{72}$	70
risch	$\frac{\frac{3x}{4}-\frac{5}{4}}{x^2}-\frac{\sqrt{15}\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{72}+\frac{13\ln(16x^2-8x+16)}{48}+\frac{2\ln(4x^2+4x+4)}{3}+\frac{8\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{15\ln(x)}{8}$	73

```
[In] int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] -5/4/x^2+3/4/x-15/8*ln(x)+2/3*ln(x^2+x+1)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3
^(1/2)+13/48*ln(2*x^2-x+2)-1/72*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{2\sqrt{5}\sqrt{3}x^2\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)-128\sqrt{3}x^2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-39x^2\log\left(2x^2-x+2\right)-96}{144x^2}$$

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] -1/144*(2*sqrt(5)*sqrt(3)*x^2*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 128*
sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) - 39*x^2*log(2*x^2 - x + 2) - 96*
x^2*log(x^2 + x + 1) + 270*x^2*log(x) - 108*x + 180)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.03

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{15\log(x)}{8}+\frac{13\log\left(x^2-\frac{x}{2}+1\right)}{48}+\frac{2\log(x^2+x+1)}{3}-\frac{\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{72}+\frac{8\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}+\frac{3x-5}{4x^2}$$

```
[In] integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] -15*log(x)/8 + 13*log(x**2 - x/2 + 1)/48 + 2*log(x**2 + x + 1)/3 - sqrt(15)
*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/72 + 8*sqrt(3)*atan(2*sqrt(3)*x/3 + sq
rt(3)/3)/9 + (3*x - 5)/(4*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.76

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\frac{1}{72}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) + \frac{8}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{3\,x-5}{4\,x^2} + \frac{13}{48}\,\log\left(2\,x^2-x+2\right) + \frac{2}{3}\,\log\left(x^2+x+1\right) - \frac{15}{8}\,\log\left(x\right)$$

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] -1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*log(2*x^2 - x + 2) + 2/3*log(x^
2 + x + 1) - 15/8*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\frac{1}{72}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) + \frac{8}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{3\,x-5}{4\,x^2} + \frac{13}{48}\,\log\left(2\,x^2-x+2\right) + \frac{2}{3}\,\log\left(x^2+x+1\right) - \frac{15}{8}\,\log\left(|x|\right)$$

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] -1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*log(2*x^2 - x + 2) + 2/3*log(x^
2 + x + 1) - 15/8*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+3x^2+x^3+2x^4\right)}\,dx=\frac{\frac{3\,x}{4}-\frac{5}{4}}{x^2}-\frac{15\,\ln\left(x\right)}{8}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-\frac{2}{3}+\frac{\sqrt{3}\,4{}\mathrm{i}}{9}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{2}{3}+\frac{\sqrt{3}\,4{}\mathrm{i}}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1{}\mathrm{i}}{4}\right)\,\left(\frac{13}{48}+\frac{\sqrt{15}\,1{}\mathrm{i}}{144}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1{}\mathrm{i}}{4}\right)\,\left(-\frac{13}{48}+\frac{\sqrt{15}\,1{}\mathrm{i}}{144}\right)$$

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^3*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] ((3*x)/4 - 5/4)/x^2 - (15*log(x))/8 - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/
2)*4i)/9 - 2/3) + log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) + lo
g(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/144 + 13/48) - log(x + (15^(1/2
)*1i)/4 - 1/4)*((15^(1/2)*1i)/144 - 13/48)
```

3.250 $\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal result 1659
Rubi [A] (verified) 1660
Mathematica [C] (verified) 1663
Maple [C] (verified) 1663
Fricas [B] (verification not implemented) 1664
Sympy [A] (verification not implemented) 1665
Maxima [F] 1666
Giac [F] 1666
Mupad [B] (verification not implemented) 1666

Optimal result

Integrand size = 35, antiderivative size = 307

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = -\frac{1}{28}\left(35-9i\sqrt{7}\right)x-\frac{1}{28}\left(35+9i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{42}\left(7-5i\sqrt{7}\right)x^3+\frac{1}{42}\left(7+5i\sqrt{7}\right)x^3+\frac{11\left(9i+5\sqrt{7}\right)\arctan\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35+i\sqrt{7}\right)}}-\frac{11\left(9i-5\sqrt{7}\right)\arctan\left(\frac{1+i\sqrt{7}+8x}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)+\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)$$

```
[Out] 1/28*x^2*(7-5*I*7^(1/2))+1/42*x^3*(7-5*I*7^(1/2))+1/28*x^2*(7+5*I*7^(1/2))+
1/42*x^3*(7+5*I*7^(1/2))-1/28*x*(35-9*I*7^(1/2))-1/28*x*(35+9*I*7^(1/2))+3/
112*ln(4+4*x^2+x*(1-I*7^(1/2)))*(7-11*I*7^(1/2))+3/112*ln(4+4*x^2+x*(1+I*7^
(1/2)))*(7+11*I*7^(1/2))-11/4*arctan((1+8*x+I*7^(1/2))/(70-2*I*7^(1/2))^(1/
2))*(9*I-5*7^(1/2))/(490-14*I*7^(1/2))^(1/2)+11/4*arctan((1+8*x-I*7^(1/2))/
(70+2*I*7^(1/2))^(1/2))*(9*I+5*7^(1/2))/(490+14*I*7^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2112, 814, 648, 632, 210, 642}

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{11(5\sqrt{7}+9i)\arctan\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35+i\sqrt{7}\right)}} - \frac{11(-5\sqrt{7}+9i)\arctan\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}} + \frac{1}{42}\left(7+5i\sqrt{7}\right)x^3 + \frac{1}{42}\left(7-5i\sqrt{7}\right)x^3 + \frac{1}{28}\left(7+5i\sqrt{7}\right)x^2 + \frac{1}{28}\left(7-5i\sqrt{7}\right)x^2 + \frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right) + \frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right) - \frac{1}{28}\left(35+9i\sqrt{7}\right)x - \frac{1}{28}\left(35-9i\sqrt{7}\right)x$$

```
[In] Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] -1/28*((35 - (9*I)*Sqrt[7])*x) - ((35 + (9*I)*Sqrt[7])*x)/28 + ((7 - (5*I)*
Sqrt[7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 + ((7 - (5*I)*Sqrt[7])*x^3)
/42 + ((7 + (5*I)*Sqrt[7])*x^3)/42 + (11*(9*I + 5*Sqrt[7])*ArcTan[(1 - I*Sq
rt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(4*Sqrt[14*(35 + I*Sqrt[7])]) - (11
*(9*I - 5*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/
(4*Sqrt[14*(35 - I*Sqrt[7])]) + (3*(7 - (11*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt
[7])*x + 4*x^2])/112 + (3*(7 + (11*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x +
4*x^2])/112
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2112

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[x^m*((b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x)/(2*a + (b + q)*x + 2*a*x^2)), x], x] - Dist[1/q, Int[x^m*((b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2)),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\int \frac{x^3\left(9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}-\frac{i\int \frac{x^3\left(9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x\right)}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}\\
&= \frac{i\int \left(\frac{1}{4}\left(-9+5i\sqrt{7}\right)+\frac{1}{2}\left(5-i\sqrt{7}\right)x+\frac{1}{2}\left(5-i\sqrt{7}\right)x^2+\frac{2\left(9-5i\sqrt{7}\right)-3\left(11+i\sqrt{7}\right)x}{2\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}\right)dx}{\sqrt{7}}\\
&\quad-\frac{i\int \left(\frac{1}{4}\left(-9-5i\sqrt{7}\right)+\frac{1}{2}\left(5+i\sqrt{7}\right)x+\frac{1}{2}\left(5+i\sqrt{7}\right)x^2+\frac{2\left(9+5i\sqrt{7}\right)-3\left(11-i\sqrt{7}\right)x}{2\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)}\right)dx}{\sqrt{7}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x-\frac{1}{28}\left(35+9i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2 \\
&\quad+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{42}\left(7-5i\sqrt{7}\right)x^3+\frac{1}{42}\left(7+5i\sqrt{7}\right)x^3 \\
&\quad+\frac{i\int\frac{2\left(9-5i\sqrt{7}\right)-3\left(11+i\sqrt{7}\right)x}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{2\sqrt{7}}-\frac{i\int\frac{2\left(9+5i\sqrt{7}\right)-3\left(11-i\sqrt{7}\right)x}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{2\sqrt{7}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x-\frac{1}{28}\left(35+9i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2 \\
&\quad+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{42}\left(7-5i\sqrt{7}\right)x^3+\frac{1}{42}\left(7+5i\sqrt{7}\right)x^3 \\
&\quad+\frac{1}{56}\left(11\left(35-9i\sqrt{7}\right)\right)\int\frac{1}{4+(1+i\sqrt{7})\,x+4x^2}\,dx \\
&\quad+\frac{1}{56}\left(11\left(35+9i\sqrt{7}\right)\right)\int\frac{1}{4+(1-i\sqrt{7})\,x+4x^2}\,dx \\
&\quad+\frac{1}{112}\left(3\left(7-11i\sqrt{7}\right)\right)\int\frac{1-i\sqrt{7}+8x}{4+(1-i\sqrt{7})\,x+4x^2}\,dx \\
&\quad+\frac{1}{112}\left(3\left(7+11i\sqrt{7}\right)\right)\int\frac{1+i\sqrt{7}+8x}{4+(1+i\sqrt{7})\,x+4x^2}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x-\frac{1}{28}\left(35+9i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2 \\
&\quad+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{42}\left(7-5i\sqrt{7}\right)x^3+\frac{1}{42}\left(7+5i\sqrt{7}\right)x^3 \\
&\quad+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right) \\
&\quad+\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right) \\
&\quad-\frac{1}{28}\left(11\left(35-9i\sqrt{7}\right)\right)\text{Subst}\left(\int\frac{1}{-2\,(35-i\sqrt{7})-x^2}\,dx,x,1+i\sqrt{7}+8x\right) \\
&\quad-\frac{1}{28}\left(11\left(35+9i\sqrt{7}\right)\right)\text{Subst}\left(\int\frac{1}{-2\,(35+i\sqrt{7})-x^2}\,dx,x,1-i\sqrt{7}+8x\right)
\end{aligned}
$$

$$= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x - \frac{1}{28}\left(35+9i\sqrt{7}\right)x + \frac{1}{28}\left(7-5i\sqrt{7}\right)x^2 + \frac{1}{28}\left(7+5i\sqrt{7}\right)x^2 + \frac{1}{42}\left(7-5i\sqrt{7}\right)x^3 + \frac{1}{42}\left(7+5i\sqrt{7}\right)x^3 + \frac{11(9i+5\sqrt{7})\tan^{-1}\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2(35+i\sqrt{7})}}\right)}{4\sqrt{14(35+i\sqrt{7})}} - \frac{11(9i-5\sqrt{7})\tan^{-1}\left(\frac{1+i\sqrt{7}+8x}{\sqrt{2(35-i\sqrt{7})}}\right)}{4\sqrt{14(35-i\sqrt{7})}} + \frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right) + \frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.36

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{1}{6}\left(x(-15+3x+2x^2) + 3\mathrm{RootSum}\left[2+\#1+5\#1^2+\#1^3+2\#1^4\&, \frac{10\log(x-\#1)+\log(x-\#1)\#1+19\log(x-\#1)\#1^2+3\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\right]\right)$$

```
[In] Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
[Out] (x*(-15 + 3*x + 2*x^2) + 3*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (10*
Log[x - #1] + Log[x - #1]*#1 + 19*Log[x - #1]*#1^2 + 3*Log[x - #1]*#1^3)/(1
 + 10*#1 + 3*#1^2 + 8*#1^3) & ])/6
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.24

method	result	size
default	$\frac{x^3}{3}+\frac{x^2}{2}-\frac{5x}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(3\_R^3+19\_R^2+\_R+10\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}\right)}{2}$	74
risch	$\frac{x^3}{3}+\frac{x^2}{2}-\frac{5x}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(3\_R^3+19\_R^2+\_R+10\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}\right)}{2}$	74

```
[In] int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3+1/2*x^2-5/2*x+1/2*sum((3*_R^3+19*_R^2+_R+10)/(8*_R^3+3*_R^2+10*_R+1
)*ln(x-_R),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1202 vs. 2(199) = 398.

Time = 1.06 (sec) , antiderivative size = 1202, normalized size of antiderivative = 3.92

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 1/112*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55
/32) - 21)*log(23324*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/
32) + 3/16)^3 - 23765*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55
/32) + 3/16)^2 + 7744*x + 19470*I*sqrt(7) - 33040*sqrt(2101/1568*I*sqrt(7)
- 55/32) + 38950) - 1/112*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55
/32) - 21)*log(-23324*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55
/32) + 3/16)^3 + 49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) -
55/32) + 3/16)^2*(-561*I*sqrt(7) + 952*sqrt(2101/1568*I*sqrt(7) - 55/32) -
869) + 1/256*(53312*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/3
2) + 3/16)^2 - 11781*I*sqrt(7) + 19992*sqrt(2101/1568*I*sqrt(7) - 55/32) -
36681)*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) + 17493*
(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 7744*
x - 15708*I*sqrt(7) + 26656*sqrt(2101/1568*I*sqrt(7) - 55/32) - 29132) + 1/
112*(2*sqrt(7)*sqrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) -
 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7)
- 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55
/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99
/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2) + 28*sqrt(210
1/1568*I*sqrt(7) - 55/32) + 28*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 21)*log
(-49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2*(-561*I*sqrt(7) + 952*sqrt(2101/1568*I*sqrt(7) - 55/32) - 869) - 1/256*(5
3312*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 -
11781*I*sqrt(7) + 19992*sqrt(2101/1568*I*sqrt(7) - 55/32) - 36681)*(33*I*sq
rt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) + 6272*(33/112*I*sqrt(7
) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 1/256*((17*sqrt(7)*(-
33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7))*(3
3*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7)*(-3
3*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) + 73728*sqrt(7))*s
```

```
qrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)
^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*
I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 8
4*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2) + 15488*x - 3762*I*sqrt(7) +
6384*sqrt(2101/1568*I*sqrt(7) - 55/32) - 5946) - 1/112*(2*sqrt(7)*sqrt(-336
*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*
(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/5
6*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7
) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2
101/1568*I*sqrt(7) - 55/32) - 1859/2) - 28*sqrt(2101/1568*I*sqrt(7) - 55/32
) - 28*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*log(-49/4*(-33/112*I*sqrt(7
) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2*(-561*I*sqrt(7) + 952*
sqrt(2101/1568*I*sqrt(7) - 55/32) - 869) - 1/256*(53312*(33/112*I*sqrt(7) -
 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 11781*I*sqrt(7) + 19992*
sqrt(2101/1568*I*sqrt(7) - 55/32) - 36681)*(33*I*sqrt(7) + 56*sqrt(-2101/15
68*I*sqrt(7) - 55/32) - 21) + 6272*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I
*sqrt(7) - 55/32) + 3/16)^2 - 1/256*((17*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(2
101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7))*(33*I*sqrt(7) + 56*sqrt(-2
101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(21
01/1568*I*sqrt(7) - 55/32) - 21) + 73728*sqrt(7))*sqrt(-336*(33/112*I*sqrt(
7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(
7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7)
+ 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101
/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(
7) - 55/32) - 1859/2) + 15488*x - 3762*I*sqrt(7) + 6384*sqrt(2101/1568*I*sq
rt(7) - 55/32) - 5946) - 5/2*x
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.20

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{x^3}{3}+\frac{x^2}{2}-\frac{5x}{2} + \text{RootSum}\left(1372t^4-1029t^3+3136t^2+2688t+512, \left(t \mapsto t\log\left(\frac{5831t^3}{1936}-\frac{23765t^2}{7744}+\frac{2065t}{242}+x+\frac{41}{12}\right.\right.\right.$$

```
[In] integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x**3/3 + x**2/2 - 5*x/2 + RootSum(1372*_t**4 - 1029*_t**3 + 3136*_t**2 + 26
88*_t + 512, Lambda(_t, _t*log(5831*_t**3/1936 - 23765*_t**2/7744 + 2065*_t
/242 + x + 415/121)))
```

Maxima [F]

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{(2\,x^3+3\,x^2+x+5)x^3}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/2*x + 1/2*integrate((3*x^3 + 19*x^2 + x + 10)/(2*x^4
+ x^3 + 5*x^2 + x + 2), x)
```

Giac [F]

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{(2\,x^3+3\,x^2+x+5)x^3}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x^3/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Mupad [B] (verification not implemented)

Time = 9.86 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.42

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \left(\sum_{k=1}^{4}\ln\left(-29\,x + \mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)\left(-\frac{289\,x}{4}+\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)\left(\frac{581\,x}{16} - \right.\right.\right.\right.$$
$$\left.\left.+7\right)\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)\right) - \frac{5\,x}{2}+\frac{x^2}{2}+\frac{x^3}{3}$$

```
[In] int((x^3*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] symsum(log(root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*(
root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*((581*x)/16
- root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*((147*x)/4
 - 49/16) + 1141/64) - (289*x)/4 + 47/4) - 29*x + 7)*root(z^4 - (3*z^3)/4 +
 (16*z^2)/7 + (96*z)/49 + 128/343, z, k), k, 1, 4) - (5*x)/2 + x^2/2 + x^3/
3
```

3.251 $\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal result 1667
Rubi [A] (verified) 1668
Mathematica [C] (verified) 1671
Maple [C] (verified) 1671
Fricas [B] (verification not implemented) 1672
Sympy [B] (verification not implemented) 1673
Maxima [F] 1676
Giac [F] 1676
Mupad [B] (verification not implemented) 1677

Optimal result

Integrand size = 35, antiderivative size = 269

$$\begin{aligned}\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx &= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2 \\ &\quad+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2-\frac{(53i+\sqrt{7})\arctan\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}} \\ &\quad+\frac{(53i-\sqrt{7})\arctan\left(\frac{1+i\sqrt{7}+8x}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}} \\ &\quad-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right) \\ &\quad-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)\end{aligned}$$

```
[Out] 1/14*x*(7-5*I*7^(1/2))+1/28*x^2*(7-5*I*7^(1/2))+1/14*x*(7+5*I*7^(1/2))+1/28
*x^2*(7+5*I*7^(1/2))-1/56*ln(4+4*x^2+x*(1+I*7^(1/2)))*(35-9*I*7^(1/2))-1/56
*ln(4+4*x^2+x*(1-I*7^(1/2)))*(35+9*I*7^(1/2))+1/2*arctan((1+8*x+I*7^(1/2))/
(70-2*I*7^(1/2))^(1/2))*(53*I-7^(1/2))/(490-14*I*7^(1/2))^(1/2)-1/2*arctan(
(1+8*x-I*7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(53*I+7^(1/2))/(490+14*I*7^(1/2))
^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2112, 814, 648, 632, 210, 642}

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = -\frac{\left(\sqrt{7}+53i\right)\arctan\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}} + \frac{\left(-\sqrt{7}+53i\right)\arctan\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}} + \frac{1}{28}\left(7+5i\sqrt{7}\right)x^2 + \frac{1}{28}\left(7-5i\sqrt{7}\right)x^2 - \frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right) - \frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right) + \frac{1}{14}\left(7+5i\sqrt{7}\right)x + \frac{1}{14}\left(7-5i\sqrt{7}\right)x$$

```
[In] Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 + ((7 - (5*I)*Sqrt[
7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 - ((53*I + Sqrt[7])*ArcTan[(1 -
I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[14*(35 + I*Sqrt[7])]) +
 ((53*I - Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/
(2*Sqrt[14*(35 - I*Sqrt[7])]) - ((35 + (9*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7
])*x + 4*x^2])/56 - ((35 - (9*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2
])/56
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2112

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[x^m*((b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x)/(2*a + (b + q)*x + 2*a*x^2)), x], x] - Dist[1/q, Int[x^m*((b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2)),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\text{integral} = \frac{i\int \frac{x^2\left(9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{x^2\left(9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x\right)}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$

$$= \frac{i\int \left(\frac{1}{2}\left(5-i\sqrt{7}\right)+\frac{1}{2}\left(5-i\sqrt{7}\right)x+\frac{i\left(2\left(5i+\sqrt{7}\right)+\left(9i+5\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\right)dx}{\sqrt{7}}$$

$$- \frac{i\int \left(\frac{1}{2}\left(5+i\sqrt{7}\right)+\frac{1}{2}\left(5+i\sqrt{7}\right)x-\frac{i\left(-2\left(5i-\sqrt{7}\right)-\left(9i-5\sqrt{7}\right)x\right)}{4+\left(1+i\sqrt{7}\right)x+4x^2}\right)dx}{\sqrt{7}}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2 + \frac{1}{28}\left(7+5i\sqrt{7}\right)x^2 - \frac{\int \frac{2\left(5i+\sqrt{7}\right)+\left(9i+5\sqrt{7}\right)x}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{\int \frac{-2\left(5i-\sqrt{7}\right)-\left(9i-5\sqrt{7}\right)x}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2 + \frac{1}{28}\left(7+5i\sqrt{7}\right)x^2 - \frac{1}{56}\left(35-9i\sqrt{7}\right)\int \frac{1+i\sqrt{7}+8x}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx - \frac{1}{56}\left(35+9i\sqrt{7}\right)\int \frac{1-i\sqrt{7}+8x}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx - \frac{1}{28}\left(7-53i\sqrt{7}\right)\int \frac{1}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx - \frac{1}{28}\left(7+53i\sqrt{7}\right)\int \frac{1}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2 + \frac{1}{28}\left(7+5i\sqrt{7}\right)x^2 - \frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right) - \frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right) - \frac{1}{14}\left(-7+53i\sqrt{7}\right)\text{Subst}\left(\int \frac{1}{-2\left(35-i\sqrt{7}\right)-x^2}\,dx, x, 1+i\sqrt{7}+8x\right) + \frac{1}{14}\left(7+53i\sqrt{7}\right)\text{Subst}\left(\int \frac{1}{-2\left(35+i\sqrt{7}\right)-x^2}\,dx, x, 1-i\sqrt{7}+8x\right)$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2 - \frac{\left(53i+\sqrt{7}\right)\tan^{-1}\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}} + \frac{\left(53i-\sqrt{7}\right)\tan^{-1}\left(\frac{1+i\sqrt{7}+8x}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}} - \frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right) - \frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.38

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$$
$$= x+\frac{x^2}{2}-\text{RootSum}\Big[2+\#1+5\#1^2+\#1^3+2\#1^4\&,\frac{2\log(x-\#1)+3\log(x-\#1)\#1+\log(x-\#1)\#1^2+5\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\Big]$$

```
[In] Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x + x^2/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (2*Log[x - #1] + 3*
Log[x - #1]*#1 + Log[x - #1]*#1^2 + 5*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2
 + 8*#1^3) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.25

method	result	size
default	$\frac{x^2}{2}+x+\left(\sum_{\_R=\text{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{(-5\_R^3-\_R^2-3\_R-2)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}\right)$	67
risch	$\frac{x^2}{2}+x+\left(\sum_{\_R=\text{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{(-5\_R^3-\_R^2-3\_R-2)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}\right)$	67

```
[In] int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+x+sum((-5*_R^3-_R^2-3*_R-2)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=Roo
tOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1145 vs. $2(171) = 342$.

Time = 1.02 (sec) , antiderivative size = 1145, normalized size of antiderivative = 4.26

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/2*x^2 - 1/56*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35)*log(
49/4*(135*I*sqrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 1459)*(9/56*I*s
qrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 10290*(-9/56*I*sqrt(
7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 - 25725*(-9/56*I*sqrt(7)
- 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 3/64*(3920*(-9/56*I*sqrt(7
) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sq
rt(-37/392*I*sqrt(7) + 79/56) + 5587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt
(7) + 79/56) + 35) + 8384*x + 6615/2*I*sqrt(7) + 10290*sqrt(-37/392*I*sqrt(
7) + 79/56) + 13373/2) + 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*
I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt
(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79
/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14
*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2) + 2*sqrt(37/392*I*s
qrt(7) + 79/56) + 2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5)*log(-49/4*(135*I*s
qrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 1459)*(9/56*I*sqrt(7) - 1/2*
sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 24304*(-9/56*I*sqrt(7) - 1/2*sqrt
(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 3/64*(3920*(-9/56*I*sqrt(7) - 1/2*sq
rt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sqrt(-37/392
*I*sqrt(7) + 79/56) + 5587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/5
6) + 35) + 7/64*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/5
6) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5
/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*
I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*
sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2)*((135*I*sqrt(7) + 420*sqrt(-37/392*
I*sqrt(7) + 79/56) - 1459)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56
) + 35) - 17856*I*sqrt(7) - 55552*sqrt(-37/392*I*sqrt(7) + 79/56) + 67776)
+ 16768*x - 4941*I*sqrt(7) - 15372*sqrt(-37/392*I*sqrt(7) + 79/56) - 9391)
- 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/
8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 -
 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(
7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-3
7/392*I*sqrt(7) + 79/56) + 11/2) - 2*sqrt(37/392*I*sqrt(7) + 79/56) - 2*sqr
t(-37/392*I*sqrt(7) + 79/56) + 5)*log(-49/4*(135*I*sqrt(7) + 420*sqrt(-37/3
```

```
92*I*sqrt(7) + 79/56) - 1459)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) +
 79/56) - 5/8)^2 + 24304*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79
/56) - 5/8)^2 - 3/64*(3920*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) +
79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sqrt(-37/392*I*sqrt(7) + 79/56) + 5
587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 7/64*sqrt(-1
2*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56
*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt
(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/
392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7)
+ 79/56) + 11/2)*((135*I*sqrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 14
59)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 17856*I*sqrt(
7) - 55552*sqrt(-37/392*I*sqrt(7) + 79/56) + 67776) + 16768*x - 4941*I*sqrt
(7) - 15372*sqrt(-37/392*I*sqrt(7) + 79/56) - 9391) - 1/56*(9*I*sqrt(7) + 2
8*sqrt(-37/392*I*sqrt(7) + 79/56) + 35)*log(10290*(-9/56*I*sqrt(7) - 1/2*sq
rt(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 + 1421*(-9/56*I*sqrt(7) - 1/2*sqrt(-
37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 8384*x + 3267/2*I*sqrt(7) + 5082*sqrt(
-37/392*I*sqrt(7) + 79/56) + 13793/2) + x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3662 vs. 2(219) = 438.

Time = 1.59 (sec) , antiderivative size = 3662, normalized size of antiderivative = 13.61

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x**2/2 + x + (-5/8 + sqrt(79/448 + sqrt(77)/49))*log(x**2 + x*(-1459*sqrt(1
4)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 - 15*s
qrt(77)*sqrt(553 + 64*sqrt(77))/2096 - 10391*sqrt(553 + 64*sqrt(77))/268288
 + 1459*sqrt(77)/8384 + 522933/268288 + 45*sqrt(14)*sqrt(553 + 64*sqrt(77))
*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576) - 51089
5297*sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/71
978450944 - 6009493*sqrt(22)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 76
48*sqrt(77))/1124663296 - 38714551*sqrt(77)*sqrt(553 + 64*sqrt(77))/2249326
592 - 4417610843*sqrt(553 + 64*sqrt(77))/35989225472 + 153195*sqrt(22)*sqrt
(553 + 64*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(7
7))/2249326592 + 8313499*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(-333*sqrt(55
3 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/71978450944 + 290832444193/359892
25472 + 2303470247*sqrt(77)/2249326592) + (-5/8 - sqrt(79/448 + sqrt(77)/49
))*log(x**2 + x*(-45*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(333*sqrt(553 + 6
4*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 - 1459*sqrt(14)*sqrt(333*sqrt(5
53 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 + 10391*sqrt(553 + 64*sqr
t(77))/268288 + 1459*sqrt(77)/8384 + 522933/268288 + 15*sqrt(77)*sqrt(553 +
```

64*sqrt(77))/2096) - 510895297*sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/71978450944 - 6009493*sqrt(22)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/1124663296 - 8313499*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/71978450944 - 153195*sqrt(22)*sqrt(553 + 64*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/2249326592 + 4417610843*sqrt(553 + 64*sqrt(77))/35989225472 + 38714551*sqrt(77)*sqrt(553 + 64*sqrt(77))/2249326592 + 290832444193/35989225472 + 2303470247*sqrt(77)/2249326592) + 2*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/1568 + 5/14 + 3*sqrt(77)/49)*atan(1073152*x/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))) - 45*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))) - 1459*sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))) + 20782*sqrt(553 + 64*sqrt(77))/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))) + 93376*sqrt(77)/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))) + 1045866/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(7

7))) + 3840*sqrt(77)*sqrt(553 + 64*sqrt(77))/(4313*sqrt(2)*sqrt(-sqrt(14)*s
qrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77
)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7
648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975
 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqr
t(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))
)) + 2*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt
(77))/1568 + 5/14 + 3*sqrt(77)/49)*atan(1073152*x/(-1459*sqrt(2)*sqrt(553 +
 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 76
48*sqrt(77)) + 560 + 96*sqrt(77)) + 4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(-333*s
qrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*s
qrt(7)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt
(77)) + 560 + 96*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648
*sqrt(77))) + 45*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(-333*sqrt(553 + 64*s
qrt(77)) + 21975 + 7648*sqrt(77))/(-1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sq
rt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 5
60 + 96*sqrt(77)) + 4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqr
t(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqr
t(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96
*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))) + 10
45866/(-1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(-333*sqrt(
553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 4313*sqr
t(2)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(7
7)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 6
4*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(-333*sqrt(55
3 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))) + 93376*sqrt(77)/(-1459*sqrt(2)*
sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) +
21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 4313*sqrt(2)*sqrt(-sqrt(14)*s
qrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(7
7)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 +
 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21
975 + 7648*sqrt(77))) - 20782*sqrt(553 + 64*sqrt(77))/(-1459*sqrt(2)*sqrt(5
53 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975
+ 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(-3
33*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) +
30*sqrt(7)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*
sqrt(77)) + 560 + 96*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 +
7648*sqrt(77))) - 3840*sqrt(77)*sqrt(553 + 64*sqrt(77))/(-1459*sqrt(2)*sqrt
(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 2197
5 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(
-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))
+ 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 764
8*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975
+ 7648*sqrt(77))) - 1459*sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975
 + 7648*sqrt(77))/(-1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqr

```
t(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)
) + 4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 +
 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(-333*
sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(
-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)))
```

Maxima [F]

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{(2\,x^3+3\,x^2+x+5)x^2}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/2*x^2 + x - integrate((5*x^3 + x^2 + 3*x + 2)/(2*x^4 + x^3 + 5*x^2 + x +
2), x)
```

Giac [F]

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{(2\,x^3+3\,x^2+x+5)x^2}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x^2/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.70

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = x+\frac{x^2}{2} + \left(\sum_{k=1}^{4}\ln\left(-\frac{179\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}{8} - 7\,x - \frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)\,x\,459}{8} - \frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^2\,x\,665}{8} - \frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^3\,x\,147}{4} - \frac{35\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^2}{32} + \frac{49\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^3}{16} - 15\right)\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)\right)$$

```
[In] int((x^2*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] x + x^2/2 + symsum(log((49*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/3
43, z, k)^3)/16 - 7*x - (459*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128
/343, z, k)*x)/8 - (665*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343,
 z, k)^2*x)/8 - (147*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z,
 k)^3*x)/4 - (35*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^
2)/32 - (179*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k))/8 -
 15)*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k), k, 1, 4)
```

3.252 $\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal result 1678
Rubi [A] (verified) 1679
Mathematica [C] (verified) 1681
Maple [C] (verified) 1682
Fricas [B] (verification not implemented) 1682
Sympy [A] (verification not implemented) 1684
Maxima [F] 1684
Giac [F] 1684
Mupad [B] (verification not implemented) 1685

Optimal result

Integrand size = 33, antiderivative size = 230

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x - \frac{(19i+7\sqrt{7})\arctan\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}} + \frac{(19i-7\sqrt{7})\arctan\left(\frac{1+i\sqrt{7}+8x}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}} + \frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right) + \frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)$$

```
[Out] 1/14*x*(7-5*I*7^(1/2))+1/28*ln(4+4*x^2+x*(1+I*7^(1/2)))*(7-5*I*7^(1/2))+1/1
4*x*(7+5*I*7^(1/2))+1/28*ln(4+4*x^2+x*(1-I*7^(1/2)))*(7+5*I*7^(1/2))+arctan
((1+8*x+I*7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(19*I-7*7^(1/2))/(490-14*I*7^(1/
2))^(1/2)-arctan((1+8*x-I*7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(19*I+7*7^(1/2))
/(490+14*I*7^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2112, 787, 648, 632, 210, 642}

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = -\frac{(7\sqrt{7}+19i)\arctan\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}} + \frac{(-7\sqrt{7}+19i)\arctan\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}} + \frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right) + \frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right) + \frac{1}{14}\left(7+5i\sqrt{7}\right)x + \frac{1}{14}\left(7-5i\sqrt{7}\right)x$$

```
[In] Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 - ((19*I + 7*Sqrt[7
])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/Sqrt[14*(35 + I*
Sqrt[7])] + ((19*I - 7*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I
*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7])] + ((7 + (5*I)*Sqrt[7])*Log[4 + (1 -
I*Sqrt[7])*x + 4*x^2])/28 + ((7 - (5*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x
+ 4*x^2])/28
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2112

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[x^m*((b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x)/(2*a + (b + q)*x + 2*a*x^2)), x], x] - Dist[1/q, Int[x^m*((b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2)),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\text{integral} = \frac{i\int \frac{x\left(9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{x\left(9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x\right)}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x + \frac{1}{14}\left(7+5i\sqrt{7}\right)x + \frac{i\int \frac{-4\left(10-2i\sqrt{7}\right)+\left(-\left(\left(1-i\sqrt{7}\right)\left(10-2i\sqrt{7}\right)\right)+4\left(9-5i\sqrt{7}\right)\right)x}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{4\sqrt{7}} - \frac{i\int \frac{-4\left(10+2i\sqrt{7}\right)+\left(-\left(\left(1+i\sqrt{7}\right)\left(10+2i\sqrt{7}\right)\right)+4\left(9+5i\sqrt{7}\right)\right)x}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{4\sqrt{7}}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x-\frac{1}{28}\left(-7+5i\sqrt{7}\right)\int\frac{1+i\sqrt{7}+8x}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx$$
$$+\frac{1}{28}\left(7+5i\sqrt{7}\right)\int\frac{1-i\sqrt{7}+8x}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx$$
$$+\frac{1}{14}\left(-49+19i\sqrt{7}\right)\int\frac{1}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx$$
$$-\frac{1}{14}\left(49+19i\sqrt{7}\right)\int\frac{1}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx$$
$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)$$
$$+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)$$
$$+\frac{1}{7}\left(49-19i\sqrt{7}\right)\text{Subst}\left(\int\frac{1}{-2\left(35-i\sqrt{7}\right)-x^2}\,dx,x,1+i\sqrt{7}+8x\right)$$
$$+\frac{1}{7}\left(49+19i\sqrt{7}\right)\text{Subst}\left(\int\frac{1}{-2\left(35+i\sqrt{7}\right)-x^2}\,dx,x,1-i\sqrt{7}+8x\right)$$
$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x-\frac{\left(19i+7\sqrt{7}\right)\tan^{-1}\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}}$$
$$+\frac{\left(19i-7\sqrt{7}\right)\tan^{-1}\left(\frac{1+i\sqrt{7}+8x}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)$$
$$+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.41

$$\int\frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$$
$$= x+2\text{RootSum}\Big[2+\#1+5\#1^2+\#1^3$$
$$+2\#1^4\&,\frac{-\log(x-\#1)+2\log(x-\#1)\#1-2\log(x-\#1)\#1^2+\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\Big]$$

```
[In] Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x + 2*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (-Log[x - #1] + 2*Log[x -
 #1]*#1 - 2*Log[x - #1]*#1^2 + Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1
^3) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.27

method	result	size
default	$x+2\left(\sum_{\_R=\operatorname{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{(\_R^3-2\_R^2+2\_R-1)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}\right)$	62
risch	$x+2\left(\sum_{\_R=\operatorname{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{(\_R^3-2\_R^2+2\_R-1)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}\right)$	62

```
[In] int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] x+2*sum((_R^3-2*_R^2+2*_R-1)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=RootOf(2*_
Z^4+_Z^3+5*_Z^2+_Z+2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1190 vs. 2(151) = 302.

Time = 1.00 (sec) , antiderivative size = 1190, normalized size of antiderivative = 5.17

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/28*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)*log(49/4*(55*I*
sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1/14) + 147)*(5/28*I*sqrt(7) - 1/2*sqr
t(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/9
8*I*sqrt(7) - 1/14) + 1/4)^3 + 3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqr
t(7) - 1/14) + 1/4)^2 + 11/16*(196*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt
(7) - 1/14) + 1/4)^2 + 35*I*sqrt(7) + 98*sqrt(53/98*I*sqrt(7) - 1/14) + 15)
*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 304*x + 1155/2*I*s
qrt(7) + 1617*sqrt(53/98*I*sqrt(7) - 1/14) + 1903/2) + 1/28*(2*sqrt(7)*sqrt
(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/
28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7
```

```
) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*
sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/
2) + 7*sqrt(53/98*I*sqrt(7) - 1/14) + 7*sqrt(-53/98*I*sqrt(7) - 1/14) + 7)*
log(-49/4*(55*I*sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1/14) + 147)*(5/28*I*s
qrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 2744*(-5/28*I*sqrt(7)
 - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 11/16*(196*(-5/28*I*sqrt(7)
- 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 35*I*sqrt(7) + 98*sqrt(53/98*
I*sqrt(7) - 1/14) + 15)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) -
7) + 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/
4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/
56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*
sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7)
 - 1/14) - 27/2)*((11*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14
) - 7) + 224*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)
 + 224*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 3456*s
qrt(7)) + 608*x - 220*I*sqrt(7) - 616*sqrt(53/98*I*sqrt(7) - 1/14) + 636) -
 1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/1
4) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)
^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7
) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*
sqrt(7) - 1/14) - 27/2) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 7*sqrt(-53/98*I*
sqrt(7) - 1/14) - 7)*log(-49/4*(55*I*sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1
/14) + 147)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 -
2744*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 11/16*(
196*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 35*I*sqr
t(7) + 98*sqrt(53/98*I*sqrt(7) - 1/14) + 15)*(-5*I*sqrt(7) + 14*sqrt(-53/98
*I*sqrt(7) - 1/14) - 7) - 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I
*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7)
- 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21
)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7
*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((11*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(5
3/98*I*sqrt(7) - 1/14) - 7) + 224*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/98*I
*sqrt(7) - 1/14) - 7) + 224*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7)
- 1/14) - 7) + 3456*sqrt(7)) + 608*x - 220*I*sqrt(7) - 616*sqrt(53/98*I*sqr
t(7) - 1/14) + 636) - 1/28*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) -
 7)*log(3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^3 -
 1029*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 304*x
- 715/2*I*sqrt(7) - 1001*sqrt(53/98*I*sqrt(7) - 1/14) - 2871/2) + x
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.21

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = x + \operatorname{RootSum}\left(343t^4-343t^3+294t^2-336t+128,\left(t\mapsto t\log\left(\frac{3773t^3}{304}-\frac{1029t^2}{304}+\frac{1001t}{152}+x-\frac{121}{19}\right)\right)\right)$$

```
[In] integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
[Out] x + RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(_t, _t
*log(3773*_t**3/304 - 1029*_t**2/304 + 1001*_t/152 + x - 121/19)))
```

Maxima [F]

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{(2\,x^3+3\,x^2+x+5)x}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
[Out] x + 2*integrate((x^3 - 2*x^2 + 2*x - 1)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Giac [F]

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{(2\,x^3+3\,x^2+x+5)x}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.80

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = x+\left(\sum_{k=1}^{4}\ln\left(\frac{115\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)}{8}+15\,x-\frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)\,x\,137}{8}+\frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^2\,x\,133}{8}-\frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^3\,x\,147}{4}-\frac{189\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^2}{16}+\frac{49\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^3}{16}-4\right)\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)\right)$$

```
[In] int((x*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] x + symsum(log((115*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)
)/8 + 15*x - (137*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)*x
)/8 + (133*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2*x)/8 -
 (147*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3*x)/4 - (189
*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2)/16 + (49*root(z
^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3)/16 - 4)*root(z^4 - z^3
 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k), k, 1, 4)
```

3.253 $\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4} dx$

Optimal result 1686
Rubi [A] (verified) 1687
Mathematica [C] (verified) 1689
Maple [C] (verified) 1689
Fricas [B] (verification not implemented) 1690
Sympy [A] (verification not implemented) 1691
Maxima [F] 1691
Giac [F] 1692
Mupad [B] (verification not implemented) 1692

Optimal result

Integrand size = 32, antiderivative size = 198

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4} dx = \frac{(19i+7\sqrt{7}) \arctan\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2(35+i\sqrt{7})}}\right)}{\sqrt{14(35+i\sqrt{7})}} - \frac{(19i-7\sqrt{7}) \arctan\left(\frac{1+i\sqrt{7}+8x}{\sqrt{2(35-i\sqrt{7})}}\right)}{\sqrt{14(35-i\sqrt{7})}} + \frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right) + \frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)$$

```
[Out] 1/28*ln(4+4*x^2+x*(1+I*7^(1/2)))*(7-5*I*7^(1/2))+1/28*ln(4+4*x^2+x*(1-I*7^(
1/2)))*(7+5*I*7^(1/2))-arctan((1+8*x+I*7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(19
*I-7*7^(1/2))/(490-14*I*7^(1/2))^(1/2)+arctan((1+8*x-I*7^(1/2))/(70+2*I*7^(
1/2))^(1/2))*(19*I+7*7^(1/2))/(490+14*I*7^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2111, 648, 632, 210, 642}

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = \frac{(7\sqrt{7}+19i)\arctan\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2(35+i\sqrt{7})}}\right)}{\sqrt{14(35+i\sqrt{7})}} - \frac{(-7\sqrt{7}+19i)\arctan\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2(35-i\sqrt{7})}}\right)}{\sqrt{14(35-i\sqrt{7})}} + \frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right) + \frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)$$

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] ((19*I + 7*Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])
/Sqrt[14*(35 + I*Sqrt[7])] - ((19*I - 7*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*
x)/Sqrt[2*(35 - I*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7])] + ((7 + (5*I)*Sqrt[
7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x^2])/28 + ((7 - (5*I)*Sqrt[7])*Log[4 + (
1 + I*Sqrt[7])*x + 4*x^2])/28
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2111

```
Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4),
 x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B =
 Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[
(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*
x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A -
2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rubi steps

$$\text{integral} = \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$

$$= -\left(\frac{1}{28}\left(-7+5i\sqrt{7}\right)\int \frac{1+i\sqrt{7}+8x}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx\right) + \frac{1}{28}\left(7+5i\sqrt{7}\right)\int \frac{1-i\sqrt{7}+8x}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx - \frac{1}{14}\left(-49+19i\sqrt{7}\right)\int \frac{1}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx + \frac{1}{14}\left(49+19i\sqrt{7}\right)\int \frac{1}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx$$

$$= \frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right) + \frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right) - \frac{1}{7}\left(49-19i\sqrt{7}\right)\text{Subst}\left(\int \frac{1}{-2\left(35-i\sqrt{7}\right)-x^2}\,dx, x, 1+i\sqrt{7}+8x\right) - \frac{1}{7}\left(49+19i\sqrt{7}\right)\text{Subst}\left(\int \frac{1}{-2\left(35+i\sqrt{7}\right)-x^2}\,dx, x, 1-i\sqrt{7}+8x\right)$$

$$= \frac{(19i+7\sqrt{7})\tan^{-1}\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}} - \frac{(19i-7\sqrt{7})\tan^{-1}\left(\frac{1+i\sqrt{7}+8x}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}} + \frac{1}{28}\Big(7 + 5i\sqrt{7}\Big)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right) + \frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.45

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx$$
$$= \text{RootSum}\Big[2+\#1+5\#1^2+\#1^3 + 2\#1^4\&, \frac{5\log(x-\#1)+\log(x-\#1)\#1+3\log(x-\#1)\#1^2+2\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\Big]$$

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
[Out] RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (5*Log[x - #1] + Log[x - #1]*#1
 + 3*Log[x - #1]*#1^2 + 2*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) &
 ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.29

method	result	size
default	$\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)} \frac{\left(2\_R^3+3\_R^2+\_R+5\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}$	58
risch	$\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)} \frac{\left(2\_R^3+3\_R^2+\_R+5\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}$	58

```
[In] int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
[Out] sum((2*_R^3+3*_R^2+_R+5)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=RootOf(2*_Z^4+
_Z^3+5*_Z^2+_Z+2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1189 vs. 2(129) = 258.

Time = 1.00 (sec) , antiderivative size = 1189, normalized size of antiderivative = 6.01

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/1
4) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)
^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7
) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*
sqrt(7) - 1/14) - 27/2) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 7*sqrt(-53/98*I*
sqrt(7) - 1/14) - 7)*log(49/4*(105*I*sqrt(7) + 294*sqrt(53/98*I*sqrt(7) - 1
/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 +
4900*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 1/16*(4
116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 735*I*sq
rt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-5*I*sqrt(7) + 14*sqrt(-53
/98*I*sqrt(7) - 1/14) - 7) + 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/9
8*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(
7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) +
 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7)
- 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((21*sqrt(7)*(5*I*sqrt(7) + 14*sqr
t(53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/9
8*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(
7) - 1/14) - 7) + 7040*sqrt(7)) + 608*x + 325*I*sqrt(7) + 910*sqrt(53/98*I*
sqrt(7) - 1/14) - 1247) + 1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sq
rt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98
*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) -
 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*
sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2) + 7*sqrt(53/98*I*sqrt(7) -
 1/14) + 7*sqrt(-53/98*I*sqrt(7) - 1/14) + 7)*log(49/4*(105*I*sqrt(7) + 294
*sqrt(53/98*I*sqrt(7) - 1/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sq
rt(7) - 1/14) + 1/4)^2 + 4900*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) -
 1/14) + 1/4)^2 + 1/16*(4116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) -
1/14) + 1/4)^2 + 735*I*sqrt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-
5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 1/16*sqrt(-21*(5/28*I
*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7)
 - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(
53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/
14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((21*sqrt
(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7))*(-5*
```

```
I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7)*(5*I*sqrt(7
) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 7040*sqrt(7)) + 608*x + 325*I*sq
rt(7) + 910*sqrt(53/98*I*sqrt(7) - 1/14) - 1247) - 1/28*(-5*I*sqrt(7) + 14*
sqrt(-53/98*I*sqrt(7) - 1/14) - 7)*log(-49/4*(105*I*sqrt(7) + 294*sqrt(53/9
8*I*sqrt(7) - 1/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/
14) + 1/4)^2 + 7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1
/4)^3 - 7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 -
 1/16*(4116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 +
735*I*sqrt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-5*I*sqrt(7) + 14*
sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 304*x - 2205/2*I*sqrt(7) - 3087*sqrt(5
3/98*I*sqrt(7) - 1/14) - 3025/2) - 1/28*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt
(7) - 1/14) - 7)*log(-7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/
14) + 1/4)^3 + 2303*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1
/4)^2 + 304*x + 1555/2*I*sqrt(7) + 2177*sqrt(53/98*I*sqrt(7) - 1/14) + 5823
/2)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.23

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx$$
$$= \operatorname{RootSum}\left(343t^4-343t^3+294t^2-336t+128,\left(t\mapsto t\log\left(-\frac{7203t^3}{304}+\frac{2303t^2}{304}-\frac{2177t}{152}+x+\frac{250}{19}\right)\right)\right)$$

```
[In] integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(_t, _t*log
(-7203*_t**3/304 + 2303*_t**2/304 - 2177*_t/152 + x + 250/19)))
```

Maxima [F]

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{2\,x^3+3\,x^2+x+5}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Giac [F]

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{2\,x^3+3\,x^2+x+5}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Mupad [B] (verification not implemented)

Time = 9.83 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.91

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = \sum_{k=1}^{4} \ln\left(-\frac{193\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)}{8}+4\,x - \frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)\,x\,137}{8} + \frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^2\,x\,651}{16} - \frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^3\,x\,147}{4} + \frac{273\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^2}{16} + \frac{49\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^3}{16}+7\right)\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)$$

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
[Out] symsum(log(4*x - (193*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z,
k))/8 - (137*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)*x)/8 +
 (651*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2*x)/16 - (14
7*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3*x)/4 + (273*roo
t(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2)/16 + (49*root(z^4 -
 z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3)/16 + 7)*root(z^4 - z^3 + (
6*z^2)/7 - (48*z)/49 + 128/343, z, k), k, 1, 4)
```

3.254 $\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx$

Optimal result 1693
Rubi [A] (verified) 1694
Mathematica [C] (verified) 1696
Maple [C] (verified) 1697
Fricas [B] (verification not implemented) 1697
Sympy [A] (verification not implemented) 1699
Maxima [F] 1699
Giac [F] 1699
Mupad [B] (verification not implemented) 1700

Optimal result

Integrand size = 35, antiderivative size = 245

$$\int \frac{5+x+3x^2+2x^3}{x\,(2+x+5x^2+x^3+2x^4)}\,dx = -\frac{\left(53+i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}} + \frac{\left(53-i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}} + \frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x) + \frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x) - \frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right) - \frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)$$

```
[Out] 1/28*ln(x)*(35-9*I*7^(1/2))-1/56*ln(4*I+4*I*x^2+x*(I-7^(1/2)))*(35-9*I*7^(1
/2))+1/28*ln(x)*(35+9*I*7^(1/2))-1/56*ln(4*I+4*I*x^2+x*(I+7^(1/2)))*(35+9*I
*7^(1/2))-1/2*arctanh((I+8*I*x-7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(53+I*7^(1/
2))/(490-14*I*7^(1/2))^(1/2)+1/2*arctanh((I+8*I*x+7^(1/2))/(70+2*I*7^(1/2))
^(1/2))*(53-I*7^(1/2))/(490+14*I*7^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2112, 814, 648, 632, 212, 642}

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx = -\frac{(53+i\sqrt{7})\,\text{arctanh}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2(35-i\sqrt{7})}}\right)}{2\sqrt{14(35-i\sqrt{7})}} + \frac{(53-i\sqrt{7})\,\text{arctanh}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2(35+i\sqrt{7})}}\right)}{2\sqrt{14(35+i\sqrt{7})}} - \frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right) - \frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right) + \frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x) + \frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x)$$

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -1/2*((53 + I*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[
7])]])/Sqrt[14*(35 - I*Sqrt[7])] + ((53 - I*Sqrt[7])*ArcTanh[(I + Sqrt[7] +
 (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[14*(35 + I*Sqrt[7])]) + ((35 -
 (9*I)*Sqrt[7])*Log[x])/28 + ((35 + (9*I)*Sqrt[7])*Log[x])/28 - ((35 - (9*I
)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/56 - ((35 + (9*I)*Sqrt[7
])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/56
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2112

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[x^m*((b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x)/(2*a + (b + q)*x + 2*a*x^2)), x], x] - Dist[1/q, Int[x^m*((b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2)),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{x\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{x\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}} \\
&= -\frac{i\int \left(\frac{9+5i\sqrt{7}}{4x}+\frac{3\left(11i+\sqrt{7}\right)-2\left(9i-5\sqrt{7}\right)x}{2\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)}\right)dx}{\sqrt{7}} + \frac{i\int \left(\frac{9-5i\sqrt{7}}{4x}+\frac{3\left(11i-\sqrt{7}\right)-2\left(9i+5\sqrt{7}\right)x}{2\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)}\right)dx}{\sqrt{7}} \\
&= \frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x)+\frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x) \\
&\quad - \frac{i\int \frac{3\left(11i+\sqrt{7}\right)-2\left(9i-5\sqrt{7}\right)x}{4i+\left(i-\sqrt{7}\right)x+4ix^2}\,dx}{2\sqrt{7}} + \frac{i\int \frac{3\left(11i-\sqrt{7}\right)-2\left(9i+5\sqrt{7}\right)x}{4i+\left(i+\sqrt{7}\right)x+4ix^2}\,dx}{2\sqrt{7}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x)+\frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x)\\
&\quad -\frac{1}{56}\left(35-9i\sqrt{7}\right)\int\frac{i-\sqrt{7}+8ix}{4i+\left(i-\sqrt{7}\right)x+4ix^2}\,dx\\
&\quad -\frac{1}{56}\left(35+9i\sqrt{7}\right)\int\frac{i+\sqrt{7}+8ix}{4i+\left(i+\sqrt{7}\right)x+4ix^2}\,dx\\
&\quad -\frac{1}{28}\left(-7i+53\sqrt{7}\right)\int\frac{1}{4i+\left(i+\sqrt{7}\right)x+4ix^2}\,dx\\
&\quad +\frac{1}{28}\left(7i+53\sqrt{7}\right)\int\frac{1}{4i+\left(i-\sqrt{7}\right)x+4ix^2}\,dx\\
&= \frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x)+\frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x)\\
&\quad -\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)\\
&\quad -\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)\\
&\quad -\frac{1}{14}\left(7i-53\sqrt{7}\right)\text{Subst}\left(\int\frac{1}{2\left(35+i\sqrt{7}\right)-x^2}\,dx,x,i+\sqrt{7}+8ix\right)\\
&\quad -\frac{1}{14}\left(7i+53\sqrt{7}\right)\text{Subst}\left(\int\frac{1}{2\left(35-i\sqrt{7}\right)-x^2}\,dx,x,i-\sqrt{7}+8ix\right)\\
&= -\frac{\left(53+i\sqrt{7}\right)\tanh^{-1}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{\left(53-i\sqrt{7}\right)\tanh^{-1}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}}\\
&\quad +\frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x)+\frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x)\\
&\quad -\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)\\
&\quad -\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.41

$$
\begin{aligned}
&\int\frac{5+x+3x^2+2x^3}{x\left(2+x+5x^2+x^3+2x^4\right)}\,dx=\frac{5\log(x)}{2}-\frac{1}{2}\text{RootSum}\Big[2+\#1+5\#1^2+\#1^3\\
&\quad +2\#1^4\&,\frac{3\log(x-\#1)+19\log(x-\#1)\#1+\log(x-\#1)\#1^2+10\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\Big]
\end{aligned}
$$

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (5*Log[x])/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (3*Log[x - #1] +
 19*Log[x - #1]*#1 + Log[x - #1]*#1^2 + 10*Log[x - #1]*#1^3)/(1 + 10*#1 + 3
*#1^2 + 8*#1^3) & ]/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.21

method	result
risch	$\frac{5\ln(x)}{2}+\left(\sum_{\_R=\mathrm{RootOf}(686\_Z^4+1715\_Z^3+1372\_Z^2+448\_Z+256)}\_R\ln\left(2058\_R^3+20825\_R^2+25844\_R\right.\right.$
default	$\frac{5\ln(x)}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{\left(-10\_R^3-\_R^2-19\_R-3\right)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}\right)}{2}$

```
[In] int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/2*ln(x)+sum(_R*ln(2058*_R^3+20825*_R^2+25844*_R+8384*x+6816),_R=RootOf(68
6*_Z^4+1715*_Z^3+1372*_Z^2+448*_Z+256))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1143 vs. 2(148) = 296.

Time = 1.02 (sec) , antiderivative size = 1143, normalized size of antiderivative = 4.67

$$\int \frac{5+x+3x^2+2x^3}{x\left(2+x+5x^2+x^3+2x^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/56*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35)*log(49/4*(27*
I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56) + 1385)*(9/56*I*sqrt(7) - 1/
2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 2058*(-9/56*I*sqrt(7) - 1/2*sqr
t(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 - 5145*(-9/56*I*sqrt(7) - 1/2*sqrt(-3
7/392*I*sqrt(7) + 79/56) - 5/8)^2 + 1/64*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(
-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 945*I*sqrt(7) - 2940*sqrt(-37/392*I*s
qrt(7) + 79/56) - 28507)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56)
+ 35) + 8384*x + 1323/2*I*sqrt(7) + 2058*sqrt(-37/392*I*sqrt(7) + 79/56) +
16089/2) + 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79
/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) -
```

```
5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-
9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 1
0*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2) + 2*sqrt(37/392*I*sqrt(7) + 79/56
) + 2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5)*log(-49/4*(27*I*sqrt(7) + 84*sqr
t(-37/392*I*sqrt(7) + 79/56) + 1385)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sq
rt(7) + 79/56) - 5/8)^2 - 15680*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(
7) + 79/56) - 5/8)^2 - 1/64*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqr
t(7) + 79/56) - 5/8)^2 - 945*I*sqrt(7) - 2940*sqrt(-37/392*I*sqrt(7) + 79/5
6) - 28507)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 7/64*
sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12
*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9
*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*s
qrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*s
qrt(7) + 79/56) + 11/2)*((27*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56)
 + 1385)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 11520*I*
sqrt(7) + 35840*sqrt(-37/392*I*sqrt(7) + 79/56) - 35072) + 16768*x + 3492*I
*sqrt(7) + 10864*sqrt(-37/392*I*sqrt(7) + 79/56) + 5484) - 1/8*(2*sqrt(-12*
(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I
*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7
) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/39
2*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) +
79/56) + 11/2) - 2*sqrt(37/392*I*sqrt(7) + 79/56) - 2*sqrt(-37/392*I*sqrt(7
) + 79/56) + 5)*log(-49/4*(27*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56
) + 1385)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1
5680*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/64
*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 94
5*I*sqrt(7) - 2940*sqrt(-37/392*I*sqrt(7) + 79/56) - 28507)*(-9*I*sqrt(7) +
 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 7/64*sqrt(-12*(9/56*I*sqrt(7) -
1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqr
t(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/39
2*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/5
6) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2)*((2
7*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56) + 1385)*(-9*I*sqrt(7) + 28
*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 11520*I*sqrt(7) + 35840*sqrt(-37/39
2*I*sqrt(7) + 79/56) - 35072) + 16768*x + 3492*I*sqrt(7) + 10864*sqrt(-37/3
92*I*sqrt(7) + 79/56) + 5484) - 1/56*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(
7) + 79/56) + 35)*log(2058*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) +
79/56) - 5/8)^3 + 20825*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/
56) - 5/8)^2 + 8384*x - 8307/2*I*sqrt(7) - 12922*sqrt(-37/392*I*sqrt(7) + 7
9/56) - 18673/2) + 5/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.24

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx = \frac{5\log(x)}{2} + \text{RootSum}\left(686t^4+1715t^3+1372t^2+448t+256, \left(t\mapsto t\log\left(-\frac{160344611t^4}{532759184}-\frac{16880402t^3}{33297449}+\frac{401052\cdots}{21310\cdots}\right.\right.\right.$$

```
[In] integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] 5*log(x)/2 + RootSum(686*_t**4 + 1715*_t**3 + 1372*_t**2 + 448*_t + 256, La
mbda(_t, _t*log(-160344611*_t**4/532759184 - 16880402*_t**3/33297449 + 4010
520787*_t**2/2131036736 + 1537535671*_t/532759184 + x + 46660495/66594898))
)
```

Maxima [F]

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx = \int \frac{2\,x^3+3\,x^2+x+5}{(2\,x^4+x^3+5\,x^2+x+2)x}\,dx$$

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
```

```
[Out] -1/2*integrate((10*x^3 + x^2 + 19*x + 3)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
+ 5/2*log(x)
```

Giac [F]

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx = \int \frac{2\,x^3+3\,x^2+x+5}{(2\,x^4+x^3+5\,x^2+x+2)x}\,dx$$

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x), x)
```

Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.97

$$\int \frac{5+x+3x^2+2x^3}{x\,(2+x+5x^2+x^3+2x^4)}\,dx = \frac{5\,\ln(x)}{2} + \left(\sum_{k=1}^{4}\ln\left(\frac{223\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}{8} - \frac{31\,x}{2} + \frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)\,x\,71}{16} - \frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^2\,x\,4463}{64} + \frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^3\,x\,1449}{16} + \frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^4\,x\,3675}{32} + \frac{257\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^2}{32} + \frac{1673\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^3}{64} - \frac{441\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^4}{32} + 10\right)\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)\right)$$

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] (5*log(x))/2 + symsum(log((223*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 1
28/343, z, k))/8 - (31*x)/2 + (71*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49
+ 128/343, z, k)*x)/16 - (4463*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 1
28/343, z, k)^2*x)/64 + (1449*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 12
8/343, z, k)^3*x)/16 + (3675*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128
/343, z, k)^4*x)/32 + (257*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/3
43, z, k)^2)/32 + (1673*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343,
 z, k)^3)/64 - (441*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z,
k)^4)/32 + 10)*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k), k
, 1, 4)
```

3.255 $\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx$

Optimal result 1701
Rubi [A] (verified) 1702
Mathematica [C] (verified) 1705
Maple [C] (verified) 1705
Fricas [B] (verification not implemented) 1706
Sympy [B] (verification not implemented) 1707
Maxima [F] 1726
Giac [F] 1726
Mupad [B] (verification not implemented) 1727

Optimal result

Integrand size = 35, antiderivative size = 281

$$\begin{aligned}\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx &= -\frac{35-9i\sqrt{7}}{28x}-\frac{35+9i\sqrt{7}}{28x}\\ &+\frac{11\left(9+5i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}}\\ &-\frac{11\left(9-5i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35+i\sqrt{7}\right)}}\\ &-\frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x)-\frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x)\\ &+\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)\\ &+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)\end{aligned}$$

```
[Out] 1/28*(-35+9*I*7^(1/2))/x+1/28*(-35-9*I*7^(1/2))/x-3/56*ln(x)*(7-11*I*7^(1/2
))+3/112*ln(4*I+4*I*x^2+x*(I+7^(1/2)))*(7-11*I*7^(1/2))-3/56*ln(x)*(7+11*I*
7^(1/2))+3/112*ln(4*I+4*I*x^2+x*(I-7^(1/2)))*(7+11*I*7^(1/2))+11/4*arctanh(
(I+8*I*x-7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(9+5*I*7^(1/2))/(490-14*I*7^(1/2)
)^(1/2)-11/4*arctanh((I+8*I*x+7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(9-5*I*7^(1/
2))/(490+14*I*7^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2112, 814, 648, 632, 212, 642}

$$\begin{aligned}\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+5x^2+x^3+2x^4\right)}\,dx =\ & \frac{11\left(9+5i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}}\\ & -\frac{11\left(9-5i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35+i\sqrt{7}\right)}}\\ & +\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)\\ & +\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)\\ & -\frac{35+9i\sqrt{7}}{28x}-\frac{35-9i\sqrt{7}}{28x}\\ & -\frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x)-\frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x)\end{aligned}$$

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -1/28*(35 - (9*I)*Sqrt[7])/x - (35 + (9*I)*Sqrt[7])/(28*x) + (11*(9 + (5*I)
*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(4*Sqr
t[14*(35 - I*Sqrt[7])]) - (11*(9 - (5*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8
*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(4*Sqrt[14*(35 + I*Sqrt[7])]) - (3*(7 - (
11*I)*Sqrt[7])*Log[x])/56 - (3*(7 + (11*I)*Sqrt[7])*Log[x])/56 + (3*(7 + (1
1*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/112 + (3*(7 - (11*I)*
Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/112
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2112

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[x^m*((b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x)/(2*a + (b + q)*x + 2*a*x^2)), x], x] - Dist[1/q, Int[x^m*((b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2)),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{x^2\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{x^2\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}} \\
&= -\frac{i\int \left(\frac{9+5i\sqrt{7}}{4x^2}+\frac{3\left(11-i\sqrt{7}\right)}{8x}+\frac{-7\left(9i-5\sqrt{7}\right)-6\left(11i+\sqrt{7}\right)x}{4\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)}\right)dx}{\sqrt{7}} \\
&\quad + \frac{i\int \left(\frac{9-5i\sqrt{7}}{4x^2}+\frac{3\left(11+i\sqrt{7}\right)}{8x}+\frac{-7\left(9i+5\sqrt{7}\right)-6\left(11i-\sqrt{7}\right)x}{4\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)}\right)dx}{\sqrt{7}}
\end{aligned}$$

$$=-\frac{35-9i\sqrt{7}}{28x}-\frac{35+9i\sqrt{7}}{28x}-\frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x)-\frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x)-\frac{i\int\frac{-7\left(9i-5\sqrt{7}\right)-6\left(11i+\sqrt{7}\right)x}{4i+\left(i-\sqrt{7}\right)x+4ix^2}\,dx}{4\sqrt{7}}+\frac{i\int\frac{-7\left(9i+5\sqrt{7}\right)-6\left(11i-\sqrt{7}\right)x}{4i+\left(i+\sqrt{7}\right)x+4ix^2}\,dx}{4\sqrt{7}}$$

$$=-\frac{35-9i\sqrt{7}}{28x}-\frac{35+9i\sqrt{7}}{28x}-\frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x)-\frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x)-\frac{1}{56}\left(11\left(35i-9\sqrt{7}\right)\right)\int\frac{1}{4i+(i+\sqrt{7})\,x+4ix^2}\,dx+\frac{1}{112}\left(3\left(7-11i\sqrt{7}\right)\right)\int\frac{i+\sqrt{7}+8ix}{4i+(i+\sqrt{7})\,x+4ix^2}\,dx+\frac{1}{112}\left(3\left(7+11i\sqrt{7}\right)\right)\int\frac{i-\sqrt{7}+8ix}{4i+(i-\sqrt{7})\,x+4ix^2}\,dx-\frac{1}{56}\left(11\left(35i+9\sqrt{7}\right)\right)\int\frac{1}{4i+(i-\sqrt{7})\,x+4ix^2}\,dx$$

$$=-\frac{35-9i\sqrt{7}}{28x}-\frac{35+9i\sqrt{7}}{28x}-\frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x)-\frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x)+\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)+\frac{1}{28}\left(11\left(35i-9\sqrt{7}\right)\right)\text{Subst}\left(\int\frac{1}{2\,(35+i\sqrt{7})-x^2}\,dx,x,i+\sqrt{7}+8ix\right)+\frac{1}{28}\left(11\left(35i+9\sqrt{7}\right)\right)\text{Subst}\left(\int\frac{1}{2\,(35-i\sqrt{7})-x^2}\,dx,x,i-\sqrt{7}+8ix\right)$$

$$=-\frac{35-9i\sqrt{7}}{28x}-\frac{35+9i\sqrt{7}}{28x}+\frac{11(9+5i\sqrt{7})\tanh^{-1}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\,(35-i\sqrt{7})}}-\frac{11(9-5i\sqrt{7})\tanh^{-1}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{4\sqrt{14\,(35+i\sqrt{7})}}-\frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x)-\frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x)+\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.39

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx = -\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{1}{4}\text{RootSum}\Big[2+\#1+5\#1^2+\#1^3+2\#1^4\&,\frac{-35\log(x-\#1)+13\log(x-\#1)\#1-17\log(x-\#1)\#1^2+6\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\Big]$$

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(2*x) - (3*Log[x])/4 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (-35*
Log[x - #1] + 13*Log[x - #1]*#1 - 17*Log[x - #1]*#1^2 + 6*Log[x - #1]*#1^3)
/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.21

method	result
risch	$-\frac{5}{2x}+\frac{\left(\sum_{\_R=\text{RootOf}\left(686\_Z^4-1029\_Z^3+6272\_Z^2+10752\_Z+4096\right)}\_R\ln\left(-45962\_R^3+98735\_R^2-497168\_R+61952x-38\right.\right.}{2}$
default	$-\frac{5}{2x}-\frac{3\ln(x)}{4}+\frac{\left(\sum_{\_R=\text{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(6\_R^3-17\_R^2+13\_R-35\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}\right)}{4}$

```
[In] int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] -5/2/x+1/2*sum(_R*ln(-45962*_R^3+98735*_R^2-497168*_R+61952*x-384256),_R=Ro
otOf(686*_Z^4-1029*_Z^3+6272*_Z^2+10752*_Z+4096))-3/4*ln(x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1245 vs. $2(172) = 344$.

Time = 1.01 (sec) , antiderivative size = 1245, normalized size of antiderivative = 4.43

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx = \text{Too large to display}$$

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] -1/224*(2*x*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*log
(91924*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^3
- 49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2*(-2211*I*sqrt(7) + 3752*sqrt(2101/1568*I*sqrt(7) - 55/32) - 3839) - 1/256
*(210112*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2 - 46431*I*sqrt(7) + 78792*sqrt(2101/1568*I*sqrt(7) - 55/32) - 117483)*(33
*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - 68943*(33/112*I*
sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 15488*x + 61908
*I*sqrt(7) - 105056*sqrt(2101/1568*I*sqrt(7) - 55/32) + 123428) + 2*x*(-33*
I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21)*log(-91924*(33/112*I
*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^3 + 98735*(33/112*
I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 15488*x - 146
487/2*I*sqrt(7) + 124292*sqrt(2101/1568*I*sqrt(7) - 55/32) - 285347/2) + (4
*sqrt(7)*sqrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32
) + 3/16)^2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/3
2) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) -
 21)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*s
qrt(7) - 84*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2)*x - x*(33*I*sqrt(7)
 + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - x*(-33*I*sqrt(7) + 56*sqrt
(2101/1568*I*sqrt(7) - 55/32) - 21) - 84*x)*log(49/4*(-33/112*I*sqrt(7) - 1
/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2*(-2211*I*sqrt(7) + 3752*sqr
t(2101/1568*I*sqrt(7) - 55/32) - 3839) + 1/256*(210112*(33/112*I*sqrt(7) -
1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 46431*I*sqrt(7) + 78792*s
qrt(2101/1568*I*sqrt(7) - 55/32) - 117483)*(33*I*sqrt(7) + 56*sqrt(-2101/15
68*I*sqrt(7) - 55/32) - 21) - 29792*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*
I*sqrt(7) - 55/32) + 3/16)^2 + 1/256*((67*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(
2101/1568*I*sqrt(7) - 55/32) - 21) - 2432*sqrt(7))*(33*I*sqrt(7) + 56*sqrt(
-2101/1568*I*sqrt(7) - 55/32) - 21) - 2432*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt
(2101/1568*I*sqrt(7) - 55/32) - 21) + 147456*sqrt(7))*sqrt(-336*(33/112*I*s
qrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*(-33/112*I*s
qrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt
(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(
2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*s
```

```
qrt(7) - 55/32) - 1859/2) + 30976*x + 22671/2*I*sqrt(7) - 19236*sqrt(2101/1
568*I*sqrt(7) - 55/32) + 53979/2) - (4*sqrt(7)*sqrt(-336*(33/112*I*sqrt(7)
- 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(7)
- 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 5
6*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101/15
68*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(7)
- 55/32) - 1859/2)*x + x*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/
32) - 21) + x*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) +
 84*x)*log(49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32)
 + 3/16)^2*(-2211*I*sqrt(7) + 3752*sqrt(2101/1568*I*sqrt(7) - 55/32) - 3839
) + 1/256*(210112*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32)
 + 3/16)^2 - 46431*I*sqrt(7) + 78792*sqrt(2101/1568*I*sqrt(7) - 55/32) - 11
7483)*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - 29792*(
33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/256*
((67*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) -
2432*sqrt(7))*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) -
 2432*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) +
 147456*sqrt(7))*sqrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7)
 - 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7
) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) -
55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) +
99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2) + 30976*x +
 22671/2*I*sqrt(7) - 19236*sqrt(2101/1568*I*sqrt(7) - 55/32) + 53979/2) + 1
68*x*log(x) + 560)/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25507 vs. 2(241) = 482.

Time = 18.69 (sec) , antiderivative size = 25507, normalized size of antiderivative = 90.77

$$\int \frac{5 + x + 3x^2 + 2x^3}{x^2\left(2 + x + 5x^2 + x^3 + 2x^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] -3*log(x)/4 + (3/16 - sqrt(-55/256 + 11*sqrt(77)/196))*log(x**2 + x*(108964
79943156192*sqrt(77)/(-39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 81
5992034457600 + 6974290892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454628
044800*sqrt(77)) + 1720992726634016*sqrt(7)*sqrt(-245 + 64*sqrt(77))/(-3936
5093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 815992034457600 + 69742908928
00*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454628044800*sqrt(77)) + 39603456
8160*sqrt(14)*sqrt(-245 + 64*sqrt(77))*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*
sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983
777)/(-39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 815992034457600 +
6974290892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454628044800*sqrt(77))
```

+ 1300300581888*sqrt(154)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) -
21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)/(-39365
093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 815992034457600 + 697429089280
0*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454628044800*sqrt(77)) - 278094051
039*sqrt(22)*sqrt(-245 + 64*sqrt(77))*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*s
qrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 59837
77)/(-39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 815992034457600 + 6
974290892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454628044800*sqrt(77))
- 29480043023893*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21
120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)/(-3936509
3785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 815992034457600 + 6974290892800*
sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454628044800*sqrt(77)) - 49949438061
3858*sqrt(11)*sqrt(-245 + 64*sqrt(77))/(-39365093785600*sqrt(7)*sqrt(-245 +
64*sqrt(77)) - 815992034457600 + 6974290892800*sqrt(11)*sqrt(-245 + 64*sqr
t(77)) + 225454628044800*sqrt(77)) - 133336449027059894/(-39365093785600*sq
rt(7)*sqrt(-245 + 64*sqrt(77)) - 815992034457600 + 6974290892800*sqrt(11)*s
qrt(-245 + 64*sqrt(77)) + 225454628044800*sqrt(77))) + 624761078719362006842
35707503295184*sqrt(77)/(-128202751499601473382069043200000*sqrt(77) - 9780
9811145429330359222272000 0*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 3531716786284
21216922828800000*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 13763884316485317499 5
608862720000) + 3325655347490676642136637231706384*sqrt(7)*sqrt(-245 + 64*s
qrt(77))/(-128202751499601473382069043200000*sqrt(77) - 978098111454293303 59
2222720000*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 353171678628421216922828800000
0*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 137638843164853174995608862720000) +
1259144806367748744302867373 6328*sqrt(154)*sqrt(-62589*sqrt(11)*sqrt(-245 +
64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) +
5983777)/(-128202751499601473382069043200000*sqrt(77) - 978098111454293303 59
2222720000*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 353171678628421216922828800000
0*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 137638843164853174995608862720000) +
12752626869860132520630997497 36*sqrt(14)*sqrt(-245 + 64*sqrt(77))*sqrt(-625
89*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77
)) - 103712*sqrt(77) + 5983777)/(-128202751499601473382069043200000*sqrt(77)
- 9780981114542933035922272 0000*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 353171
67862842121692282880 0000*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 13763884316485
3174995608862720000) - 1213346648248587045336001776855*sqrt(22)*sqrt(-245 +
64*sqrt(77))*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)
*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)/(-128202751499601473
38206904320000*sqrt(77) - 97809811145429330359222272 0000*sqrt(7)*sqrt(-245
+ 64*sqrt(77)) + 35317167862842121692282880 0000*sqrt(11)*sqrt(-245 + 64*sqr
t(77)) + 137638843164853174995608862720000) - 988335241992875085140736227 42
205*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*s
qrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)/(-12820275149960147338
206904320000*sqrt(77) - 978098111454293303592222 720000*sqrt(7)*sqrt(-245 +
64*sqrt(77)) + 353171678628421216922828800000*sqrt(11)*sqrt(-245 + 64*sqrt(
77)) + 137638843164853174995608862720000) - 3523282605099669306216811941636

850*sqrt(11)*sqrt(-245 + 64*sqrt(77))/(-12820275149960147338206904320000*sq
rt(77) - 97809811145429330359222272000 0*sqrt(7)*sqrt(-245 + 64*sqrt(77)) +
35317167862842121692282880000 0*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 13763884
316485317499560886272 0000) - 4969412991524823551767711136080 17254/(-1282027
51499601473382069043200 00*sqrt(77) - 97809811145429330359222272000 0*sqrt(7)
*sqrt(-245 + 64*sqrt(77)) + 35317167862842121692282880000 0*sqrt(11)*sqrt(-2
45 + 64*sqrt(77)) + 137638843164853174995608862720000)) + (3/16 + sqrt(-55/
256 + 11*sqrt(77)/196))*log(x**2 + x*(133336449027059894/(-225454628044800*
sqrt(77) - 39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 6974290892800*
sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 815992034457600) + 1720992726634016*sqr
t(7)*sqrt(-245 + 64*sqrt(77))/(-225454628044800*sqrt(77) - 39365093785600*s
qrt(7)*sqrt(-245 + 64*sqrt(77)) + 6974290892800*sqrt(11)*sqrt(-245 + 64*sqr
t(77)) + 815992034457600) + 278094051039*sqrt(22)*sqrt(-245 + 64*sqrt(77))*
sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt
(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)/(-225454628044800*sqrt(77) - 39365
093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 6974290892800*sqrt(11)*sqrt(-2
45 + 64*sqrt(77)) + 815992034457600) + 1300300581888*sqrt(154)*sqrt(-103712
*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-2
45 + 64*sqrt(77)) + 5983777)/(-225454628044800*sqrt(77) - 39365093785600*sq
rt(7)*sqrt(-245 + 64*sqrt(77)) + 6974290892800*sqrt(11)*sqrt(-245 + 64*sqrt
(77)) + 815992034457600) - 499494380613858*sqrt(11)*sqrt(-245 + 64*sqrt(77)
)/(-225454628044800*sqrt(77) - 39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(7
7)) + 6974290892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 815992034457600) -
396034568160*sqrt(14)*sqrt(-245 + 64*sqrt(77))*sqrt(-103712*sqrt(77) + 2112
0*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)
) + 5983777)/(-225454628044800*sqrt(77) - 39365093785600*sqrt(7)*sqrt(-245
+ 64*sqrt(77)) + 6974290892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 81599203
4457600) - 10896479943156192*sqrt(77)/(-225454628044800*sqrt(77) - 39365093
785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 6974290892800*sqrt(11)*sqrt(-245
+ 64*sqrt(77)) + 815992034457600) - 29480043023893*sqrt(2)*sqrt(-103712*sqr
t(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 +
 64*sqrt(77)) + 5983777)/(-225454628044800*sqrt(77) - 39365093785600*sqrt(7
)*sqrt(-245 + 64*sqrt(77)) + 6974290892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)
) + 815992034457600)) + 4969412991524823551767711136080 17254/(-137638843164
853174995608862720000 - 97809811145429330359222272000 0*sqrt(7)*sqrt(-245 +
64*sqrt(77)) + 35317167862842121692282880000 0*sqrt(11)*sqrt(-245 + 64*sqrt(
77)) + 12820275149960147338206904320000*sqrt(77)) + 1259144806367748744302 8
673736328*sqrt(154)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sq
rt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)/(-137638843164
853174995608862720000 - 97809811145429330359222272000 0*sqrt(7)*sqrt(-245 +
64*sqrt(77)) + 35317167862842121692282880000 0*sqrt(11)*sqrt(-245 + 64*sqrt(
77)) + 12820275149960147338206904320000*sqrt(77)) + 1213346648248587045336 0
01776855*sqrt(22)*sqrt(-245 + 64*sqrt(77))*sqrt(-103712*sqrt(77) + 21120*sq
rt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) +
5983777)/(-137638843164853174995608862720000 - 97809811145429330359222272 00

00*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 35317167862842121692282880000*sqrt(1
1)*sqrt(-245 + 64*sqrt(77)) + 128202751499601473382069043200000*sqrt(77)) +
33256553474906766421366372317063 84*sqrt(7)*sqrt(-245 + 64*sqrt(77))/(-13763
8843164853174995608862720000 - 9780981114542933035922227200000*sqrt(7)*sqrt(
-245 + 64*sqrt(77)) + 35317167862842121692282880000*sqrt(11)*sqrt(-245 + 6
4*sqrt(77)) + 128202751499601473382069043200000*sqrt(77)) - 3523282605099669
306216811941636850*sqrt(11)*sqrt(-245 + 64*sqrt(77))/(-137638843164853174999
5608862720000 - 9780981114542933035922227200000*sqrt(7)*sqrt(-245 + 64*sqrt(
77)) + 35317167862842121692282880000*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 1
28202751499601473382069043200000*sqrt(77)) - 127526268698601325206309974973 6
*sqrt(14)*sqrt(-245 + 64*sqrt(77))*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sq
rt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)
/(-1376388431648531749956088627200000 - 9780981114542933035922227200000*sqrt(
7)*sqrt(-245 + 64*sqrt(77)) + 35317167862842121692282880000*sqrt(11)*sqrt(
-245 + 64*sqrt(77)) + 128202751499601473382069043200000*sqrt(77)) - 98833524
199287508514073622742205*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt
(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)/(
-1376388431648531749956088627200000 - 9780981114542933035922227200000*sqrt(7)
*sqrt(-245 + 64*sqrt(77)) + 35317167862842121692282880000*sqrt(11)*sqrt(-2
45 + 64*sqrt(77)) + 128202751499601473382069043200000*sqrt(77)) - 6247610787
193620068423570750329518 4*sqrt(77)/(-1376388431648531749956088627200000 - 97
80981114542933035922227200000*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 35317167862
842121692282880000*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 1282027514996014733
8206904320000*sqrt(77))) + 2*sqrt((-8673280*sqrt(7)*sqrt(-245 + 64*sqrt(77)
) - 5726336*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 49674240*sqrt(77) + 5507631
36)/(-14049280*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 295034880) + 11*sqrt(2)*
sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 6
4*sqrt(77)) - 103712*sqrt(77) + 5983777)/(560*(-21 + sqrt(11)*sqrt(-245 + 6
4*sqrt(77)))))*atan(x*(-8133283840*sqrt(55)*sqrt(-245 + 64*sqrt(77))*sqrt(-
21 + sqrt(11)*sqrt(-245 + 64*sqrt(77)))/(-180730470976*sqrt(7)*sqrt(-55881
- 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77))
- 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581
*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)))
- 57446630906*sqrt(11)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589
*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77))
- 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880
*sqrt(7)*sqrt(-245 + 64*sqrt(77))) - 886040320*sqrt(77)*sqrt(-245 + 64*sqrt
(77))*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-2
45 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77
) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-24
5 + 64*sqrt(77))) - 16932307*sqrt(22)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*s
qrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 59837
77)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245
+ 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77)
+ 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245

+ 64*sqrt(77))) + 16252973*sqrt(2)*sqrt(-245 + 64*sqrt(77))*sqrt(-62589*sqr
t(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 1
03712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62
589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(7
7)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) +
880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 56909334790*sqrt(-245 + 64*sqrt(77)
)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 +
 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) +
5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 +
64*sqrt(77)))) + 170798960640*sqrt(5)*sqrt(-21 + sqrt(11)*sqrt(-245 + 64*sq
rt(77)))/(-180730470976*sqrt(7)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sq
rt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*
sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(7
7)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) - 57446630906*sqrt(11)*sqrt(-55
881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(7
7)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) +
 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77
))) - 886040320*sqrt(77)*sqrt(-245 + 64*sqrt(77))*sqrt(-55881 - 5040*sqrt(7
7) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(
7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqr
t(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) - 16932307*sq
rt(22)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-
245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sqrt(77)
 + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)
*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(
-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 16252973*sqrt
(2)*sqrt(-245 + 64*sqrt(77))*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77))
- 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)*sqrt(
-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqr
t(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777
) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt
(77))) + 56909334790*sqrt(-245 + 64*sqrt(77))*sqrt(-55881 - 5040*sqrt(77) +
 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*s
qrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-2
45 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))))) + 7157952434169
7856*sqrt(385)*sqrt(-21 + sqrt(11)*sqrt(-245 + 64*sqrt(77)))/(-251082732047
63350*sqrt(11)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11
)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 10371
2*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)
*sqrt(-245 + 64*sqrt(77))) - 5279630978565*sqrt(22)*sqrt(-62589*sqrt(11)*sq
rt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sq
rt(77) + 5983777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt
(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 10
3712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt
(7)*sqrt(-245 + 64*sqrt(77))) - 288498310874070*sqrt(-245 + 64*sqrt(77))*sq

rt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*
sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983
777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*s
qrt(77))) - 255762019205*sqrt(2)*sqrt(-245 + 64*sqrt(77))*sqrt(-62589*sqrt(
11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103
712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-6258
9*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)
) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 88
0*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 3417338724800*sqrt(14)*sqrt(-62589*sq
rt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) -
103712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-6
2589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(
77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) +
 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 76069253920*sqrt(154)*sqrt(-245 +
64*sqrt(77))*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*
sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sq
rt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*s
qrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)
*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 2556408
97900960*sqrt(77)*sqrt(-245 + 64*sqrt(77))*sqrt(-55881 - 5040*sqrt(77) + 28
*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt
(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245
+ 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 17947820059612960*
sqrt(7)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(
-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(
77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-
245 + 64*sqrt(77)))) + 8774790502440634*sqrt(55)*sqrt(-245 + 64*sqrt(77))*s
qrt(-21 + sqrt(11)*sqrt(-245 + 64*sqrt(77)))/(-25108273204763350*sqrt(11)*s
qrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64
*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 598
3777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*
sqrt(77))) - 5279630978565*sqrt(22)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqr
t(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777
)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 +
 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) +
5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 +
64*sqrt(77))) - 288498310874070*sqrt(-245 + 64*sqrt(77))*sqrt(-55881 - 5040
*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 2112
0*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(
11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) - 2557
62019205*sqrt(2)*sqrt(-245 + 64*sqrt(77))*sqrt(-62589*sqrt(11)*sqrt(-245 +
64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5
983777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(
-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(
77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-

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245 + 64*sqrt(77))) + 3417338724800*sqrt(14)*sqrt(-62589*sqrt(11)*sqrt(-245
 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77)
+ 5983777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sq
rt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sq
rt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqr
t(-245 + 64*sqrt(77))) + 76069253920*sqrt(154)*sqrt(-245 + 64*sqrt(77))*sqr
t(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*s
qrt(77)) - 103712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt
(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245
 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*
sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 255640897900960*sqrt(77
)*sqrt(-245 + 64*sqrt(77))*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-6
2589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(
77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) +
 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 1794782005961296O*sqrt(7)*sqrt(-55
881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(7
7)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) +
 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77
)))) + 22865213810736*sqrt(770)*sqrt(-21 + sqrt(11)*sqrt(-245 + 64*sqrt(77)
))*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245
+ 64*sqrt(77)) - 103712*sqrt(77) + 5983777)/(-25108273204763350*sqrt(11)*sq
rt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*
sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983
777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*s
qrt(77))) - 5279630978565*sqrt(22)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt
(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)
*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 +
64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5
983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 6
4*sqrt(77))) - 288498310874070*sqrt(-245 + 64*sqrt(77))*sqrt(-55881 - 5040*
sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120
*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(1
1)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) - 25576
2019205*sqrt(2)*sqrt(-245 + 64*sqrt(77))*sqrt(-62589*sqrt(11)*sqrt(-245 + 6
4*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 59
83777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-
245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(7
7) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-2
45 + 64*sqrt(77))) + 3417338724800*sqrt(14)*sqrt(-62589*sqrt(11)*sqrt(-245
+ 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) +
 5983777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqr
t(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqr
t(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt
(-245 + 64*sqrt(77))) + 76069253920*sqrt(154)*sqrt(-245 + 64*sqrt(77))*sqrt
(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sq
```

rt(77)) - 103712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(
2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245
+ 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*s
qrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 255640897900960*sqrt(77)
*sqrt(-245 + 64*sqrt(77))*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62
589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(7
7)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) +
880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 17947820059612960*sqrt(7)*sqrt(-558
81 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77
)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) +
581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)
))) + 1688576282291*sqrt(110)*sqrt(-245 + 64*sqrt(77))*sqrt(-21 + sqrt(11)*
sqrt(-245 + 64*sqrt(77)))*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 2
1120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)/(-251082
73204763350*sqrt(11)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*s
qrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) -
 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*s
qrt(7)*sqrt(-245 + 64*sqrt(77))) - 5279630978565*sqrt(22)*sqrt(-62589*sqrt(
11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103
712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-6258
9*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)
) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 88
0*sqrt(7)*sqrt(-245 + 64*sqrt(77))) - 288498310874070*sqrt(-245 + 64*sqrt(7
7))*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245
 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77)
+ 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245
+ 64*sqrt(77))) - 255762019205*sqrt(2)*sqrt(-245 + 64*sqrt(77))*sqrt(-62589
*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77))
 - 103712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt
(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sq
rt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)
) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 3417338724800*sqrt(14)*sqrt(-62
589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(7
7)) - 103712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*s
qrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64
*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(
77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 76069253920*sqrt(154)*sqrt(-
245 + 64*sqrt(77))*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sq
rt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)*sqrt(-55881 - 5
040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 2
1120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sq
rt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 2
55640897900960*sqrt(77)*sqrt(-245 + 64*sqrt(77))*sqrt(-55881 - 5040*sqrt(77
) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7
)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt

(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 179478200596
12960*sqrt(7)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)
*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712
*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*
sqrt(-245 + 64*sqrt(77)))) - 427612890672*sqrt(70)*sqrt(-245 + 64*sqrt(77))
*sqrt(-21 + sqrt(11)*sqrt(-245 + 64*sqrt(77)))*sqrt(-62589*sqrt(11)*sqrt(-2
45 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77
) + 5983777)/(-25108273204763350*sqrt(11)*sqrt(-55881 - 5040*sqrt(77) + 28*
sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(
-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 +
64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) - 5279630978565*sqrt(
22)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245
+ 64*sqrt(77)) - 103712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sqrt(77) +
28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sq
rt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-24
5 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) - 288498310874070*
sqrt(-245 + 64*sqrt(77))*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-625
89*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77
)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 8
80*sqrt(7)*sqrt(-245 + 64*sqrt(77))) - 255762019205*sqrt(2)*sqrt(-245 + 64*
sqrt(77))*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqr
t(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sqrt(
77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt
(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sq
rt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 3417338724
800*sqrt(14)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*
sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sq
rt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*s
qrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)
*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 7606925
3920*sqrt(154)*sqrt(-245 + 64*sqrt(77))*sqrt(-62589*sqrt(11)*sqrt(-245 + 64
*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 598
3777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-2
45 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77
) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-24
5 + 64*sqrt(77))) + 255640897900960*sqrt(77)*sqrt(-245 + 64*sqrt(77))*sqrt(
-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqr
t(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777
) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt
(77))) + 17947820059612960*sqrt(7)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)
*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 +
64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqr
t(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)))) - 237157337353167*sqrt(10)*
sqrt(-21 + sqrt(11)*sqrt(-245 + 64*sqrt(77)))*sqrt(-62589*sqrt(11)*sqrt(-24
5 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77)

+ 5983777)/(-25108273204763350*sqrt(11)*sqrt(-55881 - 5040*sqrt(77) + 28*s
qrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-
245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 +
64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) - 5279630978565*sqrt(2
2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245
+ 64*sqrt(77)) - 103712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sqrt(77) + 2
8*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqr
t(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245
+ 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) - 288498310874070*s
qrt(-245 + 64*sqrt(77))*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-6258
9*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)
) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 88
0*sqrt(7)*sqrt(-245 + 64*sqrt(77))) - 255762019205*sqrt(2)*sqrt(-245 + 64*s
qrt(77))*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt
(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sqrt(7
7) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(
7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqr
t(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 34173387248
00*sqrt(14)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*s
qrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sqr
t(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sq
rt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*
sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 76069253
920*sqrt(154)*sqrt(-245 + 64*sqrt(77))*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*
sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983
777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-24
5 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77)
+ 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245
+ 64*sqrt(77))) + 255640897900960*sqrt(77)*sqrt(-245 + 64*sqrt(77))*sqrt(-
55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt
(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)
+ 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(
77))) + 17947820059612960*sqrt(7)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*
sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 6
4*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt
(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)))) - 5980030865385984*sqrt(35)*
sqrt(-245 + 64*sqrt(77))*sqrt(-21 + sqrt(11)*sqrt(-245 + 64*sqrt(77)))/(-25
108273204763350*sqrt(11)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-625
89*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77
)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 8
80*sqrt(7)*sqrt(-245 + 64*sqrt(77))) - 5279630978565*sqrt(22)*sqrt(-62589*s
qrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) -
103712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-
62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt
(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77))

+ 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) - 288498310874070*sqrt(-245 + 64*sq
rt(77))*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(
-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(
77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-
245 + 64*sqrt(77))) - 255762019205*sqrt(2)*sqrt(-245 + 64*sqrt(77))*sqrt(-6
2589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(
77)) - 103712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*
sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 6
4*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt
(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 3417338724800*sqrt(14)*sqrt
(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sq
rt(77)) - 103712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(
2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245
+ 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*s
qrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 76069253920*sqrt(154)*sq
rt(-245 + 64*sqrt(77))*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 2112
0*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)*sqrt(-55881
 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77))
 - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 58
1*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)))
 + 255640897900960*sqrt(77)*sqrt(-245 + 64*sqrt(77))*sqrt(-55881 - 5040*sqr
t(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sq
rt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*
sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 17947820
059612960*sqrt(7)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt
(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 10
3712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt
(7)*sqrt(-245 + 64*sqrt(77)))) - 901946781583931138*sqrt(5)*sqrt(-21 + sqrt
(11)*sqrt(-245 + 64*sqrt(77)))/(-25108273204763350*sqrt(11)*sqrt(-55881 - 5
040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 2
1120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sq
rt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) - 5
279630978565*sqrt(22)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120
*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)*sqrt(-55881
- 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77))
- 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581
*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)))
- 288498310874070*sqrt(-245 + 64*sqrt(77))*sqrt(-55881 - 5040*sqrt(77) + 28
*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt
(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245
+ 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))) - 255762019205*sqrt(
2)*sqrt(-245 + 64*sqrt(77))*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) -
 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)*sqrt(-
55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt
(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)

+ 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(
77))) + 3417338724800*sqrt(14)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)
) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)*sqr
t(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*s
qrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 59837
77) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sq
rt(77))) + 76069253920*sqrt(154)*sqrt(-245 + 64*sqrt(77))*sqrt(-62589*sqrt(
11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103
712*sqrt(77) + 5983777)*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-6258
9*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)
) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 88
0*sqrt(7)*sqrt(-245 + 64*sqrt(77))) + 255640897900960*sqrt(77)*sqrt(-245 +
64*sqrt(77))*sqrt(-55881 - 5040*sqrt(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*
sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*
sqrt(77) + 5983777) + 581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*s
qrt(-245 + 64*sqrt(77))) + 17947820059612960*sqrt(7)*sqrt(-55881 - 5040*sqr
t(77) + 28*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sq
rt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777) + 581*sqrt(11)*
sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))))) + 2*sqrt
(11*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77))
+ 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)/(560*(21 + sqrt(11)*sq
rt(-245 + 64*sqrt(77)))) + (5726336*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 867
3280*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 49674240*sqrt(77) + 550763136)/(295
034880 + 14049280*sqrt(11)*sqrt(-245 + 64*sqrt(77))))*atan(x*(8133283840*sq
rt(55)*sqrt(-245 + 64*sqrt(77))*sqrt(21 + sqrt(11)*sqrt(-245 + 64*sqrt(77))
)/(-56909334790*sqrt(-245 + 64*sqrt(77))*sqrt(581*sqrt(11)*sqrt(-245 + 64*s
qrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 2
8*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) +
62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) - 180730470976*sqrt(7)*
sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqr
t(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*s
qrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) +
5983777)) - 57446630906*sqrt(11)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)
) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(
2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*s
qrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) + 886040320*sqrt(77)*sqrt(-245
+ 64*sqrt(77))*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sq
rt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sq
rt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245
+ 64*sqrt(77)) + 5983777)) + 16932307*sqrt(22)*sqrt(-103712*sqrt(77) + 2112
0*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)
) + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(
-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(
77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 6
4*sqrt(77)) + 5983777)) + 16252973*sqrt(2)*sqrt(-245 + 64*sqrt(77))*sqrt(-1

03712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sq
rt(-245 + 64*sqrt(77)) + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)
) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(
2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*s
qrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777))) + 170798960640*sqrt(5)*sqrt(2
1 + sqrt(11)*sqrt(-245 + 64*sqrt(77)))/(-56909334790*sqrt(-245 + 64*sqrt(77
))*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*
sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 2112
0*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)
) + 5983777)) - 180730470976*sqrt(7)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(
77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sq
rt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 6258
9*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) - 57446630906*sqrt(11)*sqrt
(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77
)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(
7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 598
3777)) + 886040320*sqrt(77)*sqrt(-245 + 64*sqrt(77))*sqrt(581*sqrt(11)*sqrt
(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77)
 + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*
sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) + 16932307*
sqrt(22)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 6
2589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)*sqrt(581*sqrt(11)*sqrt(-2
45 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) +
55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqr
t(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) + 16252973*sqr
t(2)*sqrt(-245 + 64*sqrt(77))*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-2
45 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)*sqrt
(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77
)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(
7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 598
3777)))) + 237157337353167*sqrt(10)*sqrt(21 + sqrt(11)*sqrt(-245 + 64*sqrt(
77)))*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 6258
9*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)/(-25108273204763350*sqrt(11)
*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sq
rt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*
sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77))
+ 5983777)) - 3417338724800*sqrt(14)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*
sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 598377
7)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*
sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 2112
0*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)
) + 5983777)) - 255640897900960*sqrt(77)*sqrt(-245 + 64*sqrt(77))*sqrt(581*
sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) +
5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sq
rt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)

) - 255762019205*sqrt(2)*sqrt(-245 + 64*sqrt(77))*sqrt(-103712*sqrt(77) + 2
1120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(
77)) + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sq
rt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sq
rt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245
+ 64*sqrt(77)) + 5983777)) + 288498310874070*sqrt(-245 + 64*sqrt(77))*sqrt(
581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)
) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7
)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983
777)) + 17947820059612960*sqrt(7)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)
) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(
2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*s
qrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) + 76069253920*sqrt(154)*sqrt(-
245 + 64*sqrt(77))*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqr
t(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)*sqrt(581*sqrt(1
1)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*s
qrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-24
5 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) + 52
79630978565*sqrt(22)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*s
qrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)*sqrt(581*sqrt
(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040
*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-
245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777))) +
 71579524341697856*sqrt(385)*sqrt(21 + sqrt(11)*sqrt(-245 + 64*sqrt(77)))/(
-25108273204763350*sqrt(11)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 88
0*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqr
t(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11
)*sqrt(-245 + 64*sqrt(77)) + 5983777)) - 3417338724800*sqrt(14)*sqrt(-10371
2*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-
245 + 64*sqrt(77)) + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) +
880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*s
qrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(
11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) - 255640897900960*sqrt(77)*sqrt(-2
45 + 64*sqrt(77))*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*
sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*
sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-24
5 + 64*sqrt(77)) + 5983777)) - 255762019205*sqrt(2)*sqrt(-245 + 64*sqrt(77)
)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sq
rt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 64
*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 +
 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77))
+ 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) + 288498310874070*sqr
t(-245 + 64*sqrt(77))*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt
(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103
712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt

(-245 + 64*sqrt(77)) + 5983777)) + 17947820059612960*sqrt(7)*sqrt(581*sqrt(
11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*
sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-2
45 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) + 7
6069253920*sqrt(154)*sqrt(-245 + 64*sqrt(77))*sqrt(-103712*sqrt(77) + 21120
*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77))
 + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-
245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(7
7) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64
*sqrt(77)) + 5983777)) + 5279630978565*sqrt(22)*sqrt(-103712*sqrt(77) + 211
20*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77
)) + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt
(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt
(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 +
64*sqrt(77)) + 5983777))) + 1688576282291*sqrt(110)*sqrt(-245 + 64*sqrt(77)
)*sqrt(21 + sqrt(11)*sqrt(-245 + 64*sqrt(77)))*sqrt(-103712*sqrt(77) + 2112
0*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)
) + 5983777)/(-25108273204763350*sqrt(11)*sqrt(581*sqrt(11)*sqrt(-245 + 64*
sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 +
28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) +
 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) - 3417338724800*sqrt(1
4)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*s
qrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 6
4*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881
+ 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77))
 + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) - 255640897900960*sq
rt(77)*sqrt(-245 + 64*sqrt(77))*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77))
+ 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)
*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqr
t(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) - 255762019205*sqrt(2)*sqrt(-245
 + 64*sqrt(77))*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(7
7)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)*sqrt(581*sqrt(11)*
sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt
(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 +
 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) + 28849
8310874070*sqrt(-245 + 64*sqrt(77))*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(7
7)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqr
t(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589
*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) + 17947820059612960*sqrt(7)*
sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqr
t(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*s
qrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) +
 5983777)) + 76069253920*sqrt(154)*sqrt(-245 + 64*sqrt(77))*sqrt(-103712*sq
rt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245
+ 64*sqrt(77)) + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*

sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(
-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*
sqrt(-245 + 64*sqrt(77)) + 5983777)) + 5279630978565*sqrt(22)*sqrt(-103712*
sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-24
5 + 64*sqrt(77)) + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 88
0*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqr
t(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11
)*sqrt(-245 + 64*sqrt(77)) + 5983777))) + 5980030865385984*sqrt(35)*sqrt(-2
45 + 64*sqrt(77))*sqrt(21 + sqrt(11)*sqrt(-245 + 64*sqrt(77)))/(-2510827320
4763350*sqrt(11)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*s
qrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*s
qrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245
 + 64*sqrt(77)) + 5983777)) - 3417338724800*sqrt(14)*sqrt(-103712*sqrt(77)
+ 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sq
rt(77)) + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)
*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712
*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-2
45 + 64*sqrt(77)) + 5983777)) - 255640897900960*sqrt(77)*sqrt(-245 + 64*sqr
t(77))*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 +
 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) +
21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt
(77)) + 5983777)) - 255762019205*sqrt(2)*sqrt(-245 + 64*sqrt(77))*sqrt(-103
712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt
(-245 + 64*sqrt(77)) + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77))
+ 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)
*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqr
t(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) + 288498310874070*sqrt(-245 + 64
*sqrt(77))*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-2
45 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77
) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*
sqrt(77)) + 5983777)) + 17947820059612960*sqrt(7)*sqrt(581*sqrt(11)*sqrt(-2
45 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) +
55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqr
t(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) + 76069253920*
sqrt(154)*sqrt(-245 + 64*sqrt(77))*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sq
rt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)
*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sq
rt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*
sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77))
+ 5983777)) + 5279630978565*sqrt(22)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*
sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 598377
7)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*
sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 2112
0*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)
) + 5983777))) - 427612890672*sqrt(70)*sqrt(-245 + 64*sqrt(77))*sqrt(21 + s

qrt(11)*sqrt(-245 + 64*sqrt(77)))*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqr
t(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)/
(-25108273204763350*sqrt(11)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 8
80*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sq
rt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(1
1)*sqrt(-245 + 64*sqrt(77)) + 5983777)) - 3417338724800*sqrt(14)*sqrt(-1037
12*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(
-245 + 64*sqrt(77)) + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) +
 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*
sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt
(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) - 255640897900960*sqrt(77)*sqrt(-
245 + 64*sqrt(77))*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)
*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712
*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-2
45 + 64*sqrt(77)) + 5983777)) - 255762019205*sqrt(2)*sqrt(-245 + 64*sqrt(77
))*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*s
qrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 6
4*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881
 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77))
 + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) + 288498310874070*sq
rt(-245 + 64*sqrt(77))*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqr
t(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-10
3712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqr
t(-245 + 64*sqrt(77)) + 5983777)) + 17947820059612960*sqrt(7)*sqrt(581*sqrt
(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040
*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-
245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) +
76069253920*sqrt(154)*sqrt(-245 + 64*sqrt(77))*sqrt(-103712*sqrt(77) + 2112
0*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)
) + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(
-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(
77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 6
4*sqrt(77)) + 5983777)) + 5279630978565*sqrt(22)*sqrt(-103712*sqrt(77) + 21
120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(7
7)) + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqr
t(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqr
t(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 +
 64*sqrt(77)) + 5983777))) - 8774790502440634*sqrt(55)*sqrt(-245 + 64*sqrt(
77))*sqrt(21 + sqrt(11)*sqrt(-245 + 64*sqrt(77)))/(-25108273204763350*sqrt(
11)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64
*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 211
20*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77
)) + 5983777)) - 3417338724800*sqrt(14)*sqrt(-103712*sqrt(77) + 21120*sqrt(
7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 598
3777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 +

64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 2
1120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(
77)) + 5983777)) - 255640897900960*sqrt(77)*sqrt(-245 + 64*sqrt(77))*sqrt(5
81*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))
 + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)
*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 59837
77)) - 255762019205*sqrt(2)*sqrt(-245 + 64*sqrt(77))*sqrt(-103712*sqrt(77)
 + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sq
rt(77)) + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)
*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712
*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-2
45 + 64*sqrt(77)) + 5983777)) + 288498310874070*sqrt(-245 + 64*sqrt(77))*sq
rt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(
77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqr
t(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5
983777)) + 17947820059612960*sqrt(7)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(
77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sq
rt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 6258
9*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) + 76069253920*sqrt(154)*sqr
t(-245 + 64*sqrt(77))*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*
sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)*sqrt(581*sqr
t(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 504
0*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(
-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) +
 5279630978565*sqrt(22)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 6
4*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)*sqrt(581*s
qrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5
040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqr
t(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777))
) - 22865213810736*sqrt(770)*sqrt(21 + sqrt(11)*sqrt(-245 + 64*sqrt(77)))*s
qrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(
11)*sqrt(-245 + 64*sqrt(77)) + 5983777)/(-25108273204763350*sqrt(11)*sqrt(5
81*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77))
 + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)
*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 59837
77)) - 3417338724800*sqrt(14)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-2
45 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)*sqrt
(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77
)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(
7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 598
3777)) - 255640897900960*sqrt(77)*sqrt(-245 + 64*sqrt(77))*sqrt(581*sqrt(11
)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sq
rt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245
 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) - 255
762019205*sqrt(2)*sqrt(-245 + 64*sqrt(77))*sqrt(-103712*sqrt(77) + 21120*sq

rt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) +
5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245
 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77)
+ 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sq
rt(77)) + 5983777)) + 288498310874070*sqrt(-245 + 64*sqrt(77))*sqrt(581*sqr
t(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 504
0*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(
-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) +
 17947820059612960*sqrt(7)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880
*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt
(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)
*sqrt(-245 + 64*sqrt(77)) + 5983777)) + 76069253920*sqrt(154)*sqrt(-245 + 6
4*sqrt(77))*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77))
+ 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)*sqrt(581*sqrt(11)*sqrt
(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77)
 + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*
sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) + 527963097
8565*sqrt(22)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)
) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)*sqrt(581*sqrt(11)*sq
rt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(7
7) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 6
4*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777))) - 901946
781583931138*sqrt(5)*sqrt(21 + sqrt(11)*sqrt(-245 + 64*sqrt(77)))/(-2510827
3204763350*sqrt(11)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7
)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-10371
2*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-
245 + 64*sqrt(77)) + 5983777)) - 3417338724800*sqrt(14)*sqrt(-103712*sqrt(7
7) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64
*sqrt(77)) + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt
(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103
712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt
(-245 + 64*sqrt(77)) + 5983777)) - 255640897900960*sqrt(77)*sqrt(-245 + 64*
sqrt(77))*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-24
5 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77)
 + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*s
qrt(77)) + 5983777)) - 255762019205*sqrt(2)*sqrt(-245 + 64*sqrt(77))*sqrt(-
103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*s
qrt(-245 + 64*sqrt(77)) + 5983777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77
)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt
(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*
sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) + 288498310874070*sqrt(-245 +
 64*sqrt(77))*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt
(-245 + 64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt
(77) + 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 +
64*sqrt(77)) + 5983777)) + 17947820059612960*sqrt(7)*sqrt(581*sqrt(11)*sqrt

```
(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 5040*sqrt(77)
 + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 21120*sqrt(7)*sqrt(-245 + 64*
sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 5983777)) + 760692539
20*sqrt(154)*sqrt(-245 + 64*sqrt(77))*sqrt(-103712*sqrt(77) + 21120*sqrt(7)
*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 59837
77)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 + 64
*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 211
20*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77
)) + 5983777)) + 5279630978565*sqrt(22)*sqrt(-103712*sqrt(77) + 21120*sqrt(
7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 598
3777)*sqrt(581*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 880*sqrt(7)*sqrt(-245 +
64*sqrt(77)) + 5040*sqrt(77) + 55881 + 28*sqrt(2)*sqrt(-103712*sqrt(77) + 2
1120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 62589*sqrt(11)*sqrt(-245 + 64*sqrt(
77)) + 5983777)))) - 5/(2*x)
```

Maxima [F]

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+5x^2+x^3+2x^4\right)}\,dx=\int \frac{2\,x^3+3\,x^2+x+5}{(2\,x^4+x^3+5\,x^2+x+2)x^2}\,dx$$

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] -5/2/x + 1/4*integrate((6*x^3 - 17*x^2 + 13*x - 35)/(2*x^4 + x^3 + 5*x^2 +
x + 2), x) - 3/4*log(x)
```

Giac [F]

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+5x^2+x^3+2x^4\right)}\,dx=\int \frac{2\,x^3+3\,x^2+x+5}{(2\,x^4+x^3+5\,x^2+x+2)x^2}\,dx$$

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.67 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.86

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+5x^2+x^3+2x^4\right)}\,dx = \left(\sum_{k=1}^{4}\ln\left(\frac{1199\,\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)}{32}+25\,x+\frac{\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)\,x\,4169}{32}+\frac{\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)^2\,x\,43993}{256}+\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)^3\,x\,28+\frac{\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)^4\,x\,3675}{32}+\frac{11647\,\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)^2}{128}+\frac{7273\,\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)^3}{128}-\frac{441\,\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)^4}{32}+\frac{21}{4}\right)\,\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)\right)-\frac{3\,\ln\left(x\right)}{4}-\frac{5}{2\,x}$$

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^2*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] symsum(log((1199*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z
, k))/32 + 25*x + (4169*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128
/343, z, k)*x)/32 + (43993*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 +
128/343, z, k)^2*x)/256 + 28*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49
+ 128/343, z, k)^3*x + (3675*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49
+ 128/343, z, k)^4*x)/32 + (11647*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z
)/49 + 128/343, z, k)^2)/128 + (7273*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (9
6*z)/49 + 128/343, z, k)^3)/128 - (441*root(z^4 - (3*z^3)/4 + (16*z^2)/7 +
(96*z)/49 + 128/343, z, k)^4)/32 + 21/4)*root(z^4 - (3*z^3)/4 + (16*z^2)/7
+ (96*z)/49 + 128/343, z, k), k, 1, 4) - (3*log(x))/4 - 5/(2*x)
```

3.256 $\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx$

Optimal result 1728
Rubi [A] (verified) 1729
Mathematica [C] (verified) 1732
Maple [C] (verified) 1732
Fricas [B] (verification not implemented) 1733
Sympy [A] (verification not implemented) 1734
Maxima [F] 1735
Giac [F] 1735
Mupad [B] (verification not implemented) 1736

Optimal result

Integrand size = 35, antiderivative size = 317

$$\begin{aligned}\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx &= -\frac{35-9i\sqrt{7}}{56x^2}-\frac{35+9i\sqrt{7}}{56x^2} \\ &+\frac{3(7-11i\sqrt{7})}{56x}+\frac{3(7+11i\sqrt{7})}{56x} \\ &+\frac{(355-73i\sqrt{7})\operatorname{arctanh}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2(35-i\sqrt{7})}}\right)}{8\sqrt{14(35-i\sqrt{7})}} \\ &-\frac{(355+73i\sqrt{7})\operatorname{arctanh}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2(35+i\sqrt{7})}}\right)}{8\sqrt{14(35+i\sqrt{7})}} \\ &-\frac{1}{16}\left(35-9i\sqrt{7}\right)\log(x)-\frac{1}{16}\left(35+9i\sqrt{7}\right)\log(x) \\ &+\frac{1}{32}\left(35-9i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right) \\ &+\frac{1}{32}\left(35+9i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)\end{aligned}$$

```
[Out] 1/56*(-35+9*I*7^(1/2))/x^2-1/16*ln(x)*(35-9*I*7^(1/2))+1/32*ln(4*I+4*I*x^2+
x*(I-7^(1/2)))*(35-9*I*7^(1/2))+1/56*(-35-9*I*7^(1/2))/x^2-1/16*ln(x)*(35+9
*I*7^(1/2))+1/32*ln(4*I+4*I*x^2+x*(I+7^(1/2)))*(35+9*I*7^(1/2))+3/56*(7-11*
I*7^(1/2))/x+3/56*(7+11*I*7^(1/2))/x+1/8*arctanh((I+8*I*x-7^(1/2))/(70-2*I*
7^(1/2))^(1/2))*(355-73*I*7^(1/2))/(490-14*I*7^(1/2))^(1/2)-1/8*arctanh((I+
8*I*x+7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(355+73*I*7^(1/2))/(490+14*I*7^(1/2)
)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2112, 814, 648, 632, 212, 642}

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx = \frac{(355-73i\sqrt{7})\operatorname{arctanh}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2(35-i\sqrt{7})}}\right)}{8\sqrt{14(35-i\sqrt{7})}} - \frac{(355+73i\sqrt{7})\operatorname{arctanh}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2(35+i\sqrt{7})}}\right)}{8\sqrt{14(35+i\sqrt{7})}} - \frac{35+9i\sqrt{7}}{56x^2} - \frac{35-9i\sqrt{7}}{56x^2} + \frac{1}{32}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right) + \frac{1}{32}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right) + \frac{3(7+11i\sqrt{7})}{56x} + \frac{3(7-11i\sqrt{7})}{56x} - \frac{1}{16}\left(35+9i\sqrt{7}\right)\log(x) - \frac{1}{16}\left(35-9i\sqrt{7}\right)\log(x)$$

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -1/56*(35 - (9*I)*Sqrt[7])/x^2 - (35 + (9*I)*Sqrt[7])/(56*x^2) + (3*(7 - (1
1*I)*Sqrt[7]))/(56*x) + (3*(7 + (11*I)*Sqrt[7]))/(56*x) + ((355 - (73*I)*Sq
rt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(8*Sqrt[1
4*(35 - I*Sqrt[7])]) - ((355 + (73*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8*I)
*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(8*Sqrt[14*(35 + I*Sqrt[7])]) - ((35 - (9*I)
*Sqrt[7])*Log[x])/16 - ((35 + (9*I)*Sqrt[7])*Log[x])/16 + ((35 - (9*I)*Sqrt
[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/32 + ((35 + (9*I)*Sqrt[7])*Log
[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/32
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2112

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[x^m*((b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x)/(2*a + (b + q)*x + 2*a*x^2)), x], x] - Dist[1/q, Int[x^m*((b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2)),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\text{integral} = \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{x^3\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{x^3\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}}$$

$$= -\frac{i\int \left(\frac{9+5i\sqrt{7}}{4x^3}+\frac{3\left(11-i\sqrt{7}\right)}{8x^2}-\frac{7i\left(-9i+5\sqrt{7}\right)}{16x}+\frac{-223i-61\sqrt{7}+14\left(9i-5\sqrt{7}\right)x}{8\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)}\right)dx}{\sqrt{7}}$$

$$+\frac{i\int \left(\frac{9-5i\sqrt{7}}{4x^3}+\frac{3\left(11+i\sqrt{7}\right)}{8x^2}+\frac{7i\left(9i+5\sqrt{7}\right)}{16x}+\frac{-223i+61\sqrt{7}+14\left(9i+5\sqrt{7}\right)x}{8\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)}\right)dx}{\sqrt{7}}$$

$$
\begin{aligned}
&= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} - \frac{1}{16}\left(35-9i\sqrt{7}\right)\log(x) \\
&\quad - \frac{1}{16}\left(35+9i\sqrt{7}\right)\log(x) - \frac{i\int \frac{-223i-61\sqrt{7}+14\left(9i-5\sqrt{7}\right)x}{4i+\left(i-\sqrt{7}\right)x+4ix^2}\,dx}{8\sqrt{7}} + \frac{i\int \frac{-223i+61\sqrt{7}+14\left(9i+5\sqrt{7}\right)x}{4i+\left(i+\sqrt{7}\right)x+4ix^2}\,dx}{8\sqrt{7}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} - \frac{1}{16}\left(35-9i\sqrt{7}\right)\log(x) \\
&\quad - \frac{1}{16}\left(35+9i\sqrt{7}\right)\log(x) + \frac{1}{112}\left(511i-355\sqrt{7}\right)\int \frac{1}{4i+\left(i-\sqrt{7}\right)x+4ix^2}\,dx \\
&\quad - \frac{1}{32}\left(-35+9i\sqrt{7}\right)\int \frac{i-\sqrt{7}+8ix}{4i+\left(i-\sqrt{7}\right)x+4ix^2}\,dx \\
&\quad + \frac{1}{32}\left(35+9i\sqrt{7}\right)\int \frac{i+\sqrt{7}+8ix}{4i+\left(i+\sqrt{7}\right)x+4ix^2}\,dx \\
&\quad + \frac{1}{112}\left(511i+355\sqrt{7}\right)\int \frac{1}{4i+\left(i+\sqrt{7}\right)x+4ix^2}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} - \frac{1}{16}\left(35-9i\sqrt{7}\right)\log(x) \\
&\quad - \frac{1}{16}\left(35+9i\sqrt{7}\right)\log(x) + \frac{1}{32}\left(35-9i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right) \\
&\quad + \frac{1}{32}\left(35+9i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right) \\
&\quad + \frac{1}{56}\left(-511i+355\sqrt{7}\right)\text{Subst}\left(\int \frac{1}{2\left(35-i\sqrt{7}\right)-x^2}\,dx, x, i-\sqrt{7}+8ix\right) \\
&\quad - \frac{1}{56}\left(511i+355\sqrt{7}\right)\text{Subst}\left(\int \frac{1}{2\left(35+i\sqrt{7}\right)-x^2}\,dx, x, i+\sqrt{7}+8ix\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} \\
&\quad + \frac{\left(355-73i\sqrt{7}\right)\tanh^{-1}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{8\sqrt{14\left(35-i\sqrt{7}\right)}} - \frac{\left(355+73i\sqrt{7}\right)\tanh^{-1}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{8\sqrt{14\left(35+i\sqrt{7}\right)}} \\
&\quad - \frac{1}{16}\left(35-9i\sqrt{7}\right)\log(x) - \frac{1}{16}\left(35+9i\sqrt{7}\right)\log(x) \\
&\quad + \frac{1}{32}\left(35-9i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right) \\
&\quad + \frac{1}{32}\left(35+9i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.37

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx$$
$$= -\frac{5}{4x^2}+\frac{3}{4x}-\frac{35\log(x)}{8}+\frac{1}{8}\text{RootSum}\Big[2+\#1+5\#1^2+\#1^3 +2\#1^4\&, \frac{61\log(x-\#1)+141\log(x-\#1)\#1+47\log(x-\#1)\#1^2+70\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\Big]$$

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(4*x^2) + 3/(4*x) - (35*Log[x])/8 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#
1^4 & , (61*Log[x - #1] + 141*Log[x - #1]*#1 + 47*Log[x - #1]*#1^2 + 70*Log
[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ]/8
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.20

method	result
risch	$\frac{\frac{3x}{4}-\frac{5}{4}}{x^2}+\frac{\left(\sum_{\_R=\text{RootOf}(686\_Z^4-12005\_Z^3+73696\_Z^2-50176\_Z+65536)} \_R\ln\left(-2261742\_R^3+41411909\_R^2-249593568\_R+\right.\right.}{4}$
default	$-\frac{5}{4x^2}+\frac{3}{4x}-\frac{35\ln(x)}{8}+\frac{\left(\sum_{\_R=\text{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)} \frac{(70\_R^3+47\_R^2+141\_R+61)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}\right)}{8}$

```
[In] int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] (3/4*x-5/4)/x^2+1/4*sum(_R*ln(-2261742*_R^3+41411909*_R^2-249593568*_R+1545
97376*x+130505728),_R=RootOf(686*_Z^4-12005*_Z^3+73696*_Z^2-50176*_Z+65536)
)-35/8*ln(x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1274 vs. 2(196) = 392.

Time = 0.98 (sec) , antiderivative size = 1274, normalized size of antiderivative = 4.02

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx = \text{Too large to display}$$

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] -1/448*(14*x^2*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35
)*log(-49/4*(207711*I*sqrt(7) + 369264*sqrt(-9803/6272*I*sqrt(7) + 2815/896
) - 957269)*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35
/32)^2 + 9046968*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/89
6) + 35/32)^3 - 39580485*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) +
 2815/896) + 35/32)^2 - 21/1024*(13785856*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803
/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 16963065*I*sqrt(7) + 30156560*sqrt
(-9803/6272*I*sqrt(7) + 2815/896) - 68488563)*(-9*I*sqrt(7) + 16*sqrt(9803/
6272*I*sqrt(7) + 2815/896) - 35) + 9662336*x - 68336919/4*I*sqrt(7) - 30371
964*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 257023549/4) + 14*x^2*(9*I*sqrt
(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35)*log(-9046968*(-9/32*I*
sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^3 + 41411909*(
-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 96
62336*x + 70198191/4*I*sqrt(7) + 31199196*sqrt(-9803/6272*I*sqrt(7) + 2815/
896) - 240366533/4) + 1960*x^2*log(x) + (4*sqrt(7)*sqrt(-1344*(9/32*I*sqrt(
7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1344*(-9/32*I*sq
rt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 7/8*(9*I*sqr
t(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 105)*(-9*I*sqrt(7) + 16*s
qrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/2*I*sqrt(7) - 1960*sqrt(-9
803/6272*I*sqrt(7) + 2815/896) + 1661/2)*x^2 - 7*x^2*(9*I*sqrt(7) + 16*sqrt
(-9803/6272*I*sqrt(7) + 2815/896) - 35) - 7*x^2*(-9*I*sqrt(7) + 16*sqrt(980
3/6272*I*sqrt(7) + 2815/896) - 35) - 980*x^2)*log(49/4*(207711*I*sqrt(7) +
369264*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 957269)*(9/32*I*sqrt(7) - 1/
2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1831424*(-9/32*I*sqrt(7
) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 21/1024*(1378585
6*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 +
 16963065*I*sqrt(7) + 30156560*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 6848
8563)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) + 1/102
4*sqrt(-1344*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 3
5/32)^2 - 1344*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896)
 + 35/32)^2 - 7/8*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) +
 105)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/
2*I*sqrt(7) - 1960*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*(7*(2307
```

```
9*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35) - 1
49504*sqrt(7))*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35
) - 1046528*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896)
 - 35) - 116260864*sqrt(7)) + 19324672*x - 465318*I*sqrt(7) - 827232*sqrt(-
9803/6272*I*sqrt(7) + 2815/896) + 666914) - (4*sqrt(7)*sqrt(-1344*(9/32*I*s
qrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1344*(-9/32*
I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 7/8*(9*I
*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 105)*(-9*I*sqrt(7) +
16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/2*I*sqrt(7) - 1960*sqr
t(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*x^2 + 7*x^2*(9*I*sqrt(7) + 16*
sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35) + 7*x^2*(-9*I*sqrt(7) + 16*sqrt
(9803/6272*I*sqrt(7) + 2815/896) - 35) + 980*x^2)*log(49/4*(207711*I*sqrt(7
) + 369264*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 957269)*(9/32*I*sqrt(7)
- 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1831424*(-9/32*I*sq
rt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 21/1024*(137
85856*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)
^2 + 16963065*I*sqrt(7) + 30156560*sqrt(-9803/6272*I*sqrt(7) + 2815/896) -
68488563)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 1
/1024*sqrt(-1344*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896)
 + 35/32)^2 - 1344*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/
896) + 35/32)^2 - 7/8*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/89
6) + 105)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2
205/2*I*sqrt(7) - 1960*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*(7*(
23079*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35)
 - 149504*sqrt(7))*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896)
- 35) - 1046528*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/
896) - 35) - 116260864*sqrt(7)) + 19324672*x - 465318*I*sqrt(7) - 827232*sq
rt(-9803/6272*I*sqrt(7) + 2815/896) + 666914) - 336*x + 560)/x^2
```

Sympy [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.22

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx = -\frac{35\log(x)}{8} + \operatorname{RootSum}\left(2744t^4-12005t^3+18424t^2-3136t+1024,\left(t\mapsto t\log\left(-\frac{20101387287723t^4}{91907904361586}+\frac{944515214\ldots}{4595395218\ldots}\right.\right.\right. + \frac{3x-5}{4x^2}$$

```
[In] integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] -35*log(x)/8 + RootSum(2744*_t**4 - 12005*_t**3 + 18424*_t**2 - 3136*_t + 1
024, Lambda(_t, _t*log(-20101387287723*_t**4/91907904361586 + 944515214496*
_t**3/45953952180793 + 16572327093911939*_t**2/5882105879141504 - 456447174
```

```
9800865*_t/735263234892688 + x + 70084064010625/91907904361586))) + (3*x -
5)/(4*x**2)
```

Maxima [F]

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx = \int \frac{2\,x^3+3\,x^2+x+5}{(2\,x^4+x^3+5\,x^2+x+2)x^3}\,dx$$

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
[Out] 1/4*(3*x - 5)/x^2 + 1/8*integrate((70*x^3 + 47*x^2 + 141*x + 61)/(2*x^4 + x
^3 + 5*x^2 + x + 2), x) - 35/8*log(x)
```

Giac [F]

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx = \int \frac{2\,x^3+3\,x^2+x+5}{(2\,x^4+x^3+5\,x^2+x+2)x^3}\,dx$$

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.78

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+5x^2+x^3+2x^4\right)}\,dx = \left(\sum_{k=1}^{4}\ln\left(-\frac{8939\,\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)}{128}-\frac{69\,x}{8}+\frac{\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)\,x\,14945}{128}-\frac{\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^2\,x\,269991}{1024}-\frac{\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^3\,x\,1393}{8}+\frac{\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^4\,x\,3675}{32}-\frac{35697\,\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^2}{512}-\frac{18487\,\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^3}{256}-\frac{441\,\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^4}{32}+\frac{245}{8}\right)\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)\right)-\frac{35\,\ln\left(x\right)}{8}+\frac{\frac{3\,x}{4}-\frac{5}{4}}{x^2}$$

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^3*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] symsum(log((14945*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7 + 128/343, z
, k)*x)/128 - (69*x)/8 - (8939*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7
 + 128/343, z, k))/128 - (269991*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)
/7 + 128/343, z, k)^2*x)/1024 - (1393*root(z^4 - (35*z^3)/8 + (47*z^2)/7 -
(8*z)/7 + 128/343, z, k)^3*x)/8 + (3675*root(z^4 - (35*z^3)/8 + (47*z^2)/7
- (8*z)/7 + 128/343, z, k)^4*x)/32 - (35697*root(z^4 - (35*z^3)/8 + (47*z^2
)/7 - (8*z)/7 + 128/343, z, k)^2)/512 - (18487*root(z^4 - (35*z^3)/8 + (47*
z^2)/7 - (8*z)/7 + 128/343, z, k)^3)/256 - (441*root(z^4 - (35*z^3)/8 + (47
*z^2)/7 - (8*z)/7 + 128/343, z, k)^4)/32 + 245/8)*root(z^4 - (35*z^3)/8 + (
```

```
47*z^2)/7 - (8*z)/7 + 128/343, z, k), k, 1, 4) - (35*log(x))/8 + ((3*x)/4 -
 5/4)/x^2
```

3.257 $\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx$

Optimal result 1738
Rubi [A] (verified) 1738
Mathematica [C] (verified) 1739
Maple [C] (verified) 1739
Fricas [B] (verification not implemented) 1740
Sympy [C] (verification not implemented) 1740
Maxima [F] 1740
Giac [F] 1741
Mupad [B] (verification not implemented) 1741

Optimal result

Integrand size = 40, antiderivative size = 19

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \frac{\arctan\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

```
[Out] arctan(c*x^3/(b*x^2+a))/c
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2119, 211}

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \frac{\arctan\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

```
[In] Int[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6),x]
```

```
[Out] ArcTan[(c*x^3)/(a + b*x^2)]/c
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
```

```
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= (3a^2)\,\text{Subst}\left(\int \frac{1}{a^2+9a^2c^2x^2}\,dx, x, \frac{x^3}{3a+3bx^2}\right)\\ &= \frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 4.58

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \frac{1}{2}\text{RootSum}\Big[a^2+2ab\#1^2+b^2\#1^4 + c^2\#1^6\&, \frac{3a\log(x-\#1)\#1+b\log(x-\#1)\#1^3}{2ab+2b^2\#1^2+3c^2\#1^4}\&\Big]$$

```
[In] Integrate[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6),x]
[Out] RootSum[a^2 + 2*a*b*#1^2 + b^2*#1^4 + c^2*#1^6 & , (3*a*Log[x - #1]*#1 + b*
Log[x - #1]*#1^3)/(2*a*b + 2*b^2*#1^2 + 3*c^2*#1^4) & ]/2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.58

method	result	size
parallelrisch	$\frac{i\ln\left(ib\,x^2+c\,x^3+ia\right)-i\ln\left(-ib\,x^2+c\,x^3-ia\right)}{2c}$	49
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(c^2\_Z^6+b^2\_Z^4+2a\_Z^2b+a^2\right)}\frac{\left(\_R^4b+3\_R^2a\right)\ln\left(x-\_R\right)}{3\_R^5c^2+2\_R^3b^2+2ab\_R}\right)}{2}$	75
risch	$-\frac{\arctan\left(\frac{c\,x^5b}{a^2}-\frac{c\,x^3}{a}+\frac{b^3x^3}{a^2c}+\frac{b^2x}{ac}\right)}{c}-\frac{\arctan\left(-\frac{c\,x^3}{a}+\frac{cx}{b}-\frac{b^2x}{ac}\right)}{c}+\frac{\arctan\left(\frac{cx}{b}\right)}{c}$	96

```
[In] int(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE
)
```

[Out] 1/2*(I*ln(c*x^3+I*b*x^2+I*a)-I*ln(-I*b*x^2+c*x^3-I*a))/c

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(19) = 38$.

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 4.37

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \frac{\arctan\left(\frac{cx}{b}\right)-\arctan\left(\frac{bc^2x^5+ab^2x+(b^3-ac^2)x^3}{a^2c}\right)+\arctan\left(\frac{bc^2x^3+(b^3-ac^2)x}{abc}\right)}{c}$$

[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")

[Out] (arctan(c*x/b) - arctan((b*c^2*x^5 + a*b^2*x + (b^3 - a*c^2)*x^3)/(a^2*c)) + arctan((b*c^2*x^3 + (b^3 - a*c^2)*x)/(a*b*c)))/c

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \frac{-\frac{i\log\left(-\frac{ia}{c}-\frac{ibx^2}{c}+x^3\right)}{2}+\frac{i\log\left(\frac{ia}{c}+\frac{ibx^2}{c}+x^3\right)}{2}}{c}$$

[In] integrate(x**2*(b*x**2+3*a)/(c**2*x**6+b**2*x**4+2*a*b*x**2+a**2),x)

[Out] (-I*log(-I*a/c - I*b*x**2/c + x**3)/2 + I*log(I*a/c + I*b*x**2/c + x**3)/2)/c

Maxima [F]

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \int \frac{(bx^2+3\,a)x^2}{c^2x^6+b^2x^4+2\,abx^2+a^2}\,dx$$

[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")

[Out] integrate((b*x^2 + 3*a)*x^2/(c^2*x^6 + b^2*x^4 + 2*a*b*x^2 + a^2), x)

Giac [F]

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \int \frac{(bx^2+3\,a)x^2}{c^2x^6+b^2x^4+2\,abx^2+a^2}\,dx$$

```
[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="gia
c")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 252, normalized size of antiderivative = 13.26

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \frac{\mathrm{atan}\left(\frac{27\,a\,c^5\,x^3}{27\,a^2\,c^4-4\,a\,b^3\,c^2}-\frac{27\,b\,c^5\,x^5}{27\,a^2\,c^4-4\,a\,b^3\,c^2}-\frac{31\,b^3\,c^3\,x^3}{27\,a^2\,c^4-4\,a\,b^3\,c^2}+\frac{4\,b^6\,c\,x^3}{27\,a^3\,c^4-4\,a^2\,b^3\,c^2}+\frac{4\,b^5\,c\,x}{27\,a^2\,c^4-4\,a\,b^3\,c^2}+\frac{4\,b^4\,c^3\,x^5}{27\,a^3\,c^4-4\,a^2\,b^3\,c^2}-\cdots\right.}{c}$$

```
[In] int((x^2*(3*a + b*x^2))/(a^2 + b^2*x^4 + c^2*x^6 + 2*a*b*x^2),x)
```

```
[Out] (atan((27*a*c^5*x^3)/(27*a^2*c^4 - 4*a*b^3*c^2) - (27*b*c^5*x^5)/(27*a^2*c^
4 - 4*a*b^3*c^2) - (31*b^3*c^3*x^3)/(27*a^2*c^4 - 4*a*b^3*c^2) + (4*b^6*c*x
^3)/(27*a^3*c^4 - 4*a^2*b^3*c^2) + (4*b^5*c*x)/(27*a^2*c^4 - 4*a*b^3*c^2) +
 (4*b^4*c^3*x^5)/(27*a^3*c^4 - 4*a^2*b^3*c^2) - (27*a*b^2*c^3*x)/(27*a^2*c^
4 - 4*a*b^3*c^2)) + atan((c*x^3)/a - (c*x)/b + (b^2*x)/(a*c)) + atan((c*x)/
b))/c
```

3.258 $\int \frac{1-3x^4}{(-2+x)(1+x^2)^2} dx$

Optimal result 1742
Rubi [A] (verified) 1742
Mathematica [A] (verified) 1744
Maple [A] (verified) 1744
Fricas [A] (verification not implemented) 1744
Sympy [A] (verification not implemented) 1745
Maxima [A] (verification not implemented) 1745
Giac [A] (verification not implemented) 1745
Mupad [B] (verification not implemented) 1746

Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{1-3x^4}{(-2+x)(1+x^2)^2} dx = -\frac{1-2x}{5(1+x^2)} - \frac{46\arctan(x)}{25} - \frac{47}{25}\log(2-x) - \frac{14}{25}\log(1+x^2)$$

```
[Out] 1/5*(-1+2*x)/(x^2+1)-46/25*arctan(x)-47/25*ln(2-x)-14/25*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1661, 1643, 649, 209, 266}

$$\int \frac{1-3x^4}{(-2+x)(1+x^2)^2} dx = -\frac{46\arctan(x)}{25} - \frac{1-2x}{5(x^2+1)} - \frac{14}{25}\log(x^2+1) - \frac{47}{25}\log(2-x)$$

```
[In] Int[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2),x]
```

```
[Out] -1/5*(1 - 2*x)/(1 + x^2) - (46*ArcTan[x])/25 - (47*Log[2 - x])/25 - (14*Log
[1 + x^2])/25
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1-2x}{5\left(1+x^2\right)}-\frac{1}{2}\int \frac{-\frac{18}{5}-\frac{4x}{5}+6x^2}{(-2+x)\left(1+x^2\right)}\,dx\\
&= -\frac{1-2x}{5\left(1+x^2\right)}-\frac{1}{2}\int \left(\frac{94}{25(-2+x)}+\frac{4(23+14x)}{25\left(1+x^2\right)}\right)\,dx\\
&= -\frac{1-2x}{5\left(1+x^2\right)}-\frac{47}{25}\log(2-x)-\frac{2}{25}\int \frac{23+14x}{1+x^2}\,dx\\
&= -\frac{1-2x}{5\left(1+x^2\right)}-\frac{47}{25}\log(2-x)-\frac{28}{25}\int \frac{x}{1+x^2}\,dx-\frac{46}{25}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{1-2x}{5\left(1+x^2\right)}-\frac{46}{25}\tan^{-1}(x)-\frac{47}{25}\log(2-x)-\frac{14}{25}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

$$\int \frac{1-3x^4}{(-2+x)\left(1+x^2\right)^2}\,dx = \frac{3+2(-2+x)}{5\left(5+4(-2+x)+(-2+x)^2\right)} - \frac{46\arctan(x)}{25} - \frac{14}{25}\log\left(5+4(-2+x)+(-2+x)^2\right) - \frac{47}{25}\log(-2+x)$$

[In] Integrate[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2),x]

[Out] (3 + 2*(-2 + x))/(5*(5 + 4*(-2 + x) + (-2 + x)^2)) - (46*ArcTan[x])/25 - (14*Log[5 + 4*(-2 + x) + (-2 + x)^2])/25 - (47*Log[-2 + x])/25

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

method	result
risch	$\frac{\frac{2x}{5}-\frac{1}{5}}{x^2+1}-\frac{14\ln\left(x^2+1\right)}{25}-\frac{46\arctan\left(x\right)}{25}-\frac{47\ln\left(x-2\right)}{25}$
default	$-\frac{2\left(-5x+\frac{5}{2}\right)}{25\left(x^2+1\right)}-\frac{14\ln\left(x^2+1\right)}{25}-\frac{46\arctan\left(x\right)}{25}-\frac{47\ln\left(x-2\right)}{25}$
parallelrisch	$-\frac{-23i\ln(x-i)x^2+23i\ln(x+i)x^2+47\ln(x-2)x^2+14\ln(x-i)x^2+14\ln(x+i)x^2+5-23i\ln(x-i)+23i\ln(x+i)+47\ln(x-2)+14\ln(}{25(x^2+1)}$

[In] int((-3*x^4+1)/(x-2)/(x^2+1)^2,x,method=_RETURNVERBOSE)

[Out] (2/5*x-1/5)/(x^2+1)-14/25*ln(x^2+1)-46/25*arctan(x)-47/25*ln(x-2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{1-3x^4}{(-2+x)\left(1+x^2\right)^2}\,dx = -\frac{46\left(x^2+1\right)\arctan\left(x\right)+14\left(x^2+1\right)\log\left(x^2+1\right)+47\left(x^2+1\right)\log\left(x-2\right)-10\,x+5}{25\left(x^2+1\right)}$$

[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="fricas")

[Out] -1/25*(46*(x^2 + 1)*arctan(x) + 14*(x^2 + 1)*log(x^2 + 1) + 47*(x^2 + 1)*log(x - 2) - 10*x + 5)/(x^2 + 1)

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{1-3x^4}{(-2+x)(1+x^2)^2}\,dx = -\frac{1-2x}{5x^2+5} - \frac{47\log(x-2)}{25} - \frac{14\log(x^2+1)}{25} - \frac{46\,\mathrm{atan}(x)}{25}$$

```
[In] integrate((-3*x**4+1)/(-2+x)/(x**2+1)**2,x)
[Out] -(1 - 2*x)/(5*x**2 + 5) - 47*log(x - 2)/25 - 14*log(x**2 + 1)/25 - 46*atan(
x)/25
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{1-3x^4}{(-2+x)(1+x^2)^2}\,dx = \frac{2x-1}{5(x^2+1)} - \frac{46}{25}\arctan(x) - \frac{14}{25}\log(x^2+1) - \frac{47}{25}\log(x-2)$$

```
[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/5*(2*x - 1)/(x^2 + 1) - 46/25*arctan(x) - 14/25*log(x^2 + 1) - 47/25*log(
x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1-3x^4}{(-2+x)(1+x^2)^2}\,dx = \frac{2x-1}{5(x^2+1)} - \frac{46}{25}\arctan(x) - \frac{14}{25}\log(x^2+1) - \frac{47}{25}\log(|x-2|)$$

```
[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="giac")
[Out] 1/5*(2*x - 1)/(x^2 + 1) - 46/25*arctan(x) - 14/25*log(x^2 + 1) - 47/25*log(
abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{1-3x^4}{(-2+x)\,(1+x^2)^2}\,dx = \frac{\frac{2\,x}{5}-\frac{1}{5}}{x^2+1} - \frac{47\,\ln\left(x-2\right)}{25} + \ln\left(x-\mathrm{i}\right)\,\left(-\frac{14}{25}+\frac{23}{25}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\,\left(-\frac{14}{25}-\frac{23}{25}\mathrm{i}\right)$$

```
[In] int(-(3*x^4 - 1)/((x^2 + 1)^2*(x - 2)),x)
```

```
[Out] ((2*x)/5 - 1/5)/(x^2 + 1) - log(x - 1i)*(14/25 - 23i/25) - log(x + 1i)*(14/
25 + 23i/25) - (47*log(x - 2))/25
```

3.259 $\int \frac{-9-9x+2x^2}{-9x+x^3}\,dx$

Optimal result 1747
Rubi [A] (verified) 1747
Mathematica [A] (verified) 1748
Maple [A] (verified) 1748
Fricas [A] (verification not implemented) 1749
Sympy [A] (verification not implemented) 1749
Maxima [A] (verification not implemented) 1749
Giac [A] (verification not implemented) 1749
Mupad [B] (verification not implemented) 1750

Optimal result

Integrand size = 20, antiderivative size = 17

$$\int \frac{-9-9x+2x^2}{-9x+x^3}\,dx = -\log(3-x)+\log(x)+2\log(3+x)$$

```
[Out] -ln(3-x)+ln(x)+2*ln(3+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1607, 1816}

$$\int \frac{-9-9x+2x^2}{-9x+x^3}\,dx = -\log(3-x)+\log(x)+2\log(x+3)$$

```
[In] Int[(-9 - 9*x + 2*x^2)/(-9*x + x^3),x]
```

```
[Out] -Log[3 - x] + Log[x] + 2*Log[3 + x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{-9-9x+2x^2}{x(-9+x^2)}\,dx\\ &= \int \left(\frac{1}{3-x}+\frac{1}{x}+\frac{2}{3+x}\right)dx\\ &= -\log(3-x)+\log(x)+2\log(3+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-9-9x+2x^2}{-9x+x^3}\,dx = -\log(3-x)+\log(x)+2\log(3+x)$$

```
[In] Integrate[(-9 - 9*x + 2*x^2)/(-9*x + x^3),x]
[Out] -Log[3 - x] + Log[x] + 2*Log[3 + x]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\ln(x)-\ln(-3+x)+2\ln(3+x)$	16
norman	$\ln(x)-\ln(-3+x)+2\ln(3+x)$	16
risch	$\ln(x)-\ln(-3+x)+2\ln(3+x)$	16
parallelrisch	$\ln(x)-\ln(-3+x)+2\ln(3+x)$	16
meijerg	$\frac{\ln\left(1-\frac{x^2}{9}\right)}{2}+\ln(x)-\ln(3)+\frac{i\pi}{2}+3\ \operatorname{arctanh}\left(\frac{x}{3}\right)$	28

```
[In] int((2*x^2-9*x-9)/(x^3-9*x),x,method=_RETURNVERBOSE)
[Out] ln(x)-ln(-3+x)+2*ln(3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-9-9x+2x^2}{-9x+x^3}\,dx = 2\,\log\left(x+3\right)-\log\left(x-3\right)+\log\left(x\right)$$

```
[In] integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="fricas")
[Out] 2*log(x + 3) - log(x - 3) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{-9-9x+2x^2}{-9x+x^3}\,dx = \log\left(x\right)-\log\left(x-3\right)+2\log\left(x+3\right)$$

```
[In] integrate((2*x**2-9*x-9)/(x**3-9*x),x)
[Out] log(x) - log(x - 3) + 2*log(x + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-9-9x+2x^2}{-9x+x^3}\,dx = 2\,\log\left(x+3\right)-\log\left(x-3\right)+\log\left(x\right)$$

```
[In] integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="maxima")
[Out] 2*log(x + 3) - log(x - 3) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{-9-9x+2x^2}{-9x+x^3}\,dx = 2\,\log\left(|x+3|\right)-\log\left(|x-3|\right)+\log\left(|x|\right)$$

```
[In] integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="giac")
[Out] 2*log(abs(x + 3)) - log(abs(x - 3)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 10.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{-9-9x+2x^2}{-9x+x^3}\,dx = 2\,\ln(x+3) - 2\,\mathrm{atanh}\left(\frac{1296}{18\,x+162} - 7\right)$$

```
[In] int((9*x - 2*x^2 + 9)/(9*x - x^3),x)
[Out] 2*log(x + 3) - 2*atanh(1296/(18*x + 162) - 7)
```

3.260 $\int \frac{1+2x^2+x^5}{-x+x^3} \, dx$

Optimal result 1751
Rubi [A] (verified) 1751
Mathematica [A] (verified) 1752
Maple [A] (verified) 1752
Fricas [A] (verification not implemented) 1753
Sympy [A] (verification not implemented) 1753
Maxima [A] (verification not implemented) 1753
Giac [A] (verification not implemented) 1753
Mupad [B] (verification not implemented) 1754

Optimal result

Integrand size = 20, antiderivative size = 25

$$\int \frac{1+2x^2+x^5}{-x+x^3} \, dx = x+\frac{x^3}{3}+2\log(1-x)-\log(x)+\log(1+x)$$

```
[Out] x+1/3*x^3+2*ln(1-x)-ln(x)+ln(1+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1607, 1816}

$$\int \frac{1+2x^2+x^5}{-x+x^3} \, dx = \frac{x^3}{3}+x+2\log(1-x)-\log(x)+\log(x+1)$$

```
[In] Int[(1 + 2*x^2 + x^5)/(-x + x^3),x]
```

```
[Out] x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1+2x^2+x^5}{x\,(-1+x^2)}\,dx \\
&= \int \left(1+\frac{2}{-1+x}-\frac{1}{x}+x^2+\frac{1}{1+x}\right) dx \\
&= x+\frac{x^3}{3}+2\log(1-x)-\log(x)+\log(1+x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1+2x^2+x^5}{-x+x^3}\,dx = x+\frac{x^3}{3}+2\log(1-x)-\log(x)+\log(1+x)$$

```
[In] Integrate[(1 + 2*x^2 + x^5)/(-x + x^3),x]
[Out] x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{x^3}{3}+x-\ln(x)+\ln(x+1)+2\ln(x-1)$	22
norman	$\frac{x^3}{3}+x-\ln(x)+\ln(x+1)+2\ln(x-1)$	22
risch	$\frac{x^3}{3}+x-\ln(x)+\ln(x+1)+2\ln(x-1)$	22
parallelrisch	$\frac{x^3}{3}+x-\ln(x)+\ln(x+1)+2\ln(x-1)$	22
meijerg	$\frac{3\ln(-x^2+1)}{2}-\ln(x)-\frac{i\pi}{2}+\frac{i\left(-\frac{2ix(5x^2+15)}{15}+2i\,\text{arctanh}(x)\right)}{2}$	40

```
[In] int((x^5+2*x^2+1)/(x^3-x),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3+x-ln(x)+ln(x+1)+2*ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1+2x^2+x^5}{-x+x^3}\,dx = \frac{1}{3}\,x^3 + x + \log\left(x+1\right) + 2\,\log\left(x-1\right) - \log\left(x\right)$$

```
[In] integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="fricas")
[Out] 1/3*x^3 + x + log(x + 1) + 2*log(x - 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1+2x^2+x^5}{-x+x^3}\,dx = \frac{x^3}{3} + x - \log\left(x\right) + 2\log\left(x-1\right) + \log\left(x+1\right)$$

```
[In] integrate((x**5+2*x**2+1)/(x**3-x),x)
[Out] x**3/3 + x - log(x) + 2*log(x - 1) + log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1+2x^2+x^5}{-x+x^3}\,dx = \frac{1}{3}\,x^3 + x + \log\left(x+1\right) + 2\,\log\left(x-1\right) - \log\left(x\right)$$

```
[In] integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="maxima")
[Out] 1/3*x^3 + x + log(x + 1) + 2*log(x - 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1+2x^2+x^5}{-x+x^3}\,dx = \frac{1}{3}\,x^3 + x + \log\left(|x+1|\right) + 2\,\log\left(|x-1|\right) - \log\left(|x|\right)$$

```
[In] integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="giac")
[Out] 1/3*x^3 + x + log(abs(x + 1)) + 2*log(abs(x - 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{1+2x^2+x^5}{-x+x^3}\,dx = x+2\,\ln(x-1)+\frac{x^3}{3}+\mathrm{atan}\left(\frac{48\mathrm{i}}{11\,(22\,x-2)}+\frac{13}{11}\mathrm{i}\right)\,2\mathrm{i}$$

```
[In] int(-(2*x^2 + x^5 + 1)/(x - x^3),x)
```

```
[Out] x + 2*log(x - 1) + atan(48i/(11*(22*x - 2)) + 13i/11)*2i + x^3/3
```

3.261 $\int \frac{3+2x^2}{(-1+x)^2x} \, dx$

Optimal result 1755
Rubi [A] (verified) 1755
Mathematica [A] (verified) 1756
Maple [A] (verified) 1756
Fricas [A] (verification not implemented) 1756
Sympy [A] (verification not implemented) 1757
Maxima [A] (verification not implemented) 1757
Giac [A] (verification not implemented) 1757
Mupad [B] (verification not implemented) 1757

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{3+2x^2}{(-1+x)^2x} \, dx = \frac{5}{1-x} - \log(1-x) + 3\log(x)$$

```
[Out] 5/(1-x)-ln(1-x)+3*ln(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {908}

$$\int \frac{3+2x^2}{(-1+x)^2x} \, dx = \frac{5}{1-x} - \log(1-x) + 3\log(x)$$

```
[In] Int[(3 + 2*x^2)/((-1 + x)^2*x),x]
```

```
[Out] 5/(1 - x) - Log[1 - x] + 3*Log[x]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{1-x} + \frac{5}{(-1+x)^2} + \frac{3}{x}\right) \, dx \\ &= \frac{5}{1-x} - \log(1-x) + 3\log(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{3+2x^2}{(-1+x)^2x}\,dx=-\frac{5}{-1+x}-\log(1-x)+3\log(x)$$

```
[In] Integrate[(3 + 2*x^2)/((-1 + x)^2*x),x]
[Out] -5/(-1 + x) - Log[1 - x] + 3*Log[x]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$3\ln(x)-\frac{5}{x-1}-\ln(x-1)$	19
norman	$3\ln(x)-\frac{5}{x-1}-\ln(x-1)$	19
risch	$3\ln(x)-\frac{5}{x-1}-\ln(x-1)$	19
parallelrisch	$\frac{3\ln(x)x-\ln(x-1)x-5-3\ln(x)+\ln(x-1)}{x-1}$	29
meijerg	$\frac{2x}{1-x}-\ln(1-x)+\frac{6x}{-2x+2}+3+3\ln(x)+3i\pi$	39

```
[In] int((2*x^2+3)/(x-1)^2/x,x,method=_RETURNVERBOSE)
[Out] 3*ln(x)-5/(x-1)-ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{3+2x^2}{(-1+x)^2x}\,dx=-\frac{(x-1)\log(x-1)-3(x-1)\log(x)+5}{x-1}$$

```
[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="fricas")
[Out] -((x - 1)*log(x - 1) - 3*(x - 1)*log(x) + 5)/(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \frac{3+2x^2}{(-1+x)^2x}\,dx = 3\log(x) - \log(x-1) - \frac{5}{x-1}$$

```
[In] integrate((2*x**2+3)/(-1+x)**2/x,x)
[Out] 3*log(x) - log(x - 1) - 5/(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{3+2x^2}{(-1+x)^2x}\,dx = -\frac{5}{x-1} - \log(x-1) + 3\log(x)$$

```
[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="maxima")
[Out] -5/(x - 1) - log(x - 1) + 3*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{3+2x^2}{(-1+x)^2x}\,dx = -\frac{5}{x-1} + 2\log(|x-1|) + 3\log\left(\left|-\frac{1}{x-1} - 1\right|\right)$$

```
[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="giac")
[Out] -5/(x - 1) + 2*log(abs(x - 1)) + 3*log(abs(-1/(x - 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{3+2x^2}{(-1+x)^2x}\,dx = 3\ln(x) - \ln(x-1) - \frac{5}{x-1}$$

```
[In] int((2*x^2 + 3)/(x*(x - 1)^2),x)
[Out] 3*log(x) - log(x - 1) - 5/(x - 1)
```

3.262 $\int \frac{-1+2x^2}{(-1+4x)(1+x^2)} \, dx$

Optimal result 1758
Rubi [A] (verified) 1758
Mathematica [A] (verified) 1759
Maple [A] (verified) 1760
Fricas [A] (verification not implemented) 1760
Sympy [A] (verification not implemented) 1760
Maxima [A] (verification not implemented) 1761
Giac [A] (verification not implemented) 1761
Mupad [B] (verification not implemented) 1761

Optimal result

Integrand size = 22, antiderivative size = 27

$$\int \frac{-1+2x^2}{(-1+4x)(1+x^2)} \, dx = \frac{3\arctan(x)}{17} - \frac{7}{34}\log(1-4x) + \frac{6}{17}\log\left(1+x^2\right)$$

```
[Out] 3/17*arctan(x)-7/34*ln(1-4*x)+6/17*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1643, 649, 209, 266}

$$\int \frac{-1+2x^2}{(-1+4x)(1+x^2)} \, dx = \frac{3\arctan(x)}{17} + \frac{6}{17}\log\left(x^2+1\right) - \frac{7}{34}\log(1-4x)$$

```
[In] Int[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)),x]
```

```
[Out] (3*ArcTan[x])/17 - (7*Log[1 - 4*x])/34 + (6*Log[1 + x^2])/17
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{14}{17(-1+4x)}+\frac{3(1+4x)}{17\,(1+x^2)}\right)\,dx \\
&= -\frac{7}{34}\log(1-4x)+\frac{3}{17}\int \frac{1+4x}{1+x^2}\,dx \\
&= -\frac{7}{34}\log(1-4x)+\frac{3}{17}\int \frac{1}{1+x^2}\,dx+\frac{12}{17}\int \frac{x}{1+x^2}\,dx \\
&= \frac{3}{17}\tan^{-1}(x)-\frac{7}{34}\log(1-4x)+\frac{6}{17}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{-1+2x^2}{(-1+4x)\,(1+x^2)}\,dx = \frac{3\arctan(x)}{17}-\frac{7}{34}\log(-1+4x) + \frac{6}{17}\log\left(17+2(-1+4x)+(-1+4x)^2\right)$$

```
[In] Integrate[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)),x]
[Out] (3*ArcTan[x])/17 - (7*Log[-1 + 4*x])/34 + (6*Log[17 + 2*(-1 + 4*x) + (-1 +
4*x)^2])/17
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{6 \ln(x^2+1)}{17}+\frac{3 \arctan(x)}{17}-\frac{7 \ln(-1+4x)}{34}$	22
risch	$\frac{6 \ln(x^2+1)}{17}+\frac{3 \arctan(x)}{17}-\frac{7 \ln(-1+4x)}{34}$	22
parallelrisch	$-\frac{7 \ln(x-\frac{1}{4})}{34}+\frac{6 \ln(x-i)}{17}-\frac{3i \ln(x-i)}{34}+\frac{6 \ln(x+i)}{17}+\frac{3i \ln(x+i)}{34}$	38

```
[In] int((2*x^2-1)/(-1+4*x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 6/17*ln(x^2+1)+3/17*arctan(x)-7/34*ln(-1+4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{-1+2x^2}{(-1+4x)(1+x^2)}\,dx = \frac{3}{17}\,\arctan\left(x\right)+\frac{6}{17}\,\log\left(x^2+1\right)-\frac{7}{34}\,\log\left(4\,x-1\right)$$

```
[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="fricas")
[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(4*x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{-1+2x^2}{(-1+4x)(1+x^2)}\,dx = -\frac{7\log\left(x-\frac{1}{4}\right)}{34}+\frac{6\log\left(x^2+1\right)}{17}+\frac{3\,\mathrm{atan}\left(x\right)}{17}$$

```
[In] integrate((2*x**2-1)/(-1+4*x)/(x**2+1),x)
[Out] -7*log(x - 1/4)/34 + 6*log(x**2 + 1)/17 + 3*atan(x)/17
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{-1+2x^2}{(-1+4x)(1+x^2)}\,dx = \frac{3}{17}\arctan(x) + \frac{6}{17}\log\left(x^2+1\right) - \frac{7}{34}\log(4x-1)$$

```
[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(4*x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{-1+2x^2}{(-1+4x)(1+x^2)}\,dx = \frac{3}{17}\arctan(x) + \frac{6}{17}\log\left(x^2+1\right) - \frac{7}{34}\log(|4x-1|)$$

```
[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="giac")
```

```
[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(abs(4*x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{-1+2x^2}{(-1+4x)(1+x^2)}\,dx = -\frac{7\ln\left(x-\frac{1}{4}\right)}{34} + \ln(x-\mathrm{i})\left(\frac{6}{17}-\frac{3}{34}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(\frac{6}{17}+\frac{3}{34}\mathrm{i}\right)$$

```
[In] int((2*x^2 - 1)/((4*x - 1)*(x^2 + 1)),x)
```

```
[Out] log(x - 1i)*(6/17 - 3i/34) - (7*log(x - 1/4))/34 + log(x + 1i)*(6/17 + 3i/3
4)
```

3.263 $\int \frac{-3+2x-3x^2+x^3}{1+x^2} \, dx$

Optimal result 1762
Rubi [A] (verified) 1762
Mathematica [A] (verified) 1763
Maple [A] (verified) 1763
Fricas [A] (verification not implemented) 1764
Sympy [A] (verification not implemented) 1764
Maxima [A] (verification not implemented) 1764
Giac [A] (verification not implemented) 1764
Mupad [B] (verification not implemented) 1765

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{-3+2x-3x^2+x^3}{1+x^2} \, dx = -3x+\frac{x^2}{2}+\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -3*x+1/2*x^2+1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1824, 266}

$$\int \frac{-3+2x-3x^2+x^3}{1+x^2} \, dx = \frac{x^2}{2}+\frac{1}{2}\log\left(x^2+1\right)-3x$$

```
[In] Int[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2),x]
```

```
[Out] -3*x + x^2/2 + Log[1 + x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-3+x+\frac{x}{1+x^2}\right) dx\\ &= -3x+\frac{x^2}{2}+\int \frac{x}{1+x^2}\,dx\\ &= -3x+\frac{x^2}{2}+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx = -3x+\frac{x^2}{2}+\frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2),x]
[Out] -3*x + x^2/2 + Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$-3x+\frac{x^2}{2}+\frac{\ln(x^2+1)}{2}$	18
norman	$-3x+\frac{x^2}{2}+\frac{\ln(x^2+1)}{2}$	18
meijerg	$-3x+\frac{x^2}{2}+\frac{\ln(x^2+1)}{2}$	18
risch	$-3x+\frac{x^2}{2}+\frac{\ln(x^2+1)}{2}$	18
parallelrisch	$-3x+\frac{x^2}{2}+\frac{\ln(x^2+1)}{2}$	18

```
[In] int((x^3-3*x^2+2*x-3)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -3*x+1/2*x^2+1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx = \frac{1}{2}\,x^2 - 3\,x + \frac{1}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="fricas")
[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx = \frac{x^2}{2} - 3x + \frac{\log\left(x^2+1\right)}{2}$$

```
[In] integrate((x**3-3*x**2+2*x-3)/(x**2+1),x)
[Out] x**2/2 - 3*x + log(x**2 + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx = \frac{1}{2}\,x^2 - 3\,x + \frac{1}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="maxima")
[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx = \frac{1}{2}\,x^2 - 3\,x + \frac{1}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="giac")
[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx = \frac{\ln(x^2+1)}{2} - 3\,x + \frac{x^2}{2}$$

```
[In] int((2*x - 3*x^2 + x^3 - 3)/(x^2 + 1),x)
[Out] log(x^2 + 1)/2 - 3*x + x^2/2
```

3.264 $\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2}\,dx$

Optimal result 1766
Rubi [A] (verified) 1766
Mathematica [A] (verified) 1768
Maple [A] (verified) 1768
Fricas [A] (verification not implemented) 1768
Sympy [A] (verification not implemented) 1769
Maxima [A] (verification not implemented) 1769
Giac [A] (verification not implemented) 1769
Mupad [B] (verification not implemented) 1769

Optimal result

Integrand size = 26, antiderivative size = 27

$$\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2}\,dx = \frac{x^3}{3}-3\arctan(3+x)+\frac{1}{2}\log\left(10+6x+x^2\right)$$

```
[Out] 1/3*x^3-3*arctan(3+x)+1/2*ln(x^2+6*x+10)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1671, 648, 632, 210, 642}

$$\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2}\,dx = -3\arctan(x+3)+\frac{x^3}{3}+\frac{1}{2}\log\left(x^2+6x+10\right)$$

```
[In] Int[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2),x]
```

```
[Out] x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(x^2+\frac{x}{10+6x+x^2}\right) dx \\
&= \frac{x^3}{3}+\int \frac{x}{10+6x+x^2}\,dx \\
&= \frac{x^3}{3}+\frac{1}{2}\int \frac{6+2x}{10+6x+x^2}\,dx-3\int \frac{1}{10+6x+x^2}\,dx \\
&= \frac{x^3}{3}+\frac{1}{2}\log\left(10+6x+x^2\right)+6\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,6+2x\right) \\
&= \frac{x^3}{3}-3\tan^{-1}(3+x)+\frac{1}{2}\log\left(10+6x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2}\,dx=\frac{x^3}{3}-3\arctan(3+x)+\frac{1}{2}\log\left(10+6x+x^2\right)$$

```
[In] Integrate[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2),x]
[Out] x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{x^3}{3}-3\arctan\left(3+x\right)+\frac{\ln\left(x^2+6x+10\right)}{2}$	24
risch	$\frac{x^3}{3}-3\arctan\left(3+x\right)+\frac{\ln\left(x^2+6x+10\right)}{2}$	24
parallelrisch	$\frac{x^3}{3}+\frac{\ln(x+3-i)}{2}+\frac{3i\ln(x+3-i)}{2}+\frac{\ln(x+3+i)}{2}-\frac{3i\ln(x+3+i)}{2}$	41

```
[In] int((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3-3*arctan(3+x)+1/2*ln(x^2+6*x+10)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2}\,dx=\frac{1}{3}\,x^3-3\,\arctan\left(x+3\right)+\frac{1}{2}\,\log\left(x^2+6\,x+10\right)$$

```
[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="fricas")
[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2}\,dx = \frac{x^3}{3} + \frac{\log(x^2+6x+10)}{2} - 3\operatorname{atan}(x+3)$$

```
[In] integrate((x**4+6*x**3+10*x**2+x)/(x**2+6*x+10),x)
[Out] x**3/3 + log(x**2 + 6*x + 10)/2 - 3*atan(x + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2}\,dx = \frac{1}{3}\,x^3 - 3\,\arctan(x+3) + \frac{1}{2}\,\log\left(x^2+6\,x+10\right)$$

```
[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="maxima")
[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2}\,dx = \frac{1}{3}\,x^3 - 3\,\arctan(x+3) + \frac{1}{2}\,\log\left(x^2+6\,x+10\right)$$

```
[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="giac")
[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2}\,dx = \frac{\ln(x^2+6\,x+10)}{2} - 3\operatorname{atan}(x+3) + \frac{x^3}{3}$$

```
[In] int((x + 10*x^2 + 6*x^3 + x^4)/(6*x + x^2 + 10),x)
[Out] log(6*x + x^2 + 10)/2 - 3*atan(x + 3) + x^3/3
```

3.265 $\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx$

Optimal result 1770
Rubi [A] (verified) 1770
Mathematica [A] (verified) 1771
Maple [A] (verified) 1771
Fricas [A] (verification not implemented) 1772
Sympy [A] (verification not implemented) 1772
Maxima [A] (verification not implemented) 1772
Giac [A] (verification not implemented) 1773
Mupad [B] (verification not implemented) 1773

Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx = \frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(3+x)$$

```
[Out] 1/8*ln(1-x)-1/5*ln(2-x)+1/12*ln(3-x)-1/120*ln(3+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2083}

$$\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx = \frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(x+3)$$

```
[In] Int[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1),x]
```

```
[Out] Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{12(-3+x)} - \frac{1}{5(-2+x)} + \frac{1}{8(-1+x)} - \frac{1}{120(3+x)} \right) dx \\ &= \frac{1}{8}\log(1-x) - \frac{1}{5}\log(2-x) + \frac{1}{12}\log(3-x) - \frac{1}{120}\log(3+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx = \frac{1}{8}\log(1-x) - \frac{1}{5}\log(2-x) + \frac{1}{12}\log(3-x) - \frac{1}{120}\log(3+x)$$

```
[In] Integrate[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1),x]
[Out] Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\ln(-3+x)}{12}-\frac{\ln(3+x)}{120}+\frac{\ln(x-1)}{8}-\frac{\ln(x-2)}{5}$	26
norman	$\frac{\ln(-3+x)}{12}-\frac{\ln(3+x)}{120}+\frac{\ln(x-1)}{8}-\frac{\ln(x-2)}{5}$	26
risch	$\frac{\ln(-3+x)}{12}-\frac{\ln(3+x)}{120}+\frac{\ln(x-1)}{8}-\frac{\ln(x-2)}{5}$	26
parallelrisch	$\frac{\ln(-3+x)}{12}-\frac{\ln(3+x)}{120}+\frac{\ln(x-1)}{8}-\frac{\ln(x-2)}{5}$	26

```
[In] int(1/(x^4-3*x^3-7*x^2+27*x-18),x,method=_RETURNVERBOSE)
[Out] 1/12*ln(-3+x)-1/120*ln(3+x)+1/8*ln(x-1)-1/5*ln(x-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx = -\frac{1}{120}\log{(x+3)}+\frac{1}{8}\log{(x-1)} -\frac{1}{5}\log{(x-2)}+\frac{1}{12}\log{(x-3)}$$

```
[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="fricas")
[Out] -1/120*log(x + 3) + 1/8*log(x - 1) - 1/5*log(x - 2) + 1/12*log(x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

$$\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx = \frac{\log{(x-3)}}{12}-\frac{\log{(x-2)}}{5}+\frac{\log{(x-1)}}{8}-\frac{\log{(x+3)}}{120}$$

```
[In] integrate(1/(x**4-3*x**3-7*x**2+27*x-18),x)
[Out] log(x - 3)/12 - log(x - 2)/5 + log(x - 1)/8 - log(x + 3)/120
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx = -\frac{1}{120}\log{(x+3)}+\frac{1}{8}\log{(x-1)} -\frac{1}{5}\log{(x-2)}+\frac{1}{12}\log{(x-3)}$$

```
[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="maxima")
[Out] -1/120*log(x + 3) + 1/8*log(x - 1) - 1/5*log(x - 2) + 1/12*log(x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx = -\frac{1}{120}\log(|x+3|)+\frac{1}{8}\log(|x-1|) - \frac{1}{5}\log(|x-2|)+\frac{1}{12}\log(|x-3|)$$

```
[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="giac")
[Out] -1/120*log(abs(x + 3)) + 1/8*log(abs(x - 1)) - 1/5*log(abs(x - 2)) + 1/12*l
og(abs(x - 3))
```

Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx = \frac{\ln(x-1)}{8}-\frac{\ln(x-2)}{5}+\frac{\ln(x-3)}{12}-\frac{\ln(x+3)}{120}$$

```
[In] int(-1/(7*x^2 - 27*x + 3*x^3 - x^4 + 18),x)
[Out] log(x - 1)/8 - log(x - 2)/5 + log(x - 3)/12 - log(x + 3)/120
```

3.266 $\int \frac{1+x^3}{-2+x}\,dx$

Optimal result 1774
Rubi [A] (verified) 1774
Mathematica [A] (verified) 1775
Maple [A] (verified) 1775
Fricas [A] (verification not implemented) 1775
Sympy [A] (verification not implemented) 1776
Maxima [A] (verification not implemented) 1776
Giac [A] (verification not implemented) 1776
Mupad [B] (verification not implemented) 1776

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{1+x^3}{-2+x}\,dx = 4x + x^2 + \frac{x^3}{3} + 9\log(2-x)$$

```
[Out] 4*x+x^2+1/3*x^3+9*ln(2-x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1864}

$$\int \frac{1+x^3}{-2+x}\,dx = \frac{x^3}{3} + x^2 + 4x + 9\log(2-x)$$

```
[In] Int[(1 + x^3)/(-2 + x),x]
[Out] 4*x + x^2 + x^3/3 + 9*Log[2 - x]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(4 + \frac{9}{-2+x} + 2x + x^2\right) dx \\ &= 4x + x^2 + \frac{x^3}{3} + 9\log(2-x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1+x^3}{-2+x}\,dx = -\frac{44}{3}+4x+x^2+\frac{x^3}{3}+9\log(-2+x)$$

```
[In] Integrate[(1 + x^3)/(-2 + x),x]
[Out] -44/3 + 4*x + x^2 + x^3/3 + 9*Log[-2 + x]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x^3}{3}+x^2+4x+9\ln\left(x-2\right)$	19
norman	$\frac{x^3}{3}+x^2+4x+9\ln\left(x-2\right)$	19
risch	$\frac{x^3}{3}+x^2+4x+9\ln\left(x-2\right)$	19
parallelrisch	$\frac{x^3}{3}+x^2+4x+9\ln\left(x-2\right)$	19
meijerg	$9\ln\left(1-\frac{x}{2}\right)+\frac{x(x^2+3x+12)}{3}$	21

```
[In] int((x^3+1)/(x-2),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3+x^2+4*x+9*ln(x-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1+x^3}{-2+x}\,dx = \frac{1}{3}\,x^3+x^2+4\,x+9\,\log\left(x-2\right)$$

```
[In] integrate((x^3+1)/(-2+x),x, algorithm="fricas")
[Out] 1/3*x^3 + x^2 + 4*x + 9*log(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1+x^3}{-2+x}\,dx = \frac{x^3}{3} + x^2 + 4x + 9\log(x-2)$$

```
[In] integrate((x**3+1)/(-2+x),x)
[Out] x**3/3 + x**2 + 4*x + 9*log(x - 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1+x^3}{-2+x}\,dx = \frac{1}{3}\,x^3 + x^2 + 4\,x + 9\,\log(x-2)$$

```
[In] integrate((x^3+1)/(-2+x),x, algorithm="maxima")
[Out] 1/3*x^3 + x^2 + 4*x + 9*log(x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1+x^3}{-2+x}\,dx = \frac{1}{3}\,x^3 + x^2 + 4\,x + 9\,\log(|x-2|)$$

```
[In] integrate((x^3+1)/(-2+x),x, algorithm="giac")
[Out] 1/3*x^3 + x^2 + 4*x + 9*log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1+x^3}{-2+x}\,dx = 4\,x + 9\,\ln(x-2) + x^2 + \frac{x^3}{3}$$

```
[In] int((x^3 + 1)/(x - 2),x)
[Out] 4*x + 9*log(x - 2) + x^2 + x^3/3
```

3.267 $\int \frac{3x-4x^2+3x^3}{1+x^2} dx$

Optimal result 1777
Rubi [A] (verified) 1777
Mathematica [A] (verified) 1778
Maple [A] (verified) 1778
Fricas [A] (verification not implemented) 1779
Sympy [A] (verification not implemented) 1779
Maxima [A] (verification not implemented) 1779
Giac [A] (verification not implemented) 1779
Mupad [B] (verification not implemented) 1780

Optimal result

Integrand size = 22, antiderivative size = 15

$$\int \frac{3x-4x^2+3x^3}{1+x^2} dx = -4x + \frac{3x^2}{2} + 4\arctan(x)$$

```
[Out] -4*x+3/2*x^2+4*arctan(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1608, 1816, 209}

$$\int \frac{3x-4x^2+3x^3}{1+x^2} dx = 4\arctan(x) + \frac{3x^2}{2} - 4x$$

```
[In] Int[(3*x - 4*x^2 + 3*x^3)/(1 + x^2),x]
```

```
[Out] -4*x + (3*x^2)/2 + 4*ArcTan[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(3-4x+3x^2)}{1+x^2}\,dx\\ &= \int \left(-4+3x+\frac{4}{1+x^2}\right)dx\\ &= -4x+\frac{3x^2}{2}+4\int \frac{1}{1+x^2}\,dx\\ &= -4x+\frac{3x^2}{2}+4\tan^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx = -4x+\frac{3x^2}{2}+4\arctan(x)$$

```
[In] Integrate[(3*x - 4*x^2 + 3*x^3)/(1 + x^2),x]
[Out] -4*x + (3*x^2)/2 + 4*ArcTan[x]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-4x+\frac{3x^2}{2}+4\arctan(x)$	14
meijerg	$-4x+\frac{3x^2}{2}+4\arctan(x)$	14
risch	$-4x+\frac{3x^2}{2}+4\arctan(x)$	14
parallelrisch	$-4x+\frac{3x^2}{2}+2i\ln(x+i)-2i\ln(x-i)$	26

```
[In] int((3*x^3-4*x^2+3*x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -4*x+3/2*x^2+4*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx = \frac{3}{2}\,x^2 - 4\,x + 4\,\arctan\left(x\right)$$

```
[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="fricas")
[Out] 3/2*x^2 - 4*x + 4*arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx = \frac{3x^2}{2} - 4x + 4\,\mathrm{atan}\left(x\right)$$

```
[In] integrate((3*x**3-4*x**2+3*x)/(x**2+1),x)
[Out] 3*x**2/2 - 4*x + 4*atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx = \frac{3}{2}\,x^2 - 4\,x + 4\,\arctan\left(x\right)$$

```
[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="maxima")
[Out] 3/2*x^2 - 4*x + 4*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx = \frac{3}{2}\,x^2 - 4\,x + 4\,\arctan\left(x\right)$$

```
[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="giac")
[Out] 3/2*x^2 - 4*x + 4*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx = 4\,\mathrm{atan}(x) - 4\,x + \frac{3\,x^2}{2}$$

```
[In] int((3*x - 4*x^2 + 3*x^3)/(x^2 + 1),x)
[Out] 4*atan(x) - 4*x + (3*x^2)/2
```

3.268 $\int \frac{5+3x}{1-x-x^2+x^3} dx$

Optimal result 1781
Rubi [A] (verified) 1781
Mathematica [A] (verified) 1782
Maple [A] (verified) 1782
Fricas [B] (verification not implemented) 1783
Sympy [B] (verification not implemented) 1783
Maxima [A] (verification not implemented) 1783
Giac [B] (verification not implemented) 1784
Mupad [B] (verification not implemented) 1784

Optimal result

Integrand size = 21, antiderivative size = 12

$$\int \frac{5+3x}{1-x-x^2+x^3} dx = \frac{4}{1-x} + \operatorname{arctanh}(x)$$

```
[Out] 4/(1-x)+arctanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2099, 212}

$$\int \frac{5+3x}{1-x-x^2+x^3} dx = \operatorname{arctanh}(x) + \frac{4}{1-x}$$

```
[In] Int[(5 + 3*x)/(1 - x - x^2 + x^3),x]
```

```
[Out] 4/(1 - x) + ArcTanh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{4}{(-1+x)^2}+\frac{1}{1-x^2}\right)\,dx \\
&= \frac{4}{1-x}+\int \frac{1}{1-x^2}\,dx \\
&= \frac{4}{1-x}+\tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \frac{5+3x}{1-x-x^2+x^3}\,dx = -\frac{4}{-1+x}-\frac{1}{2}\log(-1+x)+\frac{1}{2}\log(1+x)$$

```
[In] Integrate[(5 + 3*x)/(1 - x - x^2 + x^3),x]
[Out] -4/(-1 + x) - Log[-1 + x]/2 + Log[1 + x]/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

method	result	size
default	$\frac{\ln(x+1)}{2}-\frac{4}{x-1}-\frac{\ln(x-1)}{2}$	21
norman	$\frac{\ln(x+1)}{2}-\frac{4}{x-1}-\frac{\ln(x-1)}{2}$	21
risch	$\frac{\ln(x+1)}{2}-\frac{4}{x-1}-\frac{\ln(x-1)}{2}$	21
parallelrisch	$-\frac{\ln(x-1)x-\ln(x+1)x+8-\ln(x-1)+\ln(x+1)}{2(x-1)}$	33

```
[In] int((5+3*x)/(x^3-x^2-x+1),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x+1)-4/(x-1)-1/2*ln(x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{5+3x}{1-x-x^2+x^3}\,dx = \frac{(x-1)\log(x+1)-(x-1)\log(x-1)-8}{2\,(x-1)}$$

```
[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="fricas")
[Out] 1/2*((x - 1)*log(x + 1) - (x - 1)*log(x - 1) - 8)/(x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$.

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{5+3x}{1-x-x^2+x^3}\,dx = -\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2} - \frac{4}{x-1}$$

```
[In] integrate((5+3*x)/(x**3-x**2-x+1),x)
[Out] -log(x - 1)/2 + log(x + 1)/2 - 4/(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{5+3x}{1-x-x^2+x^3}\,dx = -\frac{4}{x-1} + \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

```
[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="maxima")
[Out] -4/(x - 1) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{5+3x}{1-x-x^2+x^3}\,dx = -\frac{4}{x-1} + \frac{1}{2}\log\left(|x+1|\right) - \frac{1}{2}\log\left(|x-1|\right)$$

```
[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="giac")
[Out] -4/(x - 1) + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{5+3x}{1-x-x^2+x^3}\,dx = \operatorname{atanh}(x) - \frac{4}{x-1}$$

```
[In] int(-(3*x + 5)/(x + x^2 - x^3 - 1),x)
[Out] atanh(x) - 4/(x - 1)
```

3.269 $\int \frac{-1-x-x^3+x^4}{-x^2+x^3} dx$

Optimal result . . . 1785
Rubi [A] (verified) . . . 1785
Mathematica [A] (verified) . . . 1786
Maple [A] (verified) . . . 1786
Fricas [A] (verification not implemented) . . . 1787
Sympy [A] (verification not implemented) . . . 1787
Maxima [A] (verification not implemented) . . . 1787
Giac [A] (verification not implemented) . . . 1787
Mupad [B] (verification not implemented) . . . 1788

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{-1-x-x^3+x^4}{-x^2+x^3} dx = -\frac{1}{x}+\frac{x^2}{2}-2\log(1-x)+2\log(x)$$

```
[Out] -1/x+1/2*x^2-2*ln(1-x)+2*ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1607, 1634}

$$\int \frac{-1-x-x^3+x^4}{-x^2+x^3} dx = \frac{x^2}{2}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

```
[In] Int[(-1 - x - x^3 + x^4)/(-x^2 + x^3),x]
```

```
[Out] -x^(-1) + x^2/2 - 2*Log[1 - x] + 2*Log[x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
```

```
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{-1-x-x^3+x^4}{(-1+x)x^2}\,dx\\ &= \int \left(-\frac{2}{-1+x}+\frac{1}{x^2}+\frac{2}{x}+x\right)dx\\ &= -\frac{1}{x}+\frac{x^2}{2}-2\log(1-x)+2\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{-1-x-x^3+x^4}{-x^2+x^3}\,dx = -\frac{1}{x}+\frac{x^2}{2}-2\log(1-x)+2\log(x)$$

```
[In] Integrate[(-1 - x - x^3 + x^4)/(-x^2 + x^3),x]
[Out] -x^(-1) + x^2/2 - 2*Log[1 - x] + 2*Log[x]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{x^2}{2}-\frac{1}{x}+2\ln(x)-2\ln(x-1)$	22
risch	$\frac{x^2}{2}-\frac{1}{x}+2\ln(x)-2\ln(x-1)$	22
norman	$\frac{-1+\frac{x^3}{2}}{x}+2\ln(x)-2\ln(x-1)$	23
parallelrisch	$\frac{x^3+4\ln(x)x-4\ln(x-1)x-2}{2x}$	23
meijerg	$2\ln(x)+2i\pi-\frac{1}{x}+\frac{x(6+3x)}{6}-x-2\ln(1-x)$	34

```
[In] int((x^4-x^3-x-1)/(x^3-x^2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2-1/x+2*ln(x)-2*ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{-1-x-x^3+x^4}{-x^2+x^3}\,dx = \frac{x^3-4\,x\log\left(x-1\right)+4\,x\log\left(x\right)-2}{2\,x}$$

```
[In] integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="fricas")
[Out] 1/2*(x^3 - 4*x*log(x - 1) + 4*x*log(x) - 2)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{-1-x-x^3+x^4}{-x^2+x^3}\,dx = \frac{x^2}{2}+2\log\left(x\right)-2\log\left(x-1\right)-\frac{1}{x}$$

```
[In] integrate((x**4-x**3-x-1)/(x**3-x**2),x)
[Out] x**2/2 + 2*log(x) - 2*log(x - 1) - 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{-1-x-x^3+x^4}{-x^2+x^3}\,dx = \frac{1}{2}\,x^2-\frac{1}{x}-2\,\log\left(x-1\right)+2\,\log\left(x\right)$$

```
[In] integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/x - 2*log(x - 1) + 2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{-1-x-x^3+x^4}{-x^2+x^3}\,dx = \frac{1}{2}\,x^2-\frac{1}{x}-2\,\log\left(|x-1|\right)+2\,\log\left(|x|\right)$$

```
[In] integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="giac")
[Out] 1/2*x^2 - 1/x - 2*log(abs(x - 1)) + 2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{-1-x-x^3+x^4}{-x^2+x^3}\,dx = 4\,\mathrm{atanh}(2\,x-1)-\frac{1}{x}+\frac{x^2}{2}$$

```
[In] int((x + x^3 - x^4 + 1)/(x^2 - x^3),x)
[Out] 4*atanh(2*x - 1) - 1/x + x^2/2
```

3.270 $\int \frac{2+x+x^2+x^3}{2+3x^2+x^4} \, dx$

Optimal result 1789
Rubi [A] (verified) 1789
Mathematica [A] (verified) 1791
Maple [A] (verified) 1791
Fricas [A] (verification not implemented) 1791
Sympy [A] (verification not implemented) 1792
Maxima [A] (verification not implemented) 1792
Giac [A] (verification not implemented) 1792
Mupad [B] (verification not implemented) 1792

Optimal result

Integrand size = 22, antiderivative size = 13

$$\int \frac{2+x+x^2+x^3}{2+3x^2+x^4} \, dx = \arctan(x)+\frac{1}{2}\log\left(2+x^2\right)$$

```
[Out] arctan(x)+1/2*ln(x^2+2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1687, 1163, 209, 1261, 640, 31}

$$\int \frac{2+x+x^2+x^3}{2+3x^2+x^4} \, dx = \arctan(x)+\frac{1}{2}\log\left(x^2+2\right)$$

```
[In] Int[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4),x]
```

```
[Out] ArcTan[x] + Log[2 + x^2]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 640

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(1+x^2)}{2+3x^2+x^4}\,dx + \int \frac{2+x^2}{2+3x^2+x^4}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{2+3x+x^2}\,dx, x, x^2\right) + \int \frac{1}{1+x^2}\,dx\\ &= \tan^{-1}(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2+x}\,dx, x, x^2\right)\\ &= \tan^{-1}(x) + \frac{1}{2}\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx = \arctan(x)+\frac{1}{2}\log\left(2+x^2\right)$$

```
[In] Integrate[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4),x]
[Out] ArcTan[x] + Log[2 + x^2]/2
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\arctan\left(x\right)+\frac{\ln\left(x^{2}+2\right)}{2}$	12
risch	$\arctan\left(x\right)+\frac{\ln\left(x^{2}+2\right)}{2}$	12
parallelrisch	$\frac{i\ln\left(x+i\right)}{2}-\frac{i\ln\left(x-i\right)}{2}+\frac{\ln\left(x^{2}+2\right)}{2}$	26

```
[In] int((x^3+x^2+x+2)/(x^4+3*x^2+2),x,method=_RETURNVERBOSE)
[Out] arctan(x)+1/2*ln(x^2+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx = \arctan\left(x\right)+\frac{1}{2}\,\log\left(x^2+2\right)$$

```
[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="fricas")
[Out] arctan(x) + 1/2*log(x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx = \frac{\log\left(x^2+2\right)}{2} + \operatorname{atan}\left(x\right)$$

```
[In] integrate((x**3+x**2+x+2)/(x**4+3*x**2+2),x)
[Out] log(x**2 + 2)/2 + atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx = \arctan\left(x\right) + \frac{1}{2}\,\log\left(x^2+2\right)$$

```
[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="maxima")
[Out] arctan(x) + 1/2*log(x^2 + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx = \arctan\left(x\right) + \frac{1}{2}\,\log\left(x^2+2\right)$$

```
[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="giac")
[Out] arctan(x) + 1/2*log(x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx = \frac{\ln\left(x^2+2\right)}{2} + \operatorname{atan}(x)$$

```
[In] int((x + x^2 + x^3 + 2)/(3*x^2 + x^4 + 2),x)
[Out] log(x^2 + 2)/2 + atan(x)
```

3.271 $\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3} dx$

Optimal result 1793
Rubi [A] (verified) 1793
Mathematica [A] (verified) 1795
Maple [A] (verified) 1795
Fricas [A] (verification not implemented) 1795
Sympy [A] (verification not implemented) 1796
Maxima [A] (verification not implemented) 1796
Giac [A] (verification not implemented) 1796
Mupad [B] (verification not implemented) 1797

Optimal result

Integrand size = 31, antiderivative size = 35

$$\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3} dx = -\frac{1}{(2+x^2)^2} - \frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{1}{2}\log(2+x^2)$$

```
[Out] -1/(x^2+2)^2+1/2*ln(x^2+2)-1/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1828, 1600, 649, 209, 266}

$$\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3} dx = -\frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{(x^2+2)^2} + \frac{1}{2}\log(x^2+2)$$

```
[In] Int[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3,x]
```

```
[Out] -(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{(2+x^2)^2} - \frac{1}{8}\int \frac{16-16x+8x^2-8x^3}{(2+x^2)^2}\,dx \\
&= -\frac{1}{(2+x^2)^2} - \frac{1}{8}\int \frac{8-8x}{2+x^2}\,dx \\
&= -\frac{1}{(2+x^2)^2} - \int \frac{1}{2+x^2}\,dx + \int \frac{x}{2+x^2}\,dx \\
&= -\frac{1}{(2+x^2)^2} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{1}{2}\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3}\,dx=-\frac{1}{(2+x^2)^2}-\frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(2+x^2\right)$$

```
[In] Integrate[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3,x]
[Out] -(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

method	result
default	$-\frac{1}{(x^2+2)^2}+\frac{\ln(x^2+2)}{2}-\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}$
risch	$-\frac{1}{(x^2+2)^2}+\frac{\ln(x^2+2)}{2}-\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}$
meijerg	$-\frac{\sqrt{2}\left(\frac{x\sqrt{2}\left(\frac{3x^2}{2}+5\right)}{4\left(1+\frac{x^2}{2}\right)^2}+\frac{3\arctan\left(\frac{x\sqrt{2}}{2}\right)}{2}\right)}{8}-\frac{x^2\left(\frac{9x^2}{2}+6\right)}{24\left(1+\frac{x^2}{2}\right)^2}+\frac{\ln\left(1+\frac{x^2}{2}\right)}{2}-\frac{\sqrt{2}\left(-\frac{x\sqrt{2}\left(\frac{25x^2}{2}+15\right)}{20\left(1+\frac{x^2}{2}\right)^2}+\frac{3\arctan\left(\frac{x\sqrt{2}}{2}\right)}{2}\right)}{8}+\frac{x^{4}}{8\left(1+\right.}$

```
[In] int((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x,method=_RETURNVERBOSE)
[Out] -1/(x^2+2)^2+1/2*ln(x^2+2)-1/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

$$\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3}\,dx =-\frac{\sqrt{2}(x^4+4\,x^2+4)\arctan\left(\frac{1}{2}\sqrt{2}x\right)-(x^4+4\,x^2+4)\log\left(x^2+2\right)+2}{2\,(x^4+4\,x^2+4)}$$

```
[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="fricas")
[Out] -1/2*(sqrt(2)*(x^4 + 4*x^2 + 4)*arctan(1/2*sqrt(2)*x) - (x^4 + 4*x^2 + 4)*l
og(x^2 + 2) + 2)/(x^4 + 4*x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3}\,dx = \frac{\log(x^2+2)}{2} - \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2} - \frac{1}{x^4+4x^2+4}$$

```
[In] integrate((x**5-x**4+4*x**3-4*x**2+8*x-4)/(x**2+2)**3,x)
[Out] log(x**2 + 2)/2 - sqrt(2)*atan(sqrt(2)*x/2)/2 - 1/(x**4 + 4*x**2 + 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3}\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{x^4+4x^2+4} + \frac{1}{2}\log(x^2+2)$$

```
[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="maxima")
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/(x^4 + 4*x^2 + 4) + 1/2*log(x^2 + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3}\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{(x^2+2)^2} + \frac{1}{2}\log(x^2+2)$$

```
[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="giac")
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/(x^2 + 2)^2 + 1/2*log(x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3}\,dx = \frac{\ln(x^2+2)}{2} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2} - \frac{1}{x^4+4\,x^2+4}$$

```
[In] int((8*x - 4*x^2 + 4*x^3 - x^4 + x^5 - 4)/(x^2 + 2)^3,x)
[Out] log(x^2 + 2)/2 - (2^(1/2)*atan((2^(1/2)*x)/2))/2 - 1/(4*x^2 + x^4 + 4)
```

3.272 $\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx$

Optimal result 1798
Rubi [A] (verified) 1798
Mathematica [A] (verified) 1799
Maple [A] (verified) 1799
Fricas [A] (verification not implemented) 1800
Sympy [A] (verification not implemented) 1800
Maxima [A] (verification not implemented) 1800
Giac [A] (verification not implemented) 1800
Mupad [B] (verification not implemented) 1801

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx = -\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(2+x)$$

```
[Out] -ln(1-x)+1/2*ln(x)+3/2*ln(2+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1608, 1642}

$$\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx = -\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(x+2)$$

```
[In] Int[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3),x]
```

```
[Out] -Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{-1-3x+x^2}{x\left(-2+x+x^2\right)}\,dx\\ &= \int \left(\frac{1}{1-x}+\frac{1}{2x}+\frac{3}{2(2+x)}\right)dx\\ &= -\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(2+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx = -\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(2+x)$$

```
[In] Integrate[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3),x]
[Out] -Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\ln(x)}{2}+\frac{3\ln(x+2)}{2}-\ln\left(x-1\right)$	18
norman	$\frac{\ln(x)}{2}+\frac{3\ln(x+2)}{2}-\ln\left(x-1\right)$	18
risch	$\frac{\ln(x)}{2}+\frac{3\ln(x+2)}{2}-\ln\left(x-1\right)$	18
parallelrisch	$\frac{\ln(x)}{2}+\frac{3\ln(x+2)}{2}-\ln\left(x-1\right)$	18

```
[In] int((x^2-3*x-1)/(x^3+x^2-2*x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x)+3/2*ln(x+2)-ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx = \frac{3}{2}\,\log\left(x+2\right) - \log\left(x-1\right) + \frac{1}{2}\,\log\left(x\right)$$

```
[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="fricas")
[Out] 3/2*log(x + 2) - log(x - 1) + 1/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx = \frac{\log\left(x\right)}{2} - \log\left(x-1\right) + \frac{3\log\left(x+2\right)}{2}$$

```
[In] integrate((x**2-3*x-1)/(x**3+x**2-2*x),x)
[Out] log(x)/2 - log(x - 1) + 3*log(x + 2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx = \frac{3}{2}\,\log\left(x+2\right) - \log\left(x-1\right) + \frac{1}{2}\,\log\left(x\right)$$

```
[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="maxima")
[Out] 3/2*log(x + 2) - log(x - 1) + 1/2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx = \frac{3}{2}\,\log\left(|x+2|\right) - \log\left(|x-1|\right) + \frac{1}{2}\,\log\left(|x|\right)$$

```
[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="giac")
[Out] 3/2*log(abs(x + 2)) - log(abs(x - 1)) + 1/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx = \frac{3\,\ln(x+2)}{2} - \ln(x-1) + \frac{\ln(x)}{2}$$

```
[In] int(-(3*x - x^2 + 1)/(x^2 - 2*x + x^3),x)
[Out] (3*log(x + 2))/2 - log(x - 1) + log(x)/2
```

3.273 $\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3} dx$

Optimal result 1802
Rubi [A] (verified) 1802
Mathematica [A] (verified) 1803
Maple [A] (verified) 1803
Fricas [A] (verification not implemented) 1804
Sympy [A] (verification not implemented) 1804
Maxima [A] (verification not implemented) 1804
Giac [A] (verification not implemented) 1805
Mupad [B] (verification not implemented) 1805

Optimal result

Integrand size = 33, antiderivative size = 23

$$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3} dx = \frac{x^2}{2}+\log(x)-\frac{1}{2}\log\left(3-2x+x^2\right)$$

```
[Out] 1/2*x^2+ln(x)-1/2*ln(x^2-2*x+3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1608, 1642, 642}

$$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3} dx = \frac{x^2}{2}-\frac{1}{2}\log\left(x^2-2x+3\right)+\log(x)$$

```
[In] Int[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3),x]
```

```
[Out] x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
```

```
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{3-x+3x^2-2x^3+x^4}{x(3-2x+x^2)}\,dx \\
&= \int \left(\frac{1}{x}+x+\frac{1-x}{3-2x+x^2}\right)\,dx \\
&= \frac{x^2}{2}+\log(x)+\int \frac{1-x}{3-2x+x^2}\,dx \\
&= \frac{x^2}{2}+\log(x)-\frac{1}{2}\log\left(3-2x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx=\frac{x^2}{2}+\log(x)-\frac{1}{2}\log\left(3-2x+x^2\right)$$

```
[In] Integrate[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3),x]
[Out] x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x^2}{2}+\ln(x)-\frac{\ln(x^2-2x+3)}{2}$	20
norman	$\frac{x^2}{2}+\ln(x)-\frac{\ln(x^2-2x+3)}{2}$	20
risch	$\frac{x^2}{2}+\ln(x)-\frac{\ln(x^2-2x+3)}{2}$	20
parallelrisch	$\frac{x^2}{2}+\ln(x)-\frac{\ln(x^2-2x+3)}{2}$	20

```
[In] int((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x,method=_RETURNVERBOSE)
```

[Out] 1/2*x^2+ln(x)-1/2*ln(x^2-2*x+3)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(x^2-2\,x+3\right) + \log\left(x\right)$$

[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="fricas")

[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(x)

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx = \frac{x^2}{2} + \log\left(x\right) - \frac{\log\left(x^2-2x+3\right)}{2}$$

[In] integrate((x**4-2*x**3+3*x**2-x+3)/(x**3-2*x**2+3*x),x)

[Out] x**2/2 + log(x) - log(x**2 - 2*x + 3)/2

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(x^2-2\,x+3\right) + \log\left(x\right)$$

[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="maxima")

[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(x)

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx=\frac{1}{2}\,x^2-\frac{1}{2}\,\log\left(x^2-2\,x+3\right)+\log\left(|x|\right)$$

```
[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="giac")
[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx=\ln\left(x\right)-\frac{\ln\left(x^2-2\,x+3\right)}{2}+\frac{x^2}{2}$$

```
[In] int((3*x^2 - x - 2*x^3 + x^4 + 3)/(3*x - 2*x^2 + x^3),x)
[Out] log(x) - log(x^2 - 2*x + 3)/2 + x^2/2
```

3.274 $\int \frac{-1+x+x^3}{(1+x^2)^2} \, dx$

Optimal result 1806
Rubi [A] (verified) 1806
Mathematica [A] (verified) 1807
Maple [A] (verified) 1807
Fricas [A] (verification not implemented) 1808
Sympy [A] (verification not implemented) 1808
Maxima [A] (verification not implemented) 1809
Giac [A] (verification not implemented) 1809
Mupad [B] (verification not implemented) 1809

Optimal result

Integrand size = 14, antiderivative size = 29

$$\int \frac{-1+x+x^3}{(1+x^2)^2} \, dx = -\frac{x}{2(1+x^2)} - \frac{\arctan(x)}{2} + \frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -1/2*x/(x^2+1)-1/2*arctan(x)+1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1828, 649, 209, 266}

$$\int \frac{-1+x+x^3}{(1+x^2)^2} \, dx = -\frac{\arctan(x)}{2} - \frac{x}{2(x^2+1)} + \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] Int[(-1 + x + x^3)/(1 + x^2)^2,x]
```

```
[Out] -1/2*x/(1 + x^2) - ArcTan[x]/2 + Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x}{2(1+x^2)} - \frac{1}{2}\int \frac{1-2x}{1+x^2}\,dx\\ &= -\frac{x}{2(1+x^2)} - \frac{1}{2}\int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx\\ &= -\frac{x}{2(1+x^2)} - \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{-1+x+x^3}{(1+x^2)^2}\,dx = \frac{1}{2}\left(-\frac{x}{1+x^2} - \arctan(x) + \log\left(1+x^2\right)\right)$$

```
[In] Integrate[(-1 + x + x^3)/(1 + x^2)^2,x]
[Out] (-(x/(1 + x^2)) - ArcTan[x] + Log[1 + x^2])/2
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{x}{2(x^2+1)}-\frac{\arctan(x)}{2}+\frac{\ln(x^2+1)}{2}$	24
risch	$-\frac{x}{2(x^2+1)}-\frac{\arctan(x)}{2}+\frac{\ln(x^2+1)}{2}$	24
meijerg	$\frac{\ln(x^2+1)}{2}-\frac{x}{2x^2+2}-\frac{\arctan(x)}{2}$	26
parallelrisch	$\frac{i\ln(x-i)x^2-i\ln(x+i)x^2+2\ln(x-i)x^2+2\ln(x+i)x^2+i\ln(x-i)-i\ln(x+i)+2\ln(x-i)+2\ln(x+i)-2x}{4x^2+4}$	86

```
[In] int((x^3+x-1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] -1/2*x/(x^2+1)-1/2*arctan(x)+1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{-1+x+x^3}{(1+x^2)^2}\,dx=-\frac{(x^2+1)\arctan(x)-(x^2+1)\log(x^2+1)+x}{2(x^2+1)}$$

```
[In] integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="fricas")
[Out] -1/2*((x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int \frac{-1+x+x^3}{(1+x^2)^2}\,dx=-\frac{x}{2x^2+2}+\frac{\log(x^2+1)}{2}-\frac{\operatorname{atan}(x)}{2}$$

```
[In] integrate((x**3+x-1)/(x**2+1)**2,x)
[Out] -x/(2*x**2 + 2) + log(x**2 + 1)/2 - atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{-1+x+x^3}{(1+x^2)^2}\,dx = -\frac{x}{2\,(x^2+1)} - \frac{1}{2}\arctan{(x)} + \frac{1}{2}\log{(x^2+1)}$$

```
[In] integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2*x/(x^2 + 1) - 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{-1+x+x^3}{(1+x^2)^2}\,dx = -\frac{x}{2\,(x^2+1)} - \frac{1}{2}\arctan{(x)} + \frac{1}{2}\log{(x^2+1)}$$

```
[In] integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*x/(x^2 + 1) - 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{-1+x+x^3}{(1+x^2)^2}\,dx = \frac{\ln{(x^2+1)}}{2} - \frac{\mathrm{atan}(x)}{2} - \frac{x}{2\,(x^2+1)}$$

```
[In] int((x + x^3 - 1)/(x^2 + 1)^2,x)
[Out] log(x^2 + 1)/2 - atan(x)/2 - x/(2*(x^2 + 1))
```

3.275 $\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)} dx$

Optimal result 1810
Rubi [A] (verified) 1810
Mathematica [A] (verified) 1812
Maple [A] (verified) 1812
Fricas [A] (verification not implemented) 1813
Sympy [A] (verification not implemented) 1813
Maxima [A] (verification not implemented) 1813
Giac [A] (verification not implemented) 1814
Mupad [B] (verification not implemented) 1814

Optimal result

Integrand size = 33, antiderivative size = 44

$$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)} dx = -\frac{3}{1+x} - \frac{2\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log(x) - 2\log(1+x) + \log\left(1-x+x^2\right)$$

```
[Out] -3/(1+x)+ln(x)-2*ln(1+x)+ln(x^2-x+1)-2/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2
)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 6857, 648, 632, 210, 642}

$$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)} dx = -\frac{2\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log\left(x^2-x+1\right) - \frac{3}{x+1} + \log(x) - 2\log(x+1)$$

```
[In] Int[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)),x]
```

```
[Out] -3/(1 + x) - (2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*Log[1 + x]
+ Log[1 - x + x^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1+2x-x^2+8x^3+x^4}{x(1+x)\left(1+x^3\right)}\,dx\\
&= \int \left(\frac{1}{x}+\frac{3}{(1+x)^2}-\frac{2}{1+x}+\frac{2x}{1-x+x^2}\right)\,dx\\
&= -\frac{3}{1+x}+\log(x)-2\log(1+x)+2\int \frac{x}{1-x+x^2}\,dx\\
&= -\frac{3}{1+x}+\log(x)-2\log(1+x)+\int \frac{1}{1-x+x^2}\,dx+\int \frac{-1+2x}{1-x+x^2}\,dx
\end{aligned}
$$

$$= -\frac{3}{1+x} + \log(x) - 2\log(1+x) + \log\left(1-x+x^2\right) - 2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1 + 2x\right)$$

$$= -\frac{3}{1+x} - \frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log(x) - 2\log(1+x) + \log\left(1-x+x^2\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx = -\frac{3}{1+x} + \frac{2\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log(x) - 2\log(1+x) + \log\left(1-x+x^2\right)$$

```
[In] Integrate[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)),x]
[Out] -3/(1 + x) + (2*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*Log[1 + x]
 + Log[1 - x + x^2]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

method	result	size
default	$\ln(x) - \frac{3}{x+1} - 2\ln(x+1) + \ln(x^2-x+1) + \frac{2\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	42
risch	$-\frac{3}{x+1} + \ln(4x^2-4x+4) + \frac{2\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3} - 2\ln(x+1) + \ln(x)$	44

```
[In] int((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x,method=_RETURNVERBOSE)
[Out] ln(x)-3/(x+1)-2*ln(x+1)+ln(x^2-x+1)+2/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

$$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx = \frac{2\sqrt{3}(x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+3(x+1)\log\left(x^2-x+1\right)-6(x+1)\log(x+1)+3(x+1)\log(x)}{3(x+1)}$$

```
[In] integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*(x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*(x + 1)*log(x^2 -
x + 1) - 6*(x + 1)*log(x + 1) + 3*(x + 1)*log(x) - 9)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx = \log(x)-2\log(x+1)+\log\left(x^2-x+1\right) + \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{3}{x+1}$$

```
[In] integrate((x**4+8*x**3-x**2+2*x+1)/(x**2+x)/(x**3+1),x)
```

```
[Out] log(x) - 2*log(x + 1) + log(x**2 - x + 1) + 2*sqrt(3)*atan(2*sqrt(3)*x/3 -
sqrt(3)/3)/3 - 3/(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{3}{x+1} + \log\left(x^2-x+1\right)-2\log(x+1)+\log(x)$$

```
[In] integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/(x + 1) + log(x^2 - x + 1) -
2*log(x + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3}{x+1} + \log\left(x^2-x+1\right) - 2\log\left(|x+1|\right) + \log\left(|x|\right)$$

```
[In] integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/(x + 1) + log(x^2 - x + 1) -
2*log(abs(x + 1)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx = \ln(x) - 2\ln(x+1) - \frac{3}{x+1} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-1+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(1+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)$$

```
[In] int((2*x - x^2 + 8*x^3 + x^4 + 1)/((x^3 + 1)*(x + x^2)),x)
[Out] log(x) - 2*log(x + 1) - 3/(x + 1) - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)
*1i)/3 - 1) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/3 + 1)
```

3.276 $\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)} dx$

Optimal result 1815
Rubi [A] (verified) 1815
Mathematica [A] (verified) 1817
Maple [A] (verified) 1817
Fricas [A] (verification not implemented) 1817
Sympy [A] (verification not implemented) 1818
Maxima [A] (verification not implemented) 1818
Giac [A] (verification not implemented) 1818
Mupad [B] (verification not implemented) 1819

Optimal result

Integrand size = 29, antiderivative size = 46

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)} dx = -\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)+\frac{5\arctan\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)$$

```
[Out] 1/2*ln(x^2+2*x+3)+5/2*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)-arctan(1/5*x*5^(1/2
))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {6857, 209, 648, 632, 210, 642}

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)} dx = -\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)+\frac{5\arctan\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(x^2+2x+3\right)$$

```
[In] Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{5}{5+x^2}+\frac{6+x}{3+2x+x^2}\right) dx\\
&= -\left(5\int \frac{1}{5+x^2}\,dx\right)+\int \frac{6+x}{3+2x+x^2}\,dx\\
&= -\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\int \frac{2+2x}{3+2x+x^2}\,dx+5\int \frac{1}{3+2x+x^2}\,dx\\
&= -\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\log\left(3+2x+x^2\right)-10\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2+2x\right)\\
&= -\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx=-\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)+\frac{5\arctan\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)$$

```
[In] Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{\ln(x^2+2x+3)}{2}+\frac{5\arctan\left(\frac{(x+1)\sqrt{2}}{2}\right)\sqrt{2}}{2}-\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}$	39
default	$-\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}+\frac{\ln(x^2+2x+3)}{2}+\frac{5\sqrt{2}\arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}$	41

```
[In] int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2+2*x+3)+5/2*arctan(1/2*(x+1)*2^(1/2))*2^(1/2)-arctan(1/5*x*5^(1/2
))*5^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx=\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="fricas")
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{\log(x^2+2x+3)}{2} - \sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right) + \frac{5\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}+\frac{\sqrt{2}}{2}\right)}{2}$$

```
[In] integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)
[Out] log(x**2 + 2*x + 3)/2 - sqrt(5)*atan(sqrt(5)*x/5) + 5*sqrt(2)*atan(sqrt(2)*
x/2 + sqrt(2)/2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right) - \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) + \frac{1}{2}\log\left(x^2+2x+3\right)$$

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="maxima")
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right) - \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) + \frac{1}{2}\log\left(x^2+2x+3\right)$$

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="giac")
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.91

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)}{2}+\frac{\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)}{2}+\sqrt{5}\,\mathrm{atan}\left(\frac{2000\sqrt{5}}{2000\,x+1120}-\frac{224\sqrt{5}\,x}{2000\,x+1120}\right)-\frac{\sqrt{2}\,\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)\,5\mathrm{i}}{4}+\frac{\sqrt{2}\,\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)\,5\mathrm{i}}{4}$$

```
[In] int((x^2 - 5*x + x^3 + 15)/((x^2 + 5)*(2*x + x^2 + 3)),x)
```

```
[Out] log(x - 2^(1/2)*1i + 1)/2 + log(x + 2^(1/2)*1i + 1)/2 + 5^(1/2)*atan((2000*
5^(1/2))/(2000*x + 1120) - (224*5^(1/2)*x)/(2000*x + 1120)) - (2^(1/2)*log(
x - 2^(1/2)*1i + 1)*5i)/4 + (2^(1/2)*log(x + 2^(1/2)*1i + 1)*5i)/4
```

3.277 $\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2} dx$

Optimal result 1820
Rubi [A] (verified) 1820
Mathematica [A] (verified) 1821
Maple [A] (verified) 1822
Fricas [B] (verification not implemented) 1822
Sympy [A] (verification not implemented) 1822
Maxima [A] (verification not implemented) 1823
Giac [A] (verification not implemented) 1823
Mupad [B] (verification not implemented) 1823

Optimal result

Integrand size = 44, antiderivative size = 33

$$\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2} dx = -\frac{3}{1+x^2}+\frac{1}{2+x+x^2}+\log(1+x^2)-\log(2+x+x^2)$$

```
[Out] -3/(x^2+1)+1/(x^2+x+2)+ln(x^2+1)-ln(x^2+x+2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6874, 267, 266, 643, 642}

$$\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2} dx = -\frac{3}{x^2+1}+\frac{1}{x^2+x+2}+\log(x^2+1)-\log(x^2+x+2)$$

```
[In] Int[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2*(2 +
x + x^2)^2),x]
```

```
[Out] -3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^2]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{6x}{(1+x^2)^2}+\frac{2x}{1+x^2}+\frac{-1-2x}{(2+x+x^2)^2}+\frac{-1-2x}{2+x+x^2}\right)dx \\
&= 2\int \frac{x}{1+x^2}\,dx+6\int \frac{x}{(1+x^2)^2}\,dx+\int \frac{-1-2x}{(2+x+x^2)^2}\,dx+\int \frac{-1-2x}{2+x+x^2}\,dx \\
&= -\frac{3}{1+x^2}+\frac{1}{2+x+x^2}+\log\left(1+x^2\right)-\log\left(2+x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\begin{aligned}
&\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2\,(2+x+x^2)^2}\,dx \\
&= -\frac{3}{1+x^2}+\frac{1}{2+x+x^2}+\log\left(1+x^2\right)-\log\left(2+x+x^2\right)
\end{aligned}$$

```
[In] Integrate[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2
*(2 + x + x^2)^2),x]
```

```
[Out] -3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^2]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

method	result
default	$-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}+\ln\left(x^2+1\right)-\ln\left(x^2+x+2\right)$
norman	$\frac{-2x^2-3x-5}{(x^2+x+2)(x^2+1)}-\ln\left(x^2+x+2\right)+\ln\left(x^2+1\right)$
risch	$\frac{-2x^2-3x-5}{(x^2+x+2)(x^2+1)}-\ln\left(x^2+x+2\right)+\ln\left(x^2+1\right)$
parallelrisch	$\frac{\ln(x^2+1)x^4-\ln(x^2+x+2)x^4-5+\ln(x^2+1)x^3-\ln(x^2+x+2)x^3+3\ln(x^2+1)x^2-3\ln(x^2+x+2)x^2+\ln(x^2+1)x-\ln(x^2+x+2)x-}{(x^2+1)(x^2+x+2)}$

```
[In] int((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,method=
_RETURNVERBOSE)
```

```
[Out] -3/(x^2+1)+1/(x^2+x+2)+ln(x^2+1)-ln(x^2+x+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(33) = 66.

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.18

$$\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx = -\frac{2\,x^2+(x^4+x^3+3\,x^2+x+2)\log\left(x^2+x+2\right)-(x^4+x^3+3\,x^2+x+2)\log\left(x^2+1\right)+3\,x+5}{x^4+x^3+3\,x^2+x+2}$$

```
[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="fricas")
```

```
[Out] -(2*x^2 + (x^4 + x^3 + 3*x^2 + x + 2)*log(x^2 + x + 2) - (x^4 + x^3 + 3*x^2
 + x + 2)*log(x^2 + 1) + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx = \frac{-2x^2-3x-5}{x^4+x^3+3x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

```
[In] integrate((x**6+7*x**5+15*x**4+32*x**3+23*x**2+25*x-3)/(x**2+1)**2/(x**2+x+
2)**2,x)
```

```
[Out] (-2*x**2 - 3*x - 5)/(x**4 + x**3 + 3*x**2 + x + 2) + log(x**2 + 1) - log(x*
*2 + x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx = -\frac{2\,x^2+3\,x+5}{x^4+x^3+3\,x^2+x+2} - \log\left(x^2+x+2\right) + \log\left(x^2+1\right)$$

```
[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="maxima")
```

```
[Out] -(2*x^2 + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2) - log(x^2 + x + 2) + log(x^2
 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx = -\frac{2\,x^2+3\,x+5}{x^4+x^3+3\,x^2+x+2} - \log\left(x^2+x+2\right) + \log\left(x^2+1\right)$$

```
[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="giac")
```

```
[Out] -(2*x^2 + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2) - log(x^2 + x + 2) + log(x^2
 + 1)
```

Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx = -\frac{2\,x^2+3\,x+5}{x^4+x^3+3\,x^2+x+2} + \mathrm{atan}\left(\frac{\frac{x\,224\mathrm{i}}{11}+\frac{224}{11}\mathrm{i}}{44\,x^2+16\,x+60} - \frac{3}{11}\mathrm{i}\right)\,2\mathrm{i}$$

```
[In] int((25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6 - 3)/((x^2 + 1)^2*(x + x
^2 + 2)^2),x)
```

```
[Out] atan(((x*224i)/11 + 224i/11)/(16*x + 44*x^2 + 60) - 3i/11)*2i - (3*x + 2*x^
2 + 5)/(x + 3*x^2 + x^3 + x^4 + 2)
```

3.278 $\int \frac{1}{(1+x^2)(4+x^2)} \, dx$

Optimal result 1824
Rubi [A] (verified) 1824
Mathematica [A] (verified) 1825
Maple [A] (verified) 1825
Fricas [A] (verification not implemented) 1825
Sympy [A] (verification not implemented) 1826
Maxima [A] (verification not implemented) 1826
Giac [A] (verification not implemented) 1826
Mupad [B] (verification not implemented) 1826

Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \frac{1}{(1+x^2)(4+x^2)} \, dx = -\frac{1}{6}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{3}$$

```
[Out] -1/6*arctan(1/2*x)+1/3*arctan(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {400, 209}

$$\int \frac{1}{(1+x^2)(4+x^2)} \, dx = \frac{\arctan(x)}{3}-\frac{1}{6}\arctan\left(\frac{x}{2}\right)$$

```
[In] Int[1/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] -1/6*ArcTan[x/2] + ArcTan[x]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\int \frac{1}{1+x^2}\,dx - \frac{1}{3}\int \frac{1}{4+x^2}\,dx\\ &= -\frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)(4+x^2)}\,dx = \frac{1}{6}\arctan\left(\frac{2}{x}\right) + \frac{\arctan(x)}{3}$$

```
[In] Integrate[1/((1 + x^2)*(4 + x^2)),x]
[Out] ArcTan[2/x]/6 + ArcTan[x]/3
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{\arctan\left(\frac{x}{2}\right)}{6}+\frac{\arctan(x)}{3}$	12
risch	$-\frac{\arctan\left(\frac{x}{2}\right)}{6}+\frac{\arctan(x)}{3}$	12
parallelrisch	$\frac{i\ln(x+i)}{6}-\frac{i\ln(x-i)}{6}+\frac{i\ln(x-2i)}{12}-\frac{i\ln(x+2i)}{12}$	34

```
[In] int(1/(x^2+1)/(x^2+4),x,method=_RETURNVERBOSE)
[Out] -1/6*arctan(1/2*x)+1/3*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{(1+x^2)(4+x^2)}\,dx = -\frac{1}{6}\arctan\left(\frac{1}{2}\,x\right) + \frac{1}{3}\arctan\left(x\right)$$

```
[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="fricas")
[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{1}{(1+x^2)\,(4+x^2)}\,dx = -\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{6} + \frac{\operatorname{atan}(x)}{3}$$

```
[In] integrate(1/(x**2+1)/(x**2+4),x)
[Out] -atan(x/2)/6 + atan(x)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{(1+x^2)\,(4+x^2)}\,dx = -\frac{1}{6}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

```
[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="maxima")
[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{(1+x^2)\,(4+x^2)}\,dx = -\frac{1}{6}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

```
[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="giac")
[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{(1+x^2)\,(4+x^2)}\,dx = \frac{\operatorname{atan}(x)}{3} - \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{6}$$

```
[In] int(1/((x^2 + 1)*(x^2 + 4)),x)
[Out] atan(x)/3 - atan(x/2)/6
```

3.279 $\int \frac{a+bx^3}{1+x^2} \, dx$

Optimal result 1827
Rubi [A] (verified) 1827
Mathematica [A] (verified) 1828
Maple [A] (verified) 1828
Fricas [A] (verification not implemented) 1829
Sympy [C] (verification not implemented) 1829
Maxima [A] (verification not implemented) 1830
Giac [A] (verification not implemented) 1830
Mupad [B] (verification not implemented) 1830

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{a+bx^3}{1+x^2} \, dx = \frac{bx^2}{2} + a\arctan(x) - \frac{1}{2}b\log\left(1+x^2\right)$$

```
[Out] 1/2*b*x^2+a*arctan(x)-1/2*b*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1824, 649, 209, 266}

$$\int \frac{a+bx^3}{1+x^2} \, dx = a\arctan(x) + \frac{bx^2}{2} - \frac{1}{2}b\log\left(x^2+1\right)$$

```
[In] Int[(a + b*x^3)/(1 + x^2),x]
```

```
[Out] (b*x^2)/2 + a*ArcTan[x] - (b*Log[1 + x^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(bx + \frac{a-bx}{1+x^2}\right) dx \\ &= \frac{bx^2}{2} + \int \frac{a-bx}{1+x^2}\,dx \\ &= \frac{bx^2}{2} + a\int \frac{1}{1+x^2}\,dx - b\int \frac{x}{1+x^2}\,dx \\ &= \frac{bx^2}{2} + a\tan^{-1}(x) - \frac{1}{2}b\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^3}{1+x^2}\,dx = a\arctan(x) + \frac{1}{2}b\left(x^2 - \log\left(1+x^2\right)\right)$$

```
[In] Integrate[(a + b*x^3)/(1 + x^2),x]
[Out] a*ArcTan[x] + (b*(x^2 - Log[1 + x^2]))/2
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{b\,x^{2}}{2}+a\arctan\left(x\right)-\frac{b\ln\left(x^{2}+1\right)}{2}$	21
meijerg	$\frac{b\left(x^{2}-\ln\left(x^{2}+1\right)\right)}{2}+a\arctan\left(x\right)$	21
risch	$\frac{b\,x^{2}}{2}+a\arctan\left(x\right)-\frac{b\ln\left(x^{2}+1\right)}{2}$	21
parallelrisch	$\frac{b\,x^{2}}{2}-\frac{\ln\left(x-i\right)b}{2}-\frac{i\ln\left(x-i\right)a}{2}-\frac{\ln\left(x+i\right)b}{2}+\frac{i\ln\left(x+i\right)a}{2}$	42

```
[In] int((b*x^3+a)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*b*x^2+a*arctan(x)-1/2*b*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^3}{1+x^2}\,dx=\frac{1}{2}\,bx^{2}+a\arctan\left(x\right)-\frac{1}{2}\,b\log\left(x^{2}+1\right)$$

```
[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="fricas")
[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{a+bx^3}{1+x^2}\,dx=\frac{bx^{2}}{2}+\left(-\frac{ia}{2}-\frac{b}{2}\right)\log\left(x-i\right)+\left(\frac{ia}{2}-\frac{b}{2}\right)\log\left(x+i\right)$$

```
[In] integrate((b*x**3+a)/(x**2+1),x)
[Out] b*x**2/2 + (-I*a/2 - b/2)*log(x - I) + (I*a/2 - b/2)*log(x + I)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^3}{1+x^2}\,dx = \frac{1}{2}\,bx^2 + a\arctan\left(x\right) - \frac{1}{2}\,b\log\left(x^2+1\right)$$

```
[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="maxima")
[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^3}{1+x^2}\,dx = \frac{1}{2}\,bx^2 + a\arctan\left(x\right) - \frac{1}{2}\,b\log\left(x^2+1\right)$$

```
[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="giac")
[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^3}{1+x^2}\,dx = \frac{b\,x^2}{2} - \frac{b\,\ln\left(x^2+1\right)}{2} + a\,\mathrm{atan}(x)$$

```
[In] int((a + b*x^3)/(x^2 + 1),x)
[Out] (b*x^2)/2 - (b*log(x^2 + 1))/2 + a*atan(x)
```

3.280 $\int \frac{x+x^2}{(4+x)(-4+x^2)}\,dx$

Optimal result 1831
Rubi [A] (verified) 1831
Mathematica [A] (verified) 1832
Maple [A] (verified) 1832
Fricas [A] (verification not implemented) 1833
Sympy [A] (verification not implemented) 1833
Maxima [A] (verification not implemented) 1833
Giac [A] (verification not implemented) 1834
Mupad [B] (verification not implemented) 1834

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{x+x^2}{(4+x)\left(-4+x^2\right)}\,dx=-\frac{1}{2}\text{arctanh}\left(\frac{x}{2}\right)+\log(4+x)$$

```
[Out] -1/2*arctanh(1/2*x)+ln(4+x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1607, 1643, 213}

$$\int \frac{x+x^2}{(4+x)\left(-4+x^2\right)}\,dx=\log(x+4)-\frac{1}{2}\text{arctanh}\left(\frac{x}{2}\right)$$

```
[In] Int[(x + x^2)/((4 + x)*(-4 + x^2)),x]
```

```
[Out] -1/2*ArcTanh[x/2] + Log[4 + x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(1+x)}{(4+x)\,(-4+x^2)}\,dx\\ &= \int \left(\frac{1}{4+x}+\frac{1}{-4+x^2}\right)dx\\ &= \log(4+x)+\int \frac{1}{-4+x^2}\,dx\\ &= -\frac{1}{2}\tanh^{-1}\left(\frac{x}{2}\right)+\log(4+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{x+x^2}{(4+x)\,(-4+x^2)}\,dx = \frac{1}{4}\log(2-x)-\frac{1}{4}\log(2+x)+\log(4+x)$$

```
[In] Integrate[(x + x^2)/((4 + x)*(-4 + x^2)),x]
[Out] Log[2 - x]/4 - Log[2 + x]/4 + Log[4 + x]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{\ln(x+2)}{4}+\ln\left(x+4\right)+\frac{\ln(x-2)}{4}$	18
norman	$-\frac{\ln(x+2)}{4}+\ln\left(x+4\right)+\frac{\ln(x-2)}{4}$	18
risch	$-\frac{\ln(x+2)}{4}+\ln\left(x+4\right)+\frac{\ln(x-2)}{4}$	18
parallelrisch	$-\frac{\ln(x+2)}{4}+\ln\left(x+4\right)+\frac{\ln(x-2)}{4}$	18

```
[In] int((x^2+x)/(x+4)/(x^2-4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(x+2)+ln(x+4)+1/4*ln(x-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{x+x^2}{(4+x)(-4+x^2)}\,dx = \log(x+4) - \frac{1}{4}\log(x+2) + \frac{1}{4}\log(x-2)$$

```
[In] integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="fricas")
[Out] log(x + 4) - 1/4*log(x + 2) + 1/4*log(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{x+x^2}{(4+x)(-4+x^2)}\,dx = \frac{\log(x-2)}{4} - \frac{\log(x+2)}{4} + \log(x+4)$$

```
[In] integrate((x**2+x)/(4+x)/(x**2-4),x)
[Out] log(x - 2)/4 - log(x + 2)/4 + log(x + 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{x+x^2}{(4+x)(-4+x^2)}\,dx = \log(x+4) - \frac{1}{4}\log(x+2) + \frac{1}{4}\log(x-2)$$

```
[In] integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="maxima")
[Out] log(x + 4) - 1/4*log(x + 2) + 1/4*log(x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{x+x^2}{(4+x)(-4+x^2)}\,dx = \log(|x+4|) - \frac{1}{4}\log(|x+2|) + \frac{1}{4}\log(|x-2|)$$

```
[In] integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="giac")
[Out] log(abs(x + 4)) - 1/4*log(abs(x + 2)) + 1/4*log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{x+x^2}{(4+x)(-4+x^2)}\,dx = \ln(x+4) + \frac{\operatorname{atanh}\left(\frac{90}{7(21x+48)} - \frac{8}{7}\right)}{2}$$

```
[In] int((x + x^2)/((x^2 - 4)*(x + 4)),x)
[Out] log(x + 4) + atanh(90/(7*(21*x + 48)) - 8/7)/2
```

3.281 $\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx$

Optimal result 1835
Rubi [A] (verified) 1835
Mathematica [A] (verified) 1836
Maple [A] (verified) 1836
Fricas [A] (verification not implemented) 1837
Sympy [A] (verification not implemented) 1837
Maxima [A] (verification not implemented) 1837
Giac [A] (verification not implemented) 1837
Mupad [B] (verification not implemented) 1838

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx = 3\arctan(x) - \sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 3*arctan(x)-arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {536, 209}

$$\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx = 3\arctan(x) - \sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

```
[In] Int[(4 + x^2)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(2\int \frac{1}{2+x^2}\,dx\right)+3\int \frac{1}{1+x^2}\,dx\\ &= 3\tan^{-1}(x)-\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{4+x^2}{(1+x^2)\,(2+x^2)}\,dx = 3\arctan(x)-\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

```
[In] Integrate[(4 + x^2)/((1 + x^2)*(2 + x^2)),x]
[Out] 3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result	size
default	$3\arctan\left(x\right)-\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	18
risch	$3\arctan\left(x\right)-\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	18

```
[In] int((x^2+4)/(x^2+1)/(x^2+2),x,method=_RETURNVERBOSE)
[Out] 3*arctan(x)-arctan(1/2*x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx = -\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 3\arctan(x)$$

```
[In] integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="fricas")
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx = 3\operatorname{atan}(x) - \sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

```
[In] integrate((x**2+4)/(x**2+1)/(x**2+2),x)
[Out] 3*atan(x) - sqrt(2)*atan(sqrt(2)*x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx = -\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 3\arctan(x)$$

```
[In] integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="maxima")
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx = -\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 3\arctan(x)$$

```
[In] integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="giac")
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx = 3\,\mathrm{atan}(x) - \sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

```
[In] int((x^2 + 4)/((x^2 + 1)*(x^2 + 2)),x)
[Out] 3*atan(x) - 2^(1/2)*atan((2^(1/2)*x)/2)
```

3.282 $\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx$

Optimal result 1839
Rubi [A] (verified) 1839
Mathematica [A] (verified) 1840
Maple [A] (verified) 1840
Fricas [A] (verification not implemented) 1841
Sympy [A] (verification not implemented) 1841
Maxima [A] (verification not implemented) 1842
Giac [B] (verification not implemented) 1842
Mupad [B] (verification not implemented) 1842

Optimal result

Integrand size = 26, antiderivative size = 37

$$\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx = \frac{5}{2(1-x)}+x+2\arctan(x)+\frac{1}{2}\log(1-x)+\frac{3}{4}\log\left(1+x^2\right)$$

```
[Out] 5/2/(1-x)+x+2*arctan(x)+1/2*ln(1-x)+3/4*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1643, 649, 209, 266}

$$\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx = 2\arctan(x)+\frac{3}{4}\log\left(x^2+1\right)+x+\frac{5}{2(1-x)}+\frac{1}{2}\log(1-x)$$

```
[In] Int[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] 5/(2*(1 - x)) + x + 2*ArcTan[x] + Log[1 - x]/2 + (3*Log[1 + x^2])/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(1+\frac{5}{2(-1+x)^2}+\frac{1}{2(-1+x)}+\frac{4+3x}{2\left(1+x^2\right)}\right)dx \\
&= \frac{5}{2(1-x)}+x+\frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{4+3x}{1+x^2}\,dx \\
&= \frac{5}{2(1-x)}+x+\frac{1}{2}\log(1-x)+\frac{3}{2}\int \frac{x}{1+x^2}\,dx+2\int \frac{1}{1+x^2}\,dx \\
&= \frac{5}{2(1-x)}+x+2\tan^{-1}(x)+\frac{1}{2}\log(1-x)+\frac{3}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{5-4x+3x^2+x^4}{(-1+x)^2\left(1+x^2\right)}\,dx=\frac{5}{2-2x}+x+2\arctan(x)+\frac{1}{2}\log(-1+x)+\frac{3}{4}\log\left(1+x^2\right)$$

```
[In] Integrate[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)),x]
[Out] 5/(2 - 2*x) + x + 2*ArcTan[x] + Log[-1 + x]/2 + (3*Log[1 + x^2])/4
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result
default	$x+\frac{3\ln\left(x^{2}+1\right)}{4}+2\arctan\left(x\right)-\frac{5}{2(x-1)}+\frac{\ln\left(x-1\right)}{2}$
risch	$x-\frac{5}{2(x-1)}+\frac{\ln\left(x-1\right)}{2}+\frac{3\ln\left(16x^{2}+16\right)}{4}+2\arctan\left(x\right)$
parallelrisch	$\frac{-4i\ln\left(x-i\right)x+4i\ln\left(x+i\right)x+2\ln\left(x-1\right)x+4i\ln\left(x-i\right)+3\ln\left(x-i\right)x-4i\ln\left(x+i\right)+3\ln\left(x+i\right)x+4x^{2}-14-2\ln\left(x-1\right)-3\ln\left(x-i\right)-3\ln}{4x-4}$

```
[In] int((x^4+3*x^2-4*x+5)/(x-1)^2/(x^2+1),x,method=_RETURNVERBOSE)
[Out] x+3/4*ln(x^2+1)+2*arctan(x)-5/2/(x-1)+1/2*ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{5-4x+3x^2+x^4}{(-1+x)^2\left(1+x^2\right)}\,dx = \frac{4\,x^{2}+8\,(x-1)\arctan\left(x\right)+3\,(x-1)\log\left(x^{2}+1\right)+2\,(x-1)\log\left(x-1\right)-4\,x-10}{4\,(x-1)}$$

```
[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
[Out] 1/4*(4*x^2 + 8*(x - 1)*arctan(x) + 3*(x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x
 - 1) - 4*x - 10)/(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{5-4x+3x^2+x^4}{(-1+x)^2\left(1+x^2\right)}\,dx = x+\frac{\log\left(x-1\right)}{2}+\frac{3\log\left(x^{2}+1\right)}{4}+2\,\mathrm{atan}\left(x\right)-\frac{5}{2x-2}$$

```
[In] integrate((x**4+3*x**2-4*x+5)/(-1+x)**2/(x**2+1),x)
[Out] x + log(x - 1)/2 + 3*log(x**2 + 1)/4 + 2*atan(x) - 5/(2*x - 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx = x-\frac{5}{2(x-1)}+2\arctan(x)+\frac{3}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x-1)$$

```
[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
[Out] x - 5/2/(x - 1) + 2*arctan(x) + 3/4*log(x^2 + 1) + 1/2*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(29) = 58.

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx = \frac{1}{2}\pi-2\pi\left\lfloor\frac{\pi+4\arctan(x)}{4\pi}+\frac{1}{2}\right\rfloor+x-\frac{5}{2(x-1)}+2\arctan(x)+\frac{3}{4}\log\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)+2\log(|x-1|)-1$$

```
[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="giac")
[Out] 1/2*pi - 2*pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + x - 5/2/(x - 1) + 2*
arctan(x) + 3/4*log(2/(x - 1) + 2/(x - 1)^2 + 1) + 2*log(abs(x - 1)) - 1
```

Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx = x+\frac{\ln(x-1)}{2}-\frac{5}{2\ (x-1)}+\ln(x-\mathrm{i})\ \left(\frac{3}{4}-\mathrm{i}\right)+\ln(x+1\mathrm{i})\ \left(\frac{3}{4}+1\mathrm{i}\right)$$

```
[In] int((3*x^2 - 4*x + x^4 + 5)/((x^2 + 1)*(x - 1)^2),x)
[Out] x + log(x - 1)/2 + log(x - 1i)*(3/4 - 1i) + log(x + 1i)*(3/4 + 1i) - 5/(2*(
x - 1))
```

3.283 $\int \frac{1+x^4}{2+x^2} dx$

Optimal result 1843
Rubi [A] (verified) 1843
Mathematica [A] (verified) 1844
Maple [A] (verified) 1844
Fricas [A] (verification not implemented) 1845
Sympy [A] (verification not implemented) 1845
Maxima [A] (verification not implemented) 1845
Giac [A] (verification not implemented) 1845
Mupad [B] (verification not implemented) 1846

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{1+x^4}{2+x^2} dx = -2x + \frac{x^3}{3} + \frac{5 \arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -2*x+1/3*x^3+5/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1168, 209}

$$\int \frac{1+x^4}{2+x^2} dx = \frac{5 \arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{x^3}{3} - 2x$$

```
[In] Int[(1 + x^4)/(2 + x^2),x]
```

```
[Out] -2*x + x^3/3 + (5*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
```

```
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-2+x^2+\frac{5}{2+x^2}\right) dx \\ &= -2x+\frac{x^3}{3}+5\int \frac{1}{2+x^2}\, dx \\ &= -2x+\frac{x^3}{3}+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{2+x^2}\, dx = -2x+\frac{x^3}{3}+\frac{5\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[In] Integrate[(1 + x^4)/(2 + x^2),x]
[Out] -2*x + x^3/3 + (5*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

method	result	size
default	$-2x+\frac{x^3}{3}+\frac{5\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}$	22
risch	$-2x+\frac{x^3}{3}+\frac{5\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}$	22
meijerg	$\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}+\sqrt{2}\left(-\frac{x\sqrt{2}\left(-\frac{5x^2}{2}+15\right)}{15}+2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)$	41

```
[In] int((x^4+1)/(x^2+2),x,method=_RETURNVERBOSE)
[Out] -2*x+1/3*x^3+5/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1+x^4}{2+x^2}\,dx = \frac{1}{3}\,x^3 + \frac{5}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 2\,x$$

```
[In] integrate((x^4+1)/(x^2+2),x, algorithm="fricas")
[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{2+x^2}\,dx = \frac{x^3}{3} - 2x + \frac{5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

```
[In] integrate((x**4+1)/(x**2+2),x)
[Out] x**3/3 - 2*x + 5*sqrt(2)*atan(sqrt(2)*x/2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1+x^4}{2+x^2}\,dx = \frac{1}{3}\,x^3 + \frac{5}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 2\,x$$

```
[In] integrate((x^4+1)/(x^2+2),x, algorithm="maxima")
[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1+x^4}{2+x^2}\,dx = \frac{1}{3}\,x^3 + \frac{5}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 2\,x$$

```
[In] integrate((x^4+1)/(x^2+2),x, algorithm="giac")
[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1+x^4}{2+x^2}\,dx = \frac{5\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2} - 2\,x + \frac{x^3}{3}$$

```
[In] int((x^4 + 1)/(x^2 + 2),x)
[Out] (5*2^(1/2)*atan((2^(1/2)*x)/2))/2 - 2*x + x^3/3
```

3.284 $\int \frac{2+2x+x^4}{x^4+x^5} \, dx$

Optimal result 1847
Rubi [A] (verified) 1847
Mathematica [A] (verified) 1848
Maple [A] (verified) 1848
Fricas [A] (verification not implemented) 1849
Sympy [A] (verification not implemented) 1849
Maxima [A] (verification not implemented) 1849
Giac [A] (verification not implemented) 1849
Mupad [B] (verification not implemented) 1850

Optimal result

Integrand size = 18, antiderivative size = 12

$$\int \frac{2+2x+x^4}{x^4+x^5} \, dx = -\frac{2}{3x^3} + \log(1+x)$$

```
[Out] -2/3/x^3+ln(1+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1607, 1634}

$$\int \frac{2+2x+x^4}{x^4+x^5} \, dx = \log(x+1) - \frac{2}{3x^3}$$

```
[In] Int[(2 + 2*x + x^4)/(x^4 + x^5),x]
```

```
[Out] -2/(3*x^3) + Log[1 + x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
```

```
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{2+2x+x^4}{x^4(1+x)}\,dx \\ &= \int \left(\frac{2}{x^4}+\frac{1}{1+x}\right)dx \\ &= -\frac{2}{3x^3}+\log(1+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{2+2x+x^4}{x^4+x^5}\,dx = -\frac{2}{3x^3}+\log(1+x)$$

```
[In] Integrate[(2 + 2*x + x^4)/(x^4 + x^5),x]
[Out] -2/(3*x^3) + Log[1 + x]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{2}{3x^3}+\ln(x+1)$	11
norman	$-\frac{2}{3x^3}+\ln(x+1)$	11
meijerg	$-\frac{2}{3x^3}+\ln(x+1)$	11
risch	$-\frac{2}{3x^3}+\ln(x+1)$	11
parallelrisch	$\frac{3\ln(x+1)x^3-2}{3x^3}$	17

```
[In] int((x^4+2*x+2)/(x^5+x^4),x,method=_RETURNVERBOSE)
[Out] -2/3/x^3+ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{2+2x+x^4}{x^4+x^5}\,dx = \frac{3\,x^3\log\left(x+1\right)-2}{3\,x^3}$$

```
[In] integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="fricas")
[Out] 1/3*(3*x^3*log(x + 1) - 2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{2+2x+x^4}{x^4+x^5}\,dx = \log\left(x+1\right) - \frac{2}{3x^3}$$

```
[In] integrate((x**4+2*x+2)/(x**5+x**4),x)
[Out] log(x + 1) - 2/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{2+2x+x^4}{x^4+x^5}\,dx = -\frac{2}{3\,x^3} + \log\left(x+1\right)$$

```
[In] integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="maxima")
[Out] -2/3/x^3 + log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{2+2x+x^4}{x^4+x^5}\,dx = -\frac{2}{3\,x^3} + \log\left(|x+1|\right)$$

```
[In] integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="giac")
[Out] -2/3/x^3 + log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{2+2x+x^4}{x^4+x^5}\,dx = \ln(x+1) - \frac{2}{3\,x^3}$$

```
[In] int((2*x + x^4 + 2)/(x^4 + x^5),x)
[Out] log(x + 1) - 2/(3*x^3)
```

3.285 $\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3} \, dx$

Optimal result 1851
Rubi [A] (verified) 1851
Mathematica [A] (verified) 1852
Maple [A] (verified) 1852
Fricas [A] (verification not implemented) 1852
Sympy [A] (verification not implemented) 1853
Maxima [A] (verification not implemented) 1853
Giac [A] (verification not implemented) 1853
Mupad [B] (verification not implemented) 1853

Optimal result

Integrand size = 26, antiderivative size = 21

$$\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3} \, dx = 2\log(1-x) - \log(2-x) + \log(1+x)$$

```
[Out] 2*ln(1-x)-ln(2-x)+ln(1+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2099}

$$\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3} \, dx = 2\log(1-x) - \log(2-x) + \log(x+1)$$

```
[In] Int[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] - Log[2 - x] + Log[1 + x]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{2-x}+\frac{2}{-1+x}+\frac{1}{1+x}\right) \, dx \\ &= 2\log(1-x)-\log(2-x)+\log(1+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx = 2\log(1-x)-\log(2-x)+\log(1+x)$$

```
[In] Integrate[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3),x]
[Out] 2*Log[1 - x] - Log[2 - x] + Log[1 + x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$\ln(x+1)+2\ln(x-1)-\ln(x-2)$	18
norman	$\ln(x+1)+2\ln(x-1)-\ln(x-2)$	18
risch	$\ln(x+1)+2\ln(x-1)-\ln(x-2)$	18
parallelrisch	$\ln(x+1)+2\ln(x-1)-\ln(x-2)$	18

```
[In] int((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x,method=_RETURNVERBOSE)
[Out] ln(x+1)+2*ln(x-1)-ln(x-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx = \log(x+1)+2\,\log(x-1)-\log(x-2)$$

```
[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x, algorithm="fricas")
[Out] log(x + 1) + 2*log(x - 1) - log(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx = -\log\left(x-2\right)+2\log\left(x-1\right)+\log\left(x+1\right)$$

```
[In] integrate((2*x**2-5*x-1)/(x**3-2*x**2-x+2),x)
[Out] -log(x - 2) + 2*log(x - 1) + log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx = \log\left(x+1\right)+2\,\log\left(x-1\right)-\log\left(x-2\right)$$

```
[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x, algorithm="maxima")
[Out] log(x + 1) + 2*log(x - 1) - log(x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx = \log\left(|x+1|\right)+2\,\log\left(|x-1|\right)-\log\left(|x-2|\right)$$

```
[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x, algorithm="giac")
[Out] log(abs(x + 1)) + 2*log(abs(x - 1)) - log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx = 2\,\ln\left(x-1\right)-2\,\mathrm{atanh}\left(\frac{144}{11\,\left(22\,x-50\right)}+\frac{13}{11}\right)$$

```
[In] int((5*x - 2*x^2 + 1)/(x + 2*x^2 - x^3 - 2),x)
[Out] 2*log(x - 1) - 2*atanh(144/(11*(22*x - 50)) + 13/11)
```

3.286 $\int \frac{2+x+x^3}{1+2x^2+x^4} \, dx$

Optimal result 1854
Rubi [A] (verified) 1854
Mathematica [A] (verified) 1855
Maple [A] (verified) 1856
Fricas [A] (verification not implemented) 1856
Sympy [A] (verification not implemented) 1856
Maxima [A] (verification not implemented) 1857
Giac [A] (verification not implemented) 1857
Mupad [B] (verification not implemented) 1857

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{2+x+x^3}{1+2x^2+x^4} \, dx = \frac{x}{1+x^2} + \arctan(x) + \frac{1}{2} \log \left(1+x^2\right)$$

```
[Out] x/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {28, 1828, 649, 209, 266}

$$\int \frac{2+x+x^3}{1+2x^2+x^4} \, dx = \arctan(x) + \frac{x}{x^2+1} + \frac{1}{2} \log \left(x^2+1\right)$$

```
[In] Int[(2 + x + x^3)/(1 + 2*x^2 + x^4),x]
```

```
[Out] x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{2+x+x^3}{(1+x^2)^2}\,dx \\
&= \frac{x}{1+x^2} - \frac{1}{2}\int \frac{-2-2x}{1+x^2}\,dx \\
&= \frac{x}{1+x^2} + \int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx \\
&= \frac{x}{1+x^2} + \tan^{-1}(x) + \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{2+x+x^3}{1+2x^2+x^4}\,dx = \frac{x}{1+x^2} + \arctan(x) + \frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[(2 + x + x^3)/(1 + 2*x^2 + x^4),x]
[Out] x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{x}{x^2+1}+\arctan(x)+\frac{\ln(x^2+1)}{2}$	21
risch	$\frac{x}{x^2+1}+\arctan(x)+\frac{\ln(x^2+1)}{2}$	21
parallelrisch	$\frac{i\ln(x+i)x^2-i\ln(x-i)x^2+\ln(x+i)x^2+\ln(x-i)x^2+i\ln(x+i)-i\ln(x-i)+\ln(x+i)+\ln(x-i)+2x}{2x^2+2}$	80

```
[In] int((x^3+x+2)/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
[Out] x/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{2+x+x^3}{1+2x^2+x^4}\,dx=\frac{2\,(x^2+1)\arctan(x)+(x^2+1)\log(x^2+1)+2\,x}{2\,(x^2+1)}$$

```
[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] 1/2*(2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) + 2*x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{2+x+x^3}{1+2x^2+x^4}\,dx=\frac{x}{x^2+1}+\frac{\log(x^2+1)}{2}+\operatorname{atan}(x)$$

```
[In] integrate((x**3+x+2)/(x**4+2*x**2+1),x)
[Out] x/(x**2 + 1) + log(x**2 + 1)/2 + atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{2+x+x^3}{1+2x^2+x^4}\,dx = \frac{x}{x^2+1} + \arctan(x) + \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="maxima")
[Out] x/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{2+x+x^3}{1+2x^2+x^4}\,dx = \frac{x}{x^2+1} + \arctan(x) + \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="giac")
[Out] x/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{2+x+x^3}{1+2x^2+x^4}\,dx = \frac{\ln\left(x^2+1\right)}{2} + \mathrm{atan}(x) + \frac{x}{x^2+1}$$

```
[In] int((x + x^3 + 2)/(2*x^2 + x^4 + 1),x)
[Out] log(x^2 + 1)/2 + atan(x) + x/(x^2 + 1)
```

3.287 $\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx$

Optimal result 1858
Rubi [A] (verified) 1858
Mathematica [A] (verified) 1859
Maple [A] (verified) 1860
Fricas [A] (verification not implemented) 1860
Sympy [A] (verification not implemented) 1860
Maxima [A] (verification not implemented) 1861
Giac [A] (verification not implemented) 1861
Mupad [B] (verification not implemented) 1861

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx = -\frac{1}{2\left(1+x^2\right)}+\arctan(x)+\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -1/2/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {28, 1828, 649, 209, 266}

$$\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx = \arctan(x)-\frac{1}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)$$

```
[In] Int[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4),x]
```

```
[Out] -1/2*1/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+2x+x^2+x^3}{(1+x^2)^2}\,dx\\ &= -\frac{1}{2(1+x^2)}-\frac{1}{2}\int \frac{-2-2x}{1+x^2}\,dx\\ &= -\frac{1}{2(1+x^2)}+\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= -\frac{1}{2(1+x^2)}+\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx = -\frac{1}{2(1+x^2)}+\arctan(x)+\frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4),x]
[Out] -1/2*1/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{1}{2(x^2+1)}+\arctan(x)+\frac{\ln(x^2+1)}{2}$	21
risch	$-\frac{1}{2(x^2+1)}+\arctan(x)+\frac{\ln(x^2+1)}{2}$	21
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(x+i)x^2-\ln(x-i)x^2-\ln(x+i)x^2+1+i\ln(x-i)-i\ln(x+i)-\ln(x-i)-\ln(x+i)}{2(x^2+1)}$	84

```
[In] int((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx=\frac{2\,(x^2+1)\arctan(x)+(x^2+1)\log(x^2+1)-1}{2\,(x^2+1)}$$

```
[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] 1/2*(2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 1)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx=\frac{\log(x^2+1)}{2}+\operatorname{atan}(x)-\frac{1}{2x^2+2}$$

```
[In] integrate((x**3+x**2+2*x+1)/(x**4+2*x**2+1),x)
[Out] log(x**2 + 1)/2 + atan(x) - 1/(2*x**2 + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx = -\frac{1}{2\,(x^2+1)} + \arctan(x) + \frac{1}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="maxima")
[Out] -1/2/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx = -\frac{1}{2\,(x^2+1)} + \arctan(x) + \frac{1}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="giac")
[Out] -1/2/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx = \frac{\ln(x^2+1)}{2} + \mathrm{atan}(x) - \frac{1}{2\,\left(x^2+1\right)}$$

```
[In] int((2*x + x^2 + x^3 + 1)/(2*x^2 + x^4 + 1),x)
[Out] log(x^2 + 1)/2 + atan(x) - 1/(2*(x^2 + 1))
```

3.288 $\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx$

Optimal result 1862
Rubi [A] (verified) 1862
Mathematica [A] (verified) 1864
Maple [A] (verified) 1864
Fricas [A] (verification not implemented) 1864
Sympy [A] (verification not implemented) 1865
Maxima [A] (verification not implemented) 1865
Giac [A] (verification not implemented) 1865
Mupad [B] (verification not implemented) 1866

Optimal result

Integrand size = 20, antiderivative size = 36

$$\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx = 3\arctan(x) - \frac{3\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + 2\log\left(1+x^2\right) - 2\log\left(2+x^2\right)$$

```
[Out] 3*arctan(x)+2*ln(x^2+1)-2*ln(x^2+2)-3/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1024, 400, 209, 455, 36, 31}

$$\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx = 3\arctan(x) - \frac{3\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + 2\log\left(x^2+1\right) - 2\log\left(x^2+2\right)$$

```
[In] Int[(3 + 4*x)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*Log[2 + x^
2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\int \frac{1}{(1+x^2)(2+x^2)}\,dx + 4\int \frac{x}{(1+x^2)(2+x^2)}\,dx \\
&= 2\text{Subst}\left(\int \frac{1}{(1+x)(2+x)}\,dx, x, x^2\right) + 3\int \frac{1}{1+x^2}\,dx - 3\int \frac{1}{2+x^2}\,dx \\
&= 3\tan^{-1}(x) - \frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + 2\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) - 2\text{Subst}\left(\int \frac{1}{2+x}\,dx, x, x^2\right) \\
&= 3\tan^{-1}(x) - \frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + 2\log\left(1+x^2\right) - 2\log\left(2+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx = 3\arctan(x) - \frac{3\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + 2\log\left(1+x^2\right) - 2\log\left(2+x^2\right)$$

```
[In] Integrate[(3 + 4*x)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*Log[2 + x^
2]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

method	result	size
default	$3\arctan\left(x\right)+2\ln\left(x^{2}+1\right)-2\ln\left(x^{2}+2\right)-\frac{3\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}$	34
risch	$3\arctan\left(x\right)+2\ln\left(x^{2}+1\right)-2\ln\left(x^{2}+2\right)-\frac{3\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}$	34

```
[In] int((3+4*x)/(x^2+1)/(x^2+2),x,method=_RETURNVERBOSE)
```

```
[Out] 3*arctan(x)+2*ln(x^2+1)-2*ln(x^2+2)-3/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx = -\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 3\arctan\left(x\right) - 2\log\left(x^2+2\right) + 2\log\left(x^2+1\right)$$

```
[In] integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="fricas")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x
^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx = 2\log\left(x^2+1\right) - 2\log\left(x^2+2\right) + 3\,\text{atan}(x) - \frac{3\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

```
[In] integrate((3+4*x)/(x**2+1)/(x**2+2),x)
[Out] 2*log(x**2 + 1) - 2*log(x**2 + 2) + 3*atan(x) - 3*sqrt(2)*atan(sqrt(2)*x/2)
/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx = -\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 3\arctan(x) - 2\log\left(x^2+2\right) + 2\log\left(x^2+1\right)$$

```
[In] integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="maxima")
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x
^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx = -\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 3\arctan(x) - 2\log\left(x^2+2\right) + 2\log\left(x^2+1\right)$$

```
[In] integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="giac")
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x
^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.56

$$\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx = \ln(x-\mathrm{i})\left(2-\frac{3}{2}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(2+\frac{3}{2}\mathrm{i}\right) + \ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(-2+\frac{\sqrt{2}\,3\mathrm{i}}{4}\right)-\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(2+\frac{\sqrt{2}\,3\mathrm{i}}{4}\right)$$

```
[In] int((4*x + 3)/((x^2 + 1)*(x^2 + 2)),x)
[Out] log(x - 1i)*(2 - 3i/2) + log(x + 1i)*(2 + 3i/2) + log(x - 2^(1/2)*1i)*((2^(
1/2)*3i)/4 - 2) - log(x + 2^(1/2)*1i)*((2^(1/2)*3i)/4 + 2)
```

3.289 $\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx$

Optimal result 1867
Rubi [A] (verified) 1867
Mathematica [A] (verified) 1869
Maple [A] (verified) 1869
Fricas [A] (verification not implemented) 1869
Sympy [A] (verification not implemented) 1870
Maxima [A] (verification not implemented) 1870
Giac [A] (verification not implemented) 1870
Mupad [B] (verification not implemented) 1870

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx = -\frac{1}{3}\arctan\left(\frac{x}{2}\right)+\frac{2\arctan(x)}{3}+\frac{1}{6}\log\left(1+x^2\right)-\frac{1}{6}\log\left(4+x^2\right)$$

```
[Out] -1/3*arctan(1/2*x)+2/3*arctan(x)+1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1024, 400, 209, 455, 36, 31}

$$\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx = -\frac{1}{3}\arctan\left(\frac{x}{2}\right)+\frac{2\arctan(x)}{3}+\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

```
[In] Int[(2 + x)/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] -1/3*ArcTan[x/2] + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\int \frac{1}{(1+x^2)(4+x^2)}\,dx + \int \frac{x}{(1+x^2)(4+x^2)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)(4+x)}\,dx, x, x^2\right) + \frac{2}{3}\int \frac{1}{1+x^2}\,dx - \frac{2}{3}\int \frac{1}{4+x^2}\,dx \\
&= -\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right) + \frac{2}{3}\tan^{-1}(x) + \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) - \frac{1}{6}\text{Subst}\left(\int \frac{1}{4+x}\,dx, x, x^2\right) \\
&= -\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right) + \frac{2}{3}\tan^{-1}(x) + \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{6}\log\left(4+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx = -\frac{1}{3}\arctan\left(\frac{x}{2}\right)+\frac{2\arctan(x)}{3}+\frac{1}{6}\log\left(1+x^2\right)-\frac{1}{6}\log\left(4+x^2\right)$$

```
[In] Integrate[(2 + x)/((1 + x^2)*(4 + x^2)),x]
[Out] -1/3*ArcTan[x/2] + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{\arctan\left(\frac{x}{2}\right)}{3}+\frac{2\arctan(x)}{3}+\frac{\ln(x^2+1)}{6}-\frac{\ln(x^2+4)}{6}$	28
risch	$-\frac{\arctan\left(\frac{x}{2}\right)}{3}+\frac{2\arctan(x)}{3}+\frac{\ln(x^2+1)}{6}-\frac{\ln(x^2+4)}{6}$	28
parallelrisch	$-\frac{\ln(x-2i)}{6}+\frac{i\ln(x-2i)}{6}+\frac{\ln(x-i)}{6}-\frac{i\ln(x-i)}{3}+\frac{\ln(x+i)}{6}+\frac{i\ln(x+i)}{3}-\frac{\ln(x+2i)}{6}-\frac{i\ln(x+2i)}{6}$	62

```
[In] int((x+2)/(x^2+1)/(x^2+4),x,method=_RETURNVERBOSE)
[Out] -1/3*arctan(1/2*x)+2/3*arctan(x)+1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx = -\frac{1}{3}\arctan\left(\frac{1}{2}x\right)+\frac{2}{3}\arctan(x)-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

```
[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="fricas")
[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx = \frac{\log(x^2+1)}{6} - \frac{\log(x^2+4)}{6} - \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{3} + \frac{2\operatorname{atan}(x)}{3}$$

```
[In] integrate((2+x)/(x**2+1)/(x**2+4),x)
[Out] log(x**2 + 1)/6 - log(x**2 + 4)/6 - atan(x/2)/3 + 2*atan(x)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx = -\frac{1}{3}\arctan\left(\frac{1}{2}x\right) + \frac{2}{3}\arctan(x) - \frac{1}{6}\log\left(x^2+4\right) + \frac{1}{6}\log\left(x^2+1\right)$$

```
[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="maxima")
[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx = -\frac{1}{3}\arctan\left(\frac{1}{2}x\right) + \frac{2}{3}\arctan(x) - \frac{1}{6}\log\left(x^2+4\right) + \frac{1}{6}\log\left(x^2+1\right)$$

```
[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="giac")
[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx = \ln(x-\mathrm{i})\left(\frac{1}{6}-\frac{1}{3}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(\frac{1}{6}+\frac{1}{3}\mathrm{i}\right) + \ln(x-2\mathrm{i})\left(-\frac{1}{6}+\frac{1}{6}\mathrm{i}\right) + \ln(x+2\mathrm{i})\left(-\frac{1}{6}-\frac{1}{6}\mathrm{i}\right)$$

```
[In] int((x + 2)/((x^2 + 1)*(x^2 + 4)),x)
[Out] log(x - 1i)*(1/6 - 1i/3) + log(x + 1i)*(1/6 + 1i/3) - log(x - 2i)*(1/6 - 1i
/6) - log(x + 2i)*(1/6 + 1i/6)
```

3.290 $\int \frac{2-x+x^3}{-7-6x+x^2} dx$

Optimal result 1871
Rubi [A] (verified) 1871
Mathematica [A] (verified) 1872
Maple [A] (verified) 1872
Fricas [A] (verification not implemented) 1873
Sympy [A] (verification not implemented) 1873
Maxima [A] (verification not implemented) 1873
Giac [A] (verification not implemented) 1874
Mupad [B] (verification not implemented) 1874

Optimal result

Integrand size = 19, antiderivative size = 29

$$\int \frac{2-x+x^3}{-7-6x+x^2} dx = 6x + \frac{x^2}{2} + \frac{169}{4}\log(7-x) - \frac{1}{4}\log(1+x)$$

```
[Out] 6*x+1/2*x^2+169/4*ln(7-x)-1/4*ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1671, 646, 31}

$$\int \frac{2-x+x^3}{-7-6x+x^2} dx = \frac{x^2}{2} + 6x + \frac{169}{4}\log(7-x) - \frac{1}{4}\log(x+1)$$

```
[In] Int[(2 - x + x^3)/(-7 - 6*x + x^2),x]
```

```
[Out] 6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(6+x+\frac{2(22+21x)}{-7-6x+x^2}\right) dx\\ &= 6x+\frac{x^2}{2}+2\int \frac{22+21x}{-7-6x+x^2}\, dx\\ &= 6x+\frac{x^2}{2}-\frac{1}{4}\int \frac{1}{1+x}\, dx+\frac{169}{4}\int \frac{1}{-7+x}\, dx\\ &= 6x+\frac{x^2}{2}+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{2-x+x^3}{-7-6x+x^2}\, dx = 6x+\frac{x^2}{2}+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(1+x)$$

```
[In] Integrate[(2 - x + x^3)/(-7 - 6*x + x^2),x]
[Out] 6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{x^2}{2}+6x+\frac{169\ln(x-7)}{4}-\frac{\ln(x+1)}{4}$	22
norman	$\frac{x^2}{2}+6x+\frac{169\ln(x-7)}{4}-\frac{\ln(x+1)}{4}$	22
risch	$\frac{x^2}{2}+6x+\frac{169\ln(x-7)}{4}-\frac{\ln(x+1)}{4}$	22
parallelrisch	$\frac{x^2}{2}+6x+\frac{169\ln(x-7)}{4}-\frac{\ln(x+1)}{4}$	22

```
[In] int((x^3-x+2)/(x^2-6*x-7),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+6*x+169/4*ln(x-7)-1/4*ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{2-x+x^3}{-7-6x+x^2}\,dx = \frac{1}{2}\,x^2+6\,x-\frac{1}{4}\,\log\left(x+1\right)+\frac{169}{4}\,\log\left(x-7\right)$$

```
[In] integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="fricas")
[Out] 1/2*x^2 + 6*x - 1/4*log(x + 1) + 169/4*log(x - 7)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{2-x+x^3}{-7-6x+x^2}\,dx = \frac{x^2}{2}+6x+\frac{169\log\left(x-7\right)}{4}-\frac{\log\left(x+1\right)}{4}$$

```
[In] integrate((x**3-x+2)/(x**2-6*x-7),x)
[Out] x**2/2 + 6*x + 169*log(x - 7)/4 - log(x + 1)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{2-x+x^3}{-7-6x+x^2}\,dx = \frac{1}{2}\,x^2+6\,x-\frac{1}{4}\,\log\left(x+1\right)+\frac{169}{4}\,\log\left(x-7\right)$$

```
[In] integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="maxima")
[Out] 1/2*x^2 + 6*x - 1/4*log(x + 1) + 169/4*log(x - 7)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{2-x+x^3}{-7-6x+x^2}\,dx = \frac{1}{2}\,x^2+6\,x-\frac{1}{4}\,\log\left(|x+1|\right)+\frac{169}{4}\,\log\left(|x-7|\right)$$

```
[In] integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="giac")
[Out] 1/2*x^2 + 6*x - 1/4*log(abs(x + 1)) + 169/4*log(abs(x - 7))
```

Mupad [B] (verification not implemented)

Time = 9.81 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{2-x+x^3}{-7-6x+x^2}\,dx = 6\,x-\frac{\ln\left(x+1\right)}{4}+\frac{169\,\ln\left(x-7\right)}{4}+\frac{x^2}{2}$$

```
[In] int(-(x^3 - x + 2)/(6*x - x^2 + 7),x)
[Out] 6*x - log(x + 1)/4 + (169*log(x - 7))/4 + x^2/2
```

3.291 $\int \frac{-1+x^5}{-1+x^2}\,dx$

Optimal result 1875
Rubi [A] (verified) 1875
Mathematica [A] (verified) 1876
Maple [A] (verified) 1876
Fricas [A] (verification not implemented) 1877
Sympy [A] (verification not implemented) 1877
Maxima [A] (verification not implemented) 1877
Giac [A] (verification not implemented) 1877
Mupad [B] (verification not implemented) 1878

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{-1+x^5}{-1+x^2}\,dx = \frac{x^2}{2}+\frac{x^4}{4}+\log(1+x)$$

```
[Out] 1/2*x^2+1/4*x^4+ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1824, 641, 31}

$$\int \frac{-1+x^5}{-1+x^2}\,dx = \frac{x^4}{4}+\frac{x^2}{2}+\log(x+1)$$

```
[In] Int[(-1 + x^5)/(-1 + x^2),x]
```

```
[Out] x^2/2 + x^4/4 + Log[1 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && IntegerQ[m + p]))
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(x + x^3 - \frac{1-x}{-1+x^2}\right) dx \\
&= \frac{x^2}{2} + \frac{x^4}{4} - \int \frac{1-x}{-1+x^2}\, dx \\
&= \frac{x^2}{2} + \frac{x^4}{4} - \int \frac{1}{-1-x}\, dx \\
&= \frac{x^2}{2} + \frac{x^4}{4} + \log(1+x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^5}{-1+x^2}\, dx = \frac{x^2}{2} + \frac{x^4}{4} + \log(1+x)$$

```
[In] Integrate[(-1 + x^5)/(-1 + x^2),x]
[Out] x^2/2 + x^4/4 + Log[1 + x]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{x^2}{2} + \frac{x^4}{4} + \ln(x+1)$	16
norman	$\frac{x^2}{2} + \frac{x^4}{4} + \ln(x+1)$	16
parallelrisch	$\frac{x^2}{2} + \frac{x^4}{4} + \ln(x+1)$	16
risch	$\frac{x^4}{4} + \frac{x^2}{2} + \frac{1}{4} + \ln(x+1)$	17
meijerg	$\operatorname{arctanh}(x) + \frac{x^2(3x^2+6)}{12} + \frac{\ln(-x^2+1)}{2}$	26

```
[In] int((x^5-1)/(x^2-1),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+1/4*x^4+ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{-1+x^5}{-1+x^2}\,dx = \frac{1}{4}\,x^4 + \frac{1}{2}\,x^2 + \log\left(x+1\right)$$

```
[In] integrate((x^5-1)/(x^2-1),x, algorithm="fricas")
[Out] 1/4*x^4 + 1/2*x^2 + log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{-1+x^5}{-1+x^2}\,dx = \frac{x^4}{4} + \frac{x^2}{2} + \log\left(x+1\right)$$

```
[In] integrate((x**5-1)/(x**2-1),x)
[Out] x**4/4 + x**2/2 + log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{-1+x^5}{-1+x^2}\,dx = \frac{1}{4}\,x^4 + \frac{1}{2}\,x^2 + \log\left(x+1\right)$$

```
[In] integrate((x^5-1)/(x^2-1),x, algorithm="maxima")
[Out] 1/4*x^4 + 1/2*x^2 + log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{-1+x^5}{-1+x^2}\,dx = \frac{1}{4}\,x^4 + \frac{1}{2}\,x^2 + \log\left(|x+1|\right)$$

```
[In] integrate((x^5-1)/(x^2-1),x, algorithm="giac")
[Out] 1/4*x^4 + 1/2*x^2 + log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 9.67 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{-1+x^5}{-1+x^2}\,dx = \ln{(x+1)} + \frac{x^2}{2} + \frac{x^4}{4}$$

```
[In] int((x^5 - 1)/(x^2 - 1),x)
[Out] log(x + 1) + x^2/2 + x^4/4
```

3.292 $\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx$

Optimal result 1879
Rubi [A] (verified) 1879
Mathematica [A] (verified) 1881
Maple [A] (verified) 1881
Fricas [A] (verification not implemented) 1881
Sympy [A] (verification not implemented) 1882
Maxima [A] (verification not implemented) 1882
Giac [A] (verification not implemented) 1882
Mupad [B] (verification not implemented) 1883

Optimal result

Integrand size = 22, antiderivative size = 41

$$\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx = -2x+\frac{x^2}{2}+\frac{11\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(1+x+x^2\right)$$

```
[Out] -2*x+1/2*x^2+3/2*ln(x^2+x+1)+11/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1671, 648, 632, 210, 642}

$$\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx = \frac{11\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x^2}{2}+\frac{3}{2}\log\left(x^2+x+1\right)-2x$$

```
[In] Int[(5 + 2*x - x^2 + x^3)/(1 + x + x^2),x]
```

```
[Out] -2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1 + x + x^2]
)/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-2+x+\frac{7+3x}{1+x+x^2}\right) dx \\
&= -2x+\frac{x^2}{2}+\int \frac{7+3x}{1+x+x^2}\, dx \\
&= -2x+\frac{x^2}{2}+\frac{3}{2}\int \frac{1+2x}{1+x+x^2}\, dx+\frac{11}{2}\int \frac{1}{1+x+x^2}\, dx \\
&= -2x+\frac{x^2}{2}+\frac{3}{2}\log\left(1+x+x^2\right)-11\text{Subst}\left(\int \frac{1}{-3-x^2}\, dx, x, 1+2x\right) \\
&= -2x+\frac{x^2}{2}+\frac{11\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx = -2x+\frac{x^2}{2}+\frac{11\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(1+x+x^2\right)$$

```
[In] Integrate[(5 + 2*x - x^2 + x^3)/(1 + x + x^2),x]
[Out] -2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1 + x + x^2]
)/2
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

method	result	size
default	$-2x+\frac{x^2}{2}+\frac{3\ln\left(x^2+x+1\right)}{2}+\frac{11\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	35
risch	$-2x+\frac{x^2}{2}+\frac{3\ln\left(4x^2+4x+4\right)}{2}+\frac{11\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	39

```
[In] int((x^3-x^2+2*x+5)/(x^2+x+1),x,method=_RETURNVERBOSE)
[Out] -2*x+1/2*x^2+3/2*ln(x^2+x+1)+11/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx = \frac{1}{2}x^2+\frac{11}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-2x+\frac{3}{2}\log\left(x^2+x+1\right)$$

```
[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="fricas")
[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 +
x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx = \frac{x^2}{2} - 2x + \frac{3\log(x^2+x+1)}{2} + \frac{11\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((x**3-x**2+2*x+5)/(x**2+x+1),x)
[Out] x**2/2 - 2*x + 3*log(x**2 + x + 1)/2 + 11*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt
(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx = \frac{1}{2}x^2 + \frac{11}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 2x + \frac{3}{2}\log\left(x^2+x+1\right)$$

```
[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="maxima")
[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 +
x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx = \frac{1}{2}x^2 + \frac{11}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 2x + \frac{3}{2}\log\left(x^2+x+1\right)$$

```
[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="giac")
[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 +
x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx=\frac{3\,\ln\left(x^2+x+1\right)}{2}-2\,x+\frac{11\,\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{x^2}{2}$$

```
[In] int((2*x - x^2 + x^3 + 5)/(x + x^2 + 1),x)
[Out] (3*log(x + x^2 + 1))/2 - 2*x + (11*3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3
))/3 + x^2/2
```

3.293 $\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2} \, dx$

Optimal result 1884
Rubi [A] (verified) 1884
Mathematica [A] (verified) 1886
Maple [A] (verified) 1886
Fricas [A] (verification not implemented) 1886
Sympy [A] (verification not implemented) 1887
Maxima [A] (verification not implemented) 1887
Giac [A] (verification not implemented) 1887
Mupad [B] (verification not implemented) 1887

Optimal result

Integrand size = 24, antiderivative size = 41

$$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2} \, dx = \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}+6\arctan(2-x)+\frac{3}{4}\log\left(5-4x+x^2\right)$$

```
[Out] 3/2*x+1/2*x^2+1/6*x^3-6*arctan(-2+x)+3/4*ln(x^2-4*x+5)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1671, 648, 632, 210, 642}

$$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2} \, dx = 6\arctan(2-x)+\frac{x^3}{6}+\frac{x^2}{2}+\frac{3}{4}\log\left(x^2-4x+5\right)+\frac{3x}{2}$$

```
[In] Int[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2),x]
```

```
[Out] (3*x)/2 + x^2/2 + x^3/6 + 6*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{2}+x+\frac{x^2}{2}-\frac{3(6-x)}{10-8x+2x^2}\right) dx\\
&= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-3\int \frac{6-x}{10-8x+2x^2}\, dx\\
&= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}+\frac{3}{4}\int \frac{-8+4x}{10-8x+2x^2}\, dx-12\int \frac{1}{10-8x+2x^2}\, dx\\
&= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}+\frac{3}{4}\log\left(5-4x+x^2\right)+24\text{Subst}\left(\int \frac{1}{-16-x^2}\, dx,x,-8+4x\right)\\
&= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}+6\tan^{-1}(2-x)+\frac{3}{4}\log\left(5-4x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx=\frac{1}{2}\left(3x+x^2+\frac{x^3}{3}+12\arctan(2-x)+\frac{3}{2}\log\left(5-4x+x^2\right)\right)$$

```
[In] Integrate[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2),x]
[Out] (3*x + x^2 + x^3/3 + 12*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2])/2)/2
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-6\arctan\left(x-2\right)+\frac{3\ln\left(x^2-4x+5\right)}{4}$	32
risch	$\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-6\arctan\left(x-2\right)+\frac{3\ln\left(x^2-4x+5\right)}{4}$	32
parallelrisch	$\frac{x^3}{6}+\frac{x^2}{2}+\frac{3x}{2}+\frac{3\ln(x-2-i)}{4}+3i\ln\left(x-2-i\right)+\frac{3\ln(x-2+i)}{4}-3i\ln\left(x-2+i\right)$	49

```
[In] int((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x,method=_RETURNVERBOSE)
[Out] 3/2*x+1/2*x^2+1/6*x^3-6*arctan(x-2)+3/4*ln(x^2-4*x+5)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx=\frac{1}{6}\,x^3+\frac{1}{2}\,x^2+\frac{3}{2}\,x-6\,\arctan\left(x-2\right)+\frac{3}{4}\,\log\left(x^2-4\,x+5\right)$$

```
[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="fricas")
[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx = \frac{x^3}{6} + \frac{x^2}{2} + \frac{3x}{2} + \frac{3\log\left(x^2-4x+5\right)}{4} - 6\,\mathrm{atan}\,(x-2)$$

```
[In] integrate((x**4-2*x**3+x-3)/(2*x**2-8*x+10),x)
[Out] x**3/6 + x**2/2 + 3*x/2 + 3*log(x**2 - 4*x + 5)/4 - 6*atan(x - 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx = \frac{1}{6}\,x^3 + \frac{1}{2}\,x^2 + \frac{3}{2}\,x - 6\,\arctan\left(x-2\right) + \frac{3}{4}\,\log\left(x^2-4\,x+5\right)$$

```
[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="maxima")
[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx = \frac{1}{6}\,x^3 + \frac{1}{2}\,x^2 + \frac{3}{2}\,x - 6\,\arctan\left(x-2\right) + \frac{3}{4}\,\log\left(x^2-4\,x+5\right)$$

```
[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="giac")
[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)
```

Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx = \frac{3\,x}{2} - 6\,\mathrm{atan}(x-2) + \frac{3\,\ln\left(x^2-4\,x+5\right)}{4} + \frac{x^2}{2} + \frac{x^3}{6}$$

```
[In] int((x - 2*x^3 + x^4 - 3)/(2*x^2 - 8*x + 10),x)
[Out] (3*x)/2 - 6*atan(x - 2) + (3*log(x^2 - 4*x + 5))/4 + x^2/2 + x^3/6
```

3.294 $\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx$

Optimal result 1888
Rubi [A] (verified) 1888
Mathematica [A] (verified) 1889
Maple [A] (verified) 1889
Fricas [A] (verification not implemented) 1889
Sympy [A] (verification not implemented) 1890
Maxima [A] (verification not implemented) 1890
Giac [A] (verification not implemented) 1890
Mupad [B] (verification not implemented) 1890

Optimal result

Integrand size = 29, antiderivative size = 30

$$\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx = x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x)$$

```
[Out] x+7/2*ln(1-x)-25*ln(2-x)+61/2*ln(3-x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1626}

$$\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx = x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x)$$

```
[In] Int[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]
[Out] x + (7*Log[1 - x])/2 - 25*Log[2 - x] + (61*Log[3 - x])/2
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(1+\frac{61}{2(-3+x)}-\frac{25}{-2+x}+\frac{7}{2(-1+x)}\right)\,dx \\ &= x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx = x+\frac{61}{2}\log(-3+x)-25\log(-2+x)+\frac{7}{2}\log(-1+x)$$

```
[In] Integrate[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]
[Out] x + (61*Log[-3 + x])/2 - 25*Log[-2 + x] + (7*Log[-1 + x])/2
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

method	result	size
default	$x+\frac{61\ln(-3+x)}{2}+\frac{7\ln(x-1)}{2}-25\ln(x-2)$	21
norman	$x+\frac{61\ln(-3+x)}{2}+\frac{7\ln(x-1)}{2}-25\ln(x-2)$	21
risch	$x+\frac{61\ln(-3+x)}{2}+\frac{7\ln(x-1)}{2}-25\ln(x-2)$	21
parallelrisch	$x+\frac{61\ln(-3+x)}{2}+\frac{7\ln(x-1)}{2}-25\ln(x-2)$	21

```
[In] int((x^3+3*x^2+2*x+1)/(-3+x)/(x-2)/(x-1),x,method=_RETURNVERBOSE)
[Out] x+61/2*ln(-3+x)+7/2*ln(x-1)-25*ln(x-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx = x+\frac{7}{2}\log(x-1)-25\log(x-2)+\frac{61}{2}\log(x-3)$$

```
[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")
[Out] x + 7/2*log(x - 1) - 25*log(x - 2) + 61/2*log(x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx = x+\frac{61\log(x-3)}{2}-25\log(x-2)+\frac{7\log(x-1)}{2}$$

```
[In] integrate((x**3+3*x**2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x)
[Out] x + 61*log(x - 3)/2 - 25*log(x - 2) + 7*log(x - 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx = x+\frac{7}{2}\log(x-1)-25\log(x-2)+\frac{61}{2}\log(x-3)$$

```
[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")
[Out] x + 7/2*log(x - 1) - 25*log(x - 2) + 61/2*log(x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx = x+\frac{7}{2}\log(|x-1|)-25\log(|x-2|)+\frac{61}{2}\log(|x-3|)$$

```
[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")
[Out] x + 7/2*log(abs(x - 1)) - 25*log(abs(x - 2)) + 61/2*log(abs(x - 3))
```

Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx = x+\frac{7\ln(x-1)}{2}-25\ln(x-2)+\frac{61\ln(x-3)}{2}$$

```
[In] int((2*x + 3*x^2 + x^3 + 1)/((x - 1)*(x - 2)*(x - 3)),x)
[Out] x + (7*log(x - 1))/2 - 25*log(x - 2) + (61*log(x - 3))/2
```

3.295 $\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3} dx$

Optimal result 1891
Rubi [A] (verified) 1891
Mathematica [A] (verified) 1892
Maple [A] (verified) 1892
Fricas [A] (verification not implemented) 1892
Sympy [A] (verification not implemented) 1893
Maxima [A] (verification not implemented) 1893
Giac [A] (verification not implemented) 1893
Mupad [B] (verification not implemented) 1893

Optimal result

Integrand size = 30, antiderivative size = 35

$$\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3} dx = -2x+\frac{x^2}{2}+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(2+x)+20\log(3+x)$$

```
[Out] -2*x+1/2*x^2+13/3*ln(4-x)-22/3*ln(2+x)+20*ln(3+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2099}

$$\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3} dx = \frac{x^2}{2}-2x+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(x+2)+20\log(x+3)$$

```
[In] Int[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3),x]
[Out] -2*x + x^2/2 + (13*Log[4 - x])/3 - (22*Log[2 + x])/3 + 20*Log[3 + x]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-2+\frac{13}{3(-4+x)}+x-\frac{22}{3(2+x)}+\frac{20}{3+x}\right) dx \\ &= -2x+\frac{x^2}{2}+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(2+x)+20\log(3+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx = -2x+\frac{x^2}{2}+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(2+x)+20\log(3+x)$$

```
[In] Integrate[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3),x]
[Out] -2*x + x^2/2 + (13*Log[4 - x])/3 - (22*Log[2 + x])/3 + 20*Log[3 + x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x^2}{2}-2x-\frac{22\ln(x+2)}{3}+20\ln(3+x)+\frac{13\ln(x-4)}{3}$	28
norman	$\frac{x^2}{2}-2x-\frac{22\ln(x+2)}{3}+20\ln(3+x)+\frac{13\ln(x-4)}{3}$	28
risch	$\frac{x^2}{2}-2x-\frac{22\ln(x+2)}{3}+20\ln(3+x)+\frac{13\ln(x-4)}{3}$	28
parallelrisch	$\frac{x^2}{2}-2x-\frac{22\ln(x+2)}{3}+20\ln(3+x)+\frac{13\ln(x-4)}{3}$	28

```
[In] int((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2-2*x-22/3*ln(x+2)+20*ln(3+x)+13/3*ln(x-4)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx = \frac{1}{2}\,x^2-2\,x+20\,\log\left(x+3\right)-\frac{22}{3}\,\log\left(x+2\right)+\frac{13}{3}\,\log\left(x-4\right)$$

```
[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="fricas")
[Out] 1/2*x^2 - 2*x + 20*log(x + 3) - 22/3*log(x + 2) + 13/3*log(x - 4)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx = \frac{x^2}{2} - 2x + \frac{13\log{(x-4)}}{3} - \frac{22\log{(x+2)}}{3} + 20\log{(x+3)}$$

```
[In] integrate((x**4-x**3+x**2-7*x+2)/(x**3+x**2-14*x-24),x)
[Out] x**2/2 - 2*x + 13*log(x - 4)/3 - 22*log(x + 2)/3 + 20*log(x + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx = \frac{1}{2}\,x^2 - 2\,x + 20\,\log\left(x+3\right) - \frac{22}{3}\,\log\left(x+2\right) + \frac{13}{3}\,\log\left(x-4\right)$$

```
[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="maxima")
[Out] 1/2*x^2 - 2*x + 20*log(x + 3) - 22/3*log(x + 2) + 13/3*log(x - 4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx = \frac{1}{2}\,x^2 - 2\,x + 20\,\log\left(|x+3|\right) - \frac{22}{3}\,\log\left(|x+2|\right) + \frac{13}{3}\,\log\left(|x-4|\right)$$

```
[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="giac")
[Out] 1/2*x^2 - 2*x + 20*log(abs(x + 3)) - 22/3*log(abs(x + 2)) + 13/3*log(abs(x
- 4))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx = 20\,\ln\left(x+3\right) - \frac{22\,\ln\left(x+2\right)}{3} - 2\,x + \frac{13\,\ln\left(x-4\right)}{3} + \frac{x^2}{2}$$

```
[In] int(-(x^2 - 7*x - x^3 + x^4 + 2)/(14*x - x^2 - x^3 + 24),x)
[Out] 20*log(x + 3) - (22*log(x + 2))/3 - 2*x + (13*log(x - 4))/3 + x^2/2
```

3.296 $\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx$

Optimal result 1894
Rubi [A] (verified) 1894
Mathematica [A] (verified) 1895
Maple [A] (verified) 1895
Fricas [A] (verification not implemented) 1895
Sympy [A] (verification not implemented) 1896
Maxima [A] (verification not implemented) 1896
Giac [A] (verification not implemented) 1896
Mupad [B] (verification not implemented) 1896

Optimal result

Integrand size = 19, antiderivative size = 34

$$\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx = \frac{3}{2(1-x)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(1+x)$$

```
[Out] 3/2/(1-x)-5/4*ln(1-x)+2*ln(x)-3/4*ln(1+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1626}

$$\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx = \frac{3}{2(1-x)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(x+1)$$

```
[In] Int[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]
[Out] 3/(2*(1 - x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{3}{2(-1+x)^2}-\frac{5}{4(-1+x)}+\frac{2}{x}-\frac{3}{4(1+x)}\right)dx \\ &= \frac{3}{2(1-x)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx=-\frac{3}{2(-1+x)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(1+x)$$

```
[In] Integrate[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]
[Out] -3/(2*(-1 + x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
default	$2\ln\left(x\right)-\frac{3\ln\left(x+1\right)}{4}-\frac{3}{2\left(x-1\right)}-\frac{5\ln\left(x-1\right)}{4}$	25
norman	$2\ln\left(x\right)-\frac{3\ln\left(x+1\right)}{4}-\frac{3}{2\left(x-1\right)}-\frac{5\ln\left(x-1\right)}{4}$	25
risch	$2\ln\left(x\right)-\frac{3\ln\left(x+1\right)}{4}-\frac{3}{2\left(x-1\right)}-\frac{5\ln\left(x-1\right)}{4}$	25
parallelrisch	$\frac{8\ln\left(x\right)x-5\ln\left(x-1\right)x-3\ln\left(x+1\right)x-6-8\ln\left(x\right)+5\ln\left(x-1\right)+3\ln\left(x+1\right)}{4x-4}$	45

```
[In] int((x^2+2)/(x-1)^2/x/(x+1),x,method=_RETURNVERBOSE)
[Out] 2*ln(x)-3/4*ln(x+1)-3/2/(x-1)-5/4*ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx=-\frac{3\,(x-1)\log\left(x+1\right)+5\,(x-1)\log\left(x-1\right)-8\,(x-1)\log\left(x\right)+6}{4\,(x-1)}$$

```
[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="fricas")
[Out] -1/4*(3*(x - 1)*log(x + 1) + 5*(x - 1)*log(x - 1) - 8*(x - 1)*log(x) + 6)/(
x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx = 2\log{(x)} - \frac{5\log{(x-1)}}{4} - \frac{3\log{(x+1)}}{4} - \frac{3}{2x-2}$$

```
[In] integrate((x**2+2)/(-1+x)**2/x/(1+x),x)
[Out] 2*log(x) - 5*log(x - 1)/4 - 3*log(x + 1)/4 - 3/(2*x - 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx = -\frac{3}{2\,(x-1)} - \frac{3}{4}\,\log\left(x+1\right) - \frac{5}{4}\,\log\left(x-1\right) + 2\,\log\left(x\right)$$

```
[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="maxima")
[Out] -3/2/(x - 1) - 3/4*log(x + 1) - 5/4*log(x - 1) + 2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx = -\frac{3}{2\,(x-1)} + 2\,\log\left(\left|-\frac{1}{x-1}-1\right|\right) - \frac{3}{4}\,\log\left(\left|-\frac{2}{x-1}-1\right|\right)$$

```
[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="giac")
[Out] -3/2/(x - 1) + 2*log(abs(-1/(x - 1) - 1)) - 3/4*log(abs(-2/(x - 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx = 2\,\ln\left(x\right) - \frac{3\,\ln\left(x+1\right)}{4} - \frac{5\,\ln\left(x-1\right)}{4} - \frac{3}{2\,\left(x-1\right)}$$

```
[In] int((x^2 + 2)/(x*(x - 1)^2*(x + 1)),x)
[Out] 2*log(x) - (3*log(x + 1))/4 - (5*log(x - 1))/4 - 3/(2*(x - 1))
```

3.297 $\int \frac{3+x^2+x^3}{\left(2+x^2\right)^2} dx$

Optimal result 1897
Rubi [A] (verified) 1897
Mathematica [A] (verified) 1898
Maple [A] (verified) 1899
Fricas [A] (verification not implemented) 1899
Sympy [A] (verification not implemented) 1899
Maxima [A] (verification not implemented) 1900
Giac [A] (verification not implemented) 1900
Mupad [B] (verification not implemented) 1900

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int \frac{3+x^2+x^3}{\left(2+x^2\right)^2} dx=\frac{4+x}{4\left(2+x^2\right)}+\frac{5\arctan\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{1}{2}\log\left(2+x^2\right)$$

```
[Out] 1/4*(4+x)/(x^2+2)+1/2*ln(x^2+2)+5/8*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1828, 649, 209, 266}

$$\int \frac{3+x^2+x^3}{\left(2+x^2\right)^2} dx=\frac{5\arctan\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{x+4}{4\left(x^2+2\right)}+\frac{1}{2}\log\left(x^2+2\right)$$

```
[In] Int[(3 + x^2 + x^3)/(2 + x^2)^2,x]
```

```
[Out] (4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{4+x}{4(2+x^2)}-\frac{1}{4}\int\frac{-5-4x}{2+x^2}\,dx\\ &= \frac{4+x}{4(2+x^2)}+\frac{5}{4}\int\frac{1}{2+x^2}\,dx+\int\frac{x}{2+x^2}\,dx\\ &= \frac{4+x}{4(2+x^2)}+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{1}{2}\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int\frac{3+x^2+x^3}{(2+x^2)^2}\,dx=\frac{4+x}{4(2+x^2)}+\frac{5\arctan\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{1}{2}\log\left(2+x^2\right)$$

```
[In] Integrate[(3 + x^2 + x^3)/(2 + x^2)^2,x]
[Out] (4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2 + x^2]/2
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\frac{x}{4}+1}{x^2+2}+\frac{\ln(x^2+2)}{2}+\frac{5\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{8}$	35
risch	$\frac{\frac{x}{4}+1}{x^2+2}+\frac{\ln(x^2+2)}{2}+\frac{5\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{8}$	35
meijerg	$\frac{3\sqrt{2}\left(\frac{x\sqrt{2}}{x^2+2}+\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{8}-\frac{x^2}{4\left(1+\frac{x^2}{2}\right)}+\frac{\ln\left(1+\frac{x^2}{2}\right)}{2}+\frac{\sqrt{2}\left(-\frac{x\sqrt{2}}{2\left(1+\frac{x^2}{2}\right)}+\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{4}$	79

```
[In] int((x^3+x^2+3)/(x^2+2)^2,x,method=_RETURNVERBOSE)
[Out] (1/4*x+1)/(x^2+2)+1/2*ln(x^2+2)+5/8*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{3+x^2+x^3}{(2+x^2)^2}\,dx=\frac{5\sqrt{2}(x^2+2)\arctan\left(\frac{1}{2}\sqrt{2}x\right)+4\left(x^2+2\right)\log\left(x^2+2\right)+2x+8}{8\left(x^2+2\right)}$$

```
[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="fricas")
[Out] 1/8*(5*sqrt(2)*(x^2 + 2)*arctan(1/2*sqrt(2)*x) + 4*(x^2 + 2)*log(x^2 + 2) +
 2*x + 8)/(x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{3+x^2+x^3}{(2+x^2)^2}\,dx=\frac{x+4}{4x^2+8}+\frac{\log\left(x^2+2\right)}{2}+\frac{5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

```
[In] integrate((x**3+x**2+3)/(x**2+2)**2,x)
[Out] (x + 4)/(4*x**2 + 8) + log(x**2 + 2)/2 + 5*sqrt(2)*atan(sqrt(2)*x/2)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{3+x^2+x^3}{(2+x^2)^2}\,dx = \frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{x+4}{4(x^2+2)} + \frac{1}{2}\log\left(x^2+2\right)$$

```
[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="maxima")
[Out] 5/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(x + 4)/(x^2 + 2) + 1/2*log(x^2 + 2
)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{3+x^2+x^3}{(2+x^2)^2}\,dx = \frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{x+4}{4(x^2+2)} + \frac{1}{2}\log\left(x^2+2\right)$$

```
[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="giac")
[Out] 5/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(x + 4)/(x^2 + 2) + 1/2*log(x^2 + 2
)
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{3+x^2+x^3}{(2+x^2)^2}\,dx = \frac{\ln\left(x^2+2\right)}{2} + \frac{5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8} + \frac{x}{4\,(x^2+2)} + \frac{1}{x^2+2}$$

```
[In] int((x^2 + x^3 + 3)/(x^2 + 2)^2,x)
[Out] log(x^2 + 2)/2 + (5*2^(1/2)*atan((2^(1/2)*x)/2))/8 + x/(4*(x^2 + 2)) + 1/(x
^2 + 2)
```

3.298 $\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx$

Optimal result 1901
Rubi [A] (verified) 1901
Mathematica [A] (verified) 1903
Maple [A] (verified) 1903
Fricas [A] (verification not implemented) 1903
Sympy [A] (verification not implemented) 1904
Maxima [A] (verification not implemented) 1904
Giac [A] (verification not implemented) 1904
Mupad [B] (verification not implemented) 1905

Optimal result

Integrand size = 36, antiderivative size = 49

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx = -\frac{15033\arctan(5-x)}{1025} - \frac{4607\arctan\left(\frac{1}{4}(-1+x)\right)}{4100} + \frac{1003\log(26-10x+x^2)}{1025} + \frac{22\log(17-2x+x^2)}{1025}$$

```
[Out] 15033/1025*arctan(-5+x)-4607/4100*arctan(-1/4+1/4*x)+1003/1025*ln(x^2-10*x+
26)+22/1025*ln(x^2-2*x+17)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {6860, 648, 632, 210, 642}

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx = -\frac{15033\arctan(5-x)}{1025} - \frac{4607\arctan\left(\frac{x-1}{4}\right)}{4100} + \frac{1003\log(x^2-10x+26)}{1025} + \frac{22\log(x^2-2x+17)}{1025}$$

```
[In] Int[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2)),x]
```

```
[Out] (-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1003*Log[26
 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{5003+2006x}{1025\left(26-10x+x^2\right)}+\frac{-4651+44x}{1025\left(17-2x+x^2\right)}\right)dx\\
&= \frac{\int \frac{5003+2006x}{26-10x+x^2}\,dx}{1025}+\frac{\int \frac{-4651+44x}{17-2x+x^2}\,dx}{1025}\\
&= \frac{22\int \frac{-2+2x}{17-2x+x^2}\,dx}{1025}+\frac{1003\int \frac{-10+2x}{26-10x+x^2}\,dx}{1025}-\frac{4607\int \frac{1}{17-2x+x^2}\,dx}{1025}+\frac{15033\int \frac{1}{26-10x+x^2}\,dx}{1025}\\
&= \frac{1003\log\left(26-10x+x^2\right)}{1025}+\frac{22\log\left(17-2x+x^2\right)}{1025}\\
&\quad+\frac{9214\text{Subst}\left(\int \frac{1}{-64-x^2}\,dx,x,-2+2x\right)}{1025}-\frac{30066\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,-10+2x\right)}{1025}\\
&= -\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{1}{4}(-1+x)\right)}{4100}\\
&\quad+\frac{1003\log\left(26-10x+x^2\right)}{1025}+\frac{22\log\left(17-2x+x^2\right)}{1025}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx = -\frac{15033\arctan(5-x)}{1025}-\frac{4607\arctan\left(\frac{1}{4}(-1+x)\right)}{4100}+\frac{1003\log\left(26-10x+x^2\right)}{1025}+\frac{22\log\left(17-2x+x^2\right)}{1025}$$

```
[In] Integrate[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2))
,x]
```

```
[Out] (-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1003*Log[26
 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

method	result
default	$\frac{15033\arctan(-5+x)}{1025}-\frac{4607\arctan\left(-\frac{1}{4}+\frac{x}{4}\right)}{4100}+\frac{1003\ln\left(x^2-10x+26\right)}{1025}+\frac{22\ln\left(x^2-2x+17\right)}{1025}$
risch	$\frac{15033\arctan(-5+x)}{1025}-\frac{4607\arctan\left(-\frac{1}{4}+\frac{x}{4}\right)}{4100}+\frac{1003\ln\left(x^2-10x+26\right)}{1025}+\frac{22\ln\left(x^2-2x+17\right)}{1025}$
parallelrisch	$\frac{1003\ln(x-5-i)}{1025}-\frac{15033i\ln(x-5-i)}{2050}+\frac{1003\ln(x-5+i)}{1025}+\frac{15033i\ln(x-5+i)}{2050}+\frac{22\ln(x-1-4i)}{1025}+\frac{4607i\ln(x-1-4i)}{8200}-$

```
[In] int((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x,method=_RETURNVERBOS
E)
```

```
[Out] 15033/1025*arctan(-5+x)-4607/4100*arctan(-1/4+1/4*x)+1003/1025*ln(x^2-10*x+
26)+22/1025*ln(x^2-2*x+17)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx = \frac{15033}{1025}\arctan(x-5)-\frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right)+\frac{22}{1025}\log\left(x^2-2x+17\right)+\frac{1003}{1025}\log\left(x^2-10x+26\right)$$

```
[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="fr
icas")
```

```
[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2
- 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx = \frac{1003\log(x^2-10x+26)}{1025}+\frac{22\log(x^2-2x+17)}{1025} - \frac{4607\operatorname{atan}\left(\frac{x}{4}-\frac{1}{4}\right)}{4100}+\frac{15033\operatorname{atan}(x-5)}{1025}$$

```
[In] integrate((2*x**3-4*x**2+70*x-35)/(x**2-10*x+26)/(x**2-2*x+17),x)
```

```
[Out] 1003*log(x**2 - 10*x + 26)/1025 + 22*log(x**2 - 2*x + 17)/1025 - 4607*atan(
x/4 - 1/4)/4100 + 15033*atan(x - 5)/1025
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx = \frac{15033}{1025}\arctan(x-5)-\frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right) + \frac{22}{1025}\log\left(x^2-2x+17\right)+\frac{1003}{1025}\log\left(x^2-10x+26\right)$$

```
[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="ma
xima")
```

```
[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2
- 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx = \frac{15033}{1025}\arctan(x-5)-\frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right) + \frac{22}{1025}\log\left(x^2-2x+17\right)+\frac{1003}{1025}\log\left(x^2-10x+26\right)$$

```
[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="gi
ac")
```

```
[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2
- 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx = \ln(x-1-4i)\left(\frac{22}{1025}+\frac{4607}{8200}i\right) + \ln(x-1+4i)\left(\frac{22}{1025}-\frac{4607}{8200}i\right) + \ln(x-5-i)\left(\frac{1003}{1025}-\frac{15033}{2050}i\right) + \ln(x-5+1i)\left(\frac{1003}{1025}+\frac{15033}{2050}i\right)$$

```
[In] int((70*x - 4*x^2 + 2*x^3 - 35)/((x^2 - 2*x + 17)*(x^2 - 10*x + 26)),x)
```

```
[Out] log(x - (1 + 4i))*(22/1025 + 4607i/8200) + log(x - (1 - 4i))*(22/1025 - 460
7i/8200) + log(x - (5 + 1i))*(1003/1025 - 15033i/2050) + log(x - (5 - 1i))*
(1003/1025 + 15033i/2050)
```

3.299 $\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx$

Optimal result 1906
Rubi [A] (verified) 1906
Mathematica [A] (verified) 1907
Maple [A] (verified) 1907
Fricas [A] (verification not implemented) 1907
Sympy [A] (verification not implemented) 1908
Maxima [A] (verification not implemented) 1908
Giac [A] (verification not implemented) 1908
Mupad [B] (verification not implemented) 1908

Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx = -\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(4+x)$$

```
[Out] -11/14*ln(3-x)+3/2*ln(5-x)+2/7*ln(4+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1626}

$$\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx = -\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(x+4)$$

```
[In] Int[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)),x]
[Out] (-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{3}{2(-5+x)}-\frac{11}{14(-3+x)}+\frac{2}{7(4+x)}\right)dx\\ &= -\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(4+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx = -\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(4+x)$$

```
[In] Integrate[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)),x]
[Out] (-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{3\ln(-5+x)}{2}-\frac{11\ln(-3+x)}{14}+\frac{2\ln(x+4)}{7}$	20
norman	$\frac{3\ln(-5+x)}{2}-\frac{11\ln(-3+x)}{14}+\frac{2\ln(x+4)}{7}$	20
risch	$\frac{3\ln(-5+x)}{2}-\frac{11\ln(-3+x)}{14}+\frac{2\ln(x+4)}{7}$	20
parallelrisch	$\frac{3\ln(-5+x)}{2}-\frac{11\ln(-3+x)}{14}+\frac{2\ln(x+4)}{7}$	20

```
[In] int((x^2+2)/(-5+x)/(-3+x)/(x+4),x,method=_RETURNVERBOSE)
[Out] 3/2*ln(-5+x)-11/14*ln(-3+x)+2/7*ln(x+4)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx = \frac{2}{7}\log\left(x+4\right)-\frac{11}{14}\log\left(x-3\right)+\frac{3}{2}\log\left(x-5\right)$$

```
[In] integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="fricas")
[Out] 2/7*log(x + 4) - 11/14*log(x - 3) + 3/2*log(x - 5)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)} \, dx = \frac{3\log{(x-5)}}{2} - \frac{11\log{(x-3)}}{14} + \frac{2\log{(x+4)}}{7}$$

```
[In] integrate((x**2+2)/(-5+x)/(-3+x)/(4+x),x)
[Out] 3*log(x - 5)/2 - 11*log(x - 3)/14 + 2*log(x + 4)/7
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)} \, dx = \frac{2}{7}\log{(x+4)} - \frac{11}{14}\log{(x-3)} + \frac{3}{2}\log{(x-5)}$$

```
[In] integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="maxima")
[Out] 2/7*log(x + 4) - 11/14*log(x - 3) + 3/2*log(x - 5)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)} \, dx = \frac{2}{7}\log{(|x+4|)} - \frac{11}{14}\log{(|x-3|)} + \frac{3}{2}\log{(|x-5|)}$$

```
[In] integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="giac")
[Out] 2/7*log(abs(x + 4)) - 11/14*log(abs(x - 3)) + 3/2*log(abs(x - 5))
```

Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)} \, dx = \frac{2\ln{(x+4)}}{7} - \frac{11\ln{(x-3)}}{14} + \frac{3\ln{(x-5)}}{2}$$

```
[In] int((x^2 + 2)/((x - 3)*(x + 4)*(x - 5)),x)
[Out] (2*log(x + 4))/7 - (11*log(x - 3))/14 + (3*log(x - 5))/2
```

3.300 $\int \frac{x^4}{(-1+x)(2+x^2)}\,dx$

Optimal result 1909
Rubi [A] (verified) 1909
Mathematica [A] (verified) 1910
Maple [A] (verified) 1911
Fricas [A] (verification not implemented) 1911
Sympy [A] (verification not implemented) 1911
Maxima [A] (verification not implemented) 1912
Giac [A] (verification not implemented) 1912
Mupad [B] (verification not implemented) 1912

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{x^4}{(-1+x)\left(2+x^2\right)}\,dx = x+\frac{x^2}{2}-\frac{2}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{1}{3}\log(1-x)-\frac{2}{3}\log\left(2+x^2\right)$$

```
[Out] x+1/2*x^2+1/3*ln(1-x)-2/3*ln(x^2+2)-2/3*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1643, 649, 209, 266}

$$\int \frac{x^4}{(-1+x)\left(2+x^2\right)}\,dx = -\frac{2}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{x^2}{2}-\frac{2}{3}\log\left(x^2+2\right)+x+\frac{1}{3}\log(1-x)$$

```
[In] Int[x^4/((-1 + x)*(2 + x^2)),x]
```

```
[Out] x + x^2/2 - (2*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 + Log[1 - x]/3 - (2*Log[2 + x^2
])/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(1+\frac{1}{3(-1+x)}+x-\frac{4(1+x)}{3\left(2+x^2\right)}\right) dx \\
&= x+\frac{x^2}{2}+\frac{1}{3}\log(1-x)-\frac{4}{3}\int \frac{1+x}{2+x^2}\,dx \\
&= x+\frac{x^2}{2}+\frac{1}{3}\log(1-x)-\frac{4}{3}\int \frac{1}{2+x^2}\,dx-\frac{4}{3}\int \frac{x}{2+x^2}\,dx \\
&= x+\frac{x^2}{2}-\frac{2}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{1}{3}\log(1-x)-\frac{2}{3}\log\left(2+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$
\int \frac{x^4}{(-1+x)\left(2+x^2\right)}\,dx = \frac{1}{6}\left(-9+6x+3x^2-4\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+2\log(-1+x)-4\log\left(2+x^2\right)\right)
$$

```
[In] Integrate[x^4/((-1 + x)*(2 + x^2)),x]
[Out] (-9 + 6*x + 3*x^2 - 4*Sqrt[2]*ArcTan[x/Sqrt[2]] + 2*Log[-1 + x] - 4*Log[2 +
 x^2])/6
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{x^{2}}{2}+x-\frac{2\ln\left(x^{2}+2\right)}{3}-\frac{2\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}+\frac{\ln\left(x-1\right)}{3}$	34
risch	$\frac{x^{2}}{2}+x-\frac{2\ln\left(x^{2}+2\right)}{3}-\frac{2\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}+\frac{\ln\left(x-1\right)}{3}$	34

```
[In] int(x^4/(x-1)/(x^2+2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+x-2/3*ln(x^2+2)-2/3*arctan(1/2*x*2^(1/2))*2^(1/2)+1/3*ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^4}{(-1+x)\left(2+x^2\right)}\,dx=\frac{1}{2}\,x^2-\frac{2}{3}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+x-\frac{2}{3}\,\log\left(x^2+2\right)+\frac{1}{3}\,\log\left(x-1\right)$$

```
[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="fricas")
[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*lo
g(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{x^4}{(-1+x)\left(2+x^2\right)}\,dx=\frac{x^2}{2}+x+\frac{\log\left(x-1\right)}{3}-\frac{2\log\left(x^2+2\right)}{3}-\frac{2\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{3}$$

```
[In] integrate(x**4/(-1+x)/(x**2+2),x)
[Out] x**2/2 + x + log(x - 1)/3 - 2*log(x**2 + 2)/3 - 2*sqrt(2)*atan(sqrt(2)*x/2)
/3
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^4}{(-1+x)(2+x^2)}\,dx = \frac{1}{2}x^2 - \frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + x - \frac{2}{3}\log\left(x^2+2\right) + \frac{1}{3}\log\left(x-1\right)$$

```
[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="maxima")
[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*lo
g(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{(-1+x)(2+x^2)}\,dx = \frac{1}{2}x^2 - \frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + x - \frac{2}{3}\log\left(x^2+2\right) + \frac{1}{3}\log\left(|x-1|\right)$$

```
[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="giac")
[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*lo
g(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{x^4}{(-1+x)(2+x^2)}\,dx = x + \frac{\ln(x-1)}{3} + \ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(-\frac{2}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}\right) - \ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(\frac{2}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}\right) + \frac{x^2}{2}$$

```
[In] int(x^4/((x^2 + 2)*(x - 1)),x)
[Out] x + log(x - 1)/3 + log(x - 2^(1/2)*1i)*((2^(1/2)*1i)/3 - 2/3) - log(x + 2^(
1/2)*1i)*((2^(1/2)*1i)/3 + 2/3) + x^2/2
```

3.301 $\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3} dx$

Optimal result 1913
Rubi [A] (verified) 1913
Mathematica [A] (verified) 1914
Maple [A] (verified) 1914
Fricas [A] (verification not implemented) 1914
Sympy [A] (verification not implemented) 1915
Maxima [A] (verification not implemented) 1915
Giac [A] (verification not implemented) 1915
Mupad [B] (verification not implemented) 1915

Optimal result

Integrand size = 24, antiderivative size = 16

$$\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3} dx = -\frac{3}{1+x} + 2\log(1-x)$$

```
[Out] -3/(1+x)+2*ln(1-x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2099}

$$\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3} dx = 2\log(1-x) - \frac{3}{x+1}$$

```
[In] Int[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3),x]
```

```
[Out] -3/(1 + x) + 2*Log[1 - x]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{2}{-1+x} + \frac{3}{(1+x)^2}\right) dx \\ &= -\frac{3}{1+x} + 2\log(1-x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3}\,dx=-\frac{3}{1+x}+2\log(-1+x)$$

```
[In] Integrate[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3),x]
[Out] -3/(1 + x) + 2*Log[-1 + x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$2\ln\left(x-1\right)-\frac{3}{x+1}$	15
norman	$2\ln\left(x-1\right)-\frac{3}{x+1}$	15
risch	$2\ln\left(x-1\right)-\frac{3}{x+1}$	15
parallelrisch	$\frac{2\ln(x-1)x-3+2\ln(x-1)}{x+1}$	22

```
[In] int((2*x^2+7*x-1)/(x^3+x^2-x-1),x,method=_RETURNVERBOSE)
[Out] 2*ln(x-1)-3/(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3}\,dx=\frac{2\left(x+1\right)\log\left(x-1\right)-3}{x+1}$$

```
[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="fricas")
[Out] (2*(x + 1)*log(x - 1) - 3)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3}\,dx = 2\log\left(x-1\right) - \frac{3}{x+1}$$

```
[In] integrate((2*x**2+7*x-1)/(x**3+x**2-x-1),x)
[Out] 2*log(x - 1) - 3/(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3}\,dx = -\frac{3}{x+1} + 2\,\log\left(x-1\right)$$

```
[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="maxima")
[Out] -3/(x + 1) + 2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3}\,dx = -\frac{3}{x+1} + 2\,\log\left(|x-1|\right)$$

```
[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="giac")
[Out] -3/(x + 1) + 2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3}\,dx = 2\,\ln\left(x-1\right) - \frac{3}{x+1}$$

```
[In] int(-(7*x + 2*x^2 - 1)/(x - x^2 - x^3 + 1),x)
[Out] 2*log(x - 1) - 3/(x + 1)
```

3.302 $\int \frac{1+2x}{-1+3x-3x^2+x^3} dx$

Optimal result 1916
Rubi [A] (verified) 1916
Mathematica [A] (verified) 1917
Maple [A] (verified) 1917
Fricas [A] (verification not implemented) 1917
Sympy [A] (verification not implemented) 1918
Maxima [A] (verification not implemented) 1918
Giac [A] (verification not implemented) 1918
Mupad [B] (verification not implemented) 1918

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1+2x}{-1+3x-3x^2+x^3} dx = -\frac{3}{2(1-x)^2} + \frac{2}{1-x}$$

```
[Out] -3/2/(1-x)^2+2/(1-x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2099}

$$\int \frac{1+2x}{-1+3x-3x^2+x^3} dx = \frac{2}{1-x} - \frac{3}{2(1-x)^2}$$

```
[In] Int[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3),x]
[Out] -3/(2*(1 - x)^2) + 2/(1 - x)
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{3}{(-1+x)^3} + \frac{2}{(-1+x)^2}\right) dx \\ &= -\frac{3}{2(1-x)^2} + \frac{2}{1-x} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx=\frac{1-4x}{2(-1+x)^2}$$

```
[In] Integrate[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3),x]
[Out] (1 - 4*x)/(2*(-1 + x)^2)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

method	result	size
norman	$\frac{-2x+\frac{1}{2}}{(x-1)^2}$	12
default	$-\frac{2}{x-1}-\frac{3}{2(x-1)^2}$	16
risch	$\frac{-2x+\frac{1}{2}}{x^2-2x+1}$	17
gosper	$-\frac{-1+4x}{2(x^2-2x+1)}$	18
parallelrisch	$\frac{1-4x}{2x^2-4x+2}$	18

```
[In] int((1+2*x)/(x^3-3*x^2+3*x-1),x,method=_RETURNVERBOSE)
[Out] (-2*x+1/2)/(x-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx=-\frac{4\,x-1}{2\,(x^2-2\,x+1)}$$

```
[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="fricas")
[Out] -1/2*(4*x - 1)/(x^2 - 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx = \frac{1-4x}{2x^2-4x+2}$$

```
[In] integrate((1+2*x)/(x**3-3*x**2+3*x-1),x)
[Out] (1 - 4*x)/(2*x**2 - 4*x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx = -\frac{4\,x-1}{2\,(x^2-2\,x+1)}$$

```
[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="maxima")
[Out] -1/2*(4*x - 1)/(x^2 - 2*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

$$\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx = -\frac{4\,x-1}{2\,(x-1)^2}$$

```
[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="giac")
[Out] -1/2*(4*x - 1)/(x - 1)^2
```

Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

$$\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx = -\frac{4\,x-1}{2\,(x-1)^2}$$

```
[In] int((2*x + 1)/(3*x - 3*x^2 + x^3 - 1),x)
[Out] -(4*x - 1)/(2*(x - 1)^2)
```

3.303 $\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3} \, dx$

Optimal result 1919
Rubi [A] (verified) 1919
Mathematica [A] (verified) 1920
Maple [A] (verified) 1920
Fricas [A] (verification not implemented) 1920
Sympy [A] (verification not implemented) 1921
Maxima [A] (verification not implemented) 1921
Giac [A] (verification not implemented) 1921
Mupad [B] (verification not implemented) 1921

Optimal result

Integrand size = 24, antiderivative size = 15

$$\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3} \, dx = \frac{1}{1-x} - \frac{2}{(1+x)^2}$$

```
[Out] 1/(1-x)-2/(1+x)^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1634}

$$\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3} \, dx = \frac{1}{1-x} - \frac{2}{(x+1)^2}$$

```
[In] Int[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3),x]
```

```
[Out] (1 - x)^(-1) - 2/(1 + x)^2
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{(-1+x)^2} + \frac{4}{(1+x)^3}\right) \, dx \\ &= \frac{1}{1-x} - \frac{2}{(1+x)^2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx = -\frac{1}{-1+x}-\frac{2}{(1+x)^2}$$

```
[In] Integrate[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3),x]
[Out] -(-1 + x)^(-1) - 2/(1 + x)^2
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{1}{x-1}-\frac{2}{(x+1)^2}$	16
gosper	$-\frac{x^2+4x-1}{(x-1)(x+1)^2}$	21
norman	$\frac{-x^2-4x+1}{(x-1)(x+1)^2}$	22
risch	$\frac{-x^2-4x+1}{(x-1)(x+1)^2}$	22
parallelrisch	$\frac{-x^2-4x+1}{(x-1)(x+1)^2}$	22

```
[In] int((x^3+7*x^2-5*x+5)/(x-1)^2/(x+1)^3,x,method=_RETURNVERBOSE)
[Out] -1/(x-1)-2/(x+1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx = -\frac{x^2+4\,x-1}{x^3+x^2-x-1}$$

```
[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="fricas")
[Out] -(x^2 + 4*x - 1)/(x^3 + x^2 - x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx = \frac{-x^2-4x+1}{x^3+x^2-x-1}$$

```
[In] integrate((x**3+7*x**2-5*x+5)/(-1+x)**2/(1+x)**3,x)
[Out] (-x**2 - 4*x + 1)/(x**3 + x**2 - x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx = -\frac{x^2+4\,x-1}{x^3+x^2-x-1}$$

```
[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="maxima")
[Out] -(x^2 + 4*x - 1)/(x^3 + x^2 - x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

$$\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx = -\frac{1}{x-1}+\frac{\frac{4}{x-1}+1}{2\left(\frac{2}{x-1}+1\right)^2}$$

```
[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="giac")
[Out] -1/(x - 1) + 1/2*(4/(x - 1) + 1)/(2/(x - 1) + 1)^2
```

Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx = -\frac{1}{x-1}-\frac{2}{\left(x+1\right)^2}$$

```
[In] int((7*x^2 - 5*x + x^3 + 5)/((x - 1)^2*(x + 1)^3),x)
[Out] - 1/(x - 1) - 2/(x + 1)^2
```

3.304 $\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3} \, dx$

Optimal result . . . 1922
Rubi [A] (verified) . . . 1922
Mathematica [A] (verified) . . . 1924
Maple [A] (verified) . . . 1924
Fricas [A] (verification not implemented) . . . 1924
Sympy [A] (verification not implemented) . . . 1925
Maxima [A] (verification not implemented) . . . 1925
Giac [A] (verification not implemented) . . . 1925
Mupad [B] (verification not implemented) . . . 1925

Optimal result

Integrand size = 26, antiderivative size = 31

$$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3} \, dx = -\frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log(1+x) + \log\left(1+x+x^2\right)$$

```
[Out] ln(1+x)+ln(x^2+x+1)-2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2099, 648, 632, 210, 642}

$$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3} \, dx = -\frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} + \log\left(x^2+x+1\right) + \log(x+1)$$

```
[In] Int[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3),x]
```

```
[Out] (-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x + x^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{1+x}+\frac{2x}{1+x+x^2}\right) dx \\
&= \log(1+x)+2\int \frac{x}{1+x+x^2}\,dx \\
&= \log(1+x)-\int \frac{1}{1+x+x^2}\,dx+\int \frac{1+2x}{1+x+x^2}\,dx \\
&= \log(1+x)+\log\left(1+x+x^2\right)+2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right) \\
&= -\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+x)+\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx = -\frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log(1+x) + \log\left(1+x+x^2\right)$$

```
[In] Integrate[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3),x]
[Out] (-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x + x^2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
default	$\ln(x+1)+\ln(x^2+x+1)-\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	29
risch	$\ln(4x^2+4x+4)-\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\ln(x+1)$	33

```
[In] int((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x,method=_RETURNVERBOSE)
[Out] ln(x+1)+ln(x^2+x+1)-2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \log\left(x^2+x+1\right) + \log(x+1)$$

```
[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="fricas")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.10

$$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx = \log(x+1)$$

```
[In] integrate((3*x**2+3*x+1)/(x**3+2*x**2+2*x+1),x)
[Out] log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \log\left(x^2+x+1\right) + \log(x+1)$$

```
[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="maxima")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \log\left(x^2+x+1\right) + \log(|x+1|)$$

```
[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="giac")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(abs(x +
 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.84

$$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx = \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) + \ln(x+1) + \frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{3} - \frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{3}$$

```
[In] int((3*x + 3*x^2 + 1)/(2*x + 2*x^2 + x^3 + 1),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2) + log(x + (3^(1/2)*1i)/2 + 1/2) + log(x + 1)
+ (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*1i)/3 - (3^(1/2)*log(x + (3^(1/2)*
1i)/2 + 1/2)*1i)/3
```

3.305 $\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} dx$

Optimal result 1927
Rubi [A] (verified) 1927
Mathematica [A] (verified) 1928
Maple [A] (verified) 1928
Fricas [A] (verification not implemented) 1929
Sympy [A] (verification not implemented) 1929
Maxima [A] (verification not implemented) 1929
Giac [A] (verification not implemented) 1929
Mupad [B] (verification not implemented) 1930

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} dx = \frac{1}{10}\log(1-2x) + \frac{\log(x)}{2} - \frac{1}{10}\log(2+x)$$

```
[Out] 1/10*ln(1-2*x)+1/2*ln(x)-1/10*ln(2+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1608, 1642}

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} dx = \frac{1}{10}\log(1-2x) + \frac{\log(x)}{2} - \frac{1}{10}\log(x+2)$$

```
[In] Int[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
```

```
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{-1+2x+x^2}{x\left(-2+3x+2x^2\right)}\,dx\\ &= \int \left(\frac{1}{2x}-\frac{1}{10(2+x)}+\frac{1}{5(-1+2x)}\right)dx\\ &= \frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(2+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx=\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(2+x)$$

```
[In] Integrate[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
parallelrisch	$\frac{\ln(x)}{2}-\frac{\ln(x+2)}{10}+\frac{\ln\left(x-\frac{1}{2}\right)}{10}$	18
default	$\frac{\ln(x)}{2}-\frac{\ln(x+2)}{10}+\frac{\ln(2x-1)}{10}$	20
norman	$\frac{\ln(x)}{2}-\frac{\ln(x+2)}{10}+\frac{\ln(2x-1)}{10}$	20
risch	$\frac{\ln(x)}{2}-\frac{\ln(x+2)}{10}+\frac{\ln(2x-1)}{10}$	20

```
[In] int((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x)-1/10*ln(x+2)+1/10*ln(x-1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx=\frac{1}{10}\,\log\left(2\,x-1\right)-\frac{1}{10}\,\log\left(x+2\right)+\frac{1}{2}\,\log\left(x\right)$$

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="fricas")
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx=\frac{\log\left(x\right)}{2}+\frac{\log\left(x-\frac{1}{2}\right)}{10}-\frac{\log\left(x+2\right)}{10}$$

```
[In] integrate((x**2+2*x-1)/(2*x**3+3*x**2-2*x),x)
[Out] log(x)/2 + log(x - 1/2)/10 - log(x + 2)/10
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx=\frac{1}{10}\,\log\left(2\,x-1\right)-\frac{1}{10}\,\log\left(x+2\right)+\frac{1}{2}\,\log\left(x\right)$$

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="maxima")
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx=\frac{1}{10}\,\log\left(|2\,x-1|\right)-\frac{1}{10}\,\log\left(|x+2|\right)+\frac{1}{2}\,\log\left(|x|\right)$$

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="giac")
[Out] 1/10*log(abs(2*x - 1)) - 1/10*log(abs(x + 2)) + 1/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx=\frac{\mathrm{atanh}\left(\frac{24}{145\,\left(\frac{29\,x}{100}-\frac{11}{50}\right)}+\frac{35}{29}\right)}{5}+\frac{\ln\left(x\right)}{2}$$

```
[In] int((2*x + x^2 - 1)/(3*x^2 - 2*x + 2*x^3),x)
```

```
[Out] atanh(24/(145*((29*x)/100 - 11/50)) + 35/29)/5 + log(x)/2
```

3.306 $\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3} \, dx$

Optimal result 1931
Rubi [A] (verified) 1931
Mathematica [A] (verified) 1932
Maple [A] (verified) 1932
Fricas [A] (verification not implemented) 1932
Sympy [A] (verification not implemented) 1933
Maxima [A] (verification not implemented) 1933
Giac [A] (verification not implemented) 1933
Mupad [B] (verification not implemented) 1933

Optimal result

Integrand size = 29, antiderivative size = 30

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3} \, dx = \frac{2}{1-x} + x + \frac{x^2}{2} + \log(1-x) - \log(1+x)$$

```
[Out] 2/(1-x)+x+1/2*x^2+ln(1-x)-ln(1+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2099}

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3} \, dx = \frac{x^2}{2} + x + \frac{2}{1-x} + \log(1-x) - \log(x+1)$$

```
[In] Int[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
```

```
[Out] 2/(1 - x) + x + x^2/2 + Log[1 - x] - Log[1 + x]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(1+\frac{1}{-1-x}+\frac{2}{(-1+x)^2}+\frac{1}{-1+x}+x\right) \, dx \\ &= \frac{2}{1-x}+x+\frac{x^2}{2}+\log(1-x)-\log(1+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx = -\frac{2}{-1+x}+\frac{1}{2}(1+x)^2+\log(1-x)-\log(1+x)$$

```
[In] Integrate[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
[Out] -2/(-1 + x) + (1 + x)^2/2 + Log[1 - x] - Log[1 + x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x^2}{2}+x-\ln\left(x+1\right)+\ln\left(x-1\right)-\frac{2}{x-1}$	25
risch	$\frac{x^2}{2}+x-\ln\left(x+1\right)+\ln\left(x-1\right)-\frac{2}{x-1}$	25
norman	$\frac{\frac{1}{2}x^2+\frac{1}{2}x^3-3}{x-1}-\ln\left(x+1\right)+\ln\left(x-1\right)$	30
parallelrisch	$\frac{x^3+2\ln(x-1)x-2\ln(x+1)x+x^2-6-2\ln(x-1)+2\ln(x+1)}{2x-2}$	42

```
[In] int((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+x-ln(x+1)+ln(x-1)-2/(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx = \frac{x^3+x^2-2\,(x-1)\log\left(x+1\right)+2\,(x-1)\log\left(x-1\right)-2\,x-4}{2\,(x-1)}$$

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="fricas")
[Out] 1/2*(x^3 + x^2 - 2*(x - 1)*log(x + 1) + 2*(x - 1)*log(x - 1) - 2*x - 4)/(x
- 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx = \frac{x^2}{2} + x + \log(x-1) - \log(x+1) - \frac{2}{x-1}$$

```
[In] integrate((x**4-2*x**2+4*x+1)/(x**3-x**2-x+1),x)
[Out] x**2/2 + x + log(x - 1) - log(x + 1) - 2/(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx = \frac{1}{2}\,x^2 + x - \frac{2}{x-1} - \log(x+1) + \log(x-1)$$

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="maxima")
[Out] 1/2*x^2 + x - 2/(x - 1) - log(x + 1) + log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx = \frac{1}{2}\,x^2 + x - \frac{2}{x-1} - \log(|x+1|) + \log(|x-1|)$$

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="giac")
[Out] 1/2*x^2 + x - 2/(x - 1) - log(abs(x + 1)) + log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx = x - \frac{2}{x-1} + \frac{x^2}{2} + \mathrm{atan}(x\,1\mathrm{i})\;2\mathrm{i}$$

```
[In] int(-(4*x - 2*x^2 + x^4 + 1)/(x + x^2 - x^3 - 1),x)
[Out] x + atan(x*1i)*2i - 2/(x - 1) + x^2/2
```

3.307 $\int \frac{4-x+2x^2}{4x+x^3} dx$

Optimal result . . . 1934
Rubi [A] (verified) . . . 1934
Mathematica [A] (verified) . . . 1935
Maple [A] (verified) . . . 1936
Fricas [A] (verification not implemented) . . . 1936
Sympy [A] (verification not implemented) . . . 1936
Maxima [A] (verification not implemented) . . . 1937
Giac [A] (verification not implemented) . . . 1937
Mupad [B] (verification not implemented) . . . 1937

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \frac{4-x+2x^2}{4x+x^3} dx = -\frac{1}{2}\arctan\left(\frac{x}{2}\right) + \log(x) + \frac{1}{2}\log\left(4+x^2\right)$$

```
[Out] -1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1607, 1816, 649, 209, 266}

$$\int \frac{4-x+2x^2}{4x+x^3} dx = -\frac{1}{2}\arctan\left(\frac{x}{2}\right) + \frac{1}{2}\log\left(x^2+4\right) + \log(x)$$

```
[In] Int[(4 - x + 2*x^2)/(4*x + x^3),x]
```

```
[Out] -1/2*ArcTan[x/2] + Log[x] + Log[4 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{4-x+2x^2}{x\left(4+x^2\right)}\,dx \\
&= \int \left(\frac{1}{x}+\frac{-1+x}{4+x^2}\right)dx \\
&= \log(x)+\int \frac{-1+x}{4+x^2}\,dx \\
&= \log(x)-\int \frac{1}{4+x^2}\,dx+\int \frac{x}{4+x^2}\,dx \\
&= -\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\log(x)+\frac{1}{2}\log\left(4+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx=-\frac{1}{2}\arctan\left(\frac{x}{2}\right)+\log(x)+\frac{1}{2}\log\left(4+x^2\right)$$

```
[In] Integrate[(4 - x + 2*x^2)/(4*x + x^3),x]
[Out] -1/2*ArcTan[x/2] + Log[x] + Log[4 + x^2]/2
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln\left(x\right)+\frac{\ln\left(x^{2}+4\right)}{2}$	18
risch	$-\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln\left(x\right)+\frac{\ln\left(x^{2}+4\right)}{2}$	18
meijerg	$\frac{\ln\left(1+\frac{x^{2}}{4}\right)}{2}+\ln\left(x\right)-\ln\left(2\right)-\frac{\arctan\left(\frac{x}{2}\right)}{2}$	24
parallelrisch	$\ln\left(x\right)+\frac{\ln(x-2i)}{2}+\frac{i\ln(x-2i)}{4}+\frac{\ln(x+2i)}{2}-\frac{i\ln(x+2i)}{4}$	34

```
[In] int((2*x^2-x+4)/(x^3+4*x),x,method=_RETURNVERBOSE)
[Out] -1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx=-\frac{1}{2}\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(x\right)$$

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="fricas")
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx=\log\left(x\right)+\frac{\log\left(x^2+4\right)}{2}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

```
[In] integrate((2*x**2-x+4)/(x**3+4*x),x)
[Out] log(x) + log(x**2 + 4)/2 - atan(x/2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx = -\frac{1}{2}\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{2}\,\log\left(x^2+4\right)+\log\left(x\right)$$

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="maxima")
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx = -\frac{1}{2}\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{2}\,\log\left(x^2+4\right)+\log\left(|x|\right)$$

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="giac")
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.81 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx = \ln\left(x\right)+\ln\left(x-2\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{1}{4}\mathrm{i}\right)+\ln\left(x+2\mathrm{i}\right)\,\left(\frac{1}{2}-\frac{1}{4}\mathrm{i}\right)$$

```
[In] int((2*x^2 - x + 4)/(4*x + x^3),x)
[Out] log(x - 2i)*(1/2 + 1i/4) + log(x + 2i)*(1/2 - 1i/4) + log(x)
```

3.308 $\int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)}\,dx$

Optimal result 1938
Rubi [A] (verified) 1938
Mathematica [A] (verified) 1941
Maple [A] (verified) 1941
Fricas [A] (verification not implemented) 1941
Sympy [A] (verification not implemented) 1942
Maxima [A] (verification not implemented) 1942
Giac [A] (verification not implemented) 1943
Mupad [B] (verification not implemented) 1943

Optimal result

Integrand size = 32, antiderivative size = 103

$$\int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)}\,dx = \frac{1+x}{8(1+x^2)^2}-\frac{3(1-x)}{8(1+x^2)}+\frac{3x}{16(1+x^2)}+\frac{7\arctan(x)}{16}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{8}\log(1-x)-\log(x)+\frac{15}{16}\log\left(1+x^2\right)-\frac{1}{2}\log\left(1+x+x^2\right)$$

```
[Out] 1/8*(1+x)/(x^2+1)^2-3/8*(1-x)/(x^2+1)+3/16*x/(x^2+1)+7/16*arctan(x)+1/8*ln(
1-x)-ln(x)+15/16*ln(x^2+1)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*
3^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6860, 653, 205, 209, 649, 266, 648, 632, 210, 642}

$$\int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)}\,dx = \frac{7\arctan(x)}{16}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{3(1-x)}{8(x^2+1)}+\frac{3x}{16(x^2+1)}+\frac{x+1}{8(x^2+1)^2}+\frac{15}{16}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{1}{8}\log(1-x)-\log(x)$$

```
[In] Int[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

```
[Out] (1 + x)/(8*(1 + x^2)^2) - (3*(1 - x))/(8*(1 + x^2)) + (3*x)/(16*(1 + x^2))
+ (7*ArcTan[x])/16 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/8 - Log
[x] + (15*Log[1 + x^2])/16 - Log[1 + x + x^2]/2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{8(-1+x)}-\frac{1}{x}+\frac{1-x}{2\left(1+x^2\right)^3}+\frac{3(1+x)}{4\left(1+x^2\right)^2}+\frac{-1+15x}{8\left(1+x^2\right)}+\frac{-1-x}{1+x+x^2}\right)dx\\
&= \frac{1}{8}\log(1-x)-\log(x)+\frac{1}{8}\int\frac{-1+15x}{1+x^2}\,dx\\
&\quad+\frac{1}{2}\int\frac{1-x}{\left(1+x^2\right)^3}\,dx+\frac{3}{4}\int\frac{1+x}{\left(1+x^2\right)^2}\,dx+\int\frac{-1-x}{1+x+x^2}\,dx\\
&= \frac{1+x}{8\left(1+x^2\right)^2}-\frac{3(1-x)}{8\left(1+x^2\right)}+\frac{1}{8}\log(1-x)-\log(x)-\frac{1}{8}\int\frac{1}{1+x^2}\,dx+\frac{3}{8}\int\frac{1}{\left(1+x^2\right)^2}\,dx\\
&\quad+\frac{3}{8}\int\frac{1}{1+x^2}\,dx-\frac{1}{2}\int\frac{1}{1+x+x^2}\,dx-\frac{1}{2}\int\frac{1+2x}{1+x+x^2}\,dx+\frac{15}{8}\int\frac{x}{1+x^2}\,dx\\
&= \frac{1+x}{8\left(1+x^2\right)^2}-\frac{3(1-x)}{8\left(1+x^2\right)}+\frac{3x}{16\left(1+x^2\right)}+\frac{1}{4}\tan^{-1}(x)+\frac{1}{8}\log(1-x)-\log(x)+\frac{15}{16}\log\left(1\right.\\
&\quad\left.+x^2\right)-\frac{1}{2}\log\left(1+x+x^2\right)+\frac{3}{16}\int\frac{1}{1+x^2}\,dx+\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= \frac{1+x}{8\left(1+x^2\right)^2}-\frac{3(1-x)}{8\left(1+x^2\right)}+\frac{3x}{16\left(1+x^2\right)}+\frac{7}{16}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}\\
&\quad+\frac{1}{8}\log(1-x)-\log(x)+\frac{15}{16}\log\left(1+x^2\right)-\frac{1}{2}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.90

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = \frac{1}{48}\left(\frac{6(1+x)}{\left(1+x^2\right)^2}+\frac{9(-2+3x)}{1+x^2}+21\arctan(x)-16\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+20\log(1-x)-48\log(x)+45\log\left(1+x^2\right)-10\log\left(1+x+x^2\right)-14\log\left(1-x^3\right)\right)$$

```
[In] Integrate[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
[Out] ((6*(1 + x))/(1 + x^2)^2 + (9*(-2 + 3*x))/(1 + x^2) + 21*ArcTan[x] - 16*Sqr
t[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 20*Log[1 - x] - 48*Log[x] + 45*Log[1 + x^2
] - 10*Log[1 + x + x^2] - 14*Log[1 - x^3])/48
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.70

method	result
risch	$\frac{\frac{9}{16}x^3-\frac{3}{8}x^2+\frac{11}{16}x-\frac{1}{4}}{\left(x^2+1\right)^2}+\frac{\ln(x-1)}{8}-\ln(x)+\frac{15\ln(49x^2+49)}{16}+\frac{7\arctan(x)}{16}-\frac{\ln(x^2+x+1)}{2}-\frac{\sqrt{3}\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$
default	$-\ln(x)+\frac{\frac{9}{2}x^3-3x^2+\frac{11}{2}x-2}{8\left(x^2+1\right)^2}+\frac{15\ln(x^2+1)}{16}+\frac{7\arctan(x)}{16}-\frac{\ln(x^2+x+1)}{2}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(x-1)}{8}$

```
[In] int((x^3+x^2+1)/(x-1)/x/(x^2+1)^3/(x^2+x+1),x,method=_RETURNVERBOSE)
[Out] (9/16*x^3-3/8*x^2+11/16*x-1/4)/(x^2+1)^2+1/8*ln(x-1)-ln(x)+15/16*ln(49*x^2+
49)+7/16*arctan(x)-1/2*ln(x^2+x+1)-1/3*3^(1/2)*arctan(2/3*(x+1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.32

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx$$

$$= \frac{27\,x^3-16\,\sqrt{3}(x^4+2\,x^2+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-18\,x^2+21\,(x^4+2\,x^2+1)\arctan(x)-24\,(x^4+2}{}$$

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/48*(27*x^3 - 16*sqrt(3)*(x^4 + 2*x^2 + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) -
 18*x^2 + 21*(x^4 + 2*x^2 + 1)*arctan(x) - 24*(x^4 + 2*x^2 + 1)*log(x^2 + x
 + 1) + 45*(x^4 + 2*x^2 + 1)*log(x^2 + 1) + 6*(x^4 + 2*x^2 + 1)*log(x - 1)
- 48*(x^4 + 2*x^2 + 1)*log(x) + 33*x - 12)/(x^4 + 2*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = -\log\left(x\right) + \frac{\log\left(x-1\right)}{8} + \frac{15\log\left(x^2+1\right)}{16} - \frac{\log\left(x^2+x+1\right)}{2} + \frac{7\,\mathrm{atan}\left(x\right)}{16} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3} + \frac{9x^3-6x^2+11x-4}{16x^4+32x^2+16}$$

```
[In] integrate((x**3+x**2+1)/(-1+x)/x/(x**2+1)**3/(x**2+x+1),x)
```

```
[Out] -log(x) + log(x - 1)/8 + 15*log(x**2 + 1)/16 - log(x**2 + x + 1)/2 + 7*atan
(x)/16 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3 + (9*x**3 - 6*x**2 + 11*
x - 4)/(16*x**4 + 32*x**2 + 16)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{9\,x^3-6\,x^2+11\,x-4}{16\,(x^4+2\,x^2+1)} + \frac{7}{16}\,\arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+x+1\right) + \frac{15}{16}\,\log\left(x^2+1\right) + \frac{1}{8}\,\log\left(x-1\right) - \log\left(x\right)$$

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4
)/(x^4 + 2*x^2 + 1) + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2
 + 1) + 1/8*log(x - 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.72

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{9x^3-6x^2+11x-4}{16\left(x^2+1\right)^2} + \frac{7}{16}\arctan(x) - \frac{1}{2}\log\left(x^2+x+1\right) + \frac{15}{16}\log\left(x^2+1\right) + \frac{1}{8}\log\left(|x-1|\right) - \log\left(|x|\right)$$

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4
)/(x^2 + 1)^2 + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2 + 1)
+ 1/8*log(abs(x - 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = \frac{\ln(x-1)}{8} - \ln(x) + \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \frac{\frac{9x^3}{16}-\frac{3x^2}{8}+\frac{11x}{16}-\frac{1}{4}}{x^4+2x^2+1} + \ln(x-\mathrm{i})\left(\frac{15}{16}-\frac{7}{32}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(\frac{15}{16}+\frac{7}{32}\mathrm{i}\right)$$

```
[In] int((x^2 + x^3 + 1)/(x*(x^2 + 1)^3*(x - 1)*(x + x^2 + 1)),x)
```

```
[Out] log(x - 1)/8 + log(x - 1i)*(15/16 - 7i/32) + log(x + 1i)*(15/16 + 7i/32) -
log(x) + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/2) - log(x + (3^
(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/2) + ((11*x)/16 - (3*x^2)/8 + (9*x^3
)/16 - 1/4)/(2*x^2 + x^4 + 1)
```

3.309 $\int \frac{1-3x+2x^2-x^3}{\left(1+x^2\right)^2}\,dx$

Optimal result 1944
Rubi [A] (verified) 1944
Mathematica [A] (verified) 1945
Maple [A] (verified) 1945
Fricas [A] (verification not implemented) 1946
Sympy [A] (verification not implemented) 1946
Maxima [A] (verification not implemented) 1947
Giac [A] (verification not implemented) 1947
Mupad [B] (verification not implemented) 1947

Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \frac{1-3x+2x^2-x^3}{\left(1+x^2\right)^2}\,dx=\frac{2-x}{2\left(1+x^2\right)}+\frac{3\arctan(x)}{2}-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] 1/2*(2-x)/(x^2+1)+3/2*arctan(x)-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1828, 649, 209, 266}

$$\int \frac{1-3x+2x^2-x^3}{\left(1+x^2\right)^2}\,dx=\frac{3\arctan(x)}{2}+\frac{2-x}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)$$

```
[In] Int[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2,x]
```

```
[Out] (2 - x)/(2*(1 + x^2)) + (3*ArcTan[x])/2 - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2-x}{2(1+x^2)} - \frac{1}{2}\int \frac{-3+2x}{1+x^2}\,dx\\ &= \frac{2-x}{2(1+x^2)} + \frac{3}{2}\int \frac{1}{1+x^2}\,dx - \int \frac{x}{1+x^2}\,dx\\ &= \frac{2-x}{2(1+x^2)} + \frac{3}{2}\tan^{-1}(x) - \frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2}\,dx = \frac{1}{2}\left(\frac{2-x}{1+x^2} + 3\arctan(x) - \log\left(1+x^2\right)\right)$$

```
[In] Integrate[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2,x]
[Out] ((2 - x)/(1 + x^2) + 3*ArcTan[x] - Log[1 + x^2])/2
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{1-\frac{x}{2}}{x^2+1}-\frac{\ln(x^2+1)}{2}+\frac{3\arctan(x)}{2}$	27
default	$-\frac{\frac{x}{2}-1}{x^2+1}-\frac{\ln(x^2+1)}{2}+\frac{3\arctan(x)}{2}$	28
meijerg	$-\frac{x^2}{x^2+1}-\frac{\ln(x^2+1)}{2}-\frac{x}{x^2+1}+\frac{3\arctan(x)}{2}+\frac{x}{2x^2+2}$	47
parallelrisch	$-\frac{3i\ln(x-i)x^2-3i\ln(x+i)x^2+2\ln(x-i)x^2+2\ln(x+i)x^2-4+3i\ln(x-i)-3i\ln(x+i)+2\ln(x-i)+2\ln(x+i)+2x}{4(x^2+1)}$	87

```
[In] int((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] (1-1/2*x)/(x^2+1)-1/2*ln(x^2+1)+3/2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2}\,dx = \frac{3\,(x^2+1)\arctan(x)-(x^2+1)\log(x^2+1)-x+2}{2\,(x^2+1)}$$

```
[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*(3*(x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) - x + 2)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2}\,dx = -\frac{x-2}{2x^2+2}-\frac{\log(x^2+1)}{2}+\frac{3\,\mathrm{atan}(x)}{2}$$

```
[In] integrate((-x**3+2*x**2-3*x+1)/(x**2+1)**2,x)
[Out] -(x - 2)/(2*x**2 + 2) - log(x**2 + 1)/2 + 3*atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1-3x+2x^2-x^3}{\left(1+x^2\right)^2}\,dx = -\frac{x-2}{2\left(x^2+1\right)} + \frac{3}{2}\arctan\left(x\right) - \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2*(x - 2)/(x^2 + 1) + 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1-3x+2x^2-x^3}{\left(1+x^2\right)^2}\,dx = -\frac{x-2}{2\left(x^2+1\right)} + \frac{3}{2}\arctan\left(x\right) - \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*(x - 2)/(x^2 + 1) + 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{1-3x+2x^2-x^3}{\left(1+x^2\right)^2}\,dx = \frac{3\,\mathrm{atan}(x)}{2} - \frac{\ln\left(x^2+1\right)}{2} - \frac{x}{2\,\left(x^2+1\right)} + \frac{1}{x^2+1}$$

```
[In] int(-(3*x - 2*x^2 + x^3 - 1)/(x^2 + 1)^2,x)
[Out] (3*atan(x))/2 - log(x^2 + 1)/2 - x/(2*(x^2 + 1)) + 1/(x^2 + 1)
```

3.310 $\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2} dx$

Optimal result 1948
Rubi [A] (verified) 1948
Mathematica [A] (verified) 1950
Maple [A] (verified) 1950
Fricas [A] (verification not implemented) 1950
Sympy [A] (verification not implemented) 1951
Maxima [A] (verification not implemented) 1951
Giac [A] (verification not implemented) 1951
Mupad [B] (verification not implemented) 1951

Optimal result

Integrand size = 26, antiderivative size = 33

$$\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2} dx = -\frac{1+2x}{2(1+x^2)} - 2\arctan(x) + \log(x) - \frac{1}{2}\log(1+x^2)$$

```
[Out] 1/2*(-1-2*x)/(x^2+1)-2*arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1819, 815, 649, 209, 266}

$$\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2} dx = -2\arctan(x) - \frac{2x+1}{2(x^2+1)} - \frac{1}{2}\log(x^2+1) + \log(x)$$

```
[In] Int[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
```

```
[Out] -1/2*(1 + 2*x)/(1 + x^2) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1+2x}{2\left(1+x^2\right)}-\frac{1}{2}\int\frac{-2+4x}{x\left(1+x^2\right)}\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}-\frac{1}{2}\int\left(-\frac{2}{x}+\frac{2(2+x)}{1+x^2}\right)\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}+\log(x)-\int\frac{2+x}{1+x^2}\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}+\log(x)-2\int\frac{1}{1+x^2}\,dx-\int\frac{x}{1+x^2}\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}-2\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx=\frac{-1-2x}{2\left(1+x^2\right)}-2\arctan(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
[Out] (-1 - 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	si
default	$\ln(x)-\frac{x+\frac{1}{2}}{x^2+1}-\frac{\ln(x^2+1)}{2}-2\arctan(x)$	28
risch	$\frac{-x-\frac{1}{2}}{x^2+1}+\ln(x)-\frac{\ln(4x^2+4)}{2}-2\arctan(x)$	31
meijerg	$-\frac{2x}{2x^2+2}-2\arctan(x)+\frac{x^2}{x^2+1}-\frac{x^2}{2x^2+2}-\frac{\ln(x^2+1)}{2}+\frac{1}{2}+\ln(x)$	64
parallelrisch	$\frac{2i\ln(x-i)x^2-2i\ln(x+i)x^2+2\ln(x)x^2-\ln(x-i)x^2-\ln(x+i)x^2-1+2i\ln(x-i)-2i\ln(x+i)+2\ln(x)-\ln(x-i)-\ln(x+i)-2x}{2x^2+2}$	98

```
[In] int((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] ln(x)-(x+1/2)/(x^2+1)-1/2*ln(x^2+1)-2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx$$
$$=-\frac{4\left(x^2+1\right)\arctan(x)+\left(x^2+1\right)\log\left(x^2+1\right)-2\left(x^2+1\right)\log(x)+2x+1}{2\left(x^2+1\right)}$$

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="fricas")
[Out] -1/2*(4*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1)*log(x) +
 2*x + 1)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx = -\frac{2x+1}{2x^2+2}+\log\left(x\right)-\frac{\log\left(x^2+1\right)}{2}-2\,\mathrm{atan}\left(x\right)$$

```
[In] integrate((-x**3+2*x**2-3*x+1)/x/(x**2+1)**2,x)
[Out] -(2*x + 1)/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 - 2*atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx = -\frac{2\,x+1}{2\left(x^2+1\right)}-2\,\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^2+1\right)+\log\left(x\right)$$

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx = -\frac{2\,x+1}{2\left(x^2+1\right)}-2\,\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^2+1\right)+\log\left(|x|\right)$$

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx = \ln\left(x\right)-\frac{x+\frac{1}{2}}{x^2+1}+\ln\left(x-\mathrm{i}\right)\left(-\frac{1}{2}+1\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{2}-\mathrm{i}\right)$$

```
[In] int(-(3*x - 2*x^2 + x^3 - 1)/(x*(x^2 + 1)^2),x)
[Out] log(x) - log(x + 1i)*(1/2 + 1i) - log(x - 1i)*(1/2 - 1i) - (x + 1/2)/(x^2 +
 1)
```

3.311 $\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} \, dx$

Optimal result 1952
Rubi [A] (verified) 1952
Mathematica [A] (verified) 1953
Maple [A] (verified) 1953
Fricas [A] (verification not implemented) 1954
Sympy [A] (verification not implemented) 1954
Maxima [A] (verification not implemented) 1954
Giac [A] (verification not implemented) 1954
Mupad [B] (verification not implemented) 1955

Optimal result

Integrand size = 26, antiderivative size = 25

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} \, dx = x+\frac{x^2}{2}-\log(x)+\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] x+1/2*x^2-ln(x)+1/2*ln(-x^2+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1607, 1816, 266}

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} \, dx = \frac{x^2}{2}+\frac{1}{2}\log\left(1-x^2\right)+x-\log(x)$$

```
[In] Int[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
```

```
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1-x-x^2+x^3+x^4}{x\left(-1+x^2\right)}\,dx\\ &= \int \left(1-\frac{1}{x}+x+\frac{x}{-1+x^2}\right)dx\\ &= x+\frac{x^2}{2}-\log(x)+\int \frac{x}{-1+x^2}\,dx\\ &= x+\frac{x^2}{2}-\log(x)+\frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx = x+\frac{x^2}{2}-\log(x)+\frac{1}{2}\log\left(1-x^2\right)$$

```
[In] Integrate[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{x^2}{2}+x-\ln(x)+\frac{\ln(x^2-1)}{2}$	20
default	$\frac{x^2}{2}+x-\ln(x)+\frac{\ln(x+1)}{2}+\frac{\ln(x-1)}{2}$	24
norman	$\frac{x^2}{2}+x-\ln(x)+\frac{\ln(x+1)}{2}+\frac{\ln(x-1)}{2}$	24
parallelrisch	$\frac{x^2}{2}+x-\ln(x)+\frac{\ln(x+1)}{2}+\frac{\ln(x-1)}{2}$	24
meijerg	$\frac{\ln(-x^2+1)}{2}-\ln(x)-\frac{i\pi}{2}+\frac{x^2}{2}-\frac{i(2ix-2i\,\mathrm{arctanh}(x))}{2}+\mathrm{arctanh}(x)$	40

```
[In] int((x^4+x^3-x^2-x+1)/(x^3-x),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+x-ln(x)+1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx = \frac{1}{2}\,x^2 + x + \frac{1}{2}\,\log\left(x^2-1\right) - \log\left(x\right)$$

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="fricas")
[Out] 1/2*x^2 + x + 1/2*log(x^2 - 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx = \frac{x^2}{2} + x - \log\left(x\right) + \frac{\log\left(x^2-1\right)}{2}$$

```
[In] integrate((x**4+x**3-x**2-x+1)/(x**3-x),x)
[Out] x**2/2 + x - log(x) + log(x**2 - 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx = \frac{1}{2}\,x^2 + x + \frac{1}{2}\,\log\left(x+1\right) + \frac{1}{2}\,\log\left(x-1\right) - \log\left(x\right)$$

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="maxima")
[Out] 1/2*x^2 + x + 1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx = \frac{1}{2}\,x^2 + x + \frac{1}{2}\,\log\left(|x+1|\right) + \frac{1}{2}\,\log\left(|x-1|\right) - \log\left(|x|\right)$$

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="giac")
[Out] 1/2*x^2 + x + 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx = x+\frac{\ln\left(x^2-1\right)}{2}-\ln\left(x\right)+\frac{x^2}{2}$$

```
[In] int(-(x^3 - x^2 - x + x^4 + 1)/(x - x^3),x)
[Out] x + log(x^2 - 1)/2 - log(x) + x^2/2
```

3.312 $\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx$

Optimal result 1956
Rubi [A] (verified) 1956
Mathematica [A] (verified) 1957
Maple [A] (verified) 1957
Fricas [A] (verification not implemented) 1958
Sympy [A] (verification not implemented) 1958
Maxima [A] (verification not implemented) 1958
Giac [A] (verification not implemented) 1959
Mupad [B] (verification not implemented) 1959

Optimal result

Integrand size = 25, antiderivative size = 36

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx = 6\arctan(x)-5\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{2}\log\left(1+x^2\right)+\log\left(2+x^2\right)$$

```
[Out] 6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6857, 649, 209, 266}

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx = 6\arctan(x)-5\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)$$

```
[In] Int[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{6-x}{1+x^2}+\frac{2(-5+x)}{2+x^2}\right) dx \\ &= 2\int \frac{-5+x}{2+x^2}\,dx+\int \frac{6-x}{1+x^2}\,dx \\ &= 2\int \frac{x}{2+x^2}\,dx+6\int \frac{1}{1+x^2}\,dx-10\int \frac{1}{2+x^2}\,dx-\int \frac{x}{1+x^2}\,dx \\ &= 6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{2}\log\left(1+x^2\right)+\log\left(2+x^2\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx = 6\arctan(x)-5\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{2}\log\left(1+x^2\right)+\log\left(2+x^2\right)$$

```
[In] Integrate[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$6\arctan\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}+\ln\left(x^{2}+2\right)-5\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	32
risch	$6\arctan\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}+\ln\left(x^{2}+2\right)-5\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	32

```
[In] int((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x,method=_RETURNVERBOSE)
[Out] 6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx = -5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan(x)+\log(x^2+2)-\frac{1}{2}\log(x^2+1)$$

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="fricas")
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx = -\frac{\log(x^2+1)}{2}+\log(x^2+2)+6\operatorname{atan}(x)-5\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

```
[In] integrate((x**3-4*x**2+2)/(x**2+1)/(x**2+2),x)
[Out] -log(x**2 + 1)/2 + log(x**2 + 2) + 6*atan(x) - 5*sqrt(2)*atan(sqrt(2)*x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx = -5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan(x)+\log(x^2+2)-\frac{1}{2}\log(x^2+1)$$

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="maxima")
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx = -5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 6\arctan(x) + \log\left(x^2+2\right) - \frac{1}{2}\log\left(x^2+1\right)$$

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.56

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx = \ln(x-\mathrm{i})\left(-\frac{1}{2}-3\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{2}+3\mathrm{i}\right) + \ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(1+\frac{\sqrt{2}\,5\mathrm{i}}{2}\right) - \ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(-1+\frac{\sqrt{2}\,5\mathrm{i}}{2}\right)$$

```
[In] int((x^3 - 4*x^2 + 2)/((x^2 + 1)*(x^2 + 2)),x)
```

```
[Out] log(x - 2^(1/2)*1i)*((2^(1/2)*5i)/2 + 1) - log(x + 1i)*(1/2 - 3i) - log(x -
 1i)*(1/2 + 3i) - log(x + 2^(1/2)*1i)*((2^(1/2)*5i)/2 - 1)
```

3.313 $\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2} dx$

Optimal result 1960
Rubi [A] (verified) 1960
Mathematica [A] (verified) 1961
Maple [A] (verified) 1961
Fricas [A] (verification not implemented) 1962
Sympy [A] (verification not implemented) 1962
Maxima [A] (verification not implemented) 1962
Giac [A] (verification not implemented) 1963
Mupad [B] (verification not implemented) 1963

Optimal result

Integrand size = 23, antiderivative size = 29

$$\int \frac{1+x^2+x^4}{(1+x^2)\,(4+x^2)^2}\,dx = -\frac{13x}{24\,(4+x^2)}+\frac{25}{144}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{9}$$

```
[Out] -13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6857, 209, 205}

$$\int \frac{1+x^2+x^4}{(1+x^2)\,(4+x^2)^2}\,dx = \frac{25}{144}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{9}-\frac{13x}{24\,(x^2+4)}$$

```
[In] Int[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
```

```
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{9(1+x^2)} - \frac{13}{3(4+x^2)^2} + \frac{8}{9(4+x^2)}\right) dx \\
&= \frac{1}{9}\int \frac{1}{1+x^2}\,dx + \frac{8}{9}\int \frac{1}{4+x^2}\,dx - \frac{13}{3}\int \frac{1}{(4+x^2)^2}\,dx \\
&= -\frac{13x}{24(4+x^2)} + \frac{4}{9}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9}\tan^{-1}(x) - \frac{13}{24}\int \frac{1}{4+x^2}\,dx \\
&= -\frac{13x}{24(4+x^2)} + \frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9}\tan^{-1}(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx = -\frac{13x}{24(4+x^2)} + \frac{25}{144}\arctan\left(\frac{x}{2}\right) + \frac{\arctan(x)}{9}$$

```
[In] Integrate[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result
default	$-\frac{13x}{24(x^2+4)} + \frac{25\arctan\left(\frac{x}{2}\right)}{144} + \frac{\arctan(x)}{9}$
risch	$-\frac{13x}{24(x^2+4)} + \frac{25\arctan\left(\frac{x}{2}\right)}{144} + \frac{\arctan(x)}{9}$
parallelrisch	$-\frac{25i\ln(x-2i)x^2+16i\ln(x-i)x^2-16i\ln(x+i)x^2-25i\ln(x+2i)x^2+100i\ln(x-2i)+64i\ln(x-i)-64i\ln(x+i)-100i\ln(x+2i)+1}{288(x^2+4)}$

```
[In] int((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x,method=_RETURNVERBOSE)
[Out] -13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx = \frac{25\,(x^2+4)\arctan\left(\frac{1}{2}\,x\right)+16\,(x^2+4)\arctan\left(x\right)-78\,x}{144\,(x^2+4)}$$

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="fricas")
[Out] 1/144*(25*(x^2 + 4)*arctan(1/2*x) + 16*(x^2 + 4)*arctan(x) - 78*x)/(x^2 + 4
)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx = -\frac{13x}{24x^2+96}+\frac{25\,\mathrm{atan}\left(\frac{x}{2}\right)}{144}+\frac{\mathrm{atan}\left(x\right)}{9}$$

```
[In] integrate((x**4+x**2+1)/(x**2+1)/(x**2+4)**2,x)
[Out] -13*x/(24*x**2 + 96) + 25*atan(x/2)/144 + atan(x)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx = -\frac{13\,x}{24\,(x^2+4)}+\frac{25}{144}\,\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{9}\,\arctan\left(x\right)$$

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="maxima")
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{1+x^2+x^4}{(1+x^2)\,(4+x^2)^2}\,dx = -\frac{13\,x}{24\,(x^2+4)} + \frac{25}{144}\,\arctan\left(\frac{1}{2}\,x\right) + \frac{1}{9}\,\arctan\left(x\right)$$

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="giac")
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1+x^2+x^4}{(1+x^2)\,(4+x^2)^2}\,dx = \frac{25\,\mathrm{atan}\left(\frac{x}{2}\right)}{144} + \frac{\mathrm{atan}(x)}{9} - \frac{13\,x}{24\,\left(x^2+4\right)}$$

```
[In] int((x^2 + x^4 + 1)/((x^2 + 1)*(x^2 + 4)^2),x)
[Out] (25*atan(x/2))/144 + atan(x)/9 - (13*x)/(24*(x^2 + 4))
```

3.314 $\int \frac{1+x^2+x^3}{2x^2+x^3+x^4} \, dx$

Optimal result 1964
Rubi [A] (verified) 1964
Mathematica [A] (verified) 1966
Maple [A] (verified) 1966
Fricas [A] (verification not implemented) 1966
Sympy [A] (verification not implemented) 1967
Maxima [A] (verification not implemented) 1967
Giac [A] (verification not implemented) 1967
Mupad [B] (verification not implemented) 1968

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4} \, dx = -\frac{1}{2x}+\frac{\arctan\left(\frac{1+2x}{\sqrt{7}}\right)}{4\sqrt{7}}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)$$

```
[Out] -1/2/x-1/4*ln(x)+5/8*ln(x^2+x+2)+1/28*arctan(1/7*(1+2*x)*7^(1/2))*7^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1608, 1642, 648, 632, 210, 642}

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4} \, dx = \frac{\arctan\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}+\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}$$

```
[In] Int[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

```
[Out] -1/2*1/x + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
+ x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1+x^2+x^3}{x^2\left(2+x+x^2\right)}\,dx \\
&= \int \left(\frac{1}{2x^2}-\frac{1}{4x}+\frac{3+5x}{4\left(2+x+x^2\right)}\right)dx \\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{1}{4}\int \frac{3+5x}{2+x+x^2}\,dx \\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{1}{8}\int \frac{1}{2+x+x^2}\,dx+\frac{5}{8}\int \frac{1+2x}{2+x+x^2}\,dx \\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx,x,1+2x\right) \\
&= -\frac{1}{2x}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{7}}\right)}{4\sqrt{7}}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx = -\frac{1}{2x}+\frac{\arctan\left(\frac{1+2x}{\sqrt{7}}\right)}{4\sqrt{7}}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)$$

```
[In] Integrate[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
[Out] -1/2*1/x + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
+ x^2])/8
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{1}{2x}-\frac{\ln(x)}{4}+\frac{5\ln(x^2+x+2)}{8}+\frac{\arctan\left(\frac{(1+2x)\sqrt{7}}{7}\right)\sqrt{7}}{28}$	36
risch	$-\frac{1}{2x}+\frac{5\ln(4x^2+4x+8)}{8}+\frac{\arctan\left(\frac{(1+2x)\sqrt{7}}{7}\right)\sqrt{7}}{28}-\frac{\ln(x)}{4}$	40

```
[In] int((x^3+x^2+1)/(x^4+x^3+2*x^2),x,method=_RETURNVERBOSE)
[Out] -1/2/x-1/4*ln(x)+5/8*ln(x^2+x+2)+1/28*arctan(1/7*(1+2*x)*7^(1/2))*7^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx = \frac{2\sqrt{7}x\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)+35x\log\left(x^2+x+2\right)-14x\log(x)-28}{56x}$$

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="fricas")
[Out] 1/56*(2*sqrt(7)*x*arctan(1/7*sqrt(7)*(2*x + 1)) + 35*x*log(x^2 + x + 2) - 1
4*x*log(x) - 28)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx = -\frac{\log(x)}{4} + \frac{5\log(x^2+x+2)}{8} + \frac{\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x}{7}+\frac{\sqrt{7}}{7}\right)}{28} - \frac{1}{2x}$$

```
[In] integrate((x**3+x**2+1)/(x**4+x**3+2*x**2),x)
```

```
[Out] -log(x)/4 + 5*log(x**2 + x + 2)/8 + sqrt(7)*atan(2*sqrt(7)*x/7 + sqrt(7)/7)
/28 - 1/(2*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx = \frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right) - \frac{1}{2x} + \frac{5}{8}\log\left(x^2+x+2\right) - \frac{1}{4}\log(x)$$

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="maxima")
```

```
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2) -
 1/4*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx = \frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right) - \frac{1}{2x} + \frac{5}{8}\log\left(x^2+x+2\right) - \frac{1}{4}\log(|x|)$$

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="giac")
```

```
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2) -
 1/4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx = -\frac{\ln(x)}{4} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)\left(-\frac{5}{8}+\frac{\sqrt{7}\,1\mathrm{i}}{56}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)\left(\frac{5}{8}+\frac{\sqrt{7}\,1\mathrm{i}}{56}\right) - \frac{1}{2\,x}$$

```
[In] int((x^2 + x^3 + 1)/(2*x^2 + x^3 + x^4),x)
```

```
[Out] log(x + (7^(1/2)*1i)/2 + 1/2)*((7^(1/2)*1i)/56 + 5/8) - log(x - (7^(1/2)*1i
)/2 + 1/2)*((7^(1/2)*1i)/56 - 5/8) - log(x)/4 - 1/(2*x)
```

3.315 $\int \frac{1-12x+x^2+x^3}{-12+x+x^2} \, dx$

Optimal result 1969
Rubi [A] (verified) 1969
Mathematica [A] (verified) 1970
Maple [A] (verified) 1970
Fricas [A] (verification not implemented) 1971
Sympy [A] (verification not implemented) 1971
Maxima [A] (verification not implemented) 1971
Giac [A] (verification not implemented) 1971
Mupad [B] (verification not implemented) 1972

Optimal result

Integrand size = 20, antiderivative size = 22

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2} \, dx = \frac{x^2}{2} - \frac{2}{7}\text{arctanh}\left(\frac{1}{7}(1+2x)\right)$$

```
[Out] 1/2*x^2-2/7*arctanh(1/7+2/7*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1671, 630, 31}

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2} \, dx = \frac{x^2}{2} + \frac{1}{7}\log(3-x) - \frac{1}{7}\log(x+4)$$

```
[In] Int[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
```

```
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(x+\frac{1}{-12+x+x^2}\right)dx\\ &= \frac{x^2}{2}+\int \frac{1}{-12+x+x^2}\,dx\\ &= \frac{x^2}{2}+\frac{1}{7}\int \frac{1}{-3+x}\,dx-\frac{1}{7}\int \frac{1}{4+x}\,dx\\ &= \frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(4+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx=\frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(4+x)$$

```
[In] Integrate[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x^2}{2}+\frac{\ln(-3+x)}{7}-\frac{\ln(x+4)}{7}$	19
norman	$\frac{x^2}{2}+\frac{\ln(-3+x)}{7}-\frac{\ln(x+4)}{7}$	19
risch	$\frac{x^2}{2}+\frac{\ln(-3+x)}{7}-\frac{\ln(x+4)}{7}$	19
parallelrisch	$\frac{x^2}{2}+\frac{\ln(-3+x)}{7}-\frac{\ln(x+4)}{7}$	19

```
[In] int((x^3+x^2-12*x+1)/(x^2+x-12),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+1/7*ln(-3+x)-1/7*ln(x+4)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{7}\,\log\left(x+4\right) + \frac{1}{7}\,\log\left(x-3\right)$$

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="fricas")
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx = \frac{x^2}{2} + \frac{\log\left(x-3\right)}{7} - \frac{\log\left(x+4\right)}{7}$$

```
[In] integrate((x**3+x**2-12*x+1)/(x**2+x-12),x)
[Out] x**2/2 + log(x - 3)/7 - log(x + 4)/7
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{7}\,\log\left(x+4\right) + \frac{1}{7}\,\log\left(x-3\right)$$

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{7}\,\log\left(|x+4|\right) + \frac{1}{7}\,\log\left(|x-3|\right)$$

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="giac")
[Out] 1/2*x^2 - 1/7*log(abs(x + 4)) + 1/7*log(abs(x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx = \frac{x^2}{2}-\frac{2\,\mathrm{atanh}\left(\frac{2\,x}{7}+\frac{1}{7}\right)}{7}$$

```
[In] int((x^2 - 12*x + x^3 + 1)/(x + x^2 - 12),x)
[Out] x^2/2 - (2*atanh((2*x)/7 + 1/7))/7
```

3.316 $\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3} dx$

Optimal result 1973
Rubi [A] (verified) 1973
Mathematica [A] (verified) 1974
Maple [A] (verified) 1974
Fricas [A] (verification not implemented) 1975
Sympy [A] (verification not implemented) 1975
Maxima [A] (verification not implemented) 1975
Giac [A] (verification not implemented) 1975
Mupad [B] (verification not implemented) 1976

Optimal result

Integrand size = 25, antiderivative size = 17

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3} dx = 2\log(1-x)+\log(x)+3\log(3+x)$$

```
[Out] 2*ln(1-x)+ln(x)+3*ln(3+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1608, 1642}

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3} dx = 2\log(1-x)+\log(x)+3\log(x+3)$$

```
[In] Int[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{-3+5x+6x^2}{x\left(-3+2x+x^2\right)}\,dx\\ &= \int \left(\frac{2}{-1+x}+\frac{1}{x}+\frac{3}{3+x}\right)dx\\ &= 2\log(1-x)+\log(x)+3\log(3+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = 2\log(1-x)+\log(x)+3\log(3+x)$$

```
[In] Integrate[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\ln(x)+3\ln(3+x)+2\ln(x-1)$	16
norman	$\ln(x)+3\ln(3+x)+2\ln(x-1)$	16
risch	$\ln(x)+3\ln(3+x)+2\ln(x-1)$	16
parallelrisch	$\ln(x)+3\ln(3+x)+2\ln(x-1)$	16

```
[In] int((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x,method=_RETURNVERBOSE)
[Out] ln(x)+3*ln(3+x)+2*ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = 3\,\log\left(x+3\right)+2\,\log\left(x-1\right)+\log\left(x\right)$$

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="fricas")
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = \log\left(x\right)+2\log\left(x-1\right)+3\log\left(x+3\right)$$

```
[In] integrate((6*x**2+5*x-3)/(x**3+2*x**2-3*x),x)
[Out] log(x) + 2*log(x - 1) + 3*log(x + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = 3\,\log\left(x+3\right)+2\,\log\left(x-1\right)+\log\left(x\right)$$

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="maxima")
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = 3\,\log\left(|x+3|\right)+2\,\log\left(|x-1|\right)+\log\left(|x|\right)$$

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="giac")
[Out] 3*log(abs(x + 3)) + 2*log(abs(x - 1)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = 2\,\ln\left(x-1\right)+3\,\ln\left(x+3\right)+\ln\left(x\right)$$

```
[In] int((5*x + 6*x^2 - 3)/(2*x^2 - 3*x + x^3),x)
[Out] 2*log(x - 1) + 3*log(x + 3) + log(x)
```

3.317 $\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx$

Optimal result 1977
Rubi [A] (verified) 1977
Mathematica [A] (verified) 1978
Maple [A] (verified) 1978
Fricas [A] (verification not implemented) 1979
Sympy [A] (verification not implemented) 1979
Maxima [A] (verification not implemented) 1979
Giac [A] (verification not implemented) 1979
Mupad [B] (verification not implemented) 1980

Optimal result

Integrand size = 22, antiderivative size = 14

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = \frac{1}{x}+2\log(x)+3\log(2+x)$$

```
[Out] 1/x+2*ln(x)+3*ln(2+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1607, 907}

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = \frac{1}{x}+2\log(x)+3\log(x+2)$$

```
[In] Int[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
```

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{-2+3x+5x^2}{x^2(2+x)}\,dx\\ &= \int \left(-\frac{1}{x^2}+\frac{2}{x}+\frac{3}{2+x}\right)dx\\ &= \frac{1}{x}+2\log(x)+3\log(2+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = \frac{1}{x}+2\log(x)+3\log(2+x)$$

```
[In] Integrate[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{1}{x}+2\ln\left(x\right)+3\ln\left(x+2\right)$	15
norman	$\frac{1}{x}+2\ln\left(x\right)+3\ln\left(x+2\right)$	15
risch	$\frac{1}{x}+2\ln\left(x\right)+3\ln\left(x+2\right)$	15
parallelrisch	$\frac{2\ln(x)x+3\ln(x+2)x+1}{x}$	19
meijerg	$3\ln\left(1+\frac{x}{2}\right)+2\ln\left(x\right)-2\ln\left(2\right)+\frac{1}{x}$	21

```
[In] int((5*x^2+3*x-2)/(x^3+2*x^2),x,method=_RETURNVERBOSE)
[Out] 1/x+2*ln(x)+3*ln(x+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = \frac{3\,x\log\left(x+2\right)+2\,x\log\left(x\right)+1}{x}$$

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="fricas")
[Out] (3*x*log(x + 2) + 2*x*log(x) + 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = 2\log\left(x\right)+3\log\left(x+2\right)+\frac{1}{x}$$

```
[In] integrate((5*x**2+3*x-2)/(x**3+2*x**2),x)
[Out] 2*log(x) + 3*log(x + 2) + 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = \frac{1}{x}+3\,\log\left(x+2\right)+2\,\log\left(x\right)$$

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="maxima")
[Out] 1/x + 3*log(x + 2) + 2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = \frac{1}{x}+3\,\log\left(|x+2|\right)+2\,\log\left(|x|\right)$$

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="giac")
[Out] 1/x + 3*log(abs(x + 2)) + 2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = 3\,\ln(x+2)+2\,\ln(x)+\frac{1}{x}$$

```
[In] int((3*x + 5*x^2 - 2)/(2*x^2 + x^3),x)
[Out] 3*log(x + 2) + 2*log(x) + 1/x
```

3.318 $\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} dx$

Optimal result 1981
Rubi [A] (verified) 1981
Mathematica [A] (verified) 1982
Maple [A] (verified) 1982
Fricas [A] (verification not implemented) 1982
Sympy [A] (verification not implemented) 1983
Maxima [A] (verification not implemented) 1983
Giac [A] (verification not implemented) 1983
Mupad [B] (verification not implemented) 1983

Optimal result

Integrand size = 24, antiderivative size = 19

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} dx = \log(1-x)-2\log(2+x)-3\log(3+x)$$

```
[Out] ln(1-x)-2*ln(2+x)-3*ln(3+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2099}

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} dx = \log(1-x)-2\log(x+2)-3\log(x+3)$$

```
[In] Int[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
```

```
[Out] Log[1 - x] - 2*Log[2 + x] - 3*Log[3 + x]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{-1+x}-\frac{2}{2+x}-\frac{3}{3+x}\right) dx \\ &= \log(1-x)-2\log(2+x)-3\log(3+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx=-2\left(-\frac{1}{2}\log(1-x)+\log(2+x)+\frac{3}{2}\log(3+x)\right)$$

```
[In] Integrate[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
[Out] -2*(-1/2*Log[1 - x] + Log[2 + x] + (3*Log[3 + x])/2)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$-2\ln(x+2)-3\ln(3+x)+\ln(x-1)$	18
norman	$-2\ln(x+2)-3\ln(3+x)+\ln(x-1)$	18
risch	$-2\ln(x+2)-3\ln(3+x)+\ln(x-1)$	18
parallelrisch	$-2\ln(x+2)-3\ln(3+x)+\ln(x-1)$	18

```
[In] int((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x,method=_RETURNVERBOSE)
[Out] -2*ln(x+2)-3*ln(3+x)+ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx=-3\,\log(x+3)-2\,\log(x+2)+\log(x-1)$$

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="fricas")
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = \log(x-1) - 2\log(x+2) - 3\log(x+3)$$

```
[In] integrate((-4*x**2-2*x+18)/(x**3+4*x**2+x-6),x)
[Out] log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = -3\,\log(x+3) - 2\,\log(x+2) + \log(x-1)$$

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="maxima")
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = -3\,\log(|x+3|) - 2\,\log(|x+2|) + \log(|x-1|)$$

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="giac")
[Out] -3*log(abs(x + 3)) - 2*log(abs(x + 2)) + log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = \ln(x-1) - 2\,\ln(x+2) - 3\,\ln(x+3)$$

```
[In] int(-(2*x + 4*x^2 - 18)/(x + 4*x^2 + x^3 - 6),x)
[Out] log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

3.319 $\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} \, dx$

Optimal result 1984
Rubi [A] (verified) 1984
Mathematica [A] (verified) 1986
Maple [A] (verified) 1986
Fricas [A] (verification not implemented) 1986
Sympy [A] (verification not implemented) 1987
Maxima [A] (verification not implemented) 1987
Giac [A] (verification not implemented) 1987
Mupad [B] (verification not implemented) 1987

Optimal result

Integrand size = 24, antiderivative size = 23

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} \, dx = -\frac{3}{2}\arctan\left(\frac{x}{2}\right)+\arctan(x)+\frac{1}{2}\log\left(4+x^2\right)$$

```
[Out] -3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1687, 1180, 209, 1261, 640, 31}

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} \, dx = -\frac{3}{2}\arctan\left(\frac{x}{2}\right)+\arctan(x)+\frac{1}{2}\log\left(x^2+4\right)$$

```
[In] Int[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
```

```
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 640

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1-2x^2}{4+5x^2+x^4}\,dx+\int \frac{x(1+x^2)}{4+5x^2+x^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{4+5x+x^2}\,dx,x,x^2\right)-3\int \frac{1}{4+x^2}\,dx+\int \frac{1}{1+x^2}\,dx\\
&= -\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{4+x}\,dx,x,x^2\right)\\
&= -\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)+\frac{1}{2}\log\left(4+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx = -\frac{3}{2}\arctan\left(\frac{x}{2}\right)+\arctan(x)+\frac{1}{2}\log\left(4+x^2\right)$$

```
[In] Integrate[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{3\arctan\left(\frac{x}{2}\right)}{2}+\arctan\left(x\right)+\frac{\ln\left(x^{2}+4\right)}{2}$	18
risch	$-\frac{3\arctan\left(\frac{x}{2}\right)}{2}+\arctan\left(x\right)+\frac{\ln\left(x^{2}+4\right)}{2}$	18
parallelrisch	$\frac{\ln(x-2i)}{2}+\frac{3i\ln(x-2i)}{4}-\frac{i\ln(x-i)}{2}+\frac{i\ln(x+i)}{2}+\frac{\ln(x+2i)}{2}-\frac{3i\ln(x+2i)}{4}$	48

```
[In] int((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x,method=_RETURNVERBOSE)
[Out] -3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx = -\frac{3}{2}\arctan\left(\frac{1}{2}\,x\right)+\arctan\left(x\right)+\frac{1}{2}\log\left(x^2+4\right)$$

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="fricas")
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx = \frac{\log\left(x^2+4\right)}{2} - \frac{3\,\mathrm{atan}\left(\frac{x}{2}\right)}{2} + \mathrm{atan}\left(x\right)$$

```
[In] integrate((x**3-2*x**2+x+1)/(x**4+5*x**2+4),x)
[Out] log(x**2 + 4)/2 - 3*atan(x/2)/2 + atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx = -\frac{3}{2}\arctan\left(\frac{1}{2}x\right) + \arctan\left(x\right) + \frac{1}{2}\log\left(x^2+4\right)$$

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="maxima")
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx = -\frac{3}{2}\arctan\left(\frac{1}{2}x\right) + \arctan\left(x\right) + \frac{1}{2}\log\left(x^2+4\right)$$

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="giac")
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx = -\mathrm{atan}\left(\frac{1305}{4\,\left(144\,x-162\right)} + \frac{9}{8}\right) + \ln\left(x-2\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{3}{4}\mathrm{i}\right) + \ln\left(x+2\mathrm{i}\right)\,\left(\frac{1}{2}-\frac{3}{4}\mathrm{i}\right)$$

```
[In] int((x - 2*x^2 + x^3 + 1)/(5*x^2 + x^4 + 4),x)
[Out] log(x - 2i)*(1/2 + 3i/4) + log(x + 2i)*(1/2 - 3i/4) - atan(1305/(4*(144*x -
 162)) + 9/8)
```

3.320 $\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5} dx$

Optimal result 1988
Rubi [A] (verified) 1988
Mathematica [A] (verified) 1990
Maple [A] (verified) 1990
Fricas [A] (verification not implemented) 1991
Sympy [A] (verification not implemented) 1991
Maxima [A] (verification not implemented) 1992
Giac [A] (verification not implemented) 1992
Mupad [B] (verification not implemented) 1993

Optimal result

Integrand size = 43, antiderivative size = 63

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5} dx = \frac{3988\arctan\left(\frac{1+2x}{\sqrt{19}}\right)}{13685\sqrt{19}} - \frac{3146\log(7-3x)}{80155} - \frac{334}{323}\log(1+2x) + \frac{4822\log(2+5x)}{4879} + \frac{11049\log(5+x+x^2)}{260015}$$

```
[Out] -3146/80155*ln(7-3*x)-334/323*ln(1+2*x)+4822/4879*ln(2+5*x)+11049/260015*ln
(x^2+x+5)+3988/260015*arctan(1/19*(1+2*x)*19^(1/2))*19^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2099, 648, 632, 210, 642}

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5} dx = \frac{3988\arctan\left(\frac{2x+1}{\sqrt{19}}\right)}{13685\sqrt{19}} + \frac{11049\log(x^2+x+5)}{260015} - \frac{3146\log(7-3x)}{80155} - \frac{334}{323}\log(2x+1) + \frac{4822\log(5x+2)}{4879}$$

```
[In] Int[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13*x^4 +
 30*x^5),x]
```

```
[Out] (3988*ArcTan[(1 + 2*x)/Sqrt[19]])/(13685*Sqrt[19]) - (3146*Log[7 - 3*x])/80
155 - (334*Log[1 + 2*x])/323 + (4822*Log[2 + 5*x])/4879 + (11049*Log[5 + x
+ x^2])/260015
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{668}{323(1+2x)}-\frac{9438}{80155(-7+3x)}+\frac{24110}{4879(2+5x)}+\frac{48935+22098x}{260015\,(5+x+x^2)}\right) dx\\
&= -\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}+\frac{\int \frac{48935+22098x}{5+x+x^2}\,dx}{260015}\\
&= -\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}\\
&\quad+\frac{11049\int \frac{1+2x}{5+x+x^2}\,dx}{260015}+\frac{1994\int \frac{1}{5+x+x^2}\,dx}{13685}
\end{aligned}$$

$$=-\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}+\frac{11049\log\left(5+x+x^2\right)}{260015}-\frac{3988\text{Subst}\left(\int\frac{1}{-19-x^2}\,dx,x,1+2x\right)}{13685}$$

$$=\frac{3988\tan^{-1}\left(\frac{1+2x}{\sqrt{19}}\right)}{13685\sqrt{19}}-\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}+\frac{11049\log\left(5+x+x^2\right)}{260015}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int\frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx$$
$$=\frac{163508\sqrt{19}\arctan\left(\frac{1+2x}{\sqrt{19}}\right)-418418\log(7-3x)-11023670\log(1+2x)+10536070\log(2+5x)+453009}{10660615}$$

```
[In] Integrate[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13
*x^4 + 30*x^5),x]
```

```
[Out] (163508*Sqrt[19]*ArcTan[(1 + 2*x)/Sqrt[19]] - 418418*Log[7 - 3*x] - 1102367
0*Log[1 + 2*x] + 10536070*Log[2 + 5*x] + 453009*Log[5 + x + x^2])/10660615
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

method	result
default	$\frac{4822\ln(2+5x)}{4879}-\frac{334\ln(1+2x)}{323}+\frac{11049\ln(x^2+x+5)}{260015}+\frac{3988\arctan\left(\frac{(1+2x)\sqrt{19}}{19}\right)\sqrt{19}}{260015}-\frac{3146\ln(3x-7)}{80155}$
risch	$-\frac{3146\ln(3x-7)}{80155}-\frac{334\ln(1+2x)}{323}+\frac{11049\ln(15904144x^2+15904144x+79520720)}{260015}+\frac{3988\sqrt{19}\,\arctan\left(\frac{(3988x+1994)\sqrt{19}}{37886}\right)}{260015}+\frac{482}{}$

```
[In] int((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,method=
_RETURNVERBOSE)
```

```
[Out] 4822/4879*ln(2+5*x)-334/323*ln(1+2*x)+11049/260015*ln(x^2+x+5)+3988/260015*
arctan(1/19*(1+2*x)*19^(1/2))*19^(1/2)-3146/80155*ln(3*x-7)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = \frac{3988}{260015}\,\sqrt{19}\arctan\left(\frac{1}{19}\,\sqrt{19}(2\,x+1)\right)+\frac{11049}{260015}\,\log\left(x^2+x+5\right)+\frac{4822}{4879}\,\log\left(5\,x+2\right)-\frac{3146}{80155}\,\log\left(3\,x-7\right)-\frac{334}{323}\,\log\left(2\,x+1\right)$$

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="fricas")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(
2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = -\frac{3146\log\left(x-\frac{7}{3}\right)}{80155}+\frac{4822\log\left(x+\frac{2}{5}\right)}{4879}-\frac{334\log\left(x+\frac{1}{2}\right)}{323}+\frac{11049\log\left(x^2+x+5\right)}{260015}+\frac{3988\sqrt{19}\operatorname{atan}\left(\frac{2\sqrt{19}x}{19}+\frac{\sqrt{19}}{19}\right)}{260015}$$

```
[In] integrate((4*x**3-27*x**2+5*x-32)/(30*x**5-13*x**4+50*x**3-286*x**2-299*x-7
0),x)
```

```
[Out] -3146*log(x - 7/3)/80155 + 4822*log(x + 2/5)/4879 - 334*log(x + 1/2)/323 +
11049*log(x**2 + x + 5)/260015 + 3988*sqrt(19)*atan(2*sqrt(19)*x/19 + sqrt(
19)/19)/260015
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = \frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2\,x+1)\right)+\frac{11049}{260015}\log\left(x^2+x+5\right) + \frac{4822}{4879}\log\left(5\,x+2\right)-\frac{3146}{80155}\log\left(3\,x-7\right)-\frac{334}{323}\log\left(2\,x+1\right)$$

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="maxima")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(
2*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = \frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2\,x+1)\right)+\frac{11049}{260015}\log\left(x^2+x+5\right) + \frac{4822}{4879}\log\left(|5\,x+2|\right)-\frac{3146}{80155}\log\left(|3\,x-7|\right)-\frac{334}{323}\log\left(|2\,x+1|\right)$$

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="giac")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(abs(5*x + 2)) - 3146/80155*log(abs(3*x - 7)) - 33
4/323*log(abs(2*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = \frac{4822\ln\left(x+\frac{2}{5}\right)}{4879}-\frac{334\ln\left(x+\frac{1}{2}\right)}{323}-\frac{3146\ln\left(x-\frac{7}{3}\right)}{80155} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{19}\,1\mathrm{i}}{2}\right)\left(-\frac{11049}{260015}+\frac{\sqrt{19}\,1994\mathrm{i}}{260015}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{19}\,1\mathrm{i}}{2}\right)\left(\frac{11049}{260015}+\frac{\sqrt{19}\,1994\mathrm{i}}{260015}\right)$$

```
[In] int(-(5*x - 27*x^2 + 4*x^3 - 32)/(299*x + 286*x^2 - 50*x^3 + 13*x^4 - 30*x^
5 + 70),x)
```

```
[Out] (4822*log(x + 2/5))/4879 - (334*log(x + 1/2))/323 - (3146*log(x - 7/3))/801
55 - log(x - (19^(1/2)*1i)/2 + 1/2)*((19^(1/2)*1994i)/260015 - 11049/260015
) + log(x + (19^(1/2)*1i)/2 + 1/2)*((19^(1/2)*1994i)/260015 + 11049/260015)
```

3.321 $\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6} \, dx$

Optimal result 1994
Rubi [A] (verified) 1994
Mathematica [A] (verified) 1996
Maple [A] (verified) 1996
Fricas [A] (verification not implemented) 1997
Sympy [A] (verification not implemented) 1997
Maxima [A] (verification not implemented) 1998
Giac [A] (verification not implemented) 1998
Mupad [B] (verification not implemented) 1999

Optimal result

Integrand size = 50, antiderivative size = 69

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6} \, dx = \frac{5828}{9075(2-5x)} - \frac{313+502x}{1452(1+2x^2)} + \frac{503\arctan\left(\sqrt{2}x\right)}{7986\sqrt{2}} - \frac{59096\log(2-5x)}{99825} + \frac{2843\log(1+2x^2)}{7986}$$

```
[Out] 5828/9075/(2-5*x)+1/1452*(-313-502*x)/(2*x^2+1)-59096/99825*ln(2-5*x)+2843/
7986*ln(2*x^2+1)+503/15972*arctan(x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2099, 653, 209, 649, 266}

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6} \, dx = \frac{272\sqrt{2}\arctan\left(\sqrt{2}x\right)}{1331} - \frac{251\arctan\left(\sqrt{2}x\right)}{726\sqrt{2}} - \frac{502x+313}{1452(2x^2+1)} + \frac{2843\log(2x^2+1)}{7986} + \frac{5828}{9075(2-5x)} - \frac{59096\log(2-5x)}{99825}$$

```
[In] Int[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 - 8
0*x^5 + 100*x^6),x]
```

```
[Out] 5828/(9075*(2 - 5*x)) - (313 + 502*x)/(1452*(1 + 2*x^2)) - (251*ArcTan[Sqrt
[2]*x])/(726*Sqrt[2]) + (272*Sqrt[2]*ArcTan[Sqrt[2]*x])/1331 - (59096*Log[2
 - 5*x])/99825 + (2843*Log[1 + 2*x^2])/7986
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{5828}{1815(-2+5x)^2}-\frac{59096}{19965(-2+5x)}+\frac{-251+313x}{363\left(1+2x^2\right)^2}+\frac{2(816+2843x)}{3993\left(1+2x^2\right)}\right)dx\\
&= \frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}+\frac{2\int\frac{816+2843x}{1+2x^2}\,dx}{3993}+\frac{1}{363}\int\frac{-251+313x}{\left(1+2x^2\right)^2}\,dx\\
&= \frac{5828}{9075(2-5x)}-\frac{313+502x}{1452\left(1+2x^2\right)}-\frac{59096\log(2-5x)}{99825}\\
&\quad-\frac{251}{726}\int\frac{1}{1+2x^2}\,dx+\frac{544\int\frac{1}{1+2x^2}\,dx}{1331}+\frac{5686\int\frac{x}{1+2x^2}\,dx}{3993}
\end{aligned}$$

$$
= \frac{5828}{9075(2-5x)} - \frac{313+502x}{1452\,(1+2x^2)} - \frac{251\tan^{-1}\left(\sqrt{2}x\right)}{726\sqrt{2}} + \frac{272\sqrt{2}\tan^{-1}\left(\sqrt{2}x\right)}{1331} - \frac{59096\log(2-5x)}{99825} + \frac{2843\log\left(1+2x^2\right)}{7986}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$
\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \frac{-\frac{33(2554+4675x+36458x^2)}{-2+5x-4x^2+10x^3} + 12575\sqrt{2}\arctan\left(\sqrt{2}x\right) - 236384\log(2-5x) + 142150\log\left(1+2x^2\right)}{399300}
$$

```
[In] Integrate[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x
^4 - 80*x^5 + 100*x^6),x]
```

```
[Out] ((-33*(2554 + 4675*x + 36458*x^2))/(-2 + 5*x - 4*x^2 + 10*x^3) + 12575*Sqrt
[2]*ArcTan[Sqrt[2]*x] - 236384*Log[2 - 5*x] + 142150*Log[1 + 2*x^2])/399300
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{5828}{9075(5x-2)}-\frac{59096\ln(5x-2)}{99825}+\frac{-\frac{2761x}{4}-\frac{3443}{8}}{3993x^2+\frac{3993}{2}}+\frac{2843\ln\left(2x^2+1\right)}{7986}+\frac{503\arctan\left(x\sqrt{2}\right)\sqrt{2}}{15972}$	54
risch	$\frac{-\frac{18229}{60500}x^2-\frac{17}{440}x-\frac{1277}{60500}}{x^3-\frac{2}{5}x^2+\frac{1}{2}x-\frac{1}{5}}-\frac{59096\ln(5x-2)}{99825}+\frac{2843\ln\left(\frac{253009}{2}+253009x^2\right)}{7986}+\frac{503\arctan\left(x\sqrt{2}\right)\sqrt{2}}{15972}$	57

```
[In] int((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*x+4),x
,method=_RETURNVERBOSE)
```

```
[Out] -5828/9075/(5*x-2)-59096/99825*ln(5*x-2)+1/3993*(-2761/4*x-3443/8)/(x^2+1/2
)+2843/7986*ln(2*x^2+1)+503/15972*arctan(x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.49

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \frac{12575\,\sqrt{2}(10\,x^3-4\,x^2+5\,x-2)\arctan\left(\sqrt{2}x\right)-1203114\,x^2+142150\,(10\,x^3-4\,x^2+5\,x-2)\log\left(2\,x^2\right.}{399300\,(10\,x^3-4\,x^2+5\,x-2)}$$

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="fricas")
```

```
[Out] 1/399300*(12575*sqrt(2)*(10*x^3 - 4*x^2 + 5*x - 2)*arctan(sqrt(2)*x) - 1203
114*x^2 + 142150*(10*x^3 - 4*x^2 + 5*x - 2)*log(2*x^2 + 1) - 236384*(10*x^3
 - 4*x^2 + 5*x - 2)*log(5*x - 2) - 154275*x - 84282)/(10*x^3 - 4*x^2 + 5*x
- 2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \frac{-36458x^2-4675x-2554}{121000x^3-48400x^2+60500x-24200} - \frac{59096\log\left(x-\frac{2}{5}\right)}{99825} + \frac{2843\log\left(x^2+\frac{1}{2}\right)}{7986} + \frac{503\sqrt{2}\operatorname{atan}\left(\sqrt{2}x\right)}{15972}$$

```
[In] integrate((12*x**5-7*x**3-13*x**2+8)/(100*x**6-80*x**5+116*x**4-80*x**3+41*
x**2-20*x+4),x)
```

```
[Out] (-36458*x**2 - 4675*x - 2554)/(121000*x**3 - 48400*x**2 + 60500*x - 24200)
- 59096*log(x - 2/5)/99825 + 2843*log(x**2 + 1/2)/7986 + 503*sqrt(2)*atan(s
qrt(2)*x)/15972
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}x\right) - \frac{36458\,x^2+4675\,x+2554}{12100\,(10\,x^3-4\,x^2+5\,x-2)} + \frac{2843}{7986}\log\left(2\,x^2+1\right) - \frac{59096}{99825}\log\left(5\,x-2\right)$$

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="maxima")
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/(
10*x^3 - 4*x^2 + 5*x - 2) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(5*x
- 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}x\right) - \frac{36458\,x^2+4675\,x+2554}{12100\,(2\,x^2+1)(5\,x-2)} + \frac{2843}{7986}\log\left(2\,x^2+1\right) - \frac{59096}{99825}\log\left(|5\,x-2|\right)$$

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="giac")
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/(
(2*x^2 + 1)*(5*x - 2)) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(abs(5*x
 - 2))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx$$
$$= -\frac{59096\,\ln\left(x-\frac{2}{5}\right)}{99825} - \frac{\frac{18229\,x^2}{60500}+\frac{17\,x}{440}+\frac{1277}{60500}}{x^3-\frac{2\,x^2}{5}+\frac{x}{2}-\frac{1}{5}}$$
$$-\ln\left(x-\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)\left(-\frac{2843}{7986}+\frac{\sqrt{2}\,503\mathrm{i}}{31944}\right)+\ln\left(x+\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)\left(\frac{2843}{7986}+\frac{\sqrt{2}\,503\mathrm{i}}{31944}\right)$$

```
[In] int(-(13*x^2 + 7*x^3 - 12*x^5 - 8)/(41*x^2 - 20*x - 80*x^3 + 116*x^4 - 80*x
^5 + 100*x^6 + 4),x)
```

```
[Out] log(x + (2^(1/2)*1i)/2)*((2^(1/2)*503i)/31944 + 2843/7986) - ((17*x)/440 +
(18229*x^2)/60500 + 1277/60500)/(x/2 - (2*x^2)/5 + x^3 - 1/5) - log(x - (2^
(1/2)*1i)/2)*((2^(1/2)*503i)/31944 - 2843/7986) - (59096*log(x - 2/5))/9982
5
```

3.322 $\int \frac{9+x^4}{x^2(9+x^2)} \, dx$

Optimal result 2000
Rubi [A] (verified) 2000
Mathematica [A] (verified) 2001
Maple [A] (verified) 2001
Fricas [A] (verification not implemented) 2002
Sympy [A] (verification not implemented) 2002
Maxima [A] (verification not implemented) 2002
Giac [A] (verification not implemented) 2002
Mupad [B] (verification not implemented) 2003

Optimal result

Integrand size = 16, antiderivative size = 17

$$\int \frac{9+x^4}{x^2\left(9+x^2\right)} \, dx = -\frac{1}{x}+x-\frac{10}{3}\arctan\left(\frac{x}{3}\right)$$

```
[Out] -1/x+x-10/3*arctan(1/3*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1276, 209}

$$\int \frac{9+x^4}{x^2\left(9+x^2\right)} \, dx = -\frac{10}{3}\arctan\left(\frac{x}{3}\right)+x-\frac{1}{x}$$

```
[In] Int[(9 + x^4)/(x^2*(9 + x^2)),x]
```

```
[Out] -x^(-1) + x - (10*ArcTan[x/3])/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
```

```
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(1+\frac{1}{x^2}-\frac{10}{9+x^2}\right) dx \\
&= -\frac{1}{x}+x-10\int \frac{1}{9+x^2}\,dx \\
&= -\frac{1}{x}+x-\frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{9+x^4}{x^2\,(9+x^2)}\,dx=-\frac{1}{x}+x-\frac{10}{3}\arctan\left(\frac{x}{3}\right)$$

```
[In] Integrate[(9 + x^4)/(x^2*(9 + x^2)),x]
[Out] -x^(-1) + x - (10*ArcTan[x/3])/3
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{1}{x}+x-\frac{10\arctan\left(\frac{x}{3}\right)}{3}$	14
meijerg	$-\frac{1}{x}+x-\frac{10\arctan\left(\frac{x}{3}\right)}{3}$	14
risch	$-\frac{1}{x}+x-\frac{10\arctan\left(\frac{x}{3}\right)}{3}$	14
parallelrisch	$\frac{5i\ln(x-3i)x-5i\ln(x+3i)x+3x^2-3}{3x}$	31

```
[In] int((x^4+9)/x^2/(x^2+9),x,method=_RETURNVERBOSE)
[Out] -1/x+x-10/3*arctan(1/3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{9+x^4}{x^2\left(9+x^2\right)}\,dx=\frac{3\,x^2-10\,x\arctan\left(\frac{1}{3}\,x\right)-3}{3\,x}$$

```
[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="fricas")
[Out] 1/3*(3*x^2 - 10*x*arctan(1/3*x) - 3)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{9+x^4}{x^2\left(9+x^2\right)}\,dx=x-\frac{10\,\mathrm{atan}\left(\frac{x}{3}\right)}{3}-\frac{1}{x}$$

```
[In] integrate((x**4+9)/x**2/(x**2+9),x)
[Out] x - 10*atan(x/3)/3 - 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{9+x^4}{x^2\left(9+x^2\right)}\,dx=x-\frac{1}{x}-\frac{10}{3}\,\arctan\left(\frac{1}{3}\,x\right)$$

```
[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="maxima")
[Out] x - 1/x - 10/3*arctan(1/3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{9+x^4}{x^2\left(9+x^2\right)}\,dx=x-\frac{1}{x}-\frac{10}{3}\,\arctan\left(\frac{1}{3}\,x\right)$$

```
[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="giac")
[Out] x - 1/x - 10/3*arctan(1/3*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{9+x^4}{x^2\,(9+x^2)}\,dx = x - \frac{10\,\mathrm{atan}\left(\frac{x}{3}\right)}{3} - \frac{1}{x}$$

```
[In] int((x^4 + 9)/(x^2*(x^2 + 9)),x)
[Out] x - (10*atan(x/3))/3 - 1/x
```

3.323 $\int \frac{2x+x^4}{1+x^2} \, dx$

Optimal result . . . 2004
Rubi [A] (verified) . . . 2004
Mathematica [A] (verified) . . . 2005
Maple [A] (verified) . . . 2006
Fricas [A] (verification not implemented) . . . 2006
Sympy [A] (verification not implemented) . . . 2006
Maxima [A] (verification not implemented) . . . 2007
Giac [A] (verification not implemented) . . . 2007
Mupad [B] (verification not implemented) . . . 2007

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{2x+x^4}{1+x^2} \, dx = -x+\frac{x^3}{3}+\arctan(x)+\log\left(1+x^2\right)$$

```
[Out] -x+1/3*x^3+arctan(x)+ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1607, 1816, 649, 209, 266}

$$\int \frac{2x+x^4}{1+x^2} \, dx = \arctan(x)+\frac{x^3}{3}+\log\left(x^2+1\right)-x$$

```
[In] Int[(2*x + x^4)/(1 + x^2),x]
```

```
[Out] -x + x^3/3 + ArcTan[x] + Log[1 + x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(2+x^3)}{1+x^2}\,dx \\
&= \int \left(-1+x^2+\frac{1+2x}{1+x^2}\right) dx \\
&= -x+\frac{x^3}{3}+\int \frac{1+2x}{1+x^2}\,dx \\
&= -x+\frac{x^3}{3}+2\int \frac{x}{1+x^2}\,dx+\int \frac{1}{1+x^2}\,dx \\
&= -x+\frac{x^3}{3}+\tan^{-1}(x)+\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{2x+x^4}{1+x^2}\,dx = -x+\frac{x^3}{3}+\arctan(x)+\log\left(1+x^2\right)$$

```
[In] Integrate[(2*x + x^4)/(1 + x^2),x]
[Out] -x + x^3/3 + ArcTan[x] + Log[1 + x^2]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$-x+\frac{x^{3}}{3}+\arctan\left(x\right)+\ln\left(x^{2}+1\right)$	18
risch	$-x+\frac{x^{3}}{3}+\arctan\left(x\right)+\ln\left(x^{2}+1\right)$	18
meijerg	$-\frac{x\left(-5x^{2}+15\right)}{15}+\arctan\left(x\right)+\ln\left(x^{2}+1\right)$	20
parallelrisch	$\frac{x^{3}}{3}-x+\ln\left(x-i\right)-\frac{i\ln\left(x-i\right)}{2}+\ln\left(x+i\right)+\frac{i\ln\left(x+i\right)}{2}$	36

```
[In] int((x^4+2*x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -x+1/3*x^3+arctan(x)+ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{2x+x^4}{1+x^2}\,dx=\frac{1}{3}\,x^{3}-x+\arctan\left(x\right)+\log\left(x^{2}+1\right)$$

```
[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="fricas")
[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{2x+x^4}{1+x^2}\,dx=\frac{x^{3}}{3}-x+\log\left(x^{2}+1\right)+\operatorname{atan}\left(x\right)$$

```
[In] integrate((x**4+2*x)/(x**2+1),x)
[Out] x**3/3 - x + log(x**2 + 1) + atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{2x+x^4}{1+x^2}\,dx = \frac{1}{3}\,x^3 - x + \arctan\left(x\right) + \log\left(x^2+1\right)$$

```
[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="maxima")
[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{2x+x^4}{1+x^2}\,dx = \frac{1}{3}\,x^3 - x + \arctan\left(x\right) + \log\left(x^2+1\right)$$

```
[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="giac")
[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 9.58 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{2x+x^4}{1+x^2}\,dx = \ln\left(x^2+1\right) - x + \mathrm{atan}(x) + \frac{x^3}{3}$$

```
[In] int((2*x + x^4)/(x^2 + 1),x)
[Out] log(x^2 + 1) - x + atan(x) + x^3/3
```

3.324 $\int \frac{-x+x^3}{(-1+x)^2(1+x^2)} dx$

Optimal result 2008
Rubi [A] (verified) 2008
Mathematica [A] (verified) 2009
Maple [A] (verified) 2010
Fricas [A] (verification not implemented) 2010
Sympy [A] (verification not implemented) 2010
Maxima [A] (verification not implemented) 2011
Giac [B] (verification not implemented) 2011
Mupad [B] (verification not implemented) 2011

Optimal result

Integrand size = 20, antiderivative size = 9

$$\int \frac{-x+x^3}{(-1+x)^2(1+x^2)} dx = \arctan(x) + \log(1-x)$$

```
[Out] arctan(x)+ln(1-x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1600, 1607, 1643, 209}

$$\int \frac{-x+x^3}{(-1+x)^2(1+x^2)} dx = \arctan(x) + \log(1-x)$$

```
[In] Int[(-x + x^3)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] ArcTan[x] + Log[1 - x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x+x^2}{(-1+x)\left(1+x^2\right)}\,dx \\
&= \int \frac{x(1+x)}{(-1+x)\left(1+x^2\right)}\,dx \\
&= \int \left(\frac{1}{-1+x}+\frac{1}{1+x^2}\right)dx \\
&= \log(1-x)+\int \frac{1}{1+x^2}\,dx \\
&= \tan^{-1}(x)+\log(1-x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{-x+x^3}{(-1+x)^2\left(1+x^2\right)}\,dx = \arctan(x)+\log(1-x)$$

```
[In] Integrate[(-x + x^3)/((-1 + x)^2*(1 + x^2)),x]
[Out] ArcTan[x] + Log[1 - x]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
default	$\arctan(x)+\ln(x-1)$	8
risch	$\arctan(x)+\ln(x-1)$	8
parallelrisch	$\frac{i\ln(x+i)}{2}-\frac{i\ln(x-i)}{2}+\ln(x-1)$	22

```
[In] int((x^3-x)/(x-1)^2/(x^2+1),x,method=_RETURNVERBOSE)
[Out] arctan(x)+ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{-x+x^3}{(-1+x)^2(1+x^2)}\,dx = \arctan(x)+\log(x-1)$$

```
[In] integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
[Out] arctan(x) + log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{-x+x^3}{(-1+x)^2(1+x^2)}\,dx = \log(x-1)+\operatorname{atan}(x)$$

```
[In] integrate((x**3-x)/(-1+x)**2/(x**2+1),x)
[Out] log(x - 1) + atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{-x+x^3}{(-1+x)^2(1+x^2)}\,dx = \arctan(x) + \log(x-1)$$

```
[In] integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
[Out] arctan(x) + log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(9) = 18.

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 3.11

$$\int \frac{-x+x^3}{(-1+x)^2(1+x^2)}\,dx = \frac{1}{4}\pi - \pi\left\lfloor\frac{\pi+4\arctan(x)}{4\pi}+\frac{1}{2}\right\rfloor + \arctan(x) + \log(|x-1|)$$

```
[In] integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="giac")
[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + arctan(x) + log(abs(x
- 1))
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int \frac{-x+x^3}{(-1+x)^2(1+x^2)}\,dx = \ln(x-1) - \mathrm{atan}\left(\frac{5}{4\,x+2}-\frac{1}{2}\right)$$

```
[In] int(-(x - x^3)/((x^2 + 1)*(x - 1)^2),x)
[Out] log(x - 1) - atan(5/(4*x + 2) - 1/2)
```

3.325 $\int \frac{2+5x+3x^2+2x^3}{1+x+x^2} dx$

Optimal result 2012
Rubi [A] (verified) 2012
Mathematica [A] (verified) 2013
Maple [A] (verified) 2013
Fricas [A] (verification not implemented) 2014
Sympy [A] (verification not implemented) 2014
Maxima [A] (verification not implemented) 2014
Giac [A] (verification not implemented) 2014
Mupad [B] (verification not implemented) 2015

Optimal result

Integrand size = 24, antiderivative size = 12

$$\int \frac{2+5x+3x^2+2x^3}{1+x+x^2} dx = x+x^2+\log\left(1+x+x^2\right)$$

```
[Out] x+x^2+ln(x^2+x+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1671, 642}

$$\int \frac{2+5x+3x^2+2x^3}{1+x+x^2} dx = x^2+\log\left(x^2+x+1\right)+x$$

```
[In] Int[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2),x]
```

```
[Out] x + x^2 + Log[1 + x + x^2]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(1+2x+\frac{1+2x}{1+x+x^2}\right)\,dx\\ &= x+x^2+\int \frac{1+2x}{1+x+x^2}\,dx\\ &= x+x^2+\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{2+5x+3x^2+2x^3}{1+x+x^2}\,dx = x+x^2+\log\left(1+x+x^2\right)$$

```
[In] Integrate[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2),x]
[Out] x + x^2 + Log[1 + x + x^2]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$x+x^2+\ln\left(x^2+x+1\right)$	13
norman	$x+x^2+\ln\left(x^2+x+1\right)$	13
risch	$x+x^2+\ln\left(x^2+x+1\right)$	13
parallelrisch	$x+x^2+\ln\left(x^2+x+1\right)$	13

```
[In] int((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x,method=_RETURNVERBOSE)
[Out] x+x^2+ln(x^2+x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{2+5x+3x^2+2x^3}{1+x+x^2}\,dx = x^2+x+\log\left(x^2+x+1\right)$$

```
[In] integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="fricas")
[Out] x^2 + x + log(x^2 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{2+5x+3x^2+2x^3}{1+x+x^2}\,dx = x^2+x+\log\left(x^2+x+1\right)$$

```
[In] integrate((2*x**3+3*x**2+5*x+2)/(x**2+x+1),x)
[Out] x**2 + x + log(x**2 + x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{2+5x+3x^2+2x^3}{1+x+x^2}\,dx = x^2+x+\log\left(x^2+x+1\right)$$

```
[In] integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="maxima")
[Out] x^2 + x + log(x^2 + x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{2+5x+3x^2+2x^3}{1+x+x^2}\,dx = x^2+x+\log\left(x^2+x+1\right)$$

```
[In] integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="giac")
[Out] x^2 + x + log(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{2+5x+3x^2+2x^3}{1+x+x^2}\,dx = x+\ln\left(x^2+x+1\right)+x^2$$

```
[In] int((5*x + 3*x^2 + 2*x^3 + 2)/(x + x^2 + 1),x)
[Out] x + log(x + x^2 + 1) + x^2
```

3.326 $\int \frac{3-4x-5x^2+3x^3}{x^3(-1+x+x^2)}\,dx$

Optimal result 2016
Rubi [A] (verified) 2016
Mathematica [A] (verified) 2017
Maple [A] (verified) 2018
Fricas [A] (verification not implemented) 2018
Sympy [A] (verification not implemented) 2018
Maxima [A] (verification not implemented) 2019
Giac [A] (verification not implemented) 2019
Mupad [B] (verification not implemented) 2020

Optimal result

Integrand size = 27, antiderivative size = 65

$$\int \frac{3-4x-5x^2+3x^3}{x^3(-1+x+x^2)}\,dx = \frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)$$

```
[Out] 3/2/x^2-1/x+3*ln(x)-1/10*ln(1+2*x-5^(1/2))*(15-5^(1/2))-1/10*ln(1+2*x+5^(1/
2))*(15+5^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1642, 646, 31}

$$\int \frac{3-4x-5x^2+3x^3}{x^3(-1+x+x^2)}\,dx = \frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)$$

```
[In] Int[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)),x]
```

```
[Out] 3/(2*x^2) - x^(-1) + 3*Log[x] - ((15 - Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/10
- ((15 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{3}{x^3}+\frac{1}{x^2}+\frac{3}{x}+\frac{-1-3x}{-1+x+x^2}\right) dx \\
&= \frac{3}{2x^2}-\frac{1}{x}+3\log(x)+\int \frac{-1-3x}{-1+x+x^2}\, dx \\
&= \frac{3}{2x^2}-\frac{1}{x}+3\log(x)+\frac{1}{10}\left(-15+\sqrt{5}\right)\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\, dx-\frac{1}{10}\left(15+\sqrt{5}\right)\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x}\, dx \\
&= \frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{3-4x-5x^2+3x^3}{x^3\left(-1+x+x^2\right)}\, dx = \frac{1}{10}\left(\frac{15}{x^2}-\frac{10}{x}+\left(-15+\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x\right)+30\log(x)-\left(15+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)\right)$$

```
[In] Integrate[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)),x]
[Out] (15/x^2 - 10/x + (-15 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x] + 30*Log[x] - (15
+ Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63

method	result	size
default	$-\frac{1}{x}+\frac{3}{2x^{2}}+3\ln\left(x\right)-\frac{3\ln\left(x^{2}+x-1\right)}{2}-\frac{\operatorname{arctanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)\sqrt{5}}{5}$	41
risch	$\frac{-x+\frac{3}{2}}{x^{2}}-\frac{3\ln\left(1+2x-\sqrt{5}\right)}{2}+\frac{\ln\left(1+2x-\sqrt{5}\right)\sqrt{5}}{10}-\frac{3\ln\left(1+2x+\sqrt{5}\right)}{2}-\frac{\ln\left(1+2x+\sqrt{5}\right)\sqrt{5}}{10}+3\ln\left(x\right)$	69

```
[In] int((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x,method=_RETURNVERBOSE)
[Out] -1/x+3/2/x^2+3*ln(x)-3/2*ln(x^2+x-1)-1/5*arctanh(1/5*(1+2*x)*5^(1/2))*5^(1/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{3-4x-5x^2+3x^3}{x^3\left(-1+x+x^2\right)}\,dx = \frac{\sqrt{5}x^{2}\log\left(\frac{2\,x^{2}-\sqrt{5}(2\,x+1)+2\,x+3}{x^{2}+x-1}\right)-15\,x^{2}\log\left(x^{2}+x-1\right)+30\,x^{2}\log\left(x\right)-10\,x+15}{10\,x^{2}}$$

```
[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="fricas")
[Out] 1/10*(sqrt(5)*x^2*log((2*x^2 - sqrt(5)*(2*x + 1) + 2*x + 3)/(x^2 + x - 1))
- 15*x^2*log(x^2 + x - 1) + 30*x^2*log(x) - 10*x + 15)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.52

$$\begin{aligned}\int \frac{3-4x-5x^2+3x^3}{x^3\left(-1+x+x^2\right)}\,dx &= 3\log\left(x\right)\\ &\quad+\left(-\frac{3}{2}+\frac{\sqrt{5}}{10}\right)\log\left(x-\frac{405}{202}-\frac{35\sqrt{5}}{202}+\frac{110\left(-\frac{3}{2}+\frac{\sqrt{5}}{10}\right)^{2}}{101}\right)\\ &\quad+\left(-\frac{3}{2}-\frac{\sqrt{5}}{10}\right)\log\left(x-\frac{405}{202}+\frac{35\sqrt{5}}{202}+\frac{110\left(-\frac{3}{2}-\frac{\sqrt{5}}{10}\right)^{2}}{101}\right)\\ &\quad+\frac{3-2x}{2x^{2}}\end{aligned}$$

```
[In] integrate((3*x**3-5*x**2-4*x+3)/x**3/(x**2+x-1),x)
```

```
[Out] 3*log(x) + (-3/2 + sqrt(5)/10)*log(x - 405/202 - 35*sqrt(5)/202 + 110*(-3/2
 + sqrt(5)/10)**2/101) + (-3/2 - sqrt(5)/10)*log(x - 405/202 + 35*sqrt(5)/2
02 + 110*(-3/2 - sqrt(5)/10)**2/101) + (3 - 2*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{3-4x-5x^2+3x^3}{x^3\left(-1+x+x^2\right)}\,dx = \frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}+1}{2\,x+\sqrt{5}+1}\right)-\frac{2\,x-3}{2\,x^2} - \frac{3}{2}\,\log\left(x^2+x-1\right)+3\,\log\left(x\right)$$

```
[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/2*(2*x - 3)/x
^2 - 3/2*log(x^2 + x - 1) + 3*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{3-4x-5x^2+3x^3}{x^3\left(-1+x+x^2\right)}\,dx = \frac{1}{10}\,\sqrt{5}\log\left(\frac{\left|2\,x-\sqrt{5}+1\right|}{\left|2\,x+\sqrt{5}+1\right|}\right)-\frac{2\,x-3}{2\,x^2} - \frac{3}{2}\,\log\left(\left|x^2+x-1\right|\right)+3\,\log\left(\left|x\right|\right)$$

```
[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5)*log(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) - 1/2*(2*x
- 3)/x^2 - 3/2*log(abs(x^2 + x - 1)) + 3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{3-4x-5x^2+3x^3}{x^3\left(-1+x+x^2\right)}\,dx = 3\,\ln\left(x\right)-\frac{x-\frac{3}{2}}{x^2}+\ln\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\,\left(\frac{\sqrt{5}}{10}-\frac{3}{2}\right)-\ln\left(x+\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\,\left(\frac{\sqrt{5}}{10}+\frac{3}{2}\right)$$

```
[In] int(-(4*x + 5*x^2 - 3*x^3 - 3)/(x^3*(x + x^2 - 1)),x)
[Out] 3*log(x) - (x - 3/2)/x^2 + log(x - 5^(1/2)/2 + 1/2)*(5^(1/2)/10 - 3/2) - lo
g(x + 5^(1/2)/2 + 1/2)*(5^(1/2)/10 + 3/2)
```

3.327 $\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2} dx$

Optimal result 2021
Rubi [A] (verified) 2021
Mathematica [A] (verified) 2023
Maple [A] (verified) 2023
Fricas [A] (verification not implemented) 2023
Sympy [A] (verification not implemented) 2024
Maxima [A] (verification not implemented) 2024
Giac [A] (verification not implemented) 2024
Mupad [B] (verification not implemented) 2024

Optimal result

Integrand size = 26, antiderivative size = 28

$$\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2} dx = -\frac{1}{2+2x+x^2} - \arctan(1+x) + \log(2+2x+x^2)$$

```
[Out] -1/(x^2+2*x+2)-arctan(1+x)+ln(x^2+2*x+2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1674, 648, 631, 210, 642}

$$\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2} dx = -\arctan(x+1) - \frac{1}{x^2+2x+2} + \log(x^2+2x+2)$$

```
[In] Int[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]
```

```
[Out] -(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{2+2x+x^2}+\frac{1}{4}\int \frac{4+8x}{2+2x+x^2}\,dx \\
&= -\frac{1}{2+2x+x^2}-\int \frac{1}{2+2x+x^2}\,dx+\int \frac{2+2x}{2+2x+x^2}\,dx \\
&= -\frac{1}{2+2x+x^2}+\log\left(2+2x+x^2\right)+\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+x\right) \\
&= -\frac{1}{2+2x+x^2}-\tan^{-1}(1+x)+\log\left(2+2x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2}\,dx = -\frac{1}{2+2x+x^2} - \arctan(1+x) + \log\left(2+2x+x^2\right)$$

```
[In] Integrate[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]
[Out] -(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result
default	$-\frac{1}{x^2+2x+2} - \arctan\left(x+1\right) + \ln\left(x^2+2x+2\right)$
risch	$-\frac{1}{x^2+2x+2} - \arctan\left(x+1\right) + \ln\left(x^2+2x+2\right)$
parallelrisch	$\frac{2i\ln(x+1-i)-i\ln(x+1+i)x^2-2i\ln(x+1+i)+2\ln(x+1-i)x^2+2i\ln(x+1-i)x+2\ln(x+1+i)x^2-2+i\ln(x+1-i)x^2+4\ln(x+1-}{2x^2+4x+4}$

```
[In] int((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x,method=_RETURNVERBOSE)
[Out] -1/(x^2+2*x+2)-arctan(x+1)+ln(x^2+2*x+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2}\,dx = -\frac{(x^2+2\,x+2)\arctan\left(x+1\right) - (x^2+2\,x+2)\log\left(x^2+2\,x+2\right) + 1}{x^2+2\,x+2}$$

```
[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="fricas")
[Out] -((x^2 + 2*x + 2)*arctan(x + 1) - (x^2 + 2*x + 2)*log(x^2 + 2*x + 2) + 1)/(
x^2 + 2*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2}\,dx = \log\left(x^2+2x+2\right) - \operatorname{atan}(x+1) - \frac{1}{x^2+2x+2}$$

```
[In] integrate((2*x**3+5*x**2+8*x+4)/(x**2+2*x+2)**2,x)
[Out] log(x**2 + 2*x + 2) - atan(x + 1) - 1/(x**2 + 2*x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2}\,dx = -\frac{1}{x^2+2\,x+2} - \arctan(x+1) + \log\left(x^2+2\,x+2\right)$$

```
[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="maxima")
[Out] -1/(x^2 + 2*x + 2) - arctan(x + 1) + log(x^2 + 2*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2}\,dx = -\frac{1}{x^2+2\,x+2} - \arctan(x+1) + \log\left(x^2+2\,x+2\right)$$

```
[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="giac")
[Out] -1/(x^2 + 2*x + 2) - arctan(x + 1) + log(x^2 + 2*x + 2)
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2}\,dx = \ln\left(x^2+2\,x+2\right) - \operatorname{atan}(x+1) - \frac{1}{x^2+2\,x+2}$$

```
[In] int((8*x + 5*x^2 + 2*x^3 + 4)/(2*x + x^2 + 2)^2,x)
[Out] log(2*x + x^2 + 2) - atan(x + 1) - 1/(2*x + x^2 + 2)
```

3.328 $\int \frac{(-1+x)^4x^4}{1+x^2}\,dx$

Optimal result 2025
Rubi [A] (verified) 2025
Mathematica [A] (verified) 2026
Maple [A] (verified) 2026
Fricas [A] (verification not implemented) 2027
Sympy [A] (verification not implemented) 2027
Maxima [A] (verification not implemented) 2027
Giac [A] (verification not implemented) 2027
Mupad [B] (verification not implemented) 2028

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{(-1+x)^4x^4}{1+x^2}\,dx = 4x-\frac{4x^3}{3}+x^5-\frac{2x^6}{3}+\frac{x^7}{7}-4\arctan(x)$$

```
[Out] 4*x-4/3*x^3+x^5-2/3*x^6+1/7*x^7-4*arctan(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1643, 209}

$$\int \frac{(-1+x)^4x^4}{1+x^2}\,dx = -4\arctan(x)+\frac{x^7}{7}-\frac{2x^6}{3}+x^5-\frac{4x^3}{3}+4x$$

```
[In] Int[((-1 + x)^4*x^4)/(1 + x^2),x]
```

```
[Out] 4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(4-4x^2+5x^4-4x^5+x^6-\frac{4}{1+x^2}\right) dx \\
&= 4x-\frac{4x^3}{3}+x^5-\frac{2x^6}{3}+\frac{x^7}{7}-4\int \frac{1}{1+x^2}\,dx \\
&= 4x-\frac{4x^3}{3}+x^5-\frac{2x^6}{3}+\frac{x^7}{7}-4\tan^{-1}(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$
\int \frac{(-1+x)^4x^4}{1+x^2}\,dx = 4x-\frac{4x^3}{3}+x^5-\frac{2x^6}{3}+\frac{x^7}{7}-4\arctan(x)
$$

```
[In] Integrate[((-1 + x)^4*x^4)/(1 + x^2),x]
[Out] 4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result
default	$4x-\frac{4x^3}{3}+x^5-\frac{2x^6}{3}+\frac{x^7}{7}-4\arctan\left(x\right)$
risch	$4x-\frac{4x^3}{3}+x^5-\frac{2x^6}{3}+\frac{x^7}{7}-4\arctan\left(x\right)$
parallelrisch	$\frac{x^7}{7}-\frac{2x^6}{3}+x^5-\frac{4x^3}{3}+4x+2i\ln\left(x-i\right)-2i\ln\left(x+i\right)$
meijerg	$-\frac{x\left(-45x^6+63x^4-105x^2+315\right)}{315}-4\arctan\left(x\right)-\frac{x^2\left(4x^4-6x^2+12\right)}{6}+\frac{2x\left(21x^4-35x^2+105\right)}{35}+\frac{x^2\left(-3x^2+6\right)}{3}-\frac{x(-}{}$

```
[In] int((x-1)^4*x^4/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 4*x-4/3*x^3+x^5-2/3*x^6+1/7*x^7-4*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{(-1+x)^4x^4}{1+x^2}\,dx = \frac{1}{7}\,x^7 - \frac{2}{3}\,x^6 + x^5 - \frac{4}{3}\,x^3 + 4\,x - 4\,\arctan\left(x\right)$$

```
[In] integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="fricas")
[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{(-1+x)^4x^4}{1+x^2}\,dx = \frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4\,\mathrm{atan}\left(x\right)$$

```
[In] integrate((-1+x)**4*x**4/(x**2+1),x)
[Out] x**7/7 - 2*x**6/3 + x**5 - 4*x**3/3 + 4*x - 4*atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{(-1+x)^4x^4}{1+x^2}\,dx = \frac{1}{7}\,x^7 - \frac{2}{3}\,x^6 + x^5 - \frac{4}{3}\,x^3 + 4\,x - 4\,\arctan\left(x\right)$$

```
[In] integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="maxima")
[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{(-1+x)^4x^4}{1+x^2}\,dx = \frac{1}{7}\,x^7 - \frac{2}{3}\,x^6 + x^5 - \frac{4}{3}\,x^3 + 4\,x - 4\,\arctan\left(x\right)$$

```
[In] integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="giac")
[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{(-1+x)^4 x^4}{1+x^2}\,dx = 4\,x - 4\,\mathrm{atan}(x) - \frac{4\,x^3}{3} + x^5 - \frac{2\,x^6}{3} + \frac{x^7}{7}$$

```
[In] int((x^4*(x - 1)^4)/(x^2 + 1),x)
[Out] 4*x - 4*atan(x) - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7
```

3.329 $\int \frac{-20x+4x^2}{9-10x^2+x^4} \, dx$

Optimal result 2029
Rubi [A] (verified) 2029
Mathematica [A] (verified) 2031
Maple [A] (verified) 2031
Fricas [A] (verification not implemented) 2032
Sympy [A] (verification not implemented) 2032
Maxima [A] (verification not implemented) 2032
Giac [A] (verification not implemented) 2032
Mupad [B] (verification not implemented) 2033

Optimal result

Integrand size = 22, antiderivative size = 31

$$\int \frac{-20x+4x^2}{9-10x^2+x^4} \, dx = \log(1-x) - \frac{1}{2}\log(3-x) + \frac{3}{2}\log(1+x) - 2\log(3+x)$$

```
[Out] ln(1-x)-1/2*ln(3-x)+3/2*ln(1+x)-2*ln(3+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1607, 1676, 12, 1121, 630, 31, 1144, 213}

$$\int \frac{-20x+4x^2}{9-10x^2+x^4} \, dx = -\frac{3}{2}\text{arctanh}\left(\frac{x}{3}\right) + \frac{\text{arctanh}(x)}{2} + \frac{5}{4}\log\left(1-x^2\right) - \frac{5}{4}\log\left(9-x^2\right)$$

```
[In] Int[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4),x]
```

```
[Out] (-3*ArcTanh[x/3])/2 + ArcTanh[x]/2 + (5*Log[1 - x^2])/4 - (5*Log[9 - x^2])/
4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\text{integral} = \int \frac{x(-20+4x)}{9-10x^2+x^4}\,dx$$

$$
\begin{aligned}
&= \int -\frac{20x}{9-10x^2+x^4}\,dx + \int \frac{4x^2}{9-10x^2+x^4}\,dx \\
&= 4\int \frac{x^2}{9-10x^2+x^4}\,dx - 20\int \frac{x}{9-10x^2+x^4}\,dx \\
&= -\left(\frac{1}{2}\int \frac{1}{-1+x^2}\,dx\right) + \frac{9}{2}\int \frac{1}{-9+x^2}\,dx - 10\text{Subst}\left(\int \frac{1}{9-10x+x^2}\,dx, x, x^2\right) \\
&= -\frac{3}{2}\tanh^{-1}\left(\frac{x}{3}\right) + \frac{1}{2}\tanh^{-1}(x) - \frac{5}{4}\text{Subst}\left(\int \frac{1}{-9+x}\,dx, x, x^2\right) \\
&\quad + \frac{5}{4}\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, x^2\right) \\
&= -\frac{3}{2}\tanh^{-1}\left(\frac{x}{3}\right) + \frac{1}{2}\tanh^{-1}(x) + \frac{5}{4}\log\left(1-x^2\right) - \frac{5}{4}\log\left(9-x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$
\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx = 4\left(\frac{1}{4}\log(1-x) - \frac{1}{8}\log(3-x) + \frac{3}{8}\log(1+x) - \frac{1}{2}\log(3+x)\right)
$$

```
[In] Integrate[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4),x]
[Out] 4*(Log[1 - x]/4 - Log[3 - x]/8 + (3*Log[1 + x])/8 - Log[3 + x]/2)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\ln(-3+x)}{2}+\frac{3\ln(x+1)}{2}-2\ln\left(3+x\right)+\ln\left(x-1\right)$	24
norman	$-\frac{\ln(-3+x)}{2}+\frac{3\ln(x+1)}{2}-2\ln\left(3+x\right)+\ln\left(x-1\right)$	24
risch	$-\frac{\ln(-3+x)}{2}+\frac{3\ln(x+1)}{2}-2\ln\left(3+x\right)+\ln\left(x-1\right)$	24
parallelrisch	$-\frac{\ln(-3+x)}{2}+\frac{3\ln(x+1)}{2}-2\ln\left(3+x\right)+\ln\left(x-1\right)$	24

```
[In] int((4*x^2-20*x)/(x^4-10*x^2+9),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(-3+x)+3/2*ln(x+1)-2*ln(3+x)+ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{-20x + 4x^2}{9 - 10x^2 + x^4} \, dx = -2 \log(x + 3) + \frac{3}{2} \log(x + 1) + \log(x - 1) - \frac{1}{2} \log(x - 3)$$

```
[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="fricas")
[Out] -2*log(x + 3) + 3/2*log(x + 1) + log(x - 1) - 1/2*log(x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{-20x + 4x^2}{9 - 10x^2 + x^4} \, dx = -\frac{\log(x - 3)}{2} + \log(x - 1) + \frac{3\log(x + 1)}{2} - 2\log(x + 3)$$

```
[In] integrate((4*x**2-20*x)/(x**4-10*x**2+9),x)
[Out] -log(x - 3)/2 + log(x - 1) + 3*log(x + 1)/2 - 2*log(x + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{-20x + 4x^2}{9 - 10x^2 + x^4} \, dx = -2 \log(x + 3) + \frac{3}{2} \log(x + 1) + \log(x - 1) - \frac{1}{2} \log(x - 3)$$

```
[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="maxima")
[Out] -2*log(x + 3) + 3/2*log(x + 1) + log(x - 1) - 1/2*log(x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{-20x + 4x^2}{9 - 10x^2 + x^4} \, dx = -2 \log(|x + 3|) + \frac{3}{2} \log(|x + 1|) + \log(|x - 1|) - \frac{1}{2} \log(|x - 3|)$$

```
[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="giac")
[Out] -2*log(abs(x + 3)) + 3/2*log(abs(x + 1)) + log(abs(x - 1)) - 1/2*log(abs(x
- 3))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx = \ln(x-1)+\frac{3\,\ln(x+1)}{2}-\frac{\ln(x-3)}{2}-2\,\ln(x+3)$$

```
[In] int(-(20*x - 4*x^2)/(x^4 - 10*x^2 + 9),x)
[Out] log(x - 1) + (3*log(x + 1))/2 - log(x - 3)/2 - 2*log(x + 3)
```

3.330 $\int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)}\,dx$

Optimal result 2034
Rubi [A] (verified) 2034
Mathematica [A] (verified) 2035
Maple [A] (verified) 2035
Fricas [A] (verification not implemented) 2036
Sympy [A] (verification not implemented) 2036
Maxima [A] (verification not implemented) 2036
Giac [A] (verification not implemented) 2037
Mupad [B] (verification not implemented) 2037

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)}\,dx = -\frac{1}{x} + \arctan(x) + 2\log(1-x) - \log\left(1+x^2\right)$$

```
[Out] -1/x+arctan(x)+2*ln(1-x)-ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6857, 649, 209, 266}

$$\int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)}\,dx = \arctan(x) - \log\left(x^2+1\right) - \frac{1}{x} + 2\log(1-x)$$

```
[In] Int[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)),x]
[Out] -x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{2}{-1+x}+\frac{1}{x^2}+\frac{1-2x}{1+x^2}\right) dx \\
&= -\frac{1}{x}+2\log(1-x)+\int \frac{1-2x}{1+x^2}\, dx \\
&= -\frac{1}{x}+2\log(1-x)-2\int \frac{x}{1+x^2}\, dx+\int \frac{1}{1+x^2}\, dx \\
&= -\frac{1}{x}+\tan^{-1}(x)+2\log(1-x)-\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{-1+x+4x^3}{(-1+x)x^2\left(1+x^2\right)}\, dx = -\frac{1}{x}+\arctan(x)+2\log(1-x)-\log\left(1+x^2\right)$$

```
[In] Integrate[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)),x]
[Out] -x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$-\ln\left(x^2+1\right)+\arctan\left(x\right)-\frac{1}{x}+2\ln\left(x-1\right)$	23
risch	$-\ln\left(x^2+1\right)+\arctan\left(x\right)-\frac{1}{x}+2\ln\left(x-1\right)$	23
parallelrisch	$\frac{-i\ln(x-i)x+i\ln(x+i)x+4\ln(x-1)x-2\ln(x-i)x-2\ln(x+i)x-2}{2x}$	49

```
[In] int((4*x^3+x-1)/(x-1)/x^2/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -ln(x^2+1)+arctan(x)-1/x+2*ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{-1+x+4x^3}{(-1+x)x^2\left(1+x^2\right)}\,dx = \frac{x\arctan\left(x\right) - x\log\left(x^2+1\right) + 2\,x\log\left(x-1\right) - 1}{x}$$

```
[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="fricas")
[Out] (x*arctan(x) - x*log(x^2 + 1) + 2*x*log(x - 1) - 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{-1+x+4x^3}{(-1+x)x^2\left(1+x^2\right)}\,dx = 2\log\left(x-1\right) - \log\left(x^2+1\right) + \operatorname{atan}\left(x\right) - \frac{1}{x}$$

```
[In] integrate((4*x**3+x-1)/(-1+x)/x**2/(x**2+1),x)
[Out] 2*log(x - 1) - log(x**2 + 1) + atan(x) - 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{-1+x+4x^3}{(-1+x)x^2\left(1+x^2\right)}\,dx = -\frac{1}{x} + \arctan\left(x\right) - \log\left(x^2+1\right) + 2\,\log\left(x-1\right)$$

```
[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="maxima")
[Out] -1/x + arctan(x) - log(x^2 + 1) + 2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{-1+x+4x^3}{(-1+x)x^2\left(1+x^2\right)}\,dx=-\frac{1}{x}+\arctan\left(x\right)-\log\left(x^2+1\right)+2\,\log\left(\left|x-1\right|\right)$$

```
[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="giac")
[Out] -1/x + arctan(x) - log(x^2 + 1) + 2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{-1+x+4x^3}{(-1+x)x^2\left(1+x^2\right)}\,dx=2\,\ln\left(x-1\right)-\frac{1}{x}+\ln\left(x-\mathrm{i}\right)\,\left(-1-\frac{1}{2}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(-1+\frac{1}{2}\mathrm{i}\right)$$

```
[In] int((x + 4*x^3 - 1)/(x^2*(x^2 + 1)*(x - 1)),x)
[Out] 2*log(x - 1) - log(x - 1i)*(1 + 1i/2) - log(x + 1i)*(1 - 1i/2) - 1/x
```

3.331 $\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3} dx$

Optimal result 2038
Rubi [A] (verified) 2038
Mathematica [A] (verified) 2039
Maple [A] (verified) 2039
Fricas [A] (verification not implemented) 2040
Sympy [A] (verification not implemented) 2040
Maxima [A] (verification not implemented) 2041
Giac [A] (verification not implemented) 2041
Mupad [B] (verification not implemented) 2041

Optimal result

Integrand size = 26, antiderivative size = 23

$$\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3} dx = -\frac{1}{4(1+x^2)^2}+\frac{2}{1+x^2}+\arctan(x)$$

```
[Out] -1/4/(x^2+1)^2+2/(x^2+1)+arctan(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1828, 12, 209}

$$\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3} dx = \arctan(x)+\frac{2}{x^2+1}-\frac{1}{4(x^2+1)^2}$$

```
[In] Int[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]
[Out] -1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{4(1+x^2)^2} - \frac{1}{4}\int \frac{-4+16x-4x^2}{(1+x^2)^2}\,dx \\ &= -\frac{1}{4(1+x^2)^2} + \frac{2}{1+x^2} + \frac{1}{8}\int \frac{8}{1+x^2}\,dx \\ &= -\frac{1}{4(1+x^2)^2} + \frac{2}{1+x^2} + \int \frac{1}{1+x^2}\,dx \\ &= -\frac{1}{4(1+x^2)^2} + \frac{2}{1+x^2} + \tan^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3}\,dx = -\frac{1}{4(1+x^2)^2} + \frac{2}{1+x^2} + \arctan(x)$$

```
[In] Integrate[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]
[Out] -1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2x^2+\frac{7}{4}}{(x^2+1)^2}+\arctan(x)$	19
risch	$\frac{2x^2+\frac{7}{4}}{(x^2+1)^2}+\arctan(x)$	19
parallelrisch	$-\frac{2i\ln(x-i)x^4-2i\ln(x+i)x^4-3+4i\ln(x-i)x^2-4i\ln(x+i)x^2+4x^4+2i\ln(x-i)-2i\ln(x+i)}{4(x^2+1)^2}$	77
meijerg	$\frac{x(3x^2+5)}{8(x^2+1)^2}+\arctan(x)-\frac{x(25x^2+15)}{40(x^2+1)^2}-\frac{x^4}{(x^2+1)^2}-\frac{x(-3x^2+3)}{12(x^2+1)^2}-\frac{3x^2(x^2+2)}{4(x^2+1)^2}$	84

```
[In] int((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x,method=_RETURNVERBOSE)
[Out] (2*x^2+7/4)/(x^2+1)^2+arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3}\,dx=\frac{8\,x^2+4\,(x^4+2\,x^2+1)\arctan(x)+7}{4\,(x^4+2\,x^2+1)}$$

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="fricas")
[Out] 1/4*(8*x^2 + 4*(x^4 + 2*x^2 + 1)*arctan(x) + 7)/(x^4 + 2*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3}\,dx=\frac{8x^2+7}{4x^4+8x^2+4}+\operatorname{atan}(x)$$

```
[In] integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**2+1)**3,x)
[Out] (8*x**2 + 7)/(4*x**4 + 8*x**2 + 4) + atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3}\,dx = \frac{8\,x^2+7}{4\,(x^4+2\,x^2+1)} + \arctan(x)$$

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="maxima")
[Out] 1/4*(8*x^2 + 7)/(x^4 + 2*x^2 + 1) + arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3}\,dx = \frac{8\,x^2+7}{4\,(x^2+1)^2} + \arctan(x)$$

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="giac")
[Out] 1/4*(8*x^2 + 7)/(x^2 + 1)^2 + arctan(x)
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3}\,dx = \operatorname{atan}(x) + \frac{2\,x^2+\frac{7}{4}}{x^4+2\,x^2+1}$$

```
[In] int((2*x^2 - 3*x - 4*x^3 + x^4 + 1)/(x^2 + 1)^3,x)
[Out] atan(x) + (2*x^2 + 7/4)/(2*x^2 + x^4 + 1)
```

3.332 $\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6} \, dx$

Optimal result 2042
Rubi [A] (verified) 2042
Mathematica [A] (verified) 2043
Maple [A] (verified) 2043
Fricas [A] (verification not implemented) 2044
Sympy [A] (verification not implemented) 2044
Maxima [A] (verification not implemented) 2044
Giac [A] (verification not implemented) 2045
Mupad [B] (verification not implemented) 2045

Optimal result

Integrand size = 36, antiderivative size = 23

$$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6} \, dx = -\frac{1}{4(1+x^2)^2}+\frac{2}{1+x^2}+\arctan(x)$$

```
[Out] -1/4/(x^2+1)^2+2/(x^2+1)+arctan(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2098, 267, 209}

$$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6} \, dx = \arctan(x)+\frac{2}{x^2+1}-\frac{1}{4(x^2+1)^2}$$

```
[In] Int[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6),x]
```

```
[Out] -1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 2098

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{x}{(1+x^2)^3} - \frac{4x}{(1+x^2)^2} + \frac{1}{1+x^2}\right) dx\\ &= -\left(4\int \frac{x}{(1+x^2)^2}\,dx\right) + \int \frac{x}{(1+x^2)^3}\,dx + \int \frac{1}{1+x^2}\,dx\\ &= -\frac{1}{4(1+x^2)^2} + \frac{2}{1+x^2} + \tan^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx = -\frac{1}{4(1+x^2)^2} + \frac{2}{1+x^2} + \arctan(x)$$

```
[In] Integrate[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6),x]
[Out] -1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2x^2+\frac{7}{4}}{(x^2+1)^2}+\arctan(x)$	19
risch	$\frac{2x^2+\frac{7}{4}}{x^4+2x^2+1}+\arctan(x)$	24
parallelrisch	$-\frac{2i\ln(x-i)x^4-2i\ln(x+i)x^4-3+4i\ln(x-i)x^2-4i\ln(x+i)x^2+4x^4+2i\ln(x-i)-2i\ln(x+i)}{4(x^4+2x^2+1)}$	82

```
[In] int((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x,method=_RETURNVERBOSE)
[Out] (2*x^2+7/4)/(x^2+1)^2+arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx = \frac{8\,x^2+4\,(x^4+2\,x^2+1)\arctan(x)+7}{4\,(x^4+2\,x^2+1)}$$

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="fricas"
)
[Out] 1/4*(8*x^2 + 4*(x^4 + 2*x^2 + 1)*arctan(x) + 7)/(x^4 + 2*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx = \frac{8x^2+7}{4x^4+8x^2+4}+\operatorname{atan}(x)$$

```
[In] integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**6+3*x**4+3*x**2+1),x)
[Out] (8*x**2 + 7)/(4*x**4 + 8*x**2 + 4) + atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx = \frac{8\,x^2+7}{4\,(x^4+2\,x^2+1)}+\arctan(x)$$

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="maxima"
)
[Out] 1/4*(8*x^2 + 7)/(x^4 + 2*x^2 + 1) + arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx=\frac{8\,x^2+7}{4\,(x^2+1)^2}+\arctan(x)$$

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="giac")
[Out] 1/4*(8*x^2 + 7)/(x^2 + 1)^2 + arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx=\mathrm{atan}(x)+\frac{2\,x^2+\frac{7}{4}}{x^4+2\,x^2+1}$$

```
[In] int((2*x^2 - 3*x - 4*x^3 + x^4 + 1)/(3*x^2 + 3*x^4 + x^6 + 1),x)
[Out] atan(x) + (2*x^2 + 7/4)/(2*x^2 + x^4 + 1)
```

3.333 $\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx$

Optimal result 2046
Rubi [A] (verified) 2046
Mathematica [A] (verified) 2047
Maple [A] (verified) 2047
Fricas [A] (verification not implemented) 2048
Sympy [A] (verification not implemented) 2048
Maxima [A] (verification not implemented) 2048
Giac [A] (verification not implemented) 2048
Mupad [B] (verification not implemented) 2049

Optimal result

Integrand size = 26, antiderivative size = 13

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx=-\frac{1}{x}+\log\left(1+x+x^2\right)$$

```
[Out] -1/x+ln(x^2+x+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1608, 1642, 642}

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx=\log\left(x^2+x+1\right)-\frac{1}{x}$$

```
[In] Int[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4),x]
```

```
[Out] -x^(-1) + Log[1 + x + x^2]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+x+2x^2+2x^3}{x^2(1+x+x^2)}\,dx\\ &= \int \left(\frac{1}{x^2}+\frac{1+2x}{1+x+x^2}\right)dx\\ &= -\frac{1}{x}+\int \frac{1+2x}{1+x+x^2}\,dx\\ &= -\frac{1}{x}+\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx = -\frac{1}{x}+\log\left(1+x+x^2\right)$$

```
[In] Integrate[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4),x]
[Out] -x^(-1) + Log[1 + x + x^2]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{1}{x}+\ln\left(x^2+x+1\right)$	14
norman	$-\frac{1}{x}+\ln\left(x^2+x+1\right)$	14
risch	$-\frac{1}{x}+\ln\left(x^2+x+1\right)$	14
parallelrisch	$\frac{\ln(x^2+x+1)x-1}{x}$	16

```
[In] int((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x,method=_RETURNVERBOSE)
[Out] -1/x+ln(x^2+x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx = \frac{x\log\left(x^2+x+1\right)-1}{x}$$

```
[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="fricas")
[Out] (x*log(x^2 + x + 1) - 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx = \log\left(x^2+x+1\right)-\frac{1}{x}$$

```
[In] integrate((2*x**3+2*x**2+x+1)/(x**4+x**3+x**2),x)
[Out] log(x**2 + x + 1) - 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx = -\frac{1}{x}+\log\left(x^2+x+1\right)$$

```
[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="maxima")
[Out] -1/x + log(x^2 + x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx = -\frac{1}{x}+\log\left(x^2+x+1\right)$$

```
[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="giac")
[Out] -1/x + log(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx = \ln\left(x^2+x+1\right)-\frac{1}{x}$$

```
[In] int((x + 2*x^2 + 2*x^3 + 1)/(x^2 + x^3 + x^4),x)
[Out] log(x + x^2 + 1) - 1/x
```

3.334 $\int \frac{x^2(c+dx)^2}{a+bx^3} dx$

Optimal result . . . 2050
Rubi [A] (verified) . . . 2050
Mathematica [A] (verified) . . . 2053
Maple [C] (verified) . . . 2054
Fricas [C] (verification not implemented) . . . 2054
Sympy [A] (verification not implemented) . . . 2057
Maxima [A] (verification not implemented) . . . 2057
Giac [A] (verification not implemented) . . . 2058
Mupad [B] (verification not implemented) . . . 2058

Optimal result

Integrand size = 20, antiderivative size = 206

$$\int \frac{x^2(c+dx)^2}{a+bx^3} dx = \frac{2cdx}{b} + \frac{d^2x^2}{2b} + \frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}} - \frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}} + \frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}} + \frac{c^2\log(a+bx^3)}{3b}$$

```
[Out] 2*c*d*x/b+1/2*d^2*x^2/b-1/3*a^(1/3)*d*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^
(1/3)*x)/b^(5/3)+1/6*a^(1/3)*d*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/b^(5/3)+1/3*c^2*ln(b*x^3+a)/b+1/3*a^(1/3)*d*(2*b^(1/3
)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)*3^
(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used

= {1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx = \frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}} + \frac{\sqrt[3]{a}d\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)}{\sqrt{3}b^{5/3}} - \frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}} + \frac{c^2\log\left(a+bx^3\right)}{3b} + \frac{2cdx}{b} + \frac{d^2x^2}{2b}$$

```
[In] Int[(x^2*(c + d*x)^2)/(a + b*x^3),x]
```

```
[Out] (2*c*d*x)/b + (d^2*x^2)/(2*b) + (a^(1/3)*d*(2*b^(1/3)*c + a^(1/3)*d)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a^(1/3)*d
*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^(1/3)
*d*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(5/3)) + (c^2*Log[a + b*x^3])/(3*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{2cd}{b}+\frac{d^2x}{b}-\frac{2acd+ad^2x-bc^2x^2}{b\left(a+bx^3\right)}\right)dx\\
&= \frac{2cdx}{b}+\frac{d^2x^2}{2b}-\frac{\int \frac{2acd+ad^2x-bc^2x^2}{a+bx^3}\,dx}{b}\\
&= \frac{2cdx}{b}+\frac{d^2x^2}{2b}-\frac{\int \frac{2acd+ad^2x}{a+bx^3}\,dx}{b}+c^2\int \frac{x^2}{a+bx^3}\,dx
\end{aligned}
$$

$$= \frac{2cdx}{b} + \frac{d^2x^2}{2b} + \frac{c^2\log(a+bx^3)}{3b} - \frac{\int \frac{\sqrt[3]{a}\left(4a\sqrt[3]{b}cd+a^{4/3}d^2\right)+\sqrt[3]{b}\left(-2a\sqrt[3]{b}cd+a^{4/3}d^2\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{3a^{2/3}b^{4/3}} - \frac{\left(\sqrt[3]{a}d\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3b}$$

$$= \frac{2cdx}{b} + \frac{d^2x^2}{2b} - \frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}} + \frac{c^2\log(a+bx^3)}{3b} + \frac{\left(\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{6b^{5/3}} - \frac{\left(a^{2/3}d\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{2b^{4/3}}$$

$$= \frac{2cdx}{b} + \frac{d^2x^2}{2b} - \frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}} + \frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}} + \frac{c^2\log(a+bx^3)}{3b} - \frac{\left(\sqrt[3]{a}d\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{5/3}}$$

$$= \frac{2cdx}{b} + \frac{d^2x^2}{2b} + \frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}} - \frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}} + \frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}} + \frac{c^2\log(a+bx^3)}{3b}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.94

$$\int \frac{x^2(c+dx)^2}{a+bx^3}dx$$

$$= \frac{12b^{2/3}cdx + 3b^{2/3}d^2x^2 + 2\sqrt{3}\sqrt[3]{a}d\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\sqrt[3]{a}d\left(-2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}\right.}{6b^{5/3}}$$

[In] Integrate[(x^2*(c + d*x)^2)/(a + b*x^3),x]

[Out] (12*b^(2/3)*c*d*x + 3*b^(2/3)*d^2*x^2 + 2*Sqrt[3]*a^(1/3)*d*(2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*a^(1/3)*d*(-2*b^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x] - a^(1/3)*d*(-2*b^(1/3)*c + a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(2/3)*c^2*Log[a + b*x^3])/(6*b^(5/3))

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.78 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.33

method	result
risch	$\frac{d^2x^2}{2b}+\frac{2cdx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^3b+a\right)}\frac{\left(\_R^2bc^2-\_Rad^2-2acd\right)\ln\left(x-\_R\right)}{\_R^2}}{3b^2}$
default	$\frac{d\left(\frac{1}{2}dx^2+2cx\right)}{b}+\frac{-2acd\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-ad^2\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\right.}{}\right.}{b}$

[In] int(x^2*(d*x+c)^2/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] 1/2*d^2*x^2/b+2*c*d*x/b+1/3/b^2*sum((_R^2*b*c^2-_R*a*d^2-2*a*c*d)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 4545, normalized size of antiderivative = 22.06

$$\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx=\text{Too large to display}$$

[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="fricas")

[Out] 1/12*(6*d^2*x^2 + 24*c*d*x - 2*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1

```
/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*
c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) -
2*c^2/b)*b*log(1/4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*s
qrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3
)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2
*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b
^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b
^3 + 3*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/
(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 +
(b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (
8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a
*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*c^2 + 5*b*c
^4 + 4*a*c*d^3 + (8*b*c^3*d + a*d^4)*x) + ((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4
 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^
5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5
)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 +
 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqr
t(3) + 1) - 2*c^2/b)*b + 6*c^2 + 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(c^4/b
^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^
3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a
^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 -
 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(
1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4
+ 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5
 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)
^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 +
2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt
(3) + 1) - 2*c^2/b)*b^2*c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3))*log(-1/4*(2*(1/2)
^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8
*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*
b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3
)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^
2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 - 3*(2*(1/2)^(2/3)*(c^4/
b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d
^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 +
a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5
- 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^
(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*c^2 - 5*b*c^4 - 4*a*c*d^3 + 2*(8*b*c^3
*d + a*d^4)*x + 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3
)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4
 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1
/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*
c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) -
2*c^2/b)*b^3 - 6*b^2*c^2)*sqrt(-((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d
^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c
```

```
^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) +
(1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3
)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1)
- 2*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I
*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d
^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*
(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 +
(b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b
^2*c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3)) + ((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 +
2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 -
 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(
1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*
a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3
) + 1) - 2*c^2/b)*b + 6*c^2 - 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(c^4/b^2
- (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*
a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*
d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*
(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3
)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2
*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 -
3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1
/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a
*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3)
 + 1) - 2*c^2/b)*b^2*c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3))*log(-1/4*(2*(1/2)^(2
/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*
c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c
^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a
*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d
^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 - 3*(2*(1/2)^(2/3)*(c^4/b^2
 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)
*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2
*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3
*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/
3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*c^2 - 5*b*c^4 - 4*a*c*d^3 + 2*(8*b*c^3*d
+ a*d^4)*x - 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b
^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 +
2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)
^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2
/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c
^2/b)*b^3 - 6*b^2*c^2)*sqrt(-((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)
/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4
+ 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/
2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c
^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2
*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sq
```

```
rt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)
*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*
c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^
2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*
c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3)))/b
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.67

$$\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx$$
$$=\operatorname{RootSum}\left(27t^3b^5-27t^2b^4c^2+t\left(18ab^2cd^3+9b^3c^4\right)-a^2d^6+2abc^3d^3-b^2c^6,\left(t\mapsto t\log\left(x+\frac{9t^2b^3-18\cdots}{a\cdots}\right.\right.\right.$$
$$+\frac{2cdx}{b}+\frac{d^2x^2}{2b}$$

```
[In] integrate(x**2*(d*x+c)**2/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**5 - 27*_t**2*b**4*c**2 + _t*(18*a*b**2*c*d**3 + 9*b**3*
c**4) - a**2*d**6 + 2*a*b*c**3*d**3 - b**2*c**6, Lambda(_t, _t*log(x + (9*_
t**2*b**3 - 18*_t*b**2*c**2 + 4*a*c*d**3 + 5*b*c**4)/(a*d**4 + 8*b*c**3*d))
)) + 2*c*d*x/b + d**2*x**2/(2*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.97

$$\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx=-\frac{\sqrt{3}\left(ad^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,acd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab}+\frac{d^2x^2+4\,cdx}{2\,b}$$
$$+\frac{\left(2\,bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,acd\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,acd\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(a*d^2*(a/b)^(2/3) + 2*a*c*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/2*(d^2*x^2 + 4*c*d*x)/b + 1/6*(2*
b*c^2*(a/b)^(2/3) - a*d^2*(a/b)^(1/3) + 2*a*c*d)*log(x^2 - x*(a/b)^(1/3) +
(a/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*(b*c^2*(a/b)^(2/3) + a*d^2*(a/b)^(1/3)
 - 2*a*c*d)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.01

$$\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx = \frac{c^2\log\left(|bx^3+a|\right)}{3\,b} - \frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}bcd - \left(-ab^2\right)^{\frac{2}{3}}d^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3} + \frac{bd^2x^2+4\,bcdx}{2\,b^2} - \frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}bcd + \left(-ab^2\right)^{\frac{2}{3}}d^2\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3} + \frac{\left(ab^4d^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,ab^4cd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^5}$$

```
[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*c^2*log(abs(b*x^3 + a))/b - 1/3*sqrt(3)*(2*(-a*b^2)^(1/3)*b*c*d - (-a*b
^2)^(2/3)*d^2)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 +
1/2*(b*d^2*x^2 + 4*b*c*d*x)/b^2 - 1/6*(2*(-a*b^2)^(1/3)*b*c*d + (-a*b^2)^(2
/3)*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 + 1/3*(a*b^4*d^2*(-a/
b)^(1/3) + 2*a*b^4*c*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5)
```

Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.73

$$\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx = \left(\sum_{k=1}^{3}\ln\left(\frac{a\left(b\,c^4+\mathrm{root}(27\,b^5\,z^3-27\,b^4\,c^2\,z^2+18\,a\,b^2\,c\,d^3\,z+9\,b^3\,c^4\,z+2\,a\,b\,c^3\,d^3-b^2\,c^6-a^2\,d^6,z,k)^2\,b^3 \cdots}{\cdots} - 27\,b^4\,c^2\,z^2+18\,a\,b^2\,c\,d^3\,z+9\,b^3\,c^4\,z+2\,a\,b\,c^3\,d^3-b^2\,c^6-a^2\,d^6,z,k)\right)\right) + \frac{d^2\,x^2}{2\,b} + \frac{2\,c\,d\,x}{b}$$

```
[In] int((x^2*(c + d*x)^2)/(a + b*x^3),x)
```

```
[Out] symsum(log((a*(b*c^4 + 9*root(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*
z + 9*b^3*c^4*z + 2*a*b*c^3*d^3 - b^2*c^6 - a^2*d^6, z, k)^2*b^3 - 6*root(2
```

```
7*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*c^3*d^3
 - b^2*c^6 - a^2*d^6, z, k)*b^2*c^2 + 2*a*c*d^3 + a*d^4*x + 2*b*c^3*d*x - 6
*root(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*
c^3*d^3 - b^2*c^6 - a^2*d^6, z, k)*b^2*c*d*x))/b)*root(27*b^5*z^3 - 27*b^4*
c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*c^3*d^3 - b^2*c^6 - a^2*d^
6, z, k), k, 1, 3) + (d^2*x^2)/(2*b) + (2*c*d*x)/b
```

3.335 $\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2} dx$

Optimal result 2060
Rubi [A] (verified) 2060
Mathematica [A] (verified) 2062
Maple [A] (verified) 2062
Fricas [A] (verification not implemented) 2062
Sympy [A] (verification not implemented) 2063
Maxima [F] 2063
Giac [A] (verification not implemented) 2063
Mupad [B] (verification not implemented) 2063

Optimal result

Integrand size = 27, antiderivative size = 45

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2} dx = \frac{5-7x^2}{8(3+2x^2+x^4)} + \frac{9\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] 1/8*(-7*x^2+5)/(x^4+2*x^2+3)+9/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1608, 1677, 1674, 12, 632, 210}

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2} dx = \frac{9\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{5-7x^2}{8(x^4+2x^2+3)}$$

```
[In] Int[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x(-1+2x^2+4x^4)}{(3+2x^2+x^4)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{-1+2x+4x^2}{(3+2x+x^2)^2}\,dx, x, x^2\right) \\
&= \frac{5-7x^2}{8\,(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \frac{18}{3+2x+x^2}\,dx, x, x^2\right) \\
&= \frac{5-7x^2}{8\,(3+2x^2+x^4)} + \frac{9}{8}\text{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx, x, x^2\right)
\end{aligned}
$$

$$= \frac{5-7x^2}{8(3+2x^2+x^4)} - \frac{9}{4}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2(1+x^2)\right)$$

$$= \frac{5-7x^2}{8(3+2x^2+x^4)} + \frac{9\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx = \frac{5-7x^2}{8(3+2x^2+x^4)} + \frac{9\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[In] Integrate[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]
[Out] (5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{-\frac{7x^2}{8}+\frac{5}{8}}{x^4+2x^2+3} + \frac{9\arctan\left(\frac{(x^2+1)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	38
default	$\frac{-\frac{7x^2}{4}+\frac{5}{4}}{2x^4+4x^2+6} + \frac{9\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16}$	41

```
[In] int((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
[Out] (-7/8*x^2+5/8)/(x^4+2*x^2+3)+9/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx = \frac{9\sqrt{2}(x^4+2x^2+3)\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) - 14x^2 + 10}{16(x^4+2x^2+3)}$$

```
[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
[Out] 1/16*(9*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 14*x^2 +
10)/(x^4 + 2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx = \frac{5-7x^2}{8x^4+16x^2+24} + \frac{9\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

```
[In] integrate((4*x**5+2*x**3-x)/(x**4+2*x**2+3)**2,x)
[Out] (5 - 7*x**2)/(8*x**4 + 16*x**2 + 24) + 9*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt
(2)/2)/16
```

Maxima [F]

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx = \int \frac{4\,x^5+2\,x^3-x}{(x^4+2\,x^2+3)^2}\,dx$$

```
[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
[Out] -1/8*(7*x^2 - 5)/(x^4 + 2*x^2 + 3) + 9/4*integrate(x/(x^4 + 2*x^2 + 3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx = \frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) - \frac{7\,x^2-5}{8\,(x^4+2\,x^2+3)}$$

```
[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="giac")
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/8*(7*x^2 - 5)/(x^4 + 2*x^2 +
 3)
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx = \frac{9\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16} - \frac{\frac{7\,x^2}{8}-\frac{5}{8}}{x^4+2\,x^2+3}$$

```
[In] int((2*x^3 - x + 4*x^5)/(2*x^2 + x^4 + 3)^2,x)
[Out] (9*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 - ((7*x^2)/8 - 5/8)/(2*x^2
 + x^4 + 3)
```

3.336 $\int \frac{x+x^5}{(1+2x^2+2x^4)^3} dx$

Optimal result 2064
Rubi [A] (verified) 2064
Mathematica [A] (verified) 2066
Maple [A] (verified) 2066
Fricas [A] (verification not implemented) 2067
Sympy [A] (verification not implemented) 2067
Maxima [F] 2067
Giac [A] (verification not implemented) 2068
Mupad [B] (verification not implemented) 2068

Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3} dx = \frac{3+4x^2}{16(1+2x^2+2x^4)^2} + \frac{1+2x^2}{2(1+2x^2+2x^4)} + \arctan(1+2x^2)$$

```
[Out] 1/16*(4*x^2+3)/(2*x^4+2*x^2+1)^2+1/2*(2*x^2+1)/(2*x^4+2*x^2+1)+arctan(2*x^2
+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1607, 1677, 1674, 12, 628, 631, 210}

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3} dx = \arctan(2x^2+1) + \frac{2x^2+1}{2(2x^4+2x^2+1)} + \frac{4x^2+3}{16(2x^4+2x^2+1)^2}$$

```
[In] Int[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]
```

```
[Out] (3 + 4*x^2)/(16*(1 + 2*x^2 + 2*x^4)^2) + (1 + 2*x^2)/(2*(1 + 2*x^2 + 2*x^4)
) + ArcTan[1 + 2*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x(1+x^4)}{(1+2x^2+2x^4)^3}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{(1+2x+2x^2)^3}\,dx, x, x^2\right) \\
&= \frac{3+4x^2}{16\,(1+2x^2+2x^4)^2} + \frac{1}{16}\text{Subst}\left(\int \frac{16}{(1+2x+2x^2)^2}\,dx, x, x^2\right) \\
&= \frac{3+4x^2}{16\,(1+2x^2+2x^4)^2} + \text{Subst}\left(\int \frac{1}{(1+2x+2x^2)^2}\,dx, x, x^2\right) \\
&= \frac{3+4x^2}{16\,(1+2x^2+2x^4)^2} + \frac{1+2x^2}{2\,(1+2x^2+2x^4)} + \text{Subst}\left(\int \frac{1}{1+2x+2x^2}\,dx, x, x^2\right) \\
&= \frac{3+4x^2}{16\,(1+2x^2+2x^4)^2} + \frac{1+2x^2}{2\,(1+2x^2+2x^4)} - \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+2x^2\right) \\
&= \frac{3+4x^2}{16\,(1+2x^2+2x^4)^2} + \frac{1+2x^2}{2\,(1+2x^2+2x^4)} + \tan^{-1}\left(1+2x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$
\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \frac{11+36x^2+48x^4+32x^6}{16\,(1+2x^2+2x^4)^2} + \arctan\left(1+2x^2\right)
$$

```
[In] Integrate[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]
[Out] (11 + 36*x^2 + 48*x^4 + 32*x^6)/(16*(1 + 2*x^2 + 2*x^4)^2) + ArcTan[1 + 2*x
^2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

method	result
default	$\frac{2x^6+3x^4+\frac{9}{4}x^2+\frac{11}{16}}{(2x^4+2x^2+1)^2}+\arctan\left(2x^2+1\right)$
risch	$\frac{2x^6+3x^4+\frac{9}{4}x^2+\frac{11}{16}}{(2x^4+2x^2+1)^2}+\arctan\left(2x^2+1\right)$
parallelrisch	$-\frac{-8i\ln\left(x^2+\frac{1}{2}+\frac{i}{2}\right)+8i\ln\left(x^2+\frac{1}{2}-\frac{i}{2}\right)-5+64i\ln\left(x^2+\frac{1}{2}-\frac{i}{2}\right)x^4-32i\ln\left(x^2+\frac{1}{2}+\frac{i}{2}\right)x^8+24x^8+32i\ln\left(x^2+\frac{1}{2}-\frac{i}{2}\right)x^2+64i\ln\left(x^2+\frac{1}{2}-\right.}{16(2x^4+2x^2+1)^2}$

```
[In] int((x^5+x)/(2*x^4+2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(x^6+3/2*x^4+9/8*x^2+11/32)/(2*x^4+2*x^2+1)^2+arctan(2*x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \frac{32\,x^6+48\,x^4+36\,x^2+16\,(4\,x^8+8\,x^6+8\,x^4+4\,x^2+1)\arctan\left(2\,x^2+1\right)+11}{16\,(4\,x^8+8\,x^6+8\,x^4+4\,x^2+1)}$$

```
[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 16*(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1)*arc
tan(2*x^2 + 1) + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \frac{32x^6+48x^4+36x^2+11}{64x^8+128x^6+128x^4+64x^2+16}+\operatorname{atan}\left(2x^2+1\right)$$

```
[In] integrate((x**5+x)/(2*x**4+2*x**2+1)**3,x)
```

```
[Out] (32*x**6 + 48*x**4 + 36*x**2 + 11)/(64*x**8 + 128*x**6 + 128*x**4 + 64*x**2
 + 16) + atan(2*x**2 + 1)
```

Maxima [F]

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \int \frac{x^5+x}{(2\,x^4+2\,x^2+1)^3}\,dx$$

```
[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1) +
2*integrate(x/(2*x^4 + 2*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.74 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \frac{32\,x^6+48\,x^4+36\,x^2+11}{16\,(2\,x^4+2\,x^2+1)^2}+\arctan\left(2\,x^2+1\right)$$

```
[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(2*x^4 + 2*x^2 + 1)^2 + arctan(2*x^2 +
 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \operatorname{atan}\left(2\,x^2+1\right)+\frac{\frac{x^6}{2}+\frac{3\,x^4}{4}+\frac{9\,x^2}{16}+\frac{11}{64}}{x^8+2\,x^6+2\,x^4+x^2+\frac{1}{4}}$$

```
[In] int((x + x^5)/(2*x^2 + 2*x^4 + 1)^3,x)
```

```
[Out] atan(2*x^2 + 1) + ((9*x^2)/16 + (3*x^4)/4 + x^6/2 + 11/64)/(x^2 + 2*x^4 + 2
*x^6 + x^8 + 1/4)
```

3.337 $\int \frac{a+bx+cx^2}{d+ex^2+fx^4} dx$

Optimal result . . . 2069
Rubi [A] (verified) . . . 2069
Mathematica [A] (verified) . . . 2071
Maple [C] (verified) . . . 2072
Fricas [C] (verification not implemented) . . . 2072
Sympy [F(-1)] . . . 2073
Maxima [F] . . . 2073
Giac [B] (verification not implemented) . . . 2073
Mupad [B] (verification not implemented) . . . 2074

Optimal result

Integrand size = 25, antiderivative size = 209

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4} dx = \frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}} - \frac{b\operatorname{arctanh}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

```
[Out] -b*arctanh((2*f*x^2+e)/(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)+1/2*arctan(x*
2^(1/2)*f^(1/2)/(e-(-4*d*f+e^2)^(1/2))^(1/2))*(c+(2*a*f-c*e)/(-4*d*f+e^2)^(
1/2))*2^(1/2)/f^(1/2)/(e-(-4*d*f+e^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*f^(
1/2)/(e+(-4*d*f+e^2)^(1/2))^(1/2))*(c+(-2*a*f+c*e)/(-4*d*f+e^2)^(1/2))*2^(1
/2)/f^(1/2)/(e+(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1687, 1180, 211, 12, 1121, 632, 212}

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4} dx = \frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(\frac{ce-2af}{\sqrt{e^2-4df}}+c\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{2}\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}} - \frac{b\operatorname{arctanh}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

```
[In] Int[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x]
```

```
[Out] ((c - (c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e -
Sqrt[e^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqrt[e - Sqrt[e^2 - 4*d*f]]) + ((c +
(c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e + Sqrt[e
^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqrt[e + Sqrt[e^2 - 4*d*f]]) - (b*ArcTanh[(
e + 2*f*x^2)/Sqrt[e^2 - 4*d*f]])/Sqrt[e^2 - 4*d*f]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
```

```
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{bx}{d+ex^2+fx^4}\,dx + \int \frac{a+cx^2}{d+ex^2+fx^4}\,dx\\
&= b\int \frac{x}{d+ex^2+fx^4}\,dx + \frac{1}{2}\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\int \frac{1}{\frac{e}{2}-\frac{1}{2}\sqrt{e^2-4df}+fx^2}\,dx\\
&\quad+\frac{1}{2}\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\int \frac{1}{\frac{e}{2}+\frac{1}{2}\sqrt{e^2-4df}+fx^2}\,dx\\
&= \frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}+\frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}}\\
&\quad+\frac{1}{2}b\text{Subst}\left(\int \frac{1}{d+ex+fx^2}\,dx,x,x^2\right)\\
&= \frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}+\frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}}\\
&\quad-b\text{Subst}\left(\int \frac{1}{e^2-4df-x^2}\,dx,x,e+2fx^2\right)\\
&= \frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}\\
&\quad+\frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}}-\frac{b\tanh^{-1}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.12

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx$$

$$=\frac{\frac{\sqrt{2}\left(2af+c\left(-e+\sqrt{e^2-4df}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}+\frac{\sqrt{2}\left(-2af+c\left(e+\sqrt{e^2-4df}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}}+b\log\left(-e+\sqrt{e^2-\right.}{2\sqrt{e^2-4df}}$$

```
[In] Integrate[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sqrt[f]*x)/S
qrt[e - Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e - Sqrt[e^2 - 4*d*f]]) + (Sqrt[
2]*(-2*a*f + c*(e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e +
 Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e + Sqrt[e^2 - 4*d*f]]) + b*Log[-e + Sq
rt[e^2 - 4*d*f] - 2*f*x^2] - b*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x^2])/(2*Sqr
t[e^2 - 4*d*f])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(f\_Z^4+e\_Z^2+d\right)}\frac{\left(c\_R^2+b\_R+a\right)\ln\left(x-\_R\right)}{2\_R^3 f+\_R e}\right)}{2}$
default	$4f\left(-\frac{\sqrt{-4df+e^2}\left(\frac{b\ln\left(-2fx^2+\sqrt{-4df+e^2}-e\right)}{2}+\frac{\left(-\sqrt{-4df+e^2}\,c-2af+ec\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{fx\sqrt{2}}{\sqrt{\left(\sqrt{-4df+e^2}-e\right)f}}\right)}{2\sqrt{\left(\sqrt{-4df+e^2}-e\right)f}}\right)}{4f(4df-e^2)}-\frac{\sqrt{-4df+e^2}\left(-\frac{b\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right.$

```
[In] int((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum((_R^2*c+_R*b+a)/(2*_R^3*f+_R*e)*ln(x-_R),_R=RootOf(_Z^4*f+_Z^2*e+d)
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.02 (sec) , antiderivative size = 578003, normalized size of antiderivative = 2765.56

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx=\text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)/(f*x**4+e*x**2+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx=\int \frac{cx^2+bx+a}{fx^4+ex^2+d}\,dx$$

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)/(f*x^4 + e*x^2 + d), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1714 vs. 2(171) = 342.

Time = 1.12 (sec) , antiderivative size = 1714, normalized size of antiderivative = 8.20

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx=\text{Too large to display}$$

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="giac")
[Out] -1/2*(e^2*f^2 - 4*d*f^3 - 2*e*f^3 + f^4)*sqrt(e^2 - 4*d*f)*b*log(x^2 + 1/2*
(e - sqrt(e^2 - 4*d*f))/f)/((e^4 - 8*d*e^2*f - 2*e^3*f + 16*d^2*f^2 + 8*d*e
*f^2 + e^2*f^2 - 4*d*f^3)*f^2) + 1/4*((sqrt(2)*sqrt(e*f + sqrt(e^2 - 4*d*f)
*f)*e^4 - 8*sqrt(2)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*e^2*f - 2*sqrt(2)*sqr
t(e*f + sqrt(e^2 - 4*d*f)*f)*e^3*f - 2*e^4*f + 16*sqrt(2)*sqrt(e*f + sqrt(e
^2 - 4*d*f)*f)*d^2*f^2 + 8*sqrt(2)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*e*f^2
+ sqrt(2)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*e^2*f^2 + 16*d*e^2*f^2 + 2*e^3*f^
2 - 4*sqrt(2)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*f^3 - 32*d^2*f^3 - 8*d*e*f^
3 - sqrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*e^3 + 4*sqrt(
2)*sqrt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*e*f + 2*sqrt(2)*sqrt
(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*e^2*f - sqrt(2)*sqrt(e^2 - 4*
d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*e*f^2 + 2*(e^2 - 4*d*f)*e^2*f - 8*(e^2
 - 4*d*f)*d*f^2 - 2*(e^2 - 4*d*f)*e*f^2)*a - 2*(2*d*e^2*f^2 - 8*d^2*f^3 - s
qrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*e^2 + 4*sqrt(2)*
sqrt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d^2*f + 2*sqrt(2)*sqrt(e^
```

```
2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*e*f - sqrt(2)*sqrt(e^2 - 4*d*f
)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*f^2 - 2*(e^2 - 4*d*f)*d*f^2)*c)*arctan(
2*sqrt(1/2)*x/sqrt((e + sqrt(e^2 - 4*d*f))/f))/((d*e^4 - 8*d^2*e^2*f - 2*d*
e^3*f + 16*d^3*f^2 + 8*d^2*e*f^2 + d*e^2*f^2 - 4*d^2*f^3)*abs(f)) + 1/4*((s
qrt(2)*sqrt(e*f - sqrt(e^2 - 4*d*f)*f)*e^4 - 8*sqrt(2)*sqrt(e*f - sqrt(e^2
- 4*d*f)*f)*d*e^2*f - 2*sqrt(2)*sqrt(e*f - sqrt(e^2 - 4*d*f)*f)*e^3*f + 2*e
^4*f + 16*sqrt(2)*sqrt(e*f - sqrt(e^2 - 4*d*f)*f)*d^2*f^2 + 8*sqrt(2)*sqrt(
e*f - sqrt(e^2 - 4*d*f)*f)*d*e*f^2 + sqrt(2)*sqrt(e*f - sqrt(e^2 - 4*d*f)*f
)*e^2*f^2 - 16*d*e^2*f^2 + 2*e^3*f^2 - 4*sqrt(2)*sqrt(e*f - sqrt(e^2 - 4*d*
f)*f)*d*f^3 + 32*d^2*f^3 - 8*d*e*f^3 - sqrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f -
 sqrt(e^2 - 4*d*f)*f)*e^3 + 4*sqrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f - sqrt(e^2
 - 4*d*f)*f)*d*e*f + 2*sqrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f - sqrt(e^2 - 4*d*
f)*f)*e^2*f - sqrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f - sqrt(e^2 - 4*d*f)*f)*e*f
^2 - 2*(e^2 - 4*d*f)*e^2*f + 8*(e^2 - 4*d*f)*d*f^2 - 2*(e^2 - 4*d*f)*e*f^2)
*a - 2*(2*d*e^2*f^2 - 8*d^2*f^3 - sqrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f - sqrt
(e^2 - 4*d*f)*f)*d*e^2 + 4*sqrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f - sqrt(e^2 -
4*d*f)*f)*d^2*f + 2*sqrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f - sqrt(e^2 - 4*d*f)*
f)*d*e*f - sqrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f - sqrt(e^2 - 4*d*f)*f)*d*f^2
- 2*(e^2 - 4*d*f)*d*f^2)*c)*arctan(2*sqrt(1/2)*x/sqrt((e - sqrt(e^2 - 4*d*f
))/f))/((d*e^4 - 8*d^2*e^2*f - 2*d*e^3*f + 16*d^3*f^2 + 8*d^2*e*f^2 + d*e^2
*f^2 - 4*d^2*f^3)*abs(f)) + 1/4*(e^5*f - 8*d*e^3*f^2 - 2*e^4*f^2 + 16*d^2*e
*f^3 + 8*d*e^2*f^3 + e^3*f^3 - 4*d*e*f^4 + (e^4*f - 6*d*e^2*f^2 - 2*e^3*f^2
 + 8*d^2*f^3 + 4*d*e*f^3 + e^2*f^3 - 2*d*f^4)*sqrt(e^2 - 4*d*f))*b*log(x^2
+ 1/2*(e + sqrt(e^2 - 4*d*f))/f)/((d*e^4 - 8*d^2*e^2*f - 2*d*e^3*f + 16*d^3
*f^2 + 8*d^2*e*f^2 + d*e^2*f^2 - 4*d^2*f^3)*f^2)
```

Mupad [B] (verification not implemented)

Time = 10.02 (sec) , antiderivative size = 3942, normalized size of antiderivative = 18.86

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx = \text{Too large to display}$$

```
[In] int((a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x)
```

```
[Out] symsum(log(a*b^2*f^2 - a^2*c*f^2 + b^3*f^2*x - c^3*d*f - 8*root(16*d*e^4*f*
z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d
^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 +
32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z +
 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a
^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*
f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^3*e^3*f^2*x + a*c^2*e*f - 16*roo
t(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z
^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d
^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*
```

c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^
2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2
*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*a*d*f^3 - 4*root(
16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2
*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^
2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*
c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e
*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*a^2*f^3*x + 4*root(16
*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 -
 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f
^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2*
d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c*
d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f
 + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*a*e^2*f^2 + 16*root(1
6*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
- 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*
f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2
*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c
*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*
f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*b*d*f^3*x + 2*root(1
6*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
- 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*
f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2
*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c
*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*
f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*b^2*e*f^2*x + 4*root(1
6*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
- 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*
f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2
*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c
*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*
f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*c^2*d*f^2*x - 2*root(1
6*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
- 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*
f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2
*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c
*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*
f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*c^2*e^2*f*x + 32*root(
16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2
*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^
2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*
c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e
*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^3*d*e*f^3*x - 4*root(

```
16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2
*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^
2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*
c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e
*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*b*e^2*f^2*x + 4*roo
t(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z
^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d
^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*
c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^
2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2
*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*a*b*e*f^2 - 8*root(
16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2
*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^
2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*
c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e
*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*b*c*d*f^2 - 2*a*b*c*f
^2*x + b*c^2*e*f*x + 4*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*
f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*
a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2
 + 4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z -
 16*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d
*e + b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2,
 z, k)*a*c*e*f^2*x)*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3
*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2
*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 +
4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16
*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e
+ b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z,
 k), k, 1, 4)
```

3.338 $\int \frac{(d+ex)^2}{a+bx^2+cx^4} dx$

Optimal result . . . 2077
Rubi [A] (verified) . . . 2077
Mathematica [A] (verified) . . . 2079
Maple [C] (verified) . . . 2080
Fricas [C] (verification not implemented) . . . 2080
Sympy [F(-1)] . . . 2081
Maxima [F] . . . 2081
Giac [B] (verification not implemented) . . . 2081
Mupad [B] (verification not implemented) . . . 2082

Optimal result

Integrand size = 22, antiderivative size = 224

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4} dx = \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{2de\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -2*d*e*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)+1/2*arcta
n(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e^2+(-b*e^2+2*c*d^2)/(-4
*a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*
2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e^2+(b*e^2-2*c*d^2)/(-4*a*c+
b^2)^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1687, 1180, 211, 12, 1121, 632, 212}

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4} dx = \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}+e^2\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{2de\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[In] Int[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((e^2 + (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
((e^2 - (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) -
(2*d*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
```

```
&&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\text{integral} = \int \frac{2dex}{a+bx^2+cx^4}\,dx + \int \frac{d^2+e^2x^2}{a+bx^2+cx^4}\,dx$$
$$= (2de)\int \frac{x}{a+bx^2+cx^4}\,dx + \frac{1}{2}\left(e^2 - \frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx$$
$$+ \frac{1}{2}\left(e^2 + \frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx$$
$$= \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$+ (de)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$- (2de)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)$$
$$= \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+ \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{2de\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx$$
$$= \frac{\frac{\sqrt{2}\left(2cd^2+\left(-b+\sqrt{b^2-4ac}\right)e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(-2cd^2+\left(b+\sqrt{b^2-4ac}\right)e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} + 2de\log\left(-b+\sqrt{b\right.}}{2\sqrt{b^2-4ac}}$$

```
[In] Integrate[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
[Out] ((Sqrt[2]*(2*c*d^2 + (-b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
```

```
qrt[2]*(-2*c*d^2 + (b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 2*d*e
*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - 2*d*e*Log[b + Sqrt[b^2 - 4*a*c] +
2*c*x^2])/(2*Sqrt[b^2 - 4*a*c])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.72 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.24

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\_R^2e^2+2\_Rde+d^2\right)\ln\left(x-\_R\right)}{2c\_R^3+b\_R}\right)}{2}$
default	$4c\left(-\frac{\sqrt{-4ac+b^2}\left(-de\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(e^2\sqrt{-4ac+b^2}+be^2-2cd^2\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4(4ac-b^2)c}-\frac{\sqrt{-4ac+b^2}\left(de\right.}{}\right.$

```
[In] int((e*x+d)^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_
Z^2*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 80.46 (sec) , antiderivative size = 540080, normalized size of antiderivative = 2411.07

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx=\text{Timed out}$$

```
[In] integrate((e*x+d)**2/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx=\int \frac{(ex+d)^2}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((e*x + d)^2/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1724 vs. $2(186) = 372$.

Time = 1.19 (sec) , antiderivative size = 1724, normalized size of antiderivative = 7.70

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] -(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c)*d*e*log(x^2 + 1/2*(b
 - sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c
^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/2*(b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^
2*b*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4 + (b^4*c - 6*a*b^2*c^2 - 2*b^3*
c^2 + 8*a^2*c^3 + 4*a*b*c^3 + b^2*c^3 - 2*a*c^4)*sqrt(b^2 - 4*a*c))*d*e*log
(x^2 + 1/2*(b + sqrt(b^2 - 4*a*c))/c)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 1
6*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2) + 1/4*((sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c + 16*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 +
16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3
- 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c
```

```
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 + 2*(b^2
- 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*d^2 - 2*(2*
a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 -
 4*a*c)*a*c^2)*e^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/(
(a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4
*a^2*c^3)*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c
^2 - 2*(b^2 - 4*a*c)*b*c^2)*d^2 - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e^2)*arctan(2*sqrt(1/
2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 1
6*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 3046, normalized size of antiderivative = 13.60

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x)^2/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log(3*c^2*d^4*e^2 - a*c*e^6 - 8*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^
2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*
a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z
^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*
z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 +
 a^2*e^8, z, k)^3*b^3*c^2*x + 4*c^2*d^3*e^3*x + 4*root(16*a*b^4*c*z^4 - 128
*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^
4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*
a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a
*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4
```

+ c^2*d^8 + a^2*e^8, z, k)^2*b^2*c^2*d^2 + b*c*d^2*e^4 - 4*root(16*a*b^4*c*
z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a
^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4
*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*
e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2
*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*c^3*d^4*x - 16*root(16*a*b^4*c*z^4 - 12
8*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e
^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4
*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*
a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4
 + c^2*d^8 + a^2*e^8, z, k)^2*a*c^3*d^2 + 32*root(16*a*b^4*c*z^4 - 128*a^2*
b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2
 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3
*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*
d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2
*d^8 + a^2*e^8, z, k)^3*a*b*c^3*x + 4*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2
*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a
*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^
2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z
 + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 +
a^2*e^8, z, k)*a*c^2*e^4*x - 2*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 +
256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*
d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b
^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*
c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8
, z, k)*b^2*c*e^4*x + 2*b*c*d*e^5*x - 16*root(16*a*b^4*c*z^4 - 128*a^2*b^2*
c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 1
6*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4
*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^
5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8
 + a^2*e^8, z, k)*a*c^2*d*e^3 + 8*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4
 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c
^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 +
8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2
*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*
e^8, z, k)*b*c^2*d^3*e + 32*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256
*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4
*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*
c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d
^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z
, k)^2*a*c^3*d*e*x + 12*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3
*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2
 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^
5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e
^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)

```
*b*c^2*d^2*e^2*x - 8*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^
3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 +
192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e
*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2
+ 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*
b^2*c^2*d*e*x)*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4
- 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^
2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 3
2*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*
c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k), k, 1, 4
)
```

3.339 $\int \frac{x^2}{(a+bx)(c+dx)}\,dx$

Optimal result 2085
Rubi [A] (verified) 2085
Mathematica [A] (verified) 2086
Maple [A] (verified) 2086
Fricas [A] (verification not implemented) 2086
Sympy [B] (verification not implemented) 2087
Maxima [A] (verification not implemented) 2087
Giac [A] (verification not implemented) 2087
Mupad [B] (verification not implemented) 2088

Optimal result

Integrand size = 18, antiderivative size = 56

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = \frac{x}{bd}+\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}$$

```
[Out] x/b/d+a^2*ln(b*x+a)/b^2/(-a*d+b*c)-c^2*ln(d*x+c)/d^2/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {84}

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = \frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}+\frac{x}{bd}$$

```
[In] Int[x^2/((a + b*x)*(c + d*x)),x]
[Out] x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x])/(d^2*(b
*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{bd}+\frac{a^2}{b(bc-ad)(a+bx)}+\frac{c^2}{d(-bc+ad)(c+dx)}\right)dx\\ &= \frac{x}{bd}+\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = \frac{x}{bd} + \frac{a^2\log(a+bx)}{b^2(bc-ad)} - \frac{c^2\log(c+dx)}{d^2(bc-ad)}$$

```
[In] Integrate[x^2/((a + b*x)*(c + d*x)),x]
[Out] x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x])/(d^2*(b
*c - a*d))
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{x}{bd}+\frac{c^2\ln(dx+c)}{d^2(da-bc)}-\frac{a^2\ln(bx+a)}{b^2(da-bc)}$	57
norman	$\frac{x}{bd}+\frac{c^2\ln(dx+c)}{d^2(da-bc)}-\frac{a^2\ln(bx+a)}{b^2(da-bc)}$	57
risch	$\frac{x}{bd}+\frac{c^2\ln(-dx-c)}{d^2(da-bc)}-\frac{a^2\ln(bx+a)}{b^2(da-bc)}$	60
parallelrisch	$-\frac{a^2\ln(bx+a)d^2-c^2\ln(dx+c)b^2-ab\,d^2x+x\,b^2cd}{b^2d^2(da-bc)}$	62

```
[In] int(x^2/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
[Out] x/b/d+1/d^2*c^2/(a*d-b*c)*ln(d*x+c)-1/b^2*a^2/(a*d-b*c)*ln(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = \frac{a^2d^2\log(bx+a) - b^2c^2\log(dx+c) + (b^2cd-abd^2)x}{b^3cd^2-ab^2d^3}$$

```
[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] (a^2*d^2*log(b*x + a) - b^2*c^2*log(d*x + c) + (b^2*c*d - a*b*d^2)*x)/(b^3*
c*d^2 - a*b^2*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(44) = 88$.

Time = 0.61 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.39

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = -\frac{a^2\log\left(x+\frac{\frac{a^4d^3}{b(ad-bc)}-\frac{2a^3cd^2}{ad-bc}+\frac{a^2bc^2d}{ad-bc}+a^2cd+abc^2}{a^2d^2+b^2c^2}\right)}{b^2\,(ad-bc)} + \frac{c^2\log\left(x+\frac{-\frac{a^2bc^2d}{ad-bc}+a^2cd+\frac{2ab^2c^3}{ad-bc}+abc^2-\frac{b^3c^4}{d(ad-bc)}}{a^2d^2+b^2c^2}\right)}{d^2\,(ad-bc)} + \frac{x}{bd}$$

```
[In] integrate(x**2/(b*x+a)/(d*x+c),x)
```

```
[Out] -a**2*log(x + (a**4*d**3/(b*(a*d - b*c)) - 2*a**3*c*d**2/(a*d - b*c) + a**2
*b*c**2*d/(a*d - b*c) + a**2*c*d + a*b*c**2)/(a**2*d**2 + b**2*c**2))/(b**2
*(a*d - b*c)) + c**2*log(x + (-a**2*b*c**2*d/(a*d - b*c) + a**2*c*d + 2*a*b
**2*c**3/(a*d - b*c) + a*b*c**2 - b**3*c**4/(d*(a*d - b*c)))/(a**2*d**2 + b
**2*c**2))/(d**2*(a*d - b*c)) + x/(b*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = \frac{a^2\log(bx+a)}{b^3c-ab^2d} - \frac{c^2\log(dx+c)}{bcd^2-ad^3} + \frac{x}{bd}$$

```
[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] a^2*log(b*x + a)/(b^3*c - a*b^2*d) - c^2*log(d*x + c)/(b*c*d^2 - a*d^3) + x
/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = \frac{a^2\log(|bx+a|)}{b^3c-ab^2d} - \frac{c^2\log(|dx+c|)}{bcd^2-ad^3} + \frac{x}{bd}$$

```
[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] a^2*log(abs(b*x + a))/(b^3*c - a*b^2*d) - c^2*log(abs(d*x + c))/(b*c*d^2 -
a*d^3) + x/(b*d)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(a+bx)(c+dx)} \, dx = -\frac{a^2 \, d^2 \, \ln \left(a+b \, x\right)-b^2 \, c^2 \, \ln \left(c+d \, x\right)-a \, b \, d^2 \, x+b^2 \, c \, d \, x}{b^2 \, d^2 \, \left(a \, d-b \, c\right)}$$

```
[In] int(x^2/((a + b*x)*(c + d*x)),x)
```

```
[Out] -(a^2*d^2*log(a + b*x) - b^2*c^2*log(c + d*x) - a*b*d^2*x + b^2*c*d*x)/(b^2
*d^2*(a*d - b*c))
```

3.340 $\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx$

Optimal result 2089
Rubi [A] (verified) 2089
Mathematica [A] (verified) 2090
Maple [A] (verified) 2091
Fricas [A] (verification not implemented) 2091
Sympy [F(-1)] 2092
Maxima [A] (verification not implemented) 2092
Giac [A] (verification not implemented) 2092
Mupad [B] (verification not implemented) 2093

Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx=-\frac{\sqrt{a}c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc^2+ad^2)}+\frac{c^2\log(c+dx)}{d(bc^2+ad^2)}+\frac{ad\log(a+bx^2)}{2b(bc^2+ad^2)}$$

```
[Out] c^2*ln(d*x+c)/d/(a*d^2+b*c^2)+1/2*a*d*ln(b*x^2+a)/b/(a*d^2+b*c^2)-c*arctan(
x*b^(1/2)/a^(1/2))*a^(1/2)/(a*d^2+b*c^2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1643, 649, 211, 266}

$$\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx=-\frac{\sqrt{a}c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(ad^2+bc^2)}+\frac{ad\log(a+bx^2)}{2b(ad^2+bc^2)}+\frac{c^2\log(c+dx)}{d(ad^2+bc^2)}$$

```
[In] Int[x^2/((c + d*x)*(a + b*x^2)),x]
```

```
[Out] -((Sqrt[a]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c^2 + a*d^2))) + (c^2
*Log[c + d*x])/(d*(b*c^2 + a*d^2)) + (a*d*Log[a + b*x^2])/(2*b*(b*c^2 + a*d
^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{c^2}{(bc^2+ad^2)(c+dx)} - \frac{a(c-dx)}{(bc^2+ad^2)(a+bx^2)}\right) dx\\ &= \frac{c^2\log(c+dx)}{d(bc^2+ad^2)} - \frac{a\int \frac{c-dx}{a+bx^2}\,dx}{bc^2+ad^2}\\ &= \frac{c^2\log(c+dx)}{d(bc^2+ad^2)} - \frac{(ac)\int \frac{1}{a+bx^2}\,dx}{bc^2+ad^2} + \frac{(ad)\int \frac{x}{a+bx^2}\,dx}{bc^2+ad^2}\\ &= -\frac{\sqrt{a}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc^2+ad^2)} + \frac{c^2\log(c+dx)}{d(bc^2+ad^2)} + \frac{ad\log(a+bx^2)}{2b(bc^2+ad^2)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx = \frac{-2\sqrt{a}\sqrt{b}cd\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + 2bc^2\log(c+dx) + ad^2\log(a+bx^2)}{2b^2c^2d + 2abd^3}$$

```
[In] Integrate[x^2/((c + d*x)*(a + b*x^2)),x]
[Out] (-2*Sqrt[a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*b*c^2*Log[c + d*x]
+ a*d^2*Log[a + b*x^2])/(2*b^2*c^2*d + 2*a*b*d^3)
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

method	result
default	$-\frac{a\left(-\frac{d\ln\left(b\,x^{2}+a\right)}{2b}+\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{a\,d^{2}+b\,c^{2}}+\frac{c^{2}\ln(dx+c)}{d(a\,d^{2}+b\,c^{2})}$
risch	$\frac{\ln\left(\left(-3a^{2}bc\,d^{3}+5a\,b^{2}c^{3}d+\sqrt{-ab}\,a^{2}d^{4}-5\sqrt{-ab}\,ab\,c^{2}d^{2}+2\sqrt{-ab}\,b^{2}c^{4}\right)x-5a^{2}b\,c^{2}d^{2}+3\sqrt{-ab}\,a^{2}c\,d^{3}-5\sqrt{-ab}\,ab\,c^{3}d+a^{3}d^{4}+2a\,b^{2}c^{4}\right)c\sqrt{}}{2(a\,d^{2}+b\,c^{2})b}$

```
[In] int(x^2/(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -a/(a*d^2+b*c^2)*(-1/2*d*ln(b*x^2+a)/b+c/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)
))+c^2*ln(d*x+c)/d/(a*d^2+b*c^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.69

$$\int \frac{x^2}{(c+dx)\,(a+bx^2)}\,dx = \left[\frac{bcd\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+ad^2\log\left(bx^2+a\right)+2\,bc^2\log\left(dx+c\right)}{2\left(b^2c^2d+abd^3\right)},\ -\frac{2\,bcd\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-ad^2\log\left(bx^2+a\right)-2\,bc^2\log\left(dx+c\right)}{2\left(b^2c^2d+abd^3\right)}\right]$$

```
[In] integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(b*c*d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + a*
d^2*log(b*x^2 + a) + 2*b*c^2*log(d*x + c))/(b^2*c^2*d + a*b*d^3), -1/2*(2*b
*c*d*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - a*d^2*log(b*x^2 + a) - 2*b*c^2*log
(d*x + c))/(b^2*c^2*d + a*b*d^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+dx)\,(a+bx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(d*x+c)/(b*x**2+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(c+dx)\,(a+bx^2)}\,dx = \frac{ad\log\left(bx^2+a\right)}{2\,(b^2c^2+abd^2)} + \frac{c^2\log\left(dx+c\right)}{bc^2d+ad^3} - \frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(bc^2+ad^2)\sqrt{ab}}$$

```
[In] integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="maxima")
[Out] 1/2*a*d*log(b*x^2 + a)/(b^2*c^2 + a*b*d^2) + c^2*log(d*x + c)/(b*c^2*d + a*
d^3) - a*c*arctan(b*x/sqrt(a*b))/((b*c^2 + a*d^2)*sqrt(a*b))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(c+dx)\,(a+bx^2)}\,dx = \frac{ad\log\left(bx^2+a\right)}{2\,(b^2c^2+abd^2)} + \frac{c^2\log\left(|dx+c|\right)}{bc^2d+ad^3} - \frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(bc^2+ad^2)\sqrt{ab}}$$

```
[In] integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="giac")
[Out] 1/2*a*d*log(b*x^2 + a)/(b^2*c^2 + a*b*d^2) + c^2*log(abs(d*x + c))/(b*c^2*d
 + a*d^3) - a*c*arctan(b*x/sqrt(a*b))/((b*c^2 + a*d^2)*sqrt(a*b))
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.61

$$\int \frac{x^2}{(c+dx)\,(a+bx^2)}\,dx$$
$$= \frac{\ln\left(a\,c+a\,d\,x+\frac{\left(c\,\sqrt{-a\,b^3}+a\,b\,d\right)\,\left(x\,\left(2\,b^2\,c^2-5\,a\,b\,d^2\right)-5\,a\,b\,c\,d+\frac{2\,b^2\,d\,\left(c\,\sqrt{-a\,b^3}+a\,b\,d\right)\,\left(-b\,x\,c^2+4\,a\,c\,d+3\,a\,x\,d^2\right)}{2\,b^3\,c^2+2\,a\,b^2\,d^2}\right)}{2\,b^3\,c^2+2\,a\,b^2\,d^2}\right)\,\left(c\,\sqrt{-a\,b^3}+\cdots\right.}{2\,b^3\,c^2+2\,a\,b^2\,d^2}$$
$$-\frac{\ln\left(a\,c+a\,d\,x+\frac{\left(c\,\sqrt{-a\,b^3}-a\,b\,d\right)\,\left(b\,x\,\left(5\,a\,d^2-2\,b\,c^2\right)+5\,a\,b\,c\,d+\frac{d\,\left(c\,\sqrt{-a\,b^3}-a\,b\,d\right)\,\left(-b\,x\,c^2+4\,a\,c\,d+3\,a\,x\,d^2\right)}{b\,c^2+a\,d^2}\right)}{2\,b^2\,\left(b\,c^2+a\,d^2\right)}\right)\,\left(c\,\sqrt{-a\,b^3}-a\cdots\right.}{2\,\left(b^3\,c^2+a\,b^2\,d^2\right)}$$
$$+\frac{c^2\,\ln\left(c+d\,x\right)}{b\,c^2\,d+a\,d^3}$$

```
[In] int(x^2/((a + b*x^2)*(c + d*x)),x)
```

```
[Out] (log(a*c + a*d*x + ((c*(-a*b^3)^(1/2) + a*b*d)*(x*(2*b^2*c^2 - 5*a*b*d^2) -
 5*a*b*c*d + (2*b^2*d*(c*(-a*b^3)^(1/2) + a*b*d)*(4*a*c*d + 3*a*d^2*x - b*c
^2*x))/(2*b^3*c^2 + 2*a*b^2*d^2)))/(2*b^3*c^2 + 2*a*b^2*d^2))*(c*(-a*b^3)^(
1/2) + a*b*d))/(2*b^3*c^2 + 2*a*b^2*d^2) - (log(a*c + a*d*x + ((c*(-a*b^3)^
(1/2) - a*b*d)*(b*x*(5*a*d^2 - 2*b*c^2) + 5*a*b*c*d + (d*(c*(-a*b^3)^(1/2)
- a*b*d)*(4*a*c*d + 3*a*d^2*x - b*c^2*x))/(a*d^2 + b*c^2)))/(2*b^2*(a*d^2 +
 b*c^2)))*(c*(-a*b^3)^(1/2) - a*b*d))/(2*(b^3*c^2 + a*b^2*d^2)) + (c^2*log(
c + d*x))/(a*d^3 + b*c^2*d)
```

3.341 $\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx$

Optimal result 2094
Rubi [A] (verified) 2095
Mathematica [A] (verified) 2098
Maple [C] (verified) 2098
Fricas [C] (verification not implemented) 2099
Sympy [F(-1)] 2099
Maxima [A] (verification not implemented) 2099
Giac [A] (verification not implemented) 2100
Mupad [B] (verification not implemented) 2101

Optimal result

Integrand size = 20, antiderivative size = 264

$$\int \frac{x^2}{(c+dx)\left(a+bx^3\right)}\,dx = -\frac{\sqrt[3]{a}d\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}\left(b^{2/3}c^2+\sqrt[3]{a}\sqrt[3]{b}cd+a^{2/3}d^2\right)} + \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)} - \frac{c^2\log(c+dx)}{bc^3-ad^3} - \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}\left(bc^3-ad^3\right)} + \frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)}$$

```
[Out] 1/3*a^(1/3)*d*(b^(1/3)*c+a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)/(-a*d^3+b
*c^3)-c^2*ln(d*x+c)/(-a*d^3+b*c^3)-1/6*a^(1/3)*d*(b^(1/3)*c+a^(1/3)*d)*ln(a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)/(-a*d^3+b*c^3)+1/3*c^2*ln(b*x
^3+a)/(-a*d^3+b*c^3)-1/3*a^(1/3)*d*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)
*3^(1/2))/b^(2/3)/(b^(2/3)*c^2+a^(1/3)*b^(1/3)*c*d+a^(2/3)*d^2)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx=-\frac{\sqrt[3]{a}d\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}\left(a^{2/3}d^2+\sqrt[3]{a}\sqrt[3]{b}cd+b^{2/3}c^2\right)}-\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}\left(bc^3-ad^3\right)}+\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)}+\frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)}-\frac{c^2\log(c+dx)}{bc^3-ad^3}$$

```
[In] Int[x^2/((c + d*x)*(a + b*x^3)),x]
```

```
[Out] -((a^(1/3)*d*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^
(2/3)*(b^(2/3)*c^2 + a^(1/3)*b^(1/3)*c*d + a^(2/3)*d^2))) + (a^(1/3)*d*(b^(
1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(2/3)*(b*c^3 - a*d^3)) -
 (c^2*Log[c + d*x])/(b*c^3 - a*d^3) - (a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(2/3)*(b*c^3 - a*d^3)) +
 (c^2*Log[a + b*x^3])/(3*(b*c^3 - a*d^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{c^2 d}{(bc^3-ad^3)(c+dx)}+\frac{acd-ad^2x+bc^2x^2}{(bc^3-ad^3)(a+bx^3)}\right) dx\\ &= -\frac{c^2\log(c+dx)}{bc^3-ad^3}+\frac{\int \frac{acd-ad^2x+bc^2x^2}{a+bx^3}\,dx}{bc^3-ad^3}\\ &= -\frac{c^2\log(c+dx)}{bc^3-ad^3}+\frac{\int \frac{acd-ad^2x}{a+bx^3}\,dx}{bc^3-ad^3}+\frac{(bc^2)\int \frac{x^2}{a+bx^3}\,dx}{bc^3-ad^3}\end{aligned}$$

$$= -\frac{c^2\log(c+dx)}{bc^3-ad^3}+\frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)}+\frac{\int\frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}cd-a^{4/3}d^2\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}cd-a^{4/3}d^2\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}\left(bc^3-ad^3\right)}$$
$$+\frac{\left(\sqrt[3]{a}d\left(c+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\left(bc^3-ad^3\right)}$$

$$= \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)}-\frac{c^2\log(c+dx)}{bc^3-ad^3}$$
$$+\frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)}+\frac{\left(a^{2/3}d\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{b}\left(b^{2/3}c^2+\sqrt[3]{a}\sqrt[3]{b}cd+a^{2/3}d^2\right)}$$
$$-\frac{\left(\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{2/3}\left(bc^3-ad^3\right)}$$

$$= \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)}-\frac{c^2\log(c+dx)}{bc^3-ad^3}$$
$$-\frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}\left(bc^3-ad^3\right)}$$
$$+\frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)}+\frac{\left(\sqrt[3]{a}d\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{2/3}\left(b^{2/3}c^2+\sqrt[3]{a}\sqrt[3]{b}cd+a^{2/3}d^2\right)}$$

$$= -\frac{\sqrt[3]{a}d\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}\left(b^{2/3}c^2+\sqrt[3]{a}\sqrt[3]{b}cd+a^{2/3}d^2\right)}+\frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)}$$
$$-\frac{c^2\log(c+dx)}{bc^3-ad^3}-\frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}\left(bc^3-ad^3\right)}$$
$$+\frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(c+dx)\,(a+bx^3)}\,dx$$

$$= \frac{2\sqrt{3}\sqrt[3]{a}d\left(-\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-6b^{2/3}c^2\log(c+dx)}{6b^{2/3}\,(bc^3}$$

```
[In] Integrate[x^2/((c + d*x)*(a + b*x^3)),x]
```

```
[Out] (2*Sqrt[3]*a^(1/3)*d*(-(b^(1/3)*c) + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3
)*x] - 6*b^(2/3)*c^2*Log[c + d*x] - a^(1/3)*b^(1/3)*c*d*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*d^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2] + 2*b^(2/3)*c^2*Log[a + b*x^3])/(6*b^(2/3)*(b*c^3 - a*d^3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.84 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.67

method	result
risch	$\frac{c^2\ln(-dx-c)}{a\,d^3-b\,c^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a\,b^2d^3-b^3c^3\right)\_Z^3+3b^2c^2\_Z^2-3bc\_Z\right)}\_R\ln\left(\left(\left(-4a\,b^2d^4-2b^3c^3d\right)\_R^3-3\_R^2b^2c^2d+8\_Rbc\right.\right.\right.}{3}$
default	$\frac{c^2\ln(dx+c)}{a\,d^3-b\,c^3}+\frac{-acd\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+a\,d^2\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right.\right.}{6b\left(\right.}\right.}{a\,d^3-b\,c^3}$

```
[In] int(x^2/(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] c^2/(a*d^3-b*c^3)*ln(-d*x-c)+1/3*sum(_R*ln(((-4*a*b^2*d^4-2*b^3*c^3*d)*_R^3
-3*_R^2*b^2*c^2*d+8*_R*b*c*d-3*d)*x+(-5*a*b^2*c*d^3-b^3*c^4)*_R^3+(a*b*d^3-
b^2*c^3)*_R^2+5*b*c^2*_R-3*c),_R=RootOf(1+(a*b^2*d^3-b^3*c^3)*_Z^3+3*b^2*c^
2*_Z^2-3*b*c*_Z))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 5975, normalized size of antiderivative = 22.63

$$\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(d*x+c)/(b*x**3+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx = &-\frac{c^2\log(dx+c)}{bc^3-ad^3} \\ &- \frac{\sqrt{3}\left(ad^2\left(\frac{a}{b}\right)^{\frac{2}{3}} - acd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}} - abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &+ \frac{\left(2\,bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}} - ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}} - acd\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}} - abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} \\ &+ \frac{\left(bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}} + ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}} + acd\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}} - abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}\end{aligned}$$

```
[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -c^2*log(d*x + c)/(b*c^3 - a*d^3) - 1/3*sqrt(3)*(a*d^2*(a/b)^(2/3) - a*c*d*
(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c^3*
(a/b)^(2/3) - a*b*d^3*(a/b)^(2/3))*(a/b)^(1/3)) + 1/6*(2*b*c^2*(a/b)^(2/3)
- a*d^2*(a/b)^(1/3) - a*c*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*c^
3*(a/b)^(2/3) - a*b*d^3*(a/b)^(2/3)) + 1/3*(b*c^2*(a/b)^(2/3) + a*d^2*(a/b)
^(1/3) + a*c*d)*log(x + (a/b)^(1/3))/(b^2*c^3*(a/b)^(2/3) - a*b*d^3*(a/b)^(
2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx$$
$$= -\frac{c^2d\log(|dx+c|)}{bc^3d-ad^4}+\frac{c^2\log(|bx^3+a|)}{3(bc^3-ad^3)}+\frac{(-ab^2)^{\frac{1}{3}}\,d\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2-\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\,bcd+\sqrt{3}\,(-ab^2)^{\frac{2}{3}}\,d^2}$$
$$+\frac{\left(ab^2c^3d^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2bd^5\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^2c^4d+a^2bcd^4\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(ab^3c^6-2\,a^2b^2c^3d^3+a^3bd^6\right)}$$
$$+\frac{\left((-ab^2)^{\frac{1}{3}}\,bcd-(-ab^2)^{\frac{2}{3}}\,d^2\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^3c^3-ab^2d^3\right)}$$

```
[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -c^2*d*log(abs(d*x + c))/(b*c^3*d - a*d^4) + 1/3*c^2*log(abs(b*x^3 + a))/(b
*c^3 - a*d^3) + (-a*b^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/(sqrt(3)*b^2*c^2 - sqrt(3)*(-a*b^2)^(1/3)*b*c*d + sqrt(3)*(-a*b
^2)^(2/3)*d^2) + 1/3*(a*b^2*c^3*d^2*(-a/b)^(1/3) - a^2*b*d^5*(-a/b)^(1/3) -
 a*b^2*c^4*d + a^2*b*c*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3*
c^6 - 2*a^2*b^2*c^3*d^3 + a^3*b*d^6) + 1/6*((-a*b^2)^(1/3)*b*c*d - (-a*b^2)
^(2/3)*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^3*c^3 - a*b^2*d^3)
```

Mupad [B] (verification not implemented)

Time = 9.58 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.16

$$\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(-a\,b\,d\,\left(c+d\,x+\mathrm{root}\left(27\,a\,b^2\,d^3\,z^3-27\,b^3\,c^3\,z^3+27\,b^2\,c^2\,z^2-9\,b\,c\,z+1,z,k\right)^2 b^2\,c^3\,3+\mathrm{root}\left(27\,a\,b^2\,d^3\,z^3-27\,b^3\,c^3\,z^3+27\,b^2\,c^2\,z^2-9\,b\,c\,z+1,z,k\right)\right)\right) + \frac{c^2\,\ln(c+d\,x)}{a\,d^3-b\,c^3}$$

```
[In] int(x^2/((a + b*x^3)*(c + d*x)),x)
```

```
[Out] symsum(log(-a*b*d*(c + d*x + 3*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*
b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^2*b^2*c^3 + 9*root(27*a*b^2*d^3*z^3 - 27*b
^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*b^3*c^4 - 5*root(27*a*b^
2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)*b*c^2 - 3*
root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k
)^2*a*b*d^3 - 8*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9
*b*c*z + 1, z, k)*b*c*d*x + 45*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*
b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*a*b^2*c*d^3 + 36*root(27*a*b^2*d^3*z^3 -
 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*a*b^2*d^4*x + 9*roo
t(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^2
*b^2*c^2*d*x + 18*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 -
 9*b*c*z + 1, z, k)^3*b^3*c^3*d*x))*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3
+ 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k), k, 1, 3) + (c^2*log(c + d*x))/(a*d^3
 - b*c^3)
```

3.342 $\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx$

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Giac [A] (verification not implemented) 2109
Mupad [B] (verification not implemented) 2110

Optimal result

Integrand size = 20, antiderivative size = 417

$$
\begin{aligned}
\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx = {} & \frac{\sqrt{a}d^3\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,(bc^4+ad^4)} - \frac{c\left(\sqrt{b}c^2-\sqrt{a}d^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(bc^4+ad^4)} \\
& + \frac{c\left(\sqrt{b}c^2-\sqrt{a}d^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(bc^4+ad^4)} + \frac{c^2d\log(c+dx)}{bc^4+ad^4} \\
& + \frac{c\left(\sqrt{b}c^2+\sqrt{a}d^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(bc^4+ad^4)} \\
& - \frac{c\left(\sqrt{b}c^2+\sqrt{a}d^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(bc^4+ad^4)} \\
& - \frac{c^2d\log\left(a+bx^4\right)}{4\,(bc^4+ad^4)}
\end{aligned}
$$

```
[Out] c^2*d*ln(d*x+c)/(a*d^4+b*c^4)-1/4*c^2*d*ln(b*x^4+a)/(a*d^4+b*c^4)+1/2*d^3*a
rctan(x^2*b^(1/2)/a^(1/2))*a^(1/2)/(a*d^4+b*c^4)/b^(1/2)+1/4*c*arctan(-1+b^
(1/4)*x*2^(1/2)/a^(1/4))*(-d^2*a^(1/2)+b^(1/2)*c^2)/a^(1/4)/b^(1/4)/(a*d^4+
b*c^4)*2^(1/2)+1/4*c*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(-d^2*a^(1/2)+b^(1
/2)*c^2)/a^(1/4)/b^(1/4)/(a*d^4+b*c^4)*2^(1/2)+1/8*c*ln(-a^(1/4)*b^(1/4)*x*
2^(1/2)+a^(1/2)+x^2*b^(1/2))*(d^2*a^(1/2)+b^(1/2)*c^2)/a^(1/4)/b^(1/4)/(a*d
^4+b*c^4)*2^(1/2)-1/8*c*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(
d^2*a^(1/2)+b^(1/2)*c^2)/a^(1/4)/b^(1/4)/(a*d^4+b*c^4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6857, 1475, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 211, 266}

$$\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx = \frac{\sqrt{a}d^3\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,(ad^4+bc^4)}-\frac{c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}c^2-\sqrt{a}d^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(ad^4+bc^4)}$$
$$+\frac{c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}c^2-\sqrt{a}d^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(ad^4+bc^4)}$$
$$-\frac{c^2d\log(a+bx^4)}{4\,(ad^4+bc^4)}+\frac{c^2d\log(c+dx)}{ad^4+bc^4}$$
$$+\frac{c\left(\sqrt{a}d^2+\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(ad^4+bc^4)}$$
$$-\frac{c\left(\sqrt{a}d^2+\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(ad^4+bc^4)}$$

```
[In] Int[x^2/((c + d*x)*(a + b*x^4)),x]
```

```
[Out] (Sqrt[a]*d^3*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[b]*(b*c^4 + a*d^4)) - (
c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*S
qrt[2]*a^(1/4)*b^(1/4)*(b*c^4 + a*d^4)) + (c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c^4 +
a*d^4)) + (c^2*d*Log[c + d*x])/(b*c^4 + a*d^4) + (c*(Sqrt[b]*c^2 + Sqrt[a]*
d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(
1/4)*b^(1/4)*(b*c^4 + a*d^4)) - (c*(Sqrt[b]*c^2 + Sqrt[a]*d^2)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c
^4 + a*d^4)) - (c^2*d*Log[a + b*x^4])/(4*(b*c^4 + a*d^4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1475

```
Int[((A_) + (B_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*
(x_)^(n2_))^(p_.), x_Symbol] :> Dist[A, Int[(d + e*x^n)^q*(a + c*x^(2*n))^p
, x], x] + Dist[B, Int[x^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ
[{a, c, d, e, A, B, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[m - n + 1, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c^2d^2}{(bc^4+ad^4)(c+dx)}+\frac{(c-dx)(-ad^2+bc^2x^2)}{(bc^4+ad^4)(a+bx^4)}\right)dx\\
&= \frac{c^2d\log(c+dx)}{bc^4+ad^4}+\frac{\int \frac{(c-dx)(-ad^2+bc^2x^2)}{a+bx^4}\,dx}{bc^4+ad^4}\\
&= \frac{c^2d\log(c+dx)}{bc^4+ad^4}+\frac{c\int \frac{-ad^2+bc^2x^2}{a+bx^4}\,dx}{bc^4+ad^4}-\frac{d\int \frac{x(-ad^2+bc^2x^2)}{a+bx^4}\,dx}{bc^4+ad^4}\\
&= \frac{c^2d\log(c+dx)}{bc^4+ad^4}-\frac{d\text{Subst}\left(\int \frac{-ad^2+bc^2x}{a+bx^2}\,dx,x,x^2\right)}{2(bc^4+ad^4)}\\
&\quad+\frac{\left(c\left(c^2-\frac{\sqrt{a}d^2}{\sqrt{b}}\right)\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2(bc^4+ad^4)}-\frac{\left(c\left(c^2+\frac{\sqrt{a}d^2}{\sqrt{b}}\right)\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2(bc^4+ad^4)}\\
&= \frac{c^2d\log(c+dx)}{bc^4+ad^4}-\frac{(bc^2d)\,\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx,x,x^2\right)}{2(bc^4+ad^4)}+\frac{(ad^3)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{2(bc^4+ad^4)}\\
&\quad+\frac{\left(c\left(c^2-\frac{\sqrt{a}d^2}{\sqrt{b}}\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4(bc^4+ad^4)}+\frac{\left(c\left(c^2-\frac{\sqrt{a}d^2}{\sqrt{b}}\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4(bc^4+ad^4)}\\
&\quad+\frac{\left(\sqrt[4]{b}c\left(c^2+\frac{\sqrt{a}d^2}{\sqrt{b}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}(bc^4+ad^4)}+\frac{\left(\sqrt[4]{b}c\left(c^2+\frac{\sqrt{a}d^2}{\sqrt{b}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}(bc^4+ad^4)}
\end{aligned}$$

$$= \frac{\sqrt{a}d^3\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,(bc^4+ad^4)} + \frac{c^2d\log(c+dx)}{bc^4+ad^4} + \frac{\sqrt[4]{b}c\left(c^2+\frac{\sqrt{a}d^2}{\sqrt{b}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(bc^4+ad^4)} - \frac{\sqrt[4]{b}c\left(c^2+\frac{\sqrt{a}d^2}{\sqrt{b}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(bc^4+ad^4)} - \frac{c^2d\log(a+bx^4)}{4\,(bc^4+ad^4)} + \frac{\left(\sqrt[4]{b}c\left(c^2-\frac{\sqrt{a}d^2}{\sqrt{b}}\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(bc^4+ad^4)} - \frac{\left(\sqrt[4]{b}c\left(c^2-\frac{\sqrt{a}d^2}{\sqrt{b}}\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(bc^4+ad^4)}$$

$$= \frac{\sqrt{a}d^3\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,(bc^4+ad^4)} - \frac{\sqrt[4]{b}c\left(c^2-\frac{\sqrt{a}d^2}{\sqrt{b}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(bc^4+ad^4)} + \frac{\sqrt[4]{b}c\left(c^2-\frac{\sqrt{a}d^2}{\sqrt{b}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(bc^4+ad^4)} + \frac{c^2d\log(c+dx)}{bc^4+ad^4} + \frac{\sqrt[4]{b}c\left(c^2+\frac{\sqrt{a}d^2}{\sqrt{b}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(bc^4+ad^4)} - \frac{\sqrt[4]{b}c\left(c^2+\frac{\sqrt{a}d^2}{\sqrt{b}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(bc^4+ad^4)} - \frac{c^2d\log(a+bx^4)}{4\,(bc^4+ad^4)}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(c+dx)\,(a+bx^4)}\,dx = \frac{-2\left(\sqrt{2}b^{3/4}c^3-\sqrt{2}\sqrt{a}\sqrt[4]{b}cd^2+2a^{3/4}d^3\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}b^{3/4}c^3-\sqrt{2}\sqrt{a}\sqrt[4]{b}cd^2-2a^{3/4}d^3\right)\text{ar}$$

```
[In] Integrate[x^2/((c + d*x)*(a + b*x^4)),x]
```

```
[Out] (-2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]*Sqrt[a]*b^(1/4)*c*d^2 + 2*a^(3/4)*d^3)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]*S
qrt[a]*b^(1/4)*c*d^2 - 2*a^(3/4)*d^3)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
```

```
)] + b^(1/4)*c*(8*a^(1/4)*b^(1/4)*c*d*Log[c + d*x] + Sqrt[2]*(Sqrt[b]*c^2 +
 Sqrt[a]*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Sqrt
[2]*Sqrt[b]*c^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Sq
rt[2]*Sqrt[a]*d^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] -
2*a^(1/4)*b^(1/4)*c*d*Log[a + b*x^4]))/(8*a^(1/4)*Sqrt[b]*(b*c^4 + a*d^4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.84 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.54

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(1+\left(a^{2} b^{2} d^{4}+a\, b^{3} c^{4}\right)\_Z^{4}+4 a\, b^{2} c^{2} d\,\_Z^{3}+2 a\,\_Z^{2} d^{2} b\right)}\_R \ln\left(\left(\left(\left(5 a^{2} b^{2} d^{6}-3 a\, b^{3} c^{4} d^{2}\right)\_R^{4}+10\,\_R^{3} a\, b^{2} c^{2} d^{3}+\left(9 a b\, d^{4}+\right.\right.\right.\right.}{4}$
default	$\frac{-\frac{d^{2} c \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8}+\frac{a\, d^{3} \arctan\left(x^{2} \sqrt{\frac{b}{a}}\right)}{2 \sqrt{a b}}+\frac{c^{3} \sqrt{2}\, \left(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}}\right.\right.}{}}{a\, d^{4}+b\, c^{4}}$

```
[In] int(x^2/(d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R*ln(((5*a^2*b^2*d^6-3*a*b^3*c^4*d^2)*_R^4+10*_R^3*a*b^2*c^2*d^3+(
9*a*b*d^4+b^2*c^4)*_R^2-5*_R*b*c^2*d+4*d^2)*x+(6*a^2*b^2*c*d^5-2*a*b^3*c^5*
d)*_R^4+6*a*b^2*c^3*d^2*_R^3+8*a*b*c*d^3*_R^2-b*c^3*_R+4*c*d),_R=RootOf(1+(
a^2*b^2*d^4+a*b^3*c^4)*_Z^4+4*a*b^2*c^2*d*_Z^3+2*a*_Z^2*d^2*b))+c^2*d*ln(d*
x+c)/(a*d^4+b*c^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 23.56 (sec) , antiderivative size = 259898, normalized size of antiderivative = 623.26

$$\int \frac{x^2}{(c+dx)\left(a+bx^4\right)}\,dx=\text{Too large to display}$$

```
[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(d*x+c)/(b*x**4+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx = \frac{c^2 d\log(dx+c)}{bc^4+ad^4}$$

$$-\frac{\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}c^2d+\sqrt{a}b^{\frac{3}{2}}c^3+abcd^2\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}c^2d-\sqrt{a}b^{\frac{3}{2}}c^3-abcd^2\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}-\frac{2\left(\sqrt{\cdots}\right.}{\cdots}}{8\left(bc^4\cdots\right.}$$

```
[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="maxima")
[Out] c^2*d*log(d*x + c)/(b*c^4 + a*d^4) - 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*
c^2*d + sqrt(a)*b^(3/2)*c^3 + a*b*c*d^2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*
b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*c
^2*d - sqrt(a)*b^(3/2)*c^3 - a*b*c*d^2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b
^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) - 2*(sqrt(2)*a^(3/4)*b^(7/4)*c^3 - sq
rt(2)*a^(5/4)*b^(5/4)*c*d^2 - 2*a^(3/2)*b*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b
)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)
*sqrt(b))*b^(5/4)) - 2*(sqrt(2)*a^(3/4)*b^(7/4)*c^3 - sqrt(2)*a^(5/4)*b^(5/
4)*c*d^2 + 2*a^(3/2)*b*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/
4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)))
/(b*c^4 + a*d^4)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx = \frac{c^2d^2\log(|dx+c|)}{bc^4d+ad^5} - \frac{c^2d\log(|bx^4+a|)}{4(bc^4+ad^4)} - \frac{\left(\sqrt{2}ab^2d-(ab^3)^{\frac{3}{4}}c\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^3c^2+\sqrt{2}\sqrt{ab}ab^2d^2-2(ab^3)^{\frac{1}{4}}ab^2cd\right)} + \frac{\left(\sqrt{2}ab^2d+(ab^3)^{\frac{3}{4}}c\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^3c^2+\sqrt{2}\sqrt{ab}ab^2d^2+2(ab^3)^{\frac{1}{4}}ab^2cd\right)} - \frac{\left((ab^3)^{\frac{1}{4}}abcd^2+(ab^3)^{\frac{3}{4}}c^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^3c^4+\sqrt{2}a^2b^2d^4\right)} + \frac{\left((ab^3)^{\frac{1}{4}}abcd^2+(ab^3)^{\frac{3}{4}}c^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^3c^4+\sqrt{2}a^2b^2d^4\right)}$$

```
[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] c^2*d^2*log(abs(d*x + c))/(b*c^4*d + a*d^5) - 1/4*c^2*d*log(abs(b*x^4 + a))
/(b*c^4 + a*d^4) - 1/2*(sqrt(2)*a*b^2*d - (a*b^3)^(3/4)*c)*arctan(1/2*sqrt(
2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^2 + sqrt(2)*sq
rt(a*b)*a*b^2*d^2 - 2*(a*b^3)^(1/4)*a*b^2*c*d) + 1/2*(sqrt(2)*a*b^2*d + (a*
b^3)^(3/4)*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(
sqrt(2)*a*b^3*c^2 + sqrt(2)*sqrt(a*b)*a*b^2*d^2 + 2*(a*b^3)^(1/4)*a*b^2*c*d
) - 1/4*((a*b^3)^(1/4)*a*b*c*d^2 + (a*b^3)^(3/4)*c^3)*log(x^2 + sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^3*c^4 + sqrt(2)*a^2*b^2*d^4) + 1/4*((a
*b^3)^(1/4)*a*b*c*d^2 + (a*b^3)^(3/4)*c^3)*log(x^2 - sqrt(2)*x*(a/b)^(1/4)
+ sqrt(a/b))/(sqrt(2)*a*b^3*c^4 + sqrt(2)*a^2*b^2*d^4)
```

Mupad [B] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 823, normalized size of antiderivative = 1.97

$$\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx$$

$$= \left(\sum_{k=1}^{4} \ln\left(a\,b^2\,d\left(c\,d+d^2\,x-\mathrm{root}\left(256\,a^2\,b^2\,d^4\,z^4+256\,a\,b^3\,c^4\,z^4+256\,a\,b^2\,c^2\,d\,z^3+32\,a\,b\,d^2\,z^2+1,z,k\right)\,b\,c^3+\mathrm{ro}\right.\right.\right.$$

$$\left.+256\,a\,b^3\,c^4\,z^4+256\,a\,b^2\,c^2\,d\,z^3+32\,a\,b\,d^2\,z^2+1,z,k)\right)+\frac{c^2\,d\,\ln\left(c+d\,x\right)}{b\,c^4+a\,d^4}$$

```
[In] int(x^2/((a + b*x^4)*(c + d*x)),x)
```

```
[Out] symsum(log(a*b^2*d*(c*d + d^2*x - root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*
z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)*b*c^3 + 4*root(256*a^
2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 +
1, z, k)^2*b^2*c^4*x + 36*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 25
6*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^2*a*b*d^4*x - 128*root(256*a^
2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 +
1, z, k)^4*a*b^3*c^5*d - 5*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 2
56*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)*b*c^2*d*x + 96*root(256*a^2*
b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1,
 z, k)^3*a*b^2*c^3*d^2 + 384*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 +
 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^4*a^2*b^2*c*d^5 + 320*root
(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2
*z^2 + 1, z, k)^4*a^2*b^2*d^6*x + 32*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c
^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^2*a*b*c*d^3 + 160*
root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b
*d^2*z^2 + 1, z, k)^3*a*b^2*c^2*d^3*x - 192*root(256*a^2*b^2*d^4*z^4 + 256*
a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^4*a*b^3*c^4
*d^2*x))*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3
 + 32*a*b*d^2*z^2 + 1, z, k), k, 1, 4) + (c^2*d*log(c + d*x))/(a*d^4 + b*c^
4)
```

3.343 $\int \frac{x}{(1-x)(1+x)^2} \, dx$

Optimal result 2111
Rubi [A] (verified) 2111
Mathematica [A] (verified) 2112
Maple [A] (verified) 2112
Fricas [B] (verification not implemented) 2113
Sympy [A] (verification not implemented) 2113
Maxima [A] (verification not implemented) 2113
Giac [A] (verification not implemented) 2113
Mupad [B] (verification not implemented) 2114

Optimal result

Integrand size = 14, antiderivative size = 16

$$\int \frac{x}{(1-x)(1+x)^2} \, dx = \frac{1}{2(1+x)} + \frac{\operatorname{arctanh}(x)}{2}$$

```
[Out] 1/2/(1+x)+1/2*arctanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {78, 213}

$$\int \frac{x}{(1-x)(1+x)^2} \, dx = \frac{\operatorname{arctanh}(x)}{2} + \frac{1}{2(x+1)}$$

```
[In] Int[x/((1 - x)*(1 + x)^2),x]
```

```
[Out] 1/(2*(1 + x)) + ArcTanh[x]/2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2(1+x)^2}-\frac{1}{2\left(-1+x^2\right)}\right) dx \\
&= \frac{1}{2(1+x)}-\frac{1}{2}\int \frac{1}{-1+x^2}\,dx \\
&= \frac{1}{2(1+x)}+\frac{1}{2}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{x}{(1-x)(1+x)^2}\,dx=\frac{1}{4}\left(\frac{2}{1+x}-\log(1-x)+\log(1+x)\right)$$

```
[In] Integrate[x/((1 - x)*(1 + x)^2),x]
[Out] (2/(1 + x) - Log[1 - x] + Log[1 + x])/4
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

method	result	size
default	$\frac{1}{2x+2}+\frac{\ln(x+1)}{4}-\frac{\ln(x-1)}{4}$	21
norman	$\frac{1}{2x+2}+\frac{\ln(x+1)}{4}-\frac{\ln(x-1)}{4}$	21
risch	$\frac{1}{2x+2}+\frac{\ln(x+1)}{4}-\frac{\ln(x-1)}{4}$	21
parallelrisch	$-\frac{\ln(x-1)x-\ln(x+1)x-2+\ln(x-1)-\ln(x+1)}{4(x+1)}$	33

```
[In] int(x/(1-x)/(x+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/2/(x+1)+1/4*ln(x+1)-1/4*ln(x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{x}{(1-x)(1+x)^2} dx = \frac{(x+1)\log(x+1) - (x+1)\log(x-1) + 2}{4(x+1)}$$

```
[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="fricas")
[Out] 1/4*((x + 1)*log(x + 1) - (x + 1)*log(x - 1) + 2)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{x}{(1-x)(1+x)^2} dx = -\frac{\log(x-1)}{4} + \frac{\log(x+1)}{4} + \frac{1}{2x+2}$$

```
[In] integrate(x/(1-x)/(1+x)**2,x)
[Out] -log(x - 1)/4 + log(x + 1)/4 + 1/(2*x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{x}{(1-x)(1+x)^2} dx = \frac{1}{2(x+1)} + \frac{1}{4}\log(x+1) - \frac{1}{4}\log(x-1)$$

```
[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="maxima")
[Out] 1/2/(x + 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \frac{x}{(1-x)(1+x)^2} dx = \frac{1}{2(x+1)} - \frac{1}{4}\log\left(\left|-\frac{2}{x+1} + 1\right|\right)$$

```
[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="giac")
[Out] 1/2/(x + 1) - 1/4*log(abs(-2/(x + 1) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{(1-x)(1+x)^2} \, dx = \frac{\operatorname{atanh}(x)}{2} + \frac{1}{2\,(x+1)}$$

```
[In] int(-x/((x - 1)*(x + 1)^2),x)
[Out] atanh(x)/2 + 1/(2*(x + 1))
```

3.344 $\int \frac{x^2}{(1-x^2)(1+x^2)^2} dx$

Optimal result 2115
Rubi [A] (verified) 2115
Mathematica [A] (verified) 2116
Maple [A] (verified) 2116
Fricas [B] (verification not implemented) 2117
Sympy [A] (verification not implemented) 2117
Maxima [A] (verification not implemented) 2117
Giac [A] (verification not implemented) 2117
Mupad [B] (verification not implemented) 2118

Optimal result

Integrand size = 20, antiderivative size = 19

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2} dx = -\frac{x}{4(1+x^2)} + \frac{\text{arctanh}(x)}{4}$$

```
[Out] -1/4*x/(x^2+1)+1/4*arctanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {482, 212}

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2} dx = \frac{\text{arctanh}(x)}{4} - \frac{x}{4(x^2+1)}$$

```
[In] Int[x^2/((1 - x^2)*(1 + x^2)^2),x]
```

```
[Out] -1/4*x/(1 + x^2) + ArcTanh[x]/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
```

```
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x}{4(1+x^2)}+\frac{1}{4}\int\frac{1}{1-x^2}\,dx\\ &= -\frac{x}{4(1+x^2)}+\frac{1}{4}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int\frac{x^2}{(1-x^2)(1+x^2)^2}\,dx=\frac{1}{8}\left(-\frac{2x}{1+x^2}-\log(1-x)+\log(1+x)\right)$$

```
[In] Integrate[x^2/((1 - x^2)*(1 + x^2)^2),x]
[Out] ((-2*x)/(1 + x^2) - Log[1 - x] + Log[1 + x])/8
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\ln(x+1)}{8}-\frac{x}{4(x^2+1)}-\frac{\ln(x-1)}{8}$	24
norman	$\frac{\ln(x+1)}{8}-\frac{x}{4(x^2+1)}-\frac{\ln(x-1)}{8}$	24
risch	$\frac{\ln(x+1)}{8}-\frac{x}{4(x^2+1)}-\frac{\ln(x-1)}{8}$	24
parallelrisch	$-\frac{\ln(x-1)x^2-\ln(x+1)x^2+\ln(x-1)-\ln(x+1)+2x}{8(x^2+1)}$	41

```
[In] int(x^2/(-x^2+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/8*ln(x+1)-1/4*x/(x^2+1)-1/8*ln(x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(15) = 30.

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2} dx = \frac{(x^2+1)\log(x+1) - (x^2+1)\log(x-1) - 2x}{8(x^2+1)}$$

```
[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/8*((x^2 + 1)*log(x + 1) - (x^2 + 1)*log(x - 1) - 2*x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2} dx = -\frac{x}{4x^2+4} - \frac{\log(x-1)}{8} + \frac{\log(x+1)}{8}$$

```
[In] integrate(x**2/(-x**2+1)/(x**2+1)**2,x)
[Out] -x/(4*x**2 + 4) - log(x - 1)/8 + log(x + 1)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2} dx = -\frac{x}{4(x^2+1)} + \frac{1}{8}\log(x+1) - \frac{1}{8}\log(x-1)$$

```
[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/4*x/(x^2 + 1) + 1/8*log(x + 1) - 1/8*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2} dx = -\frac{1}{4\left(x+\frac{1}{x}\right)} + \frac{1}{16}\log\left(\left|x+\frac{1}{x}+2\right|\right) - \frac{1}{16}\log\left(\left|x+\frac{1}{x}-2\right|\right)$$

```
[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/4/(x + 1/x) + 1/16*log(abs(x + 1/x + 2)) - 1/16*log(abs(x + 1/x - 2))
```

Mupad [B] (verification not implemented)

Time = 9.91 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx = \frac{\operatorname{atanh}(x)}{4} - \frac{x}{4(x^2+1)}$$

```
[In] int(-x^2/((x^2 - 1)*(x^2 + 1)^2),x)
[Out] atanh(x)/4 - x/(4*(x^2 + 1))
```

3.345 $\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx$

Optimal result . . . 2119
Rubi [A] (verified) . . . 2119
Mathematica [A] (verified) . . . 2121
Maple [A] (verified) . . . 2122
Fricas [A] (verification not implemented) . . . 2122
Sympy [A] (verification not implemented) . . . 2122
Maxima [A] (verification not implemented) . . . 2123
Giac [A] (verification not implemented) . . . 2123
Mupad [B] (verification not implemented) . . . 2124

Optimal result

Integrand size = 20, antiderivative size = 97

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = -\frac{x}{6(1+x^3)} + \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{12}\log(1-x) - \frac{1}{36}\log(1+x) + \frac{1}{72}\log\left(1-x+x^2\right) + \frac{1}{24}\log\left(1+x+x^2\right)$$

```
[Out] -1/6*x/(x^3+1)-1/12*ln(1-x)-1/36*ln(1+x)+1/72*ln(x^2-x+1)+1/24*ln(x^2+x+1)+
1/36*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1+2*x)*3^(1/2))*3
^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {482, 536, 206, 31, 648, 632, 210, 642}

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{x}{6(x^3+1)} + \frac{1}{72}\log\left(x^2-x+1\right) + \frac{1}{24}\log\left(x^2+x+1\right) - \frac{1}{12}\log(1-x) - \frac{1}{36}\log(x+1)$$

```
[In] Int[x^3/((1 - x^3)*(1 + x^3)^2),x]
```

```
[Out] -1/6*x/(1 + x^3) + ArcTan[(1 - 2*x)/Sqrt[3]]/(12*Sqrt[3]) + ArcTan[(1 + 2*x
)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x]/12 - Log[1 + x]/36 + Log[1 - x + x^2]/7
2 + Log[1 + x + x^2]/24
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x}{6(1+x^3)} + \frac{1}{6}\int \frac{1+2x^3}{(1-x^3)(1+x^3)}\,dx \\
&= -\frac{x}{6(1+x^3)} - \frac{1}{12}\int \frac{1}{1+x^3}\,dx + \frac{1}{4}\int \frac{1}{1-x^3}\,dx \\
&= -\frac{x}{6(1+x^3)} - \frac{1}{36}\int \frac{1}{1+x}\,dx - \frac{1}{36}\int \frac{2-x}{1-x+x^2}\,dx + \frac{1}{12}\int \frac{1}{1-x}\,dx + \frac{1}{12}\int \frac{2+x}{1+x+x^2}\,dx \\
&= -\frac{x}{6(1+x^3)} - \frac{1}{12}\log(1-x) - \frac{1}{36}\log(1+x) + \frac{1}{72}\int \frac{-1+2x}{1-x+x^2}\,dx \\
&\quad - \frac{1}{24}\int \frac{1}{1-x+x^2}\,dx + \frac{1}{24}\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1+x+x^2}\,dx \\
&= -\frac{x}{6(1+x^3)} - \frac{1}{12}\log(1-x) - \frac{1}{36}\log(1+x) + \frac{1}{72}\log\left(1-x+x^2\right) + \frac{1}{24}\log\left(1+x+x^2\right) \\
&\quad + \frac{1}{12}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right) \\
&= -\frac{x}{6(1+x^3)} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{12}\log(1-x) \\
&\quad - \frac{1}{36}\log(1+x) + \frac{1}{72}\log\left(1-x+x^2\right) + \frac{1}{24}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$
\begin{aligned}
\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = \frac{1}{72}\Bigg(&-\frac{12x}{1+x^3} - 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 6\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) \\
&-6\log(1-x) - 2\log(1+x) + \log\left(1-x+x^2\right) + 3\log\left(1+x+x^2\right)\Bigg)
\end{aligned}
$$

```
[In] Integrate[x^3/((1 - x^3)*(1 + x^3)^2),x]
```

```
[Out] ((-12*x)/(1 + x^3) - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 6*Sqrt[3]*ArcTa
n[(1 + 2*x)/Sqrt[3]] - 6*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] + 3*L
og[1 + x + x^2])/72
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{x}{6(x^3+1)}-\frac{\ln(x-1)}{12}-\frac{\ln(x+1)}{36}+\frac{\ln(x^2+x+1)}{24}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}+\frac{\ln(x^2-x+1)}{72}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{36}$
default	$\frac{1}{18x+18}-\frac{\ln(x+1)}{36}+\frac{-2x-2}{36x^2-36x+36}+\frac{\ln(x^2-x+1)}{72}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{36}+\frac{\ln(x^2+x+1)}{24}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}-$

```
[In] int(x^3/(-x^3+1)/(x^3+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*x/(x^3+1)-1/12*ln(x-1)-1/36*ln(x+1)+1/24*ln(x^2+x+1)+1/12*3^(1/2)*arct
an(2/3*(x+1/2)*3^(1/2))+1/72*ln(x^2-x+1)-1/36*3^(1/2)*arctan(2/3*(x-1/2)*3^
(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(1-x^3)\,(1+x^3)^2}\,dx = \frac{6\,\sqrt{3}(x^3+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-2\,\sqrt{3}(x^3+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+3\,(x^3+1)\log\left(x^2+x+1\right)+}{72\,(x^3+1)}$$

```
[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="fricas")
```

```
[Out] 1/72*(6*sqrt(3)*(x^3 + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*(x^3 +
1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*(x^3 + 1)*log(x^2 + x + 1) + (x^3 + 1)
*log(x^2 - x + 1) - 2*(x^3 + 1)*log(x + 1) - 6*(x^3 + 1)*log(x - 1) - 12*x)
/(x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{(1-x^3)\,(1+x^3)^2}\,dx = -\frac{x}{6x^3+6}-\frac{\log(x-1)}{12}-\frac{\log(x+1)}{36}+\frac{\log(x^2-x+1)}{72}+\frac{\log(x^2+x+1)}{24}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{36}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

```
[In] integrate(x**3/(-x**3+1)/(x**3+1)**2,x)
```

```
[Out] -x/(6*x**3 + 6) - log(x - 1)/12 - log(x + 1)/36 + log(x**2 - x + 1)/72 + lo
g(x**2 + x + 1)/24 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/36 + sqrt(3)*a
tan(2*sqrt(3)*x/3 + sqrt(3)/3)/12
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{36}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{x}{6(x^3+1)} + \frac{1}{24}\log\left(x^2+x+1\right) + \frac{1}{72}\log\left(x^2-x+1\right) - \frac{1}{36}\log\left(x+1\right) - \frac{1}{12}\log\left(x-1\right)$$

```
[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/36*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/6*x/(x^3 + 1) + 1/24*log(x^2 + x + 1) + 1/72*log(x^2 - x +
 1) - 1/36*log(x + 1) - 1/12*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{36}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{x}{6(x^3+1)} + \frac{1}{24}\log\left(x^2+x+1\right) + \frac{1}{72}\log\left(x^2-x+1\right) - \frac{1}{36}\log\left(|x+1|\right) - \frac{1}{12}\log\left(|x-1|\right)$$

```
[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/36*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/6*x/(x^3 + 1) + 1/24*log(x^2 + x + 1) + 1/72*log(x^2 - x +
 1) - 1/36*log(abs(x + 1)) - 1/12*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = -\frac{\ln(x-1)}{12} - \frac{\ln(x+1)}{36} - \frac{x}{6(x^3+1)} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right) + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{72}+\frac{\sqrt{3}\,1\mathrm{i}}{72}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{72}+\frac{\sqrt{3}\,1\mathrm{i}}{72}\right)$$

```
[In] int(-x^3/((x^3 - 1)*(x^3 + 1)^2),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/24 + 1/24) - log(x + 1)/36 - x/
(6*(x^3 + 1)) - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/24 - 1/24) - lo
g(x - 1)/12 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/72 + 1/72) - log(
x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/72 - 1/72)
```

3.346 $\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)} \, dx$

Optimal result . . . 2125
Rubi [A] (verified) . . . 2125
Mathematica [A] (verified) . . . 2126
Maple [A] (verified) . . . 2126
Fricas [A] (verification not implemented) . . . 2127
Sympy [A] (verification not implemented) . . . 2127
Maxima [A] (verification not implemented) . . . 2127
Giac [A] (verification not implemented) . . . 2127
Mupad [B] (verification not implemented) . . . 2128

Optimal result

Integrand size = 26, antiderivative size = 15

$$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)} \, dx = 3\arctan(x) + \frac{1}{2}\log\left(3+x^2\right)$$

```
[Out] 3*arctan(x)+1/2*ln(x^2+3)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {6857, 209, 266}

$$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)} \, dx = 3\arctan(x) + \frac{1}{2}\log\left(x^2+3\right)$$

```
[In] Int[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

```
[Out] 3*ArcTan[x] + Log[3 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{3}{1+x^2}+\frac{x}{3+x^2}\right)dx\\ &= 3\int \frac{1}{1+x^2}\,dx+\int \frac{x}{3+x^2}\,dx\\ &= 3\tan^{-1}(x)+\frac{1}{2}\log\left(3+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx = 3\arctan(x)+\frac{1}{2}\log\left(3+x^2\right)$$

```
[In] Integrate[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
[Out] 3*ArcTan[x] + Log[3 + x^2]/2
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$3\arctan\left(x\right)+\frac{\ln\left(x^{2}+3\right)}{2}$	14
risch	$3\arctan\left(x\right)+\frac{\ln\left(x^{2}+3\right)}{2}$	14
parallelrisch	$\frac{3i\ln\left(x+i\right)}{2}-\frac{3i\ln\left(x-i\right)}{2}+\frac{\ln\left(x^{2}+3\right)}{2}$	26

```
[In] int((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x,method=_RETURNVERBOSE)
[Out] 3*arctan(x)+1/2*ln(x^2+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx = 3\arctan(x) + \frac{1}{2}\log\left(x^2+3\right)$$

```
[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="fricas")
[Out] 3*arctan(x) + 1/2*log(x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx = \frac{\log(x^2+3)}{2} + 3\,\mathrm{atan}(x)$$

```
[In] integrate((x**3+3*x**2+x+9)/(x**2+1)/(x**2+3),x)
[Out] log(x**2 + 3)/2 + 3*atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx = 3\arctan(x) + \frac{1}{2}\log\left(x^2+3\right)$$

```
[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="maxima")
[Out] 3*arctan(x) + 1/2*log(x^2 + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx = 3\arctan(x) + \frac{1}{2}\log\left(x^2+3\right)$$

```
[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="giac")
[Out] 3*arctan(x) + 1/2*log(x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx = \frac{\ln(x^2+3)}{2} + 3\,\mathrm{atan}(x)$$

```
[In] int((x + 3*x^2 + x^3 + 9)/((x^2 + 1)*(x^2 + 3)),x)
[Out] log(x^2 + 3)/2 + 3*atan(x)
```

3.347 $\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx$

Optimal result 2129
Rubi [A] (verified) 2129
Mathematica [A] (verified) 2130
Maple [A] (verified) 2130
Fricas [A] (verification not implemented) 2131
Sympy [A] (verification not implemented) 2131
Maxima [A] (verification not implemented) 2131
Giac [A] (verification not implemented) 2131
Mupad [B] (verification not implemented) 2132

Optimal result

Integrand size = 24, antiderivative size = 13

$$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx = \arctan(x) + \frac{1}{2}\log\left(3+x^2\right)$$

```
[Out] arctan(x)+1/2*ln(x^2+3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6857, 209, 266}

$$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx = \arctan(x) + \frac{1}{2}\log\left(x^2+3\right)$$

```
[In] Int[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

```
[Out] ArcTan[x] + Log[3 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{1+x^2}+\frac{x}{3+x^2}\right) dx\\ &= \int \frac{1}{1+x^2}\,dx+\int \frac{x}{3+x^2}\,dx\\ &= \tan^{-1}(x)+\frac{1}{2}\log\left(3+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx = \arctan(x)+\frac{1}{2}\log\left(3+x^2\right)$$

```
[In] Integrate[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
[Out] ArcTan[x] + Log[3 + x^2]/2
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\arctan\left(x\right)+\frac{\ln\left(x^{2}+3\right)}{2}$	12
risch	$\arctan\left(x\right)+\frac{\ln\left(x^{2}+3\right)}{2}$	12
parallelrisch	$\frac{i\ln\left(x+i\right)}{2}-\frac{i\ln\left(x-i\right)}{2}+\frac{\ln\left(x^{2}+3\right)}{2}$	26

```
[In] int((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x,method=_RETURNVERBOSE)
[Out] arctan(x)+1/2*ln(x^2+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx = \arctan(x) + \frac{1}{2}\log\left(x^2+3\right)$$

```
[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="fricas")
[Out] arctan(x) + 1/2*log(x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx = \frac{\log\left(x^2+3\right)}{2} + \operatorname{atan}(x)$$

```
[In] integrate((x**3+x**2+x+3)/(x**2+1)/(x**2+3),x)
[Out] log(x**2 + 3)/2 + atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx = \arctan(x) + \frac{1}{2}\log\left(x^2+3\right)$$

```
[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="maxima")
[Out] arctan(x) + 1/2*log(x^2 + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx = \arctan(x) + \frac{1}{2}\log\left(x^2+3\right)$$

```
[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="giac")
[Out] arctan(x) + 1/2*log(x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{3+x+x^2+x^3}{(1+x^2)\,(3+x^2)}\,dx = \frac{\ln(x^2+3)}{2} + \operatorname{atan}(x)$$

```
[In] int((x + x^2 + x^3 + 3)/((x^2 + 1)*(x^2 + 3)),x)
[Out] log(x^2 + 3)/2 + atan(x)
```

3.348 $\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx$

Optimal result 2133
Rubi [A] (verified) 2133
Mathematica [A] (verified) 2134
Maple [A] (verified) 2134
Fricas [A] (verification not implemented) 2135
Sympy [A] (verification not implemented) 2135
Maxima [A] (verification not implemented) 2135
Giac [A] (verification not implemented) 2136
Mupad [B] (verification not implemented) 2136

Optimal result

Integrand size = 30, antiderivative size = 29

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx = -3\arctan(x)+\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(1+x^2\right)$$

```
[Out] -3*arctan(x)+3/2*ln(x^2+1)+arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6857, 649, 209, 266}

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx = -3\arctan(x)+\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(x^2+1\right)$$

```
[In] Int[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{3(-1+x)}{1+x^2}+\frac{2}{2+x^2}\right) dx\\ &= 2\int \frac{1}{2+x^2}\,dx+3\int \frac{-1+x}{1+x^2}\,dx\\ &= \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-3\int \frac{1}{1+x^2}\,dx+3\int \frac{x}{1+x^2}\,dx\\ &= -3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx=-3\arctan(x)+\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

method	result	size
default	$-3\arctan\left(x\right)+\frac{3\ln\left(x^{2}+1\right)}{2}+\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	25
risch	$-3\arctan\left(x\right)+\frac{3\ln\left(x^{2}+1\right)}{2}+\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	25

```
[In] int((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x,method=_RETURNVERBOSE)
[Out] -3*arctan(x)+3/2*ln(x^2+1)+arctan(1/2*x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx = \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 3\,\arctan\left(x\right) + \frac{3}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="fricas")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx = \frac{3\log\left(x^2+1\right)}{2} - 3\operatorname{atan}\left(x\right) + \sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

```
[In] integrate((3*x**3-x**2+6*x-4)/(x**2+1)/(x**2+2),x)
[Out] 3*log(x**2 + 1)/2 - 3*atan(x) + sqrt(2)*atan(sqrt(2)*x/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx = \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 3\,\arctan\left(x\right) + \frac{3}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="maxima")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx = \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 3\arctan(x) + \frac{3}{2}\log\left(x^2+1\right)$$

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="giac")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 9.89 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.76

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx = -\sqrt{2}\,\mathrm{atan}\left(\frac{24\sqrt{2}}{24x-64} + \frac{32\sqrt{2}\,x}{24x-64}\right) + \ln(x-\mathrm{i})\left(\frac{3}{2}+\frac{3}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)$$

```
[In] int((6*x - x^2 + 3*x^3 - 4)/((x^2 + 1)*(x^2 + 2)),x)
[Out] log(x - 1i)*(3/2 + 3i/2) + log(x + 1i)*(3/2 - 3i/2) - 2^(1/2)*atan((24*2^(1
/2))/(24*x - 64) + (32*2^(1/2)*x)/(24*x - 64))
```

3.349 $\int \frac{1}{(4-4x+x^2)(5-4x+x^2)} dx$

Optimal result . . . 2137
Rubi [A] (verified) . . . 2137
Mathematica [A] (verified) . . . 2138
Maple [A] (verified) . . . 2139
Fricas [A] (verification not implemented) . . . 2139
Sympy [A] (verification not implemented) . . . 2139
Maxima [A] (verification not implemented) . . . 2140
Giac [A] (verification not implemented) . . . 2140
Mupad [B] (verification not implemented) . . . 2140

Optimal result

Integrand size = 21, antiderivative size = 14

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)} dx = \frac{1}{2-x} + \arctan(2-x)$$

```
[Out] 1/(2-x)-arctan(-2+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {27, 707, 632, 210}

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)} dx = \arctan(2-x) + \frac{1}{2-x}$$

```
[In] Int[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
```

```
[Out] (2 - x)^(-1) + ArcTan[2 - x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 707

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[-2*b*d*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(d^2*(m
+ 1)*(b^2 - 4*a*c))), x] + Dist[b^2*((m + 2*p + 3)/(d^2*(m + 1)*(b^2 - 4*a*
c))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{(-2+x)^2\,(5-4x+x^2)}\,dx\\ &= \frac{1}{2-x} - \int \frac{1}{5-4x+x^2}\,dx\\ &= \frac{1}{2-x} + 2\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, -4+2x\right)\\ &= \frac{1}{2-x} + \tan^{-1}(2-x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)\,(5-4x+x^2)}\,dx = -\frac{1}{-2+x} + \arctan(2-x)$$

```
[In] Integrate[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
[Out] -(-2 + x)^(-1) + ArcTan[2 - x]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$-\arctan\left(x-2\right)-\frac{1}{x-2}$	15
risch	$-\arctan\left(x-2\right)-\frac{1}{x-2}$	15
parallelrisch	$\frac{i\ln(x-2-i)x-i\ln(x-2+i)x-2i\ln(x-2-i)+2i\ln(x-2+i)-x}{2x-4}$	50

```
[In] int(1/(x^2-4*x+4)/(x^2-4*x+5),x,method=_RETURNVERBOSE)
[Out] -arctan(x-2)-1/(x-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\frac{(x-2)\arctan\left(x-2\right)+1}{x-2}$$

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="fricas")
[Out] -((x - 2)*arctan(x - 2) + 1)/(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\operatorname{atan}\left(x-2\right)-\frac{1}{x-2}$$

```
[In] integrate(1/(x**2-4*x+4)/(x**2-4*x+5),x)
[Out] -atan(x - 2) - 1/(x - 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\frac{1}{x-2} - \arctan(x-2)$$

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="maxima")
[Out] -1/(x - 2) - arctan(x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\frac{1}{x-2} - \arctan(x-2)$$

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="giac")
[Out] -1/(x - 2) - arctan(x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\mathrm{atan}(x-2) - \frac{1}{x-2}$$

```
[In] int(1/((x^2 - 4*x + 4)*(x^2 - 4*x + 5)),x)
[Out] - atan(x - 2) - 1/(x - 2)
```

3.350 $\int \frac{-3+x+x^2}{(-3+x)x^2} \, dx$

Optimal result 2141
Rubi [A] (verified) 2141
Mathematica [A] (verified) 2142
Maple [A] (verified) 2142
Fricas [A] (verification not implemented) 2143
Sympy [A] (verification not implemented) 2143
Maxima [A] (verification not implemented) 2143
Giac [A] (verification not implemented) 2143
Mupad [B] (verification not implemented) 2144

Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{-3+x+x^2}{(-3+x)x^2} \, dx = -\frac{1}{x} + \log(3-x)$$

```
[Out] -1/x+ln(3-x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {907}

$$\int \frac{-3+x+x^2}{(-3+x)x^2} \, dx = \log(3-x) - \frac{1}{x}$$

```
[In] Int[(-3 + x + x^2)/((-3 + x)*x^2),x]
```

```
[Out] -x^(-1) + Log[3 - x]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{-3+x}+\frac{1}{x^2}\right)\,dx$$
$$= -\frac{1}{x}+\log(3-x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx = -\frac{1}{x}+\log(3-x)$$

```
[In] Integrate[(-3 + x + x^2)/((-3 + x)*x^2),x]
[Out] -x^(-1) + Log[3 - x]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$\ln\left(-3+x\right)-\frac{1}{x}$	11
norman	$\ln\left(-3+x\right)-\frac{1}{x}$	11
risch	$\ln\left(-3+x\right)-\frac{1}{x}$	11
meijerg	$\ln\left(1-\frac{x}{3}\right)-\frac{1}{x}$	13
parallelrisch	$\frac{\ln(-3+x)x-1}{x}$	13

```
[In] int((x^2+x-3)/(-3+x)/x^2,x,method=_RETURNVERBOSE)
[Out] ln(-3+x)-1/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx = \frac{x\log(x-3)-1}{x}$$

```
[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="fricas")
[Out] (x*log(x - 3) - 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx = \log(x-3) - \frac{1}{x}$$

```
[In] integrate((x**2+x-3)/(-3+x)/x**2,x)
[Out] log(x - 3) - 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx = -\frac{1}{x} + \log(x-3)$$

```
[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="maxima")
[Out] -1/x + log(x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx = -\frac{1}{x} + \log(|x-3|)$$

```
[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="giac")
[Out] -1/x + log(abs(x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx = \ln(x-3) - \frac{1}{x}$$

```
[In] int((x + x^2 - 3)/(x^2*(x - 3)),x)
[Out] log(x - 3) - 1/x
```

3.351 $\int \frac{1+x+4x^2}{x+4x^3} dx$

Optimal result 2145
Rubi [A] (verified) 2145
Mathematica [A] (verified) 2146
Maple [A] (verified) 2146
Fricas [A] (verification not implemented) 2147
Sympy [A] (verification not implemented) 2147
Maxima [A] (verification not implemented) 2147
Giac [A] (verification not implemented) 2147
Mupad [B] (verification not implemented) 2148

Optimal result

Integrand size = 18, antiderivative size = 11

$$\int \frac{1+x+4x^2}{x+4x^3} dx = \frac{1}{2}\arctan(2x) + \log(x)$$

```
[Out] 1/2*arctan(2*x)+ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1607, 1816, 209}

$$\int \frac{1+x+4x^2}{x+4x^3} dx = \frac{1}{2}\arctan(2x) + \log(x)$$

```
[In] Int[(1 + x + 4*x^2)/(x + 4*x^3),x]
```

```
[Out] ArcTan[2*x]/2 + Log[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+x+4x^2}{x\left(1+4x^2\right)}\,dx \\ &= \int \left(\frac{1}{x}+\frac{1}{1+4x^2}\right)dx \\ &= \log(x)+\int \frac{1}{1+4x^2}\,dx \\ &= \frac{1}{2}\tan^{-1}(2x)+\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx = \frac{1}{2}\arctan(2x)+\log(x)$$

```
[In] Integrate[(1 + x + 4*x^2)/(x + 4*x^3),x]
[Out] ArcTan[2*x]/2 + Log[x]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\arctan(2x)}{2}+\ln(x)$	10
risch	$\frac{\arctan(2x)}{2}+\ln(x)$	10
meijerg	$\frac{\arctan(2x)}{2}+\ln(x)+\ln(2)$	12
parallelrisch	$\ln(x)-\frac{i\ln\left(x-\frac{i}{2}\right)}{4}+\frac{i\ln\left(x+\frac{i}{2}\right)}{4}$	20

```
[In] int((4*x^2+x+1)/(4*x^3+x),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(2*x)+ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx = \frac{1}{2}\arctan\left(2\,x\right)+\log\left(x\right)$$

```
[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="fricas")
[Out] 1/2*arctan(2*x) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx = \log\left(x\right)+\frac{\operatorname{atan}\left(2x\right)}{2}$$

```
[In] integrate((4*x**2+x+1)/(4*x**3+x),x)
[Out] log(x) + atan(2*x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx = \frac{1}{2}\arctan\left(2\,x\right)+\log\left(x\right)$$

```
[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="maxima")
[Out] 1/2*arctan(2*x) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx = \frac{1}{2}\arctan\left(2\,x\right)+\log\left(|x|\right)$$

```
[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="giac")
[Out] 1/2*arctan(2*x) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx=\ln\left(x\right)-\frac{\mathrm{atan}\left(\frac{17}{32\,\left(\frac{x}{16}-\frac{1}{8}\right)}+4\right)}{2}$$

```
[In] int((x + 4*x^2 + 1)/(x + 4*x^3),x)
[Out] log(x) - atan(17/(32*(x/16 - 1/8)) + 4)/2
```

3.352 $\int \frac{1-x+3x^2}{-x^2+x^3} \, dx$

Optimal result 2149
Rubi [A] (verified) 2149
Mathematica [A] (verified) 2150
Maple [A] (verified) 2150
Fricas [A] (verification not implemented) 2151
Sympy [A] (verification not implemented) 2151
Maxima [A] (verification not implemented) 2151
Giac [A] (verification not implemented) 2151
Mupad [B] (verification not implemented) 2152

Optimal result

Integrand size = 22, antiderivative size = 12

$$\int \frac{1-x+3x^2}{-x^2+x^3} \, dx = \frac{1}{x} + 3\log(1-x)$$

```
[Out] 1/x+3*ln(1-x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1607, 907}

$$\int \frac{1-x+3x^2}{-x^2+x^3} \, dx = \frac{1}{x} + 3\log(1-x)$$

```
[In] Int[(1 - x + 3*x^2)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + 3*Log[1 - x]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1-x+3x^2}{(-1+x)x^2}\,dx\\ &= \int \left(\frac{3}{-1+x}-\frac{1}{x^2}\right)dx\\ &= \frac{1}{x}+3\log(1-x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1-x+3x^2}{-x^2+x^3}\,dx=\frac{1}{x}+3\log(1-x)$$

```
[In] Integrate[(1 - x + 3*x^2)/(-x^2 + x^3),x]
[Out] x^(-1) + 3*Log[1 - x]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{1}{x}+3\ln\left(x-1\right)$	11
norman	$\frac{1}{x}+3\ln\left(x-1\right)$	11
risch	$\frac{1}{x}+3\ln\left(x-1\right)$	11
meijerg	$\frac{1}{x}+3\ln\left(1-x\right)$	13
parallelrisch	$\frac{3\ln(x-1)x+1}{x}$	14

```
[In] int((3*x^2-x+1)/(x^3-x^2),x,method=_RETURNVERBOSE)
[Out] 1/x+3*ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1-x+3x^2}{-x^2+x^3}\,dx = \frac{3\,x\log{(x-1)}+1}{x}$$

```
[In] integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="fricas")
[Out] (3*x*log(x - 1) + 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1-x+3x^2}{-x^2+x^3}\,dx = 3\log{(x-1)} + \frac{1}{x}$$

```
[In] integrate((3*x**2-x+1)/(x**3-x**2),x)
[Out] 3*log(x - 1) + 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1-x+3x^2}{-x^2+x^3}\,dx = \frac{1}{x} + 3\,\log\left(x-1\right)$$

```
[In] integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="maxima")
[Out] 1/x + 3*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1-x+3x^2}{-x^2+x^3}\,dx = \frac{1}{x} + 3\,\log\left(|x-1|\right)$$

```
[In] integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="giac")
[Out] 1/x + 3*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1-x+3x^2}{-x^2+x^3}\,dx = 3\,\ln\left(x-1\right)+\frac{1}{x}$$

```
[In] int(-(3*x^2 - x + 1)/(x^2 - x^3),x)
[Out] 3*log(x - 1) + 1/x
```

3.353 $\int \frac{4+3x+x^2}{x+x^2} dx$

Optimal result . . . 2153
Rubi [A] (verified) . . . 2153
Mathematica [A] (verified) . . . 2154
Maple [A] (verified) . . . 2154
Fricas [A] (verification not implemented) . . . 2155
Sympy [A] (verification not implemented) . . . 2155
Maxima [A] (verification not implemented) . . . 2155
Giac [A] (verification not implemented) . . . 2155
Mupad [B] (verification not implemented) . . . 2156

Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{4+3x+x^2}{x+x^2} dx = x + 4\log(x) - 2\log(1+x)$$

```
[Out] x+4*ln(x)-2*ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1607, 907}

$$\int \frac{4+3x+x^2}{x+x^2} dx = x + 4\log(x) - 2\log(x+1)$$

```
[In] Int[(4 + 3*x + x^2)/(x + x^2),x]
```

```
[Out] x + 4*Log[x] - 2*Log[1 + x]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{4+3x+x^2}{x(1+x)}\,dx\\ &= \int \left(1+\frac{4}{x}-\frac{2}{1+x}\right)dx\\ &= x+4\log(x)-2\log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{4+3x+x^2}{x+x^2}\,dx = x+4\log(x)-2\log(1+x)$$

```
[In] Integrate[(4 + 3*x + x^2)/(x + x^2),x]
[Out] x + 4*Log[x] - 2*Log[1 + x]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$x+4\ln(x)-2\ln(x+1)$	13
norman	$x+4\ln(x)-2\ln(x+1)$	13
meijerg	$x+4\ln(x)-2\ln(x+1)$	13
risch	$x+4\ln(x)-2\ln(x+1)$	13
parallelrisch	$x+4\ln(x)-2\ln(x+1)$	13

```
[In] int((x^2+3*x+4)/(x^2+x),x,method=_RETURNVERBOSE)
[Out] x+4*ln(x)-2*ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{4+3x+x^2}{x+x^2}\,dx = x - 2\,\log\left(x+1\right) + 4\,\log\left(x\right)$$

```
[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="fricas")
[Out] x - 2*log(x + 1) + 4*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{4+3x+x^2}{x+x^2}\,dx = x + 4\log\left(x\right) - 2\log\left(x+1\right)$$

```
[In] integrate((x**2+3*x+4)/(x**2+x),x)
[Out] x + 4*log(x) - 2*log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{4+3x+x^2}{x+x^2}\,dx = x - 2\,\log\left(x+1\right) + 4\,\log\left(x\right)$$

```
[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="maxima")
[Out] x - 2*log(x + 1) + 4*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{4+3x+x^2}{x+x^2}\,dx = x - 2\,\log\left(|x+1|\right) + 4\,\log\left(|x|\right)$$

```
[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="giac")
[Out] x - 2*log(abs(x + 1)) + 4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{4+3x+x^2}{x+x^2}\,dx = x - 2\,\ln(x+1) + 4\,\ln(x)$$

```
[In] int((3*x + x^2 + 4)/(x + x^2),x)
[Out] x - 2*log(x + 1) + 4*log(x)
```

3.354 $\int \frac{4+x+3x^2}{x+x^3}\,dx$

Optimal result . . . 2157
Rubi [A] (verified) . . . 2157
Mathematica [A] (verified) . . . 2158
Maple [A] (verified) . . . 2159
Fricas [A] (verification not implemented) . . . 2159
Sympy [A] (verification not implemented) . . . 2159
Maxima [A] (verification not implemented) . . . 2160
Giac [A] (verification not implemented) . . . 2160
Mupad [B] (verification not implemented) . . . 2160

Optimal result

Integrand size = 16, antiderivative size = 17

$$\int \frac{4+x+3x^2}{x+x^3}\,dx = \arctan(x)+4\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] arctan(x)+4*ln(x)-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1607, 1816, 649, 209, 266}

$$\int \frac{4+x+3x^2}{x+x^3}\,dx = \arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+4\log(x)$$

```
[In] Int[(4 + x + 3*x^2)/(x + x^3),x]
```

```
[Out] ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{4+x+3x^2}{x(1+x^2)}\,dx \\
&= \int \left(\frac{4}{x}+\frac{1-x}{1+x^2}\right)dx \\
&= 4\log(x)+\int \frac{1-x}{1+x^2}\,dx \\
&= 4\log(x)+\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx \\
&= \tan^{-1}(x)+4\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{4+x+3x^2}{x+x^3}\,dx = \arctan(x)+4\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[(4 + x + 3*x^2)/(x + x^3),x]
[Out] ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\arctan(x)+4\ln(x)-\frac{\ln(x^2+1)}{2}$	16
meijerg	$\arctan(x)+4\ln(x)-\frac{\ln(x^2+1)}{2}$	16
risch	$\arctan(x)+4\ln(x)-\frac{\ln(x^2+1)}{2}$	16
parallelrisch	$4\ln(x)-\frac{\ln(x-i)}{2}-\frac{i\ln(x-i)}{2}-\frac{\ln(x+i)}{2}+\frac{i\ln(x+i)}{2}$	36

```
[In] int((3*x^2+x+4)/(x^3+x),x,method=_RETURNVERBOSE)
[Out] arctan(x)+4*ln(x)-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{4+x+3x^2}{x+x^3}\,dx = \arctan(x) - \frac{1}{2}\,\log\left(x^2+1\right) + 4\,\log(x)$$

```
[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="fricas")
[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{4+x+3x^2}{x+x^3}\,dx = 4\log(x) - \frac{\log(x^2+1)}{2} + \operatorname{atan}(x)$$

```
[In] integrate((3*x**2+x+4)/(x**3+x),x)
[Out] 4*log(x) - log(x**2 + 1)/2 + atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{4+x+3x^2}{x+x^3}\,dx = \arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + 4\log(x)$$

```
[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="maxima")
[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{4+x+3x^2}{x+x^3}\,dx = \arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + 4\log(|x|)$$

```
[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="giac")
[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.92 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{4+x+3x^2}{x+x^3}\,dx = 4\ln(x) + \ln(x-\mathrm{i})\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

```
[In] int((x + 3*x^2 + 4)/(x + x^3),x)
[Out] 4*log(x) - log(x + 1i)*(1/2 - 1i/2) - log(x - 1i)*(1/2 + 1i/2)
```

3.355 $\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)} \, dx$

Optimal result 2161
Rubi [A] (verified) 2161
Mathematica [A] (verified) 2162
Maple [A] (verified) 2162
Fricas [A] (verification not implemented) 2163
Sympy [A] (verification not implemented) 2163
Maxima [A] (verification not implemented) 2163
Giac [A] (verification not implemented) 2163
Mupad [B] (verification not implemented) 2164

Optimal result

Integrand size = 25, antiderivative size = 13

$$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)} \, dx = -\arctan(x)+2\log(1+4x)$$

```
[Out] -arctan(x)+2*ln(1+4*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1643, 210}

$$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)} \, dx = 2\log(4x+1)-\arctan(x)$$

```
[In] Int[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]
```

```
[Out] -ArcTan[x] + 2*Log[1 + 4*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
```

```
d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{8}{1+4x}+\frac{1}{-1-x^2}\right)\,dx\\
&= 2\log(1+4x)+\int \frac{1}{-1-x^2}\,dx\\
&= -\tan^{-1}(x)+2\log(1+4x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{7-4x+8x^2}{(1+4x)\,(1+x^2)}\,dx = -\arctan(x)+2\log(1+4x)$$

```
[In] Integrate[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]
[Out] -ArcTan[x] + 2*Log[1 + 4*x]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$-\arctan\left(x\right)+2\ln\left(1+4x\right)$	14
risch	$-\arctan\left(x\right)+2\ln\left(1+4x\right)$	14
parallelrisch	$2\ln\left(x+\frac{1}{4}\right)+\frac{i\ln(x-i)}{2}-\frac{i\ln(x+i)}{2}$	24

```
[In] int((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -arctan(x)+2*ln(1+4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx = -\arctan(x) + 2\log(4x+1)$$

```
[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="fricas")
[Out] -arctan(x) + 2*log(4*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx = 2\log\left(x+\frac{1}{4}\right) - \operatorname{atan}(x)$$

```
[In] integrate((8*x**2-4*x+7)/(1+4*x)/(x**2+1),x)
[Out] 2*log(x + 1/4) - atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx = -\arctan(x) + 2\log(4x+1)$$

```
[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="maxima")
[Out] -arctan(x) + 2*log(4*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx = -\arctan(x) + 2\log(|4x+1|)$$

```
[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="giac")
[Out] -arctan(x) + 2*log(abs(4*x + 1))
```

Mupad [B] (verification not implemented)

Time = 10.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{7-4x+8x^2}{(1+4x)\,(1+x^2)}\,dx = \operatorname{atan}\left(\frac{4\,x+1}{x-4}\right)+2\,\ln\left(x+\frac{1}{4}\right)$$

```
[In] int((8*x^2 - 4*x + 7)/((4*x + 1)*(x^2 + 1)),x)
[Out] atan((4*x + 1)/(x - 4)) + 2*log(x + 1/4)
```

3.356 $\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx$

Optimal result 2165
Rubi [A] (verified) 2165
Mathematica [A] (verified) 2166
Maple [A] (verified) 2166
Fricas [A] (verification not implemented) 2167
Sympy [A] (verification not implemented) 2167
Maxima [A] (verification not implemented) 2167
Giac [A] (verification not implemented) 2167
Mupad [B] (verification not implemented) 2168

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{x^2}{(-1+x)\,(1+2x+x^2)}\,dx = \frac{1}{2(1+x)} + \frac{1}{4}\log(1-x) + \frac{3}{4}\log(1+x)$$

```
[Out] 1/2/(1+x)+1/4*ln(1-x)+3/4*ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {27, 90}

$$\int \frac{x^2}{(-1+x)\,(1+2x+x^2)}\,dx = \frac{1}{2(x+1)} + \frac{1}{4}\log(1-x) + \frac{3}{4}\log(x+1)$$

```
[In] Int[x^2/((-1 + x)*(1 + 2*x + x^2)),x]
```

```
[Out] 1/(2*(1 + x)) + Log[1 - x]/4 + (3*Log[1 + x])/4
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{x^2}{(-1+x)(1+x)^2}\,dx \\ &= \int \left(\frac{1}{4(-1+x)} - \frac{1}{2(1+x)^2} + \frac{3}{4(1+x)}\right)dx \\ &= \frac{1}{2(1+x)} + \frac{1}{4}\log(1-x) + \frac{3}{4}\log(1+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx = \frac{1}{4}\left(\frac{2}{1+x} + \log(-1+x) + 3\log(1+x)\right)$$

```
[In] Integrate[x^2/((-1 + x)*(1 + 2*x + x^2)),x]
[Out] (2/(1 + x) + Log[-1 + x] + 3*Log[1 + x])/4
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{1}{2x+2}+\frac{3\ln(x+1)}{4}+\frac{\ln(x-1)}{4}$	21
norman	$\frac{1}{2x+2}+\frac{3\ln(x+1)}{4}+\frac{\ln(x-1)}{4}$	21
risch	$\frac{1}{2x+2}+\frac{3\ln(x+1)}{4}+\frac{\ln(x-1)}{4}$	21
parallelrisch	$\frac{\ln(x-1)x+3\ln(x+1)x+2+\ln(x-1)+3\ln(x+1)}{4+4x}$	33

```
[In] int(x^2/(x-1)/(x^2+2*x+1),x,method=_RETURNVERBOSE)
[Out] 1/2/(x+1)+3/4*ln(x+1)+1/4*ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx = \frac{3(x+1)\log(x+1)+(x+1)\log(x-1)+2}{4(x+1)}$$

```
[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="fricas")
[Out] 1/4*(3*(x + 1)*log(x + 1) + (x + 1)*log(x - 1) + 2)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx = \frac{\log(x-1)}{4}+\frac{3\log(x+1)}{4}+\frac{1}{2x+2}$$

```
[In] integrate(x**2/(-1+x)/(x**2+2*x+1),x)
[Out] log(x - 1)/4 + 3*log(x + 1)/4 + 1/(2*x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx = \frac{1}{2(x+1)}+\frac{3}{4}\log(x+1)+\frac{1}{4}\log(x-1)$$

```
[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="maxima")
[Out] 1/2/(x + 1) + 3/4*log(x + 1) + 1/4*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx = \frac{1}{2(x+1)}+\frac{3}{4}\log(|x+1|)+\frac{1}{4}\log(|x-1|)$$

```
[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="giac")
[Out] 1/2/(x + 1) + 3/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(-1+x)\left(1+2x+x^2\right)}\,dx=\frac{\ln\left(x-1\right)}{4}+\frac{3\,\ln\left(x+1\right)}{4}+\frac{1}{2\,\left(x+1\right)}$$

```
[In] int(x^2/((x - 1)*(2*x + x^2 + 1)),x)
[Out] log(x - 1)/4 + (3*log(x + 1))/4 + 1/(2*(x + 1))
```

3.357 $\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx$

Optimal result 2169
Rubi [A] (verified) 2169
Mathematica [A] (verified) 2170
Maple [A] (verified) 2170
Fricas [A] (verification not implemented) 2171
Sympy [A] (verification not implemented) 2171
Maxima [A] (verification not implemented) 2171
Giac [A] (verification not implemented) 2172
Mupad [B] (verification not implemented) 2172

Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = -\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(3+2x)$$

```
[Out] -9/32/(1-2*x)+41/128*ln(1-2*x)-25/128*ln(3+2*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {907}

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = -\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

```
[In] Int[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
[Out] -9/(32*(1 - 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-\frac{9}{16(-1+2x)^2} + \frac{41}{64(-1+2x)} - \frac{25}{64(3+2x)}\right) dx \\ &= -\frac{9}{32(1-2x)} + \frac{41}{128}\log(1-2x) - \frac{25}{128}\log(3+2x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = \frac{9}{32(-1+2x)} + \frac{41}{128}\log(1-2x) - \frac{25}{128}\log(3+2x)$$

```
[In] Integrate[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
[Out] 9/(32*(-1 + 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{9}{64\left(x-\frac{1}{2}\right)}+\frac{41\ln\left(2x-1\right)}{128}-\frac{25\ln\left(2x+3\right)}{128}$	25
default	$-\frac{25\ln\left(2x+3\right)}{128}+\frac{9}{32\left(2x-1\right)}+\frac{41\ln\left(2x-1\right)}{128}$	27
norman	$\frac{9x}{16\left(2x-1\right)}+\frac{41\ln\left(2x-1\right)}{128}-\frac{25\ln\left(2x+3\right)}{128}$	28
parallelrisch	$\frac{82\ln\left(x-\frac{1}{2}\right)x-50\ln\left(x+\frac{3}{2}\right)x-41\ln\left(x-\frac{1}{2}\right)+25\ln\left(x+\frac{3}{2}\right)+72x}{256x-128}$	40

```
[In] int((x^2+3*x-4)/(2*x-1)^2/(2*x+3),x,method=_RETURNVERBOSE)
[Out] 9/64/(x-1/2)+41/128*ln(2*x-1)-25/128*ln(2*x+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = -\frac{25\,(2\,x-1)\log\left(2\,x+3\right)-41\,(2\,x-1)\log\left(2\,x-1\right)-36}{128\,(2\,x-1)}$$

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="fricas")
[Out] -1/128*(25*(2*x - 1)*log(2*x + 3) - 41*(2*x - 1)*log(2*x - 1) - 36)/(2*x -
1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = \frac{41\log\left(x-\frac{1}{2}\right)}{128}-\frac{25\log\left(x+\frac{3}{2}\right)}{128}+\frac{9}{64x-32}$$

```
[In] integrate((x**2+3*x-4)/(-1+2*x)**2/(3+2*x),x)
[Out] 41*log(x - 1/2)/128 - 25*log(x + 3/2)/128 + 9/(64*x - 32)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = \frac{9}{32\,(2\,x-1)}-\frac{25}{128}\,\log\left(2\,x+3\right)+\frac{41}{128}\,\log\left(2\,x-1\right)$$

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="maxima")
[Out] 9/32/(2*x - 1) - 25/128*log(2*x + 3) + 41/128*log(2*x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = \frac{9}{32\,(2\,x-1)} - \frac{1}{8}\,\log\left(\frac{|2\,x-1|}{2\,(2\,x-1)^2}\right) - \frac{25}{128}\,\log\left(\left|-\frac{4}{2\,x-1}-1\right|\right)$$

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="giac")
[Out] 9/32/(2*x - 1) - 1/8*log(1/2*abs(2*x - 1)/(2*x - 1)^2) - 25/128*log(abs(-4/
(2*x - 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 9.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = \frac{41\,\ln\left(x-\frac{1}{2}\right)}{128} - \frac{25\,\ln\left(x+\frac{3}{2}\right)}{128} + \frac{9}{64\,\left(x-\frac{1}{2}\right)}$$

```
[In] int((3*x + x^2 - 4)/((2*x - 1)^2*(2*x + 3)),x)
[Out] (41*log(x - 1/2))/128 - (25*log(x + 3/2))/128 + 9/(64*(x - 1/2))
```

3.358 $\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)} dx$

Optimal result 2173
Rubi [A] (verified) 2173
Mathematica [A] (verified) 2174
Maple [A] (verified) 2174
Fricas [A] (verification not implemented) 2175
Sympy [A] (verification not implemented) 2175
Maxima [A] (verification not implemented) 2175
Giac [A] (verification not implemented) 2176
Mupad [B] (verification not implemented) 2176

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)} dx = -3\arctan(x)+2\log(1-x)+\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -3*arctan(x)+2*ln(1-x)+1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1643, 649, 209, 266}

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)} dx = -3\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)+2\log(1-x)$$

```
[In] Int[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + 2*Log[1 - x] + Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{2}{-1+x}+\frac{-3+x}{1+x^2}\right) dx\\ &= 2\log(1-x)+\int \frac{-3+x}{1+x^2}\,dx\\ &= 2\log(1-x)-3\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= -3\tan^{-1}(x)+2\log(1-x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)}\,dx = -3\arctan(x)+\frac{1}{2}\log\left(2+2(-1+x)+(-1+x)^2\right)+2\log(-1+x)$$

```
[In] Integrate[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
[Out] -3*ArcTan[x] + Log[2 + 2*(-1 + x) + (-1 + x)^2]/2 + 2*Log[-1 + x]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\ln(x^2+1)}{2}-3\arctan(x)+2\ln(x-1)$	20
risch	$2\ln(x-1)+\frac{\ln(9x^2+9)}{2}-3\arctan(x)$	22
parallelrisch	$2\ln(x-1)+\frac{\ln(x-i)}{2}+\frac{3i\ln(x-i)}{2}+\frac{\ln(x+i)}{2}-\frac{3i\ln(x+i)}{2}$	38

```
[In] int((3*x^2-4*x+5)/(x-1)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2+1)-3*arctan(x)+2*ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = -3\,\arctan(x) + \frac{1}{2}\,\log\left(x^2+1\right) + 2\,\log(x-1)$$

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="fricas")
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = 2\log(x-1) + \frac{\log(x^2+1)}{2} - 3\,\mathrm{atan}(x)$$

```
[In] integrate((3*x**2-4*x+5)/(-1+x)/(x**2+1),x)
[Out] 2*log(x - 1) + log(x**2 + 1)/2 - 3*atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = -3\,\arctan(x) + \frac{1}{2}\,\log\left(x^2+1\right) + 2\,\log(x-1)$$

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="maxima")
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = -3\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)+2\log\left(|x-1|\right)$$

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="giac")
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = 2\ln(x-1)+\ln(x-\mathrm{i})\left(\frac{1}{2}+\frac{3}{2}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(\frac{1}{2}-\frac{3}{2}\mathrm{i}\right)$$

```
[In] int((3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)),x)
[Out] 2*log(x - 1) + log(x - 1i)*(1/2 + 3i/2) + log(x + 1i)*(1/2 - 3i/2)
```

3.359 $\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx$

Optimal result 2177
Rubi [A] (verified) 2177
Mathematica [A] (verified) 2178
Maple [A] (verified) 2178
Fricas [A] (verification not implemented) 2179
Sympy [A] (verification not implemented) 2179
Maxima [A] (verification not implemented) 2180
Giac [B] (verification not implemented) 2180
Mupad [B] (verification not implemented) 2180

Optimal result

Integrand size = 21, antiderivative size = 24

$$\int \frac{-1-2x+x^2}{(-1+x)^2\,(1+x^2)}\,dx=\frac{1}{-1+x}+\arctan(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] 1/(-1+x)+arctan(x)+ln(1-x)-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1643, 649, 209, 266}

$$\int \frac{-1-2x+x^2}{(-1+x)^2\,(1+x^2)}\,dx=\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)$$

```
[In] Int[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{1}{(-1+x)^2}+\frac{1}{-1+x}+\frac{1-x}{1+x^2}\right) dx \\ &= \frac{1}{-1+x}+\log(1-x)+\int \frac{1-x}{1+x^2}\, dx \\ &= \frac{1}{-1+x}+\log(1-x)+\int \frac{1}{1+x^2}\, dx-\int \frac{x}{1+x^2}\, dx \\ &= \frac{1}{-1+x}+\tan^{-1}(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{-1-2x+x^2}{(-1+x)^2\left(1+x^2\right)}\, dx = \frac{1}{-1+x}+\arctan(x)+\log(-1+x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[-1 + x] - Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\ln(x^2+1)}{2}+\arctan(x)+\ln(x-1)+\frac{1}{x-1}$	21
risch	$-\frac{\ln(x^2+1)}{2}+\arctan(x)+\ln(x-1)+\frac{1}{x-1}$	21
parallelrisch	$-\frac{i\ln(x-i)x-i\ln(x+i)x-i\ln(x-i)+i\ln(x+i)-2\ln(x-1)x+\ln(x-i)x+\ln(x+i)x-2+2\ln(x-1)-\ln(x-i)-\ln(x+i)}{2(x-1)}$	85

```
[In] int((x^2-2*x-1)/(x-1)^2/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(x^2+1)+arctan(x)+ln(x-1)+1/(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \frac{2(x-1)\arctan(x)-(x-1)\log(x^2+1)+2(x-1)\log(x-1)+2}{2(x-1)}$$

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
[Out] 1/2*(2*(x - 1)*arctan(x) - (x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x - 1) + 2)
/(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \log(x-1)-\frac{\log(x^2+1)}{2}+\operatorname{atan}(x)+\frac{1}{x-1}$$

```
[In] integrate((x**2-2*x-1)/(-1+x)**2/(x**2+1),x)
[Out] log(x - 1) - log(x**2 + 1)/2 + atan(x) + 1/(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \frac{1}{x-1} + \arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + \log(x-1)$$

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
[Out] 1/(x - 1) + arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(22) = 44.

Time = 0.39 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \frac{1}{4}\pi - \pi\left\lfloor\frac{\pi+4\arctan(x)}{4\pi}+\frac{1}{2}\right\rfloor + \frac{1}{x-1} + \arctan(x) - \frac{1}{2}\log\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)$$

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="giac")
[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/(x - 1) + arctan(x)
- 1/2*log(2/(x - 1) + 2/(x - 1)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \ln(x-1) + \frac{1}{x-1} + \ln(x-\mathrm{i})\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

```
[In] int(-(2*x - x^2 + 1)/((x^2 + 1)*(x - 1)^2),x)
[Out] log(x - 1) - log(x - 1i)*(1/2 + 1i/2) - log(x + 1i)*(1/2 - 1i/2) + 1/(x - 1
)
```

3.360 $\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)} \, dx$

Optimal result 2181
Rubi [A] (verified) 2181
Mathematica [A] (verified) 2183
Maple [A] (verified) 2183
Fricas [A] (verification not implemented) 2184
Sympy [A] (verification not implemented) 2184
Maxima [A] (verification not implemented) 2184
Giac [A] (verification not implemented) 2185
Mupad [B] (verification not implemented) 2185

Optimal result

Integrand size = 28, antiderivative size = 49

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)} \, dx = -\frac{261}{221}\arctan(1-2x)-\frac{1026}{221}\arctan(3-x) +\frac{56}{221}\log\left(10-6x+x^2\right)+\frac{109}{442}\log\left(1-2x+2x^2\right)$$

```
[Out] 261/221*arctan(-1+2*x)+1026/221*arctan(-3+x)+56/221*ln(x^2-6*x+10)+109/442*
ln(2*x^2-2*x+1)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6860, 648, 632, 210, 642, 631}

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)} \, dx = -\frac{261}{221}\arctan(1-2x)-\frac{1026}{221}\arctan(3-x) +\frac{56}{221}\log\left(x^2-6x+10\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)$$

```
[In] Int[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]
```

```
[Out] (-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x +
x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{2(345+56x)}{221\,(10-6x+x^2)}+\frac{2(76+109x)}{221\,(1-2x+2x^2)}\right) dx \\
&= \frac{2}{221}\int \frac{345+56x}{10-6x+x^2}\,dx+\frac{2}{221}\int \frac{76+109x}{1-2x+2x^2}\,dx \\
&= \frac{109}{442}\int \frac{-2+4x}{1-2x+2x^2}\,dx+\frac{56}{221}\int \frac{-6+2x}{10-6x+x^2}\,dx \\
&\quad +\frac{261}{221}\int \frac{1}{1-2x+2x^2}\,dx+\frac{1026}{221}\int \frac{1}{10-6x+x^2}\,dx
\end{aligned}$$

$$= \frac{56}{221}\log\left(10-6x+x^2\right)+\frac{109}{442}\log\left(1-2x+2x^2\right) + \frac{261}{221}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-2x\right)-\frac{2052}{221}\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, -6+2x\right)$$

$$= -\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)+\frac{56}{221}\log\left(10-6x+x^2\right)+\frac{109}{442}\log\left(1-2x + 2x^2\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = -\frac{261}{221}\arctan(1-2x)-\frac{1026}{221}\arctan(3-x) + \frac{56}{221}\log\left(10-6x+x^2\right)+\frac{109}{442}\log\left(1-2x+2x^2\right)$$

```
[In] Integrate[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]
```

```
[Out] (-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x +
x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result
default	$\frac{261\arctan(2x-1)}{221}+\frac{1026\arctan(-3+x)}{221}+\frac{56\ln(x^2-6x+10)}{221}+\frac{109\ln(2x^2-2x+1)}{442}$
risch	$\frac{56\ln(x^2-6x+10)}{221}+\frac{1026\arctan(-3+x)}{221}+\frac{109\ln(4x^2-4x+2)}{442}+\frac{261\arctan(2x-1)}{221}$
parallelrisch	$\frac{56\ln(x-3-i)}{221}-\frac{513i\ln(x-3-i)}{221}+\frac{56\ln(x-3+i)}{221}+\frac{513i\ln(x-3+i)}{221}+\frac{109\ln(x-\frac{1}{2}-\frac{i}{2})}{442}-\frac{261i\ln(x-\frac{1}{2}-\frac{i}{2})}{442}+\frac{109\ln}{}$

```
[In] int((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 261/221*arctan(2*x-1)+1026/221*arctan(-3+x)+56/221*ln(x^2-6*x+10)+109/442*l
n(2*x^2-2*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = \frac{261}{221}\,\arctan\left(2\,x-1\right)+\frac{1026}{221}\,\arctan\left(x-3\right)+\frac{109}{442}\,\log\left(2\,x^2-2\,x+1\right)+\frac{56}{221}\,\log\left(x^2-6\,x+10\right)$$

```
[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="fricas")
```

```
[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x
+ 1) + 56/221*log(x^2 - 6*x + 10)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = \frac{56\log\left(x^2-6x+10\right)}{221}+\frac{109\log\left(x^2-x+\frac{1}{2}\right)}{442}+\frac{1026\,\mathrm{atan}\left(x-3\right)}{221}+\frac{261\,\mathrm{atan}\left(2x-1\right)}{221}$$

```
[In] integrate((x**3+5)/(x**2-6*x+10)/(1/2-x+x**2),x)
```

```
[Out] 56*log(x**2 - 6*x + 10)/221 + 109*log(x**2 - x + 1/2)/442 + 1026*atan(x - 3
)/221 + 261*atan(2*x - 1)/221
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = \frac{261}{221}\,\arctan\left(2\,x-1\right)+\frac{1026}{221}\,\arctan\left(x-3\right)+\frac{109}{442}\,\log\left(2\,x^2-2\,x+1\right)+\frac{56}{221}\,\log\left(x^2-6\,x+10\right)$$

```
[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="maxima")
```

```
[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x
+ 1) + 56/221*log(x^2 - 6*x + 10)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = \frac{261}{221}\arctan\left(2\,x-1\right)+\frac{1026}{221}\arctan\left(x-3\right)+\frac{109}{442}\log\left(2\,x^2-2\,x+1\right)+\frac{56}{221}\log\left(x^2-6\,x+10\right)$$

```
[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="giac")
[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x
+ 1) + 56/221*log(x^2 - 6*x + 10)
```

Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = \ln\left(x-3-\mathrm{i}\right)\left(\frac{56}{221}-\frac{513}{221}\mathrm{i}\right)+\ln\left(x-3+1\mathrm{i}\right)\left(\frac{56}{221}+\frac{513}{221}\mathrm{i}\right)+\ln\left(x-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{109}{442}-\frac{261}{442}\mathrm{i}\right)+\ln\left(x-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{109}{442}+\frac{261}{442}\mathrm{i}\right)$$

```
[In] int((x^3 + 5)/((x^2 - x + 1/2)*(x^2 - 6*x + 10)),x)
[Out] log(x - (3 + 1i))*(56/221 - 513i/221) + log(x - (3 - 1i))*(56/221 + 513i/22
1) + log(x - (1/2 + 1i/2))*(109/442 - 261i/442) + log(x - (1/2 - 1i/2))*(10
9/442 + 261i/442)
```

3.361 $\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx$

Optimal result 2186
Rubi [A] (verified) 2186
Mathematica [A] (verified) 2187
Maple [A] (verified) 2187
Fricas [A] (verification not implemented) 2187
Sympy [A] (verification not implemented) 2188
Maxima [A] (verification not implemented) 2188
Giac [A] (verification not implemented) 2188
Mupad [B] (verification not implemented) 2188

Optimal result

Integrand size = 24, antiderivative size = 25

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 4\log(1-x)-14\log(2-x)+11\log(3-x)$$

```
[Out] 4*ln(1-x)-14*ln(2-x)+11*ln(3-x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{1626\}$

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 4\log(1-x)-14\log(2-x)+11\log(3-x)$$

```
[In] Int[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] 4*Log[1 - x] - 14*Log[2 - x] + 11*Log[3 - x]
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{11}{-3+x}-\frac{14}{-2+x}+\frac{4}{-1+x}\right)dx \\ &= 4\log(1-x)-14\log(2-x)+11\log(3-x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 11\log(-3+x)-14\log(-2+x)+4\log(-1+x)$$

```
[In] Integrate[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]
[Out] 11*Log[-3 + x] - 14*Log[-2 + x] + 4*Log[-1 + x]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$11\ln(-3+x)+4\ln(x-1)-14\ln(x-2)$	20
norman	$11\ln(-3+x)+4\ln(x-1)-14\ln(x-2)$	20
risch	$11\ln(-3+x)+4\ln(x-1)-14\ln(x-2)$	20
parallelrisch	$11\ln(-3+x)+4\ln(x-1)-14\ln(x-2)$	20

```
[In] int((x^2+3*x+4)/(-3+x)/(x-2)/(x-1),x,method=_RETURNVERBOSE)
[Out] 11*ln(-3+x)+4*ln(x-1)-14*ln(x-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 4\log(x-1)-14\log(x-2)+11\log(x-3)$$

```
[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")
[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 11\log(x-3) - 14\log(x-2) + 4\log(x-1)$$

```
[In] integrate((x**2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x)
[Out] 11*log(x - 3) - 14*log(x - 2) + 4*log(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 4\log(x-1) - 14\log(x-2) + 11\log(x-3)$$

```
[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")
[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 4\log(|x-1|) - 14\log(|x-2|) + 11\log(|x-3|)$$

```
[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")
[Out] 4*log(abs(x - 1)) - 14*log(abs(x - 2)) + 11*log(abs(x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 4\ln(x-1) - 14\ln(x-2) + 11\ln(x-3)$$

```
[In] int((3*x + x^2 + 4)/((x - 1)*(x - 2)*(x - 3)),x)
[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

3.362 $\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)}\,dx$

Optimal result 2189
Rubi [A] (verified) 2189
Mathematica [A] (verified) 2191
Maple [A] (verified) 2191
Fricas [A] (verification not implemented) 2191
Sympy [A] (verification not implemented) 2192
Maxima [A] (verification not implemented) 2192
Giac [A] (verification not implemented) 2192
Mupad [B] (verification not implemented) 2193

Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\,(1+x+x^2)}\,dx = -\frac{79}{273(5+x)}+\frac{451\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2793\sqrt{3}}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}-\frac{481\log\left(1+x+x^2\right)}{5586}$$

```
[Out] -79/273/(5+x)+200/3211*ln(3-2*x)+2731/24843*ln(5+x)-481/5586*ln(x^2+x+1)+45
1/8379*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6860, 648, 632, 210, 642}

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\,(1+x+x^2)}\,dx = \frac{451\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}-\frac{481\log\left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(x+5)}{24843}$$

```
[In] Int[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] -79/(273*(5 + x)) + (451*ArcTan[(1 + 2*x)/Sqrt[3]])/(2793*Sqrt[3]) + (200*L
og[3 - 2*x])/3211 + (2731*Log[5 + x])/24843 - (481*Log[1 + x + x^2])/5586
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{79}{273(5+x)^2}+\frac{2731}{24843(5+x)}+\frac{400}{3211(-3+2x)}+\frac{-15-481x}{2793\left(1+x+x^2\right)}\right)dx \\
&= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}+\frac{\int\frac{-15-481x}{1+x+x^2}\,dx}{2793} \\
&= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}+\frac{451\int\frac{1}{1+x+x^2}\,dx}{5586}-\frac{481\int\frac{1+2x}{1+x+x^2}\,dx}{5586} \\
&= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843} \\
&\quad -\frac{481\log\left(1+x+x^2\right)}{5586}-\frac{451\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+2x\right)}{2793} \\
&= -\frac{79}{273(5+x)}+\frac{451\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2793\sqrt{3}}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}-\frac{481\log\left(1+x+x^2\right)}{5586}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx$$
$$= \frac{-\frac{819546}{5+x}+152438\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+176400\log(3-2x)+311334\log(5+x)-243867\log\left(1+x+x^2\right)}{2832102}$$

```
[In] Integrate[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] (-819546/(5 + x) + 152438*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 176400*Log[3
- 2*x] + 311334*Log[5 + x] - 243867*Log[1 + x + x^2])/2832102
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{79}{273(5+x)}+\frac{2731\ln(5+x)}{24843}-\frac{481\ln(x^2+x+1)}{5586}+\frac{451\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{8379}+\frac{200\ln(2x-3)}{3211}$	48
risch	$-\frac{79}{273(5+x)}-\frac{481\ln(4x^2+4x+4)}{5586}+\frac{451\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{8379}+\frac{200\ln(2x-3)}{3211}+\frac{2731\ln(5+x)}{24843}$	52

```
[In] int((1+16*x)/(5+x)^2/(2*x-3)/(x^2+x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -79/273/(5+x)+2731/24843*ln(5+x)-481/5586*ln(x^2+x+1)+451/8379*arctan(1/3*(
1+2*x)*3^(1/2))*3^(1/2)+200/3211*ln(2*x-3)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx$$
$$= \frac{152438\sqrt{3}(x+5)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-243867(x+5)\log\left(x^2+x+1\right)+176400(x+5)\log(2x-3}{2832102(x+5)}$$

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/2832102*(152438*sqrt(3)*(x + 5)*arctan(1/3*sqrt(3)*(2*x + 1)) - 243867*(x
 + 5)*log(x^2 + x + 1) + 176400*(x + 5)*log(2*x - 3) + 311334*(x + 5)*log(x
 + 5) - 819546)/(x + 5)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

$$\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)}\,dx = \frac{200\log\left(x-\frac{3}{2}\right)}{3211} + \frac{2731\log(x+5)}{24843} - \frac{481\log(x^2+x+1)}{5586} + \frac{451\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{8379} - \frac{79}{273x+1365}$$

```
[In] integrate((1+16*x)/(5+x)**2/(-3+2*x)/(x**2+x+1),x)
[Out] 200*log(x - 3/2)/3211 + 2731*log(x + 5)/24843 - 481*log(x**2 + x + 1)/5586
+ 451*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/8379 - 79/(273*x + 1365)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)}\,dx = \frac{451}{8379}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{79}{273(x+5)} - \frac{481}{5586}\log\left(x^2+x+1\right) + \frac{200}{3211}\log(2x-3) + \frac{2731}{24843}\log(x+5)$$

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="maxima")
[Out] 451/8379*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 79/273/(x + 5) - 481/5586*
log(x^2 + x + 1) + 200/3211*log(2*x - 3) + 2731/24843*log(x + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)}\,dx = \frac{451}{8379}\sqrt{3}\arctan\left(-\sqrt{3}\left(\frac{14}{x+5}-3\right)\right) - \frac{79}{273(x+5)} - \frac{481}{5586}\log\left(-\frac{9}{x+5}+\frac{21}{(x+5)^2}+1\right) + \frac{200}{3211}\log\left(\left|-\frac{13}{x+5}+2\right|\right)$$

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 451/8379*sqrt(3)*arctan(-sqrt(3)*(14/(x + 5) - 3)) - 79/273/(x + 5) - 481/5
586*log(-9/(x + 5) + 21/(x + 5)^2 + 1) + 200/3211*log(abs(-13/(x + 5) + 2))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\,(1+x+x^2)}\,dx = \frac{200\,\ln\left(x-\frac{3}{2}\right)}{3211}+\frac{2731\,\ln\left(x+5\right)}{24843}-\frac{79}{273\,\left(x+5\right)} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{481}{5586}+\frac{\sqrt{3}\,451\mathrm{i}}{16758}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{481}{5586}+\frac{\sqrt{3}\,451\mathrm{i}}{16758}\right)$$

```
[In] int((16*x + 1)/((2*x - 3)*(x + 5)^2*(x + x^2 + 1)),x)
```

```
[Out] (200*log(x - 3/2))/3211 + (2731*log(x + 5))/24843 - 79/(273*(x + 5)) - log(
x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*451i)/16758 + 481/5586) + log(x + (3^(1
/2)*1i)/2 + 1/2)*((3^(1/2)*451i)/16758 - 481/5586)
```

3.363 $\int \frac{-1+x^3}{1+x+x^2} \, dx$

Optimal result 2194
Rubi [A] (verified) 2194
Mathematica [A] (verified) 2195
Maple [A] (verified) 2195
Fricas [A] (verification not implemented) 2195
Sympy [A] (verification not implemented) 2196
Maxima [A] (verification not implemented) 2196
Giac [A] (verification not implemented) 2196
Mupad [B] (verification not implemented) 2196

Optimal result

Integrand size = 14, antiderivative size = 11

$$\int \frac{-1+x^3}{1+x+x^2} \, dx = -x + \frac{x^2}{2}$$

```
[Out] -x+1/2*x^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1600}

$$\int \frac{-1+x^3}{1+x+x^2} \, dx = \frac{x^2}{2} - x$$

```
[In] Int[(-1 + x^3)/(1 + x + x^2),x]
```

```
[Out] -x + x^2/2
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int (-1+x) \, dx \\ &= -x + \frac{x^2}{2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^3}{1+x+x^2} \, dx = -x + \frac{x^2}{2}$$

```
[In] Integrate[(-1 + x^3)/(1 + x + x^2),x]
[Out] -x + x^2/2
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x(x-2)}{2}$	7
default	$-x+\frac{1}{2}x^2$	10
norman	$-x+\frac{1}{2}x^2$	10
risch	$-x+\frac{1}{2}x^2$	10
parallelrisch	$-x+\frac{1}{2}x^2$	10
parts	$-x+\frac{1}{2}x^2$	10

```
[In] int((x^3-1)/(x^2+x+1),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(x-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{-1+x^3}{1+x+x^2} \, dx = \frac{1}{2} \, x^2 - x$$

```
[In] integrate((x^3-1)/(x^2+x+1),x, algorithm="fricas")
[Out] 1/2*x^2 - x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.45

$$\int \frac{-1+x^3}{1+x+x^2}\,dx = \frac{x^2}{2}-x$$

```
[In] integrate((x**3-1)/(x**2+x+1),x)
[Out] x**2/2 - x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{-1+x^3}{1+x+x^2}\,dx = \frac{1}{2}\,x^2-x$$

```
[In] integrate((x^3-1)/(x^2+x+1),x, algorithm="maxima")
[Out] 1/2*x^2 - x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{-1+x^3}{1+x+x^2}\,dx = \frac{1}{2}\,x^2-x$$

```
[In] integrate((x^3-1)/(x^2+x+1),x, algorithm="giac")
[Out] 1/2*x^2 - x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{-1+x^3}{1+x+x^2}\,dx = \frac{x\,(x-2)}{2}$$

```
[In] int((x^3 - 1)/(x + x^2 + 1),x)
[Out] (x*(x - 2))/2
```

3.364 $\int \frac{-3+x^3}{-7-6x+x^2} \, dx$

Optimal result 2197
Rubi [A] (verified) 2197
Mathematica [A] (verified) 2198
Maple [A] (verified) 2198
Fricas [A] (verification not implemented) 2199
Sympy [A] (verification not implemented) 2199
Maxima [A] (verification not implemented) 2199
Giac [A] (verification not implemented) 2200
Mupad [B] (verification not implemented) 2200

Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{-3+x^3}{-7-6x+x^2} \, dx = 6x+\frac{x^2}{2}+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(1+x)$$

```
[Out] 6*x+1/2*x^2+85/2*ln(7-x)+1/2*ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1671, 646, 31}

$$\int \frac{-3+x^3}{-7-6x+x^2} \, dx = \frac{x^2}{2}+6x+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(x+1)$$

```
[In] Int[(-3 + x^3)/(-7 - 6*x + x^2),x]
```

```
[Out] 6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(6+x+\frac{39+43x}{-7-6x+x^2}\right) dx\\ &= 6x+\frac{x^2}{2}+\int \frac{39+43x}{-7-6x+x^2}\,dx\\ &= 6x+\frac{x^2}{2}+\frac{1}{2}\int \frac{1}{1+x}\,dx+\frac{85}{2}\int \frac{1}{-7+x}\,dx\\ &= 6x+\frac{x^2}{2}+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{-3+x^3}{-7-6x+x^2}\,dx = 6x+\frac{x^2}{2}+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(1+x)$$

```
[In] Integrate[(-3 + x^3)/(-7 - 6*x + x^2),x]
[Out] 6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{x^2}{2}+6x+\frac{85\ln(x-7)}{2}+\frac{\ln(x+1)}{2}$	22
norman	$\frac{x^2}{2}+6x+\frac{85\ln(x-7)}{2}+\frac{\ln(x+1)}{2}$	22
risch	$\frac{x^2}{2}+6x+\frac{85\ln(x-7)}{2}+\frac{\ln(x+1)}{2}$	22
parallelrisch	$\frac{x^2}{2}+6x+\frac{85\ln(x-7)}{2}+\frac{\ln(x+1)}{2}$	22

```
[In] int((x^3-3)/(x^2-6*x-7),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+6*x+85/2*ln(x-7)+1/2*ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{-3+x^3}{-7-6x+x^2}\,dx=\frac{1}{2}\,x^2+6\,x+\frac{1}{2}\,\log\left(x+1\right)+\frac{85}{2}\,\log\left(x-7\right)$$

```
[In] integrate((x^3-3)/(x^2-6*x-7),x, algorithm="fricas")
[Out] 1/2*x^2 + 6*x + 1/2*log(x + 1) + 85/2*log(x - 7)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{-3+x^3}{-7-6x+x^2}\,dx=\frac{x^2}{2}+6x+\frac{85\log\left(x-7\right)}{2}+\frac{\log\left(x+1\right)}{2}$$

```
[In] integrate((x**3-3)/(x**2-6*x-7),x)
[Out] x**2/2 + 6*x + 85*log(x - 7)/2 + log(x + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{-3+x^3}{-7-6x+x^2}\,dx=\frac{1}{2}\,x^2+6\,x+\frac{1}{2}\,\log\left(x+1\right)+\frac{85}{2}\,\log\left(x-7\right)$$

```
[In] integrate((x^3-3)/(x^2-6*x-7),x, algorithm="maxima")
[Out] 1/2*x^2 + 6*x + 1/2*log(x + 1) + 85/2*log(x - 7)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{-3+x^3}{-7-6x+x^2}\,dx = \frac{1}{2}\,x^2 + 6\,x + \frac{1}{2}\,\log\left(|x+1|\right) + \frac{85}{2}\,\log\left(|x-7|\right)$$

```
[In] integrate((x^3-3)/(x^2-6*x-7),x, algorithm="giac")
[Out] 1/2*x^2 + 6*x + 1/2*log(abs(x + 1)) + 85/2*log(abs(x - 7))
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{-3+x^3}{-7-6x+x^2}\,dx = 6\,x + \frac{\ln\left(x+1\right)}{2} + \frac{85\,\ln\left(x-7\right)}{2} + \frac{x^2}{2}$$

```
[In] int(-(x^3 - 3)/(6*x - x^2 + 7),x)
[Out] 6*x + log(x + 1)/2 + (85*log(x - 7))/2 + x^2/2
```

3.365 $\int \frac{1+x^3}{(13+4x+x^2)^2} dx$

Optimal result . . . 2201
Rubi [A] (verified) . . . 2201
Mathematica [A] (verified) . . . 2203
Maple [A] (verified) . . . 2203
Fricas [A] (verification not implemented) . . . 2203
Sympy [A] (verification not implemented) . . . 2204
Maxima [A] (verification not implemented) . . . 2204
Giac [A] (verification not implemented) . . . 2204
Mupad [B] (verification not implemented) . . . 2205

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \frac{1+x^3}{(13+4x+x^2)^2} dx = \frac{67+47x}{18(13+4x+x^2)} - \frac{61}{54}\arctan\left(\frac{2+x}{3}\right) + \frac{1}{2}\log\left(13+4x+x^2\right)$$

```
[Out] 1/18*(67+47*x)/(x^2+4*x+13)-61/54*arctan(2/3+1/3*x)+1/2*ln(x^2+4*x+13)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1674, 648, 632, 210, 642}

$$\int \frac{1+x^3}{(13+4x+x^2)^2} dx = -\frac{61}{54}\arctan\left(\frac{x+2}{3}\right) + \frac{47x+67}{18(x^2+4x+13)} + \frac{1}{2}\log\left(x^2+4x+13\right)$$

```
[In] Int[(1 + x^3)/(13 + 4*x + x^2)^2,x]
```

```
[Out] (67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + Log[13 + 4*
x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{67+47x}{18\left(13+4x+x^2\right)}+\frac{1}{36}\int \frac{-50+36x}{13+4x+x^2}\,dx \\
&= \frac{67+47x}{18\left(13+4x+x^2\right)}+\frac{1}{2}\int \frac{4+2x}{13+4x+x^2}\,dx-\frac{61}{18}\int \frac{1}{13+4x+x^2}\,dx \\
&= \frac{67+47x}{18\left(13+4x+x^2\right)}+\frac{1}{2}\log\left(13+4x+x^2\right)+\frac{61}{9}\text{Subst}\left(\int \frac{1}{-36-x^2}\,dx,x,4+2x\right) \\
&= \frac{67+47x}{18\left(13+4x+x^2\right)}-\frac{61}{54}\tan^{-1}\left(\frac{2+x}{3}\right)+\frac{1}{2}\log\left(13+4x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1+x^3}{\left(13+4x+x^2\right)^2}\,dx = \frac{67+47x}{18\left(13+4x+x^2\right)} - \frac{61}{54}\arctan\left(\frac{2+x}{3}\right) + \frac{1}{2}\log\left(13+4x+x^2\right)$$

```
[In] Integrate[(1 + x^3)/(13 + 4*x + x^2)^2,x]
```

```
[Out] (67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + Log[13 + 4*
x + x^2]/2
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result
default	$\frac{\frac{47x}{18}+\frac{67}{18}}{x^2+4x+13}+\frac{\ln\left(x^2+4x+13\right)}{2}-\frac{61\arctan\left(\frac{2}{3}+\frac{x}{3}\right)}{54}$
risch	$\frac{\frac{47x}{18}+\frac{67}{18}}{x^2+4x+13}+\frac{\ln\left(x^2+4x+13\right)}{2}-\frac{61\arctan\left(\frac{2}{3}+\frac{x}{3}\right)}{54}$
parallelrisch	$\frac{-793i\ln(x+2+3i)x^2+10309i\ln(x+2-3i)+3172i\ln(x+2-3i)x+702\ln(x+2-3i)x^2-3172i\ln(x+2+3i)x+702\ln(x+2+3i)x^2+}{1404x^2}$

```
[In] int((x^3+1)/(x^2+4*x+13)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (47/18*x+67/18)/(x^2+4*x+13)+1/2*ln(x^2+4*x+13)-61/54*arctan(2/3+1/3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int \frac{1+x^3}{\left(13+4x+x^2\right)^2}\,dx = -\frac{61\left(x^2+4\,x+13\right)\arctan\left(\frac{1}{3}\,x+\frac{2}{3}\right)-27\left(x^2+4\,x+13\right)\log\left(x^2+4\,x+13\right)-141\,x-201}{54\left(x^2+4\,x+13\right)}$$

```
[In] integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="fricas")
```

```
[Out] -1/54*(61*(x^2 + 4*x + 13)*arctan(1/3*x + 2/3) - 27*(x^2 + 4*x + 13)*log(x^
2 + 4*x + 13) - 141*x - 201)/(x^2 + 4*x + 13)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{1+x^3}{(13+4x+x^2)^2} \, dx = \frac{47x+67}{18x^2+72x+234} + \frac{\log(x^2+4x+13)}{2} - \frac{61\operatorname{atan}\left(\frac{x}{3}+\frac{2}{3}\right)}{54}$$

```
[In] integrate((x**3+1)/(x**2+4*x+13)**2,x)
[Out] (47*x + 67)/(18*x**2 + 72*x + 234) + log(x**2 + 4*x + 13)/2 - 61*atan(x/3 +
 2/3)/54
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{1+x^3}{(13+4x+x^2)^2} \, dx = \frac{47\,x+67}{18\,(x^2+4\,x+13)} - \frac{61}{54}\arctan\left(\frac{1}{3}\,x+\frac{2}{3}\right) + \frac{1}{2}\log\left(x^2+4\,x+13\right)$$

```
[In] integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="maxima")
[Out] 1/18*(47*x + 67)/(x^2 + 4*x + 13) - 61/54*arctan(1/3*x + 2/3) + 1/2*log(x^2
 + 4*x + 13)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{1+x^3}{(13+4x+x^2)^2} \, dx = \frac{47\,x+67}{18\,(x^2+4\,x+13)} - \frac{61}{54}\arctan\left(\frac{1}{3}\,x+\frac{2}{3}\right) + \frac{1}{2}\log\left(x^2+4\,x+13\right)$$

```
[In] integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="giac")
[Out] 1/18*(47*x + 67)/(x^2 + 4*x + 13) - 61/54*arctan(1/3*x + 2/3) + 1/2*log(x^2
 + 4*x + 13)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int \frac{1+x^3}{(13+4x+x^2)^2}\,dx = \frac{\ln\left(x^2+4\,x+13\right)}{2} - \frac{61\,\mathrm{atan}\left(\frac{x}{3}+\frac{2}{3}\right)}{54} + \frac{47\,x}{18\,\left(x^2+4\,x+13\right)} + \frac{67}{18\,\left(x^2+4\,x+13\right)}$$

```
[In] int((x^3 + 1)/(4*x + x^2 + 13)^2,x)
[Out] log(4*x + x^2 + 13)/2 - (61*atan(x/3 + 2/3))/54 + (47*x)/(18*(4*x + x^2 + 1
3)) + 67/(18*(4*x + x^2 + 13))
```

3.366 $\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2} dx$

Optimal result 2206
Rubi [A] (verified) 2206
Mathematica [A] (verified) 2208
Maple [A] (verified) 2208
Fricas [A] (verification not implemented) 2208
Sympy [A] (verification not implemented) 2209
Maxima [A] (verification not implemented) 2209
Giac [A] (verification not implemented) 2209
Mupad [B] (verification not implemented) 2210

Optimal result

Integrand size = 43, antiderivative size = 32

$$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2} dx = \frac{1}{4+x^2}+\frac{1}{2}\arctan\left(\frac{x}{2}\right)+2\arctan(x)-2\log(x)+\log\left(4+x^2\right)$$

```
[Out] 1/(x^2+4)+1/2*arctan(1/2*x)+2*arctan(x)-2*ln(x)+ln(x^2+4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {6857, 209, 267, 649, 266}

$$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2} dx = \frac{1}{2}\arctan\left(\frac{x}{2}\right)+2\arctan(x)+\frac{1}{x^2+4}+\log\left(x^2+4\right)-2\log(x)$$

```
[In] Int[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 + x^2)*(4 + x^2)^
2),x]
```

```
[Out] (4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 + x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{2}{x}+\frac{2}{1+x^2}-\frac{2x}{(4+x^2)^2}+\frac{1+2x}{4+x^2}\right) dx \\
&= -2\log(x)+2\int \frac{1}{1+x^2}\,dx-2\int \frac{x}{(4+x^2)^2}\,dx+\int \frac{1+2x}{4+x^2}\,dx \\
&= \frac{1}{4+x^2}+2\tan^{-1}(x)-2\log(x)+2\int \frac{x}{4+x^2}\,dx+\int \frac{1}{4+x^2}\,dx \\
&= \frac{1}{4+x^2}+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+2\tan^{-1}(x)-2\log(x)+\log\left(4+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x\left(1+x^2\right)\left(4+x^2\right)^2}\,dx = \frac{1}{4+x^2}+\frac{1}{2}\arctan\left(\frac{x}{2}\right)+2\arctan(x)-2\log(x)+\log\left(4+x^2\right)$$

```
[In] Integrate[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 + x^2)*(4 +
 x^2)^2),x]
```

```
[Out] (4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 + x^2]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result
default	$\frac{1}{x^2+4}+\frac{\arctan\left(\frac{x}{2}\right)}{2}+2\arctan\left(x\right)-2\ln\left(x\right)+\ln\left(x^2+4\right)$
risch	$\frac{1}{x^2+4}+\frac{\arctan\left(\frac{x}{2}\right)}{2}+2\arctan\left(x\right)-2\ln\left(x\right)+\ln\left(x^2+4\right)$
parallelrisch	$-\frac{4i\ln(x-2i)-i\ln(x+2i)x^2+16i\ln(x-i)+4i\ln(x-i)x^2+8\ln(x)x^2-4\ln(x-2i)x^2-4\ln(x+2i)x^2-4i\ln(x+2i)-4i\ln(x+i)x^2-16}{4(x^2+4)}$

```
[In] int((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/(x^2+4)+1/2*arctan(1/2*x)+2*arctan(x)-2*ln(x)+ln(x^2+4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x\left(1+x^2\right)\left(4+x^2\right)^2}\,dx = \frac{\left(x^2+4\right)\arctan\left(\frac{1}{2}x\right)+4\left(x^2+4\right)\arctan\left(x\right)+2\left(x^2+4\right)\log\left(x^2+4\right)-4\left(x^2+4\right)\log\left(x\right)+2}{2\left(x^2+4\right)}$$

```
[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algor
ithm="fricas")
```

```
[Out] 1/2*((x^2 + 4)*arctan(1/2*x) + 4*(x^2 + 4)*arctan(x) + 2*(x^2 + 4)*log(x^2
+ 4) - 4*(x^2 + 4)*log(x) + 2)/(x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2}\,dx = -2\log(x)+\log(x^2+4)+\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}+2\operatorname{atan}(x)+\frac{1}{x^2+4}$$

```
[In] integrate((3*x**5-10*x**4+21*x**3-42*x**2+36*x-32)/x/(x**2+1)/(x**2+4)**2,x
)
```

```
[Out] -2*log(x) + log(x**2 + 4) + atan(x/2)/2 + 2*atan(x) + 1/(x**2 + 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2}\,dx = \frac{1}{x^2+4}+\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+2\arctan(x)+\log(x^2+4)-2\log(x)$$

```
[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algor
ithm="maxima")
```

```
[Out] 1/(x^2 + 4) + 1/2*arctan(1/2*x) + 2*arctan(x) + log(x^2 + 4) - 2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2}\,dx = \frac{1}{x^2+4}+\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+2\arctan(x)+\log(x^2+4)-2\log(|x|)$$

```
[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algor
ithm="giac")
```

```
[Out] 1/(x^2 + 4) + 1/2*arctan(1/2*x) + 2*arctan(x) + log(x^2 + 4) - 2*log(abs(x)
)
```

Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x\,(1+x^2)\,(4+x^2)^2}\,dx$$
$$=\frac{1}{x^2+4}-2\,\ln\left(x\right)-2\,\mathrm{atan}\left(\frac{328000}{7\,\left(36288\,x-19584\right)}+\frac{34}{63}\right)$$
$$+\ln\left(x-2\mathrm{i}\right)\,\left(1-\frac{1}{4}\mathrm{i}\right)+\ln\left(x+2\mathrm{i}\right)\,\left(1+\frac{1}{4}\mathrm{i}\right)$$

```
[In] int((36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5 - 32)/(x*(x^2 + 1)*(x^2 + 4)^2
),x)
```

```
[Out] log(x - 2i)*(1 - 1i/4) + log(x + 2i)*(1 + 1i/4) - 2*atan(328000/(7*(36288*x
 - 19584)) + 34/63) - 2*log(x) + 1/(x^2 + 4)
```

3.367 $\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx$

Optimal result . . . 2211
Rubi [A] (verified) . . . 2211
Mathematica [A] (verified) . . . 2215
Maple [C] (verified) . . . 2215
Fricas [C] (verification not implemented) . . . 2216
Sympy [A] (verification not implemented) . . . 2216
Maxima [A] (verification not implemented) . . . 2217
Giac [A] (verification not implemented) . . . 2217
Mupad [B] (verification not implemented) . . . 2218

Optimal result

Integrand size = 26, antiderivative size = 148

$$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx = \frac{x^2}{2}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}7^{3/4}}+\frac{\arctan\left(1+\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}7^{3/4}}-\frac{\operatorname{arctanh}(x^2)}{2}-\frac{\log\left(\sqrt{7}-\sqrt{2}\sqrt[4]{7}x+x^2\right)}{4\sqrt{2}7^{3/4}}+\frac{\log\left(\sqrt{7}+\sqrt{2}\sqrt[4]{7}x+x^2\right)}{4\sqrt{2}7^{3/4}}$$

```
[Out] 1/2*x^2-1/2*arctanh(x^2)+1/28*arctan(-1+1/7*x*2^(1/2)*7^(3/4))*7^(1/4)*2^(1
/2)+1/28*arctan(1+1/7*x*2^(1/2)*7^(3/4))*7^(1/4)*2^(1/2)-1/56*ln(x^2-7^(1/4
)*x*2^(1/2)+7^(1/2))*7^(1/4)*2^(1/2)+1/56*ln(x^2+7^(1/4)*x*2^(1/2)+7^(1/2))
*7^(1/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1804, 1417, 217, 1179, 642, 1176, 631, 210, 1598, 1492, 281, 327, 213}

$$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}7^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{7}}+1\right)}{2\sqrt{2}7^{3/4}}-\frac{\operatorname{arctanh}(x^2)}{2}+\frac{x^2}{2}-\frac{\log\left(x^2-\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}7^{3/4}}+\frac{\log\left(x^2+\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}7^{3/4}}$$

```
[In] Int[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8),x]
```

```
[Out] x^2/2 - ArcTan[1 - (Sqrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) + ArcTan[1 + (S
qrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) - ArcTanh[x^2]/2 - Log[Sqrt[7] - Sqr
t[2]*7^(1/4)*x + x^2]/(4*Sqrt[2]*7^(3/4)) + Log[Sqrt[7] + Sqrt[2]*7^(1/4)*x
 + x^2]/(4*Sqrt[2]*7^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1417

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_))^(p_.), x_Symbol] :> Int[(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /;
FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 1492

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n
_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*
(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p
]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1804

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x]] /;
 FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] &&  !PolyQ[Pq, x^n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{-1+x^4}{-7+6x^4+x^8}+\frac{x(7x^4+x^8)}{-7+6x^4+x^8}\right)dx\\
&= \int \frac{-1+x^4}{-7+6x^4+x^8}\,dx+\int \frac{x(7x^4+x^8)}{-7+6x^4+x^8}\,dx\\
&= \int \frac{1}{7+x^4}\,dx+\int \frac{x^5(7+x^4)}{-7+6x^4+x^8}\,dx\\
&= \frac{\int \frac{\sqrt{7}-x^2}{7+x^4}\,dx}{2\sqrt{7}}+\frac{\int \frac{\sqrt{7}+x^2}{7+x^4}\,dx}{2\sqrt{7}}+\int \frac{x^5}{-1+x^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,x^2\right)-\frac{\int \frac{\sqrt{2}\sqrt[4]{7}+2x}{-\sqrt{7}-\sqrt{2}\sqrt[4]{7}x-x^2}\,dx}{4\sqrt{2}7^{3/4}}\\
&\quad-\frac{\int \frac{\sqrt{2}\sqrt[4]{7}-2x}{-\sqrt{7}+\sqrt{2}\sqrt[4]{7}x-x^2}\,dx}{4\sqrt{2}7^{3/4}}+\frac{\int \frac{1}{\sqrt{7}-\sqrt{2}\sqrt[4]{7}x+x^2}\,dx}{4\sqrt{7}}+\frac{\int \frac{1}{\sqrt{7}+\sqrt{2}\sqrt[4]{7}x+x^2}\,dx}{4\sqrt{7}}\\
&= \frac{x^2}{2}-\frac{\log\left(\sqrt{7}-\sqrt{2}\sqrt[4]{7}x+x^2\right)}{4\sqrt{2}7^{3/4}}+\frac{\log\left(\sqrt{7}+\sqrt{2}\sqrt[4]{7}x+x^2\right)}{4\sqrt{2}7^{3/4}}\\
&\quad+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,x^2\right)+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}7^{3/4}}\\
&\quad-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}7^{3/4}}\\
&= \frac{x^2}{2}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}7^{3/4}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}7^{3/4}}\\
&\quad-\frac{1}{2}\tanh^{-1}\left(x^2\right)-\frac{\log\left(\sqrt{7}-\sqrt{2}\sqrt[4]{7}x+x^2\right)}{4\sqrt{2}7^{3/4}}+\frac{\log\left(\sqrt{7}+\sqrt{2}\sqrt[4]{7}x+x^2\right)}{4\sqrt{2}7^{3/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07

$$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx = \frac{1}{56}\left(28x^2-2\sqrt{2}\sqrt[4]{7}\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)+2\sqrt{2}\sqrt[4]{7}\arctan\left(1+\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)+14\log(1-x)+14\log(1+x)-14\log\left(1+x^2\right)-\sqrt{2}\sqrt[4]{7}\log\left(7-\sqrt{2}7^{3/4}x+\sqrt{7}x^2\right)+\sqrt{2}\sqrt[4]{7}\log\left(7+\sqrt{2}7^{3/4}x+\sqrt{7}x^2\right)\right)$$

```
[In] Integrate[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8),x]
```

```
[Out] (28*x^2 - 2*Sqrt[2]*7^(1/4)*ArcTan[1 - (Sqrt[2]*x)/7^(1/4)] + 2*Sqrt[2]*7^(
1/4)*ArcTan[1 + (Sqrt[2]*x)/7^(1/4)] + 14*Log[1 - x] + 14*Log[1 + x] - 14*L
og[1 + x^2] - Sqrt[2]*7^(1/4)*Log[7 - Sqrt[2]*7^(3/4)*x + Sqrt[7]*x^2] + Sq
rt[2]*7^(1/4)*Log[7 + Sqrt[2]*7^(3/4)*x + Sqrt[7]*x^2])/56
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.30

method	result
risch	$\frac{x^2}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(343\_Z^4+1\right)}\_R\ln\left(x+7\_R\right)\right)}{4}+\frac{\ln\left(x^2-1\right)}{4}-\frac{\ln\left(x^2+1\right)}{4}$
default	$\frac{x^2}{2}+\frac{\ln(x+1)}{4}+\frac{7^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+7^{\frac{1}{4}}x\sqrt{2}+\sqrt{7}}{x^2-7^{\frac{1}{4}}x\sqrt{2}+\sqrt{7}}\right)+2\arctan\left(1+\frac{x\sqrt{2}\,7^{\frac{3}{4}}}{7}\right)+2\arctan\left(-1+\frac{x\sqrt{2}\,7^{\frac{3}{4}}}{7}\right)\right)}{56}-\frac{\ln\left(x^2+1\right)}{4}+\frac{\ln(x-1)}{4}$

```
[In] int((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+1/4*sum(_R*ln(x+7*_R),_R=RootOf(343*_Z^4+1))+1/4*ln(x^2-1)-1/4*ln(x
^2+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.72

$$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx = \left(\frac{1}{2744}i+\frac{1}{2744}\right)\cdot 343^{\frac{3}{4}}\sqrt{2}\log\left((i+1)\cdot 343^{\frac{3}{4}}\sqrt{2}+98\,x\right) - \left(\frac{1}{2744}i-\frac{1}{2744}\right)\cdot 343^{\frac{3}{4}}\sqrt{2}\log\left(-(i-1)\cdot 343^{\frac{3}{4}}\sqrt{2}+98\,x\right) + \left(\frac{1}{2744}i-\frac{1}{2744}\right)\cdot 343^{\frac{3}{4}}\sqrt{2}\log\left((i-1)\cdot 343^{\frac{3}{4}}\sqrt{2}+98\,x\right) - \left(\frac{1}{2744}i+\frac{1}{2744}\right)\cdot 343^{\frac{3}{4}}\sqrt{2}\log\left(-(i+1)\cdot 343^{\frac{3}{4}}\sqrt{2}+98\,x\right) + \frac{1}{2}\,x^2-\frac{1}{4}\,\log\left(x^2+1\right)+\frac{1}{4}\,\log\left(x^2-1\right)$$

```
[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="fricas")
```

```
[Out] (1/2744*I + 1/2744)*343^(3/4)*sqrt(2)*log((I + 1)*343^(3/4)*sqrt(2) + 98*x)
 - (1/2744*I - 1/2744)*343^(3/4)*sqrt(2)*log(-(I - 1)*343^(3/4)*sqrt(2) + 9
8*x) + (1/2744*I - 1/2744)*343^(3/4)*sqrt(2)*log((I - 1)*343^(3/4)*sqrt(2)
+ 98*x) - (1/2744*I + 1/2744)*343^(3/4)*sqrt(2)*log(-(I + 1)*343^(3/4)*sqrt
(2) + 98*x) + 1/2*x^2 - 1/4*log(x^2 + 1) + 1/4*log(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx = \frac{x^2}{2}+\frac{\log\left(x^2-1\right)}{4}-\frac{\log\left(x^2+1\right)}{4} - \frac{\sqrt{2}\cdot\sqrt[4]{7}\log\left(x^2-\sqrt{2}\cdot\sqrt[4]{7}x+\sqrt{7}\right)}{56} + \frac{\sqrt{2}\cdot\sqrt[4]{7}\log\left(x^2+\sqrt{2}\cdot\sqrt[4]{7}x+\sqrt{7}\right)}{56} + \frac{\sqrt{2}\cdot\sqrt[4]{7}\,\mathrm{atan}\left(\frac{\sqrt{2}\cdot 7^{\frac{3}{4}}x}{7}-1\right)}{28} + \frac{\sqrt{2}\cdot\sqrt[4]{7}\,\mathrm{atan}\left(\frac{\sqrt{2}\cdot 7^{\frac{3}{4}}x}{7}+1\right)}{28}$$

```
[In] integrate((x**9+7*x**5+x**4-1)/(x**8+6*x**4-7),x)
```

```
[Out] x**2/2 + log(x**2 - 1)/4 - log(x**2 + 1)/4 - sqrt(2)*7**(1/4)*log(x**2 - sq
rt(2)*7**(1/4)*x + sqrt(7))/56 + sqrt(2)*7**(1/4)*log(x**2 + sqrt(2)*7**(1/
4)*x + sqrt(7))/56 + sqrt(2)*7**(1/4)*atan(sqrt(2)*7**(3/4)*x/7 - 1)/28 + s
qrt(2)*7**(1/4)*atan(sqrt(2)*7**(3/4)*x/7 + 1)/28
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

$$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx = \frac{1}{2}\,x^2 + \frac{1}{28}\cdot 7^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2\,x+7^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{28}\cdot 7^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2\,x-7^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{56}\cdot 7^{\frac{1}{4}}\sqrt{2}\log\left(x^2+7^{\frac{1}{4}}\sqrt{2}x+\sqrt{7}\right) - \frac{1}{56}\cdot 7^{\frac{1}{4}}\sqrt{2}\log\left(x^2-7^{\frac{1}{4}}\sqrt{2}x+\sqrt{7}\right) - \frac{1}{4}\,\log\left(x^2+1\right) + \frac{1}{4}\,\log\left(x+1\right) + \frac{1}{4}\,\log\left(x-1\right)$$

```
[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 1/28*7^(1/4)*sqrt(2)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x + 7^(1/4)*s
qrt(2))) + 1/28*7^(1/4)*sqrt(2)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x - 7^(1/4)*
sqrt(2))) + 1/56*7^(1/4)*sqrt(2)*log(x^2 + 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1
/56*7^(1/4)*sqrt(2)*log(x^2 - 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/4*log(x^2 +
1) + 1/4*log(x + 1) + 1/4*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx = \frac{1}{2}\,x^2 + \frac{1}{28}\cdot 28^{\frac{1}{4}}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2\,x+7^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{28}\cdot 28^{\frac{1}{4}}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2\,x-7^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{56}\cdot 28^{\frac{1}{4}}\log\left(x^2+7^{\frac{1}{4}}\sqrt{2}x+\sqrt{7}\right) - \frac{1}{56}\cdot 28^{\frac{1}{4}}\log\left(x^2-7^{\frac{1}{4}}\sqrt{2}x+\sqrt{7}\right) - \frac{1}{4}\,\log\left(x^2+1\right) + \frac{1}{4}\,\log\left(|x+1|\right) + \frac{1}{4}\,\log\left(|x-1|\right)$$

```
[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/28*28^(1/4)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x + 7^(1/4)*sqrt(2))
) + 1/28*28^(1/4)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x - 7^(1/4)*sqrt(2))) + 1/
56*28^(1/4)*log(x^2 + 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/56*28^(1/4)*log(x^2
- 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/4*log(x^2 + 1) + 1/4*log(abs(x + 1)) + 1
/4*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

$$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx$$
$$= \frac{\operatorname{atan}(x^2\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{x^2}{2}+\sqrt{2}\,7^{1/4}\operatorname{atan}\left(\frac{\sqrt{2}\,7^{1/4}\,x\left(\frac{89653248}{2401}+\frac{89653248}{2401}\mathrm{i}\right)}{-\frac{1048576}{49}+\frac{\sqrt{7}\,179306496\mathrm{i}}{2401}}+\frac{\sqrt{2}\,7^{3/4}\,x\left(-\frac{524288}{343}+\frac{524288}{343}\mathrm{i}\right)}{-\frac{1048576}{49}+\frac{\sqrt{7}\,179306496\mathrm{i}}{2401}}\right)\left(\frac{1}{28}+\frac{1}{28}\mathrm{i}\right)+\sqrt{2}\,7^{1/4}\operatorname{atan}\left(\frac{\sqrt{2}\,7^{1/4}\,x\left(\frac{89653248}{2401}-\frac{89653248}{2401}\mathrm{i}\right)}{\frac{1048576}{49}+\frac{\sqrt{7}\,179306496\mathrm{i}}{2401}}+\frac{\sqrt{2}\,7^{3/4}\,x\,(\cdots}{\frac{1048576}{49}\cdots}\right.$$

```
[In] int((x^4 + 7*x^5 + x^9 - 1)/(6*x^4 + x^8 - 7),x)
```

```
[Out] (atan(x^2*1i)*1i)/2 + x^2/2 + 2^(1/2)*7^(1/4)*atan((2^(1/2)*7^(1/4)*x*(8965
3248/2401 + 89653248i/2401))/((7^(1/2)*179306496i)/2401 - 1048576/49) - (2^
(1/2)*7^(3/4)*x*(524288/343 - 524288i/343))/((7^(1/2)*179306496i)/2401 - 10
48576/49))*(1/28 + 1i/28) - 2^(1/2)*7^(1/4)*atan((2^(1/2)*7^(1/4)*x*(896532
48/2401 - 89653248i/2401))/((7^(1/2)*179306496i)/2401 + 1048576/49) - (2^(1
/2)*7^(3/4)*x*(524288/343 + 524288i/343))/((7^(1/2)*179306496i)/2401 + 1048
576/49))*(1/28 - 1i/28)
```

3.368 $\int \frac{1+x^3+x^6}{x+x^5} dx$

Optimal result . . . 2219
Rubi [A] (verified) . . . 2219
Mathematica [A] (verified) . . . 2222
Maple [C] (verified) . . . 2223
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Sympy [A] (verification not implemented) . . . 2224
Maxima [A] (verification not implemented) . . . 2224
Giac [A] (verification not implemented) . . . 2225
Mupad [B] (verification not implemented) . . . 2225

Optimal result

Integrand size = 16, antiderivative size = 112

$$\int \frac{1+x^3+x^6}{x+x^5} dx = \frac{x^2}{2} - \frac{\arctan(x^2)}{2} - \frac{\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}x\right)}{2\sqrt{2}} + \log(x) + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}} - \frac{1}{4}\log\left(1+x^4\right)$$

```
[Out] 1/2*x^2-1/2*arctan(x^2)+ln(x)-1/4*ln(x^4+1)+1/4*arctan(-1+x*2^(1/2))*2^(1/2
)+1/4*arctan(1+x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/8*ln(1+
x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {1607, 1847, 303, 1176, 631, 210, 1179, 642, 1848, 1262, 649, 209, 266}

$$\int \frac{1+x^3+x^6}{x+x^5} dx = -\frac{\arctan(x^2)}{2} - \frac{\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\arctan\left(\sqrt{2}x+1\right)}{2\sqrt{2}} - \frac{1}{4}\log\left(x^4+1\right) + \frac{x^2}{2} + \frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}} - \frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}} + \log(x)$$

```
[In] Int[(1 + x^3 + x^6)/(x + x^5),x]
```

```
[Out] x^2/2 - ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt
[2]*x]/(2*Sqrt[2]) + Log[x] + Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1
+ Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + x^4]/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+x^3+x^6}{x\left(1+x^4\right)}\,dx\\ &= \int \left(\frac{x^2}{1+x^4}+\frac{1+x^6}{x\left(1+x^4\right)}\right)dx\\ &= \int \frac{x^2}{1+x^4}\,dx+\int \frac{1+x^6}{x\left(1+x^4\right)}\,dx\end{aligned}$$

$$
\begin{aligned}
&= -\left(\frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx\right) + \frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx + \int \left(\frac{1}{x} + x + \frac{x(-1-x^2)}{1+x^4}\right)\,dx \\
&= \frac{x^2}{2} + \log(x) + \frac{1}{4}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx \\
&\quad + \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{4\sqrt{2}} + \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{4\sqrt{2}} + \int \frac{x(-1-x^2)}{1+x^4}\,dx \\
&= \frac{x^2}{2} + \log(x) + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} \\
&\quad - \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{-1-x}{1+x^2}\,dx, x, x^2\right) \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}x\right)}{2\sqrt{2}} \\
&= \frac{x^2}{2} - \frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}} + \log(x) + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} \\
&\quad - \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right) \\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, x^2\right) \\
&= \frac{x^2}{2} - \frac{1}{2}\tan^{-1}\left(x^2\right) - \frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}} + \log(x) \\
&\quad + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}} - \frac{1}{4}\log\left(1+x^4\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.90

$$
\begin{aligned}
\int \frac{1+x^3+x^6}{x+x^5}\,dx = \frac{1}{8}\Big(&4x^2 - 2\left(-2+\sqrt{2}\right)\arctan\left(1-\sqrt{2}x\right) + 2\left(2+\sqrt{2}\right)\arctan\left(1+\sqrt{2}x\right) \\
&+ 8\log(x) + \sqrt{2}\log\left(1-\sqrt{2}x+x^2\right) - \sqrt{2}\log\left(1+\sqrt{2}x+x^2\right) \\
&- 2\log\left(1+x^4\right)\Big)
\end{aligned}
$$

```
[In] Integrate[(1 + x^3 + x^6)/(x + x^5),x]
```

```
[Out] (4*x^2 - 2*(-2 + Sqrt[2])*ArcTan[1 - Sqrt[2]*x] + 2*(2 + Sqrt[2])*ArcTan[1
+ Sqrt[2]*x] + 8*Log[x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2]*Log[1
+ Sqrt[2]*x + x^2] - 2*Log[1 + x^4])/8
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.48

method	result
risch	$\frac{x^2}{2}+\ln(x)+\frac{\left(\sum_{\_R=\operatorname{RootOf}(\_Z^4+4\_Z^3+8\_Z^2+4\_Z+1)}\_R\ln\left(-\_R^3-5\_R^2-10\_R+3x-5\right)\right)}{4}$
default	$\frac{x^2}{2}+\ln(x)-\frac{\arctan(x^2)}{2}+\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{8}-\frac{\ln(x^4+1)}{4}$
meijerg	$\frac{x^2}{2}-\frac{\arctan(x^2)}{2}+\frac{x^3\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^4)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}+\frac{x^3}{}$

```
[In] int((x^6+x^3+1)/(x^5+x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+ln(x)+1/4*sum(_R*ln(-_R^3-5*_R^2-10*_R+3*x-5),_R=RootOf(_Z^4+4*_Z^3
+8*_Z^2+4*_Z+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 515, normalized size of antiderivative = 4.60

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx = \text{Too large to display}$$

```
[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/4*(2*sqrt(1/4*I) + I + 1)*log((2*sqrt(1/4*I) + I + 1)^3 - 5*(2*
sqrt(1/4*I) + I + 1)^2 + 3*x + 20*sqrt(1/4*I) + 10*I + 5) - 1/4*(2*sqrt(-1/
4*I) - I + 1)*log(-(2*sqrt(1/4*I) + I + 1)^3 - (2*sqrt(1/4*I) + I + 2)*(2*s
qrt(-1/4*I) - I + 1)^2 + 4*(2*sqrt(1/4*I) + I + 1)^2 - ((2*sqrt(1/4*I) + I
+ 1)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(-1/4*I) - I + 1) + 3*x - 16*sqrt(
1/4*I) - 8*I - 9) + 1/4*(sqrt(1/4*I) + sqrt(-1/4*I) - 2*sqrt(-3/16*(2*sqrt(
1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) -
3/16*(2*sqrt(-1/4*I) - I + 1)^2 + sqrt(1/4*I) + 1/2*I - 1/2) - 1)*log(1/2*(
2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1)^2 + 1/2*(2*sqrt(1/4*I) + I
+ 1)^2 + 1/2*((2*sqrt(1/4*I) + I + 1)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(
-1/4*I) - I + 1) + 2*sqrt(-3/16*(2*sqrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4
*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) - 3/16*(2*sqrt(-1/4*I) - I + 1)^2 + s
qrt(1/4*I) + 1/2*I - 1/2)*((2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1)
 + 2*sqrt(1/4*I) + I - 1) + 3*x - 2*sqrt(1/4*I) - I + 2) + 1/4*(sqrt(1/4*I)
```

```
 + sqrt(-1/4*I) + 2*sqrt(-3/16*(2*sqrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*
I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) - 3/16*(2*sqrt(-1/4*I) - I + 1)^2 + sq
rt(1/4*I) + 1/2*I - 1/2) - 1)*log(1/2*(2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*
I) - I + 1)^2 + 1/2*(2*sqrt(1/4*I) + I + 1)^2 + 1/2*((2*sqrt(1/4*I) + I + 1
)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(-1/4*I) - I + 1) - 2*sqrt(-3/16*(2*s
qrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1
) - 3/16*(2*sqrt(-1/4*I) - I + 1)^2 + sqrt(1/4*I) + 1/2*I - 1/2)*((2*sqrt(1
/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1) + 2*sqrt(1/4*I) + I - 1) + 3*x - 2*
sqrt(1/4*I) - I + 2) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx = \frac{x^2}{2} + \log(x) + \operatorname{RootSum}\left(256t^4+256t^3+128t^2+16t+1, \left(t \mapsto t\log\left(\frac{1792t^4}{73} + \frac{704t^3}{219} - \frac{3152t^2}{219} - \frac{2584t}{219} + x - \frac{344}{219}\right)\right)\right)$$

```
[In] integrate((x**6+x**3+1)/(x**5+x),x)
```

```
[Out] x**2/2 + log(x) + RootSum(256*_t**4 + 256*_t**3 + 128*_t**2 + 16*_t + 1, La
mbda(_t, _t*log(1792*_t**4/73 + 704*_t**3/219 - 3152*_t**2/219 - 2584*_t/21
9 + x - 344/219)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{1+x^3+x^6}{x+x^5}\,dx = {} & \frac{1}{4}\sqrt{2}\left(\sqrt{2}+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) \\ & - \frac{1}{4}\sqrt{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) \\ & - \frac{1}{8}\sqrt{2}\left(\sqrt{2}+1\right)\log\left(x^2+\sqrt{2}x+1\right) \\ & - \frac{1}{8}\sqrt{2}\left(\sqrt{2}-1\right)\log\left(x^2-\sqrt{2}x+1\right) + \frac{1}{2}x^2 + \log(x)\end{aligned}$$

```
[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2) + 1)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/4*sqrt(2)
*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*(sqrt(2) +
 1)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*(sqrt(2) - 1)*log(x^2 - sqrt(2)*
x + 1) + 1/2*x^2 + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.82

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx = \frac{1}{2}\,x^2 + \frac{1}{4}\left(\sqrt{2}+2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{4}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{1}{8}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{8}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{1}{4}\,\log\left(x^4+1\right) + \log\left(|x|\right)$$

```
[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/4*(sqrt(2) + 2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(sqrt
(2) - 2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt(2
)*x + 1) + 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*log(x^4 + 1) + log(ab
s(x))
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.52

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx = \ln\left(x\right) + \left(\sum_{k=1}^{4}\ln\left(\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}+\frac{z}{16}+\frac{1}{256},z,k\right)\ \left(8\,\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}+\frac{z}{16}+\frac{1}{256},z,k\right)+x+\mathrm{root}\left(z^4+z^3+\right.\right.\right.\right. \cdots + z^3+\frac{z^2}{2}+\frac{z}{16}+\frac{1}{256},z,k\left.\left.\right)\right)\Bigg) + \frac{x^2}{2}$$

```
[In] int((x^3 + x^6 + 1)/(x + x^5),x)
```

```
[Out] log(x) + symsum(log(root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)*(8*root(z^
4 + z^3 + z^2/2 + z/16 + 1/256, z, k) + x + 96*root(z^4 + z^3 + z^2/2 + z/1
6 + 1/256, z, k)*x + 240*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)^2*x +
 320*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)^3*x - 16*root(z^4 + z^3 +
 z^2/2 + z/16 + 1/256, z, k)^2 + 8))*root(z^4 + z^3 + z^2/2 + z/16 + 1/256,
 z, k), k, 1, 4) + x^2/2
```

3.369 $\int \frac{1+x^2}{-x+x^2} \, dx$

Optimal result 2226
Rubi [A] (verified) 2226
Mathematica [A] (verified) 2227
Maple [A] (verified) 2227
Fricas [A] (verification not implemented) 2228
Sympy [A] (verification not implemented) 2228
Maxima [A] (verification not implemented) 2228
Giac [A] (verification not implemented) 2228
Mupad [B] (verification not implemented) 2229

Optimal result

Integrand size = 15, antiderivative size = 14

$$\int \frac{1+x^2}{-x+x^2} \, dx = x+2\log(1-x)-\log(x)$$

```
[Out] x+2*ln(1-x)-ln(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1607, 908}

$$\int \frac{1+x^2}{-x+x^2} \, dx = x+2\log(1-x)-\log(x)$$

```
[In] Int[(1 + x^2)/(-x + x^2),x]
```

```
[Out] x + 2*Log[1 - x] - Log[x]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+x^2}{(-1+x)x}\,dx\\ &= \int \left(1+\frac{2}{-1+x}-\frac{1}{x}\right)\,dx\\ &= x+2\log(1-x)-\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{-x+x^2}\,dx = x+2\log(1-x)-\log(x)$$

```
[In] Integrate[(1 + x^2)/(-x + x^2),x]
[Out] x + 2*Log[1 - x] - Log[x]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$x-\ln(x)+2\ln(x-1)$	13
norman	$x-\ln(x)+2\ln(x-1)$	13
risch	$x-\ln(x)+2\ln(x-1)$	13
parallelrisch	$x-\ln(x)+2\ln(x-1)$	13
meijerg	$2\ln(1-x)-\ln(x)-i\pi+x$	19

```
[In] int((x^2+1)/(x^2-x),x,method=_RETURNVERBOSE)
[Out] x-ln(x)+2*ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{-x+x^2} \, dx = x + 2 \, \log \left(x - 1\right) - \log \left(x\right)$$

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="fricas")
[Out] x + 2*log(x - 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1+x^2}{-x+x^2} \, dx = x - \log \left(x\right) + 2 \log \left(x - 1\right)$$

```
[In] integrate((x**2+1)/(x**2-x),x)
[Out] x - log(x) + 2*log(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{-x+x^2} \, dx = x + 2 \, \log \left(x - 1\right) - \log \left(x\right)$$

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="maxima")
[Out] x + 2*log(x - 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{-x+x^2} \, dx = x + 2 \, \log \left(|x - 1|\right) - \log \left(|x|\right)$$

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="giac")
[Out] x + 2*log(abs(x - 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{-x+x^2} \, dx = x + 2 \ln (x-1) - \ln (x)$$

```
[In] int(-(x^2 + 1)/(x - x^2),x)
[Out] x + 2*log(x - 1) - log(x)
```

3.370 $\int \frac{1+x^3}{-x+x^3} dx$

Optimal result 2230
Rubi [A] (verified) 2230
Mathematica [A] (verified) 2231
Maple [A] (verified) 2231
Fricas [A] (verification not implemented) 2232
Sympy [A] (verification not implemented) 2232
Maxima [A] (verification not implemented) 2232
Giac [A] (verification not implemented) 2232
Mupad [B] (verification not implemented) 2233

Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{1+x^3}{-x+x^3} dx = x + \log(1-x) - \log(x)$$

```
[Out] x+ln(1-x)-ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1607, 1816}

$$\int \frac{1+x^3}{-x+x^3} dx = x + \log(1-x) - \log(x)$$

```
[In] Int[(1 + x^3)/(-x + x^3),x]
```

```
[Out] x + Log[1 - x] - Log[x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1+x^3}{x\,(-1+x^2)}\,dx \\
&= \int \left(1+\frac{1}{-1+x}-\frac{1}{x}\right)\,dx \\
&= x+\log(1-x)-\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+x^3}{-x+x^3}\,dx = x+\log(1-x)-\log(x)$$

```
[In] Integrate[(1 + x^3)/(-x + x^3),x]
[Out] x + Log[1 - x] - Log[x]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$x-\ln\left(x\right)+\ln\left(x-1\right)$	11
norman	$x-\ln\left(x\right)+\ln\left(x-1\right)$	11
risch	$x-\ln\left(x\right)+\ln\left(x-1\right)$	11
parallelrisch	$x-\ln\left(x\right)+\ln\left(x-1\right)$	11
meijerg	$\frac{\ln\left(-x^2+1\right)}{2}-\ln\left(x\right)-\frac{i\pi}{2}-\frac{i(2ix-2i\,\mathrm{arctanh}(x))}{2}$	33

```
[In] int((x^3+1)/(x^3-x),x,method=_RETURNVERBOSE)
[Out] x-ln(x)+ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^3}{-x+x^3}\,dx = x + \log\left(x-1\right) - \log\left(x\right)$$

```
[In] integrate((x^3+1)/(x^3-x),x, algorithm="fricas")
[Out] x + log(x - 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1+x^3}{-x+x^3}\,dx = x - \log\left(x\right) + \log\left(x-1\right)$$

```
[In] integrate((x**3+1)/(x**3-x),x)
[Out] x - log(x) + log(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^3}{-x+x^3}\,dx = x + \log\left(x-1\right) - \log\left(x\right)$$

```
[In] integrate((x^3+1)/(x^3-x),x, algorithm="maxima")
[Out] x + log(x - 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+x^3}{-x+x^3}\,dx = x + \log\left(|x-1|\right) - \log\left(|x|\right)$$

```
[In] integrate((x^3+1)/(x^3-x),x, algorithm="giac")
[Out] x + log(abs(x - 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^3}{-x+x^3}\,dx = x-2\,\mathrm{atanh}(2\,x-1)$$

```
[In] int(-(x^3 + 1)/(x - x^3),x)
[Out] x - 2*atanh(2*x - 1)
```

3.371 $\int \frac{1+x^3}{-x^2+x^3} dx$

Optimal result . . . 2234
Rubi [A] (verified) . . . 2234
Mathematica [A] (verified) . . . 2235
Maple [A] (verified) . . . 2235
Fricas [A] (verification not implemented) . . . 2236
Sympy [A] (verification not implemented) . . . 2236
Maxima [A] (verification not implemented) . . . 2236
Giac [A] (verification not implemented) . . . 2236
Mupad [B] (verification not implemented) . . . 2237

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1+x^3}{-x^2+x^3} dx = \frac{1}{x} + x + 2\log(1-x) - \log(x)$$

```
[Out] 1/x+x+2*ln(1-x)-ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1607, 1634}

$$\int \frac{1+x^3}{-x^2+x^3} dx = x + \frac{1}{x} + 2\log(1-x) - \log(x)$$

```
[In] Int[(1 + x^3)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
```

```
xpon[Px, x], 2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1+x^3}{(-1+x)x^2}\,dx \\
&= \int \left(1+\frac{2}{-1+x}-\frac{1}{x^2}-\frac{1}{x}\right)dx \\
&= \frac{1}{x}+x+2\log(1-x)-\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1+x^3}{-x^2+x^3}\,dx = \frac{1}{x}+x+2\log(1-x)-\log(x)$$

```
[In] Integrate[(1 + x^3)/(-x^2 + x^3),x]
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$x+\frac{1}{x}-\ln\left(x\right)+2\ln\left(x-1\right)$	16
risch	$x+\frac{1}{x}-\ln\left(x\right)+2\ln\left(x-1\right)$	16
norman	$\frac{x^2+1}{x}-\ln\left(x\right)+2\ln\left(x-1\right)$	21
meijerg	$2\ln\left(1-x\right)-\ln\left(x\right)-i\pi+\frac{1}{x}+x$	22
parallelrisch	$-\frac{\ln(x)x-2\ln(x-1)x-x^2-1}{x}$	24

```
[In] int((x^3+1)/(x^3-x^2),x,method=_RETURNVERBOSE)
[Out] x+1/x-ln(x)+2*ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{1+x^3}{-x^2+x^3}\,dx=\frac{x^2+2\,x\log\left(x-1\right)-x\log\left(x\right)+1}{x}$$

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="fricas")
[Out] (x^2 + 2*x*log(x - 1) - x*log(x) + 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1+x^3}{-x^2+x^3}\,dx=x-\log\left(x\right)+2\log\left(x-1\right)+\frac{1}{x}$$

```
[In] integrate((x**3+1)/(x**3-x**2),x)
[Out] x - log(x) + 2*log(x - 1) + 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^3}{-x^2+x^3}\,dx=x+\frac{1}{x}+2\,\log\left(x-1\right)-\log\left(x\right)$$

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="maxima")
[Out] x + 1/x + 2*log(x - 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1+x^3}{-x^2+x^3}\,dx=x+\frac{1}{x}+2\,\log\left(|x-1|\right)-\log\left(|x|\right)$$

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="giac")
[Out] x + 1/x + 2*log(abs(x - 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^3}{-x^2+x^3}\,dx = x+2\,\ln\left(x-1\right)-\ln\left(x\right)+\frac{1}{x}$$

```
[In] int(-(x^3 + 1)/(x^2 - x^3),x)
[Out] x + 2*log(x - 1) - log(x) + 1/x
```

3.372 $\int \frac{-1+x^5}{-x+x^3} \, dx$

Optimal result 2238
Rubi [A] (verified) 2238
Mathematica [A] (verified) 2239
Maple [A] (verified) 2239
Fricas [A] (verification not implemented) 2240
Sympy [A] (verification not implemented) 2240
Maxima [A] (verification not implemented) 2240
Giac [A] (verification not implemented) 2240
Mupad [B] (verification not implemented) 2241

Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \frac{-1+x^5}{-x+x^3} \, dx = x+\frac{x^3}{3}+\log(x)-\log(1+x)$$

```
[Out] x+1/3*x^3+ln(x)-ln(1+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1607, 1816}

$$\int \frac{-1+x^5}{-x+x^3} \, dx = \frac{x^3}{3}+x+\log(x)-\log(x+1)$$

```
[In] Int[(-1 + x^5)/(-x + x^3),x]
```

```
[Out] x + x^3/3 + Log[x] - Log[1 + x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{-1+x^5}{x(-1+x^2)}\,dx \\
&= \int \left(1+\frac{1}{-1-x}+\frac{1}{x}+x^2\right)dx \\
&= x+\frac{x^3}{3}+\log(x)-\log(1+x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^5}{-x+x^3}\,dx = x+\frac{x^3}{3}+\log(x)-\log(1+x)$$

```
[In] Integrate[(-1 + x^5)/(-x + x^3),x]
[Out] x + x^3/3 + Log[x] - Log[1 + x]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$x+\frac{x^3}{3}+\ln\left(x\right)-\ln\left(x+1\right)$	16
norman	$x+\frac{x^3}{3}+\ln\left(x\right)-\ln\left(x+1\right)$	16
risch	$x+\frac{x^3}{3}+\ln\left(x\right)-\ln\left(x+1\right)$	16
parallelrisch	$x+\frac{x^3}{3}+\ln\left(x\right)-\ln\left(x+1\right)$	16
meijerg	$-\frac{\ln\left(-x^2+1\right)}{2}+\ln\left(x\right)+\frac{i\pi}{2}+\frac{i\left(-\frac{2ix\left(5x^2+15\right)}{15}+2i\,\mathrm{arctanh}(x)\right)}{2}$	38

```
[In] int((x^5-1)/(x^3-x),x,method=_RETURNVERBOSE)
[Out] x+1/3*x^3+ln(x)-ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^5}{-x+x^3}\,dx = \frac{1}{3}\,x^3 + x - \log\left(x+1\right) + \log\left(x\right)$$

```
[In] integrate((x^5-1)/(x^3-x),x, algorithm="fricas")
[Out] 1/3*x^3 + x - log(x + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{-1+x^5}{-x+x^3}\,dx = \frac{x^3}{3} + x + \log\left(x\right) - \log\left(x+1\right)$$

```
[In] integrate((x**5-1)/(x**3-x),x)
[Out] x**3/3 + x + log(x) - log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^5}{-x+x^3}\,dx = \frac{1}{3}\,x^3 + x - \log\left(x+1\right) + \log\left(x\right)$$

```
[In] integrate((x^5-1)/(x^3-x),x, algorithm="maxima")
[Out] 1/3*x^3 + x - log(x + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^5}{-x+x^3}\,dx = \frac{1}{3}\,x^3 + x - \log\left(|x+1|\right) + \log\left(|x|\right)$$

```
[In] integrate((x^5-1)/(x^3-x),x, algorithm="giac")
[Out] 1/3*x^3 + x - log(abs(x + 1)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^5}{-x+x^3}\,dx = x - 2\,\mathrm{atanh}(2\,x+1) + \frac{x^3}{3}$$

```
[In] int(-(x^5 - 1)/(x - x^3),x)
[Out] x - 2*atanh(2*x + 1) + x^3/3
```

3.373 $\int \frac{1+x^4}{x^3+x^5} dx$

Optimal result 2242
Rubi [A] (verified) 2242
Mathematica [A] (verified) 2243
Maple [A] (verified) 2243
Fricas [A] (verification not implemented) 2244
Sympy [A] (verification not implemented) 2244
Maxima [A] (verification not implemented) 2244
Giac [A] (verification not implemented) 2245
Mupad [B] (verification not implemented) 2245

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{1+x^4}{x^3+x^5} \, dx = -\frac{1}{2x^2} - \log(x) + \log\left(1+x^2\right)$$

```
[Out] -1/2/x^2-ln(x)+ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1607, 1266, 908}

$$\int \frac{1+x^4}{x^3+x^5} \, dx = -\frac{1}{2x^2} + \log\left(x^2+1\right) - \log(x)$$

```
[In] Int[(1 + x^4)/(x^3 + x^5),x]
```

```
[Out] -1/2*1/x^2 - Log[x] + Log[1 + x^2]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1+x^4}{x^3(1+x^2)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{x^2(1+x)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{2}{1+x}\right)\,dx, x, x^2\right) \\
&= -\frac{1}{2x^2}-\log(x)+\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{x^3+x^5}\,dx = -\frac{1}{2x^2}-\log(x)+\log\left(1+x^2\right)$$

```
[In] Integrate[(1 + x^4)/(x^3 + x^5),x]
[Out] -1/2*1/x^2 - Log[x] + Log[1 + x^2]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{1}{2x^2}-\ln(x)+\ln(x^2+1)$	17
norman	$-\frac{1}{2x^2}-\ln(x)+\ln(x^2+1)$	17
meijerg	$-\frac{1}{2x^2}-\ln(x)+\ln(x^2+1)$	17
risch	$-\frac{1}{2x^2}-\ln(x)+\ln(x^2+1)$	17
parallelrisch	$-\frac{2\ln(x)x^2-2\ln(x^2+1)x^2+1}{2x^2}$	26

```
[In] int((x^4+1)/(x^5+x^3),x,method=_RETURNVERBOSE)
[Out] -1/2/x^2-ln(x)+ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{1+x^4}{x^3+x^5}\,dx = \frac{2\,x^2\log\left(x^2+1\right)-2\,x^2\log\left(x\right)-1}{2\,x^2}$$

```
[In] integrate((x^4+1)/(x^5+x^3),x, algorithm="fricas")
[Out] 1/2*(2*x^2*log(x^2 + 1) - 2*x^2*log(x) - 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1+x^4}{x^3+x^5}\,dx = -\log\left(x\right)+\log\left(x^2+1\right)-\frac{1}{2x^2}$$

```
[In] integrate((x**4+1)/(x**5+x**3),x)
[Out] -log(x) + log(x**2 + 1) - 1/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1+x^4}{x^3+x^5}\,dx = -\frac{1}{2\,x^2}+\log\left(x^2+1\right)-\log\left(x\right)$$

```
[In] integrate((x^4+1)/(x^5+x^3),x, algorithm="maxima")
[Out] -1/2/x^2 + log(x^2 + 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1+x^4}{x^3+x^5}\,dx = \frac{x^2-1}{2\,x^2} + \log\left(x^2+1\right) - \frac{1}{2}\,\log\left(x^2\right)$$

```
[In] integrate((x^4+1)/(x^5+x^3),x, algorithm="giac")
[Out] 1/2*(x^2 - 1)/x^2 + log(x^2 + 1) - 1/2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1+x^4}{x^3+x^5}\,dx = \ln\left(x^2+1\right) - \ln\left(x\right) - \frac{1}{2\,x^2}$$

```
[In] int((x^4 + 1)/(x^3 + x^5),x)
[Out] log(x^2 + 1) - log(x) - 1/(2*x^2)
```

3.374 $\int \frac{1+x^2}{x+2x^2+x^3} \, dx$

Optimal result 2246
Rubi [A] (verified) 2246
Mathematica [A] (verified) 2247
Maple [A] (verified) 2247
Fricas [A] (verification not implemented) 2248
Sympy [A] (verification not implemented) 2248
Maxima [A] (verification not implemented) 2248
Giac [A] (verification not implemented) 2249
Mupad [B] (verification not implemented) 2249

Optimal result

Integrand size = 18, antiderivative size = 10

$$\int \frac{1+x^2}{x+2x^2+x^3} \, dx = \frac{2}{1+x} + \log(x)$$

```
[Out] 2/(1+x)+ln(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1608, 27, 908}

$$\int \frac{1+x^2}{x+2x^2+x^3} \, dx = \frac{2}{x+1} + \log(x)$$

```
[In] Int[(1 + x^2)/(x + 2*x^2 + x^3),x]
```

```
[Out] 2/(1 + x) + Log[x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
```

```
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+x^2}{x\,(1+2x+x^2)}\,dx\\ &= \int \frac{1+x^2}{x(1+x)^2}\,dx\\ &= \int \left(\frac{1}{x}-\frac{2}{(1+x)^2}\right)dx\\ &= \frac{2}{1+x}+\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx = \frac{2}{1+x}+\log(x)$$

```
[In] Integrate[(1 + x^2)/(x + 2*x^2 + x^3),x]
[Out] 2/(1 + x) + Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{2}{x+1}+\ln\left(x\right)$	11
norman	$\frac{2}{x+1}+\ln\left(x\right)$	11
risch	$\frac{2}{x+1}+\ln\left(x\right)$	11
parallelrisch	$\frac{\ln(x)x+2+\ln(x)}{x+1}$	15

```
[In] int((x^2+1)/(x^3+2*x^2+x),x,method=_RETURNVERBOSE)
[Out] 2/(x+1)+ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx = \frac{(x+1)\log{(x)}+2}{x+1}$$

```
[In] integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="fricas")
[Out] ((x + 1)*log(x) + 2)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx = \log{(x)} + \frac{2}{x+1}$$

```
[In] integrate((x**2+1)/(x**3+2*x**2+x),x)
[Out] log(x) + 2/(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx = \frac{2}{x+1} + \log{(x)}$$

```
[In] integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="maxima")
[Out] 2/(x + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx=\frac{2}{x+1}+\log\left(|x|\right)$$

```
[In] integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="giac")
[Out] 2/(x + 1) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx=\ln\left(x\right)+\frac{2}{x+1}$$

```
[In] int((x^2 + 1)/(x + 2*x^2 + x^3),x)
[Out] log(x) + 2/(x + 1)
```

3.375 $\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx$

Optimal result 2250
Rubi [A] (verified) 2250
Mathematica [A] (verified) 2251
Maple [A] (verified) 2251
Fricas [A] (verification not implemented) 2252
Sympy [A] (verification not implemented) 2252
Maxima [A] (verification not implemented) 2252
Giac [A] (verification not implemented) 2253
Mupad [B] (verification not implemented) 2253

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = 19x+\frac{3x^2}{2}+\frac{x^3}{3}+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(2+x)$$

```
[Out] 19*x+3/2*x^2+1/3*x^3+3126/35*ln(5-x)-1/10*ln(x)-31/14*ln(2+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1608, 1642}

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

```
[In] Int[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x
])/14
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+x^5}{x\left(-10-3x+x^2\right)}\,dx \\ &= \int \left(19+\frac{3126}{35(-5+x)}-\frac{1}{10x}+3x+x^2-\frac{31}{14(2+x)}\right)dx \\ &= 19x+\frac{3x^2}{2}+\frac{x^3}{3}+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(2+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = 19x+\frac{3x^2}{2}+\frac{x^3}{3}+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(2+x)$$

```
[In] Integrate[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x
])/14
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{\ln(x)}{10}+\frac{3126\ln(-5+x)}{35}-\frac{31\ln(x+2)}{14}$	31
norman	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{\ln(x)}{10}+\frac{3126\ln(-5+x)}{35}-\frac{31\ln(x+2)}{14}$	31
risch	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{\ln(x)}{10}+\frac{3126\ln(-5+x)}{35}-\frac{31\ln(x+2)}{14}$	31
parallelrisch	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{\ln(x)}{10}+\frac{3126\ln(-5+x)}{35}-\frac{31\ln(x+2)}{14}$	31

```
[In] int((x^5+1)/(x^3-3*x^2-10*x),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3+3/2*x^2+19*x-1/10*ln(x)+3126/35*ln(-5+x)-31/14*ln(x+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \frac{1}{3}\,x^3 + \frac{3}{2}\,x^2 + 19\,x - \frac{31}{14}\,\log\left(x+2\right) + \frac{3126}{35}\,\log\left(x-5\right) - \frac{1}{10}\,\log\left(x\right)$$

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="fricas")
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log
(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \frac{x^3}{3} + \frac{3x^2}{2} + 19x - \frac{\log\left(x\right)}{10} + \frac{3126\log\left(x-5\right)}{35} - \frac{31\log\left(x+2\right)}{14}$$

```
[In] integrate((x**5+1)/(x**3-3*x**2-10*x),x)
[Out] x**3/3 + 3*x**2/2 + 19*x - log(x)/10 + 3126*log(x - 5)/35 - 31*log(x + 2)/1
4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \frac{1}{3}\,x^3 + \frac{3}{2}\,x^2 + 19\,x - \frac{31}{14}\,\log\left(x+2\right) + \frac{3126}{35}\,\log\left(x-5\right) - \frac{1}{10}\,\log\left(x\right)$$

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="maxima")
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log
(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \frac{1}{3}\,x^3+\frac{3}{2}\,x^2+19\,x-\frac{31}{14}\,\log\left(|x+2|\right) + \frac{3126}{35}\,\log\left(|x-5|\right)-\frac{1}{10}\,\log\left(|x|\right)$$

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(abs(x + 2)) + 3126/35*log(abs(x - 5))
- 1/10*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = 19\,x-\frac{31\,\ln\left(x+2\right)}{14}+\frac{3126\,\ln\left(x-5\right)}{35}-\frac{\ln\left(x\right)}{10}+\frac{3\,x^2}{2}+\frac{x^3}{3}$$

```
[In] int(-(x^5 + 1)/(10*x + 3*x^2 - x^3),x)
```

```
[Out] 19*x - (31*log(x + 2))/14 + (3126*log(x - 5))/35 - log(x)/10 + (3*x^2)/2 +
x^3/3
```

3.376 $\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)} dx$

Optimal result 2254
Rubi [A] (verified) 2254
Mathematica [A] (verified) 2256
Maple [A] (verified) 2256
Fricas [A] (verification not implemented) 2256
Sympy [A] (verification not implemented) 2257
Maxima [A] (verification not implemented) 2257
Giac [A] (verification not implemented) 2257
Mupad [B] (verification not implemented) 2258

Optimal result

Integrand size = 29, antiderivative size = 46

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)} dx = -\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)+\frac{5\arctan\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)$$

```
[Out] 1/2*ln(x^2+2*x+3)+5/2*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)-arctan(1/5*x*5^(1/2
))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {6857, 209, 648, 632, 210, 642}

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)} dx = -\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)+\frac{5\arctan\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(x^2+2x+3\right)$$

```
[In] Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{5}{5+x^2}+\frac{6+x}{3+2x+x^2}\right)dx\\
&= -\left(5\int \frac{1}{5+x^2}\,dx\right)+\int \frac{6+x}{3+2x+x^2}\,dx\\
&= -\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\int \frac{2+2x}{3+2x+x^2}\,dx+5\int \frac{1}{3+2x+x^2}\,dx\\
&= -\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\log\left(3+2x+x^2\right)-10\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2+2x\right)\\
&= -\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx=-\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)+\frac{5\arctan\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)$$

```
[In] Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{\ln(x^2+2x+3)}{2}+\frac{5\arctan\left(\frac{(x+1)\sqrt{2}}{2}\right)\sqrt{2}}{2}-\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}$	39
default	$-\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}+\frac{\ln(x^2+2x+3)}{2}+\frac{5\sqrt{2}\arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}$	41

```
[In] int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x^2+2*x+3)+5/2*arctan(1/2*(x+1)*2^(1/2))*2^(1/2)-arctan(1/5*x*5^(1/2
))*5^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx=\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="fricas")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{\log(x^2+2x+3)}{2} - \sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}x}{5}\right) + \frac{5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}+\frac{\sqrt{2}}{2}\right)}{2}$$

```
[In] integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)
[Out] log(x**2 + 2*x + 3)/2 - sqrt(5)*atan(sqrt(5)*x/5) + 5*sqrt(2)*atan(sqrt(2)*
x/2 + sqrt(2)/2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right) - \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) + \frac{1}{2}\log\left(x^2+2x+3\right)$$

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="maxima")
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right) - \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) + \frac{1}{2}\log\left(x^2+2x+3\right)$$

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="giac")
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.91

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)}{2}+\frac{\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)}{2} + \sqrt{5}\,\mathrm{atan}\left(\frac{2000\sqrt{5}}{2000\,x+1120}-\frac{224\sqrt{5}\,x}{2000\,x+1120}\right) - \frac{\sqrt{2}\,\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)\,5\mathrm{i}}{4}+\frac{\sqrt{2}\,\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)\,5\mathrm{i}}{4}$$

```
[In] int((x^2 - 5*x + x^3 + 15)/((x^2 + 5)*(2*x + x^2 + 3)),x)
```

```
[Out] log(x - 2^(1/2)*1i + 1)/2 + log(x + 2^(1/2)*1i + 1)/2 + 5^(1/2)*atan((2000*
5^(1/2))/(2000*x + 1120) - (224*5^(1/2)*x)/(2000*x + 1120)) - (2^(1/2)*log(
x - 2^(1/2)*1i + 1)*5i)/4 + (2^(1/2)*log(x + 2^(1/2)*1i + 1)*5i)/4
```

3.377 $\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx$

Optimal result 2259
Rubi [A] (verified) 2259
Mathematica [A] (verified) 2260
Maple [A] (verified) 2260
Fricas [A] (verification not implemented) 2261
Sympy [A] (verification not implemented) 2261
Maxima [A] (verification not implemented) 2261
Giac [A] (verification not implemented) 2261
Mupad [B] (verification not implemented) 2262

Optimal result

Integrand size = 22, antiderivative size = 19

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx=-\frac{1}{8}\log(3+x)+\frac{1}{8}\log(1+3x)$$

```
[Out] -1/8*ln(3+x)+1/8*ln(1+3*x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6820, 630, 31}

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx=\frac{1}{8}\log(3x+1)-\frac{1}{8}\log(x+3)$$

```
[In] Int[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]
[Out] -1/8*Log[3 + x] + Log[1 + 3*x]/8
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{3+10x+3x^2}\,dx \\
&= \frac{3}{8}\int \frac{1}{1+3x}\,dx - \frac{3}{8}\int \frac{1}{9+3x}\,dx \\
&= -\frac{1}{8}\log(3+x) + \frac{1}{8}\log(1+3x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = -\frac{1}{8}\log(3+x) + \frac{1}{8}\log(1+3x)$$

```
[In] Integrate[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]
[Out] -1/8*Log[3 + x] + Log[1 + 3*x]/8
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$-\frac{\ln(3+x)}{8}+\frac{\ln\left(\frac{1}{3}+x\right)}{8}$	14
default	$-\frac{\ln(3+x)}{8}+\frac{\ln(1+3x)}{8}$	16
norman	$-\frac{\ln(3+x)}{8}+\frac{\ln(1+3x)}{8}$	16
risch	$-\frac{\ln(3+x)}{8}+\frac{\ln(1+3x)}{8}$	16

```
[In] int(1/(x^2+1)/(3+10*x/(x^2+1)),x,method=_RETURNVERBOSE)
[Out] -1/8*ln(3+x)+1/8*ln(1/3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = \frac{1}{8}\,\log\left(3\,x+1\right) - \frac{1}{8}\,\log\left(x+3\right)$$

```
[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="fricas")
[Out] 1/8*log(3*x + 1) - 1/8*log(x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = \frac{\log\left(x+\frac{1}{3}\right)}{8} - \frac{\log\left(x+3\right)}{8}$$

```
[In] integrate(1/(x**2+1)/(3+10*x/(x**2+1)),x)
[Out] log(x + 1/3)/8 - log(x + 3)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = \frac{1}{8}\,\log\left(3\,x+1\right) - \frac{1}{8}\,\log\left(x+3\right)$$

```
[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="maxima")
[Out] 1/8*log(3*x + 1) - 1/8*log(x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = \frac{1}{8}\,\log\left(|3\,x+1|\right) - \frac{1}{8}\,\log\left(|x+3|\right)$$

```
[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="giac")
[Out] 1/8*log(abs(3*x + 1)) - 1/8*log(abs(x + 3))
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = -\frac{\operatorname{atanh}\left(\frac{3\,x}{4}+\frac{5}{4}\right)}{4}$$

```
[In] int(1/((x^2 + 1)*((10*x)/(x^2 + 1) + 3)),x)
[Out] -atanh((3*x)/4 + 5/4)/4
```

3.378 $\int \frac{x^3}{13+\frac{2}{x}+15x} \, dx$

Optimal result 2263
Rubi [A] (verified) 2263
Mathematica [A] (verified) 2264
Maple [A] (verified) 2265
Fricas [A] (verification not implemented) 2265
Sympy [A] (verification not implemented) 2265
Maxima [A] (verification not implemented) 2266
Giac [A] (verification not implemented) 2266
Mupad [B] (verification not implemented) 2266

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{x^3}{13+\frac{2}{x}+15x} \, dx = \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} - \frac{16}{567}\log(2+3x) + \frac{\log(1+5x)}{4375}$$

```
[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1400, 715, 646, 31}

$$\int \frac{x^3}{13+\frac{2}{x}+15x} \, dx = \frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} - \frac{16}{567}\log(3x+2) + \frac{\log(5x+1)}{4375}$$

```
[In] Int[x^3/(13 + 2/x + 15*x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
```

```
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1400

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^4}{2+13x+15x^2}\,dx \\
&= \int \left(\frac{139}{3375}-\frac{13x}{225}+\frac{x^2}{15}-\frac{278+1417x}{3375\,(2+13x+15x^2)}\right)dx \\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{\int \frac{278+1417x}{2+13x+15x^2}\,dx}{3375} \\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}+\frac{3}{875}\int \frac{1}{3+15x}\,dx-\frac{80}{189}\int \frac{1}{10+15x}\,dx \\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$
\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx=\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}
$$

```
[In] Integrate[x^3/(13 + 2/x + 15*x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}+\frac{\ln\left(x+\frac{1}{5}\right)}{4375}-\frac{16\ln\left(x+\frac{2}{3}\right)}{567}$	27
default	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(3x+2)}{567}+\frac{\ln(1+5x)}{4375}$	31
norman	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(3x+2)}{567}+\frac{\ln(1+5x)}{4375}$	31
risch	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(3x+2)}{567}+\frac{\ln(1+5x)}{4375}$	31

```
[In] int(x^3/(13+2/x+15*x),x,method=_RETURNVERBOSE)
[Out] 1/45*x^3-13/450*x^2+139/3375*x+1/4375*ln(x+1/5)-16/567*ln(x+2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx = \frac{1}{45}\,x^3 - \frac{13}{450}\,x^2 + \frac{139}{3375}\,x + \frac{1}{4375}\,\log\left(5\,x+1\right) - \frac{16}{567}\,\log\left(3\,x+2\right)$$

```
[In] integrate(x^3/(13+2/x+15*x),x, algorithm="fricas")
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx = \frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\log\left(x+\frac{1}{5}\right)}{4375} - \frac{16\log\left(x+\frac{2}{3}\right)}{567}$$

```
[In] integrate(x**3/(13+2/x+15*x),x)
[Out] x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x + 2/3)/56
7
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx = \frac{1}{45}\,x^3 - \frac{13}{450}\,x^2 + \frac{139}{3375}\,x + \frac{1}{4375}\,\log\left(5\,x+1\right) - \frac{16}{567}\,\log\left(3\,x+2\right)$$

```
[In] integrate(x^3/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx = \frac{1}{45}\,x^3 - \frac{13}{450}\,x^2 + \frac{139}{3375}\,x + \frac{1}{4375}\,\log\left(|5\,x+1|\right) - \frac{16}{567}\,\log\left(|3\,x+2|\right)$$

```
[In] integrate(x^3/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log(
abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

$$\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx = \frac{139\,x}{3375} - \frac{16\,\ln\left(x+\frac{2}{3}\right)}{567} + \frac{\ln\left(x+\frac{1}{5}\right)}{4375} - \frac{13\,x^2}{450} + \frac{x^3}{45}$$

```
[In] int(x^3/(15*x + 2/x + 13),x)
```

```
[Out] (139*x)/3375 - (16*log(x + 2/3))/567 + log(x + 1/5)/4375 - (13*x^2)/450 + x
^3/45
```

3.379 $\int \frac{x^2}{13+\frac{2}{x}+15x} dx$

Optimal result 2267
Rubi [A] (verified) 2267
Mathematica [A] (verified) 2268
Maple [A] (verified) 2269
Fricas [A] (verification not implemented) 2269
Sympy [A] (verification not implemented) 2269
Maxima [A] (verification not implemented) 2270
Giac [A] (verification not implemented) 2270
Mupad [B] (verification not implemented) 2270

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{x^2}{13+\frac{2}{x}+15x} dx = -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)$$

```
[Out] -13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1400, 715, 646, 31}

$$\int \frac{x^2}{13+\frac{2}{x}+15x} dx = \frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

```
[In] Int[x^2/(13 + 2/x + 15*x),x]
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
```

```
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1400

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3}{2+13x+15x^2}\,dx \\
&= \int \left(-\frac{13}{225}+\frac{x}{15}+\frac{26+139x}{225\,(2+13x+15x^2)}\right)dx \\
&= -\frac{13x}{225}+\frac{x^2}{30}+\frac{1}{225}\int \frac{26+139x}{2+13x+15x^2}\,dx \\
&= -\frac{13x}{225}+\frac{x^2}{30}-\frac{3}{175}\int \frac{1}{3+15x}\,dx+\frac{40}{63}\int \frac{1}{10+15x}\,dx \\
&= -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx = -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)$$

```
[In] Integrate[x^2/(13 + 2/x + 15*x),x]
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{x^{2}}{30}-\frac{13 x}{225}-\frac{\ln\left(x+\frac{1}{5}\right)}{875}+\frac{8 \ln\left(x+\frac{2}{3}\right)}{189}$	22
default	$-\frac{13 x}{225}+\frac{x^{2}}{30}+\frac{8 \ln(3 x+2)}{189}-\frac{\ln(1+5 x)}{875}$	26
norman	$-\frac{13 x}{225}+\frac{x^{2}}{30}+\frac{8 \ln(3 x+2)}{189}-\frac{\ln(1+5 x)}{875}$	26
risch	$-\frac{13 x}{225}+\frac{x^{2}}{30}+\frac{8 \ln(3 x+2)}{189}-\frac{\ln(1+5 x)}{875}$	26

```
[In] int(x^2/(13+2/x+15*x),x,method=_RETURNVERBOSE)
[Out] 1/30*x^2-13/225*x-1/875*ln(x+1/5)+8/189*ln(x+2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx=\frac{1}{30}\,x^{2}-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

```
[In] integrate(x^2/(13+2/x+15*x),x, algorithm="fricas")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx=\frac{x^{2}}{30}-\frac{13x}{225}-\frac{\log\left(x+\frac{1}{5}\right)}{875}+\frac{8\log\left(x+\frac{2}{3}\right)}{189}$$

```
[In] integrate(x**2/(13+2/x+15*x),x)
[Out] x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx = \frac{1}{30}\,x^2 - \frac{13}{225}\,x - \frac{1}{875}\,\log\left(5\,x+1\right) + \frac{8}{189}\,\log\left(3\,x+2\right)$$

```
[In] integrate(x^2/(13+2/x+15*x),x, algorithm="maxima")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx = \frac{1}{30}\,x^2 - \frac{13}{225}\,x - \frac{1}{875}\,\log\left(|5\,x+1|\right) + \frac{8}{189}\,\log\left(|3\,x+2|\right)$$

```
[In] integrate(x^2/(13+2/x+15*x),x, algorithm="giac")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx = \frac{8\,\ln\left(x+\frac{2}{3}\right)}{189} - \frac{13\,x}{225} - \frac{\ln\left(x+\frac{1}{5}\right)}{875} + \frac{x^2}{30}$$

```
[In] int(x^2/(15*x + 2/x + 13),x)
[Out] (8*log(x + 2/3))/189 - (13*x)/225 - log(x + 1/5)/875 + x^2/30
```

3.380 $\int \frac{x}{13+\frac{2}{x}+15x}\,dx$

Optimal result 2271
Rubi [A] (verified) 2271
Mathematica [A] (verified) 2272
Maple [A] (verified) 2273
Fricas [A] (verification not implemented) 2273
Sympy [A] (verification not implemented) 2273
Maxima [A] (verification not implemented) 2274
Giac [A] (verification not implemented) 2274
Mupad [B] (verification not implemented) 2274

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \frac{x}{13+\frac{2}{x}+15x}\,dx=\frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)$$

```
[Out] 1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1400, 717, 646, 31}

$$\int \frac{x}{13+\frac{2}{x}+15x}\,dx=\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

```
[In] Int[x/(13 + 2/x + 15*x),x]
```

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
```

```
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1400

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{x^2}{2+13x+15x^2}\,dx \\ &= \frac{x}{15}+\frac{1}{15}\int \frac{-2-13x}{2+13x+15x^2}\,dx \\ &= \frac{x}{15}+\frac{3}{35}\int \frac{1}{3+15x}\,dx-\frac{20}{21}\int \frac{1}{10+15x}\,dx \\ &= \frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{13+\frac{2}{x}+15x}\,dx=\frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)$$

```
[In] Integrate[x/(13 + 2/x + 15*x),x]
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$\frac{x}{15}+\frac{\ln\left(x+\frac{1}{5}\right)}{175}-\frac{4\ln\left(x+\frac{2}{3}\right)}{63}$	17
default	$\frac{x}{15}-\frac{4\ln(3x+2)}{63}+\frac{\ln(1+5x)}{175}$	21
norman	$\frac{x}{15}-\frac{4\ln(3x+2)}{63}+\frac{\ln(1+5x)}{175}$	21
risch	$\frac{x}{15}-\frac{4\ln(3x+2)}{63}+\frac{\ln(1+5x)}{175}$	21

```
[In] int(x/(13+2/x+15*x),x,method=_RETURNVERBOSE)
[Out] 1/15*x+1/175*ln(x+1/5)-4/63*ln(x+2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x}{13+\frac{2}{x}+15x}\,dx=\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

```
[In] integrate(x/(13+2/x+15*x),x, algorithm="fricas")
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x}{13+\frac{2}{x}+15x}\,dx=\frac{x}{15}+\frac{\log\left(x+\frac{1}{5}\right)}{175}-\frac{4\log\left(x+\frac{2}{3}\right)}{63}$$

```
[In] integrate(x/(13+2/x+15*x),x)
[Out] x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x}{13+\frac{2}{x}+15x}\,dx = \frac{1}{15}\,x + \frac{1}{175}\,\log\left(5\,x+1\right) - \frac{4}{63}\,\log\left(3\,x+2\right)$$

```
[In] integrate(x/(13+2/x+15*x),x, algorithm="maxima")
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x}{13+\frac{2}{x}+15x}\,dx = \frac{1}{15}\,x + \frac{1}{175}\,\log\left(|5\,x+1|\right) - \frac{4}{63}\,\log\left(|3\,x+2|\right)$$

```
[In] integrate(x/(13+2/x+15*x),x, algorithm="giac")
[Out] 1/15*x + 1/175*log(abs(5*x + 1)) - 4/63*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{x}{13+\frac{2}{x}+15x}\,dx = \frac{x}{15} - \frac{4\,\ln\left(x+\frac{2}{3}\right)}{63} + \frac{\ln\left(x+\frac{1}{5}\right)}{175}$$

```
[In] int(x/(15*x + 2/x + 13),x)
[Out] x/15 - (4*log(x + 2/3))/63 + log(x + 1/5)/175
```

3.381 $\int \frac{1}{13+\frac{2}{x}+15x} dx$

Optimal result . . . 2275
Rubi [A] (verified) . . . 2275
Mathematica [A] (verified) . . . 2276
Maple [A] (verified) . . . 2276
Fricas [A] (verification not implemented) . . . 2277
Sympy [A] (verification not implemented) . . . 2277
Maxima [A] (verification not implemented) . . . 2277
Giac [A] (verification not implemented) . . . 2277
Mupad [B] (verification not implemented) . . . 2278

Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{1}{13+\frac{2}{x}+15x} dx = \frac{2}{21}\log(2+3x) - \frac{1}{35}\log(1+5x)$$

```
[Out] 2/21*ln(2+3*x)-1/35*ln(1+5*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1364, 646, 31}

$$\int \frac{1}{13+\frac{2}{x}+15x} dx = \frac{2}{21}\log(3x+2) - \frac{1}{35}\log(5x+1)$$

```
[In] Int[(13 + 2/x + 15*x)^(-1),x]
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1364

```
Int[((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] :> Int[(b
 + a*x^n + c*x^(2*n))^p/x^(n*p), x] /; FreeQ[{a, b, c, n}, x] && EqQ[mn, -n
] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x}{2+13x+15x^2}\,dx\\ &= -\left(\frac{3}{7}\int \frac{1}{3+15x}\,dx\right)+\frac{10}{7}\int \frac{1}{10+15x}\,dx\\ &= \frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{13+\frac{2}{x}+15x}\,dx=\frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)$$

```
[In] Integrate[(13 + 2/x + 15*x)^(-1),x]
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$-\frac{\ln\left(x+\frac{1}{5}\right)}{35}+\frac{2\ln\left(x+\frac{2}{3}\right)}{21}$	14
default	$\frac{2\ln(3x+2)}{21}-\frac{\ln(1+5x)}{35}$	18
norman	$\frac{2\ln(3x+2)}{21}-\frac{\ln(1+5x)}{35}$	18
risch	$\frac{2\ln(3x+2)}{21}-\frac{\ln(1+5x)}{35}$	18

```
[In] int(1/(13+2/x+15*x),x,method=_RETURNVERBOSE)
[Out] -1/35*ln(x+1/5)+2/21*ln(x+2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{13+\frac{2}{x}+15x}\,dx = -\frac{1}{35}\,\log\left(5\,x+1\right) + \frac{2}{21}\,\log\left(3\,x+2\right)$$

```
[In] integrate(1/(13+2/x+15*x),x, algorithm="fricas")
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{13+\frac{2}{x}+15x}\,dx = -\frac{\log\left(x+\frac{1}{5}\right)}{35} + \frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

```
[In] integrate(1/(13+2/x+15*x),x)
[Out] -log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{13+\frac{2}{x}+15x}\,dx = -\frac{1}{35}\,\log\left(5\,x+1\right) + \frac{2}{21}\,\log\left(3\,x+2\right)$$

```
[In] integrate(1/(13+2/x+15*x),x, algorithm="maxima")
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{13+\frac{2}{x}+15x}\,dx = -\frac{1}{35}\,\log\left(|5\,x+1|\right) + \frac{2}{21}\,\log\left(|3\,x+2|\right)$$

```
[In] integrate(1/(13+2/x+15*x),x, algorithm="giac")
[Out] -1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{13+\frac{2}{x}+15x}\,dx=\frac{2\,\ln\left(x+\frac{2}{3}\right)}{21}-\frac{\ln\left(x+\frac{1}{5}\right)}{35}$$

```
[In] int(1/(15*x + 2/x + 13),x)
[Out] (2*log(x + 2/3))/21 - log(x + 1/5)/35
```

3.382 $\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal result 2279
Rubi [A] (verified) 2279
Mathematica [A] (verified) 2280
Maple [A] (verified) 2280
Fricas [A] (verification not implemented) 2281
Sympy [A] (verification not implemented) 2281
Maxima [A] (verification not implemented) 2281
Giac [A] (verification not implemented) 2281
Mupad [B] (verification not implemented) 2282

Optimal result

Integrand size = 16, antiderivative size = 21

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx=-\frac{1}{7}\log(2+3x)+\frac{1}{7}\log(1+5x)$$

```
[Out] -1/7*ln(2+3*x)+1/7*ln(1+5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1400, 630, 31}

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx=\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

```
[In] Int[1/(x*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/7*Log[2 + 3*x] + Log[1 + 5*x]/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1400

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{2+13x+15x^2}\,dx\\ &= \frac{15}{7}\int \frac{1}{3+15x}\,dx - \frac{15}{7}\int \frac{1}{10+15x}\,dx\\ &= -\frac{1}{7}\log(2+3x)+\frac{1}{7}\log(1+5x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{1}{7}\log(2+3x)+\frac{1}{7}\log(1+5x)$$

```
[In] Integrate[1/(x*(13 + 2/x + 15*x)),x]
[Out] -1/7*Log[2 + 3*x] + Log[1 + 5*x]/7
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{\ln\left(x+\frac{1}{5}\right)}{7}-\frac{\ln\left(x+\frac{2}{3}\right)}{7}$	14
default	$\frac{\ln(1+5x)}{7}-\frac{\ln(3x+2)}{7}$	18
norman	$\frac{\ln(1+5x)}{7}-\frac{\ln(3x+2)}{7}$	18
risch	$\frac{\ln(1+5x)}{7}-\frac{\ln(3x+2)}{7}$	18

```
[In] int(1/x/(13+2/x+15*x),x,method=_RETURNVERBOSE)
[Out] 1/7*ln(x+1/5)-1/7*ln(x+2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx = \frac{1}{7}\,\log\left(5\,x+1\right)-\frac{1}{7}\,\log\left(3\,x+2\right)$$

```
[In] integrate(1/x/(13+2/x+15*x),x, algorithm="fricas")
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx = \frac{\log\left(x+\frac{1}{5}\right)}{7}-\frac{\log\left(x+\frac{2}{3}\right)}{7}$$

```
[In] integrate(1/x/(13+2/x+15*x),x)
[Out] log(x + 1/5)/7 - log(x + 2/3)/7
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx = \frac{1}{7}\,\log\left(5\,x+1\right)-\frac{1}{7}\,\log\left(3\,x+2\right)$$

```
[In] integrate(1/x/(13+2/x+15*x),x, algorithm="maxima")
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx = \frac{1}{7}\,\log\left(|5\,x+1|\right)-\frac{1}{7}\,\log\left(|3\,x+2|\right)$$

```
[In] integrate(1/x/(13+2/x+15*x),x, algorithm="giac")
[Out] 1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{30\,x}{7}+\frac{13}{7}\right)}{7}$$

```
[In] int(1/(x*(15*x + 2/x + 13)),x)
[Out] -(2*atanh((30*x)/7 + 13/7))/7
```

3.383 $\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal result 2283
Rubi [A] (verified) 2283
Mathematica [A] (verified) 2284
Maple [A] (verified) 2285
Fricas [A] (verification not implemented) 2285
Sympy [A] (verification not implemented) 2285
Maxima [A] (verification not implemented) 2286
Giac [A] (verification not implemented) 2286
Mupad [B] (verification not implemented) 2286

Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx=\frac{\log(x)}{2}+\frac{3}{14}\log(2+3x)-\frac{5}{7}\log(1+5x)$$

```
[Out] 1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1400, 719, 29, 646, 31}

$$\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx=\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

```
[In] Int[1/(x^2*(13 + 2/x + 15*x)),x]
```

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1400

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x\,(2+13x+15x^2)}\,dx \\
&= \frac{1}{2}\int \frac{1}{x}\,dx + \frac{1}{2}\int \frac{-13-15x}{2+13x+15x^2}\,dx \\
&= \frac{\log(x)}{2} + \frac{45}{14}\int \frac{1}{10+15x}\,dx - \frac{75}{7}\int \frac{1}{3+15x}\,dx \\
&= \frac{\log(x)}{2} + \frac{3}{14}\log(2+3x) - \frac{5}{7}\log(1+5x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx = \frac{\log(x)}{2} + \frac{3}{14}\log(2+3x) - \frac{5}{7}\log(1+5x)$$

```
[In] Integrate[1/(x^2*(13 + 2/x + 15*x)),x]
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{\ln(x)}{2}-\frac{5\ln\left(x+\frac{1}{5}\right)}{7}+\frac{3\ln\left(x+\frac{2}{3}\right)}{14}$	18
default	$\frac{\ln(x)}{2}+\frac{3\ln(3x+2)}{14}-\frac{5\ln(1+5x)}{7}$	22
norman	$\frac{\ln(x)}{2}+\frac{3\ln(3x+2)}{14}-\frac{5\ln(1+5x)}{7}$	22
risch	$\frac{\ln(x)}{2}+\frac{3\ln(3x+2)}{14}-\frac{5\ln(1+5x)}{7}$	22

```
[In] int(1/x^2/(13+2/x+15*x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x)-5/7*ln(x+1/5)+3/14*ln(x+2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx=-\frac{5}{7}\log\left(5\,x+1\right)+\frac{3}{14}\log\left(3\,x+2\right)+\frac{1}{2}\log\left(x\right)$$

```
[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="fricas")
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx=\frac{\log\left(x\right)}{2}-\frac{5\log\left(x+\frac{1}{5}\right)}{7}+\frac{3\log\left(x+\frac{2}{3}\right)}{14}$$

```
[In] integrate(1/x**2/(13+2/x+15*x),x)
[Out] log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{5}{7}\,\log\left(5\,x+1\right)+\frac{3}{14}\,\log\left(3\,x+2\right)+\frac{1}{2}\,\log\left(x\right)$$

```
[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="maxima")
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{5}{7}\,\log\left(|5\,x+1|\right)+\frac{3}{14}\,\log\left(|3\,x+2|\right)+\frac{1}{2}\,\log\left(|x|\right)$$

```
[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="giac")
[Out] -5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx = \frac{3\,\ln\left(x+\frac{2}{3}\right)}{14}-\frac{5\,\ln\left(x+\frac{1}{5}\right)}{7}+\frac{\ln\left(x\right)}{2}$$

```
[In] int(1/(x^2*(15*x + 2/x + 13)),x)
[Out] (3*log(x + 2/3))/14 - (5*log(x + 1/5))/7 + log(x)/2
```

3.384 $\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal result 2287
Rubi [A] (verified) 2287
Mathematica [A] (verified) 2288
Maple [A] (verified) 2288
Fricas [A] (verification not implemented) 2289
Sympy [A] (verification not implemented) 2289
Maxima [A] (verification not implemented) 2289
Giac [A] (verification not implemented) 2290
Mupad [B] (verification not implemented) 2290

Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx=-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

```
[Out] -1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1400, 723, 814}

$$\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx=-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

```
[In] Int[1/(x^3*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1400

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^2\left(2+13x+15x^2\right)}\,dx \\
&= -\frac{1}{2x}+\frac{1}{2}\int \frac{-13-15x}{x\left(2+13x+15x^2\right)}\,dx \\
&= -\frac{1}{2x}+\frac{1}{2}\int \left(-\frac{13}{2x}-\frac{27}{14(2+3x)}+\frac{250}{7(1+5x)}\right)dx \\
&= -\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

```
[In] Integrate[1/(x^3*(13 + 2/x + 15*x)),x]
[Out] -1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(3x+2)}{28}+\frac{25\ln(1+5x)}{7}$	27
norman	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(3x+2)}{28}+\frac{25\ln(1+5x)}{7}$	27
risch	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(3x+2)}{28}+\frac{25\ln(1+5x)}{7}$	27
parallelrisch	$-\frac{91\ln(x)x-100\ln\left(x+\frac{1}{5}\right)x+9\ln\left(x+\frac{2}{3}\right)x+14}{28x}$	27

```
[In] int(1/x^3/(13+2/x+15*x),x,method=_RETURNVERBOSE)
[Out] -1/2/x-13/4*ln(x)-9/28*ln(3*x+2)+25/7*ln(1+5*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx=\frac{100\,x\log\left(5\,x+1\right)-9\,x\log\left(3\,x+2\right)-91\,x\log\left(x\right)-14}{28\,x}$$

```
[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="fricas")
[Out] 1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx=-\frac{13\log\left(x\right)}{4}+\frac{25\log\left(x+\frac{1}{5}\right)}{7}-\frac{9\log\left(x+\frac{2}{3}\right)}{28}-\frac{1}{2x}$$

```
[In] integrate(1/x**3/(13+2/x+15*x),x)
[Out] -13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx=-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(5\,x+1\right)-\frac{9}{28}\,\log\left(3\,x+2\right)-\frac{13}{4}\,\log\left(x\right)$$

```
[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="maxima")
[Out] -1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx=-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(|5\,x+1|\right)-\frac{9}{28}\,\log\left(|3\,x+2|\right)-\frac{13}{4}\,\log\left(|x|\right)$$

```
[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="giac")
[Out] -1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx=\frac{25\,\ln\left(x+\frac{1}{5}\right)}{7}-\frac{9\,\ln\left(x+\frac{2}{3}\right)}{28}-\frac{13\,\ln\left(x\right)}{4}-\frac{1}{2\,x}$$

```
[In] int(1/(x^3*(15*x + 2/x + 13)),x)
[Out] (25*log(x + 1/5))/7 - (9*log(x + 2/3))/28 - (13*log(x))/4 - 1/(2*x)
```

3.385 $\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal result 2291
Rubi [A] (verified) 2291
Mathematica [A] (verified) 2292
Maple [A] (verified) 2293
Fricas [A] (verification not implemented) 2293
Sympy [A] (verification not implemented) 2293
Maxima [A] (verification not implemented) 2294
Giac [A] (verification not implemented) 2294
Mupad [B] (verification not implemented) 2294

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

```
[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1400, 723, 814}

$$\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

```
[In] Int[1/(x^4*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1400

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x^3\left(2+13x+15x^2\right)}\,dx\\ &= -\frac{1}{4x^2}+\frac{1}{2}\int \frac{-13-15x}{x^2\left(2+13x+15x^2\right)}\,dx\\ &= -\frac{1}{4x^2}+\frac{1}{2}\int \left(-\frac{13}{2x^2}+\frac{139}{4x}+\frac{81}{28(2+3x)}-\frac{1250}{7(1+5x)}\right)dx\\ &= -\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

```
[In] Integrate[1/(x^4*(13 + 2/x + 15*x)),x]
[Out] -1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{\frac{13x}{4}-\frac{1}{4}}{x^2}+\frac{139\ln(x)}{8}+\frac{27\ln(3x+2)}{56}-\frac{125\ln(1+5x)}{7}$	31
default	$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\ln(x)}{8}+\frac{27\ln(3x+2)}{56}-\frac{125\ln(1+5x)}{7}$	32
norman	$\frac{-\frac{1}{4}x+\frac{13}{4}x^2}{x^3}+\frac{139\ln(x)}{8}+\frac{27\ln(3x+2)}{56}-\frac{125\ln(1+5x)}{7}$	35
parallelrisch	$\frac{973\ln(x)x^2-1000\ln\left(x+\frac{1}{5}\right)x^2+27\ln\left(x+\frac{2}{3}\right)x^2-14+182x}{56x^2}$	36

```
[In] int(1/x^4/(13+2/x+15*x),x,method=_RETURNVERBOSE)
```

```
[Out] (13/4*x-1/4)/x^2+139/8*ln(x)+27/56*ln(3*x+2)-125/7*ln(1+5*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{1000\,x^2\log\left(5\,x+1\right)-27\,x^2\log\left(3\,x+2\right)-973\,x^2\log\left(x\right)-182\,x+14}{56\,x^2}$$

```
[In] integrate(1/x^4/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] -1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(x) - 182*x
 + 14)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx = \frac{139\log\left(x\right)}{8}-\frac{125\log\left(x+\frac{1}{5}\right)}{7}+\frac{27\log\left(x+\frac{2}{3}\right)}{56}+\frac{13x-1}{4x^2}$$

```
[In] integrate(1/x**4/(13+2/x+15*x),x)
```

```
[Out] 139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**2
)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx = \frac{13\,x-1}{4\,x^2} - \frac{125}{7}\,\log\left(5\,x+1\right) + \frac{27}{56}\,\log\left(3\,x+2\right) + \frac{139}{8}\,\log\left(x\right)$$

```
[In] integrate(1/x^4/(13+2/x+15*x),x, algorithm="maxima")
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 139/8*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx = \frac{13\,x-1}{4\,x^2} - \frac{125}{7}\,\log\left(|5\,x+1|\right) + \frac{27}{56}\,\log\left(|3\,x+2|\right) + \frac{139}{8}\,\log\left(|x|\right)$$

```
[In] integrate(1/x^4/(13+2/x+15*x),x, algorithm="giac")
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(abs(5*x + 1)) + 27/56*log(abs(3*x + 2)) + 13
9/8*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx = \frac{27\,\ln\left(x+\frac{2}{3}\right)}{56} - \frac{125\,\ln\left(x+\frac{1}{5}\right)}{7} + \frac{139\,\ln\left(x\right)}{8} + \frac{\frac{13\,x}{4}-\frac{1}{4}}{x^2}$$

```
[In] int(1/(x^4*(15*x + 2/x + 13)),x)
[Out] (27*log(x + 2/3))/56 - (125*log(x + 1/5))/7 + (139*log(x))/8 + ((13*x)/4 -
1/4)/x^2
```

3.386 $\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal result . . . 2295
Rubi [A] (verified) . . . 2295
Mathematica [A] (verified) . . . 2296
Maple [A] (verified) . . . 2297
Fricas [A] (verification not implemented) . . . 2297
Sympy [A] (verification not implemented) . . . 2297
Maxima [A] (verification not implemented) . . . 2298
Giac [A] (verification not implemented) . . . 2298
Mupad [B] (verification not implemented) . . . 2298

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx=-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)$$

```
[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln(1+5*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1400, 723, 814}

$$\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx=-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

```
[In] Int[1/(x^5*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
```

```
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1400

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^4 (2+13x+15x^2)}\,dx \\
&= -\frac{1}{6x^3}+\frac{1}{2}\int \frac{-13-15x}{x^3 (2+13x+15x^2)}\,dx \\
&= -\frac{1}{6x^3}+\frac{1}{2}\int \left(-\frac{13}{2x^3}+\frac{139}{4x^2}-\frac{1417}{8x}-\frac{243}{56(2+3x)}+\frac{6250}{7(1+5x)}\right)dx \\
&= -\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$
\int \frac{1}{x^5 \left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)
$$

```
[In] Integrate[1/(x^5*(13 + 2/x + 15*x)),x]
[Out] -1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{-\frac{139}{8}x^2+\frac{13}{8}x-\frac{1}{6}}{x^3}-\frac{1417\ln(x)}{16}-\frac{81\ln(3x+2)}{112}+\frac{625\ln(1+5x)}{7}$	36
default	$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\ln(x)}{16}-\frac{81\ln(3x+2)}{112}+\frac{625\ln(1+5x)}{7}$	37
norman	$\frac{-\frac{1}{6}x+\frac{13}{8}x^2-\frac{139}{8}x^3}{x^4}-\frac{1417\ln(x)}{16}-\frac{81\ln(3x+2)}{112}+\frac{625\ln(1+5x)}{7}$	40
parallelrisch	$-\frac{29757\ln(x)x^3-30000\ln(x+\frac{1}{5})x^3+243\ln(x+\frac{2}{3})x^3+56+5838x^2-546x}{336x^3}$	41

```
[In] int(1/x^5/(13+2/x+15*x),x,method=_RETURNVERBOSE)
[Out] (-139/8*x^2+13/8*x-1/6)/x^3-1417/16*ln(x)-81/112*ln(3*x+2)+625/7*ln(1+5*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx = \frac{30000\,x^3\log\left(5\,x+1\right)-243\,x^3\log\left(3\,x+2\right)-29757\,x^3\log\left(x\right)-5838\,x^2+546\,x-56}{336\,x^3}$$

```
[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="fricas")
[Out] 1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*log(x) - 5
838*x^2 + 546*x - 56)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{1417\log\left(x\right)}{16}+\frac{625\log\left(x+\frac{1}{5}\right)}{7}-\frac{81\log\left(x+\frac{2}{3}\right)}{112}+\frac{-417x^2+39x-4}{24x^3}$$

```
[In] integrate(1/x**5/(13+2/x+15*x),x)
[Out] -1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 + (-417*x**2 + 3
9*x - 4)/(24*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\,\log\left(5\,x+1\right)-\frac{81}{112}\,\log\left(3\,x+2\right)-\frac{1417}{16}\,\log\left(x\right)$$

```
[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(3*x + 2) -
 1417/16*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\,\log\left(|5\,x+1|\right)-\frac{81}{112}\,\log\left(|3\,x+2|\right)-\frac{1417}{16}\,\log\left(|x|\right)$$

```
[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(abs(5*x + 1)) - 81/112*log(abs(3
*x + 2)) - 1417/16*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx = \frac{625\,\ln\left(x+\frac{1}{5}\right)}{7}-\frac{81\,\ln\left(x+\frac{2}{3}\right)}{112}-\frac{1417\,\ln\left(x\right)}{16}-\frac{\frac{139\,x^2}{8}-\frac{13\,x}{8}+\frac{1}{6}}{x^3}$$

```
[In] int(1/(x^5*(15*x + 2/x + 13)),x)
```

```
[Out] (625*log(x + 1/5))/7 - (81*log(x + 2/3))/112 - (1417*log(x))/16 - ((139*x^2
)/8 - (13*x)/8 + 1/6)/x^3
```

3.387 $\int \frac{x^2}{2-(1+x^2)^4} \, dx$

Optimal result 2299
Rubi [A] (verified) 2299
Mathematica [C] (verified) 2301
Maple [C] (verified) 2301
Fricas [B] (verification not implemented) 2302
Sympy [A] (verification not implemented) 2303
Maxima [F] 2303
Giac [F] 2304
Mupad [B] (verification not implemented) 2304

Optimal result

Integrand size = 17, antiderivative size = 157

$$\int \frac{x^2}{2-(1+x^2)^4} \, dx = \frac{i\sqrt{1-i\sqrt[4]{2}}\arctan\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} - \frac{i\sqrt{1+i\sqrt[4]{2}}\arctan\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} - \frac{\sqrt{1+\sqrt[4]{2}}\arctan\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} + \frac{\sqrt{-1+\sqrt[4]{2}}\text{arctanh}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}$$

```
[Out] 1/8*I*arctan(x/(1-I*2^(1/4))^(1/2))*(1-I*2^(1/4))^(1/2)*2^(1/4)-1/8*I*arcta
n(x/(1+I*2^(1/4))^(1/2))*(1+I*2^(1/4))^(1/2)*2^(1/4)+1/8*arctanh(x/(-1+2^(1
/4))^(1/2))*(-1+2^(1/4))^(1/2)*2^(1/4)-1/8*arctan(x/(1+2^(1/4))^(1/2))*(1+2
^(1/4))^(1/2)*2^(1/4)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6872, 212, 209, 1997, 211}

$$\int \frac{x^2}{2-(1+x^2)^4} \, dx = \frac{i\sqrt{1-i\sqrt[4]{2}}\arctan\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} - \frac{i\sqrt{1+i\sqrt[4]{2}}\arctan\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} - \frac{\sqrt{1+\sqrt[4]{2}}\arctan\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} + \frac{\sqrt{\sqrt[4]{2}-1}\text{arctanh}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\ 2^{3/4}}$$

```
[In] Int[x^2/(2 - (1 + x^2)^4),x]
```

```
[Out] ((I/4)*Sqrt[1 - I*2^(1/4)]*ArcTan[x/Sqrt[1 - I*2^(1/4)]])/2^(3/4) - ((I/4)*
Sqrt[1 + I*2^(1/4)]*ArcTan[x/Sqrt[1 + I*2^(1/4)]])/2^(3/4) - (Sqrt[1 + 2^(1
/4)]*ArcTan[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/4)) + (Sqrt[-1 + 2^(1/4)]*ArcTanh
[x/Sqrt[-1 + 2^(1/4)]])/(4*2^(3/4))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 6872

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\text{integral} = \int \left(\frac{-\sqrt[4]{2}+\sqrt{2}}{8\left(-1+\sqrt[4]{2}-x^2\right)}+\frac{-\sqrt[4]{2}-\sqrt{2}}{8\left(1+\sqrt[4]{2}+x^2\right)}+\frac{-\sqrt[4]{2}-i\sqrt{2}}{8\left(\sqrt[4]{2}-i\left(1+x^2\right)\right)}+\frac{-\sqrt[4]{2}+i\sqrt{2}}{8\left(\sqrt[4]{2}+i\left(1+x^2\right)\right)}\right) dx$$

$$= -\frac{\left(1-\sqrt[4]{2}\right)\int \frac{1}{-1+\sqrt[4]{2}-x^2}\,dx}{4\ 2^{3/4}}-\frac{\left(1-i\sqrt[4]{2}\right)\int \frac{1}{\sqrt[4]{2}+i(1+x^2)}\,dx}{4\ 2^{3/4}}-\frac{\left(1+i\sqrt[4]{2}\right)\int \frac{1}{\sqrt[4]{2}-i(1+x^2)}\,dx}{4\ 2^{3/4}}-\frac{\left(1+\sqrt[4]{2}\right)\int \frac{1}{1+\sqrt[4]{2}+x^2}\,dx}{4\ 2^{3/4}}$$

$$= -\frac{\sqrt{1+\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} + \frac{\sqrt{-1+\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} - \frac{\left(1-i\sqrt[4]{2}\right)\int \frac{1}{i+\sqrt[4]{2}+ix^2}\,dx}{4\ 2^{3/4}} - \frac{\left(1+i\sqrt[4]{2}\right)\int \frac{1}{-i+\sqrt[4]{2}-ix^2}\,dx}{4\ 2^{3/4}}$$

$$= \frac{i\sqrt{1-i\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} - \frac{i\sqrt{1+i\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} - \frac{\sqrt{1+\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} + \frac{\sqrt{-1+\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{2-(1+x^2)^4}\,dx = -\frac{1}{8}\text{RootSum}\left[-1+4\#1^2+6\#1^4+4\#1^6+\#1^8\&,\frac{\log(x-\#1)\#1}{1+3\#1^2+3\#1^4+\#1^6}\&\right]$$

```
[In] Integrate[x^2/(2 - (1 + x^2)^4),x]
[Out] -1/8*RootSum[-1 + 4*#1^2 + 6*#1^4 + 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(1 +
 3*#1^2 + 3*#1^4 + #1^6) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.34

method	result	size
default	$-\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2-1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7+3\_R^5+3\_R^3+\_R}\right)}{8}$	54
risch	$-\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2-1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7+3\_R^5+3\_R^3+\_R}\right)}{8}$	54

```
[In] int(x^2/(2-(x^2+1)^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*sum(_R^2/(_R^7+3*_R^5+3*_R^3+_R)*ln(x-_R),_R=RootOf(_Z^8+4*_Z^6+6*_Z^4
+4*_Z^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. $2(97) = 194$.

Time = 1.02 (sec) , antiderivative size = 1506, normalized size of antiderivative = 9.59

$$\int \frac{x^2}{2-(1+x^2)^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^
(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log
(1/4*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2))^2 + sqrt(
2)*(2^(3/4) + sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*(2^(
3/4) - sqrt(2)) + 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(
2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)*((sqrt(2)*(2^(3/4
) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqrt(2)) -
 4*sqrt(2)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*sqrt(2) +
 sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt
(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2)*sqrt(1/2*sqrt
(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) -
 sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log(-1/4*((sqrt(2)*(2^(3/4) +
sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2))^2 + sqrt(2)*(2^(3/4) + sqrt(2))^2 -
 (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*(2^(3/4) - sqrt(2)) + 4*sqrt(-
3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) -
3/16*(2^(3/4) - sqrt(2))^2 + 1)*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2
^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqrt(2)) - 4*sqrt(2)) - 4*sqrt(2)*(2
^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqr
t(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqr
t(2))^2 + 1)) + 6*x) - 1/16*sqrt(2)*sqrt(1/2*sqrt(2) - sqrt(-3/16*(2^(3/4)
+ sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4)
- sqrt(2))^2 + 1))*log(1/4*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4
) - sqrt(2))^2 + sqrt(2)*(2^(3/4) + sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2
))^2 - 4*sqrt(2))*(2^(3/4) - sqrt(2)) - 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2
+ 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2
+ 1)*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)
*(2^(3/4) + sqrt(2)) - 4*sqrt(2)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(
2))*sqrt(1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sq
rt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16
*sqrt(2)*sqrt(1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4)
 + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log(-1/4
```

```
*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2))^2 + sqrt(2)*(
2^(3/4) + sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*(2^(3/4)
 - sqrt(2)) - 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*
(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)*((sqrt(2)*(2^(3/4) +
sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqrt(2)) - 4*s
qrt(2)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*sqrt(2) - sqr
t(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2))
 - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2^(3/4) - sqrt(2))*l
og(1/4*((2^(3/4) + sqrt(2))^3 + (2^(3/4) + sqrt(2) + 1)*(2^(3/4) - sqrt(2))
^2 - ((2^(3/4) + sqrt(2))^2 - 4)*(2^(3/4) - sqrt(2)) - 4*2^(3/4) - 4*sqrt(2
) - 6)*sqrt(2^(3/4) - sqrt(2)) + 3*x) - 1/16*sqrt(2^(3/4) - sqrt(2))*log(-1
/4*((2^(3/4) + sqrt(2))^3 + (2^(3/4) + sqrt(2) + 1)*(2^(3/4) - sqrt(2))^2 -
 ((2^(3/4) + sqrt(2))^2 - 4)*(2^(3/4) - sqrt(2)) - 4*2^(3/4) - 4*sqrt(2) -
6)*sqrt(2^(3/4) - sqrt(2)) + 3*x) - sqrt(-1/256*2^(3/4) - 1/256*sqrt(2))*lo
g(4*((2^(3/4) + sqrt(2))^3 - (2^(3/4) + sqrt(2))^2 - 10)*sqrt(-1/256*2^(3/4
) - 1/256*sqrt(2)) + 3*x) + sqrt(-1/256*2^(3/4) - 1/256*sqrt(2))*log(-4*((2
^(3/4) + sqrt(2))^3 - (2^(3/4) + sqrt(2))^2 - 10)*sqrt(-1/256*2^(3/4) - 1/2
56*sqrt(2)) + 3*x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.26

$$\int \frac{x^2}{2-(1+x^2)^4}\,dx = -\operatorname{RootSum}\left(1073741824t^8-65536t^4+1024t^2-1,\left(t\mapsto t\log\left(-\frac{67108864t^7}{3}-\frac{262144t^5}{3}-\frac{40t}{3}+x\right)\right)\right)$$

```
[In] integrate(x**2/(2-(x**2+1)**4),x)
[Out] -RootSum(1073741824*_t**8 - 65536*_t**4 + 1024*_t**2 - 1, Lambda(_t, _t*log
(-67108864*_t**7/3 - 262144*_t**5/3 - 40*_t/3 + x)))
```

Maxima [F]

$$\int \frac{x^2}{2-(1+x^2)^4}\,dx = \int -\frac{x^2}{(x^2+1)^4-2}\,dx$$

```
[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="maxima")
[Out] -integrate(x^2/((x^2 + 1)^4 - 2), x)
```

Giac [F]

$$\int \frac{x^2}{2-(1+x^2)^4}\,dx=\int -\frac{x^2}{(x^2+1)^4-2}\,dx$$

```
[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="giac")
[Out] integrate(-x^2/((x^2 + 1)^4 - 2), x)
```

Mupad [B] (verification not implemented)

Time = 9.91 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{2-(1+x^2)^4}\,dx=\sum_{k=1}^{8}\ln\left(-\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(56\,x\right.\right.$$
$$\left.-\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(40\right.\right.\right.$$
$$\left.-1\right)\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)$$

```
[In] int(-x^2/((x^2 + 1)^4 - 2),x)
[Out] symsum(log(- root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)*(56*x
 - root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)*(root(z^8 - z^4
/16384 + z^2/1048576 - 1/1073741824, z, k)*(4096*x - root(z^8 - z^4/16384 +
 z^2/1048576 - 1/1073741824, z, k)^2*(262144*x + 67108864*root(z^8 - z^4/16
384 + z^2/1048576 - 1/1073741824, z, k)^2*x)) + 256)) - 1)*root(z^8 - z^4/1
6384 + z^2/1048576 - 1/1073741824, z, k), k, 1, 8)
```

3.388 $\int \frac{x^2}{2-(1-x^2)^4} dx$

Optimal result . . . 2305
Rubi [A] (verified) . . . 2305
Mathematica [C] (verified) . . . 2307
Maple [C] (verified) . . . 2307
Fricas [B] (verification not implemented) . . . 2308
Sympy [A] (verification not implemented) . . . 2309
Maxima [F] . . . 2309
Giac [F] . . . 2310
Mupad [B] (verification not implemented) . . . 2310

Optimal result

Integrand size = 19, antiderivative size = 157

$$\int \frac{x^2}{2-(1-x^2)^4} dx = -\frac{\sqrt{-1+\sqrt[4]{2}}\arctan\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} - \frac{i\sqrt{1-i\sqrt[4]{2}}\text{arctanh}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} + \frac{i\sqrt{1+i\sqrt[4]{2}}\text{arctanh}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} + \frac{\sqrt{1+\sqrt[4]{2}}\text{arctanh}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}$$

```
[Out] -1/8*I*arctanh(x/(1-I*2^(1/4))^(1/2))*(1-I*2^(1/4))^(1/2)*2^(1/4)+1/8*I*arc
tanh(x/(1+I*2^(1/4))^(1/2))*(1+I*2^(1/4))^(1/2)*2^(1/4)-1/8*arctan(x/(-1+2^
(1/4))^(1/2))*(-1+2^(1/4))^(1/2)*2^(1/4)+1/8*arctanh(x/(1+2^(1/4))^(1/2))*(
1+2^(1/4))^(1/2)*2^(1/4)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6872, 212, 209, 1997, 214}

$$\int \frac{x^2}{2-(1-x^2)^4} dx = -\frac{\sqrt{\sqrt[4]{2}-1}\arctan\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\ 2^{3/4}} - \frac{i\sqrt{1-i\sqrt[4]{2}}\text{arctanh}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} + \frac{i\sqrt{1+i\sqrt[4]{2}}\text{arctanh}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} + \frac{\sqrt{1+\sqrt[4]{2}}\text{arctanh}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}$$

```
[In] Int[x^2/(2 - (1 - x^2)^4),x]
```

```
[Out] -1/4*(Sqrt[-1 + 2^(1/4)]*ArcTan[x/Sqrt[-1 + 2^(1/4)]])/2^(3/4) - ((I/4)*Sqr
t[1 - I*2^(1/4)]*ArcTanh[x/Sqrt[1 - I*2^(1/4)]])/2^(3/4) + ((I/4)*Sqrt[1 +
I*2^(1/4)]*ArcTanh[x/Sqrt[1 + I*2^(1/4)]])/2^(3/4) + (Sqrt[1 + 2^(1/4)]*Arc
Tanh[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/4))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 6872

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt[4]{2}+\sqrt{2}}{8\left(1+\sqrt[4]{2}-x^2\right)}+\frac{\sqrt[4]{2}-\sqrt{2}}{8\left(-1+\sqrt[4]{2}+x^2\right)}+\frac{\sqrt[4]{2}+i\sqrt{2}}{8\left(\sqrt[4]{2}-i\left(1-x^2\right)\right)}+\frac{\sqrt[4]{2}-i\sqrt{2}}{8\left(\sqrt[4]{2}+i\left(1-x^2\right)\right)}\right)dx$$

$$= \frac{\left(1-\sqrt[4]{2}\right)\int \frac{1}{-1+\sqrt[4]{2}+x^2}\,dx}{4\ 2^{3/4}}+\frac{\left(1-i\sqrt[4]{2}\right)\int \frac{1}{\sqrt[4]{2}+i(1-x^2)}\,dx}{4\ 2^{3/4}}+\frac{\left(1+i\sqrt[4]{2}\right)\int \frac{1}{\sqrt[4]{2}-i(1-x^2)}\,dx}{4\ 2^{3/4}}+\frac{\left(1+\sqrt[4]{2}\right)\int \frac{1}{1+\sqrt[4]{2}-x^2}\,dx}{4\ 2^{3/4}}$$

$$= -\frac{\sqrt{-1+\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} + \frac{\sqrt{1+\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} + \frac{\left(1-i\sqrt[4]{2}\right)\int \frac{1}{i+\sqrt[4]{2}-ix^2}\,dx}{4\ 2^{3/4}} + \frac{\left(1+i\sqrt[4]{2}\right)\int \frac{1}{-i+\sqrt[4]{2}+ix^2}\,dx}{4\ 2^{3/4}}$$

$$= -\frac{\sqrt{-1+\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} - \frac{i\sqrt{1-i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} + \frac{i\sqrt{1+i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} + \frac{\sqrt{1+\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{2-(1-x^2)^4}\,dx = -\frac{1}{8}\text{RootSum}\left[-1-4\#1^2+6\#1^4-4\#1^6 + \#1^8\&, \frac{\log(x-\#1)\#1}{-1+3\#1^2-3\#1^4+\#1^6}\&\right]$$

```
[In] Integrate[x^2/(2 - (1 - x^2)^4),x]
[Out] -1/8*RootSum[-1 - 4*#1^2 + 6*#1^4 - 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(-1
+ 3*#1^2 - 3*#1^4 + #1^6) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.36

method	result	size
default	$-\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2-1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}\right)}{8}$	56
risch	$-\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2-1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}\right)}{8}$	56

```
[In] int(x^2/(2-(-x^2+1)^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*sum(_R^2/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8-4*_Z^6+6*_Z^4
-4*_Z^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1546 vs. 2(97) = 194.

Time = 0.99 (sec) , antiderivative size = 1546, normalized size of antiderivative = 9.85

$$\int \frac{x^2}{2-(1-x^2)^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(-1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2
^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*lo
g(1/4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2))^2 - sqrt
(2)*(2^(3/4) - sqrt(2))^2 - (sqrt(2)*(2^(3/4) - sqrt(2))^2 - 4*sqrt(2))*(2^
(3/4) + sqrt(2)) + 4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sq
rt(2)) + sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-3/16*(2^(3/4) + sqr
t(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqr
t(2))^2 + 1) - 4*sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-1/2*sqrt(2)
 + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sq
rt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2)*sqrt(-1/2*s
qrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4
) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log(-1/4*((sqrt(2)*(2^(3/4)
 - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2))^2 - sqrt(2)*(2^(3/4) - sqrt(2))^
2 - (sqrt(2)*(2^(3/4) - sqrt(2))^2 - 4*sqrt(2))*(2^(3/4) + sqrt(2)) + 4*((s
qrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2)) + sqrt(2)*(2^(3/4
) - sqrt(2)) - 4*sqrt(2))*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) +
 sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1) - 4*sqrt(2)
*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) +
 sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) -
 sqrt(2))^2 + 1)) + 6*x) - 1/16*sqrt(2)*sqrt(-1/2*sqrt(2) - sqrt(-3/16*(2^(
3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(
3/4) - sqrt(2))^2 + 1))*log(1/4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2
^(3/4) + sqrt(2))^2 - sqrt(2)*(2^(3/4) - sqrt(2))^2 - (sqrt(2)*(2^(3/4) - s
qrt(2))^2 - 4*sqrt(2))*(2^(3/4) + sqrt(2)) - 4*((sqrt(2)*(2^(3/4) - sqrt(2)
) - sqrt(2))*(2^(3/4) + sqrt(2)) + sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))
*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt
(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1) - 4*sqrt(2)*(2^(3/4) - sqrt(2)) - 4*
sqrt(2))*sqrt(-1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4
) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x)
+ 1/16*sqrt(2)*sqrt(-1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(
2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*l
```

```
og(-1/4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2))^2 - sq
rt(2)*(2^(3/4) - sqrt(2))^2 - (sqrt(2)*(2^(3/4) - sqrt(2))^2 - 4*sqrt(2))*(
2^(3/4) + sqrt(2)) - 4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) +
sqrt(2)) + sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-3/16*(2^(3/4) + s
qrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - s
qrt(2))^2 + 1) - 4*sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-1/2*sqrt(
2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) -
sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2^(3/4) + sq
rt(2))*log(1/4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) + sqrt(2))^2*(2^(3/4) - sq
rt(2) - 1) - ((2^(3/4) - sqrt(2))^2 - 4)*(2^(3/4) + sqrt(2)) - 4*2^(3/4) +
4*sqrt(2) + 6)*sqrt(2^(3/4) + sqrt(2)) + 3*x) - 1/16*sqrt(2^(3/4) + sqrt(2)
)*log(-1/4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) + sqrt(2))^2*(2^(3/4) - sqrt(2
) - 1) - ((2^(3/4) - sqrt(2))^2 - 4)*(2^(3/4) + sqrt(2)) - 4*2^(3/4) + 4*sq
rt(2) + 6)*sqrt(2^(3/4) + sqrt(2)) + 3*x) - sqrt(-1/256*2^(3/4) + 1/256*sqr
t(2))*log(4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) - sqrt(2))^2 + 10)*sqrt(-1/25
6*2^(3/4) + 1/256*sqrt(2)) + 3*x) + sqrt(-1/256*2^(3/4) + 1/256*sqrt(2))*lo
g(-4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) - sqrt(2))^2 + 10)*sqrt(-1/256*2^(3/
4) + 1/256*sqrt(2)) + 3*x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.26

$$\int \frac{x^2}{2-(1-x^2)^4}\,dx = -\operatorname{RootSum}\left(1073741824t^8-65536t^4-1024t^2-1,\left(t\mapsto t\log\left(-\frac{67108864t^7}{3}+\frac{262144t^5}{3}+\frac{40t}{3}+x\right)\right)\right)$$

```
[In] integrate(x**2/(2-(-x**2+1)**4),x)
[Out] -RootSum(1073741824*_t**8 - 65536*_t**4 - 1024*_t**2 - 1, Lambda(_t, _t*log
(-67108864*_t**7/3 + 262144*_t**5/3 + 40*_t/3 + x)))
```

Maxima [F]

$$\int \frac{x^2}{2-(1-x^2)^4}\,dx = \int -\frac{x^2}{(x^2-1)^4-2}\,dx$$

```
[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="maxima")
[Out] -integrate(x^2/((x^2 - 1)^4 - 2), x)
```

Giac [F]

$$\int \frac{x^2}{2-(1-x^2)^4}\,dx=\int -\frac{x^2}{(x^2-1)^4-2}\,dx$$

```
[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="giac")
[Out] integrate(-x^2/((x^2 - 1)^4 - 2), x)
```

Mupad [B] (verification not implemented)

Time = 9.94 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{2-(1-x^2)^4}\,dx=\sum_{k=1}^{8}\ln\left(-\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(56\,x+\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(40\right.\right.\right.\right.\right.$$
$$\left.-1\right)\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)$$

```
[In] int(-x^2/((x^2 - 1)^4 - 2),x)
[Out] symsum(log(- root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)*(56*x
 + root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)*(root(z^8 - z^4
/16384 - z^2/1048576 - 1/1073741824, z, k)*(4096*x + root(z^8 - z^4/16384 -
 z^2/1048576 - 1/1073741824, z, k)^2*(262144*x - 67108864*root(z^8 - z^4/16
384 - z^2/1048576 - 1/1073741824, z, k)^2*x)) + 256)) - 1)*root(z^8 - z^4/1
6384 - z^2/1048576 - 1/1073741824, z, k), k, 1, 8)
```

3.389 $\int \frac{x^2}{2+(1+x^2)^4} \, dx$

Optimal result 2311
Rubi [A] (verified) 2312
Mathematica [C] (verified) 2314
Maple [C] (verified) 2314
Fricas [B] (verification not implemented) 2314
Sympy [A] (verification not implemented) 2317
Maxima [F] 2317
Giac [F] 2317
Mupad [B] (verification not implemented) 2317

Optimal result

Integrand size = 15, antiderivative size = 188

$$\begin{aligned}\int \frac{x^2}{2+(1+x^2)^4} \, dx = & \frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\arctan\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}} \\ & - \frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\arctan\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}} \\ & - \frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\arctan\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}} \\ & + \frac{1}{8}i\left(\sqrt[4]{-2}+\sqrt{2}\right)\sqrt{\frac{1+i}{(1+i)+2^{3/4}}}\arctan\left(\sqrt{\frac{1+i}{(1+i)+2^{3/4}}}x\right)\end{aligned}$$

```
[Out] 1/8*(-1)^(1/4)*arctan(x/(1-(-2)^(1/4))^(1/2))*(1-(-2)^(1/4))^(1/2)*2^(1/4)-
1/8*(-1)^(3/4)*2^(1/4)*arctan(x/(1+I*(-2)^(1/4))^(1/2))*(1+I*(-2)^(1/4))^(1
/2)-1/8*(-1)^(1/4)*arctan(x/(1+(-2)^(1/4))^(1/2))*(1+(-2)^(1/4))^(1/2)*2^(1
/4)+1/8*I*arctan(x*((1+I)/(1+I+2^(3/4)))^(1/2))*((-2)^(1/4)+2^(1/2))*((1+I)
/(1+I+2^(3/4)))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6872, 210, 209, 1997, 211}

$$\begin{aligned}\int \frac{x^2}{2+(1+x^2)^4}\,dx &= \frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\arctan\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}} \\ &\quad -\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\arctan\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}} \\ &\quad -\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\arctan\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}} \\ &\quad +\frac{1}{8}i\left(\sqrt[4]{-2}+\sqrt{2}\right)\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}\arctan\left(\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}x\right)\end{aligned}$$

```
[In] Int[x^2/(2 + (1 + x^2)^4),x]
```

```
[Out] ((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTan[x/Sqrt[1 - (-2)^(1/4)]])/(4*2^(3/4)
) - ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTan[x/Sqrt[1 + I*(-2)^(1/4)]])/(4
*2^(3/4)) - ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4)]*ArcTan[x/Sqrt[1 + (-2)^(1/4)]]
)/(4*2^(3/4)) + (I/8)*((-2)^(1/4) + Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^(3/4
))]*ArcTan[Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 6872

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\text{integral} = \int \left(\frac{-\sqrt[4]{-2}+i\sqrt{2}}{8\left(-1+\sqrt[4]{-2}-x^2\right)}+\frac{-\sqrt[4]{-2}-i\sqrt{2}}{8\left(1+\sqrt[4]{-2}+x^2\right)}+\frac{-\sqrt[4]{-2}+\sqrt{2}}{8\left(\sqrt[4]{-2}-i\left(1+x^2\right)\right)}+\frac{-\sqrt[4]{-2}-\sqrt{2}}{8\left(\sqrt[4]{-2}+i\left(1+x^2\right)\right)}\right)dx$$

$$=\frac{1}{8}\left(-\sqrt[4]{-2}-\sqrt{2}\right)\int \frac{1}{\sqrt[4]{-2}+i\left(1+x^2\right)}\,dx+\frac{1}{8}\left(-\sqrt[4]{-2}-i\sqrt{2}\right)\int \frac{1}{1+\sqrt[4]{-2}+x^2}\,dx+\frac{1}{8}\left(-\sqrt[4]{-2}+i\sqrt{2}\right)\int \frac{1}{-1+\sqrt[4]{-2}-x^2}\,dx+\frac{1}{8}\left(-\sqrt[4]{-2}+\sqrt{2}\right)\int \frac{1}{\sqrt[4]{-2}-i\left(1+x^2\right)}\,dx$$

$$=\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{1}{8}\left(-\sqrt[4]{-2}-\sqrt{2}\right)\int \frac{1}{i+\sqrt[4]{-2}+ix^2}\,dx+\frac{1}{8}\left(-\sqrt[4]{-2}+\sqrt{2}\right)\int \frac{1}{-i+\sqrt[4]{-2}-ix^2}\,dx$$

$$=\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{1}{8}i\left(\sqrt[4]{-2}+\sqrt{2}\right)\sqrt{\frac{1+i}{(1+i)+2^{3/4}}}\tan^{-1}\left(\sqrt{\frac{1+i}{(1+i)+2^{3/4}}}x\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.32

$$\int \frac{x^2}{2+(1+x^2)^4}\,dx = \frac{1}{8}\text{RootSum}\left[3+4\#1^2+6\#1^4+4\#1^6+\#1^8\&,\frac{\log(x-\#1)\#1}{1+3\#1^2+3\#1^4+\#1^6}\&\right]$$

```
[In] Integrate[x^2/(2 + (1 + x^2)^4),x]
[Out] RootSum[3 + 4*#1^2 + 6*#1^4 + 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(1 + 3*#1^
2 + 3*#1^4 + #1^6) & ]/8
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.29

method	result	size
default	$\frac{\sum_{\_R=\text{RootOf}(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+3)}\frac{\_R^2\ln(x-\_R)}{\_R^7+3\_R^5+3\_R^3+\_R}}{8}$	54
risch	$\frac{\sum_{\_R=\text{RootOf}(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+3)}\frac{\_R^2\ln(x-\_R)}{\_R^7+3\_R^5+3\_R^3+\_R}}{8}$	54

```
[In] int(x^2/(2+(x^2+1)^4),x,method=_RETURNVERBOSE)
[Out] 1/8*sum(_R^2/(_R^7+3*_R^5+3*_R^3+_R)*ln(x-_R),_R=RootOf(_Z^8+4*_Z^6+6*_Z^4+
4*_Z^2+3))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2271 vs. 2(118) = 236.

Time = 0.98 (sec) , antiderivative size = 2271, normalized size of antiderivative = 12.08

$$\int \frac{x^2}{2+(1+x^2)^4}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)
))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sq
rt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2
))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((163
84*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) +
 128*sqrt(1/8192*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192
*I*sqrt(2))) - sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 -
16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - sqrt(-12
288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt
(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*
sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(
2) + 128*sqrt(-1/8192*I*sqrt(2))) - sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*
I*sqrt(2))) - sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))
*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*
(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*s
qrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32
*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)
*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*
(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*s
qrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32
*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(
-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/
8192*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
 - sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 16384*sqrt(2
)*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - sqrt(-12288*(1/256*I
*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sq
rt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-
I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqr
t(-1/8192*I*sqrt(2))) - sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))
- sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))*sqrt(sqrt(-
12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sq
rt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*
I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192
*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) - 1/16*sqrt(2)*sqrt(-sqrt(
-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*s
qrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192
*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/819
2*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((16384*sqrt(2)*(-1/256*I*sqr
t(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(
2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - sqrt(2))*
(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 16384*sqrt(2)*(-1/256*I*
sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1
/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I
*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) +
128*sqrt(1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*
```

```
sqrt(2))) - sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - sqrt(2)*(I
*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))*sqrt(-sqrt(-12288*(1/25
6*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2
*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2))
 + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)*sqrt(-sqrt(-12288*(1/2
56*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/
2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))
)*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)
) + 32*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2
*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + 163
84*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - sqrt(2))*(1/256*I*s
qrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 16384*sqrt(2)*(-1/256*I*sqrt(2) -
1/2*sqrt(-1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/
8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))
^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1
/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
- sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - sqrt(2)*(I*sqrt(2) +
 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))*sqrt(-sqrt(-12288*(1/256*I*sqrt(2
) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8
192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(
2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt
(-1/8192*I*sqrt(2))) + 2*x) - sqrt(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt
(2)))*log(8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^3 - 3
2768*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*s
qrt(1/8192*I*sqrt(2))) + 32768*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)
))^2*(-I*sqrt(2) - 128*sqrt(-1/8192*I*sqrt(2)) + 1) - 2*I*sqrt(2) - 256*sqr
t(-1/8192*I*sqrt(2)) + 3)*sqrt(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2))
) + x) + sqrt(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))*log(-8*(8388608
*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^3 - 32768*(-1/256*I*sqrt(
2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)
)) + 32768*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) - 1
28*sqrt(-1/8192*I*sqrt(2)) + 1) - 2*I*sqrt(2) - 256*sqrt(-1/8192*I*sqrt(2))
 + 3)*sqrt(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2))) + x) + sqrt(-1/256
*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))*log(8*(8388608*(-1/256*I*sqrt(2)
- 1/2*sqrt(-1/8192*I*sqrt(2)))^3 - 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/81
92*I*sqrt(2)))^2 - 2*I*sqrt(2) - 256*sqrt(-1/8192*I*sqrt(2)) + 5)*sqrt(-1/2
56*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))) + x) - sqrt(-1/256*I*sqrt(2) -
1/2*sqrt(-1/8192*I*sqrt(2)))*log(-8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(-
1/8192*I*sqrt(2)))^3 - 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)
))^2 - 2*I*sqrt(2) - 256*sqrt(-1/8192*I*sqrt(2)) + 5)*sqrt(-1/256*I*sqrt(2)
 - 1/2*sqrt(-1/8192*I*sqrt(2))) + x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.21

$$\int \frac{x^2}{2+(1+x^2)^4}\,dx$$
$$= \text{RootSum}\left(1073741824t^8+65536t^4+1024t^2+3,\left(t\mapsto t\log\left(67108864t^7-262144t^5+4096t^3+40t+x\right)\right)\right)$$

```
[In] integrate(x**2/(2+(x**2+1)**4),x)
[Out] RootSum(1073741824*_t**8 + 65536*_t**4 + 1024*_t**2 + 3, Lambda(_t, _t*log(
67108864*_t**7 - 262144*_t**5 + 4096*_t**3 + 40*_t + x)))
```

Maxima [F]

$$\int \frac{x^2}{2+(1+x^2)^4}\,dx = \int \frac{x^2}{(x^2+1)^4+2}\,dx$$

```
[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="maxima")
[Out] integrate(x^2/((x^2 + 1)^4 + 2), x)
```

Giac [F]

$$\int \frac{x^2}{2+(1+x^2)^4}\,dx = \int \frac{x^2}{(x^2+1)^4+2}\,dx$$

```
[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="giac")
[Out] integrate(x^2/((x^2 + 1)^4 + 2), x)
```

Mupad [B] (verification not implemented)

Time = 10.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{2+(1+x^2)^4}\,dx = \sum_{k=1}^{8}\ln\left(\text{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(40\,x\right.\right.$$
$$+\text{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(\text{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\right.\left(\right.$$
$$\left.-3\right)\text{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)$$

```
[In] int(x^2/((x^2 + 1)^4 + 2),x)
```

```
[Out] symsum(log(root(z^8 + z^4/16384 + z^2/1048576 + 3/1073741824, z, k)*(40*x +
 root(z^8 + z^4/16384 + z^2/1048576 + 3/1073741824, z, k)*(root(z^8 + z^4/1
6384 + z^2/1048576 + 3/1073741824, z, k)*(4096*x - root(z^8 + z^4/16384 + z
^2/1048576 + 3/1073741824, z, k)^2*(786432*x - 67108864*root(z^8 + z^4/1638
4 + z^2/1048576 + 3/1073741824, z, k)^2*x)) - 768)) - 3)*root(z^8 + z^4/163
84 + z^2/1048576 + 3/1073741824, z, k), k, 1, 8)
```

3.390 $\int \frac{x^2}{2+(1-x^2)^4}\,dx$

Optimal result 2319
Rubi [A] (verified) 2320
Mathematica [C] (verified) 2322
Maple [C] (verified) 2322
Fricas [B] (verification not implemented) 2322
Sympy [A] (verification not implemented) 2325
Maxima [F] 2325
Giac [F] 2325
Mupad [B] (verification not implemented) 2325

Optimal result

Integrand size = 17, antiderivative size = 188

$$\int \frac{x^2}{2+(1-x^2)^4}\,dx = -\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\,\text{arctanh}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}} + \frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\,\text{arctanh}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}} + \frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\,\text{arctanh}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}} - \frac{1}{8}i\left(\sqrt[4]{-2}+\sqrt{2}\right)\sqrt{\frac{1+i}{(1+i)+2^{3/4}}}\,\text{arctanh}\left(\sqrt{\frac{1+i}{(1+i)+2^{3/4}}}\,x\right)$$

```
[Out] -1/8*(-1)^(1/4)*arctanh(x/(1-(-2)^(1/4))^(1/2))*(1-(-2)^(1/4))^(1/2)*2^(1/4
)+1/8*(-1)^(3/4)*2^(1/4)*arctanh(x/(1+I*(-2)^(1/4))^(1/2))*(1+I*(-2)^(1/4))
^(1/2)+1/8*(-1)^(1/4)*arctanh(x/(1+(-2)^(1/4))^(1/2))*(1+(-2)^(1/4))^(1/2)*
2^(1/4)-1/8*I*arctanh(x*((1+I)/(1+I+2^(3/4)))^(1/2))*((-2)^(1/4)+2^(1/2))*(
(1+I)/(1+I+2^(3/4)))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6872, 212, 213, 1997, 214}

$$\int \frac{x^2}{2+(1-x^2)^4}\,dx = -\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\,\text{arctanh}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}} + \frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\,\text{arctanh}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}} + \frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\,\text{arctanh}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}} - \frac{1}{8}i\left(\sqrt[4]{-2}+\sqrt{2}\right)\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}\,\text{arctanh}\left(\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}x\right)$$

```
[In] Int[x^2/(2 + (1 - x^2)^4),x]
```

```
[Out] -1/4*((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTanh[x/Sqrt[1 - (-2)^(1/4)]])/2^(3
/4) + ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTanh[x/Sqrt[1 + I*(-2)^(1/4)]])
/(4*2^(3/4)) + ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4)]*ArcTanh[x/Sqrt[1 + (-2)^(1/
4)]])/(4*2^(3/4)) - (I/8)*((-2)^(1/4) + Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^
(3/4))]*ArcTanh[Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 6872

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sqrt[4]{-2}+i\sqrt{2}}{8\left(1+\sqrt[4]{-2}-x^2\right)}+\frac{\sqrt[4]{-2}-i\sqrt{2}}{8\left(-1+\sqrt[4]{-2}+x^2\right)}+\frac{\sqrt[4]{-2}-\sqrt{2}}{8\left(\sqrt[4]{-2}-i\left(1-x^2\right)\right)}\right.\\
&\qquad\left.+\frac{\sqrt[4]{-2}+\sqrt{2}}{8\left(\sqrt[4]{-2}+i\left(1-x^2\right)\right)}\right)dx\\
&= \frac{1}{8}\left(\sqrt[4]{-2}-\sqrt{2}\right)\int \frac{1}{\sqrt[4]{-2}-i\left(1-x^2\right)}\,dx+\frac{1}{8}\left(\sqrt[4]{-2}-i\sqrt{2}\right)\int \frac{1}{-1+\sqrt[4]{-2}+x^2}\,dx\\
&\quad+\frac{1}{8}\left(\sqrt[4]{-2}+i\sqrt{2}\right)\int \frac{1}{1+\sqrt[4]{-2}-x^2}\,dx+\frac{1}{8}\left(\sqrt[4]{-2}+\sqrt{2}\right)\int \frac{1}{\sqrt[4]{-2}+i\left(1-x^2\right)}\,dx\\
&= -\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}\\
&\quad+\frac{1}{8}\left(\sqrt[4]{-2}-\sqrt{2}\right)\int \frac{1}{-i+\sqrt[4]{-2}+ix^2}\,dx+\frac{1}{8}\left(\sqrt[4]{-2}+\sqrt{2}\right)\int \frac{1}{i+\sqrt[4]{-2}-ix^2}\,dx\\
&= -\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}\\
&\quad+\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}\\
&\quad+\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}\\
&\quad-\frac{1}{8}i\left(\sqrt[4]{-2}+\sqrt{2}\right)\sqrt{\frac{1+i}{(1+i)+2^{3/4}}}\tanh^{-1}\left(\sqrt{\frac{1+i}{(1+i)+2^{3/4}}}x\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.32

$$\int \frac{x^2}{2+(1-x^2)^4} dx = \frac{1}{8}\text{RootSum}\left[3-4\#1^2+6\#1^4-4\#1^6 + \#1^8\&, \frac{\log(x-\#1)\#1}{-1+3\#1^2-3\#1^4+\#1^6}\&\right]$$

```
[In] Integrate[x^2/(2 + (1 - x^2)^4),x]
[Out] RootSum[3 - 4*#1^2 + 6*#1^4 - 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(-1 + 3*#1
^2 - 3*#1^4 + #1^6) & ]/8
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.30

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+3)}\frac{\_R^2\ln(x-\_R)}{\_R^7-3\_R^5+3\_R^3-\_R}\right)}{8}$	56
risch	$\frac{\left(\sum_{\_R=\text{RootOf}(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+3)}\frac{\_R^2\ln(x-\_R)}{\_R^7-3\_R^5+3\_R^3-\_R}\right)}{8}$	56

```
[In] int(x^2/(2+(-x^2+1)^4),x,method=_RETURNVERBOSE)
[Out] 1/8*sum(_R^2/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8-4*_Z^6+6*_Z^4-
4*_Z^2+3))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2259 vs. 2(118) = 236.

Time = 0.99 (sec) , antiderivative size = 2259, normalized size of antiderivative = 12.02

$$\int \frac{x^2}{2+(1-x^2)^4} dx = \text{Too large to display}$$

```
[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2
)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sq
rt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2
))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((163
84*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) +
128*sqrt(-1/8192*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*
I*sqrt(2))) + sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 +
16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - sqrt(-122
88*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt
(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sq
rt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2
) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2))*(-I*sqrt(2) + 128*sqrt(-1/8192*I
*sqrt(2))) + sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))) - sqrt(2))*s
qrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(
-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqr
t(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*s
qrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)*s
qrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(
-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqr
t(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*s
qrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(-1
/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(-1/81
92*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) +
sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + 16384*sqrt(2)*
(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - sqrt(-12288*(1/256*I*sq
rt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt
(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sq
rt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(1
/8192*I*sqrt(2))) + sqrt(2))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + s
qrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))) - sqrt(2))*sqrt(sqrt(-1228
8*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(
2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqr
t(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*s
qrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) - 1/16*sqrt(2)*sqrt(-sqrt(-122
88*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt
(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sq
rt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*
sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((16384*sqrt(2)*(-1/256*I*sqrt(2)
 - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
 + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2))*(1/25
6*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + 16384*sqrt(2)*(-1/256*I*sqrt
(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sq
rt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt
(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sq
rt(-1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2
```

))) + sqrt(2))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + sqrt(2)*(I*sqrt
(2) + 128*sqrt(1/8192*I*sqrt(2)))) - sqrt(2))*sqrt(-sqrt(-12288*(1/256*I*sq
rt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt
(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sq
rt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*
sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)*sqrt(-sqrt(-12288*(1/256*I*s
qrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqr
t(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*s
qrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32
*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(
1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + 16384*(sq
rt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2))*(1/256*I*sqrt(2)
- 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + 16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sq
rt(1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I
sqrt(2)))^2 - 12288(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/
8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I
sqrt(2))) - 1)((sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2
))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + sqrt(2)*(I*sqrt(2) + 128*sq
rt(1/8192*I*sqrt(2)))) - sqrt(2))*sqrt(-sqrt(-12288*(1/256*I*sqrt(2) - 1/2*
sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sq
rt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*
sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192
*I*sqrt(2))) + 2*x) - sqrt(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))*l
og(8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^3 + 32768*(1/
256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) - 128*sqrt(1/819
2*I*sqrt(2)) - 1) - 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2
*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 2*I*sqrt(2) - 256*sqrt(1/8192
*I*sqrt(2)) - 3)*sqrt(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))) + x) +
sqrt(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))*log(-8*(8388608*(-1/25
6*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^3 + 32768*(1/256*I*sqrt(2) - 1/2*
sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) - 128*sqrt(1/8192*I*sqrt(2)) - 1) -
32768*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*s
qrt(-1/8192*I*sqrt(2))) - 2*I*sqrt(2) - 256*sqrt(1/8192*I*sqrt(2)) - 3)*sqr
t(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))) + x) + sqrt(-1/256*I*sqrt(
2) - 1/2*sqrt(1/8192*I*sqrt(2)))*log(8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqr
t(1/8192*I*sqrt(2)))^3 + 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2
)))^2 - 2*I*sqrt(2) - 256*sqrt(1/8192*I*sqrt(2)) - 5)*sqrt(-1/256*I*sqrt(2)
- 1/2*sqrt(1/8192*I*sqrt(2))) + x) - sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(1/81
92*I*sqrt(2)))*log(-8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2
)))^3 + 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 2*I*sqrt(
2) - 256*sqrt(1/8192*I*sqrt(2)) - 5)*sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(1/819
2*I*sqrt(2))) + x)

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.21

$$\int \frac{x^2}{2+(1-x^2)^4}\,dx$$
$$= \mathrm{RootSum}\left(1073741824t^8+65536t^4-1024t^2+3,\left(t\mapsto t\log\left(67108864t^7+262144t^5+4096t^3-40t+x\right)\right.\right.$$

```
[In] integrate(x**2/(2+(-x**2+1)**4),x)
[Out] RootSum(1073741824*_t**8 + 65536*_t**4 - 1024*_t**2 + 3, Lambda(_t, _t*log(
67108864*_t**7 + 262144*_t**5 + 4096*_t**3 - 40*_t + x)))
```

Maxima [F]

$$\int \frac{x^2}{2+(1-x^2)^4}\,dx = \int \frac{x^2}{(x^2-1)^4+2}\,dx$$

```
[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="maxima")
[Out] integrate(x^2/((x^2 - 1)^4 + 2), x)
```

Giac [F]

$$\int \frac{x^2}{2+(1-x^2)^4}\,dx = \int \frac{x^2}{(x^2-1)^4+2}\,dx$$

```
[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="giac")
[Out] integrate(x^2/((x^2 - 1)^4 + 2), x)
```

Mupad [B] (verification not implemented)

Time = 10.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{2+(1-x^2)^4}\,dx = \sum_{k=1}^{8}\ln\left(\mathrm{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(40\,x\right.\right.$$
$$-\mathrm{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(\mathrm{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\right.\Big(\ldots$$
$$\left.-3\right)\mathrm{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)$$

```
[In] int(x^2/((x^2 - 1)^4 + 2),x)
```

```
[Out] symsum(log(root(z^8 + z^4/16384 - z^2/1048576 + 3/1073741824, z, k)*(40*x -
 root(z^8 + z^4/16384 - z^2/1048576 + 3/1073741824, z, k)*(root(z^8 + z^4/1
6384 - z^2/1048576 + 3/1073741824, z, k)*(4096*x + root(z^8 + z^4/16384 - z
^2/1048576 + 3/1073741824, z, k)^2*(786432*x + 67108864*root(z^8 + z^4/1638
4 - z^2/1048576 + 3/1073741824, z, k)^2*x)) - 768)) - 3)*root(z^8 + z^4/163
84 - z^2/1048576 + 3/1073741824, z, k), k, 1, 8)
```

3.391 $\int \frac{1-x^2}{a+b(1-x^2)^4}\,dx$

Optimal result . . . 2327
Rubi [A] (verified) . . . 2328
Mathematica [C] (verified) . . . 2333
Maple [C] (verified) . . . 2333
Fricas [C] (verification not implemented) . . . 2334
Sympy [A] (verification not implemented) . . . 2334
Maxima [F] . . . 2334
Giac [F] . . . 2335
Mupad [B] (verification not implemented) . . . 2335

Optimal result

Integrand size = 23, antiderivative size = 663

$$\begin{aligned}
\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx =& -\frac{\arctan\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}} \\
&+ \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}b^{3/8}} \\
&+ \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt[8]{b}x+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}} \\
&- \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt[8]{b}x+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}}
\end{aligned}$$

```
[Out] -1/4*arctan(b^(1/8)*x/((-a)^(1/4)-b^(1/4))^(1/2))/b^(3/8)/(-a)^(1/2)/((-a)^
(1/4)-b^(1/4))^(1/2)+1/4*arctanh(b^(1/8)*x/((-a)^(1/4)+b^(1/4))^(1/2))/b^(3
/8)/(-a)^(1/2)/((-a)^(1/4)+b^(1/4))^(1/2)-1/8*arctan((-b^(1/8)*x*2^(1/2)+(b
^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))/(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1
/2))^(1/2))*(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a
```

```
)^(1/2)/((-a)^(1/2)+b^(1/2))^(1/2)+1/8*arctan((b^(1/8)*x*2^(1/2)+(b^(1/4)+(
(-a)^(1/2)+b^(1/2))^(1/2))^(1/2))/(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/
2))*(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/
((-a)^(1/2)+b^(1/2))^(1/2)+1/16*ln(b^(1/4)*x^2+((-a)^(1/2)+b^(1/2))^(1/2)-b
^(1/8)*x*2^(1/2)*(b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))*(b^(1/4)+((-a)
^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/((-a)^(1/2)+b^(1/2)
)^(1/2)-1/16*ln(b^(1/4)*x^2+((-a)^(1/2)+b^(1/2))^(1/2)+b^(1/8)*x*2^(1/2)*(b
^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))*(b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/
2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/((-a)^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6872, 2015, 1180, 211, 214, 1183, 648, 632, 210, 642}

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}$$
$$+ \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}} + \frac{\text{arctanh}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$
$$+ \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(-\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}$$
$$- \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}$$

```
[In] Int[(1 - x^2)/(a + b*(1 - x^2)^4),x]
```

```
[Out] -1/4*ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(Sqrt[-a]*Sqrt[(-a)^(1/
4) - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(
Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqr
t[-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*
b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt
```

```
[-a] + Sqrt[b]] + b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b
]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + Arc
Tanh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4) +
b^(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt
[-a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*
x + b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) - (
Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] + Sqr
t[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*
Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 2015

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rule 6872

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{\sqrt{b}(1-x^2)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b} - b\left(1-x^2\right)^2\right)} - \frac{\sqrt{b}(1-x^2)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b} + b\left(1-x^2\right)^2\right)} \right) dx$$

$$= -\frac{\sqrt{b}\int \frac{1-x^2}{\sqrt{-a}\sqrt{b}-b(1-x^2)^2}\,dx}{2\sqrt{-a}} - \frac{\sqrt{b}\int \frac{1-x^2}{\sqrt{-a}\sqrt{b}+b(1-x^2)^2}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\sqrt{b}\int \frac{1-x^2}{\left(\sqrt{-a}-\sqrt{b}\right)\sqrt{b}+2bx^2-bx^4}\,dx}{2\sqrt{-a}} - \frac{\sqrt{b}\int \frac{1-x^2}{\left(\sqrt{-a}+\sqrt{b}\right)\sqrt{b}-2bx^2+bx^4}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}-\left(1+\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}x}{\sqrt[8]{b}}+x^2}dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt{\sqrt{-a}+\sqrt{b}}\sqrt[8]{b}}$$

$$-\frac{\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}+\left(1+\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}+\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}x}{\sqrt[8]{b}}+x^2}dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt{\sqrt{-a}+\sqrt{b}}\sqrt[8]{b}}$$

$$+\frac{\sqrt{b}\int \frac{1}{-\sqrt[4]{-a}b^{3/4}+b-bx^2}dx}{4\sqrt{-a}}+\frac{\sqrt{b}\int \frac{1}{\sqrt[4]{-a}b^{3/4}+b-bx^2}dx}{4\sqrt{-a}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}b^{3/8}}$$

$$+\frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right)\int \frac{1}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}x}{\sqrt[8]{b}}+x^2}dx}{8\sqrt{-a}\sqrt{b}}$$

$$+\frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right)\int \frac{1}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}+\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}x}{\sqrt[8]{b}}+x^2}dx}{8\sqrt{-a}\sqrt{b}}$$

$$+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\int \frac{-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}+2x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}x}{\sqrt[8]{b}}+x^2}dx}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}}$$

$$-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}+2x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}+\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}x}{\sqrt[8]{b}}+x^2}dx}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}b^{3/8}}$$
$$+ \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt[8]{b}x+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}}$$
$$- \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt[8]{b}x+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}}$$
$$- \frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right)\text{Subst}\left(\int\frac{1}{2\left(1-\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}\right)-x^2}dx, x, -\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}+2x\right)}{4\sqrt{-a}\sqrt{b}}$$
$$- \frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right)\text{Subst}\left(\int\frac{1}{2\left(1-\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}\right)-x^2}dx, x, \frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}+2x\right)}{4\sqrt{-a}\sqrt{b}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}}$$
$$+ \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}b^{3/8}}$$
$$+ \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt[8]{b}x+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}}$$
$$- \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt[8]{b}x+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.10

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4} \, dx = -\frac{\text{RootSum}\left[a+b-4b\#1^2+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{\log(x-\#1)}{\#1-2\#1^3+\#1^5}\&\right]}{8b}$$

```
[In] Integrate[(1 - x^2)/(a + b*(1 - x^2)^4),x]
```

```
[Out] -1/8*RootSum[a + b - 4*b*#1^2 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , Log[x - #1
]/(#1 - 2*#1^3 + #1^5) & ]/b
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^8-4b\_Z^6+6\_Z^4b-4\_Z^2b+a+b\right)}\frac{\left(-\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}}{8b}$	69
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^8-4b\_Z^6+6\_Z^4b-4\_Z^2b+a+b\right)}\frac{\left(-\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}}{8b}$	69

```
[In] int((-x^2+1)/(a+b*(-x^2+1)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*sum((-_R^2+1)/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8*b-4*_Z^
6*b+6*_Z^4*b-4*_Z^2*b+a+b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 322185, normalized size of antiderivative = 485.95

$$\int \frac{1-x^2}{a+b(1-x^2)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.20

$$\int \frac{1-x^2}{a+b(1-x^2)^4}\,dx = -\operatorname{RootSum}\left(t^8\cdot\left(16777216a^5b^3+16777216a^4b^4\right)+1048576t^6a^3b^3+24576t^4a^2b^2+256t^2ab+1,\left(t\mapsto t\log\right.\right.$$

```
[In] integrate((-x**2+1)/(a+b*(-x**2+1)**4),x)
```

```
[Out] -RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 1048576*_t**6*a*
*3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, Lambda(_t, _t*log(-629
1456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**4 + 65536*_t**5*a**3*b**2 - 32
7680*_t**5*a**2*b**3 - 512*_t**3*a**2*b - 5632*_t**3*a*b**2 - 32*_t*b + x))
)
```

Maxima [F]

$$\int \frac{1-x^2}{a+b(1-x^2)^4}\,dx = \int -\frac{x^2-1}{(x^2-1)^4b+a}\,dx$$

```
[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

Giac [F]

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx=\int -\frac{x^2-1}{\left(x^2-1\right)^4b+a}\,dx$$

```
[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="giac")
[Out] integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 9.83 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.49

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx$$
$$=\sum_{k=1}^{8}\ln\Big(a\,b^5\,\Big(\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+$$
$$+1\Big)\,\Big(\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k$$
$$+\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\big)^2\,a\,b$$
$$-\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\big)^5\,a^3$$
$$+1\Big)\Big)\,\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4$$
$$+256\,a\,b\,z^2+1,z,k\big)$$

```
[In] int(-(x^2 - 1)/(a + b*(x^2 - 1)^4),x)
[Out] symsum(log(a*b^5*(64*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 104
8576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b + 1)*(4
096*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6
+ 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^4*a^2*b^2 + 128*root(16777216*
a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^
4 + 256*a*b*z^2 + 1, z, k)^2*a*b - 32768*root(16777216*a^5*b^3*z^8 + 167772
16*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1,
 z, k)^5*a^3*b^2*x + 1))*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 +
 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k), k, 1, 8)
```

3.392 $\int \frac{1-x^2}{a+b(-1+x^2)^4} dx$

Optimal result 2336
Rubi [A] (verified) 2337
Mathematica [C] (verified) 2342
Maple [C] (verified) 2342
Fricas [C] (verification not implemented) 2343
Sympy [A] (verification not implemented) 2343
Maxima [F] 2343
Giac [F] 2344
Mupad [B] (verification not implemented) 2344

Optimal result

Integrand size = 21, antiderivative size = 663

$$\begin{aligned}
&\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4} dx \\
&= -\frac{\arctan\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}} \\
&+ \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}b^{3/8}} \\
&+ \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt[8]{b}x+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}} \\
&- \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt[8]{b}x+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}}
\end{aligned}$$

```
[Out] -1/4*arctan(b^(1/8)*x/((-a)^(1/4)-b^(1/4))^(1/2))/b^(3/8)/(-a)^(1/2)/((-a)^
(1/4)-b^(1/4))^(1/2)+1/4*arctanh(b^(1/8)*x/((-a)^(1/4)+b^(1/4))^(1/2))/b^(3
/8)/(-a)^(1/2)/((-a)^(1/4)+b^(1/4))^(1/2)-1/8*arctan((-b^(1/8)*x*2^(1/2)+(b
^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))/(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1
/2))^(1/2))*(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a
```

```
)^(1/2)/((-a)^(1/2)+b^(1/2))^(1/2)+1/8*arctan((b^(1/8)*x*2^(1/2)+(b^(1/4)+(
(-a)^(1/2)+b^(1/2))^(1/2))^(1/2))/(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/
2))*(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/
((-a)^(1/2)+b^(1/2))^(1/2)+1/16*ln(b^(1/4)*x^2+((-a)^(1/2)+b^(1/2))^(1/2)-b
^(1/8)*x*2^(1/2)*(b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))*(b^(1/4)+((-a)
^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/((-a)^(1/2)+b^(1/2)
)^(1/2)-1/16*ln(b^(1/4)*x^2+((-a)^(1/2)+b^(1/2))^(1/2)+b^(1/8)*x*2^(1/2)*(b
^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))*(b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/
2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/((-a)^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6872, 2015, 1180, 211, 214, 1183, 648, 632, 210, 642}

$$
\begin{aligned}
&\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx \\
&= -\frac{\arctan\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}} \\
&+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}} \\
&+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(-\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}} \\
&-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}
\end{aligned}
$$

```
[In] Int[(1 - x^2)/(a + b*(-1 + x^2)^4),x]
```

```
[Out] -1/4*ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(Sqrt[-a]*Sqrt[(-a)^(1/
4) - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(
Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqr
t[-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*
b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt
```

```
[-a] + Sqrt[b]] + b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b
]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + Arc
Tanh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4) +
b^(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt
[-a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*
x + b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) - (
Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] + Sqr
t[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*
Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 2015

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rule 6872

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{-1+x^2}{a+b\left(-1+x^2\right)^4}\,dx \\
&= -\int \left(-\frac{\sqrt{b}(-1+x^2)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}-b\left(-1+x^2\right)^2\right)}-\frac{\sqrt{b}(-1+x^2)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}+b\left(-1+x^2\right)^2\right)}\right)dx \\
&= \frac{\sqrt{b}\int \frac{-1+x^2}{\sqrt{-a}\sqrt{b}-b(-1+x^2)^2}\,dx}{2\sqrt{-a}}+\frac{\sqrt{b}\int \frac{-1+x^2}{\sqrt{-a}\sqrt{b}+b(-1+x^2)^2}\,dx}{2\sqrt{-a}} \\
&= \frac{\sqrt{b}\int \frac{-1+x^2}{\left(\sqrt{-a}-\sqrt{b}\right)\sqrt{b}+2bx^2-bx^4}\,dx}{2\sqrt{-a}}+\frac{\sqrt{b}\int \frac{-1+x^2}{\left(\sqrt{-a}+\sqrt{b}\right)\sqrt{b}-2bx^2+bx^4}\,dx}{2\sqrt{-a}}
\end{aligned}
$$

$$=\frac{\int \frac{-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}-\left(-1-\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}x}{\sqrt[8]{b}}+x^2}\,dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt{\sqrt{-a}+\sqrt{b}}\sqrt[8]{b}}$$

$$+\frac{\int \frac{-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}+\left(-1-\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}+\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}x}{\sqrt[8]{b}}+x^2}\,dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt{\sqrt{-a}+\sqrt{b}}\sqrt[8]{b}}$$

$$+\frac{\sqrt{b}\int \frac{1}{-\sqrt[4]{-a}b^{3/4}+b-bx^2}\,dx}{4\sqrt{-a}}+\frac{\sqrt{b}\int \frac{1}{\sqrt[4]{-a}b^{3/4}+b-bx^2}\,dx}{4\sqrt{-a}}$$

$$=-\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}b^{3/8}}$$

$$+\frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right)\int \frac{1}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}x}{\sqrt[8]{b}}+x^2}\,dx}{8\sqrt{-a}\sqrt{b}}$$

$$+\frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right)\int \frac{1}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}+\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}x}{\sqrt[8]{b}}+x^2}\,dx}{8\sqrt{-a}\sqrt{b}}$$

$$+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\int \frac{-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}+2x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}x}{\sqrt[8]{b}}+x^2}\,dx}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}}$$

$$-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}+2x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}+\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}x}{\sqrt[8]{b}}+x^2}\,dx}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}b^{3/8}}$$

$$+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt[8]{b}x+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}}$$

$$-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt[8]{b}x+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}}$$

$$-\frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right)\text{Subst}\left(\int\frac{1}{2\left(1-\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}\right)-x^2}dx,x,-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}+2x\right)}{4\sqrt{-a}\sqrt{b}}$$

$$-\frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right)\text{Subst}\left(\int\frac{1}{2\left(1-\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}\right)-x^2}dx,x,\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}+2x\right)}{4\sqrt{-a}\sqrt{b}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}}$$

$$+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}b^{3/8}}$$

$$+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt[8]{b}x+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}}$$

$$-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt[8]{b}x+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.10

$$\int \frac{1-x^2}{a+b(-1+x^2)^4} \, dx$$
$$= -\frac{\text{RootSum}\left[a+b-4b\#1^2+6b\#1^4-4b\#1^6+b\#1^8\&, \frac{\log(x-\#1)}{\#1-2\#1^3+\#1^5}\&\right]}{8b}$$

```
[In] Integrate[(1 - x^2)/(a + b*(-1 + x^2)^4),x]
```

```
[Out] -1/8*RootSum[a + b - 4*b*#1^2 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , Log[x - #1
]/(#1 - 2*#1^3 + #1^5) & ]/b
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^8-4b\_Z^6+6\_Z^4b-4\_Z^2b+a+b\right)} \frac{\left(-\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}}{8b}$	69
risch	$\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^8-4b\_Z^6+6\_Z^4b-4\_Z^2b+a+b\right)} \frac{\left(-\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}}{8b}$	69

```
[In] int((-x^2+1)/(a+b*(x^2-1)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*sum((-_R^2+1)/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8*b-4*_Z^
6*b+6*_Z^4*b-4*_Z^2*b+a+b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 322185, normalized size of antiderivative = 485.95

$$\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.20

$$\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx = -\operatorname{RootSum}\left(t^8\cdot\left(16777216a^5b^3+16777216a^4b^4\right)+1048576t^6a^3b^3+24576t^4a^2b^2+256t^2ab+1,\left(t\mapsto t\,\mathrm{l}\right.\right.$$

```
[In] integrate((-x**2+1)/(a+b*(x**2-1)**4),x)
```

```
[Out] -RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 1048576*_t**6*a*
*3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, Lambda(_t, _t*log(-629
1456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**4 + 65536*_t**5*a**3*b**2 - 32
7680*_t**5*a**2*b**3 - 512*_t**3*a**2*b - 5632*_t**3*a*b**2 - 32*_t*b + x))
)
```

Maxima [F]

$$\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx = \int -\frac{x^2-1}{\left(x^2-1\right)^4b+a}\,dx$$

```
[In] integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

Giac [F]

$$\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx=\int -\frac{x^2-1}{(x^2-1)^4b+a}\,dx$$

```
[In] integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.49

$$\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx$$

$$=\sum_{k=1}^{8} \ln\Big(a\,b^5\,\Big(\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1$$

$$+1\Big)\,\Big(\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\big)$$

$$+\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\big)^2\,a\,b$$

$$-\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\big)^5\,a^3$$

$$+1\Big)\Big)\,\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4$$

$$+256\,a\,b\,z^2+1,z,k\big)$$

```
[In] int(-(x^2 - 1)/(a + b*(x^2 - 1)^4),x)
```

```
[Out] symsum(log(a*b^5*(64*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 104
8576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b + 1)*(4
096*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6
+ 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^4*a^2*b^2 + 128*root(16777216*
a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^
4 + 256*a*b*z^2 + 1, z, k)^2*a*b - 32768*root(16777216*a^5*b^3*z^8 + 167772
16*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1,
 z, k)^5*a^3*b^2*x + 1))*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 +
 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k), k, 1, 8)
```

3.393 $\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx$

Optimal result 2345
Rubi [F] 2345
Mathematica [C] (verified) 2346
Maple [C] (verified) 2346
Fricas [C] (verification not implemented) 2347
Sympy [A] (verification not implemented) 2347
Maxima [F] 2348
Giac [F] 2348
Mupad [B] (verification not implemented) 2348

Optimal result

Integrand size = 25, antiderivative size = 168

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\,x}{\sqrt[6]{b}}\right)}{3\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\,b^{5/6}} + \frac{\arctan\left(\frac{\sqrt{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}}\,x}{\sqrt[6]{b}}\right)}{3\sqrt{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}}\,b^{5/6}} + \frac{\arctan\left(\frac{\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}\,x}{\sqrt[6]{b}}\right)}{3\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}\,b^{5/6}}$$

```
[Out] 1/3*arctan(x*(a^(1/3)+b^(1/3))^(1/2)/b^(1/6))/b^(5/6)/(a^(1/3)+b^(1/3))^(1/
2)+1/3*arctan(x*(-(-1)^(1/3)*a^(1/3)+b^(1/3))^(1/2)/b^(1/6))/b^(5/6)/(-(-1)
^(1/3)*a^(1/3)+b^(1/3))^(1/2)+1/3*arctan(x*((-1)^(2/3)*a^(1/3)+b^(1/3))^(1/
2)/b^(1/6))/b^(5/6)/((-1)^(2/3)*a^(1/3)+b^(1/3))^(1/2)
```

Rubi [F]

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx$$

```
[In] Int[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]
```

```
[Out] Defer[Int][(a*x^6 + b*(1 + x^2)^3)^(-1), x] + 2*Defer[Int][x^2/(a*x^6 + b*(
1 + x^2)^3), x] + Defer[Int][x^4/(a*x^6 + b*(1 + x^2)^3), x]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6}+\frac{2x^2}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6}+\frac{x^4}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6}\right)dx$$

$$=2\int \frac{x^2}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6}\,dx+\int \frac{1}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6}\,dx+\int \frac{x^4}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6}\,dx$$

$$=2\int \frac{x^2}{ax^6+b\left(1+x^2\right)^3}\,dx+\int \frac{1}{ax^6+b\left(1+x^2\right)^3}\,dx+\int \frac{x^4}{ax^6+b\left(1+x^2\right)^3}\,dx$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.57

$$\int \frac{\left(1+x^2\right)^2}{ax^6+b\left(1+x^2\right)^3}\,dx=\frac{1}{6}\text{RootSum}\left[b+3b\#1^2+3b\#1^4+a\#1^6+b\#1^6\&,\frac{\log(x-\#1)+2\log(x-\#1)\#1^2+\log(x-\#1)\#1^4}{b\#1+2b\#1^3+a\#1^5+b\#1^5}\&\right]$$

```
[In] Integrate[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]
```

```
[Out] RootSum[b + 3*b*#1^2 + 3*b*#1^4 + a*#1^6 + b*#1^6 & , (Log[x - #1] + 2*Log[
x - #1]*#1^2 + Log[x - #1]*#1^4)/(b*#1 + 2*b*#1^3 + a*#1^5 + b*#1^5) & ]/6
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.82 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^6+3\_Z^4b+3\_Z^2b+b\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^5a+\_R^5b+2\_R^3b+b\_R}\right)}{6}$	67
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^6+3\_Z^4b+3\_Z^2b+b\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^5a+\_R^5b+2\_R^3b+b\_R}\right)}{6}$	67

```
[In] int((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sum((_R^4+2*_R^2+1)/(_R^5*a+_R^5*b+2*_R^3*b+_R*b)*ln(x-_R),_R=RootOf((a
+b)*_Z^6+3*_Z^4*b+3*_Z^2*b+b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 5653, normalized size of antiderivative = 33.65

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.25

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \operatorname{RootSum}\left(t^6\cdot\left(46656ab^5+46656b^6\right)+3888t^4b^4+108t^2b^2+1,\left(t\mapsto t\log\left(6tb+x\right)\right)\right)$$

```
[In] integrate((x**2+1)**2/(a*x**6+b*(x**2+1)**3),x)
```

```
[Out] RootSum(_t**6*(46656*a*b**5 + 46656*b**6) + 3888*_t**4*b**4 + 108*_t**2*b**
2 + 1, Lambda(_t, _t*log(6*_t*b + x)))
```

Maxima [F]

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \int \frac{(x^2+1)^2}{ax^6+(x^2+1)^3b}\,dx$$

```
[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="maxima")
[Out] integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b), x)
```

Giac [F]

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \int \frac{(x^2+1)^2}{ax^6+(x^2+1)^3b}\,dx$$

```
[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 10.48 (sec) , antiderivative size = 504, normalized size of antiderivative = 3.00

$$\begin{aligned}
&\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx \\
&= \sum_{k=1}^{6} \ln\Big(-a^3\,(a \\
&+ b)\,\Big(-\mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^2\,b^2\,60 \\
&- \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^4\,b^4\,864 \\
&- \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^4\,a\,b^3\,864 \\
&+ \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^3\,b^3\,x\,504 \\
&+ \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^5\,b^5\,x\,7776 \\
&+ \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)\,a\,x\,2 \\
&+ \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)\,b\,x\,8 \\
&+ \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^2\,a\,b\,12 \\
&- \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^3\,a\,b^2\,x\,144 \\
&+ \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^5\,a\,b^4\,x\,7776 \\
&- 1\Big)\,3\Big)\,\mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)
\end{aligned}$$

```
[In] int((x^2 + 1)^2/(b*(x^2 + 1)^3 + a*x^6),x)
```

```
[Out] symsum(log(-3*a^3*(a + b)*(504*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*
b^4*z^4 + 108*b^2*z^2 + 1, z, k)^3*b^3*x - 864*root(46656*a*b^5*z^6 + 46656
*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^4*b^4 - 864*root(46656*a*b
^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^4*a*b^3 - 60
*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z,
k)^2*b^2 + 7776*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b
^2*z^2 + 1, z, k)^5*b^5*x + 2*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b
^4*z^4 + 108*b^2*z^2 + 1, z, k)*a*x + 8*root(46656*a*b^5*z^6 + 46656*b^6*z^
6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)*b*x + 12*root(46656*a*b^5*z^6 + 4
6656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^2*a*b - 144*root(46656
*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^3*a*b^2*
x + 7776*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2
+ 1, z, k)^5*a*b^4*x - 1))*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*
z^4 + 108*b^2*z^2 + 1, z, k), k, 1, 6)
```

3.394 $\int \frac{(d+ex)^3}{a+cx^4}\,dx$

Optimal result 2350
Rubi [A] (verified) 2351
Mathematica [A] (verified) 2354
Maple [C] (verified) 2355
Fricas [C] (verification not implemented) 2355
Sympy [A] (verification not implemented) 2355
Maxima [A] (verification not implemented) 2356
Giac [A] (verification not implemented) 2357
Mupad [B] (verification not implemented) 2358

Optimal result

Integrand size = 17, antiderivative size = 320

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx = \frac{3d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} - \frac{d\left(\sqrt{c}d^2+3\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{d\left(\sqrt{c}d^2+3\sqrt{a}e^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} - \frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{e^3\log\left(a+cx^4\right)}{4c}$$

```
[Out] 1/4*e^3*ln(c*x^4+a)/c+3/2*d^2*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(1/2)/c^(1/2)
-1/8*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*e^2*a^(1/2)+d
^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/8*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1
/2)+x^2*c^(1/2))*(-3*e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*d
*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c
^(3/4)*2^(1/2)+1/4*d*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e^2*a^(1/2)+d^2
*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {1890, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 211, 266}

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx = -\frac{d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(3\sqrt{a}e^2+\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{a}e^2+\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}} - \frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{3d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} + \frac{e^3\log\left(a+cx^4\right)}{4c}$$

```
[In] Int[(d + e*x)^3/(a + c*x^4),x]
```

```
[Out] (3*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*(Sqrt[c]*d
^2 + 3*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*c^(3/4)) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^
2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/
4)*c^(3/4)) + (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + (e^3*Log[a + c*x^
4])/(4*c)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d^3+3de^2x^2}{a+cx^4}+\frac{x(3d^2e+e^3x^2)}{a+cx^4}\right)dx\\
&= \int \frac{d^3+3de^2x^2}{a+cx^4}\,dx+\int \frac{x(3d^2e+e^3x^2)}{a+cx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{3d^2e+e^3x}{a+cx^2}\,dx,x,x^2\right)+\frac{\left(d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}-3e^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c}\\
&\quad+\frac{\left(d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+3e^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c}\\
&= \frac{1}{2}(3d^2e)\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)+\frac{1}{2}e^3\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx,x,x^2\right)\\
&\quad+\frac{\left(d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+3e^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c}+\frac{\left(d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+3e^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c}\\
&\quad-\frac{\left(d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}c^{3/4}}\\
&\quad-\frac{\left(d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}c^{3/4}}
\end{aligned}
$$

$$= \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} - \frac{d\left(\sqrt{c}d^2 - 3\sqrt{a}e^2\right)\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{d\left(\sqrt{c}d^2 - 3\sqrt{a}e^2\right)\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{e^3\log\left(a + cx^4\right)}{4c} + \frac{\left(d\left(\sqrt{c}d^2 + 3\sqrt{a}e^2\right)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} - \frac{\left(d\left(\sqrt{c}d^2 + 3\sqrt{a}e^2\right)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}$$

$$= \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} - \frac{d\left(\sqrt{c}d^2 + 3\sqrt{a}e^2\right)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{d\left(\sqrt{c}d^2 + 3\sqrt{a}e^2\right)\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} - \frac{d\left(\sqrt{c}d^2 - 3\sqrt{a}e^2\right)\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{d\left(\sqrt{c}d^2 - 3\sqrt{a}e^2\right)\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{e^3\log\left(a + cx^4\right)}{4c}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx = \frac{-2\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{2}\sqrt{c}d^2 + 6\sqrt[4]{a}\sqrt[4]{c}de + 3\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 2\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{2}\sqrt{c}d^2 - 6\sqrt[4]{a}\sqrt[4]{c}de + 3\right.}{}$$

```
[In] Integrate[(d + e*x)^3/(a + c*x^4),x]
```

```
[Out] (-2*a^(1/4)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 6*a^(1/4)*c^(1/4)*d*e + 3*Sqrt
[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*c^(1/4
)*d*(Sqrt[2]*Sqrt[c]*d^2 - 6*a^(1/4)*c^(1/4)*d*e + 3*Sqrt[2]*Sqrt[a]*e^2)*A
rcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d
^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
] + Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 2*a*e^3*Log[a + c*x^4])/(8*a*c)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.77 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.17

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(\_R^3e^3+3\_R^2de^2+3\_Rd^2e+d^3\right)\ln\left(x-\_R\right)}{\_R^3}}{4c}$
default	$\frac{d^3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{3d^2e\arctan\left(x^2\sqrt{\frac{c}{a}}\right)}{2\sqrt{ac}}+\frac{3de^2\sqrt{2}\left(\ln\right.}{}$

[In] int((e*x+d)^3/(c*x^4+a),x,method=_RETURNVERBOSE)

[Out] 1/4/c*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.63 (sec) , antiderivative size = 141845, normalized size of antiderivative = 443.27

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx = \text{Too large to display}$$

[In] integrate((e*x+d)^3/(c*x^4+a),x, algorithm="fricas")

[Out] Too large to include

Sympy [A] (verification not implemented)

Time = 28.21 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx$$
$$=\mathrm{RootSum}\left(256t^4a^3c^4-256t^3a^3c^3e^3+t^2\cdot\left(96a^3c^2e^6+480a^2c^3d^4e^2\right)+t\left(-16a^3ce^9+192a^2c^2d^4e^5-48ac^3\right.\right.$$

[In] integrate((e*x+d)**3/(c*x**4+a),x)

[Out] RootSum(256*_t**4*a**3*c**4 - 256*_t**3*a**3*c**3*e**3 + _t**2*(96*a**3*c**2*e**6 + 480*a**2*c**3*d**4*e**2) + _t*(-16*a**3*c*e**9 + 192*a**2*c**2*d**4*e**5 - 48*a*c**3*d**8*e) + a**3*e**12 + 3*a**2*c*d**4*e**8 + 3*a*c**2*d**

```
8*e**4 + c**3*d**12, Lambda(_t, _t*log(x + (1728*_t**3*a**4*c**3*e**6 + 960
*_t**3*a**3*c**4*d**4*e**2 - 1296*_t**2*a**4*c**2*e**9 - 2016*_t**2*a**3*c*
*3*d**4*e**5 + 48*_t**2*a**2*c**4*d**8*e + 324*_t*a**4*c*e**12 + 4716*_t*a*
*3*c**2*d**4*e**8 + 1452*_t*a**2*c**3*d**8*e**4 + 4*_t*a*c**4*d**12 - 27*a*
*4*e**15 + 1119*a**3*c*d**4*e**11 - 609*a**2*c**2*d**8*e**7 - 91*a*c**3*d**
12*e**3)/(729*a**3*c*d**3*e**12 - 1053*a**2*c**2*d**7*e**8 - 117*a*c**3*d**
11*e**4 + c**4*d**15))))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx = \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^3+cd^3-3\,\sqrt{a}\sqrt{c}de^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^3-cd^3+3\,\sqrt{a}\sqrt{c}de^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{5}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{5}{4}}d^3+3\,\sqrt{2}a^{\frac{3}{4}}c^{\frac{3}{4}}de^2-6\,\sqrt{a}cd^2e\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{5}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{5}{4}}d^3+3\,\sqrt{2}a^{\frac{3}{4}}c^{\frac{3}{4}}de^2+6\,\sqrt{a}cd^2e\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{5}{4}}}$$

```
[In] integrate((e*x+d)^3/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*e^3 + c*d^3 - 3*sqrt(a)*sqrt(c)*d*e^2)
*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) +
 1/8*sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*e^3 - c*d^3 + 3*sqrt(a)*sqrt(c)*d*e^2
)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4))
+ 1/4*(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + 3*sqrt(2)*a^(3/4)*c^(3/4)*d*e^2 - 6*sq
rt(a)*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/s
qrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) + 1/4*(sqrt(2
)*a^(1/4)*c^(5/4)*d^3 + 3*sqrt(2)*a^(3/4)*c^(3/4)*d*e^2 + 6*sqrt(a)*c*d^2*e
)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*s
qrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx = \frac{e^3\log\left(|cx^4+a|\right)}{4\,c} + \frac{\sqrt{2}\Big(3\,\sqrt{2}\sqrt{ac}c^2d^2e+(ac^3)^{\frac{1}{4}}\,c^2d^3+3\,(ac^3)^{\frac{3}{4}}\,de^2\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} + \frac{\sqrt{2}\Big(3\,\sqrt{2}\sqrt{ac}c^2d^2e+(ac^3)^{\frac{1}{4}}\,c^2d^3+3\,(ac^3)^{\frac{3}{4}}\,de^2\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} + \frac{\sqrt{2}\Big((ac^3)^{\frac{1}{4}}\,c^2d^3-3\,(ac^3)^{\frac{3}{4}}\,de^2\Big)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3} - \frac{\sqrt{2}\Big((ac^3)^{\frac{1}{4}}\,c^2d^3-3\,(ac^3)^{\frac{3}{4}}\,de^2\Big)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$

```
[In] integrate((e*x+d)^3/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*e^3*log(abs(c*x^4 + a))/c + 1/4*sqrt(2)*(3*sqrt(2)*sqrt(a*c)*c^2*d^2*e
+ (a*c^3)^(1/4)*c^2*d^3 + 3*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x +
sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*(3*sqrt(2)*sqrt(a*c
)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + 3*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3
)^(1/4)*c^2*d^3 - 3*(a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) +
sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - 3*(a*c^3)^(3/4)*d
*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)
```

Mupad [B] (verification not implemented)

Time = 9.67 (sec) , antiderivative size = 894, normalized size of antiderivative = 2.79

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx=\sum_{k=1}^{4}\ln\Big(-c\,d^2\,\Big(-3\,c\,d^5\,e^2+5\,a\,d\,e^6+3\,a\,e^7\,x$$

$$+\mathrm{root}\big(256\,a^3\,c^4\,z^4-256\,a^3\,c^3\,e^3\,z^3+480\,a^2\,c^3\,d^4\,e^2\,z^2+96\,a^3\,c^2\,e^6\,z^2+192\,a^2\,c^2\,d^4\,e^5\,z-48\,a\,c^3\,d^8\,e\,z-1$$

$$+\mathrm{root}\big(256\,a^3\,c^4\,z^4-256\,a^3\,c^3\,e^3\,z^3+480\,a^2\,c^3\,d^4\,e^2\,z^2+96\,a^3\,c^2\,e^6\,z^2+192\,a^2\,c^2\,d^4\,e^5\,z-48\,a\,c^3\,d^8\,e\,z-16\,a^3\,c$$

$$-5\,c\,d^4\,e^3\,x$$

$$-\mathrm{root}\big(256\,a^3\,c^4\,z^4-256\,a^3\,c^3\,e^3\,z^3+480\,a^2\,c^3\,d^4\,e^2\,z^2+96\,a^3\,c^2\,e^6\,z^2+192\,a^2\,c^2\,d^4\,e^5\,z-48\,a\,c^3\,d^8\,e\,z-1$$

$$+\mathrm{root}\big(256\,a^3\,c^4\,z^4-256\,a^3\,c^3\,e^3\,z^3+480\,a^2\,c^3\,d^4\,e^2\,z^2+96\,a^3\,c^2\,e^6\,z^2+192\,a^2\,c^2\,d^4\,e^5\,z-48\,a\,c^3\,d^8\,e\,z-16\,a^3\,c$$

$$-\mathrm{root}\big(256\,a^3\,c^4\,z^4-256\,a^3\,c^3\,e^3\,z^3+480\,a^2\,c^3\,d^4\,e^2\,z^2+96\,a^3\,c^2\,e^6\,z^2+192\,a^2\,c^2\,d^4\,e^5\,z-48\,a\,c^3\,d^8\,e\,z-16\,a^3\,c$$

$$-256\,a^3\,c^3\,e^3\,z^3+480\,a^2\,c^3\,d^4\,e^2\,z^2+96\,a^3\,c^2\,e^6\,z^2+192\,a^2\,c^2\,d^4\,e^5\,z-48\,a\,c^3\,d^8\,e\,z$$

$$-16\,a^3\,c\,e^9\,z+3\,a^2\,c\,d^4\,e^8+3\,a\,c^2\,d^8\,e^4+c^3\,d^{12}+a^3\,e^{12},z,k\big)$$

```
[In] int((d + e*x)^3/(a + c*x^4),x)
```

```
[Out] symsum(log(-2*c*d^2*(5*a*d*e^6 - 3*c*d^5*e^2 + 3*a*e^7*x + 8*root(256*a^3*c
^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2
 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*
e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)^2*a*c^2*d + 2*root(256*a
^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6
*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*
d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)*c^2*d^4*x - 5*c*d^4*
e^3*x - 24*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2
*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a
^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)
^2*a*c^2*e*x + 32*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*
d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z
 - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12
, z, k)*a*c*d*e^3 - 6*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*
c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8
*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*
e^12, z, k)*a*c*e^4*x))*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^
2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d
^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^
3*e^12, z, k), k, 1, 4)
```

3.395 $\int \frac{(d+ex)^2}{a+cx^4} \, dx$

Optimal result . . . 2359
Rubi [A] (verified) . . . 2360
Mathematica [A] (verified) . . . 2363
Maple [C] (verified) . . . 2363
Fricas [C] (verification not implemented) . . . 2364
Sympy [A] (verification not implemented) . . . 2364
Maxima [A] (verification not implemented) . . . 2364
Giac [A] (verification not implemented) . . . 2365
Mupad [B] (verification not implemented) . . . 2366

Optimal result

Integrand size = 17, antiderivative size = 291

$$\begin{aligned}\int \frac{(d+ex)^2}{a+cx^4} \, dx = & \frac{de \arctan \left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} - \frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right) \arctan \left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} \\ & + \frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right) \arctan \left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} \\ & - \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right) \log \left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} \\ & + \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right) \log \left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}\end{aligned}$$

```
[Out] d*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(1/2)/c^(1/2)-1/8*ln(-a^(1/4)*c^(1/4)*x*2
^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1
/2)+1/8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2
*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*
(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^
(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}} - \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{de\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

```
[In] Int[(d + e*x)^2/(a + c*x^4),x]
```

```
[Out] (d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - ((Sqrt[c]*d^2 + Sqr
t[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/
4)) + ((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sq
rt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \int \left(\frac{2dex}{a+cx^4} + \frac{d^2+e^2x^2}{a+cx^4}\right) dx$$

$$= (2de)\int \frac{x}{a+cx^4}\,dx + \int \frac{d^2+e^2x^2}{a+cx^4}\,dx$$

$$= (de)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right) + \frac{\left(\frac{\sqrt{c}d^2}{\sqrt{a}} - e^2\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c} + \frac{\left(\frac{\sqrt{c}d^2}{\sqrt{a}} + e^2\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c}$$

$$= \frac{de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} + \frac{\left(\frac{\sqrt{c}d^2}{\sqrt{a}} + e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2}\,dx}{4c} + \frac{\left(\frac{\sqrt{c}d^2}{\sqrt{a}} + e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2}\,dx}{4c}$$
$$- \frac{\left(\sqrt{c}d^2 - \sqrt{a}e^2\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2}\,dx}{4\sqrt{2}a^{3/4}c^{3/4}} - \frac{\left(\sqrt{c}d^2 - \sqrt{a}e^2\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} - 2x}{-\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2}\,dx}{4\sqrt{2}a^{3/4}c^{3/4}}$$

$$= \frac{de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} - \frac{\left(\sqrt{c}d^2 - \sqrt{a}e^2\right)\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}$$
$$+ \frac{\left(\sqrt{c}d^2 - \sqrt{a}e^2\right)\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}$$
$$+ \frac{\left(\sqrt{c}d^2 + \sqrt{a}e^2\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}$$
$$- \frac{\left(\sqrt{c}d^2 + \sqrt{a}e^2\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}$$

$$= \frac{de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} - \frac{\left(\sqrt{c}d^2 + \sqrt{a}e^2\right)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}$$
$$+ \frac{\left(\sqrt{c}d^2 + \sqrt{a}e^2\right)\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}$$
$$- \frac{\left(\sqrt{c}d^2 - \sqrt{a}e^2\right)\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}$$
$$+ \frac{\left(\sqrt{c}d^2 - \sqrt{a}e^2\right)\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx = \frac{-2\left(\sqrt{2}\sqrt{c}d^2+4\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt{c}d^2-4\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2\right)\text{arctan}}{8a^{3/}}$$

[In] Integrate[(d + e*x)^2/(a + c*x^4),x]

[Out] (-2*(Sqrt[2]*Sqrt[c]*d^2 + 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*Arc
Tan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*Sqrt[c]*d^2 - 4*a^(1/4)*c
^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] -
Sqrt[2]*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
)/(8*a^(3/4)*c^(3/4))

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.81 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.15

method	result
risch	$\frac{\sum_{\_R=\text{RootOf}\left(c\_Z^4+a\right)}\frac{\left(\_R^2e^2+2\_Rde+d^2\right)\ln\left(x-\_R\right)}{\_R^3}}{4c}$
default	$\frac{d^2\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{ed\arctan\left(x^2\sqrt{\frac{c}{a}}\right)}{\sqrt{ac}}+\frac{e^2\sqrt{2}\left(\ln\left(\frac{x^2-}{x^2-}\right.\right.}{}$

[In] int((e*x+d)^2/(c*x^4+a),x,method=_RETURNVERBOSE)

[Out] 1/4/c*sum((_R^2*e^2+2*_R*d*e+d^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 86139, normalized size of antiderivative = 296.01

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^2/(c*x^4+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx$$
$$= \text{RootSum}\left(256t^4a^3c^3 + 192t^2a^2c^2d^2e^2 + t\left(32a^2cde^5 - 32ac^2d^5e\right) + a^2e^8 + 2acd^4e^4 + c^2d^8, \left(t \mapsto t\log\left(x + \right.\right.\right.$$

```
[In] integrate((e*x+d)**2/(c*x**4+a),x)
[Out] RootSum(256*_t**4*a**3*c**3 + 192*_t**2*a**2*c**2*d**2*e**2 + _t*(32*a**2*c
*d*e**5 - 32*a*c**2*d**5*e) + a**2*e**8 + 2*a*c*d**4*e**4 + c**2*d**8, Lamb
da(_t, _t*log(x + (64*_t**3*a**4*c**2*e**6 + 448*_t**3*a**3*c**3*d**4*e**2
- 160*_t**2*a**3*c**2*d**3*e**5 + 32*_t**2*a**2*c**3*d**7*e + 60*_t*a**3*c*
d**2*e**8 + 256*_t*a**2*c**2*d**6*e**4 + 4*_t*a*c**3*d**10 + 6*a**3*d*e**11
 - 24*a**2*c*d**5*e**7 - 30*a*c**2*d**9*e**3)/(a**3*e**12 - 33*a**2*c*d**4*
e**8 - 33*a*c**2*d**8*e**4 + c**3*d**12))))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx = \frac{\sqrt{2}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2-4\sqrt{a}\sqrt{c}de\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{3}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2+4\sqrt{a}\sqrt{c}de\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{3}{4}}}$$

```
[In] integrate((e*x+d)^2/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c
^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - 1/8*sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*
e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4
)) + 1/4*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 4*sqr
t(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4)
)/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 1/4*(sqr
t(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 4*sqrt(a)*sqrt(c)*
d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a
)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx = \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}c^2de+(ac^3)^{\frac{1}{4}}c^2d^2+(ac^3)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}c^2de+(ac^3)^{\frac{1}{4}}c^2d^2+(ac^3)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d^2-(ac^3)^{\frac{3}{4}}e^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3} - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d^2-(ac^3)^{\frac{3}{4}}e^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$

```
[In] integrate((e*x+d)^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d*e + (a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^
(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c
^3) + 1/4*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d*e + (a*c^3)^(1/4)*c^2*d^2 + (a
*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4)
)/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2
 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*
c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a*c^3)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.91

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx = \sum_{k=1}^{4} \ln\Big(3\,c^2\,d^4\,e^2 - a\,c\,e^6 + 4\,c^2\,d^3\,e^3\,x - \mathrm{root}\big(256\,a^3\,c^3\,z^4 + 192\,a^2\,c^2\,d^2\,e^2\,z^2 + 32\,a^2\,c\,d\,e^5\,z - 32\,a\,c^2\,d^5\,e\,z + 2\,a\,c\,d^4\,e^4 + c^2\,d^8 + a^2\,e^8, z, k\big)\;c^3\,d^4\,x\,4 - \mathrm{root}\big(256\,a^3\,c^3\,z^4 + 192\,a^2\,c^2\,d^2\,e^2\,z^2 + 32\,a^2\,c\,d\,e^5\,z - 32\,a\,c^2\,d^5\,e\,z + 2\,a\,c\,d^4\,e^4 + c^2\,d^8 + a^2\,e^8, z, k\big)^2\,a\,c^3\,d \cdots + \mathrm{root}\big(256\,a^3\,c^3\,z^4 + 192\,a^2\,c^2\,d^2\,e^2\,z^2 + 32\,a^2\,c\,d\,e^5\,z - 32\,a\,c^2\,d^5\,e\,z + 2\,a\,c\,d^4\,e^4 + c^2\,d^8 + a^2\,e^8, z, k\big)\;a\,c^2\,e^4\,x\,4 - \mathrm{root}\big(256\,a^3\,c^3\,z^4 + 192\,a^2\,c^2\,d^2\,e^2\,z^2 + 32\,a^2\,c\,d\,e^5\,z - 32\,a\,c^2\,d^5\,e\,z + 2\,a\,c\,d^4\,e^4 + c^2\,d^8 + a^2\,e^8, z, k\big)\;a\,c^2\,d\,e^3\,16 + \mathrm{root}\big(256\,a^3\,c^3\,z^4 + 192\,a^2\,c^2\,d^2\,e^2\,z^2 + 32\,a^2\,c\,d\,e^5\,z - 32\,a\,c^2\,d^5\,e\,z + 2\,a\,c\,d^4\,e^4 + c^2\,d^8 + a^2\,e^8, z, k\big)^2\,a\,c^3\,d \cdots + 192\,a^2\,c^2\,d^2\,e^2\,z^2 + 32\,a^2\,c\,d\,e^5\,z - 32\,a\,c^2\,d^5\,e\,z + 2\,a\,c\,d^4\,e^4 + c^2\,d^8 + a^2\,e^8, z, k\big)$$

```
[In] int((d + e*x)^2/(a + c*x^4),x)
```

```
[Out] symsum(log(3*c^2*d^4*e^2 - a*c*e^6 + 4*c^2*d^3*e^3*x - 4*root(256*a^3*c^3*z
^4 + 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*
d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*c^3*d^4*x - 16*root(256*a^3*c^3*z^4 + 19
2*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4
 + c^2*d^8 + a^2*e^8, z, k)^2*a*c^3*d^2 + 4*root(256*a^3*c^3*z^4 + 192*a^2*
c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2
*d^8 + a^2*e^8, z, k)*a*c^2*e^4*x - 16*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d
^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8
+ a^2*e^8, z, k)*a*c^2*d*e^3 + 32*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^
2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2
*e^8, z, k)^2*a*c^3*d*e*x)*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^2*z^2 +
 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2*e^8, z
, k), k, 1, 4)
```

3.396 $\int \frac{d+ex}{a+cx^4} \, dx$

Optimal result 2367
Rubi [A] (verified) 2367
Mathematica [A] (verified) 2370
Maple [C] (verified) 2370
Fricas [C] (verification not implemented) 2371
Sympy [A] (verification not implemented) 2371
Maxima [A] (verification not implemented) 2371
Giac [A] (verification not implemented) 2372
Mupad [B] (verification not implemented) 2373

Optimal result

Integrand size = 15, antiderivative size = 219

$$\int \frac{d+ex}{a+cx^4} \, dx = \frac{e \arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} - \frac{d \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d \arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{d \log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d \log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

```
[Out] 1/4*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)+1/4*d*ar
ctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)-1/8*d*ln(-a^(1/4)
*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)+1/8*d*ln(a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)+1/2*e*
arctan(x^2*c^(1/2)/a^(1/2))/a^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1890, 217, 1179, 642, 1176, 631, 210, 281, 211}

$$\int \frac{d+ex}{a+cx^4} \, dx = -\frac{d \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d \arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{d \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{e \arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}$$

[In] Int[(d + e*x)/(a + c*x^4),x]

[Out] (e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - (d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4))

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 217

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 281

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 631

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d}{a+cx^4}+\frac{ex}{a+cx^4}\right) dx \\
&= d\int \frac{1}{a+cx^4}\,dx + e\int \frac{x}{a+cx^4}\,dx \\
&= \frac{d\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}} + \frac{d\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}} + \frac{1}{2}e\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right) \\
&= \frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} + \frac{d\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} + \frac{d\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} \\
&\quad - \frac{d\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{d\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} \\
&= \frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} - \frac{d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} \\
&\quad + \frac{d\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{d\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}
\end{aligned}$$

$$= \frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} - \frac{d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.84

$$\int \frac{d+ex}{a+cx^4}\,dx$$

$$= \frac{-2\left(\sqrt{2}\sqrt[4]{c}d+2\sqrt[4]{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt[4]{c}d-2\sqrt[4]{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+\sqrt{2}\sqrt[4]{c}d(-\log(}{8a^{3/4}\sqrt{c}}$$

```
[In] Integrate[(d + e*x)/(a + c*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*c^(1/4)*d + 2*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4
)] + 2*(Sqrt[2]*c^(1/4)*d - 2*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(
1/4)] + Sqrt[2]*c^(1/4)*d*(-Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(8*a^(3/
4)*Sqrt[c])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.15

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(\_Re+d\right)\ln\left(x-\_R\right)}{\_R^3}}{4c}$	32
default	$\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{e\arctan\left(x^2\sqrt{\frac{c}{a}}\right)}{2\sqrt{ac}}$	124

```
[In] int((e*x+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/c*sum((_R*e+d)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 41851, normalized size of antiderivative = 191.10

$$\int \frac{d+ex}{a+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.57

$$\int \frac{d+ex}{a+cx^4}\,dx = \operatorname{RootSum}\left(256t^4a^3c^2+32t^2a^2ce^2-16tacd^2e+ae^4+cd^4, \left(t\mapsto t\log\left(x+\frac{-128t^3a^3ce^2-16t^2a^2cd^2e-\cdots}{4ade^4-\cdots}\right.\right.\right.$$

```
[In] integrate((e*x+d)/(c*x**4+a),x)
[Out] RootSum(256*_t**4*a**3*c**2 + 32*_t**2*a**2*c*e**2 - 16*_t*a*c*d**2*e + a*e
**4 + c*d**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*c*e**2 - 16*_t**2*a**2
*c*d**2*e - 8*_t*a**2*e**4 - 4*_t*a*c*d**4 + 5*a*d**2*e**3)/(4*a*d*e**4 - c
*d**5))))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.95

$$\int \frac{d+ex}{a+cx^4}\,dx = \frac{\sqrt{2}d\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}d\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d-2\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{1}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d+2\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{1}{4}}}$$

```
[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*d*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/
4)*c^(1/4)) - 1/8*sqrt(2)*d*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + s
qrt(a))/(a^(3/4)*c^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(1/4)*d - 2*sqrt(a)*e)*a
rctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt
(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(1/4
)*d + 2*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4
))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.97

$$\int \frac{d+ex}{a+cx^4}\,dx = \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\,d\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac} - \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\,d\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ac}ce-(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^2} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ac}ce-(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^2}$$

```
[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a
*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(a*c) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*c)*c*e - (a*c^3)^(1/4)*c*d)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2) - 1/4*sqrt(2)*
(sqrt(2)*sqrt(a*c)*c*e - (a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2)
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.73

$$\int \frac{d+ex}{a+cx^4}\,dx$$
$$= \begin{cases} -\frac{2\,d+3\,e\,x}{6\,c\,x^3} & \text{if } \\ \frac{\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/4}\,x}{a^{1/4}}-1\right)\,\left(2\,a^{1/4}\,e+\sqrt{2}\,c^{1/4}\,d\right)}{4\,a^{3/4}\,\sqrt{c}}-\frac{\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/4}\,x}{a^{1/4}}+1\right)\,\left(4\,a^{1/4}\,e-2\,\sqrt{2}\,c^{1/4}\,d\right)}{8\,a^{3/4}\,\sqrt{c}}+\frac{\sqrt{2}\,d\,\ln\left(\frac{\sqrt{a}+\sqrt{c}\,x^2+\sqrt{2}\,a^{1/4}\,c^{1/4}\,x}{\sqrt{a}+\sqrt{c}\,x^2-\sqrt{2}\,a^{1/4}\,c^{1/4}\,x}\right)}{8\,a^{3/4}\,c^{1/4}} & \text{if } \end{cases}$$

```
[In] int((d + e*x)/(a + c*x^4),x)
```

```
[Out] piecewise(a == 0, -(2*d + 3*e*x)/(6*c*x^3), a ~= 0, (atan((2^(1/2)*c^(1/4)*
x)/a^(1/4) - 1)*(2*a^(1/4)*e + 2^(1/2)*c^(1/4)*d))/(4*a^(3/4)*c^(1/2)) - (a
tan((2^(1/2)*c^(1/4)*x)/a^(1/4) + 1)*(4*a^(1/4)*e - 2*2^(1/2)*c^(1/4)*d))/(
8*a^(3/4)*c^(1/2)) + (2^(1/2)*d*log((a^(1/2) + c^(1/2)*x^2 + 2^(1/2)*a^(1/4
)*c^(1/4)*x)/(a^(1/2) + c^(1/2)*x^2 - 2^(1/2)*a^(1/4)*c^(1/4)*x)))/(8*a^(3/
4)*c^(1/4)))
```

3.397 $\int \frac{1}{a+cx^4}\,dx$

Optimal result 2374
Rubi [A] (verified) 2374
Mathematica [A] (verified) 2376
Maple [C] (verified) 2377
Fricas [C] (verification not implemented) 2377
Sympy [A] (verification not implemented) 2378
Maxima [A] (verification not implemented) 2378
Giac [A] (verification not implemented) 2378
Mupad [B] (verification not implemented) 2379

Optimal result

Integrand size = 9, antiderivative size = 185

$$\int \frac{1}{a+cx^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

```
[Out] 1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)+1/4*arctan
(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)-1/8*ln(-a^(1/4)*c^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)+1/8*ln(a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{a+cx^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

```
[In] Int[(a + c*x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(3/4)*c^(1/4)) + Ar
cTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - Log[Sqr
```

```
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4))
 + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4
)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}} + \frac{\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.72

$$\int \frac{1}{a+cx^4}\,dx = \frac{-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)+\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

```
[In] Integrate[(a + c*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4)
*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(
1/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.15

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}}{4c}$	27
default	$\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}$	102

```
[In] int(1/(c*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4/c*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.61

$$\int \frac{1}{a+cx^4}\,dx = \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)+\frac{1}{4}i\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(i\,a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right) - \frac{1}{4}i\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(-i\,a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right) - \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)$$

```
[In] integrate(1/(c*x^4+a),x, algorithm="fricas")
[Out] 1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/(a^3*c))^(1/4) + x) + 1/4*I*(-1/(a^3*c))^(
1/4)*log(I*a*(-1/(a^3*c))^(1/4) + x) - 1/4*I*(-1/(a^3*c))^(1/4)*log(-I*a*(-
1/(a^3*c))^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4) +
x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.11

$$\int \frac{1}{a+cx^4}\,dx = \operatorname{RootSum}\left(256t^4a^3c+1,\left(t\mapsto t\log\left(4ta+x\right)\right)\right)$$

```
[In] integrate(1/(c*x**4+a),x)
[Out] RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.91

$$\int \frac{1}{a+cx^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}}$$

```
[In] integrate(1/(c*x^4+a),x, algorithm="maxima")
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt
(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/4*sqrt(2)*arctan(1/2
*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sq
rt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/8*sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4
)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 1/8*sqrt(2)*log(sqrt(c)*x^2 - sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.97

$$\int \frac{1}{a+cx^4}\,dx = \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac} - \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}$$

```
[In] integrate(1/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + s
qrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2
 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.18

$$\int \frac{1}{a+cx^4}\,dx=-\frac{\operatorname{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)+\operatorname{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{2\,(-a)^{3/4}\,c^{1/4}}$$

```
[In] int(1/(a + c*x^4),x)
```

```
[Out] -(atan((c^(1/4)*x)/(-a)^(1/4)) + atanh((c^(1/4)*x)/(-a)^(1/4)))/(2*(-a)^(3/
4)*c^(1/4))
```

3.398 $\int \frac{1}{(d+ex)(a+cx^4)}\,dx$

Optimal result . . . 2380
Rubi [A] (verified) . . . 2381
Mathematica [A] (verified) . . . 2385
Maple [C] (verified) . . . 2385
Fricas [C] (verification not implemented) . . . 2386
Sympy [F(-1)] . . . 2386
Maxima [A] (verification not implemented) . . . 2386
Giac [A] (verification not implemented) . . . 2387
Mupad [B] (verification not implemented) . . . 2388

Optimal result

Integrand size = 17, antiderivative size = 416

$$\begin{aligned}\int \frac{1}{(d+ex)(a+cx^4)}\,dx = &-\frac{\sqrt{c}d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(cd^4+ae^4)}\\ &-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)}\\ &+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)}+\frac{e^3\log(d+ex)}{cd^4+ae^4}\\ &-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)}\\ &+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)}\\ &-\frac{e^3\log\left(a+cx^4\right)}{4\,(cd^4+ae^4)}\end{aligned}$$

```
[Out] e^3*ln(e*x+d)/(a*e^4+c*d^4)-1/4*e^3*ln(c*x^4+a)/(a*e^4+c*d^4)-1/2*d^2*e*arc
tan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)/a^(1/2)-1/8*c^(1/4)*d*ln(-a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(
3/4)/(a*e^4+c*d^4)*2^(1/2)+1/8*c^(1/4)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/
2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)*2^(1/2)+1/
4*c^(1/4)*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/
a^(3/4)/(a*e^4+c*d^4)*2^(1/2)+1/4*c^(1/4)*d*arctan(1+c^(1/4)*x*2^(1/2)/a^(1
/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {6857, 1890, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 211, 266}

$$\int \frac{1}{(d+ex)(a+cx^4)}\,dx = -\frac{\sqrt[4]{c}d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)}+\frac{\sqrt[4]{c}d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)}-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)}+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)}-\frac{\sqrt{c}d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^4+cd^4\right)}-\frac{e^3\log\left(a+cx^4\right)}{4\left(ae^4+cd^4\right)}+\frac{e^3\log(d+ex)}{ae^4+cd^4}$$

```
[In] Int[1/((d + e*x)*(a + c*x^4)),x]
```

```
[Out] -1/2*(Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)
) - (c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(Sqrt[c]*d^2 + Sq
rt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d
^4 + a*e^4)) + (e^3*Log[d + e*x])/(c*d^4 + a*e^4) - (c^(1/4)*d*(Sqrt[c]*d^2
 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*
Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(
c*d^4 + a*e^4)) - (e^3*Log[a + c*x^4])/(4*(c*d^4 + a*e^4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{e^4}{(cd^4+ae^4)(d+ex)}+\frac{c(d^3-d^2ex+de^2x^2-e^3x^3)}{(cd^4+ae^4)(a+cx^4)}\right)dx\\
&= \frac{e^3\log(d+ex)}{cd^4+ae^4}+\frac{c\int \frac{d^3-d^2ex+de^2x^2-e^3x^3}{a+cx^4}\,dx}{cd^4+ae^4}\\
&= \frac{e^3\log(d+ex)}{cd^4+ae^4}+\frac{c\int \left(\frac{d^3+de^2x^2}{a+cx^4}+\frac{x(-d^2e-e^3x^2)}{a+cx^4}\right)dx}{cd^4+ae^4}\\
&= \frac{e^3\log(d+ex)}{cd^4+ae^4}+\frac{c\int \frac{d^3+de^2x^2}{a+cx^4}\,dx}{cd^4+ae^4}+\frac{c\int \frac{x(-d^2e-e^3x^2)}{a+cx^4}\,dx}{cd^4+ae^4}\\
&= \frac{e^3\log(d+ex)}{cd^4+ae^4}+\frac{c\text{Subst}\left(\int \frac{-d^2e-e^3x}{a+cx^2}\,dx,x,x^2\right)}{2(cd^4+ae^4)}\\
&\quad+\frac{\left(d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}-e^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2(cd^4+ae^4)}+\frac{\left(d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2(cd^4+ae^4)}
\end{aligned}$$

$$= \frac{e^3\log(d+ex)}{cd^4+ae^4} - \frac{(cd^2e)\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2\,(cd^4+ae^4)} - \frac{(ce^3)\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2\,(cd^4+ae^4)}$$
$$+ \frac{\left(d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\,(cd^4+ae^4)} + \frac{\left(d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\,(cd^4+ae^4)}$$
$$- \frac{\left(\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)}$$
$$- \frac{\left(\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)}$$

$$= -\frac{\sqrt{c}d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(cd^4+ae^4)} + \frac{e^3\log(d+ex)}{cd^4+ae^4}$$
$$- \frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)}$$
$$+ \frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)} - \frac{e^3\log\left(a+cx^4\right)}{4\,(cd^4+ae^4)}$$
$$+ \frac{\left(\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)}$$
$$- \frac{\left(\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)}$$

$$= -\frac{\sqrt{c}d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(cd^4+ae^4)} - \frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)}$$
$$+ \frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)} + \frac{e^3\log(d+ex)}{cd^4+ae^4}$$
$$- \frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)}$$
$$+ \frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)} - \frac{e^3\log\left(a+cx^4\right)}{4\,(cd^4+ae^4)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d+ex)\,(a+cx^4)}\,dx$$
$$= \frac{-2\sqrt[4]{c}d\left(\sqrt{2}\sqrt{c}d^2-2\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{c}d\left(\sqrt{2}\sqrt{c}d^2+2\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e\right.}{}$$

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)),x]
```

```
[Out] (-2*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 - 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a
]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*c^(1/4)*d*(Sqrt[2]*Sqrt[
c]*d^2 + 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)] + 8*a^(3/4)*e^3*Log[d + e*x] - Sqrt[2]*c^(3/4)*d^3*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*Sqrt[a]*c^(1/4)
*d*e^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*c^(
3/4)*d^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*S
qrt[a]*c^(1/4)*d*e^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
 - 2*a^(3/4)*e^3*Log[a + c*x^4])/(8*a^(3/4)*(c*d^4 + a*e^4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.89 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.51

method	result
risch	$\frac{e^3\ln(ex+d)}{e^4a+d^4c}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a^4e^4+a^3c\,d^4\right)\_Z^4+4a^3e^3\_Z^3+6\_Z^2a^2e^2+4\_Zae\right)}\_R\ln\left(\left(\left(5e^6a^3-3a^2d^4e^2c\right)\_R^3+\left(15a^2e^5-\right.\right.\right.\right.}{}$
default	$\frac{c\left(\frac{d^3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{d^2e\arctan\left(x^2\sqrt{\frac{c}{a}}\right)}{2\sqrt{ac}}+\frac{d\,e^2\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{}}\right.\right.}{}\right.}{e^4a+d^4c}$

```
[In] int(1/(e*x+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*ln(e*x+d)/(a*e^4+c*d^4)+1/4*sum(_R*ln(((5*a^3*e^6-3*a^2*c*d^4*e^2)*_R^3
+(15*a^2*e^5-3*a*c*d^4*e)*_R^2+(15*a*e^4-c*d^4)*_R+5*e^3)*x+(6*a^3*d*e^5-2*
a^2*c*d^5*e)*_R^3+(13*a^2*d*e^4-a*c*d^5)*_R^2+8*a*d*e^3*_R+d*e^2),_R=RootOf
(1+(a^4*e^4+a^3*c*d^4)*_Z^4+4*a^3*e^3*_Z^3+6*_Z^2*a^2*e^2+4*_Z*a*e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 97.11 (sec) , antiderivative size = 352864, normalized size of antiderivative = 848.23

$$\int \frac{1}{(d+ex)\,(a+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\,(a+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(c*x**4+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d+ex)\,(a+cx^4)}\,dx = \frac{e^3\log(ex+d)}{cd^4+ae^4} - \frac{c\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^3-cd^3+\sqrt{a}\sqrt{c}de^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^3+cd^3-\sqrt{a}\sqrt{c}de^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}} - \frac{2(\sqrt{}}{}\right.}{8(c}$$

```
[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="maxima")
[Out] e^3*log(e*x + d)/(c*d^4 + a*e^4) - 1/8*c*(sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*
e^3 - c*d^3 + sqrt(a)*sqrt(c)*d*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1
/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*e^3 +
 c*d^3 - sqrt(a)*sqrt(c)*d*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x
 + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + sqrt(2)*a^
(3/4)*c^(3/4)*d*e^2 + 2*sqrt(a)*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x +
```

```
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(
c))*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + sqrt(2)*a^(3/4)*c^(3/4)*d*e
^2 - 2*sqrt(a)*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c
^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(c*
d^4 + a*e^4)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex)(a+cx^4)}\,dx = \frac{e^4\log(|ex+d|)}{cd^4e+ae^5} - \frac{e^3\log(|cx^4+a|)}{4(cd^4+ae^4)} + \frac{(ac^3)^{\frac{1}{4}}\,cd\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^2d^2+\sqrt{2}\sqrt{ac}ace^2-2\,(ac^3)^{\frac{1}{4}}\,acde\right)} + \frac{(ac^3)^{\frac{1}{4}}\,cd\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^2d^2+\sqrt{2}\sqrt{ac}ace^2+2\,(ac^3)^{\frac{1}{4}}\,acde\right)} + \frac{\left((ac^3)^{\frac{1}{4}}\,c^2d^3-(ac^3)^{\frac{3}{4}}\,de^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^4+\sqrt{2}a^2c^2e^4\right)} - \frac{\left((ac^3)^{\frac{1}{4}}\,c^2d^3-(ac^3)^{\frac{3}{4}}\,de^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^4+\sqrt{2}a^2c^2e^4\right)}$$

```
[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] e^4*log(abs(e*x + d))/(c*d^4*e + a*e^5) - 1/4*e^3*log(abs(c*x^4 + a))/(c*d^
4 + a*e^4) + 1/2*(a*c^3)^(1/4)*c*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^
(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^2*d^2 + sqrt(2)*sqrt(a*c)*a*c*e^2 - 2*(a*c
^3)^(1/4)*a*c*d*e) + 1/2*(a*c^3)^(1/4)*c*d*arctan(1/2*sqrt(2)*(2*x - sqrt(2
)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^2*d^2 + sqrt(2)*sqrt(a*c)*a*c*e^2
+ 2*(a*c^3)^(1/4)*a*c*d*e) + 1/4*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e
^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + sqrt(
2)*a^2*c^2*e^4) - 1/4*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2
 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + sqrt(2)*a^2*c^2*
e^4)
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 874, normalized size of antiderivative = 2.10

$$\int \frac{1}{(d+ex)\,(a+cx^4)}\,dx$$
$$= \left(\sum_{k=1}^{4} \ln\left(\mathrm{root}\left(256\,a^3\,c\,d^4\,z^4+256\,a^4\,e^4\,z^4+256\,a^3\,e^3\,z^3+96\,a^2\,e^2\,z^2+16\,a\,e\,z+1,z,k\right)\,c^4\,e\,\left(d\,e^2+5\,e^3\,x+\mathrm{roo}\right.\right.\right.$$
$$\left.\left. + 256\,a^4\,e^4\,z^4+256\,a^3\,e^3\,z^3+96\,a^2\,e^2\,z^2+16\,a\,e\,z+1,z,k\right)\right) + \frac{e^3\,\ln\left(d+e\,x\right)}{c\,d^4+a\,e^4}$$

```
[In] int(1/((a + c*x^4)*(d + e*x)),x)
```

```
[Out] symsum(log(root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*
a^2*e^2*z^2 + 16*a*e*z + 1, z, k)*c^4*e*(d*e^2 + 5*e^3*x + 240*root(256*a^3
*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z
+ 1, z, k)^2*a^2*e^5*x + 320*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256
*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^3*e^6*x + 32*root(2
56*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*
a*e*z + 1, z, k)*a*d*e^3 + 60*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 25
6*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)*a*e^4*x - 4*root(256*a
^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*
z + 1, z, k)*c*d^4*x - 16*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^
3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^2*a*c*d^5 + 208*root(256*a
^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*
z + 1, z, k)^2*a^2*d*e^4 + 384*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 2
56*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^3*d*e^5 - 128*roo
t(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 +
16*a*e*z + 1, z, k)^3*a^2*c*d^5*e - 192*root(256*a^3*c*d^4*z^4 + 256*a^4*e^
4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^2*c*d^4*
e^2*x - 48*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*
a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^2*a*c*d^4*e*x))*root(256*a^3*c*d^4*z^4 +
256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k), k
, 1, 4) + (e^3*log(d + e*x))/(a*e^4 + c*d^4)
```

3.399 $\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx$

Optimal result . . . 2389
Rubi [A] (verified) . . . 2390
Mathematica [A] (verified) . . . 2394
Maple [A] (verified) . . . 2395
Fricas [F(-1)] . . . 2396
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Maxima [A] (verification not implemented) . . . 2396
Giac [A] (verification not implemented) . . . 2397
Mupad [B] (verification not implemented) . . . 2398

Optimal result

Integrand size = 17, antiderivative size = 552

$$\begin{aligned}
\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx
&= -\frac{e^3}{(cd^4+ae^4)(d+ex)} - \frac{\sqrt{c}de(cd^4-ae^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\,(cd^4+ae^4)^2} \\
&\quad - \frac{\sqrt[4]{c}\left(\sqrt{c}d^2(cd^4-3ae^4)+\sqrt{a}e^2(3cd^4-ae^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^2} \\
&\quad + \frac{\sqrt[4]{c}\left(\sqrt{c}d^2(cd^4-3ae^4)+\sqrt{a}e^2(3cd^4-ae^4)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^2} \\
&\quad + \frac{4cd^3e^3\log(d+ex)}{(cd^4+ae^4)^2} \\
&\quad - \frac{\sqrt[4]{c}\left(\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)^2} \\
&\quad + \frac{\sqrt[4]{c}\left(\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)^2} \\
&\quad - \frac{cd^3e^3\log(a+cx^4)}{(cd^4+ae^4)^2}
\end{aligned}$$

```
[Out] -e^3/(a*e^4+c*d^4)/(e*x+d)+4*c*d^3*e^3*ln(e*x+d)/(a*e^4+c*d^4)^2-c*d^3*e^3*
ln(c*x^4+a)/(a*e^4+c*d^4)^2-d*e*(-a*e^4+c*d^4)*arctan(x^2*c^(1/2)/a^(1/2))*
c^(1/2)/(a*e^4+c*d^4)^2/a^(1/2)-1/8*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a
^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4)*c^(
1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/8*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1
```

```
/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4
)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/4*c^(1/4)*arctan(-1+c^(1/4)*x*
2^(1/2)/a^(1/4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4)*c^(1/2)
)/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/4*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^
(1/4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(3/4)/
(a*e^4+c*d^4)^2*2^(1/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {6857, 1890, 1262, 649, 211, 266, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx$$
$$= -\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e^2(3cd^4-ae^4)+\sqrt{c}d^2(cd^4-3ae^4)\right)}{2\sqrt{2}a^{3/4}(ae^4+cd^4)^2}$$
$$+\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e^2(3cd^4-ae^4)+\sqrt{c}d^2(cd^4-3ae^4)\right)}{2\sqrt{2}a^{3/4}(ae^4+cd^4)^2}$$
$$-\frac{\sqrt[4]{c}\left(\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^4+cd^4)^2}$$
$$+\frac{\sqrt[4]{c}\left(\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^4+cd^4)^2}$$
$$-\frac{\sqrt{c}de\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)(cd^4-ae^4)}{\sqrt{a}(ae^4+cd^4)^2}-\frac{e^3}{(d+ex)(ae^4+cd^4)}$$
$$-\frac{cd^3e^3\log(a+cx^4)}{(ae^4+cd^4)^2}+\frac{4cd^3e^3\log(d+ex)}{(ae^4+cd^4)^2}$$

```
[In] Int[1/((d + e*x)^2*(a + c*x^4)),x]
```

```
[Out] -(e^3/((c*d^4 + a*e^4)*(d + e*x))) - (Sqrt[c]*d*e*(c*d^4 - a*e^4)*ArcTan[(S
qrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^2) - (c^(1/4)*(Sqrt[c]*d^2*(
c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*(Sqrt[c]*d
^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (4*c*d^3*e^3*Lo
g[d + e*x])/(c*d^4 + a*e^4)^2 - (c^(1/4)*(Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - S
qrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*(Sqrt[c]*d^2*(c
*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1
```

```
/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c*d^
3*e^3*Log[a + c*x^4])/(c*d^4 + a*e^4)^2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^4}{(cd^4+ae^4)(d+ex)^2} + \frac{4cd^3e^4}{(cd^4+ae^4)^2(d+ex)} + \frac{c(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)x+e^2(3cd^4-ae^4)x^2-4cd^3e^3x^3)}{(cd^4+ae^4)^2(a+cx^4)}\right)dx$$

$$= -\frac{e^3}{(cd^4+ae^4)(d+ex)} + \frac{4cd^3e^3\log(d+ex)}{(cd^4+ae^4)^2} + \frac{c\int \frac{d^2(cd^4-3ae^4)-2de(cd^4-ae^4)x+e^2(3cd^4-ae^4)x^2-4cd^3e^3x^3}{a+cx^4}\,dx}{(cd^4+ae^4)^2}$$

$$= -\frac{e^3}{(cd^4+ae^4)(d+ex)} + \frac{4cd^3e^3\log(d+ex)}{(cd^4+ae^4)^2} + \frac{c\int \left(\frac{x(-2de(cd^4-ae^4)-4cd^3e^3x^2)}{a+cx^4} + \frac{d^2(cd^4-3ae^4)+e^2(3cd^4-ae^4)x^2}{a+cx^4}\right) dx}{(cd^4+ae^4)^2}$$

$$= -\frac{e^3}{(cd^4+ae^4)(d+ex)} + \frac{4cd^3e^3\log(d+ex)}{(cd^4+ae^4)^2} + \frac{c\int \frac{x(-2de(cd^4-ae^4)-4cd^3e^3x^2)}{a+cx^4} dx}{(cd^4+ae^4)^2} + \frac{c\int \frac{d^2(cd^4-3ae^4)+e^2(3cd^4-ae^4)x^2}{a+cx^4} dx}{(cd^4+ae^4)^2}$$

$$= -\frac{e^3}{(cd^4+ae^4)(d+ex)} + \frac{4cd^3e^3\log(d+ex)}{(cd^4+ae^4)^2} + \frac{c\text{Subst}\left(\int \frac{-2de(cd^4-ae^4)-4cd^3e^3x}{a+cx^2} dx, x, x^2\right)}{2(cd^4+ae^4)^2} - \frac{\left(3cd^4e^2-ae^6-\frac{\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4} dx}{2(cd^4+ae^4)^2} + \frac{\left(3cd^4e^2-ae^6+\frac{\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4} dx}{2(cd^4+ae^4)^2}$$

$$= -\frac{e^3}{(cd^4+ae^4)(d+ex)} + \frac{4cd^3e^3\log(d+ex)}{(cd^4+ae^4)^2} - \frac{(2c^2d^3e^3)\text{Subst}\left(\int \frac{x}{a+cx^2} dx, x, x^2\right)}{(cd^4+ae^4)^2} - \frac{(cde(cd^4-ae^4))\text{Subst}\left(\int \frac{1}{a+cx^2} dx, x, x^2\right)}{(cd^4+ae^4)^2} + \frac{\left(\sqrt[4]{c}\left(3cd^4e^2-ae^6-\frac{\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2} dx}{4\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^2} + \frac{\left(\sqrt[4]{c}\left(3cd^4e^2-ae^6-\frac{\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2} dx}{4\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^2} + \frac{\left(3cd^4e^2-ae^6+\frac{\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2} dx}{4(cd^4+ae^4)^2} + \frac{\left(3cd^4e^2-ae^6+\frac{\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2} dx}{4(cd^4+ae^4)^2}$$

$$= -\frac{e^3}{(cd^4+ae^4)(d+ex)} - \frac{\sqrt{c}de(cd^4-ae^4)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\,(cd^4+ae^4)^2} + \frac{4cd^3e^3\log(d+ex)}{(cd^4+ae^4)^2}$$
$$+ \frac{\sqrt[4]{c}\left(3cd^4e^2-ae^6-\frac{\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(cd^4+ae^4)^2}$$
$$- \frac{\sqrt[4]{c}\left(3cd^4e^2-ae^6-\frac{\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(cd^4+ae^4)^2}$$
$$- \frac{cd^3e^3\log(a+cx^4)}{(cd^4+ae^4)^2} + \frac{\left(\sqrt[4]{c}\left(3cd^4e^2-ae^6+\frac{\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(cd^4+ae^4)^2}$$
$$- \frac{\left(\sqrt[4]{c}\left(3cd^4e^2-ae^6+\frac{\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(cd^4+ae^4)^2}$$

$$= -\frac{e^3}{(cd^4+ae^4)(d+ex)} - \frac{\sqrt{c}de(cd^4-ae^4)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\,(cd^4+ae^4)^2}$$
$$- \frac{\sqrt[4]{c}\left(3cd^4e^2-ae^6+\frac{\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(cd^4+ae^4)^2}$$
$$+ \frac{\sqrt[4]{c}\left(3cd^4e^2-ae^6+\frac{\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(cd^4+ae^4)^2} + \frac{4cd^3e^3\log(d+ex)}{(cd^4+ae^4)^2}$$
$$+ \frac{\sqrt[4]{c}\left(3cd^4e^2-ae^6-\frac{\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(cd^4+ae^4)^2}$$
$$- \frac{\sqrt[4]{c}\left(3cd^4e^2-ae^6-\frac{\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(cd^4+ae^4)^2}$$
$$- \frac{cd^3e^3\log(a+cx^4)}{(cd^4+ae^4)^2}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 524, normalized size of antiderivative = 0.95

$$\int\frac{1}{(d+ex)^2(a+cx^4)}\,dx$$
$$= \frac{-\frac{8e^3(cd^4+ae^4)}{d+ex} + \frac{2\sqrt[4]{c}(-\sqrt{c}d^2+\sqrt{a}e^2)\left(\sqrt{2}cd^4-4\sqrt[4]{a}c^{3/4}d^3e+4\sqrt{2}\sqrt{a}\sqrt{c}d^2e^2-4a^{3/4}\sqrt[4]{c}de^3+\sqrt{2}ae^4\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{2\sqrt[4]{c}(\sqrt{}}{}$$

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^4)),x]
```

```
[Out] ((-8*e^3*(c*d^4 + a*e^4))/(d + e*x) + (2*c^(1/4)*(-(Sqrt[c]*d^2) + Sqrt[a]*
e^2)*(Sqrt[2]*c*d^4 - 4*a^(1/4)*c^(3/4)*d^3*e + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*d
^2*e^2 - 4*a^(3/4)*c^(1/4)*d*e^3 + Sqrt[2]*a*e^4)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/a^(3/4) + (2*c^(1/4)*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[2]*c
*d^4 + 4*a^(1/4)*c^(3/4)*d^3*e + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*d^2*e^2 + 4*a^(3
/4)*c^(1/4)*d*e^3 + Sqrt[2]*a*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/a^(3/4) + 32*c*d^3*e^3*Log[d + e*x] - (Sqrt[2]*c^(1/4)*(c^(3/2)*d^6 - 3*Sq
rt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4) + (Sqrt[2]*c^(1/4)*(c^(3/2)*d^6 -
 3*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4) - 8*c*d^3*e^3*Log[a + c*x^4]
)/(8*(c*d^4 + a*e^4)^2)
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.64

method	result
default	$-\frac{c\left(\frac{\left(3a\,d^{2}e^{4}-c\,d^{6}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{\left(-2ad\,e^{5}+2c\,d^{5}e\right)\arctan\left(x^{2}\sqrt{\frac{c}{a}}\right.}{2\sqrt{ac}}\right.}{\left(e^{4}a+d^{4}c\right)^{2}}$
risch	$-\frac{e^{3}}{\left(e^{4}a+d^{4}c\right)\left(ex+d\right)}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a^{5}e^{8}+2d^{4}c\,a^{4}e^{4}+c^{2}d^{8}a^{3}\right)\_Z^{4}+16a^{3}c\,d^{3}e^{3}\_Z^{3}+20a^{2}c\,d^{2}e^{2}\_Z^{2}+8acde\_Z+c\right)}\_R\ln\left(\left(\left((5a^{5}e^{1}\right.\right.\right.\right.}{}$

```
[In] int(1/(e*x+d)^2/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -c/(a*e^4+c*d^4)^2*(1/8*(3*a*d^2*e^4-c*d^6)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+
(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))
+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/2*(
-2*a*d*e^5+2*c*d^5*e)/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*(a*e^6-3*c*d^
4*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x
^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*
arctan(2^(1/2)/(a/c)^(1/4)*x-1))+d^3*e^3*ln(c*x^4+a))-e^3/(a*e^4+c*d^4)/(e*
x+d)+4*c*d^3*e^3*ln(e*x+d)/(a*e^4+c*d^4)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.02

$$\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx = \frac{4\,cd^3e^3\log(ex+d)}{c^2d^8+2\,acd^4e^4+a^2e^8} - \frac{e^3}{cd^5+ade^4+(cd^4e+ae^5)x} - \frac{c\left(\frac{\sqrt{2}\left(4\sqrt{2}a^{\frac{3}{4}}c^{\frac{5}{4}}d^3e^3-c^2d^6+3\sqrt{a}c^{\frac{3}{2}}d^4e^2+3\,acd^2e^4-a^{\frac{3}{2}}\sqrt{c}e^6\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}} + \frac{\sqrt{2}\left(4\sqrt{2}a^{\frac{3}{4}}c^{\frac{5}{4}}d^3e^3+c^2d^6-3\sqrt{a}c^{\frac{3}{2}}d^4e^2-3\right.}{a}\right.}{}$$

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="maxima")
[Out] 4*c*d^3*e^3*log(e*x + d)/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) - e^3/(c*d^5 +
 a*d*e^4 + (c*d^4*e + a*e^5)*x) - 1/8*c*(sqrt(2)*(4*sqrt(2)*a^(3/4)*c^(5/4)
*d^3*e^3 - c^2*d^6 + 3*sqrt(a)*c^(3/2)*d^4*e^2 + 3*a*c*d^2*e^4 - a^(3/2)*sq
rt(c)*e^6)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*
c^(5/4)) + sqrt(2)*(4*sqrt(2)*a^(3/4)*c^(5/4)*d^3*e^3 + c^2*d^6 - 3*sqrt(a)
*c^(3/2)*d^4*e^2 - 3*a*c*d^2*e^4 + a^(3/2)*sqrt(c)*e^6)*log(sqrt(c)*x^2 - s
qrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*
c^(9/4)*d^6 + 3*sqrt(2)*a^(3/4)*c^(7/4)*d^4*e^2 - 3*sqrt(2)*a^(5/4)*c^(5/4)
*d^2*e^4 - sqrt(2)*a^(7/4)*c^(3/4)*e^6 + 4*sqrt(a)*c^2*d^5*e - 4*a^(3/2)*c*
```

```
d*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt
(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*
c^(9/4)*d^6 + 3*sqrt(2)*a^(3/4)*c^(7/4)*d^4*e^2 - 3*sqrt(2)*a^(5/4)*c^(5/4)
*d^2*e^4 - sqrt(2)*a^(7/4)*c^(3/4)*e^6 - 4*sqrt(a)*c^2*d^5*e + 4*a^(3/2)*c*
d*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt
(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(c^2*d^8 + 2*a*c*d^4
*e^4 + a^2*e^8)
```

Giac [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.21

$$\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx = \frac{4\,cd^3e^4\log(|ex+d|)}{c^2d^8e+2\,acd^4e^5+a^2e^9} - \frac{cd^3e^3\log(|cx^4+a|)}{c^2d^8+2\,acd^4e^4+a^2e^8}$$
$$+ \frac{\left((ac^3)^{\frac14}c^2d^2-(ac^3)^{\frac34}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{2\left(\sqrt{2}ac^3d^4+\sqrt{2}a^2c^2e^4+4\sqrt{2}\sqrt{ac}ac^2d^2e^2-4\,(ac^3)^{\frac14}ac^2d^3e-4\,(ac^3)^{\frac34}ade^3\right)}$$
$$+ \frac{\left((ac^3)^{\frac14}c^2d^2-(ac^3)^{\frac34}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{2\left(\sqrt{2}ac^3d^4+\sqrt{2}a^2c^2e^4+4\sqrt{2}\sqrt{ac}ac^2d^2e^2+4\,(ac^3)^{\frac14}ac^2d^3e+4\,(ac^3)^{\frac34}ade^3\right)}$$
$$+ \frac{\left(\sqrt{2}(ac^3)^{\frac14}c^3d^6-3\sqrt{2}(ac^3)^{\frac14}ac^2d^2e^4-3\sqrt{2}(ac^3)^{\frac34}cd^4e^2+\sqrt{2}(ac^3)^{\frac34}ae^6\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{8\left(ac^4d^8+2\,a^2c^3d^4e^4+a^3c^2e^8\right)}$$
$$- \frac{\left(\sqrt{2}(ac^3)^{\frac14}c^3d^6-3\sqrt{2}(ac^3)^{\frac14}ac^2d^2e^4-3\sqrt{2}(ac^3)^{\frac34}cd^4e^2+\sqrt{2}(ac^3)^{\frac34}ae^6\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{8\left(ac^4d^8+2\,a^2c^3d^4e^4+a^3c^2e^8\right)}$$
$$- \frac{cd^4e^3+ae^7}{(cd^4+ae^4)^2(ex+d)}$$

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 4*c*d^3*e^4*log(abs(e*x + d))/(c^2*d^8*e + 2*a*c*d^4*e^5 + a^2*e^9) - c*d^3
*e^3*log(abs(c*x^4 + a))/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) + 1/2*((a*c^3)
^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)
^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 + sqrt(2)*a^2*c^2*e^4 + 4*sqrt(2)*s
qrt(a*c)*a*c^2*d^2*e^2 - 4*(a*c^3)^(1/4)*a*c^2*d^3*e - 4*(a*c^3)^(3/4)*a*d*
e^3) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 + sqrt(2)*a^2*c^
2*e^4 + 4*sqrt(2)*sqrt(a*c)*a*c^2*d^2*e^2 + 4*(a*c^3)^(1/4)*a*c^2*d^3*e + 4
*(a*c^3)^(3/4)*a*d*e^3) + 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^6 - 3*sqrt(2)*(a
```

```
*c^3)^(1/4)*a*c^2*d^2*e^4 - 3*sqrt(2)*(a*c^3)^(3/4)*c*d^4*e^2 + sqrt(2)*(a*
c^3)^(3/4)*a*e^6)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4*d^8 +
 2*a^2*c^3*d^4*e^4 + a^3*c^2*e^8) - 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^6 - 3*
sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^2*e^4 - 3*sqrt(2)*(a*c^3)^(3/4)*c*d^4*e^2 + s
qrt(2)*(a*c^3)^(3/4)*a*e^6)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a
*c^4*d^8 + 2*a^2*c^3*d^4*e^4 + a^3*c^2*e^8) - (c*d^4*e^3 + a*e^7)/((c*d^4 +
 a*e^4)^2*(e*x + d))
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 2436, normalized size of antiderivative = 4.41

$$\int \frac{1}{(d+ex)^2\,(a+cx^4)}\,dx=\text{Too large to display}$$

```
[In] int(1/((a + c*x^4)*(d + e*x)^2),x)
```

```
[Out] symsum(log((c^5*d*e^6 + c^5*e^7*x + 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3
*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2
*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^4*c^4*e^13 + 256*root(512*a^4*c*d^4*e^4*
z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*
a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a^2*c^5*d^3*e^8 + 496*root(51
2*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^
3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^2*c^6*d^8*e
^5 + 528*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4
 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)
^3*a^3*c^5*d^4*e^9 - 128*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 +
 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*
d*e*z + c, z, k)^4*a^2*c^7*d^13*e^2 + 128*root(512*a^4*c*d^4*e^4*z^4 + 256*
a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*
e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^3*c^6*d^9*e^6 + 640*root(512*a^4*c*d^
4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3
+ 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^4*c^5*d^5*e^10 + 32*r
oot(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^
3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)*a*c^5*d^2
*e^7 - 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^
4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k
)^3*a*c^7*d^12*e + 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 25
6*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e
*z + c, z, k)*c^6*d^5*e^4*x - 4*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^
8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 +
32*a*c*d*e*z + c, z, k)^2*c^7*d^10*e*x + 64*root(512*a^4*c*d^4*e^4*z^4 + 25
6*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^
2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a*c^6*d^7*e^4 + 384*root(512*a^4*c*d^
4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3
```

+ 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^5*c^4*d*e^14 + 320*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^5*c^4*e^15*x + 248*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a*c^6*d^6*e^5*x - 64*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a*c^7*d^11*e^2*x + 32*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)*a*c^5*d*e^8*x + 316*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a^2*c^5*d^2*e^9*x + 640*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^2*c^6*d^7*e^6*x + 704*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^3*c^5*d^3*e^10*x - 192*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^2*c^7*d^12*e^3*x - 64*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^3*c^6*d^8*e^7*x + 448*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^4*c^5*d^4*e^11*x)/(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4))*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k), k, 1, 4) - e^3/(c*d^5 + a*d*e^4 + a*e^5*x + c*d^4*e*x) + (4*c*d^3*e^3*log(d + e*x))/(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)

3.400 $\int \frac{1}{(d+ex)^3(a+cx^4)}\,dx$

Optimal result 2400
Rubi [A] (verified) 2401
Mathematica [A] (verified) 2407
Maple [A] (verified) 2407
Fricas [F(-1)] 2408
Sympy [F(-1)] 2408
Maxima [A] (verification not implemented) 2408
Giac [A] (verification not implemented) 2410
Mupad [B] (verification not implemented) 2410

Optimal result

Integrand size = 17, antiderivative size = 680

$$\int \frac{1}{(d+ex)^3(a+cx^4)}\,dx = -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} - \frac{\sqrt{c}e(3c^2d^8-12acd^4e^4+a^2e^8)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(cd^4+ae^4)^3}$$
$$-\frac{c^{3/4}d\left(c^2d^8-12acd^4e^4+3a^2e^8+2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^3}$$
$$+\frac{c^{3/4}d\left(c^2d^8-12acd^4e^4+3a^2e^8+2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^3}$$
$$+\frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^3}$$
$$-\frac{c^{3/4}d\left(c^2d^8-12acd^4e^4+3a^2e^8-2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^3}$$
$$+\frac{c^{3/4}d\left(c^2d^8-12acd^4e^4+3a^2e^8-2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^3}$$
$$-\frac{cd^2e^3(5cd^4-3ae^4)\log(a+cx^4)}{2(cd^4+ae^4)^3}$$

```
[Out] -1/2*e^3/(a*e^4+c*d^4)/(e*x+d)^2-4*c*d^3*e^3/(a*e^4+c*d^4)^2/(e*x+d)+2*c*d^
2*e^3*(-3*a*e^4+5*c*d^4)*ln(e*x+d)/(a*e^4+c*d^4)^3-1/2*c*d^2*e^3*(-3*a*e^4+
5*c*d^4)*ln(c*x^4+a)/(a*e^4+c*d^4)^3-1/2*e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^
8)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^3/a^(1/2)-1/8*c^(3/4)*
d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c^2*d^8-12*a*c*d^4*e^
```

```
4+3*a^2*e^8-2*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*
d^4)^3*2^(1/2)+1/8*c^(3/4)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/
2))*(c^2*d^8-12*a*c*d^4*e^4+3*a^2*e^8-2*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*
c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(3/4)*d*arctan(-1+c^(1/4)*x*
2^(1/2)/a^(1/4))*(c^2*d^8-12*a*c*d^4*e^4+3*a^2*e^8+2*d^2*e^2*(-5*a*e^4+3*c*
d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(3/4)*d*arctan(
1+c^(1/4)*x*2^(1/2)/a^(1/4))*(c^2*d^8-12*a*c*d^4*e^4+3*a^2*e^8+2*d^2*e^2*(-
5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {6857, 1890, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 211, 266}

$$\begin{aligned}
\int \frac{1}{(d+ex)^3(a+cx^4)}dx = &-\frac{\sqrt{c}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)(a^2e^8-12acd^4e^4+3c^2d^8)}{2\sqrt{a}(ae^4+cd^4)^3}\\
&-\frac{c^{3/4}d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(3a^2e^8-12acd^4e^4+2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)+c^2d^8)}{2\sqrt{2}a^{3/4}(ae^4+cd^4)^3}\\
&+\frac{c^{3/4}d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(3a^2e^8-12acd^4e^4+2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)+c^2d^8)}{2\sqrt{2}a^{3/4}(ae^4+cd^4)^3}\\
&-\frac{c^{3/4}d(3a^2e^8-12acd^4e^4-2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)+c^2d^8)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^4+cd^4)^3}\\
&+\frac{c^{3/4}d(3a^2e^8-12acd^4e^4-2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)+c^2d^8)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^4+cd^4)^3}\\
&-\frac{e^3}{2(d+ex)^2(ae^4+cd^4)}-\frac{4cd^3e^3}{(d+ex)(ae^4+cd^4)^2}\\
&-\frac{cd^2e^3(5cd^4-3ae^4)\log(a+cx^4)}{2(ae^4+cd^4)^3}+\frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{(ae^4+cd^4)^3}
\end{aligned}$$

```
[In] Int[1/((d + e*x)^3*(a + c*x^4)),x]
```

```
[Out] -1/2*e^3/((c*d^4 + a*e^4)*(d + e*x)^2) - (4*c*d^3*e^3)/((c*d^4 + a*e^4)^2*(
d + e*x)) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[
c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*(c^2*d^8 - 12*
a*c*d^4*e^4 + 3*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3
) + (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^
2*e^2*(3*c*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (2*c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*Log[d + e
```

```
*x])/(c*d^4 + a*e^4)^3 - (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 -
 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(3/
4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c
*d^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4
*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*Log[a
+ c*x^4])/(2*(c*d^4 + a*e^4)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^4}{(cd^4+ae^4)(d+ex)^3} + \frac{4cd^3e^4}{(cd^4+ae^4)^2(d+ex)^2} + \frac{2cd^2e^4(5cd^4-3ae^4)}{(cd^4+ae^4)^3(d+ex)} \right.$$
$$+ \frac{c(d(c^2d^8-12acd^4e^4+3a^2e^8)-e(3c^2d^8-12acd^4e^4+a^2e^8)x+2cd^3e^2(3cd^4-5ae^4)x^2-2cd^2e^3(5cd^4 -}{(cd^4+ae^4)^3(a+cx^4)}$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{\left(cd^4+ae^4\right)^3} + \frac{c\int \frac{d(c^2d^8-12acd^4e^4+3a^2e^8)-e(3c^2d^8-12acd^4e^4+a^2e^8)x+2cd^3e^2(3cd^4-5ae^4)x^2-2cd^2e^3(5cd^4-3ae^4)x^3}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^3}$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{\left(cd^4+ae^4\right)^3} + \frac{c\int \left(\frac{d(c^2d^8-12acd^4e^4+3a^2e^8)+2cd^3e^2(3cd^4-5ae^4)x^2}{a+cx^4} + \frac{x\left(-e(3c^2d^8-12acd^4e^4+a^2e^8)-2cd^2e^3(5cd^4-3ae^4)x^2\right)}{a+cx^4}\right)dx}{\left(cd^4+ae^4\right)^3}$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{\left(cd^4+ae^4\right)^3} + \frac{c\int \frac{d(c^2d^8-12acd^4e^4+3a^2e^8)+2cd^3e^2(3cd^4-5ae^4)x^2}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^3} + \frac{c\int \frac{x\left(-e(3c^2d^8-12acd^4e^4+a^2e^8)-2cd^2e^3(5cd^4-3ae^4)x^2\right)}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^3}$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{\left(cd^4+ae^4\right)^3} + \frac{c\text{Subst}\left(\int \frac{-e(3c^2d^8-12acd^4e^4+a^2e^8)-2cd^2e^3(5cd^4-3ae^4)x}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^4+ae^4\right)^3} - \frac{\left(cd\left(6cd^6e^2-10ad^2e^6-\frac{c^2d^8-12acd^4e^4+3a^2e^8}{\sqrt{a}\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\left(cd^4+ae^4\right)^3} + \frac{\left(cd\left(6cd^6e^2-10ad^2e^6+\frac{c^2d^8-12acd^4e^4+3a^2e^8}{\sqrt{a}\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^4+ae^4\right)^3}$$

$$= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^3}$$
$$- \frac{(c^2d^2e^3(5cd^4-3ae^4))\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{(cd^4+ae^4)^3}$$
$$- \frac{(ce(3c^2d^8-12acd^4e^4+a^2e^8))\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2(cd^4+ae^4)^3}$$
$$+ \frac{\left(c^{5/4}d\left(6cd^6e^2-10ad^2e^6-\frac{c^2d^8-12acd^4e^4+3a^2e^8}{\sqrt{a}\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3}$$
$$+ \frac{\left(c^{5/4}d\left(6cd^6e^2-10ad^2e^6-\frac{c^2d^8-12acd^4e^4+3a^2e^8}{\sqrt{a}\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3}$$
$$+ \frac{\left(cd\left(6cd^6e^2-10ad^2e^6+\frac{c^2d^8-12acd^4e^4+3a^2e^8}{\sqrt{a}\sqrt{c}}\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4(cd^4+ae^4)^3}$$
$$+ \frac{\left(cd\left(6cd^6e^2-10ad^2e^6+\frac{c^2d^8-12acd^4e^4+3a^2e^8}{\sqrt{a}\sqrt{c}}\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4(cd^4+ae^4)^3}$$

$$= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} - \frac{\sqrt{c}e(3c^2d^8-12acd^4e^4+a^2e^8)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(cd^4+ae^4)^3} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^3} + \frac{c^{5/4}d\left(6cd^6e^2-10ad^2e^6-\frac{c^2d^8-12acd^4e^4+3a^2e^8}{\sqrt{a}\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} - \frac{c^{5/4}d\left(6cd^6e^2-10ad^2e^6-\frac{c^2d^8-12acd^4e^4+3a^2e^8}{\sqrt{a}\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} - \frac{cd^2e^3(5cd^4-3ae^4)\log(a+cx^4)}{2(cd^4+ae^4)^3} + \frac{\left(c^{5/4}d\left(6cd^6e^2-10ad^2e^6+\frac{c^2d^8-12acd^4e^4+3a^2e^8}{\sqrt{a}\sqrt{c}}\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} - \frac{\left(c^{5/4}d\left(6cd^6e^2-10ad^2e^6+\frac{c^2d^8-12acd^4e^4+3a^2e^8}{\sqrt{a}\sqrt{c}}\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3}$$

$$= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} - \frac{\sqrt{c}e(3c^2d^8-12acd^4e^4+a^2e^8)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(cd^4+ae^4)^3} - \frac{c^{5/4}d\left(6cd^6e^2-10ad^2e^6+\frac{c^2d^8-12acd^4e^4+3a^2e^8}{\sqrt{a}\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} + \frac{c^{5/4}d\left(6cd^6e^2-10ad^2e^6+\frac{c^2d^8-12acd^4e^4+3a^2e^8}{\sqrt{a}\sqrt{c}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^3} + \frac{c^{5/4}d\left(6cd^6e^2-10ad^2e^6-\frac{c^2d^8-12acd^4e^4+3a^2e^8}{\sqrt{a}\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} - \frac{c^{5/4}d\left(6cd^6e^2-10ad^2e^6-\frac{c^2d^8-12acd^4e^4+3a^2e^8}{\sqrt{a}\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} - \frac{cd^2e^3(5cd^4-3ae^4)\log(a+cx^4)}{2(cd^4+ae^4)^3}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 738, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex)^3\,(a+cx^4)}\,dx$$
$$= \frac{-4a^{3/4}e^3(cd^4+ae^4)^2-32a^{3/4}cd^3e^3(cd^4+ae^4)(d+ex)-2\sqrt{c}\big(\sqrt{2}c^{9/4}d^9-6\sqrt[4]{a}c^2d^8e+6\sqrt{2}\sqrt{a}c^{7/4}d^7e^2 \cdots}{}$$

```
[In] Integrate[1/((d + e*x)^3*(a + c*x^4)),x]
```

```
[Out] (-4*a^(3/4)*e^3*(c*d^4 + a*e^4)^2 - 32*a^(3/4)*c*d^3*e^3*(c*d^4 + a*e^4)*(d
 + e*x) - 2*Sqrt[c]*(Sqrt[2]*c^(9/4)*d^9 - 6*a^(1/4)*c^2*d^8*e + 6*Sqrt[2]*
Sqrt[a]*c^(7/4)*d^7*e^2 - 12*Sqrt[2]*a*c^(5/4)*d^5*e^4 + 24*a^(5/4)*c*d^4*e
^5 - 10*Sqrt[2]*a^(3/2)*c^(3/4)*d^3*e^6 + 3*Sqrt[2]*a^2*c^(1/4)*d*e^8 - 2*a
^(9/4)*e^9)*(d + e*x)^2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[c]
*(Sqrt[2]*c^(9/4)*d^9 + 6*a^(1/4)*c^2*d^8*e + 6*Sqrt[2]*Sqrt[a]*c^(7/4)*d^7
*e^2 - 12*Sqrt[2]*a*c^(5/4)*d^5*e^4 - 24*a^(5/4)*c*d^4*e^5 - 10*Sqrt[2]*a^(
3/2)*c^(3/4)*d^3*e^6 + 3*Sqrt[2]*a^2*c^(1/4)*d*e^8 + 2*a^(9/4)*e^9)*(d + e*
x)^2*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 16*a^(3/4)*c*d^2*e^3*(5*c*d^
4 - 3*a*e^4)*(d + e*x)^2*Log[d + e*x] - Sqrt[2]*c^(3/4)*d*(c^2*d^8 - 6*Sqrt
[a]*c^(3/2)*d^6*e^2 - 12*a*c*d^4*e^4 + 10*a^(3/2)*Sqrt[c]*d^2*e^6 + 3*a^2*e
^8)*(d + e*x)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sq
rt[2]*c^(3/4)*d*(c^2*d^8 - 6*Sqrt[a]*c^(3/2)*d^6*e^2 - 12*a*c*d^4*e^4 + 10*
a^(3/2)*Sqrt[c]*d^2*e^6 + 3*a^2*e^8)*(d + e*x)^2*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2] + 4*a^(3/4)*c*d^2*e^3*(-5*c*d^4 + 3*a*e^4)*(d
+ e*x)^2*Log[a + c*x^4])/(8*a^(3/4)*(c*d^4 + a*e^4)^3*(d + e*x)^2)
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.66

method	result
default	$\frac{c\left(\frac{\left(3a^2d\,e^8-12ac\,d^5e^4+c^2d^9\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{\left(-a^2e^9+12ac\,d^4e^5-3c^2 \cdots}{2\sqrt{a \cdots}} \cdots\right.}{\cdots}$
risch	$\frac{-\frac{4d^3c\,e^4x}{a^2e^8+2ac\,d^4e^4+c^2d^8}-\frac{\left(e^4a+9d^4c\right)e^3}{2\left(a^2e^8+2ac\,d^4e^4+c^2d^8\right)}}{(ex+d)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^6e^{12}+3a^5c\,d^4e^8+3a^4c^2d^8e^4+a^3c^3d^{12}\right)\_Z^4+\left(-24a^4c\,d^2e^7+40a^3c^2 \cdots\right.\right.}\cdots\right.}{\cdots}$

```
[In] int(1/(e*x+d)^3/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] c/(a*e^4+c*d^4)^3*(1/8*(3*a^2*d*e^8-12*a*c*d^5*e^4+c^2*d^9)*(a/c)^(1/4)/a*2
^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/
2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(
1/4)*x-1))+1/2*(-a^2*e^9+12*a*c*d^4*e^5-3*c^2*d^8*e)/(a*c)^(1/2)*arctan(x^2
*(c/a)^(1/2))+1/8*(-10*a*c*d^3*e^6+6*c^2*d^7*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln
((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(
1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))
+1/4*(6*a*c*d^2*e^7-10*c^2*d^6*e^3)/c*ln(c*x^4+a))-1/2*e^3/(a*e^4+c*d^4)/(e
*x+d)^2-4*c*d^3*e^3/(a*e^4+c*d^4)^2/(e*x+d)-2*c*d^2*e^3*(3*a*e^4-5*c*d^4)/(
a*e^4+c*d^4)^3*ln(e*x+d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\,(a+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\,(a+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 817, normalized size of antiderivative = 1.20

$$\int \frac{1}{(d+ex)^3\,(a+cx^4)}\,dx =$$

$$-\frac{c\left(\frac{\sqrt{2}\left(10\,\sqrt{2}a^{\frac{3}{4}}c^{\frac{9}{4}}d^6e^3-6\,\sqrt{2}a^{\frac{7}{4}}c^{\frac{5}{4}}d^2e^7-c^3d^9+6\,\sqrt{a}c^{\frac{5}{2}}d^7e^2+12\,ac^2d^5e^4-10\,a^{\frac{3}{2}}c^{\frac{3}{2}}d^3e^6-3\,a^2cde^8\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}+\frac{\sqrt{2}(1}{}\right.}{}$$

$$+\frac{2\,(5\,c^2d^6e^3-3\,acd^2e^7)\log{(ex+d)}}{c^3d^{12}+3\,ac^2d^8e^4+3\,a^2cd^4e^8+a^3e^{12}}$$

$$-\frac{8\,cd^3e^4x+9\,cd^4e^3+ae^7}{2\,(c^2d^{10}+2\,acd^6e^4+a^2d^2e^8+(c^2d^8e^2+2\,acd^4e^6+a^2e^{10})x^2+2\,(c^2d^9e+2\,acd^5e^5+a^2de^9)x)}$$

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/8*c*(sqrt(2)*(10*sqrt(2)*a^(3/4)*c^(9/4)*d^6*e^3 - 6*sqrt(2)*a^(7/4)*c^(
5/4)*d^2*e^7 - c^3*d^9 + 6*sqrt(a)*c^(5/2)*d^7*e^2 + 12*a*c^2*d^5*e^4 - 10*
a^(3/2)*c^(3/2)*d^3*e^6 - 3*a^2*c*d*e^8)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*
c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(10*sqrt(2)*a^(3/4)*c^(9/4
)*d^6*e^3 - 6*sqrt(2)*a^(7/4)*c^(5/4)*d^2*e^7 + c^3*d^9 - 6*sqrt(a)*c^(5/2)
*d^7*e^2 - 12*a*c^2*d^5*e^4 + 10*a^(3/2)*c^(3/2)*d^3*e^6 + 3*a^2*c*d*e^8)*l
og(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2
*(sqrt(2)*a^(1/4)*c^(13/4)*d^9 + 6*sqrt(2)*a^(3/4)*c^(11/4)*d^7*e^2 - 12*sq
rt(2)*a^(5/4)*c^(9/4)*d^5*e^4 - 10*sqrt(2)*a^(7/4)*c^(7/4)*d^3*e^6 + 3*sqrt
(2)*a^(9/4)*c^(5/4)*d*e^8 + 6*sqrt(a)*c^3*d^8*e - 24*a^(3/2)*c^2*d^4*e^5 +
2*a^(5/2)*c*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))
/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(sqrt(2
)*a^(1/4)*c^(13/4)*d^9 + 6*sqrt(2)*a^(3/4)*c^(11/4)*d^7*e^2 - 12*sqrt(2)*a^
(5/4)*c^(9/4)*d^5*e^4 - 10*sqrt(2)*a^(7/4)*c^(7/4)*d^3*e^6 + 3*sqrt(2)*a^(9
/4)*c^(5/4)*d*e^8 - 6*sqrt(a)*c^3*d^8*e + 24*a^(3/2)*c^2*d^4*e^5 - 2*a^(5/2
)*c*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sq
rt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(c^3*d^12 + 3*a*c^
2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + 2*(5*c^2*d^6*e^3 - 3*a*c*d^2*e^7)
*log(e*x + d)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) - 1
/2*(8*c*d^3*e^4*x + 9*c*d^4*e^3 + a*e^7)/(c^2*d^10 + 2*a*c*d^6*e^4 + a^2*d^
2*e^8 + (c^2*d^8*e^2 + 2*a*c*d^4*e^6 + a^2*e^10)*x^2 + 2*(c^2*d^9*e + 2*a*c
*d^5*e^5 + a^2*d*e^9)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 941, normalized size of antiderivative = 1.38

$$\int \frac{1}{(d+ex)^3 (a+cx^4)} dx = \text{Too large to display}$$

[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="giac")

[Out] 1/4*(2*a*c^2*e^3 + sqrt(2)*(a*c^3)^(1/4)*c^2*d^3 - 3*sqrt(2)*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3*d^6 + 9*a^2*c^2*d^2*e^4 - 3*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^5*e - 3*sqrt(2)*(a*c^3)^(1/4)*a^2*c*d*e^5 + 9*sqrt(a*c)*a*c^2*d^4*e^2 + sqrt(a*c)*a^2*c*e^6 - 8*sqrt(2)*(a*c^3)^(3/4)*a*d^3*e^3) - 1/4*(2*a*c^2*e^3 - sqrt(2)*(a*c^3)^(1/4)*c^2*d^3 + 3*sqrt(2)*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3*d^6 + 9*a^2*c^2*d^2*e^4 + 3*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^5*e + 3*sqrt(2)*(a*c^3)^(1/4)*a^2*c*d*e^5 - 9*sqrt(a*c)*a*c^2*d^4*e^2 + sqrt(a*c)*a^2*c*e^6 + 8*sqrt(2)*(a*c^3)^(3/4)*a*d^3*e^3) + 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^9 - 12*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^5*e^4 + 3*sqrt(2)*(a*c^3)^(1/4)*a^2*c*d*e^8 - 6*sqrt(2)*(a*c^3)^(3/4)*c*d^7*e^2 + 10*sqrt(2)*(a*c^3)^(3/4)*a*d^3*e^6)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4*d^12 + 3*a^2*c^3*d^8*e^4 + 3*a^3*c^2*d^4*e^8 + a^4*c*e^12) - 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^9 - 12*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^5*e^4 + 3*sqrt(2)*(a*c^3)^(1/4)*a^2*c*d*e^8 - 6*sqrt(2)*(a*c^3)^(3/4)*c*d^7*e^2 + 10*sqrt(2)*(a*c^3)^(3/4)*a*d^3*e^6)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4*d^12 + 3*a^2*c^3*d^8*e^4 + 3*a^3*c^2*d^4*e^8 + a^4*c*e^12) - 1/2*(5*c^2*d^6*e^3 - 3*a*c*d^2*e^7)*log(abs(c*x^4 + a))/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + 2*(5*c^2*d^6*e^4 - 3*a*c*d^2*e^8)*log(abs(e*x + d))/(c^3*d^12*e + 3*a*c^2*d^8*e^5 + 3*a^2*c*d^4*e^9 + a^3*e^13) - 1/2*(9*c^2*d^8*e^3 + 10*a*c*d^4*e^7 + a^2*e^11 + 8*(c^2*d^7*e^4 + a*c*d^3*e^8)*x)/((c*d^4 + a*e^4)^3*(e*x + d)^2)

Mupad [B] (verification not implemented)

Time = 9.89 (sec) , antiderivative size = 1955, normalized size of antiderivative = 2.88

$$\int \frac{1}{(d+ex)^3 (a+cx^4)} dx = \text{Too large to display}$$

[In] int(1/((a + c*x^4)*(d + e*x)^3),x)

[Out] symsum(log((c^7*d^5*e^6 + a*c^6*d*e^10)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) + root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a

^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((208*a*c^7*d^7*e^7 - 48*a^2*c^6*d^3*e^11)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) + root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((144*a*c^8*d^13*e^4 + 16*a^4*c^5*d*e^16 + 2608*a^2*c^7*d^9*e^8 - 592*a^3*c^6*d^5*e^12)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) - root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((896*a^4*c^6*d^7*e^13 - 1120*a^3*c^7*d^11*e^9 - 1024*a^2*c^8*d^15*e^5 + 976*a^5*c^5*d^3*e^17 + 16*a*c^9*d^19*e)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) - root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((384*a^7*c^4*d*e^22 - 128*a^2*c^9*d^21*e^2 - 128*a^3*c^8*d^17*e^6 + 768*a^4*c^7*d^13*e^10 + 1792*a^5*c^6*d^9*e^14 + 1408*a^6*c^5*d^5*e^18)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) + (x*(320*a^7*c^4*e^23 - 192*a^2*c^9*d^20*e^3 - 448*a^3*c^8*d^16*e^7 + 128*a^4*c^7*d^12*e^11 + 1152*a^5*c^6*d^8*e^15 + 1088*a^6*c^5*d^4*e^19))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)) + (x*(80*a*c^9*d^18*e^2 - 1536*a^2*c^8*d^14*e^6 - 2016*a^3*c^7*d^10*e^10 + 896*a^4*c^6*d^6*e^14 + 1296*a^5*c^5*d^2*e^18))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)) + (x*(36*a^4*c^5*e^17 - 4*c^9*d^16*e + 792*a*c^8*d^12*e^5 + 1632*a^2*c^7*d^8*e^9 - 152*a^3*c^6*d^4*e^13))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)) + (x*(40*c^8*d^10*e^4 - 16*a*c^7*d^6*e^8 + 72*a^2*c^6*d^2*e^12))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)) + (x*(a*c^6*e^11 + c^7*d^4*e^7))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8))*root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k), k, 1, 4) - ((a*e^7 + 9*c*d^4*e^3)/(2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (4*c*d^3*e^4*x)/(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4))/(d^2 + e^2*x^2 + 2*d*e*x) + (log(d + e*x)*(10*c^2*d^6*e^3 - 6*a*c*d^2*e^7))/(a^3*e^12 + c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)

3.401 $\int \frac{(d+ex)^3}{(a+cx^4)^2} dx$

Optimal result . . . 2412
Rubi [A] (verified) . . . 2413
Mathematica [A] (verified) . . . 2417
Maple [C] (verified) . . . 2417
Fricas [C] (verification not implemented) . . . 2418
Sympy [F(-1)] . . . 2418
Maxima [A] (verification not implemented) . . . 2418
Giac [A] (verification not implemented) . . . 2419
Mupad [B] (verification not implemented) . . . 2420

Optimal result

Integrand size = 17, antiderivative size = 349

$$\begin{aligned}\int \frac{(d+ex)^3}{(a+cx^4)^2} dx = & -\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac(a+cx^4)} + \frac{3d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} \\ & - \frac{3d(\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} \\ & + \frac{3d(\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} \\ & - \frac{3d(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}} \\ & + \frac{3d(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}\end{aligned}$$

```
[Out] 1/4*(-a*e^3+c*x*(3*d*e^2*x^2+3*d^2*e*x+d^3))/a/c/(c*x^4+a)+3/4*d^2*e*arctan
(x^2*c^(1/2)/a^(1/2))/a^(3/2)/c^(1/2)-3/32*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+
a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+3/3
2*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(
1/2))/a^(7/4)/c^(3/4)*2^(1/2)+3/16*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(
e^2*a^(1/2)+d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+3/16*d*arctan(1+c^(1/4)*x*
2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {1868, 27, 12, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx = -\frac{3d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{8\sqrt{2}a^{7/4}c^{3/4}} + \frac{3d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{8\sqrt{2}a^{7/4}c^{3/4}} + \frac{3d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}} + \frac{3d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac\,(a+cx^4)}$$

```
[In] Int[(d + e*x)^3/(a + c*x^4)^2,x]
```

```
[Out] -1/4*(a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(a*c*(a + c*x^4)) + (3*d
^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[c]) - (3*d*(Sqrt[c]*d^2
 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)
*c^(3/4)) + (3*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)
/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - (3*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*
c^(3/4)) + (3*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
```

```
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac\,(a + cx^4)} - \frac{\int \frac{-3d^3 - 6d^2ex - 3de^2x^2}{a + cx^4}\,dx}{4a} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac\,(a + cx^4)} - \frac{\int -\frac{3d(d+ex)^2}{a + cx^4}\,dx}{4a} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac\,(a + cx^4)} + \frac{(3d)\int \frac{(d+ex)^2}{a + cx^4}\,dx}{4a} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac\,(a + cx^4)} + \frac{(3d)\int \left(\frac{2dex}{a + cx^4} + \frac{d^2 + e^2x^2}{a + cx^4}\right)dx}{4a} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac\,(a + cx^4)} + \frac{(3d)\int \frac{d^2 + e^2x^2}{a + cx^4}\,dx}{4a} + \frac{(3d^2e)\int \frac{x}{a + cx^4}\,dx}{2a} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac\,(a + cx^4)} + \frac{(3d^2e)\,\text{Subst}\left(\int \frac{1}{a + cx^2}\,dx, x, x^2\right)}{4a} \\
&\quad + \frac{\left(3d\left(\frac{\sqrt{c}d^2}{\sqrt{a}} - e^2\right)\right)\int \frac{\sqrt{a}\sqrt{c} - cx^2}{a + cx^4}\,dx}{8ac} + \frac{\left(3d\left(\frac{\sqrt{c}d^2}{\sqrt{a}} + e^2\right)\right)\int \frac{\sqrt{a}\sqrt{c} + cx^2}{a + cx^4}\,dx}{8ac}
\end{aligned}$$

$$=-\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac\left(a+cx^4\right)}+\frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}$$
$$+\frac{\left(3d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16ac}+\frac{\left(3d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16ac}$$
$$-\frac{\left(3d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}c^{3/4}}$$
$$-\frac{\left(3d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}c^{3/4}}$$

$$=-\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac\left(a+cx^4\right)}+\frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}$$
$$-\frac{3d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}$$
$$+\frac{3d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}$$
$$+\frac{\left(3d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}$$
$$-\frac{\left(3d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}$$

$$=-\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac\left(a+cx^4\right)}+\frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}$$
$$-\frac{3d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}$$
$$+\frac{3d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}$$
$$-\frac{3d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}$$
$$+\frac{3d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx = \frac{-\frac{8a(ae^3-cdx(d^2+3dex+3e^2x^2))}{a+cx^4}-6\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{2}\sqrt{c}d^2+4\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+6\sqrt[4]{a}\sqrt[4]{c}}{}$$

```
[In] Integrate[(d + e*x)^3/(a + c*x^4)^2,x]
```

```
[Out] ((-8*a*(a*e^3 - c*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(a + c*x^4) - 6*a^(1/4)
*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e
^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*a^(1/4)*c^(1/4)*d*(Sqrt[2]*
Sqrt[c]*d^2 - 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*c^(1/4)*(-(a^(1/4)*Sqrt[c]*d^3) + a^(3/
4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2
]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - a^(3/4)*d*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(32*a^2*c)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.85 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.29

method	result
risch	$\frac{\frac{3d\,e^2x^3}{4a}+\frac{3e\,x^2d^2}{4a}+\frac{d^3x}{4a}-\frac{e^3}{4c}}{c\,x^4+a}+\frac{3d\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+a\right)}\frac{\left(e^2\_R^2+2ed\_R+d^2\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{16ac}$
default	$d^3\left(\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+3d^2e\left(\frac{x^2}{4a(c\,x}\right.$

```
[In] int((e*x+d)^3/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (3/4*d*e^2/a*x^3+3/4*e/a*x^2*d^2+1/4*d^3/a*x-1/4*e^3/c)/(c*x^4+a)+3/16*d/a/
c*sum((_R^2*e^2+2*_R*d*e+d^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.49 (sec) , antiderivative size = 91191, normalized size of antiderivative = 261.29

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**3/(c*x**4+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx = \frac{3\,d\left(\frac{\sqrt{2}(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{2\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2-4\sqrt{a}\sqrt{}\right.}{a^{\frac{3}{4}}\sqrt{\sqrt{}}}\right.}{32\,a} + \frac{3\,cde^2x^3+3\,cd^2ex^2+cd^3x-ae^3}{4\,(ac^2x^4+a^2c)}$$

```
[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="maxima")
[Out] 3/32*d*(sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/
4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*
e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4
)) + 2*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 4*sqrt(
```

```
a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(sqrt(2)
*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 4*sqrt(a)*sqrt(c)*d*e)
*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sq
rt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)))/a + 1/4*(3*c*d*e^2*x^3 + 3
*c*d^2*e*x^2 + c*d^3*x - a*e^3)/(a*c^2*x^4 + a^2*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx = \frac{3\,cde^2x^3+3\,cd^2ex^2+cd^3x-ae^3}{4\,(cx^4+a)ac} + \frac{3\,\sqrt{2}\Big(2\,\sqrt{2}\sqrt{ac}c^2d^2e+(ac^3)^{\frac{1}{4}}\,c^2d^3+(ac^3)^{\frac{3}{4}}\,de^2\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{3\,\sqrt{2}\Big(2\,\sqrt{2}\sqrt{ac}c^2d^2e+(ac^3)^{\frac{1}{4}}\,c^2d^3+(ac^3)^{\frac{3}{4}}\,de^2\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{3\,\sqrt{2}\Big((ac^3)^{\frac{1}{4}}\,c^2d^3-(ac^3)^{\frac{3}{4}}\,de^2\Big)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3} - \frac{3\,\sqrt{2}\Big((ac^3)^{\frac{1}{4}}\,c^2d^3-(ac^3)^{\frac{3}{4}}\,de^2\Big)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}$$

```
[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(3*c*d*e^2*x^3 + 3*c*d^2*e*x^2 + c*d^3*x - a*e^3)/((c*x^4 + a)*a*c) + 3
/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + (a*c^3
)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/
(a^2*c^3) + 3/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1/4)*c^2
*d^3 + (a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(a^2*c^3) + 3/32*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4
)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 3/32*sqrt
(2)*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2 - sqrt(2)*x*(a/c)
^(1/4) + sqrt(a/c))/(a^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.92

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx$$
$$= \left(\sum_{k=1}^{4} \ln\left(\frac{c\,d^2\left(27\,c\,d^5\,e^2 - 9\,a\,d\,e^6 + 36\,c\,d^4\,e^3\,x - \mathrm{root}(65536\,a^7\,c^3\,z^4 + 27648\,a^4\,c^2\,d^4\,e^2\,z^2 + 3456\,a^3\,c\,d^4\,e\right.}{}\right.\right.$$
$$+ 27648\,a^4\,c^2\,d^4\,e^2\,z^2 + 3456\,a^3\,c\,d^4\,e^5\,z - 3456\,a^2\,c^2\,d^8\,e\,z + 162\,a\,c\,d^8\,e^4 + 81\,a^2\,d^4\,e^8$$
$$\left.+ 81\,c^2\,d^{12}, z, k)\right) + \frac{\frac{d^3\,x}{4\,a} - \frac{e^3}{4\,c} + \frac{3\,d^2\,e\,x^2}{4\,a} + \frac{3\,d\,e^2\,x^3}{4\,a}}{c\,x^4 + a}$$

[In] int((d + e*x)^3/(a + c*x^4)^2,x)

[Out] symsum(log((3*c*d^2*(27*c*d^5*e^2 - 9*a*d*e^6 + 36*c*d^4*e^3*x - 256*root(6
5536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*
a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)^2*a
^3*c^2*d - 48*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3
*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81
*c^2*d^12, z, k)*a*c^2*d^4*x + 48*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^
4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 +
 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)*a^2*c*e^4*x + 512*root(65536*a^7*c^3*z
^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*
z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)^2*a^3*c^2*e*x - 1
92*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*
z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12,
z, k)*a^2*c*d*e^3))/(64*a^3))*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^
2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*
a^2*d^4*e^8 + 81*c^2*d^12, z, k), k, 1, 4) + ((d^3*x)/(4*a) - e^3/(4*c) + (
3*d^2*e*x^2)/(4*a) + (3*d*e^2*x^3)/(4*a))/(a + c*x^4)

3.402 $\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx$

Optimal result 2421
Rubi [A] (verified) 2422
Mathematica [A] (verified) 2425
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Mupad [B] (verification not implemented) 2429

Optimal result

Integrand size = 17, antiderivative size = 322

$$\begin{aligned}\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx=&\ \frac{x(d+ex)^2}{4a(a+cx^4)}+\frac{de\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}-\frac{\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}\\&+\frac{\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}\\&-\frac{\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}\\&+\frac{\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}\end{aligned}$$

```
[Out] 1/4*x*(e*x+d)^2/a/(c*x^4+a)+1/2*d*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(3/2)/c^(
1/2)-1/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+
3*d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+1/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^
(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+1/1
6*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/
c^(3/4)*2^(1/2)+1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+3*d^2
*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e^2+3\sqrt{c}d^2\right)}{8\sqrt{2}a^{7/4}c^{3/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e^2+3\sqrt{c}d^2\right)}{8\sqrt{2}a^{7/4}c^{3/4}} + \frac{de\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} - \frac{\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}} + \frac{\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}} + \frac{x(d+ex)^2}{4a\left(a+cx^4\right)}$$

```
[In] Int[(d + e*x)^2/(a + c*x^4)^2,x]
```

```
[Out] (x*(d + e*x)^2)/(4*a*(a + c*x^4)) + (d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*
a^(3/2)*Sqrt[c]) - ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - ((
3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^
(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
```

```
0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(d+ex)^2}{4a(a+cx^4)} - \frac{\int \frac{-3d^2-4dex-e^2x^2}{a+cx^4}\,dx}{4a}$$

$$= \frac{x(d+ex)^2}{4a(a+cx^4)} - \frac{\int \left(-\frac{4dex}{a+cx^4} + \frac{-3d^2-e^2x^2}{a+cx^4}\right) dx}{4a}$$

$$= \frac{x(d+ex)^2}{4a(a+cx^4)} - \frac{\int \frac{-3d^2-e^2x^2}{a+cx^4}\,dx}{4a} + \frac{(de)\int \frac{x}{a+cx^4}\,dx}{a}$$

$$= \frac{x(d+ex)^2}{4a(a+cx^4)} + \frac{(de)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2a} + \frac{\left(\frac{3\sqrt{c}d^2}{\sqrt{a}} - e^2\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8ac} + \frac{\left(\frac{3\sqrt{c}d^2}{\sqrt{a}} + e^2\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8ac}$$

$$= \frac{x(d+ex)^2}{4a(a+cx^4)} + \frac{de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} + \frac{\left(\frac{3\sqrt{c}d^2}{\sqrt{a}} + e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2}\,dx}{16ac} + \frac{\left(\frac{3\sqrt{c}d^2}{\sqrt{a}} + e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2}\,dx}{16ac} - \frac{\left(3\sqrt{c}d^2 - \sqrt{a}e^2\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2}\,dx}{16\sqrt{2}a^{7/4}c^{3/4}} - \frac{\left(3\sqrt{c}d^2 - \sqrt{a}e^2\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} - 2x}{-\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2}\,dx}{16\sqrt{2}a^{7/4}c^{3/4}}$$

$$= \frac{x(d+ex)^2}{4a(a+cx^4)} + \frac{de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} - \frac{\left(3\sqrt{c}d^2 - \sqrt{a}e^2\right)\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}} + \frac{\left(3\sqrt{c}d^2 - \sqrt{a}e^2\right)\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}} + \frac{\left(3\sqrt{c}d^2 + \sqrt{a}e^2\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} - \frac{\left(3\sqrt{c}d^2 + \sqrt{a}e^2\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}$$

$$= \frac{x(d+ex)^2}{4a(a+cx^4)} + \frac{de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} - \frac{(3\sqrt{c}d^2+\sqrt{a}e^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}$$
$$+ \frac{(3\sqrt{c}d^2+\sqrt{a}e^2)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}$$
$$- \frac{(3\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}$$
$$+ \frac{(3\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx$$
$$= \frac{\frac{8ax(d+ex)^2}{a+cx^4} - \frac{2\sqrt[4]{a}\left(3\sqrt{2}\sqrt{c}d^2+8\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{2\sqrt[4]{a}\left(3\sqrt{2}\sqrt{c}d^2-8\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1+\right.}{c^{3/4}}}{32a}$$

```
[In] Integrate[(d + e*x)^2/(a + c*x^4)^2,x]
```

```
[Out] ((8*a*x*(d + e*x)^2)/(a + c*x^4) - (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*d^2 + 8*a^
(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/c^(3/4) + (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*d^2 - 8*a^(1/4)*c^(1/4)*d*e
+ Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (
Sqrt[2]*(-3*a^(1/4)*Sqrt[c]*d^2 + a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[c]*d^2 - a^(
3/4)*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/
(32*a^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.82 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.28

method	result
risch	$\frac{\frac{e^2x^3}{4a}+\frac{ed\,x^2}{2a}+\frac{d^2x}{4a}}{c\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(e^2\_R^2+4ed\_R+3d^2\right)\ln\left(x-\_R\right)}{\_R^3}}{16ac}$
default	$d^2\left(\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+2ed\left(\frac{x^2}{4a(c\,x^4+}\right.$

```
[In] int((e*x+d)^2/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4*e^2/a*x^3+1/2*e*d/a*x^2+1/4/a*d^2*x)/(c*x^4+a)+1/16/a/c*sum((_R^2*e^2+
4*_R*d*e+3*d^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.74 (sec) , antiderivative size = 90963, normalized size of antiderivative = 282.49

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx$$
$$= \operatorname{RootSum}\left(65536t^4a^7c^3+11264t^2a^4c^2d^2e^2+t\left(256a^3cde^5-2304a^2c^2d^5e\right)+a^2e^8+82acd^4e^4+81c^2d^8,\left(t\right.\right.$$
$$+\frac{d^2x+2dex^2+e^2x^3}{4a^2+4acx^4}$$

```
[In] integrate((e*x+d)**2/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**3 + 11264*_t**2*a**4*c**2*d**2*e**2 + _t*(256*a
**3*c*d*e**5 - 2304*a**2*c**2*d**5*e) + a**2*e**8 + 82*a*c*d**4*e**4 + 81*c
**2*d**8, Lambda(_t, _t*log(x + (4096*_t**3*a**7*c**2*e**6 + 356352*_t**3*a
**6*c**3*d**4*e**2 - 23552*_t**2*a**5*c**2*d**3*e**5 + 27648*_t**2*a**4*c**
```

```
3*d**7*e + 912*_t*a**4*c*d**2*e**8 + 43584*_t*a**3*c**2*d**6*e**4 + 3888*_t
*a**2*c**3*d**10 + 12*a**3*d*e**11 - 1088*a**2*c*d**5*e**7 - 7020*a*c**2*d*
*9*e**3)/(a**3*e**12 - 649*a**2*c*d**4*e**8 - 5841*a*c**2*d**8*e**4 + 729*c
**3*d**12)))) + (d**2*x + 2*d*e*x**2 + e**2*x**3)/(4*a**2 + 4*a*c*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx = \frac{e^2x^3+2\,dex^2+d^2x}{4\,(acx^4+a^2)} + \frac{\frac{\sqrt{2}(3\,\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}(3\,\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{2\left(3\,\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2-8\,\sqrt{c}\right.}{a^{\frac{3}{4}}\sqrt{}}}{32\,a}$$

```
[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(e^2*x^3 + 2*d*e*x^2 + d^2*x)/(a*c*x^4 + a^2) + 1/32*(sqrt(2)*(3*sqrt(c
)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))
/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2
- sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 2*(3*sqrt(2)*a^(
1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 8*sqrt(a)*sqrt(c)*d*e)*arc
tan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c
)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*c^(3/4)*
d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 8*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(
2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*
sqrt(sqrt(a)*sqrt(c))*c^(3/4)))/a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx = \frac{e^2x^3+2\,dex^2+d^2x}{4\,(cx^4+a)a} + \frac{\sqrt{2}\left(4\,\sqrt{2}\sqrt{ac}c^2de+3\,(ac^3)^{\frac{1}{4}}\,c^2d^2+(ac^3)^{\frac{3}{4}}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{\sqrt{2}\left(4\,\sqrt{2}\sqrt{ac}c^2de+3\,(ac^3)^{\frac{1}{4}}\,c^2d^2+(ac^3)^{\frac{3}{4}}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2-(ac^3)^{\frac{3}{4}}\,e^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3} - \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2-(ac^3)^{\frac{3}{4}}\,e^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}$$

```
[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(e^2*x^3 + 2*d*e*x^2 + d^2*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(4*sqrt(2)
*sqrt(a*c)*c^2*d*e + 3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)*e^2)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/16*sqrt(2)
*(4*sqrt(2)*sqrt(a*c)*c^2*d*e + 3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)*e^2
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1
/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 + sqrt(2)
*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d
^2 - (a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c
^3)
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx = \frac{\frac{d^2\,x}{4\,a}+\frac{e^2\,x^3}{4\,a}+\frac{d\,e\,x^2}{2\,a}}{c\,x^4+a}+\left(\sum_{k=1}^{4}\ln\left(\frac{39\,c^2\,d^4\,e^2-a\,c\,e^6}{64\,a^3} -\mathrm{root}\left(65536\,a^7\,c^3\,z^4+11264\,a^4\,c^2\,d^2\,e^2\,z^2-2304\,a^2\,c^2\,d^5\,e\,z+256\,a^3\,c\,d\,e^5\,z+82\,a\,c\,d^4\,e^4+81\,c^2\,d^8+a^2\,e^8,z\right.\right.\right.$$
$$\left.\left.+\frac{5\,c^2\,d^3\,e^3\,x}{8\,a^3}\right)\,\mathrm{root}\left(65536\,a^7\,c^3\,z^4+11264\,a^4\,c^2\,d^2\,e^2\,z^2-2304\,a^2\,c^2\,d^5\,e\,z\right.$$
$$\left.\left.+256\,a^3\,c\,d\,e^5\,z+82\,a\,c\,d^4\,e^4+81\,c^2\,d^8+a^2\,e^8,z,k\right)\right)$$

```
[In] int((d + e*x)^2/(a + c*x^4)^2,x)
```

```
[Out] ((d^2*x)/(4*a) + (e^2*x^3)/(4*a) + (d*e*x^2)/(2*a))/(a + c*x^4) + symsum(lo
g((39*c^2*d^4*e^2 - a*c*e^6)/(64*a^3) - root(65536*a^7*c^3*z^4 + 11264*a^4*
c^2*d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a^3*c*d*e^5*z + 82*a*c*d^4*e^4
 + 81*c^2*d^8 + a^2*e^8, z, k)*(root(65536*a^7*c^3*z^4 + 11264*a^4*c^2*d^2*
e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a^3*c*d*e^5*z + 82*a*c*d^4*e^4 + 81*c^
2*d^8 + a^2*e^8, z, k)*(12*c^3*d^2 - 16*c^3*d*e*x) + (x*(18*a*c^3*d^4 - 2*a
^2*c^2*e^4))/(8*a^3) + (2*c^2*d*e^3)/a) + (5*c^2*d^3*e^3*x)/(8*a^3))*root(6
5536*a^7*c^3*z^4 + 11264*a^4*c^2*d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a
^3*c*d*e^5*z + 82*a*c*d^4*e^4 + 81*c^2*d^8 + a^2*e^8, z, k), k, 1, 4)
```

3.403 $\int \frac{d+ex}{\left(a+cx^4\right)^2}\,dx$

Optimal result 2430
Rubi [A] (verified) 2430
Mathematica [A] (verified) 2433
Maple [C] (verified) 2434
Fricas [C] (verification not implemented) 2434
Sympy [A] (verification not implemented) 2435
Maxima [A] (verification not implemented) 2435
Giac [A] (verification not implemented) 2436
Mupad [B] (verification not implemented) 2436

Optimal result

Integrand size = 15, antiderivative size = 241

$$\int \frac{d+ex}{\left(a+cx^4\right)^2}\,dx=\frac{x(d+ex)}{4a\left(a+cx^4\right)}+\frac{e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}-\frac{3d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3d\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

```
[Out] 1/4*x*(e*x+d)/a/(c*x^4+a)+3/16*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/
4)/c^(1/4)*2^(1/2)+3/16*d*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/
4)*2^(1/2)-3/32*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4
)/c^(1/4)*2^(1/2)+3/32*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/
a^(7/4)/c^(1/4)*2^(1/2)+1/4*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(3/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules

used = {1869, 1890, 217, 1179, 642, 1176, 631, 210, 281, 211}

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx = -\frac{3d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{x(d+ex)}{4a\,(a+cx^4)}$$

```
[In] Int[(d + e*x)/(a + c*x^4)^2,x]
```

```
[Out] (x*(d + e*x))/(4*a*(a + c*x^4)) + (e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3
/2)*Sqrt[c]) - (3*d*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(
7/4)*c^(1/4)) + (3*d*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^
(7/4)*c^(1/4)) - (3*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(d+ex)}{4a\left(a+cx^4\right)}-\frac{\int \frac{-3d-2ex}{a+cx^4}\,dx}{4a}\\
&= \frac{x(d+ex)}{4a\left(a+cx^4\right)}-\frac{\int \left(-\frac{3d}{a+cx^4}-\frac{2ex}{a+cx^4}\right)\,dx}{4a}
\end{aligned}$$

$$= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{(3d)\int \frac{1}{a+cx^4}\,dx}{4a} + \frac{e\int \frac{x}{a+cx^4}\,dx}{2a}$$

$$= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{(3d)\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{8a^{3/2}} + \frac{(3d)\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{8a^{3/2}} + \frac{e\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{4a}$$

$$= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} + \frac{(3d)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} + \frac{(3d)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} - \frac{(3d)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{(3d)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

$$= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{(3d)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{(3d)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

$$= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3d\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.93

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx$$

$$= \frac{\frac{8a^{3/4}x(d+ex)}{a+cx^4} - \frac{2\left(3\sqrt{2}\sqrt[4]{c}d+4\sqrt[4]{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}} + \frac{2\left(3\sqrt{2}\sqrt[4]{c}d-4\sqrt[4]{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}} - \frac{3\sqrt{2}d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{\sqrt[4]{c}}}{32a^{7/4}}$$

```
[In] Integrate[(d + e*x)/(a + c*x^4)^2,x]
```

```
[Out] ((8*a^(3/4)*x*(d + e*x))/(a + c*x^4) - (2*(3*Sqrt[2]*c^(1/4)*d + 4*a^(1/4)*
e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] + (2*(3*Sqrt[2]*c^(1/4)
*d - 4*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] - (3*Sqr
t[2]*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (3
*Sqrt[2]*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))
/(32*a^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.87 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.27

method	result
risch	$\frac{\frac{e\,x^2}{4a}+\frac{dx}{4a}}{c\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(2e\_R+3d\right)\ln\left(x-\_R\right)}{\_R^3}}{16ac}$
default	$d\left(\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+e\left(\frac{x^2}{4a(c\,x^4+a)}+\right.$

```
[In] int((e*x+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4*e/a*x^2+1/4*d/a*x)/(c*x^4+a)+1/16/a/c*sum((2*_R*e+3*d)/_R^3*ln(x-_R),_
R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 43065, normalized size of antiderivative = 178.69

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.64

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx$$
$$=\operatorname{RootSum}\left(65536t^4a^7c^2+2048t^2a^4ce^2-1152ta^2cd^2e+16ae^4+81cd^4,\left(t\mapsto t\log\left(x+\frac{-32768t^3a^6ce^2-\cdots}{\cdots}\right.\right.\right.$$
$$+\frac{dx+ex^2}{4a^2+4acx^4}$$

```
[In] integrate((e*x+d)/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**2 + 2048*_t**2*a**4*c*e**2 - 1152*_t*a**2*c*d**
2*e + 16*a*e**4 + 81*c*d**4, Lambda(_t, _t*log(x + (-32768*_t**3*a**6*c*e**
2 - 4608*_t**2*a**4*c*d**2*e - 512*_t*a**3*e**4 - 1296*_t*a**2*c*d**4 + 360
*a*d**2*e**3)/(192*a*d*e**4 - 243*c*d**5)))) + (d*x + e*x**2)/(4*a**2 + 4*a
*c*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.99

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx=\frac{ex^2+dx}{4\,(acx^4+a^2)}$$
$$+\frac{\frac{3\sqrt{2}d\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{3\sqrt{2}d\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d-4\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{1}{4}}}+\cdots}{32\,a}$$

```
[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(e*x^2 + d*x)/(a*c*x^4 + a^2) + 1/32*(3*sqrt(2)*d*log(sqrt(c)*x^2 + sqr
t(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 3*sqrt(2)*d*log(sqrt(
c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) + 2*(3*sqrt
(2)*a^(1/4)*c^(1/4)*d - 4*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt
(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*
c^(1/4)) + 2*(3*sqrt(2)*a^(1/4)*c^(1/4)*d + 4*sqrt(a)*e)*arctan(1/2*sqrt(2)
*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sq
rt(sqrt(a)*sqrt(c))*c^(1/4)))/a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.99

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx = \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}}\,d\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c} - \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}}\,d\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c} + \frac{ex^2+dx}{4\,(cx^4+a)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}ce+3\,(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^2} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}ce+3\,(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^2}$$

```
[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a^2*c) - 3/32*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqr
t(a/c))/(a^2*c) + 1/4*(e*x^2 + d*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(2*sqrt(
2)*sqrt(a*c)*c*e + 3*(a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^2) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c*e
+ 3*(a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)
^(1/4))/(a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.17

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx = \left(\sum_{k=1}^{4}\ln\left(\frac{c^2\left(3\,d\,e^2+2\,e^3\,x-\mathrm{root}(65536\,a^7\,c^2\,z^4+2048\,a^4\,c\,e^2\,z^2-1152\,a^2\,c\,d^2\,e\,z+81\,c\,d^4+16\,a\,e^4,z,k\right.}{\cdots}\right.\right.$$

$$\left.\left.+2048\,a^4\,c\,e^2\,z^2-1152\,a^2\,c\,d^2\,e\,z+81\,c\,d^4+16\,a\,e^4,z,k)\right)\right)+\frac{\frac{e\,x^2}{4\,a}+\frac{d\,x}{4\,a}}{c\,x^4+a}$$

```
[In] int((d + e*x)/(a + c*x^4)^2,x)
```

```
[Out] symsum(log((c^2*(3*d*e^2 + 2*e^3*x - 192*root(65536*a^7*c^2*z^4 + 2048*a^4*
c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k)^2*a^3*c*d + 128
*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^
4 + 16*a*e^4, z, k)^2*a^3*c*e*x - 36*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^
2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k)*a*c*d^2*x))/(16*a^3
))*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*
d^4 + 16*a*e^4, z, k), k, 1, 4) + ((e*x^2)/(4*a) + (d*x)/(4*a))/(a + c*x^4)
```

3.404 $\int \frac{1}{(a+cx^4)^2}\,dx$

Optimal result 2438
Rubi [A] (verified) 2438
Mathematica [A] (verified) 2441
Maple [C] (verified) 2441
Fricas [C] (verification not implemented) 2442
Sympy [A] (verification not implemented) 2442
Maxima [A] (verification not implemented) 2442
Giac [A] (verification not implemented) 2443
Mupad [B] (verification not implemented) 2444

Optimal result

Integrand size = 9, antiderivative size = 202

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4a(a+cx^4)} - \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

```
[Out] 1/4*x/a/(c*x^4+a)+3/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4)
*2^(1/2)+3/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4)*2^(1/2)-3
/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1/4)*2^(1
/2)+3/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1/4)*
2^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {205, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{(a+cx^4)^2}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{x}{4a(a+cx^4)}$$

```
[In] Int[(a + c*x^4)^(-2),x]
```

```
[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \frac{x}{4a(a+cx^4)} + \frac{3\int \frac{1}{a+cx^4}\,dx}{4a}$$

$$= \frac{x}{4a(a+cx^4)} + \frac{3\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{8a^{3/2}} + \frac{3\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{8a^{3/2}}$$

$$= \frac{x}{4a(a+cx^4)} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} - \frac{3\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

$$= \frac{x}{4a(a+cx^4)} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

$$= \frac{x}{4a(a+cx^4)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{\frac{8a^{3/4}x}{a+cx^4} - \frac{6\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} - \frac{3\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{\sqrt[4]{c}} + \frac{3\sqrt{2}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\right.}{\sqrt[4]{c}}}{32a^{7/4}}$$

```
[In] Integrate[(a + c*x^4)^(-2),x]
```

```
[Out] ((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4)
 - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/
4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(
1/4))/(32*a^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.76 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.23

method	result	size
risch	$\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16ac}$	46
default	$\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}$	118

```
[In] int(1/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x/a/(c*x^4+a)+3/16/a/c*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+cx^4)^2}\,dx$$
$$= \frac{3\,(acx^4+a^2)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)-3\,(-i\,acx^4-i\,a^2)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(i\,a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)-3\,(i\,acx^4}{16\,(acx^4+a^2}$$

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(3*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*log(a^2*(-1/(a^7*c))^(1/4) + x)
- 3*(-I*a*c*x^4 - I*a^2)*(-1/(a^7*c))^(1/4)*log(I*a^2*(-1/(a^7*c))^(1/4) +
x) - 3*(I*a*c*x^4 + I*a^2)*(-1/(a^7*c))^(1/4)*log(-I*a^2*(-1/(a^7*c))^(1/4)
 + x) - 3*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*log(-a^2*(-1/(a^7*c))^(1/4) +
x) + 4*x)/(a*c*x^4 + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.19

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4a^2+4acx^4}+\operatorname{RootSum}\left(65536t^4a^7c+81,\left(t\mapsto t\log\left(\frac{16ta^2}{3}+x\right)\right)\right)$$

```
[In] integrate(1/(c*x**4+a)**2,x)
```

```
[Out] x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda(_t, _t*lo
g(16*_t*a**2/3 + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4\,(acx^4+a^2)}$$
$$+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c}x^2-\sqrt{2}\right.}{a^{\frac{3}{4}}c}\right.}{32\,a}$$

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x/(a*c*x^4 + a^2) + 3/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + s
qrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c
))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c
)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*lo
g(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4\,(cx^4+a)a} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c} - \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c}$$

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
+ sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt(2)*(a*c^3)^(1/4)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/32*sq
rt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c) -
3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
2*c)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.29

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4\,a\,(c\,x^4+a)}+\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{7/4}\,c^{1/4}}+\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{7/4}\,c^{1/4}}$$

```
[In] int(1/(a + c*x^4)^2,x)
```

```
[Out] x/(4*a*(a + c*x^4)) + (3*atan((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4
)) + (3*atanh((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4))
```

3.405 $\int \frac{1}{(d+ex)(a+cx^4)^2}\,dx$

Optimal result 2446
Rubi [A] (verified) 2447
Mathematica [A] (verified) 2454
Maple [A] (verified) 2455
Fricas [F(-1)] 2456
Sympy [F(-1)] 2456
Maxima [A] (verification not implemented) 2456
Giac [A] (verification not implemented) 2457
Mupad [B] (verification not implemented) 2458

Optimal result

Integrand size = 17, antiderivative size = 855

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^2}\,dx = \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} - \frac{\sqrt{c}d^2e^5\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)^2} - \frac{\sqrt{c}d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^4+ae^4\right)} - \frac{\sqrt[4]{c}de^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2} - \frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)} + \frac{\sqrt[4]{c}de^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2} + \frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)} + \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\sqrt[4]{c}de^4\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2} - \frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)} + \frac{\sqrt[4]{c}de^4\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2} + \frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)} - \frac{e^7\log\left(a+cx^4\right)}{4\left(cd^4+ae^4\right)^2}$$

```
[Out] 1/4*(a*e^3+c*x*(d*e^2*x^2-d^2*e*x+d^3))/a/(a*e^4+c*d^4)/(c*x^4+a)+e^7*ln(e*
x+d)/(a*e^4+c*d^4)^2-1/4*e^7*ln(c*x^4+a)/(a*e^4+c*d^4)^2-1/4*d^2*e*arctan(x
^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^4+c*d^4)-1/2*d^2*e^5*arctan(x^2*c^
(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^2/a^(1/2)-1/8*c^(1/4)*d*e^4*ln(-a^(1/4
)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)
/(a*e^4+c*d^4)^2*2^(1/2)+1/8*c^(1/4)*d*e^4*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(
1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2
)+1/4*c^(1/4)*d*e^4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c
^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/4*c^(1/4)*d*e^4*arctan(1+c^(1/4)*
```

```
x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2
)-1/32*c^(1/4)*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a
^(1/2)+3*d^2*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)*2^(1/2)+1/32*c^(1/4)*d*ln(a^(1/
4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7
/4)/(a*e^4+c*d^4)*2^(1/2)+1/16*c^(1/4)*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4
))*(e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)*2^(1/2)+1/16*c^(1/4)*d
*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/(a
*e^4+c*d^4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 855, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {6874, 1868, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 266}

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^2}\,dx = \frac{\log(d+ex)e^7}{\left(cd^4+ae^4\right)^2}-\frac{\log\left(cx^4+a\right)e^7}{4\left(cd^4+ae^4\right)^2}-\frac{\sqrt{c}d^2\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{2\sqrt{a}\left(cd^4+ae^4\right)^2}$$
$$-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}$$
$$+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^4}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}$$
$$-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}$$
$$+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}$$
$$-\frac{\sqrt{c}d^2\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{4a^{3/2}\left(cd^4+ae^4\right)}+\frac{ae^3+cx(d^3-exd^2+e^2x^2d)}{4a\left(cd^4+ae^4\right)\left(cx^4+a\right)}$$
$$-\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)}$$
$$+\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)}$$
$$-\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)}$$
$$+\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)}$$

```
[In] Int[1/((d + e*x)*(a + c*x^4)^2),x]
```

```
[Out] (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(4*a*(c*d^4 + a*e^4)*(a + c*x^4))
 - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^
4)^2) - (Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a
*e^4)) - (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(3*Sqr
t[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2)
+ (c^(1/4)*d*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (e^7*Log[d + e*x])/(c*d^4 +
a*e^4)^2 - (c^(1/4)*d*e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) -
(c^(1/4)*d*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^4
*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(3*Sqrt[c]*d^2
 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16
*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) - (e^7*Log[a + c*x^4])/(4*(c*d^4 + a*e^4)
^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
```

```
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^8}{(cd^4+ae^4)^2(d+ex)} + \frac{c(d^3-d^2ex+de^2x^2-e^3x^3)}{(cd^4+ae^4)(a+cx^4)^2} - \frac{ce^4(-d^3+d^2ex-de^2x^2+e^3x^3)}{(cd^4+ae^4)^2(a+cx^4)}\right) dx$$

$$= \frac{e^7\log(d+ex)}{(cd^4+ae^4)^2} - \frac{(ce^4)\int \frac{-d^3+d^2ex-de^2x^2+e^3x^3}{a+cx^4}\,dx}{(cd^4+ae^4)^2} + \frac{c\int \frac{d^3-d^2ex+de^2x^2-e^3x^3}{(a+cx^4)^2}\,dx}{cd^4+ae^4}$$

$$= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a(cd^4+ae^4)(a+cx^4)} + \frac{e^7\log(d+ex)}{(cd^4+ae^4)^2} - \frac{(ce^4)\int \left(\frac{-d^3-de^2x^2}{a+cx^4}+\frac{x(d^2e+e^3x^2)}{a+cx^4}\right) dx}{(cd^4+ae^4)^2} - \frac{c\int \frac{-3d^3+2d^2ex-de^2x^2}{a+cx^4}\,dx}{4a(cd^4+ae^4)}$$

$$= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a(cd^4+ae^4)(a+cx^4)} + \frac{e^7\log(d+ex)}{(cd^4+ae^4)^2} - \frac{(ce^4)\int \frac{-d^3-de^2x^2}{a+cx^4}\,dx}{(cd^4+ae^4)^2} - \frac{(ce^4)\int \frac{x(d^2e+e^3x^2)}{a+cx^4}\,dx}{(cd^4+ae^4)^2} - \frac{c\int \left(\frac{2d^2ex}{a+cx^4}+\frac{-3d^3-de^2x^2}{a+cx^4}\right) dx}{4a(cd^4+ae^4)}$$

$$= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a(cd^4+ae^4)(a+cx^4)} + \frac{e^7\log(d+ex)}{(cd^4+ae^4)^2} - \frac{(ce^4)\,\text{Subst}\left(\int \frac{d^2e+e^3x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^4+ae^4)^2} + \frac{\left(de^4\left(\frac{\sqrt{c}d^2}{\sqrt{a}}-e^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2(cd^4+ae^4)^2} + \frac{\left(de^4\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2(cd^4+ae^4)^2} - \frac{c\int \frac{-3d^3-de^2x^2}{a+cx^4}\,dx}{4a(cd^4+ae^4)} - \frac{(cd^2e)\int \frac{x}{a+cx^4}\,dx}{2a(cd^4+ae^4)}$$

$$= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)}+\frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2}$$
$$-\frac{\left(cd^2e^5\right)\mathrm{Subst}\left(\int\frac{1}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)^2}-\frac{\left(ce^7\right)\mathrm{Subst}\left(\int\frac{x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)^2}$$
$$+\frac{\left(de^4\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^4+ae^4\right)^2}+\frac{\left(de^4\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^4+ae^4\right)^2}$$
$$-\frac{\left(\sqrt[4]{c}de^4\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}$$
$$-\frac{\left(\sqrt[4]{c}de^4\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}-\frac{\left(cd^2e\right)\mathrm{Subst}\left(\int\frac{1}{a+cx^2}\,dx,x,x^2\right)}{4a\left(cd^4+ae^4\right)}$$
$$+\frac{\left(d\left(\frac{3\sqrt{c}d^2}{\sqrt{a}}-e^2\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a\left(cd^4+ae^4\right)}+\frac{\left(d\left(\frac{3\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8a\left(cd^4+ae^4\right)}$$

$$= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} - \frac{\sqrt{c}d^2e^5\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)^2} - \frac{\sqrt{c}d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^4+ae^4\right)}$$

$$+ \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\sqrt[4]{c}de^4\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}$$

$$+ \frac{\sqrt[4]{c}de^4\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2} - \frac{e^7\log\left(a+cx^4\right)}{4\left(cd^4+ae^4\right)^2}$$

$$+ \frac{\left(\sqrt[4]{c}de^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}$$

$$- \frac{\left(\sqrt[4]{c}de^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}$$

$$+ \frac{\left(d\left(\frac{3\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a\left(cd^4+ae^4\right)} + \frac{\left(d\left(\frac{3\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a\left(cd^4+ae^4\right)}$$

$$- \frac{\left(\sqrt[4]{c}d\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)}$$

$$- \frac{\left(\sqrt[4]{c}d\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)}$$

$$= \frac{ae^3 + cx(d^3 - d^2ex + de^2x^2)}{4a(cd^4 + ae^4)(a + cx^4)} - \frac{\sqrt{c}d^2e^5 \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(cd^4 + ae^4)^2}$$

$$- \frac{\sqrt{c}d^2e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}(cd^4 + ae^4)} - \frac{\sqrt[4]{c}de^4(\sqrt{c}d^2 + \sqrt{a}e^2)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^4 + ae^4)^2}$$

$$+ \frac{\sqrt[4]{c}de^4(\sqrt{c}d^2 + \sqrt{a}e^2)\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^4 + ae^4)^2} + \frac{e^7\log(d + ex)}{(cd^4 + ae^4)^2}$$

$$- \frac{\sqrt[4]{c}de^4(\sqrt{c}d^2 - \sqrt{a}e^2)\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^4 + ae^4)^2}$$

$$- \frac{\sqrt[4]{c}d(3\sqrt{c}d^2 - \sqrt{a}e^2)\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(cd^4 + ae^4)}$$

$$+ \frac{\sqrt[4]{c}de^4(\sqrt{c}d^2 - \sqrt{a}e^2)\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^4 + ae^4)^2}$$

$$+ \frac{\sqrt[4]{c}d(3\sqrt{c}d^2 - \sqrt{a}e^2)\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(cd^4 + ae^4)} - \frac{e^7\log(a + cx^4)}{4(cd^4 + ae^4)^2}$$

$$+ \frac{\left(\sqrt[4]{c}d(3\sqrt{c}d^2 + \sqrt{a}e^2)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^4 + ae^4)}$$

$$- \frac{\left(\sqrt[4]{c}d(3\sqrt{c}d^2 + \sqrt{a}e^2)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^4 + ae^4)}$$

$$
\begin{aligned}
&= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a\,(cd^4+ae^4)\,(a+cx^4)} - \frac{\sqrt{c}d^2e^5\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(cd^4+ae^4)^2} \\
&\quad - \frac{\sqrt{c}d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\,(cd^4+ae^4)} - \frac{\sqrt[4]{c}de^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^2} \\
&\quad - \frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\,(cd^4+ae^4)} \\
&\quad + \frac{\sqrt[4]{c}de^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^2} \\
&\quad + \frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\,(cd^4+ae^4)} + \frac{e^7\log(d+ex)}{(cd^4+ae^4)^2} \\
&\quad - \frac{\sqrt[4]{c}de^4\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)^2} \\
&\quad - \frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\,(cd^4+ae^4)} \\
&\quad + \frac{\sqrt[4]{c}de^4\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)^2} \\
&\quad + \frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\,(cd^4+ae^4)} - \frac{e^7\log\left(a+cx^4\right)}{4\,(cd^4+ae^4)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 558, normalized size of antiderivative = 0.65

$$
\int \frac{1}{(d+ex)\,(a+cx^4)^2}\,dx
$$

$$
= \frac{\frac{8(cd^4+ae^4)\left(ae^3+cdx\left(d^2-dex+e^2x^2\right)\right)}{a(a+cx^4)} - \frac{2\sqrt[4]{c}d\left(3\sqrt{2}c^{3/2}d^6-4\sqrt[4]{a}c^{5/4}d^5e+\sqrt{2}\sqrt{a}cd^4e^2+7\sqrt{2}a\sqrt{c}d^2e^4-12a^{5/4}\sqrt[4]{c}de^5+5\sqrt{2}a^{3/2}e^6\right)\arctan}{a^{7/4}}}{}
$$

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)^2),x]
```

```
[Out] ((8*(c*d^4 + a*e^4)*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(a*(a + c*x^4)
) - (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 - 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt[2]
*Sqrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 - 12*a^(5/4)*c^(1/4)*d*e^5
 + 5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4)
+ (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 + 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt[2]*S
```

qrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 + 12*a^(5/4)*c^(1/4)*d*e^5 + 5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + 32*e^7*Log[d + e*x] + (Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^7 + Sqrt[a]*c*d^5*e^2 - 7*a*Sqrt[c]*d^3*e^4 + 5*a^(3/2)*d*e^6)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^7 - Sqrt[a]*c*d^5*e^2 + 7*a*Sqrt[c]*d^3*e^4 - 5*a^(3/2)*d*e^6)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 8*e^7*Log[a + c*x^4])/(32*(c*d^4 + a*e^4)^2)

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.50

method	result
default	$\frac{c\left(\frac{\frac{d e^2\left(e^4 a+d^4 c\right)x^3}{4a}-\frac{d^2 e\left(e^4 a+d^4 c\right)x^2}{4a}+\frac{d^3\left(e^4 a+d^4 c\right)x}{4a}+\frac{e^3\left(e^4 a+d^4 c\right)}{4c}}{c x^4+a}+\frac{\frac{\left(7a d^3 e^4+3c d^7\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\cdots\right.\right.}{8a}}{\cdots}\right.}{\cdots}$
risch	$\frac{\frac{c d e^2 x^3}{4a\left(e^4 a+d^4 c\right)}-\frac{d^2 c e x^2}{4a\left(e^4 a+d^4 c\right)}+\frac{d^3 c x}{4a\left(e^4 a+d^4 c\right)}+\frac{e^3}{4e^4 a+4d^4 c}}{c x^4+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^9 e^8+2a^8 c d^4 e^4+a^7 c^2 d^8\right)\_Z^4+16a^7 e^7\_Z^3+\left(96a^5 e^6+20a^4\cdots\right.\right.}\cdots\right.}{\cdots}$

[In] int(1/(e*x+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)

[Out] c/(a*e^4+c*d^4)^2*((1/4*d*e^2*(a*e^4+c*d^4)/a*x^3-1/4*d^2*e*(a*e^4+c*d^4)/a*x^2+1/4*d^3*(a*e^4+c*d^4)/a*x+1/4*e^3*(a*e^4+c*d^4)/c)/(c*x^4+a)+1/4/a*(1/8*(7*a*d^3*e^4+3*c*d^7)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/2*(-6*a*d^2*e^5-2*c*d^6*e)/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*(5*a*d*e^6+c*d^5*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))-a*e^7/c*ln(c*x^4+a)))+e^7*ln(e*x+d)/(a*e^4+c*d^4)^2

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(a+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(a+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 601, normalized size of antiderivative = 0.70

$$\int \frac{1}{(d+ex)(a+cx^4)^2}\,dx = \frac{e^7\log(ex+d)}{c^2d^8+2\,acd^4e^4+a^2e^8} - \frac{c\left(\frac{\sqrt{2}\left(4\sqrt{2}a^{\frac{7}{4}}c^{\frac{1}{4}}e^7-3\,c^2d^7+\sqrt{a}c^{\frac{3}{2}}d^5e^2-7\,acd^3e^4+5\,a^{\frac{3}{2}}\sqrt{c}de^6\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}+\frac{\sqrt{2}\left(4\sqrt{2}a^{\frac{7}{4}}c^{\frac{1}{4}}e^7+3\,c^2d^7-\sqrt{a}c^{\frac{3}{2}}d^5e^2+7\,a\cdots\right)\cdots}{\cdots}\cdots\right)}{\cdots} + \frac{cde^2x^3-cd^2ex^2+cd^3x+ae^3}{4\,(a^2cd^4+a^3e^4+(ac^2d^4+a^2ce^4)x^4)}$$

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] e^7*log(e*x + d)/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) - 1/32*c*(sqrt(2)*(4*s
qrt(2)*a^(7/4)*c^(1/4)*e^7 - 3*c^2*d^7 + sqrt(a)*c^(3/2)*d^5*e^2 - 7*a*c*d^
3*e^4 + 5*a^(3/2)*sqrt(c)*d*e^6)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*
x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(4*sqrt(2)*a^(7/4)*c^(1/4)*e^7 + 3
*c^2*d^7 - sqrt(a)*c^(3/2)*d^5*e^2 + 7*a*c*d^3*e^4 - 5*a^(3/2)*sqrt(c)*d*e^
6)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4))
```

```
 - 2*(3*sqrt(2)*a^(1/4)*c^(9/4)*d^7 + sqrt(2)*a^(3/4)*c^(7/4)*d^5*e^2 + 7*s
qrt(2)*a^(5/4)*c^(5/4)*d^3*e^4 + 5*sqrt(2)*a^(7/4)*c^(3/4)*d*e^6 + 4*sqrt(a
)*c^2*d^6*e + 12*a^(3/2)*c*d^2*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c
^(5/4)) - 2*(3*sqrt(2)*a^(1/4)*c^(9/4)*d^7 + sqrt(2)*a^(3/4)*c^(7/4)*d^5*e^
2 + 7*sqrt(2)*a^(5/4)*c^(5/4)*d^3*e^4 + 5*sqrt(2)*a^(7/4)*c^(3/4)*d*e^6 - 4
*sqrt(a)*c^2*d^6*e - 12*a^(3/2)*c*d^2*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x
- sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqr
t(c))*c^(5/4)))/(a*c^2*d^8 + 2*a^2*c*d^4*e^4 + a^3*e^8) + 1/4*(c*d*e^2*x^3
- c*d^2*e*x^2 + c*d^3*x + a*e^3)/(a^2*c*d^4 + a^3*e^4 + (a*c^2*d^4 + a^2*c*
e^4)*x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 795, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+ex)(a+cx^4)^2}\,dx = \frac{e^8\log(|ex+d|)}{c^2d^8e+2\,acd^4e^5+a^2e^9} - \frac{e^7\log(|cx^4+a|)}{4\,(c^2d^8+2\,acd^4e^4+a^2e^8)}$$
$$+ \frac{\left(4\sqrt{2}\sqrt{ac}c^2d^2e+3\,(ac^3)^{\frac14}c^2d^3+5\,(ac^3)^{\frac34}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{8\left(\sqrt{2}a^2c^3d^4+\sqrt{2}a^3c^2e^4+4\sqrt{2}\sqrt{ac}a^2c^2d^2e^2-4\,(ac^3)^{\frac14}a^2c^2d^3e-4\,(ac^3)^{\frac34}a^2de^3\right)}$$
$$+ \frac{\left(4\sqrt{2}\sqrt{ac}c^2d^2e+3\,(ac^3)^{\frac14}c^2d^3+5\,(ac^3)^{\frac34}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{8\left(\sqrt{2}a^2c^3d^4+\sqrt{2}a^3c^2e^4+4\sqrt{2}\sqrt{ac}a^2c^2d^2e^2+4\,(ac^3)^{\frac14}a^2c^2d^3e+4\,(ac^3)^{\frac34}a^2de^3\right)}$$
$$+ \frac{\left(3\sqrt{2}(ac^3)^{\frac14}c^3d^7+7\sqrt{2}(ac^3)^{\frac14}ac^2d^3e^4-\sqrt{2}(ac^3)^{\frac34}cd^5e^2-5\sqrt{2}(ac^3)^{\frac34}ade^6\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\surd\right.}{32\,(a^2c^4d^8+2\,a^3c^3d^4e^4+a^4c^2e^8)}$$
$$- \frac{\left(3\sqrt{2}(ac^3)^{\frac14}c^3d^7+7\sqrt{2}(ac^3)^{\frac14}ac^2d^3e^4-\sqrt{2}(ac^3)^{\frac34}cd^5e^2-5\sqrt{2}(ac^3)^{\frac34}ade^6\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\surd\right.}{32\,(a^2c^4d^8+2\,a^3c^3d^4e^4+a^4c^2e^8)}$$
$$+ \frac{acd^4e^3+a^2e^7+(c^2d^5e^2+acde^6)x^3-(c^2d^6e+acd^2e^5)x^2+(c^2d^7+acd^3e^4)x}{4\,(cd^4+ae^4)^2(cx^4+a)a}$$

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] e^8*log(abs(e*x + d))/(c^2*d^8*e + 2*a*c*d^4*e^5 + a^2*e^9) - 1/4*e^7*log(a
bs(c*x^4 + a))/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) + 1/8*(4*sqrt(2)*sqrt(a*
c)*c^2*d^2*e + 3*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*
sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^3*d^4 + sqr
t(2)*a^3*c^2*e^4 + 4*sqrt(2)*sqrt(a*c)*a^2*c^2*d^2*e^2 - 4*(a*c^3)^(1/4)*a^
```

```
2*c^2*d^3*e - 4*(a*c^3)^(3/4)*a^2*d*e^3) + 1/8*(4*sqrt(2)*sqrt(a*c)*c^2*d^2
*e + 3*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^3*d^4 + sqrt(2)*a^3*c
^2*e^4 + 4*sqrt(2)*sqrt(a*c)*a^2*c^2*d^2*e^2 + 4*(a*c^3)^(1/4)*a^2*c^2*d^3*
e + 4*(a*c^3)^(3/4)*a^2*d*e^3) + 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*c^3*d^7 + 7*
sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^3*e^4 - sqrt(2)*(a*c^3)^(3/4)*c*d^5*e^2 - 5*s
qrt(2)*(a*c^3)^(3/4)*a*d*e^6)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/
(a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a^4*c^2*e^8) - 1/32*(3*sqrt(2)*(a*c^3)^(
1/4)*c^3*d^7 + 7*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^3*e^4 - sqrt(2)*(a*c^3)^(3/4
)*c*d^5*e^2 - 5*sqrt(2)*(a*c^3)^(3/4)*a*d*e^6)*log(x^2 - sqrt(2)*x*(a/c)^(1
/4) + sqrt(a/c))/(a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a^4*c^2*e^8) + 1/4*(a*c
*d^4*e^3 + a^2*e^7 + (c^2*d^5*e^2 + a*c*d*e^6)*x^3 - (c^2*d^6*e + a*c*d^2*e
^5)*x^2 + (c^2*d^7 + a*c*d^3*e^4)*x)/((c*d^4 + a*e^4)^2*(c*x^4 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 1591, normalized size of antiderivative = 1.86

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + c*x^4)^2*(d + e*x)),x)
```

```
[Out] e^3/(4*(a^2*e^4 + c^2*d^4*x^4 + a*c*d^4 + a*c*e^4*x^4)) + symsum(log((81*c^
5*d^5*e^6 + 64*a*c^4*d*e^10)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)
) + root(131072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z
^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*
a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*(root(131072*a
^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^
7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4
096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*(root(131072*a^8*c*d^4*e^4*z^4
 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*
c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 8
1*c*d^4 + 256*a*e^4, z, k)*(root(131072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d
^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 2
4576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e
^4, z, k)*((98304*a^9*c^4*d*e^14 - 32768*a^6*c^7*d^13*e^2 + 32768*a^7*c^6*d
^9*e^6 + 163840*a^8*c^5*d^5*e^10)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4
*e^4)) + (x*(81920*a^9*c^4*e^15 - 49152*a^6*c^7*d^12*e^3 - 16384*a^7*c^6*d^
8*e^7 + 114688*a^8*c^5*d^4*e^11))/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4
*e^4))) + (52224*a^7*c^4*d*e^13 - 3072*a^4*c^7*d^13*e + 13312*a^5*c^6*d^9*e
^5 + 68608*a^6*c^5*d^5*e^9)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))
 + (x*(61440*a^7*c^4*e^14 - 8192*a^4*c^7*d^12*e^2 - 4096*a^5*c^6*d^8*e^6 +
65536*a^6*c^5*d^4*e^10))/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))) +
 (8704*a^5*c^4*d*e^12 + 3584*a^3*c^6*d^9*e^4 + 15360*a^4*c^5*d^5*e^8)/(256*
```

```
(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + (x*(15360*a^5*c^4*e^13 - 576*a
^2*c^7*d^12*e + 1920*a^3*c^6*d^8*e^5 + 18880*a^4*c^5*d^4*e^9))/(256*(a^6*e^
8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))) + (192*a*c^6*d^9*e^3 + 704*a^3*c^4*d*e
^11 + 1536*a^2*c^5*d^5*e^7)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))
 + (x*(1280*a^3*c^4*e^12 + 256*a*c^6*d^8*e^4 + 2240*a^2*c^5*d^4*e^8))/(256*
(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))) + (81*c^5*d^4*e^7*x)/(256*(a^6*
e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)))*root(131072*a^8*c*d^4*e^4*z^4 + 6553
6*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*
e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^
4 + 256*a*e^4, z, k), k, 1, 4) + (e^7*log(d + e*x))/(a^2*e^8 + c^2*d^8 + 2*
a*c*d^4*e^4) + (c*d^3*x)/(4*(a^3*e^4 + a^2*c*d^4 + a*c^2*d^4*x^4 + a^2*c*e^
4*x^4)) - (c*d^2*e*x^2)/(4*(a^3*e^4 + a^2*c*d^4 + a*c^2*d^4*x^4 + a^2*c*e^4
*x^4)) + (c*d*e^2*x^3)/(4*(a^3*e^4 + a^2*c*d^4 + a*c^2*d^4*x^4 + a^2*c*e^4*
x^4))
```

3.406 $\int \frac{1}{(d+ex)^2(a+cx^4)^2} dx$

Optimal result 2461
Rubi [A] (verified) 2462
Mathematica [A] (verified) 2472
Maple [A] (verified) 2473
Fricas [F(-1)] 2473
Sympy [F(-1)] 2474
Maxima [A] (verification not implemented) 2474
Giac [A] (verification not implemented) 2475
Mupad [B] (verification not implemented) 2476

Optimal result

Integrand size = 17, antiderivative size = 1141

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^2}\,dx$$
$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)\,x+e^2(3cd^4-ae^4)\,x^2))}{4a\left(cd^4+ae^4\right)^2(a+cx^4)}$$
$$-\frac{\sqrt{c}de^5(3cd^4-ae^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\left(cd^4+ae^4\right)^3} - \frac{\sqrt{c}de(cd^4-ae^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(cd^4+ae^4\right)^2}$$
$$-\frac{\sqrt[4]{c}\left(3\sqrt{c}d^2(cd^4-3ae^4)+\sqrt{a}e^2(3cd^4-ae^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}$$
$$-\frac{\sqrt[4]{c}e^4\left(\sqrt{c}d^2(5cd^4-3ae^4)+\sqrt{a}e^2(7cd^4-ae^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}$$
$$+\frac{\sqrt[4]{c}\left(3\sqrt{c}d^2(cd^4-3ae^4)+\sqrt{a}e^2(3cd^4-ae^4)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}$$
$$+\frac{\sqrt[4]{c}e^4\left(\sqrt{c}d^2(5cd^4-3ae^4)+\sqrt{a}e^2(7cd^4-ae^4)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}$$
$$+\frac{8cd^3e^7\log(d+ex)}{\left(cd^4+ae^4\right)^3}$$
$$-\frac{\sqrt[4]{c}\left(3\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}$$
$$-\frac{\sqrt[4]{c}e^4\left(\sqrt{c}d^2(5cd^4-3ae^4)-\sqrt{a}e^2(7cd^4-ae^4)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}$$
$$+\frac{\sqrt[4]{c}\left(3\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}$$
$$+\frac{\sqrt[4]{c}e^4\left(\sqrt{c}d^2(5cd^4-3ae^4)-\sqrt{a}e^2(7cd^4-ae^4)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}$$
$$-\frac{2cd^3e^7\log\left(a+cx^4\right)}{\left(cd^4+ae^4\right)^3}$$

```
[Out] -e^7/(a*e^4+c*d^4)^2/(e*x+d)+1/4*c*(4*a*d^3*e^3+x*(d^2*(-3*a*e^4+c*d^4)-2*d
*e*(-a*e^4+c*d^4)*x+e^2*(-a*e^4+3*c*d^4)*x^2))/a/(a*e^4+c*d^4)^2/(c*x^4+a)+
8*c*d^3*e^7*ln(e*x+d)/(a*e^4+c*d^4)^3-2*c*d^3*e^7*ln(c*x^4+a)/(a*e^4+c*d^4)
```

```
^3-1/2*d*e*(-a*e^4+c*d^4)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^
4+c*d^4)^2-d*e^5*(-a*e^4+3*c*d^4)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^
4+c*d^4)^3/a^(1/2)-1/32*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c
^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(7
/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/32*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/
2)+x^2*c^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+c*d^4)*c^(1/
2))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/16*c^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2
)/a^(1/4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^
(7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/16*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/
4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(7/4)/(
a*e^4+c*d^4)^2*2^(1/2)-1/8*c^(1/4)*e^4*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2
)+x^2*c^(1/2))*(-e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+5*c*d^4)*c^(1/2
))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/8*c^(1/4)*e^4*ln(a^(1/4)*c^(1/4)*x*2^(
1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+5*c*
d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(1/4)*e^4*arctan(-1+c^(
1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+5*c*d^4
)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(1/4)*e^4*arctan(1+c^(1/4)
*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+5*c*d^4)*c^
(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 1141, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules

used = {6874, 1868, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 266}

$$
\begin{aligned}
&\int \frac{1}{(d+ex)^2 (a+cx^4)^2}\,dx \\
&= \frac{8cd^3\log(d+ex)e^7}{(cd^4+ae^4)^3} - \frac{2cd^3\log\left(cx^4+a\right)e^7}{(cd^4+ae^4)^3} \\
&- \frac{e^7}{(cd^4+ae^4)^2(d+ex)} - \frac{\sqrt{c}d(3cd^4-ae^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{\sqrt{a}\,(cd^4+ae^4)^3} \\
&- \frac{\sqrt[4]{c}\left(\sqrt{c}(5cd^4-3ae^4)\,d^2+\sqrt{a}e^2(7cd^4-ae^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^3} \\
&+ \frac{\sqrt[4]{c}\left(\sqrt{c}(5cd^4-3ae^4)\,d^2+\sqrt{a}e^2(7cd^4-ae^4)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^4}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^3} \\
&- \frac{\sqrt[4]{c}\left(\sqrt{c}d^2(5cd^4-3ae^4)-\sqrt{a}e^2(7cd^4-ae^4)\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)^3} \\
&+ \frac{\sqrt[4]{c}\left(\sqrt{c}d^2(5cd^4-3ae^4)-\sqrt{a}e^2(7cd^4-ae^4)\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)^3} \\
&- \frac{\sqrt{c}d(cd^4-ae^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{2a^{3/2}\,(cd^4+ae^4)^2} \\
&+ \frac{c(4ad^3e^3+x((cd^4-3ae^4)\,d^2-2e(cd^4-ae^4)\,xd+e^2(3cd^4-ae^4)\,x^2))}{4a\,(cd^4+ae^4)^2\,(cx^4+a)} \\
&- \frac{\sqrt[4]{c}\left(3\sqrt{c}(cd^4-3ae^4)\,d^2+\sqrt{a}e^2(3cd^4-ae^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\,(cd^4+ae^4)^2} \\
&+ \frac{\sqrt[4]{c}\left(3\sqrt{c}(cd^4-3ae^4)\,d^2+\sqrt{a}e^2(3cd^4-ae^4)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\,(cd^4+ae^4)^2} \\
&- \frac{\sqrt[4]{c}\left(3\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\,(cd^4+ae^4)^2} \\
&+ \frac{\sqrt[4]{c}\left(3\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\,(cd^4+ae^4)^2}
\end{aligned}
$$

```
[In] Int[1/((d + e*x)^2*(a + c*x^4)^2),x]
```

```
[Out] -(e^7/((c*d^4 + a*e^4)^2*(d + e*x))) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3*
a*e^4) - 2*d*e*(c*d^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4
 + a*e^4)^2*(a + c*x^4)) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]
*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^3) - (Sqrt[c]*d*e*(c*d^4 - a*e^4)*
```

```
ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^2) - (c^(1/4)*(3*
Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/
4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3
) + (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^
4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*
e^4)^2) + (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*
d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(
c*d^4 + a*e^4)^3) + (8*c*d^3*e^7*Log[d + e*x])/(c*d^4 + a*e^4)^3 - (c^(1/4)
*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4
+ a*e^4)^2) - (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(
7*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a
*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(
Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a
*e^4)^3) - (2*c*d^3*e^7*Log[a + c*x^4])/(c*d^4 + a*e^4)^3
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
```

```
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^8}{(cd^4+ae^4)^2(d+ex)^2} + \frac{8cd^3e^8}{(cd^4+ae^4)^3(d+ex)} + \frac{c(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)x+e^2(3cd^4-ae^4)x^2-4cd^3e^3x^3)}{(cd^4+ae^4)^2(a+cx^4)^2} + \frac{ce^4(d^2(5cd^4-3ae^4)-2de(3cd^4-ae^4)x+e^2(7cd^4-ae^4)x^2-8cd^3e^3x^3)}{(cd^4+ae^4)^3(a+cx^4)}\right)dx$$

$$= -\frac{e^7}{(cd^4+ae^4)^2(d+ex)} + \frac{8cd^3e^7\log(d+ex)}{(cd^4+ae^4)^3} + \frac{(ce^4)\int \frac{d^2(5cd^4-3ae^4)-2de(3cd^4-ae^4)x+e^2(7cd^4-ae^4)x^2-8cd^3e^3x^3}{a+cx^4}\,dx}{(cd^4+ae^4)^3} + \frac{c\int \frac{d^2(cd^4-3ae^4)-2de(cd^4-ae^4)x+e^2(3cd^4-ae^4)x^2-4cd^3e^3x^3}{(a+cx^4)^2}\,dx}{(cd^4+ae^4)^2}$$

$$= -\frac{e^7}{(cd^4+ae^4)^2(d+ex)} + \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)x+e^2(3cd^4-ae^4)x^2))}{4a(cd^4+ae^4)^2(a+cx^4)} + \frac{8cd^3e^7\log(d+ex)}{(cd^4+ae^4)^3} + \frac{(ce^4)\int\left(\frac{x(-2de(3cd^4-ae^4)-8cd^3e^3x^2)}{a+cx^4}+\frac{d^2(5cd^4-3ae^4)+e^2(7cd^4-ae^4)x^2}{a+cx^4}\right)dx}{(cd^4+ae^4)^3} - \frac{c\int\frac{-3d^2(cd^4-3ae^4)+4de(cd^4-ae^4)x-e^2(3cd^4-ae^4)x^2}{a+cx^4}dx}{4a(cd^4+ae^4)^2}$$

$$= -\frac{e^7}{(cd^4+ae^4)^2(d+ex)} + \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)x+e^2(3cd^4-ae^4)x^2))}{4a(cd^4+ae^4)^2(a+cx^4)} + \frac{8cd^3e^7\log(d+ex)}{(cd^4+ae^4)^3} + \frac{(ce^4)\int\frac{x(-2de(3cd^4-ae^4)-8cd^3e^3x^2)}{a+cx^4}dx}{(cd^4+ae^4)^3} + \frac{(ce^4)\int\frac{d^2(5cd^4-3ae^4)+e^2(7cd^4-ae^4)x^2}{a+cx^4}dx}{(cd^4+ae^4)^3} - \frac{c\int\left(\frac{4de(cd^4-ae^4)x}{a+cx^4}+\frac{-3d^2(cd^4-3ae^4)-e^2(3cd^4-ae^4)x^2}{a+cx^4}\right)dx}{4a(cd^4+ae^4)^2}$$

$$= -\frac{e^7}{(cd^4+ae^4)^2(d+ex)} + \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)x+e^2(3cd^4-ae^4)x^2))}{4a(cd^4+ae^4)^2(a+cx^4)} + \frac{8cd^3e^7\log(d+ex)}{(cd^4+ae^4)^3} + \frac{(ce^4)\,\mathrm{Subst}\left(\int\frac{-2de(3cd^4-ae^4)-8cd^3e^3x}{a+cx^2}dx,x,x^2\right)}{2(cd^4+ae^4)^3} - \frac{c\int\frac{-3d^2(cd^4-3ae^4)-e^2(3cd^4-ae^4)x^2}{a+cx^4}dx}{4a(cd^4+ae^4)^2} - \frac{(cde(cd^4-ae^4))\int\frac{x}{a+cx^4}dx}{a(cd^4+ae^4)^2} - \frac{\left(e^4\left(7cd^4e^2-ae^6-\frac{\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}dx}{2(cd^4+ae^4)^3} + \frac{\left(e^4\left(7cd^4e^2-ae^6+\frac{\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}dx}{2(cd^4+ae^4)^3}$$

$$
\begin{aligned}
&= -\frac{e^7}{(cd^4+ae^4)^2(d+ex)} \\
&+ \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)x+e^2(3cd^4-ae^4)x^2))}{4a(cd^4+ae^4)^2(a+cx^4)} \\
&+ \frac{8cd^3e^7\log(d+ex)}{(cd^4+ae^4)^3} - \frac{(4c^2d^3e^7)\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx,x,x^2\right)}{(cd^4+ae^4)^3} \\
&- \frac{(cde^5(3cd^4-ae^4))\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{(cd^4+ae^4)^3} \\
&- \frac{(cde(cd^4-ae^4))\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{2a(cd^4+ae^4)^2} \\
&- \frac{\left(3cd^4e^2-ae^6-\frac{3\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a(cd^4+ae^4)^2} \\
&+ \frac{\left(3cd^4e^2-ae^6+\frac{3\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8a(cd^4+ae^4)^2} \\
&+ \frac{\left(\sqrt[4]{c}e^4\left(7cd^4e^2-ae^6-\frac{\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} \\
&+ \frac{\left(\sqrt[4]{c}e^4\left(7cd^4e^2-ae^6-\frac{\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} \\
&+ \frac{\left(e^4\left(7cd^4e^2-ae^6+\frac{\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4(cd^4+ae^4)^3} \\
&+ \frac{\left(e^4\left(7cd^4e^2-ae^6+\frac{\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4(cd^4+ae^4)^3}
\end{aligned}
$$

$$= -\frac{e^7}{(cd^4+ae^4)^2(d+ex)} + \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)x+e^2(3cd^4-ae^4)x^2))}{4a(cd^4+ae^4)^2(a+cx^4)} - \frac{\sqrt{c}de^5(3cd^4-ae^4)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}(cd^4+ae^4)^3} - \frac{\sqrt{c}de(cd^4-ae^4)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}(cd^4+ae^4)^2} + \frac{8cd^3e^7\log(d+ex)}{(cd^4+ae^4)^3} + \frac{\sqrt[4]{c}e^4\left(7cd^4e^2-ae^6-\frac{\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} - \frac{\sqrt[4]{c}e^4\left(7cd^4e^2-ae^6-\frac{\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} - \frac{2cd^3e^7\log(a+cx^4)}{(cd^4+ae^4)^3} + \frac{\left(\sqrt[4]{c}\left(3cd^4e^2-ae^6-\frac{3\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}dx}{16\sqrt{2}a^{5/4}(cd^4+ae^4)^2} + \frac{\left(\sqrt[4]{c}\left(3cd^4e^2-ae^6-\frac{3\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}dx}{16\sqrt{2}a^{5/4}(cd^4+ae^4)^2} + \frac{\left(3cd^4e^2-ae^6+\frac{3\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}dx}{16a(cd^4+ae^4)^2} + \frac{\left(3cd^4e^2-ae^6+\frac{3\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}dx}{16a(cd^4+ae^4)^2} + \frac{\left(\sqrt[4]{c}e^4\left(7cd^4e^2-ae^6+\frac{\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} - \frac{\left(\sqrt[4]{c}e^4\left(7cd^4e^2-ae^6+\frac{\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3}$$

$$= -\frac{e^7}{(cd^4+ae^4)^2(d+ex)} + \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)x+e^2(3cd^4-ae^4)x^2))}{4a(cd^4+ae^4)^2(a+cx^4)} - \frac{\sqrt{c}de^5(3cd^4-ae^4)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}(cd^4+ae^4)^3} - \frac{\sqrt{c}de(cd^4-ae^4)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}(cd^4+ae^4)^2} - \frac{\sqrt[4]{c}e^4\left(7cd^4e^2-ae^6+\frac{\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} + \frac{\sqrt[4]{c}e^4\left(7cd^4e^2-ae^6+\frac{\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} + \frac{8cd^3e^7\log(d+ex)}{(cd^4+ae^4)^3} + \frac{\sqrt[4]{c}\left(3cd^4e^2-ae^6-\frac{3\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}(cd^4+ae^4)^2} + \frac{\sqrt[4]{c}e^4\left(7cd^4e^2-ae^6-\frac{\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} - \frac{\sqrt[4]{c}\left(3cd^4e^2-ae^6-\frac{3\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}(cd^4+ae^4)^2} - \frac{\sqrt[4]{c}e^4\left(7cd^4e^2-ae^6-\frac{\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} - \frac{2cd^3e^7\log(a+cx^4)}{(cd^4+ae^4)^3} + \frac{\left(\sqrt[4]{c}\left(3cd^4e^2-ae^6+\frac{3\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}(cd^4+ae^4)^2} - \frac{\left(\sqrt[4]{c}\left(3cd^4e^2-ae^6+\frac{3\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}(cd^4+ae^4)^2}$$

$$
\begin{aligned}
&= -\frac{e^7}{(cd^4+ae^4)^2(d+ex)} \\
&+ \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)x+e^2(3cd^4-ae^4)x^2))}{4a(cd^4+ae^4)^2(a+cx^4)} \\
&- \frac{\sqrt{c}de^5(3cd^4-ae^4)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}(cd^4+ae^4)^3} - \frac{\sqrt{c}de(cd^4-ae^4)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}(cd^4+ae^4)^2} \\
&- \frac{\sqrt[4]{c}\left(3cd^4e^2-ae^6+\frac{3\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}(cd^4+ae^4)^2} \\
&- \frac{\sqrt[4]{c}e^4\left(7cd^4e^2-ae^6+\frac{\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} \\
&+ \frac{\sqrt[4]{c}\left(3cd^4e^2-ae^6+\frac{3\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}(cd^4+ae^4)^2} \\
&+ \frac{\sqrt[4]{c}e^4\left(7cd^4e^2-ae^6+\frac{\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} + \frac{8cd^3e^7\log(d+ex)}{(cd^4+ae^4)^3} \\
&+ \frac{\sqrt[4]{c}\left(3cd^4e^2-ae^6-\frac{3\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}(cd^4+ae^4)^2} \\
&+ \frac{\sqrt[4]{c}e^4\left(7cd^4e^2-ae^6-\frac{\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} \\
&- \frac{\sqrt[4]{c}\left(3cd^4e^2-ae^6-\frac{3\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}(cd^4+ae^4)^2} \\
&- \frac{\sqrt[4]{c}e^4\left(7cd^4e^2-ae^6-\frac{\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^3} \\
&- \frac{2cd^3e^7\log(a+cx^4)}{(cd^4+ae^4)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 807, normalized size of antiderivative = 0.71

$$\int \frac{1}{(d+ex)^2 \left(a+cx^4\right)^2} \, dx$$

$$= \frac{-\frac{32e^7(cd^4+ae^4)}{d+ex} + \frac{8c(cd^4+ae^4)(cd^4x(d^2-2dex+3e^2x^2)+ae^3(4d^3-3d^2ex+2de^2x^2-e^3x^3))}{a(a+cx^4)} + \frac{2\sqrt[4]{c}\left(-3\sqrt{2}c^{5/2}d^{10}+8\sqrt[4]{a}c^{9/4}d^9e-3\sqrt{2}\sqrt{a}\right.}{}}{}$$

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^4)^2),x]
```

```
[Out] ((-32*e^7*(c*d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(c*d^4*x*(d^2 -
 2*d*e*x + 3*e^2*x^2) + a*e^3*(4*d^3 - 3*d^2*e*x + 2*d*e^2*x^2 - e^3*x^3)))
/(a*(a + c*x^4)) + (2*c^(1/4)*(-3*Sqrt[2]*c^(5/2)*d^10 + 8*a^(1/4)*c^(9/4)*
d^9*e - 3*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 - 14*Sqrt[2]*a*c^(3/2)*d^6*e^4 + 48*a
^(5/4)*c^(5/4)*d^5*e^5 - 30*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 21*Sqrt[2]*a^2*Sqrt
[c]*d^2*e^8 - 24*a^(9/4)*c^(1/4)*d*e^9 + 5*Sqrt[2]*a^(5/2)*e^10)*ArcTan[1 -
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (2*c^(1/4)*(3*Sqrt[2]*c^(5/2)*d^10
 + 8*a^(1/4)*c^(9/4)*d^9*e + 3*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 + 14*Sqrt[2]*a*c
^(3/2)*d^6*e^4 + 48*a^(5/4)*c^(5/4)*d^5*e^5 + 30*Sqrt[2]*a^(3/2)*c*d^4*e^6
- 21*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 - 24*a^(9/4)*c^(1/4)*d*e^9 - 5*Sqrt[2]*a^(
5/2)*e^10)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + 256*c*d^3*e^7
*Log[d + e*x] - (Sqrt[2]*c^(1/4)*(3*c^(5/2)*d^10 - 3*Sqrt[a]*c^2*d^8*e^2 +
14*a*c^(3/2)*d^6*e^4 - 30*a^(3/2)*c*d^4*e^6 - 21*a^2*Sqrt[c]*d^2*e^8 + 5*a^
(5/2)*e^10)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4)
 + (Sqrt[2]*c^(1/4)*(3*c^(5/2)*d^10 - 3*Sqrt[a]*c^2*d^8*e^2 + 14*a*c^(3/2)*
d^6*e^4 - 30*a^(3/2)*c*d^4*e^6 - 21*a^2*Sqrt[c]*d^2*e^8 + 5*a^(5/2)*e^10)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 64*c*d^3*e
^7*Log[a + c*x^4])/(32*(c*d^4 + a*e^4)^3)
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 534, normalized size of antiderivative = 0.47

method	result
default	$-\frac{c\left(\frac{\frac{e^{2}\left(a^{2}e^{8}-2ac\,d^{4}e^{4}-3c^{2}d^{8}\right)x^{3}}{4a}-\frac{ed\left(a^{2}e^{8}-c^{2}d^{8}\right)x^{2}}{2a}+\frac{d^{2}\left(3a^{2}e^{8}+2ac\,d^{4}e^{4}-c^{2}d^{8}\right)x}{4a}-d^{3}e^{3}\left(e^{4}a+d^{4}c\right)}{c\,x^{4}+a}+\frac{\left(21a^{2}d^{2}e^{8}-14ac\,d^{6}e^{4}-3c^{2}d^{10}\right)\left(\right.}{}\right.}{}$
risch	$\frac{-\frac{e^{3}c\left(5e^{4}a-3d^{4}c\right)x^{4}}{4a\left(e^{4}a+d^{4}c\right)^{2}}+\frac{cd\,e^{2}x^{3}}{4a\left(e^{4}a+d^{4}c\right)}-\frac{d^{2}ce\,x^{2}}{4a\left(e^{4}a+d^{4}c\right)}+\frac{d^{3}cx}{4a\left(e^{4}a+d^{4}c\right)}-\frac{e^{3}\left(e^{4}a-d^{4}c\right)}{\left(e^{4}a+d^{4}c\right)^{2}}}{(ex+d)(c\,x^{4}+a)}+\frac{8d^{3}e^{7}c\ln(ex+d)}{a^{3}e^{12}+3a^{2}c\,d^{4}e^{8}+3a\,c^{2}d^{8}e^{4}+c^{3}d^{12}}+\frac{\left(\_R=\right.}{}$

```
[In] int(1/(e*x+d)^2/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -c/(a*e^4+c*d^4)^3*((1/4*e^2*(a^2*e^8-2*a*c*d^4*e^4-3*c^2*d^8)/a*x^3-1/2*e*
d*(a^2*e^8-c^2*d^8)/a*x^2+1/4*d^2*(3*a^2*e^8+2*a*c*d^4*e^4-c^2*d^8)/a*x-d^3
*e^3*(a*e^4+c*d^4))/(c*x^4+a)+1/4/a*(1/8*(21*a^2*d^2*e^8-14*a*c*d^6*e^4-3*c
^2*d^10)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/
(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+
2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/2*(-12*a^2*d*e^9+24*a*c*d^5*e^5+4*c^2*
d^9*e)/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*(5*a^2*e^10-30*a*c*d^4*e^6-3
*c^2*d^8*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1
/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*
x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+8*a*d^3*e^7*ln(c*x^4+a)))-e^7/(a*e^
4+c*d^4)^2/(e*x+d)+8*c*d^3*e^7*ln(e*x+d)/(a*e^4+c*d^4)^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^2} \, dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 961, normalized size of antiderivative = 0.84

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^2} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
[Out] 8*c*d^3*e^7*log(e*x + d)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^
3*e^12) - 1/32*c*(sqrt(2)*(32*sqrt(2)*a^(7/4)*c^(5/4)*d^3*e^7 - 3*c^3*d^10
+ 3*sqrt(a)*c^(5/2)*d^8*e^2 - 14*a*c^2*d^6*e^4 + 30*a^(3/2)*c^(3/2)*d^4*e^6
 + 21*a^2*c*d^2*e^8 - 5*a^(5/2)*sqrt(c)*e^10)*log(sqrt(c)*x^2 + sqrt(2)*a^(
1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(32*sqrt(2)*a^(7/4)*c
^(5/4)*d^3*e^7 + 3*c^3*d^10 - 3*sqrt(a)*c^(5/2)*d^8*e^2 + 14*a*c^2*d^6*e^4
- 30*a^(3/2)*c^(3/2)*d^4*e^6 - 21*a^2*c*d^2*e^8 + 5*a^(5/2)*sqrt(c)*e^10)*l
og(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2
*(3*sqrt(2)*a^(1/4)*c^(13/4)*d^10 + 3*sqrt(2)*a^(3/4)*c^(11/4)*d^8*e^2 + 14
*sqrt(2)*a^(5/4)*c^(9/4)*d^6*e^4 + 30*sqrt(2)*a^(7/4)*c^(7/4)*d^4*e^6 - 21*
sqrt(2)*a^(9/4)*c^(5/4)*d^2*e^8 - 5*sqrt(2)*a^(11/4)*c^(3/4)*e^10 + 8*sqrt(
a)*c^3*d^9*e + 48*a^(3/2)*c^2*d^5*e^5 - 24*a^(5/2)*c*d*e^9)*arctan(1/2*sqrt
(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)
*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(3*sqrt(2)*a^(1/4)*c^(13/4)*d^10 + 3*sq
rt(2)*a^(3/4)*c^(11/4)*d^8*e^2 + 14*sqrt(2)*a^(5/4)*c^(9/4)*d^6*e^4 + 30*sq
rt(2)*a^(7/4)*c^(7/4)*d^4*e^6 - 21*sqrt(2)*a^(9/4)*c^(5/4)*d^2*e^8 - 5*sqrt
(2)*a^(11/4)*c^(3/4)*e^10 - 8*sqrt(a)*c^3*d^9*e - 48*a^(3/2)*c^2*d^5*e^5 +
24*a^(5/2)*c*d*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/
4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(a*c^3*
d^12 + 3*a^2*c^2*d^8*e^4 + 3*a^3*c*d^4*e^8 + a^4*e^12) + 1/4*(4*a*c*d^4*e^3
 - 4*a^2*e^7 + (3*c^2*d^4*e^3 - 5*a*c*e^7)*x^4 + (c^2*d^5*e^2 + a*c*d*e^6)*
x^3 - (c^2*d^6*e + a*c*d^2*e^5)*x^2 + (c^2*d^7 + a*c*d^3*e^4)*x)/(a^2*c^2*d
^9 + 2*a^3*c*d^5*e^4 + a^4*d*e^8 + (a*c^3*d^8*e + 2*a^2*c^2*d^4*e^5 + a^3*c
*e^9)*x^5 + (a*c^3*d^9 + 2*a^2*c^2*d^5*e^4 + a^3*c*d*e^8)*x^4 + (a^2*c^2*d^
8*e + 2*a^3*c*d^4*e^5 + a^4*e^9)*x)
```

Giac [A] (verification not implemented)

none

Time = 6.61 (sec) , antiderivative size = 1145, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^2} dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 8*c*d^3*e^8*log(abs(e*x + d))/(c^3*d^12*e + 3*a*c^2*d^8*e^5 + 3*a^2*c*d^4*e
^9 + a^3*e^13) - 2*c*d^3*e^7*log(abs(c*x^4 + a))/(c^3*d^12 + 3*a*c^2*d^8*e^
4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + 1/8*(3*sqrt(2)*a*c^2*d*e^3 + 5*sqrt(2)*sq
rt(a*c)*c^2*d^3*e + 3*(a*c^3)^(1/4)*c^2*d^4 - 5*(a*c^3)^(1/4)*a*c*e^4 + 6*(
a*c^3)^(3/4)*d^2*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^
(1/4))/(sqrt(2)*a^2*c^3*d^6 + 9*sqrt(2)*a^3*c^2*d^2*e^4 + 9*sqrt(2)*sqrt(a*
c)*a^2*c^2*d^4*e^2 + sqrt(2)*sqrt(a*c)*a^3*c*e^6 - 6*(a*c^3)^(1/4)*a^2*c^2*
d^5*e - 6*(a*c^3)^(1/4)*a^3*c*d*e^5 - 16*(a*c^3)^(3/4)*a^2*d^3*e^3) - 1/8*(
3*sqrt(2)*a*c^2*d*e^3 - 5*sqrt(2)*sqrt(a*c)*c^2*d^3*e - 3*(a*c^3)^(1/4)*c^2
*d^4 + 5*(a*c^3)^(1/4)*a*c*e^4 - 6*(a*c^3)^(3/4)*d^2*e^2)*arctan(1/2*sqrt(2
)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^3*d^6 + 9*sqrt(2)
*a^3*c^2*d^2*e^4 + 9*sqrt(2)*sqrt(a*c)*a^2*c^2*d^4*e^2 + sqrt(2)*sqrt(a*c)*
a^3*c*e^6 + 6*(a*c^3)^(1/4)*a^2*c^2*d^5*e + 6*(a*c^3)^(1/4)*a^3*c*d*e^5 + 1
6*(a*c^3)^(3/4)*a^2*d^3*e^3) + 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*c^4*d^10 + 14*
sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^6*e^4 - 21*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^2*
e^8 - 3*sqrt(2)*(a*c^3)^(3/4)*c^2*d^8*e^2 - 30*sqrt(2)*(a*c^3)^(3/4)*a*c*d^
4*e^6 + 5*sqrt(2)*(a*c^3)^(3/4)*a^2*e^10)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) +
 sqrt(a/c))/(a^2*c^5*d^12 + 3*a^3*c^4*d^8*e^4 + 3*a^4*c^3*d^4*e^8 + a^5*c^2
*e^12) - 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*c^4*d^10 + 14*sqrt(2)*(a*c^3)^(1/4)*
a*c^3*d^6*e^4 - 21*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^2*e^8 - 3*sqrt(2)*(a*c^3
)^(3/4)*c^2*d^8*e^2 - 30*sqrt(2)*(a*c^3)^(3/4)*a*c*d^4*e^6 + 5*sqrt(2)*(a*c
^3)^(3/4)*a^2*e^10)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^5*d
^12 + 3*a^3*c^4*d^8*e^4 + 3*a^4*c^3*d^4*e^8 + a^5*c^2*e^12) + 1/4*(3*c^2*d^
4*e^3*x^4 - 5*a*c*e^7*x^4 + c^2*d^5*e^2*x^3 + a*c*d*e^6*x^3 - c^2*d^6*e*x^2
 - a*c*d^2*e^5*x^2 + c^2*d^7*x + a*c*d^3*e^4*x + 4*a*c*d^4*e^3 - 4*a^2*e^7)
/((a*c^2*d^8 + 2*a^2*c*d^4*e^4 + a^3*e^8)*(c*e*x^5 + c*d*x^4 + a*e*x + a*d)
)
```

Mupad [B] (verification not implemented)

Time = 10.59 (sec) , antiderivative size = 2246, normalized size of antiderivative = 1.97

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^2} \, dx = \text{Too large to display}$$

[In] int(1/((a + c*x^4)^2*(d + e*x)^2),x)

[Out] symsum(log(root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((120*a*c^8*d^14*e^3 + 2664*a^2*c^7*d^10*e^7 - 10904*a^3*c^6*d^6*e^11 + 19320*a^4*c^5*d^2*e^15)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((4096*a^3*c^8*d^15*e^4 + 54272*a^4*c^7*d^11*e^8 - 2048*a^5*c^6*d^7*e^12 + 144384*a^6*c^5*d^3*e^16)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*(root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((98304*a^11*c^4*d*e^22 - 32768*a^6*c^9*d^21*e^2 - 32768*a^7*c^8*d^17*e^6 + 196608*a^8*c^7*d^13*e^10 + 458752*a^9*c^6*d^9*e^14 + 360448*a^10*c^5*d^5*e^18)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(81920*a^11*c^4*e^23 - 49152*a^6*c^9*d^20*e^3 - 114688*a^7*c^8*d^16*e^7 + 32768*a^8*c^7*d^12*e^11 + 294912*a^9*c^6*d^8*e^15 + 278528*a^10*c^5*d^4*e^19))/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8))) + (5120*a^9*c^4*e^21 - 3072*a^4*c^9*d^20*e + 17408*a^5*c^8*d^16*e^5 + 337920*a^6*c^7*d^12*e^9 + 616448*a^7*c^6*d^8*e^13 + 304128*a^8*c^5*d^4*e^17)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(356352*a^6*c^7*d^11*e^10 - 32768*a^5*c^8*d^15*e^6 - 10240*a^4*c^9*d^19*e^2 + 770048*a^7*c^6*d^7*e^14 + 391168*a^8*c^5*d^3*e^18))/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8))) + (x*(768*a^3*c^8*d^14*e^5 - 576*a^2*c^9*d^18*e + 105088*a^4*c^7*d^10*e^9 + 221952*a^5*c^6*d^6*e^13 + 183744*a^6*c^5*d^2*e^17))/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8))) + (x*(200*a*c^8*d^13*e^4 + 19400*a^4*c^5*d*e^16 +

```
7512*a^2*c^7*d^9*e^8 + 2136*a^3*c^6*d^5*e^12))/(256*(a^8*e^16 + a^4*c^4*d^1
6 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8))) + (81*c^7*
d^9*e^6 - 254*a*c^6*d^5*e^10 + 625*a^2*c^5*d*e^14)/(256*(a^8*e^16 + a^4*c^4
*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(6
25*a^2*c^5*e^15 + 81*c^7*d^8*e^7 - 894*a*c^6*d^4*e^11))/(256*(a^8*e^16 + a^
4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)))*r
oot(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d
^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2
*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d
*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k), k, 1, 4) + ((x^4*(3*c^2*d^4*e^3 -
 5*a*c*e^7))/(4*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (a*e^7 - c*d^4*e^3
)/(a*e^4 + c*d^4)^2 + (c*d^3*x)/(4*a*(a*e^4 + c*d^4)) - (c*d^2*e*x^2)/(4*a*
(a*e^4 + c*d^4)) + (c*d*e^2*x^3)/(4*a*(a*e^4 + c*d^4)))/(a*d + a*e*x + c*d*
x^4 + c*e*x^5) + (8*c*d^3*e^7*log(d + e*x))/(a^3*e^12 + c^3*d^12 + 3*a*c^2*
d^8*e^4 + 3*a^2*c*d^4*e^8)
```

3.407 $\int \frac{1}{(d+ex)^3 (a+cx^4)^2} \, dx$

Optimal result 2479
Rubi [A] (verified) 2480
Mathematica [A] (verified) 2488
Maple [A] (verified) 2489
Fricas [F(-1)] 2489
Sympy [F(-1)] 2490
Maxima [A] (verification not implemented) 2490
Giac [A] (verification not implemented) 2491
Mupad [B] (verification not implemented) 2492

Optimal result

Integrand size = 17, antiderivative size = 1384

$$\int \frac{1}{(d+ex)^3\left(a+cx^4\right)^2}\,dx = -\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2}-\frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}$$
$$+\frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d(c^2d^8-12acd^4e^4+3a^2e^8)-e(3c^2d^8-12acd^4e^4+a^2e^8)\,x+2cd^3e^2(3cd^4-}{4a\left(cd^4+ae^4\right)^3\left(a+cx^4\right)}$$
$$-\frac{\sqrt{c}e^5(21c^2d^8-26acd^4e^4+a^2e^8)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)^4}$$
$$-\frac{\sqrt{c}e(3c^2d^8-12acd^4e^4+a^2e^8)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^4+ae^4\right)^3}$$
$$-\frac{c^{3/4}d\left(3c^2d^8-36acd^4e^4+9a^2e^8+2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3}$$
$$-\frac{c^{3/4}de^4\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)+3(5c^2d^8-10acd^4e^4+a^2e^8)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4}$$
$$+\frac{c^{3/4}d\left(3c^2d^8-36acd^4e^4+9a^2e^8+2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3}$$
$$+\frac{c^{3/4}de^4\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)+3(5c^2d^8-10acd^4e^4+a^2e^8)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4}$$
$$+\frac{12cd^2e^7(3cd^4-ae^4)\log(d+ex)}{\left(cd^4+ae^4\right)^4}$$
$$-\frac{c^{3/4}d\left(3c^2d^8-36acd^4e^4+9a^2e^8-2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3}$$
$$+\frac{c^{3/4}de^4\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)-3(5c^2d^8-10acd^4e^4+a^2e^8)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4}$$
$$+\frac{c^{3/4}d\left(3c^2d^8-36acd^4e^4+9a^2e^8-2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3}$$
$$-\frac{c^{3/4}de^4\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)-3(5c^2d^8-10acd^4e^4+a^2e^8)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4}$$
$$-\frac{3cd^2e^7(3cd^4-ae^4)\log\left(a+cx^4\right)}{\left(cd^4+ae^4\right)^4}$$

```
[Out] -1/2*e^7/(a*e^4+c*d^4)^2/(e*x+d)^2-8*c*d^3*e^7/(a*e^4+c*d^4)^3/(e*x+d)+1/4*
c*(2*a*d^2*e^3*(-3*a*e^4+5*c*d^4)+x*(d*(3*a^2*e^8-12*a*c*d^4*e^4+c^2*d^8)-e
```

```
*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*x+2*c*d^3*e^2*(-5*a*e^4+3*c*d^4)*x^2))/
a/(a*e^4+c*d^4)^3/(c*x^4+a)+12*c*d^2*e^7*(-a*e^4+3*c*d^4)*ln(e*x+d)/(a*e^4+
c*d^4)^4-3*c*d^2*e^7*(-a*e^4+3*c*d^4)*ln(c*x^4+a)/(a*e^4+c*d^4)^4-1/4*e*(a^
2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)
/(a*e^4+c*d^4)^3-1/2*e^5*(a^2*e^8-26*a*c*d^4*e^4+21*c^2*d^8)*arctan(x^2*c^(
1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^4/a^(1/2)-1/32*c^(3/4)*d*ln(-a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c^2*d^8-36*a*c*d^4*e^4+9*a^2*e^8-2*
d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)
+1/32*c^(3/4)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c^2*d^
8-36*a*c*d^4*e^4+9*a^2*e^8-2*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^
(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/16*c^(3/4)*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^
(1/4))*(3*c^2*d^8-36*a*c*d^4*e^4+9*a^2*e^8+2*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(
1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/16*c^(3/4)*d*arctan(1+c^(1/
4)*x*2^(1/2)/a^(1/4))*(3*c^2*d^8-36*a*c*d^4*e^4+9*a^2*e^8+2*d^2*e^2*(-5*a*e
^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/8*c^(3/4)*d*
e^4*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*a^2*e^8+30*a*c*d
^4*e^4-15*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*
e^4+c*d^4)^4*2^(1/2)-1/8*c^(3/4)*d*e^4*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*c^(1/2))*(-3*a^2*e^8+30*a*c*d^4*e^4-15*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c
*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)+1/4*c^(3/4)*d*e^4*ar
ctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*a^2*e^8-30*a*c*d^4*e^4+15*c^2*d^8+4*d
^2*e^2*(-5*a*e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)+
1/4*c^(3/4)*d*e^4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*a^2*e^8-30*a*c*d^4
*e^4+15*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^
4+c*d^4)^4*2^(1/2)
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 1384, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules

used = {6874, 1868, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 266}

$$
\begin{aligned}
\int \frac{1}{(d+ex)^3\left(a+cx^4\right)^2}\,dx = {} & \frac{12cd^2(3cd^4-ae^4)\log(d+ex)e^7}{\left(cd^4+ae^4\right)^4} \\
& - \frac{3cd^2(3cd^4-ae^4)\log\left(cx^4+a\right)e^7}{\left(cd^4+ae^4\right)^4} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)} \\
& - \frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2} - \frac{\sqrt{c}(21c^2d^8-26ace^4d^4+a^2e^8)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{2\sqrt{a}\left(cd^4+ae^4\right)^4} \\
& - \frac{c^{3/4}d\left(4\sqrt{a}\sqrt{c}d^2(7cd^4-5ae^4)e^2+3(5c^2d^8-10ace^4d^4+a^2e^8)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4} \\
& + \frac{c^{3/4}d\left(4\sqrt{a}\sqrt{c}d^2(7cd^4-5ae^4)e^2+3(5c^2d^8-10ace^4d^4+a^2e^8)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^4}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4} \\
& + \frac{c^{3/4}d\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)-3(5c^2d^8-10ace^4d^4+a^2e^8)\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4} \\
& - \frac{c^{3/4}d\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)-3(5c^2d^8-10ace^4d^4+a^2e^8)\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4} \\
& - \frac{\sqrt{c}(3c^2d^8-12ace^4d^4+a^2e^8)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{4a^{3/2}\left(cd^4+ae^4\right)^3} \\
& + \frac{c(2ad^2(5cd^4-3ae^4)e^3+x(2ce^2(3cd^4-5ae^4)x^2d^3+(c^2d^8-12ace^4d^4+3a^2e^8)d-e(3c^2d^8-12ace^4d^4}{4a\left(cd^4+ae^4\right)^3\left(cx^4+a\right)} \\
& - \frac{c^{3/4}d\left(3c^2d^8-36ace^4d^4+2\sqrt{a}\sqrt{c}e^2(3cd^4-5ae^4)d^2+9a^2e^8\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3} \\
& + \frac{c^{3/4}d\left(3c^2d^8-36ace^4d^4+2\sqrt{a}\sqrt{c}e^2(3cd^4-5ae^4)d^2+9a^2e^8\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3} \\
& - \frac{c^{3/4}d\left(3c^2d^8-36ace^4d^4-2\sqrt{a}\sqrt{c}e^2(3cd^4-5ae^4)d^2+9a^2e^8\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3} \\
& + \frac{c^{3/4}d\left(3c^2d^8-36ace^4d^4-2\sqrt{a}\sqrt{c}e^2(3cd^4-5ae^4)d^2+9a^2e^8\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3}
\end{aligned}
$$

```
[In] Int[1/((d + e*x)^3*(a + c*x^4)^2),x]
```

```
[Out] -1/2*e^7/((c*d^4 + a*e^4)^2*(d + e*x)^2) - (8*c*d^3*e^7)/((c*d^4 + a*e^4)^3
*(d + e*x)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e^4) + x*(d*(c^2*d^8 - 12*a*c*
d^4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 2*c*d^3
*e^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^4)) - (Sqrt
```

```
[c]*e^5*(21*c^2*d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a
]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4
+ a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a*e^4)^3) - (
c^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e
^2*(3*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*
c*d^4 - 5*a*e^4) + 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 - (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(3/4)
*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c
*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4
)*(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 -
 5*a*e^4) + 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (12*c*d^2*e^7*(
3*c*d^4 - a*e^4)*Log[d + e*x])/(c*d^4 + a*e^4)^4 - (c^(3/4)*d*(3*c^2*d^8 -
36*a*c*d^4*e^4 + 9*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4
)*(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 -
 5*a*e^4) - 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) +
(c^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*
e^2*(3*c*d^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sq
rt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) - 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8
))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/
4)*(c*d^4 + a*e^4)^4) - (3*c*d^2*e^7*(3*c*d^4 - a*e^4)*Log[a + c*x^4])/(c*d
^4 + a*e^4)^4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, c, d, e, p, q}, x]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^8}{(cd^4+ae^4)^2(d+ex)^3} + \frac{8cd^3e^8}{(cd^4+ae^4)^3(d+ex)^2} + \frac{12cd^2e^8(3cd^4-ae^4)}{(cd^4+ae^4)^4(d+ex)} + \frac{c(d(c^2d^8-12acd^4e^4+3a^2e^8)-e(3c^2d^8-12acd^4e^4+a^2e^8)x+2cd^3e^2(3cd^4-5ae^4)x^2-2cd^2e^3(5cd^4-}{(cd^4+ae^4)^3(a+cx^4)^2} + \frac{ce^4(3d(5c^2d^8-10acd^4e^4+a^2e^8)-e(21c^2d^8-26acd^4e^4+a^2e^8)x+4cd^3e^2(7cd^4-5ae^4)x^2-12cd^2e^3(3}{(cd^4+ae^4)^4(a+cx^4)}\right.$$

$$= -\frac{e^7}{2(cd^4+ae^4)^2(d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3(d+ex)} + \frac{12cd^2e^7(3cd^4-ae^4)\log(d+ex)}{(cd^4+ae^4)^4} + \frac{(ce^4)\int \frac{3d(5c^2d^8-10acd^4e^4+a^2e^8)-e(21c^2d^8-26acd^4e^4+a^2e^8)x+4cd^3e^2(7cd^4-5ae^4)x^2-12cd^2e^3(3cd^4-ae^4)x^3}{a+cx^4}\,dx}{(cd^4+ae^4)^4} + \frac{c\int \frac{d(c^2d^8-12acd^4e^4+3a^2e^8)-e(3c^2d^8-12acd^4e^4+a^2e^8)x+2cd^3e^2(3cd^4-5ae^4)x^2-2cd^2e^3(5cd^4-3ae^4)x^3}{(a+cx^4)^2}\,dx}{(cd^4+ae^4)^3}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}$$
$$+ \frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d(c^2d^8-12acd^4e^4+3a^2e^8)-e(3c^2d^8-12acd^4e^4+a^2e^8)\,x+2cd^3e}{4a\left(cd^4+ae^4\right)^3(a+cx^4)}$$
$$+ \frac{12cd^2e^7(3cd^4-ae^4)\log(d+ex)}{\left(cd^4+ae^4\right)^4}$$
$$+ \frac{(ce^4)\int\left(\frac{3d(5c^2d^8-10acd^4e^4+a^2e^8)+4cd^3e^2(7cd^4-5ae^4)x^2}{a+cx^4}+\frac{x(-e(21c^2d^8-26acd^4e^4+a^2e^8)-12cd^2e^3(3cd^4-ae^4)x^2)}{a+cx^4}\right)\,d}{\left(cd^4+ae^4\right)^4}$$
$$- \frac{c\int\frac{-3d(c^2d^8-12acd^4e^4+3a^2e^8)+2e(3c^2d^8-12acd^4e^4+a^2e^8)x-2cd^3e^2(3cd^4-5ae^4)x^2}{a+cx^4}\,dx}{4a\left(cd^4+ae^4\right)^3}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}$$
$$+ \frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d(c^2d^8-12acd^4e^4+3a^2e^8)-e(3c^2d^8-12acd^4e^4+a^2e^8)\,x+2cd^3e}{4a\left(cd^4+ae^4\right)^3(a+cx^4)}$$
$$+ \frac{12cd^2e^7(3cd^4-ae^4)\log(d+ex)}{\left(cd^4+ae^4\right)^4}$$
$$+ \frac{(ce^4)\int\frac{3d(5c^2d^8-10acd^4e^4+a^2e^8)+4cd^3e^2(7cd^4-5ae^4)x^2}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^4}$$
$$+ \frac{(ce^4)\int\frac{x(-e(21c^2d^8-26acd^4e^4+a^2e^8)-12cd^2e^3(3cd^4-ae^4)x^2)}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^4}$$
$$- \frac{c\int\left(\frac{2e(3c^2d^8-12acd^4e^4+a^2e^8)x}{a+cx^4}+\frac{-3d(c^2d^8-12acd^4e^4+3a^2e^8)-2cd^3e^2(3cd^4-5ae^4)x^2}{a+cx^4}\right)\,dx}{4a\left(cd^4+ae^4\right)^3}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}$$
$$+ \frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d(c^2d^8-12acd^4e^4+3a^2e^8)-e(3c^2d^8-12acd^4e^4+a^2e^8)\,x+2cd^3e^2(}{4a\left(cd^4+ae^4\right)^3(a+cx^4)}$$
$$+ \frac{12cd^2e^7(3cd^4-ae^4)\log(d+ex)}{\left(cd^4+ae^4\right)^4}$$
$$+ \frac{(ce^4)\,\text{Subst}\left(\int \frac{-e(21c^2d^8-26acd^4e^4+a^2e^8)-12cd^2e^3(3cd^4-ae^4)x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)^4}$$
$$- \frac{c\int \frac{-3d(c^2d^8-12acd^4e^4+3a^2e^8)-2cd^3e^2(3cd^4-5ae^4)x^2}{a+cx^4}\,dx}{4a\left(cd^4+ae^4\right)^3}$$
$$- \frac{(ce(3c^2d^8-12acd^4e^4+a^2e^8))\int \frac{x}{a+cx^4}\,dx}{2a\left(cd^4+ae^4\right)^3}$$
$$- \frac{\left(\sqrt{c}de^4\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)-3(5c^2d^8-10acd^4e^4+a^2e^8)\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\sqrt{a}\left(cd^4+ae^4\right)^4}$$
$$+ \frac{\left(cde^4\left(4d^2e^2(7cd^4-5ae^4)+\frac{3(5c^2d^8-10acd^4e^4+a^2e^8)}{\sqrt{a}\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^4+ae^4\right)^4}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}$$

$$+ \frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d(c^2d^8-12acd^4e^4+3a^2e^8)-e(3c^2d^8-12acd^4e^4+a^2e^8)\,x+2cd^3e}{4a\left(cd^4+ae^4\right)^3(a+cx^4)}$$

$$+ \frac{12cd^2e^7(3cd^4-ae^4)\log(d+ex)}{\left(cd^4+ae^4\right)^4} - \frac{(6c^2d^2e^7(3cd^4-ae^4))\,\mathrm{Subst}\left(\int\frac{x}{a+cx^2}\,dx,x,x^2\right)}{\left(cd^4+ae^4\right)^4}$$

$$- \frac{(ce^5(21c^2d^8-26acd^4e^4+a^2e^8))\,\mathrm{Subst}\left(\int\frac{1}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)^4}$$

$$- \frac{(ce(3c^2d^8-12acd^4e^4+a^2e^8))\,\mathrm{Subst}\left(\int\frac{1}{a+cx^2}\,dx,x,x^2\right)}{4a\left(cd^4+ae^4\right)^3}$$

$$+ \frac{\left(c^{3/4}de^4\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)-3(5c^2d^8-10acd^4e^4+a^2e^8)\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4}$$

$$+ \frac{\left(c^{3/4}de^4\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)-3(5c^2d^8-10acd^4e^4+a^2e^8)\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4}$$

$$+ \frac{\left(cde^4\left(4d^2e^2(7cd^4-5ae^4)+\frac{3\left(5c^2d^8-10acd^4e^4+a^2e^8\right)}{\sqrt{a}\sqrt{c}}\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^4+ae^4\right)^4}$$

$$+ \frac{\left(cde^4\left(4d^2e^2(7cd^4-5ae^4)+\frac{3\left(5c^2d^8-10acd^4e^4+a^2e^8\right)}{\sqrt{a}\sqrt{c}}\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^4+ae^4\right)^4}$$

$$- \frac{\left(cd\left(6cd^6e^2-10ad^2e^6-\frac{3\left(c^2d^8-12acd^4e^4+3a^2e^8\right)}{\sqrt{a}\sqrt{c}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a\left(cd^4+ae^4\right)^3}$$

$$+ \frac{\left(cd\left(6cd^6e^2-10ad^2e^6+\frac{3\left(c^2d^8-12acd^4e^4+3a^2e^8\right)}{\sqrt{a}\sqrt{c}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8a\left(cd^4+ae^4\right)^3}$$

$$= \text{Too large to display}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 996, normalized size of antiderivative = 0.72

$$\int \frac{1}{(d+ex)^3 (a+cx^4)^2} \, dx$$

$$= \frac{-\frac{16e^7(cd^4+ae^4)^2}{(d+ex)^2} - \frac{256cd^3e^7(cd^4+ae^4)}{d+ex} + \frac{8c(cd^4+ae^4)(-a^2e^7(6d^2-3dex+e^2x^2)+c^2d^7x(d^2-3dex+6e^2x^2)+2acd^3e^3(5d^3-6d^2ex+6de^2x^2}{a(a+cx^4)}}{}$$

```
[In] Integrate[1/((d + e*x)^3*(a + c*x^4)^2),x]
```

```
[Out] ((-16*e^7*(c*d^4 + a*e^4)^2)/(d + e*x)^2 - (256*c*d^3*e^7*(c*d^4 + a*e^4))/
(d + e*x) + (8*c*(c*d^4 + a*e^4)*(-(a^2*e^7*(6*d^2 - 3*d*e*x + e^2*x^2)) +
c^2*d^7*x*(d^2 - 3*d*e*x + 6*e^2*x^2) + 2*a*c*d^3*e^3*(5*d^3 - 6*d^2*e*x +
6*d*e^2*x^2 - 5*e^3*x^3)))/(a*(a + c*x^4)) - (6*Sqrt[c]*(Sqrt[2]*c^(13/4)*d
^13 - 4*a^(1/4)*c^3*d^12*e + 2*Sqrt[2]*Sqrt[a]*c^(11/4)*d^11*e^2 + 9*Sqrt[2
]*a*c^(9/4)*d^9*e^4 - 44*a^(5/4)*c^2*d^8*e^5 + 36*Sqrt[2]*a^(3/2)*c^(7/4)*d
^7*e^6 - 49*Sqrt[2]*a^2*c^(5/4)*d^5*e^8 + 84*a^(9/4)*c*d^4*e^9 - 30*Sqrt[2]
*a^(5/2)*c^(3/4)*d^3*e^10 + 7*Sqrt[2]*a^3*c^(1/4)*d*e^12 - 4*a^(13/4)*e^13)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (6*Sqrt[c]*(Sqrt[2]*c^(
13/4)*d^13 + 4*a^(1/4)*c^3*d^12*e + 2*Sqrt[2]*Sqrt[a]*c^(11/4)*d^11*e^2 + 9
*Sqrt[2]*a*c^(9/4)*d^9*e^4 + 44*a^(5/4)*c^2*d^8*e^5 + 36*Sqrt[2]*a^(3/2)*c^
(7/4)*d^7*e^6 - 49*Sqrt[2]*a^2*c^(5/4)*d^5*e^8 - 84*a^(9/4)*c*d^4*e^9 - 30*
Sqrt[2]*a^(5/2)*c^(3/4)*d^3*e^10 + 7*Sqrt[2]*a^3*c^(1/4)*d*e^12 + 4*a^(13/4
)*e^13)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + 384*c*d^2*e^7*(3
*c*d^4 - a*e^4)*Log[d + e*x] - (3*Sqrt[2]*c^(3/4)*(c^3*d^13 - 2*Sqrt[a]*c^(
5/2)*d^11*e^2 + 9*a*c^2*d^9*e^4 - 36*a^(3/2)*c^(3/2)*d^7*e^6 - 49*a^2*c*d^5
*e^8 + 30*a^(5/2)*Sqrt[c]*d^3*e^10 + 7*a^3*d*e^12)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) + (3*Sqrt[2]*c^(3/4)*(c^3*d^13 - 2*
Sqrt[a]*c^(5/2)*d^11*e^2 + 9*a*c^2*d^9*e^4 - 36*a^(3/2)*c^(3/2)*d^7*e^6 - 4
9*a^2*c*d^5*e^8 + 30*a^(5/2)*Sqrt[c]*d^3*e^10 + 7*a^3*d*e^12)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 96*c*d^2*e^7*(3*c*d^4
- a*e^4)*Log[a + c*x^4])/(32*(c*d^4 + a*e^4)^4)
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 680, normalized size of antiderivative = 0.49

method	result
default	$c\left(\frac{-\frac{c\,d^{3}e^{2}\left(5a^{2}e^{8}+2ac\,d^{4}e^{4}-3c^{2}d^{8}\right)x^{3}}{2a}-\frac{e\left(a^{3}e^{12}-11a^{2}c\,d^{4}e^{8}-9a\,c^{2}d^{8}e^{4}+3c^{3}d^{12}\right)x^{2}}{4a}+\frac{d\left(3a^{3}e^{12}-9a^{2}c\,d^{4}e^{8}-11a\,c^{2}d^{8}e^{4}+c^{3}d^{12}\right)x}{4a}-\frac{d^{2}e^{3}\left(3\ldots\right.}{\ldots}}{c\,x^{4}+a}\ldots\right.$
risch	Expression too large to display

```
[In] int(1/(e*x+d)^3/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c/(a*e^4+c*d^4)^4*((-1/2*c*d^3*e^2*(5*a^2*e^8+2*a*c*d^4*e^4-3*c^2*d^8)/a*x^
3-1/4*e*(a^3*e^12-11*a^2*c*d^4*e^8-9*a*c^2*d^8*e^4+3*c^3*d^12)/a*x^2+1/4*d*
(3*a^3*e^12-9*a^2*c*d^4*e^8-11*a*c^2*d^8*e^4+c^3*d^12)/a*x-1/2*d^2*e^3*(3*a
^2*e^8-2*a*c*d^4*e^4-5*c^2*d^8))/(c*x^4+a)+3/4/a*(1/8*(7*a^3*d*e^12-49*a^2*
c*d^5*e^8+9*a*c^2*d^9*e^4+c^3*d^13)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1
/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arcta
n(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/2*(-2*a^3*e
^13+42*a^2*c*d^4*e^9-22*a*c^2*d^8*e^5-2*c^3*d^12*e)/(a*c)^(1/2)*arctan(x^2*
(c/a)^(1/2))+1/8*(-30*a^2*c*d^3*e^10+36*a*c^2*d^7*e^6+2*c^3*d^11*e^2)/c/(a/
c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/
4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/
2)/(a/c)^(1/4)*x-1))+1/4*(16*a^2*c*d^2*e^11-48*a*c^2*d^6*e^7)/c*ln(c*x^4+a)
))-1/2*e^7/(a*e^4+c*d^4)^2/(e*x+d)^2-8*c*d^3*e^7/(a*e^4+c*d^4)^3/(e*x+d)-12
*e^7*c*d^2*(a*e^4-3*c*d^4)/(a*e^4+c*d^4)^4*ln(e*x+d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\,(a+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3 (a+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 1394, normalized size of antiderivative = 1.01

$$\int \frac{1}{(d+ex)^3 (a+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -3/32*c*(sqrt(2)*(48*sqrt(2)*a^(7/4)*c^(9/4)*d^6*e^7 - 16*sqrt(2)*a^(11/4)*
c^(5/4)*d^2*e^11 - c^4*d^13 + 2*sqrt(a)*c^(7/2)*d^11*e^2 - 9*a*c^3*d^9*e^4
+ 36*a^(3/2)*c^(5/2)*d^7*e^6 + 49*a^2*c^2*d^5*e^8 - 30*a^(5/2)*c^(3/2)*d^3*
e^10 - 7*a^3*c*d*e^12)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a
))/(a^(3/4)*c^(5/4)) + sqrt(2)*(48*sqrt(2)*a^(7/4)*c^(9/4)*d^6*e^7 - 16*sqr
t(2)*a^(11/4)*c^(5/4)*d^2*e^11 + c^4*d^13 - 2*sqrt(a)*c^(7/2)*d^11*e^2 + 9*
a*c^3*d^9*e^4 - 36*a^(3/2)*c^(5/2)*d^7*e^6 - 49*a^2*c^2*d^5*e^8 + 30*a^(5/2
)*c^(3/2)*d^3*e^10 + 7*a^3*c*d*e^12)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1
/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(17/4)*d^13 + 2*s
qrt(2)*a^(3/4)*c^(15/4)*d^11*e^2 + 9*sqrt(2)*a^(5/4)*c^(13/4)*d^9*e^4 + 36*
sqrt(2)*a^(7/4)*c^(11/4)*d^7*e^6 - 49*sqrt(2)*a^(9/4)*c^(9/4)*d^5*e^8 - 30*
sqrt(2)*a^(11/4)*c^(7/4)*d^3*e^10 + 7*sqrt(2)*a^(13/4)*c^(5/4)*d*e^12 + 4*s
qrt(a)*c^4*d^12*e + 44*a^(3/2)*c^3*d^8*e^5 - 84*a^(5/2)*c^2*d^4*e^9 + 4*a^(
7/2)*c*e^13)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqr
t(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(sqrt(2)*a^
(1/4)*c^(17/4)*d^13 + 2*sqrt(2)*a^(3/4)*c^(15/4)*d^11*e^2 + 9*sqrt(2)*a^(5/
4)*c^(13/4)*d^9*e^4 + 36*sqrt(2)*a^(7/4)*c^(11/4)*d^7*e^6 - 49*sqrt(2)*a^(9
/4)*c^(9/4)*d^5*e^8 - 30*sqrt(2)*a^(11/4)*c^(7/4)*d^3*e^10 + 7*sqrt(2)*a^(1
3/4)*c^(5/4)*d*e^12 - 4*sqrt(a)*c^4*d^12*e - 44*a^(3/2)*c^3*d^8*e^5 + 84*a^
(5/2)*c^2*d^4*e^9 - 4*a^(7/2)*c*e^13)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqr
t(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))
*c^(5/4)))/(a*c^4*d^16 + 4*a^2*c^3*d^12*e^4 + 6*a^3*c^2*d^8*e^8 + 4*a^4*c*d
^4*e^12 + a^5*e^16) + 12*(3*c^2*d^6*e^7 - a*c*d^2*e^11)*log(e*x + d)/(c^4*d
^16 + 4*a*c^3*d^12*e^4 + 6*a^2*c^2*d^8*e^8 + 4*a^3*c*d^4*e^12 + a^4*e^16) +
 1/4*(10*a*c^2*d^8*e^3 - 40*a^2*c*d^4*e^7 - 2*a^3*e^11 + 6*(c^3*d^7*e^4 - 7
```

```
*a*c^2*d^3*e^8)*x^5 + 3*(3*c^3*d^8*e^3 - 14*a*c^2*d^4*e^7 - a^2*c*e^11)*x^4
 + (c^3*d^9*e^2 + 2*a*c^2*d^5*e^6 + a^2*c*d*e^10)*x^3 - (c^3*d^10*e + 2*a*c
^2*d^6*e^5 + a^2*c*d^2*e^9)*x^2 + (c^3*d^11 + 8*a*c^2*d^7*e^4 - 41*a^2*c*d^
3*e^8)*x)/(a^2*c^3*d^14 + 3*a^3*c^2*d^10*e^4 + 3*a^4*c*d^6*e^8 + a^5*d^2*e^
12 + (a*c^4*d^12*e^2 + 3*a^2*c^3*d^8*e^6 + 3*a^3*c^2*d^4*e^10 + a^4*c*e^14)
*x^6 + 2*(a*c^4*d^13*e + 3*a^2*c^3*d^9*e^5 + 3*a^3*c^2*d^5*e^9 + a^4*c*d*e^
13)*x^5 + (a*c^4*d^14 + 3*a^2*c^3*d^10*e^4 + 3*a^3*c^2*d^6*e^8 + a^4*c*d^2*
e^12)*x^4 + (a^2*c^3*d^12*e^2 + 3*a^3*c^2*d^8*e^6 + 3*a^4*c*d^4*e^10 + a^5*
e^14)*x^2 + 2*(a^2*c^3*d^13*e + 3*a^3*c^2*d^9*e^5 + 3*a^4*c*d^5*e^9 + a^5*d
*e^13)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 1557, normalized size of antiderivative = 1.12

$$\int \frac{1}{(d+ex)^3\,(a+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/8*(4*sqrt(2)*a*c^3*d^2*e^3 + 2*sqrt(2)*sqrt(a*c)*c^3*d^4*e + 2*sqrt(2)*sq
rt(a*c)*a*c^2*e^5 + (a*c^3)^(1/4)*c^3*d^5 - 9*(a*c^3)^(1/4)*a*c^2*d*e^4 + 2
*(a*c^3)^(3/4)*c*d^3*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(sqrt(2)*a^2*c^4*d^8 + 34*sqrt(2)*a^3*c^3*d^4*e^4 + sqrt(2)*a^4*
c^2*e^8 + 16*sqrt(2)*sqrt(a*c)*a^2*c^3*d^6*e^2 + 16*sqrt(2)*sqrt(a*c)*a^3*c
^2*d^2*e^6 - 8*(a*c^3)^(1/4)*a^2*c^3*d^7*e - 40*(a*c^3)^(1/4)*a^3*c^2*d^3*e
^5 - 40*(a*c^3)^(3/4)*a^2*c*d^5*e^3 - 8*(a*c^3)^(3/4)*a^3*d*e^7) - 3/8*(4*s
qrt(2)*a*c^3*d^2*e^3 - 2*sqrt(2)*sqrt(a*c)*c^3*d^4*e - 2*sqrt(2)*sqrt(a*c)*
a*c^2*e^5 - (a*c^3)^(1/4)*c^3*d^5 + 9*(a*c^3)^(1/4)*a*c^2*d*e^4 - 2*(a*c^3)
^(3/4)*c*d^3*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4
))/(sqrt(2)*a^2*c^4*d^8 + 34*sqrt(2)*a^3*c^3*d^4*e^4 + sqrt(2)*a^4*c^2*e^8
+ 16*sqrt(2)*sqrt(a*c)*a^2*c^3*d^6*e^2 + 16*sqrt(2)*sqrt(a*c)*a^3*c^2*d^2*e
^6 + 8*(a*c^3)^(1/4)*a^2*c^3*d^7*e + 40*(a*c^3)^(1/4)*a^3*c^2*d^3*e^5 + 40*
(a*c^3)^(3/4)*a^2*c*d^5*e^3 + 8*(a*c^3)^(3/4)*a^3*d*e^7) + 3/32*(sqrt(2)*(a
*c^3)^(1/4)*c^4*d^13 + 9*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^9*e^4 - 49*sqrt(2)*(
a*c^3)^(1/4)*a^2*c^2*d^5*e^8 + 7*sqrt(2)*(a*c^3)^(1/4)*a^3*c*d*e^12 - 2*sqr
t(2)*(a*c^3)^(3/4)*c^2*d^11*e^2 - 36*sqrt(2)*(a*c^3)^(3/4)*a*c*d^7*e^6 + 30
*sqrt(2)*(a*c^3)^(3/4)*a^2*d^3*e^10)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt
(a/c))/(a^2*c^5*d^16 + 4*a^3*c^4*d^12*e^4 + 6*a^4*c^3*d^8*e^8 + 4*a^5*c^2*d
^4*e^12 + a^6*c*e^16) - 3/32*(sqrt(2)*(a*c^3)^(1/4)*c^4*d^13 + 9*sqrt(2)*(a
*c^3)^(1/4)*a*c^3*d^9*e^4 - 49*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^5*e^8 + 7*sq
rt(2)*(a*c^3)^(1/4)*a^3*c*d*e^12 - 2*sqrt(2)*(a*c^3)^(3/4)*c^2*d^11*e^2 - 3
6*sqrt(2)*(a*c^3)^(3/4)*a*c*d^7*e^6 + 30*sqrt(2)*(a*c^3)^(3/4)*a^2*d^3*e^10
```

```
)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^5*d^16 + 4*a^3*c^4*d^
12*e^4 + 6*a^4*c^3*d^8*e^8 + 4*a^5*c^2*d^4*e^12 + a^6*c*e^16) - 3*(3*c^2*d^
6*e^7 - a*c*d^2*e^11)*log(abs(c*x^4 + a))/(c^4*d^16 + 4*a*c^3*d^12*e^4 + 6*
a^2*c^2*d^8*e^8 + 4*a^3*c*d^4*e^12 + a^4*e^16) + 12*(3*c^2*d^6*e^8 - a*c*d^
2*e^12)*log(abs(e*x + d))/(c^4*d^16*e + 4*a*c^3*d^12*e^5 + 6*a^2*c^2*d^8*e^
9 + 4*a^3*c*d^4*e^13 + a^4*e^17) + 1/4*(10*a*c^3*d^12*e^3 - 30*a^2*c^2*d^8*
e^7 - 42*a^3*c*d^4*e^11 - 2*a^4*e^15 + 6*(c^4*d^11*e^4 - 6*a*c^3*d^7*e^8 -
7*a^2*c^2*d^3*e^12)*x^5 + 3*(3*c^4*d^12*e^3 - 11*a*c^3*d^8*e^7 - 15*a^2*c^2
*d^4*e^11 - a^3*c*e^15)*x^4 + (c^4*d^13*e^2 + 3*a*c^3*d^9*e^6 + 3*a^2*c^2*d
^5*e^10 + a^3*c*d*e^14)*x^3 - (c^4*d^14*e + 3*a*c^3*d^10*e^5 + 3*a^2*c^2*d^
6*e^9 + a^3*c*d^2*e^13)*x^2 + (c^4*d^15 + 9*a*c^3*d^11*e^4 - 33*a^2*c^2*d^7
*e^8 - 41*a^3*c*d^3*e^12)*x)/((c*d^4 + a*e^4)^4*(c*x^4 + a)*(e*x + d)^2*a)
```

Mupad [B] (verification not implemented)

Time = 10.85 (sec) , antiderivative size = 3256, normalized size of antiderivative = 2.35

$$\int \frac{1}{(d+ex)^3\,(a+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + c*x^4)^2*(d + e*x)^3),x)
```

```
[Out] symsum(log(root(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*e^8*z^4 + 2
62144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11*e^16*z^4 -
 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712*a^5*c^2*d
^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 + 58752*a^3*c
^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4, z, k)*((
108*a*c^10*d^19*e^3 + 3888*a^2*c^9*d^15*e^7 - 99576*a^3*c^8*d^11*e^11 + 591
408*a^4*c^7*d^7*e^15 - 79380*a^5*c^6*d^3*e^19)/(256*(a^10*e^24 + a^4*c^6*d^
24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c
^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16)) + root(262144*a^10*c*d^4*e^12*z^4 + 39
3216*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16
*z^4 + 65536*a^11*e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^
6*e^7*z^3 + 755712*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*
a^6*c*e^10*z^2 + 58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^
2*e^4 + 81*c^3*d^4, z, k)*((6912*a^8*c^5*d*e^24 + 4608*a^3*c^10*d^21*e^4 +
154368*a^4*c^9*d^17*e^8 - 331776*a^5*c^8*d^13*e^12 + 5976576*a^6*c^7*d^9*e^
16 - 612864*a^7*c^6*d^5*e^20)/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*
e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15
*a^8*c^2*d^8*e^16)) + root(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*
e^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11
*e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 75571
2*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 +
58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^
4, z, k)*((18432*a^5*c^10*d^23*e^5 - 3072*a^4*c^11*d^27*e + 1170432*a^6*c^9
```

```
*d^19*e^9 + 2863104*a^7*c^8*d^15*e^13 + 1797120*a^8*c^7*d^11*e^17 - 423936*
a^9*c^6*d^7*e^21 - 506880*a^10*c^5*d^3*e^25)/(256*(a^10*e^24 + a^4*c^6*d^24
 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3
*d^12*e^12 + 15*a^8*c^2*d^8*e^16)) + root(262144*a^10*c*d^4*e^12*z^4 + 3932
16*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z
^4 + 65536*a^11*e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*
e^7*z^3 + 755712*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^
6*c*e^10*z^2 + 58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*
e^4 + 81*c^3*d^4, z, k)*((98304*a^13*c^4*d*e^30 - 32768*a^6*c^11*d^29*e^2 -
 98304*a^7*c^10*d^25*e^6 + 98304*a^8*c^9*d^21*e^10 + 819200*a^9*c^8*d^17*e^
14 + 1474560*a^10*c^7*d^13*e^18 + 1277952*a^11*c^6*d^9*e^22 + 557056*a^12*c
^5*d^5*e^26)/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*
d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16
)) + (x*(81920*a^13*c^4*e^31 - 49152*a^6*c^11*d^28*e^3 - 212992*a^7*c^10*d^
24*e^7 - 245760*a^8*c^9*d^20*e^11 + 245760*a^9*c^8*d^16*e^15 + 901120*a^10*
c^7*d^12*e^19 + 933888*a^11*c^6*d^8*e^23 + 442368*a^12*c^5*d^4*e^27))/(256*
(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*
c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16))) - (x*(12288*a^
4*c^11*d^26*e^2 + 98304*a^5*c^10*d^22*e^6 - 1413120*a^6*c^9*d^18*e^10 - 403
0464*a^7*c^8*d^14*e^14 - 2813952*a^8*c^7*d^10*e^18 + 393216*a^9*c^6*d^6*e^2
2 + 675840*a^10*c^5*d^2*e^26))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4
*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 1
5*a^8*c^2*d^8*e^16))) + (x*(20736*a^8*c^5*e^25 - 576*a^2*c^11*d^24*e - 576*
a^3*c^10*d^20*e^5 + 484992*a^4*c^9*d^16*e^9 + 2468736*a^5*c^8*d^12*e^13 + 4
093632*a^6*c^7*d^8*e^17 - 228672*a^7*c^6*d^4*e^21))/(256*(a^10*e^24 + a^4*c
^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*
a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16))) + (x*(216*a*c^10*d^18*e^4 + 2505
6*a^2*c^9*d^14*e^8 - 2160*a^3*c^8*d^10*e^12 + 59616*a^4*c^7*d^6*e^16 + 8661
6*a^5*c^6*d^2*e^20))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*
a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*
d^8*e^16))) + (81*c^9*d^13*e^6 - 2430*a*c^8*d^9*e^10 + 1296*a^3*c^6*d*e^18
+ 3969*a^2*c^7*d^5*e^14)/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20
+ 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*
c^2*d^8*e^16)) + (x*(1296*a^3*c^6*e^19 + 81*c^9*d^12*e^7 - 6318*a*c^8*d^8*e
^11 + 5265*a^2*c^7*d^4*e^15))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*
e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15
*a^8*c^2*d^8*e^16)))*root(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*e
^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11*
e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712
*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 + 5
8752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4
, z, k), k, 1, 4) - ((a^2*e^11 - 5*c^2*d^8*e^3 + 20*a*c*d^4*e^7)/(2*(a*e^4
+ c*d^4)*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (3*x^5*(c^3*d^7*e^4 - 7*a*c
^2*d^3*e^8))/(2*a*(a^3*e^12 + c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)
) + (3*x^4*(a^2*c*e^11 - 3*c^3*d^8*e^3 + 14*a*c^2*d^4*e^7))/(4*a*(a^3*e^12
```

```
+ c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)) + (c*d^2*e*x^2)/(4*a*(a*e^
4 + c*d^4)) - (c*d*e^2*x^3)/(4*a*(a*e^4 + c*d^4)) - (d*x*(c^3*d^10 + 8*a*c^
2*d^6*e^4 - 41*a^2*c*d^2*e^8))/(4*a*(a*e^4 + c*d^4)*(a^2*e^8 + c^2*d^8 + 2*
a*c*d^4*e^4)))/(a*d^2 + a*e^2*x^2 + c*d^2*x^4 + c*e^2*x^6 + 2*a*d*e*x + 2*c
*d*e*x^5) + (log(d + e*x)*(36*c^2*d^6*e^7 - 12*a*c*d^2*e^11))/(a^4*e^16 + c
^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)
```

3.408 $\int \frac{(d+ex)^3}{(a+cx^4)^3} dx$

Optimal result 2495
Rubi [A] (verified) 2496
Mathematica [A] (verified) 2500
Maple [C] (verified) 2500
Fricas [C] (verification not implemented) 2501
Sympy [A] (verification not implemented) 2501
Maxima [A] (verification not implemented) 2502
Giac [A] (verification not implemented) 2503
Mupad [B] (verification not implemented) 2504

Optimal result

Integrand size = 17, antiderivative size = 394

$$\int \frac{(d+ex)^3}{(a+cx^4)^3} dx = \frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2(a+cx^4)} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac(a+cx^4)^2} + \frac{9d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{3d(7\sqrt{c}d^2+5\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} + \frac{3d(7\sqrt{c}d^2+5\sqrt{a}e^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} - \frac{3d(7\sqrt{c}d^2-5\sqrt{a}e^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}} + \frac{3d(7\sqrt{c}d^2-5\sqrt{a}e^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}$$

```
[Out] 1/32*x*(15*d*e^2*x^2+18*d^2*e*x+7*d^3)/a^2/(c*x^4+a)+1/8*(-a*e^3+c*x*(3*d*e
^2*x^2+3*d^2*e*x+d^3))/a/c/(c*x^4+a)^2+9/16*d^2*e*arctan(x^2*c^(1/2)/a^(1/2
))/a^(5/2)/c^(1/2)-3/256*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2
))*(-5*e^2*a^(1/2)+7*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^(1/2)+3/256*d*ln(a^(1/
4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-5*e^2*a^(1/2)+7*d^2*c^(1/2))/a^
(11/4)/c^(3/4)*2^(1/2)+3/128*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(5*e^2*
a^(1/2)+7*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^(1/2)+3/128*d*arctan(1+c^(1/4)*x*
2^(1/2)/a^(1/4))*(5*e^2*a^(1/2)+7*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx = -\frac{3d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}e^2+7\sqrt{c}d^2\right)}{64\sqrt{2}a^{11/4}c^{3/4}} + \frac{3d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}e^2+7\sqrt{c}d^2\right)}{64\sqrt{2}a^{11/4}c^{3/4}} + \frac{9d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{3d\left(7\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}} + \frac{3d\left(7\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}} + \frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2(a+cx^4)} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac(a+cx^4)^2}$$

```
[In] Int[(d + e*x)^3/(a + c*x^4)^3,x]
```

```
[Out] (x*(7*d^3 + 18*d^2*e*x + 15*d*e^2*x^2))/(32*a^2*(a + c*x^4)) - (a*e^3 - c*x
*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(8*a*c*(a + c*x^4)^2) + (9*d^2*e*ArcTan[(
Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[c]) - (3*d*(7*Sqrt[c]*d^2 + 5*Sqrt[
a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/
4)) + (3*d*(7*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) - (3*d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^
2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(
11/4)*c^(3/4)) + (3*d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac\left(a+cx^4\right)^2}-\frac{\int\frac{-7d^3-18d^2ex-15de^2x^2}{(a+cx^4)^2}\,dx}{8a}\\
&=\frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2\left(a+cx^4\right)}-\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac\left(a+cx^4\right)^2}+\frac{\int\frac{21d^3+36d^2ex+15de^2x^2}{a+cx^4}\,dx}{32a^2}\\
&=\frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2\left(a+cx^4\right)}-\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac\left(a+cx^4\right)^2}+\frac{\int\left(\frac{36d^2ex}{a+cx^4}+\frac{21d^3+15de^2x^2}{a+cx^4}\right)dx}{32a^2}\\
&=\frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2\left(a+cx^4\right)}-\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac\left(a+cx^4\right)^2}\\
&\quad+\frac{\int\frac{21d^3+15de^2x^2}{a+cx^4}\,dx}{32a^2}+\frac{(9d^2e)\int\frac{x}{a+cx^4}\,dx}{8a^2}\\
&=\frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2\left(a+cx^4\right)}-\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac\left(a+cx^4\right)^2}\\
&\quad+\frac{(9d^2e)\,\text{Subst}\left(\int\frac{1}{a+cx^2}\,dx,x,x^2\right)}{16a^2}+\frac{\left(3d\left(\frac{7\sqrt{c}d^2}{\sqrt{a}}-5e^2\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{64a^2c}\\
&\quad+\frac{\left(3d\left(\frac{7\sqrt{c}d^2}{\sqrt{a}}+5e^2\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{64a^2c}
\end{aligned}$$

$$= \frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2(a+cx^4)} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac(a+cx^4)^2} + \frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{\left(3d\left(\frac{7\sqrt{c}d^2}{\sqrt{a}}-5e^2\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}dx}{128\sqrt{2}a^{9/4}c^{3/4}} - \frac{\left(3d\left(\frac{7\sqrt{c}d^2}{\sqrt{a}}-5e^2\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}dx}{128\sqrt{2}a^{9/4}c^{3/4}} + \frac{\left(3d\left(\frac{7\sqrt{c}d^2}{\sqrt{a}}+5e^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}dx}{128a^2c} + \frac{\left(3d\left(\frac{7\sqrt{c}d^2}{\sqrt{a}}+5e^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}dx}{128a^2c}$$

$$= \frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2(a+cx^4)} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac(a+cx^4)^2} + \frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{3d\left(\frac{7\sqrt{c}d^2}{\sqrt{a}}-5e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{9/4}c^{3/4}} + \frac{3d\left(\frac{7\sqrt{c}d^2}{\sqrt{a}}-5e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{9/4}c^{3/4}} + \frac{\left(3d\left(7\sqrt{c}d^2+5\sqrt{a}e^2\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} - \frac{\left(3d\left(7\sqrt{c}d^2+5\sqrt{a}e^2\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}}$$

$$= \frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2(a+cx^4)} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac(a+cx^4)^2} + \frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{3d\left(7\sqrt{c}d^2+5\sqrt{a}e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} + \frac{3d\left(7\sqrt{c}d^2+5\sqrt{a}e^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} - \frac{3d\left(\frac{7\sqrt{c}d^2}{\sqrt{a}}-5e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{9/4}c^{3/4}} + \frac{3d\left(\frac{7\sqrt{c}d^2}{\sqrt{a}}-5e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{9/4}c^{3/4}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx$$

$$= \frac{\frac{8adx(7d^2+18dex+15e^2x^2)}{a+cx^4} - \frac{32a^2(ae^3-cdx(d^2+3dex+3e^2x^2))}{c(a+cx^4)^2} - \frac{6\sqrt[4]{a}d\left(7\sqrt{2}\sqrt{c}d^2+24\sqrt[4]{a}\sqrt[4]{c}de+5\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \cdots}{\cdots}$$

[In] Integrate[(d + e*x)^3/(a + c*x^4)^3,x]

[Out] ((8*a*d*x*(7*d^2 + 18*d*e*x + 15*e^2*x^2))/(a + c*x^4) - (32*a^2*(a*e^3 - c*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(c*(a + c*x^4)^2) - (6*a^(1/4)*d*(7*Sqrt[2]*Sqrt[c]*d^2 + 24*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (6*a^(1/4)*d*(7*Sqrt[2]*Sqrt[c]*d^2 - 24*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (3*Sqrt[2]*(-7*a^(1/4)*Sqrt[c]*d^3 + 5*a^(3/4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (3*Sqrt[2]*(7*a^(1/4)*Sqrt[c]*d^3 - 5*a^(3/4)*d*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(256*a^3)

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.81 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.36

method	result
risch	$\frac{\frac{15cd\,e^2x^7}{32a^2}+\frac{9c\,d^2e\,x^6}{16a^2}+\frac{7d^3c\,x^5}{32a^2}+\frac{27d\,e^2x^3}{32a}+\frac{15e\,x^2d^2}{16a}+\frac{11d^3x}{32a}-\frac{e^3}{8c}}{(c\,x^4+a)^2}+\frac{3d\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+a)}\frac{\left(5e^2\_R^2+12ed\_R+7d^2\right)\ln\left(x-\_R\right)}{\_R^3}\cdots\right)}{128a^2c}$
default	$d^3\left(\frac{x}{8a(c\,x^4+a)^2}+\frac{\frac{7x}{32a(c\,x^4+a)}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2}}{a}\right)+3d^2e\,\cdots$

[In] int((e*x+d)^3/(c*x^4+a)^3,x,method=_RETURNVERBOSE)

[Out] (15/32*c*d*e^2/a^2*x^7+9/16*c*d^2*e/a^2*x^6+7/32*d^3*c/a^2*x^5+27/32*d*e^2/a*x^3+15/16*e/a*x^2*d^2+11/32*d^3/a*x-1/8*e^3/c)/(c*x^4+a)^2+3/128/a^2*d/c*sum((5*_R^2*e^2+12*_R*d*e+7*d^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.23 (sec) , antiderivative size = 95566, normalized size of antiderivative = 242.55

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 131.88 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx$$
$$= \text{RootSum}\Big(268435456t^4a^{11}c^3 + 63111168t^2a^6c^2d^4e^2 + t\big(4147200a^4cd^4e^5 - 8128512a^3c^2d^8e\big) + 50625a^2d^4$$
$$+ \frac{-4a^2e^3 + 11acd^3x + 30acd^2ex^2 + 27acde^2x^3 + 7c^2d^3x^5 + 18c^2d^2ex^6 + 15c^2de^2x^7}{32a^4c + 64a^3c^2x^4 + 32a^2c^3x^8}$$

```
[In] integrate((e*x+d)**3/(c*x**4+a)**3,x)
[Out] RootSum(268435456*_t**4*a**11*c**3 + 63111168*_t**2*a**6*c**2*d**4*e**2 + _
t*(4147200*a**4*c*d**4*e**5 - 8128512*a**3*c**2*d**8*e) + 50625*a**2*d**4*e
**8 + 245106*a*c*d**8*e**4 + 194481*c**2*d**12, Lambda(_t, _t*log(x + (2621
44000*_t**3*a**10*c**2*e**6 + 3714056192*_t**3*a**9*c**3*d**4*e**2 - 539688
960*_t**2*a**7*c**2*d**4*e**5 + 202309632*_t**2*a**6*c**3*d**8*e + 77328000
*_t*a**5*c*d**4*e**8 + 660699648*_t*a**4*c**2*d**8*e**4 + 19361664*_t*a**3*
c**3*d**12 + 3037500*a**3*d**4*e**11 - 26360640*a**2*c*d**8*e**7 - 60566940
*a*c**2*d**12*e**3)/(421875*a**3*d**3*e**12 - 29598075*a**2*c*d**7*e**8 - 5
8012227*a*c**2*d**11*e**4 + 3176523*c**3*d**15)))) + (-4*a**2*e**3 + 11*a*c
*d**3*x + 30*a*c*d**2*e*x**2 + 27*a*c*d*e**2*x**3 + 7*c**2*d**3*x**5 + 18*c
**2*d**2*e*x**6 + 15*c**2*d*e**2*x**7)/(32*a**4*c + 64*a**3*c**2*x**4 + 32*
a**2*c**3*x**8)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx = \frac{15\,c^2de^2x^7+18\,c^2d^2ex^6+7\,c^2d^3x^5+27\,acde^2x^3+30\,acd^2ex^2+11\,acd^3x-4\,a^2e^3}{32\,(a^2c^3x^8+2\,a^3c^2x^4+a^4c)} + \frac{3\,d\left(\frac{\sqrt{2}(7\,\sqrt{c}d^2-5\,\sqrt{a}e^2)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}(7\,\sqrt{c}d^2-5\,\sqrt{a}e^2)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{2\left(7\,\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+5\,\sqrt{2}a^{\frac{3}{4}}c\right.}{}\right.}{256\,a^2}$$

```
[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(15*c^2*d*e^2*x^7 + 18*c^2*d^2*e*x^6 + 7*c^2*d^3*x^5 + 27*a*c*d*e^2*x^
3 + 30*a*c*d^2*e*x^2 + 11*a*c*d^3*x - 4*a^2*e^3)/(a^2*c^3*x^8 + 2*a^3*c^2*x
^4 + a^4*c) + 3/256*d*(sqrt(2)*(7*sqrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c)*
x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(7*s
qrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + s
qrt(a))/(a^(3/4)*c^(3/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt(2)*a^
(3/4)*c^(1/4)*e^2 - 24*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x
 + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sq
rt(c))*c^(3/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt(2)*a^(3/4)*c^(1
/4)*e^2 + 24*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)
*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(
3/4)))/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx = \frac{3\sqrt{2}\left(12\sqrt{2}\sqrt{ac}c^2d^2e+7\,(ac^3)^{\frac{1}{4}}\,c^2d^3+5\,(ac^3)^{\frac{3}{4}}\,de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3} + \frac{3\sqrt{2}\left(12\sqrt{2}\sqrt{ac}c^2d^2e+7\,(ac^3)^{\frac{1}{4}}\,c^2d^3+5\,(ac^3)^{\frac{3}{4}}\,de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3} + \frac{3\sqrt{2}\left(7\,(ac^3)^{\frac{1}{4}}\,c^2d^3-5\,(ac^3)^{\frac{3}{4}}\,de^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3} - \frac{3\sqrt{2}\left(7\,(ac^3)^{\frac{1}{4}}\,c^2d^3-5\,(ac^3)^{\frac{3}{4}}\,de^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3} + \frac{15\,c^2de^2x^7+18\,c^2d^2ex^6+7\,c^2d^3x^5+27\,acde^2x^3+30\,acd^2ex^2+11\,acd^3x-4\,a^2e^3}{32\,(cx^4+a)^2a^2c}$$

```
[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*c^3)^(1/4)*c^2*d^3 + 5
*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^
(1/4))/(a^3*c^3) + 3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*c^3
)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c^2*d^
3 - 5*(a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
3*c^3) - 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c^2*d^3 - 5*(a*c^3)^(3/4)*d*e^2)*lo
g(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) + 1/32*(15*c^2*d*e^2*x
^7 + 18*c^2*d^2*e*x^6 + 7*c^2*d^3*x^5 + 27*a*c*d*e^2*x^3 + 30*a*c*d^2*e*x^2
 + 11*a*c*d^3*x - 4*a^2*e^3)/((c*x^4 + a)^2*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.83

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx = \frac{\frac{11\,d^3\,x}{32\,a}-\frac{e^3}{8\,c}+\frac{7\,c\,d^3\,x^5}{32\,a^2}+\frac{15\,d^2\,e\,x^2}{16\,a}+\frac{27\,d\,e^2\,x^3}{32\,a}+\frac{9\,c\,d^2\,e\,x^6}{16\,a^2}+\frac{15\,c\,d\,e^2\,x^7}{32\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}$$
$$+\left(\sum_{k=1}^{4}\ln\left(\frac{c\,d^2\,\Big(6867\,c\,d^5\,e^2-1125\,a\,d\,e^6+7992\,c\,d^4\,e^3\,x-\mathrm{root}(268435456\,a^{11}\,c^3\,z^4+63111168\,a^6\,c^2\,d^4\,e\cdots}{}\right.\right.$$
$$+63111168\,a^6\,c^2\,d^4\,e^2\,z^2-8128512\,a^3\,c^2\,d^8\,e\,z+4147200\,a^4\,c\,d^4\,e^5\,z+245106\,a\,c\,d^8\,e^4$$
$$\left.+50625\,a^2\,d^4\,e^8+194481\,c^2\,d^{12},z,k)\right)$$

```
[In] int((d + e*x)^3/(a + c*x^4)^3,x)
```

```
[Out] ((11*d^3*x)/(32*a) - e^3/(8*c) + (7*c*d^3*x^5)/(32*a^2) + (15*d^2*e*x^2)/(1
6*a) + (27*d*e^2*x^3)/(32*a) + (9*c*d^2*e*x^6)/(16*a^2) + (15*c*d*e^2*x^7)/
(32*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + symsum(log((3*c*d^2*(6867*c*d^5*e^2
 - 1125*a*d*e^6 + 7992*c*d^4*e^3*x - 114688*root(268435456*a^11*c^3*z^4 + 6
3111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e
^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)^2*a^
5*c^2*d + 9600*root(268435456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 -
 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 5
0625*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)*a^3*c*e^4*x - 18816*root(26843545
6*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4
147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^
2*d^12, z, k)*a^2*c^2*d^4*x + 196608*root(268435456*a^11*c^3*z^4 + 63111168
*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z +
245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)^2*a^5*c^2*e
*x - 46080*root(268435456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 812
8512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625
*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)*a^3*c*d*e^3))/(32768*a^6))*root(26843
5456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z
+ 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481
*c^2*d^12, z, k), k, 1, 4)
```

3.409 $\int \frac{(d+ex)^2}{(a+cx^4)^3} dx$

Optimal result 2505
Rubi [A] (verified) 2506
Mathematica [A] (verified) 2509
Maple [C] (verified) 2510
Fricas [C] (verification not implemented) 2510
Sympy [A] (verification not implemented) 2511
Maxima [A] (verification not implemented) 2511
Giac [A] (verification not implemented) 2512
Mupad [B] (verification not implemented) 2513

Optimal result

Integrand size = 17, antiderivative size = 360

$$
\begin{aligned}
\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx &= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}+\frac{x(7d^2+12dex+5e^2x^2)}{32a^2\left(a+cx^4\right)}+\frac{3de\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} \\
&\quad-\frac{\left(21\sqrt{c}d^2+5\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} \\
&\quad+\frac{\left(21\sqrt{c}d^2+5\sqrt{a}e^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} \\
&\quad-\frac{\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}} \\
&\quad+\frac{\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}
\end{aligned}
$$

```
[Out] 1/8*x*(e*x+d)^2/a/(c*x^4+a)^2+1/32*x*(5*e^2*x^2+12*d*e*x+7*d^2)/a^2/(c*x^4+
a)+3/8*d*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/c^(1/2)-1/256*ln(-a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/
4)/c^(3/4)*2^(1/2)+1/256*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*
(-5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^(1/2)+1/128*arctan(-1+c^
(1/4)*x*2^(1/2)/a^(1/4))*(5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^
(1/2)+1/128*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(5*e^2*a^(1/2)+21*d^2*c^(1/
2))/a^(11/4)/c^(3/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}e^2+21\sqrt{c}d^2\right)}{64\sqrt{2}a^{11/4}c^{3/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}e^2+21\sqrt{c}d^2\right)}{64\sqrt{2}a^{11/4}c^{3/4}} + \frac{3de\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} - \frac{\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}} + \frac{\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{x(d+ex)^2}{8a(a+cx^4)^2}$$

```
[In] Int[(d + e*x)^2/(a + c*x^4)^3,x]
```

```
[Out] (x*(d + e*x)^2)/(8*a*(a + c*x^4)^2) + (x*(7*d^2 + 12*d*e*x + 5*e^2*x^2))/(3
2*a^2*(a + c*x^4)) + (3*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(8*a^(5/2)*Sqrt[
c]) - ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*Arc
Tan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) - ((21*
Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(1
1/4)*c^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} - \frac{\int \frac{-7d^2-12dex-5e^2x^2}{\left(a+cx^4\right)^2}\,dx}{8a}$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2\left(a+cx^4\right)} + \frac{\int \frac{21d^2+24dex+5e^2x^2}{a+cx^4}\,dx}{32a^2}$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2\left(a+cx^4\right)} + \frac{\int \left(\frac{24dex}{a+cx^4} + \frac{21d^2+5e^2x^2}{a+cx^4}\right)dx}{32a^2}$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2\left(a+cx^4\right)} + \frac{\int \frac{21d^2+5e^2x^2}{a+cx^4}\,dx}{32a^2} + \frac{(3de)\int \frac{x}{a+cx^4}\,dx}{4a^2}$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2\left(a+cx^4\right)} + \frac{(3de)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{8a^2} + \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}} - 5e^2\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{64a^2c} + \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}} + 5e^2\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{64a^2c}$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2\left(a+cx^4\right)} + \frac{3de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} - \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}} - 5e^2\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{128\sqrt{2}a^{9/4}c^{3/4}} - \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}} - 5e^2\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{128\sqrt{2}a^{9/4}c^{3/4}} + \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}} + 5e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{128a^2c} + \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}} + 5e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{128a^2c}$$

$$= \frac{x(d+ex)^2}{8a(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{3de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} - \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}}-5e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{9/4}c^{3/4}} + \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}}-5e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{9/4}c^{3/4}} + \frac{(21\sqrt{c}d^2+5\sqrt{a}e^2)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} - \frac{(21\sqrt{c}d^2+5\sqrt{a}e^2)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}}$$

$$= \frac{x(d+ex)^2}{8a(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{3de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} - \frac{(21\sqrt{c}d^2+5\sqrt{a}e^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} + \frac{(21\sqrt{c}d^2+5\sqrt{a}e^2)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} - \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}}-5e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{9/4}c^{3/4}} + \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}}-5e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{9/4}c^{3/4}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx$$

$$= \frac{\frac{32a^2x(d+ex)^2}{(a+cx^4)^2} + \frac{8ax(7d^2+12dex+5e^2x^2)}{a+cx^4} - \frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{c}d^2+48\sqrt[4]{a}\sqrt[4]{c}de+5\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{c}d^2-\right.}{}}{}$$

```
[In] Integrate[(d + e*x)^2/(a + c*x^4)^3,x]
```

```
[Out] ((32*a^2*x*(d + e*x)^2)/(a + c*x^4)^2 + (8*a*x*(7*d^2 + 12*d*e*x + 5*e^2*x^
2))/(a + c*x^4) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[c]*d^2 + 48*a^(1/4)*c^(1/4)*d
*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4
) + (2*a^(1/4)*(21*Sqrt[2]*Sqrt[c]*d^2 - 48*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]
*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (Sqrt[2]*(
-21*a^(1/4)*Sqrt[c]*d^2 + 5*a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[c]*d^2 - 5*a^(3/4
)*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(25
6*a^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.84 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.35

method	result
risch	$\frac{\frac{5ce^2x^7}{32a^2}+\frac{3dcex^6}{8a^2}+\frac{7cd^2x^5}{32a^2}+\frac{9e^2x^3}{32a}+\frac{5edx^2}{8a}+\frac{11d^2x}{32a}}{(cx^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+a)}\frac{\left(5e^2\_R^2+24ed\_R+21d^2\right)\ln(x-\_R)}{\_R^3}}{128a^2c}$
default	$d^2\left(\frac{x}{8a(cx^4+a)^2}+\frac{\frac{7x}{32a(cx^4+a)}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2}}{a}\right)+2ed\Bigg($

```
[In] int((e*x+d)^2/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (5/32*c*e^2/a^2*x^7+3/8*d*c*e/a^2*x^6+7/32*c*d^2/a^2*x^5+9/32*e^2/a*x^3+5/8
*e*d/a*x^2+11/32/a*d^2*x)/(c*x^4+a)^2+1/128/a^2/c*sum((5*_R^2*e^2+24*_R*d*e
+21*d^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.15 (sec) , antiderivative size = 91420, normalized size of antiderivative = 253.94

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 14.65 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx$$
$$= \operatorname{RootSum}\Big(268435456t^4a^{11}c^3 + 25755648t^2a^6c^2d^2e^2 + t\big(307200a^4cde^5 - 5419008a^3c^2d^5e\big) + 625a^2e^8 + 1$$
$$+ \frac{11ad^2x + 20adex^2 + 9ae^2x^3 + 7cd^2x^5 + 12cdex^6 + 5ce^2x^7}{32a^4 + 64a^3cx^4 + 32a^2c^2x^8}$$

```
[In] integrate((e*x+d)**2/(c*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*c**3 + 25755648*_t**2*a**6*c**2*d**2*e**2 + _
t*(307200*a**4*c*d*e**5 - 5419008*a**3*c**2*d**5*e) + 625*a**2*e**8 + 11190
6*a*c*d**4*e**4 + 194481*c**2*d**8, Lambda(_t, _t*log(x + (262144000*_t**3*
a**10*c**2*e**6 + 46110081024*_t**3*a**9*c**3*d**4*e**2 - 1645608960*_t**2*
a**7*c**2*d**3*e**5 + 3641573376*_t**2*a**6*c**3*d**7*e + 32688000*_t*a**5*
c*d**2*e**8 + 3128219136*_t*a**4*c**2*d**6*e**4 + 522764928*_t*a**3*c**3*d*
*10 + 225000*a**3*d*e**11 - 43338240*a**2*c*d**5*e**7 - 523431720*a*c**2*d*
*9*e**3)/(15625*a**3*e**12 - 21357225*a**2*c*d**4*e**8 - 376741449*a*c**2*d
**8*e**4 + 85766121*c**3*d**12)))) + (11*a*d**2*x + 20*a*d*e*x**2 + 9*a*e**
2*x**3 + 7*c*d**2*x**5 + 12*c*d*e*x**6 + 5*c*e**2*x**7)/(32*a**4 + 64*a**3*
c*x**4 + 32*a**2*c**2*x**8)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx = \frac{5\,ce^2x^7 + 12\,cdex^6 + 7\,cd^2x^5 + 9\,ae^2x^3 + 20\,adex^2 + 11\,ad^2x}{32\,(a^2c^2x^8 + 2\,a^3cx^4 + a^4)}$$
$$+ \frac{\frac{\sqrt{2}(21\sqrt{c}d^2 - 5\sqrt{a}e^2)\log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}(21\sqrt{c}d^2 - 5\sqrt{a}e^2)\log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{2\left(21\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2 + 5\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}\right.}{}}{256\,a^2}$$

```
[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(5*c*e^2*x^7 + 12*c*d*e*x^6 + 7*c*d^2*x^5 + 9*a*e^2*x^3 + 20*a*d*e*x^2
 + 11*a*d^2*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 1/256*(sqrt(2)*(21*sqrt(
c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(
a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(21*sqrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c
)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 2*(21*sqrt
```

```
(2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 48*sqrt(a)*sqrt(c
)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt
(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(21*sqrt(2)*a^(1/
4)*c^(3/4)*d^2 + 5*sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 48*sqrt(a)*sqrt(c)*d*e)*ar
ctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(
c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)))/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx = \frac{5\,ce^2x^7+12\,cdex^6+7\,cd^2x^5+9\,ae^2x^3+20\,adex^2+11\,ad^2x}{32\,(cx^4+a)^2a^2} + \frac{\sqrt{2}\left(24\,\sqrt{2}\sqrt{ac}c^2de+21\,(ac^3)^{\frac{1}{4}}\,c^2d^2+5\,(ac^3)^{\frac{3}{4}}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3} + \frac{\sqrt{2}\left(24\,\sqrt{2}\sqrt{ac}c^2de+21\,(ac^3)^{\frac{1}{4}}\,c^2d^2+5\,(ac^3)^{\frac{3}{4}}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3} + \frac{\sqrt{2}\left(21\,(ac^3)^{\frac{1}{4}}\,c^2d^2-5\,(ac^3)^{\frac{3}{4}}\,e^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3} - \frac{\sqrt{2}\left(21\,(ac^3)^{\frac{1}{4}}\,c^2d^2-5\,(ac^3)^{\frac{3}{4}}\,e^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3}$$

```
[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(5*c*e^2*x^7 + 12*c*d*e*x^6 + 7*c*d^2*x^5 + 9*a*e^2*x^3 + 20*a*d*e*x^2
 + 11*a*d^2*x)/((c*x^4 + a)^2*a^2) + 1/128*sqrt(2)*(24*sqrt(2)*sqrt(a*c)*c^
2*d*e + 21*(a*c^3)^(1/4)*c^2*d^2 + 5*(a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 1/128*sqrt(2)*(24*sqrt
(2)*sqrt(a*c)*c^2*d*e + 21*(a*c^3)^(1/4)*c^2*d^2 + 5*(a*c^3)^(3/4)*e^2)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 1/256*
sqrt(2)*(21*(a*c^3)^(1/4)*c^2*d^2 - 5*(a*c^3)^(3/4)*e^2)*log(x^2 + sqrt(2)*
x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) - 1/256*sqrt(2)*(21*(a*c^3)^(1/4)*c^2*
d^2 - 5*(a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
3*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx = \frac{\frac{11\,d^2\,x}{32\,a}+\frac{9\,e^2\,x^3}{32\,a}+\frac{7\,c\,d^2\,x^5}{32\,a^2}+\frac{5\,c\,e^2\,x^7}{32\,a^2}+\frac{5\,d\,e\,x^2}{8\,a}+\frac{3\,c\,d\,e\,x^6}{8\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8} + \left(\sum_{k=1}^{4}\ln\left(-\frac{c\left(125\,a\,e^6-9891\,c\,d^4\,e^2+\mathrm{root}(268435456\,a^{11}\,c^3\,z^4+25755648\,a^6\,c^2\,d^2\,e^2\,z^2-5419008\,a^3\right.}{}\right.\right.$$

$$+\,25755648\,a^6\,c^2\,d^2\,e^2\,z^2-5419008\,a^3\,c^2\,d^5\,e\,z+307200\,a^4\,c\,d\,e^5\,z+111906\,a\,c\,d^4\,e^4$$

$$\left.+\,194481\,c^2\,d^8+625\,a^2\,e^8,z,k)\right)$$

```
[In] int((d + e*x)^2/(a + c*x^4)^3,x)
```

```
[Out] ((11*d^2*x)/(32*a) + (9*e^2*x^3)/(32*a) + (7*c*d^2*x^5)/(32*a^2) + (5*c*e^2
*x^7)/(32*a^2) + (5*d*e*x^2)/(8*a) + (3*c*d*e*x^6)/(8*a^2))/(a^2 + c^2*x^8
+ 2*a*c*x^4) + symsum(log(-(c*(125*a*e^6 - 9891*c*d^4*e^2 + 344064*root(268
435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*
z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^
8, z, k)^2*a^5*c^2*d^2 - 8784*c*d^3*e^3*x - 3200*root(268435456*a^11*c^3*z^
4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d
*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)*a^3*c*e^4
*x + 56448*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 541
9008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c
^2*d^8 + 625*a^2*e^8, z, k)*a^2*c^2*d^4*x + 30720*root(268435456*a^11*c^3*z
^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*
d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)*a^3*c*d*
e^3 - 393216*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5
419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481
*c^2*d^8 + 625*a^2*e^8, z, k)^2*a^5*c^2*d*e*x))/(32768*a^6))*root(268435456
*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 30
7200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z,
k), k, 1, 4)
```

3.410 $\int \frac{d+ex}{(a+cx^4)^3}\,dx$

Optimal result 2514
Rubi [A] (verified) 2515
Mathematica [A] (verified) 2518
Maple [C] (verified) 2519
Fricas [C] (verification not implemented) 2519
Sympy [A] (verification not implemented) 2519
Maxima [A] (verification not implemented) 2520
Giac [A] (verification not implemented) 2520
Mupad [B] (verification not implemented) 2521

Optimal result

Integrand size = 15, antiderivative size = 266

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx = \frac{x(d+ex)}{8a(a+cx^4)^2}+\frac{x(7d+6ex)}{32a^2(a+cx^4)}+\frac{3e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{21d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{21d\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} - \frac{21d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}$$

```
[Out] 1/8*x*(e*x+d)/a/(c*x^4+a)^2+1/32*x*(6*e*x+7*d)/a^2/(c*x^4+a)+21/128*d*arcta
n(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(11/4)/c^(1/4)*2^(1/2)+21/128*d*arctan(1+
c^(1/4)*x*2^(1/2)/a^(1/4))/a^(11/4)/c^(1/4)*2^(1/2)-21/256*d*ln(-a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)+21/256*d*ln(a
^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)+3/16
*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1869, 1890, 217, 1179, 642, 1176, 631, 210, 281, 211}

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx = -\frac{21d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{21d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{3e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}-\frac{21d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{21d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{x(7d+6ex)}{32a^2(a+cx^4)}+\frac{x(d+ex)}{8a(a+cx^4)^2}$$

```
[In] Int[(d + e*x)/(a + c*x^4)^3,x]
```

```
[Out] (x*(d + e*x))/(8*a*(a + c*x^4)^2) + (x*(7*d + 6*e*x))/(32*a^2*(a + c*x^4))
+ (3*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[c]) - (21*d*ArcTan[1
 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*d*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) - (21*d*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/
4)*c^(1/4)) + (21*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(128*Sqrt[2]*a^(11/4)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\text{integral} = \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} - \frac{\int \frac{-7d-6ex}{\left(a+cx^4\right)^2}\,dx}{8a}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{\int \frac{21d+12ex}{a+cx^4}\,dx}{32a^2}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{\int \left(\frac{21d}{a+cx^4} + \frac{12ex}{a+cx^4}\right)\,dx}{32a^2}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{(21d)\int \frac{1}{a+cx^4}\,dx}{32a^2} + \frac{(3e)\int \frac{x}{a+cx^4}\,dx}{8a^2}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{(21d)\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{64a^{5/2}} + \frac{(21d)\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{64a^{5/2}} + \frac{(3e)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{16a^2}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{3e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} + \frac{(21d)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{128a^{5/2}\sqrt{c}} + \frac{(21d)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{128a^{5/2}\sqrt{c}} - \frac{(21d)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} - \frac{(21d)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{3e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{21d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{(21d)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} - \frac{(21d)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{3e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}$$
$$- \frac{21d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21d\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}$$
$$- \frac{21d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.94

$$\int \frac{d+ex}{\left(a+cx^4\right)^3}\,dx$$
$$= \frac{\frac{32a^{7/4}x(d+ex)}{\left(a+cx^4\right)^2} + \frac{8a^{3/4}x(7d+6ex)}{a+cx^4} - \frac{6\left(7\sqrt{2}\sqrt[4]{c}d+8\sqrt[4]{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}} + \frac{6\left(7\sqrt{2}\sqrt[4]{c}d-8\sqrt[4]{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}} - 21}{256a^{11/4}}$$

```
[In] Integrate[(d + e*x)/(a + c*x^4)^3,x]
```

```
[Out] ((32*a^(7/4)*x*(d + e*x))/(a + c*x^4)^2 + (8*a^(3/4)*x*(7*d + 6*e*x))/(a +
c*x^4) - (6*(7*Sqrt[2]*c^(1/4)*d + 8*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)
*x)/a^(1/4)])/Sqrt[c] + (6*(7*Sqrt[2]*c^(1/4)*d - 8*a^(1/4)*e)*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] - (21*Sqrt[2]*d*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (21*Sqrt[2]*d*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.84 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\frac{3ec\,x^{6}}{16a^{2}}+\frac{7cd\,x^{5}}{32a^{2}}+\frac{5e\,x^{2}}{16a}+\frac{11dx}{32a}}{\left(c\,x^{4}+a\right)^{2}}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+a\right)}\frac{\left(4e\_R+7d\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{128a^{2}c}$
default	$d\left(\frac{x}{8a\left(c\,x^{4}+a\right)^{2}}+\frac{\frac{7x}{32a\left(c\,x^{4}+a\right)}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^{2}}}{a}\right)+e\left(\frac{}{8a}\right.$

```
[In] int((e*x+d)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (3/16*e*c/a^2*x^6+7/32/a^2*c*d*x^5+5/16*e/a*x^2+11/32*d/a*x)/(c*x^4+a)^2+3/
128/a^2/c*sum((4*_R*e+7*d)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 43180, normalized size of antiderivative = 162.33

$$\int\frac{d+ex}{\left(a+cx^{4}\right)^{3}}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.72

$$\int\frac{d+ex}{\left(a+cx^{4}\right)^{3}}\,dx$$

$$=\operatorname{RootSum}\left(268435456t^{4}a^{11}c^{2}+4718592t^{2}a^{6}ce^{2}-2709504ta^{3}cd^{2}e+20736ae^{4}+194481cd^{4},\left(t\mapsto t\log\left(\right.\right.\right.$$

$$+\frac{11adx+10aex^{2}+7cdx^{5}+6cex^{6}}{32a^{4}+64a^{3}cx^{4}+32a^{2}c^{2}x^{8}}$$

```
[In] integrate((e*x+d)/(c*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*c**2 + 4718592*_t**2*a**6*c*e**2 - 2709504*_t
*a**3*c*d**2*e + 20736*a*e**4 + 194481*c*d**4, Lambda(_t, _t*log(x + (-6710
8864*_t**3*a**9*c*e**2 - 9633792*_t**2*a**6*c*d**2*e - 589824*_t*a**4*e**4
- 2765952*_t*a**3*c*d**4 + 423360*a*d**2*e**3)/(193536*a*d*e**4 - 453789*c*
d**5)))) + (11*a*d*x + 10*a*e*x**2 + 7*c*d*x**5 + 6*c*e*x**6)/(32*a**4 + 64
*a**3*c*x**4 + 32*a**2*c**2*x**8)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.01

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx = \frac{6\,cex^6+7\,cdx^5+10\,aex^2+11\,adx}{32\,(a^2c^2x^8+2\,a^3cx^4+a^4)} + \frac{3\left(\frac{7\,\sqrt{2}d\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{7\,\sqrt{2}d\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} + \frac{2\left(7\,\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d-8\,\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{1}{4}}} + \cdots\right)}{256\,a^2}$$

```
[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(6*c*e*x^6 + 7*c*d*x^5 + 10*a*e*x^2 + 11*a*d*x)/(a^2*c^2*x^8 + 2*a^3*c
*x^4 + a^4) + 3/256*(7*sqrt(2)*d*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*
x + sqrt(a))/(a^(3/4)*c^(1/4)) - 7*sqrt(2)*d*log(sqrt(c)*x^2 - sqrt(2)*a^(1
/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(1/4)*d
 - 8*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4)) + 2*(7*sqrt(
2)*a^(1/4)*c^(1/4)*d + 8*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c
^(1/4)))/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.96

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx = \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}}\,d\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c} - \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}}\,d\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ac}ce+7\,(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^2} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ac}ce+7\,(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^2} + \frac{6\,cex^6+7\,cdx^5+10\,aex^2+11\,adx}{32\,(cx^4+a)^2a^2}$$

```
[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/256*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))
/(a^3*c) - 21/256*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) +
 sqrt(a/c))/(a^3*c) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c*e + 7*(a*c^3)^(1
/4)*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c
^2) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c*e + 7*(a*c^3)^(1/4)*c*d)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^2) + 1/32*(6*c*
e*x^6 + 7*c*d*x^5 + 10*a*e*x^2 + 11*a*d*x)/((c*x^4 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.18

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx = \frac{\frac{5\,e\,x^2}{16\,a}+\frac{11\,d\,x}{32\,a}+\frac{7\,c\,d\,x^5}{32\,a^2}+\frac{3\,c\,e\,x^6}{16\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8} + \left(\sum_{k=1}^{4}\ln\left(\frac{c^2\left(63\,d\,e^2+36\,e^3\,x-\mathrm{root}(268435456\,a^{11}\,c^2\,z^4+4718592\,a^6\,c\,e^2\,z^2-2709504\,a^3\,c\,d^2\,e\,z+1\right.}{}\right.\right.$$

$$\left.+4718592\,a^6\,c\,e^2\,z^2-2709504\,a^3\,c\,d^2\,e\,z+194481\,c\,d^4+20736\,a\,e^4,z,k)\right)$$

```
[In] int((d + e*x)/(a + c*x^4)^3,x)
```

```
[Out] ((5*e*x^2)/(16*a) + (11*d*x)/(32*a) + (7*c*d*x^5)/(32*a^2) + (3*c*e*x^6)/(1
6*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + symsum(log((3*c^2*(63*d*e^2 + 36*e^3*
```

```
x - 7168*root(268435456*a^11*c^2*z^4 + 4718592*a^6*c*e^2*z^2 - 2709504*a^3*
c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4, z, k)^2*a^5*c*d - 1176*root(2684354
56*a^11*c^2*z^4 + 4718592*a^6*c*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*
d^4 + 20736*a*e^4, z, k)*a^2*c*d^2*x + 4096*root(268435456*a^11*c^2*z^4 + 4
718592*a^6*c*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4,
z, k)^2*a^5*c*e*x))/(2048*a^6))*root(268435456*a^11*c^2*z^4 + 4718592*a^6*c
*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4, z, k), k, 1,
 4)
```

3.411 $\int \frac{1}{(a+cx^4)^3} dx$

Optimal result 2523
Rubi [A] (verified) 2523
Mathematica [A] (verified) 2526
Maple [C] (verified) 2526
Fricas [C] (verification not implemented) 2527
Sympy [A] (verification not implemented) 2527
Maxima [A] (verification not implemented) 2528
Giac [A] (verification not implemented) 2528
Mupad [B] (verification not implemented) 2529

Optimal result

Integrand size = 9, antiderivative size = 219

$$\int \frac{1}{(a+cx^4)^3} dx = \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} - \frac{21\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} - \frac{21\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}$$

```
[Out] 1/8*x/a/(c*x^4+a)^2+7/32*x/a^2/(c*x^4+a)+21/128*arctan(-1+c^(1/4)*x*2^(1/2)
/a^(1/4))/a^(11/4)/c^(1/4)*2^(1/2)+21/128*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4
))/a^(11/4)/c^(1/4)*2^(1/2)-21/256*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^
2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)+21/256*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(
1/2)+x^2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used

$= \{205, 217, 1179, 642, 1176, 631, 210\}$

$$\int \frac{1}{(a+cx^4)^3}\,dx = -\frac{21\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} - \frac{21\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{7x}{32a^2(a+cx^4)} + \frac{x}{8a(a+cx^4)^2}$$

```
[In] Int[(a + c*x^4)^(-3),x]
```

```
[Out] x/(8*a*(a + c*x^4)^2) + (7*x)/(32*a^2*(a + c*x^4)) - (21*ArcTan[1 - (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) - (21*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4)) +
(21*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^
(11/4)*c^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \frac{x}{8a\left(a+cx^4\right)^2}+\frac{7\int \frac{1}{\left(a+cx^4\right)^2}\,dx}{8a}$$

$$= \frac{x}{8a\left(a+cx^4\right)^2}+\frac{7x}{32a^2\left(a+cx^4\right)}+\frac{21\int \frac{1}{a+cx^4}\,dx}{32a^2}$$

$$= \frac{x}{8a\left(a+cx^4\right)^2}+\frac{7x}{32a^2\left(a+cx^4\right)}+\frac{21\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{64a^{5/2}}+\frac{21\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{64a^{5/2}}$$

$$= \frac{x}{8a\left(a+cx^4\right)^2}+\frac{7x}{32a^2\left(a+cx^4\right)}+\frac{21\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{128a^{5/2}\sqrt{c}}$$

$$+\frac{21\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{128a^{5/2}\sqrt{c}}-\frac{21\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}-\frac{21\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}$$

$$= \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} - \frac{21\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} - \frac{21\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}$$

$$= \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} - \frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} - \frac{21\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+cx^4)^3}\,dx = \frac{\frac{32a^{7/4}x}{(a+cx^4)^2} + \frac{56a^{3/4}x}{a+cx^4} - \frac{42\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{42\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} - \frac{21\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{\sqrt[4]{c}} + \frac{21\sqrt{2}\log\left(\right.}{}}{256a^{11/4}}$$

```
[In] Integrate[(a + c*x^4)^(-3),x]
```

```
[Out] ((32*a^(7/4)*x)/(a + c*x^4)^2 + (56*a^(3/4)*x)/(a + c*x^4) - (42*Sqrt[2]*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) + (42*Sqrt[2]*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) - (21*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (21*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.26

method	result	size
risch	$\frac{\frac{7c\,x^5}{32a^2}+\frac{11x}{32a}}{(c\,x^4+a)^2}+\frac{21\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+a)}\frac{\ln(x-\_R)}{\_R^3}\right)}{128a^2c}$	57
default	$\frac{x}{8a(c\,x^4+a)^2}+\frac{\frac{7x}{32a(c\,x^4+a)}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2}}{a}$	139

```
[In] int(1/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (7/32*c/a^2*x^5+11/32*x/a)/(c*x^4+a)^2+21/128/a^2/c*sum(1/_R^3*ln(x-_R),_R=
RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.16

$$\int \frac{1}{(a+cx^4)^3}\,dx$$
$$=\frac{28\,cx^5+21\,(a^2c^2x^8+2\,a^3cx^4+a^4)\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}\log\left(a^3\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}+x\right)-21\,(-i\,a^2c^2x^8-2i\,a^3cx^4-i\,a^4)\left(-\frac{}{a}\right.}{}$$

```
[In] integrate(1/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(28*c*x^5 + 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c))^(1/4)*
log(a^3*(-1/(a^11*c))^(1/4) + x) - 21*(-I*a^2*c^2*x^8 - 2*I*a^3*c*x^4 - I*a
^4)*(-1/(a^11*c))^(1/4)*log(I*a^3*(-1/(a^11*c))^(1/4) + x) - 21*(I*a^2*c^2*
x^8 + 2*I*a^3*c*x^4 + I*a^4)*(-1/(a^11*c))^(1/4)*log(-I*a^3*(-1/(a^11*c))^(
1/4) + x) - 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c))^(1/4)*log(-a
^3*(-1/(a^11*c))^(1/4) + x) + 44*a*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.29

$$\int \frac{1}{(a+cx^4)^3}\,dx=\frac{11ax+7cx^5}{32a^4+64a^3cx^4+32a^2c^2x^8}+\operatorname{RootSum}\left(268435456t^4a^{11}c+194481,\left(t\mapsto t\log\left(\frac{128ta^3}{21}+x\right)\right)\right)$$

```
[In] integrate(1/(c*x**4+a)**3,x)
```

```
[Out] (11*a*x + 7*c*x**5)/(32*a**4 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8) + RootSu
m(268435456*_t**4*a**11*c + 194481, Lambda(_t, _t*log(128*_t*a**3/21 + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+cx^4\right)^3}\,dx=\frac{7\,cx^5+11\,ax}{32\,\left(a^2c^2x^8+2\,a^3cx^4+a^4\right)}+\frac{21\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{256\,a^2}$$

```
[In] integrate(1/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(7*c*x^5 + 11*a*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 21/256*(2*sqrt(
2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*
sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2
*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + s
qrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+cx^4\right)^3}\,dx=\frac{21\,\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c}+\frac{21\,\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c}+\frac{21\,\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c}-\frac{21\,\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c}+\frac{7\,cx^5+11\,ax}{32\,\left(cx^4+a\right)^2a^2}$$

```
[In] integrate(1/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))
/(a/c)^(1/4))/(a^3*c) + 21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*
x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c) + 21/256*sqrt(2)*(a*c^3)^(1/4
)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) - 21/256*sqrt(2)*(a*
c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) + 1/32*(7*c
*x^5 + 11*a*x)/((c*x^4 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.37

$$\int \frac{1}{(a+cx^4)^3}\,dx=\frac{\frac{11\,x}{32\,a}+\frac{7\,c\,x^5}{32\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}-\frac{21\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{{(-a)}^{1/4}}\right)}{64\,{(-a)}^{11/4}\,c^{1/4}}-\frac{21\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{{(-a)}^{1/4}}\right)}{64\,{(-a)}^{11/4}\,c^{1/4}}$$

```
[In] int(1/(a + c*x^4)^3,x)
```

```
[Out] ((11*x)/(32*a) + (7*c*x^5)/(32*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (21*atan
((c^(1/4)*x)/(-a)^(1/4)))/(64*(-a)^(11/4)*c^(1/4)) - (21*atanh((c^(1/4)*x)/
(-a)^(1/4)))/(64*(-a)^(11/4)*c^(1/4))
```

3.412 $\int \frac{1}{(d+ex)(a+cx^4)^3} dx$

Optimal result 2531
Rubi [A] (verified) 2532
Mathematica [A] (verified) 2540
Maple [A] (verified) 2541
Fricas [F(-1)] 2541
Sympy [F(-1)] 2542
Maxima [A] (verification not implemented) 2542
Giac [A] (verification not implemented) 2543
Mupad [B] (verification not implemented) 2544

Optimal result

Integrand size = 17, antiderivative size = 1352

$$
\begin{aligned}
\int \frac{1}{(d+ex)\left(a+cx^4\right)^3}\,dx &= \frac{cx(7d^3-6d^2ex+5de^2x^2)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)}+\frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2}\\
&+\frac{e^4(ae^3+cx(d^3-d^2ex+de^2x^2))}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}-\frac{\sqrt{c}d^2e^9\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)^3}\\
&-\frac{\sqrt{c}d^2e^5\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^4+ae^4\right)^2}-\frac{3\sqrt{c}d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\left(cd^4+ae^4\right)}\\
&-\frac{\sqrt[4]{c}de^8\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}\\
&-\frac{\sqrt[4]{c}de^4\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}\\
&-\frac{\sqrt[4]{c}d\left(21\sqrt{c}d^2+5\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\left(cd^4+ae^4\right)}\\
&+\frac{\sqrt[4]{c}de^8\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}\\
&+\frac{\sqrt[4]{c}de^4\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}\\
&+\frac{\sqrt[4]{c}d\left(21\sqrt{c}d^2+5\sqrt{a}e^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\left(cd^4+ae^4\right)}\\
&+\frac{e^{11}\log(d+ex)}{\left(cd^4+ae^4\right)^3}\\
&-\frac{\sqrt[4]{c}de^8\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}\\
&-\frac{\sqrt[4]{c}de^4\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}\\
&-\frac{\sqrt[4]{c}d\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\left(cd^4+ae^4\right)}\\
&+\frac{\sqrt[4]{c}de^8\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}\\
&+\frac{\sqrt[4]{c}de^4\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}\\
&+\frac{\sqrt[4]{c}d\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\left(cd^4+ae^4\right)}\\
&-\frac{e^{11}\log\left(a+cx^4\right)}{\cdots}
\end{aligned}
$$

```
[Out] 1/32*c*x*(5*d*e^2*x^2-6*d^2*e*x+7*d^3)/a^2/(a*e^4+c*d^4)/(c*x^4+a)+1/8*(a*e
^3+c*x*(d*e^2*x^2-d^2*e*x+d^3))/a/(a*e^4+c*d^4)/(c*x^4+a)^2+1/4*e^4*(a*e^3+
c*x*(d*e^2*x^2-d^2*e*x+d^3))/a/(a*e^4+c*d^4)^2/(c*x^4+a)+e^11*ln(e*x+d)/(a*
e^4+c*d^4)^3-1/4*e^11*ln(c*x^4+a)/(a*e^4+c*d^4)^3-1/4*d^2*e^5*arctan(x^2*c^
(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^4+c*d^4)^2-3/16*d^2*e*arctan(x^2*c^(1/2
)/a^(1/2))*c^(1/2)/a^(5/2)/(a*e^4+c*d^4)-1/2*d^2*e^9*arctan(x^2*c^(1/2)/a^(
1/2))*c^(1/2)/(a*e^4+c*d^4)^3/a^(1/2)-1/8*c^(1/4)*d*e^8*ln(-a^(1/4)*c^(1/4)
*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c
*d^4)^3*2^(1/2)+1/8*c^(1/4)*d*e^8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(
1/4)*d*e^8*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a
^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(1/4)*d*e^8*arctan(1+c^(1/4)*x*2^(1/2)
/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)-1/32*c^
(1/4)*d*e^4*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2
)+3*d^2*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/32*c^(1/4)*d*e^4*ln(a^(1
/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+3*d^2*c^(1/2))/a^(
7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/16*c^(1/4)*d*e^4*arctan(-1+c^(1/4)*x*2^(1/2)
/a^(1/4))*(e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/16*
c^(1/4)*d*e^4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+3*d^2*c^(1/2
))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)-1/256*c^(1/4)*d*ln(-a^(1/4)*c^(1/4)*x*2^
(1/2)+a^(1/2)+x^2*c^(1/2))*(-5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/4)/(a*e^4+
c*d^4)*2^(1/2)+1/256*c^(1/4)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(
1/2))*(-5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)*2^(1/2)+1/128*
c^(1/4)*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(5*e^2*a^(1/2)+21*d^2*c^(1/2
))/a^(11/4)/(a*e^4+c*d^4)*2^(1/2)+1/128*c^(1/4)*d*arctan(1+c^(1/4)*x*2^(1/2
)/a^(1/4))*(5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 1352, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules

used = {6874, 1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 266}

$$\int \frac{1}{(d+ex)(a+cx^4)^3}\,dx = \frac{\log(d+ex)e^{11}}{(cd^4+ae^4)^3} - \frac{\log(cx^4+a)\,e^{11}}{4(cd^4+ae^4)^3} - \frac{\sqrt{c}d^2\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^9}{2\sqrt{a}(cd^4+ae^4)^3}$$
$$- \frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^8}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^3}$$
$$+ \frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^8}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^3}$$
$$- \frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^8}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^3}$$
$$+ \frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^8}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^3}$$
$$- \frac{\sqrt{c}d^2\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{4a^{3/2}(cd^4+ae^4)^2} + \frac{(ae^3+cx(d^3-exd^2+e^2x^2d))\,e^4}{4a(cd^4+ae^4)^2(cx^4+a)}$$
$$- \frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{8\sqrt{2}a^{7/4}(cd^4+ae^4)^2}$$
$$+ \frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^4}{8\sqrt{2}a^{7/4}(cd^4+ae^4)^2}$$
$$- \frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{16\sqrt{2}a^{7/4}(cd^4+ae^4)^2}$$
$$+ \frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{16\sqrt{2}a^{7/4}(cd^4+ae^4)^2}$$
$$- \frac{3\sqrt{c}d^2\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{16a^{5/2}(cd^4+ae^4)} + \frac{ae^3+cx(d^3-exd^2+e^2x^2d)}{8a(cd^4+ae^4)(cx^4+a)^2}$$
$$- \frac{\sqrt[4]{c}d\left(21\sqrt{c}d^2+5\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}(cd^4+ae^4)}$$
$$+ \frac{\sqrt[4]{c}d\left(21\sqrt{c}d^2+5\sqrt{a}e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}(cd^4+ae^4)}$$
$$- \frac{\sqrt[4]{c}d\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{128\sqrt{2}a^{11/4}(cd^4+ae^4)}$$
$$+ \frac{\sqrt[4]{c}d\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{128\sqrt{2}a^{11/4}(cd^4+ae^4)}$$
$$+ \frac{cx(7d^3-6exd^2+5e^2x^2d)}{32a^2(cd^4+ae^4)(cx^4+a)}$$

```
[In] Int[1/((d + e*x)*(a + c*x^4)^3),x]
```

```
[Out] (c*x*(7*d^3 - 6*d^2*e*x + 5*d*e^2*x^2))/(32*a^2*(c*d^4 + a*e^4)*(a + c*x^4)
) + (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(8*a*(c*d^4 + a*e^4)*(a + c*x
^4)^2) + (e^4*(a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2)))/(4*a*(c*d^4 + a*e^
4)^2*(a + c*x^4)) - (Sqrt[c]*d^2*e^9*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt
[a]*(c*d^4 + a*e^4)^3) - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4
*a^(3/2)*(c*d^4 + a*e^4)^2) - (3*Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]
])/(16*a^(5/2)*(c*d^4 + a*e^4)) - (c^(1/4)*d*e^8*(Sqrt[c]*d^2 + Sqrt[a]*e^2
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^
4)^3) - (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(21*S
qrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sq
rt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^8*(Sqrt[c]*d^2 + Sqrt[a]*e^2
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^
4)^3) + (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(21*S
qrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sq
rt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (e^11*Log[d + e*x])/(c*d^4 + a*e^4)^3 - (
c^(1/4)*d*e^8*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (c^(1/4)*d*e
^4*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(21*Sqrt[
c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^8*(Sqrt[c]*d^2 -
 Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sq
rt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 - Sqrt[a]*
e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^
(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(
c*d^4 + a*e^4)) - (e^11*Log[a + c*x^4])/(4*(c*d^4 + a*e^4)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{e^{12}}{(cd^4+ae^4)^3(d+ex)}+\frac{c(d^3-d^2ex+de^2x^2-e^3x^3)}{(cd^4+ae^4)(a+cx^4)^3}\right.\\ &\qquad \left.-\frac{ce^4(-d^3+d^2ex-de^2x^2+e^3x^3)}{(cd^4+ae^4)^2(a+cx^4)^2}-\frac{ce^8(-d^3+d^2ex-de^2x^2+e^3x^3)}{(cd^4+ae^4)^3(a+cx^4)}\right)dx\\ &= \frac{e^{11}\log(d+ex)}{(cd^4+ae^4)^3}-\frac{(ce^8)\int\frac{-d^3+d^2ex-de^2x^2+e^3x^3}{a+cx^4}\,dx}{(cd^4+ae^4)^3}\\ &\quad -\frac{(ce^4)\int\frac{-d^3+d^2ex-de^2x^2+e^3x^3}{(a+cx^4)^2}\,dx}{(cd^4+ae^4)^2}+\frac{c\int\frac{d^3-d^2ex+de^2x^2-e^3x^3}{(a+cx^4)^3}\,dx}{cd^4+ae^4}\end{aligned}$$

$$= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2}+\frac{e^4(ae^3+cx(d^3-d^2ex+de^2x^2))}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}+\frac{e^{11}\log(d+ex)}{\left(cd^4+ae^4\right)^3}-\frac{(ce^8)\int\left(\frac{-d^3-de^2x^2}{a+cx^4}+\frac{x(d^2e+e^3x^2)}{a+cx^4}\right)dx}{\left(cd^4+ae^4\right)^3}+\frac{(ce^4)\int\frac{3d^3-2d^2ex+de^2x^2}{a+cx^4}dx}{4a\left(cd^4+ae^4\right)^2}-\frac{c\int\frac{-7d^3+6d^2ex-5de^2x^2}{(a+cx^4)^2}dx}{8a\left(cd^4+ae^4\right)}$$

$$= \frac{cx(7d^3-6d^2ex+5de^2x^2)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)}+\frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2}+\frac{e^4(ae^3+cx(d^3-d^2ex+de^2x^2))}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}+\frac{e^{11}\log(d+ex)}{\left(cd^4+ae^4\right)^3}-\frac{(ce^8)\int\frac{-d^3-de^2x^2}{a+cx^4}dx}{\left(cd^4+ae^4\right)^3}-\frac{(ce^8)\int\frac{x(d^2e+e^3x^2)}{a+cx^4}dx}{\left(cd^4+ae^4\right)^3}+\frac{(ce^4)\int\left(-\frac{2d^2ex}{a+cx^4}+\frac{3d^3+de^2x^2}{a+cx^4}\right)dx}{4a\left(cd^4+ae^4\right)^2}+\frac{c\int\frac{21d^3-12d^2ex+5de^2x^2}{a+cx^4}dx}{32a^2\left(cd^4+ae^4\right)}$$

$$= \frac{cx(7d^3-6d^2ex+5de^2x^2)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)}+\frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2}+\frac{e^4(ae^3+cx(d^3-d^2ex+de^2x^2))}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}+\frac{e^{11}\log(d+ex)}{\left(cd^4+ae^4\right)^3}-\frac{(ce^8)\,\mathrm{Subst}\left(\int\frac{d^2e+e^3x}{a+cx^2}dx,x,x^2\right)}{2\left(cd^4+ae^4\right)^3}+\frac{\left(de^8\left(\frac{\sqrt{c}d^2}{\sqrt{a}}-e^2\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}dx}{2\left(cd^4+ae^4\right)^3}+\frac{\left(de^8\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}dx}{2\left(cd^4+ae^4\right)^3}+\frac{(ce^4)\int\frac{3d^3+de^2x^2}{a+cx^4}dx}{4a\left(cd^4+ae^4\right)^2}-\frac{(cd^2e^5)\int\frac{x}{a+cx^4}dx}{2a\left(cd^4+ae^4\right)^2}+\frac{c\int\left(-\frac{12d^2ex}{a+cx^4}+\frac{21d^3+5de^2x^2}{a+cx^4}\right)dx}{32a^2\left(cd^4+ae^4\right)}$$

$$= \frac{cx(7d^3-6d^2ex+5de^2x^2)}{32a^2(cd^4+ae^4)(a+cx^4)} + \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{8a(cd^4+ae^4)(a+cx^4)^2}$$
$$+ \frac{e^4(ae^3+cx(d^3-d^2ex+de^2x^2))}{4a(cd^4+ae^4)^2(a+cx^4)} + \frac{e^{11}\log(d+ex)}{(cd^4+ae^4)^3}$$
$$- \frac{(cd^2e^9)\operatorname{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2(cd^4+ae^4)^3} - \frac{(ce^{11})\operatorname{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^4+ae^4)^3}$$
$$+ \frac{\left(de^8\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4(cd^4+ae^4)^3} + \frac{\left(de^8\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4(cd^4+ae^4)^3}$$
$$- \frac{\left(\sqrt[4]{c}de^8\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^3}$$
$$- \frac{\left(\sqrt[4]{c}de^8\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^3}$$
$$- \frac{(cd^2e^5)\operatorname{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{4a(cd^4+ae^4)^2} + \frac{\left(de^4\left(\frac{3\sqrt{c}d^2}{\sqrt{a}}-e^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a(cd^4+ae^4)^2}$$
$$+ \frac{\left(de^4\left(\frac{3\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8a(cd^4+ae^4)^2} + \frac{c\int \frac{21d^3+5de^2x^2}{a+cx^4}\,dx}{32a^2(cd^4+ae^4)} - \frac{(3cd^2e)\int \frac{x}{a+cx^4}\,dx}{8a^2(cd^4+ae^4)}$$

$$= \frac{cx(7d^3-6d^2ex+5de^2x^2)}{32a^2(cd^4+ae^4)(a+cx^4)} + \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{8a(cd^4+ae^4)(a+cx^4)^2}$$
$$+ \frac{e^4(ae^3+cx(d^3-d^2ex+de^2x^2))}{4a(cd^4+ae^4)^2(a+cx^4)} - \frac{\sqrt{c}d^2e^9\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(cd^4+ae^4)^3} - \frac{\sqrt{c}d^2e^5\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}(cd^4+ae^4)^2}$$
$$+ \frac{e^{11}\log(d+ex)}{(cd^4+ae^4)^3} - \frac{\sqrt[4]{c}de^8(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^3}$$
$$+ \frac{\sqrt[4]{c}de^8(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^3} - \frac{e^{11}\log(a+cx^4)}{4(cd^4+ae^4)^3}$$
$$+ \frac{\left(\sqrt[4]{c}de^8(\sqrt{c}d^2+\sqrt{a}e^2)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^3}$$
$$- \frac{\left(\sqrt[4]{c}de^8(\sqrt{c}d^2+\sqrt{a}e^2)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^3}$$
$$+ \frac{\left(de^4\left(\frac{3\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a(cd^4+ae^4)^2} + \frac{\left(de^4\left(\frac{3\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a(cd^4+ae^4)^2}$$
$$- \frac{\left(\sqrt[4]{c}de^4(3\sqrt{c}d^2-\sqrt{a}e^2)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}(cd^4+ae^4)^2}$$
$$- \frac{\left(\sqrt[4]{c}de^4(3\sqrt{c}d^2-\sqrt{a}e^2)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}(cd^4+ae^4)^2}$$
$$- \frac{(3cd^2e)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{16a^2(cd^4+ae^4)} + \frac{\left(d\left(\frac{21\sqrt{c}d^2}{\sqrt{a}}-5e^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{64a^2(cd^4+ae^4)}$$
$$+ \frac{\left(d\left(\frac{21\sqrt{c}d^2}{\sqrt{a}}+5e^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{64a^2(cd^4+ae^4)}$$

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Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 835, normalized size of antiderivative = 0.62

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^3}\,dx$$

$$= \frac{\frac{32\left(cd^4+ae^4\right)^2\left(ae^3+cdx\left(d^2-dex+e^2x^2\right)\right)}{a\left(a+cx^4\right)^2}+\frac{8\left(cd^4+ae^4\right)\left(8a^2e^7+c^2d^5x\left(7d^2-6dex+5e^2x^2\right)+acde^4x\left(15d^2-14dex+13e^2x^2\right)\right)}{a^2\left(a+cx^4\right)}-\frac{2\sqrt[4]{c}d\left(21\sqrt{2}c\cdots\right.}{\cdots}}{\cdots}$$

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)^3),x]
```

```
[Out] ((32*(c*d^4 + a*e^4)^2*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(a*(a + c*x
^4)^2) + (8*(c*d^4 + a*e^4)*(8*a^2*e^7 + c^2*d^5*x*(7*d^2 - 6*d*e*x + 5*e^2
*x^2) + a*c*d*e^4*x*(15*d^2 - 14*d*e*x + 13*e^2*x^2)))/(a^2*(a + c*x^4)) -
(2*c^(1/4)*d*(21*Sqrt[2]*c^(5/2)*d^10 - 24*a^(1/4)*c^(9/4)*d^9*e + 5*Sqrt[2
]*Sqrt[a]*c^2*d^8*e^2 + 66*Sqrt[2]*a*c^(3/2)*d^6*e^4 - 80*a^(5/4)*c^(5/4)*d
^5*e^5 + 18*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 77*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 - 12
0*a^(9/4)*c^(1/4)*d*e^9 + 45*Sqrt[2]*a^(5/2)*e^10)*ArcTan[1 - (Sqrt[2]*c^(1
/4)*x)/a^(1/4)])/a^(11/4) + (2*c^(1/4)*d*(21*Sqrt[2]*c^(5/2)*d^10 + 24*a^(1
/4)*c^(9/4)*d^9*e + 5*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 + 66*Sqrt[2]*a*c^(3/2)*d^
6*e^4 + 80*a^(5/4)*c^(5/4)*d^5*e^5 + 18*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 77*Sqrt
[2]*a^2*Sqrt[c]*d^2*e^8 + 120*a^(9/4)*c^(1/4)*d*e^9 + 45*Sqrt[2]*a^(5/2)*e^
10)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + 256*e^11*Log[d + e*
x] + (Sqrt[2]*c^(1/4)*(-21*c^(5/2)*d^11 + 5*Sqrt[a]*c^2*d^9*e^2 - 66*a*c^(3
/2)*d^7*e^4 + 18*a^(3/2)*c*d^5*e^6 - 77*a^2*Sqrt[c]*d^3*e^8 + 45*a^(5/2)*d*
e^10)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4) + (S
qrt[2]*c^(1/4)*(21*c^(5/2)*d^11 - 5*Sqrt[a]*c^2*d^9*e^2 + 66*a*c^(3/2)*d^7*
e^4 - 18*a^(3/2)*c*d^5*e^6 + 77*a^2*Sqrt[c]*d^3*e^8 - 45*a^(5/2)*d*e^10)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4) - 64*e^11*Lo
g[a + c*x^4])/(256*(c*d^4 + a*e^4)^3)
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 677, normalized size of antiderivative = 0.50

method	result
default	$c\left(\frac{\frac{c d e^{2}\left(13 a^{2} e^{8}+18 a c d^{4} e^{4}+5 c^{2} d^{8}\right) x^{7}}{32 a^{2}}-\frac{c d^{2} e\left(7 a^{2} e^{8}+10 a c d^{4} e^{4}+3 c^{2} d^{8}\right) x^{6}}{16 a^{2}}+\frac{d^{3} c\left(15 a^{2} e^{8}+22 a c d^{4} e^{4}+7 c^{2} d^{8}\right) x^{5}}{32 a^{2}}+\left(\frac{1}{4} a e^{11}+\frac{1}{4} d^{4} e^{7} c\right) x^{4}+\frac{d e^{2}(}{}}{\left(c x^{4}+a\right)^{2}}\right.$
risch	Expression too large to display

```
[In] int(1/(e*x+d)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c/(a*e^4+c*d^4)^3*((1/32*c*d*e^2*(13*a^2*e^8+18*a*c*d^4*e^4+5*c^2*d^8)/a^2*
x^7-1/16*c*d^2*e*(7*a^2*e^8+10*a*c*d^4*e^4+3*c^2*d^8)/a^2*x^6+1/32*d^3*c*(1
5*a^2*e^8+22*a*c*d^4*e^4+7*c^2*d^8)/a^2*x^5+(1/4*a*e^11+1/4*d^4*e^7*c)*x^4+
1/32*d*e^2*(17*a^2*e^8+26*a*c*d^4*e^4+9*c^2*d^8)/a*x^3-1/16*d^2*e*(9*a^2*e^
8+14*a*c*d^4*e^4+5*c^2*d^8)/a*x^2+1/32*d^3*(19*a^2*e^8+30*a*c*d^4*e^4+11*c^
2*d^8)/a*x+1/8*e^3*(3*a^2*e^8+4*a*c*d^4*e^4+c^2*d^8)/c)/(c*x^4+a)^2+1/32/a^
2*(1/8*(77*a^2*d^3*e^8+66*a*c*d^7*e^4+21*c^2*d^11)*(a/c)^(1/4)/a*2^(1/2)*(l
n((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^
(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
)+1/2*(-60*a^2*d^2*e^9-40*a*c*d^6*e^5-12*c^2*d^10*e)/(a*c)^(1/2)*arctan(x^2
*(c/a)^(1/2))+1/8*(45*a^2*d*e^10+18*a*c*d^5*e^6+5*c^2*d^9*e^2)/c/(a/c)^(1/4
)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^
(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c
)^(1/4)*x-1))-8*a^2*e^11/c*ln(c*x^4+a)))+e^11*ln(e*x+d)/(a*e^4+c*d^4)^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\,(a+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(c*x**4+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 1015, normalized size of antiderivative = 0.75

$$\int \frac{1}{(d+ex)\,(a+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="maxima")
[Out] e^11*log(e*x + d)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12)
 - 1/256*c*(sqrt(2)*(32*sqrt(2)*a^(11/4)*c^(1/4)*e^11 - 21*c^3*d^11 + 5*sqr
t(a)*c^(5/2)*d^9*e^2 - 66*a*c^2*d^7*e^4 + 18*a^(3/2)*c^(3/2)*d^5*e^6 - 77*a
^2*c*d^3*e^8 + 45*a^(5/2)*sqrt(c)*d*e^10)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)
*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(32*sqrt(2)*a^(11/4)*c^(1
/4)*e^11 + 21*c^3*d^11 - 5*sqrt(a)*c^(5/2)*d^9*e^2 + 66*a*c^2*d^7*e^4 - 18*
a^(3/2)*c^(3/2)*d^5*e^6 + 77*a^2*c*d^3*e^8 - 45*a^(5/2)*sqrt(c)*d*e^10)*log
(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(
21*sqrt(2)*a^(1/4)*c^(13/4)*d^11 + 5*sqrt(2)*a^(3/4)*c^(11/4)*d^9*e^2 + 66*
sqrt(2)*a^(5/4)*c^(9/4)*d^7*e^4 + 18*sqrt(2)*a^(7/4)*c^(7/4)*d^5*e^6 + 77*s
qrt(2)*a^(9/4)*c^(5/4)*d^3*e^8 + 45*sqrt(2)*a^(11/4)*c^(3/4)*d*e^10 + 24*sq
rt(a)*c^3*d^10*e + 80*a^(3/2)*c^2*d^6*e^5 + 120*a^(5/2)*c*d^2*e^9)*arctan(1
/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(
a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(21*sqrt(2)*a^(1/4)*c^(13/4)*d^1
1 + 5*sqrt(2)*a^(3/4)*c^(11/4)*d^9*e^2 + 66*sqrt(2)*a^(5/4)*c^(9/4)*d^7*e^4
 + 18*sqrt(2)*a^(7/4)*c^(7/4)*d^5*e^6 + 77*sqrt(2)*a^(9/4)*c^(5/4)*d^3*e^8
+ 45*sqrt(2)*a^(11/4)*c^(3/4)*d*e^10 - 24*sqrt(a)*c^3*d^10*e - 80*a^(3/2)*c
^2*d^6*e^5 - 120*a^(5/2)*c*d^2*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c
^(5/4)))/(a^2*c^3*d^12 + 3*a^3*c^2*d^8*e^4 + 3*a^4*c*d^4*e^8 + a^5*e^12) +
1/32*(8*a^2*c*e^7*x^4 + 4*a^2*c*d^4*e^3 + 12*a^3*e^7 + (5*c^3*d^5*e^2 + 13*
a*c^2*d*e^6)*x^7 - 2*(3*c^3*d^6*e + 7*a*c^2*d^2*e^5)*x^6 + (7*c^3*d^7 + 15*
a*c^2*d^3*e^4)*x^5 + (9*a*c^2*d^5*e^2 + 17*a^2*c*d*e^6)*x^3 - 2*(5*a*c^2*d^
6*e + 9*a^2*c*d^2*e^5)*x^2 + (11*a*c^2*d^7 + 19*a^2*c*d^3*e^4)*x)/(a^4*c^2*
d^8 + 2*a^5*c*d^4*e^4 + a^6*e^8 + (a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a^4*c^
2*e^8)*x^8 + 2*(a^3*c^3*d^8 + 2*a^4*c^2*d^4*e^4 + a^5*c*e^8)*x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 1311, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] e^12*log(abs(e*x + d))/(c^3*d^12*e + 3*a*c^2*d^8*e^5 + 3*a^2*c*d^4*e^9 + a^
3*e^13) - 1/4*e^11*log(abs(c*x^4 + a))/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*
c*d^4*e^8 + a^3*e^12) - 1/64*(75*sqrt(2)*a*c^2*d^2*e^3 - 51*sqrt(2)*sqrt(a*
c)*c^2*d^4*e - 21*(a*c^3)^(1/4)*c^2*d^5 - 45*(a*c^3)^(1/4)*a*c*d*e^4 - 122*
(a*c^3)^(3/4)*d^3*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)
^(1/4))/(sqrt(2)*a^3*c^3*d^6 + 9*sqrt(2)*a^4*c^2*d^2*e^4 + 9*sqrt(2)*sqrt(a
*c)*a^3*c^2*d^4*e^2 + sqrt(2)*sqrt(a*c)*a^4*c*e^6 - 6*(a*c^3)^(1/4)*a^3*c^2
*d^5*e - 6*(a*c^3)^(1/4)*a^4*c*d*e^5 - 16*(a*c^3)^(3/4)*a^3*d^3*e^3) + 1/64
*(75*sqrt(2)*a*c^2*d^2*e^3 + 51*sqrt(2)*sqrt(a*c)*c^2*d^4*e + 21*(a*c^3)^(1
/4)*c^2*d^5 + 45*(a*c^3)^(1/4)*a*c*d*e^4 + 122*(a*c^3)^(3/4)*d^3*e^2)*arcta
n(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^6
 + 9*sqrt(2)*a^4*c^2*d^2*e^4 + 9*sqrt(2)*sqrt(a*c)*a^3*c^2*d^4*e^2 + sqrt(2
)*sqrt(a*c)*a^4*c*e^6 + 6*(a*c^3)^(1/4)*a^3*c^2*d^5*e + 6*(a*c^3)^(1/4)*a^4
*c*d*e^5 + 16*(a*c^3)^(3/4)*a^3*d^3*e^3) + 1/256*(21*sqrt(2)*(a*c^3)^(1/4)*
c^4*d^11 + 66*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^7*e^4 + 77*sqrt(2)*(a*c^3)^(1/4
)*a^2*c^2*d^3*e^8 - 5*sqrt(2)*(a*c^3)^(3/4)*c^2*d^9*e^2 - 18*sqrt(2)*(a*c^3
)^(3/4)*a*c*d^5*e^6 - 45*sqrt(2)*(a*c^3)^(3/4)*a^2*d*e^10)*log(x^2 + sqrt(2
)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^5*d^12 + 3*a^4*c^4*d^8*e^4 + 3*a^5*c^3*
d^4*e^8 + a^6*c^2*e^12) - 1/256*(21*sqrt(2)*(a*c^3)^(1/4)*c^4*d^11 + 66*sqr
t(2)*(a*c^3)^(1/4)*a*c^3*d^7*e^4 + 77*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^3*e^8
 - 5*sqrt(2)*(a*c^3)^(3/4)*c^2*d^9*e^2 - 18*sqrt(2)*(a*c^3)^(3/4)*a*c*d^5*e
^6 - 45*sqrt(2)*(a*c^3)^(3/4)*a^2*d*e^10)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) +
 sqrt(a/c))/(a^3*c^5*d^12 + 3*a^4*c^4*d^8*e^4 + 3*a^5*c^3*d^4*e^8 + a^6*c^2
*e^12) + 1/32*(4*a^2*c^2*d^8*e^3 + 16*a^3*c*d^4*e^7 + 12*a^4*e^11 + (5*c^4*
d^9*e^2 + 18*a*c^3*d^5*e^6 + 13*a^2*c^2*d*e^10)*x^7 - 2*(3*c^4*d^10*e + 10*
a*c^3*d^6*e^5 + 7*a^2*c^2*d^2*e^9)*x^6 + (7*c^4*d^11 + 22*a*c^3*d^7*e^4 + 1
5*a^2*c^2*d^3*e^8)*x^5 + 8*(a^2*c^2*d^4*e^7 + a^3*c*e^11)*x^4 + (9*a*c^3*d^
9*e^2 + 26*a^2*c^2*d^5*e^6 + 17*a^3*c*d*e^10)*x^3 - 2*(5*a*c^3*d^10*e + 14*
a^2*c^2*d^6*e^5 + 9*a^3*c*d^2*e^9)*x^2 + (11*a*c^3*d^11 + 30*a^2*c^2*d^7*e^
4 + 19*a^3*c*d^3*e^8)*x)/((c*d^4 + a*e^4)^3*(c*x^4 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 10.55 (sec) , antiderivative size = 2720, normalized size of antiderivative = 2.01

$$\int \frac{1}{(d+ex)(a+cx^4)^3}\,dx = \text{Too large to display}$$

[In] int(1/((a + c*x^4)^3*(d + e*x)),x)

[Out] symsum(log((194481*c^7*d^13*e^6 + 871362*a*c^6*d^9*e^10 + 425984*a^3*c^4*d*e^18 + 1148881*a^2*c^5*d^5*e^14)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + root(805306368*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 + 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + 676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*(root(805306368*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 + 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + 676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*(root(805306368*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 + 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + 676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*(root(805306368*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 + 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + 676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*((402653184*a^15*c^4*d*e^22 - 134217728*a^10*c^9*d^21*e^2 - 134217728*a^11*c^8*d^17*e^6 + 805306368*a^12*c^7*d^13*e^10 + 1879048192*a^13*c^6*d^9*e^14 + 1476395008*a^14*c^5*d^5*e^18)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + (x*(335544320*a^15*c^4*e^23 - 201326592*a^10*c^9*d^20*e^3 - 469762048*a^11*c^8*d^16*e^7 + 134217728*a^12*c^7*d^12*e^11 + 1207959552*a^13*c^6*d^8*e^15 + 1140850688*a^14*c^5*d^4*e^19))/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8))) + (211288064*a^12*c^4*d*e^21 - 11010048*a^7*c^9*d^21*e + 20447232*a^8*c^8*d^17*e^5 + 204472320*a^9*c^7*d^13*e^9 + 514850816*a^10*c^6*d^9*e^13 + 553123840*a^11*c^5*d^5*e^17)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + (x*(251658240*a^12*c^4*e^22 - 28311552*a^7*c^9*d^20*e^2 - 67108864*a^8*c^8*d^16*e^6 + 18874368*a^9*c^7*d^12*e^10 + 377487360*a^10*c^6*d^8*e^14 + 571473920*a^11*c^5*d^4* e^18))/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8))) + (36962304*a^9*c^4*d*e^20 + 11010048*a^5*c^8*d^17*e^4 + 57999360*a^6*c^7*d^13*e^8 + 138805248*a^7*c^6*d^9*e^12 + 141361152*a^8*c^5*d^5*e^16)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + (x*(62914560*a^9*c^4*e^21 - 1806336*a^4*c^9*d^20*e + 2670592*a^5*c^8*d^16*e^5 + 43032576*a^6*c^7*d^12*e^9 + 143179776*a^7*c^6*d^8*e^13 + 171732992*a^8*c^5*d^4*e^17))/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8))) + (4030464*a^6*c^4*d*e^19 + 576576*a^2*c^8*d^17*e^3 + 5061824*a^3*c^7*d^13*e^7 + 15959232*a^4*c^6*d^9*e^11 + 17863744*a^5*c^5*d^5*e^15)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + (x*(5242880*a^6*c^4*e^20 + 755136*a^2*c^8*d^16*e^4 + 6023488*a^3*c^7*d^12*e^8 + 19579200*a^4*c^6*d^8*e^12 + 22240704*a^5*c^5*d^4*e^16))/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8))) + (x*(194481*c^7*d^12*e^7 + 871362*a*c^6*d^8*e^11 + 970321*a^2*c^5*d^4*e^15))/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)))*root(805306368*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 + 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + 676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k), k, 1, 4) + ((3*a*e^7 + c*d^4*e^3)/(8*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x^5*(7*c^3*d^7 + 15*a*c^2*d^3*e^4))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x^2*(5*c^2*d^6*e + 9*a*c*d^2*e^5))/(16*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (c*e^7*x^4)/(4*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x*(11*c^2*d^7 + 19*a*c*d^3*e^4))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x^6*(3*c^3*d^6*e + 7*a*c^2*d^2*e^5))/(16*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (e^2*x^3*(9*c^2*d^5 + 17*a*c*d*e^4))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (e^2*x^7*(5*c^3*d^5 + 13*a*c^2*d*e^4))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)))/(a^2 + c^2*x^8 + 2*a*c*x^4) + (e^11*log(d + e*x))/(a^3*e^12 + c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)

3.413 $\int \frac{1}{(d+ex)^2 (a+cx^4)^3} \, dx$

Optimal result 2547
Rubi [A] (verified) 2549
Mathematica [A] (verified) 2557
Maple [A] (verified) 2558
Fricas [F(-1)] 2558
Sympy [F(-1)] 2559
Maxima [A] (verification not implemented) 2559
Giac [A] (verification not implemented) 2560
Mupad [B] (verification not implemented) 2561

Optimal result

Integrand size = 17, antiderivative size = 1830

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^3}\,dx$$
$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{cx(7d^2(cd^4-3ae^4)-12de(cd^4-ae^4)\,x+5e^2(3cd^4-ae^4)\,x^2)}{32a^2\left(cd^4+ae^4\right)^2(a+cx^4)}$$
$$+ \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)\,x+e^2(3cd^4-ae^4)\,x^2))}{8a\left(cd^4+ae^4\right)^2(a+cx^4)^2}$$
$$+ \frac{ce^4(8ad^3e^3+x(d^2(5cd^4-3ae^4)-2de(3cd^4-ae^4)\,x+e^2(7cd^4-ae^4)\,x^2))}{4a\left(cd^4+ae^4\right)^3(a+cx^4)}$$
$$- \frac{\sqrt{c}de^9(5cd^4-ae^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\left(cd^4+ae^4\right)^4}$$
$$- \frac{\sqrt{c}de^5(3cd^4-ae^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(cd^4+ae^4\right)^3} - \frac{3\sqrt{c}de(cd^4-ae^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{8a^{5/2}\left(cd^4+ae^4\right)^2}$$
$$- \frac{\sqrt[4]{c}\left(21\sqrt{c}d^2(cd^4-3ae^4)+5\sqrt{a}e^2(3cd^4-ae^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\left(cd^4+ae^4\right)^2}$$
$$- \frac{\sqrt[4]{c}e^4\left(3\sqrt{c}d^2(5cd^4-3ae^4)+\sqrt{a}e^2(7cd^4-ae^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3}$$
$$- \frac{\sqrt[4]{c}e^8\left(3\sqrt{c}d^2(3cd^4-ae^4)+\sqrt{a}e^2(11cd^4-ae^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4}$$
$$+ \frac{\sqrt[4]{c}\left(21\sqrt{c}d^2(cd^4-3ae^4)+5\sqrt{a}e^2(3cd^4-ae^4)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\left(cd^4+ae^4\right)^2}$$
$$+ \frac{\sqrt[4]{c}e^4\left(3\sqrt{c}d^2(5cd^4-3ae^4)+\sqrt{a}e^2(7cd^4-ae^4)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3}$$
$$+ \frac{\sqrt[4]{c}e^8\left(3\sqrt{c}d^2(3cd^4-ae^4)+\sqrt{a}e^2(11cd^4-ae^4)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4}$$
$$+ \frac{12cd^3e^{11}\log(d+ex)}{\left(cd^4+ae^4\right)^4}$$
$$- \frac{\sqrt[4]{c}e^8\left(9c^{3/2}d^6-11\sqrt{a}cd^4e^2-3a\sqrt{c}d^2e^4+a^{3/2}e^6\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4}$$
$$- \frac{\sqrt[4]{c}\left(21\sqrt{c}d^2(cd^4-3ae^4)-5\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\left(cd^4+ae^4\right)^2}$$
$$- \frac{\sqrt[4]{c}e^4\left(3\sqrt{c}d^2(5cd^4-3ae^4)-\sqrt{a}e^2(7cd^4-ae^4)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3}$$
$$+ \frac{\sqrt[4]{c}e^8\left(9c^{3/2}d^6-11\sqrt{a}cd^4e^2-3a\sqrt{c}d^2e^4+a^{3/2}e^6\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4}$$

```
[Out] -e^11/(a*e^4+c*d^4)^3/(e*x+d)+1/32*c*x*(7*d^2*(-3*a*e^4+c*d^4)-12*d*e*(-a*e
^4+c*d^4)*x+5*e^2*(-a*e^4+3*c*d^4)*x^2)/a^2/(a*e^4+c*d^4)^2/(c*x^4+a)+1/8*c
*(4*a*d^3*e^3+x*(d^2*(-3*a*e^4+c*d^4)-2*d*e*(-a*e^4+c*d^4)*x+e^2*(-a*e^4+3*
c*d^4)*x^2))/a/(a*e^4+c*d^4)^2/(c*x^4+a)^2+1/4*c*e^4*(8*a*d^3*e^3+x*(d^2*(-
3*a*e^4+5*c*d^4)-2*d*e*(-a*e^4+3*c*d^4)*x+e^2*(-a*e^4+7*c*d^4)*x^2))/a/(a*e
^4+c*d^4)^3/(c*x^4+a)+12*c*d^3*e^11*ln(e*x+d)/(a*e^4+c*d^4)^4-3*c*d^3*e^11*
ln(c*x^4+a)/(a*e^4+c*d^4)^4-1/2*d*e^5*(-a*e^4+3*c*d^4)*arctan(x^2*c^(1/2)/a
^(1/2))*c^(1/2)/a^(3/2)/(a*e^4+c*d^4)^3-3/8*d*e*(-a*e^4+c*d^4)*arctan(x^2*c
^(1/2)/a^(1/2))*c^(1/2)/a^(5/2)/(a*e^4+c*d^4)^2-d*e^9*(-a*e^4+5*c*d^4)*arct
an(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^4/a^(1/2)-1/8*c^(1/4)*e^8*ln(
-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(9*c^(3/2)*d^6+a^(3/2)*e^6-
11*c*d^4*e^2*a^(1/2)-3*a*d^2*e^4*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)+1
/8*c^(1/4)*e^8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(9*c^(3/2)
*d^6+a^(3/2)*e^6-11*c*d^4*e^2*a^(1/2)-3*a*d^2*e^4*c^(1/2))/a^(3/4)/(a*e^4+c
*d^4)^4*2^(1/2)-1/256*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(
1/2))*(-5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+21*d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(
11/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/256*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^
(1/2)+x^2*c^(1/2))*(-5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+21*d^2*(-3*a*e^4+c*d^4)
*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/128*c^(1/4)*arctan(-1+c^(1/4)*
x*2^(1/2)/a^(1/4))*(5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+21*d^2*(-3*a*e^4+c*d^4)*
c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/128*c^(1/4)*arctan(1+c^(1/4)*x*
2^(1/2)/a^(1/4))*(5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+21*d^2*(-3*a*e^4+c*d^4)*c^
(1/2))/a^(11/4)/(a*e^4+c*d^4)^2*2^(1/2)-1/32*c^(1/4)*e^4*ln(-a^(1/4)*c^(1/4
)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+7*c*d^4)*a^(1/2)+3*d^2*(-3*a
*e^4+5*c*d^4)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/32*c^(1/4)*e^4*ln(
a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+7*c*d^4)*a^(1/
2)+3*d^2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/16*c
^(1/4)*e^4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+7*c*d^4)*a^(1/
2)+3*d^2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/16*c
^(1/4)*e^4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+7*c*d^4)*a^(1/2
)+3*d^2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(
1/4)*e^8*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+11*c*d^4)*a^(1/2
)+3*d^2*(-a*e^4+3*c*d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)+1/4*c^(1/
4)*e^8*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+11*c*d^4)*a^(1/2)+3
*d^2*(-a*e^4+3*c*d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)
```

Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 1830, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.882, Rules

used = {6874, 1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 266}

$$
\begin{aligned}
&\int \frac{1}{(d+ex)^2 (a+cx^4)^3} dx \\
&= \frac{12cd^3 \log(d+ex)e^{11}}{(cd^4+ae^4)^4} - \frac{3cd^3 \log(cx^4+a) e^{11}}{(cd^4+ae^4)^4} \\
&- \frac{e^{11}}{(cd^4+ae^4)^3 (d+ex)} - \frac{\sqrt{c}d(5cd^4-ae^4) \arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) e^9}{\sqrt{a}\,(cd^4+ae^4)^4} \\
&- \frac{\sqrt[4]{c}\left(3\sqrt{c}(3cd^4-ae^4)\,d^2+\sqrt{a}e^2(11cd^4-ae^4)\right) \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) e^8}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^4} \\
&+ \frac{\sqrt[4]{c}\left(3\sqrt{c}(3cd^4-ae^4)\,d^2+\sqrt{a}e^2(11cd^4-ae^4)\right) \arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right) e^8}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^4} \\
&- \frac{\sqrt[4]{c}\left(9c^{3/2}d^6-11\sqrt{a}ce^2d^4-3a\sqrt{c}e^4d^2+a^{3/2}e^6\right) \log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right) e^8}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)^4} \\
&+ \frac{\sqrt[4]{c}\left(9c^{3/2}d^6-11\sqrt{a}ce^2d^4-3a\sqrt{c}e^4d^2+a^{3/2}e^6\right) \log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right) e^8}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)^4} \\
&- \frac{\sqrt{c}d(3cd^4-ae^4) \arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) e^5}{2a^{3/2}\,(cd^4+ae^4)^3} \\
&+ \frac{c(8ad^3e^3+x((5cd^4-3ae^4)\,d^2-2e(3cd^4-ae^4)\,xd+e^2(7cd^4-ae^4)\,x^2))\,e^4}{4a\,(cd^4+ae^4)^3\,(cx^4+a)} \\
&- \frac{\sqrt[4]{c}\left(3\sqrt{c}(5cd^4-3ae^4)\,d^2+\sqrt{a}e^2(7cd^4-ae^4)\right) \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) e^4}{8\sqrt{2}a^{7/4}\,(cd^4+ae^4)^3} \\
&+ \frac{\sqrt[4]{c}\left(3\sqrt{c}(5cd^4-3ae^4)\,d^2+\sqrt{a}e^2(7cd^4-ae^4)\right) \arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right) e^4}{8\sqrt{2}a^{7/4}\,(cd^4+ae^4)^3} \\
&- \frac{\sqrt[4]{c}\left(3\sqrt{c}d^2(5cd^4-3ae^4)-\sqrt{a}e^2(7cd^4-ae^4)\right) \log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right) e^4}{16\sqrt{2}a^{7/4}\,(cd^4+ae^4)^3} \\
&+ \frac{\sqrt[4]{c}\left(3\sqrt{c}d^2(5cd^4-3ae^4)-\sqrt{a}e^2(7cd^4-ae^4)\right) \log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right) e^4}{16\sqrt{2}a^{7/4}\,(cd^4+ae^4)^3} \\
&- \frac{3\sqrt{c}d(cd^4-ae^4) \arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) e}{8a^{5/2}\,(cd^4+ae^4)^2} \\
&+ \frac{c(4ad^3e^3+x((cd^4-3ae^4)\,d^2-2e(cd^4-ae^4)\,xd+e^2(3cd^4-ae^4)\,x^2))}{8a\,(cd^4+ae^4)^2\,(cx^4+a)^2} \\
&- \frac{\sqrt[4]{c}\left(21\sqrt{c}(cd^4-3ae^4)\,d^2+5\sqrt{a}e^2(3cd^4-ae^4)\right) \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\,(cd^4+ae^4)^2} \\
&+ \frac{\sqrt[4]{c}\left(21\sqrt{c}(cd^4-3ae^4)\,d^2+5\sqrt{a}e^2(3cd^4-ae^4)\right) \arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}\,(cd^4+ae^4)^2}
\end{aligned}
$$

```
[In] Int[1/((d + e*x)^2*(a + c*x^4)^3),x]
```

```
[Out] -(e^11/((c*d^4 + a*e^4)^3*(d + e*x))) + (c*x*(7*d^2*(c*d^4 - 3*a*e^4) - 12*
d*e*(c*d^4 - a*e^4)*x + 5*e^2*(3*c*d^4 - a*e^4)*x^2))/(32*a^2*(c*d^4 + a*e^
4)^2*(a + c*x^4)) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3*a*e^4) - 2*d*e*(c*d
^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(8*a*(c*d^4 + a*e^4)^2*(a + c*
x^4)^2) + (c*e^4*(8*a*d^3*e^3 + x*(d^2*(5*c*d^4 - 3*a*e^4) - 2*d*e*(3*c*d^4
 - a*e^4)*x + e^2*(7*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^
4)) - (Sqrt[c]*d*e^9*(5*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt
[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x
^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^3) - (3*Sqrt[c]*d*e*(c*d^4 - a*e^4
)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(8*a^(5/2)*(c*d^4 + a*e^4)^2) - (c^(1/4)*(
21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[
1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^2) -
 (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a
*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 +
 a*e^4)^3) - (c^(1/4)*e^8*(3*Sqrt[c]*d^2*(3*c*d^4 - a*e^4) + Sqrt[a]*e^2*(1
1*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/
4)*(c*d^4 + a*e^4)^4) + (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + 5*Sqrt
[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqr
t[2]*a^(11/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3
*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*e^8*(3*Sqrt[c]*d^2*
(3*c*d^4 - a*e^4) + Sqrt[a]*e^2*(11*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (12*c*d^3*e^11*Lo
g[d + e*x])/(c*d^4 + a*e^4)^4 - (c^(1/4)*e^8*(9*c^(3/2)*d^6 - 11*Sqrt[a]*c*
d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) - (c^(1/4)*
(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*
d^4 + a*e^4)^2) - (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]
*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*e^8*(9*c^(3/2)*d^6 -
 11*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^
4) + (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - 5*Sqrt[a]*e^2*(3*c*d^4 -
a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2
]*a^(11/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*
e^4) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (3*c*d^3*e^11*L
og[a + c*x^4])/(c*d^4 + a*e^4)^4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^{12}}{(cd^4+ae^4)^3 (d+ex)^2}+\frac{12cd^3e^{12}}{(cd^4+ae^4)^4 (d+ex)}+\frac{c(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)x+e^2(3cd^4-ae^4)x^2-4cd^3e^3x^3)}{(cd^4+ae^4)^2 (a+cx^4)^3}+\frac{ce^4(d^2(5cd^4-3ae^4)-2de(3cd^4-ae^4)x+e^2(7cd^4-ae^4)x^2-8cd^3e^3x^3)}{(cd^4+ae^4)^3 (a+cx^4)^2}+\frac{ce^8(3d^2(3cd^4-ae^4)-2de(5cd^4-ae^4)x+e^2(11cd^4-ae^4)x^2-12cd^3e^3x^3)}{(cd^4+ae^4)^4 (a+cx^4)}\right) dx$$

$$= -\frac{e^{11}}{(cd^4+ae^4)^3 (d+ex)}+\frac{12cd^3e^{11}\log(d+ex)}{(cd^4+ae^4)^4}+\frac{(ce^8)\int \frac{3d^2(3cd^4-ae^4)-2de(5cd^4-ae^4)x+e^2(11cd^4-ae^4)x^2-12cd^3e^3x^3}{a+cx^4}\,dx}{(cd^4+ae^4)^4}+\frac{(ce^4)\int \frac{d^2(5cd^4-3ae^4)-2de(3cd^4-ae^4)x+e^2(7cd^4-ae^4)x^2-8cd^3e^3x^3}{(a+cx^4)^2}\,dx}{(cd^4+ae^4)^3}+\frac{c\int \frac{d^2(cd^4-3ae^4)-2de(cd^4-ae^4)x+e^2(3cd^4-ae^4)x^2-4cd^3e^3x^3}{(a+cx^4)^3}\,dx}{(cd^4+ae^4)^2}$$

$$= -\frac{e^{11}}{(cd^4+ae^4)^3 (d+ex)}+\frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)x+e^2(3cd^4-ae^4)x^2))}{8a(cd^4+ae^4)^2 (a+cx^4)^2}+\frac{ce^4(8ad^3e^3+x(d^2(5cd^4-3ae^4)-2de(3cd^4-ae^4)x+e^2(7cd^4-ae^4)x^2))}{4a(cd^4+ae^4)^3 (a+cx^4)}+\frac{12cd^3e^{11}\log(d+ex)}{(cd^4+ae^4)^4}+\frac{(ce^8)\int \left(\frac{x(-2de(5cd^4-ae^4)-12cd^3e^3x^2)}{a+cx^4}+\frac{3d^2(3cd^4-ae^4)+e^2(11cd^4-ae^4)x^2}{a+cx^4}\right) dx}{(cd^4+ae^4)^4}-\frac{(ce^4)\int \frac{-3d^2(5cd^4-3ae^4)+4de(3cd^4-ae^4)x-e^2(7cd^4-ae^4)x^2}{a+cx^4}\,dx}{4a(cd^4+ae^4)^3}-\frac{c\int \frac{-7d^2(cd^4-3ae^4)+12de(cd^4-ae^4)x-5e^2(3cd^4-ae^4)x^2}{(a+cx^4)^2}\,dx}{8a(cd^4+ae^4)^2}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{cx(7d^2(cd^4-3ae^4)-12de(cd^4-ae^4)\,x+5e^2(3cd^4-ae^4)\,x^2)}{32a^2\left(cd^4+ae^4\right)^2(a+cx^4)} + \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)\,x+e^2(3cd^4-ae^4)\,x^2))}{8a\left(cd^4+ae^4\right)^2(a+cx^4)^2} + \frac{ce^4(8ad^3e^3+x(d^2(5cd^4-3ae^4)-2de(3cd^4-ae^4)\,x+e^2(7cd^4-ae^4)\,x^2))}{4a\left(cd^4+ae^4\right)^3(a+cx^4)} + \frac{12cd^3e^{11}\log(d+ex)}{\left(cd^4+ae^4\right)^4} + \frac{(ce^8)\int\frac{x\left(-2de(5cd^4-ae^4)-12cd^3e^3x^2\right)}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^4} + \frac{(ce^8)\int\frac{3d^2(3cd^4-ae^4)+e^2(11cd^4-ae^4)x^2}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^4} - \frac{(ce^4)\int\left(\frac{4de(3cd^4-ae^4)x}{a+cx^4}+\frac{-3d^2(5cd^4-3ae^4)-e^2(7cd^4-ae^4)x^2}{a+cx^4}\right)dx}{4a\left(cd^4+ae^4\right)^3} + \frac{c\int\frac{21d^2(cd^4-3ae^4)-24de(cd^4-ae^4)x+5e^2(3cd^4-ae^4)x^2}{a+cx^4}\,dx}{32a^2\left(cd^4+ae^4\right)^2}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{cx(7d^2(cd^4-3ae^4)-12de(cd^4-ae^4)\,x+5e^2(3cd^4-ae^4)\,x^2)}{32a^2\left(cd^4+ae^4\right)^2(a+cx^4)} + \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)\,x+e^2(3cd^4-ae^4)\,x^2))}{8a\left(cd^4+ae^4\right)^2(a+cx^4)^2} + \frac{ce^4(8ad^3e^3+x(d^2(5cd^4-3ae^4)-2de(3cd^4-ae^4)\,x+e^2(7cd^4-ae^4)\,x^2))}{4a\left(cd^4+ae^4\right)^3(a+cx^4)} + \frac{12cd^3e^{11}\log(d+ex)}{\left(cd^4+ae^4\right)^4} + \frac{(ce^8)\,\mathrm{Subst}\left(\int\frac{-2de(5cd^4-ae^4)-12cd^3e^3x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)^4} - \frac{(ce^4)\int\frac{-3d^2(5cd^4-3ae^4)-e^2(7cd^4-ae^4)x^2}{a+cx^4}\,dx}{4a\left(cd^4+ae^4\right)^3} - \frac{(cde^5(3cd^4-ae^4))\int\frac{x}{a+cx^4}\,dx}{a\left(cd^4+ae^4\right)^3} + \frac{c\int\left(-\frac{24de(cd^4-ae^4)x}{a+cx^4}+\frac{21d^2(cd^4-3ae^4)+5e^2(3cd^4-ae^4)x^2}{a+cx^4}\right)dx}{32a^2\left(cd^4+ae^4\right)^2} - \frac{\left(e^8\left(11cd^4e^2-ae^6-\frac{3\sqrt{c}d^2(3cd^4-ae^4)}{\sqrt{a}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\left(cd^4+ae^4\right)^4} + \frac{\left(e^8\left(11cd^4e^2-ae^6+\frac{3\sqrt{c}d^2(3cd^4-ae^4)}{\sqrt{a}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^4+ae^4\right)^4}$$

$$= -\frac{e^{11}}{(cd^4+ae^4)^3(d+ex)} + \frac{cx(7d^2(cd^4-3ae^4)-12de(cd^4-ae^4)x+5e^2(3cd^4-ae^4)x^2)}{32a^2(cd^4+ae^4)^2(a+cx^4)}$$

$$+ \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)x+e^2(3cd^4-ae^4)x^2))}{8a(cd^4+ae^4)^2(a+cx^4)^2}$$

$$+ \frac{ce^4(8ad^3e^3+x(d^2(5cd^4-3ae^4)-2de(3cd^4-ae^4)x+e^2(7cd^4-ae^4)x^2))}{4a(cd^4+ae^4)^3(a+cx^4)}$$

$$+ \frac{12cd^3e^{11}\log(d+ex)}{(cd^4+ae^4)^4} - \frac{(6c^2d^3e^{11})\,\mathrm{Subst}\left(\int\frac{x}{a+cx^2}\,dx,x,x^2\right)}{(cd^4+ae^4)^4}$$

$$- \frac{(cde^9(5cd^4-ae^4))\,\mathrm{Subst}\left(\int\frac{1}{a+cx^2}\,dx,x,x^2\right)}{(cd^4+ae^4)^4}$$

$$- \frac{(cde^5(3cd^4-ae^4))\,\mathrm{Subst}\left(\int\frac{1}{a+cx^2}\,dx,x,x^2\right)}{2a(cd^4+ae^4)^3} + \frac{c\int\frac{21d^2(cd^4-3ae^4)+5e^2(3cd^4-ae^4)x^2}{a+cx^4}\,dx}{32a^2(cd^4+ae^4)^2}$$

$$- \frac{(3cde(cd^4-ae^4))\int\frac{x}{a+cx^4}\,dx}{4a^2(cd^4+ae^4)^2} - \frac{\left(e^4\left(7cd^4e^2-ae^6-\frac{3\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a(cd^4+ae^4)^3}$$

$$+ \frac{\left(e^4\left(7cd^4e^2-ae^6+\frac{3\sqrt{c}d^2(5cd^4-3ae^4)}{\sqrt{a}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8a(cd^4+ae^4)^3}$$

$$+ \frac{\left(\sqrt[4]{c}e^8\left(11cd^4e^2-ae^6-\frac{3\sqrt{c}d^2(3cd^4-ae^4)}{\sqrt{a}}\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^4}$$

$$+ \frac{\left(\sqrt[4]{c}e^8\left(11cd^4e^2-ae^6-\frac{3\sqrt{c}d^2(3cd^4-ae^4)}{\sqrt{a}}\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}(cd^4+ae^4)^4}$$

$$+ \frac{\left(e^8\left(11cd^4e^2-ae^6+\frac{3\sqrt{c}d^2(3cd^4-ae^4)}{\sqrt{a}}\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4(cd^4+ae^4)^4}$$

$$+ \frac{\left(e^8\left(11cd^4e^2-ae^6+\frac{3\sqrt{c}d^2(3cd^4-ae^4)}{\sqrt{a}}\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4(cd^4+ae^4)^4}$$

= Too large to display

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 1115, normalized size of antiderivative = 0.61

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3}\,dx$$

$$= \frac{-\frac{256e^{11}(cd^4+ae^4)}{d+ex} + \frac{8c(cd^4+ae^4)(c^2d^8x(7d^2-12dex+15e^2x^2)+2acd^4e^4x(13d^2-24dex+33e^2x^2)+a^2e^7(64d^3-45d^2ex+28de^2x^2-13e^3x^3))}{a^2(a+cx^4)}}{}$$

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^4)^3),x]
[Out] ((-256*e^11*(c*d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(c^2*d^8*x*(7
*d^2 - 12*d*e*x + 15*e^2*x^2) + 2*a*c*d^4*e^4*x*(13*d^2 - 24*d*e*x + 33*e^2
*x^2) + a^2*e^7*(64*d^3 - 45*d^2*e*x + 28*d*e^2*x^2 - 13*e^3*x^3)))/(a^2*(a
 + c*x^4)) + (32*c*(c*d^4 + a*e^4)^2*(c*d^4*x*(d^2 - 2*d*e*x + 3*e^2*x^2) +
 a*e^3*(4*d^3 - 3*d^2*e*x + 2*d*e^2*x^2 - e^3*x^3)))/(a*(a + c*x^4)^2) - (6
*c^(1/4)*(7*Sqrt[2]*c^(7/2)*d^14 - 16*a^(1/4)*c^(13/4)*d^13*e + 5*Sqrt[2]*S
qrt[a]*c^3*d^12*e^2 + 33*Sqrt[2]*a*c^(5/2)*d^10*e^4 - 80*a^(5/4)*c^(9/4)*d^
9*e^5 + 27*Sqrt[2]*a^(3/2)*c^2*d^8*e^6 + 77*Sqrt[2]*a^2*c^(3/2)*d^6*e^8 - 2
40*a^(9/4)*c^(5/4)*d^5*e^9 + 135*Sqrt[2]*a^(5/2)*c*d^4*e^10 - 77*Sqrt[2]*a^
3*Sqrt[c]*d^2*e^12 + 80*a^(13/4)*c^(1/4)*d*e^13 - 15*Sqrt[2]*a^(7/2)*e^14)*
ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + (6*c^(1/4)*(7*Sqrt[2]*c
^(7/2)*d^14 + 16*a^(1/4)*c^(13/4)*d^13*e + 5*Sqrt[2]*Sqrt[a]*c^3*d^12*e^2 +
 33*Sqrt[2]*a*c^(5/2)*d^10*e^4 + 80*a^(5/4)*c^(9/4)*d^9*e^5 + 27*Sqrt[2]*a^
(3/2)*c^2*d^8*e^6 + 77*Sqrt[2]*a^2*c^(3/2)*d^6*e^8 + 240*a^(9/4)*c^(5/4)*d^
5*e^9 + 135*Sqrt[2]*a^(5/2)*c*d^4*e^10 - 77*Sqrt[2]*a^3*Sqrt[c]*d^2*e^12 -
80*a^(13/4)*c^(1/4)*d*e^13 - 15*Sqrt[2]*a^(7/2)*e^14)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/a^(11/4) + 3072*c*d^3*e^11*Log[d + e*x] - (3*Sqrt[2]*c^
(1/4)*(7*c^(7/2)*d^14 - 5*Sqrt[a]*c^3*d^12*e^2 + 33*a*c^(5/2)*d^10*e^4 - 27
*a^(3/2)*c^2*d^8*e^6 + 77*a^2*c^(3/2)*d^6*e^8 - 135*a^(5/2)*c*d^4*e^10 - 77
*a^3*Sqrt[c]*d^2*e^12 + 15*a^(7/2)*e^14)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/a^(11/4) + (3*Sqrt[2]*c^(1/4)*(7*c^(7/2)*d^14 - 5*Sqr
t[a]*c^3*d^12*e^2 + 33*a*c^(5/2)*d^10*e^4 - 27*a^(3/2)*c^2*d^8*e^6 + 77*a^2
*c^(3/2)*d^6*e^8 - 135*a^(5/2)*c*d^4*e^10 - 77*a^3*Sqrt[c]*d^2*e^12 + 15*a^
(7/2)*e^14)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4
) - 768*c*d^3*e^11*Log[a + c*x^4])/(256*(c*d^4 + a*e^4)^4)
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 829, normalized size of antiderivative = 0.45

method	result
default	$-\frac{c\left(\frac{\frac{c\,e^{2}\left(13a^{3}e^{12}-53a^{2}c\,d^{4}e^{8}-81a\,c^{2}d^{8}e^{4}-15c^{3}d^{12}\right)x^{7}}{32a^{2}}-\frac{dce\left(7a^{3}e^{12}-5a^{2}c\,d^{4}e^{8}-15a\,c^{2}d^{8}e^{4}-3c^{3}d^{12}\right)x^{6}}{8a^{2}}+\frac{c\,d^{2}\left(45a^{3}e^{12}+19a^{2}c\,d^{4}e^{8}-33a\,c^{2}\right.}{32a^{2}}}{}\right.}{}$
risch	Expression too large to display

```
[In] int(1/(e*x+d)^2/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -c/(a*e^4+c*d^4)^4*((1/32*c*e^2*(13*a^3*e^12-53*a^2*c*d^4*e^8-81*a*c^2*d^8*
e^4-15*c^3*d^12)/a^2*x^7-1/8*d*c*e*(7*a^3*e^12-5*a^2*c*d^4*e^8-15*a*c^2*d^8
*e^4-3*c^3*d^12)/a^2*x^6+1/32*c*d^2*(45*a^3*e^12+19*a^2*c*d^4*e^8-33*a*c^2*
d^8*e^4-7*c^3*d^12)/a^2*x^5+(-2*a*c*d^3*e^11-2*c^2*d^7*e^7)*x^4+1/32*e^2*(1
7*a^3*e^12-57*a^2*c*d^4*e^8-101*a*c^2*d^8*e^4-27*c^3*d^12)/a*x^3-1/8*d*e*(9
*a^3*e^12-3*a^2*c*d^4*e^8-17*a*c^2*d^8*e^4-5*c^3*d^12)/a*x^2+1/32*d^2*(57*a
^3*e^12+39*a^2*c*d^4*e^8-29*a*c^2*d^8*e^4-11*c^3*d^12)/a*x-5/2*a^2*d^3*e^11
-3*a*d^7*e^7*c-1/2*d^11*e^3*c^2)/(c*x^4+a)^2+3/32/a^2*(1/8*(77*a^3*d^2*e^12
-77*a^2*c*d^6*e^8-33*a*c^2*d^10*e^4-7*c^3*d^14)*(a/c)^(1/4)/a*2^(1/2)*(ln((
x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/
2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1
/2*(-40*a^3*d*e^13+120*a^2*c*d^5*e^9+40*a*c^2*d^9*e^5+8*c^3*d^13*e)/(a*c)^(
1/2)*arctan(x^2*(c/a)^(1/2))+1/8*(15*a^3*e^14-135*a^2*c*d^4*e^10-27*a*c^2*d
^8*e^6-5*c^3*d^12*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)
+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/
c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+32*a^2*d^3*e^11*ln(c*x^4+a
)))-e^11/(a*e^4+c*d^4)^3/(e*x+d)+12*c*d^3*e^11*ln(e*x+d)/(a*e^4+c*d^4)^4
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\,(a+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3} dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 1564, normalized size of antiderivative = 0.85

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3} dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 12*c*d^3*e^11*log(e*x + d)/(c^4*d^16 + 4*a*c^3*d^12*e^4 + 6*a^2*c^2*d^8*e^8
 + 4*a^3*c*d^4*e^12 + a^4*e^16) - 3/256*c*(sqrt(2)*(128*sqrt(2)*a^(11/4)*c^
(5/4)*d^3*e^11 - 7*c^4*d^14 + 5*sqrt(a)*c^(7/2)*d^12*e^2 - 33*a*c^3*d^10*e^
4 + 27*a^(3/2)*c^(5/2)*d^8*e^6 - 77*a^2*c^2*d^6*e^8 + 135*a^(5/2)*c^(3/2)*d
^4*e^10 + 77*a^3*c*d^2*e^12 - 15*a^(7/2)*sqrt(c)*e^14)*log(sqrt(c)*x^2 + sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(128*sqrt(2)
*a^(11/4)*c^(5/4)*d^3*e^11 + 7*c^4*d^14 - 5*sqrt(a)*c^(7/2)*d^12*e^2 + 33*a
*c^3*d^10*e^4 - 27*a^(3/2)*c^(5/2)*d^8*e^6 + 77*a^2*c^2*d^6*e^8 - 135*a^(5/
2)*c^(3/2)*d^4*e^10 - 77*a^3*c*d^2*e^12 + 15*a^(7/2)*sqrt(c)*e^14)*log(sqrt
(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(7*sqr
t(2)*a^(1/4)*c^(17/4)*d^14 + 5*sqrt(2)*a^(3/4)*c^(15/4)*d^12*e^2 + 33*sqrt(
2)*a^(5/4)*c^(13/4)*d^10*e^4 + 27*sqrt(2)*a^(7/4)*c^(11/4)*d^8*e^6 + 77*sqr
t(2)*a^(9/4)*c^(9/4)*d^6*e^8 + 135*sqrt(2)*a^(11/4)*c^(7/4)*d^4*e^10 - 77*s
qrt(2)*a^(13/4)*c^(5/4)*d^2*e^12 - 15*sqrt(2)*a^(15/4)*c^(3/4)*e^14 + 16*sq
rt(a)*c^4*d^13*e + 80*a^(3/2)*c^3*d^9*e^5 + 240*a^(5/2)*c^2*d^5*e^9 - 80*a^
(7/2)*c*d*e^13)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(7*sqrt(
2)*a^(1/4)*c^(17/4)*d^14 + 5*sqrt(2)*a^(3/4)*c^(15/4)*d^12*e^2 + 33*sqrt(2)
*a^(5/4)*c^(13/4)*d^10*e^4 + 27*sqrt(2)*a^(7/4)*c^(11/4)*d^8*e^6 + 77*sqrt(
2)*a^(9/4)*c^(9/4)*d^6*e^8 + 135*sqrt(2)*a^(11/4)*c^(7/4)*d^4*e^10 - 77*sqr
t(2)*a^(13/4)*c^(5/4)*d^2*e^12 - 15*sqrt(2)*a^(15/4)*c^(3/4)*e^14 - 16*sqrt
(a)*c^4*d^13*e - 80*a^(3/2)*c^3*d^9*e^5 - 240*a^(5/2)*c^2*d^5*e^9 + 80*a^(7
/2)*c*d*e^13)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sq
rt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(a^2*c^4*d^16
 + 4*a^3*c^3*d^12*e^4 + 6*a^4*c^2*d^8*e^8 + 4*a^5*c*d^4*e^12 + a^6*e^16) +
```

```
1/32*(16*a^2*c^2*d^8*e^3 + 80*a^3*c*d^4*e^7 - 32*a^4*e^11 + 3*(5*c^4*d^8*e^
3 + 22*a*c^3*d^4*e^7 - 15*a^2*c^2*e^11)*x^8 + 3*(c^4*d^9*e^2 + 6*a*c^3*d^5*
e^6 + 5*a^2*c^2*d*e^10)*x^7 - (5*c^4*d^10*e + 22*a*c^3*d^6*e^5 + 17*a^2*c^2
*d^2*e^9)*x^6 + (7*c^4*d^11 + 26*a*c^3*d^7*e^4 + 19*a^2*c^2*d^3*e^8)*x^5 +
3*(9*a*c^3*d^8*e^3 + 46*a^2*c^2*d^4*e^7 - 27*a^3*c*e^11)*x^4 + (7*a*c^3*d^9
*e^2 + 26*a^2*c^2*d^5*e^6 + 19*a^3*c*d*e^10)*x^3 - 3*(3*a*c^3*d^10*e + 10*a
^2*c^2*d^6*e^5 + 7*a^3*c*d^2*e^9)*x^2 + (11*a*c^3*d^11 + 34*a^2*c^2*d^7*e^4
 + 23*a^3*c*d^3*e^8)*x)/(a^4*c^3*d^13 + 3*a^5*c^2*d^9*e^4 + 3*a^6*c*d^5*e^8
 + a^7*d*e^12 + (a^2*c^5*d^12*e + 3*a^3*c^4*d^8*e^5 + 3*a^4*c^3*d^4*e^9 + a
^5*c^2*e^13)*x^9 + (a^2*c^5*d^13 + 3*a^3*c^4*d^9*e^4 + 3*a^4*c^3*d^5*e^8 +
a^5*c^2*d*e^12)*x^8 + 2*(a^3*c^4*d^12*e + 3*a^4*c^3*d^8*e^5 + 3*a^5*c^2*d^4
*e^9 + a^6*c*e^13)*x^5 + 2*(a^3*c^4*d^13 + 3*a^4*c^3*d^9*e^4 + 3*a^5*c^2*d^
5*e^8 + a^6*c*d*e^12)*x^4 + (a^4*c^3*d^12*e + 3*a^5*c^2*d^8*e^5 + 3*a^6*c*d
^4*e^9 + a^7*e^13)*x)
```

Giac [A] (verification not implemented)

none

Time = 46.91 (sec) , antiderivative size = 1809, normalized size of antiderivative = 0.99

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 12*c*d^3*e^12*log(abs(e*x + d))/(c^4*d^16*e + 4*a*c^3*d^12*e^5 + 6*a^2*c^2*
d^8*e^9 + 4*a^3*c*d^4*e^13 + a^4*e^17) - 3*c*d^3*e^11*log(abs(c*x^4 + a))/(
c^4*d^16 + 4*a*c^3*d^12*e^4 + 6*a^2*c^2*d^8*e^8 + 4*a^3*c*d^4*e^12 + a^4*e^
16) - 3/64*(32*sqrt(2)*a*c^3*d^3*e^3 - 20*sqrt(2)*sqrt(a*c)*c^3*d^5*e - 20*
sqrt(2)*sqrt(a*c)*a*c^2*d*e^5 - 7*(a*c^3)^(1/4)*c^3*d^6 - 3*(a*c^3)^(1/4)*a
*c^2*d^2*e^4 - 53*(a*c^3)^(3/4)*c*d^4*e^2 + 15*(a*c^3)^(3/4)*a*e^6)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^4*d^8 +
 34*sqrt(2)*a^4*c^3*d^4*e^4 + sqrt(2)*a^5*c^2*e^8 + 16*sqrt(2)*sqrt(a*c)*a^
3*c^3*d^6*e^2 + 16*sqrt(2)*sqrt(a*c)*a^4*c^2*d^2*e^6 - 8*(a*c^3)^(1/4)*a^3*
c^3*d^7*e - 40*(a*c^3)^(1/4)*a^4*c^2*d^3*e^5 - 40*(a*c^3)^(3/4)*a^3*c*d^5*e
^3 - 8*(a*c^3)^(3/4)*a^4*d*e^7) + 3/64*(32*sqrt(2)*a*c^3*d^3*e^3 + 20*sqrt(
2)*sqrt(a*c)*c^3*d^5*e + 20*sqrt(2)*sqrt(a*c)*a*c^2*d*e^5 + 7*(a*c^3)^(1/4)
*c^3*d^6 + 3*(a*c^3)^(1/4)*a*c^2*d^2*e^4 + 53*(a*c^3)^(3/4)*c*d^4*e^2 - 15*
(a*c^3)^(3/4)*a*e^6)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(
1/4))/(sqrt(2)*a^3*c^4*d^8 + 34*sqrt(2)*a^4*c^3*d^4*e^4 + sqrt(2)*a^5*c^2*e
^8 + 16*sqrt(2)*sqrt(a*c)*a^3*c^3*d^6*e^2 + 16*sqrt(2)*sqrt(a*c)*a^4*c^2*d^
2*e^6 + 8*(a*c^3)^(1/4)*a^3*c^3*d^7*e + 40*(a*c^3)^(1/4)*a^4*c^2*d^3*e^5 +
40*(a*c^3)^(3/4)*a^3*c*d^5*e^3 + 8*(a*c^3)^(3/4)*a^4*d*e^7) + 3/256*(7*sqrt
(2)*(a*c^3)^(1/4)*c^5*d^14 + 33*sqrt(2)*(a*c^3)^(1/4)*a*c^4*d^10*e^4 + 77*s
```

```
qrt(2)*(a*c^3)^(1/4)*a^2*c^3*d^6*e^8 - 77*sqrt(2)*(a*c^3)^(1/4)*a^3*c^2*d^2
*e^12 - 5*sqrt(2)*(a*c^3)^(3/4)*c^3*d^12*e^2 - 27*sqrt(2)*(a*c^3)^(3/4)*a*c
^2*d^8*e^6 - 135*sqrt(2)*(a*c^3)^(3/4)*a^2*c*d^4*e^10 + 15*sqrt(2)*(a*c^3)^
(3/4)*a^3*e^14)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^6*d^16
+ 4*a^4*c^5*d^12*e^4 + 6*a^5*c^4*d^8*e^8 + 4*a^6*c^3*d^4*e^12 + a^7*c^2*e^1
6) - 3/256*(7*sqrt(2)*(a*c^3)^(1/4)*c^5*d^14 + 33*sqrt(2)*(a*c^3)^(1/4)*a*c
^4*d^10*e^4 + 77*sqrt(2)*(a*c^3)^(1/4)*a^2*c^3*d^6*e^8 - 77*sqrt(2)*(a*c^3)
^(1/4)*a^3*c^2*d^2*e^12 - 5*sqrt(2)*(a*c^3)^(3/4)*c^3*d^12*e^2 - 27*sqrt(2)
*(a*c^3)^(3/4)*a*c^2*d^8*e^6 - 135*sqrt(2)*(a*c^3)^(3/4)*a^2*c*d^4*e^10 + 1
5*sqrt(2)*(a*c^3)^(3/4)*a^3*e^14)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(a^3*c^6*d^16 + 4*a^4*c^5*d^12*e^4 + 6*a^5*c^4*d^8*e^8 + 4*a^6*c^3*d^4*
e^12 + a^7*c^2*e^16) + 1/32*(16*a^2*c^3*d^12*e^3 + 96*a^3*c^2*d^8*e^7 + 48*
a^4*c*d^4*e^11 - 32*a^5*e^15 + 3*(5*c^5*d^12*e^3 + 27*a*c^4*d^8*e^7 + 7*a^2
*c^3*d^4*e^11 - 15*a^3*c^2*e^15)*x^8 + 3*(c^5*d^13*e^2 + 7*a*c^4*d^9*e^6 +
11*a^2*c^3*d^5*e^10 + 5*a^3*c^2*d*e^14)*x^7 - (5*c^5*d^14*e + 27*a*c^4*d^10
*e^5 + 39*a^2*c^3*d^6*e^9 + 17*a^3*c^2*d^2*e^13)*x^6 + (7*c^5*d^15 + 33*a*c
^4*d^11*e^4 + 45*a^2*c^3*d^7*e^8 + 19*a^3*c^2*d^3*e^12)*x^5 + 3*(9*a*c^4*d^
12*e^3 + 55*a^2*c^3*d^8*e^7 + 19*a^3*c^2*d^4*e^11 - 27*a^4*c*e^15)*x^4 + (7
*a*c^4*d^13*e^2 + 33*a^2*c^3*d^9*e^6 + 45*a^3*c^2*d^5*e^10 + 19*a^4*c*d*e^1
4)*x^3 - 3*(3*a*c^4*d^14*e + 13*a^2*c^3*d^10*e^5 + 17*a^3*c^2*d^6*e^9 + 7*a
^4*c*d^2*e^13)*x^2 + (11*a*c^4*d^15 + 45*a^2*c^3*d^11*e^4 + 57*a^3*c^2*d^7*
e^8 + 23*a^4*c*d^3*e^12)*x)/((c*d^4 + a*e^4)^4*(c*x^4 + a)^2*(e*x + d)*a^2)
```

Mupad [B] (verification not implemented)

Time = 11.18 (sec) , antiderivative size = 3572, normalized size of antiderivative = 1.95

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^3}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + c*x^4)^3*(d + e*x)^2),x)
```

```
[Out] symsum(log((194481*c^9*d^17*e^6 + 1527012*a*c^8*d^13*e^10 + 4100625*a^4*c^5
*d*e^22 + 1926342*a^2*c^7*d^9*e^14 - 3102300*a^3*c^6*d^5*e^18)/(1048576*(a^
14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c
^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)) + root(1610612
736*a^13*c^2*d^8*e^8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^
14*c*d^4*e^12*z^4 + 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 +
 3221225472*a^11*c*d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*
a^6*c^3*d^10*e^2*z^2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5
*e^5*z + 5419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*
d^4*e^4 + 4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k)*(root(1610612736*a^13*c
^2*d^8*e^8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^14*c*d^4*e
^12*z^4 + 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 + 322122547
2*a^11*c*d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^
```

10*e^2*z^2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5
419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 +
4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k)*(root(1610612736*a^13*c^2*d^8*e^8
*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^14*c*d^4*e^12*z^4 +
268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 + 3221225472*a^11*c*d
^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^10*e^2*z^2
 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5419008*a^3
*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 + 4100625*a^
2*c*e^8 + 194481*c^3*d^8, z, k)*((23592960*a^14*c^4*e^29 - 11010048*a^7*c^1
1*d^28*e + 33030144*a^8*c^10*d^24*e^5 + 504889344*a^9*c^9*d^20*e^9 + 310326
0672*a^10*c^8*d^16*e^13 + 6799491072*a^11*c^7*d^12*e^17 + 6101139456*a^12*c
^6*d^8*e^21 + 1967652864*a^13*c^5*d^4*e^25)/(1048576*(a^14*e^24 + a^8*c^6*d
^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^
11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)) + root(1610612736*a^13*c^2*d^8*e^
8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^14*c*d^4*e^12*z^4 +
 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 + 3221225472*a^11*c*
d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^10*e^2*z^
2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5419008*a^
3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 + 4100625*a
^2*c*e^8 + 194481*c^3*d^8, z, k)*((402653184*a^17*c^4*d*e^30 - 134217728*a^
10*c^11*d^29*e^2 - 402653184*a^11*c^10*d^25*e^6 + 402653184*a^12*c^9*d^21*e
^10 + 3355443200*a^13*c^8*d^17*e^14 + 6039797760*a^14*c^7*d^13*e^18 + 52344
91392*a^15*c^6*d^9*e^22 + 2281701376*a^16*c^5*d^5*e^26)/(1048576*(a^14*e^24
 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16
*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)) + (x*(335544320*a^17*
c^4*e^31 - 201326592*a^10*c^11*d^28*e^3 - 872415232*a^11*c^10*d^24*e^7 - 10
06632960*a^12*c^9*d^20*e^11 + 1006632960*a^13*c^8*d^16*e^15 + 3690987520*a^
14*c^7*d^12*e^19 + 3825205248*a^15*c^6*d^8*e^23 + 1811939328*a^16*c^5*d^4*e
^27))/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^
20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^1
6))) + (x*(2554331136*a^10*c^8*d^15*e^14 - 144703488*a^8*c^10*d^23*e^6 - 15
4140672*a^9*c^9*d^19*e^10 - 34603008*a^7*c^11*d^27*e^2 + 7659847680*a^11*c^
7*d^11*e^18 + 7556038656*a^12*c^6*d^7*e^22 + 2494562304*a^13*c^5*d^3*e^26))
/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^
4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)))
+ (12681216*a^5*c^10*d^23*e^4 + 127107072*a^6*c^9*d^19*e^8 + 674168832*a^7*
c^8*d^15*e^12 + 1018626048*a^8*c^7*d^11*e^16 - 446201856*a^9*c^6*d^7*e^20 +
 906854400*a^10*c^5*d^3*e^24)/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c
*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^
12 + 15*a^12*c^2*d^8*e^16)) + (x*(516096*a^5*c^10*d^22*e^5 - 1806336*a^4*c^
11*d^26*e + 90427392*a^6*c^9*d^18*e^9 + 896090112*a^7*c^8*d^14*e^13 + 19609
06752*a^8*c^7*d^10*e^17 + 1732829184*a^9*c^6*d^6*e^21 + 1183887360*a^10*c^5
*d^2*e^25))/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*
c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d
^8*e^16))) + (387072*a^2*c^10*d^22*e^3 + 8004096*a^3*c^9*d^18*e^7 + 4937932

8*a^4*c^8*d^14*e^11 + 49572864*a^5*c^7*d^10*e^15 - 156930048*a^6*c^6*d^6*e^19 + 125452800*a^7*c^5*d^2*e^23)/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)) + (x*(126360000*a^7*c^5*d*e^24 + 561600*a^2*c^10*d^21*e^4 + 9609408*a^3*c^9*d^17*e^8 + 75731328*a^4*c^8*d^13*e^12 + 114991488*a^5*c^7*d^9*e^16 - 80136000*a^6*c^6*d^5*e^20))/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16))) + (x*(4100625*a^4*c^5*e^23 + 194481*c^9*d^16*e^7 + 1527012*a*c^8*d^12*e^11 - 167994*a^2*c^7*d^8*e^15 - 13988700*a^3*c^6*d^4*e^19))/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)))*root(1610612736*a^13*c^2*d^8*e^8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^14*c*d^4*e^12*z^4 + 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 + 3221225472*a^11*c*d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^10*e^2*z^2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 + 4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k), k, 1, 4) + ((c^2*d^8*e^3 - 2*a^2*e^11 + 5*a*c*d^4*e^7)/(2*(a*e^4 + c*d^4)*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (3*x^8*(5*c^4*d^8*e^3 - 15*a^2*c^2*e^11 + 22*a*c^3*d^4*e^7))/(32*a^2*(a^3*e^12 + c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)) + (x^5*(7*c^3*d^7 + 19*a*c^2*d^3*e^4))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (3*x^2*(3*c^2*d^6*e + 7*a*c*d^2*e^5))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x^3*(7*c^2*d^5*e^2 + 19*a*c*d*e^6))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x*(11*c^2*d^7 + 23*a*c*d^3*e^4))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x^6*(5*c^3*d^6*e + 17*a*c^2*d^2*e^5))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (3*e^2*x^7*(c^3*d^5 + 5*a*c^2*d*e^4))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (3*e^2*x^4*(9*c^3*d^8*e - 27*a^2*c*e^9 + 46*a*c^2*d^4*e^5))/(32*a*(a*e^4 + c*d^4)*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)))/(a^2*d + c^2*d*x^8 + c^2*e*x^9 + a^2*e*x + 2*a*c*d*x^4 + 2*a*c*e*x^5) + (12*c*d^3*e^11*log(d + e*x))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)

3.414 $\int \frac{1}{(d+ex)^3 (a+cx^4)^3} dx$

Optimal result 2565
Rubi [A] (verified) 2567
Mathematica [A] (verified) 2574
Maple [A] (verified) 2575
Fricas [F(-1)] 2576
Sympy [F(-1)] 2576
Maxima [A] (verification not implemented) 2577
Giac [A] (verification not implemented) 2578
Mupad [B] (verification not implemented) 2579

Optimal result

Integrand size = 17, antiderivative size = 2204

$$\int \frac{1}{(d+ex)^3\left(a+cx^4\right)^3}\,dx = -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}$$

$$+\frac{cx(7d(c^2d^8-12acd^4e^4+3a^2e^8)-6e(3c^2d^8-12acd^4e^4+a^2e^8)\,x+10cd^3e^2(3cd^4-5ae^4)\,x^2)}{32a^2\left(cd^4+ae^4\right)^3(a+cx^4)}$$

$$+\frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d(c^2d^8-12acd^4e^4+3a^2e^8)-e(3c^2d^8-12acd^4e^4+a^2e^8)\,x+2cd^3e^2(3cd^4-}{8a\left(cd^4+ae^4\right)^3(a+cx^4)^2}$$

$$+\frac{ce^4(12ad^2e^3(3cd^4-ae^4)+x(3d(5c^2d^8-10acd^4e^4+a^2e^8)-e(21c^2d^8-26acd^4e^4+a^2e^8)\,x+4cd^3e^2(7c}{4a\left(cd^4+ae^4\right)^4(a+cx^4)}$$

$$-\frac{\sqrt{c}e^9(55c^2d^8-40acd^4e^4+a^2e^8)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)^5}$$

$$-\frac{\sqrt{c}e^5(21c^2d^8-26acd^4e^4+a^2e^8)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^4+ae^4\right)^4}$$

$$-\frac{3\sqrt{c}e(3c^2d^8-12acd^4e^4+a^2e^8)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\left(cd^4+ae^4\right)^3}$$

$$-\frac{3c^{3/4}de^8\left(15c^2d^8-16acd^4e^4+a^2e^8+2\sqrt{a}\sqrt{c}d^2e^2(11cd^4-5ae^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^5}$$

$$-\frac{c^{3/4}de^4\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)+9(5c^2d^8-10acd^4e^4+a^2e^8)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^4}$$

$$-\frac{c^{3/4}d\left(10\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)+21(c^2d^8-12acd^4e^4+3a^2e^8)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\left(cd^4+ae^4\right)^3}$$

$$+\frac{3c^{3/4}de^8\left(15c^2d^8-16acd^4e^4+a^2e^8+2\sqrt{a}\sqrt{c}d^2e^2(11cd^4-5ae^4)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^5}$$

$$+\frac{c^{3/4}de^4\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)+9(5c^2d^8-10acd^4e^4+a^2e^8)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^4}$$

$$+\frac{c^{3/4}d\left(10\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)+21(c^2d^8-12acd^4e^4+3a^2e^8)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\left(cd^4+ae^4\right)^3}$$

$$+\frac{6cd^2e^{11}(13cd^4-3ae^4)\log(d+ex)}{\left(cd^4+ae^4\right)^5}$$

$$-\frac{3c^{3/4}de^8\left(15c^2d^8-16acd^4e^4+a^2e^8-2\sqrt{a}\sqrt{c}d^2e^2(11cd^4-5ae^4)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^5}$$

$$+\frac{c^{3/4}de^4\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)-9(5c^2d^8-10acd^4e^4+a^2e^8)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^4}$$

$$+\frac{c^{3/4}d\left(10\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)-21(c^2d^8-12acd^4e^4+3a^2e^8)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{}$$

```
[Out] -1/2*e^11/(a*e^4+c*d^4)^3/(e*x+d)^2-12*c*d^3*e^11/(a*e^4+c*d^4)^4/(e*x+d)+1
/32*c*x*(7*d*(3*a^2*e^8-12*a*c*d^4*e^4+c^2*d^8)-6*e*(a^2*e^8-12*a*c*d^4*e^4
+3*c^2*d^8)*x+10*c*d^3*e^2*(-5*a*e^4+3*c*d^4)*x^2)/a^2/(a*e^4+c*d^4)^3/(c*x
^4+a)+1/8*c*(2*a*d^2*e^3*(-3*a*e^4+5*c*d^4)+x*(d*(3*a^2*e^8-12*a*c*d^4*e^4+
c^2*d^8)-e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*x+2*c*d^3*e^2*(-5*a*e^4+3*c*d
^4)*x^2))/a/(a*e^4+c*d^4)^3/(c*x^4+a)^2+1/4*c*e^4*(12*a*d^2*e^3*(-a*e^4+3*c
*d^4)+x*(3*d*(a^2*e^8-10*a*c*d^4*e^4+5*c^2*d^8)-e*(a^2*e^8-26*a*c*d^4*e^4+2
1*c^2*d^8)*x+4*c*d^3*e^2*(-5*a*e^4+7*c*d^4)*x^2))/a/(a*e^4+c*d^4)^4/(c*x^4+
a)+6*c*d^2*e^11*(-3*a*e^4+13*c*d^4)*ln(e*x+d)/(a*e^4+c*d^4)^5-3/2*c*d^2*e^1
1*(-3*a*e^4+13*c*d^4)*ln(c*x^4+a)/(a*e^4+c*d^4)^5-1/4*e^5*(a^2*e^8-26*a*c*d
^4*e^4+21*c^2*d^8)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^4+c*d^4
)^4-3/16*e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*arctan(x^2*c^(1/2)/a^(1/2))*c
^(1/2)/a^(5/2)/(a*e^4+c*d^4)^3-1/2*e^9*(a^2*e^8-40*a*c*d^4*e^4+55*c^2*d^8)*
arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^5/a^(1/2)+1/256*c^(3/4)*d
*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-63*a^2*e^8+252*a*c*d^
4*e^4-21*c^2*d^8+10*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(11/4)/(a
*e^4+c*d^4)^3*2^(1/2)-1/256*c^(3/4)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+
x^2*c^(1/2))*(-63*a^2*e^8+252*a*c*d^4*e^4-21*c^2*d^8+10*d^2*e^2*(-5*a*e^4+3
*c*d^4)*a^(1/2)*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/128*c^(3/4)*d*a
rctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(63*a^2*e^8-252*a*c*d^4*e^4+21*c^2*d^8+
10*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^3*2^(
1/2)+1/128*c^(3/4)*d*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(63*a^2*e^8-252*a*
c*d^4*e^4+21*c^2*d^8+10*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(11/4
)/(a*e^4+c*d^4)^3*2^(1/2)+1/32*c^(3/4)*d*e^4*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+
a^(1/2)+x^2*c^(1/2))*(-9*a^2*e^8+90*a*c*d^4*e^4-45*c^2*d^8+4*d^2*e^2*(-5*a*
e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^4*2^(1/2)-1/32*c^(3/4)*
d*e^4*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-9*a^2*e^8+90*a*c*
d^4*e^4-45*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a
*e^4+c*d^4)^4*2^(1/2)+1/16*c^(3/4)*d*e^4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4
))*(9*a^2*e^8-90*a*c*d^4*e^4+45*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4)*a^(1/2
)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^4*2^(1/2)+1/16*c^(3/4)*d*e^4*arctan(1+c^(1
/4)*x*2^(1/2)/a^(1/4))*(9*a^2*e^8-90*a*c*d^4*e^4+45*c^2*d^8+4*d^2*e^2*(-5*a
*e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^4*2^(1/2)-3/8*c^(3/4)*
d*e^8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(15*c^2*d^8-16*a*c
*d^4*e^4+a^2*e^8-2*d^2*e^2*(-5*a*e^4+11*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*
e^4+c*d^4)^5*2^(1/2)+3/8*c^(3/4)*d*e^8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*c^(1/2))*(15*c^2*d^8-16*a*c*d^4*e^4+a^2*e^8-2*d^2*e^2*(-5*a*e^4+11*c*d
^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^5*2^(1/2)+3/4*c^(3/4)*d*e^8*arct
an(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(15*c^2*d^8-16*a*c*d^4*e^4+a^2*e^8+2*d^2*e
^2*(-5*a*e^4+11*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^5*2^(1/2)+3/4
*c^(3/4)*d*e^8*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(15*c^2*d^8-16*a*c*d^4*e
^4+a^2*e^8+2*d^2*e^2*(-5*a*e^4+11*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*
d^4)^5*2^(1/2)
```

Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 2204, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules

used = {6874, 1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 266}

$$\int \frac{1}{(d+ex)^3(a+cx^4)^3}dx = \frac{6cd^2(13cd^4-3ae^4)\log(d+ex)e^{11}}{(cd^4+ae^4)^5}$$
$$-\frac{3cd^2(13cd^4-3ae^4)\log(cx^4+a)e^{11}}{2(cd^4+ae^4)^5}-\frac{12cd^3e^{11}}{(cd^4+ae^4)^4(d+ex)}$$
$$-\frac{e^{11}}{2(cd^4+ae^4)^3(d+ex)^2}-\frac{\sqrt{c}(55c^2d^8-40ace^4d^4+a^2e^8)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^9}{2\sqrt{a}(cd^4+ae^4)^5}$$
$$-\frac{3c^{3/4}d\left(15c^2d^8-16ace^4d^4+2\sqrt{a}\sqrt{c}e^2(11cd^4-5ae^4)d^2+a^2e^8\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^8}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^5}$$
$$+\frac{3c^{3/4}d\left(15c^2d^8-16ace^4d^4+2\sqrt{a}\sqrt{c}e^2(11cd^4-5ae^4)d^2+a^2e^8\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^8}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^5}$$
$$-\frac{3c^{3/4}d\left(15c^2d^8-16ace^4d^4-2\sqrt{a}\sqrt{c}e^2(11cd^4-5ae^4)d^2+a^2e^8\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^8}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^5}$$
$$+\frac{3c^{3/4}d\left(15c^2d^8-16ace^4d^4-2\sqrt{a}\sqrt{c}e^2(11cd^4-5ae^4)d^2+a^2e^8\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^8}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^5}$$
$$-\frac{\sqrt{c}(21c^2d^8-26ace^4d^4+a^2e^8)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{4a^{3/2}(cd^4+ae^4)^4}$$
$$+\frac{c(12ad^2(3cd^4-ae^4)e^3+x(4ce^2(7cd^4-5ae^4)x^2d^3+3(5c^2d^8-10ace^4d^4+a^2e^8)d-e(21c^2d^8-26ace^4d}{4a(cd^4+ae^4)^4(cx^4+a)}$$
$$-\frac{c^{3/4}d\left(4\sqrt{a}\sqrt{c}d^2(7cd^4-5ae^4)e^2+9(5c^2d^8-10ace^4d^4+a^2e^8)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{8\sqrt{2}a^{7/4}(cd^4+ae^4)^4}$$
$$+\frac{c^{3/4}d\left(4\sqrt{a}\sqrt{c}d^2(7cd^4-5ae^4)e^2+9(5c^2d^8-10ace^4d^4+a^2e^8)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^4}{8\sqrt{2}a^{7/4}(cd^4+ae^4)^4}$$
$$+\frac{c^{3/4}d\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)-9(5c^2d^8-10ace^4d^4+a^2e^8)\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{16\sqrt{2}a^{7/4}(cd^4+ae^4)^4}$$
$$-\frac{c^{3/4}d\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)-9(5c^2d^8-10ace^4d^4+a^2e^8)\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{16\sqrt{2}a^{7/4}(cd^4+ae^4)^4}$$
$$-\frac{3\sqrt{c}(3c^2d^8-12ace^4d^4+a^2e^8)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{16a^{5/2}(cd^4+ae^4)^3}$$
$$+\frac{c(2ad^2(5cd^4-3ae^4)e^3+x(2ce^2(3cd^4-5ae^4)x^2d^3+(c^2d^8-12ace^4d^4+3a^2e^8)d-e(3c^2d^8-12ace^4d^4+}{8a(cd^4+ae^4)^3(cx^4+a)^2}$$
$$-\frac{c^{3/4}d\left(10\sqrt{a}\sqrt{c}d^2(3cd^4-5ae^4)e^2+21(c^2d^8-12ace^4d^4+3a^2e^8)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}(cd^4+ae^4)^3}$$
$$+\frac{c^{3/4}d\left(10\sqrt{a}\sqrt{c}d^2(3cd^4-5ae^4)e^2+21(c^2d^8-12ace^4d^4+3a^2e^8)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}(cd^4+ae^4)^3}$$

```
[In] Int[1/((d + e*x)^3*(a + c*x^4)^3),x]

[Out] -1/2*e^11/((c*d^4 + a*e^4)^3*(d + e*x)^2) - (12*c*d^3*e^11)/((c*d^4 + a*e^4
)^4*(d + e*x)) + (c*x*(7*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - 6*e*(3*
c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 10*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^
2))/(32*a^2*(c*d^4 + a*e^4)^3*(a + c*x^4)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a
*e^4) + x*(d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c
*d^4*e^4 + a^2*e^8)*x + 2*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(8*a*(c*d^4
+ a*e^4)^3*(a + c*x^4)^2) + (c*e^4*(12*a*d^2*e^3*(3*c*d^4 - a*e^4) + x*(3*d
*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8) - e*(21*c^2*d^8 - 26*a*c*d^4*e^4 +
a^2*e^8)*x + 4*c*d^3*e^2*(7*c*d^4 - 5*a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^4*
(a + c*x^4)) - (Sqrt[c]*e^9*(55*c^2*d^8 - 40*a*c*d^4*e^4 + a^2*e^8)*ArcTan[
(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^5) - (Sqrt[c]*e^5*(21*c^
2*d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)
*(c*d^4 + a*e^4)^4) - (3*Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*A
rcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*(c*d^4 + a*e^4)^3) - (3*c^(3/4)*d
*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11
*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3
/4)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4
 - 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 - (Sqrt[2]
*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^4) - (c^(3/4)*d*(1
0*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) + 21*(c^2*d^8 - 12*a*c*d^4*e^
4 + 3*a^2*e^8))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/
4)*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2
*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*
c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4
*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4
*e^4 + a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4
)*(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*
a*e^4) + 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (6*c*d^2*e^11*(
13*c*d^4 - 3*a*e^4)*Log[d + e*x])/(c*d^4 + a*e^4)^5 - (3*c^(3/4)*d*e^8*(15*
c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 -
5*a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2
]*a^(3/4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7
*c*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e
^4)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) - 21*(c
^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*
e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*
c*d^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^
2*e^2*(7*c*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d
```

```
^4 + a*e^4)^4) - (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4)
 - 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) - (3*c*
d^2*e^11*(13*c*d^4 - 3*a*e^4)*Log[a + c*x^4])/(2*(c*d^4 + a*e^4)^5)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
```

```
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^{12}}{(cd^4+ae^4)^3 (d+ex)^3} + \frac{12cd^3e^{12}}{(cd^4+ae^4)^4 (d+ex)^2} + \frac{6cd^2e^{12}(13cd^4-3ae^4)}{(cd^4+ae^4)^5 (d+ex)} + \frac{c(d(c^2d^8-12acd^4e^4+3a^2e^8)-e(3c^2d^8-12acd^4e^4+a^2e^8)x+2cd^3e^2(3cd^4-5ae^4)x^2-2cd^2e^3(5cd^4-}{(cd^4+ae^4)^3 (a+cx^4)^3} + \frac{ce^4(3d(5c^2d^8-10acd^4e^4+a^2e^8)-e(21c^2d^8-26acd^4e^4+a^2e^8)x+4cd^3e^2(7cd^4-5ae^4)x^2-12cd^2e^3(3}{(cd^4+ae^4)^4 (a+cx^4)^2} + \frac{ce^8(3d(15c^2d^8-16acd^4e^4+a^2e^8)-e(55c^2d^8-40acd^4e^4+a^2e^8)x+6cd^3e^2(11cd^4-5ae^4)x^2-6cd^2e^3(}{(cd^4+ae^4)^5 (a+cx^4)}\right.$$

$$= -\frac{e^{11}}{2(cd^4+ae^4)^3 (d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4 (d+ex)} + \frac{6cd^2e^{11}(13cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^5} + \frac{(ce^8)\int \frac{3d(15c^2d^8-16acd^4e^4+a^2e^8)-e(55c^2d^8-40acd^4e^4+a^2e^8)x+6cd^3e^2(11cd^4-5ae^4)x^2-6cd^2e^3(13cd^4-3ae^4)x^3}{a+cx^4}\,dx}{(cd^4+ae^4)^5} + \frac{(ce^4)\int \frac{3d(5c^2d^8-10acd^4e^4+a^2e^8)-e(21c^2d^8-26acd^4e^4+a^2e^8)x+4cd^3e^2(7cd^4-5ae^4)x^2-12cd^2e^3(3cd^4-ae^4)x^3}{(a+cx^4)^2}\,dx}{(cd^4+ae^4)^4} + \frac{c\int \frac{d(c^2d^8-12acd^4e^4+3a^2e^8)-e(3c^2d^8-12acd^4e^4+a^2e^8)x+2cd^3e^2(3cd^4-5ae^4)x^2-2cd^2e^3(5cd^4-3ae^4)x^3}{(a+cx^4)^3}\,dx}{(cd^4+ae^4)^3}$$

$$= -\frac{e^{11}}{2(cd^4+ae^4)^3(d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4(d+ex)}$$
$$+ \frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d(c^2d^8-12acd^4e^4+3a^2e^8)-e(3c^2d^8-12acd^4e^4+a^2e^8)x+2cd^3e}{8a(cd^4+ae^4)^3(a+cx^4)^2}$$
$$+ \frac{ce^4(12ad^2e^3(3cd^4-ae^4)+x(3d(5c^2d^8-10acd^4e^4+a^2e^8)-e(21c^2d^8-26acd^4e^4+a^2e^8)x+4}{4a(cd^4+ae^4)^4(a+cx^4)}$$
$$+ \frac{6cd^2e^{11}(13cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^5}$$
$$+ \frac{(ce^8)\int\left(\frac{3d(15c^2d^8-16acd^4e^4+a^2e^8)+6cd^3e^2(11cd^4-5ae^4)x^2}{a+cx^4}+\frac{x(-e(55c^2d^8-40acd^4e^4+a^2e^8)-6cd^2e^3(13cd^4-3ae^4)x^2)}{a+cx^4}\right.}{(cd^4+ae^4)^5}$$
$$- \frac{(ce^4)\int\frac{-9d(5c^2d^8-10acd^4e^4+a^2e^8)+2e(21c^2d^8-26acd^4e^4+a^2e^8)x-4cd^3e^2(7cd^4-5ae^4)x^2}{a+cx^4}dx}{4a(cd^4+ae^4)^4}$$
$$- \frac{c\int\frac{-7d(c^2d^8-12acd^4e^4+3a^2e^8)+6e(3c^2d^8-12acd^4e^4+a^2e^8)x-10cd^3e^2(3cd^4-5ae^4)x^2}{(a+cx^4)^2}dx}{8a(cd^4+ae^4)^3}$$

$$= -\frac{e^{11}}{2(cd^4+ae^4)^3(d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4(d+ex)}$$
$$+ \frac{cx(7d(c^2d^8-12acd^4e^4+3a^2e^8)-6e(3c^2d^8-12acd^4e^4+a^2e^8)x+10cd^3e^2(3cd^4-5ae^4)x^2)}{32a^2(cd^4+ae^4)^3(a+cx^4)}$$
$$+ \frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d(c^2d^8-12acd^4e^4+3a^2e^8)-e(3c^2d^8-12acd^4e^4+a^2e^8)x+2cd^3e}{8a(cd^4+ae^4)^3(a+cx^4)^2}$$
$$+ \frac{ce^4(12ad^2e^3(3cd^4-ae^4)+x(3d(5c^2d^8-10acd^4e^4+a^2e^8)-e(21c^2d^8-26acd^4e^4+a^2e^8)x+4}{4a(cd^4+ae^4)^4(a+cx^4)}$$
$$+ \frac{6cd^2e^{11}(13cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^5}$$
$$+ \frac{(ce^8)\int\frac{3d(15c^2d^8-16acd^4e^4+a^2e^8)+6cd^3e^2(11cd^4-5ae^4)x^2}{a+cx^4}dx}{(cd^4+ae^4)^5}$$
$$+ \frac{(ce^8)\int\frac{x(-e(55c^2d^8-40acd^4e^4+a^2e^8)-6cd^2e^3(13cd^4-3ae^4)x^2)}{a+cx^4}dx}{(cd^4+ae^4)^5}$$
$$- \frac{(ce^4)\int\left(\frac{2e(21c^2d^8-26acd^4e^4+a^2e^8)x}{a+cx^4}+\frac{-9d(5c^2d^8-10acd^4e^4+a^2e^8)-4cd^3e^2(7cd^4-5ae^4)x^2}{a+cx^4}\right)dx}{4a(cd^4+ae^4)^4}$$
$$+ \frac{c\int\frac{21d(c^2d^8-12acd^4e^4+3a^2e^8)-12e(3c^2d^8-12acd^4e^4+a^2e^8)x+10cd^3e^2(3cd^4-5ae^4)x^2}{a+cx^4}dx}{32a^2(cd^4+ae^4)^3}$$

$$= -\frac{e^{11}}{2(cd^4+ae^4)^3(d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4(d+ex)}$$
$$+ \frac{cx(7d(c^2d^8-12acd^4e^4+3a^2e^8)-6e(3c^2d^8-12acd^4e^4+a^2e^8)x+10cd^3e^2(3cd^4-5ae^4)x^2)}{32a^2(cd^4+ae^4)^3(a+cx^4)}$$
$$+ \frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d(c^2d^8-12acd^4e^4+3a^2e^8)-e(3c^2d^8-12acd^4e^4+a^2e^8)x+2cd^3e^2(}{8a(cd^4+ae^4)^3(a+cx^4)^2}$$
$$+ \frac{ce^4(12ad^2e^3(3cd^4-ae^4)+x(3d(5c^2d^8-10acd^4e^4+a^2e^8)-e(21c^2d^8-26acd^4e^4+a^2e^8)x+4c}{4a(cd^4+ae^4)^4(a+cx^4)}$$
$$+ \frac{6cd^2e^{11}(13cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^5}$$
$$+ \frac{(ce^8)\operatorname{Subst}\left(\int \frac{-e(55c^2d^8-40acd^4e^4+a^2e^8)-6cd^2e^3(13cd^4-3ae^4)x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^4+ae^4)^5}$$
$$- \frac{(ce^4)\int \frac{-9d(5c^2d^8-10acd^4e^4+a^2e^8)-4cd^3e^2(7cd^4-5ae^4)x^2}{a+cx^4}\,dx}{4a(cd^4+ae^4)^4}$$
$$+ \frac{c\int \left(-\frac{12e(3c^2d^8-12acd^4e^4+a^2e^8)x}{a+cx^4} + \frac{21d(c^2d^8-12acd^4e^4+3a^2e^8)+10cd^3e^2(3cd^4-5ae^4)x^2}{a+cx^4}\right)dx}{32a^2(cd^4+ae^4)^3}$$
$$- \frac{(ce^5(21c^2d^8-26acd^4e^4+a^2e^8))\int \frac{x}{a+cx^4}\,dx}{2a(cd^4+ae^4)^4}$$
$$- \frac{\left(3cde^8\left(22cd^6e^2-10ad^2e^6-\frac{15c^2d^8-16acd^4e^4+a^2e^8}{\sqrt{a}\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2(cd^4+ae^4)^5}$$
$$+ \frac{\left(3cde^8\left(22cd^6e^2-10ad^2e^6+\frac{15c^2d^8-16acd^4e^4+a^2e^8}{\sqrt{a}\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2(cd^4+ae^4)^5}$$

= Too large to display

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 1338, normalized size of antiderivative = 0.61

$$\int \frac{1}{(d+ex)^3(a+cx^4)^3}\,dx$$

$$= \frac{-\frac{128e^{11}(cd^4+ae^4)^2}{(d+ex)^2} - \frac{3072cd^3e^{11}(cd^4+ae^4)}{d+ex} + \frac{8c(cd^4+ae^4)(a^3e^{11}(-96d^2+45dex-14e^2x^2)+c^3d^{11}x(7d^2-18dex+30e^2x^2)+ac^2d^7e^4x(43d^2}{a^2(a+cx^4)}}{}$$

```
[In] Integrate[1/((d + e*x)^3*(a + c*x^4)^3),x]
```

```
[Out] ((-128*e^11*(c*d^4 + a*e^4)^2)/(d + e*x)^2 - (3072*c*d^3*e^11*(c*d^4 + a*e^
4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(a^3*e^11*(-96*d^2 + 45*d*e*x - 14*e^2
```

```
*x^2) + c^3*d^11*x*(7*d^2 - 18*d*e*x + 30*e^2*x^2) + a*c^2*d^7*e^4*x*(43*d^
2 - 114*d*e*x + 204*e^2*x^2) + a^2*c*d^3*e^7*(288*d^3 - 303*d^2*e*x + 274*d
*e^2*x^2 - 210*e^3*x^3)))/(a^2*(a + c*x^4)) + (32*c*(c*d^4 + a*e^4)^2*(-(a^
2*e^7*(6*d^2 - 3*d*e*x + e^2*x^2)) + c^2*d^7*x*(d^2 - 3*d*e*x + 6*e^2*x^2)
+ 2*a*c*d^3*e^3*(5*d^3 - 6*d^2*e*x + 6*d*e^2*x^2 - 5*e^3*x^3)))/(a*(a + c*x
^4)^2) - (6*Sqrt[c]*(7*Sqrt[2]*c^(17/4)*d^17 - 24*a^(1/4)*c^4*d^16*e + 10*S
qrt[2]*Sqrt[a]*c^(15/4)*d^15*e^2 + 50*Sqrt[2]*a*c^(13/4)*d^13*e^4 - 176*a^(
5/4)*c^3*d^12*e^5 + 78*Sqrt[2]*a^(3/2)*c^(11/4)*d^11*e^6 + 220*Sqrt[2]*a^2*
c^(9/4)*d^9*e^8 - 960*a^(9/4)*c^2*d^8*e^9 + 702*Sqrt[2]*a^(5/2)*c^(7/4)*d^7
*e^10 - 770*Sqrt[2]*a^3*c^(5/4)*d^5*e^12 + 1200*a^(13/4)*c*d^4*e^13 - 390*S
qrt[2]*a^(7/2)*c^(3/4)*d^3*e^14 + 77*Sqrt[2]*a^4*c^(1/4)*d*e^16 - 40*a^(17/
4)*e^17)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + (6*Sqrt[c]*(7*
Sqrt[2]*c^(17/4)*d^17 + 24*a^(1/4)*c^4*d^16*e + 10*Sqrt[2]*Sqrt[a]*c^(15/4)
*d^15*e^2 + 50*Sqrt[2]*a*c^(13/4)*d^13*e^4 + 176*a^(5/4)*c^3*d^12*e^5 + 78*
Sqrt[2]*a^(3/2)*c^(11/4)*d^11*e^6 + 220*Sqrt[2]*a^2*c^(9/4)*d^9*e^8 + 960*a
^(9/4)*c^2*d^8*e^9 + 702*Sqrt[2]*a^(5/2)*c^(7/4)*d^7*e^10 - 770*Sqrt[2]*a^3
*c^(5/4)*d^5*e^12 - 1200*a^(13/4)*c*d^4*e^13 - 390*Sqrt[2]*a^(7/2)*c^(3/4)*
d^3*e^14 + 77*Sqrt[2]*a^4*c^(1/4)*d*e^16 + 40*a^(17/4)*e^17)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + 1536*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*
Log[d + e*x] - (3*Sqrt[2]*c^(3/4)*(7*c^4*d^17 - 10*Sqrt[a]*c^(7/2)*d^15*e^2
 + 50*a*c^3*d^13*e^4 - 78*a^(3/2)*c^(5/2)*d^11*e^6 + 220*a^2*c^2*d^9*e^8 -
702*a^(5/2)*c^(3/2)*d^7*e^10 - 770*a^3*c*d^5*e^12 + 390*a^(7/2)*Sqrt[c]*d^3
*e^14 + 77*a^4*d*e^16)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/a^(11/4) + (3*Sqrt[2]*c^(3/4)*(7*c^4*d^17 - 10*Sqrt[a]*c^(7/2)*d^15*e^2
 + 50*a*c^3*d^13*e^4 - 78*a^(3/2)*c^(5/2)*d^11*e^6 + 220*a^2*c^2*d^9*e^8 -
702*a^(5/2)*c^(3/2)*d^7*e^10 - 770*a^3*c*d^5*e^12 + 390*a^(7/2)*Sqrt[c]*d^3
*e^14 + 77*a^4*d*e^16)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/a^(11/4) - 384*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*Log[a + c*x^4])/(256*(c*
d^4 + a*e^4)^5)
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 1008, normalized size of antiderivative = 0.46

method	result	size
default	Expression too large to display	1008
risch	Expression too large to display	2238

```
[In] int(1/(e*x+d)^3/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c/(a*e^4+c*d^4)^5*((-3/16*c^2*d^3*e^2*(35*a^3*e^12+a^2*c*d^4*e^8-39*a*c^2*d
^8*e^4-5*c^3*d^12)/a^2*x^7-1/16*e*c*(7*a^4*e^16-130*a^3*c*d^4*e^12-80*a^2*c
^2*d^8*e^8+66*a*c^3*d^12*e^4+9*c^4*d^16)/a^2*x^6+1/32*c*d*(45*a^4*e^16-258*
a^3*c*d^4*e^12-260*a^2*c^2*d^8*e^8+50*a*c^3*d^12*e^4+7*c^4*d^16)/a^2*x^5+(-
3*a^2*c*d^2*e^15+6*a*c^2*d^6*e^11+9*c^3*d^10*e^7)*x^4-1/16*c*d^3*e^2*(125*a
```

```
^3*e^12+31*a^2*c*d^4*e^8-121*a*c^2*d^8*e^4-27*c^3*d^12)/a*x^3-3/16*e*(3*a^4
*e^16-50*a^3*c*d^4*e^12-40*a^2*c^2*d^8*e^8+18*a*c^3*d^12*e^4+5*c^4*d^16)/a*
x^2+1/32*d*(57*a^4*e^16-282*a^3*c*d^4*e^12-340*a^2*c^2*d^8*e^8+10*a*c^3*d^1
2*e^4+11*c^4*d^16)/a*x-15/4*a^3*d^2*e^15+23/4*a^2*d^6*e^11*c+43/4*a*d^10*e^
7*c^2+5/4*d^14*e^3*c^3)/(c*x^4+a)^2+3/32/a^2*(1/8*(77*a^4*d*e^16-770*a^3*c*
d^5*e^12+220*a^2*c^2*d^9*e^8+50*a*c^3*d^13*e^4+7*c^4*d^17)*(a/c)^(1/4)/a*2^
(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2
)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1
/4)*x-1))+1/2*(-20*a^4*e^17+600*a^3*c*d^4*e^13-480*a^2*c^2*d^8*e^9-88*a*c^3
*d^12*e^5-12*c^4*d^16*e)/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*(-390*a^3*
c*d^3*e^14+702*a^2*c^2*d^7*e^10+78*a*c^3*d^11*e^6+10*c^4*d^15*e^2)/c/(a/c)^
(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*
x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/
(a/c)^(1/4)*x-1))+1/4*(192*a^3*c*d^2*e^15-832*a^2*c^2*d^6*e^11)/c*ln(c*x^4+
a)))-1/2*e^11/(a*e^4+c*d^4)^3/(e*x+d)^2-12*c*d^3*e^11/(a*e^4+c*d^4)^4/(e*x+
d)-6*e^11*c*d^2*(3*a*e^4-13*c*d^4)/(a*e^4+c*d^4)^5*ln(e*x+d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\,(a+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\,(a+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 2198, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)^3 (a+cx^4)^3} dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] -3/256*c*(sqrt(2)*(832*sqrt(2)*a^(11/4)*c^(9/4)*d^6*e^11 - 192*sqrt(2)*a^(1
5/4)*c^(5/4)*d^2*e^15 - 7*c^5*d^17 + 10*sqrt(a)*c^(9/2)*d^15*e^2 - 50*a*c^4
*d^13*e^4 + 78*a^(3/2)*c^(7/2)*d^11*e^6 - 220*a^2*c^3*d^9*e^8 + 702*a^(5/2)
*c^(5/2)*d^7*e^10 + 770*a^3*c^2*d^5*e^12 - 390*a^(7/2)*c^(3/2)*d^3*e^14 - 7
7*a^4*c*d*e^16)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(
3/4)*c^(5/4)) + sqrt(2)*(832*sqrt(2)*a^(11/4)*c^(9/4)*d^6*e^11 - 192*sqrt(2
)*a^(15/4)*c^(5/4)*d^2*e^15 + 7*c^5*d^17 - 10*sqrt(a)*c^(9/2)*d^15*e^2 + 50
*a*c^4*d^13*e^4 - 78*a^(3/2)*c^(7/2)*d^11*e^6 + 220*a^2*c^3*d^9*e^8 - 702*a
^(5/2)*c^(5/2)*d^7*e^10 - 770*a^3*c^2*d^5*e^12 + 390*a^(7/2)*c^(3/2)*d^3*e^
14 + 77*a^4*c*d*e^16)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a)
)/(a^(3/4)*c^(5/4)) - 2*(7*sqrt(2)*a^(1/4)*c^(21/4)*d^17 + 10*sqrt(2)*a^(3/
4)*c^(19/4)*d^15*e^2 + 50*sqrt(2)*a^(5/4)*c^(17/4)*d^13*e^4 + 78*sqrt(2)*a^
(7/4)*c^(15/4)*d^11*e^6 + 220*sqrt(2)*a^(9/4)*c^(13/4)*d^9*e^8 + 702*sqrt(2
)*a^(11/4)*c^(11/4)*d^7*e^10 - 770*sqrt(2)*a^(13/4)*c^(9/4)*d^5*e^12 - 390*
sqrt(2)*a^(15/4)*c^(7/4)*d^3*e^14 + 77*sqrt(2)*a^(17/4)*c^(5/4)*d*e^16 + 24
*sqrt(a)*c^5*d^16*e + 176*a^(3/2)*c^4*d^12*e^5 + 960*a^(5/2)*c^3*d^8*e^9 -
1200*a^(7/2)*c^2*d^4*e^13 + 40*a^(9/2)*c*e^17)*arctan(1/2*sqrt(2)*(2*sqrt(c
)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)
*sqrt(c))*c^(5/4)) - 2*(7*sqrt(2)*a^(1/4)*c^(21/4)*d^17 + 10*sqrt(2)*a^(3/4
)*c^(19/4)*d^15*e^2 + 50*sqrt(2)*a^(5/4)*c^(17/4)*d^13*e^4 + 78*sqrt(2)*a^(
7/4)*c^(15/4)*d^11*e^6 + 220*sqrt(2)*a^(9/4)*c^(13/4)*d^9*e^8 + 702*sqrt(2)
*a^(11/4)*c^(11/4)*d^7*e^10 - 770*sqrt(2)*a^(13/4)*c^(9/4)*d^5*e^12 - 390*s
qrt(2)*a^(15/4)*c^(7/4)*d^3*e^14 + 77*sqrt(2)*a^(17/4)*c^(5/4)*d*e^16 - 24*
sqrt(a)*c^5*d^16*e - 176*a^(3/2)*c^4*d^12*e^5 - 960*a^(5/2)*c^3*d^8*e^9 + 1
200*a^(7/2)*c^2*d^4*e^13 - 40*a^(9/2)*c*e^17)*arctan(1/2*sqrt(2)*(2*sqrt(c)
*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*
sqrt(c))*c^(5/4)))/(a^2*c^5*d^20 + 5*a^3*c^4*d^16*e^4 + 10*a^4*c^3*d^12*e^8
 + 10*a^5*c^2*d^8*e^12 + 5*a^6*c*d^4*e^16 + a^7*e^20) + 6*(13*c^2*d^6*e^11
- 3*a*c*d^2*e^15)*log(e*x + d)/(c^5*d^20 + 5*a*c^4*d^16*e^4 + 10*a^2*c^3*d^
12*e^8 + 10*a^3*c^2*d^8*e^12 + 5*a^4*c*d^4*e^16 + a^5*e^20) + 1/32*(40*a^2*
c^3*d^12*e^3 + 304*a^3*c^2*d^8*e^7 - 520*a^4*c*d^4*e^11 - 16*a^5*e^15 + 6*(
5*c^5*d^11*e^4 + 34*a*c^4*d^7*e^8 - 99*a^2*c^3*d^3*e^12)*x^9 + 6*(7*c^5*d^1
2*e^3 + 49*a*c^4*d^8*e^7 - 91*a^2*c^3*d^4*e^11 - 5*a^3*c^2*e^15)*x^8 + (c^5
*d^13*e^2 + 19*a*c^4*d^9*e^6 + 35*a^2*c^3*d^5*e^10 + 17*a^3*c^2*d*e^14)*x^7
 - 4*(c^5*d^14*e + 7*a*c^4*d^10*e^5 + 11*a^2*c^3*d^6*e^9 + 5*a^3*c^2*d^2*e^
```

```
13)*x^6 + (7*c^5*d^15 + 97*a*c^4*d^11*e^4 + 461*a^2*c^3*d^7*e^8 - 1165*a^3*
c^2*d^3*e^12)*x^5 + 2*(39*a*c^4*d^12*e^3 + 293*a^2*c^3*d^8*e^7 - 539*a^3*c^
2*d^4*e^11 - 25*a^4*c*e^15)*x^4 + (5*a*c^4*d^13*e^2 + 31*a^2*c^3*d^9*e^6 +
47*a^3*c^2*d^5*e^10 + 21*a^4*c*d*e^14)*x^3 - 8*(a*c^4*d^14*e + 5*a^2*c^3*d^
10*e^5 + 7*a^3*c^2*d^6*e^9 + 3*a^4*c*d^2*e^13)*x^2 + (11*a*c^4*d^15 + 79*a^
2*c^3*d^11*e^4 + 269*a^3*c^2*d^7*e^8 - 567*a^4*c*d^3*e^12)*x)/(a^4*c^4*d^18
 + 4*a^5*c^3*d^14*e^4 + 6*a^6*c^2*d^10*e^8 + 4*a^7*c*d^6*e^12 + a^8*d^2*e^1
6 + (a^2*c^6*d^16*e^2 + 4*a^3*c^5*d^12*e^6 + 6*a^4*c^4*d^8*e^10 + 4*a^5*c^3
*d^4*e^14 + a^6*c^2*e^18)*x^10 + 2*(a^2*c^6*d^17*e + 4*a^3*c^5*d^13*e^5 + 6
*a^4*c^4*d^9*e^9 + 4*a^5*c^3*d^5*e^13 + a^6*c^2*d*e^17)*x^9 + (a^2*c^6*d^18
 + 4*a^3*c^5*d^14*e^4 + 6*a^4*c^4*d^10*e^8 + 4*a^5*c^3*d^6*e^12 + a^6*c^2*d
^2*e^16)*x^8 + 2*(a^3*c^5*d^16*e^2 + 4*a^4*c^4*d^12*e^6 + 6*a^5*c^3*d^8*e^1
0 + 4*a^6*c^2*d^4*e^14 + a^7*c*e^18)*x^6 + 4*(a^3*c^5*d^17*e + 4*a^4*c^4*d^
13*e^5 + 6*a^5*c^3*d^9*e^9 + 4*a^6*c^2*d^5*e^13 + a^7*c*d*e^17)*x^5 + 2*(a^
3*c^5*d^18 + 4*a^4*c^4*d^14*e^4 + 6*a^5*c^3*d^10*e^8 + 4*a^6*c^2*d^6*e^12 +
 a^7*c*d^2*e^16)*x^4 + (a^4*c^4*d^16*e^2 + 4*a^5*c^3*d^12*e^6 + 6*a^6*c^2*d
^8*e^10 + 4*a^7*c*d^4*e^14 + a^8*e^18)*x^2 + 2*(a^4*c^4*d^17*e + 4*a^5*c^3*
d^13*e^5 + 6*a^6*c^2*d^9*e^9 + 4*a^7*c*d^5*e^13 + a^8*d*e^17)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 2200, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)^3 (a+cx^4)^3} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -3/64*(23*sqrt(2)*a*c^4*d^6*e - 115*sqrt(2)*a^2*c^3*d^2*e^5 + 30*sqrt(2)*sq
rt(a*c)*a*c^3*d^4*e^3 + 20*sqrt(2)*sqrt(a*c)*a^2*c^2*e^7 - 65*(a*c^3)^(1/4)
*a*c^3*d^5*e^2 + 123*(a*c^3)^(1/4)*a^2*c^2*d*e^6 - 7*(a*c^3)^(3/4)*c^2*d^7
+ 65*(a*c^3)^(3/4)*a*c*d^3*e^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/
4))/(a/c)^(1/4))/(25*sqrt(2)*a^4*c^4*d^8*e^2 + 90*sqrt(2)*a^5*c^3*d^4*e^6 +
 sqrt(2)*a^6*c^2*e^10 - sqrt(2)*sqrt(a*c)*a^3*c^4*d^10 - 90*sqrt(2)*sqrt(a*
c)*a^4*c^3*d^6*e^4 - 25*sqrt(2)*sqrt(a*c)*a^5*c^2*d^2*e^8 - 80*(a*c^3)^(1/4
)*a^4*c^3*d^7*e^3 - 80*(a*c^3)^(1/4)*a^5*c^2*d^3*e^7 - 10*(a*c^3)^(3/4)*a^3
*c^2*d^9*e - 148*(a*c^3)^(3/4)*a^4*c*d^5*e^5 - 10*(a*c^3)^(3/4)*a^5*d*e^9)
+ 3/64*(23*sqrt(2)*a*c^4*d^6*e - 115*sqrt(2)*a^2*c^3*d^2*e^5 - 30*sqrt(2)*s
qrt(a*c)*a*c^3*d^4*e^3 - 20*sqrt(2)*sqrt(a*c)*a^2*c^2*e^7 + 65*(a*c^3)^(1/4
)*a*c^3*d^5*e^2 - 123*(a*c^3)^(1/4)*a^2*c^2*d*e^6 + 7*(a*c^3)^(3/4)*c^2*d^7
 - 65*(a*c^3)^(3/4)*a*c*d^3*e^4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1
/4))/(a/c)^(1/4))/(25*sqrt(2)*a^4*c^4*d^8*e^2 + 90*sqrt(2)*a^5*c^3*d^4*e^6
+ sqrt(2)*a^6*c^2*e^10 - sqrt(2)*sqrt(a*c)*a^3*c^4*d^10 - 90*sqrt(2)*sqrt(a
```

```
*c)*a^4*c^3*d^6*e^4 - 25*sqrt(2)*sqrt(a*c)*a^5*c^2*d^2*e^8 + 80*(a*c^3)^(1/
4)*a^4*c^3*d^7*e^3 + 80*(a*c^3)^(1/4)*a^5*c^2*d^3*e^7 + 10*(a*c^3)^(3/4)*a^
3*c^2*d^9*e + 148*(a*c^3)^(3/4)*a^4*c*d^5*e^5 + 10*(a*c^3)^(3/4)*a^5*d*e^9)
 + 3/128*(7*(a*c^3)^(1/4)*c^5*d^17 + 50*(a*c^3)^(1/4)*a*c^4*d^13*e^4 + 220*
(a*c^3)^(1/4)*a^2*c^3*d^9*e^8 - 770*(a*c^3)^(1/4)*a^3*c^2*d^5*e^12 + 77*(a*
c^3)^(1/4)*a^4*c*d*e^16 - 10*(a*c^3)^(3/4)*c^3*d^15*e^2 - 78*(a*c^3)^(3/4)*
a*c^2*d^11*e^6 - 702*(a*c^3)^(3/4)*a^2*c*d^7*e^10 + 390*(a*c^3)^(3/4)*a^3*d
^3*e^14)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^6*d^20
 + 5*sqrt(2)*a^4*c^5*d^16*e^4 + 10*sqrt(2)*a^5*c^4*d^12*e^8 + 10*sqrt(2)*a^
6*c^3*d^8*e^12 + 5*sqrt(2)*a^7*c^2*d^4*e^16 + sqrt(2)*a^8*c*e^20) - 3/128*(
7*(a*c^3)^(1/4)*c^5*d^17 + 50*(a*c^3)^(1/4)*a*c^4*d^13*e^4 + 220*(a*c^3)^(1
/4)*a^2*c^3*d^9*e^8 - 770*(a*c^3)^(1/4)*a^3*c^2*d^5*e^12 + 77*(a*c^3)^(1/4)
*a^4*c*d*e^16 - 10*(a*c^3)^(3/4)*c^3*d^15*e^2 - 78*(a*c^3)^(3/4)*a*c^2*d^11
*e^6 - 702*(a*c^3)^(3/4)*a^2*c*d^7*e^10 + 390*(a*c^3)^(3/4)*a^3*d^3*e^14)*l
og(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^6*d^20 + 5*sqrt(
2)*a^4*c^5*d^16*e^4 + 10*sqrt(2)*a^5*c^4*d^12*e^8 + 10*sqrt(2)*a^6*c^3*d^8*
e^12 + 5*sqrt(2)*a^7*c^2*d^4*e^16 + sqrt(2)*a^8*c*e^20) - 3/2*(13*c^2*d^6*e
^11 - 3*a*c*d^2*e^15)*log(abs(c*x^4 + a))/(c^5*d^20 + 5*a*c^4*d^16*e^4 + 10
*a^2*c^3*d^12*e^8 + 10*a^3*c^2*d^8*e^12 + 5*a^4*c*d^4*e^16 + a^5*e^20) + 6*
(13*c^2*d^6*e^12 - 3*a*c*d^2*e^16)*log(abs(e*x + d))/(c^5*d^20*e + 5*a*c^4*
d^16*e^5 + 10*a^2*c^3*d^12*e^9 + 10*a^3*c^2*d^8*e^13 + 5*a^4*c*d^4*e^17 + a
^5*e^21) + 1/32*(30*c^5*d^11*e^4*x^9 + 204*a*c^4*d^7*e^8*x^9 - 594*a^2*c^3*
d^3*e^12*x^9 + 42*c^5*d^12*e^3*x^8 + 294*a*c^4*d^8*e^7*x^8 - 546*a^2*c^3*d^
4*e^11*x^8 - 30*a^3*c^2*e^15*x^8 + c^5*d^13*e^2*x^7 + 19*a*c^4*d^9*e^6*x^7
+ 35*a^2*c^3*d^5*e^10*x^7 + 17*a^3*c^2*d*e^14*x^7 - 4*c^5*d^14*e*x^6 - 28*a
*c^4*d^10*e^5*x^6 - 44*a^2*c^3*d^6*e^9*x^6 - 20*a^3*c^2*d^2*e^13*x^6 + 7*c^
5*d^15*x^5 + 97*a*c^4*d^11*e^4*x^5 + 461*a^2*c^3*d^7*e^8*x^5 - 1165*a^3*c^2
*d^3*e^12*x^5 + 78*a*c^4*d^12*e^3*x^4 + 586*a^2*c^3*d^8*e^7*x^4 - 1078*a^3*
c^2*d^4*e^11*x^4 - 50*a^4*c*e^15*x^4 + 5*a*c^4*d^13*e^2*x^3 + 31*a^2*c^3*d^
9*e^6*x^3 + 47*a^3*c^2*d^5*e^10*x^3 + 21*a^4*c*d*e^14*x^3 - 8*a*c^4*d^14*e*
x^2 - 40*a^2*c^3*d^10*e^5*x^2 - 56*a^3*c^2*d^6*e^9*x^2 - 24*a^4*c*d^2*e^13*
x^2 + 11*a*c^4*d^15*x + 79*a^2*c^3*d^11*e^4*x + 269*a^3*c^2*d^7*e^8*x - 567
*a^4*c*d^3*e^12*x + 40*a^2*c^3*d^12*e^3 + 304*a^3*c^2*d^8*e^7 - 520*a^4*c*d
^4*e^11 - 16*a^5*e^15)/((a^2*c^4*d^16 + 4*a^3*c^3*d^12*e^4 + 6*a^4*c^2*d^8*
e^8 + 4*a^5*c*d^4*e^12 + a^6*e^16)*(c*e*x^5 + c*d*x^4 + a*e*x + a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 12.67 (sec) , antiderivative size = 6280, normalized size of antiderivative = 2.85

$$\int \frac{1}{(d+ex)^3\,(a+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + c*x^4)^3*(d + e*x)^3),x)
```

[Out] symsum(log(root(2684354560*a^12*c^9*d^36*e^4*z^5 + 32212254720*a^18*c^3*d^12*e^28*z^5 + 32212254720*a^14*c^7*d^28*e^12*z^5 + 2684354560*a^20*c*d^4*e^36*z^5 + 56371445760*a^17*c^4*d^16*e^24*z^5 + 56371445760*a^15*c^6*d^24*e^16*z^5 + 12079595520*a^19*c^2*d^8*e^32*z^5 + 12079595520*a^13*c^8*d^32*e^8*z^5 + 67645734912*a^16*c^5*d^20*e^20*z^5 + 268435456*a^11*c^10*d^40*z^5 + 268435456*a^21*e^40*z^5 + 45339770880*a^9*c^6*d^20*e^14*z^3 - 79148482560*a^13*c^2*d^4*e^30*z^3 + 791941349376*a^12*c^3*d^8*e^26*z^3 + 1239810048*a^7*c^8*d^28*e^6*z^3 - 1555444924416*a^11*c^4*d^12*e^22*z^3 + 83755008*a^6*c^9*d^32*e^2*z^3 + 81566760960*a^10*c^5*d^16*e^18*z^3 + 12177506304*a^8*c^7*d^24*e^10*z^3 + 117964800*a^14*c*e^34*z^3 - 2785204224*a^6*c^6*d^18*e^13*z^2 + 8128512*a^3*c^9*d^30*e*z^2 + 2700933120*a^10*c^2*d^2*e^29*z^2 - 543361222656*a^8*c^4*d^10*e^21*z^2 + 1048135680*a^5*c^7*d^22*e^9*z^2 + 118499328*a^4*c^8*d^26*e^5*z^2 - 55938263040*a^7*c^5*d^14*e^17*z^2 + 123990497280*a^9*c^3*d^6*e^25*z^2 + 24139215*a^2*c^7*d^20*e^8*z + 2819286*a*c^8*d^24*e^4*z + 10462847841*a^6*c^3*d^4*e^24*z - 5777473473*a^4*c^5*d^12*e^16*z - 43509753450*a^5*c^4*d^8*e^20*z - 548810316*a^3*c^6*d^16*e^12*z + 12960000*a^7*c^2*e^28*z + 194481*c^9*d^28*z - 977636142*a^2*c^4*d^6*e^19 + 233280000*a^3*c^3*d^2*e^23 - 140556060*a*c^5*d^10*e^15 - 15169518*c^6*d^14*e^11, z, k)*(root(2684354560*a^12*c^9*d^36*e^4*z^5 + 32212254720*a^18*c^3*d^12*e^28*z^5 + 32212254720*a^14*c^7*d^28*e^12*z^5 + 2684354560*a^20*c*d^4*e^36*z^5 + 56371445760*a^17*c^4*d^16*e^24*z^5 + 56371445760*a^15*c^6*d^24*e^16*z^5 + 12079595520*a^19*c^2*d^8*e^32*z^5 + 12079595520*a^13*c^8*d^32*e^8*z^5 + 67645734912*a^16*c^5*d^20*e^20*z^5 + 268435456*a^11*c^10*d^40*z^5 + 268435456*a^21*e^40*z^5 + 45339770880*a^9*c^6*d^20*e^14*z^3 - 79148482560*a^13*c^2*d^4*e^30*z^3 + 791941349376*a^12*c^3*d^8*e^26*z^3 + 1239810048*a^7*c^8*d^28*e^6*z^3 - 1555444924416*a^11*c^4*d^12*e^22*z^3 + 83755008*a^6*c^9*d^32*e^2*z^3 + 81566760960*a^10*c^5*d^16*e^18*z^3 + 12177506304*a^8*c^7*d^24*e^10*z^3 + 117964800*a^14*c*e^34*z^3 - 2785204224*a^6*c^6*d^18*e^13*z^2 + 8128512*a^3*c^9*d^30*e*z^2 + 2700933120*a^10*c^2*d^2*e^29*z^2 - 543361222656*a^8*c^4*d^10*e^21*z^2 + 1048135680*a^5*c^7*d^22*e^9*z^2 + 118499328*a^4*c^8*d^26*e^5*z^2 - 55938263040*a^7*c^5*d^14*e^17*z^2 + 123990497280*a^9*c^3*d^6*e^25*z^2 + 24139215*a^2*c^7*d^20*e^8*z + 2819286*a*c^8*d^24*e^4*z + 10462847841*a^6*c^3*d^4*e^24*z - 5777473473*a^4*c^5*d^12*e^16*z - 43509753450*a^5*c^4*d^8*e^20*z - 548810316*a^3*c^6*d^16*e^12*z + 12960000*a^7*c^2*e^28*z + 194481*c^9*d^28*z - 977636142*a^2*c^4*d^6*e^19 + 233280000*a^3*c^3*d^2*e^23 - 140556060*a*c^5*d^10*e^15 - 15169518*c^6*d^14*e^11, z, k)*(root(2684354560*a^12*c^9*d^36*e^4*z^5 + 32212254720*a^18*c^3*d^12*e^28*z^5 + 32212254720*a^14*c^7*d^28*e^12*z^5 + 2684354560*a^20*c*d^4*e^36*z^5 + 56371445760*a^17*c^4*d^16*e^24*z^5 + 56371445760*a^15*c^6*d^24*e^16*z^5 + 12079595520*a^19*c^2*d^8*e^32*z^5 + 12079595520*a^13*c^8*d^32*e^8*z^5 + 67645734912*a^16*c^5*d^20*e^20*z^5 + 268435456*a^11*c^10*d^40*z^5 + 268435456*a^21*e^40*z^5 + 45339770880*a^9*c^6*d^20*e^14*z^3 - 79148482560*a^13*c^2*d^4*e^30*z^3 + 791941349376*a^12*c^3*d^8*e^26*z^3 + 1239810048*a^7*c^8*d^28*e^6*z^3 - 1555444924416*a^11*c^4*d^12*e^22*z^3 + 83755008*a^6*c^9*d^32*e^2*z^3 + 81566760960*a^10*c^5*d^16*e^18*z^3 + 12177506304*a^8*c^7*d^24*e^10*z^3 + 117964800*a^14*c*e^34*z^3 - 278520

4224*a^6*c^6*d^18*e^13*z^2 + 8128512*a^3*c^9*d^30*e*z^2 + 2700933120*a^10*c^2*d^2*e^29*z^2 - 543361222656*a^8*c^4*d^10*e^21*z^2 + 1048135680*a^5*c^7*d^22*e^9*z^2 + 118499328*a^4*c^8*d^26*e^5*z^2 - 55938263040*a^7*c^5*d^14*e^17*z^2 + 123990497280*a^9*c^3*d^6*e^25*z^2 + 24139215*a^2*c^7*d^20*e^8*z + 2819286*a*c^8*d^24*e^4*z + 10462847841*a^6*c^3*d^4*e^24*z - 5777473473*a^4*c^5*d^12*e^16*z - 43509753450*a^5*c^4*d^8*e^20*z - 548810316*a^3*c^6*d^16*e^12*z + 12960000*a^7*c^2*e^28*z + 194481*c^9*d^28*z - 977636142*a^2*c^4*d^6*e^19 + 233280000*a^3*c^3*d^2*e^23 - 140556060*a*c^5*d^10*e^15 - 15169518*c^6*d^14*e^11, z, k)*((44040192*a^8*c^12*d^31*e^5 - 11010048*a^7*c^13*d^35*e + 994050048*a^9*c^11*d^27*e^9 + 13683916800*a^10*c^10*d^23*e^13 + 42936041472*a^11*c^9*d^19*e^17 + 52628029440*a^12*c^8*d^15*e^21 + 23429382144*a^13*c^7*d^11*e^25 - 2132803584*a^14*c^6*d^7*e^29 - 3125280768*a^15*c^5*d^3*e^33)/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24)) + root(2684354560*a^12*c^9*d^36*e^4*z^5 + 32212254720*a^18*c^3*d^12*e^28*z^5 + 32212254720*a^14*c^7*d^28*e^12*z^5 + 2684354560*a^20*c*d^4*e^36*z^5 + 56371445760*a^17*c^4*d^16*e^24*z^5 + 56371445760*a^15*c^6*d^24*e^16*z^5 + 12079595520*a^19*c^2*d^8*e^32*z^5 + 12079595520*a^13*c^8*d^32*e^8*z^5 + 67645734912*a^16*c^5*d^20*e^20*z^5 + 268435456*a^11*c^10*d^40*z^5 + 268435456*a^21*e^40*z^5 + 45339770880*a^9*c^6*d^20*e^14*z^3 - 79148482560*a^13*c^2*d^4*e^30*z^3 + 791941349376*a^12*c^3*d^8*e^26*z^3 + 1239810048*a^7*c^8*d^28*e^6*z^3 - 1555444924416*a^11*c^4*d^12*e^22*z^3 + 83755008*a^6*c^9*d^32*e^2*z^3 + 81566760960*a^10*c^5*d^16*e^18*z^3 + 12177506304*a^8*c^7*d^24*e^10*z^3 + 117964800*a^14*c*e^34*z^3 - 2785204224*a^6*c^6*d^18*e^13*z^2 + 8128512*a^3*c^9*d^30*e*z^2 + 2700933120*a^10*c^2*d^2*e^29*z^2 - 543361222656*a^8*c^4*d^10*e^21*z^2 + 1048135680*a^5*c^7*d^22*e^9*z^2 + 118499328*a^4*c^8*d^26*e^5*z^2 - 55938263040*a^7*c^5*d^14*e^17*z^2 + 123990497280*a^9*c^3*d^6*e^25*z^2 + 24139215*a^2*c^7*d^20*e^8*z + 2819286*a*c^8*d^24*e^4*z + 10462847841*a^6*c^3*d^4*e^24*z - 5777473473*a^4*c^5*d^12*e^16*z - 43509753450*a^5*c^4*d^8*e^20*z - 548810316*a^3*c^6*d^16*e^12*z + 12960000*a^7*c^2*e^28*z + 194481*c^9*d^28*z - 977636142*a^2*c^4*d^6*e^19 + 233280000*a^3*c^3*d^2*e^23 - 140556060*a*c^5*d^10*e^15 - 15169518*c^6*d^14*e^11, z, k)*((402653184*a^19*c^4*d*e^38 - 134217728*a^10*c^13*d^37*e^2 - 671088640*a^11*c^12*d^33*e^6 - 536870912*a^12*c^11*d^29*e^10 + 3758096384*a^13*c^10*d^25*e^14 + 13153337344*a^14*c^9*d^21*e^18 + 20669530112*a^15*c^8*d^17*e^22 + 18790481920*a^16*c^7*d^13*e^26 + 10200547328*a^17*c^6*d^9*e^30 + 3087007744*a^18*c^5*d^5*e^34)/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24)) + (x*(335544320*a^19*c^4*e^39 - 201326592*a^10*c^13*d^36*e^3 - 1275068416*a^11*c^12*d^32*e^7 - 2952790016*a^12*c^11*d^28*e^11 - 1879048192*a^13*c^10*d^24*e^15 + 4697620480*a^14*c^9*d^20*e^19 + 12213813248*a^15*c^8*d^16*e^23 + 13153337344*a^16*c^7*d^12*e^27 + 7784628224*a^17*c^6*d^8*e^31 + 2483027968*a^18*c^5*d^4*e^35))/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d

^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24))) - (x*(40894464*a^7*c^13*d^34*e^2 + 276824064*a^8*c^12*d^30*e^6 + 968884224*a^9*c^11*d^26*e^10 - 13010731008*a^10*c^10*d^22*e^14 - 53433335808*a^11*c^9*d^18*e^18 - 71647100928*a^12*c^8*d^14*e^22 - 34313601024*a^13*c^7*d^10*e^26 + 1837105152*a^14*c^6*d^6*e^30 + 4193255424*a^15*c^5*d^2*e^34))/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24))) + (33914880*a^12*c^5*d*e^32 + 13713408*a^5*c^12*d^29*e^4 + 225902592*a^6*c^11*d^25*e^8 + 2352070656*a^7*c^10*d^21*e^12 + 2474606592*a^8*c^9*d^17*e^16 - 21361803264*a^9*c^8*d^13*e^20 + 88707170304*a^10*c^7*d^9*e^24 - 5526503424*a^11*c^6*d^5*e^28)/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24)) + (x*(132710400*a^12*c^5*e^33 - 1806336*a^4*c^13*d^32*e - 2027520*a^5*c^12*d^28*e^5 + 162017280*a^6*c^11*d^24*e^9 + 4635316224*a^7*c^10*d^20*e^13 + 15604273152*a^8*c^9*d^16*e^17 + 39318663168*a^9*c^8*d^12*e^21 + 64184389632*a^10*c^7*d^8*e^25 - 2525073408*a^11*c^6*d^4*e^29))/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24))) + (320544*a^2*c^12*d^27*e^3 + 11448000*a^3*c^11*d^23*e^7 + 114031584*a^4*c^10*d^19*e^11 - 213750144*a^5*c^9*d^15*e^15 - 3499271712*a^6*c^8*d^11*e^19 + 9699804864*a^7*c^7*d^7*e^23 - 933615072*a^8*c^6*d^3*e^27)/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24)) + (x*(514944*a^2*c^12*d^26*e^4 + 14314752*a^3*c^11*d^22*e^8 + 266343552*a^4*c^10*d^18*e^12 + 297948672*a^5*c^9*d^14*e^16 - 2642613120*a^6*c^8*d^10*e^20 + 1782459648*a^7*c^7*d^6*e^24 + 846599040*a^8*c^6*d^2*e^28))/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24))) + (194481*c^11*d^21*e^6 + 2430324*a*c^10*d^17*e^10 + 12960000*a^5*c^6*d*e^26 - 5918346*a^2*c^9*d^13*e^14 - 83522988*a^3*c^8*d^9*e^18 + 71628705*a^4*c^7*d^5*e^22)/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24)) + (x*(12960000*a^5*c^6*e^27 + 194481*c^11*d^20*e^7 + 2430324*a*c^10*d^16*e^11 - 21081546*a^2*c^9*d^12*e^15 - 227814444*a^3*c^8*d^8*e^19 + 105734241*a^4*c^7*d^4*e^23))/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24)))*root(2684354560*a^12*c^9*d^36*e^4*z^5 + 32212254720*a^18*c^3*d^12*e^28*z^5 + 32212254720*a^14*c^7*d^28*e^12*z^5 + 2684354560*a^20*c*d^4*e^36*z^5 + 56371445760*a^17*c^4*d^16*e^24*z^5 + 56371445760*a^15*c^6*d^24*e^16*z^5 + 12079595520*a^19*c^2*d^8*e^32*z^5 + 12079595520*a^13*c^8*d^32*e^8*z^5 + 67645734912*a^16*c^5*d^20*e^20*z^5 + 268435456*a^11*c^10*d^4

0*z^5 + 268435456*a^21*e^40*z^5 + 45339770880*a^9*c^6*d^20*e^14*z^3 - 79148
482560*a^13*c^2*d^4*e^30*z^3 + 791941349376*a^12*c^3*d^8*e^26*z^3 + 1239810
048*a^7*c^8*d^28*e^6*z^3 - 1555444924416*a^11*c^4*d^12*e^22*z^3 + 83755008*
a^6*c^9*d^32*e^2*z^3 + 81566760960*a^10*c^5*d^16*e^18*z^3 + 12177506304*a^8
*c^7*d^24*e^10*z^3 + 117964800*a^14*c*e^34*z^3 - 2785204224*a^6*c^6*d^18*e^
13*z^2 + 8128512*a^3*c^9*d^30*e*z^2 + 2700933120*a^10*c^2*d^2*e^29*z^2 - 54
3361222656*a^8*c^4*d^10*e^21*z^2 + 1048135680*a^5*c^7*d^22*e^9*z^2 + 118499
328*a^4*c^8*d^26*e^5*z^2 - 55938263040*a^7*c^5*d^14*e^17*z^2 + 123990497280
*a^9*c^3*d^6*e^25*z^2 + 24139215*a^2*c^7*d^20*e^8*z + 2819286*a*c^8*d^24*e^
4*z + 10462847841*a^6*c^3*d^4*e^24*z - 5777473473*a^4*c^5*d^12*e^16*z - 435
09753450*a^5*c^4*d^8*e^20*z - 548810316*a^3*c^6*d^16*e^12*z + 12960000*a^7*
c^2*e^28*z + 194481*c^9*d^28*z - 977636142*a^2*c^4*d^6*e^19 + 233280000*a^3
*c^3*d^2*e^23 - 140556060*a*c^5*d^10*e^15 - 15169518*c^6*d^14*e^11, z, k),
k, 1, 5) - ((2*a^3*e^15 - 5*c^3*d^12*e^3 - 38*a*c^2*d^8*e^7 + 65*a^2*c*d^4*
e^11)/(4*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)^2) + (3*x^8*(5*a^3*c^2*e^15 -
7*c^5*d^12*e^3 - 49*a*c^4*d^8*e^7 + 91*a^2*c^3*d^4*e^11))/(16*a^2*(a^4*e^16
 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)) - (
x^5*(7*c^5*d^15 + 97*a*c^4*d^11*e^4 + 461*a^2*c^3*d^7*e^8 - 1165*a^3*c^2*d^
3*e^12))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)^2) - (3*x^9*(5*c^5*d^1
1*e^4 + 34*a*c^4*d^7*e^8 - 99*a^2*c^3*d^3*e^12))/(16*a^2*(a^4*e^16 + c^4*d^
16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)) + (x^2*(c^2*
d^6*e + 3*a*c*d^2*e^5))/(4*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x^3*(5
*c^2*d^5*e^2 + 21*a*c*d*e^6))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) -
(x*(11*c^4*d^15 + 79*a*c^3*d^11*e^4 - 567*a^3*c*d^3*e^12 + 269*a^2*c^2*d^7*
e^8))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)^2) + (x^6*(c^3*d^6*e + 5*a*
c^2*d^2*e^5))/(8*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x^4*(25*a^3*c*
e^15 - 39*c^4*d^12*e^3 - 293*a*c^3*d^8*e^7 + 539*a^2*c^2*d^4*e^11))/(16*a*(
a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)^2) - (e^2*x^7*(c^3*d^5 + 17*a*c^2*d*e^4)
)/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)))/(a^2*d^2 + a^2*e^2*x^2 + c^
2*d^2*x^8 + c^2*e^2*x^10 + 2*a^2*d*e*x + 2*a*c*d^2*x^4 + 2*a*c*e^2*x^6 + 2*
c^2*d*e*x^9 + 4*a*c*d*e*x^5)

3.415 $\int \frac{-1+x}{1-x+x^2} dx$

Optimal result 2584
Rubi [A] (verified) 2584
Mathematica [A] (verified) 2585
Maple [A] (verified) 2586
Fricas [A] (verification not implemented) 2586
Sympy [A] (verification not implemented) 2586
Maxima [A] (verification not implemented) 2587
Giac [A] (verification not implemented) 2587
Mupad [B] (verification not implemented) 2587

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int \frac{-1+x}{1-x+x^2} dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(1-x+x^2\right)$$

```
[Out] 1/2*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {648, 632, 210, 642}

$$\int \frac{-1+x}{1-x+x^2} dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(x^2-x+1\right)$$

```
[In] Int[(-1 + x)/(1 - x + x^2),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\right)+\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\
&= \frac{1}{2}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$
\int \frac{-1+x}{1-x+x^2}\,dx=-\frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)
$$

```
[In] Integrate[(-1 + x)/(1 - x + x^2),x]
[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\ln(x^2-x+1)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	29
risch	$\frac{\ln(4x^2-4x+4)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	31

```
[In] int((x-1)/(x^2-x+1),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-1+x}{1-x+x^2}\,dx=-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

```
[In] integrate((-1+x)/(x^2-x+1),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{-1+x}{1-x+x^2}\,dx=\frac{\log\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((-1+x)/(x**2-x+1),x)
[Out] log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-1+x}{1-x+x^2}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{2}\,\log\left(x^2-x+1\right)$$

```
[In] integrate((-1+x)/(x^2-x+1),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-1+x}{1-x+x^2}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{2}\,\log\left(x^2-x+1\right)$$

```
[In] integrate((-1+x)/(x^2-x+1),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{-1+x}{1-x+x^2}\,dx = \frac{\ln\left(x^2-x+1\right)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] int((x - 1)/(x^2 - x + 1),x)
[Out] log(x^2 - x + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

3.416 $\int \frac{-1+x^2}{1+x^3} dx$

Optimal result 2588
Rubi [A] (verified) 2588
Mathematica [A] (verified) 2589
Maple [A] (verified) 2590
Fricas [A] (verification not implemented) 2590
Sympy [A] (verification not implemented) 2590
Maxima [A] (verification not implemented) 2591
Giac [A] (verification not implemented) 2591
Mupad [B] (verification not implemented) 2591

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{-1+x^2}{1+x^3} dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(1-x+x^2\right)$$

```
[Out] 1/2*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1886, 648, 632, 210, 642}

$$\int \frac{-1+x^2}{1+x^3} dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(x^2-x+1\right)$$

```
[In] Int[(-1 + x^2)/(1 + x^3),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1886

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[q^2/a
, Int[(A + C*q*x)/(q^2 - q*x + x^2), x], x]] /; EqQ[A - B*(a/b)^(1/3) + C*(
a/b)^(2/3), 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{-1+x}{1-x+x^2}\,dx\\ &= -\left(\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\right)+\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\ &= \frac{1}{2}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{-1+x^2}{1+x^3}\,dx = -\frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)$$

```
[In] Integrate[(-1 + x^2)/(1 + x^3),x]
[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\ln(x^2-x+1)}{2}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	29
risch	$\frac{\ln(4x^2-4x+4)}{2}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	31
meijerg	$\frac{\ln(x^3+1)}{3}-\frac{x\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{1}{3}}}+\frac{x\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{1}{3}}}-\frac{x\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{1}{3}}}$	82

```
[In] int((x^2-1)/(x^3+1),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^2}{1+x^3}\,dx=-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^2-x+1\right)$$

```
[In] integrate((x^2-1)/(x^3+1),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{-1+x^2}{1+x^3}\,dx=\frac{\log\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((x**2-1)/(x**3+1),x)
[Out] log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^2}{1+x^3}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^2-x+1\right)$$

```
[In] integrate((x^2-1)/(x^3+1),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^2}{1+x^3}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^2-x+1\right)$$

```
[In] integrate((x^2-1)/(x^3+1),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{-1+x^2}{1+x^3}\,dx = \frac{\ln\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] int((x^2 - 1)/(x^3 + 1),x)
[Out] log(x^2 - x + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

3.417 $\int \frac{-4+3x}{4-2x+x^2}\,dx$

Optimal result 2592
Rubi [A] (verified) 2592
Mathematica [A] (verified) 2593
Maple [A] (verified) 2594
Fricas [A] (verification not implemented) 2594
Sympy [A] (verification not implemented) 2594
Maxima [A] (verification not implemented) 2595
Giac [A] (verification not implemented) 2595
Mupad [B] (verification not implemented) 2595

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{-4+3x}{4-2x+x^2}\,dx=\frac{\arctan\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(4-2x+x^2\right)$$

```
[Out] 3/2*ln(x^2-2*x+4)+1/3*arctan(1/3*(1-x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {648, 632, 210, 642}

$$\int \frac{-4+3x}{4-2x+x^2}\,dx=\frac{\arctan\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(x^2-2x+4\right)$$

```
[In] Int[(-4 + 3*x)/(4 - 2*x + x^2),x]
```

```
[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3}{2}\int \frac{-2+2x}{4-2x+x^2}\,dx - \int \frac{1}{4-2x+x^2}\,dx \\
&= \frac{3}{2}\log\left(4-2x+x^2\right) + 2\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, -2+2x\right) \\
&= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$
\int \frac{-4+3x}{4-2x+x^2}\,dx = -\frac{\arctan\left(\frac{-1+x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)
$$

```
[In] Integrate[(-4 + 3*x)/(4 - 2*x + x^2),x]
[Out] -(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{3\ln\left(x^{2}-2x+4\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(x-1)\sqrt{3}}{3}\right)}{3}$	27
default	$\frac{3\ln\left(x^{2}-2x+4\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-2)\sqrt{3}}{6}\right)}{3}$	29

```
[In] int((-4+3*x)/(x^2-2*x+4),x,method=_RETURNVERBOSE)
[Out] 3/2*ln(x^2-2*x+4)-1/3*3^(1/2)*arctan(1/3*(x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-4+3x}{4-2x+x^2}\,dx=-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(x-1)\right)+\frac{3}{2}\,\log\left(x^{2}-2\,x+4\right)$$

```
[In] integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{-4+3x}{4-2x+x^2}\,dx=\frac{3\log\left(x^{2}-2x+4\right)}{2}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((-4+3*x)/(x**2-2*x+4),x)
[Out] 3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*x/3 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-4+3x}{4-2x+x^2}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

```
[In] integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-4+3x}{4-2x+x^2}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

```
[In] integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{-4+3x}{4-2x+x^2}\,dx = \frac{3\,\ln\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] int((3*x - 4)/(x^2 - 2*x + 4),x)
[Out] (3*log(x^2 - 2*x + 4))/2 - (3^(1/2)*atan((3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

3.418 $\int \frac{-8+2x+3x^2}{8+x^3}\,dx$

Optimal result 2596
Rubi [A] (verified) 2596
Mathematica [A] (verified) 2597
Maple [A] (verified) 2598
Fricas [A] (verification not implemented) 2598
Sympy [A] (verification not implemented) 2598
Maxima [A] (verification not implemented) 2599
Giac [A] (verification not implemented) 2599
Mupad [B] (verification not implemented) 2599

Optimal result

Integrand size = 18, antiderivative size = 32

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx=\frac{\arctan\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(4-2x+x^2\right)$$

```
[Out] 3/2*ln(x^2-2*x+4)+1/3*arctan(1/3*(1-x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1886, 648, 632, 210, 642}

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx=\frac{\arctan\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(x^2-2x+4\right)$$

```
[In] Int[(-8 + 2*x + 3*x^2)/(8 + x^3),x]
```

```
[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1886

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[q^2/a
, Int[(A + C*q*x)/(q^2 - q*x + x^2), x], x]] /; EqQ[A - B*(a/b)^(1/3) + C*(
a/b)^(2/3), 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{-8+6x}{4-2x+x^2}\,dx \\
&= \frac{3}{2}\int \frac{-2+2x}{4-2x+x^2}\,dx - \int \frac{1}{4-2x+x^2}\,dx \\
&= \frac{3}{2}\log\left(4-2x+x^2\right) + 2\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, -2+2x\right) \\
&= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx = -\frac{\arctan\left(\frac{-1+x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)$$

```
[In] Integrate[(-8 + 2*x + 3*x^2)/(8 + x^3),x]
[Out] -(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result
risch	$\frac{3\ln\left(x^{2}-2x+4\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(x-1)\sqrt{3}}{3}\right)}{3}$
default	$\frac{3\ln\left(x^{2}-2x+4\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-2)\sqrt{3}}{6}\right)}{3}$
meijerg	$-\frac{2x\ln\left(1+\frac{\left(x^{3}\right)^{\frac{1}{3}}}{2}\right)}{3\left(x^{3}\right)^{\frac{1}{3}}}+\frac{x\ln\left(1-\frac{\left(x^{3}\right)^{\frac{1}{3}}}{2}+\frac{\left(x^{3}\right)^{\frac{2}{3}}}{4}\right)}{3\left(x^{3}\right)^{\frac{1}{3}}}-\frac{2x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{3}\right)^{\frac{1}{3}}}{4-\left(x^{3}\right)^{\frac{1}{3}}}\right)}{3\left(x^{3}\right)^{\frac{1}{3}}}+\ln\left(1+\frac{x^{3}}{8}\right)-\frac{x^{2}\ln\left(1+\frac{\left(x^{3}\right)^{\frac{1}{3}}}{2}\right)}{3\left(x^{3}\right)^{\frac{2}{3}}}+$

```
[In] int((3*x^2+2*x-8)/(x^3+8),x,method=_RETURNVERBOSE)
[Out] 3/2*ln(x^2-2*x+4)-1/3*3^(1/2)*arctan(1/3*(x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx=-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2\,x+4\right)$$

```
[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx=\frac{3\log\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((3*x**2+2*x-8)/(x**3+8),x)
[Out] 3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*x/3 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right) + \frac{3}{2}\log\left(x^2-2x+4\right)$$

```
[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right) + \frac{3}{2}\log\left(x^2-2x+4\right)$$

```
[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx = \frac{3\,\ln\left(x^2-2\,x+4\right)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] int((2*x + 3*x^2 - 8)/(x^3 + 8),x)
[Out] (3*log(x^2 - 2*x + 4))/2 - (3^(1/2)*atan((3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

3.419 $\int \frac{2+x}{-1+2x+x^2} \, dx$

Optimal result 2600
Rubi [A] (verified) 2600
Mathematica [A] (verified) 2601
Maple [A] (verified) 2601
Fricas [A] (verification not implemented) 2602
Sympy [A] (verification not implemented) 2602
Maxima [A] (verification not implemented) 2602
Giac [A] (verification not implemented) 2603
Mupad [B] (verification not implemented) 2603

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{2+x}{-1+2x+x^2} \, dx = \frac{1}{4}\left(2+\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)$$

```
[Out] 1/4*ln(1+x+2^(1/2))*(2-2^(1/2))+1/4*ln(1+x-2^(1/2))*(2+2^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {646, 31}

$$\int \frac{2+x}{-1+2x+x^2} \, dx = \frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

```
[In] Int[(2 + x)/(-1 + 2*x + x^2),x]
```

```
[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] + x
])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
```

```
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{4}\left(-2+\sqrt{2}\right)\int \frac{1}{1+\sqrt{2}+x}\,dx\right)+\frac{1}{4}\left(2+\sqrt{2}\right)\int \frac{1}{1-\sqrt{2}+x}\,dx\\ &= \frac{1}{4}\left(2+\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{2+x}{-1+2x+x^2}\,dx = \frac{1}{4}\left(\left(2+\sqrt{2}\right)\log\left(-1+\sqrt{2}-x\right)-\left(-2+\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)\right)$$

```
[In] Integrate[(2 + x)/(-1 + 2*x + x^2),x]
[Out] ((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqrt[2] + x])
/4
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\ln(x^2+2x-1)}{2}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}$	29
risch	$\frac{\ln\left(1+x-\sqrt{2}\right)}{2}+\frac{\ln\left(1+x-\sqrt{2}\right)\sqrt{2}}{4}+\frac{\ln\left(1+x+\sqrt{2}\right)}{2}-\frac{\ln\left(1+x+\sqrt{2}\right)\sqrt{2}}{4}$	48

```
[In] int((x+2)/(x^2+2*x-1),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{-1+2x+x^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}(x+1)+2\,x+3}{x^2+2\,x-1}\right)+\frac{1}{2}\log\left(x^2+2\,x-1\right)$$

```
[In] integrate((2+x)/(x^2+2*x-1),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) + 1/2*
log(x^2 + 2*x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{2+x}{-1+2x+x^2}\,dx = \left(\frac{1}{2}-\frac{\sqrt{2}}{4}\right)\log\left(x+1+\sqrt{2}\right)+\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log\left(x-\sqrt{2}+1\right)$$

```
[In] integrate((2+x)/(x**2+2*x-1),x)
[Out] (1/2 - sqrt(2)/4)*log(x + 1 + sqrt(2)) + (sqrt(2)/4 + 1/2)*log(x - sqrt(2)
+ 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{2+x}{-1+2x+x^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{2}\log\left(x^2+2\,x-1\right)$$

```
[In] integrate((2+x)/(x^2+2*x-1),x, algorithm="maxima")
[Out] 1/4*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/2*log(x^2 + 2*x -
1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{2+x}{-1+2x+x^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{|2\,x-2\,\sqrt{2}+2|}{|2\,x+2\,\sqrt{2}+2|}\right)+\frac{1}{2}\,\log\left(|x^2+2\,x-1|\right)$$

```
[In] integrate((2+x)/(x^2+2*x-1),x, algorithm="giac")
[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/2*lo
g(abs(x^2 + 2*x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \frac{2+x}{-1+2x+x^2}\,dx = \ln\left(x-\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)-\ln\left(x+\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}-\frac{1}{2}\right)$$

```
[In] int((x + 2)/(2*x + x^2 - 1),x)
[Out] log(x - 2^(1/2) + 1)*(2^(1/2)/4 + 1/2) - log(x + 2^(1/2) + 1)*(2^(1/2)/4 -
1/2)
```

3.420 $\int \frac{-4+x^2}{2-5x+x^3}\,dx$

Optimal result 2604
Rubi [A] (verified) 2604
Mathematica [A] (verified) 2605
Maple [A] (verified) 2605
Fricas [A] (verification not implemented) 2606
Sympy [A] (verification not implemented) 2606
Maxima [A] (verification not implemented) 2606
Giac [A] (verification not implemented) 2607
Mupad [B] (verification not implemented) 2607

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx=\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)$$

```
[Out] 1/4*ln(1+x+2^(1/2))*(2-2^(1/2))+1/4*ln(1+x-2^(1/2))*(2+2^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2080, 646, 31}

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx=\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

```
[In] Int[(-4 + x^2)/(2 - 5*x + x^3),x]
```

```
[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] + x
])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
```

```
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2080

```
Int[(u_.)*(P_)*(Q_)^(q_), x_Symbol] :> Module[{gcd = PolyGCD[P, Q, x]}, Int
[u*gcd^(q + 1)*PolynomialQuotient[P, gcd, x]*PolynomialQuotient[Q, gcd, x]^
q, x] /; NeQ[gcd, 1]] /; ILtQ[q, 0] && PolyQ[P, x] && PolyQ[Q, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{2+x}{-1+2x+x^2}\,dx\\ &= -\left(\frac{1}{4}\left(-2+\sqrt{2}\right)\int \frac{1}{1+\sqrt{2}+x}\,dx\right)+\frac{1}{4}\left(2+\sqrt{2}\right)\int \frac{1}{1-\sqrt{2}+x}\,dx\\ &= \frac{1}{4}\left(2+\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx = \frac{1}{4}\left(\left(2+\sqrt{2}\right)\log\left(-1+\sqrt{2}-x\right)-\left(-2+\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)\right)$$

```
[In] Integrate[(-4 + x^2)/(2 - 5*x + x^3),x]
[Out] ((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqrt[2] + x])
/4
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\ln(x^2+2x-1)}{2}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}$	29
risch	$\frac{\ln\left(1+x-\sqrt{2}\right)}{2}+\frac{\ln\left(1+x-\sqrt{2}\right)\sqrt{2}}{4}+\frac{\ln\left(1+x+\sqrt{2}\right)}{2}-\frac{\ln\left(1+x+\sqrt{2}\right)\sqrt{2}}{4}$	48

```
[In] int((x^2-4)/(x^3-5*x+2),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx = \frac{1}{4}\,\sqrt{2}\log\left(\frac{x^2-2\,\sqrt{2}(x+1)+2\,x+3}{x^2+2\,x-1}\right)+\frac{1}{2}\,\log\left(x^2+2\,x-1\right)$$

```
[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) + 1/2*
log(x^2 + 2*x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx = \left(\frac{1}{2}-\frac{\sqrt{2}}{4}\right)\log\left(x+1+\sqrt{2}\right)+\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log\left(x-\sqrt{2}+1\right)$$

```
[In] integrate((x**2-4)/(x**3-5*x+2),x)
[Out] (1/2 - sqrt(2)/4)*log(x + 1 + sqrt(2)) + (sqrt(2)/4 + 1/2)*log(x - sqrt(2)
+ 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx = \frac{1}{4}\,\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{2}\,\log\left(x^2+2\,x-1\right)$$

```
[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="maxima")
[Out] 1/4*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/2*log(x^2 + 2*x -
1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{|2x-2\sqrt{2}+2|}{|2x+2\sqrt{2}+2|}\right)+\frac{1}{2}\log\left(|x^2+2x-1|\right)$$

```
[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/2*lo
g(abs(x^2 + 2*x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx = \ln\left(x-\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)-\ln\left(x+\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}-\frac{1}{2}\right)$$

```
[In] int((x^2 - 4)/(x^3 - 5*x + 2),x)
```

```
[Out] log(x - 2^(1/2) + 1)*(2^(1/2)/4 + 1/2) - log(x + 2^(1/2) + 1)*(2^(1/2)/4 -
1/2)
```

3.421 $\int \frac{2}{-1+4x^2}\,dx$

Optimal result 2608
Rubi [A] (verified) 2608
Mathematica [B] (verified) 2609
Maple [A] (verified) 2609
Fricas [B] (verification not implemented) 2610
Sympy [B] (verification not implemented) 2610
Maxima [B] (verification not implemented) 2610
Giac [B] (verification not implemented) 2611
Mupad [B] (verification not implemented) 2611

Optimal result

Integrand size = 11, antiderivative size = 6

$$\int \frac{2}{-1+4x^2}\,dx = -\text{arctanh}(2x)$$

```
[Out] -arctanh(2*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 213}

$$\int \frac{2}{-1+4x^2}\,dx = -\text{arctanh}(2x)$$

```
[In] Int[2/(-1 + 4*x^2),x]
```

```
[Out] -ArcTanh[2*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int \frac{1}{-1+4x^2}\,dx\\ &= -\tanh^{-1}(2x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(6) = 12.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 3.83

$$\int \frac{2}{-1+4x^2}\,dx = 2\left(\frac{1}{4}\log(1-2x)-\frac{1}{4}\log(1+2x)\right)$$

```
[In] Integrate[2/(-1 + 4*x^2),x]
[Out] 2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
meijerg	$-\operatorname{arctanh}(2x)$	7
parallelrisch	$\frac{\ln\left(x-\frac{1}{2}\right)}{2}-\frac{\ln\left(x+\frac{1}{2}\right)}{2}$	14
default	$-\frac{\ln(1+2x)}{2}+\frac{\ln(2x-1)}{2}$	18
norman	$-\frac{\ln(1+2x)}{2}+\frac{\ln(2x-1)}{2}$	18
risch	$-\frac{\ln(1+2x)}{2}+\frac{\ln(2x-1)}{2}$	18

```
[In] int(2/(4*x^2-1),x,method=_RETURNVERBOSE)
[Out] -arctanh(2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \frac{2}{-1+4x^2}\,dx = -\frac{1}{2}\,\log\left(2\,x+1\right) + \frac{1}{2}\,\log\left(2\,x-1\right)$$

```
[In] integrate(2/(4*x^2-1),x, algorithm="fricas")
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{2}{-1+4x^2}\,dx = \frac{\log\left(x-\frac{1}{2}\right)}{2} - \frac{\log\left(x+\frac{1}{2}\right)}{2}$$

```
[In] integrate(2/(4*x**2-1),x)
[Out] log(x - 1/2)/2 - log(x + 1/2)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \frac{2}{-1+4x^2}\,dx = -\frac{1}{2}\,\log\left(2\,x+1\right) + \frac{1}{2}\,\log\left(2\,x-1\right)$$

```
[In] integrate(2/(4*x^2-1),x, algorithm="maxima")
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(6) = 12$.

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{2}{-1+4x^2}\,dx = -\frac{1}{2}\log\left(\left|x+\frac{1}{2}\right|\right) + \frac{1}{2}\log\left(\left|x-\frac{1}{2}\right|\right)$$

```
[In] integrate(2/(4*x^2-1),x, algorithm="giac")
[Out] -1/2*log(abs(x + 1/2)) + 1/2*log(abs(x - 1/2))
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{2}{-1+4x^2}\,dx = -\mathrm{atanh}(2\,x)$$

```
[In] int(2/(4*x^2 - 1),x)
[Out] -atanh(2*x)
```

3.422 $\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right) dx$

Optimal result 2612
Rubi [A] (verified) 2612
Mathematica [A] (verified) 2613
Maple [A] (verified) 2613
Fricas [A] (verification not implemented) 2613
Sympy [A] (verification not implemented) 2614
Maxima [A] (verification not implemented) 2614
Giac [A] (verification not implemented) 2614
Mupad [B] (verification not implemented) 2614

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right) dx=\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(1+2x)$$

```
[Out] 1/2*ln(1-2*x)-1/2*ln(1+2*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right) dx=\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(2x+1)$$

```
[In] Int[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1),x]
```

```
[Out] Log[1 - 2*x]/2 - Log[1 + 2*x]/2
```

Rubi steps

$$\text{integral}=\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(1+2x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx=2\left(\frac{1}{4}\log(1-2x)-\frac{1}{4}\log(1+2x)\right)$$

```
[In] Integrate[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1),x]
[Out] 2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{\ln\left(x-\frac{1}{2}\right)}{2}-\frac{\ln\left(x+\frac{1}{2}\right)}{2}$	14
default	$-\frac{\ln(1+2x)}{2}+\frac{\ln(2x-1)}{2}$	18
norman	$-\frac{\ln(1+2x)}{2}+\frac{\ln(2x-1)}{2}$	18
meijerg	$\frac{\ln(1-2x)}{2}-\frac{\ln(1+2x)}{2}$	18
risch	$-\frac{\ln(1+2x)}{2}+\frac{\ln(2x-1)}{2}$	18

```
[In] int(1/(2*x-1)-1/(1+2*x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x-1/2)-1/2*ln(x+1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx=-\frac{1}{2}\,\log\left(2\,x+1\right)+\frac{1}{2}\,\log\left(2\,x-1\right)$$

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="fricas")
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx=\frac{\log\left(x-\frac{1}{2}\right)}{2}-\frac{\log\left(x+\frac{1}{2}\right)}{2}$$

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x)
[Out] log(x - 1/2)/2 - log(x + 1/2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx=-\frac{1}{2}\,\log\left(2\,x+1\right)+\frac{1}{2}\,\log\left(2\,x-1\right)$$

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="maxima")
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx=-\frac{1}{2}\,\log\left(|2\,x+1|\right)+\frac{1}{2}\,\log\left(|2\,x-1|\right)$$

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="giac")
[Out] -1/2*log(abs(2*x + 1)) + 1/2*log(abs(2*x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.29

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx=-\mathrm{atanh}(2\,x)$$

```
[In] int(1/(2*x - 1) - 1/(2*x + 1),x)
[Out] -atanh(2*x)
```

3.423 $\int \frac{x}{(1-x^2)^5}\,dx$

Optimal result 2615
Rubi [A] (verified) 2615
Mathematica [A] (verified) 2616
Maple [A] (verified) 2616
Fricas [B] (verification not implemented) 2616
Sympy [B] (verification not implemented) 2617
Maxima [A] (verification not implemented) 2617
Giac [A] (verification not implemented) 2617
Mupad [B] (verification not implemented) 2618

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{x}{(1-x^2)^5}\,dx = \frac{1}{8(1-x^2)^4}$$

```
[Out] 1/8/(-x^2+1)^4
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {267}

$$\int \frac{x}{(1-x^2)^5}\,dx = \frac{1}{8(1-x^2)^4}$$

```
[In] Int[x/(1 - x^2)^5,x]
```

```
[Out] 1/(8*(1 - x^2)^4)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{8(1-x^2)^4}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{(1-x^2)^5}\,dx = \frac{1}{8\,(-1+x^2)^4}$$

```
[In] Integrate[x/(1 - x^2)^5,x]
[Out] 1/(8*(-1 + x^2)^4)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{1}{8(x^2-1)^4}$	10
default	$\frac{1}{8(x^2-1)^4}$	10
norman	$\frac{1}{8(x^2-1)^4}$	10
risch	$\frac{1}{8(x^2-1)^4}$	10
parallelrisch	$\frac{1}{8(x^2-1)^4}$	10
derivativedivides	$\frac{1}{8(-x^2+1)^4}$	12
meijerg	$\frac{x^2(-x^6+4x^4-6x^2+4)}{8(-x^2+1)^4}$	32

```
[In] int(x/(-x^2+1)^5,x,method=_RETURNVERBOSE)
[Out] 1/8/(x^2-1)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(9) = 18$.

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \frac{x}{(1-x^2)^5}\,dx = \frac{1}{8\,(x^8-4\,x^6+6\,x^4-4\,x^2+1)}$$

```
[In] integrate(x/(-x^2+1)^5,x, algorithm="fricas")
[Out] 1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{x}{(1-x^2)^5}\, dx = \frac{1}{8x^8 - 32x^6 + 48x^4 - 32x^2 + 8}$$

```
[In] integrate(x/(-x**2+1)**5,x)
[Out] 1/(8*x**8 - 32*x**6 + 48*x**4 - 32*x**2 + 8)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x}{(1-x^2)^5}\, dx = \frac{1}{8\,(x^2-1)^4}$$

```
[In] integrate(x/(-x^2+1)^5,x, algorithm="maxima")
[Out] 1/8/(x^2 - 1)^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x}{(1-x^2)^5}\, dx = \frac{1}{8\,(x^2-1)^4}$$

```
[In] integrate(x/(-x^2+1)^5,x, algorithm="giac")
[Out] 1/8/(x^2 - 1)^4
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x}{(1-x^2)^5} \, dx = \frac{1}{8 (x^2-1)^4}$$

```
[In] int(-x/(x^2 - 1)^5,x)
[Out] 1/(8*(x^2 - 1)^4)
```

3.424 $\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\right.$

Optimal result 2619
Rubi [B] (verified) 2619
Mathematica [A] (verified) 2620
Maple [A] (verified) 2620
Fricas [B] (verification not implemented) 2621
Sympy [B] (verification not implemented) 2621
Maxima [B] (verification not implemented) 2621
Giac [B] (verification not implemented) 2622
Mupad [B] (verification not implemented) 2622

Optimal result

Integrand size = 73, antiderivative size = 13

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right)dx=\frac{1}{8(1-x^2)^4}$$

```
[Out] 1/8/(-x^2+1)^4
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 81 vs. 2(13) = 26.

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 6.23, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right)dx=\frac{5}{256(x+1)}+\frac{5}{256(x+1)^2}+\frac{1}{64(x+1)^3}+\frac{1}{128(x+1)^4}+\frac{5}{256(1-x)}+\frac{5}{256(1-x)^2}+\frac{1}{64(1-x)^3}+\frac{1}{128(1-x)^4}$$

```
[In] Int[-1/32*1/(-1 + x)^5 + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(256*(-
1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) - 5/(25
6*(1 + x)^2),x]
```

```
[Out] 1/(128*(1 - x)^4) + 1/(64*(1 - x)^3) + 5/(256*(1 - x)^2) + 5/(256*(1 - x))
+ 1/(128*(1 + x)^4) + 1/(64*(1 + x)^3) + 5/(256*(1 + x)^2) + 5/(256*(1 + x)
)
```

Rubi steps

$$\text{integral} = \frac{1}{128(1-x)^4} + \frac{1}{64(1-x)^3} + \frac{5}{256(1-x)^2} + \frac{5}{256(1-x)} + \frac{1}{128(1+x)^4} + \frac{1}{64(1+x)^3} + \frac{5}{256(1+x)^2} + \frac{5}{256(1+x)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \left(-\frac{1}{32(-1+x)^5} + \frac{3}{64(-1+x)^4} - \frac{5}{128(-1+x)^3} + \frac{5}{256(-1+x)^2} - \frac{1}{32(1+x)^5} - \frac{3}{64(1+x)^4} - \frac{5}{128(1+x)^3} - \frac{5}{256(1+x)^2}\right) dx = \frac{1}{8\left(-1+x^2\right)^4}$$

```
[In] Integrate[-1/32*1/(-1 + x)^5 + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(
256*(-1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) -
 5/(256*(1 + x)^2),x]
```

```
[Out] 1/(8*(-1 + x^2)^4)
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result
gosper	$\frac{1}{8(x+1)^4(x-1)^4}$
norman	$\frac{1}{8(x+1)^4(x-1)^4}$
risch	$\frac{1}{8(x+1)^4(x-1)^4}$
parallelrisch	$\frac{1}{8(x+1)^4(x-1)^4}$
default	$\frac{1}{128(x-1)^4} - \frac{1}{64(x-1)^3} + \frac{5}{256(x-1)^2} - \frac{5}{256(x-1)} + \frac{1}{128(x+1)^4} + \frac{1}{64(x+1)^3} + \frac{5}{256(x+1)^2} + \frac{5}{256(x+1)}$
meijerg	$\frac{x(-x^3+4x^2-6x+4)}{128(1-x)^4} + \frac{x(x^2-3x+3)}{64(1-x)^3} + \frac{5x(2-x)}{256(1-x)^2} + \frac{5x}{256(1-x)} - \frac{x(x^3+4x^2+6x+4)}{128(x+1)^4} - \frac{x(x^2+3x+3)}{64(x+1)^3} - \frac{5x(x+2)}{256(x+1)^2}$

```
[In] int(-1/32/(x-1)^5+3/64/(x-1)^4-5/128/(x-1)^3+5/256/(x-1)^2-1/32/(x+1)^5-3/6
4/(x+1)^4-5/128/(x+1)^3-5/256/(x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/(x+1)^4/(x-1)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(9) = 18$.

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int\left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right)dx=\frac{1}{8\,(x^8-4\,x^6+6\,x^4-4\,x^2+1)}$$

```
[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(
1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int\left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right)dx=\frac{1}{8x^8-32x^6+48x^4-32x^2+8}$$

```
[In] integrate(-1/32/(-1+x)**5+3/64/(-1+x)**4-5/128/(-1+x)**3+5/256/(-1+x)**2-1/
32/(1+x)**5-3/64/(1+x)**4-5/128/(1+x)**3-5/256/(1+x)**2,x)
```

```
[Out] 1/(8*x**8 - 32*x**6 + 48*x**4 - 32*x**2 + 8)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(9) = 18$.

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.38

$$\int\left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right)dx=\frac{5}{256\,(x+1)}-\frac{5}{256\,(x-1)}+\frac{5}{256\,(x+1)^2}+\frac{5}{256\,(x-1)^2}+\frac{1}{64\,(x+1)^3}-\frac{1}{64\,(x-1)^3}+\frac{1}{128\,(x+1)^4}+\frac{1}{128\,(x-1)^4}$$

```
[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(
1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2 + 1/64/(x
 + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x - 1)^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(9) = 18$.

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.38

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right) dx = \frac{5}{256\,(x+1)}-\frac{5}{256\,(x-1)}+\frac{5}{256\,(x+1)^2}+\frac{5}{256\,(x-1)^2}+\frac{1}{64\,(x+1)^3}-\frac{1}{64\,(x-1)^3}+\frac{1}{128\,(x+1)^4}+\frac{1}{128\,(x-1)^4}$$

```
[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(
1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="giac")
```

```
[Out] 5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2 + 1/64/(x
 + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x - 1)^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right) dx = \frac{1}{8\,(x^2-1)^4}$$

```
[In] int(5/(256*(x - 1)^2) - 5/(256*(x + 1)^2) - 5/(128*(x - 1)^3) - 5/(128*(x +
 1)^3) + 3/(64*(x - 1)^4) - 3/(64*(x + 1)^4) - 1/(32*(x - 1)^5) - 1/(32*(x
+ 1)^5),x)
```

```
[Out] 1/(8*(x^2 - 1)^4)
```

3.425 $\int \frac{1+x^6}{-1+x^6}\,dx$

Optimal result 2623
Rubi [A] (verified) 2623
Mathematica [A] (verified) 2625
Maple [A] (verified) 2626
Fricas [A] (verification not implemented) 2626
Sympy [A] (verification not implemented) 2627
Maxima [A] (verification not implemented) 2627
Giac [A] (verification not implemented) 2627
Mupad [B] (verification not implemented) 2628

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{1+x^6}{-1+x^6}\,dx=x+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2\text{arctanh}(x)}{3}+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)$$

```
[Out] x-2/3*arctanh(x)+1/6*ln(x^2-x+1)-1/6*ln(x^2+x+1)+1/3*arctan(1/3*(1-2*x)*3^(
1/2))*3^(1/2)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {396, 216, 648, 632, 210, 642, 212}

$$\int \frac{1+x^6}{-1+x^6}\,dx=\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2\text{arctanh}(x)}{3}+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x^2+x+1\right)+x$$

```
[In] Int[(1 + x^6)/(-1 + x^6),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] -
 (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^2]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x+2\int \frac{1}{-1+x^6}\,dx\\
&= x-\frac{2}{3}\int \frac{1}{1-x^2}\,dx-\frac{2}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx\\
&= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx\\
&\quad -\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx\\
&= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)\\
&\quad +\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= x-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1\right.\\
&\quad \left.+x+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$
\int \frac{1+x^6}{-1+x^6}\,dx=\frac{1}{6}\left(6x-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+2\log(1-x)\right.
\\
\left.-2\log(1+x)+\log\left(1-x+x^2\right)-\log\left(1+x+x^2\right)\right)
$$

```
[In] Integrate[(1 + x^6)/(-1 + x^6),x]
```

```
[Out] (6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sq
rt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])
/6
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

method	result
default	$x-\frac{\ln(x^2+x+1)}{6}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(x-1)}{3}-\frac{\ln(x+1)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$
risch	$x-\frac{\ln(4x^2+4x+4)}{6}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(4x^2-4x+4)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{\ln(x+1)}{3}+\frac{\ln(x-1)}{3}$
meijerg	$\frac{x\left(\ln\left(1-(x^6)^{\frac{1}{6}}\right)-\ln\left(1+(x^6)^{\frac{1}{6}}\right)+\frac{\ln\left(1-(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{2-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{2+(x^6)^{\frac{1}{6}}}\right.\right.}{6(x^6)^{\frac{1}{6}}}$

```
[In] int((x^6+1)/(x^6-1),x,method=_RETURNVERBOSE)
```

```
[Out] x-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/3*ln(x-1)-1/3*l
n(x+1)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1+x^6}{-1+x^6}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+x-\frac{1}{6}\,\log\left(x^2+x+1\right)+\frac{1}{6}\,\log\left(x^2-x+1\right)-\frac{1}{3}\,\log\left(x+1\right)+\frac{1}{3}\,\log\left(x-1\right)$$

```
[In] integrate((x^6+1)/(x^6-1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23

$$\int \frac{1+x^6}{-1+x^6}\,dx = x + \frac{\log(x-1)}{3} - \frac{\log(x+1)}{3} + \frac{\log(x^2-x+1)}{6} - \frac{\log(x^2+x+1)}{6} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((x**6+1)/(x**6-1),x)
```

```
[Out] x + log(x - 1)/3 - log(x + 1)/3 + log(x**2 - x + 1)/6 - log(x**2 + x + 1)/6
 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - sqrt(3)*atan(2*sqrt(3)*x/3 +
 sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1+x^6}{-1+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log(x+1) + \frac{1}{3}\log(x-1)$$

```
[In] integrate((x^6+1)/(x^6-1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{1+x^6}{-1+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log(|x+1|) + \frac{1}{3}\log(|x-1|)$$

```
[In] integrate((x^6+1)/(x^6-1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(abs
(x + 1)) + 1/3*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.36

$$\int \frac{1+x^6}{-1+x^6}\,dx = x+\frac{\operatorname{atan}(x\,1\mathrm{i})\;2\mathrm{i}}{3}-\operatorname{atan}\left(\frac{x\,32\mathrm{i}}{-32+\sqrt{3}\,32\mathrm{i}}-\frac{32\,\sqrt{3}\,x}{-32+\sqrt{3}\,32\mathrm{i}}\right)\,\left(\frac{\sqrt{3}}{3}-\frac{1}{3}\mathrm{i}\right)-\operatorname{atan}\left(\frac{x\,32\mathrm{i}}{32+\sqrt{3}\,32\mathrm{i}}+\frac{32\,\sqrt{3}\,x}{32+\sqrt{3}\,32\mathrm{i}}\right)\,\left(\frac{\sqrt{3}}{3}+\frac{1}{3}\mathrm{i}\right)$$

```
[In] int((x^6 + 1)/(x^6 - 1),x)
```

```
[Out] x + (atan(x*1i)*2i)/3 - atan((x*32i)/(3^(1/2)*32i - 32) - (32*3^(1/2)*x)/(3
^(1/2)*32i - 32))*(3^(1/2)/3 - 1i/3) - atan((x*32i)/(3^(1/2)*32i + 32) + (3
2*3^(1/2)*x)/(3^(1/2)*32i + 32))*(3^(1/2)/3 + 1i/3)
```

3.426 $\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3} \, dx$

Optimal result 2629
Rubi [A] (verified) 2629
Mathematica [A] (verified) 2632
Maple [A] (verified) 2632
Fricas [A] (verification not implemented) 2632
Sympy [A] (verification not implemented) 2633
Maxima [A] (verification not implemented) 2633
Giac [A] (verification not implemented) 2633
Mupad [B] (verification not implemented) 2634

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3} \, dx = x+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2\text{arctanh}(x)}{3}+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)$$

```
[Out] x-2/3*arctanh(x)+1/6*ln(x^2-x+1)-1/6*ln(x^2+x+1)+1/3*arctan(1/3*(1-2*x)*3^(
1/2))*3^(1/2)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1607, 1598, 396, 216, 648, 632, 210, 642, 212}

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3} \, dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2\text{arctanh}(x)}{3}+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x^2+x+1\right)+x$$

```
[In] Int[(x^(-3) + x^3)/(-x^(-3) + x^3),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] -
 (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^2]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3\left(\frac{1}{x^3}+x^3\right)}{-1+x^6}\,dx \\
&= \int \frac{1+x^6}{-1+x^6}\,dx \\
&= x+2\int \frac{1}{-1+x^6}\,dx \\
&= x-\frac{2}{3}\int \frac{1}{1-x^2}\,dx-\frac{2}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx \\
&= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx \\
&\quad -\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx \\
&= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right) \\
&\quad +\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right) \\
&= x-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1\right. \\
&\quad \left.+x+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = \frac{1}{6}\left(6x-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+2\log(1-x)-2\log(1+x)+\log\left(1-x+x^2\right)-\log\left(1+x+x^2\right)\right)$$

```
[In] Integrate[(x^(-3) + x^3)/(-x^(-3) + x^3),x]
[Out] (6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sq
rt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])
/6
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

method	result	size
risch	$x+\frac{\ln(x-1)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}-\frac{\ln(x^2+x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}-\frac{\ln(x+1)}{3}$	63
default	$x-\frac{\ln(x^2+x+1)}{6}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(x-1)}{3}-\frac{\ln(x+1)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	67

```
[In] int((1/x^3+x^3)/(-1/x^3+x^3),x,method=_RETURNVERBOSE)
[Out] x+1/3*ln(x-1)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))-1/6*l
n(x^2+x+1)-1/3*3^(1/2)*arctan(2/3*(x+1/2)*3^(1/2))-1/3*ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

```
[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = x+\frac{\log(x-1)}{3}-\frac{\log(x+1)}{3}+\frac{\log(x^2-x+1)}{6}-\frac{\log(x^2+x+1)}{6} -\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((1/x**3+x**3)/(-1/x**3+x**3),x)
[Out] x + log(x - 1)/3 - log(x + 1)/3 + log(x**2 - x + 1)/6 - log(x**2 + x + 1)/6
 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - sqrt(3)*atan(2*sqrt(3)*x/3 +
 sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x -\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

```
[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x -\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(|x+1|\right)+\frac{1}{3}\log\left(|x-1|\right)$$

```
[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(abs
(x + 1)) + 1/3*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.36

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = x+\frac{\mathrm{atan}(x\,1\mathrm{i})\ 2\mathrm{i}}{3}-\mathrm{atan}\left(\frac{x\,32\mathrm{i}}{-32+\sqrt{3}\,32\mathrm{i}}-\frac{32\,\sqrt{3}\,x}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}-\frac{1}{3}\mathrm{i}\right) - \mathrm{atan}\left(\frac{x\,32\mathrm{i}}{32+\sqrt{3}\,32\mathrm{i}}+\frac{32\,\sqrt{3}\,x}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}+\frac{1}{3}\mathrm{i}\right)$$

```
[In] int(-(1/x^3 + x^3)/(1/x^3 - x^3),x)
```

```
[Out] x + (atan(x*1i)*2i)/3 - atan((x*32i)/(3^(1/2)*32i - 32) - (32*3^(1/2)*x)/(3
^(1/2)*32i - 32))*(3^(1/2)/3 - 1i/3) - atan((x*32i)/(3^(1/2)*32i + 32) + (3
2*3^(1/2)*x)/(3^(1/2)*32i + 32))*(3^(1/2)/3 + 1i/3)
```

3.427 $\int \frac{-x+x^3}{6+2x} \, dx$

Optimal result 2635
Rubi [A] (verified) 2635
Mathematica [A] (verified) 2636
Maple [A] (verified) 2636
Fricas [A] (verification not implemented) 2637
Sympy [A] (verification not implemented) 2637
Maxima [A] (verification not implemented) 2637
Giac [A] (verification not implemented) 2637
Mupad [B] (verification not implemented) 2638

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{-x+x^3}{6+2x} \, dx = 4x - \frac{3x^2}{4} + \frac{x^3}{6} - 12\log(3+x)$$

```
[Out] 4*x-3/4*x^2+1/6*x^3-12*ln(3+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1607, 786}

$$\int \frac{-x+x^3}{6+2x} \, dx = \frac{x^3}{6} - \frac{3x^2}{4} + 4x - 12\log(x+3)$$

```
[In] Int[(-x + x^3)/(6 + 2*x),x]
```

```
[Out] 4*x - (3*x^2)/4 + x^3/6 - 12*Log[3 + x]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(-1+x^2)}{6+2x}\,dx\\ &= \int \left(4-\frac{3x}{2}+\frac{x^2}{2}-\frac{12}{3+x}\right)dx\\ &= 4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\log(3+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{-x+x^3}{6+2x}\,dx = \frac{1}{2}\left(\frac{93}{2}+8x-\frac{3x^2}{2}+\frac{x^3}{3}-24\log(3+x)\right)$$

```
[In] Integrate[(-x + x^3)/(6 + 2*x),x]
[Out] (93/2 + 8*x - (3*x^2)/2 + x^3/3 - 24*Log[3 + x])/2
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\ln\left(3+x\right)$	21
risch	$4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\ln\left(3+x\right)$	21
parallelrisch	$4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\ln\left(3+x\right)$	21
norman	$4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\ln\left(6+2x\right)$	23
meijerg	$\frac{3x\left(\frac{4}{9}x^2-2x+12\right)}{8}-12\ln\left(1+\frac{x}{3}\right)-\frac{x}{2}$	26

```
[In] int((x^3-x)/(6+2*x),x,method=_RETURNVERBOSE)
[Out] 4*x-3/4*x^2+1/6*x^3-12*ln(3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-x+x^3}{6+2x} \, dx = \frac{1}{6} \, x^3 - \frac{3}{4} \, x^2 + 4 \, x - 12 \, \log\left(x + 3\right)$$

```
[In] integrate((x^3-x)/(6+2*x),x, algorithm="fricas")
[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-x+x^3}{6+2x} \, dx = \frac{x^3}{6} - \frac{3x^2}{4} + 4x - 12 \log\left(x + 3\right)$$

```
[In] integrate((x**3-x)/(6+2*x),x)
[Out] x**3/6 - 3*x**2/4 + 4*x - 12*log(x + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-x+x^3}{6+2x} \, dx = \frac{1}{6} \, x^3 - \frac{3}{4} \, x^2 + 4 \, x - 12 \, \log\left(x + 3\right)$$

```
[In] integrate((x^3-x)/(6+2*x),x, algorithm="maxima")
[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{-x+x^3}{6+2x} \, dx = \frac{1}{6} \, x^3 - \frac{3}{4} \, x^2 + 4 \, x - 12 \, \log\left({\left| x + 3 \right|}\right)$$

```
[In] integrate((x^3-x)/(6+2*x),x, algorithm="giac")
[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(abs(x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-x+x^3}{6+2x}\,dx = 4\,x - 12\,\ln\left(x+3\right) - \frac{3\,x^2}{4} + \frac{x^3}{6}$$

```
[In] int(-(x - x^3)/(2*x + 6),x)
[Out] 4*x - 12*log(x + 3) - (3*x^2)/4 + x^3/6
```

3.428 $\int \frac{x+x^3}{-1+x} dx$

Optimal result 2639
Rubi [A] (verified) 2639
Mathematica [A] (verified) 2640
Maple [A] (verified) 2640
Fricas [A] (verification not implemented) 2641
Sympy [A] (verification not implemented) 2641
Maxima [A] (verification not implemented) 2641
Giac [A] (verification not implemented) 2641
Mupad [B] (verification not implemented) 2642

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{x+x^3}{-1+x} dx = 2x + \frac{x^2}{2} + \frac{x^3}{3} + 2\log(1-x)$$

```
[Out] 2*x+1/2*x^2+1/3*x^3+2*ln(1-x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1607, 786}

$$\int \frac{x+x^3}{-1+x} dx = \frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\log(1-x)$$

```
[In] Int[(x + x^3)/(-1 + x),x]
```

```
[Out] 2*x + x^2/2 + x^3/3 + 2*Log[1 - x]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(1+x^2)}{-1+x}\,dx\\ &= \int \left(2+\frac{2}{-1+x}+x+x^2\right)dx\\ &= 2x+\frac{x^2}{2}+\frac{x^3}{3}+2\log(1-x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{x+x^3}{-1+x}\,dx=\frac{1}{6}\left(-17+12x+3x^2+2x^3+12\log(-1+x)\right)$$

```
[In] Integrate[(x + x^3)/(-1 + x),x]
[Out] (-17 + 12*x + 3*x^2 + 2*x^3 + 12*Log[-1 + x])/6
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln\left(x-1\right)$	21
norman	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln\left(x-1\right)$	21
risch	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln\left(x-1\right)$	21
parallelrisch	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln\left(x-1\right)$	21
meijerg	$\frac{x\left(4x^2+6x+12\right)}{12}+2\ln\left(1-x\right)+x$	24

```
[In] int((x^3+x)/(x-1),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3+1/2*x^2+2*x+2*ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x+x^3}{-1+x}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 + 2\,x + 2\,\log\left(x-1\right)$$

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{x+x^3}{-1+x}\,dx = \frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\log\left(x-1\right)$$

```
[In] integrate((x**3+x)/(-1+x),x)
[Out] x**3/3 + x**2/2 + 2*x + 2*log(x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x+x^3}{-1+x}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 + 2\,x + 2\,\log\left(x-1\right)$$

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="maxima")
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{x+x^3}{-1+x}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 + 2\,x + 2\,\log\left(|x-1|\right)$$

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="giac")
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x+x^3}{-1+x}\,dx = 2\,x + 2\,\ln\left(x-1\right) + \frac{x^2}{2} + \frac{x^3}{3}$$

```
[In] int((x + x^3)/(x - 1),x)
[Out] 2*x + 2*log(x - 1) + x^2/2 + x^3/3
```

3.429 $\int (ac + (bc + d)x)\, dx$

Optimal result . . . 2643
Rubi [A] (verified) . . . 2643
Mathematica [A] (verified) . . . 2644
Maple [A] (verified) . . . 2644
Fricas [A] (verification not implemented) . . . 2644
Sympy [A] (verification not implemented) . . . 2645
Maxima [A] (verification not implemented) . . . 2645
Giac [A] (verification not implemented) . . . 2645
Mupad [B] (verification not implemented) . . . 2645

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int (ac + (bc + d)x)\, dx = acx + \frac{1}{2}(bc + d)x^2$$

```
[Out] a*c*x+1/2*(b*c+d)*x^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ac + (bc + d)x)\, dx = acx + \frac{1}{2}x^2(bc + d)$$

```
[In] Int[a*c + (b*c + d)*x,x]
[Out] a*c*x + ((b*c + d)*x^2)/2
```

Rubi steps

$$\text{integral} = acx + \frac{1}{2}(bc + d)x^2$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int (ac+(bc+d)x)\,dx=acx+\frac{1}{2}bcx^2+\frac{dx^2}{2}$$

```
[In] Integrate[a*c + (b*c + d)*x,x]
[Out] a*c*x + (b*c*x^2)/2 + (d*x^2)/2
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x(bcx+2ac+dx)}{2}$	16
parallelrisch	$acx+\frac{(bc+d)x^2}{2}$	16
norman	$\left(\frac{bc}{2}+\frac{d}{2}\right)x^2+acx$	18
default	$\frac{1}{2}bc\,x^2+acx+\frac{1}{2}d\,x^2$	19
risch	$\frac{1}{2}bc\,x^2+acx+\frac{1}{2}d\,x^2$	19
parts	$\frac{1}{2}bc\,x^2+acx+\frac{1}{2}d\,x^2$	19

```
[In] int(a*c+(b*c+d)*x,x,method=_RETURNVERBOSE)
[Out] 1/2*x*(b*c*x+2*a*c+d*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int (ac+(bc+d)x)\,dx=\frac{1}{2}x^2cb+\frac{1}{2}x^2d+xca$$

```
[In] integrate(a*c+(b*c+d)*x,x, algorithm="fricas")
[Out] 1/2*x^2*c*b + 1/2*x^2*d + x*c*a
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int (ac+(bc+d)x)\,dx = acx + x^2\left(\frac{bc}{2}+\frac{d}{2}\right)$$

```
[In] integrate(a*c+(b*c+d)*x,x)
[Out] a*c*x + x**2*(b*c/2 + d/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int (ac+(bc+d)x)\,dx = acx + \frac{1}{2}\,(bc+d)x^2$$

```
[In] integrate(a*c+(b*c+d)*x,x, algorithm="maxima")
[Out] a*c*x + 1/2*(b*c + d)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int (ac+(bc+d)x)\,dx = acx + \frac{1}{2}\,(bc+d)x^2$$

```
[In] integrate(a*c+(b*c+d)*x,x, algorithm="giac")
[Out] a*c*x + 1/2*(b*c + d)*x^2
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (ac+(bc+d)x)\,dx = \left(\frac{d}{2}+\frac{b\,c}{2}\right)\,x^2 + a\,c\,x$$

```
[In] int(a*c + x*(d + b*c),x)
[Out] x^2*(d/2 + (b*c)/2) + a*c*x
```

3.430 $\int (dx + c(a + bx))\,dx$

Optimal result 2646
Rubi [A] (verified) 2646
Mathematica [A] (verified) 2647
Maple [A] (verified) 2647
Fricas [A] (verification not implemented) 2647
Sympy [A] (verification not implemented) 2648
Maxima [A] (verification not implemented) 2648
Giac [A] (verification not implemented) 2648
Mupad [B] (verification not implemented) 2648

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int (dx + c(a + bx))\,dx = \frac{dx^2}{2} + \frac{c(a + bx)^2}{2b}$$

```
[Out] 1/2*d*x^2+1/2*c*(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx + c(a + bx))\,dx = \frac{c(a + bx)^2}{2b} + \frac{dx^2}{2}$$

```
[In] Int[d*x + c*(a + b*x),x]
```

```
[Out] (d*x^2)/2 + (c*(a + b*x)^2)/(2*b)
```

Rubi steps

$$\text{integral} = \frac{dx^2}{2} + \frac{c(a + bx)^2}{2b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int (dx + c(a + bx))\,dx = acx + \frac{1}{2}bcx^2 + \frac{dx^2}{2}$$

```
[In] Integrate[d*x + c*(a + b*x),x]
[Out] a*c*x + (b*c*x^2)/2 + (d*x^2)/2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{x(bcx+2ac+dx)}{2}$	16
norman	$\left(\frac{bc}{2}+\frac{d}{2}\right)x^2+acx$	18
default	$\frac{1}{2}bc\,x^2+acx+\frac{1}{2}d\,x^2$	19
risch	$\frac{1}{2}bc\,x^2+acx+\frac{1}{2}d\,x^2$	19
parallelrisch	$\frac{1}{2}bc\,x^2+acx+\frac{1}{2}d\,x^2$	19
parts	$\frac{1}{2}bc\,x^2+acx+\frac{1}{2}d\,x^2$	19

```
[In] int(d*x+c*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(b*c*x+2*a*c+d*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int (dx + c(a + bx))\,dx = \frac{1}{2}x^2cb + \frac{1}{2}x^2d + xca$$

```
[In] integrate(d*x+c*(b*x+a),x, algorithm="fricas")
[Out] 1/2*x^2*c*b + 1/2*x^2*d + x*c*a
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int (dx + c(a + bx))\, dx = acx + x^2 \left(\frac{bc}{2} + \frac{d}{2}\right)$$

```
[In] integrate(d*x+c*(b*x+a),x)
[Out] a*c*x + x**2*(b*c/2 + d/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int (dx + c(a + bx))\, dx = \frac{1}{2}\, dx^2 + \frac{1}{2}\, (bx^2 + 2\, ax)c$$

```
[In] integrate(d*x+c*(b*x+a),x, algorithm="maxima")
[Out] 1/2*d*x^2 + 1/2*(b*x^2 + 2*a*x)*c
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int (dx + c(a + bx))\, dx = \frac{1}{2}\, dx^2 + \frac{1}{2}\, (bx^2 + 2\, ax)c$$

```
[In] integrate(d*x+c*(b*x+a),x, algorithm="giac")
[Out] 1/2*d*x^2 + 1/2*(b*x^2 + 2*a*x)*c
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int (dx + c(a + bx))\, dx = \left(\frac{d}{2} + \frac{b\, c}{2}\right)\, x^2 + a\, c\, x$$

```
[In] int(d*x + c*(a + b*x),x)
[Out] x^2*(d/2 + (b*c)/2) + a*c*x
```

3.431 $\int \frac{4+4x}{x^2(1+x^2)} \, dx$

Optimal result 2649
Rubi [A] (verified) 2649
Mathematica [A] (verified) 2650
Maple [A] (verified) 2650
Fricas [A] (verification not implemented) 2651
Sympy [A] (verification not implemented) 2651
Maxima [A] (verification not implemented) 2651
Giac [A] (verification not implemented) 2652
Mupad [B] (verification not implemented) 2652

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{4+4x}{x^2(1+x^2)} \, dx = -\frac{4}{x} - 4\arctan(x) + 4\log(x) - 2\log\left(1+x^2\right)$$

```
[Out] -4/x-4*arctan(x)+4*ln(x)-2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {815, 649, 209, 266}

$$\int \frac{4+4x}{x^2(1+x^2)} \, dx = -4\arctan(x) - 2\log\left(x^2+1\right) - \frac{4}{x} + 4\log(x)$$

```
[In] Int[(4 + 4*x)/(x^2*(1 + x^2)),x]
```

```
[Out] -4/x - 4*ArcTan[x] + 4*Log[x] - 2*Log[1 + x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{4}{x^2}+\frac{4}{x}-\frac{4(1+x)}{1+x^2}\right) dx \\
&= -\frac{4}{x}+4\log(x)-4\int \frac{1+x}{1+x^2}\,dx \\
&= -\frac{4}{x}+4\log(x)-4\int \frac{1}{1+x^2}\,dx-4\int \frac{x}{1+x^2}\,dx \\
&= -\frac{4}{x}-4\tan^{-1}(x)+4\log(x)-2\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{4+4x}{x^2\left(1+x^2\right)}\,dx = 4\left(-\frac{1}{x}-\arctan(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)\right)$$

```
[In] Integrate[(4 + 4*x)/(x^2*(1 + x^2)),x]
[Out] 4*(-x^(-1) - ArcTan[x] + Log[x] - Log[1 + x^2]/2)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
default	$-\frac{4}{x}-4\arctan\left(x\right)+4\ln\left(x\right)-2\ln\left(x^{2}+1\right)$	23
meijerg	$-\frac{4}{x}-4\arctan\left(x\right)+4\ln\left(x\right)-2\ln\left(x^{2}+1\right)$	23
risch	$-\frac{4}{x}-4\arctan\left(x\right)+4\ln\left(x\right)-2\ln\left(x^{2}+1\right)$	23
parallelrisch	$\frac{2i\ln(x-i)x-2i\ln(x+i)x+4\ln(x)x-2\ln(x-i)x-2\ln(x+i)x-4}{x}$	46

```
[In] int((4+4*x)/x^2/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -4/x-4*arctan(x)+4*ln(x)-2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{4+4x}{x^2\left(1+x^2\right)}\,dx = -\frac{2\left(2\,x\arctan\left(x\right)+x\log\left(x^2+1\right)-2\,x\log\left(x\right)+2\right)}{x}$$

```
[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="fricas")
[Out] -2*(2*x*arctan(x) + x*log(x^2 + 1) - 2*x*log(x) + 2)/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{4+4x}{x^2\left(1+x^2\right)}\,dx = 4\log\left(x\right)-2\log\left(x^2+1\right)-4\operatorname{atan}\left(x\right)-\frac{4}{x}$$

```
[In] integrate((4+4*x)/x**2/(x**2+1),x)
[Out] 4*log(x) - 2*log(x**2 + 1) - 4*atan(x) - 4/x
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{4+4x}{x^2\left(1+x^2\right)}\,dx = -\frac{4}{x}-4\arctan\left(x\right)-2\log\left(x^2+1\right)+4\log\left(x\right)$$

```
[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="maxima")
[Out] -4/x - 4*arctan(x) - 2*log(x^2 + 1) + 4*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{4+4x}{x^2(1+x^2)}\,dx = -\frac{4}{x} - 4\arctan(x) - 2\log\left(x^2+1\right) + 4\log(|x|)$$

```
[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="giac")
[Out] -4/x - 4*arctan(x) - 2*log(x^2 + 1) + 4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{4+4x}{x^2(1+x^2)}\,dx = 4\ln(x) - \frac{4}{x} + \ln(x-\mathrm{i})\,(-2+2\mathrm{i}) + \ln(x+1\mathrm{i})\,(-2-2\mathrm{i})$$

```
[In] int((4*x + 4)/(x^2*(x^2 + 1)),x)
[Out] 4*log(x) - log(x + 1i)*(2 + 2i) - log(x - 1i)*(2 - 2i) - 4/x
```

3.432 $\int \frac{24+8x}{x(-4+x^2)}\,dx$

Optimal result 2653
Rubi [A] (verified) 2653
Mathematica [A] (verified) 2654
Maple [A] (verified) 2654
Fricas [A] (verification not implemented) 2654
Sympy [A] (verification not implemented) 2655
Maxima [A] (verification not implemented) 2655
Giac [A] (verification not implemented) 2655
Mupad [B] (verification not implemented) 2655

Optimal result

Integrand size = 16, antiderivative size = 17

$$\int \frac{24+8x}{x\left(-4+x^2\right)}\,dx = 5\log(2-x)-6\log(x)+\log(2+x)$$

```
[Out] 5*ln(2-x)-6*ln(x)+ln(2+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {815}

$$\int \frac{24+8x}{x\left(-4+x^2\right)}\,dx = 5\log(2-x)-6\log(x)+\log(x+2)$$

```
[In] Int[(24 + 8*x)/(x*(-4 + x^2)),x]
```

```
[Out] 5*Log[2 - x] - 6*Log[x] + Log[2 + x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{5}{-2+x}-\frac{6}{x}+\frac{1}{2+x}\right)\,dx \\ &= 5\log(2-x)-6\log(x)+\log(2+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{24+8x}{x(-4+x^2)}\,dx = 8\left(\frac{5}{8}\log(2-x)-\frac{3\log(x)}{4}+\frac{1}{8}\log(2+x)\right)$$

```
[In] Integrate[(24 + 8*x)/(x*(-4 + x^2)),x]
[Out] 8*((5*Log[2 - x])/8 - (3*Log[x])/4 + Log[2 + x]/8)
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$-6\ln(x)+\ln(x+2)+5\ln(x-2)$	16
norman	$-6\ln(x)+\ln(x+2)+5\ln(x-2)$	16
risch	$-6\ln(x)+\ln(x+2)+5\ln(x-2)$	16
parallelrisch	$-6\ln(x)+\ln(x+2)+5\ln(x-2)$	16
meijerg	$3\ln\left(1-\frac{x^2}{4}\right)-6\ln(x)+6\ln(2)-3i\pi-4\,\operatorname{arctanh}\left(\frac{x}{2}\right)$	30

```
[In] int((24+8*x)/x/(x^2-4),x,method=_RETURNVERBOSE)
[Out] -6*ln(x)+ln(x+2)+5*ln(x-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{24+8x}{x(-4+x^2)}\,dx = \log(x+2)+5\,\log(x-2)-6\,\log(x)$$

```
[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="fricas")
[Out] log(x + 2) + 5*log(x - 2) - 6*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{24+8x}{x(-4+x^2)}\,dx = -6\log(x) + 5\log(x-2) + \log(x+2)$$

```
[In] integrate((24+8*x)/x/(x**2-4),x)
[Out] -6*log(x) + 5*log(x - 2) + log(x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{24+8x}{x(-4+x^2)}\,dx = \log(x+2) + 5\log(x-2) - 6\log(x)$$

```
[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="maxima")
[Out] log(x + 2) + 5*log(x - 2) - 6*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{24+8x}{x(-4+x^2)}\,dx = \log(|x+2|) + 5\log(|x-2|) - 6\log(|x|)$$

```
[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="giac")
[Out] log(abs(x + 2)) + 5*log(abs(x - 2)) - 6*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{24+8x}{x(-4+x^2)}\,dx = 5\ln(x-2) + \ln(x+2) - 6\ln(x)$$

```
[In] int((8*x + 24)/(x*(x^2 - 4)),x)
[Out] 5*log(x - 2) + log(x + 2) - 6*log(x)
```

3.433 $\int \frac{-1+x^2}{-2x+x^3} \, dx$

Optimal result 2656
Rubi [A] (verified) 2656
Mathematica [A] (verified) 2657
Maple [A] (verified) 2657
Fricas [A] (verification not implemented) 2658
Sympy [A] (verification not implemented) 2658
Maxima [A] (verification not implemented) 2659
Giac [A] (verification not implemented) 2659
Mupad [B] (verification not implemented) 2659

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{-1+x^2}{-2x+x^3} \, dx = \frac{\log(x)}{2} + \frac{1}{4}\log\left(2-x^2\right)$$

```
[Out] 1/2*ln(x)+1/4*ln(-x^2+2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1607, 457, 78}

$$\int \frac{-1+x^2}{-2x+x^3} \, dx = \frac{1}{4}\log\left(2-x^2\right) + \frac{\log(x)}{2}$$

```
[In] Int[(-1 + x^2)/(-2*x + x^3),x]
```

```
[Out] Log[x]/2 + Log[2 - x^2]/4
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{-1+x^2}{x\left(-2+x^2\right)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x}{(-2+x)x}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{2(-2+x)}+\frac{1}{2x}\right)\,dx, x, x^2\right) \\
&= \frac{\log(x)}{2}+\frac{1}{4}\log\left(2-x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^2}{-2x+x^3}\,dx = \frac{\log(x)}{2}+\frac{1}{4}\log\left(2-x^2\right)$$

```
[In] Integrate[(-1 + x^2)/(-2*x + x^3),x]
[Out] Log[x]/2 + Log[2 - x^2]/4
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\ln(x)}{2}+\frac{\ln(x^2-2)}{4}$	14
norman	$\frac{\ln(x)}{2}+\frac{\ln(x^2-2)}{4}$	14
risch	$\frac{\ln(x)}{2}+\frac{\ln(x^2-2)}{4}$	14
parallelrisch	$\frac{\ln(x)}{2}+\frac{\ln(x^2-2)}{4}$	14
meijerg	$\frac{\ln\left(1-\frac{x^2}{2}\right)}{4}+\frac{\ln(x)}{2}-\frac{\ln(2)}{4}+\frac{i\pi}{4}$	24

```
[In] int((x^2-1)/(x^3-2*x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x)+1/4*ln(x^2-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{-1+x^2}{-2x+x^3}\,dx=\frac{1}{4}\,\log\left(x^2-2\right)+\frac{1}{2}\,\log\left(x\right)$$

```
[In] integrate((x^2-1)/(x^3-2*x),x, algorithm="fricas")
[Out] 1/4*log(x^2 - 2) + 1/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{-1+x^2}{-2x+x^3}\,dx=\frac{\log\left(x\right)}{2}+\frac{\log\left(x^2-2\right)}{4}$$

```
[In] integrate((x**2-1)/(x**3-2*x),x)
[Out] log(x)/2 + log(x**2 - 2)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{-1+x^2}{-2x+x^3}\,dx = \frac{1}{4}\,\log\left(x^2-2\right) + \frac{1}{2}\,\log\left(x\right)$$

```
[In] integrate((x^2-1)/(x^3-2*x),x, algorithm="maxima")
[Out] 1/4*log(x^2 - 2) + 1/2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{-1+x^2}{-2x+x^3}\,dx = \frac{1}{4}\,\log\left(x^2\right) + \frac{1}{4}\,\log\left(\left|x^2-2\right|\right)$$

```
[In] integrate((x^2-1)/(x^3-2*x),x, algorithm="giac")
[Out] 1/4*log(x^2) + 1/4*log(abs(x^2 - 2))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{-1+x^2}{-2x+x^3}\,dx = \frac{\ln\left(x^2-2\right)}{4} + \frac{\ln\left(x\right)}{2}$$

```
[In] int(-(x^2 - 1)/(2*x - x^3),x)
[Out] log(x^2 - 2)/4 + log(x)/2
```

3.434 $\int \frac{1+x^2}{3x+x^3} \, dx$

Optimal result . . . 2660
Rubi [A] (verified) . . . 2660
Mathematica [A] (verified) . . . 2661
Maple [A] (verified) . . . 2661
Fricas [A] (verification not implemented) . . . 2661
Sympy [A] (verification not implemented) . . . 2662
Maxima [A] (verification not implemented) . . . 2662
Giac [A] (verification not implemented) . . . 2662
Mupad [B] (verification not implemented) . . . 2662

Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{1+x^2}{3x+x^3} \, dx = \frac{1}{3} \log \left(3x+x^3\right)$$

```
[Out] 1/3*ln(x^3+3*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1601}

$$\int \frac{1+x^2}{3x+x^3} \, dx = \frac{1}{3} \log \left(x^3+3x\right)$$

```
[In] Int[(1 + x^2)/(3*x + x^3),x]
```

```
[Out] Log[3*x + x^3]/3
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\text{integral} = \frac{1}{3} \log \left(3x+x^3\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1+x^2}{3x+x^3}\,dx = \frac{\log(x)}{3} + \frac{1}{3}\log\left(3+x^2\right)$$

```
[In] Integrate[(1 + x^2)/(3*x + x^3),x]
[Out] Log[x]/3 + Log[3 + x^2]/3
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\ln\left(x\left(x^2+3\right)\right)}{3}$	11
risch	$\frac{\ln\left(x^3+3x\right)}{3}$	11
norman	$\frac{\ln(x)}{3}+\frac{\ln\left(x^2+3\right)}{3}$	14
parallelrisch	$\frac{\ln(x)}{3}+\frac{\ln\left(x^2+3\right)}{3}$	14
meijerg	$\frac{\ln\left(1+\frac{x^2}{3}\right)}{3}+\frac{\ln(x)}{3}-\frac{\ln(3)}{6}$	20

```
[In] int((x^2+1)/(x^3+3*x),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(x*(x^2+3))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^2}{3x+x^3}\,dx = \frac{1}{3}\log\left(x^3+3\,x\right)$$

```
[In] integrate((x^2+1)/(x^3+3*x),x, algorithm="fricas")
[Out] 1/3*log(x^3 + 3*x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1+x^2}{3x+x^3}\,dx = \frac{\log\left(x^3+3x\right)}{3}$$

```
[In] integrate((x**2+1)/(x**3+3*x),x)
[Out] log(x**3 + 3*x)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^2}{3x+x^3}\,dx = \frac{1}{3}\,\log\left(x^3+3\,x\right)$$

```
[In] integrate((x^2+1)/(x^3+3*x),x, algorithm="maxima")
[Out] 1/3*log(x^3 + 3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1+x^2}{3x+x^3}\,dx = \frac{1}{3}\,\log\left(3\left|\frac{1}{3}\,x^3+x\right|\right)$$

```
[In] integrate((x^2+1)/(x^3+3*x),x, algorithm="giac")
[Out] 1/3*log(3*abs(1/3*x^3 + x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^2}{3x+x^3}\,dx = \frac{\ln\left(x^3+3\,x\right)}{3}$$

```
[In] int((x^2 + 1)/(3*x + x^3),x)
[Out] log(3*x + x^3)/3
```

3.435 $\int \frac{a+3bx^2}{ax+bx^3} dx$

Optimal result . . . 2663
Rubi [A] (verified) . . . 2663
Mathematica [A] (verified) . . . 2664
Maple [A] (verified) . . . 2664
Fricas [A] (verification not implemented) . . . 2664
Sympy [A] (verification not implemented) . . . 2665
Maxima [A] (verification not implemented) . . . 2665
Giac [A] (verification not implemented) . . . 2665
Mupad [B] (verification not implemented) . . . 2665

Optimal result

Integrand size = 20, antiderivative size = 10

$$\int \frac{a+3bx^2}{ax+bx^3} dx = \log \left(ax+bx^3\right)$$

```
[Out] ln(b*x^3+a*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1601}

$$\int \frac{a+3bx^2}{ax+bx^3} dx = \log \left(ax+bx^3\right)$$

```
[In] Int[(a + 3*b*x^2)/(a*x + b*x^3),x]
```

```
[Out] Log[a*x + b*x^3]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\text{integral} = \log \left(ax+bx^3\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx=\log(x)+\log\left(a+bx^2\right)$$

```
[In] Integrate[(a + 3*b*x^2)/(a*x + b*x^3),x]
[Out] Log[x] + Log[a + b*x^2]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\ln\left(b\,x^3+ax\right)$	11
default	$\ln\left(x(b\,x^2+a)\right)$	11
risch	$\ln\left(b\,x^3+ax\right)$	11
norman	$\ln\left(x\right)+\ln\left(b\,x^2+a\right)$	12
parallelrisch	$\ln\left(x\right)+\ln\left(b\,x^2+a\right)$	12

```
[In] int((3*b*x^2+a)/(b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] ln(b*x^3+a*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx=\log\left(bx^3+ax\right)$$

```
[In] integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="fricas")
[Out] log(b*x^3 + a*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx = \log\left(ax+bx^3\right)$$

```
[In] integrate((3*b*x**2+a)/(b*x**3+a*x),x)
[Out] log(a*x + b*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx = \log\left(bx^3+ax\right)$$

```
[In] integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="maxima")
[Out] log(b*x^3 + a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx = \log\left(\left|bx^3+ax\right|\right)$$

```
[In] integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="giac")
[Out] log(abs(b*x^3 + a*x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx = \ln\left(b\,x^3+a\,x\right)$$

```
[In] int((a + 3*b*x^2)/(a*x + b*x^3),x)
[Out] log(a*x + b*x^3)
```

3.436 $\int \frac{-2+4x}{-x+x^3} dx$

Optimal result 2666
Rubi [A] (verified) 2666
Mathematica [A] (verified) 2667
Maple [A] (verified) 2667
Fricas [A] (verification not implemented) 2668
Sympy [A] (verification not implemented) 2668
Maxima [A] (verification not implemented) 2668
Giac [A] (verification not implemented) 2668
Mupad [B] (verification not implemented) 2669

Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \frac{-2+4x}{-x+x^3} dx = \log(1-x) + 2\log(x) - 3\log(1+x)$$

```
[Out] ln(1-x)+2*ln(x)-3*ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1607, 815}

$$\int \frac{-2+4x}{-x+x^3} dx = \log(1-x) + 2\log(x) - 3\log(x+1)$$

```
[In] Int[(-2 + 4*x)/(-x + x^3),x]
```

```
[Out] Log[1 - x] + 2*Log[x] - 3*Log[1 + x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{-2+4x}{x\left(-1+x^2\right)}\,dx\\
&= \int \left(\frac{1}{-1+x}+\frac{2}{x}-\frac{3}{1+x}\right)dx\\
&= \log(1-x)+2\log(x)-3\log(1+x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-2+4x}{-x+x^3}\,dx=\log(1-x)+2\log(x)-3\log(1+x)$$

```
[In] Integrate[(-2 + 4*x)/(-x + x^3),x]
[Out] Log[1 - x] + 2*Log[x] - 3*Log[1 + x]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$2\ln(x)-3\ln(x+1)+\ln(x-1)$	16
norman	$2\ln(x)-3\ln(x+1)+\ln(x-1)$	16
risch	$2\ln(x)-3\ln(x+1)+\ln(x-1)$	16
parallelrisch	$2\ln(x)-3\ln(x+1)+\ln(x-1)$	16
meijerg	$-\ln(-x^2+1)+2\ln(x)+i\pi-4\operatorname{arctanh}(x)$	24

```
[In] int((-2+4*x)/(x^3-x),x,method=_RETURNVERBOSE)
[Out] 2*ln(x)-3*ln(x+1)+ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-2+4x}{-x+x^3}\,dx = -3\,\log\left(x+1\right)+\log\left(x-1\right)+2\,\log\left(x\right)$$

```
[In] integrate((-2+4*x)/(x^3-x),x, algorithm="fricas")
[Out] -3*log(x + 1) + log(x - 1) + 2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-2+4x}{-x+x^3}\,dx = 2\log\left(x\right)+\log\left(x-1\right)-3\log\left(x+1\right)$$

```
[In] integrate((-2+4*x)/(x**3-x),x)
[Out] 2*log(x) + log(x - 1) - 3*log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-2+4x}{-x+x^3}\,dx = -3\,\log\left(x+1\right)+\log\left(x-1\right)+2\,\log\left(x\right)$$

```
[In] integrate((-2+4*x)/(x^3-x),x, algorithm="maxima")
[Out] -3*log(x + 1) + log(x - 1) + 2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{-2+4x}{-x+x^3}\,dx = -3\,\log\left(|x+1|\right)+\log\left(|x-1|\right)+2\,\log\left(|x|\right)$$

```
[In] integrate((-2+4*x)/(x^3-x),x, algorithm="giac")
[Out] -3*log(abs(x + 1)) + log(abs(x - 1)) + 2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-2+4x}{-x+x^3}\,dx = \ln(x-1) - 3\ln(x+1) + 2\ln(x)$$

```
[In] int(-(4*x - 2)/(x - x^3),x)
[Out] log(x - 1) - 3*log(x + 1) + 2*log(x)
```

3.437 $\int \frac{4+x}{4x+x^3} \, dx$

Optimal result 2670
Rubi [A] (verified) 2670
Mathematica [A] (verified) 2671
Maple [A] (verified) 2672
Fricas [A] (verification not implemented) 2672
Sympy [A] (verification not implemented) 2672
Maxima [A] (verification not implemented) 2673
Giac [A] (verification not implemented) 2673
Mupad [B] (verification not implemented) 2673

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{4+x}{4x+x^3} \, dx = \frac{1}{2}\arctan\left(\frac{x}{2}\right) + \log(x) - \frac{1}{2}\log\left(4+x^2\right)$$

```
[Out] 1/2*arctan(1/2*x)+ln(x)-1/2*ln(x^2+4)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1607, 815, 649, 209, 266}

$$\int \frac{4+x}{4x+x^3} \, dx = \frac{1}{2}\arctan\left(\frac{x}{2}\right) - \frac{1}{2}\log\left(x^2+4\right) + \log(x)$$

```
[In] Int[(4 + x)/(4*x + x^3),x]
```

```
[Out] ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{4+x}{x\,(4+x^2)}\,dx \\
&= \int \left(\frac{1}{x}+\frac{1-x}{4+x^2}\right)dx \\
&= \log(x)+\int \frac{1-x}{4+x^2}\,dx \\
&= \log(x)+\int \frac{1}{4+x^2}\,dx-\int \frac{x}{4+x^2}\,dx \\
&= \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\log(x)-\frac{1}{2}\log\left(4+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{4+x}{4x+x^3}\,dx=\frac{1}{2}\arctan\left(\frac{x}{2}\right)+\log(x)-\frac{1}{2}\log\left(4+x^2\right)$$

```
[In] Integrate[(4 + x)/(4*x + x^3),x]
[Out] ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln\left(x\right)-\frac{\ln\left(x^{2}+4\right)}{2}$	18
risch	$\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln\left(x\right)-\frac{\ln\left(x^{2}+4\right)}{2}$	18
meijerg	$-\frac{\ln\left(1+\frac{x^{2}}{4}\right)}{2}+\ln\left(x\right)-\ln\left(2\right)+\frac{\arctan\left(\frac{x}{2}\right)}{2}$	24
parallelrisch	$\ln\left(x\right)-\frac{\ln\left(x-2i\right)}{2}-\frac{i\ln\left(x-2i\right)}{4}-\frac{\ln\left(x+2i\right)}{2}+\frac{i\ln\left(x+2i\right)}{4}$	34

```
[In] int((x+4)/(x^3+4*x),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(1/2*x)+ln(x)-1/2*ln(x^2+4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{4+x}{4x+x^3}\,dx = \frac{1}{2}\arctan\left(\frac{1}{2}\,x\right)-\frac{1}{2}\log\left(x^2+4\right)+\log\left(x\right)$$

```
[In] integrate((4+x)/(x^3+4*x),x, algorithm="fricas")
[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{4+x}{4x+x^3}\,dx = \log\left(x\right)-\frac{\log\left(x^2+4\right)}{2}+\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

```
[In] integrate((4+x)/(x**3+4*x),x)
[Out] log(x) - log(x**2 + 4)/2 + atan(x/2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{4+x}{4x+x^3}\,dx = \frac{1}{2}\arctan\left(\frac{1}{2}\,x\right) - \frac{1}{2}\log\left(x^2+4\right) + \log\left(x\right)$$

```
[In] integrate((4+x)/(x^3+4*x),x, algorithm="maxima")
[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{4+x}{4x+x^3}\,dx = \frac{1}{2}\arctan\left(\frac{1}{2}\,x\right) - \frac{1}{2}\log\left(x^2+4\right) + \log\left(|x|\right)$$

```
[In] integrate((4+x)/(x^3+4*x),x, algorithm="giac")
[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{4+x}{4x+x^3}\,dx = \ln\left(x\right) + \ln\left(x-2\mathrm{i}\right)\,\left(-\frac{1}{2}-\frac{1}{4}\mathrm{i}\right) + \ln\left(x+2\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{1}{4}\mathrm{i}\right)$$

```
[In] int((x + 4)/(4*x + x^3),x)
[Out] log(x) - log(x + 2i)*(1/2 - 1i/4) - log(x - 2i)*(1/2 + 1i/4)
```

3.438 $\int \frac{-x+2x^3}{1-x^2+x^4} \, dx$

Optimal result 2674
Rubi [A] (verified) 2674
Mathematica [A] (verified) 2675
Maple [A] (verified) 2675
Fricas [A] (verification not implemented) 2675
Sympy [A] (verification not implemented) 2676
Maxima [A] (verification not implemented) 2676
Giac [A] (verification not implemented) 2676
Mupad [B] (verification not implemented) 2676

Optimal result

Integrand size = 22, antiderivative size = 15

$$\int \frac{-x+2x^3}{1-x^2+x^4} \, dx = \frac{1}{2} \log \left(1-x^2+x^4\right)$$

```
[Out] 1/2*ln(x^4-x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1601}

$$\int \frac{-x+2x^3}{1-x^2+x^4} \, dx = \frac{1}{2} \log \left(x^4-x^2+1\right)$$

```
[In] Int[(-x + 2*x^3)/(1 - x^2 + x^4),x]
```

```
[Out] Log[1 - x^2 + x^4]/2
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2} \log \left(1-x^2+x^4\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx=\frac{1}{2}\log\left(1-x^2+x^4\right)$$

```
[In] Integrate[(-x + 2*x^3)/(1 - x^2 + x^4),x]
[Out] Log[1 - x^2 + x^4]/2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln(x^4-x^2+1)}{2}$	14
norman	$\frac{\ln(x^4-x^2+1)}{2}$	14
risch	$\frac{\ln(x^4-x^2+1)}{2}$	14
parallelrisch	$\frac{\ln(x^4-x^2+1)}{2}$	14

```
[In] int((2*x^3-x)/(x^4-x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^4-x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx=\frac{1}{2}\log\left(x^4-x^2+1\right)$$

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="fricas")
[Out] 1/2*log(x^4 - x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx = \frac{\log\left(x^4-x^2+1\right)}{2}$$

```
[In] integrate((2*x**3-x)/(x**4-x**2+1),x)
[Out] log(x**4 - x**2 + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx = \frac{1}{2}\log\left(x^4-x^2+1\right)$$

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="maxima")
[Out] 1/2*log(x^4 - x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx = \frac{1}{2}\log\left(x^4-x^2+1\right)$$

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="giac")
[Out] 1/2*log(x^4 - x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx = \frac{\ln\left(x^4-x^2+1\right)}{2}$$

```
[In] int(-(x - 2*x^3)/(x^4 - x^2 + 1),x)
[Out] log(x^4 - x^2 + 1)/2
```

3.439 $\int \frac{-3+x}{2x+3x^2+x^3}\,dx$

Optimal result 2677
Rubi [A] (verified) 2677
Mathematica [A] (verified) 2678
Maple [A] (verified) 2678
Fricas [A] (verification not implemented) 2679
Sympy [A] (verification not implemented) 2679
Maxima [A] (verification not implemented) 2679
Giac [A] (verification not implemented) 2679
Mupad [B] (verification not implemented) 2680

Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)$$

```
[Out] -3/2*ln(x)+4*ln(1+x)-5/2*ln(2+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1608, 814}

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

```
[In] Int[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
```

```
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{-3+x}{x\,(2+3x+x^2)}\,dx\\ &= \int \left(-\frac{3}{2x}+\frac{4}{1+x}-\frac{5}{2(2+x)}\right)\,dx\\ &= -\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)$$

```
[In] Integrate[(-3 + x)/(2*x + 3*x^2 + x^3),x]
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{3\ln(x)}{2}+4\ln\left(x+1\right)-\frac{5\ln(x+2)}{2}$	18
norman	$-\frac{3\ln(x)}{2}+4\ln\left(x+1\right)-\frac{5\ln(x+2)}{2}$	18
risch	$-\frac{3\ln(x)}{2}+4\ln\left(x+1\right)-\frac{5\ln(x+2)}{2}$	18
parallelrisch	$-\frac{3\ln(x)}{2}+4\ln\left(x+1\right)-\frac{5\ln(x+2)}{2}$	18

```
[In] int((-3+x)/(x^3+3*x^2+2*x),x,method=_RETURNVERBOSE)
[Out] -3/2*ln(x)+4*ln(x+1)-5/2*ln(x+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{5}{2}\,\log\left(x+2\right)+4\,\log\left(x+1\right)-\frac{3}{2}\,\log\left(x\right)$$

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="fricas")
[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{3\log\left(x\right)}{2}+4\log\left(x+1\right)-\frac{5\log\left(x+2\right)}{2}$$

```
[In] integrate((-3+x)/(x**3+3*x**2+2*x),x)
[Out] -3*log(x)/2 + 4*log(x + 1) - 5*log(x + 2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{5}{2}\,\log\left(x+2\right)+4\,\log\left(x+1\right)-\frac{3}{2}\,\log\left(x\right)$$

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="maxima")
[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{5}{2}\,\log\left(|x+2|\right)+4\,\log\left(|x+1|\right)-\frac{3}{2}\,\log\left(|x|\right)$$

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="giac")
[Out] -5/2*log(abs(x + 2)) + 4*log(abs(x + 1)) - 3/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = 4\,\ln(x+1) - \frac{5\,\ln(x+2)}{2} - \frac{3\,\ln(x)}{2}$$

```
[In] int((x - 3)/(2*x + 3*x^2 + x^3),x)
[Out] 4*log(x + 1) - (5*log(x + 2))/2 - (3*log(x))/2
```

3.440 $\int \frac{2+4x}{x^2+2x^3+x^4} dx$

Optimal result . . . 2681
Rubi [A] (verified) . . . 2681
Mathematica [A] (verified) . . . 2682
Maple [A] (verified) . . . 2682
Fricas [A] (verification not implemented) . . . 2683
Sympy [A] (verification not implemented) . . . 2683
Maxima [A] (verification not implemented) . . . 2683
Giac [A] (verification not implemented) . . . 2683
Mupad [B] (verification not implemented) . . . 2684

Optimal result

Integrand size = 20, antiderivative size = 10

$$\int \frac{2+4x}{x^2+2x^3+x^4} dx = -\frac{2}{x(1+x)}$$

```
[Out] -2/x/(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1608, 27, 75}

$$\int \frac{2+4x}{x^2+2x^3+x^4} dx = -\frac{2}{x(x+1)}$$

```
[In] Int[(2 + 4*x)/(x^2 + 2*x^3 + x^4),x]
```

```
[Out] -2/(x*(1 + x))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

```
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{2+4x}{x^2(1+2x+x^2)}\,dx\\ &= \int \frac{2+4x}{x^2(1+x)^2}\,dx\\ &= -\frac{2}{x(1+x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{2+4x}{x^2+2x^3+x^4}\,dx = -\frac{2}{x+x^2}$$

```
[In] Integrate[(2 + 4*x)/(x^2 + 2*x^3 + x^4),x]
[Out] -2/(x + x^2)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
gosper	$-\frac{2}{x(x+1)}$	11
norman	$-\frac{2}{x(x+1)}$	11
risch	$-\frac{2}{x(x+1)}$	11
parallelrisch	$-\frac{2}{x(x+1)}$	11
default	$-\frac{2}{x}+\frac{2}{x+1}$	14

```
[In] int((4*x+2)/(x^4+2*x^3+x^2),x,method=_RETURNVERBOSE)
[Out] -2/x/(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{2+4x}{x^2+2x^3+x^4}\,dx=-\frac{2}{x^2+x}$$

```
[In] integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="fricas")
[Out] -2/(x^2 + x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{2+4x}{x^2+2x^3+x^4}\,dx=-\frac{2}{x^2+x}$$

```
[In] integrate((2+4*x)/(x**4+2*x**3+x**2),x)
[Out] -2/(x**2 + x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{2+4x}{x^2+2x^3+x^4}\,dx=-\frac{2}{x^2+x}$$

```
[In] integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="maxima")
[Out] -2/(x^2 + x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{2+4x}{x^2+2x^3+x^4}\,dx=-\frac{2}{x^2+x}$$

```
[In] integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="giac")
[Out] -2/(x^2 + x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{2+4x}{x^2+2x^3+x^4}\,dx = -\frac{2}{x\,(x+1)}$$

```
[In] int((4*x + 2)/(x^2 + 2*x^3 + x^4),x)
[Out] -2/(x*(x + 1))
```

3.441 $\int \frac{1+x}{-6x+x^2+x^3} \, dx$

Optimal result 2685
Rubi [A] (verified) 2685
Mathematica [A] (verified) 2686
Maple [A] (verified) 2686
Fricas [A] (verification not implemented) 2687
Sympy [A] (verification not implemented) 2687
Maxima [A] (verification not implemented) 2687
Giac [A] (verification not implemented) 2687
Mupad [B] (verification not implemented) 2688

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{1+x}{-6x+x^2+x^3} \, dx = \frac{3}{10}\log(2-x) - \frac{\log(x)}{6} - \frac{2}{15}\log(3+x)$$

```
[Out] 3/10*ln(2-x)-1/6*ln(x)-2/15*ln(3+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1608, 814}

$$\int \frac{1+x}{-6x+x^2+x^3} \, dx = \frac{3}{10}\log(2-x) - \frac{\log(x)}{6} - \frac{2}{15}\log(x+3)$$

```
[In] Int[(1 + x)/(-6*x + x^2 + x^3),x]
```

```
[Out] (3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
```

```
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1+x}{x\left(-6+x+x^2\right)}\,dx\\ &= \int \left(\frac{3}{10(-2+x)}-\frac{1}{6x}-\frac{2}{15(3+x)}\right)dx\\ &= \frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(3+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{-6x+x^2+x^3}\,dx=\frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(3+x)$$

```
[In] Integrate[(1 + x)/(-6*x + x^2 + x^3),x]
[Out] (3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{\ln(x)}{6}-\frac{2\ln(3+x)}{15}+\frac{3\ln(x-2)}{10}$	18
norman	$-\frac{\ln(x)}{6}-\frac{2\ln(3+x)}{15}+\frac{3\ln(x-2)}{10}$	18
risch	$-\frac{\ln(x)}{6}-\frac{2\ln(3+x)}{15}+\frac{3\ln(x-2)}{10}$	18
parallelrisch	$-\frac{\ln(x)}{6}-\frac{2\ln(3+x)}{15}+\frac{3\ln(x-2)}{10}$	18

```
[In] int((x+1)/(x^3+x^2-6*x),x,method=_RETURNVERBOSE)
[Out] -1/6*ln(x)-2/15*ln(3+x)+3/10*ln(x-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1+x}{-6x+x^2+x^3}\,dx = -\frac{2}{15}\,\log\left(x+3\right) + \frac{3}{10}\,\log\left(x-2\right) - \frac{1}{6}\,\log\left(x\right)$$

```
[In] integrate((1+x)/(x^3+x^2-6*x),x, algorithm="fricas")
[Out] -2/15*log(x + 3) + 3/10*log(x - 2) - 1/6*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1+x}{-6x+x^2+x^3}\,dx = -\frac{\log\left(x\right)}{6} + \frac{3\log\left(x-2\right)}{10} - \frac{2\log\left(x+3\right)}{15}$$

```
[In] integrate((1+x)/(x**3+x**2-6*x),x)
[Out] -log(x)/6 + 3*log(x - 2)/10 - 2*log(x + 3)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1+x}{-6x+x^2+x^3}\,dx = -\frac{2}{15}\,\log\left(x+3\right) + \frac{3}{10}\,\log\left(x-2\right) - \frac{1}{6}\,\log\left(x\right)$$

```
[In] integrate((1+x)/(x^3+x^2-6*x),x, algorithm="maxima")
[Out] -2/15*log(x + 3) + 3/10*log(x - 2) - 1/6*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1+x}{-6x+x^2+x^3}\,dx = -\frac{2}{15}\,\log\left(|x+3|\right) + \frac{3}{10}\,\log\left(|x-2|\right) - \frac{1}{6}\,\log\left(|x|\right)$$

```
[In] integrate((1+x)/(x^3+x^2-6*x),x, algorithm="giac")
[Out] -2/15*log(abs(x + 3)) + 3/10*log(abs(x - 2)) - 1/6*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1+x}{-6x+x^2+x^3}\,dx=\frac{3\,\ln\left(x-2\right)}{10}-\frac{2\,\ln\left(x+3\right)}{15}-\frac{\ln\left(x\right)}{6}$$

```
[In] int((x + 1)/(x^2 - 6*x + x^3),x)
[Out] (3*log(x - 2))/10 - (2*log(x + 3))/15 - log(x)/6
```

3.442 $\int \frac{4x^2+x^3}{x+x^3} \, dx$

Optimal result 2689
Rubi [A] (verified) 2689
Mathematica [A] (verified) 2691
Maple [A] (verified) 2691
Fricas [A] (verification not implemented) 2691
Sympy [A] (verification not implemented) 2692
Maxima [A] (verification not implemented) 2692
Giac [A] (verification not implemented) 2692
Mupad [B] (verification not implemented) 2692

Optimal result

Integrand size = 17, antiderivative size = 14

$$\int \frac{4x^2+x^3}{x+x^3} \, dx = x - \arctan(x) + 2\log\left(1+x^2\right)$$

```
[Out] x-arctan(x)+2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1607, 1598, 788, 649, 209, 266}

$$\int \frac{4x^2+x^3}{x+x^3} \, dx = -\arctan(x) + 2\log\left(x^2+1\right) + x$$

```
[In] Int[(4*x^2 + x^3)/(x + x^3),x]
```

```
[Out] x - ArcTan[x] + 2*Log[1 + x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{4x^2+x^3}{x\left(1+x^2\right)}\,dx \\
&= \int \frac{x(4+x)}{1+x^2}\,dx \\
&= x+\int \frac{-1+4x}{1+x^2}\,dx \\
&= x+4\int \frac{x}{1+x^2}\,dx-\int \frac{1}{1+x^2}\,dx \\
&= x-\tan^{-1}(x)+2\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{4x^2+x^3}{x+x^3}\,dx = x-\arctan(x)+2\log\left(1+x^2\right)$$

```
[In] Integrate[(4*x^2 + x^3)/(x + x^3),x]
[Out] x - ArcTan[x] + 2*Log[1 + x^2]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$x-\arctan\left(x\right)+2\ln\left(x^{2}+1\right)$	15
meijerg	$x-\arctan\left(x\right)+2\ln\left(x^{2}+1\right)$	15
risch	$x-\arctan\left(x\right)+2\ln\left(x^{2}+1\right)$	15
parallelrisch	$x+2\ln\left(x-i\right)+\frac{i\ln\left(x-i\right)}{2}+2\ln\left(x+i\right)-\frac{i\ln\left(x+i\right)}{2}$	33

```
[In] int((x^3+4*x^2)/(x^3+x),x,method=_RETURNVERBOSE)
[Out] x-arctan(x)+2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{4x^2+x^3}{x+x^3}\,dx = x-\arctan\left(x\right)+2\,\log\left(x^{2}+1\right)$$

```
[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="fricas")
[Out] x - arctan(x) + 2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{4x^2 + x^3}{x + x^3} \, dx = x + 2\log \left(x^2 + 1\right) - \operatorname{atan}\left(x\right)$$

```
[In] integrate((x**3+4*x**2)/(x**3+x),x)
[Out] x + 2*log(x**2 + 1) - atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{4x^2 + x^3}{x + x^3} \, dx = x - \arctan\left(x\right) + 2 \log\left(x^2 + 1\right)$$

```
[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="maxima")
[Out] x - arctan(x) + 2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{4x^2 + x^3}{x + x^3} \, dx = x - \arctan\left(x\right) + 2 \log\left(x^2 + 1\right)$$

```
[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="giac")
[Out] x - arctan(x) + 2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{4x^2 + x^3}{x + x^3} \, dx = x + 2 \ln\left(x^2 + 1\right) - \operatorname{atan}(x)$$

```
[In] int((4*x^2 + x^3)/(x + x^3),x)
[Out] x + 2*log(x^2 + 1) - atan(x)
```

3.443 $\int \frac{x+2x^3}{\left(x^2+x^4\right)^3} \, dx$

Optimal result . . . 2693
Rubi [A] (verified) . . . 2693
Mathematica [A] (verified) . . . 2694
Maple [A] (verified) . . . 2694
Fricas [A] (verification not implemented) . . . 2694
Sympy [A] (verification not implemented) . . . 2695
Maxima [A] (verification not implemented) . . . 2695
Giac [A] (verification not implemented) . . . 2695
Mupad [B] (verification not implemented) . . . 2695

Optimal result

Integrand size = 17, antiderivative size = 13

$$\int \frac{x+2x^3}{\left(x^2+x^4\right)^3} \, dx = -\frac{1}{4\left(x^2+x^4\right)^2}$$

```
[Out] -1/4/(x^4+x^2)^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1602}

$$\int \frac{x+2x^3}{\left(x^2+x^4\right)^3} \, dx = -\frac{1}{4\left(x^4+x^2\right)^2}$$

```
[In] Int[(x + 2*x^3)/(x^2 + x^4)^3,x]
```

```
[Out] -1/4*1/(x^2 + x^4)^2
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{4\left(x^2+x^4\right)^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{x+2x^3}{\left(x^2+x^4\right)^3}\,dx=-\frac{1}{4x^4\left(1+x^2\right)^2}$$

```
[In] Integrate[(x + 2*x^3)/(x^2 + x^4)^3,x]
[Out] -1/4*1/(x^4*(1 + x^2)^2)
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{1}{4x^4(x^2+1)^2}$	13
norman	$-\frac{1}{4x^4(x^2+1)^2}$	13
risch	$-\frac{1}{4x^4(x^2+1)^2}$	13
parallelrisch	$-\frac{1}{4x^4(x^2+1)^2}$	13
default	$-\frac{1}{4x^4}+\frac{1}{2x^2}-\frac{1}{4(x^2+1)^2}-\frac{1}{2(x^2+1)}$	30
meijerg	$\frac{x^2(5x^2+6)}{2(x^2+1)^2}-\frac{3}{4}+\frac{1}{2x^2}-\frac{x^2(7x^2+8)}{4(x^2+1)^2}-\frac{1}{4x^4}$	51

```
[In] int((2*x^3+x)/(x^4+x^2)^3,x,method=_RETURNVERBOSE)
[Out] -1/4/x^4/(x^2+1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{x+2x^3}{\left(x^2+x^4\right)^3}\,dx=-\frac{1}{4\left(x^8+2\,x^6+x^4\right)}$$

```
[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="fricas")
[Out] -1/4/(x^8 + 2*x^6 + x^4)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{x+2x^3}{(x^2+x^4)^3}\,dx = -\frac{1}{4x^8+8x^6+4x^4}$$

```
[In] integrate((2*x**3+x)/(x**4+x**2)**3,x)
[Out] -1/(4*x**8 + 8*x**6 + 4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x+2x^3}{(x^2+x^4)^3}\,dx = -\frac{1}{4\,(x^4+x^2)^2}$$

```
[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="maxima")
[Out] -1/4/(x^4 + x^2)^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x+2x^3}{(x^2+x^4)^3}\,dx = -\frac{1}{4\,(x^4+x^2)^2}$$

```
[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="giac")
[Out] -1/4/(x^4 + x^2)^2
```

Mupad [B] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{x+2x^3}{(x^2+x^4)^3}\,dx = -\frac{1}{4\,x^8+8\,x^6+4\,x^4}$$

```
[In] int((x + 2*x^3)/(x^2 + x^4)^3,x)
[Out] -1/(4*x^4 + 8*x^6 + 4*x^8)
```

3.444 $\int \frac{ax^2+bx^3}{cx^2+dx^3} dx$

Optimal result 2696
Rubi [A] (verified) 2696
Mathematica [A] (verified) 2697
Maple [A] (verified) 2697
Fricas [A] (verification not implemented) 2698
Sympy [A] (verification not implemented) 2698
Maxima [A] (verification not implemented) 2698
Giac [A] (verification not implemented) 2699
Mupad [B] (verification not implemented) 2699

Optimal result

Integrand size = 25, antiderivative size = 26

$$\int \frac{ax^2+bx^3}{cx^2+dx^3} dx = \frac{bx}{d} - \frac{(bc-ad)\log(c+dx)}{d^2}$$

```
[Out] b*x/d-(-a*d+b*c)*ln(d*x+c)/d^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1607, 1598, 45}

$$\int \frac{ax^2+bx^3}{cx^2+dx^3} dx = \frac{bx}{d} - \frac{(bc-ad)\log(c+dx)}{d^2}$$

```
[In] Int[(a*x^2 + b*x^3)/(c*x^2 + d*x^3),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{x^2(a+bx)}{cx^2+dx^3}\,dx \\ &= \int \frac{a+bx}{c+dx}\,dx \\ &= \int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx \\ &= \frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{ax^2+bx^3}{cx^2+dx^3}\,dx = \frac{bx}{d}+\frac{(-bc+ad)\log(c+dx)}{d^2}$$

```
[In] Integrate[(a*x^2 + b*x^3)/(c*x^2 + d*x^3),x]
[Out] (b*x)/d + ((-(b*c) + a*d)*Log[c + d*x])/d^2
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{xb}{d}+\frac{(da-bc)\ln(dx+c)}{d^2}$	26
norman	$\frac{xb}{d}+\frac{(da-bc)\ln(dx+c)}{d^2}$	26
parallelrisch	$\frac{\ln(dx+c)ad-\ln(dx+c)bc+bdx}{d^2}$	29
risch	$\frac{xb}{d}+\frac{\ln(dx+c)a}{d}-\frac{\ln(dx+c)bc}{d^2}$	32

```
[In] int((b*x^3+a*x^2)/(d*x^3+c*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*x*b+(a*d-b*c)/d^2*ln(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{ax^2 + bx^3}{cx^2 + dx^3} \, dx = \frac{bdx - (bc - ad)\log(dx + c)}{d^2}$$

```
[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="fricas")
[Out] (b*d*x - (b*c - a*d)*log(d*x + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{ax^2 + bx^3}{cx^2 + dx^3} \, dx = \frac{bx}{d} + \frac{(ad - bc)\log(c + dx)}{d^2}$$

```
[In] integrate((b*x**3+a*x**2)/(d*x**3+c*x**2),x)
[Out] b*x/d + (a*d - b*c)*log(c + d*x)/d**2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{ax^2 + bx^3}{cx^2 + dx^3} \, dx = \frac{bx}{d} - \frac{(bc - ad)\log(dx + c)}{d^2}$$

```
[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="maxima")
[Out] b*x/d - (b*c - a*d)*log(d*x + c)/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{ax^2 + bx^3}{cx^2 + dx^3} \, dx = \frac{bx}{d} - \frac{(bc - ad) \log(|dx + c|)}{d^2}$$

```
[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="giac")
[Out] b*x/d - (b*c - a*d)*log(abs(d*x + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{ax^2 + bx^3}{cx^2 + dx^3} \, dx = \frac{\ln(c + d\,x)\,(a\,d - b\,c)}{d^2} + \frac{b\,x}{d}$$

```
[In] int((a*x^2 + b*x^3)/(c*x^2 + d*x^3),x)
[Out] (log(c + d*x)*(a*d - b*c))/d^2 + (b*x)/d
```

3.445 $\int \frac{x+x^2}{-2x-x^2+x^3} \, dx$

Optimal result 2700
Rubi [A] (verified) 2700
Mathematica [A] (verified) 2701
Maple [A] (verified) 2701
Fricas [A] (verification not implemented) 2701
Sympy [A] (verification not implemented) 2702
Maxima [A] (verification not implemented) 2702
Giac [A] (verification not implemented) 2702
Mupad [B] (verification not implemented) 2702

Optimal result

Integrand size = 20, antiderivative size = 6

$$\int \frac{x+x^2}{-2x-x^2+x^3} \, dx = \log(2-x)$$

```
[Out] ln(2-x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1600, 31}

$$\int \frac{x+x^2}{-2x-x^2+x^3} \, dx = \log(2-x)$$

```
[In] Int[(x + x^2)/(-2*x - x^2 + x^3),x]
```

```
[Out] Log[2 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{-2+x}\,dx\\ &= \log(2-x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx = \log(-2+x)$$

```
[In] Integrate[(x + x^2)/(-2*x - x^2 + x^3),x]
[Out] Log[-2 + x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

method	result	size
default	$\ln(x-2)$	5
norman	$\ln(x-2)$	5
risch	$\ln(x-2)$	5
parallelrisch	$\ln(x-2)$	5

```
[In] int((x^2+x)/(x^3-x^2-2*x),x,method=_RETURNVERBOSE)
[Out] ln(x-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx = \log(x-2)$$

```
[In] integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="fricas")
[Out] log(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx = \log{(x-2)}$$

```
[In] integrate((x**2+x)/(x**3-x**2-2*x),x)
[Out] log(x - 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx = \log\left(x-2\right)$$

```
[In] integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="maxima")
[Out] log(x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx = \log\left(|x-2|\right)$$

```
[In] integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="giac")
[Out] log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx = \ln\left(x-2\right)$$

```
[In] int(-(x + x^2)/(2*x + x^2 - x^3),x)
[Out] log(x - 2)
```

3.446 $\int \frac{1-5x^2}{x^3(1+x^2)}\,dx$

Optimal result . . . 2703
Rubi [A] (verified) . . . 2703
Mathematica [A] (verified) . . . 2704
Maple [A] (verified) . . . 2704
Fricas [A] (verification not implemented) . . . 2705
Sympy [A] (verification not implemented) . . . 2705
Maxima [A] (verification not implemented) . . . 2705
Giac [A] (verification not implemented) . . . 2705
Mupad [B] (verification not implemented) . . . 2706

Optimal result

Integrand size = 18, antiderivative size = 20

$$\int \frac{1-5x^2}{x^3\left(1+x^2\right)}\,dx=-\frac{1}{2x^2}-6\log(x)+3\log\left(1+x^2\right)$$

```
[Out] -1/2/x^2-6*ln(x)+3*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {457, 78}

$$\int \frac{1-5x^2}{x^3\left(1+x^2\right)}\,dx=-\frac{1}{2x^2}+3\log\left(x^2+1\right)-6\log(x)$$

```
[In] Int[(1 - 5*x^2)/(x^3*(1 + x^2)),x]
```

```
[Out] -1/2*1/x^2 - 6*Log[x] + 3*Log[1 + x^2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1-5x}{x^2(1+x)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{6}{x}+\frac{6}{1+x}\right)\,dx, x, x^2\right)\\ &= -\frac{1}{2x^2}-6\log(x)+3\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1-5x^2}{x^3\left(1+x^2\right)}\,dx = -\frac{1}{2x^2}-6\log(x)+3\log\left(1+x^2\right)$$

```
[In] Integrate[(1 - 5*x^2)/(x^3*(1 + x^2)),x]
[Out] -1/2*1/x^2 - 6*Log[x] + 3*Log[1 + x^2]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{1}{2x^2}-6\ln\left(x\right)+3\ln\left(x^2+1\right)$	19
norman	$-\frac{1}{2x^2}-6\ln\left(x\right)+3\ln\left(x^2+1\right)$	19
meijerg	$-\frac{1}{2x^2}-6\ln\left(x\right)+3\ln\left(x^2+1\right)$	19
risch	$-\frac{1}{2x^2}-6\ln\left(x\right)+3\ln\left(x^2+1\right)$	19
parallelrisch	$-\frac{12\ln(x)x^2-6\ln(x^2+1)x^2+1}{2x^2}$	26

```
[In] int((-5*x^2+1)/x^3/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2/x^2-6*ln(x)+3*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{1-5x^2}{x^3 (1+x^2)} \, dx = \frac{6 \, x^2 \log \left(x^2+1\right) - 12 \, x^2 \log \left(x\right) - 1}{2 \, x^2}$$

```
[In] integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="fricas")
[Out] 1/2*(6*x^2*log(x^2 + 1) - 12*x^2*log(x) - 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1-5x^2}{x^3 (1+x^2)} \, dx = -6 \log (x) + 3 \log \left(x^2+1\right) - \frac{1}{2x^2}$$

```
[In] integrate((-5*x**2+1)/x**3/(x**2+1),x)
[Out] -6*log(x) + 3*log(x**2 + 1) - 1/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1-5x^2}{x^3 (1+x^2)} \, dx = -\frac{1}{2 \, x^2} + 3 \, \log \left(x^2+1\right) - 3 \, \log \left(x^2\right)$$

```
[In] integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="maxima")
[Out] -1/2/x^2 + 3*log(x^2 + 1) - 3*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1-5x^2}{x^3 (1+x^2)} \, dx = \frac{6 \, x^2 - 1}{2 \, x^2} + 3 \, \log \left(x^2+1\right) - 3 \, \log \left(x^2\right)$$

```
[In] integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="giac")
[Out] 1/2*(6*x^2 - 1)/x^2 + 3*log(x^2 + 1) - 3*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1-5x^2}{x^3\,(1+x^2)}\,dx = 3\,\ln\left(x^2+1\right) - 6\,\ln\left(x\right) - \frac{1}{2\,x^2}$$

```
[In] int(-(5*x^2 - 1)/(x^3*(x^2 + 1)),x)
[Out] 3*log(x^2 + 1) - 6*log(x) - 1/(2*x^2)
```

3.447 $\int \frac{2x}{(-1+x)(5+x^2)}\,dx$

Optimal result 2707
Rubi [A] (verified) 2707
Mathematica [A] (verified) 2708
Maple [A] (verified) 2709
Fricas [A] (verification not implemented) 2709
Sympy [A] (verification not implemented) 2709
Maxima [A] (verification not implemented) 2710
Giac [A] (verification not implemented) 2710
Mupad [B] (verification not implemented) 2710

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{2x}{(-1+x)\,(5+x^2)}\,dx=\frac{1}{3}\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(5+x^2\right)$$

```
[Out] 1/3*ln(1-x)-1/6*ln(x^2+5)+1/3*arctan(1/5*x*5^(1/2))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 815, 649, 209, 266}

$$\int \frac{2x}{(-1+x)\,(5+x^2)}\,dx=\frac{1}{3}\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log(1-x)$$

```
[In] Int[(2*x)/((-1 + x)*(5 + x^2)),x]
```

```
[Out] (Sqrt[5]*ArcTan[x/Sqrt[5]])/3 + Log[1 - x]/3 - Log[5 + x^2]/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\int \frac{x}{(-1+x)\left(5+x^2\right)}\,dx \\
&= 2\int \left(\frac{1}{6(-1+x)}+\frac{5-x}{6\left(5+x^2\right)}\right)\,dx \\
&= \frac{1}{3}\log(1-x)+\frac{1}{3}\int \frac{5-x}{5+x^2}\,dx \\
&= \frac{1}{3}\log(1-x)-\frac{1}{3}\int \frac{x}{5+x^2}\,dx+\frac{5}{3}\int \frac{1}{5+x^2}\,dx \\
&= \frac{1}{3}\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(5+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{2x}{(-1+x)\left(5+x^2\right)}\,dx = 2\left(\frac{1}{6}\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{6}\log(1-x)-\frac{1}{12}\log\left(5+x^2\right)\right)$$

```
[In] Integrate[(2*x)/((-1 + x)*(5 + x^2)),x]
[Out] 2*((Sqrt[5]*ArcTan[x/Sqrt[5]])/6 + Log[1 - x]/6 - Log[5 + x^2]/12)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{\ln(x^2+5)}{6}+\frac{\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}}{3}+\frac{\ln(x-1)}{3}$	28
risch	$-\frac{\ln(x^2+5)}{6}+\frac{\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}}{3}+\frac{\ln(x-1)}{3}$	28

```
[In] int(2*x/(x-1)/(x^2+5),x,method=_RETURNVERBOSE)
[Out] -1/6*ln(x^2+5)+1/3*arctan(1/5*x*5^(1/2))*5^(1/2)+1/3*ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{2x}{(-1+x)(5+x^2)}\,dx = \frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) - \frac{1}{6}\log\left(x^2+5\right) + \frac{1}{3}\log\left(x-1\right)$$

```
[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="fricas")
[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{2x}{(-1+x)(5+x^2)}\,dx = \frac{\log\left(x-1\right)}{3} - \frac{\log\left(x^2+5\right)}{6} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}x}{5}\right)}{3}$$

```
[In] integrate(2*x/(-1+x)/(x**2+5),x)
[Out] log(x - 1)/3 - log(x**2 + 5)/6 + sqrt(5)*atan(sqrt(5)*x/5)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{2x}{(-1+x)(5+x^2)}\,dx = \frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) - \frac{1}{6}\log\left(x^2+5\right) + \frac{1}{3}\log\left(x-1\right)$$

```
[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="maxima")
[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{2x}{(-1+x)(5+x^2)}\,dx = \frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) - \frac{1}{6}\log\left(x^2+5\right) + \frac{1}{3}\log\left(|x-1|\right)$$

```
[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="giac")
[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{2x}{(-1+x)(5+x^2)}\,dx = \frac{\ln(x-1)}{3} - \ln\left(x-\sqrt{5}\,1\mathrm{i}\right)\left(\frac{1}{6}+\frac{\sqrt{5}\,1\mathrm{i}}{6}\right) + \ln\left(x+\sqrt{5}\,1\mathrm{i}\right)\left(-\frac{1}{6}+\frac{\sqrt{5}\,1\mathrm{i}}{6}\right)$$

```
[In] int((2*x)/((x^2 + 5)*(x - 1)),x)
[Out] log(x - 1)/3 - log(x - 5^(1/2)*1i)*((5^(1/2)*1i)/6 + 1/6) + log(x + 5^(1/2)
*1i)*((5^(1/2)*1i)/6 - 1/6)
```

3.448 $\int \frac{2+x^2}{2+x}\,dx$

Optimal result 2711
Rubi [A] (verified) 2711
Mathematica [A] (verified) 2712
Maple [A] (verified) 2712
Fricas [A] (verification not implemented) 2712
Sympy [A] (verification not implemented) 2713
Maxima [A] (verification not implemented) 2713
Giac [A] (verification not implemented) 2713
Mupad [B] (verification not implemented) 2713

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{2+x^2}{2+x}\,dx = -2x+\frac{x^2}{2}+6\log(2+x)$$

```
[Out] -2*x+1/2*x^2+6*ln(2+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {711}

$$\int \frac{2+x^2}{2+x}\,dx = \frac{x^2}{2}-2x+6\log(x+2)$$

```
[In] Int[(2 + x^2)/(2 + x),x]
```

```
[Out] -2*x + x^2/2 + 6*Log[2 + x]
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-2+x+\frac{6}{2+x}\right)\,dx\\ &= -2x+\frac{x^2}{2}+6\log(2+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{2+x^2}{2+x}\,dx = -6-2x+\frac{x^2}{2}+6\log(2+x)$$

```
[In] Integrate[(2 + x^2)/(2 + x),x]
[Out] -6 - 2*x + x^2/2 + 6*Log[2 + x]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$-2x+\frac{x^2}{2}+6\ln\left(x+2\right)$	16
norman	$-2x+\frac{x^2}{2}+6\ln\left(x+2\right)$	16
risch	$-2x+\frac{x^2}{2}+6\ln\left(x+2\right)$	16
parallelrisch	$-2x+\frac{x^2}{2}+6\ln\left(x+2\right)$	16
meijerg	$6\ln\left(1+\frac{x}{2}\right)-\frac{x\left(-\frac{3x}{2}+6\right)}{3}$	18

```
[In] int((x^2+2)/(x+2),x,method=_RETURNVERBOSE)
[Out] -2*x+1/2*x^2+6*ln(x+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{2+x^2}{2+x}\,dx = \frac{1}{2}\,x^2-2\,x+6\,\log\left(x+2\right)$$

```
[In] integrate((x^2+2)/(2+x),x, algorithm="fricas")
[Out] 1/2*x^2 - 2*x + 6*log(x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{2+x^2}{2+x}\,dx = \frac{x^2}{2} - 2x + 6\log{(x+2)}$$

```
[In] integrate((x**2+2)/(2+x),x)
[Out] x**2/2 - 2*x + 6*log(x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{2+x^2}{2+x}\,dx = \frac{1}{2}\,x^2 - 2\,x + 6\,\log\left(x+2\right)$$

```
[In] integrate((x^2+2)/(2+x),x, algorithm="maxima")
[Out] 1/2*x^2 - 2*x + 6*log(x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{2+x^2}{2+x}\,dx = \frac{1}{2}\,x^2 - 2\,x + 6\,\log\left(|x+2|\right)$$

```
[In] integrate((x^2+2)/(2+x),x, algorithm="giac")
[Out] 1/2*x^2 - 2*x + 6*log(abs(x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{2+x^2}{2+x}\,dx = 6\,\ln\left(x+2\right) - 2\,x + \frac{x^2}{2}$$

```
[In] int((x^2 + 2)/(x + 2),x)
[Out] 6*log(x + 2) - 2*x + x^2/2
```

3.449 $\int \frac{1}{(-3+x)(4+x^2)}\,dx$

Optimal result 2714
Rubi [A] (verified) 2714
Mathematica [A] (verified) 2715
Maple [A] (verified) 2716
Fricas [A] (verification not implemented) 2716
Sympy [A] (verification not implemented) 2716
Maxima [A] (verification not implemented) 2717
Giac [A] (verification not implemented) 2717
Mupad [B] (verification not implemented) 2717

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{1}{(-3+x)\left(4+x^2\right)}\,dx=-\frac{3}{26}\arctan\left(\frac{x}{2}\right)+\frac{1}{13}\log(3-x)-\frac{1}{26}\log\left(4+x^2\right)$$

```
[Out] -3/26*arctan(1/2*x)+1/13*ln(3-x)-1/26*ln(x^2+4)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {720, 31, 649, 209, 266}

$$\int \frac{1}{(-3+x)\left(4+x^2\right)}\,dx=-\frac{3}{26}\arctan\left(\frac{x}{2}\right)-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log(3-x)$$

```
[In] Int[1/((-3 + x)*(4 + x^2)),x]
```

```
[Out] (-3*ArcTan[x/2])/26 + Log[3 - x]/13 - Log[4 + x^2]/26
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{13}\int \frac{1}{-3+x}\,dx + \frac{1}{13}\int \frac{-3-x}{4+x^2}\,dx\\ &= \frac{1}{13}\log(3-x) - \frac{1}{13}\int \frac{x}{4+x^2}\,dx - \frac{3}{13}\int \frac{1}{4+x^2}\,dx\\ &= -\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{13}\log(3-x) - \frac{1}{26}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{1}{(-3+x)\left(4+x^2\right)}\,dx = -\frac{3}{26}\arctan\left(\frac{x}{2}\right) - \frac{1}{26}\log\left(13+6(-3+x)+(-3+x)^2\right) + \frac{1}{13}\log(-3+x)$$

```
[In] Integrate[1/((-3 + x)*(4 + x^2)),x]
[Out] (-3*ArcTan[x/2])/26 - Log[13 + 6*(-3 + x) + (-3 + x)^2]/26 + Log[-3 + x]/13
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{\ln(x^2+4)}{26}-\frac{3\arctan\left(\frac{x}{2}\right)}{26}+\frac{\ln(-3+x)}{13}$	22
risch	$\frac{\ln(-3+x)}{13}-\frac{\ln(9x^2+36)}{26}-\frac{3\arctan\left(\frac{x}{2}\right)}{26}$	24
parallelrisch	$\frac{\ln(-3+x)}{13}-\frac{\ln(x-2i)}{26}+\frac{3i\ln(x-2i)}{52}-\frac{\ln(x+2i)}{26}-\frac{3i\ln(x+2i)}{52}$	38

```
[In] int(1/(-3+x)/(x^2+4),x,method=_RETURNVERBOSE)
[Out] -1/26*ln(x^2+4)-3/26*arctan(1/2*x)+1/13*ln(-3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{(-3+x)(4+x^2)}\,dx = -\frac{3}{26}\arctan\left(\frac{1}{2}x\right)-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log\left(x-3\right)$$

```
[In] integrate(1/(-3+x)/(x^2+4),x, algorithm="fricas")
[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-3+x)(4+x^2)}\,dx = \frac{\log\left(x-3\right)}{13}-\frac{\log\left(x^2+4\right)}{26}-\frac{3\operatorname{atan}\left(\frac{x}{2}\right)}{26}$$

```
[In] integrate(1/(-3+x)/(x**2+4),x)
[Out] log(x - 3)/13 - log(x**2 + 4)/26 - 3*atan(x/2)/26
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{(-3+x)(4+x^2)}\,dx = -\frac{3}{26}\arctan\left(\frac{1}{2}\,x\right) - \frac{1}{26}\log\left(x^2+4\right) + \frac{1}{13}\log\left(x-3\right)$$

```
[In] integrate(1/(-3+x)/(x^2+4),x, algorithm="maxima")
[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(x - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-3+x)(4+x^2)}\,dx = -\frac{3}{26}\arctan\left(\frac{1}{2}\,x\right) - \frac{1}{26}\log\left(x^2+4\right) + \frac{1}{13}\log\left(|x-3|\right)$$

```
[In] integrate(1/(-3+x)/(x^2+4),x, algorithm="giac")
[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(abs(x - 3))
```

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{(-3+x)(4+x^2)}\,dx = \frac{\ln(x-3)}{13} + \ln(x-2\mathrm{i})\left(-\frac{1}{26}+\frac{3}{52}\mathrm{i}\right) + \ln(x+2\mathrm{i})\left(-\frac{1}{26}-\frac{3}{52}\mathrm{i}\right)$$

```
[In] int(1/((x^2 + 4)*(x - 3)),x)
[Out] log(x - 3)/13 - log(x - 2i)*(1/26 - 3i/52) - log(x + 2i)*(1/26 + 3i/52)
```

3.450 $\int \frac{-2+3x^6}{x(5+2x^6)}\,dx$

Optimal result 2718
Rubi [A] (verified) 2718
Mathematica [A] (verified) 2719
Maple [A] (verified) 2719
Fricas [A] (verification not implemented) 2720
Sympy [A] (verification not implemented) 2720
Maxima [A] (verification not implemented) 2720
Giac [A] (verification not implemented) 2720
Mupad [B] (verification not implemented) 2721

Optimal result

Integrand size = 20, antiderivative size = 19

$$\int \frac{-2+3x^6}{x(5+2x^6)}\,dx = -\frac{2\log(x)}{5}+\frac{19}{60}\log\left(5+2x^6\right)$$

```
[Out] -2/5*ln(x)+19/60*ln(2*x^6+5)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\int \frac{-2+3x^6}{x(5+2x^6)}\,dx = \frac{19}{60}\log\left(2x^6+5\right)-\frac{2\log(x)}{5}$$

```
[In] Int[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]
```

```
[Out] (-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{6}\text{Subst}\left(\int \frac{-2+3x}{x(5+2x)}\,dx, x, x^6\right)\\ &= \frac{1}{6}\text{Subst}\left(\int \left(-\frac{2}{5x}+\frac{19}{5(5+2x)}\right)\,dx, x, x^6\right)\\ &= -\frac{2\log(x)}{5}+\frac{19}{60}\log\left(5+2x^6\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{-2+3x^6}{x\left(5+2x^6\right)}\,dx = -\frac{2\log(x)}{5}+\frac{19}{60}\log\left(5+2x^6\right)$$

```
[In] Integrate[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]
[Out] (-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$-\frac{2\ln(x)}{5}+\frac{19\ln\left(x^6+\frac{5}{2}\right)}{60}$	14
default	$-\frac{2\ln(x)}{5}+\frac{19\ln\left(2x^6+5\right)}{60}$	16
norman	$-\frac{2\ln(x)}{5}+\frac{19\ln\left(2x^6+5\right)}{60}$	16
risch	$-\frac{2\ln(x)}{5}+\frac{19\ln\left(2x^6+5\right)}{60}$	16
meijerg	$\frac{19\ln\left(1+\frac{2x^6}{5}\right)}{60}-\frac{2\ln(x)}{5}-\frac{\ln(2)}{15}+\frac{\ln(5)}{15}$	24

```
[In] int((3*x^6-2)/x/(2*x^6+5),x,method=_RETURNVERBOSE)
[Out] -2/5*ln(x)+19/60*ln(x^6+5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{-2+3x^6}{x\,(5+2x^6)}\,dx = \frac{19}{60}\,\log\left(2\,x^6+5\right) - \frac{2}{5}\,\log\left(x\right)$$

```
[In] integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="fricas")
[Out] 19/60*log(2*x^6 + 5) - 2/5*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{-2+3x^6}{x\,(5+2x^6)}\,dx = -\frac{2\log\left(x\right)}{5} + \frac{19\log\left(2x^6+5\right)}{60}$$

```
[In] integrate((3*x**6-2)/x/(2*x**6+5),x)
[Out] -2*log(x)/5 + 19*log(2*x**6 + 5)/60
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{-2+3x^6}{x\,(5+2x^6)}\,dx = \frac{19}{60}\,\log\left(2\,x^6+5\right) - \frac{1}{15}\,\log\left(x^6\right)$$

```
[In] integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="maxima")
[Out] 19/60*log(2*x^6 + 5) - 1/15*log(x^6)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{-2+3x^6}{x\,(5+2x^6)}\,dx = \frac{19}{60}\,\log\left(2\,x^6+5\right) - \frac{1}{15}\,\log\left(x^6\right)$$

```
[In] integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="giac")
[Out] 19/60*log(2*x^6 + 5) - 1/15*log(x^6)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{-2+3x^6}{x\left(5+2x^6\right)}\,dx=\frac{19\,\ln\left(x^6+\frac{5}{2}\right)}{60}-\frac{2\,\ln\left(x\right)}{5}$$

```
[In] int((3*x^6 - 2)/(x*(2*x^6 + 5)),x)
[Out] (19*log(x^6 + 5/2))/60 - (2*log(x))/5
```

3.451 $\int \frac{3+2x}{(-2+x)(5+x)}\,dx$

Optimal result 2722
Rubi [A] (verified) 2722
Mathematica [A] (verified) 2723
Maple [A] (verified) 2723
Fricas [A] (verification not implemented) 2723
Sympy [A] (verification not implemented) 2724
Maxima [A] (verification not implemented) 2724
Giac [A] (verification not implemented) 2724
Mupad [B] (verification not implemented) 2724

Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log(2-x)+\log(5+x)$$

```
[Out] ln(2-x)+ln(5+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {78}

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log(2-x)+\log(x+5)$$

```
[In] Int[(3 + 2*x)/((-2 + x)*(5 + x)),x]
[Out] Log[2 - x] + Log[5 + x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{-2+x}+\frac{1}{5+x}\right)\,dx \\ &= \log(2-x)+\log(5+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log(-2+x)+\log(5+x)$$

```
[In] Integrate[(3 + 2*x)/((-2 + x)*(5 + x)),x]
[Out] Log[-2 + x] + Log[5 + x]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
default	$\ln\left(\left(x-2\right)\left(5+x\right)\right)$	9
norman	$\ln\left(5+x\right)+\ln\left(x-2\right)$	10
risch	$\ln\left(x^{2}+3x-10\right)$	10
parallelrisch	$\ln\left(5+x\right)+\ln\left(x-2\right)$	10

```
[In] int((2*x+3)/(x-2)/(5+x),x,method=_RETURNVERBOSE)
[Out] ln((x-2)*(5+x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log\left(x^{2}+3\,x-10\right)$$

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="fricas")
[Out] log(x^2 + 3*x - 10)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log\left(x^2+3x-10\right)$$

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x)
[Out] log(x**2 + 3*x - 10)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log\left(x+5\right)+\log\left(x-2\right)$$

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="maxima")
[Out] log(x + 5) + log(x - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log\left(|x+5|\right)+\log\left(|x-2|\right)$$

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="giac")
[Out] log(abs(x + 5)) + log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \ln\left(x^2+3\,x-10\right)$$

```
[In] int((2*x + 3)/((x - 2)*(x + 5)),x)
[Out] log(3*x + x^2 - 10)
```

3.452 $\int \frac{x^4}{4+5x^2+x^4} \, dx$

Optimal result 2725
Rubi [A] (verified) 2725
Mathematica [A] (verified) 2726
Maple [A] (verified) 2726
Fricas [A] (verification not implemented) 2727
Sympy [A] (verification not implemented) 2727
Maxima [A] (verification not implemented) 2727
Giac [A] (verification not implemented) 2728
Mupad [B] (verification not implemented) 2728

Optimal result

Integrand size = 16, antiderivative size = 18

$$\int \frac{x^4}{4+5x^2+x^4} \, dx = x - \frac{8}{3}\arctan\left(\frac{x}{2}\right) + \frac{\arctan(x)}{3}$$

```
[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1136, 1180, 209}

$$\int \frac{x^4}{4+5x^2+x^4} \, dx = -\frac{8}{3}\arctan\left(\frac{x}{2}\right) + \frac{\arctan(x)}{3} + x$$

```
[In] Int[x^4/(4 + 5*x^2 + x^4),x]
```

```
[Out] x - (8*ArcTan[x/2])/3 + ArcTan[x]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
```

```
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x-\int \frac{4+5x^2}{4+5x^2+x^4}\,dx\\ &= x+\frac{1}{3}\int \frac{1}{1+x^2}\,dx-\frac{16}{3}\int \frac{1}{4+x^2}\,dx\\ &= x-\frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{4+5x^2+x^4}\,dx=x+\frac{8}{3}\arctan\left(\frac{2}{x}\right)+\frac{\arctan(x)}{3}$$

```
[In] Integrate[x^4/(4 + 5*x^2 + x^4),x]
[Out] x + (8*ArcTan[2/x])/3 + ArcTan[x]/3
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

method	result	size
default	$x-\frac{8\arctan\left(\frac{x}{2}\right)}{3}+\frac{\arctan(x)}{3}$	13
risch	$x-\frac{8\arctan\left(\frac{x}{2}\right)}{3}+\frac{\arctan(x)}{3}$	13
parallelrisch	$x+\frac{i\ln(x+i)}{6}-\frac{i\ln(x-i)}{6}+\frac{4i\ln(x-2i)}{3}-\frac{4i\ln(x+2i)}{3}$	35

```
[In] int(x^4/(x^4+5*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan\left(x\right)$$

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="fricas")
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3} + \frac{\mathrm{atan}\left(x\right)}{3}$$

```
[In] integrate(x**4/(x**4+5*x**2+4),x)
[Out] x - 8*atan(x/2)/3 + atan(x)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan\left(x\right)$$

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="maxima")
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8}{3}\,\arctan\left(\frac{1}{2}\,x\right) + \frac{1}{3}\,\arctan\left(x\right)$$

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="giac")
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3} + \frac{\mathrm{atan}(x)}{3}$$

```
[In] int(x^4/(5*x^2 + x^4 + 4),x)
[Out] x - (8*atan(x/2))/3 + atan(x)/3
```

3.453 $\int \frac{1}{(1+x)(2+x)^2(3+x)^3} dx$

Optimal result 2729
Rubi [A] (verified) 2729
Mathematica [A] (verified) 2730
Maple [A] (verified) 2730
Fricas [B] (verification not implemented) 2731
Sympy [A] (verification not implemented) 2731
Maxima [A] (verification not implemented) 2731
Giac [A] (verification not implemented) 2732
Mupad [B] (verification not implemented) 2732

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3} dx = \frac{1}{2+x} + \frac{1}{4(3+x)^2} + \frac{5}{4(3+x)} + \frac{1}{8}\log(1+x) + 2\log(2+x) - \frac{17}{8}\log(3+x)$$

```
[Out] 1/(2+x)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(1+x)+2*ln(2+x)-17/8*ln(3+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{90\}$

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3} dx = \frac{1}{x+2} + \frac{5}{4(x+3)} + \frac{1}{4(x+3)^2} + \frac{1}{8}\log(x+1) + 2\log(x+2) - \frac{17}{8}\log(x+3)$$

```
[In] Int[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (2 + x)^(-1) + 1/(4*(3 + x)^2) + 5/(4*(3 + x)) + Log[1 + x]/8 + 2*Log[2 + x
] - (17*Log[3 + x])/8
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{8(1+x)} - \frac{1}{(2+x)^2} + \frac{2}{2+x} - \frac{1}{2(3+x)^3} - \frac{5}{4(3+x)^2} - \frac{17}{8(3+x)}\right) dx \\ &= \frac{1}{2+x} + \frac{1}{4(3+x)^2} + \frac{5}{4(3+x)} + \frac{1}{8}\log(1+x) + 2\log(2+x) - \frac{17}{8}\log(3+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{1}{8}\left(\frac{8}{2+x} + \frac{2}{(3+x)^2} + \frac{10}{3+x} + \log(-1-x) + 16\log(2+x) - 17\log(3+x)\right)$$

```
[In] Integrate[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
[Out] (8/(2 + x) + 2/(3 + x)^2 + 10/(3 + x) + Log[-1 - x] + 16*Log[2 + x] - 17*Lo
g[3 + x])/8
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result
default	$\frac{1}{x+2}+\frac{1}{4(3+x)^2}+\frac{5}{4(3+x)}+\frac{\ln(x+1)}{8}+2\ln(x+2)-\frac{17\ln(3+x)}{8}$
norman	$\frac{\frac{9}{4}x^2+\frac{25}{2}x+17}{(x+2)(3+x)^2}-\frac{17\ln(3+x)}{8}+\frac{\ln(x+1)}{8}+2\ln(x+2)$
risch	$\frac{\frac{9}{4}x^2+\frac{25}{2}x+17}{(x+2)(3+x)^2}-\frac{17\ln(3+x)}{8}+\frac{\ln(x+1)}{8}+2\ln(x+2)$
parallelrisch	$\frac{\ln(x+1)x^3+16\ln(x+2)x^3-17\ln(3+x)x^3+136+8\ln(x+1)x^2+128\ln(x+2)x^2-136\ln(3+x)x^2+21\ln(x+1)x+336\ln(x+2)x-357}{8(x+2)(3+x)^2}$

```
[In] int(1/(x+1)/(x+2)^2/(3+x)^3,x,method=_RETURNVERBOSE)
[Out] 1/(x+2)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(x+1)+2*ln(x+2)-17/8*ln(3+x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(38) = 76$.

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{18\,x^2 - 17\,(x^3+8\,x^2+21\,x+18)\log(x+3) + 16\,(x^3+8\,x^2+21\,x+18)\log(x+2) + (x^3+8\,x^2+21\,x}{8\,(x^3+8\,x^2+21\,x+18)}$$

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(18*x^2 - 17*(x^3 + 8*x^2 + 21*x + 18)*log(x + 3) + 16*(x^3 + 8*x^2 + 2
1*x + 18)*log(x + 2) + (x^3 + 8*x^2 + 21*x + 18)*log(x + 1) + 100*x + 136)/
(x^3 + 8*x^2 + 21*x + 18)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{9x^2+50x+68}{4x^3+32x^2+84x+72} + \frac{\log(x+1)}{8} + 2\log(x+2) - \frac{17\log(x+3)}{8}$$

```
[In] integrate(1/(1+x)/(2+x)**2/(3+x)**3,x)
```

```
[Out] (9*x**2 + 50*x + 68)/(4*x**3 + 32*x**2 + 84*x + 72) + log(x + 1)/8 + 2*log(
x + 2) - 17*log(x + 3)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{9\,x^2+50\,x+68}{4\,(x^3+8\,x^2+21\,x+18)} - \frac{17}{8}\,\log(x+3) + 2\,\log(x+2) + \frac{1}{8}\,\log(x+1)$$

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^2 + 50*x + 68)/(x^3 + 8*x^2 + 21*x + 18) - 17/8*log(x + 3) + 2*log
(x + 2) + 1/8*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{1}{x+2} - \frac{\frac{7}{x+2}+6}{4\left(\frac{1}{x+2}+1\right)^2} + \frac{1}{8}\log\left(\left|-\frac{1}{x+2}+1\right|\right) - \frac{17}{8}\log\left(\left|-\frac{1}{x+2}-1\right|\right)$$

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="giac")
```

```
[Out] 1/(x + 2) - 1/4*(7/(x + 2) + 6)/(1/(x + 2) + 1)^2 + 1/8*log(abs(-1/(x + 2)
+ 1)) - 17/8*log(abs(-1/(x + 2) - 1))
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{\ln(x+1)}{8} + 2\,\ln(x+2) - \frac{17\,\ln(x+3)}{8} + \frac{\frac{9\,x^2}{4}+\frac{25\,x}{2}+17}{x^3+8\,x^2+21\,x+18}$$

```
[In] int(1/((x + 1)*(x + 2)^2*(x + 3)^3),x)
```

```
[Out] log(x + 1)/8 + 2*log(x + 2) - (17*log(x + 3))/8 + ((25*x)/2 + (9*x^2)/4 + 1
7)/(21*x + 8*x^2 + x^3 + 18)
```

3.454 $\int \frac{x}{-1+x^2} \, dx$

Optimal result 2733
Rubi [A] (verified) 2733
Mathematica [A] (verified) 2734
Maple [A] (verified) 2734
Fricas [A] (verification not implemented) 2734
Sympy [A] (verification not implemented) 2735
Maxima [A] (verification not implemented) 2735
Giac [A] (verification not implemented) 2735
Mupad [B] (verification not implemented) 2735

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{x}{-1+x^2} \, dx = \frac{1}{2} \log \left(1-x^2\right)$$

```
[Out] 1/2*ln(-x^2+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {266}

$$\int \frac{x}{-1+x^2} \, dx = \frac{1}{2} \log \left(1-x^2\right)$$

```
[In] Int[x/(-1 + x^2),x]
```

```
[Out] Log[1 - x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2} \log \left(1-x^2\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{-1+x^2}\,dx = \frac{1}{2}\log\left(-1+x^2\right)$$

```
[In] Integrate[x/(-1 + x^2),x]
[Out] Log[-1 + x^2]/2
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\ln(x^2-1)}{2}$	9
risch	$\frac{\ln(x^2-1)}{2}$	9
meijerg	$\frac{\ln(-x^2+1)}{2}$	11
default	$\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	14
norman	$\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	14
parallelrisch	$\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	14

```
[In] int(x/(x^2-1),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{-1+x^2}\,dx = \frac{1}{2}\log\left(x^2-1\right)$$

```
[In] integrate(x/(x^2-1),x, algorithm="fricas")
[Out] 1/2*log(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{x}{-1+x^2}\,dx = \frac{\log(x^2-1)}{2}$$

```
[In] integrate(x/(x**2-1),x)
[Out] log(x**2 - 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{-1+x^2}\,dx = \frac{1}{2}\log\left(x^2-1\right)$$

```
[In] integrate(x/(x^2-1),x, algorithm="maxima")
[Out] 1/2*log(x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{x}{-1+x^2}\,dx = \frac{1}{2}\log\left(\left|x^2-1\right|\right)$$

```
[In] integrate(x/(x^2-1),x, algorithm="giac")
[Out] 1/2*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{-1+x^2}\,dx = \frac{\ln(x^2-1)}{2}$$

```
[In] int(x/(x^2 - 1),x)
[Out] log(x^2 - 1)/2
```

3.455 $\int \frac{1}{\left(-1+x^2\right)^2} \, dx$

Optimal result 2736
Rubi [A] (verified) 2736
Mathematica [A] (verified) 2737
Maple [C] (verified) 2737
Fricas [B] (verification not implemented) 2738
Sympy [A] (verification not implemented) 2738
Maxima [A] (verification not implemented) 2738
Giac [A] (verification not implemented) 2738
Mupad [B] (verification not implemented) 2739

Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \frac{1}{\left(-1+x^2\right)^2} \, dx=\frac{x}{2 \left(1-x^2\right)}+\frac{\text{arctanh}(x)}{2}$$

```
[Out] 1/2*x/(-x^2+1)+1/2*arctanh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {205, 213}

$$\int \frac{1}{\left(-1+x^2\right)^2} \, dx=\frac{\text{arctanh}(x)}{2}+\frac{x}{2 \left(1-x^2\right)}$$

```
[In] Int[(-1 + x^2)^(-2),x]
```

```
[Out] x/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x}{2(1-x^2)} - \frac{1}{2}\int \frac{1}{-1+x^2}\,dx \\ &= \frac{x}{2(1-x^2)} + \frac{1}{2}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{1}{(-1+x^2)^2}\,dx = \frac{1}{4}\left(-\frac{2x}{-1+x^2} - \log(1-x) + \log(1+x)\right)$$

```
[In] Integrate[(-1 + x^2)^(-2),x]
[Out] ((-2*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

method	result	size
meijerg	$-\frac{i\left(\frac{2ix}{-2x^2+2}+i\,\text{arctanh}(x)\right)}{2}$	23
norman	$-\frac{x}{2(x^2-1)} - \frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4}$	24
risch	$-\frac{x}{2(x^2-1)} - \frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4}$	24
default	$-\frac{1}{4(x+1)} + \frac{\ln(x+1)}{4} - \frac{1}{4(x-1)} - \frac{\ln(x-1)}{4}$	28
parallelrisch	$-\frac{\ln(x-1)x^2-\ln(x+1)x^2-\ln(x-1)+\ln(x+1)+2x}{4(x^2-1)}$	41

```
[In] int(1/(x^2-1)^2,x,method=_RETURNVERBOSE)
[Out] -1/2*I*(2*I*x/(-2*x^2+2)+I*arctanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{1}{(-1+x^2)^2}\,dx = \frac{(x^2-1)\log(x+1) - (x^2-1)\log(x-1) - 2x}{4(x^2-1)}$$

```
[In] integrate(1/(x^2-1)^2,x, algorithm="fricas")
[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{(-1+x^2)^2}\,dx = -\frac{x}{2x^2-2} - \frac{\log(x-1)}{4} + \frac{\log(x+1)}{4}$$

```
[In] integrate(1/(x**2-1)**2,x)
[Out] -x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{(-1+x^2)^2}\,dx = -\frac{x}{2(x^2-1)} + \frac{1}{4}\log(x+1) - \frac{1}{4}\log(x-1)$$

```
[In] integrate(1/(x^2-1)^2,x, algorithm="maxima")
[Out] -1/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{(-1+x^2)^2}\,dx = -\frac{x}{2(x^2-1)} + \frac{1}{4}\log(|x+1|) - \frac{1}{4}\log(|x-1|)$$

```
[In] integrate(1/(x^2-1)^2,x, algorithm="giac")
[Out] -1/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{(-1+x^2)^2}\,dx = \frac{\operatorname{atanh}(x)}{2} - \frac{x}{2\,(x^2-1)}$$

```
[In] int(1/(x^2 - 1)^2,x)
[Out] atanh(x)/2 - x/(2*(x^2 - 1))
```

3.456 $\int \frac{x^2}{(1+x^2)^2} dx$

Optimal result 2740
Rubi [A] (verified) 2740
Mathematica [A] (verified) 2741
Maple [A] (verified) 2741
Fricas [A] (verification not implemented) 2742
Sympy [A] (verification not implemented) 2742
Maxima [A] (verification not implemented) 2742
Giac [A] (verification not implemented) 2742
Mupad [B] (verification not implemented) 2743

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{x^2}{(1+x^2)^2} dx = -\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}$$

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {294, 209}

$$\int \frac{x^2}{(1+x^2)^2} dx = \frac{\arctan(x)}{2} - \frac{x}{2(x^2+1)}$$

```
[In] Int[x^2/(1 + x^2)^2,x]
```

```
[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{x}{2(1+x^2)} + \frac{1}{2}\int \frac{1}{1+x^2}\,dx \\ &= -\frac{x}{2(1+x^2)} + \frac{1}{2}\tan^{-1}(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+x^2)^2}\,dx = -\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}$$

```
[In] Integrate[x^2/(1 + x^2)^2,x]
[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
meijerg	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
risch	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(x+i)x^2+i\ln(x-i)-i\ln(x+i)+2x}{4(x^2+1)}$	52

```
[In] int(x^2/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(1+x^2)^2}\,dx = \frac{(x^2+1)\arctan(x) - x}{2\,(x^2+1)}$$

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{(1+x^2)^2}\,dx = -\frac{x}{2x^2+2} + \frac{\operatorname{atan}(x)}{2}$$

```
[In] integrate(x**2/(x**2+1)**2,x)
[Out] -x/(2*x**2 + 2) + atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(1+x^2)^2}\,dx = -\frac{x}{2\,(x^2+1)} + \frac{1}{2}\arctan(x)$$

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(1+x^2)^2}\,dx = -\frac{x}{2\,(x^2+1)} + \frac{1}{2}\arctan(x)$$

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(1+x^2)^2}\,dx = \frac{\operatorname{atan}(x)}{2} - \frac{x}{2\,(x^2+1)}$$

```
[In] int(x^2/(x^2 + 1)^2,x)
[Out] atan(x)/2 - x/(2*(x^2 + 1))
```

3.457 $\int \frac{1}{2+3x} \, dx$

Optimal result 2744
Rubi [A] (verified) 2744
Mathematica [A] (verified) 2745
Maple [A] (verified) 2745
Fricas [A] (verification not implemented) 2745
Sympy [A] (verification not implemented) 2746
Maxima [A] (verification not implemented) 2746
Giac [A] (verification not implemented) 2746
Mupad [B] (verification not implemented) 2746

Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{2+3x} \, dx = \frac{1}{3} \log(2+3x)$$

```
[Out] 1/3*ln(2+3*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{31\}$

$$\int \frac{1}{2+3x} \, dx = \frac{1}{3} \log(3x+2)$$

```
[In] Int[(2 + 3*x)^(-1),x]
```

```
[Out] Log[2 + 3*x]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{3} \log(2+3x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+3x}\,dx = \frac{1}{3}\log(2+3x)$$

```
[In] Integrate[(2 + 3*x)^(-1),x]
[Out] Log[2 + 3*x]/3
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
parallelrisch	$\frac{\ln\left(x+\frac{2}{3}\right)}{3}$	7
default	$\frac{\ln(3x+2)}{3}$	9
norman	$\frac{\ln(3x+2)}{3}$	9
meijerg	$\frac{\ln\left(1+\frac{3x}{2}\right)}{3}$	9
risch	$\frac{\ln(3x+2)}{3}$	9

```
[In] int(1/(3*x+2),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(x+2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{2+3x}\,dx = \frac{1}{3}\,\log\left(3\,x+2\right)$$

```
[In] integrate(1/(2+3*x),x, algorithm="fricas")
[Out] 1/3*log(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{2+3x}\,dx = \frac{\log(3x+2)}{3}$$

```
[In] integrate(1/(2+3*x),x)
[Out] log(3*x + 2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{2+3x}\,dx = \frac{1}{3}\log(3\,x+2)$$

```
[In] integrate(1/(2+3*x),x, algorithm="maxima")
[Out] 1/3*log(3*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{2+3x}\,dx = \frac{1}{3}\log(|3\,x+2|)$$

```
[In] integrate(1/(2+3*x),x, algorithm="giac")
[Out] 1/3*log(abs(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{2+3x}\,dx = \frac{\ln\left(x+\frac{2}{3}\right)}{3}$$

```
[In] int(1/(3*x + 2),x)
[Out] log(x + 2/3)/3
```

3.458 $\int \frac{1}{a^2+x^2} dx$

Optimal result 2747
Rubi [A] (verified) 2747
Mathematica [A] (verified) 2748
Maple [A] (verified) 2748
Fricas [A] (verification not implemented) 2748
Sympy [C] (verification not implemented) 2749
Maxima [A] (verification not implemented) 2749
Giac [A] (verification not implemented) 2749
Mupad [B] (verification not implemented) 2750

Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{1}{a^2+x^2} dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

```
[Out] arctan(x/a)/a
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {209}

$$\int \frac{1}{a^2+x^2} dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

```
[In] Int[(a^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/a]/a
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+x^2}\,dx=\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

```
[In] Integrate[(a^2 + x^2)^(-1),x]
[Out] ArcTan[x/a]/a
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\arctan\left(\frac{x}{a}\right)}{a}$	11
risch	$\frac{\arctan\left(\frac{x}{a}\right)}{a}$	11
parallelrisch	$-\frac{i\ln(-ia+x)-i\ln(ia+x)}{2a}$	27

```
[In] int(1/(a^2+x^2),x,method=_RETURNVERBOSE)
[Out] arctan(x/a)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+x^2}\,dx=\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

```
[In] integrate(1/(a^2+x^2),x, algorithm="fricas")
[Out] arctan(x/a)/a
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{-\frac{i\log(-ia+x)}{2} + \frac{i\log(ia+x)}{2}}{a}$$

```
[In] integrate(1/(a**2+x**2),x)
[Out] (-I*log(-I*a + x)/2 + I*log(I*a + x)/2)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

```
[In] integrate(1/(a^2+x^2),x, algorithm="maxima")
[Out] arctan(x/a)/a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

```
[In] integrate(1/(a^2+x^2),x, algorithm="giac")
[Out] arctan(x/a)/a
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{\operatorname{atan}\left(\frac{x}{a}\right)}{a}$$

```
[In] int(1/(a^2 + x^2),x)
[Out] atan(x/a)/a
```

3.459 $\int \frac{1}{a+bx^2}\,dx$

Optimal result 2751
Rubi [A] (verified) 2751
Mathematica [A] (verified) 2752
Maple [A] (verified) 2752
Fricas [A] (verification not implemented) 2752
Sympy [B] (verification not implemented) 2753
Maxima [A] (verification not implemented) 2753
Giac [A] (verification not implemented) 2753
Mupad [B] (verification not implemented) 2754

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \frac{1}{a+bx^2}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {211}

$$\int \frac{1}{a+bx^2}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[In] Int[(a + b*x^2)^(-1),x]
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\text{integral}=\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[In] Integrate[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$-\frac{\ln(bx+\sqrt{-ab})}{2\sqrt{-ab}}+\frac{\ln(-bx+\sqrt{-ab})}{2\sqrt{-ab}}$	41

```
[In] int(1/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \frac{1}{a+bx^2}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab}\right]$$

```
[In] integrate(1/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{1}{a+bx^2}\,dx = -\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

```
[In] integrate(1/(b*x**2+a),x)
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{a+bx^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

```
[In] integrate(1/(b*x^2+a),x, algorithm="maxima")
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{a+bx^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

```
[In] integrate(1/(b*x^2+a),x, algorithm="giac")
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{a+bx^2}\,dx=\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[In] int(1/(a + b*x^2),x)
[Out] atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

3.460 $\int \frac{1}{2-x+x^2} \, dx$

Optimal result 2755
Rubi [A] (verified) 2755
Mathematica [A] (verified) 2756
Maple [A] (verified) 2756
Fricas [A] (verification not implemented) 2757
Sympy [A] (verification not implemented) 2757
Maxima [A] (verification not implemented) 2757
Giac [A] (verification not implemented) 2757
Mupad [B] (verification not implemented) 2758

Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \frac{1}{2-x+x^2} \, dx = -\frac{2\arctan\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] -2/7*arctan(1/7*(1-2*x)*7^(1/2))*7^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {632, 210}

$$\int \frac{1}{2-x+x^2} \, dx = -\frac{2\arctan\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[In] Int[(2 - x + x^2)^(-1),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[7]])/Sqrt[7]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx, x, -1+2x\right)\right)$$
$$= -\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2\arctan\left(\frac{-1+2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[In] Integrate[(2 - x + x^2)^(-1),x]
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{2\sqrt{7}\,\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)}{7}$	17
risch	$\frac{2\sqrt{7}\,\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)}{7}$	17

```
[In] int(1/(x^2-x+2),x,method=_RETURNVERBOSE)
[Out] 2/7*7^(1/2)*arctan(1/7*(2*x-1)*7^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2}{7}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x-1)\right)$$

```
[In] integrate(1/(x^2-x+2),x, algorithm="fricas")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x}{7}-\frac{\sqrt{7}}{7}\right)}{7}$$

```
[In] integrate(1/(x**2-x+2),x)
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x/7 - sqrt(7)/7)/7
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2}{7}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x-1)\right)$$

```
[In] integrate(1/(x^2-x+2),x, algorithm="maxima")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2}{7}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(2\,x-1)\right)$$

```
[In] integrate(1/(x^2-x+2),x, algorithm="giac")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{\sqrt{7}\,(2\,x-1)}{7}\right)}{7}$$

```
[In] int(1/(x^2 - x + 2),x)
[Out] (2*7^(1/2)*atan((7^(1/2)*(2*x - 1))/7))/7
```

3.461 $\int x^2(4-x^2)^2 \, dx$

Optimal result 2759
Rubi [A] (verified) 2759
Mathematica [A] (verified) 2760
Maple [A] (verified) 2760
Fricas [A] (verification not implemented) 2760
Sympy [A] (verification not implemented) 2761
Maxima [A] (verification not implemented) 2761
Giac [A] (verification not implemented) 2761
Mupad [B] (verification not implemented) 2761

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int x^2(4-x^2)^2 \, dx = \frac{16x^3}{3} - \frac{8x^5}{5} + \frac{x^7}{7}$$

```
[Out] 16/3*x^3-8/5*x^5+1/7*x^7
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {276}

$$\int x^2(4-x^2)^2 \, dx = \frac{x^7}{7} - \frac{8x^5}{5} + \frac{16x^3}{3}$$

```
[In] Int[x^2*(4 - x^2)^2,x]
```

```
[Out] (16*x^3)/3 - (8*x^5)/5 + x^7/7
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int (16x^2 - 8x^4 + x^6) \, dx \\ &= \frac{16x^3}{3} - \frac{8x^5}{5} + \frac{x^7}{7} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2(4-x^2)^2\,dx=\frac{16x^3}{3}-\frac{8x^5}{5}+\frac{x^7}{7}$$

```
[In] Integrate[x^2*(4 - x^2)^2,x]
[Out] (16*x^3)/3 - (8*x^5)/5 + x^7/7
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{16}{3}x^3-\frac{8}{5}x^5+\frac{1}{7}x^7$	17
norman	$\frac{16}{3}x^3-\frac{8}{5}x^5+\frac{1}{7}x^7$	17
risch	$\frac{16}{3}x^3-\frac{8}{5}x^5+\frac{1}{7}x^7$	17
parallelrisch	$\frac{16}{3}x^3-\frac{8}{5}x^5+\frac{1}{7}x^7$	17
gosper	$\frac{x^3(15x^4-168x^2+560)}{105}$	18

```
[In] int(x^2*(-x^2+4)^2,x,method=_RETURNVERBOSE)
[Out] 16/3*x^3-8/5*x^5+1/7*x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x^2(4-x^2)^2\,dx=\frac{1}{7}\,x^7-\frac{8}{5}\,x^5+\frac{16}{3}\,x^3$$

```
[In] integrate(x^2*(-x^2+4)^2,x, algorithm="fricas")
[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int x^2 (4 - x^2)^2 \, dx = \frac{x^7}{7} - \frac{8x^5}{5} + \frac{16x^3}{3}$$

```
[In] integrate(x**2*(-x**2+4)**2,x)
[Out] x**7/7 - 8*x**5/5 + 16*x**3/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x^2 (4 - x^2)^2 \, dx = \frac{1}{7} x^7 - \frac{8}{5} x^5 + \frac{16}{3} x^3$$

```
[In] integrate(x^2*(-x^2+4)^2,x, algorithm="maxima")
[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x^2 (4 - x^2)^2 \, dx = \frac{1}{7} x^7 - \frac{8}{5} x^5 + \frac{16}{3} x^3$$

```
[In] integrate(x^2*(-x^2+4)^2,x, algorithm="giac")
[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int x^2 (4 - x^2)^2 \, dx = \frac{x^3 (15 x^4 - 168 x^2 + 560)}{105}$$

```
[In] int(x^2*(x^2 - 4)^2,x)
[Out] (x^3*(15*x^4 - 168*x^2 + 560))/105
```

3.462 $\int x(1-x^3)^2\,dx$

Optimal result 2762
Rubi [A] (verified) 2762
Mathematica [A] (verified) 2763
Maple [A] (verified) 2763
Fricas [A] (verification not implemented) 2763
Sympy [A] (verification not implemented) 2764
Maxima [A] (verification not implemented) 2764
Giac [A] (verification not implemented) 2764
Mupad [B] (verification not implemented) 2764

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int x(1-x^3)^2\,dx=\frac{x^2}{2}-\frac{2x^5}{5}+\frac{x^8}{8}$$

```
[Out] 1/2*x^2-2/5*x^5+1/8*x^8
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {276}

$$\int x(1-x^3)^2\,dx=\frac{x^8}{8}-\frac{2x^5}{5}+\frac{x^2}{2}$$

```
[In] Int[x*(1 - x^3)^2,x]
```

```
[Out] x^2/2 - (2*x^5)/5 + x^8/8
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int (x-2x^4+x^7)\,dx\\&=\frac{x^2}{2}-\frac{2x^5}{5}+\frac{x^8}{8}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(1-x^3)^2\,dx=\frac{x^2}{2}-\frac{2x^5}{5}+\frac{x^8}{8}$$

```
[In] Integrate[x*(1 - x^3)^2,x]
[Out] x^2/2 - (2*x^5)/5 + x^8/8
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{1}{2}x^2-\frac{2}{5}x^5+\frac{1}{8}x^8$	17
norman	$\frac{1}{2}x^2-\frac{2}{5}x^5+\frac{1}{8}x^8$	17
risch	$\frac{1}{2}x^2-\frac{2}{5}x^5+\frac{1}{8}x^8$	17
parallelrisch	$\frac{1}{2}x^2-\frac{2}{5}x^5+\frac{1}{8}x^8$	17
gosper	$\frac{x^2(5x^6-16x^3+20)}{40}$	18

```
[In] int(x*(-x^3+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x^2-2/5*x^5+1/8*x^8
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x(1-x^3)^2\,dx=\frac{1}{8}\,x^8-\frac{2}{5}\,x^5+\frac{1}{2}\,x^2$$

```
[In] integrate(x*(-x^3+1)^2,x, algorithm="fricas")
[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int x(1-x^3)^2\,dx=\frac{x^8}{8}-\frac{2x^5}{5}+\frac{x^2}{2}$$

```
[In] integrate(x*(-x**3+1)**2,x)
[Out] x**8/8 - 2*x**5/5 + x**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x(1-x^3)^2\,dx=\frac{1}{8}\,x^8-\frac{2}{5}\,x^5+\frac{1}{2}\,x^2$$

```
[In] integrate(x*(-x^3+1)^2,x, algorithm="maxima")
[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x(1-x^3)^2\,dx=\frac{1}{8}\,x^8-\frac{2}{5}\,x^5+\frac{1}{2}\,x^2$$

```
[In] integrate(x*(-x^3+1)^2,x, algorithm="giac")
[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int x(1-x^3)^2\,dx=\frac{x^2\,(5\,x^6-16\,x^3+20)}{40}$$

```
[In] int(x*(x^3 - 1)^2,x)
[Out] (x^2*(5*x^6 - 16*x^3 + 20))/40
```

3.463 $\int \frac{-4+5x^2+x^3}{x^2} \, dx$

Optimal result . . . 2765
Rubi [A] (verified) . . . 2765
Mathematica [A] (verified) . . . 2766
Maple [A] (verified) . . . 2766
Fricas [A] (verification not implemented) . . . 2766
Sympy [A] (verification not implemented) . . . 2767
Maxima [A] (verification not implemented) . . . 2767
Giac [A] (verification not implemented) . . . 2767
Mupad [B] (verification not implemented) . . . 2767

Optimal result

Integrand size = 14, antiderivative size = 16

$$\int \frac{-4+5x^2+x^3}{x^2} \, dx = \frac{4}{x}+5x+\frac{x^2}{2}$$

```
[Out] 4/x+5*x+1/2*x^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = $\{14\}$

$$\int \frac{-4+5x^2+x^3}{x^2} \, dx = \frac{x^2}{2}+5x+\frac{4}{x}$$

```
[In] Int[(-4 + 5*x^2 + x^3)/x^2,x]
```

```
[Out] 4/x + 5*x + x^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\text{integral} = \int \left(5-\frac{4}{x^2}+x\right) \, dx$$
$$= \frac{4}{x}+5x+\frac{x^2}{2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx = \frac{4}{x}+5x+\frac{x^2}{2}$$

```
[In] Integrate[(-4 + 5*x^2 + x^3)/x^2,x]
[Out] 4/x + 5*x + x^2/2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{4}{x}+5x+\frac{x^2}{2}$	15
risch	$\frac{4}{x}+5x+\frac{x^2}{2}$	15
gosper	$\frac{x^3+10x^2+8}{2x}$	16
parallelrisch	$\frac{x^3+10x^2+8}{2x}$	16
norman	$\frac{\frac{1}{2}x^3+5x^2+4}{x}$	17

```
[In] int((x^3+5*x^2-4)/x^2,x,method=_RETURNVERBOSE)
[Out] 4/x+5*x+1/2*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx = \frac{x^3+10\,x^2+8}{2\,x}$$

```
[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="fricas")
[Out] 1/2*(x^3 + 10*x^2 + 8)/x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx = \frac{x^2}{2}+5x+\frac{4}{x}$$

```
[In] integrate((x**3+5*x**2-4)/x**2,x)
[Out] x**2/2 + 5*x + 4/x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx = \frac{1}{2}\,x^2+5\,x+\frac{4}{x}$$

```
[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="maxima")
[Out] 1/2*x^2 + 5*x + 4/x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx = \frac{1}{2}\,x^2+5\,x+\frac{4}{x}$$

```
[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="giac")
[Out] 1/2*x^2 + 5*x + 4/x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx = \frac{x^3+10\,x^2+8}{2\,x}$$

```
[In] int((5*x^2 + x^3 - 4)/x^2,x)
[Out] (10*x^2 + x^3 + 8)/(2*x)
```

3.464 $\int \frac{-1+x}{3-4x+3x^2}\,dx$

Optimal result 2768
Rubi [A] (verified) 2768
Mathematica [A] (verified) 2769
Maple [A] (verified) 2770
Fricas [A] (verification not implemented) 2770
Sympy [A] (verification not implemented) 2770
Maxima [A] (verification not implemented) 2771
Giac [A] (verification not implemented) 2771
Mupad [B] (verification not implemented) 2771

Optimal result

Integrand size = 16, antiderivative size = 37

$$\int \frac{-1+x}{3-4x+3x^2}\,dx=\frac{\arctan\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}+\frac{1}{6}\log\left(3-4x+3x^2\right)$$

```
[Out] 1/6*ln(3*x^2-4*x+3)+1/15*arctan(1/5*(2-3*x)*5^(1/2))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {648, 632, 210, 642}

$$\int \frac{-1+x}{3-4x+3x^2}\,dx=\frac{\arctan\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}+\frac{1}{6}\log\left(3x^2-4x+3\right)$$

```
[In] Int[(-1 + x)/(3 - 4*x + 3*x^2),x]
```

```
[Out] ArcTan[(2 - 3*x)/Sqrt[5]]/(3*Sqrt[5]) + Log[3 - 4*x + 3*x^2]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{6}\int \frac{-4+6x}{3-4x+3x^2}\,dx - \frac{1}{3}\int \frac{1}{3-4x+3x^2}\,dx\\ &= \frac{1}{6}\log\left(3-4x+3x^2\right) + \frac{2}{3}\text{Subst}\left(\int \frac{1}{-20-x^2}\,dx, x, -4+6x\right)\\ &= \frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}} + \frac{1}{6}\log\left(3-4x+3x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{-1+x}{3-4x+3x^2}\,dx = -\frac{\arctan\left(\frac{-2+3x}{\sqrt{5}}\right)}{3\sqrt{5}} + \frac{1}{6}\log\left(3-4x+3x^2\right)$$

```
[In] Integrate[(-1 + x)/(3 - 4*x + 3*x^2),x]
[Out] -1/3*ArcTan[(-2 + 3*x)/Sqrt[5]]/Sqrt[5] + Log[3 - 4*x + 3*x^2]/6
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln(3x^2-4x+3)}{6}-\frac{\sqrt{5}\,\arctan\left(\frac{(6x-4)\sqrt{5}}{10}\right)}{15}$	31
risch	$\frac{\ln(9x^2-12x+9)}{6}-\frac{\sqrt{5}\,\arctan\left(\frac{(3x-2)\sqrt{5}}{5}\right)}{15}$	31

```
[In] int((x-1)/(3*x^2-4*x+3),x,method=_RETURNVERBOSE)
[Out] 1/6*ln(3*x^2-4*x+3)-1/15*5^(1/2)*arctan(1/10*(6*x-4)*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{-1+x}{3-4x+3x^2}\,dx=-\frac{1}{15}\,\sqrt{5}\arctan\left(\frac{1}{5}\,\sqrt{5}(3\,x-2)\right)+\frac{1}{6}\,\log\left(3\,x^2-4\,x+3\right)$$

```
[In] integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="fricas")
[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{-1+x}{3-4x+3x^2}\,dx=\frac{\log\left(x^2-\frac{4x}{3}+1\right)}{6}-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{3\sqrt{5}x}{5}-\frac{2\sqrt{5}}{5}\right)}{15}$$

```
[In] integrate((-1+x)/(3*x**2-4*x+3),x)
[Out] log(x**2 - 4*x/3 + 1)/6 - sqrt(5)*atan(3*sqrt(5)*x/5 - 2*sqrt(5)/5)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{-1+x}{3-4x+3x^2}\,dx = -\frac{1}{15}\,\sqrt{5}\arctan\left(\frac{1}{5}\,\sqrt{5}(3\,x-2)\right)+\frac{1}{6}\,\log\left(3\,x^2-4\,x+3\right)$$

```
[In] integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="maxima")
[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{-1+x}{3-4x+3x^2}\,dx = -\frac{1}{15}\,\sqrt{5}\arctan\left(\frac{1}{5}\,\sqrt{5}(3\,x-2)\right)+\frac{1}{6}\,\log\left(3\,x^2-4\,x+3\right)$$

```
[In] integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="giac")
[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{-1+x}{3-4x+3x^2}\,dx = \frac{\ln\left(x^2-\frac{4\,x}{3}+1\right)}{6}-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{3\,\sqrt{5}\,x}{5}-\frac{2\,\sqrt{5}}{5}\right)}{15}$$

```
[In] int((x - 1)/(3*x^2 - 4*x + 3),x)
[Out] log(x^2 - (4*x)/3 + 1)/6 - (5^(1/2)*atan((3*5^(1/2)*x)/5 - (2*5^(1/2))/5))/
15
```

3.465 $\int (2+x^3)^2 \, dx$

Optimal result 2772
Rubi [A] (verified) 2772
Mathematica [A] (verified) 2773
Maple [A] (verified) 2773
Fricas [A] (verification not implemented) 2773
Sympy [A] (verification not implemented) 2774
Maxima [A] (verification not implemented) 2774
Giac [A] (verification not implemented) 2774
Mupad [B] (verification not implemented) 2774

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int (2+x^3)^2 \, dx = 4x + x^4 + \frac{x^7}{7}$$

```
[Out] 4*x+x^4+1/7*x^7
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {200}

$$\int (2+x^3)^2 \, dx = \frac{x^7}{7} + x^4 + 4x$$

```
[In] Int[(2 + x^3)^2,x]
```

```
[Out] 4*x + x^4 + x^7/7
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int (4+4x^3+x^6) \, dx \\ &= 4x + x^4 + \frac{x^7}{7} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(2+x^3\right)^2 dx = 4x + x^4 + \frac{x^7}{7}$$

```
[In] Integrate[(2 + x^3)^2,x]
[Out] 4*x + x^4 + x^7/7
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$4x + x^4 + \frac{1}{7}x^7$	13
default	$4x + x^4 + \frac{1}{7}x^7$	13
norman	$4x + x^4 + \frac{1}{7}x^7$	13
risch	$4x + x^4 + \frac{1}{7}x^7$	13
parallelrisch	$4x + x^4 + \frac{1}{7}x^7$	13

```
[In] int((x^3+2)^2,x,method=_RETURNVERBOSE)
[Out] 4*x+x^4+1/7*x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(2+x^3\right)^2 dx = \frac{1}{7}x^7 + x^4 + 4x$$

```
[In] integrate((x^3+2)^2,x, algorithm="fricas")
[Out] 1/7*x^7 + x^4 + 4*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(2+x^3\right)^2 dx = \frac{x^7}{7} + x^4 + 4x$$

```
[In] integrate((x**3+2)**2,x)
[Out] x**7/7 + x**4 + 4*x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(2+x^3\right)^2 dx = \frac{1}{7}x^7 + x^4 + 4x$$

```
[In] integrate((x^3+2)^2,x, algorithm="maxima")
[Out] 1/7*x^7 + x^4 + 4*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(2+x^3\right)^2 dx = \frac{1}{7}x^7 + x^4 + 4x$$

```
[In] integrate((x^3+2)^2,x, algorithm="giac")
[Out] 1/7*x^7 + x^4 + 4*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \left(2+x^3\right)^2 dx = \frac{x\left(x^6+7x^3+28\right)}{7}$$

```
[In] int((x^3 + 2)^2,x)
[Out] (x*(7*x^3 + x^6 + 28))/7
```

3.466 $\int \frac{-4+x^2}{2+x} \, dx$

Optimal result 2775
Rubi [A] (verified) 2775
Mathematica [A] (verified) 2776
Maple [A] (verified) 2776
Fricas [A] (verification not implemented) 2776
Sympy [A] (verification not implemented) 2777
Maxima [A] (verification not implemented) 2777
Giac [A] (verification not implemented) 2777
Mupad [B] (verification not implemented) 2777

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{-4+x^2}{2+x} \, dx = -2x + \frac{x^2}{2}$$

```
[Out] -2*x+1/2*x^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {641}

$$\int \frac{-4+x^2}{2+x} \, dx = \frac{x^2}{2} - 2x$$

```
[In] Int[(-4 + x^2)/(2 + x),x]
```

```
[Out] -2*x + x^2/2
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int (-2+x) \, dx \\ &= -2x + \frac{x^2}{2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-4+x^2}{2+x}\,dx = -2x+\frac{x^2}{2}$$

```
[In] Integrate[(-4 + x^2)/(2 + x),x]
[Out] -2*x + x^2/2
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x(x-4)}{2}$	7
meijerg	$-\frac{x\left(-\frac{3x}{2}+6\right)}{3}$	9
default	$-2x+\frac{1}{2}x^2$	10
norman	$-2x+\frac{1}{2}x^2$	10
risch	$-2x+\frac{1}{2}x^2$	10
parallelrisch	$-2x+\frac{1}{2}x^2$	10
parts	$-2x+\frac{1}{2}x^2$	10

```
[In] int((x^2-4)/(x+2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(x-4)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{-4+x^2}{2+x}\,dx = \frac{1}{2}\,x^2-2\,x$$

```
[In] integrate((x^2-4)/(2+x),x, algorithm="fricas")
[Out] 1/2*x^2 - 2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{-4+x^2}{2+x}\,dx = \frac{x^2}{2} - 2x$$

```
[In] integrate((x**2-4)/(2+x),x)
```

```
[Out] x**2/2 - 2*x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{-4+x^2}{2+x}\,dx = \frac{1}{2}\,x^2 - 2\,x$$

```
[In] integrate((x^2-4)/(2+x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 2*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{-4+x^2}{2+x}\,dx = \frac{1}{2}\,x^2 - 2\,x$$

```
[In] integrate((x^2-4)/(2+x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 2*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{-4+x^2}{2+x}\,dx = \frac{x\,(x-4)}{2}$$

```
[In] int((x^2 - 4)/(x + 2),x)
```

```
[Out] (x*(x - 4))/2
```

3.467 $\int \frac{1}{(2+x)(1+x^2)}\,dx$

Optimal result 2778
Rubi [A] (verified) 2778
Mathematica [A] (verified) 2779
Maple [A] (verified) 2780
Fricas [A] (verification not implemented) 2780
Sympy [A] (verification not implemented) 2780
Maxima [A] (verification not implemented) 2781
Giac [A] (verification not implemented) 2781
Mupad [B] (verification not implemented) 2781

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{2\arctan(x)}{5} + \frac{1}{5}\log(2+x) - \frac{1}{10}\log\left(1+x^2\right)$$

```
[Out] 2/5*arctan(x)+1/5*ln(2+x)-1/10*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {720, 31, 649, 209, 266}

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{2\arctan(x)}{5} - \frac{1}{10}\log\left(x^2+1\right) + \frac{1}{5}\log(x+2)$$

```
[In] Int[1/((2 + x)*(1 + x^2)),x]
```

```
[Out] (2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{5}\int \frac{1}{2+x}\,dx + \frac{1}{5}\int \frac{2-x}{1+x^2}\,dx\\ &= \frac{1}{5}\log(2+x) - \frac{1}{5}\int \frac{x}{1+x^2}\,dx + \frac{2}{5}\int \frac{1}{1+x^2}\,dx\\ &= \frac{2}{5}\tan^{-1}(x) + \frac{1}{5}\log(2+x) - \frac{1}{10}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+x)\left(1+x^2\right)}\,dx = \frac{2\arctan(x)}{5} + \frac{1}{5}\log(2+x) - \frac{1}{10}\log\left(1+x^2\right)$$

```
[In] Integrate[1/((2 + x)*(1 + x^2)),x]
[Out] (2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{2\arctan(x)}{5}+\frac{\ln(x+2)}{5}-\frac{\ln(x^2+1)}{10}$	20
risch	$\frac{2\arctan(x)}{5}+\frac{\ln(x+2)}{5}-\frac{\ln(x^2+1)}{10}$	20
parallelrisch	$\frac{\ln(x+2)}{5}-\frac{\ln(x-i)}{10}-\frac{i\ln(x-i)}{5}-\frac{\ln(x+i)}{10}+\frac{i\ln(x+i)}{5}$	38

```
[In] int(1/(x+2)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 2/5*arctan(x)+1/5*ln(x+2)-1/10*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{2}{5}\arctan(x) - \frac{1}{10}\log\left(x^2+1\right) + \frac{1}{5}\log(x+2)$$

```
[In] integrate(1/(2+x)/(x^2+1),x, algorithm="fricas")
[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{\log(x+2)}{5} - \frac{\log(x^2+1)}{10} + \frac{2\,\mathrm{atan}(x)}{5}$$

```
[In] integrate(1/(2+x)/(x**2+1),x)
[Out] log(x + 2)/5 - log(x**2 + 1)/10 + 2*atan(x)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{2}{5}\arctan(x) - \frac{1}{10}\log\left(x^2+1\right) + \frac{1}{5}\log(x+2)$$

```
[In] integrate(1/(2+x)/(x^2+1),x, algorithm="maxima")
[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{2}{5}\arctan(x) - \frac{1}{10}\log\left(x^2+1\right) + \frac{1}{5}\log(|x+2|)$$

```
[In] integrate(1/(2+x)/(x^2+1),x, algorithm="giac")
[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(abs(x + 2))
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{\ln(x+2)}{5} + \ln(x-\mathrm{i})\left(-\frac{1}{10}-\frac{1}{5}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{10}+\frac{1}{5}\mathrm{i}\right)$$

```
[In] int(1/((x^2 + 1)*(x + 2)),x)
[Out] log(x + 2)/5 - log(x - 1i)*(1/10 + 1i/5) - log(x + 1i)*(1/10 - 1i/5)
```

3.468 $\int \frac{1}{(1+x)(1+x^2)} dx$

Optimal result 2782
Rubi [A] (verified) 2782
Mathematica [A] (verified) 2783
Maple [A] (verified) 2784
Fricas [A] (verification not implemented) 2784
Sympy [A] (verification not implemented) 2784
Maxima [A] (verification not implemented) 2785
Giac [A] (verification not implemented) 2785
Mupad [B] (verification not implemented) 2785

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{1}{(1+x)(1+x^2)} dx = \frac{\arctan(x)}{2} + \frac{1}{2}\log(1+x) - \frac{1}{4}\log\left(1+x^2\right)$$

```
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {720, 31, 649, 209, 266}

$$\int \frac{1}{(1+x)(1+x^2)} dx = \frac{\arctan(x)}{2} - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1)$$

```
[In] Int[1/((1 + x)*(1 + x^2)),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int \frac{1}{1+x}\,dx+\frac{1}{2}\int \frac{1-x}{1+x^2}\,dx\\ &= \frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx-\frac{1}{2}\int \frac{x}{1+x^2}\,dx\\ &= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)\left(1+x^2\right)}\,dx=\frac{\arctan(x)}{2}+\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)$$

```
[In] Integrate[1/((1 + x)*(1 + x^2)),x]
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{2}-\frac{\ln(x^2+1)}{4}$	20
risch	$\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{2}-\frac{\ln(x^2+1)}{4}$	20
parallelrisch	$\frac{\ln(x+1)}{2}-\frac{\ln(x-i)}{4}-\frac{i\ln(x-i)}{4}-\frac{\ln(x+i)}{4}+\frac{i\ln(x+i)}{4}$	38

```
[In] int(1/(x+1)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x)+1/2*ln(x+1)-1/4*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1+x)(1+x^2)}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1)$$

```
[In] integrate(1/(1+x)/(x^2+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1+x)(1+x^2)}\,dx = \frac{\log(x+1)}{2} - \frac{\log(x^2+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

```
[In] integrate(1/(1+x)/(x**2+1),x)
[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1+x)(1+x^2)}\,dx = \frac{1}{2}\arctan\left(x\right) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log\left(x+1\right)$$

```
[In] integrate(1/(1+x)/(x^2+1),x, algorithm="maxima")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{(1+x)(1+x^2)}\,dx = \frac{1}{2}\arctan\left(x\right) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log\left(|x+1|\right)$$

```
[In] integrate(1/(1+x)/(x^2+1),x, algorithm="giac")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)(1+x^2)}\,dx = \frac{\ln\left(x+1\right)}{2} + \ln\left(x-\mathrm{i}\right)\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

```
[In] int(1/((x^2 + 1)*(x + 1)),x)
[Out] log(x + 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)
```

3.469 $\int \frac{x}{(1+x)(1+x^2)} dx$

Optimal result 2786
Rubi [A] (verified) 2786
Mathematica [A] (verified) 2787
Maple [A] (verified) 2787
Fricas [A] (verification not implemented) 2788
Sympy [A] (verification not implemented) 2788
Maxima [A] (verification not implemented) 2788
Giac [A] (verification not implemented) 2789
Mupad [B] (verification not implemented) 2789

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{x}{(1+x)(1+x^2)} dx = \frac{\arctan(x)}{2} - \frac{1}{2}\log(1+x) + \frac{1}{4}\log(1+x^2)$$

```
[Out] 1/2*arctan(x)-1/2*ln(1+x)+1/4*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {815, 649, 209, 266}

$$\int \frac{x}{(1+x)(1+x^2)} dx = \frac{\arctan(x)}{2} + \frac{1}{4}\log(x^2+1) - \frac{1}{2}\log(x+1)$$

```
[In] Int[x/((1 + x)*(1 + x^2)),x]
```

```
[Out] ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2(1+x)}+\frac{1+x}{2\left(1+x^2\right)}\right) dx \\
&= -\frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1+x}{1+x^2}\,dx \\
&= -\frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{1}{2}\int \frac{x}{1+x^2}\,dx \\
&= \frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\log(1+x)+\frac{1}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)\left(1+x^2\right)}\,dx=\frac{\arctan(x)}{2}-\frac{1}{2}\log(1+x)+\frac{1}{4}\log\left(1+x^2\right)$$

```
[In] Integrate[x/((1 + x)*(1 + x^2)),x]
[Out] ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\arctan(x)}{2}-\frac{\ln(x+1)}{2}+\frac{\ln(x^2+1)}{4}$	20
risch	$\frac{\arctan(x)}{2}-\frac{\ln(x+1)}{2}+\frac{\ln(x^2+1)}{4}$	20
parallelrisch	$-\frac{\ln(x+1)}{2}+\frac{\ln(x-i)}{4}-\frac{i\ln(x-i)}{4}+\frac{\ln(x+i)}{4}+\frac{i\ln(x+i)}{4}$	38

```
[In] int(x/(x+1)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x)-1/2*ln(x+1)+1/4*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{x}{(1+x)(1+x^2)}\,dx = \frac{1}{2}\arctan(x) + \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log(x+1)$$

```
[In] integrate(x/(1+x)/(x^2+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{x}{(1+x)(1+x^2)}\,dx = -\frac{\log(x+1)}{2} + \frac{\log(x^2+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

```
[In] integrate(x/(1+x)/(x**2+1),x)
[Out] -log(x + 1)/2 + log(x**2 + 1)/4 + atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{x}{(1+x)(1+x^2)}\,dx = \frac{1}{2}\arctan(x) + \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log(x+1)$$

```
[In] integrate(x/(1+x)/(x^2+1),x, algorithm="maxima")
[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x)(1+x^2)}\,dx = \frac{1}{2}\arctan(x) + \frac{1}{4}\log(x^2+1) - \frac{1}{2}\log(|x+1|)$$

```
[In] integrate(x/(1+x)/(x^2+1),x, algorithm="giac")
[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)(1+x^2)}\,dx = -\frac{\ln(x+1)}{2} + \ln(x-\mathrm{i})\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

```
[In] int(x/((x^2 + 1)*(x + 1)),x)
[Out] log(x - 1i)*(1/4 - 1i/4) - log(x + 1)/2 + log(x + 1i)*(1/4 + 1i/4)
```

3.470 $\int \frac{2x+x^2}{(1+x)^2} dx$

Optimal result 2790
Rubi [A] (verified) 2790
Mathematica [A] (verified) 2791
Maple [A] (verified) 2791
Fricas [A] (verification not implemented) 2791
Sympy [A] (verification not implemented) 2792
Maxima [A] (verification not implemented) 2792
Giac [A] (verification not implemented) 2792
Mupad [B] (verification not implemented) 2792

Optimal result

Integrand size = 13, antiderivative size = 9

$$\int \frac{2x+x^2}{(1+x)^2} dx = \frac{x^2}{1+x}$$

```
[Out] x^2/(1+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {697}

$$\int \frac{2x+x^2}{(1+x)^2} dx = x+\frac{1}{x+1}$$

```
[In] Int[(2*x + x^2)/(1 + x)^2,x]
```

```
[Out] x + (1 + x)^(-1)
```

Rule 697

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(1-\frac{1}{(1+x)^2}\right) dx \\ &= x+\frac{1}{1+x} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{2x+x^2}{(1+x)^2}\,dx = x+\frac{1}{1+x}$$

```
[In] Integrate[(2*x + x^2)/(1 + x)^2,x]
```

```
[Out] x + (1 + x)^(-1)
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
default	$x+\frac{1}{x+1}$	8
risch	$x+\frac{1}{x+1}$	8
gosper	$\frac{x^2}{x+1}$	10
norman	$\frac{x^2}{x+1}$	10
parallelrisch	$\frac{x^2}{x+1}$	10
meijerg	$\frac{x(6+3x)}{3x+3}-\frac{2x}{x+1}$	23

```
[In] int((x^2+2*x)/(x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x+1/(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{2x+x^2}{(1+x)^2}\,dx = \frac{x^2+x+1}{x+1}$$

```
[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] (x^2 + x + 1)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int \frac{2x+x^2}{(1+x)^2}\,dx = x + \frac{1}{x+1}$$

```
[In] integrate((x**2+2*x)/(1+x)**2,x)
[Out] x + 1/(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{2x+x^2}{(1+x)^2}\,dx = x + \frac{1}{x+1}$$

```
[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="maxima")
[Out] x + 1/(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{2x+x^2}{(1+x)^2}\,dx = x + \frac{1}{x+1} + 1$$

```
[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="giac")
[Out] x + 1/(x + 1) + 1
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{2x+x^2}{(1+x)^2}\,dx = x + \frac{1}{x+1}$$

```
[In] int((2*x + x^2)/(x + 1)^2,x)
[Out] x + 1/(x + 1)
```

3.471 $\int \frac{-10+x^2}{4+9x^2+2x^4} \, dx$

Optimal result 2793
Rubi [A] (verified) 2793
Mathematica [A] (verified) 2794
Maple [A] (verified) 2794
Fricas [A] (verification not implemented) 2795
Sympy [A] (verification not implemented) 2795
Maxima [A] (verification not implemented) 2795
Giac [A] (verification not implemented) 2795
Mupad [B] (verification not implemented) 2796

Optimal result

Integrand size = 20, antiderivative size = 22

$$\int \frac{-10+x^2}{4+9x^2+2x^4} \, dx = \arctan\left(\frac{x}{2}\right) - \frac{3\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] arctan(1/2*x)-3/2*arctan(x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1180, 209}

$$\int \frac{-10+x^2}{4+9x^2+2x^4} \, dx = \arctan\left(\frac{x}{2}\right) - \frac{3\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[In] Int[(-10 + x^2)/(4 + 9*x^2 + 2*x^4),x]
```

```
[Out] ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\left(3\int \frac{1}{1+2x^2}\,dx\right) + 4\int \frac{1}{8+2x^2}\,dx$$
$$= \tan^{-1}\left(\frac{x}{2}\right) - \frac{3\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx = \arctan\left(\frac{x}{2}\right) - \frac{3\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[In] Integrate[(-10 + x^2)/(4 + 9*x^2 + 2*x^4),x]
[Out] ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
default	$\arctan\left(\frac{x}{2}\right) - \frac{3\arctan\left(x\sqrt{2}\right)\sqrt{2}}{2}$	17
risch	$\arctan\left(\frac{x}{2}\right) - \frac{3\arctan\left(x\sqrt{2}\right)\sqrt{2}}{2}$	17

```
[In] int((x^2-10)/(2*x^4+9*x^2+4),x,method=_RETURNVERBOSE)
[Out] arctan(1/2*x)-3/2*arctan(x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx = -\frac{3}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right) + \arctan\left(\frac{1}{2}x\right)$$

```
[In] integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="fricas")
[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx = \operatorname{atan}\left(\frac{x}{2}\right) - \frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}x\right)}{2}$$

```
[In] integrate((x**2-10)/(2*x**4+9*x**2+4),x)
[Out] atan(x/2) - 3*sqrt(2)*atan(sqrt(2)*x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx = -\frac{3}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right) + \arctan\left(\frac{1}{2}x\right)$$

```
[In] integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="maxima")
[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx = -\frac{3}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right) + \arctan\left(\frac{1}{2}x\right)$$

```
[In] integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="giac")
[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx = \operatorname{atan}\left(\frac{x}{2}\right) - \frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x\right)}{2}$$

```
[In] int((x^2 - 10)/(9*x^2 + 2*x^4 + 4),x)
[Out] atan(x/2) - (3*2^(1/2)*atan(2^(1/2)*x))/2
```

3.472 $\int \frac{31+5x}{11-4x+3x^2}\,dx$

Optimal result 2797
Rubi [A] (verified) 2797
Mathematica [A] (verified) 2798
Maple [A] (verified) 2799
Fricas [A] (verification not implemented) 2799
Sympy [A] (verification not implemented) 2799
Maxima [A] (verification not implemented) 2800
Giac [A] (verification not implemented) 2800
Mupad [B] (verification not implemented) 2800

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{31+5x}{11-4x+3x^2}\,dx=-\frac{103\arctan\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}+\frac{5}{6}\log\left(11-4x+3x^2\right)$$

```
[Out] 5/6*ln(3*x^2-4*x+11)-103/87*arctan(1/29*(2-3*x)*29^(1/2))*29^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {648, 632, 210, 642}

$$\int \frac{31+5x}{11-4x+3x^2}\,dx=\frac{5}{6}\log\left(3x^2-4x+11\right)-\frac{103\arctan\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}$$

```
[In] Int[(31 + 5*x)/(11 - 4*x + 3*x^2),x]
```

```
[Out] (-103*ArcTan[(2 - 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x + 3*x^2])/
6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{5}{6}\int \frac{-4+6x}{11-4x+3x^2}\,dx + \frac{103}{3}\int \frac{1}{11-4x+3x^2}\,dx \\
&= \frac{5}{6}\log\left(11-4x+3x^2\right) - \frac{206}{3}\text{Subst}\left(\int \frac{1}{-116-x^2}\,dx, x, -4+6x\right) \\
&= -\frac{103\tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}} + \frac{5}{6}\log\left(11-4x+3x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{31+5x}{11-4x+3x^2}\,dx = \frac{103\arctan\left(\frac{-2+3x}{\sqrt{29}}\right)}{3\sqrt{29}} + \frac{5}{6}\log\left(11-4x+3x^2\right)$$

```
[In] Integrate[(31 + 5*x)/(11 - 4*x + 3*x^2),x]
[Out] (103*ArcTan[(-2 + 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x + 3*x^2])/
6
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{5 \ln \left(3 x^{2}-4 x+11\right)}{6}+\frac{103 \sqrt{29}\, \arctan \left(\frac{(6 x-4) \sqrt{29}}{58}\right)}{87}$	31
risch	$\frac{5 \ln \left(9 x^{2}-12 x+33\right)}{6}+\frac{103 \sqrt{29}\, \arctan \left(\frac{(3 x-2) \sqrt{29}}{29}\right)}{87}$	31

```
[In] int((31+5*x)/(3*x^2-4*x+11),x,method=_RETURNVERBOSE)
[Out] 5/6*ln(3*x^2-4*x+11)+103/87*29^(1/2)*arctan(1/58*(6*x-4)*29^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{31+5x}{11-4x+3x^2}\,dx = \frac{103}{87}\,\sqrt{29}\arctan\left(\frac{1}{29}\,\sqrt{29}(3\,x-2)\right)+\frac{5}{6}\,\log\left(3\,x^2-4\,x+11\right)$$

```
[In] integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="fricas")
[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{31+5x}{11-4x+3x^2}\,dx = \frac{5\log\left(x^2-\frac{4x}{3}+\frac{11}{3}\right)}{6}+\frac{103\sqrt{29}\,\mathrm{atan}\left(\frac{3\sqrt{29}x}{29}-\frac{2\sqrt{29}}{29}\right)}{87}$$

```
[In] integrate((31+5*x)/(3*x**2-4*x+11),x)
[Out] 5*log(x**2 - 4*x/3 + 11/3)/6 + 103*sqrt(29)*atan(3*sqrt(29)*x/29 - 2*sqrt(2
9)/29)/87
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{31+5x}{11-4x+3x^2}\,dx = \frac{103}{87}\sqrt{29}\arctan\left(\frac{1}{29}\sqrt{29}(3\,x-2)\right)+\frac{5}{6}\log\left(3\,x^2-4\,x+11\right)$$

```
[In] integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="maxima")
[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{31+5x}{11-4x+3x^2}\,dx = \frac{103}{87}\sqrt{29}\arctan\left(\frac{1}{29}\sqrt{29}(3\,x-2)\right)+\frac{5}{6}\log\left(3\,x^2-4\,x+11\right)$$

```
[In] integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="giac")
[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{31+5x}{11-4x+3x^2}\,dx = \frac{5\,\ln\left(x^2-\frac{4\,x}{3}+\frac{11}{3}\right)}{6}+\frac{103\,\sqrt{29}\,\mathrm{atan}\left(\frac{3\,\sqrt{29}\,x}{29}-\frac{2\,\sqrt{29}}{29}\right)}{87}$$

```
[In] int((5*x + 31)/(3*x^2 - 4*x + 11),x)
[Out] (5*log(x^2 - (4*x)/3 + 11/3))/6 + (103*29^(1/2)*atan((3*29^(1/2)*x)/29 - (2
*29^(1/2))/29))/87
```

3.473 $\int \frac{-2+x^2+x^3}{x^4} \, dx$

Optimal result 2801
Rubi [A] (verified) 2801
Mathematica [A] (verified) 2802
Maple [A] (verified) 2802
Fricas [A] (verification not implemented) 2802
Sympy [A] (verification not implemented) 2803
Maxima [A] (verification not implemented) 2803
Giac [A] (verification not implemented) 2803
Mupad [B] (verification not implemented) 2803

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \frac{-2+x^2+x^3}{x^4} \, dx = \frac{2}{3x^3}-\frac{1}{x}+\log(x)$$

```
[Out] 2/3/x^3-1/x+ln(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {14}

$$\int \frac{-2+x^2+x^3}{x^4} \, dx = \frac{2}{3x^3}-\frac{1}{x}+\log(x)$$

```
[In] Int[(-2 + x^2 + x^3)/x^4,x]
```

```
[Out] 2/(3*x^3) - x^(-1) + Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-\frac{2}{x^4}+\frac{1}{x^2}+\frac{1}{x}\right) \, dx \\ &= \frac{2}{3x^3}-\frac{1}{x}+\log(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{-2+x^2+x^3}{x^4}\,dx=\frac{2}{3x^3}-\frac{1}{x}+\log(x)$$

```
[In] Integrate[(-2 + x^2 + x^3)/x^4,x]
[Out] 2/(3*x^3) - x^(-1) + Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{2}{3x^3}-\frac{1}{x}+\ln(x)$	14
norman	$\frac{\frac{2}{3}-x^2}{x^3}+\ln(x)$	15
risch	$\frac{\frac{2}{3}-x^2}{x^3}+\ln(x)$	15
parallelrisch	$\frac{3\ln(x)x^3+2-3x^2}{3x^3}$	20

```
[In] int((x^3+x^2-2)/x^4,x,method=_RETURNVERBOSE)
[Out] 2/3/x^3-1/x+ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{-2+x^2+x^3}{x^4}\,dx=\frac{3\,x^3\log(x)-3\,x^2+2}{3\,x^3}$$

```
[In] integrate((x^3+x^2-2)/x^4,x, algorithm="fricas")
[Out] 1/3*(3*x^3*log(x) - 3*x^2 + 2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{-2+x^2+x^3}{x^4}\,dx = \log(x) + \frac{2-3x^2}{3x^3}$$

```
[In] integrate((x**3+x**2-2)/x**4,x)
[Out] log(x) + (2 - 3*x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{-2+x^2+x^3}{x^4}\,dx = -\frac{3\,x^2-2}{3\,x^3} + \log(x)$$

```
[In] integrate((x^3+x^2-2)/x^4,x, algorithm="maxima")
[Out] -1/3*(3*x^2 - 2)/x^3 + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{-2+x^2+x^3}{x^4}\,dx = -\frac{3\,x^2-2}{3\,x^3} + \log(|x|)$$

```
[In] integrate((x^3+x^2-2)/x^4,x, algorithm="giac")
[Out] -1/3*(3*x^2 - 2)/x^3 + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-2+x^2+x^3}{x^4}\,dx = \ln(x) - \frac{x^2-\frac{2}{3}}{x^3}$$

```
[In] int((x^2 + x^3 - 2)/x^4,x)
[Out] log(x) - (x^2 - 2/3)/x^3
```

3.474 $\int \frac{1+x+x^3}{x^2} dx$

Optimal result 2804
Rubi [A] (verified) 2804
Mathematica [A] (verified) 2805
Maple [A] (verified) 2805
Fricas [A] (verification not implemented) 2805
Sympy [A] (verification not implemented) 2806
Maxima [A] (verification not implemented) 2806
Giac [A] (verification not implemented) 2806
Mupad [B] (verification not implemented) 2806

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \frac{1+x+x^3}{x^2} dx = -\frac{1}{x}+\frac{x^2}{2}+\log(x)$$

```
[Out] -1/x+1/2*x^2+ln(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {14}

$$\int \frac{1+x+x^3}{x^2} dx = \frac{x^2}{2}-\frac{1}{x}+\log(x)$$

```
[In] Int[(1 + x + x^3)/x^2,x]
```

```
[Out] -x^(-1) + x^2/2 + Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{x^2}+\frac{1}{x}+x\right) dx \\ &= -\frac{1}{x}+\frac{x^2}{2}+\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^3}{x^2} \, dx=-\frac{1}{x}+\frac{x^2}{2}+\log(x)$$

```
[In] Integrate[(1 + x + x^3)/x^2,x]
[Out] -x^(-1) + x^2/2 + Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{x}+\frac{x^{2}}{2}+\ln \left(x \right)$	14
risch	$-\frac{1}{x}+\frac{x^{2}}{2}+\ln \left(x \right)$	14
norman	$\frac{-1+\frac{x^{3}}{2}}{x}+\ln \left(x \right)$	15
parallelrisch	$\frac{x^{3}+2 \ln \left(x \right) x -2}{2 x}$	16

```
[In] int((x^3+x+1)/x^2,x,method=_RETURNVERBOSE)
[Out] -1/x+1/2*x^2+ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^3}{x^2} \, dx=\frac{x^{3} + 2 \, x \log \left(x\right) - 2}{2 \, x}$$

```
[In] integrate((x^3+x+1)/x^2,x, algorithm="fricas")
[Out] 1/2*(x^3 + 2*x*log(x) - 2)/x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1+x+x^3}{x^2}\,dx = \frac{x^2}{2} + \log{(x)} - \frac{1}{x}$$

```
[In] integrate((x**3+x+1)/x**2,x)
[Out] x**2/2 + log(x) - 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1+x+x^3}{x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{x} + \log\left(x\right)$$

```
[In] integrate((x^3+x+1)/x^2,x, algorithm="maxima")
[Out] 1/2*x^2 - 1/x + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1+x+x^3}{x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{x} + \log\left(|x|\right)$$

```
[In] integrate((x^3+x+1)/x^2,x, algorithm="giac")
[Out] 1/2*x^2 - 1/x + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1+x+x^3}{x^2}\,dx = \ln\left(x\right) - \frac{1}{x} + \frac{x^2}{2}$$

```
[In] int((x + x^3 + 1)/x^2,x)
[Out] log(x) - 1/x + x^2/2
```

3.475 $\int \frac{-2+x^2}{x(2+x^2)} \, dx$

Optimal result 2807
Rubi [A] (verified) 2807
Mathematica [A] (verified) 2808
Maple [A] (verified) 2808
Fricas [A] (verification not implemented) 2809
Sympy [A] (verification not implemented) 2809
Maxima [A] (verification not implemented) 2809
Giac [A] (verification not implemented) 2809
Mupad [B] (verification not implemented) 2810

Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \frac{-2+x^2}{x(2+x^2)} \, dx = -\log(x) + \log\left(2+x^2\right)$$

```
[Out] -ln(x)+ln(x^2+2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {457, 78}

$$\int \frac{-2+x^2}{x(2+x^2)} \, dx = \log\left(x^2+2\right) - \log(x)$$

```
[In] Int[(-2 + x^2)/(x*(2 + x^2)),x]
```

```
[Out] -Log[x] + Log[2 + x^2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{-2+x}{x(2+x)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x}+\frac{2}{2+x}\right)\,dx, x, x^2\right)\\ &= -\log(x)+\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-2+x^2}{x\left(2+x^2\right)}\,dx = -\log(x)+\log\left(2+x^2\right)$$

```
[In] Integrate[(-2 + x^2)/(x*(2 + x^2)),x]
[Out] -Log[x] + Log[2 + x^2]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$-\ln(x)+\ln(x^2+2)$	12
norman	$-\ln(x)+\ln(x^2+2)$	12
risch	$-\ln(x)+\ln(x^2+2)$	12
parallelrisch	$-\ln(x)+\ln(x^2+2)$	12
meijerg	$\ln\left(1+\frac{x^2}{2}\right)-\ln(x)+\frac{\ln(2)}{2}$	18

```
[In] int((x^2-2)/x/(x^2+2),x,method=_RETURNVERBOSE)
[Out] -ln(x)+ln(x^2+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-2+x^2}{x(2+x^2)} \, dx = \log(x^2+2) - \log(x)$$

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="fricas")
[Out] log(x^2 + 2) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{-2+x^2}{x(2+x^2)} \, dx = -\log(x) + \log(x^2+2)$$

```
[In] integrate((x**2-2)/x/(x**2+2),x)
[Out] -log(x) + log(x**2 + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{-2+x^2}{x(2+x^2)} \, dx = \log(x^2+2) - \frac{1}{2} \log(x^2)$$

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="maxima")
[Out] log(x^2 + 2) - 1/2*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{-2+x^2}{x(2+x^2)} \, dx = \log(x^2+2) - \frac{1}{2} \log(x^2)$$

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="giac")
[Out] log(x^2 + 2) - 1/2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-2+x^2}{x\,(2+x^2)}\,dx = \ln\left(x^2+2\right) - \ln\left(x\right)$$

```
[In] int((x^2 - 2)/(x*(x^2 + 2)),x)
[Out] log(x^2 + 2) - log(x)
```

3.476 $\int (-3+x) (-7+4x^2) \, dx$

Optimal result 2811
Rubi [A] (verified) 2811
Mathematica [A] (verified) 2812
Maple [A] (verified) 2812
Fricas [A] (verification not implemented) 2812
Sympy [A] (verification not implemented) 2813
Maxima [A] (verification not implemented) 2813
Giac [A] (verification not implemented) 2813
Mupad [B] (verification not implemented) 2813

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int (-3+x) (-7+4x^2) \, dx = 21x - 4x^3 + \frac{1}{16}(7-4x^2)^2$$

```
[Out] 21*x-4*x^3+1/16*(-4*x^2+7)^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {655}

$$\int (-3+x) (-7+4x^2) \, dx = -4x^3 + \frac{1}{16}(7-4x^2)^2 + 21x$$

```
[In] Int[(-3 + x)*(-7 + 4*x^2),x]
```

```
[Out] 21*x - 4*x^3 + (7 - 4*x^2)^2/16
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{16}(7-4x^2)^2 - 3\int (-7+4x^2) \, dx \\ &= 21x - 4x^3 + \frac{1}{16}(7-4x^2)^2 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int (-3+x) \left(-7+4x^2\right) \, dx = 21x - \frac{7x^2}{2} - 4x^3 + x^4$$

```
[In] Integrate[(-3 + x)*(-7 + 4*x^2),x]
[Out] 21*x - (7*x^2)/2 - 4*x^3 + x^4
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

method	result	size
gosper	$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$	18
default	$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$	18
norman	$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$	18
risch	$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$	18
parallelrisch	$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$	18

```
[In] int((-3+x)*(4*x^2-7),x,method=_RETURNVERBOSE)
[Out] x^4-4*x^3-7/2*x^2+21*x
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int (-3+x) \left(-7+4x^2\right) \, dx = x^4 - 4\,x^3 - \frac{7}{2}\,x^2 + 21\,x$$

```
[In] integrate((-3+x)*(4*x^2-7),x, algorithm="fricas")
[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int (-3+x)\left(-7+4x^2\right)\,dx = x^4-4x^3-\frac{7x^2}{2}+21x$$

```
[In] integrate((-3+x)*(4*x**2-7),x)
[Out] x**4 - 4*x**3 - 7*x**2/2 + 21*x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int (-3+x)\left(-7+4x^2\right)\,dx = x^4-4\,x^3-\frac{7}{2}\,x^2+21\,x$$

```
[In] integrate((-3+x)*(4*x^2-7),x, algorithm="maxima")
[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int (-3+x)\left(-7+4x^2\right)\,dx = x^4-4\,x^3-\frac{7}{2}\,x^2+21\,x$$

```
[In] integrate((-3+x)*(4*x^2-7),x, algorithm="giac")
[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int (-3+x)\left(-7+4x^2\right)\,dx = x^4-4\,x^3-\frac{7\,x^2}{2}+21\,x$$

```
[In] int((4*x^2 - 7)*(x - 3),x)
[Out] 21*x - (7*x^2)/2 - 4*x^3 + x^4
```

3.477 $\int (-2+7x)^3 \, dx$

Optimal result 2814
Rubi [A] (verified) 2814
Mathematica [A] (verified) 2815
Maple [A] (verified) 2815
Fricas [B] (verification not implemented) 2815
Sympy [B] (verification not implemented) 2816
Maxima [B] (verification not implemented) 2816
Giac [A] (verification not implemented) 2816
Mupad [B] (verification not implemented) 2816

Optimal result

Integrand size = 7, antiderivative size = 11

$$\int (-2+7x)^3 \, dx = \frac{1}{28}(2-7x)^4$$

```
[Out] 1/28*(2-7*x)^4
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\int (-2+7x)^3 \, dx = \frac{1}{28}(2-7x)^4$$

```
[In] Int[(-2 + 7*x)^3,x]
```

```
[Out] (2 - 7*x)^4/28
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{28}(2-7x)^4$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (-2+7x)^3\,dx=\frac{1}{28}(-2+7x)^4$$

```
[In] Integrate[(-2 + 7*x)^3,x]
[Out] (-2 + 7*x)^4/28
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{(-2+7x)^4}{28}$	10
gosper	$\frac{343}{4}x^4-98x^3+42x^2-8x$	20
norman	$\frac{343}{4}x^4-98x^3+42x^2-8x$	20
parallelrisch	$\frac{343}{4}x^4-98x^3+42x^2-8x$	20
risch	$\frac{343}{4}x^4-98x^3+42x^2-8x+\frac{4}{7}$	21

```
[In] int((-2+7*x)^3,x,method=_RETURNVERBOSE)
[Out] 1/28*(-2+7*x)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (-2+7x)^3\,dx=\frac{343}{4}\,x^4-98\,x^3+42\,x^2-8\,x$$

```
[In] integrate((-2+7*x)^3,x, algorithm="fricas")
[Out] 343/4*x^4 - 98*x^3 + 42*x^2 - 8*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (-2+7x)^3\,dx = \frac{343x^4}{4} - 98x^3 + 42x^2 - 8x$$

```
[In] integrate((-2+7*x)**3,x)
[Out] 343*x**4/4 - 98*x**3 + 42*x**2 - 8*x
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (-2+7x)^3\,dx = \frac{343}{4}\,x^4 - 98\,x^3 + 42\,x^2 - 8\,x$$

```
[In] integrate((-2+7*x)^3,x, algorithm="maxima")
[Out] 343/4*x^4 - 98*x^3 + 42*x^2 - 8*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2+7x)^3\,dx = \frac{1}{28}\,(7\,x-2)^4$$

```
[In] integrate((-2+7*x)^3,x, algorithm="giac")
[Out] 1/28*(7*x - 2)^4
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2+7x)^3\,dx = \frac{(7\,x-2)^4}{28}$$

```
[In] int((7*x - 2)^3,x)
[Out] (7*x - 2)^4/28
```

3.478 $\int \frac{-7+4x^2}{3+2x}\,dx$

Optimal result . . . 2817
Rubi [A] (verified) . . . 2817
Mathematica [A] (verified) . . . 2818
Maple [A] (verified) . . . 2818
Fricas [A] (verification not implemented) . . . 2818
Sympy [A] (verification not implemented) . . . 2819
Maxima [A] (verification not implemented) . . . 2819
Giac [A] (verification not implemented) . . . 2819
Mupad [B] (verification not implemented) . . . 2819

Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{-7+4x^2}{3+2x}\,dx = -3x+x^2+\log(3+2x)$$

```
[Out] -3*x+x^2+ln(3+2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {711}

$$\int \frac{-7+4x^2}{3+2x}\,dx = x^2-3x+\log(2x+3)$$

```
[In] Int[(-7 + 4*x^2)/(3 + 2*x),x]
```

```
[Out] -3*x + x^2 + Log[3 + 2*x]
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-3+2x+\frac{2}{3+2x}\right)\,dx \\ &= -3x+x^2+\log(3+2x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{-7+4x^2}{3+2x}\,dx=-\frac{27}{4}-3x+x^2+\log(3+2x)$$

```
[In] Integrate[(-7 + 4*x^2)/(3 + 2*x),x]
[Out] -27/4 - 3*x + x^2 + Log[3 + 2*x]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$x^2-3x+\ln\left(x+\frac{3}{2}\right)$	12
default	$-3x+x^2+\ln\left(2x+3\right)$	14
norman	$-3x+x^2+\ln\left(2x+3\right)$	14
risch	$-3x+x^2+\ln\left(2x+3\right)$	14
meijerg	$\ln\left(1+\frac{2x}{3}\right)-\frac{x(-2x+6)}{2}$	16

```
[In] int((4*x^2-7)/(2*x+3),x,method=_RETURNVERBOSE)
[Out] x^2-3*x+ln(x+3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{-7+4x^2}{3+2x}\,dx=x^2-3\,x+\log\left(2\,x+3\right)$$

```
[In] integrate((4*x^2-7)/(3+2*x),x, algorithm="fricas")
[Out] x^2 - 3*x + log(2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{-7+4x^2}{3+2x}\,dx = x^2 - 3x + \log(2x+3)$$

```
[In] integrate((4*x**2-7)/(3+2*x),x)
[Out] x**2 - 3*x + log(2*x + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{-7+4x^2}{3+2x}\,dx = x^2 - 3\,x + \log(2\,x+3)$$

```
[In] integrate((4*x^2-7)/(3+2*x),x, algorithm="maxima")
[Out] x^2 - 3*x + log(2*x + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{-7+4x^2}{3+2x}\,dx = x^2 - 3\,x + \log(|2\,x+3|)$$

```
[In] integrate((4*x^2-7)/(3+2*x),x, algorithm="giac")
[Out] x^2 - 3*x + log(abs(2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{-7+4x^2}{3+2x}\,dx = \ln\left(x+\frac{3}{2}\right) - 3\,x + x^2$$

```
[In] int((4*x^2 - 7)/(2*x + 3),x)
[Out] log(x + 3/2) - 3*x + x^2
```

3.479 $\int \frac{1+x}{(-1+x)x^2} dx$

Optimal result 2820
Rubi [A] (verified) 2820
Mathematica [A] (verified) 2821
Maple [A] (verified) 2821
Fricas [A] (verification not implemented) 2822
Sympy [A] (verification not implemented) 2822
Maxima [A] (verification not implemented) 2822
Giac [A] (verification not implemented) 2822
Mupad [B] (verification not implemented) 2823

Optimal result

Integrand size = 12, antiderivative size = 16

$$\int \frac{1+x}{(-1+x)x^2} dx = \frac{1}{x} + 2\log(1-x) - 2\log(x)$$

```
[Out] 1/x+2*ln(1-x)-2*ln(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {78}

$$\int \frac{1+x}{(-1+x)x^2} dx = \frac{1}{x} + 2\log(1-x) - 2\log(x)$$

```
[In] Int[(1 + x)/((-1 + x)*x^2),x]
```

```
[Out] x^(-1) + 2*Log[1 - x] - 2*Log[x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\text{integral} = \int \left(\frac{2}{-1+x} - \frac{1}{x^2} - \frac{2}{x}\right) dx$$
$$= \frac{1}{x} + 2\log(1-x) - 2\log(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{(-1+x)x^2} dx = \frac{1}{x} + 2\log(1-x) - 2\log(x)$$

```
[In] Integrate[(1 + x)/((-1 + x)*x^2),x]
[Out] x^(-1) + 2*Log[1 - x] - 2*Log[x]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{1}{x} - 2\ln(x) + 2\ln(x-1)$	15
norman	$\frac{1}{x} - 2\ln(x) + 2\ln(x-1)$	15
risch	$\frac{1}{x} - 2\ln(x) + 2\ln(x-1)$	15
parallelrisch	$-\frac{2\ln(x)x-2\ln(x-1)x-1}{x}$	20
meijerg	$2\ln(1-x) - 2\ln(x) - 2i\pi + \frac{1}{x}$	21

```
[In] int((x+1)/(x-1)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/x-2*ln(x)+2*ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1+x}{(-1+x)x^2}\,dx = \frac{2\,x\log\left(x-1\right)-2\,x\log\left(x\right)+1}{x}$$

```
[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="fricas")
[Out] (2*x*log(x - 1) - 2*x*log(x) + 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x}{(-1+x)x^2}\,dx = -2\log\left(x\right)+2\log\left(x-1\right)+\frac{1}{x}$$

```
[In] integrate((1+x)/(-1+x)/x**2,x)
[Out] -2*log(x) + 2*log(x - 1) + 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x}{(-1+x)x^2}\,dx = \frac{1}{x}+2\,\log\left(x-1\right)-2\,\log\left(x\right)$$

```
[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="maxima")
[Out] 1/x + 2*log(x - 1) - 2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{(-1+x)x^2}\,dx = \frac{1}{x}+2\,\log\left(|x-1|\right)-2\,\log\left(|x|\right)$$

```
[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="giac")
[Out] 1/x + 2*log(abs(x - 1)) - 2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1+x}{(-1+x)x^2}\,dx = \frac{1}{x} - 4\,\mathrm{atanh}(2\,x-1)$$

```
[In] int((x + 1)/(x^2*(x - 1)),x)
[Out] 1/x - 4*atanh(2*x - 1)
```

3.480 $\int \frac{1}{4x^2+4x^3+x^4} dx$

Optimal result 2824
Rubi [A] (verified) 2824
Mathematica [A] (verified) 2825
Maple [A] (verified) 2825
Fricas [A] (verification not implemented) 2826
Sympy [A] (verification not implemented) 2826
Maxima [A] (verification not implemented) 2826
Giac [A] (verification not implemented) 2827
Mupad [B] (verification not implemented) 2827

Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{1}{4x^2+4x^3+x^4} dx = \frac{1+x}{2(1-(1+x)^2)} + \frac{1}{2}\text{arctanh}(1+x)$$

```
[Out] 1/2*(1+x)/(1-(1+x)^2)+1/2*arctanh(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1120, 205, 212}

$$\int \frac{1}{4x^2+4x^3+x^4} dx = \frac{1}{2}\text{arctanh}(x+1) + \frac{x+1}{2(1-(x+1)^2)}$$

```
[In] Int[(4*x^2 + 4*x^3 + x^4)^(-1),x]
```

```
[Out] (1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{(1-x^2)^2}\,dx, x, 1+x\right)\\ &= \frac{1+x}{2\left(1-(1+x)^2\right)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, 1+x\right)\\ &= \frac{1+x}{2\left(1-(1+x)^2\right)} + \frac{1}{2}\tanh^{-1}(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{4x^2+4x^3+x^4}\,dx = \frac{1}{4}\left(-\frac{2(1+x)}{x(2+x)} - \log(x) + \log(2+x)\right)$$

```
[In] Integrate[(4*x^2 + 4*x^3 + x^4)^(-1),x]
[Out] ((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{1}{4x}-\frac{\ln(x)}{4}-\frac{1}{4(x+2)}+\frac{\ln(x+2)}{4}$	24
norman	$\frac{-\frac{1}{2}-\frac{x}{2}}{x(x+2)}-\frac{\ln(x)}{4}+\frac{\ln(x+2)}{4}$	26
risch	$\frac{-\frac{1}{2}-\frac{x}{2}}{x(x+2)}-\frac{\ln(x)}{4}+\frac{\ln(x+2)}{4}$	26
parallelrisch	$-\frac{\ln(x)x^2-\ln(x+2)x^2+2+2\ln(x)x-2\ln(x+2)x+2x}{4x(x+2)}$	43

```
[In] int(1/(x^4+4*x^3+4*x^2),x,method=_RETURNVERBOSE)
[Out] -1/4/x-1/4*ln(x)-1/4/(x+2)+1/4*ln(x+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{1}{4x^2+4x^3+x^4}\,dx = \frac{(x^2+2\,x)\log(x+2)-(x^2+2\,x)\log(x)-2\,x-2}{4\,(x^2+2\,x)}$$

```
[In] integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="fricas")
[Out] 1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{4x^2+4x^3+x^4}\,dx = \frac{-x-1}{2x^2+4x}-\frac{\log(x)}{4}+\frac{\log(x+2)}{4}$$

```
[In] integrate(1/(x**4+4*x**3+4*x**2),x)
[Out] (-x - 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{4x^2+4x^3+x^4}\,dx = -\frac{x+1}{2\,(x^2+2\,x)}+\frac{1}{4}\,\log(x+2)-\frac{1}{4}\,\log(x)$$

```
[In] integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="maxima")
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{4x^2+4x^3+x^4}\,dx=-\frac{x+1}{2\,(x^2+2\,x)}+\frac{1}{4}\,\log\left(|x+2|\right)-\frac{1}{4}\,\log\left(|x|\right)$$

```
[In] integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="giac")
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(abs(x + 2)) - 1/4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{4x^2+4x^3+x^4}\,dx=\frac{\mathrm{atanh}(x+1)}{2}-\frac{\frac{x}{2}+\frac{1}{2}}{x^2+2\,x}$$

```
[In] int(1/(4*x^2 + 4*x^3 + x^4),x)
[Out] atanh(x + 1)/2 - (x/2 + 1/2)/(2*x + x^2)
```

3.481 $\int \frac{1+x^2}{1+x} \, dx$

Optimal result 2828
Rubi [A] (verified) 2828
Mathematica [A] (verified) 2829
Maple [A] (verified) 2829
Fricas [A] (verification not implemented) 2829
Sympy [A] (verification not implemented) 2830
Maxima [A] (verification not implemented) 2830
Giac [A] (verification not implemented) 2830
Mupad [B] (verification not implemented) 2830

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{1+x^2}{1+x} \, dx = -x + \frac{x^2}{2} + 2\log(1+x)$$

```
[Out] -x+1/2*x^2+2*ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{711\}$

$$\int \frac{1+x^2}{1+x} \, dx = \frac{x^2}{2} - x + 2\log(x+1)$$

```
[In] Int[(1 + x^2)/(1 + x),x]
```

```
[Out] -x + x^2/2 + 2*Log[1 + x]
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-1 + x + \frac{2}{1+x}\right) \, dx \\ &= -x + \frac{x^2}{2} + 2\log(1+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1+x^2}{1+x}\,dx = \frac{1}{2}\left(-3-2x+x^2+4\log(1+x)\right)$$

```
[In] Integrate[(1 + x^2)/(1 + x),x]
[Out] (-3 - 2*x + x^2 + 4*Log[1 + x])/2
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$-x+\frac{x^2}{2}+2\ln\left(x+1\right)$	16
norman	$-x+\frac{x^2}{2}+2\ln\left(x+1\right)$	16
meijerg	$-\frac{x(6-3x)}{6}+2\ln\left(x+1\right)$	16
risch	$-x+\frac{x^2}{2}+2\ln\left(x+1\right)$	16
parallelrisch	$-x+\frac{x^2}{2}+2\ln\left(x+1\right)$	16

```
[In] int((x^2+1)/(x+1),x,method=_RETURNVERBOSE)
[Out] -x+1/2*x^2+2*ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{1+x}\,dx = \frac{1}{2}\,x^2-x+2\,\log\left(x+1\right)$$

```
[In] integrate((x^2+1)/(1+x),x, algorithm="fricas")
[Out] 1/2*x^2 - x + 2*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{1+x^2}{1+x}\,dx = \frac{x^2}{2} - x + 2\log\left(x+1\right)$$

```
[In] integrate((x**2+1)/(1+x),x)
[Out] x**2/2 - x + 2*log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{1+x}\,dx = \frac{1}{2}\,x^2 - x + 2\,\log\left(x+1\right)$$

```
[In] integrate((x^2+1)/(1+x),x, algorithm="maxima")
[Out] 1/2*x^2 - x + 2*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1+x^2}{1+x}\,dx = \frac{1}{2}\,x^2 - x + 2\,\log\left(|x+1|\right)$$

```
[In] integrate((x^2+1)/(1+x),x, algorithm="giac")
[Out] 1/2*x^2 - x + 2*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{1+x}\,dx = 2\,\ln\left(x+1\right) - x + \frac{x^2}{2}$$

```
[In] int((x^2 + 1)/(x + 1),x)
[Out] 2*log(x + 1) - x + x^2/2
```

3.482 $\int \frac{-1+3x-3x^2+x^3}{x^2} \, dx$

Optimal result 2831
Rubi [A] (verified) 2831
Mathematica [A] (verified) 2832
Maple [A] (verified) 2832
Fricas [A] (verification not implemented) 2832
Sympy [A] (verification not implemented) 2833
Maxima [A] (verification not implemented) 2833
Giac [A] (verification not implemented) 2833
Mupad [B] (verification not implemented) 2833

Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \frac{-1+3x-3x^2+x^3}{x^2} \, dx = \frac{1}{x} - 3x + \frac{x^2}{2} + 3\log(x)$$

```
[Out] 1/x-3*x+1/2*x^2+3*ln(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{14\}$

$$\int \frac{-1+3x-3x^2+x^3}{x^2} \, dx = \frac{x^2}{2} - 3x + \frac{1}{x} + 3\log(x)$$

```
[In] Int[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]
```

```
[Out] x^(-1) - 3*x + x^2/2 + 3*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-3 - \frac{1}{x^2} + \frac{3}{x} + x\right) \, dx \\
&= \frac{1}{x} - 3x + \frac{x^2}{2} + 3\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx=\frac{1}{x}-3x+\frac{x^2}{2}+3\log(x)$$

```
[In] Integrate[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]
[Out] x^(-1) - 3*x + x^2/2 + 3*Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{1}{x}-3x+\frac{x^2}{2}+3\ln(x)$	17
risch	$\frac{1}{x}-3x+\frac{x^2}{2}+3\ln(x)$	17
parallelrisch	$\frac{x^3+6\ln(x)x-6x^2+2}{2x}$	21
norman	$\frac{1-3x^2+\frac{1}{2}x^3}{x}+3\ln(x)$	22

```
[In] int((x^3-3*x^2+3*x-1)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/x-3*x+1/2*x^2+3*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx=\frac{x^3-6\,x^2+6\,x\log(x)+2}{2\,x}$$

```
[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="fricas")
[Out] 1/2*(x^3 - 6*x^2 + 6*x*log(x) + 2)/x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx = \frac{x^2}{2} - 3x + 3\log{(x)} + \frac{1}{x}$$

```
[In] integrate((x**3-3*x**2+3*x-1)/x**2,x)
[Out] x**2/2 - 3*x + 3*log(x) + 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx = \frac{1}{2}\,x^2 - 3\,x + \frac{1}{x} + 3\,\log\left(x\right)$$

```
[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="maxima")
[Out] 1/2*x^2 - 3*x + 1/x + 3*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx = \frac{1}{2}\,x^2 - 3\,x + \frac{1}{x} + 3\,\log\left(|x|\right)$$

```
[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="giac")
[Out] 1/2*x^2 - 3*x + 1/x + 3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx = 3\,\ln\left(x\right) - 3\,x + \frac{1}{x} + \frac{x^2}{2}$$

```
[In] int((3*x - 3*x^2 + x^3 - 1)/x^2,x)
[Out] 3*log(x) - 3*x + 1/x + x^2/2
```

3.483 $\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx$

Optimal result 2834
Rubi [A] (verified) 2834
Mathematica [A] (verified) 2835
Maple [A] (verified) 2835
Fricas [A] (verification not implemented) 2835
Sympy [A] (verification not implemented) 2836
Maxima [A] (verification not implemented) 2836
Giac [A] (verification not implemented) 2836
Mupad [B] (verification not implemented) 2836

Optimal result

Integrand size = 29, antiderivative size = 18

$$\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx=-7x+\frac{3x^2}{2}+\frac{x^3}{3}$$

```
[Out] -7*x+3/2*x^2+1/3*x^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {45}

$$\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx=\frac{x^3}{3}+\frac{3x^2}{2}-7x$$

```
[In] Int[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x),x]
[Out] -7*x + (3*x^2)/2 + x^3/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(-7+3x+x^2\right)\,dx\\&=-7x+\frac{3x^2}{2}+\frac{x^3}{3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx=-7x+\frac{3x^2}{2}+\frac{x^3}{3}$$

```
[In] Integrate[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x),x]
[Out] -7*x + (3*x^2)/2 + x^3/3
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
norman	$-7x+\frac{3}{2}x^2+\frac{1}{3}x^3$	15
risch	$-7x+\frac{3}{2}x^2+\frac{1}{3}x^3$	15
parallelrisch	$-7x+\frac{3}{2}x^2+\frac{1}{3}x^3$	15
default	$\frac{x^3}{3}+\frac{3x^2}{2}+\frac{\left(3-\sqrt{37}\right)\left(3+\sqrt{37}\right)x}{4}$	27
gosper	$-\frac{x\left(2x^2+9x-42\right)\left(-2x-3+\sqrt{37}\right)\left(2x+3+\sqrt{37}\right)}{24\left(x^2+3x-7\right)}$	40

```
[In] int((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x,method=_RETURNVERBOSE)
[Out] -7*x+3/2*x^2+1/3*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx=\frac{1}{3}\,x^3+\frac{3}{2}\,x^2-7\,x$$

```
[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="fricas")
[Out] 1/3*x^3 + 3/2*x^2 - 7*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx=\frac{x^3}{3}+\frac{3x^2}{2}-7x$$

```
[In] integrate((x+3/2-1/2*37**(1/2))*(x+3/2+1/2*37**(1/2)),x)
[Out] x**3/3 + 3*x**2/2 - 7*x
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx=\frac{1}{3}\,x^3+\frac{3}{2}\,x^2-7\,x$$

```
[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="maxima")
[Out] 1/3*x^3 + 3/2*x^2 - 7*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx=\frac{1}{3}\,x^3+\frac{3}{2}\,x^2-7\,x$$

```
[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="giac")
[Out] 1/3*x^3 + 3/2*x^2 - 7*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx=\frac{x\,\left(2\,x^2+9\,x-42\right)}{6}$$

```
[In] int((x - 37^(1/2)/2 + 3/2)*(x + 37^(1/2)/2 + 3/2),x)
[Out] (x*(9*x + 2*x^2 - 42))/6
```

3.484 $\int \frac{4+3x^2+2x^3}{(1+x)^4} dx$

Optimal result . . . 2837
Rubi [A] (verified) . . . 2837
Mathematica [A] (verified) . . . 2838
Maple [A] (verified) . . . 2838
Fricas [B] (verification not implemented) . . . 2838
Sympy [A] (verification not implemented) . . . 2839
Maxima [A] (verification not implemented) . . . 2839
Giac [A] (verification not implemented) . . . 2839
Mupad [B] (verification not implemented) . . . 2839

Optimal result

Integrand size = 18, antiderivative size = 23

$$\int \frac{4+3x^2+2x^3}{(1+x)^4}\,dx=-\frac{5}{3(1+x)^3}+\frac{3}{1+x}+2\log(1+x)$$

```
[Out] -5/3/(1+x)^3+3/(1+x)+2*ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1864}

$$\int \frac{4+3x^2+2x^3}{(1+x)^4}\,dx=\frac{3}{x+1}-\frac{5}{3(x+1)^3}+2\log(x+1)$$

```
[In] Int[(4 + 3*x^2 + 2*x^3)/(1 + x)^4,x]
[Out] -5/(3*(1 + x)^3) + 3/(1 + x) + 2*Log[1 + x]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(\frac{5}{(1+x)^4}-\frac{3}{(1+x)^2}+\frac{2}{1+x}\right)dx\\&=-\frac{5}{3(1+x)^3}+\frac{3}{1+x}+2\log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{4+3x^2+2x^3}{(1+x)^4}\,dx=-\frac{5}{3(1+x)^3}+\frac{3}{1+x}+2\log(1+x)$$

```
[In] Integrate[(4 + 3*x^2 + 2*x^3)/(1 + x)^4,x]
[Out] -5/(3*(1 + x)^3) + 3/(1 + x) + 2*Log[1 + x]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{5}{3(x+1)^3}+\frac{3}{x+1}+2\ln(x+1)$	22
norman	$\frac{3x^2+6x+\frac{4}{3}}{(x+1)^3}+2\ln(x+1)$	24
risch	$\frac{3x^2+6x+\frac{4}{3}}{(x+1)^3}+2\ln(x+1)$	24
parallelrisch	$\frac{6\ln(x+1)x^3+4+18\ln(x+1)x^2+18\ln(x+1)x+9x^2+6\ln(x+1)+18x}{3(x+1)^3}$	49
meijerg	$\frac{4x(x^2+3x+3)}{3(x+1)^3}-\frac{x(22x^2+30x+12)}{6(x+1)^3}+2\ln(x+1)+\frac{x^3}{(x+1)^3}$	51

```
[In] int((2*x^3+3*x^2+4)/(x+1)^4,x,method=_RETURNVERBOSE)
[Out] -5/3/(x+1)^3+3/(x+1)+2*ln(x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(21) = 42.

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \frac{4+3x^2+2x^3}{(1+x)^4}\,dx=\frac{9\,x^2+6\,(x^3+3\,x^2+3\,x+1)\log(x+1)+18\,x+4}{3\,(x^3+3\,x^2+3\,x+1)}$$

```
[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="fricas")
[Out] 1/3*(9*x^2 + 6*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 4)/(x^3 + 3*x^2
+ 3*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{4+3x^2+2x^3}{(1+x)^4}\,dx = \frac{9x^2+18x+4}{3x^3+9x^2+9x+3} + 2\log(x+1)$$

```
[In] integrate((2*x**3+3*x**2+4)/(1+x)**4,x)
[Out] (9*x**2 + 18*x + 4)/(3*x**3 + 9*x**2 + 9*x + 3) + 2*log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{4+3x^2+2x^3}{(1+x)^4}\,dx = \frac{9\,x^2+18\,x+4}{3\,(x^3+3\,x^2+3\,x+1)} + 2\,\log(x+1)$$

```
[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="maxima")
[Out] 1/3*(9*x^2 + 18*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + 2*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{4+3x^2+2x^3}{(1+x)^4}\,dx = \frac{9\,x^2+18\,x+4}{3\,(x+1)^3} + 2\,\log(|x+1|)$$

```
[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="giac")
[Out] 1/3*(9*x^2 + 18*x + 4)/(x + 1)^3 + 2*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{4+3x^2+2x^3}{(1+x)^4}\,dx = 2\,\ln(x+1) + \frac{3\,x^2+6\,x+\frac{4}{3}}{(x+1)^3}$$

```
[In] int((3*x^2 + 2*x^3 + 4)/(x + 1)^4,x)
[Out] 2*log(x + 1) + (6*x + 3*x^2 + 4/3)/(x + 1)^3
```

3.485 $\int \frac{x}{(1+x)^2(1+x^2)} dx$

Optimal result 2840
Rubi [A] (verified) 2840
Mathematica [A] (verified) 2841
Maple [A] (verified) 2841
Fricas [A] (verification not implemented) 2842
Sympy [A] (verification not implemented) 2842
Maxima [A] (verification not implemented) 2842
Giac [B] (verification not implemented) 2842
Mupad [B] (verification not implemented) 2843

Optimal result

Integrand size = 14, antiderivative size = 16

$$\int \frac{x}{(1+x)^2(1+x^2)} dx = \frac{1}{2(1+x)} + \frac{\arctan(x)}{2}$$

```
[Out] 1/2/(1+x)+1/2*arctan(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {815, 209}

$$\int \frac{x}{(1+x)^2(1+x^2)} dx = \frac{\arctan(x)}{2} + \frac{1}{2(x+1)}$$

```
[In] Int[x/((1 + x)^2*(1 + x^2)),x]
```

```
[Out] 1/(2*(1 + x)) + ArcTan[x]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{1}{2(1+x)^2}+\frac{1}{2\left(1+x^2\right)}\right) dx\\ &= \frac{1}{2(1+x)}+\frac{1}{2}\int \frac{1}{1+x^2}\,dx\\ &= \frac{1}{2(1+x)}+\frac{1}{2}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{(1+x)^2\left(1+x^2\right)}\,dx=\frac{1}{2}\left(\frac{1}{1+x}+\arctan(x)\right)$$

```
[In] Integrate[x/((1 + x)^2*(1 + x^2)),x]
[Out] ((1 + x)^(-1) + ArcTan[x])/2
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{1}{2x+2}+\frac{\arctan(x)}{2}$	13
risch	$\frac{1}{2x+2}+\frac{\arctan(x)}{2}$	13
parallelrisch	$-\frac{i\ln(x-i)x-i\ln(x+i)x+i\ln(x-i)-i\ln(x+i)-2}{4(x+1)}$	44

```
[In] int(x/(x+1)^2/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2/(x+1)+1/2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{(1+x)^2(1+x^2)}\,dx = \frac{(x+1)\arctan(x)+1}{2(x+1)}$$

```
[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="fricas")
[Out] 1/2*((x + 1)*arctan(x) + 1)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x}{(1+x)^2(1+x^2)}\,dx = \frac{\operatorname{atan}(x)}{2} + \frac{1}{2x+2}$$

```
[In] integrate(x/(1+x)**2/(x**2+1),x)
[Out] atan(x)/2 + 1/(2*x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{(1+x)^2(1+x^2)}\,dx = \frac{1}{2(x+1)} + \frac{1}{2}\arctan(x)$$

```
[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="maxima")
[Out] 1/2/(x + 1) + 1/2*arctan(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{x}{(1+x)^2(1+x^2)}\,dx = -\frac{1}{8}\pi - \frac{1}{2}\pi\left\lfloor -\frac{\pi-4\arctan(x)}{4\pi} + \frac{1}{2}\right\rfloor + \frac{1}{2(x+1)} + \frac{1}{2}\arctan(x)$$

```
[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="giac")
[Out] -1/8*pi - 1/2*pi*floor(-1/4*(pi - 4*arctan(x))/pi + 1/2) + 1/2/(x + 1) + 1/
2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{(1+x)^2\,(1+x^2)}\,dx=\frac{\operatorname{atan}(x)}{2}+\frac{1}{2\,(x+1)}$$

```
[In] int(x/((x^2 + 1)*(x + 1)^2),x)
[Out] atan(x)/2 + 1/(2*(x + 1))
```

3.486 $\int \frac{7-2x+3x^2-x^3+x^4}{2+x} dx$

Optimal result 2844
Rubi [A] (verified) 2844
Mathematica [A] (verified) 2845
Maple [A] (verified) 2845
Fricas [A] (verification not implemented) 2845
Sympy [A] (verification not implemented) 2846
Maxima [A] (verification not implemented) 2846
Giac [A] (verification not implemented) 2846
Mupad [B] (verification not implemented) 2846

Optimal result

Integrand size = 24, antiderivative size = 29

$$\int \frac{7-2x+3x^2-x^3+x^4}{2+x} dx = -20x+\frac{9x^2}{2}-x^3+\frac{x^4}{4}+47\log(2+x)$$

```
[Out] -20*x+9/2*x^2-x^3+1/4*x^4+47*ln(2+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1864}

$$\int \frac{7-2x+3x^2-x^3+x^4}{2+x} dx = \frac{x^4}{4}-x^3+\frac{9x^2}{2}-20x+47\log(x+2)$$

```
[In] Int[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x),x]
[Out] -20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-20+9x-3x^2+x^3+\frac{47}{2+x}\right) dx \\ &= -20x+\frac{9x^2}{2}-x^3+\frac{x^4}{4}+47\log(2+x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{7-2x+3x^2-x^3+x^4}{2+x}\,dx = -70-20x+\frac{9x^2}{2}-x^3+\frac{x^4}{4}+47\log(2+x)$$

```
[In] Integrate[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x),x]
[Out] -70 - 20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$-20x+\frac{9x^2}{2}-x^3+\frac{x^4}{4}+47\ln(x+2)$	26
norman	$-20x+\frac{9x^2}{2}-x^3+\frac{x^4}{4}+47\ln(x+2)$	26
risch	$-20x+\frac{9x^2}{2}-x^3+\frac{x^4}{4}+47\ln(x+2)$	26
parallelrisch	$-20x+\frac{9x^2}{2}-x^3+\frac{x^4}{4}+47\ln(x+2)$	26
meijerg	$47\ln\left(1+\frac{x}{2}\right)-\frac{2x\left(-\frac{15}{8}x^3+5x^2-15x+60\right)}{15}-\frac{x(x^2-3x+12)}{3}-x\left(-\frac{3x}{2}+6\right)-2x$	50

```
[In] int((x^4-x^3+3*x^2-2*x+7)/(x+2),x,method=_RETURNVERBOSE)
[Out] -20*x+9/2*x^2-x^3+1/4*x^4+47*ln(x+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{7-2x+3x^2-x^3+x^4}{2+x}\,dx = \frac{1}{4}x^4-x^3+\frac{9}{2}x^2-20x+47\log(x+2)$$

```
[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="fricas")
[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{7-2x+3x^2-x^3+x^4}{2+x}\,dx = \frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47\log\left(x+2\right)$$

```
[In] integrate((x**4-x**3+3*x**2-2*x+7)/(2+x),x)
[Out] x**4/4 - x**3 + 9*x**2/2 - 20*x + 47*log(x + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{7-2x+3x^2-x^3+x^4}{2+x}\,dx = \frac{1}{4}\,x^4 - x^3 + \frac{9}{2}\,x^2 - 20\,x + 47\,\log\left(x+2\right)$$

```
[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="maxima")
[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{7-2x+3x^2-x^3+x^4}{2+x}\,dx = \frac{1}{4}\,x^4 - x^3 + \frac{9}{2}\,x^2 - 20\,x + 47\,\log\left(|x+2|\right)$$

```
[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="giac")
[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(abs(x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{7-2x+3x^2-x^3+x^4}{2+x}\,dx = 47\,\ln\left(x+2\right) - 20\,x + \frac{9\,x^2}{2} - x^3 + \frac{x^4}{4}$$

```
[In] int((3*x^2 - 2*x - x^3 + x^4 + 7)/(x + 2),x)
[Out] 47*log(x + 2) - 20*x + (9*x^2)/2 - x^3 + x^4/4
```

3.487 $\int \frac{-1+x^3}{-1+x} \, dx$

Optimal result 2847
Rubi [A] (verified) 2847
Mathematica [A] (verified) 2848
Maple [A] (verified) 2848
Fricas [A] (verification not implemented) 2848
Sympy [A] (verification not implemented) 2849
Maxima [A] (verification not implemented) 2849
Giac [A] (verification not implemented) 2849
Mupad [B] (verification not implemented) 2849

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{-1+x^3}{-1+x} \, dx = x + \frac{x^2}{2} + \frac{x^3}{3}$$

```
[Out] x+1/2*x^2+1/3*x^3
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1600}

$$\int \frac{-1+x^3}{-1+x} \, dx = \frac{x^3}{3} + \frac{x^2}{2} + x$$

```
[In] Int[(-1 + x^3)/(-1 + x),x]
[Out] x + x^2/2 + x^3/3
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(1 + x + x^2\right) \, dx \\ &= x + \frac{x^2}{2} + \frac{x^3}{3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^3}{-1+x}\,dx = x+\frac{x^2}{2}+\frac{x^3}{3}$$

```
[In] Integrate[(-1 + x^3)/(-1 + x),x]
[Out] x + x^2/2 + x^3/3
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
default	$x+\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
norman	$x+\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
risch	$x+\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
parallelrisch	$x+\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
parts	$x+\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
gosper	$\frac{x(2x^{2}+3x+6)}{6}$	14
meijerg	$\frac{x(4x^{2}+6x+12)}{12}$	14

```
[In] int((x^3-1)/(x-1),x,method=_RETURNVERBOSE)
[Out] x+1/2*x^2+1/3*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{-1+x^3}{-1+x}\,dx = \frac{1}{3}\,x^{3}+\frac{1}{2}\,x^{2}+x$$

```
[In] integrate((x^3-1)/(-1+x),x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{-1+x^3}{-1+x}\,dx = \frac{x^3}{3}+\frac{x^2}{2}+x$$

```
[In] integrate((x**3-1)/(-1+x),x)
[Out] x**3/3 + x**2/2 + x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{-1+x^3}{-1+x}\,dx = \frac{1}{3}\,x^3+\frac{1}{2}\,x^2+x$$

```
[In] integrate((x^3-1)/(-1+x),x, algorithm="maxima")
[Out] 1/3*x^3 + 1/2*x^2 + x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{-1+x^3}{-1+x}\,dx = \frac{1}{3}\,x^3+\frac{1}{2}\,x^2+x$$

```
[In] integrate((x^3-1)/(-1+x),x, algorithm="giac")
[Out] 1/3*x^3 + 1/2*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{-1+x^3}{-1+x}\,dx = \frac{x\,\left(2\,x^2+3\,x+6\right)}{6}$$

```
[In] int((x^3 - 1)/(x - 1),x)
[Out] (x*(3*x + 2*x^2 + 6))/6
```

3.488 $\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx$

Optimal result 2850
Rubi [A] (verified) 2850
Mathematica [A] (verified) 2851
Maple [A] (verified) 2851
Fricas [A] (verification not implemented) 2852
Sympy [A] (verification not implemented) 2852
Maxima [A] (verification not implemented) 2852
Giac [A] (verification not implemented) 2852
Mupad [B] (verification not implemented) 2853

Optimal result

Integrand size = 18, antiderivative size = 17

$$\int \frac{2+2x}{(-1+x)^3\,(1+x^2)}\,dx = -\frac{1}{(1-x)^2}+\frac{1}{-1+x}+\arctan(x)$$

```
[Out] -1/(1-x)^2+1/(-1+x)+arctan(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {815, 209}

$$\int \frac{2+2x}{(-1+x)^3\,(1+x^2)}\,dx = \arctan(x)+\frac{1}{x-1}-\frac{1}{(1-x)^2}$$

```
[In] Int[(2 + 2*x)/((-1 + x)^3*(1 + x^2)),x]
```

```
[Out] -(1 - x)^(-2) + (-1 + x)^(-1) + ArcTan[x]
```

<u>Rule 209</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

<u>Rule 815</u>

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{2}{(-1+x)^3} - \frac{1}{(-1+x)^2} + \frac{1}{1+x^2}\right) dx \\ &= -\frac{1}{(1-x)^2} + \frac{1}{-1+x} + \int \frac{1}{1+x^2}\, dx \\ &= -\frac{1}{(1-x)^2} + \frac{1}{-1+x} + \tan^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{2+2x}{(-1+x)^3\,(1+x^2)}\, dx = \frac{-2+x+(-1+x)^2\arctan(x)}{(-1+x)^2}$$

```
[In] Integrate[(2 + 2*x)/((-1 + x)^3*(1 + x^2)),x]
[Out] (-2 + x + (-1 + x)^2*ArcTan[x])/(-1 + x)^2
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{x-2}{(x-1)^2} + \arctan(x)$	13
default	$\arctan(x) - \frac{1}{(x-1)^2} + \frac{1}{x-1}$	16
parallelrisch	$-\frac{i\ln(x-i)x^2 - i\ln(x+i)x^2 - 2i\ln(x-i)x + 2i\ln(x+i)x + 3 + i\ln(x-i) - i\ln(x+i) - x^2}{2(x-1)^2}$	71

```
[In] int((2*x+2)/(x-1)^3/(x^2+1),x,method=_RETURNVERBOSE)
[Out] (x-2)/(x-1)^2+arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx = \frac{(x^2-2\,x+1)\arctan(x)+x-2}{x^2-2\,x+1}$$

```
[In] integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="fricas")
[Out] ((x^2 - 2*x + 1)*arctan(x) + x - 2)/(x^2 - 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx = \frac{x-2}{x^2-2x+1}+\operatorname{atan}(x)$$

```
[In] integrate((2+2*x)/(-1+x)**3/(x**2+1),x)
[Out] (x - 2)/(x**2 - 2*x + 1) + atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx = \frac{x-2}{x^2-2\,x+1}+\arctan(x)$$

```
[In] integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="maxima")
[Out] (x - 2)/(x^2 - 2*x + 1) + arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx = \frac{x-2}{(x-1)^2}+\arctan(x)$$

```
[In] integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="giac")
[Out] (x - 2)/(x - 1)^2 + arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{2+2x}{(-1+x)^3\,(1+x^2)}\,dx = \mathrm{atan}(x) + \frac{x-2}{x^2-2\,x+1}$$

```
[In] int((2*x + 2)/((x^2 + 1)*(x - 1)^3),x)
[Out] atan(x) + (x - 2)/(x^2 - 2*x + 1)
```

3.489 $\int \frac{1}{bx+c(d+ex)^2}\,dx$

Optimal result 2854
Rubi [A] (verified) 2854
Mathematica [A] (verified) 2855
Maple [A] (verified) 2855
Fricas [A] (verification not implemented) 2856
Sympy [B] (verification not implemented) 2856
Maxima [F(-2)] 2857
Giac [A] (verification not implemented) 2857
Mupad [B] (verification not implemented) 2857

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{1}{bx+c(d+ex)^2}\,dx=-\frac{2\text{arctanh}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

```
[Out] -2*arctanh((b+2*c*e*(e*x+d))/b^(1/2)/(4*c*d*e+b)^(1/2))/b^(1/2)/(4*c*d*e+b)
^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2006, 632, 212}

$$\int \frac{1}{bx+c(d+ex)^2}\,dx=-\frac{2\text{arctanh}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

```
[In] Int[(b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(Sqrt[b]*Sq
rt[b + 4*c*d*e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2006

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{cd^2+(b+2cde)x+ce^2x^2}\,dx\\ &= -\left(2\text{Subst}\left(\int \frac{1}{b(b+4cde)-x^2}\,dx, x, b+2cde+2ce^2x\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{1}{bx+c(d+ex)^2}\,dx = -\frac{2\text{arctanh}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

```
[In] Integrate[(b*x + c*(d + e*x)^2)^(-1),x]
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(Sqrt[b]*Sq
rt[b + 4*c*d*e])
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{2\,\text{arctanh}\left(\frac{2c\,e^2x+2dce+b}{\sqrt{4bcde+b^2}}\right)}{\sqrt{4bcde+b^2}}$	43
risch	$\frac{\ln\left(-2c\,e^2x-2dce+\sqrt{b(4dce+b)}-b\right)}{\sqrt{b(4dce+b)}}-\frac{\ln\left(2c\,e^2x+2dce+\sqrt{b(4dce+b)}+b\right)}{\sqrt{b(4dce+b)}}$	81

```
[In] int(1/(b*x+c*(e*x+d)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(4*b*c*d*e+b^2)^(1/2)*arctanh((2*c*e^2*x+2*c*d*e+b)/(4*b*c*d*e+b^2)^(1/2
))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 4.04

$$\int \frac{1}{bx+c(d+ex)^2}\,dx = \left[\frac{\log\left(\frac{2\,c^2e^4x^2+2\,c^2d^2e^2+4\,bcde+b^2+2\,(2\,c^2de^3+bce^2)x-\sqrt{4\,bcde+b^2}(2\,ce^2x+2\,cde+b)}{ce^2x^2+cd^2+(2\,cde+b)x}\right)}{\sqrt{4\,bcde+b^2}}, \frac{2\sqrt{-4\,bcde-b^2}\arctan\left(\frac{\sqrt{-4\,bcde-b^2}(2\cdots}{4\,bcd\cdots}\right.}{4\,bcde+b^2}\right.$$

```
[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="fricas")
```

```
[Out] [log((2*c^2*e^4*x^2 + 2*c^2*d^2*e^2 + 4*b*c*d*e + b^2 + 2*(2*c^2*d*e^3 + b*
c*e^2)*x - sqrt(4*b*c*d*e + b^2)*(2*c*e^2*x + 2*c*d*e + b))/(c*e^2*x^2 + c*
d^2 + (2*c*d*e + b)*x))/sqrt(4*b*c*d*e + b^2), 2*sqrt(-4*b*c*d*e - b^2)*arc
tan(sqrt(-4*b*c*d*e - b^2)*(2*c*e^2*x + 2*c*d*e + b)/(4*b*c*d*e + b^2))/(4*
b*c*d*e + b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(49) = 98.

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.21

$$\int \frac{1}{bx+c(d+ex)^2}\,dx = \sqrt{\frac{1}{b\,(b+4cde)}}\log\left(x+\frac{-b^2\sqrt{\frac{1}{b(b+4cde)}}-4bcde\sqrt{\frac{1}{b(b+4cde)}}+b+2cde}{2ce^2}\right) - \sqrt{\frac{1}{b\,(b+4cde)}}\log\left(x+\frac{b^2\sqrt{\frac{1}{b(b+4cde)}}+4bcde\sqrt{\frac{1}{b(b+4cde)}}+b+2cde}{2ce^2}\right)$$

```
[In] integrate(1/(b*x+c*(e*x+d)**2),x)
```

```
[Out] sqrt(1/(b*(b + 4*c*d*e)))*log(x + (-b**2*sqrt(1/(b*(b + 4*c*d*e))) - 4*b*c*
d*e*sqrt(1/(b*(b + 4*c*d*e))) + b + 2*c*d*e)/(2*c*e**2)) - sqrt(1/(b*(b + 4
*c*d*e)))*log(x + (b**2*sqrt(1/(b*(b + 4*c*d*e))) + 4*b*c*d*e*sqrt(1/(b*(b
+ 4*c*d*e))) + b + 2*c*d*e)/(2*c*e**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{bx + c(d + ex)^2} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c*d*e+b>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{1}{bx + c(d + ex)^2} \, dx = \frac{2 \arctan\left(\frac{2 ce^2 x + 2 cde + b}{\sqrt{-4 bcde - b^2}}\right)}{\sqrt{-4 bcde - b^2}}$$

```
[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*e^2*x + 2*c*d*e + b)/sqrt(-4*b*c*d*e - b^2))/sqrt(-4*b*c*d*e
- b^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{1}{bx + c(d + ex)^2} \, dx = -\frac{2 \operatorname{atanh}\left(\frac{2 c x e^2 + 2 c d e + b}{\sqrt{b} \sqrt{b + 4 c d e}}\right)}{\sqrt{b} \sqrt{b + 4 c d e}}$$

```
[In] int(1/(c*(d + e*x)^2 + b*x),x)
```

```
[Out] -(2*atanh((b + 2*c*d*e + 2*c*e^2*x)/(b^(1/2)*(b + 4*c*d*e)^(1/2))))/(b^(1/2
)*(b + 4*c*d*e)^(1/2))
```

3.490 $\int \frac{1}{a+bx+c(d+ex)^2} dx$

Optimal result 2858
Rubi [A] (verified) 2858
Mathematica [A] (verified) 2859
Maple [A] (verified) 2859
Fricas [A] (verification not implemented) 2860
Sympy [B] (verification not implemented) 2860
Maxima [F(-2)] 2861
Giac [A] (verification not implemented) 2861
Mupad [B] (verification not implemented) 2861

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{1}{a+bx+c(d+ex)^2} dx = -\frac{2\operatorname{arctanh}\left(\frac{b+2ce(d+ex)}{\sqrt{b^2+4bcde-4ace^2}}\right)}{\sqrt{b^2+4bcde-4ace^2}}$$

```
[Out] -2*arctanh((b+2*c*e*(e*x+d))/(-4*a*c*e^2+4*b*c*d*e+b^2)^(1/2))/(-4*a*c*e^2+
4*b*c*d*e+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2006, 632, 212}

$$\int \frac{1}{a+bx+c(d+ex)^2} dx = -\frac{2\operatorname{arctanh}\left(\frac{b+2ce(d+ex)}{\sqrt{-4ace^2+b^2+4bcde}}\right)}{\sqrt{-4ace^2+b^2+4bcde}}$$

```
[In] Int[(a + b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/Sqrt[b^2 + 4*b*c*d*e - 4*a*c*e^2]])/Sqrt[
b^2 + 4*b*c*d*e - 4*a*c*e^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2006

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{a+cd^2+(b+2cde)x+ce^2x^2}\,dx\\ &= -\left(2\text{Subst}\left(\int \frac{1}{b^2+4bcde-4ace^2-x^2}\,dx, x, b+2cde+2ce^2x\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b^2+4bcde-4ace^2}}\right)}{\sqrt{b^2+4bcde-4ace^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{1}{a+bx+c(d+ex)^2}\,dx = \frac{2\arctan\left(\frac{b+2ce(d+ex)}{\sqrt{-b^2-4bcde+4ace^2}}\right)}{\sqrt{-b^2-4bcde+4ace^2}}$$

```
[In] Integrate[(a + b*x + c*(d + e*x)^2)^(-1),x]
[Out] (2*ArcTan[(b + 2*c*e*(d + e*x))/Sqrt[-b^2 - 4*b*c*d*e + 4*a*c*e^2]])/Sqrt[-
b^2 - 4*b*c*d*e + 4*a*c*e^2]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{2\arctan\left(\frac{2c\,e^2x+2dce+b}{\sqrt{4e^2ac-4bcde-b^2}}\right)}{\sqrt{4e^2ac-4bcde-b^2}}$	61
risch	$-\frac{\ln\left(2c\,e^2x+2dce+\sqrt{-4e^2ac+4bcde+b^2}+b\right)}{\sqrt{-4e^2ac+4bcde+b^2}}+\frac{\ln\left(-2c\,e^2x-2dce+\sqrt{-4e^2ac+4bcde+b^2}-b\right)}{\sqrt{-4e^2ac+4bcde+b^2}}$	113

```
[In] int(1/(a+b*x+c*(e*x+d)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(4*a*c*e^2-4*b*c*d*e-b^2)^(1/2)*arctan((2*c*e^2*x+2*c*d*e+b)/(4*a*c*e^2-4
*b*c*d*e-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.21

$$\int \frac{1}{a+bx+c(d+ex)^2}\,dx = \left[\frac{\log\left(\frac{2\,c^2e^4x^2+4\,bcde+2\,(c^2d^2-ac)\,e^2+b^2+2\,(2\,c^2de^3+bce^2)\,x-\sqrt{4\,bcde-4\,ace^2+b^2}\,(2\,ce^2x+2\,cde+b)}{ce^2x^2+cd^2+(2\,cde+b)x+a}\right)}{\sqrt{4\,bcde-4\,ace^2+b^2}}, -\frac{2\,\sqrt{-4\,bcde+4\,ace^2-b^2}\,\arctan\left(-\frac{\sqrt{-4\,bcde+4\,ace^2-b^2}\,(2\,ce^2x+2\,cde+b)}{4\,bcde-4\,ace^2+b^2}\right)}{4\,bcde-4\,ace^2+b^2}\right]$$

```
[In] integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="fricas")
```

```
[Out] [log((2*c^2*e^4*x^2 + 4*b*c*d*e + 2*(c^2*d^2 - a*c)*e^2 + b^2 + 2*(2*c^2*d*
e^3 + b*c*e^2)*x - sqrt(4*b*c*d*e - 4*a*c*e^2 + b^2)*(2*c*e^2*x + 2*c*d*e +
 b))/(c*e^2*x^2 + c*d^2 + (2*c*d*e + b)*x + a))/sqrt(4*b*c*d*e - 4*a*c*e^2
+ b^2), -2*sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2)*arctan(-sqrt(-4*b*c*d*e + 4*a
*c*e^2 - b^2)*(2*c*e^2*x + 2*c*d*e + b)/(4*b*c*d*e - 4*a*c*e^2 + b^2))/(4*b
*c*d*e - 4*a*c*e^2 + b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(63) = 126$.

Time = 0.18 (sec) , antiderivative size = 294, normalized size of antiderivative = 5.16

$$\int \frac{1}{a+bx+c(d+ex)^2}\,dx = -\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}\log\left(x+\frac{-4ace^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+b^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+4bcde\sqrt{-\frac{1}{4ace^2-b^2-4bc}}}{2ce^2}\right. + \sqrt{-\frac{1}{4ace^2-b^2-4bcde}}\log\left(x+\frac{4ace^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}-b^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}-4bcde\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}}{2ce^2}\right.$$

```
[In] integrate(1/(a+b*x+c*(e*x+d)**2),x)
```

```
[Out] -sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*log(x + (-4*a*c*e**2*sqrt(-1/(4*a
*c*e**2 - b**2 - 4*b*c*d*e)) + b**2*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)
) + 4*b*c*d*e*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2*c*d*e)/(2*c*
e**2)) + sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*log(x + (4*a*c*e**2*sqrt(
-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) - b**2*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b
*c*d*e)) - 4*b*c*d*e*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2*c*d*e
)/(2*c*e**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+bx+c(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c*e^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{1}{a+bx+c(d+ex)^2}\,dx = \frac{2\arctan\left(\frac{2ce^2x+2cde+b}{\sqrt{-4bcde+4ace^2-b^2}}\right)}{\sqrt{-4bcde+4ace^2-b^2}}$$

```
[In] integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*e^2*x + 2*c*d*e + b)/sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2))/sqrt
(-4*b*c*d*e + 4*a*c*e^2 - b^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.44

$$\int \frac{1}{a+bx+c(d+ex)^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b+2\,c\,d\,e}{\sqrt{-b^2-4\,c\,d\,b\,e+4\,a\,c\,e^2}}+\frac{2\,c\,e^2\,x}{\sqrt{-b^2-4\,c\,d\,b\,e+4\,a\,c\,e^2}}\right)}{\sqrt{-b^2-4\,c\,d\,b\,e+4\,a\,c\,e^2}}$$

```
[In] int(1/(a + c*(d + e*x)^2 + b*x),x)
```

```
[Out] (2*atan((b + 2*c*d*e)/(4*a*c*e^2 - b^2 - 4*b*c*d*e)^(1/2) + (2*c*e^2*x)/(4*
a*c*e^2 - b^2 - 4*b*c*d*e)^(1/2)))/(4*a*c*e^2 - b^2 - 4*b*c*d*e)^(1/2)
```

3.491 $\int \frac{x^2}{1+(-1+x^2)^2} \, dx$

Optimal result 2862
Rubi [A] (verified) 2863
Mathematica [C] (verified) 2865
Maple [C] (verified) 2865
Fricas [C] (verification not implemented) 2866
Sympy [A] (verification not implemented) 2866
Maxima [F] 2867
Giac [A] (verification not implemented) 2867
Mupad [B] (verification not implemented) 2868

Optimal result

Integrand size = 15, antiderivative size = 188

$$\int \frac{x^2}{1+(-1+x^2)^2} \, dx = -\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)} \arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)} \arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right) + \frac{\log\left(\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}} - \frac{\log\left(\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

```
[Out] -1/4*arctan((-2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^
(1/2)+1/4*arctan((2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/
2))^(1/2)+1/4*ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)-1/4
*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2014, 1141, 1175, 632, 210, 1178, 642}

$$\int \frac{x^2}{1+(-1+x^2)^2}\,dx = -\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right) + \frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}} - \frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

```
[In] Int[x^2/(1 + (-1 + x^2)^2),x]
```

```
[Out] -1/2*(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*x)/Sqrt[2*(-1
 + Sqrt[2])]]) + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*x
)/Sqrt[2*(-1 + Sqrt[2])]])/2 + Log[Sqrt[2] - Sqrt[2*(1 + Sqrt[2])]*x + x^2]
/(4*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(
4*Sqrt[2*(1 + Sqrt[2])])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 2014

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{2-2x^2+x^4}\,dx$$

$$= -\left(\frac{1}{2}\int \frac{\sqrt{2}-x^2}{2-2x^2+x^4}\,dx\right) + \frac{1}{2}\int \frac{\sqrt{2}+x^2}{2-2x^2+x^4}\,dx$$

$$= \frac{1}{4}\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx + \frac{1}{4}\int \frac{1}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx$$

$$+ \frac{\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{-\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x-x^2}\,dx}{4\sqrt{2\left(1+\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{-\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x-x^2}\,dx}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

$$= \frac{\log\left(\sqrt{2}-\sqrt{2(1+\sqrt{2})}x+x^2\right)}{4\sqrt{2(1+\sqrt{2})}} - \frac{\log\left(\sqrt{2}+\sqrt{2(1+\sqrt{2})}x+x^2\right)}{4\sqrt{2(1+\sqrt{2})}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{2(1-\sqrt{2})-x^2}\,dx, x, -\sqrt{2\left(1+\sqrt{2}\right)}+2x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{2(1-\sqrt{2})-x^2}\,dx, x, \sqrt{2\left(1+\sqrt{2}\right)}+2x\right)$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2(-1+\sqrt{2})}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2(-1+\sqrt{2})}} + \frac{\log\left(\sqrt{2}-\sqrt{2(1+\sqrt{2})}x+x^2\right)}{4\sqrt{2(1+\sqrt{2})}} - \frac{\log\left(\sqrt{2}+\sqrt{2(1+\sqrt{2})}x+x^2\right)}{4\sqrt{2(1+\sqrt{2})}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.21

$$\int \frac{x^2}{1+(-1+x^2)^2}\,dx = -\frac{\arctan\left(\frac{x}{\sqrt{-1-i}}\right)}{(-1-i)^{3/2}} - \frac{\arctan\left(\frac{x}{\sqrt{-1+i}}\right)}{(-1+i)^{3/2}}$$

```
[In] Integrate[x^2/(1 + (-1 + x^2)^2),x]
[Out] -(ArcTan[x/Sqrt[-1 - I]]/(-1 - I)^(3/2)) - ArcTan[x/Sqrt[-1 + I]]/(-1 + I)^
(3/2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.19

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-2\_Z^2+2\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^3-\_R}\right)}{4}$
default	$-\frac{\sqrt{2+2\sqrt{2}}\left(\sqrt{2}-1\right)\left(\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{2+2\sqrt{2}}\right)}{2}-\frac{\sqrt{2+2\sqrt{2}}\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}\right)}{4}+\frac{\sqrt{2+2\sqrt{2}}\left(\sqrt{2}-1\right)\left(\frac{\ln\left(x^2+\sqrt{2}-x\sqrt{2+2\sqrt{2}}\right)}{2}+\frac{\sqrt{2+}\right.}{4}$

[In] int(x^2/(1+(x^2-1)^2),x,method=_RETURNVERBOSE)

[Out] 1/4*sum(_R^2/(_R^3-_R)*ln(x-_R),_R=RootOf(_Z^4-2*_Z^2+2))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.28

$$\int \frac{x^2}{1+(-1+x^2)^2}\,dx = \frac{1}{4}\sqrt{i-1}\log\left(x+i\sqrt{i-1}\right)-\frac{1}{4}\sqrt{i-1}\log\left(x-i\sqrt{i-1}\right) -\frac{1}{4}\sqrt{-i-1}\log\left(x+i\sqrt{-i-1}\right)+\frac{1}{4}\sqrt{-i-1}\log\left(x-i\sqrt{-i-1}\right)$$

[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="fricas")

[Out] 1/4*sqrt(I - 1)*log(x + I*sqrt(I - 1)) - 1/4*sqrt(I - 1)*log(x - I*sqrt(I - 1)) - 1/4*sqrt(-I - 1)*log(x + I*sqrt(-I - 1)) + 1/4*sqrt(-I - 1)*log(x - I*sqrt(-I - 1))

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.13

$$\int \frac{x^2}{1+(-1+x^2)^2}\,dx = \mathrm{RootSum}\left(128t^4+16t^2+1,\left(t\mapsto t\log\left(64t^3+4t+x\right)\right)\right)$$

[In] integrate(x**2/(1+(x**2-1)**2),x)

[Out] RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*log(64*_t**3 + 4*_t + x)))

Maxima [F]

$$\int \frac{x^2}{1 + (-1 + x^2)^2}\, dx = \int \frac{x^2}{(x^2 - 1)^2 + 1}\, dx$$

```
[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="maxima")
[Out] integrate(x^2/((x^2 - 1)^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.65 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.78

$$\begin{aligned}\int \frac{x^2}{1 + (-1 + x^2)^2}\, dx = & \frac{1}{4}\sqrt{2\sqrt{2} + 2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x + 2^{\frac{1}{4}}\sqrt{\sqrt{2} + 2}\right)}{2\sqrt{-\sqrt{2} + 2}}\right) \\ & + \frac{1}{4}\sqrt{2\sqrt{2} + 2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x - 2^{\frac{1}{4}}\sqrt{\sqrt{2} + 2}\right)}{2\sqrt{-\sqrt{2} + 2}}\right) \\ & - \frac{1}{8}\sqrt{2\sqrt{2} - 2}\log\left(x^2 + 2^{\frac{1}{4}}x\sqrt{\sqrt{2} + 2} + \sqrt{2}\right) \\ & + \frac{1}{8}\sqrt{2\sqrt{2} - 2}\log\left(x^2 - 2^{\frac{1}{4}}x\sqrt{\sqrt{2} + 2} + \sqrt{2}\right)\end{aligned}$$

```
[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="giac")
[Out] 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x + 2^(1/4)*sqrt(sqrt(2) + 2)
)/sqrt(-sqrt(2) + 2)) + 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x - 2
^(1/4)*sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2*sqrt(2) - 2)*log
(x^2 + 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2)) + 1/8*sqrt(2*sqrt(2) - 2)*log
(x^2 - 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.54

$$\int \frac{x^2}{1+(-1+x^2)^2}\,dx = \operatorname{atanh}\left(32\,x\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)+\operatorname{atanh}\left(32\,x\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)$$

```
[In] int(x^2/((x^2 - 1)^2 + 1),x)
[Out] atanh(32*x*((- 2^(1/2)/32 - 1/32)^(1/2) + (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*
(- 2^(1/2)/32 - 1/32)^(1/2) + 2*(2^(1/2)/32 - 1/32)^(1/2)) + atanh(32*x*((-
 2^(1/2)/32 - 1/32)^(1/2) - (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*(- 2^(1/2)/32
- 1/32)^(1/2) - 2*(2^(1/2)/32 - 1/32)^(1/2))
```

3.492 $\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx$

Optimal result 2869
Rubi [A] (verified) 2869
Mathematica [A] (verified) 2870
Maple [A] (verified) 2871
Fricas [A] (verification not implemented) 2871
Sympy [A] (verification not implemented) 2871
Maxima [A] (verification not implemented) 2872
Giac [A] (verification not implemented) 2872
Mupad [B] (verification not implemented) 2873

Optimal result

Integrand size = 50, antiderivative size = 60

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx = \frac{2}{(3+x+x^4)^3}-\frac{3x}{(3+x+x^4)^3}+\frac{5x^2}{(3+x+x^4)^3}+\frac{x^4}{(3+x+x^4)^3}-\frac{5x^6}{(3+x+x^4)^3}$$

```
[Out] 2/(x^4+x+3)^3-3*x/(x^4+x+3)^3+5*x^2/(x^4+x+3)^3+x^4/(x^4+x+3)^3-5*x^6/(x^4+
x+3)^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2127, 1602}

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx = \frac{x^4}{(x^4+x+3)^3}-\frac{3x}{(x^4+x+3)^3}+\frac{2}{(x^4+x+3)^3}-\frac{5x^6}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}$$

```
[In] Int[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*
x^9)/(3 + x + x^4)^4),x]
```

```
[Out] 2/(3 + x + x^4)^3 - (3*x)/(3 + x + x^4)^3 + (5*x^2)/(3 + x + x^4)^3 + x^4/(
3 + x + x^4)^3 - (5*x^6)/(3 + x + x^4)^3
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2127

```
Int[(Pm_)*(Qn_)^(p_.), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x
]}, Simp[Coeff[Pm, x, m]*x^(m - n + 1)*(Qn^(p + 1)/((m + n*p + 1)*Coeff[Qn,
 x, n])), x] + Dist[1/((m + n*p + 1)*Coeff[Qn, x, n]), Int[ExpandToSum[(m +
 n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn +
(p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 < 0
] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{5x^6}{(3+x+x^4)^3}+\frac{1}{6}\int\frac{-90+216x-30x^2-72x^3+204x^4-300x^5-48x^7}{(3+x+x^4)^4}\,dx\\
&=\frac{x^4}{(3+x+x^4)^3}-\frac{5x^6}{(3+x+x^4)^3}-\frac{1}{48}\int\frac{720-1728x+240x^2+1152x^3-1584x^4+2400x^5}{(3+x+x^4)^4}\,dx\\
&=\frac{5x^2}{(3+x+x^4)^3}+\frac{x^4}{(3+x+x^4)^3}-\frac{5x^6}{(3+x+x^4)^3}\\
&\quad+\frac{1}{480}\int\frac{-7200+2880x-11520x^3+15840x^4}{(3+x+x^4)^4}\,dx\\
&=-\frac{3x}{(3+x+x^4)^3}+\frac{5x^2}{(3+x+x^4)^3}+\frac{x^4}{(3+x+x^4)^3}-\frac{5x^6}{(3+x+x^4)^3}-\frac{\int\frac{31680+126720x^3}{(3+x+x^4)^4}\,dx}{5280}\\
&=\frac{2}{(3+x+x^4)^3}-\frac{3x}{(3+x+x^4)^3}+\frac{5x^2}{(3+x+x^4)^3}+\frac{x^4}{(3+x+x^4)^3}-\frac{5x^6}{(3+x+x^4)^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.45

$$\int-\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx=\frac{2-3x+5x^2+x^4-5x^6}{(3+x+x^4)^3}$$

```
[In] Integrate[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7
 - 30*x^9)/(3 + x + x^4)^4),x]
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$	28
norman	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$	28
risch	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$	28
parallelrisch	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$	28
gosper	$-\frac{5x^6-x^4-5x^2+3x-2}{(x^4+x+3)^3}$	31

```
[In] int((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+3)^4,x
,method=_RETURNVERBOSE)
```

```
[Out] (-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx$$
$$= -\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

```
[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+
3)^4,x, algorithm="fricas")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx$$
$$= \frac{-5x^6+x^4+5x^2-3x+2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

```
[In] integrate((30*x**9-8*x**7-15*x**6-140*x**5+34*x**4-12*x**3-5*x**2+36*x-15)/
(x**4+x+3)**4,x)
```

```
[Out] (-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18*
x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx$$
$$= -\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

```
[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+
3)^4,x, algorithm="maxima")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.50

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx$$
$$= -\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{(x^4+x+3)^3}$$

```
[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+
3)^4,x, algorithm="giac")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^4 + x + 3)^3
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.45

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx$$
$$=\frac{-5\,x^6+x^4+5\,x^2-3\,x+2}{\left(x^4+x+3\right)^3}$$

```
[In] int(-(5*x^2 - 36*x + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*x^9 +
15)/(x + x^4 + 3)^4,x)
[Out] (5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3
```

3.493 $\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4} + \frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3} + \frac{30x}{(3+x+x^4}\right.$

Optimal result 2874
Rubi [F] 2874
Mathematica [A] (verified) 2875
Maple [A] (verified) 2876
Fricas [B] (verification not implemented) 2876
Sympy [B] (verification not implemented) 2877
Maxima [B] (verification not implemented) 2877
Giac [B] (verification not implemented) 2877
Mupad [B] (verification not implemented) 2878

Optimal result

Integrand size = 61, antiderivative size = 27

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right) dx = \frac{2-3x+5x^2+x^4-5x^6}{(3+x+x^4)^3}$$

```
[Out] (-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

Rubi [F]

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right) dx = \int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right) dx$$

```
[In] Int[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*x - 75
*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2,x]
```

```
[Out] -19/(4*(3 + x + x^4)^3) + (3 + x + x^4)^(-2) - (621*Defer[Int][(3 + x + x^4
)^(-4), x])/4 + 684*Defer[Int][x/(3 + x + x^4)^4, x] + 360*Defer[Int][x^2/(
3 + x + x^4)^4, x] + 44*Defer[Int][(3 + x + x^4)^(-3), x] - 320*Defer[Int][
x/(3 + x + x^4)^3, x] - 75*Defer[Int][x^2/(3 + x + x^4)^3, x] + 30*Defer[In
t][x/(3 + x + x^4)^2, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\int \frac{-47+228x+120x^2+19x^3}{(3+x+x^4)^4}\,dx \\
&\quad + 30\int \frac{x}{(3+x+x^4)^2}\,dx + \int \frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}\,dx \\
&= -\frac{19}{4(3+x+x^4)^3} + \frac{1}{(3+x+x^4)^2} + \frac{1}{4}\int \frac{176-1280x-300x^2}{(3+x+x^4)^3}\,dx \\
&\quad + \frac{3}{4}\int \frac{-207+912x+480x^2}{(3+x+x^4)^4}\,dx + 30\int \frac{x}{(3+x+x^4)^2}\,dx \\
&= -\frac{19}{4(3+x+x^4)^3} + \frac{1}{(3+x+x^4)^2} \\
&\quad + \frac{1}{4}\int \left(\frac{176}{(3+x+x^4)^3} - \frac{1280x}{(3+x+x^4)^3} - \frac{300x^2}{(3+x+x^4)^3}\right) dx \\
&\quad + \frac{3}{4}\int \left(-\frac{207}{(3+x+x^4)^4} + \frac{912x}{(3+x+x^4)^4} + \frac{480x^2}{(3+x+x^4)^4}\right) dx \\
&\quad + 30\int \frac{x}{(3+x+x^4)^2}\,dx \\
&= -\frac{19}{4(3+x+x^4)^3} + \frac{1}{(3+x+x^4)^2} + 30\int \frac{x}{(3+x+x^4)^2}\,dx \\
&\quad + 44\int \frac{1}{(3+x+x^4)^3}\,dx - 75\int \frac{x^2}{(3+x+x^4)^3}\,dx - \frac{621}{4}\int \frac{1}{(3+x+x^4)^4}\,dx \\
&\quad - 320\int \frac{x}{(3+x+x^4)^3}\,dx + 360\int \frac{x^2}{(3+x+x^4)^4}\,dx + 684\int \frac{x}{(3+x+x^4)^4}\,dx
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$
\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4} + \frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3} + \frac{30x}{(3+x+x^4)^2}\right) dx
= \frac{2-3x+5x^2+x^4-5x^6}{(3+x+x^4)^3}
$$

```
[In] Integrate[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*
x - 75*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2,x]
```

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result
norman	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$
parallelrisch	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$
gosper	$-\frac{5x^6-x^4-5x^2+3x-2}{(x^4+x+3)^3}$
default	$\frac{\frac{377432}{195075}x^7-\frac{1404328}{195075}x^6+\frac{234517}{195075}x^5+\frac{660506}{195075}x^4-\frac{208792}{195075}x^3-\frac{13339729}{390150}x^2+\frac{89881}{13005}x+\frac{121303}{21675}}{(x^4+x+3)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+\_Z+3)} \frac{(377432\_}{}\right.}{19}$
risch	$\frac{\frac{377432}{195075}x^7-\frac{1404328}{195075}x^6+\frac{234517}{195075}x^5+\frac{660506}{195075}x^4-\frac{208792}{195075}x^3-\frac{13339729}{390150}x^2+\frac{89881}{13005}x+\frac{121303}{21675}}{(x^4+x+3)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+\_Z+3)} \frac{(377432\_}{}\right.}{19}$

```
[In] int(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/(x^4+x
+3)^3+30*x/(x^4+x+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)\,dx = -\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

```
[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/
(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="fricas")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26) = 52$.

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)dx = \frac{-5x^6+x^4+5x^2-3x+2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

```
[In] integrate(3*(19*x**3+120*x**2+228*x-47)/(x**4+x+3)**4+(-8*x**3-75*x**2-320*
x+42)/(x**4+x+3)**3+30*x/(x**4+x+3)**2,x)
```

```
[Out] (-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18*
x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)dx = -\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

```
[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/
(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="maxima")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 7.30

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)\,dx$$
$$=\frac{1}{195075}\,x\left(\frac{377432\,x^2-2808656\,x+703551}{x^4+x+3}-\frac{255032\,x^2-1829456\,x+680601}{x^4+x+3}-\frac{7650\,(16\,x^2-128\,x+3}{x^4+x+3}\right.$$
$$-\frac{2\,(16\,x^3-64\,x^2+x+12)}{51\,(x^4+x+3)}$$
$$+\frac{754864\,x^7-2808656\,x^6+469034\,x^5+1321012\,x^4-417584\,x^3-13339729\,x^2+2696430\,x+2183454}{390150\,(x^4+x+3)^2}$$
$$-\frac{510064\,x^{11}-1829456\,x^{10}+453734\,x^9+1402676\,x^8-472048\,x^7-13501313\,x^6+4720744\,x^5+3747556}{390150\,(x^4+x+3)^3}$$

```
[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/
(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="giac")
```

```
[Out] 1/195075*x*((377432*x^2 - 2808656*x + 703551)/(x^4 + x + 3) - (255032*x^2 -
 1829456*x + 680601)/(x^4 + x + 3) - 7650*(16*x^2 - 128*x + 3)/(x^4 + x + 3
)) - 2/51*(16*x^3 - 64*x^2 + x + 12)/(x^4 + x + 3) + 1/390150*(754864*x^7 -
 2808656*x^6 + 469034*x^5 + 1321012*x^4 - 417584*x^3 - 13339729*x^2 + 26964
30*x + 2183454)/(x^4 + x + 3)^2 - 1/390150*(510064*x^11 - 1829456*x^10 + 45
3734*x^9 + 1402676*x^8 - 472048*x^7 - 13501313*x^6 + 4720744*x^5 + 3747556*
x^4 - 10935781*x^3 - 30736107*x^2 + 10203894*x + 4117662)/(x^4 + x + 3)^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)\,dx$$
$$=\frac{-5\,x^6+x^4+5\,x^2-3\,x+2}{(x^4+x+3)^3}$$

```
[In] int((684*x + 360*x^2 + 57*x^3 - 141)/(x + x^4 + 3)^4 - (320*x + 75*x^2 + 8*
x^3 - 42)/(x + x^4 + 3)^3 + (30*x)/(x + x^4 + 3)^2,x)
```

```
[Out] (5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3
```

3.494 $\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right) dx$

Optimal result 2879
Rubi [F] 2879
Mathematica [A] (verified) 2881
Maple [A] (verified) 2882
Fricas [B] (verification not implemented) 2882
Sympy [B] (verification not implemented) 2882
Maxima [B] (verification not implemented) 2883
Giac [B] (verification not implemented) 2883
Mupad [B] (verification not implemented) 2884

Optimal result

Integrand size = 60, antiderivative size = 27

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right) dx = \frac{2-3x+5x^2+x^4-5x^6}{(3+x+x^4)^3}$$

```
[Out] (-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

Rubi [F]

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right) dx = \int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right) dx$$

```
[In] Int[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2 - 3*x
+ 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4,x]
```

```
[Out] 7/(2*(3 + x + x^4)^3) - (63*x)/(22*(3 + x + x^4)^3) - (12*x^2)/(3 + x + x^4
)^3 - (5*x^3)/(3 + x + x^4)^3 + (3*x^4)/(2*(3 + x + x^4)^3) - (10*x^6)/(3 +
 x + x^4)^3 - 1/(2*(3 + x + x^4)^2) + (5*x^2)/(3 + x + x^4)^2 + (144*Defer[
Int][(3 + x + x^4)^(-4), x])/11 + (828*Defer[Int][x/(3 + x + x^4)^4, x])/11
 + 18*Defer[Int][x^2/(3 + x + x^4)^4, x] - 4*Defer[Int][(3 + x + x^4)^(-3),
 x] - 20*Defer[Int][x/(3 + x + x^4)^3, x]
```

Rubi steps

$$\text{integral} = -\left(3\int \frac{(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\,dx\right) + \int \frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}\,dx$$

$$= -\frac{10x^6}{(3+x+x^4)^3} + \frac{5x^2}{(3+x+x^4)^2} - \frac{1}{6}\int \frac{18+120x-24x^3}{(3+x+x^4)^3}\,dx + \frac{1}{2}\int \frac{-12+18x-30x^2-48x^3+66x^4+240x^5+90x^6-24x^7}{(3+x+x^4)^4}\,dx$$

$$= \frac{3x^4}{2(3+x+x^4)^3} - \frac{10x^6}{(3+x+x^4)^3} - \frac{1}{2(3+x+x^4)^2} + \frac{5x^2}{(3+x+x^4)^2} - \frac{1}{24}\int \frac{96+480x}{(3+x+x^4)^3}\,dx - \frac{1}{16}\int \frac{96-144x+240x^2+672x^3-504x^4-1920x^5-720x^6}{(3+x+x^4)^4}\,dx$$

$$= -\frac{5x^3}{(3+x+x^4)^3} + \frac{3x^4}{2(3+x+x^4)^3} - \frac{10x^6}{(3+x+x^4)^3} - \frac{1}{2(3+x+x^4)^2} + \frac{5x^2}{(3+x+x^4)^2} + \frac{1}{144}\int \frac{-864+1296x+4320x^2-6048x^3+4536x^4+17280x^5}{(3+x+x^4)^4}\,dx - \frac{1}{24}\int \left(\frac{96}{(3+x+x^4)^3} + \frac{480x}{(3+x+x^4)^3}\right)dx$$

$$= -\frac{12x^2}{(3+x+x^4)^3} - \frac{5x^3}{(3+x+x^4)^3} + \frac{3x^4}{2(3+x+x^4)^3} - \frac{10x^6}{(3+x+x^4)^3} - \frac{1}{2(3+x+x^4)^2} + \frac{5x^2}{(3+x+x^4)^2} - \frac{\int \frac{8640-116640x-25920x^2+60480x^3-45360x^4}{(3+x+x^4)^4}\,dx}{1440} - 4\int \frac{1}{(3+x+x^4)^3}\,dx - 20\int \frac{x}{(3+x+x^4)^3}\,dx$$

$$= -\frac{63x}{22(3+x+x^4)^3} - \frac{12x^2}{(3+x+x^4)^3} - \frac{5x^3}{(3+x+x^4)^3} + \frac{3x^4}{2(3+x+x^4)^3} - \frac{10x^6}{(3+x+x^4)^3} - \frac{1}{2(3+x+x^4)^2} + \frac{5x^2}{(3+x+x^4)^2} + \frac{\int \frac{41040+1192320x+285120x^2-665280x^3}{(3+x+x^4)^4}\,dx}{15840} - 4\int \frac{1}{(3+x+x^4)^3}\,dx - 20\int \frac{x}{(3+x+x^4)^3}\,dx$$

$$= \frac{7}{2(3+x+x^4)^3} - \frac{63x}{22(3+x+x^4)^3} - \frac{12x^2}{(3+x+x^4)^3} - \frac{5x^3}{(3+x+x^4)^3} + \frac{3x^4}{2(3+x+x^4)^3} - \frac{10x^6}{(3+x+x^4)^3} - \frac{1}{2(3+x+x^4)^2} + \frac{5x^2}{(3+x+x^4)^2} + \frac{\int \frac{829440+4769280x+1140480x^2}{(3+x+x^4)^4}\,dx}{63360} - 4\int \frac{1}{(3+x+x^4)^3}\,dx - 20\int \frac{x}{(3+x+x^4)^3}\,dx$$

$$= \frac{7}{2(3+x+x^4)^3} - \frac{63x}{22(3+x+x^4)^3} - \frac{12x^2}{(3+x+x^4)^3} - \frac{5x^3}{(3+x+x^4)^3} + \frac{3x^4}{2(3+x+x^4)^3} - \frac{10x^6}{(3+x+x^4)^3} - \frac{1}{2(3+x+x^4)^2} + \frac{5x^2}{(3+x+x^4)^2} + \frac{\int \left(\frac{829440}{(3+x+x^4)^4} + \frac{4769280x}{(3+x+x^4)^4} + \frac{1140480x^2}{(3+x+x^4)^4}\right)dx}{63360} - 4\int \frac{1}{(3+x+x^4)^3}\,dx - 20\int \frac{x}{(3+x+x^4)^3}\,dx$$

$$= \frac{7}{2(3+x+x^4)^3} - \frac{63x}{22(3+x+x^4)^3} - \frac{12x^2}{(3+x+x^4)^3} - \frac{5x^3}{(3+x+x^4)^3} + \frac{3x^4}{2(3+x+x^4)^3} - \frac{10x^6}{(3+x+x^4)^3} - \frac{1}{2(3+x+x^4)^2} + \frac{5x^2}{(3+x+x^4)^2} - 4\int \frac{1}{(3+x+x^4)^3}\,dx + \frac{144}{11}\int \frac{1}{(3+x+x^4)^4}\,dx + 18\int \frac{x^2}{(3+x+x^4)^4}\,dx - 20\int \frac{x}{(3+x+x^4)^3}\,dx + \frac{828}{11}\int \frac{x}{(3+x+x^4)^4}\,dx$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3} - \frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right)dx = \frac{2-3x+5x^2+x^4-5x^6}{(3+x+x^4)^3}$$

```
[In] Integrate[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2
- 3*x + 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4,x]
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result
norman	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$
parallelrisch	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$
gosper	$-\frac{5x^6-x^4-5x^2+3x-2}{(x^4+x+3)^3}$
risch	$-\frac{5x^6-x^4-5x^2+3x-2}{(x^4+x+3)^3}$
default	$-\frac{-\frac{34568}{195075}x^7+\frac{73672}{195075}x^6+\frac{15392}{195075}x^5-\frac{60494}{195075}x^4-\frac{68792}{195075}x^3-\frac{583927}{195075}x^2+\frac{3356}{13005}x-\frac{2069}{43350}}{(x^4+x+3)^2}+\frac{-\frac{34568}{195075}x^{11}+\frac{73672}{195075}x^{10}+\frac{15392}{195075}x^9-$

```
[In] int((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-3*x+2)
/(x^4+x+3)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right)dx = -\frac{5x^6-x^4-5x^2+3x-2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

```
[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-
3*x+2)/(x^4+x+3)^4,x, algorithm="fricas")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26) = 52$.

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right)dx = \frac{-5x^6+x^4+5x^2-3x+2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

```
[In] integrate((-30*x**5+4*x**3+10*x-3)/(x**4+x+3)**3-3*(4*x**3+1)*(-5*x**6+x**4
+5*x**2-3*x+2)/(x**4+x+3)**4,x)
```

```
[Out] (-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18*
x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(30) = 60.

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)\left(2-3x+5x^2+x^4-5x^6\right)}{(3+x+x^4)^4}\right)\,dx = -\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

```
[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-
3*x+2)/(x^4+x+3)^4,x, algorithm="maxima")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(30) = 60.

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.11

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)\left(2-3x+5x^2+x^4-5x^6\right)}{(3+x+x^4)^4}\right)\,dx = \frac{69136\,x^7-147344\,x^6-30784\,x^5+120988\,x^4+137584\,x^3+1167854\,x^2-100680\,x+18621}{390150\,(x^4+x+3)^2} - \frac{69136\,x^{11}-147344\,x^{10}-30784\,x^9+190124\,x^8+197648\,x^7+2645788\,x^6-72044\,x^5+129019\,x^4+15}{390150\,(x^4+x+3)^3}$$

```
[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-
3*x+2)/(x^4+x+3)^4,x, algorithm="giac")
```

```
[Out] 1/390150*(69136*x^7 - 147344*x^6 - 30784*x^5 + 120988*x^4 + 137584*x^3 + 11
67854*x^2 - 100680*x + 18621)/(x^4 + x + 3)^2 - 1/390150*(69136*x^11 - 1473
44*x^10 - 30784*x^9 + 190124*x^8 + 197648*x^7 + 2645788*x^6 - 72044*x^5 + 1
29019*x^4 + 1580606*x^3 + 1452132*x^2 + 887031*x - 724437)/(x^4 + x + 3)^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)\,(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right)\,dx = \frac{-5\,x^6+x^4+5\,x^2-3\,x+2}{{\left(x^4+x+3\right)}^3}$$

```
[In] int((10*x + 4*x^3 - 30*x^5 - 3)/(x + x^4 + 3)^3 - (3*(4*x^3 + 1)*(5*x^2 - 3
*x + x^4 - 5*x^6 + 2))/(x + x^4 + 3)^4,x)
```

```
[Out] (5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 2885

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                 end if
             else #result contains complex but optimal is not
                 if debug then
                         print("result contains complex but optimal is not");
                 fi;
                 return "C","Result contains complex when optimal does not.";
             fi;
         else # result do not contain complex
              # this assumes optimal do not as well. No check is needed here.
             if debug then
                    print("result do not contain complex, this assumes optimal do not as well")
             fi;
             if leaf_count_result<=2*leaf_count_optimal then
                 if debug then
                     print("leaf_count_result<=2*leaf_count_optimal");
                 fi;
                 return "A"," ";
             else
                 if debug then
                     print("leaf_count_result>2*leaf_count_optimal");
                 fi;
                 return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
             fi;
         fi;
     else #ExpnType(result) > ExpnType(optimal)
         if debug then
             print("ExpnType(result) > ExpnType(optimal)");
         fi;
         return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
     fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```